Probabilistic Assessment of Prestressed Concrete Bridge

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The structure of an existing bridge cannot be assessed using the same nominal parameters and safety factors calibrated for the design of new bridges. The direct use of probabilistic methods allows a determination of the safety and serviceability of an existing structure. The uncertainties involved with load and resistance values in the assessment of concrete bridges are usually lower than in the design stage because the strength and load effects can be estimated accurately by using information from inspections, experimental tests, traffic measurements and other supplementary data. After that, a full probabilistic analysis is performed, as with design code calibration, but for each particular case of study. A practical example is presented, including the assessment of a prestressed concrete bridge, to illustrate the abilities of probabilistic methods to assess existing structures.

The assessment of existing concrete bridges is becoming an important task for structural engineers. Approximately 72 percent of U.S. bridges are older than 25 years (55 percent in Western Europe), and most of these bridges were made of concrete [in 1989, 70 percent in Western Europe and 52 percent in the United States (*1*)]. In addition, many of these bridges have been classified as deficient (*1*). In this way, the development of new structural evaluation methods will have important safety and economical implications.

Using semiprobalistic methods, with safety factors and nominal resistance or loads that have been calibrated for the design of new structures, to assess existing structures is not always best. The safety factors and nominal parameters used in design have been obtained for a set of specifications included in the design codes, but for existing bridges these criteria can be different:

1. Safety factors include global uncertainties that are possible to find in structures designed with the same codes when substantially different construction methods and technologies are used. For example, the average error of concrete slab depths built in situ usually varies between 1 and 3 cm. These values affect significantly the evaluation of self-weight of slabs between 20 and 40 cm thick, as used in building construction. In prestressed concrete slab bridges these errors usually are not relevant because depth varies between 80 and 150 cm. Nevertheless, design codes usually specify the same dead load factor for both types of construction. For an existing structure it is possible to estimate accurately the geometry and deficiencies of the structural elements.
2. In the assessment of existing bridges, the resistance parameters can be updated with data from experimental tests (static or dynamic load, cores of concrete or steel bars, etc.). Variations of resistance in the structure are estimated accurately using Bayesian tech-

niques. In general, the uncertainties are less than those assumed in current standards.

3. Traffic loads can be measured in specific places (weigh-in-motion techniques, traffic flow control, etc), and expected maximum values of loads can be obtained for the study. For each case, it is possible to consider allowances in truck weights, increasing and heavier traffic, or other circumstances.

4. Some existing bridges have been designed with codes including structural verification criteria and rules different than current methods. In other old structures, lower nominal live loads were used in design. In most cases, these bridges would be classified as unsafe if the current design code requirements were imposed.

The safety factors and nominal parameters included in the codes are based on and calibrated with a probabilistic approach. The reliability theory provides tools to determine the parameters necessary for safe structures, if design rules are verified.

The use of probabilistic methods has been accepted as the more rational analysis for assessing existing concrete bridges. On such structural evaluations, the reliability level shall be at least the level accepted for new bridges in national or international standards. The recent European codes have been calibrated for a maximum failure probability of collapse between $P_f = 10^{-4}$ to 10^{-6} during a lifetime (*1–3*). In the United States a probability of failure $P_f = 10^{-3}$ has been accepted for a reference period of 50 years. Instead of using probability of failure, the reliability index, β, has been used more as a convenient measure of the structural safety level. For the aforementioned probabilities, the reliability index is between 3.5 (United States) and 3.8 to 5.0 (Europe).

The reliability index generally is defined as a function of the probability of failure:

$$\beta = \phi^{-1} (P_f) \tag{1}$$

where ϕ^{-1} is the inverse standard normal probability density function. Values of reliability index versus probability of failure are as follows:

β	P_f
0.0	0.500
1.0	0.159
2.0	$0.23\ 10^{-1}$
3.0	$0.14\ 10^{-2}$
4.0	$0.32\ 10^{-4}$
5.0	$0.29\ 10^{-6}$
6.0	$0.13\ 10^{-9}$

More advanced definitions and concepts related to the reliability theory can be found elsewhere (*4,5*).

Structural Assessment Procedure

The exposed method for evaluating the safety of existing concrete bridges is based on a probabilistic approach. The traditional formulation of the ultimate limit states (ULS) can be written as

$$M = R - S \tag{2}$$

where

M = safety margin
R = structural response (resistance), and
S = load effects.

M, R, and S should be modeled as random variables to evaluate the reliability index, β, as the more rational structural safety measure for the assessment of existing bridges considering its real structural capacity and actual or future load conditions. The more relevant parameters are updated for a specific case using information derived from inspections, tests, traffic measurements, and so on.

The formulation of the limit states concerning flexural behavior is presented in the following sections for the case of a simply supported beam. For continuous bridges, Sobrino and Casas proposed a method that takes into account the nonlinear behavior of concrete decks in evaluating ultimate load-carrying capacity (*6,7*).

Evaluation of ULS of Collapse Due to Moment Carrying Capacity

The failure function of the ULS of collapse due to bending moment, in a simply supported beam, can be formulated for the critical section as

$$M_u - (M_{g1} + M_{g2} + M_{q,T}) = 0 \tag{3}$$

where

M_u = ultimate bending moment capacity,
M_{g1} = bending moment due to self weight,
M_{g2} = bending moment due to dead load (pavement, fascia beams, etc.), and
$M_{q,T}$ = maximum bending moment due to traffic load for time T.

All of these should be modeled as random variables. The safety of the bridge will be ensured if the reliability index of this ULS (β_u) is greater than the minimum accepted value $\beta_{u,\min}$.

The same ULS is verified with the rules provided by the current Spanish design code. In this case, the load safety factor is obtained as

$$\gamma = \frac{M_{u,\text{nom}}}{M_{g1,\text{nom}} + M_{g2,\text{nom}} + M_{q,\text{nom}}} \tag{4}$$

where

γ = load safety factor,
$M_{u,\text{nom}}$ = nominal ultimate bending moment capacity (with design values of resistance parameters),
$M_{g1,\text{nom}}$ = bending moment due to nominal self-weight,
$M_{g2,\text{nom}}$ = bending moment due to nominal imposed dead load, and
$M_{q,\text{nom}}$ = bending moment due to nominal traffic load.

According to the current Spanish design code, high-quality control of execution requires that the value of γ be at least $\gamma > \gamma_{\min}$, with $\gamma_{\min} = 1.5$

Evaluation of Serviceability Limit State of Cracking

The evaluation of the serviceability limit state (SLS) is carried out according to the CEB-FIP model code and the proposal for the new Eurocodes (*8,9*). The verification of the SLS is based on the calculation of characteristic values for resistance and dead loads and frequent (or infrequent) values for traffic load effects. Frequent values are defined as loads with a return period of 2 weeks; infrequent loads have a return period of 1 year (*8*).

If cracking is not allowed, the failure function of the SLS can be formulated as

$$M_{\text{cr},k} - (M_{g1,k} + M_{g2,k} + M_{q,\text{freq}}) = 0 \tag{5}$$

where

$M_{\text{cr},k}$ = characteristic cracking bending moment capacity,
$M_{g1,k}$ = characteristic bending moment due to self-weight,
$M_{g2,k}$ = characteristic bending moment due to dead load (pavement, etc.), and
$M_{q,\text{freq}}$ = maximum bending moment due to frequent traffic loads.

If cracking is allowed, the crack width should be verified as

$$W_{\max} < W(g_1, g_2, q_{\text{freq}}) \tag{6}$$

where $W_{\max}$ is the accepted maximum crack width, and $W(g_1, g_2, q_{\text{freq}})$ is the crack width under dead loads and frequent live load.

Using the actual Spanish design code, the SLS should be verified as

$$M_{\text{cr,nom}} > (M_{g1,\text{nom}} + M_{g2,\text{nom}} + M_{q,\text{nom}}) \tag{7}$$

where $M_{\text{cr,nom}}$ is the nominal cracking bending moment capacity.

If cracking is allowed, the crack width should be verified as

$$W_{\max} < W(g_1, g_2, q_{,\text{nom}}) \tag{8}$$

where $W(g_1, g_2, q_{,\text{nom}})$ is the crack width under dead loads and nominal live load.

Case Study

A prestressed concrete voided slab deck is evaluated to illustrate the use of probabilistic methods in assessing existing bridges. The results of the reliability analysis are compared with those derived from the semiprobabilistic analysis with the current Spanish bridge design code.

The proposed example is not a real case, but it is similar to other cases recently studied by PEDELTA. The bridge is a simply supported beam 27.6 m in length; the typical cross section is a concrete voided slab 1.20 m deep. The dimensions are shown in Figure 1. The prestressing fo the bridge was designed to avoid tension stresses under dead load and nominal live load (using the Spanish bridge design code). The bridge deck has 10 cables 31 ϕ 0.5″ (grade Super according to BS5896).

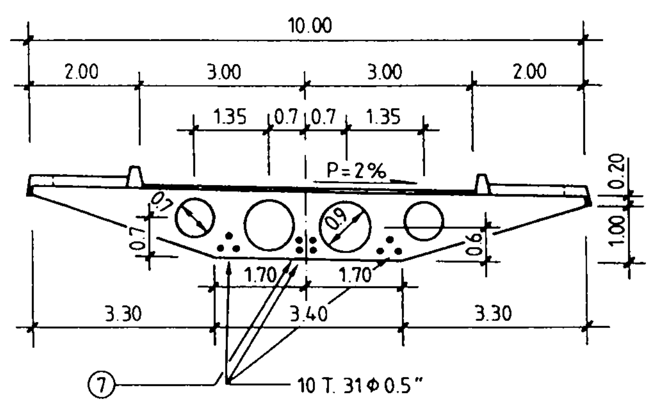

FIGURE 1 Typical cross section; prestressing cable affected during construction is marked with number 7.

In this example, the following situation is assumed:

1. It is supposed that during the construction of bridge one of the ducts is broken and it is not possible to place one of the prestressing tendons.
2. A higher quality of concrete has been obtained. A C35 concrete was specified, and the result of quality controls provided the following parameters:

$$f_{c,\text{mean}} = 42 \text{ MPa}$$

The coefficient of variation V_{fc} = 8 percent.

3. A higher quality of prestressing and reinforcement steel has been obtained. The result of quality controls provided the following parameters:

–Yield tensile stress of prestressing steel:

$$f_{yp,\text{mean}} = 1820 \text{ MPa } (f_{yp,\text{nom}} = 1620 \text{ MPa})$$

The coefficient of variation is assumed to be 3 percent.

–Yield tensile stress of reinforcement steel:

$$f_{y,\text{mean}} = 560 \text{ MPa } \quad (f_{y,\text{nom}} = 500 \text{ MPa})$$

The coefficient of variation is assumed to be 5 percent.

In both cases, the coefficient of variation is assumed from recent studies and a large data bank collected in Spain (6). These data have been updated with the results of quality control using Bayesian techniques.

4. The mean elongation of the cables during execution was practically coincident with the theoretical calculated values, with a coefficient of variation of 6 percent.
5. During the revision of the load effects in the design project, the effect of eccentric live load was not considered because of human error.

The objective of this paper is to evaluate the serviceability and load-carrying capacity of the deck with nine tendons under the real traffic loads.

Load Effects

The dead loads have been modeled as random variables using Monte Carlo techniques. The data used for the uncertainty modeling of geometry are derived from available data collected in concrete bridges in Spain (6). These models allow the consideration of common errors and uncertainties in concrete depths, real position of steel, real depth of pavements and so forth. The obtained values of bending moment at midspan due to self-weight and dead load in the case of study, for a good construction are given in Table 1. The mean thickness of the pavement is 80 mm.

TABLE 1 Bending Moment at Midspan (MN·m)—Nominal Values and Probabilistic Models (Including Model Uncertainties) for Evaluation of ULS

	Nominal Value (1)	Mean	Coefficient of Variation	Type of distribution
Self-weight	15.60	15.93	0.025	Normal
Imposed dead load	3.75	4.12	0.080	Normal
Traffic T=50 years	7.92	8.25	0.060	Gumbel

(1) Nominal values for traffic according to current Spanish code (including eccentricity effect of loads) and mean values of geometry for dead loads.

The traffic load effects have been obtained using numerical simulations with a model developed by Sobrino (6) and improved to simulate traffic using a grillage structural model. The traffic has been simulated passing over the bridge under real conditions. The traffic is 20 percent trucks with a typical highway composition of vehicles, classified by the number of axles (Figure 2). The average intensity is 12,000 vehicles per day in the period of study. The loads are derived from real measurements in Spain (6). Figures 3 and 4 show the mean and maximum truck weights obtained in measuring more than 16,000 trucks in 1 week during 1990 and 1992 near Barcelona. In some weigh stations more than 60 percent of vehicles exceed the legal limit (380 kN for more than four axles).

The maximum traffic load effects have been obtained for two situations: (a) free traffic with a maximum intensity of 1,000 vehicles per hour (2 h/day), and (b) full-stop traffic, with an average value of 4hr/week. The results of the simulations are presented in Tables 1 and

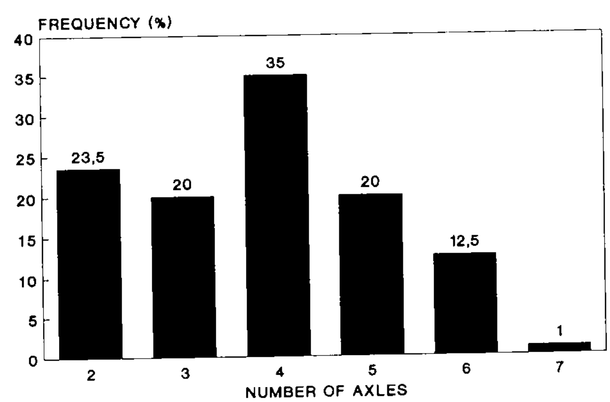

FIGURE 2 Heavy traffic composition classified by number of axles.

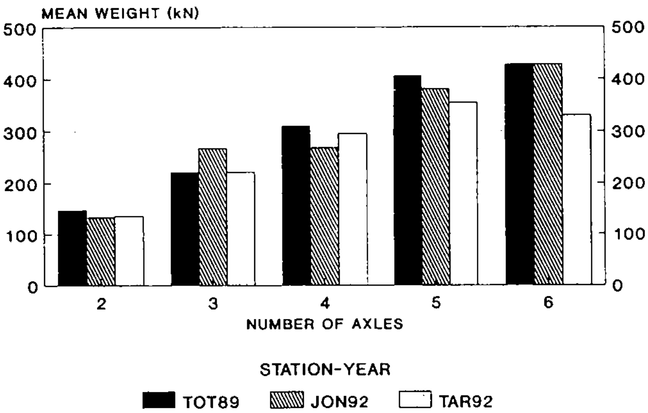

FIGURE 3 Mean truck weight observed in three weigh stations, 1990 and 1992, Spain.

2, in terms of bending moment at midspan, including the impact factor and the effect of load eccentricities on the deck.

Structural Response

The structural capacity of the bridge has been evaluated in terms of flexural response of the critical cross section. The probabilistic models for material strengths are based on a data bank collected in Spain; the values are similar to those measured in other countries for a high quality of control (6). The Monte Carlo techniques have been used to obtain the cross-sectional response, including the nonlinear behavior of steel and concrete in the calculation of the bending moment–curvature relationships and for the ultimate flexural capacity. Results are given in Table 3.

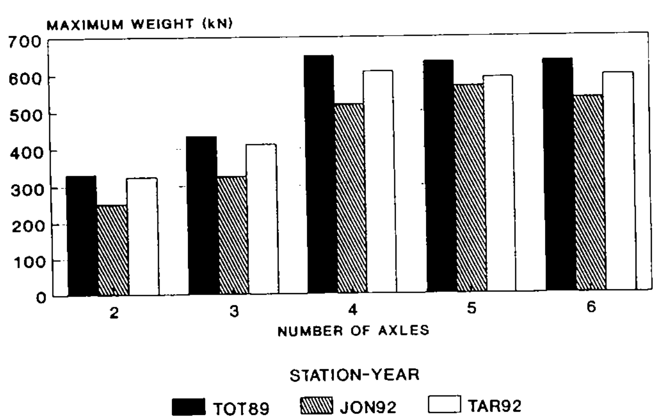

FIGURE 4 Maximum truck weight observed in three weigh stations, 1990 and 1992, Spain.

TABLE 2 Traffic Bending Moments for Evaluation of the SLS

	Bending Moment (MN·m)
Frequent loads	6.08
Infrequent loads	7.15

Note: These values were obtained by means of numerical simulations with real traffic loads and configurations.

Evaluation of Ultimate Limit Flexural Capacity

A reliability analysis has been performed to evaluate the safety of the bridge under the actual conditions with nine cables. The minimum value of $\beta_{u,min} = 5$ has been accepted. The results are shown in Figure 5. In Figure 6 the load safety factor is shown, defined as Expression 4.

The load safety factor for the girder with nine cables is $\gamma = 1.42$ (less than the minimum required $\gamma_{min} = 1.50$), but the reliability index is $\beta_u = 6.6$ (probability of failure $P_f = 10^{-11}$).

If the design condition is checked (deck with 10 prestressing cables), the measures of the safety level are

$\gamma = 1.55$

$\beta_u = 7.8$ (probability of failure $P_f = 10^{-14}$)

As a consequence of this study, it is noticed that the semiprobabilistic analysis gives very conservative criteria for the structural safety of the bridge compared with the more rational probabilistic study. The bridge should be considered safe enough under real traffic conditions for 50 years.

The safety of the bridge has also been verified with the same traffic configuration, assuming 40 percent of trucks in the traffic flow. In this case, the reliability index was very similar ($\beta_u = 7.6$) because the mean value of live load moment at midspan increases an 8 percent but the coefficient of variation is about 3.5 percent (not including the statistical or model uncertainties).

TABLE 3 Cross-Sectional Response at Midspan with 10 and 9 Prestressing Cables (MNm)

	No. of Cables	Nominal Value		Mean	Coefficient of Variation	Type of distribution
Mu	10	43.63	(1)	53.33	0.059	Lognormal
	9	40.17		48.09	0.059	Lognormal
Mcr	10	25.46	(2)	26.83	0.074	Normal
	9	22.82		24.21	0.075	Normal

(1) Nominal values are calculated with characteristic values of material strengths and partial safety factors.

(2) Nominal values calculated with characteristic values of strengths and prestressing force. The values of the mean and the Coefficient of Variation include model uncertainties.

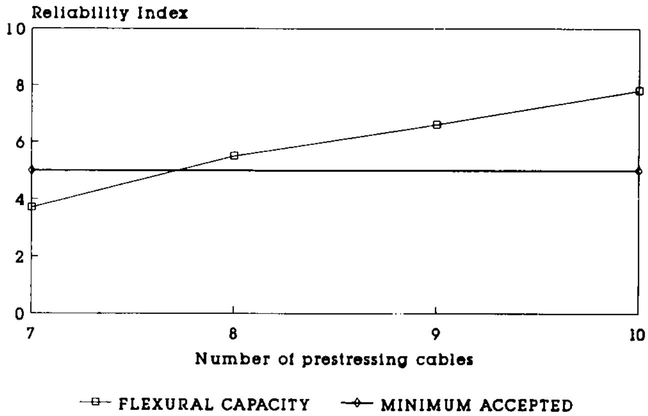

FIGURE 5 **Reliability index for ULS flexural carrying capacity; T = 50 years. Index has been calculated for different numbers of prestressing cables in deck. Bridge would be considered safe with eight cables.**

Evaluation of SLS of Cracking

The SLS of cracking has also been checked with 10 and 9 cables. In this case, the results are

$(M_{g1,k} + M_{g2,k} + M_{q,\text{freq}}) = 27.1$ MN·m

$(M_{g1,\text{nom}} + M_{g2,\text{nom}} + M_{q,\text{nom}}) = 27.6$ MN·m

Design condition with 10 cables: $M_{cr,k} = 25.5$ MN·m

Design condition with 9 cables: $M_{cr,k} = 22.8$ MN·m

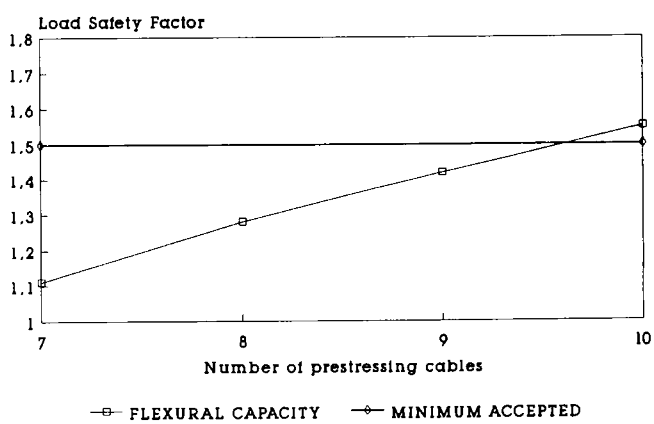

FIGURE 6 **Load safety factor for ULS of flexural carrying capacity. Safety factor is calculated by means of Expression 4 for different numbers of prestressing cables. Minimum accepted value by Spanish design code is 1.50. Bridge would be considered safe with 10 prestressing cables.**

In both cases, the SLS of cracking is not verified, and crack width should be checked under permanent and frequent traffic load effects. The crack width has been calculated using the rules of Eurocode 2 (9). The obtained values are

Design condition with 10 cables: $W_k = 0.08$ mm

Design condition with 9 cables: $W_k = 0.17$ mm

The maximum characteristic value of the crack width is, in both cases, less than the maximum design value accepted under frequent combination of loads and nonaggressive environment $W_{max} = 0.2$ mm. So the actual situation of the deck, with nine cables, should be considered satisfactory under real traffic conditions.

Conclusions

The use of probabilistic methods allows a rational evaluation of existing structures. The more relevant features of the probabilistic assessment method are presented using a simple example.

For evaluating existing prestressed concrete bridges, the use of the same safety factors and nominal design properties and loads does not allow the real condition of the structure—or further information about materials, test, and loads—to be taken into account. Because the probabilistic assessment leads to a more rational evaluation, it is an efficient tool for managing existing bridges.

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REFERENCES

1. *Bridge Management.* Road Transport Research, Organization for Economic Cooperation and Development, Paris, France, 1992.
2. European Committee for Standardization. *Eurocode 1: Basis of Design and Actions on Structures. Part 1: Basis of Design*, 6th draft. March 1993.
3. Nowak, A. S. Calibration of LRDF Bridge Design Code. *Proc., ICOSSAR '93 Conference, Structural Safety and Reliability, Vol.* 2, Austria, 1993, pp. 927–932.
4. *Conceptional Preparation of Future Codes.* Bulletin 147. Comité Euro-International du Béton, Feb., 1982.

5. Thoft-Christensen, P., and M. J. Baker. *Structural Reliability Theory and Its Applications.* Springer-Verlag, 1982.
6. Sobrino, J. A. *Evaluation of Structural Safety and Serviceability of Existing Reinforced and Prestressed Concrete Bridges* (in Spanish). Doctoral thesis. Technical University of Catalonia, Barcelona, Spain, Dec. 1993.
7. Sobrino, J. A., and J. R. Casas. Random System Response of Reinforced and Prestressed Concrete Bridges. *Proc., ICOSSAR '93 Conference, Structural Safety and Reliability,* Vol. 2, Austria, 1993, pp. 985–988.
8. European Committee for Standardization. *Eurocode 1: Basis of Design and Actions on Structures, Vol 3: Traffic Loads on Bridges.* 10th draft, 1993.
9. European Commitee for Standardization. *Eurocode 2: Design of Concrete Structures.* Revised final draft, 1989.