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Elastic Behavior of Soil-Cement Mixtures

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Introductory Remarks by the Chairman: Modern soil-cement is a remarkable construction material. Aside from its excellent structural properties, it is noteworthy that from its early infancy in South Carolina to its present world-wide use, it was guided and supervised scientifically and that more precise records have been kept on its development than on any other large-scale construction material.

But do we know as much as we should even with respect to the mechanical properties of this material? Have we been completely fair to it and studied and developed all its potentialities? Or have we treated it as a dependable slave that can carry the loads we place on it without bothering how it does it?

Reinhold looks at soil-cement as a unique construction material that is not just a poor concrete or a stabilized soil but a material that may prove to be superior in certain highway and other applications than traditional materials presently employed for them. From this point of view, Reinhold has studied the elastic behavior of soil-cement as a function of cement and clay content and has given in his paper a precise account of the results so far obtained in his important work.

● THE modulus of elasticity and Poisson's number play an important role in the dimensional design of homogeneous pavements. Questions regarding the strength of a pavement, its reaction to temperature and moisture changes, and the distance and type of joints to be used are directly related to the elasticity constants. These constants must be determined for soil-cement if we want to understand its actual behavior under stresses caused by externally applied forces. From a knowledge of their magnitudes, a better utilization of the properties of this new construction material may result as well as an expansion of its field of application.

GENERAL REMARKS CONCERNING ELASTIC BEHAVIOR

For the purpose of defining fundamental concepts and of determining the type of experiments necessary to measure the elastic behavior of soil-cement mixtures, the following general considerations are presented.

The elastic behavior of homogeneous isotropic materials is essentially characterized by two coefficients, the modulus of elasticity E (or its inverse a) and the Poisson number m (or its inverse μ = Poisson's Ratio). In the general case these coefficients are not constants. They are determined by measurements of changes in longitudinal and lateral dimensions in uniaxial tension or compression, which are evaluated by means of the stress-strain diagram.

Stresses $s = \frac{P}{A}$ are forces per unit area that are caused by externally applied loads. Strains are defined by $\epsilon = \frac{\Delta l}{l}$ i.e., by the ratio of change in a dimension to the original dimension and, hence are dimensionless. Strains may be caused by compression, tension or shear forces.

Fundamentally, there exists no proportionality between stress and strain. The law established by Hooke in 1678 "Ut tensio, sic vis":

$$\epsilon = \frac{s}{E} = \alpha \times s$$

which for a long time had been considered as a natural law covering the behavior of all materials, is strictly true only for the lower stress range of steel and for a minority of substances, within certain limits. All other materials show larger

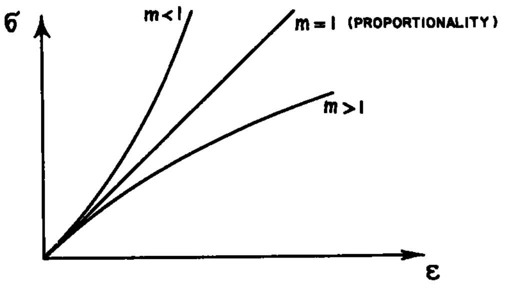

Figure 1. Graphical presentation of the equation $\epsilon = a\ s^{m}$.

or smaller deviations. Schüle, therefore, proposed the expression:

$$\epsilon = a\ s^{m}$$

This equation, which also represents only an approximation of the actual behavior of materials, already causes such mathematical difficulties in the practical use of the theory of elasticity that one is forced to linearize in accordance with Hook's law in order to obtain mathematical solutions of actual problems.

For the definition and calculation of the modulus of elasticity E from measured data, there exist two possibilities: (1) the elasticity modulus is represented by the tangent at a chosen point of the stress strain curve

$$E_1 = \frac{d\ s}{d\ \epsilon}$$

(2) the modulus of elasticity is assumed to be constant for a certain stress range

$$E_2 = \frac{s}{\epsilon}$$

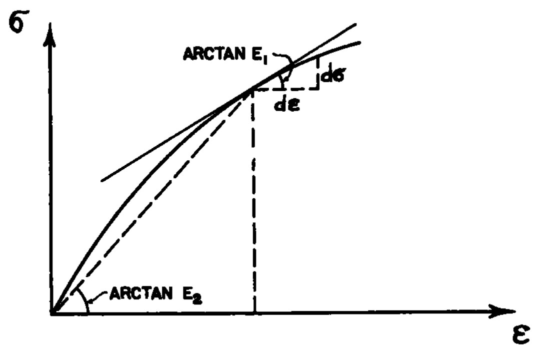

Figure 2. Graphical presentation of the modulus of elasticity.

Since calculation methods normally employed in civil engineering assume proportionality between stress and strain, the

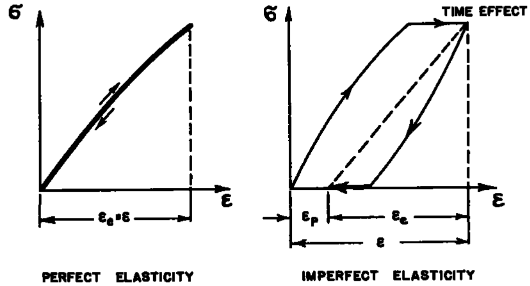

Figure 3. Perfectly and imperfectly elastic material.

modulus of elasticity is usually determined according to the second method.

Further differentiation is now made between perfect and imperfect elastic behavior. This renders the definition of the modulus of elasticity E considerably more complicated. In the case of perfect elasticity there exists a definite reversible relationship between stress and strain. More generally observed in imperfect elasticity in which the strains are a function not only of the stress but also of the type of loading, of the time, and of the repetition of load application. Thus, a difference exists between the corresponding strains and stresses in the case of loading and unloading, a phenomenon which is known as hysteresis.

The deformations ϵ_p remaining after unloading are called plastic, while the reversible deformations are called elastic ϵ_e. That portion of the deformation which is a function of time is called after effect. A measure for the degree of perfection of elasticity is:

$$\text{degree of elasticity } e = \frac{\text{elastic strain } \epsilon_e}{\text{total strain } \epsilon}$$

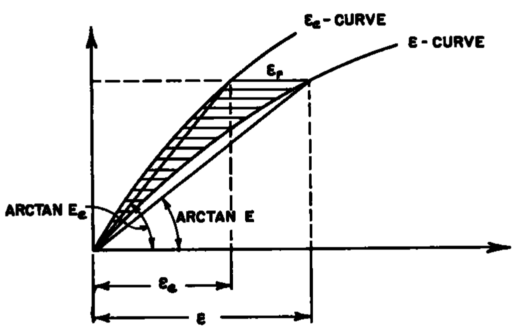

Figure 4. Graphical presentation of ϵ_e, ϵ, E_e, and E.

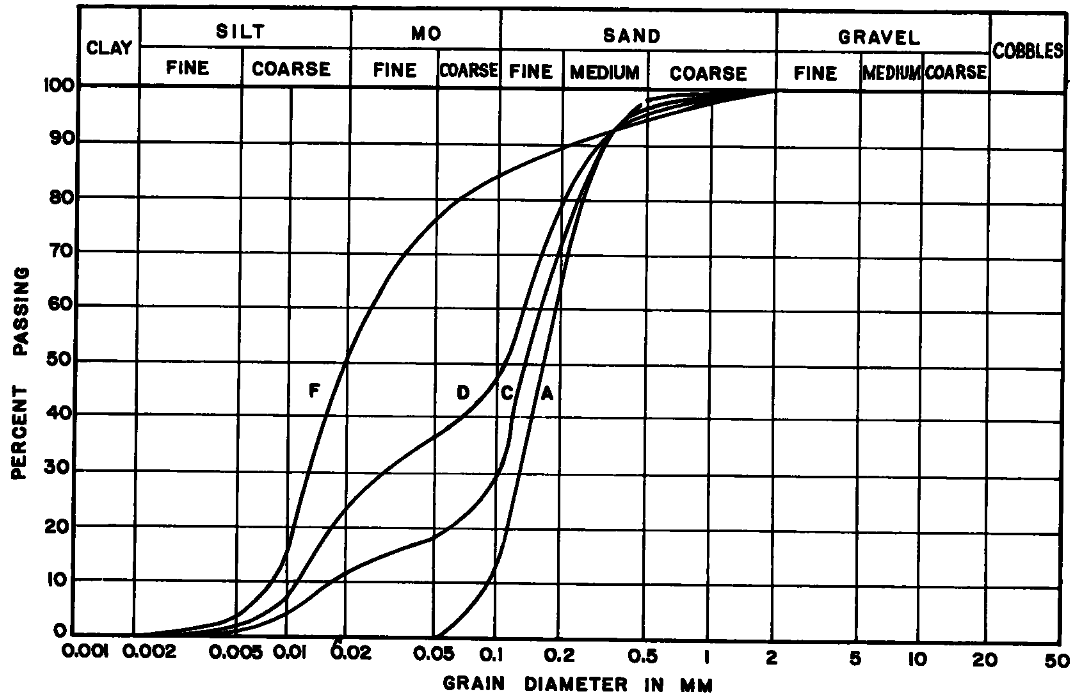

Figure 5. Granulometric distribution.

If e = 1, perfect elastic behavior exists; accordingly, this equation defines perfect elasticity.

Only the elastic strains are expressive of the elastic behavior of a material. Accordingly, the real modulus of elasticity E_e takes into account only the elastic strain while E is to be considered as a modulus of the total strain.

If ϵ_e and ϵ are plotted for equal loading stages in one stress strain diagram, then two curves are obtained (Figure 4). The cross-hatched area in Figure 4 represents the magnitude of the residual strain ϵ_p.

Strains obtained during the first application of loads of increasing magnitude are called virginal strains ϵ^0. The stress strain curve thus obtained represents extreme conditions and envelopes the $s - \epsilon$ branches that are obtained in repeated loading and unloading.

Under the influence of load repetitions, the strain lines shift in the ϵ direction. However, in most cases (for small stress increments earlier, and for large increments later) a stable state is usually reached in which the strains tend to approach constant ϵ values..

These phenomena have been thoroughly investigated in the case of portland-cement concrete. As will be shown later, they have also been observed in the case of soil-cement mixtures. Accordingly, the same methods of investigation and presentation of data can be employed for both materials.

The previous statements hold true, as a matter of principle, for both tension and compression, for longitudinal as well as lateral strains (ϵ_q). One must differentiate, therefore, between ϵq and ϵqe, ϵq and ϵq^0, and also between $m = \frac{\epsilon}{\epsilon q}$ and $m_e = \frac{e}{q_e}$. With respect to the exact determination of the stress actually present in the specimen, the tension experiment is, as a matter of principle, equivalent to the compression experiment even though the course of the stress-strain curve may be different.

In the case of brittle materials, i.e., also in the case of soil-cement, it is customary to investigate the elastic behavior under compressive stress, since by this means more-uniform uniaxial stress conditions may be realized, and also because a greater stress range can be investigated because of the higher compressive strength.

With respect to its structure, soil-cement may be considered as quasi-isotropic and homogeneous.

PREPARATION OF TEST SPECIMENS

Synthetic soils were employed for the manufacturing of the test specimens. The soils were made by mixing Griesheim sand, a fine noncohesive material of yellow color with Heppenheim clay. This clay is employed for the manufacture of roofing tile, and was available in finely ground condition.

Four different soil types were prepared which were mixed, at their optimum moisture contents, with three different proportions of portland cement. The ratio of cement to soil was taken as 1 to 6 for an economically justified upper limit, as 1 to 10 for a lower limit, and as 1 to 8 for an in-between amount.

The different soils are designated as follows:

Designation	Composition Sand %	Composition Clay %
A	100	0
C	75	25
D	50	50
F	0	100

The cement proportions are indicated as follows:

Designation	Mixing Ratio Soil/Cement
I	1:6
II	1:8
III	1:10

The granulometric composition of the four different soils is given in Figure 5. The liquid limits, plastic limits, and plasticity indexes are presented in Table 1.

Chemical investigation of the component soils A and F showed the absence of substances that could have a detrimental influence on the soil-cement. The cement employed (PZ325) fulfilled the German Standard Specifications, DIN 1164.

TABLE 1

CONSISTENCY CHARACTERISTICS OF THE SOILS

Designation	Liquid Limit	Plastic Limit	Plasticity Index
A	-	-	-[a]
C	16.80	-	-[a]
D	25.00	16.07	8.93
F	38.50	20 98	17.52

[a]Because of the granular nature of the soil, the plasticity index could not be determined.

The test specimens were made in accordance with the "Tentative Specifications for the construction of soil-cement roads."

Since, in accordance with Siebel (1), the ratio of height to width of test specimens used in strain measurements, should be about 3 to 1, specimens with the dimensions 7.07 by 7.07 by 23.21 cm. were manufactured.

In order to have a uniform basis of comparison, all specimens were cured in the same manner and for the same period of time before they were tested. The 28-day curing period was composed as follows: 2 days moist air, 12 days water immersion, 7 days dry air storage, and 7 days water immersion.

EXPERIMENTS AND TEST RESULTS

The following equipment was used for the stress-strain measurements: A 500-ton hydraulic testing machine provided with a 5-ton pressure stirrup for exact loading; a Martens mirror apparatus for the measurement of the longitudinal strains; and a tensometer for the measurement of the lateral strains.

For all longitudinal and lateral measurements the same arrangement was employed. This arrangement is briefly described in the following:

1. Mirror apparatus for measurement of longitudinal strains. The two mirror devices were fastened on two opposite sides of the prisms (normal to the compaction joints) in order to obtain dependable mean values. The gage length was 10 centimeters in the center portion of the test specimens in order to obtain exclusively the mono-axial stress condition, and to remain outside of the friction influence of the shear cone.

2. Tensometer for the measurement of lateral strains. Also arranged on the two opposite sides of the prism (parallel to the compaction joints) on the two free sides of

the prism. The gauge length was 5 cm.; the gages were fastened at the elevation of the center of the gages measuring the longitudinal strains, and at a right angle to them.

3. Measuring stirrup for a range of 5 tons. Placed above the test specimen between a plane-parallel steel plate and the ball-jointed pressure head of the testing machine. The entire experimental arrangement of the test specimens, measuring appliances, and reading scales is given in Figure 6.

Details concerning the fastening of the instruments on the test specimens are shown in Figure 7. The strain determinations were based on the ϵ curve, with one repetition of loading for measuring the time effect. Previously conducted experiments had shown that this was sufficient for the determination of the elastic portion of the strain.

In the lower stress range, in which no influence of the time effect can be noted, the curve is identical with the virginal curve envelope ϵ^0, the upper portion of which could also be plotted if one would record the quick-reading values. The unloading usually was done to a stress of 5 kg. per sq. cm. in order to avoid slipping of the instruments.

As examples of the graphic presentation of the experimental results, Figure 9 shows the ϵ and ϵ_q curves for longitudinal and lateral strains, as well as the moduli of elasticity, calculated therefrom, and the Poisson numbers for four test specimens made of the four soils types with a cement-to-soil ratio of 1 to 8. All measured values are given with reference to the condition of the test specimen before the first loading.

EVALUATION OF EXPERIMENTAL RESULTS

The stress-strain diagrams of all the different soil-cement compositions tested in compression, show the following basic characteristics: (1) The curve is always more or less concave with respect to the ϵ axis. (2) In a range up to about a third of the failure stress ($0.33\, s_D$) the curve is almost a straight line; thus, for all practical purposes one can assume linearity in this range. The plastic deformations in this range are so unimportant that one can consider the material as possessing almost perfect elasticity; e = 1. (3) At a stress of $0.6\, s_D$, the curvature increases rapidly. The curve becomes flatter and its tangent is often horizontal at the failure point.

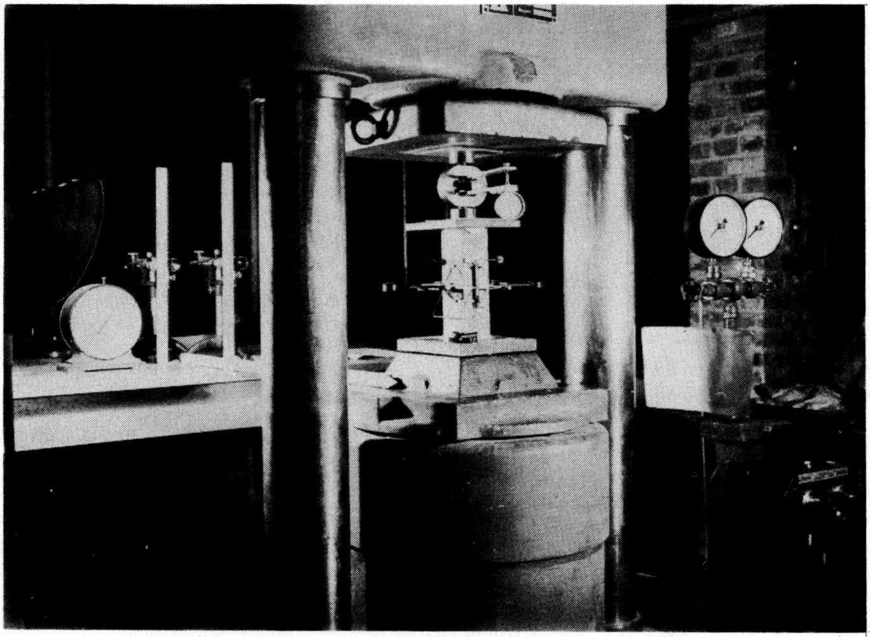

Figure 6. Arrangement for stress-strain measurements.

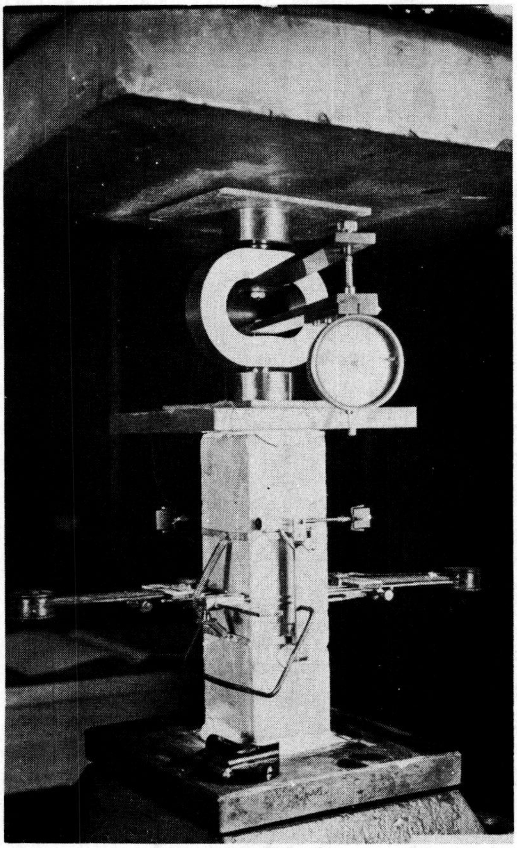

Figure 7. Details of experimental set-up.

These characteristics agree in principle with those of portland-cement concrete. Although soil-cement depending on its composition, possesses characteristics that are specifically its own, these characteristics are not alien to those of portland-cement concrete. This, of course, is not unexpected.

The stress-strain diagram for the lateral strains possesses characteristics similar to that for the longitudinal strains, with the only difference that the linearity is less developed but extends to about $0.6\, s_D$. From this point on, the lateral strains increase much more rapidly than the longitudinal strains. This is reflected in a decrease of the Poisson numbers. The reason for this behavior is according to Roŝ (2) a certain loosening of the internal structure (an indication of the beginning of failure as a result of fatigue).

Very generally, it can be said that the elastic behavior of soil-cement is a function of its strength. All factors that influence the strength properties also determine the elastic behavior of soil-cement. Fundamentally the following holds true: (1) the higher the failure stress the less curved is the stress strain diagram; (2) the higher is the range of linearity; and (3) the smaller is the proportion of plastic deformation and, therefore, the more perfect is the elasticity.

The determinant influence of the higher strength begins already with the work of compaction, as can always be seen from the density of the test specimens. Accordingly, of two specimens of the same composition, the one with the greater density will show smaller strains with the same loading increments.

In the following, the influence of the two components, cement and clay, shall be considered in greater detail. For their analysis, the total strains and the moduli of elasticity E_e shall be used. The ϵ curves are more suitable for comparison than the ϵ_e curves, since they cover a larger range (usually up to failure) and they are more differentiated.

Influence of Cement Content

Figure 11 shows the great influence of the cement content. As the cement content decidedly influences the strength of soil-cement compositions, the same way it influences the relationship between stress and strain. The point x which is plotted in the graphs gives the values for the stress of $0.33\, s_D$.

Influence of Clay Content

While in the case of cement it was the maximum content which gave the greatest strength and the related elastic properties, here it is the optimum clay content of the soil (about 25 percent). The characteristics are the same for all cement ratios. The different soils show different

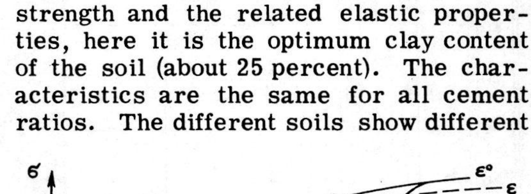

Figure 8. Presentation of ϵ^o and ϵ curves.

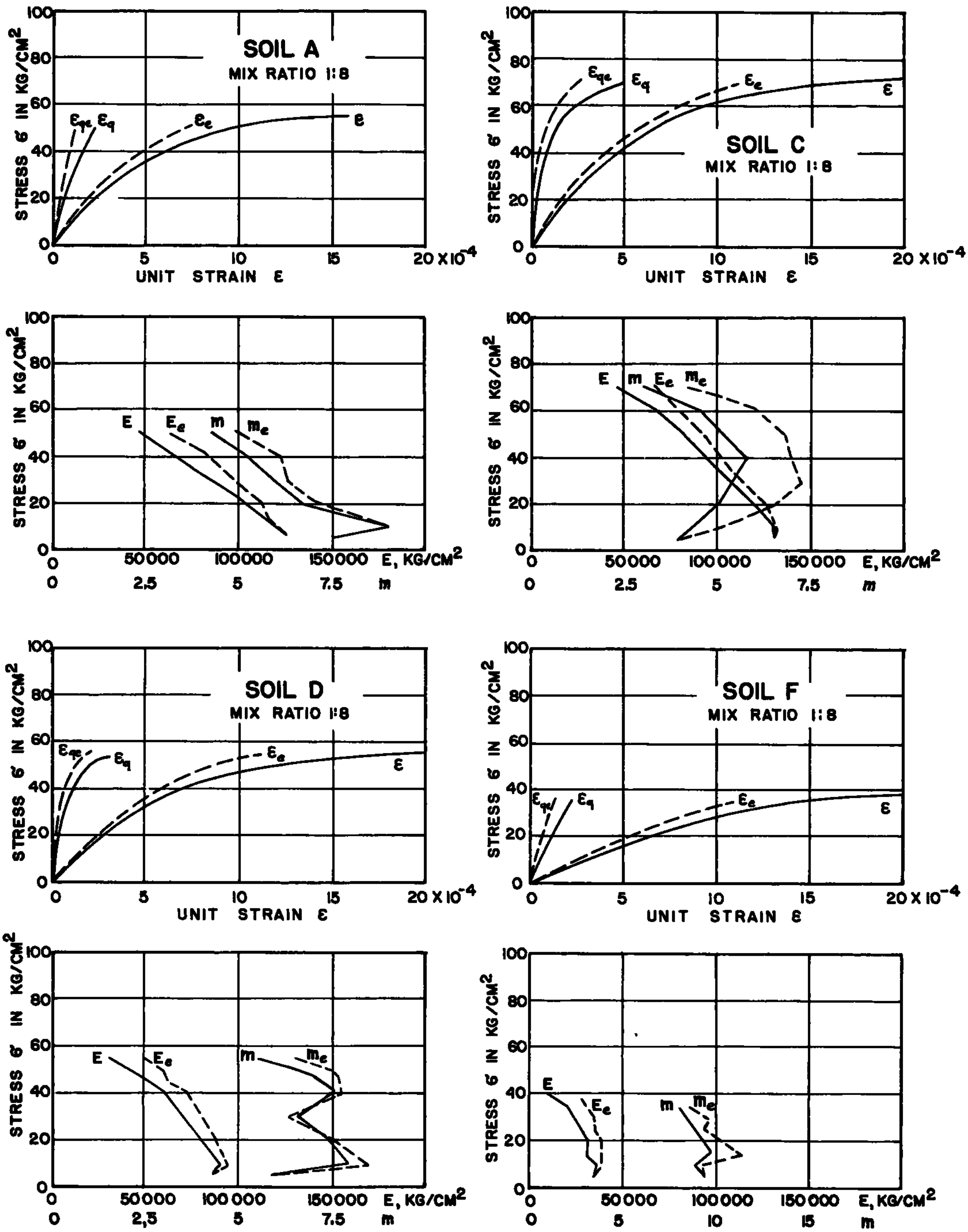

Figure 9. Graphical presentation of experimental results.

TABLE 2

Soil	Cement: Soil	0.33 s_D kg/cm²	E_e kg/cm²	m_e
A	1:6	29.8	138 500	8.35
	1:8	19.7	112 250	7.25
	1:10	14.2	91 000	7.05
C	1:6	40.0	143 000	8.00
	1:8	26.7	114 000	7.50
	1:10	19.2	93 000	7.35
D	1:6	24.2	93 000	9.00
	1:8	16.7	83 500	7.75
	1:10	13.5	66 500	10.56
F	1:6	17.85	46 000	11.1
	1:8	13.00	38 500	14.3
	1:10	10.00	29 650	18.75

angles of the linear portion of the stress-strain diagram with the strain axis. While the curve for the sandy soil starts with a relatively steep angle and then curves strongly into the upper flat part, the clay soils start with a flatter tangent, which is characteristic for them. One can say therefore:

The clay content is characterized by the angle of the tangent in the working range and by the curvature at 0.6 s_D. The influence of the clay contents is shown in Figure 12. The plotted points x give again the values for 0.33 s_D.

Practical Values for Modulus of Elasticity and Poisson's Number.

The 0.33 s_D line seems to be especially suited for the fixing of practical average values for the modulus of elasticity and for Poisson's number. First, in this stress range lies the main use of soil-cement; second, all values converge quite strongly at this point and tend to become steady after initial wide fluctuations. In Table 2, average values from the experiments are given for the stress of 0.33 s_D. The following tendencies can be recognized: (1) the modulus of elasticity decreases with decreasing cement content and with increasing clay content and (2) Poisson number ($\frac{1}{\mu}$) increases with increasing clay content; the influence of the cement content appears to be indifferent or to depend upon the clay content, because in the case of sandy soils the m-values fall with decreasing cement content while they increase strongly in the case of clay soils.

An increase of the Poisson numbers indicates increasing compressibility of the material. The material can be compressed into itself with little lateral strain. Incompressible liquids, especially water, have the smallest possible Poisson number m = 2 (μ = 0.5) which signifies volume constancy. Soil-cement with high clay content and low cement content possesses, therefore, a great compressibility in addition to a large range of elasticity.

SUMMARY AND PRACTICAL SIGNIFICANCE OF THE RESULTS

On the basis of the performed tests it could be established that soil-cement is basically similar to portland cement concrete, with respect to stress strain relationships.

Specifically, the following important results shall be stated: (1) determinant for the elastic behavior of soil-cement is its compressive strength; (2) this can be influenced by densification, cement content, water content, and clay content; (3) up to one-third of the compressive strength, one can assume a linear stress-strain diagram for soil-cement; (4) dependable values for moduli of elasticity and sufficiently accurate Poisson numbers are available for the range of practical application; and (5) they are functions of the stress and depend strongly upon the cement content and on the clay content; as average values may be taken: E = 100,000 kg. per sq. cm. and m = 7.5 ($\mu \simeq 0.13$).

On the basis of these investigations it appears possible to judge correctly the elastic behavior of soil-cement as a function of the cement and clay contents and to obtain dependable values for the dimensional design of pavement sections.

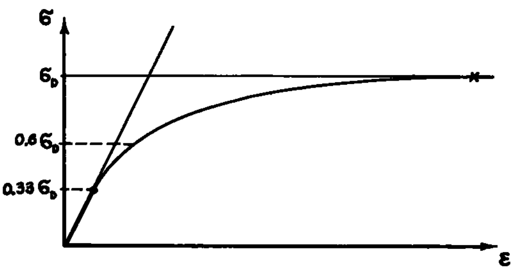

Figure 10. Characteristic stress-strain diagram of soil-cement.

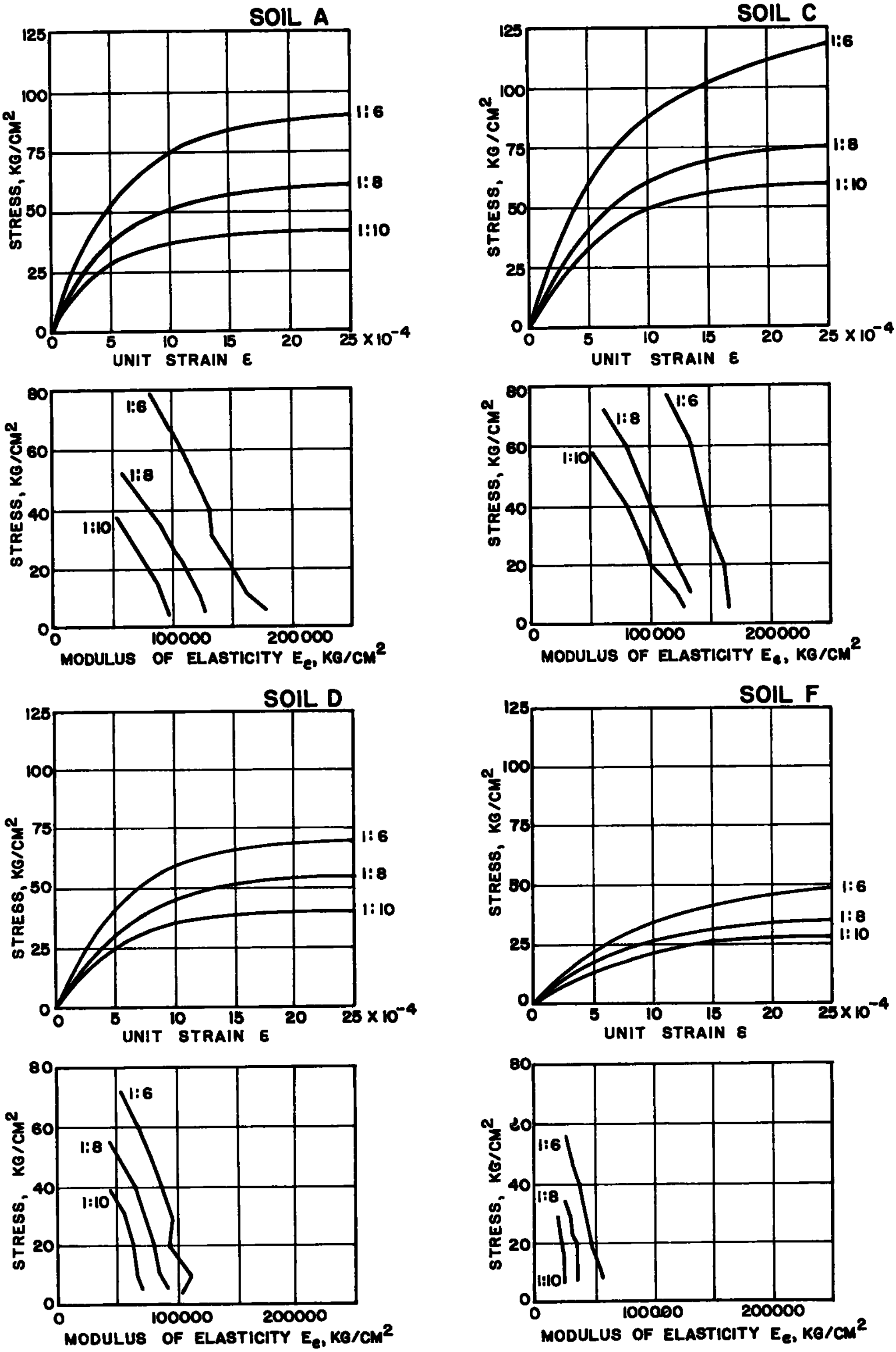

Figure 11. Influence of the cement content.

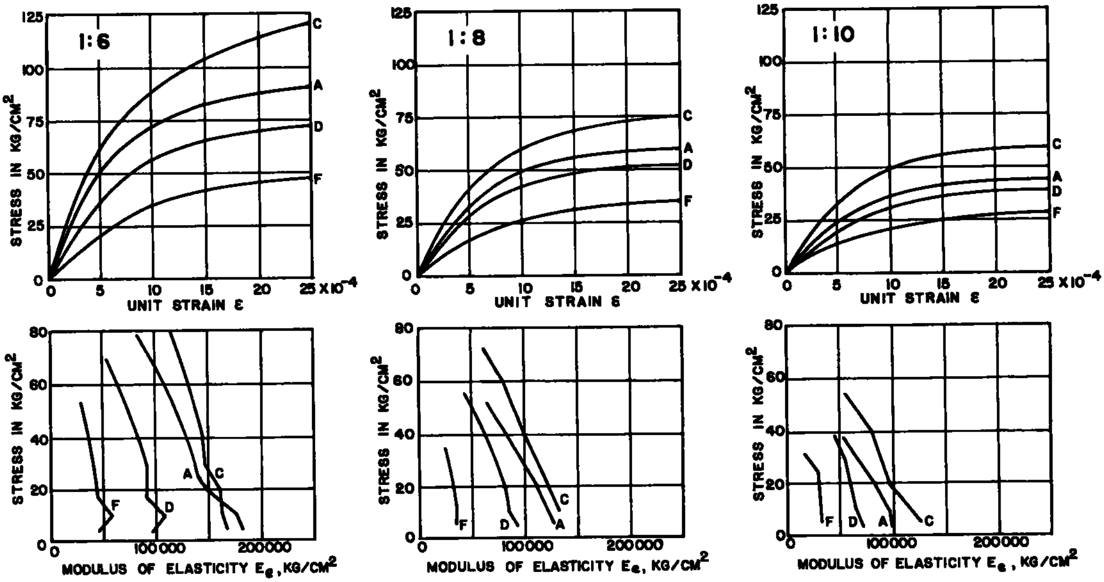

Figure 12. Influence of the clay content.

Acknowledgment

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