HIGHWAY RESEARCH BOARD

Bulletin 102

Tests on Large Culvert Pipe

National Academy of Sciences—
National Research Council

HIGHWAY RESEARCH BOARD

Officers and Members of the Executive Committee
1955

HIGHWAY RESEARCH BOARD

Bulletin 102

Tests on Large Culvert Pipe

PRESENTED AT THE

Thirty-Third Annual Meeting

January 12-15, 1954

1955

Washington, D. C.

Contents

Department of Materials and Construction

Tests of Large-Diameter Reinforced-Concrete Pipe

JOHN G. HENDRICKSON, JR., Research Engineer,
American Concrete Pipe Association, Chicago

The standard test in general use for concrete pipe of all sizes is the three-edge bearing test. This test is used to determine two values, generally accepted as indicative of the quality and strength of concrete pipe: the load to produce a 0.01-inch crack and the load to produce ultimate failure.

It is common practice to use this test on both plain and reinforced-concrete pipe in sizes ranging from 4 inches to 72 inches in diameter. Relatively few three-edge bearing tests have been performed on pipe larger than 72-inches in diameter. This was partly due to the lack of equipment with sufficient capacity to handle the larger pipe as well as the cost of manufacturing and testing the pipe to destruction.

In addition, there was some feeling that the three-edge bearing test was not applicable to large-diameter pipe. Depending to some degree on the size of the pipe and the amount of steel reinforcement, test results on large pipe did not appear to be consistent with either theory or tests on pipe of smaller sizes. It was found that increasing the steel area beyond a certain amount had little or no affect on either the 0.01-inch-crack load or the ultimate load which the pipe would support.

In the last 10 years the demand for concrete pipe in larger sizes has grown considerably. It is well known that the relationship between the supporting strength of the pipe in the ground and the strength of the same pipe in a three-edge bearing test is an empirical one developed mainly with tests on smaller pipe. The increased demand for large pipe and the lack of available design data pointed up the need for a study of the behavior of large pipe under load.

A number of tests have been run on pipe 84 inches in diameter. The three-edge bearing test was selected as the easiest method of loading the pipe. Various systems for reinforcing the pipes were used including standard types normally used in pipe production.

As a result of these tests, it is believed that the major factor causing the discrepancy between test results and expected theoretical results has been the failure to consider shearing stresses set up in the wall of the pipe. Failure due to shearing stresses occurs before the tension reinforcement has begun to carry its design load, particularly in heavily reinforced pipe. If special reinforcement is provided to take care of shearing stresses, the strength of the tension reinforcement can be used more effeciently. It is possible to greatly increase the crushing strength of large diameter pipe without an increase in the wall thickness.

● THERE are two different methods of determining the load-carrying ability of any structure. One is an analytical method wherein certain safe values of stress are assigned to the materials in the structure. The structure is then proportioned in accordance with known principles of mechanics, so that applied loads will not create stresses in the structure greater than the known safe values. The second method is to build and test a number of individual specimens whose dimensions and materials are consistent with those used in the actual structure. The second method is the one which has been used most extensively to determine the load-carrying ability of reinforced-concrete pipe.

METHODS OF TESTING

A number of different types of tests have been devised to determine the load-carrying capacity of a rigid pipe. The standard method of testing now generally used for concrete pipe is the three-edge bearing test as described in ASTM Specifications

C4, C118, C14, C75, and C76, as well as AASHO Designation T33-45. The sand-bearing test also described in these same specifications is sometimes used as an alternate. It is considered by some to be more representative of the loading to which a pipe in the ground is subjected, but this appears to be questionable. In addition, the test is cumbersome and expensive to perform. The three-edge bearing test is a simple, convenient, and rapid method of checking the strength and quality of pipe.

STRENGTH REQUIREMENTS FOR REINFORCED-CONCRETE PIPE

Load-test requirements for concrete and reinforced-concrete pipe have been a subject for discussion and study for many years. One of the earliest and most-complete studies on the load-supporting strength of reinforced-concrete pipe was made in 1907 and 1908 at the University of Illinois. The tests were reported in Engineering Experiment Station Bulletin 22 by A. N. Talbot. Following these tests, railroads began using reinforced-concrete culvert pipe for drainage structures. The pipe was designed in accordance with the formulas from Bulletin 22, but in 1919 it was said that more than 20 different specifications for reinforced-concrete culvert pipe were in existence. In 1919 the Joint Concrete Culvert Committee was formed to simplify and reduce the number of specifications. In a report issued in 1926 the committee specified the steel reinforcing to be used with concrete pipe. This report was the forerunner of the present day ASTM specifications for concrete sewer and culvert pipe.

In the current ASTM specifications there are no references to design stresses for reinforced concrete pipe. Strength requirements are given for three-edge and sand tests together with minimum wall thicknesses and minimum steel areas for each size of pipe. These quantities were determined partly by theory and partly by experience and testing. Two strength requirements are specified, one for a 0.01-inch crack and one for an ultimate.

Strength requirements are specified only for pipe 72 inches and smaller in diameter. Until recently the bulk of the pipe manufactured in this country came within this range of sizes. Strength requirements for pipe larger than 72 inches have never been specified, because there has never been sufficient test data available to determine what the requirements should be. In addition, there has been some feeling in the concrete-pipe industry that the three-edge bearing test is not applicable to large-diameter pipe. This is probably due to the fact that large pipe when tested in three-edge bearing sometimes fails to develop the strength anticipated, considering the amount of reinforcing steel and the strength of the concrete used in making the pipe.

Since the end of World War II, there has been a trend toward the use of larger sizes of concrete pipe. Wider use of heavy power equipment has reduced the problem of handling heavy sections of pipe. Increasing costs of labor and form work have placed precast concrete pipe in a better competitive position with monolithic structures. Wider recognition of the great durability and good hydraulic properties have probably had some influence on the trend toward the larger sizes.

Concurrent with the demand for larger and larger pipe came a demand for information on the load-bearing capacity of that pipe. The empirical relationship developed at Iowa State College between the strength of a pipe in a three-edge bearing test and the supporting strength of the same pipe in the ground is well known. To apply it one obviously needs to know the strength of a pipe in a three-edge bearing test. But as previously pointed out, this information is lacking for pipe larger in diameter than 72 inches.

In an effort to throw some light on this problem, the American Concrete Pipe Association, together with the Lewistown Pipe Company at Hillside, Illinois, and later with the Lock Joint Pipe Company of East Orange, New Jersey, decided to run a series of load tests on 84-inch reinforced-concrete pipe. It was considered impractical to try to develop average values for test-load requirements to be included in pipe specifications. Rather, the tests were to study the behavior of the pipe under load when reinforced in different ways. The three-edge bearing test was selected as the method of testing the pipe. Ultimately it is hoped that a design procedure for large-diameter reinforced-concrete pipe can be recommended, but this paper will report only on tests of the pipe.

TYPES OF FAILURE

Reinforced-concrete pipe is generally reinforced either by two circular cages or by one elliptical cage. In testing pipe reinforced in this manner, it has been observed that two different forms of failure usually occur. One is a tension failure of the steel reinforcing at either the top or bottom of the pipe. This type of failure occurs rarely in large-diameter pipe of the proportions generally used in present-day manufacture.

The second type of failure is by a stripping or shearing of the concrete from the tension face at the top and bottom of the pipe. As the pipe is loaded, tension cracks form in the concrete and the reinforcing steel begins to be effective. This steel is originally in the shape of a circle, but the high bending moments at the top and bottom of the pipe cause the steel near the inner face of the pipe wall to tend to move inward, so as to form a chord of the original circle. As the load is increased this action continues until the tendency of the steel to straighten is great enough to strip off the concrete. This is the type of failure usually occurring in the larger pipes.

Experience has shown that the larger the pipe the more susceptible it is to this type of failure. Forces tending to straighten the steel increase with the pipe size. This type of action does not occur in ordinary reinforced-concrete beams as the direction of the stress in the tension steel is mainly in a straight line along the axis of the steel itself.

A typical example of a pipe loaded to failure in a three-edge bearing test is shown in Figure 1.

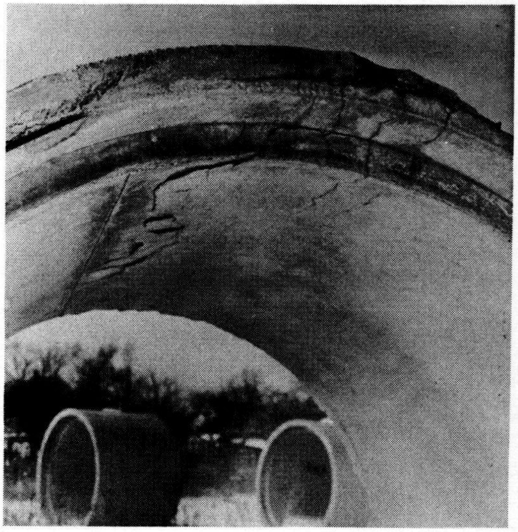


Figure 1. This pipe was tested to failure in a three-edge bearing test.

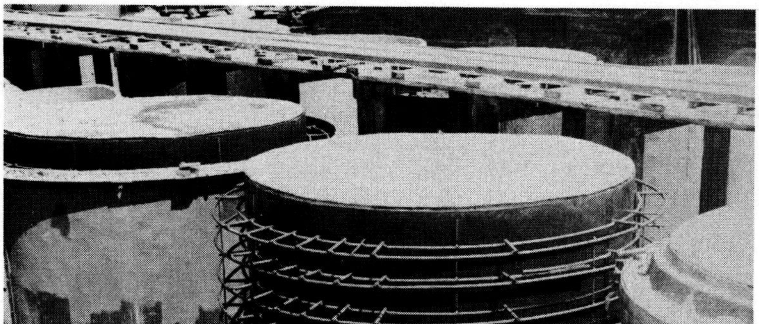


Figure 2. Reinforcing for Pipe A has been placed around the inside form of the pipe.

FIRST SERIES OF TESTS ON 84-INCH PIPE

The first series of tests on 84-inch pipe were intended to show the effectiveness of various methods of reinforcing the pipe in preventing failure due to stripping or shearing of the concrete. The pipe sections were made with standard tongue-and-groove forms. Test sections were of uniform wall thickness and showed no evidence of honeycombing or other defects. Test sections were placed in a testing frame for a standard three-edge test. The load was applied by means of a 100-ton hydraulic jack. Changes in the vertical and horizontal diameter were measured to the nearest 1/32 inch. While this may seem to be a rather crude method of measuring deflections, it was considered good enough for the purpose of these tests.

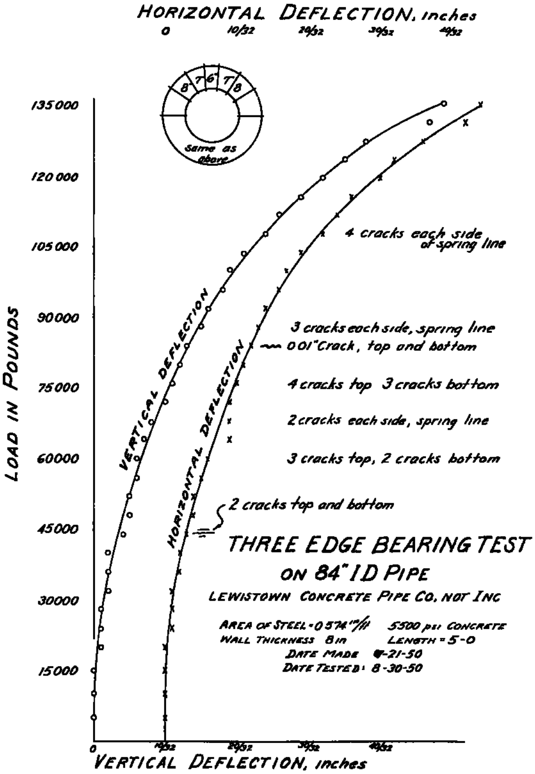


Figure 3. Load-deflection curves for Pipe A.

The first pipe tested, Pipe A, was reinforced with concentric steel hoops spaced on 6-inch centers. The two hoops were welded together with 1/2-inch ties at the top and bottom of the pipe. These ties, or stirrups, are to prevent the tendency of the curved reinforcing bars to straighten as the pipe is loaded.

This system of reinforcing, shown in Figure 2, requires that the pipe be tested with a particular axis vertical. Results of tests on this pipe are shown in Figure 3. The 0.01-inch crack occurred at a load of 84,000 lb. and the ultimate at 135,000 lb. This corresponds to 16,800 and 27,120 lb. per ft., respectively.

It is generally considered that an 84-inch extra-strength reinforced-concrete culvert pipe should have an ultimate strength of 21,000 lb. per ft. This pipe easily meets this requirement despite the fact that the steel area is less than the minimum required by ASTM specifications for that class of pipe.

The reinforcing used in Pipe B, Figure 4, was similar to that already described. The main difference was the equal spacing of 3/8-inch ties around the circumference of the pipe, so the pipe could be tested in any position. The results from the test of this pipe are shown in Figure 5 and are seen to correspond fairly closely to the first pipe tested.

Figure 6 shows a different method of tying the steel hoops together with 3/8-inch bars. The steel area is slightly greater in this pipe. Results of the tests are shown in Figure 7. The 0.01-inch-crack load and the ultimate load are consistent with the other pipe tested. The deflection in this pipe is considerably less, probably due to the method of tying the hoops together. The steel used in forming the concentric hoops in these three pipes was heavy, cold-drawn steel wire approximately 5/8 inch in diameter. Its tensile properties were about the same as those of cold-drawn wire mesh.

In Pipe D, reinforced with cold-drawn wire mesh, 3/8-inch tie bars hooked into inner and outer cages rather than welded to them, were used. As shown in Figure 8, three ties spaced 12 inches apart were placed on every circumferential bar at the top and bottom of the pipe. When tested, the pipe had a 0.01-inch-crack load of 137,000 lb. or 27,400 lb. per lin. ft. The load was not increased to ultimate failure but was stopped at 160,000 lb. as shown in Figure 9.

The high value of the 0.01-inch-crack load is thought to be due to the 6-inch

Figure 4. Reinforcing for Pipe B is shown in position.

spacing of the longitudinal wires in the wire-mesh reinforcing. If this spacing were 8 inches or 12 inches, as is the case for some styles of mesh, such a high value of the 0. 01-inch-crack load probably could not be expected. The area of reinforcing steel in this pipe is that required by ASTM Specification C76 Table 2 for 84-inch pipe.

Pipe E, reinforced as shown in Figure 10, is the same as Pipe D, except for the spacing of the tie bars or stirrups. Three

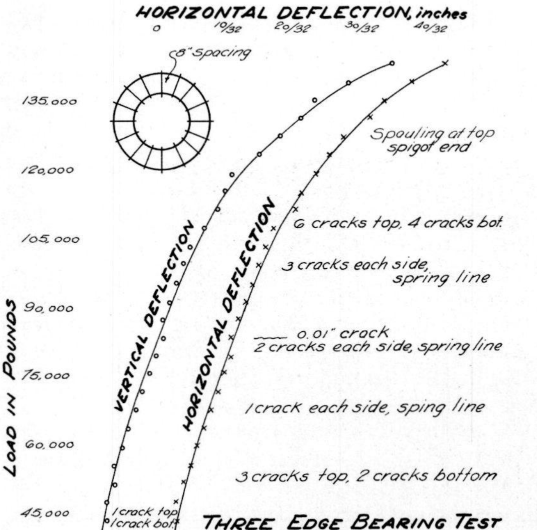


Figure 5. Load-deflection curves for Pipe B.

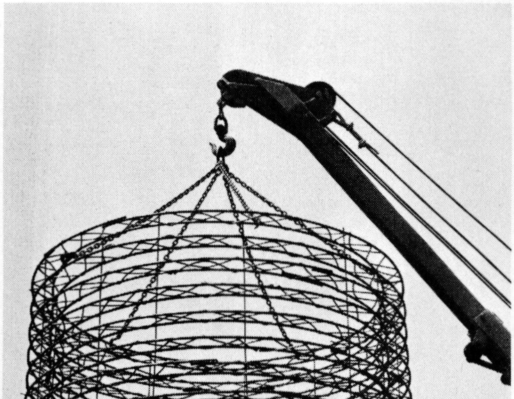


Figure 6. Reinforcing for Pipe C is being lifted into place in the forms.

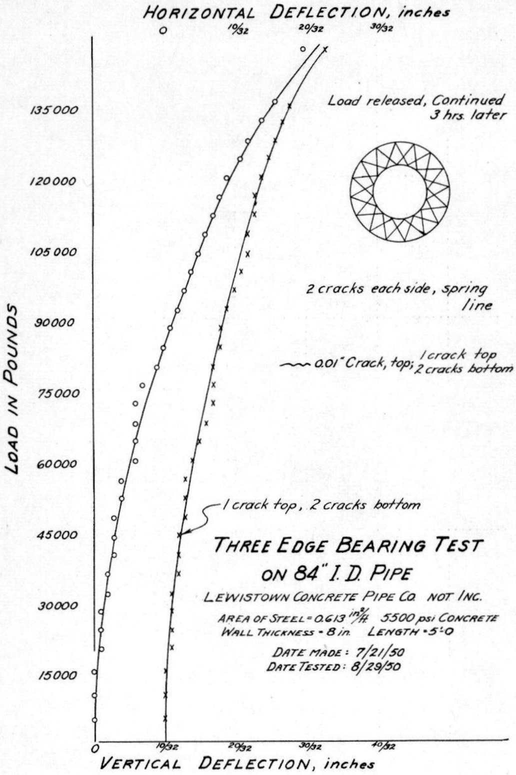


Figure 7. Load-deflection curves for Pipe C.

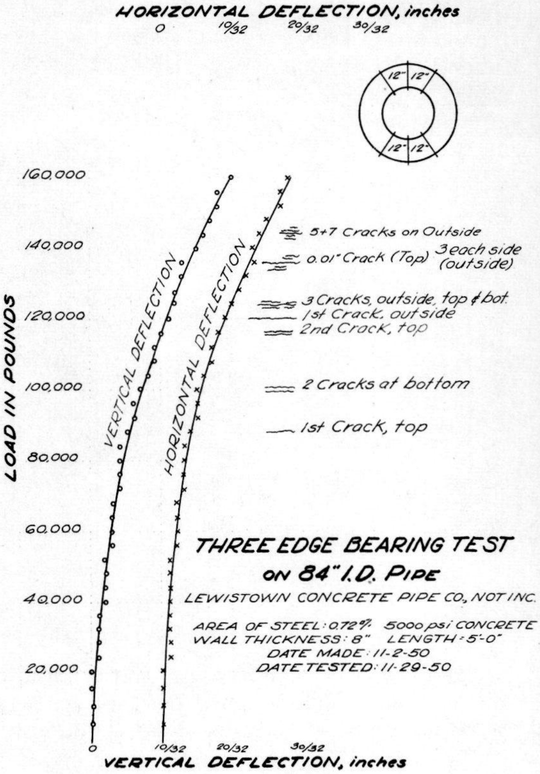


Figure 9. Load-deflection curves for Pipe D.

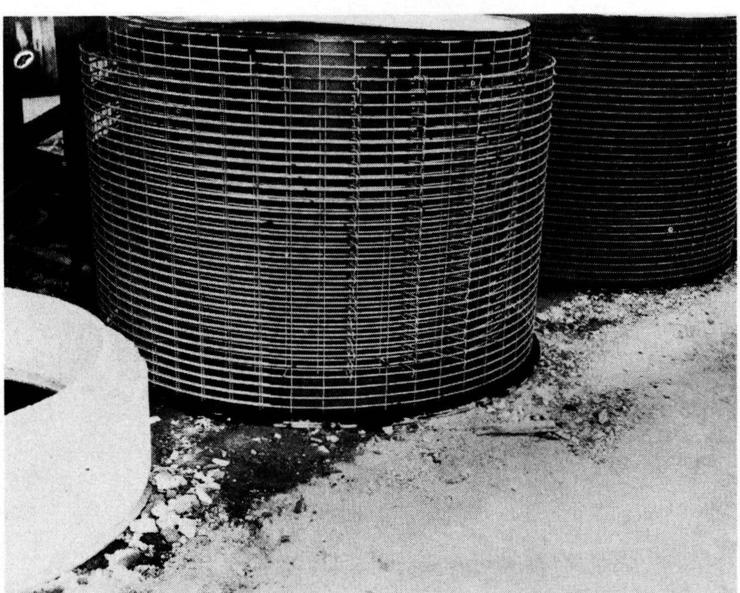


Figure 8. Pipe D was reinforced with wire mesh. Hooked tie bars provide shear reinforcing.

ties spaced alternately 12 inches and 6 inches apart were placed on every circumferential bar at the top and bottom of the pipe. Figure 11 shows that the test results correspond closely with those of Pipe D, as would be expected.

Failure occurred by a separation of the inner cage from the curved bar of the trusses and the stripping off of the concrete at the top of the pipe. Load-deflection curves are shown in Figure 14. The results of this series of tests is given in Table 1.

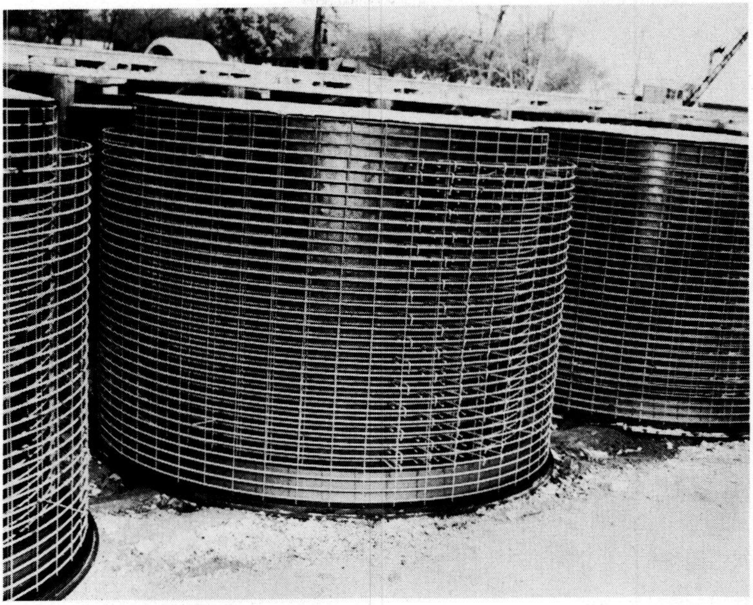


Figure 10. Pipe E reinforced as shown differed from Pipe D only in the spacing of the tie bars.

TABLE 1

LOAD TESTS ON 84-INCH REINFORCED-CONCRETE PIPE

Pipe	Steel Area sq. in. per lin. ft.	ASTM 0.01 in. crack lb. per lin. ft.	Ultimate Load lb. per lin. ft.
A	0.574	16,800	27,120
B	0.601	16,800	28,800
C	0.613	15,900	29,400
D	0.720	27,200	32,000 (No failure)
E	0.720	28,800	32,000 (No failure)
F	1.33	--	38,400

Pipe F was reinforced by a method which provided both shear reinforcing and an increased steel area in the region of high bending moments. Welded trusses of ⅝-inch round bars similar to that shown in Figure 12 were placed on 6-inch centers inside wire-mesh cages, as shown in Figure 13. The trusses were not fastened to the mesh other than for wires used to hold the assembly together. The wire mesh was the same as that used for Pipes D and E. This pipe was tested to failure without any crack as great as 0.01 inch being observed.

As a result of these tests, it was felt that the type of reinforcing used in Pipe F showed considerable promise as a method of producing a high-strength pipe without too great an increase in the smount of steel reinforcing required. The additional reinforcing steel required for greater strength could be provided by the truss, which does not need to extend around the entire circumference of the pipe. It was, therefore, planned to run a second series of tests on pipe reinforced in this manner.

SECOND SERIES OF TESTS ON 84-INCH PIPE

Nine additional pipes were cast for test purposes. Each pipe was 5 feet long with an 8-inch wall. Standard tongue-and-groove forms were used in casting the pipe.

Three pipes were standard ASTM C76 Table 2 pipe for control specimens. The reinforcing consisted of two wire-mesh cages, each providing a steel area of 0.731

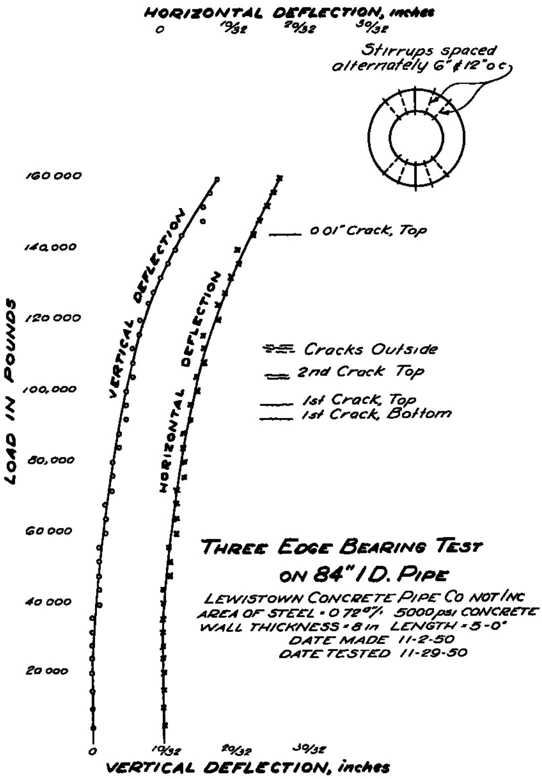


Figure 11. Load-deflection curves for Pipe E.

sq. in. per foot of pipe. These pipes are referred to as Design 1.

In Design 2, the same wire-mesh cages were used. In addition, welded trusses made up of ⅜-inch bars were placed on 6-inch centers at the top and bottom of the pipe.

Design 3 pipes were constructed with two lighter wire-mesh cages, each providing a steel area of 0.52 sq. in. per linear foot. Trusses were located on 2-inch centers to provide a total tension steel area in the top and bottom of the pipe of 1.18 sq. in. per ft. At the springline, the outer cage was doubled up to provide a tensile steel area of 1.04 sq. in. per ft.

Details of these designs are shown in Figure 15.

Standard test cylinders cured with the pipe indicated a concrete compressive strength in excess of 5,000 psi. at the time the pipe was tested.

The pipes were tested in three-edge bearing, and the results are summarized in Table 2.

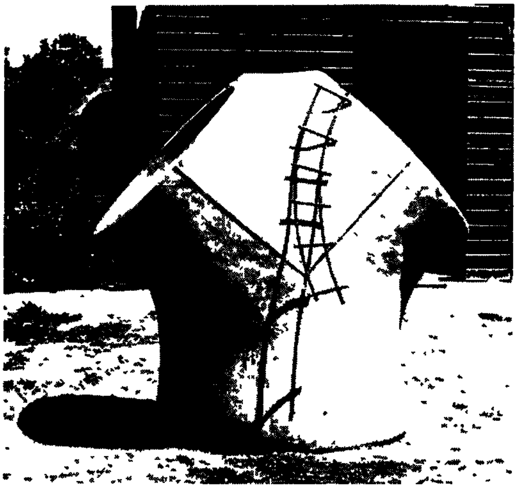


Figure 12. A single truss similar to those placed between circular wire-mesh cages in Pipe F.

In each of the pipes of Design 1, a shear failure similar to that shown in Figure 1 occurred at the ultimate load.

Of the three pipes of Design 2, two failed in shear. The third, although badly cracked, continued to take on load until a compression failure of the concrete occurred at the springline, on one side.

Two pipes in Design 3 failed through excessive elongation and, eventually, the rupture of the outside cage at a point just beyond the ends of the trusses. Failure of the concrete in compression was indicated by spalling on the inside of the pipe at this point. In the third pipe, failure at the ultimate load occurred when the concrete on the inside of the pipe began to crush, indicating excessive elongation of the steel in the outer cage. As before, the outer cage wire ruptured just beyond the ends of the trusses, but the inner cage wires and truss bars also ruptured directly below the top bearing block.

TABLE 2

LOAD TESTS ON 84-INCH REINFORCED-CONCRETE PIPE
(In Pounds per Lineal Foot)

Pipe No	Age (days)	ASTM 0 01 in crack	Ultimate Load
DESIGN NO 1			
1	34	14,000	23,200
2	30	14,000	21,400
3	41	16,000	20,000
DESIGN NO. 2			
4	33	18,000	38,100
5	34	22,000	35,050
6	36	20,000	33,980
DESIGN NO 3			
7	32	28,000	44,400
8	36	26,000	43,950
9	39	30,000	45,750

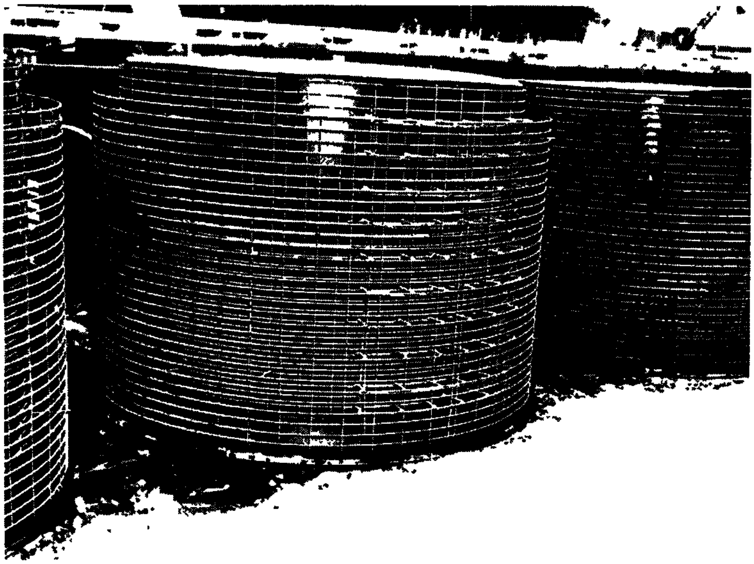


Figure 13. Reinforcing for Pipe F is shown prior to placing the outside form in position.

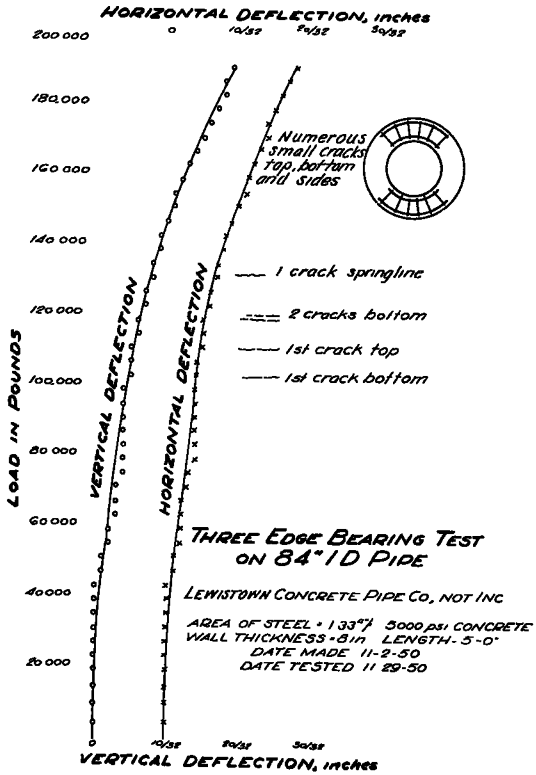


Figure 14. Load-deflection curves for Pipe F.

The top of this pipe after it had been removed from the testing machine is shown in Figure 16.

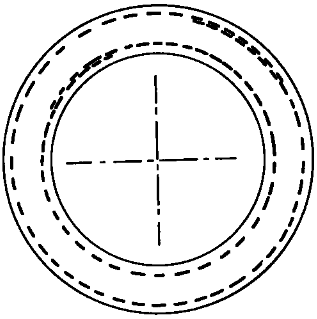
CONCLUSIONS AND COMMENTS

These tests on 84-inch reinforced-concrete pipe indicate that some type of reinforcement to prevent what has been called a shear failure is desirable in the larger sizes if the full strength of the pipe is to be developed. It should be pointed out that this fact has been recognized by many others prior to these tests. Some reinforced-concrete rings with stirrups to prevent shear failure were tested at the University of Illinois in 1908. Pipe reinforced to prevent a shear failure was tested at Iowa State College in 1925. The diameters of the specimens used in these tests were 48 and 36 inches respectively.

A method of preventing a shear failure in pipe reinforced with elliptical cages was patented by Elmer L. Johnson, of Colton, California, around 1935. In 1948 Howard F. Peckworth, after observing failures and working entirely on a theoretical basis, discovered an original method of providing shear reinforcement in large pipe. The designs of the first series of pipe reported on in this paper were developed, built, and tested by J. E. Miller, of Hillside, Illinois, and the writer.

There is some indication that the shear reinforcement used in these tests does more than increase the ultimate strength of the pipe. Pipe reinforced for shear tended to develop numerous fine cracks in the concrete on the tension face of the pipe rather than one or two wide cracks. This has the effect of increasing the load which the pipe will support before developing a 0.01-inch crack.

but stronger pipe can be offset by lower installation costs is not yet known. The type of shear reinforcing which now appears to be most usable is the hooked tie bar, such as was used in pipes D and E. A few manufacturers have used this type of reinforcing and report that once a workman develops a systematic method of installing them, the ties can be placed quite rapidly.



DESIGN NO. 1

Control Specimen - C76-II 84" x 8" Wall x 5'-0

2 Lines, each 0.731 in^2 per lin ft

Outside Cage (17"± lap) 58" x 27'-0, 2" x 12", 4/0 x 6

Inside Cage (17"± lap) 58" x 24'-0, 2" x 12", 4/0 x 6

Lap at upper quarter points

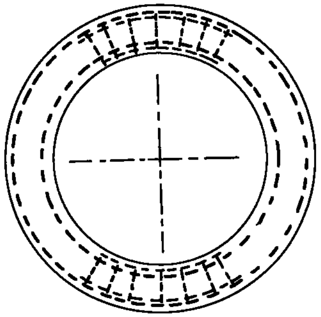
DESIGN NO 2

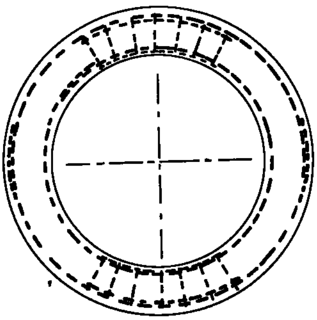
84" x 8" Wall x 5'-0 - Two lines, each 0.731 in^2 per ft plus trusses at 6" centers

Outside and Inside Cage same as Design No 1

Trusses 60° arc top and bottom

Tension steel top and bottom
Cage - 0.73
Truss - 0.22
0.95 in^2/lin ft





DESIGN NO. 3

84" x 8" Wall x 5'-0 - Two lines each 0.52 in^2 per ft plus trusses at 2" centers

Outside Cage (5'-9"± lap at each springline) 2-58" x 18'-6", 2" x 12", 2/0 x 5

Inside Cage (17"± lap at quarter points) 58" x 24'-0, 2" x 12", 2/0 x 5

Trusses top and bottom 60° arc

Tension steel top and bottom 1.18 in^2 per ft

Figure 15. Reinforcing for Designs 1, 2, and 3 are shown. All truss bars were 3/8-inch-diameter bars.

There can be little doubt that providing shear reinforcement of the type used in these tests will increase the cost of the pipe. Whether this increased cost can be offset by the more-efficient use of the main reinforcing steel or whether a more costly

An added advantage of the ties or trusses is that they form the entire system of reinforcing into a rigid unit. The steel is less likely to be displaced from its proper location during pouring and vibrating.

It appears that shear reinforcing is important in pipe of large diameter, particularly if the pipe is heavily reinforced. However, the types of shear reinforcing described in this paper have some drawbacks. It is certainly possible that better methods may be developed. It is hoped that these tests will focus attention on a phase of pipe design and manufacture where improvements beneficial to both consumer and producer are possible.

Figure 16. This is the top of a Design 3 pipe after being tested to failure. Stripping or shearing of the concrete did not occur.

References

1. Loving, M. W. - "History of the Development of ASTM Standards C76-41, C75-41, C14-41 and the Need for Trench Tests at Memphis, Tennessee in 1943" - Memorandum No. 45, American Concrete Pipe Association, October 15, 1943.

2. Mason, William R. - "Laboratory Tests on Culvert Pipe", Engineering Experiment Station Bulletin No. 18, University of Washington.

3. Packworth, Howard F. - "Concrete Pipe Handbook", American Concrete Pipe Association, February 1951.

4. Talbot, Arthur N. - "Tests of Cast-Iron and Reinforced Concrete Culvert Pipe", Engineering Experiment Station Bulletin No. 22, University of Illinois.

5. Schlick, W. J. - "Report of Tests of Concrete Pipe, Plain and with 'Special Reinforcing' " - Project 21, Preliminary Report, Engineering Experiment Station, Iowa State College.

6. Spangler, M. G. - "The Supporting Strength of Rigid Pipe Culverts", Bulletin 112, Iowa Engineering Experiment Station, Iowa State College.

Deflections of Timber-Strutted Corrugated-Metal-Pipe Culverts under Earth Fills

M. G. SPANGLER, Research Professor of Civil Engineering, and
DONALD L. PHILLIPS, Former Graduate Student, Iowa State College

An analysis is presented and a formula developed for predicting the vertical and horizontal deflection or change in diameter of timber-strutted corrugated-metal-pipe culverts under earth fills. In the analysis the strutted pipe is considered to be a composite elastic structure in which the vertical deflection of the pipe is the same as the deformation of the timber strut. The deflection of the pipe is determined by curved-beam theory and equated to the expression for strut deformation.

The loading system used in the analysis consists of a vertical load and vertical reaction on the top and bottom of the pipe and horizontal pressures at the sides. These horizontal pressures are postulated to be passive earth pressures which are mobilized by the outward movement of the sides of the pipe as it deforms. The expression for lateral pressures involves a quantity which is called the modulus of passive resistance of the side-fill soil.

Results indicated by the developed formula are compared with the performance of an actual culvert installation at Cullman, Alabama. This culvert is an 84-inch-diameter "improved multiplate" pipe under a highway embankment which is 137 feet high above the top of the pipe. Extensive observations of the performance of this structure were made and reported by the Armco Research Laboratories.

All of the factors involved in the derived equation for deflection, except the modulus of passive pressure of the side-fill soil, are available in the report or can be estimated from information given. It is possible, therefore, to compute the value of this modulus. The indicated value is 190 psi. per inch, which seems to be very high. However, the character of the soil is described as a "crumbly sandstone," and it was compacted to 100 percent AASHO density. Such a soil would have a very high bearing value and would be very resistant to deformation under pressure. It is the authors' opinion, therefore, that the calculated value may be of the same order of magnitude as the actual value. The need for extensive research to clarify the relationship between modulus of passive resistance and soil type and degree of compaction is indicated by this study.

● The Iowa Engineering Experiment Station in 1941 published Bulletin 153, entitled "The Structural Design of Flexible Pipe Culverts." In this bulletin a circular metal-pipe culvert was analyzed as a thin elastic ring acted upon by a system of external loads consisting of a vertical earth load on the top, and equal and opposite vertical reaction on the bottom, and horizontal pressures acting on both sides of the ring.

In the analysis it was postulated that the horizontal deflection of the pipe, caused by the vertical load and reaction, mobilized certain passive-resistance pressures in the soil at the sides of the pipe which acted in conjunction with the inherent strength of the pipe to resist deflection. These pressures were assumed to be proportional to the horizontal deflection of the pipe. The analysis was made for the case of a plain pipe (one installed without timber struts or horizontal tie bars or other predeforming devices).

The assumed load system employed in this early analysis is illustrated in Figure 1 and may be stated as follows: (1) The vertical load on a pipe may be determined by Marston's theory of loads on conduits and is distributed approximately uniformly over the breadth of the pipe. (2) The vertical reaction on the bottom of a pipe is equal to the vertical load and is distributed approximately uniformly over the width of bedding of the pipe. (3) The passive horizontal pressures on the sides of the pipe are distributed parabolically over the middle 100 degrees of the pipe and the maximum unit pressure is equal to the modulus of passive pressure of the sidefill material multiplied by half the horizontal deflection of the pipe.

A formula for the immediate or short

time deflection of a pipe under this system of loads was derived.

$$\Delta X = \frac{K W_C r^3}{E I + 0.061 \ e r^4} \qquad (1)$$

in which

ΔX = the horizontal deflection, in. (the vertical deflection is nearly the same)

$$K = 0.5 \sin \alpha - 0.082 \sin^2 \alpha + 0.08 \frac{\alpha}{\sin \alpha} - 0.16 \sin (\pi - \alpha) - 0.04 - \frac{\sin^2 \alpha}{\sin \alpha} + 0.318 \cos \alpha - 0.208$$

α = bedding angle

W_C = load on pipe, lb. per lin. in.

r = mean radius of pipe, in.

E = modulus of elasticity of pipe metal, psi.

I = moment of inertia of pipe wall, in.^{4} per in.

e = modulus of passive resistance of sidefill soil, psi. per in.

A series of field loading experiments was conducted in which the deflections of corrugated-metal pipes of several different diameters under a 15-foot earth fill were measured. The soil at the sides of the experimental pipes was compacted to several different densities in order to observe the influence of different values of lateral resistance pressures against the sides of the pipes. The measured deflections in these experiments were in reasonably good agreement with the deflections calculated by Equation 1.

As fill heights and pipe diameters have increased in recent years, more and more metal-pipe culverts are being installed with vertical timber struts, and there is need for an analysis of the deflections of a pipe in which these deflection-resistant members are included. The purpose of this paper is to present such an analysis. In this study the loading hypothesis stated above has been employed, along with the addition of a fourth item: The reactions at each end of the vertical strut act as concentrated loads at the inside of the top and bottom of the pipe; the magnitude of these concentrated loads depends upon the modulus of compression of the strut and the vertical deflection of the pipe.

The assumed load system for a timber strutted pipe is shown in Figure 2. It differs from that shown in Figure 1 only by the addition of the strut loads at the top and bottom. In actual construction practice, strutted pipes are usually predeformed to an out-of-round shape with the vertical diameter lengthened and the horizontal diameter shortened. This predeformation introduces bending moments in the pipe wall which are opposite in direction to the moments induced by the subsequent earth load. It also introduces an initial thrust in the timber strut due to the resilience of the pipe. These effects of predeformation have been ignored in the analysis, as a simplifying measure, in the belief that their influence on the magnitude of the deflection of a pipe under earth load is relatively minor. Further studies are needed to verify or disprove this assumption.

The deflection of a pipe which is referred to in this discussion is the change in diameter, i.e., the shortening of the vertical diameter and lengthening of the horizontal diameter, which is caused by the earth fill load. If the pipe is predeformed to an out-of-round shape prior to construction of the fill, the deflection refers to the

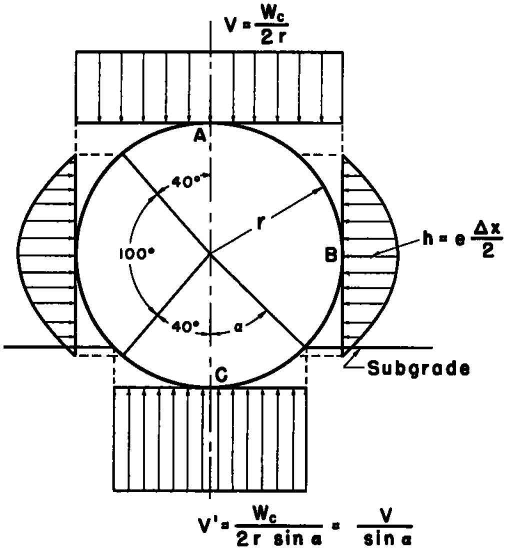


Figure 1.

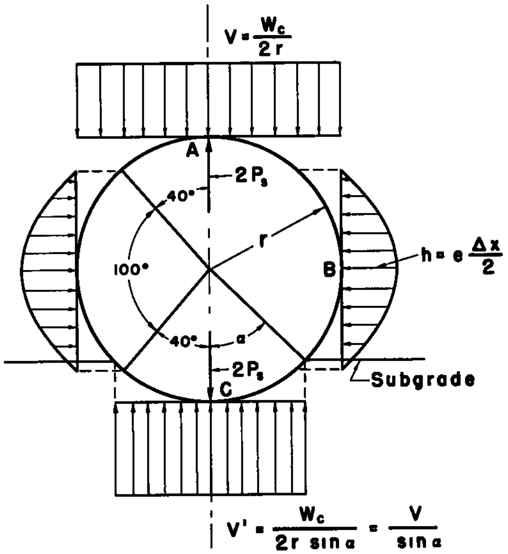


Figure 2.

change in length of the diameters from their predeformed dimensions, not the change from the initial diameter of the circular pipe.

A free-body diagram of a segment of the elastic ring under the postulated loading is shown in Figure 3.

The general equation for the bending moment at any Point D on the ring, considering clockwise moments as positive, is:

$$
\begin{aligned}
M_D = & -M_C - rP_S \sin\phi \\
& -rR_C\,(1 - \cos\phi) && 0 \leqq \phi \leqq \pi \\
& + 0.5v'r^2\sin^2\phi && 0 \leqq \phi \leqq \alpha \\
& + v'r^2 \sin\alpha\,(\sin\phi - \frac{\sin\alpha}{2}) && \alpha \leqq \phi \leqq \pi \\
& + hr^2(0.147 - 0.51\cos\phi + 0.5\cos^2\phi - 0.143\cos^4\phi) && 40^0 \leqq \phi \leqq 140^0 \\
& - 1.021hr^2\cos\phi && 140^0 \leqq \phi \leqq \pi \\
& + 0.5vr^2(1 - \sin\phi)^2 && \frac{\pi}{2} \leqq 0 \leqq \pi
\end{aligned}
\qquad (2)
$$

In which

M_D = Moment at any point D

M_C = Moment at point C

R_C = Thrust at point C

P_S = one-half the thrust carried by the strut per unit length of ring. Concentrated loads acting at points C and A.

v = Vertical unit load on ring = $\frac{W_C}{2r}$

v' = Vertical unit reaction on ring = $\frac{W_C}{2r\sin\alpha}$

W_C = Load on ring per unit of length

r = Mean radius of ring

h = Maximum horizontal unit pressure on ring = $\frac{e\,\Delta X}{2}$

e = Modulus of passive resistance of side-fill soil

ΔX = Horizontal deflection of ring

α = Bedding angle with vertical axis

ϕ = Angle between radius to any point on the ring and the vertical axis.

Since the load on the ring is symmetrical about the vertical axis, the normal sections at Points A and C will remain vertical, regardless of the character and magnitude of the angular displacements of normal sections at intermediate points. Therefore the sum of all the elementary angular displacements as ϕ varies from 0 to π will be zero, and we may write:

$$\frac{r}{E\,I}\int_0^{\pi} M d\phi = \int_0^{\pi} M d\phi = 0 \qquad (3)$$

Substituting the general moment Equation 2 in Equation 3 and integrating:

$$
\begin{aligned}
M_C = & -rR_C - 0.637rP_S + 0.345hr^2 \\
& + 0.057vr^2 + v'r^2\Big[0.08\,\alpha \\
& - 0.04\sin^2\alpha - 0.159\sin^2\alpha\,(\pi - \alpha) \\
& + 0.318\sin\alpha\,(1 + \cos\alpha)\Big]
\end{aligned}
\qquad (4)
$$

By the displacement theory, the horizontal movement of A relative to C is equal to

$$0 = \frac{r^2}{EI} \int_0^{\pi} M(1-\cos\phi)d\phi \qquad (5)$$

Substituting Equation 3 in Equation 5 yields:

$$\int_0^{\pi} M\cos\phi\, d\phi = 0 \qquad (6)$$

Substituting the general moment Equation 2 in Equation 6 and integrating, the thrust R_C may be evaluated and is found to be:

$$R_C = 0.053W_C\,(1-\sin^2\alpha) + 0.511hr \qquad (7)$$

Substituting this value of R_C in Equation 4 the moment at Point C is:

$$M_C = -0.166hr^2 - 0.637rP_S + W_Cr \left[0.053\sin^2\alpha - \frac{0.04\,\alpha}{\sin\alpha} - \frac{0.02\sin^2\alpha}{\sin\alpha} - 0.08\sin\alpha\,(\pi-\alpha) + 0.159\cos\alpha + 0.135\right] \qquad (8)$$

The value of the thrust carried by the strut, $2P_S$, may be determinated by setting the vertical deflection of the strut equal to the vertical movement of Point A with respect to Point C, that is, the vertical deflection of the ring. Assuming the bottom of the ring or Point C as fixed, the vertical deflection of the ring will be:

$$\Delta y = \frac{r^2}{EI} \int_0^{\pi} M\sin\phi d\phi \qquad (9)$$

Also the deflection of the strut will be:

$$\Delta y = \frac{2P_SL_SS_S}{A_SE_S} \qquad (10)$$

In which

P_S = half of the strut load per unit length of ring.

L_S = length of the strut

A_S = cross sectional area of the strut

S_S = longitudinal spacing of struts

E_S = equivalent modulus of compression of the strut

Then

$$\frac{2P_SL_SS_S}{A_SE_S} = \frac{r^2}{EI} \int_0^{\pi} M\sin\phi d\phi \qquad (11)$$

Substituting the general moment Equation 2 in Equation 11, integrating and reducing:

$$P_S = \left\{\frac{A_SE_Sr^2}{2L_SS_SEI + 0.296r^3A_SE_S}\right\} \left\{-0.119hr^2 + W_Cr \left[0.16\sin\alpha\,(\pi-\alpha) - 0.25\sin\alpha\,(1-\cos\alpha) + 0.125\sin^2\alpha - 0.08\,\alpha - 0.04\sin^2\alpha - \frac{0.083\cos^3\alpha + 0.26\cos\alpha - 0.167}{\sin\alpha} - 0.318\cos\alpha - 0.25\,\alpha + 0.433\right]\right\} \qquad (12)$$

Let

$$K_1 = \frac{A_SE_Sr^2}{2L_SS_SEI + 0.296r^3A_SE_S} \qquad (13)$$

and K_2 = the expression in brackets in Equation 12

$$\text{Then } P_S = K_1\,(W_CrK_2 - 0.119hr^2) \qquad (14)$$

The horizontal deflection of the ring is equal to twice the horizontal movement of Point B relative to Point C and we may write:

$$\Delta X = \frac{2r^2}{EI} \int_0^{\frac{\pi}{2}} M(1-\cos\phi)\, d\phi \qquad (15)$$

However, if the loads on the ring are symmetrical about a horizontal axis, the normal cross sections at the sides of the ring ($\phi = \frac{\pi}{2}$) will not rotate in relation to their unloaded positions and the tangents to the sides will remain vertical when the loads are applied. Under these conditions, Equation 15 may be simplified to:

$$\Delta X = \frac{2r^2}{EI} \int_0^{\frac{\pi}{2}} M\cos\phi\, d\phi \qquad (16)$$

The postulated loading shown in Figure 2 approaches this condition of symmetry with respect to the horizontal axis as the bedding angle α increases toward its maximum value of 90 degrees. Since, in practice, the bedding angle is usually large, say from 60 degrees to 90 degrees, it is believed that Equation 16 can be used without appreciable error.

Substituting the general moment Equation 2 in Equation 16 and integrating gives:

$$\Delta X = \frac{r^2}{EI}\Big[-0.122hr^2 - 0.274P_S r + W_C r\,(0.5\sin\alpha - 0.082\sin^2\alpha + 0.08\frac{\alpha}{\sin\alpha} - 0.16\sin\alpha(\pi-\alpha) - 0.04\frac{\sin^2\alpha}{\sin\alpha} + 0.318\cos\alpha - 0.208)\Big] \quad (17)$$

According to the lateral pressure hypothesis proposed in Bulletin 153.

$$h = \frac{e\,\Delta X}{2} \quad (18)$$

Substituting the value of h (Equation 18) and the value of P_S (Equation 14) in Equation 17 and letting the expression in parenthesis equal K:

$$\Delta X = \frac{W_C r^3\,(K - 0.274rK_1K_2)}{EI + er^4\,(0.061 - 0.016rK_1)} \quad (19)$$

The value of K and K_2 are nearly the same numerically. This is particularly true for larger values of the bedding angle α. Therefore for practical purposes we may rewrite Equation 19 without appreciable error, as follows:

$$\Delta X = \frac{KW_C r^3\,(1 - 0.274rK_1)}{EI + er^4\,(0.061 - 0.016rK_1)} \quad (20)$$

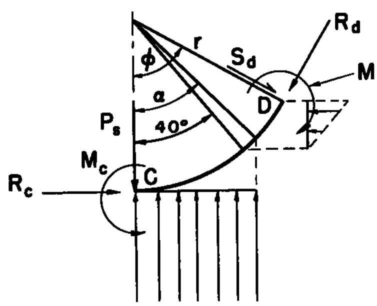


Figure 3.

In the case of a plain or unstrutted pipe, the value of E_S will be zero and K_1 (Equation 13) will be zero. Substituting $K_1 = 0$ in Equation 20 yields the same expression as Equation 1, which was derived for the unstrutted condition. This gives a partial check on the form of Equation 19.

No experimental program of loading strutted corrugated metal pipe culverts under earth fills has been carried out for the purpose of verifying Equation 20. However, the Research Laboratories of Armco Steel Corporation have recently issued a report entitled "Multi-Plate-Compression Measurements at Cullman, Alabama Installation," by John H. Timmers. The details of installation and of the performance of the middle line of three 84-inch multi-plate pipe culverts under 137 feet of highway fill are given in this report. Although all of the information needed to apply Equation 20 to this installation is not available, it is interesting to study the application of the formula for deflection in the light of the reported data.

Detailed observations of the performance of the culvert were made on several 8-foot-long bolted sections along the length of the pipe. Three of these sections (Nos. 31, 32, and 33) were located under the roadway portion of the embankment where the height of fill above the top of the pipe was 137 feet. The data relative to these three sections have been used in this study of the applicability of the formula for deflection of a strutted pipe. The horizontal deflections of these sections were 0.67, 0.63, and 0.85 inch, respectively, or an average of 0.72 inch, under the maximum height of fill and before the timber struts were removed. This average deflection increased to 0.75 inch immediately after the struts were removed.

These culvert-pipe sections had a nominal diameter of 84 inches and were fabricated of corrugated-metal plates which were 1 gage (approx. 5/16 inch) thick. The corrugations were spaced 6 inches on centers and were 1 3/4 inches deep. The moment of inertia of a cross-section of the pipe wall was 0.1288 inch per inch of length of pipe. The mean radius of the pipe, that is, the average radius to the neutral axis of the pipe wall was 43.13 inches. The modulus of elasticity of the pipe metal has been assumed to be 30 million psi.

The pipes were bedded on a 2 foot uniform thickness of creek-bed sand. Side-fill material consisting of a "crumbly sandstone" which was compacted under and around the pipes to a specified 100 percent of standard AASHO density by means of power operated tampers. Inspectors' reports indicate that the specified density was exceeded by 0.4 to 5.25 percent. With this type of soil and the compaction obtained, it seems probable that the pipe was supported completely over the width of the bottom half of the pipe and the value of the bedding angle α probably approached 90 deg.

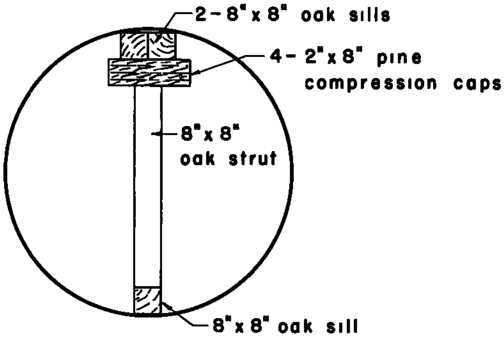


Figure 4.

The pipes were strutted along their longitudinal axes by means of vertical timbers acting between longitudinal sills at the top and bottom of the pipe. The struts and bottom sills consisted of 8-by-8-inch rough oak timbers. The top sills were made of two 8-by-8-inch rough oak timbers arranged side by side. A compression cap of relatively soft wood was inserted between the top sill and the strut. It consisted of four 2-by-8-inch rough pine planks laid flatwise. A diagram of the strut, compression cap, and sills is shown in Figure 4. There were three struts in each of pipe Sections 31, 32, and 33. The average longitudinal spacing of the struts was 32 inches on centers.

In order to apply the deflection formula for strutted pipes it is necessary to know or to estimate the "equivalent modulus of compression" of the strut as installed. This modulus is similar to the modulus of elasticity of an elastic member, i.e., the unit stress divided by the unit strain. The total compression strain of the strut assembly consists of the compression of the sills, the compression of the corrugations into the corners of the sills, the compression of the compression cap, and the shortening of the main body of the strut. It is substantially equal to the vertical deflection of the pipe, which is the shortening of the vertical diameter. Since the soft wood compression caps were stressed well beyond their elastic limit as the earth load on the pipe increased, the stress-strain diagram for the strut assembly is a curved line. The modulus of compression used in this analysis is essentially a secant modulus drawn to the point of greatest strain on the stress-strain curve.

In the Cullman project the average vertical diameter of Sections 31, 32, and 33 at the time the struts were installed was 90.53 inches and this is essentially the length of the struts. The vertical deflection of these three sections under the maximum fill of 137 feet was 0.67, 0.62, and 1.02, respectively, or an average of 0.77 inches. Therefore the average unit strain of the strut assembly was $\frac{0.77}{90.53} = 0.0085$ inches per inch. The loads on the struts were determined by means of load cells and by measurements of the compression of the soft wood caps. The average load on each strut in the three sections was approximately 54,000 lb. Therefore, the unit stress on the struts was $\frac{54000}{64} = 844$ psi. and the equivalent modulus of compression was $\frac{844}{0.0085}$ or approximately 100,000 psi.

The load on the culvert was measured by means of resistance strain gages mounted on the neutral axis of the pipe wall at the ends of the horizontal diameter in conjunction with two strut load cells which were installed between the compression caps and the main element of two struts near the junction of Sections 31 and 32 and Sections 32 and 33. These strut load cells consisted of short lengths (about 6 inches) of 6-inch pipe mounted between ¾-inch-square steel plates. The strut load was measured by means of resistance strain gages mounted on the inside of the cell walls. The average load on the three sections under the highest portion of the fill was about 86,700 lb. per lin. ft. or 7,225 lb. per lin. in.

Thus it is seen that in the case of the Cullman project, we have actual measurements or reasonable estimates of all the factors necessary for the solution of Equation 20, except the modulus of passive resistance of the side-fill soils. A recapitulation of the data is as follows:

$\Delta X = 0.72$ in.

$W_c = 7225$ lb. per lin. in.

$r = 43.13$ in.

$E = 30,000,000$ psi.

$I = 0.1288$ in.^{4} per in.

$L_s = 90.53$ in.

$A_s = 64$ sq. in.

$E_s = 100,000$ psi.

$S_s = 32$ in.

$\alpha = 90$ deg.

Substituting the above numerical values in Equation 20 yields a modulus of passive pressure, e, equal to 190 psi. per in. At first glance this appears to be an inordinately high value. It is more than twice as high as the value of this modulus which has been estimated in connection with any previous flexible pipe loading tests which are within the author's knowledge.

However, a critical study of the character of the sidefill soil material led to the belief that the indicated value of the modulus may not be excessive or out of reason. The Armco Laboratories Report describes the fill material and method of handling and placement as follows:

> The fill material was mainly a crumbly sandstone and rock for the first 25 feet. In order to secure the desired density of fill and guard against future settlement, choking was accomplished by alternately placing embankment layers of rock and earth except in the first 10 feet immediately over the pipes. The fill material was obtained from borrow pits on both sides of the fill ... The borrow pit on the north side consisted mainly of rock while the borrow pit on the south side, from which the major part of the fill was constructed, was a very crumbly friable sandstone which after blasting was easily handled. The fill was largely built using self-loading scrapers. The compaction of the embankment was handled by a combination of tandem sheepsfoot rollers and the normal equipment traffic.
>
> The compaction required under and between the pipes was 100 percent AASHO Standard. The size of the rock within 2 feet under and 3 feet over the pipe was limited to 4 inches. All this select material was tamped with power operated hand tampers. Springs that were encountered in preparing the bed were handled with 6-inch Helcor subdrain.

It is apparent from the above description that the side-fill material was of high quality from the standpoint of bearing capacity and was thoroughly compacted. A foundation engineer would consider such material to be an ideal foundation soil. Settlement of a structure founded on material of this character could be expected to be negligible.

The fact that the sides of these 84-inch corrugated-metal pipes moved outward only 0.36 inch under a vertical load on the pipe of about 86,700 lb. per lin. ft. is ample evidence that the sidefill soil material developed high resistance to lateral movement. Therefore it is the authors' conclusion that the indicated value of 190 psi. per in. is not an excessive value of the modulus of passive resistance of the soil material which prevailed at the sides of the pipes in this project.

This study of the performance of the Cullman project in relation to the analysis of a timber-strutted corrugated-metal pipe is not offered as conclusive evidence of the validity of the deflection formula, Equation 20. It is merely one straw in the wind and many more such comparisons are needed before definite conclusions as to its validity can be drawn. Particularly, detailed studies of the magnitude and character of the modulus of passive resistance of soils of various classifications and in various states of density are needed to enhance the technology of flexible culvert design.

References

1. Phillips, Donald L. Investigation of the Stresses and Deflections of a Corrugated Metal Pipe Culvert Under a High Earth Fill. Unpublished Thesis, Library, Iowa State College, Ames, Iowa, 1953.

2. Spangler, M. G. The Structural Design of Flexible Pipe Culverts. Bulletin 153, Iowa Engineering Experiment Station, Ames, Iowa, 1941.

3. Spangler, M. G. Stresses and Deflections in Flexible Pipe Culverts. Proc. Highway Research Board. Vol. 28, p. 249, 1948.

4. Timmers, John H. Multi-Plate Compression Measurements at Cullman, Alabama Installation. Armco Research Laboratories, Middletown, Ohio, 1953.

THE NATIONAL ACADEMY OF SCIENCES—NATIONAL RESEARCH COUNCIL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The ACADEMY itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the ACADEMY and the government, although the ACADEMY is not a governmental agency.

The NATIONAL RESEARCH COUNCIL was established by the ACADEMY in 1916, at the request of President Wilson, to enable scientists generally to associate their efforts with those of the limited membership of the ACADEMY in service to the nation, to society, and to science at home and abroad. Members of the NATIONAL RESEARCH COUNCIL receive their appointments from the president of the ACADEMY. They include representatives nominated by the major scientific and technical societies, representatives of the federal government designated by the President of the United States, and a number of members at large. In addition, several thousand scientists and engineers take part in the activities of the research council through membership on its various boards and committees.

Receiving funds from both public and private sources, by contribution, grant, or contract, the ACADEMY and its RESEARCH COUNCIL thus work to stimulate research and its applications, to survey the broad possibilities of science, to promote effective utilization of the scientific and technical resources of the country, to serve the government, and to further the general interests of science.

The HIGHWAY RESEARCH BOARD was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the NATIONAL RESEARCH COUNCIL. The BOARD is a cooperative organization of the highway technologists of America operating under the auspices of the ACADEMY-COUNCIL and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the BOARD are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.