## Deflections of Timber-Strutted Corrugated-Metal-Pipe Culverts under Earth Fills

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An analysis is presented and a formula developed for predicting the vertical and horizontal deflection or change in diameter of timber-strutted corrugatedmetal-pipe culverts under earth fills. In the analysis the strutted pipe is considered to be a composite elastic structure in which the vertical deflection of the pipe is the same as the deformation of the timber strut. The deflection of the pipe is determined by curved-beam theory and equated to the expression for strut deformation.

The loading system used in the analysis consists of a vertical load and vertical reaction on the top and bottom of the pipe and horizontal pressures at the sides. These horizontal pressures are postulated to be passive earth pressures which are mobilized by the outward movement of the sides of the pipe as it deforms. The expression for lateral pressures involves a quantity which is called the modulus of passive resistance of the side-fill soil.

Results indicated by the developed formula are compared with the performance of an actual culvert installation at Cullman, Alabama. This culvert is an 84inch-diameter "improved multiplate" pipe under a highway embankment which is 137 feet high above the top of the pipe. Extensive observations of the performance of this structure were made and reported by the Armco Research Laboratories.

All of the factors involved in the derived equation for deflection, except the modulus of passive pressure of the side-fill soil, are available in the report or can be estimated from information given. It is possible, therefore, to compute the value of this modulus. The indicated value is 190 psi. per inch, which seems to be very high. However, the character of the soil is described as a "crumbly sandstone," and it was compacted to 100 percent AASHO density. Such a soil would have a very high bearing value and would be very resistant to deformation under pressure. It is the authors' opinion, therefore, that the calculated value may be of the same order of magnitude as the actual value. The need for extensive research to clarify the relationship between modulus of passive resistance and soil type and degree of compaction is indicated by this study.

• The Iowa Engineering Experiment Station in 1941 published Bulletin 153, entitled "The Structural Design of Flexible Pipe Culverts." In this bulletin a circular metal-pipe culvert was analyzed as a thin elastic ring acted upon by a system of external loads consisting of a vertical earth load on the top, and equal and opposite vertical reaction on the bottom, and horizontal pressures acting on both sides of the ring.

In the analysis it was postulated that the horizontal deflection of the pipe, caused by the vertical load and reaction, mobilized certain passive-resistance pressures in the soil at the sides of the pipe which acted in conjunction with the inherent strength of the pipe to resist deflection. These pressures were assumed to be proportional to the horizontal deflection of the pipe. The analysis was made for the case of a plain pipe (one installed without timber struts or horizontal tie bars or other predeforming devices).

The assumed load system employed in this early analysis is illustrated in Figure 1 and may be stated as follows: (1) The vertical load on a pipe may be determined by Marston's theory of loads on conduits and is distributed approximately uniformly over the breadth of the pipe. (2) The vertical reaction on the bottom of a pipe is equal to the vertical load and is distributed approximately uniformly over the width of bedding of the pipe. (3) The passive horizontal pressures on the sides of the pipe are distributed parabolically over the middle 100 degrees of the pipe and the maximum unit pressure is equal to the modulus of passive pressure of the sidefill material multiplied by half the horizontal deflection of the pipe.

A formula for the immediate or short

time deflection of a pipe under this system of loads was derived.

$$\Delta X = \frac{K W_C r^3}{E I + 0.061 er^4}$$
(1)

in which

- ▲X = the horizontal deflection, in. (the vertical deflection is nearly the same)
  - $K = 0.5 \sin a 0.082 \sin^2 a$ 
    - + 0.08  $\frac{a}{\sin a}$  0.16 sin ( $\pi$  a) -0.04 -  $\frac{\sin^2 a}{\sin a}$  + 0.318 cos a

-0.208

a = bedding angle

 $W_c$  = load on pipe, lb. per lin. in.

r = mean radius of pipe, in.

- E = modulus of elasticity of pipe metal, psi.
- I = moment of inertia of pipe wall, in.<sup>4</sup> per in.
- e = modulus of passive resistance of sidefill soil, psi. per in.

A series of field loading experiments was conducted in which the deflections of corrugated-metal pipes of several different diameters under a 15-foot earth fill were measured. The soil at the sides of the experimental pipes was compacted to several different densities in order to observe the influence of different values of lateral resistance pressures against the sides of the pipes. The measured deflections in these experiments were in reasonably good agreement with the deflections calculated by Equation 1.

As fill heights and pipe diameters have increased in recent years, more and more metal-pipe culverts are being installed with vertical timber struts, and there is need for an analysis of the deflections of a pipe in which these deflection-resistant members are included. The purpose of this paper is to present such an analysis. In this study the loading hypothesis stated above has been employed, along with the addition of a fourth item: The reactions at each end of the vertical strut act as concentrated loads at the inside of the top and bottom of the pipe; the magnitude of these concentrated loads depends upon the modulus of compression of the strut and the vertical deflection of the pipe.

The assumed load system for a timber strutted pipe is shown in Figure 2. It differs from that shown in Figure 1 only by the addition of the strut loads at the top and bottom. In actual construction practice, strutted pipes are usually predeformed to an out-of-round shape with the vertical diameter lengthened and the horizontal diameter shortened. This predeformation introduces bending moments in the pipe wall which are opposite in direction to the moments induced by the subsequent earth load. It also introduces an initial thrust in the timber strut due to the resilience of the pipe. These effects of predeformation have been ignored in the analysis, as a simplifying measure, in the belief that their influence on the magnitude of the deflection of a pipe under earth load is relatively minor. Further studies are needed to verify or disprove this assumption.

The deflection of a pipe which is referred to in this discussion is the change in diameter, i.e., the shortening of the vertical diameter and lengthening of the horizontal diameter, which is caused by the earth fill load. If the pipe is predeformed to an out-of-round shape prior to construction of the fill, the deflection refers to the



Figure 1.



Figure 2.

change in length of the diameters from their predeformed dimensions, not the change from the initial diameter of the circular pipe.

A free-body diagram of a segment of the elastic ring under the postulated loading is shown in Figure 3.

The general equation for the bending moment at any Point D on the ring, considering clockwise moments as positive, is:

$$M_{D} = -M_{C} - rP_{s}\sin \phi$$
  
-rR<sub>c</sub> (1 -cos  $\phi$ ) 0  $\leq \phi \leq \pi$   
+ 0.5v'r<sup>2</sup>sin<sup>2</sup>  $\phi$  0  $\leq \phi \leq a$   
+ v'r<sup>2</sup> sin a (sin  $\phi$   
-  $\frac{\sin a}{2}$ ) a  $\leq \phi \leq \pi$   
+ hr<sup>2</sup>(0.147 - 0.51 cos  $\phi$   
+ 0.5 cos<sup>2</sup>  $\phi$   
- 0.143 cos<sup>4</sup>  $\phi$ ) 40°  $\leq \phi \leq 140°$   
- 1.021hr<sup>2</sup>cos  $\phi$  140°  $\leq \phi \leq \pi$   
+ 0.5vr<sup>2</sup>(1 -sin  $\phi$ )<sup>2</sup>  $\frac{\pi}{2} \leq 0 \leq \pi$   
(2)

- $M_D$  = Moment at any point D
- $M_{C}$  = Moment at point C
- $R_c$  = Thrust at point C
- P<sub>S</sub> = one-half the thrust carried by the strut per unit length of ring. Concentrated loads acting at points C and A.

v = Vertical unit load on ring =  $\frac{W_C}{2r}$ 

v' = Vertical unit reaction on ring =  $\frac{W_c}{2r\sin a}$ 

$$W_{c}$$
 = Load on ring per unit of length

- r = Mean radius of ring
- h = Maximum horizontal unit pressure on ring =  $e \Delta X$ 2
- e = Modulus of passive resistance of side-fill soil
- $\Delta X$  = Horizontal deflection of ring
- a = Bedding angle with vertical axis
- φ = Angle between radius to any point on the ring and the vertical axis.

Since the load on the ring is symmetrical about the vertical axis, the normal sections at Points A and C will remain vertical, regardless of the character and magnitude of the angular displacements of normal sections at intermediate points. Therefore the sum of all the elementary angular displacements as  $\phi$  varies from 0 to  $\pi$  will be zero, and we may write:

$$\frac{\mathbf{r}}{\mathbf{E} \mathbf{I}} \int_{0}^{\pi} \mathbf{M} d\phi = \int_{0}^{\pi} \mathbf{M} d\phi = 0 \quad (3)$$

Substituting the general moment Equation 2 in Equation 3 and integrating:

$$M_{C} = -rR_{C} - 0.637rP_{S} + 0.345hr^{2}$$
  
+ 0.057vr<sup>2</sup> + v'r<sup>2</sup> [0.08 a  
- 0.04sin<sup>2</sup> a - 0.159sin<sup>2</sup> a (π-a)  
+ 0.318sin a (1+cos a)] (4)

By the displacement theory, the horizontal movement of A relative to C is equal to

In which

$$0 = \underline{\mathbf{r}^2}_{\text{EI}} \int_0^{\pi} M(1 - \cos \phi) d\phi$$
 (5)

Substituting Equation 3 in Equation 5 yields:

$$\int_{0}^{u} \operatorname{Mcos} \phi \, \mathrm{d}\phi = 0 \tag{6}$$

Substituting the general moment Equation 2 in Equation 6 and integrating, the thrust  $R_c$  may be evaluated and is found to be:

$$R_c = 0.053W_c (1-\sin^2 a) + 0.511hr$$
 (7)

Substituting this value of  $R_C$  in Equation 4 the moment at Point C is:

$$M_{C} = -0.136hr^{2} - 0.637rP_{S} + W_{C}r$$

$$\begin{bmatrix} 0.053sin^{2}a - \frac{0.04a}{sin a} - \frac{0.02sin^{2}a}{sin a} \\ - 0.08sin a (\pi - a) + 0.159cos a \\ + 0.135 \end{bmatrix}$$
(8)

The value of the thrust carried by the strut,  $2P_s$ , may be determinated by setting the vertical deflection of the strut equal to the vertical movement of Point A with respect to Point C, that is, the vertical deflection of the ring. Assuming the bottom of the ring or Point C as fixed, the vertical deflection of the ring will be:

$$\Delta y = \frac{r^2}{EI} \int_{0}^{\pi} M \sin \phi d\phi \qquad (9)$$

Also the deflection of the strut will be:

$$\Delta y = \frac{2P_{s}L_{s}S_{s}}{A_{s}E_{s}}$$
(10)

In which

 $P_s$  = half of the strut load per unit length of ring.  $L_s$  = length of the strut

 $A_s$  = cross sectional area of the strut

 $S_S$  = longitudinal spacing of struts

E<sub>s</sub> = equivalent modulus of compression of the strut

Then

$$\frac{2P_{s}L_{s}S_{s}}{A_{s}E_{s}} = \frac{r^{2}}{\overline{EI}} \int_{0}^{\pi} M\sin\phi d\phi \quad (11)$$

Substituting the general moment Equation 2 in Equation 11, integrating and reducing:

$$P_{S} = \left\{ \frac{A_{S}E_{S}r^{2}}{2L_{S}S_{S}EI + 0.296r^{3}A_{S}E_{S}} \right\}$$

$$\left\{ -0.119hr^{2} + W_{C}r \\ \left[ 0.16\sin \alpha (\pi - \alpha) \\ -0.25\sin \alpha (1 - \cos \alpha) \\ + 0.125\sin^{2} \alpha \\ -0.08\alpha - 0.04\sin^{2} \alpha \\ - \frac{0.083\cos^{3} \alpha + 0.26\cos \alpha - 0.167}{\sin \alpha} \\ - 0.318\cos \alpha - 0.25\alpha + 0.433 \right] \right\}$$
(12)

Let

$$K_{1} = \frac{A_{s}E_{s}r^{2}}{2L_{s}S_{s}EI + 0.296r^{3}A_{s}E_{s}}$$
(13)

and  $K_2$  = the expression in brackets in Equation 12

Then 
$$P_s = K_1 (W_c r K_2 - 0.119 h r^2)$$
 (14)

The horizontal deflection of the ring is equal to twice the horizontal movement of Point B relative to Point C and we may write:

$$\Delta \mathbf{X} = \frac{2\mathbf{r}^2}{\mathbf{EI}} \qquad \int_{0}^{\frac{\pi}{2}} \mathbf{M} (1 - \cos \phi) \, d\phi \tag{15}$$

However, if the loads on the ring are symmetrical about a horizontal axis, the normal cross sections at the sides of the ring  $(\phi = \frac{\pi}{2})$  will not rotate in relation to their unloaded positions and the tangents to the sides will remain vertical when the loads are applied. Under these conditions, Equation 15 may be simplified to:

$$\Delta X = \frac{2r^2}{EI} \int_{0}^{\frac{T}{2}} M\cos\phi \, d\phi \qquad (16)$$

The postulated loading shown in Figure 2 approaches this condition of symmetry with respect to the horizontal axis as the bedding angle  $\alpha$  increases toward its maximum value of 90 degrees. Since, in prac-

tice, the bedding angle is usually large, say from 60 degrees to 90 degrees, it is believed that Equation 16 can be used without appreciable error.

Substituting the general moment Equation 2 in Equation 16 and integrating gives:

$$\Delta \mathbf{X} = \frac{r^2}{EI} \left[ -0.122hr^2 - 0.274P_S r + W_C r (0.5sin a - 0.082sin^2 a) + 0.08 \frac{a}{sin a} - 0.16 sin a(\pi - a) - 0.04 \frac{sin^2 a}{sin a} + 0.318cos a - 0.208) \right]$$
(17)

According to the lateral pressure hypothesis proposed in Bulletin 153.

$$h = \frac{e \Delta X}{2}$$
(18)

Substituting the value of h (Equation 18) and the value of  $P_{\rm S}$  (Equation 14) in Equation 17 and letting the expression in parenthesis equal K:

$$\Delta \mathbf{X} = \frac{\mathbf{W}_{\mathbf{C}}\mathbf{r}^{3} \ (\mathbf{K} - 0.274\mathbf{r}\mathbf{K}_{1}\mathbf{K}_{2})}{\mathbf{EI} + \mathbf{er}^{4} \ (0.061 - 0.016\mathbf{r}\mathbf{K}_{1}) \ (19)}$$

The value of K and  $K_2$  are nearly the same numerically. This is particularly true for larger values of the bedding angle a. Therefore for practical purposes we may rewrite Equation 19 without appreciable error, as follows:

$$\Delta \mathbf{X} = \frac{\mathbf{K} \mathbf{W}_{\mathbf{C}} \mathbf{r}^{3} (1 - 0.274 \mathbf{r} \mathbf{K}_{1})}{\mathbf{E} \mathbf{I} + \mathbf{e} \mathbf{r}^{4} (0.061 - 0.016 \mathbf{r} \mathbf{K}_{1})}$$
(20)



In the case of a plain or unstrutted pipe, the value of  $E_S$  will be zero and  $K_1$  (Equation 13) will be zero. Substituting  $K_1 = 0$  in Equation 20 yields the same expression as Equation 1, which was derived for the unstrutted condition. This gives a partial check on the form of Equation 19.

No experimental program of loading strutted corrugated metal pipe culverts under earth fills has been carried out for the purpose of verifying Equation 20. However, the Research Laboratories of Armco Steel Corporation have recently issued a report entitled "Multi-Plate-Compression Measurements at Cullman, Alabama Installation, "by John H. Timmers. The details of installation and of the performance of the middle line of three 84-inch multi-plate pipe culverts under 137 feet of highway fill are given in this report. Although all of the information needed to apply Equation 20 to this installation is not available, it is interesting to study the application of the formula for deflection in the light of the reported data.

Detailed observations of the performaance of the culvert were made on several 8-foot-long bolted sections along the length of the pipe. Three of these sections (Nos. 31, 32, and 33) were located under the roadway portion of the embankment where the height of fill above the top of the pipe was 137 feet. The data relative to these three sections have been used in this study of the applicability of the formula for deflection of a strutted pipe. The horizontal deflections of these sections were 0.67, 0.63, and 0.85 inch, respectively, or an average of 0.72 inch, under the maximum height of fill and before the timber struts were removed. This average deflection increased to 0.75 inch immediately after the struts were removed.

These culvert-pipe sections had a nominal diameter of 84 inches and were fabricated of corrugated-metal plates which were l gage (approx.  $\frac{5}{16}$  inch) thick. The corrugations were spaced 6 inches on centers and were l  $\frac{3}{4}$  inches deep. The moment of inertia of a cross-section of the pipe wall was 0.1288 inch per inch of length of pipe. The mean radius of the pipe, that is, the average radius to the neutral axis of the pipe wall was 43.13 inches. The modulus of elasticity of the pipe metal has been assumed to be 30 million psi.

The pipes were bedded on a 2 foot uniform thickness of creek-bed sand. Sidefill material consisting of a "crumbly sandstone" which was compacted under and around the pipes to a specified 100 percent of standard AASHO density by means of power operated tampers. Inspectors' reports indicate that the specified density was exceeded by 0.4 to 5.25 percent. With this type of soil and the compaction obtained, it seems probable that the pipe was supported completely over the width of the bottom half of the pipe and the value of the bedding angle  $\alpha$  probably approached 90 deg.



## Figure 4.

The pipes were strutted along their longitudinal axes by means of vertical timbers acting between longitudinal sills at the top and bottom of the pipe. The struts and bottom sills consisted of 8-by-8-inch rough oak timbers. The top sills were made of two 8-by-8-inch rough oak timbers arranged side by side. A compression cap of relatively soft wood was inserted between the top sill and the strut. It consisted of four 2-by-8-inch rough pine planks laid flatwise. A diagram of the strut, compression cap, and sills is shown in Figure 4. There were three struts in each of pipe Sections 31, 32, and 33. The average longitudinal spacing of the struts was 32 inches on centers.

In order to apply the deflection formula for strutted pipes it is necessary to know or to estimate the "equivalent modulus of compression" of the strut as installed. This modulus is similar to the modulus of elasticity of an elastic member, i.e., the unit stress divided by the unit strain. The total compression strain of the strut assem bly consists of the compression of the sills, the compression of the corrugations into the corners of the sills, the compression of the compression cap, and the shortening of the main body of the strut. It is substantially equal to the vertical deflection of the pipe, which is the shortening of the vertical diameter. Since the soft wood compression caps were stressed well bevond their elastic limit as the earth load

on the pipe increased, the stress-strain diagram for the strut assembly is a curved line. The modulus of compression used in this analysis is essentially a secant modulus drawn to the point of greatest strain on the stress-strain curve.

In the Cullman project the average vertical diameter of Sections 31, 32, and 33 at the time the struts were installed was 90.53 inches and this is essentially the length of the struts. The vertical deflection of these three sections under the maximum fill of 137 feet was 0.67, 0.62, and 1.02, respectively, or an average of 0.77 inches. Therefore the average unit strain of the strut assembly was 0.77 = 0.008590.53

inches per inch. The loads on the struts were determined by means of load cells and by measurements of the compression of the soft wood caps. The average load on each strut in the three sections was approximately 54,000 lb. Therefore, the unit stress on the struts was  $\frac{54000}{64} = 844$ 

ps1. and the equivalent modulus of compression was <u>844</u> or approximately 100,000 <u>0.0085</u>

psi.

The load on the culvert was measured by means of resistance strain gages mounted on the neutral axis of the pipe wall at the ends of the horizontal diameter in conjunction with two strut load cells which were installed between the compression caps and the main element of two struts near the junction of Sections 31 and 32 and Sections 32 and 33. These strut load cells consisted of short lengths (about 6 inches) of  $\delta$ -inch pipe mounted between  $\frac{3}{4}$  -inch-square steel plates. The strut load was measured by means of resistance strain gages mounted on the inside of the cell walls. The average load on the three sections under the highest portion of the fill was about 86,700 lb. per lin. ft. or 7,225 lb. per lin. in.

Thus it is seen that in the case of the Cullman project, we have actual measurements or reasonable estimates of all the factors necessary for the solution of Equation 20, except the modulus of passive resistance of the side-fill soils. A recapitulation of the data is as follows:

$$\Delta X = 0.72$$
 in.

$$W_C = 7225$$
 lb. per lin. in.

$$r = 43.13$$
 in.

E = 30,000,000 psi.I = 0.1288 in.<sup>4</sup> per in. L<sub>s</sub> = 90.53 in. A<sub>s</sub> = 64 sq. in. E<sub>s</sub> = 100,000 psi. S<sub>s</sub> = 32 in. a = 90 deg.

Substituting the above numerical values in Equation 20 yields a modulus of passive pressure, e, equal to 190 psi. per in. At first glance this appears to be an inordinately high value. It is more than twice as high as the value of this modulus which has been estimated in connection with any previous flexible pipe loading tests which are within the author's knowledge.

However, a critical study of the character of the sidefill soil material led to the belief that the indicated value of the modulus may not be excessive or out of reason. The Armco Laboratories Report describes the fill material and method of handling and placement as follows:

The fill material was mainly a crumbly sandstone and rock for the first 25 feet. In order to secure the desired density of fill and guard against future settlement, choking was accomplished by alternately placing embankment layers of rock and earth except in the first 10 feet immediately over the pipes. The fill material was obtained from borrow pits on both sides of the fill ... The borrow pit on the north side consisted mainly of rock while the borrow pit on the south side, from which the major part of the fill was constructed, was a very crumbly friable sandstone which after blasting was easily handled. The fill was largely built using self-loading scrapers. The compaction of the embankment was handled by a combination of tandem sheepsfoot rollers and the normal equipment traffic.

1. Phillips, Donald L. Investigation of the Stresses and Deflections of a Corrugated Metal Pipe Culvert Under a High Earth Fill. Unpublished Thesis, Library, Iowa State College, Ames, Iowa, 1953.

2. Spangler, M. G. The Structural Design of Flexible Pipe Culverts. Bulletin 153, Iowa Engineering Experiment Station, The compaction required under and between the pipes was 100 percent AASHO Standard. The size of the rock within 2 feet under and 3 feet over the pipe was limited to 4 inches. All this select material was tamped with power operated hand tampers. Springs that were encountered in preparing the bed were handled with 6-inch Helcor subdrain.

It is apparent from the above description that the side-fill material was of high quality from the standpoint of bearing capacity and was thoroughly compacted. A foundation engineer would consider such material to be an ideal foundation soil. Settlement of a structure founded on material of this character could be expected to be negligible.

The fact that the sides of these 84-inch corrugated-metal pipes moved outward only 0.36 inch under a vertical load on the pipe of about 86,700 lb. per lin. ft. is ample evidence that the sidefill soil material developed high resistance to lateral movement. Therefore it is the authors' conclusion that the indicated value of 190 psi. per in. is not an excessive value of the modulus of passive resistance of the soil material which prevailed at the sides of the pipes in this project.

This study of the performance of the Cullman project in relation to the analysis of a timber-strutted corrugated-metal pipe is not offered as conclusive evidence of the validity of the deflection formula, Equation 20. It is merely one straw in the wind and many more such comparisons are needed before definite conclusions as to its validity can be drawn. Particularly, detailed studies of the magnitude and character of the modulus of passive resistance of soils of various classifications and in various states of density are needed to enhance the technology of flexible culvert design.

## References

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3. Spangler, M. G. Stresses and Deflections in Flexible Pipe Culverts. Proc. Highway Research Board. Vol. 28, p. 249, 1948.

4. Timmers, John H. Multi-Plate Compression Measurements at Cullman, Alabama Installation. Armco Research Laboratories, Middletown, Ohio, 1953.

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