

Appraisal of Models for Unpaved Roads

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Current models of unpaved roads are based on statistical analyses of experimental data guided or preceded by mechanistic analyses. The complexity of these models and their questionable transferability to new data bases must be considered to evaluate their wide acceptance as a design tool. An extensive review of models was conducted. Of the many models examined, it appears that a model originally proposed by Barber, Odom, and Patrick (WES1) and modified by the U.S. Army Corps of Engineer Waterways Experiment Station (WES2) shows the greatest promise. The WES models predict rut depth and the required surface thickness for aggregate surface unpaved roads as functions of important variables such as tire pressure, axle load, number of passages, and so forth. In this paper, the WES model predictions of rut depth and surface thickness are compared with actual measurements. In a number of cases, the WES models underpredicted surface thickness by as much as 64 percent; in other cases, WES models overpredicted by as much as 290 percent. A study of the WES models shows that they are fairly sensitive to changes in the exponents of the variables and to changes in the variables themselves. Although the WES models have considerable merit, in many cases they require further tuning.

In the United States, the majority of research on road roughness has concentrated on paved asphalt and concrete roads. These studies have taken two distinct approaches. One approach starts with empirical methods followed by enhancements based on mechanistic principles (1,2). It uses comprehensive field data

from in-service roads and statistical techniques to evaluate models that are based upon mechanistic principles. A second approach employs a mechanistic model calibrated empirically with field data to evaluate damage caused by traffic. Results obtained by this approach have served as input to a major U.S. study on highway cost allocation (3).

The roughness of unpaved roads is best studied and modeled by empirical methods based on the field performance of a variety of roads to identify and quantify the factors involved. The deterioration of unpaved roads is strongly affected by the behavior of the surfacing material associated with traffic and environment (1-7).

Recently, vehicle tire structure and pressure were shown to have a strong influence on the deterioration of unpaved roads particularly their tendency to corrugate (washboard). The likelihood of unpaved roads to corrugate is not clearly understood. However, research by the U.S. Army Corps of Engineer Waterways Experiment Station (WES) and the U.S. Department of Agriculture Forest Service on central tire inflation vehicles has shown that low inflation pressures reduce or eliminate corrugating (8,9) and, in certain situations, remove (heal) the corrugations.

Roughness can be represented by a power spectral density (PSD) function that expresses the mean square amplitude of roughness as a function of wavelength. Unpaved roads have a broad PSD of roughness. That is, they have appreciable roughness in all wavelengths. However, they are rougher in the short wavelengths

[less than 1 m (3.28ft)] than are paved roads since fine-grained materials tend to develop small depressions (washboard effect), humps, and potholes (10).

The response of a moving vehicle to road roughness depends on the mechanical properties (e.g., spring stiffness) of the vehicle, vehicle speed, road quality, and so forth (11). The rate at which unpaved road surfaces become rough depends upon many factors such as vehicle speed, vehicle tire pressure, wheel load, traffic count, surface thickness, soil characteristics of road layers, and so on. A model that is used to predict the required surface thickness of an unpaved road should include many of these factors. It should produce a reliable design and be relatively easy to use. It should not be overly sensitive to small changes in its parameters so it can be easily transferred to other areas.

DEFINITIONS AND CLASSIFICATION OF UNPAVED ROADS

Paterson categorized the definitions and classification of unpaved roads (1). Paterson also separated deterioration modes into two categories: wear and abrasion of the surface under traffic loads and deformation of the road surface by traffic, water, and wind. The approach to modeling deterioration of unpaved roads has been categorized under four groups (6): (a) dry weather deterioration, (b) wet weather deterioration, (c) wet weather deterioration with weak road-surface layer, and (d) wet weather deterioration with weak roadbed material. Each group has different deterioration mechanisms.

Current literature does not contain rigorous guidelines to control deterioration. However, there is a general consensus that certain properties of the surface material are necessary to control ravelling, looseness of the top surface, and corrugating (4-6). In particular, fines appear to be a primary means of preventing ravelling, looseness, and corrugating. For example, Visser (6) recommends that the percentage of material finer than 0.075 mm (0.003 in.) be kept greater than 14; in other words, $P_{075} \geq 14$ percent, where P_{075} is the percentage of material finer than 0.075 mm (0.003 in.). The 14 percent requirement is based on Visser's review of empirical studies.

SURVEY OF MODELS FOR UNPAVED ROADS

Rolt (12) has discussed models for surface roughness, rut depth, surface looseness, and gravel loss. These models were developed with data taken from 37 test sections in Kenya. With the exception of the model for gravel loss, traffic volume is the only variable employed in the models. All other factors that affect the road sur-

face are incorporated in measurements determined by statistical regression techniques. Although this makes the models simple to use, it does not allow for the wide variation in soil properties such as grain size distribution, plastic index, or strength properties such as California bearing ratio (CBR) or resilient modulus (M_r) that might be found in other areas of Kenya or in other countries. The model for gravel loss includes climatic conditions and roadbed alignment, but it does not include variables that distinguish soil properties other than wearing course type (e.g., lateritic, volcanic, or quartzitic gravel).

Visser et al. (13) describe models developed from a study in Brazil. These models include factors such as time elapsed since last blading, wearing course type, average daily traffic (ADT), seasonal adjustments, and roadbed alignment to predict road roughness and rut depth. The models require the addition of a constant factor if the wearing course is a quartzitic gravel (a compact granular rock composed of quartz and derived from sandstone). The constant is dropped if the wearing course is lateritic gravel (a residual product of rock decay with a high content of iron oxides and aluminum hydroxides). The larger number of variables employed makes these models more adaptable, but they lack sufficient ability to account for soil properties.

Several papers that include models for unpaved roads were published in the Proceedings of the Third International Conference on Low-Volume Roads. In particular, the paper by Coghlan (14) compares two models. One of these models was developed by WES. It extends an earlier model for a single unsurfaced soil to include a more competent surfacing material overlying a subgrade soil. The WES model relates the surface thickness to the number of coverages, single or equivalent single wheel load, tire contact area, and CBR of the subgrade soil as follows:

$$t = (0.176 \log C + 0.12) \sqrt{\frac{P}{8.1(\text{CBR})} - \frac{A}{\pi}} \quad (1)$$

where

- t = design thickness (1 in. = 25.4 mm),
- C = coverages (equivalent, more or less, to number of passages, axial, wheel, vehicle, etc.),
- P = single or equivalent single wheel load (1 lb = 4.45 N),
- A = tire contact area (in^2) equal to load/tire contact pressure (1 in^2 = 645.1 mm^2), and
- CBR = California bearing ratio of the subgrade soil.

Equation 1 and subsequent models and equations presented in this paper were originally derived by various researchers using experimental data and regression techniques to obtain best fits. Consequently, the coefficients

in these equations have embedded units, and it is therefore inappropriate and inaccurate, if not impossible, to convert these to SI units. Nevertheless, results obtained from these models are presented graphically in SI units.

The constants 0.176, 0.12, and 8.1 in Equation 1 are based on experimental data. The WES model is based on load tests with a failure criterion of 76 mm (3 in.) rut or 38 mm (1.5 in.) plastic deformation of the surface. This model relates the failure criteria to the soil properties of the subgrade, but it lacks input about the properties of the surface other than the failure criteria. The effect of surface roughness is not included in the equation. The effects of surface material on rutting are embedded in the constants of the equation; therefore, the model cannot treat changes in surfacing material. Coghlan (14) concludes that "the WES model provides a workable design procedure for aggregate-surface roads that meets many tests of reasonableness" and that the model could be improved by including terms of soil support, 80-kN (18-kip) equivalent axle loads (EALs), and the structural number concept from AASHTO pavement design.

Visser and Hudson (15) describe an attempt to apply models developed from other countries to South African roads. These models include factors such as time, surfacing type, traffic volume and type, road alignment, seasonal factor, surface material passing the 0.074-mm (No. 200) sieve, and the plasticity index of the surfacing material. Using these factors makes the models more adaptable, but the authors found that further calibrations for South African conditions were required. The authors also recommend that the validity of the models be simultaneously verified on individual road sections in several countries. At this time there has been no indication that this process has been completed.

Luhr et al. (16) describe a design procedure that includes a number of models. Most notable among these is a modification of the WES model developed by Barber et al. (17). This modification results in a model used to predict the number of 80-kN (18-kip) equivalent single axle loads (ESALs) that would cause a critical rut depth. The variables included in the model are critical rut depth, the total thickness of aggregate above the subgrade, and the elastic moduli of the aggregate above the subgrade and of the subgrade. There is no indication that the model has been tested.

Alkire (18) reported on a modification to the model developed by Barber et al. (17). This modification developed two equations with one independent variable and one constant. The independent variable was the CBR value for one equation and the Clegg impact value (CIV) for the second equation. The CBR and CIV data are for the subgrade (18). Although these equations are easy to use and require the testing of only the subgrade soil, the characteristics of the surface material are em-

bedded in the constant and cannot be adjusted for local conditions.

Riverson et al. (19) discussed models based on tests conducted in Indiana. The models were developed with multiple regression analysis, using factors such as road alignment, ADT, CIV, and soil properties. The resulting models for predicting roughness number and average rut depth include independent variables for soil properties and must be sensitive to changes in the aggregate used for road surfacing. Because data used in the regression analysis are taken from nonplastic aggregate samples, the models should be tested on aggregate with some plasticity. To determine the sensitivity of constants to changes in location, the models should also be tested for transferability to areas outside Indiana.

Yapp et al. (20) examined eight major design methods used by the Forest Service as well as the method suggested by the Corps of Engineers. The authors state, "All the design methods for aggregate-surfaced and earth roads found in the literature are generally related to each other and typically can be traced back to two basic studies." Most of the models examined by Yapp et al. have been discussed in this paper. The criteria for evaluating models are in the form of the following nine questions:

1. Is the design procedure valid for aggregate-surface and earth roads?
2. Are the inputs expected to have a major role in pavement deterioration?
3. Are standard traffic units [e.g., 80-kN (18-kip) ESALs] used?
4. Can tire pressures be varied?
5. Is the material characterization "reasonable"?
6. Are risk and reliability concepts considered?
7. Can failure criteria levels be changed?
8. Is seasonal haul incorporated into the model?
9. Has there been any field experience?

Yapp et al. (20) indicate that for earth and aggregate-surface roads, the models developed by Barber et al. (17) were best suited for design. However, they caution that the models have limitations because they lack field testing and they underpredict thickness requirements for low-strength subgrade materials.

A recent study (21) done at WES attempted to remedy the lack of field testing of the Barber et al. design model (17). By constructing roads with known aggregate CBR values and thickness and operating vehicles with controlled loads and tires pressures, WES was able to add 19 data sets to the original 254 data sets used by Barber et al. Using the combined data and a separate regression analysis for each model, WES (21) developed models for predicting rut depth and required aggregate thickness. These models use the same independent var-

ables as the Barber et al. models; however, the regression analyses produced different constants.

The aggregate-surface design equation developed by Barber et al. (the WES1 model) is the primary aggregate-surface design equation currently used by the U.S. Department of Agriculture Forest Service. Based upon the reviews of alternative methods and upon recommendations of the Forest Service, an analysis by WES (21) of aggregate-surface tests (8) was performed using the WES1 model, which relates initial pavement material properties to performance using rut depth as the major failure criterion. It gives the required surface thickness t by the relationship:

$$\log t = \left[(0.1741) \frac{P_k^{0.4704} t_p^{0.5695} R^{0.2476}}{RDC_1^{0.9335} C_2^{0.2848}} \right]^{0.4995} \quad (2)$$

where

- t = aggregate depth, in. (1 in. = 25.4 mm),
- p_k = equivalent single-wheel load (ESWL), kips
(1 kip = 4.45 kN),
- t_p = tire pressure, psi (1 psi = 6.89 kPa),
- R = number of passes of ESWL,
- RD = rut depth, in. (1 in. = 25.4 mm)
- C_1 = CBR of aggregate surface, and
- C_2 = CBR of subgrade

Solving Equation 2, algebraically, for RD , the result is

$$RD = 0.1741 \frac{P_k^{0.4704} t_p^{0.5695} R^{0.2476}}{(\log t)^{2.002} C_1^{0.9335} C_2^{0.2848}} \quad (3)$$

In the report by Smith (21), the data from the test program conducted by WES (8) were added to the data upon which Equations 2 and 3 were based. Then the coefficients were recalculated by multiple linear regression. Thus, the new data (19 data sets) were added to the original data base (254 data sets) of Equations 2 and 3. A multiple linear regression was performed on these data, treating rut depth (RD) as the dependent variable, to obtain the following relationship:

$$RD = 0.1090 \frac{P_k^{0.4988} t_p^{0.5641} R^{0.2418}}{(\log t)^{1.567} C_1^{0.9169} C_2^{0.0365}} \quad (4)$$

Rather than solve Equation 4 algebraically for surface thickness t , a second multiple linear regression was performed on the data, treating t as the dependent variable, to obtain the relationship

$$\log(t) = 0.2959 \frac{P_k^{0.2016} t_p^{0.2481} R^{0.0747}}{RD^{0.2176} C_1^{0.2414} C_2^{0.0206}} \quad (5)$$

Justification for developing Equation 5 by a multiple linear regression rather than algebraically from Equations

2 and 4 is based on the premise that the algebraic manipulation may not yield the most statistically correct relationship.

The authors of the present paper believe that the Barker et al. model (WES1: Equations 2 and 3) and the U.S. Army Corps of Engineers Waterways Experiment Station Model (WES2: Equations 4 and 5) are the best suited of the models reviewed for design of unpaved roads. Therefore, in the remainder of this paper, comparisons of the predictive capabilities of these two models are presented.

COMPARISON OF PREDICTIONS OF WES1 AND WES2

Models with Test Data

In the bar chart in Figure 1, the aggregate-surface thicknesses requirements predicted by the WES1 and WES2 models (Equations 2 and 5) are compared with the actual aggregate-surface thicknesses for the 19 data sets reported by WES (8,21).

For data sets 1 to 6, Figure 1, the WES1 and WES2 models predict greater required surface thicknesses than actual thickness on the test sections for actual rut depths of 51 to 127 mm (2 to 5 in.). For data set 7, WES1 predicts a smaller required thickness and WES2 predicts a larger required thickness than the 63.5 mm (2.5 in.) used. For data set 8, both WES1 and WES2 predict less required surface thickness than the actual 146 mm (5.75 in.). For set 9, WES1 predicts a requirement less than and WES2 predicts a requirement more than the actual thickness of 146 mm (5.75 in.). For data sets 10 through 19, both WES1 and WES2 predict considerably smaller required thicknesses than the actual thicknesses. In all cases, WES1 predicts smaller required thickness than WES2 does.

The WES1 and WES2 models return of rut depth (RD) and aggregate thickness (t) were examined by means of Equations 2 and 3 and Equations 4 and 5, respectively. Equations 3 and 4 were used to calculate rut depths using the original test data (8,21). The rut depths computed were then used in Equations 2 and 5 to compute surface thicknesses. These computed surface thicknesses were then compared to the surface thicknesses originally used to find the rut depth. Ordinarily for a set of equations to be consistent, the equations should return the values (thickness) used to get the result (rut depth). The WES1 equation returns the thicknesses in all cases, since Equation 2 is obtained by direct algebraic manipulation of Equation 1. However, in no case did the WES2 equation return the original thicknesses. This is because Equations 4 and 5 were obtained by separate multiple linear regression analyses of the data. In 6 cases out of the 19 data sets, WES2 returned

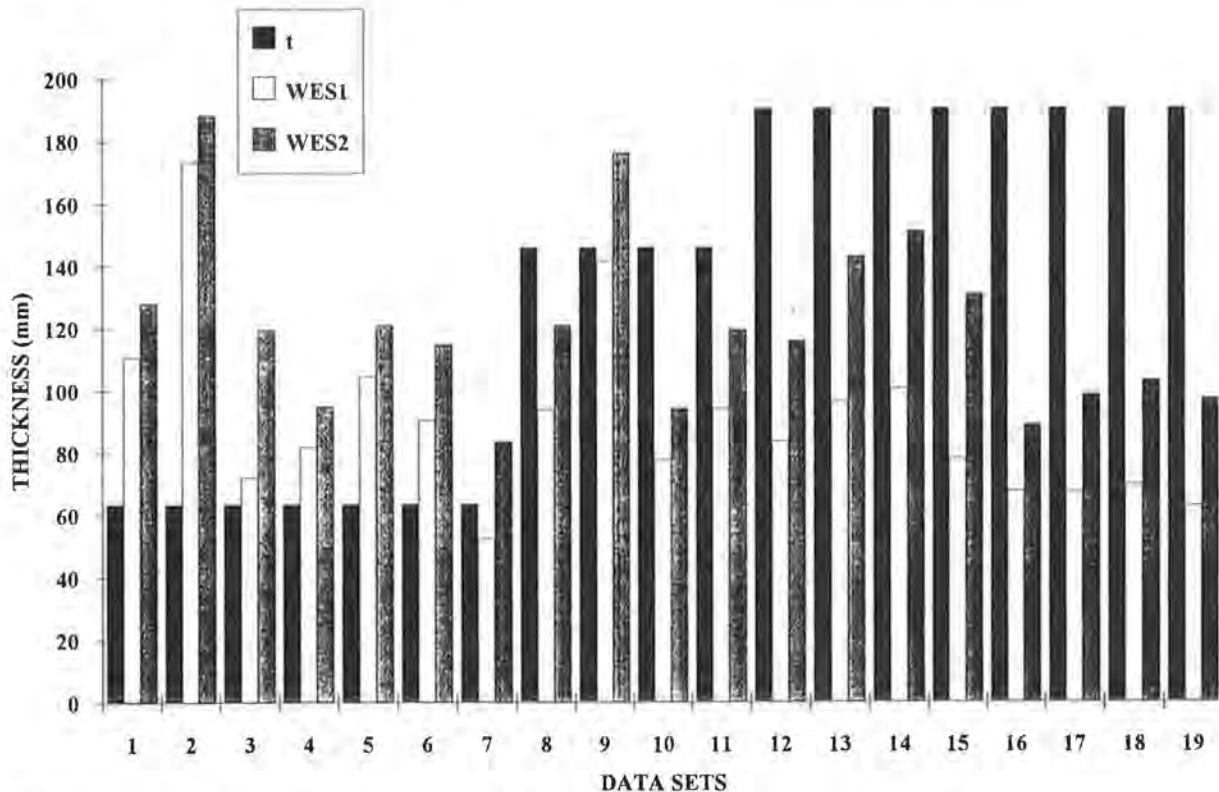


FIGURE 1 Comparison of actual aggregate thickness (t) and thickness as predicted by WES1 and WES2 (25.4 mm = 1 in.)

thickness values greater than 150 percent of the original thickness. In 9 cases, WES2 returned thicknesses from 80 to 149 percent of actual thickness, and in 4 cases WES2 returned thicknesses less than 80 percent of the original.

Figure 2 compares measured rut depths with rut depths calculated by the WES1 and WES2 models for the given 19 data sets. The figure shows that for data sets 7, 8, and 10 through 19, the WES models underestimate the measured rut depths by large amounts; for data sets 1 through 6 and set 9, the WES models overestimate the measured rut depths, the WES2 estimates being larger than those of WES1. Figure 3 is a bar chart of rut depth versus aggregate thickness. It shows that the WES1 and WES2 models consistently yield rut depths that are small compared with the measured depths for an aggregate thickness of 190 mm (7.5 in.).

Figures 4, 5, and 6 are bar charts of rut depth versus tire pressure, CBR of the surface aggregate and CBR of the subgrade, respectively. The WES models estimates of rut depth differed from the actual rut depth.

Sensitivity of Models

In addition to the comparison in Figures 1 to 6, the sensitivity of the WES models to changes in exponents

was examined. In particular, Figure 7 illustrates the changes in rut depth due to changes in the exponent of C_1 , the CBR of the surface, for the WES1 and WES2 models, respectively. In this figure, the coefficient of C_1 was reduced from 1 to 10 percent. A 10 percent change resulted in a change of 38.2 percent in the WES1 model and 37.4 percent in the WES2 model. The effects of changes in other exponents were also examined; they showed similar results. However, these exponents did not affect rut depth as much as the C_1 exponent did.

CONCLUSIONS

The WES1 and WES2 models have many of the required components to achieve a good road design. However, it appears that for accurate estimates of rut depths and required road surface thicknesses, additional soil properties may be needed. For example, it may be desirable to include soil properties such as percent material passing through sieve sizes 10, 40, and 200, liquid limit, and plasticity index, and so forth (19). These properties can be obtained by simple tests with equipment that is readily available in most soil laboratories.

Tests of this type are being conducted at the University of Wyoming to determine the effects of additives on

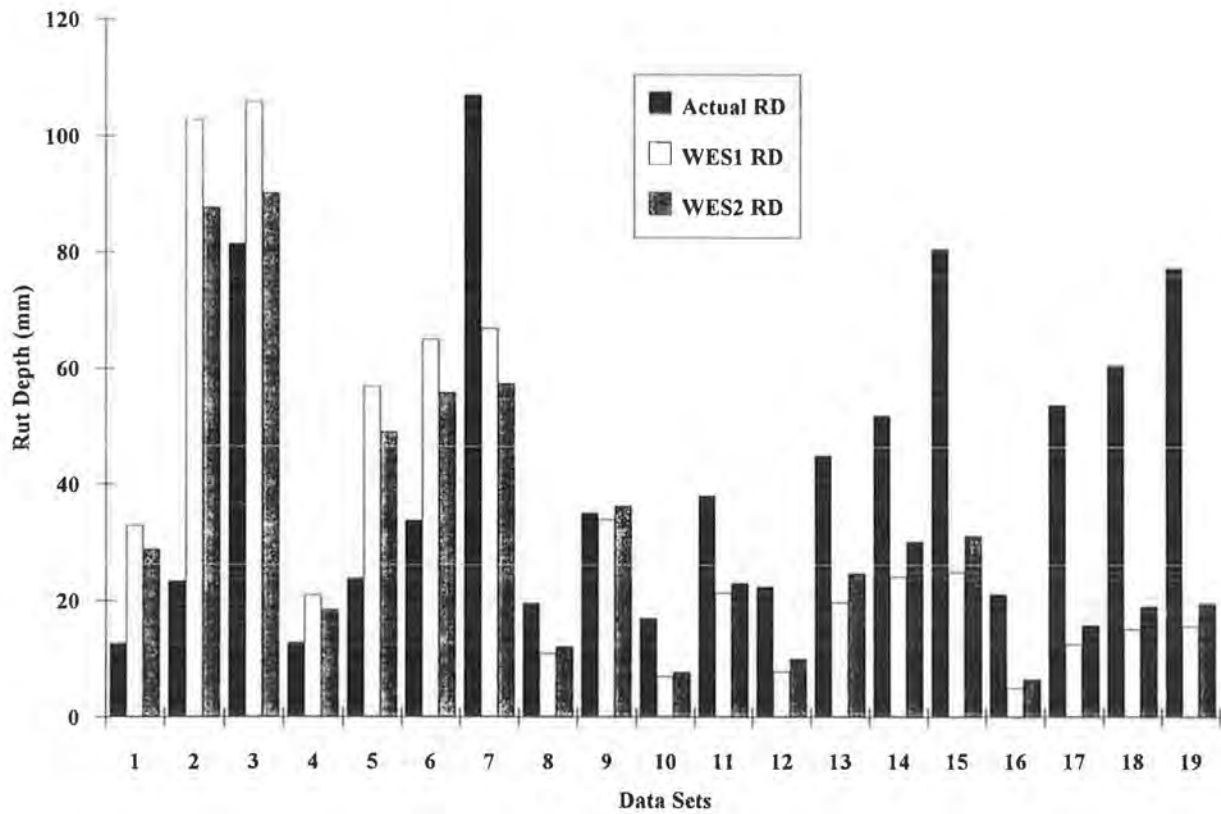


FIGURE 2 Comparison of actual RD versus WES1 and WES2 RD predictions (25.4 mm = 1 in.).

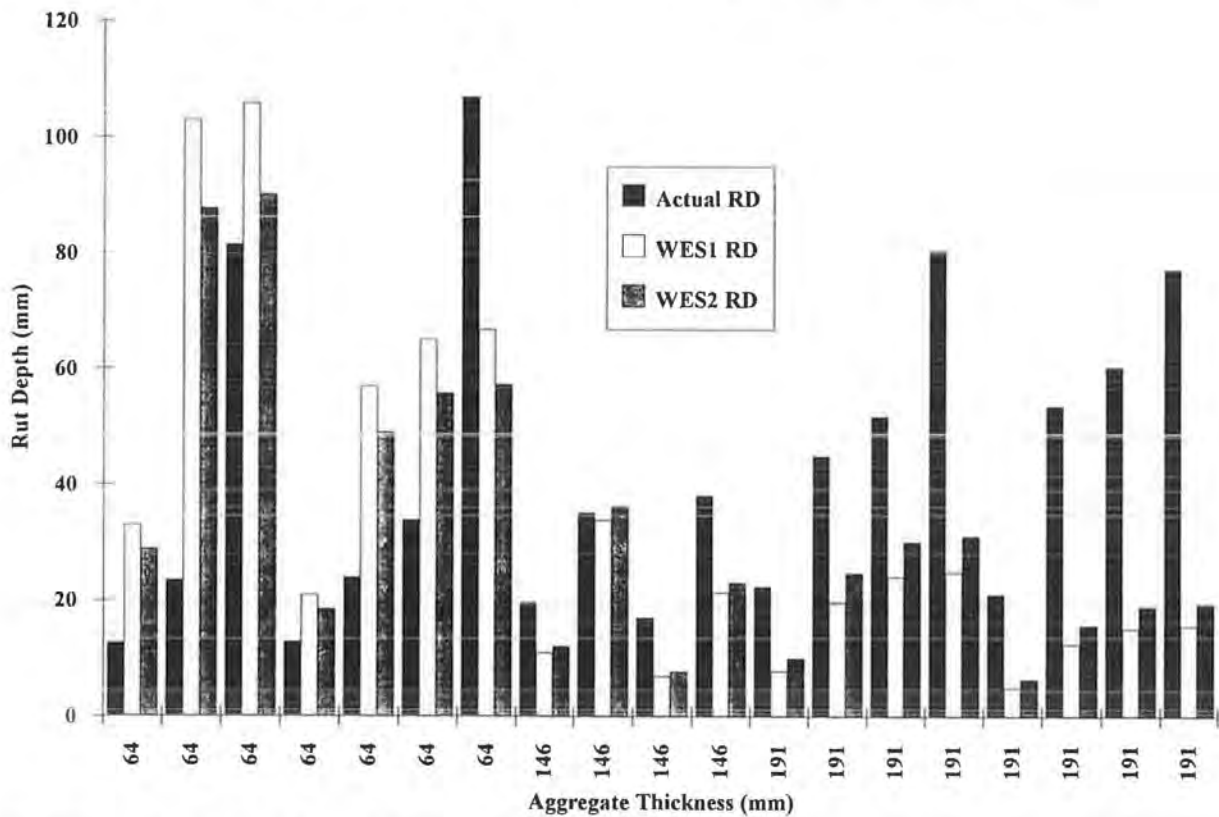


FIGURE 3 Comparison of actual, WES1, and WES2 RD versus aggregate thickness (25.4 mm = 1 in.).

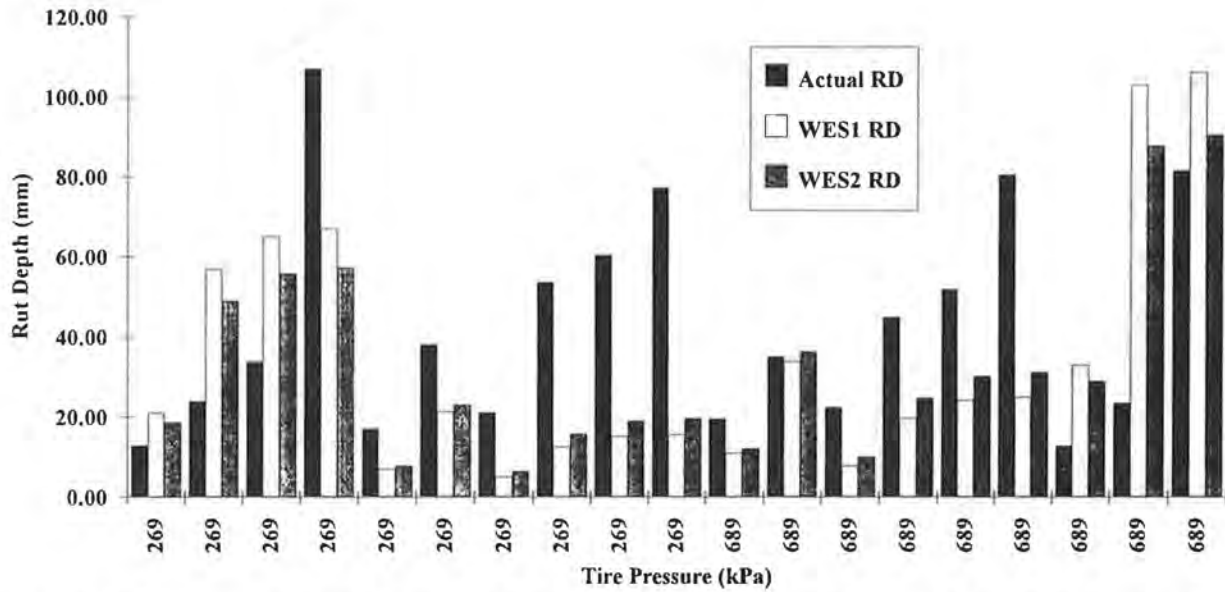


FIGURE 4 Comparison of actual, WES1, and WES2 RD versus tire pressure (25.4 mm = 1 in., 6.89 kPa = 1 psi).

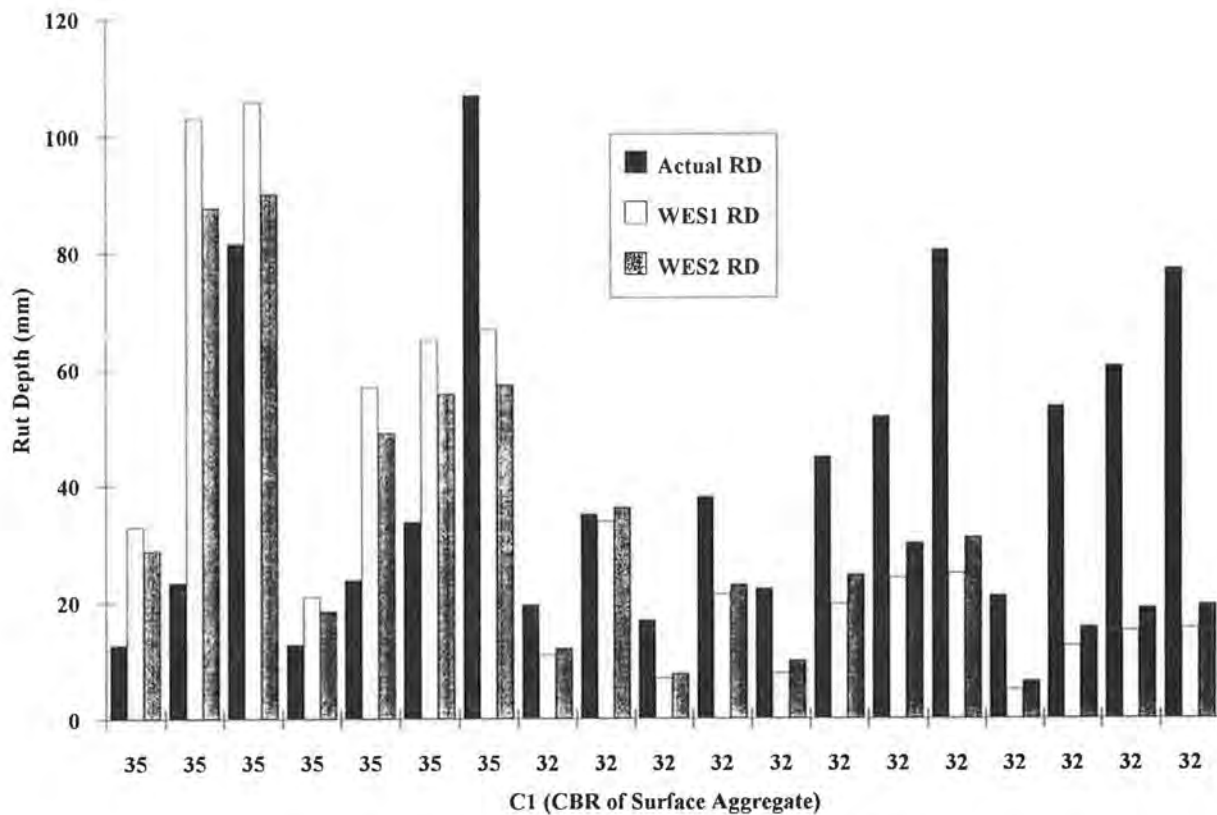


FIGURE 5 Comparison of actual, WES1, and WES2 RD versus C₁ (25.4 mm = 1 in.).

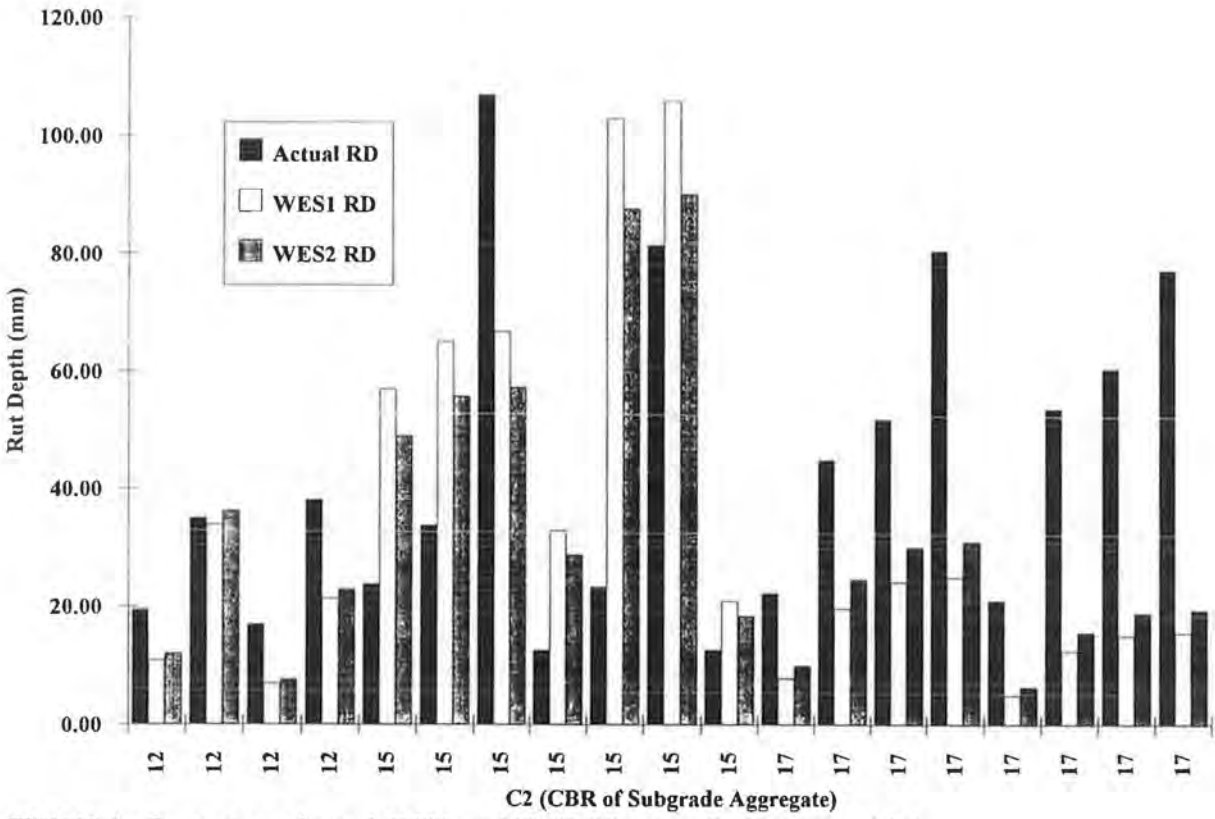


FIGURE 6 Comparison of actual, WES1, and WES2 RD versus C₂ (25.4 mm = 1 in.).

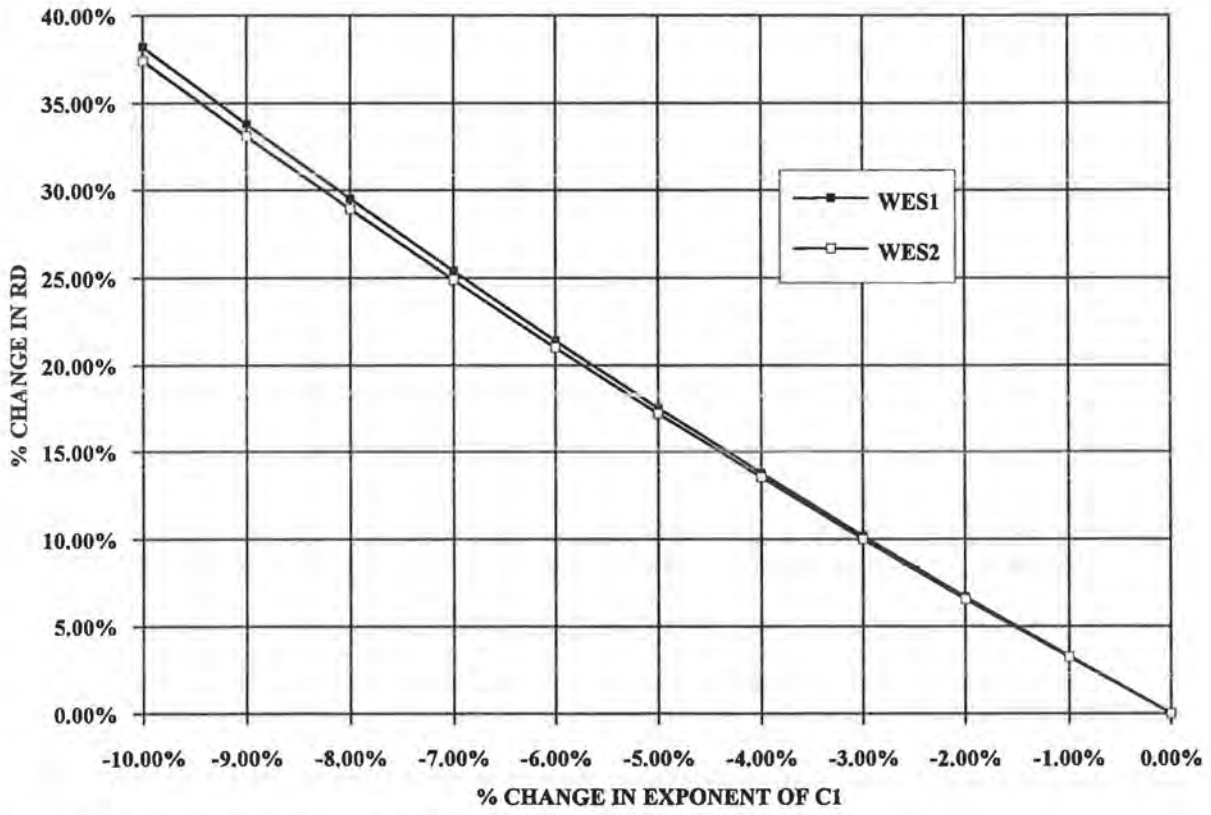


FIGURE 7 Check of sensitivity of WES1 and WES2 rut depth equations.

soils used for surfacing unpaved roads. These tests are designed to determine soil property changes associated with the use of additives. The objective of this study is to test and evaluate soil-additive mixtures as they relate to road surface performance and design.

Alkire (18) illustrates a design system frequently used by county engineers: "It is possible to estimate layer thickness for an aggregate-surface road based on experience to achieve a simple design. Interviews with county engineers suggest that this is the technique that is being used by many local governmental agencies." This may mean design by experience without resorting to mathematical models. Design by experience can work well if an engineer knows the characteristics of the soil in the geographic area reasonably well. However, an engineer may need to construct or reconstruct a road without sufficient experience or knowledge of the soil characteristics. This may lead to an inadequately designed road.

Whatever model is used, the engineer should be aware of the limitations. If the soil and climatic conditions are similar to the soil and climatic conditions for which the model was developed, the model may work well. If soil and climatic conditions are not similar, further tests may be needed to characterize the soil and modify the model for the new conditions.

Few of the papers reviewed employed dimensional analysis (22) in the development of models for unpaved roads. With the large number of variables involved, a dimensional analysis approach should lead to a more efficient model based upon a complete set of dimensionless products. A dimensional analysis approach may also facilitate the transferability of models.

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