Frost Penetration Under Bituminous Pavements

MILES S. KERSTEN, Professor of Civil Engineering, University of Minnesota, and
RODNEY W. JOHNSON, Engineer, Hazelet and Erdal, Chicago, Illinois

This study was made near Minneapolis in the winter of 1953-54. Nine sites were selected, each a bituminous-surfaced road. At each of these locations borings were made to a depth of 8 feet to obtain soil textural information, moisture contents, and a few densities. During the winter season borings were made by a machine-driven auger at about 3-week intervals to determine the frost depths. Daily maximum and minimum air temperatures were recorded.

The subgrade soils at the test sites selected represent a variety of textures. The finest-grained soil was a silty clay loam with moisture contents between 25 and 30 percent. Other soils included loam, sandy loam, sandy loam till, and fine sands. Moisture contents in some of the sands were as low as 5 percent.

Observed frost penetrations at the end of the freezing season varied from about 3 1/2 feet in the silty clay loams to more than 5 feet for some of the sandy soils.

Theoretical calculations of frost penetration were made by means of the Stefan equation, utilizing an air-surface correction factor and the layered system solution developed by the Corps of Engineers. The moisture content and density data for the soils and the air temperatures are the only information required for these solutions. (Thermal conductivity coefficients were taken from published charts).

Comparisons of the observed and calculated frost depths indicate that an air-surface correction factor of 0.8 gives best results for the sites investigated. Utilizing this value, the difference between actual and calculated depths of freeze is commonly about 1/2 foot, plus or minus. Considering the many factors which affect frost penetration, this comparison is thought to be good.

In securing average daily temperatures by averaging maximum and minimum values for 24-hour periods, it was found that the time of recording made a significant difference. For the freezing season studied, the use of a period from 6 P.M. to 6 P.M. gave a freezing index about 6 percent less than that for a midnight to midnight period.

The investigation indicates the feasibility of making reasonable frost depth predictions for known soil and temperature conditions. Additional measurements should yield further refinements. Further study of the air-surface is considered particularly desirable. Similar procedures could also be applied to a study of the thawing of soils.

● ALTHOUGH the need for reasonably accurate methods for predicting depth of frost penetration has long been recognized, the difficulties of handling numerous variables, such as those of the soil, temperature, and other factors of climate, have until recently hindered such a development. In recent years work of the Corps of Engineers, Department of the Army, has indicated some promise in the use of the Stefan equation for calculating depths of freeze and thaw utilizing air temperatures, an air-surface temperature correction factor, and moisture content and density data on the soil section.

Carlson and Kersten, in a previous paper before the Highway Research Board (1), reported on observations and calculations for some airfield pavement sections in Alaska which showed a good agreement between measured depths of freeze and thaw by thermocouple readings and borings with values calculated on the basis of the Stefan equation. The literature records few studies of a similar nature in continental United States.

This study is an attempt to apply calculations by the Stefan equation to some test locations in Minnesota and compare these calculated depths with observed depths. The calculations are based on such easily collectable data as air temperatures, soil texture, and moisture content. No ground temperature installations, such as thermocouple strings, were utilized. For comparison with calculated depths of freeze, auger borings were made periodically during the winter season and the depth of freeze in the holes noted.

All test points were on bituminous-surfaced roads which were kept free of snow. The subgrade soils at the test points represent a variety of textures and also moisture contents. It is thought that the comparisons of calculated and observed depths afford a check on the degree of accuracy that may be expected for predictions of frost depth based on such procedures. It is also hoped that studies such as this will assist in indicating the magnitude of air-surface correction factors for northern United States.

EQUATION FOR FROST DEPTH

The Stefan equation as modified in studies by the Corps of Engineers has been presented in prior publications (1, 2) and the explanation will not be repeated here. It is based on the simplification that the only heat flow that need be considered in frost penetration is that represented by the latent heat of fusion of the water in the soil. The Corps of Engineers modification includes the use of an air-surface correction factor to convert degree-days of freeze for air temperatures to those for the pavement surface. Also, the solution is applied to a layered system by considering the individual properties such as volumetric latent heat of fusion and thernal conductivity for each strata of soil. The equations as applied in this study may be stated as follows:

Consider a layered system, such as a bituminous mat, gravel base, and layers of underlying soil, numbered from the top down.

The degree-days required to freeze layer 1 is

$$F_1 = \frac{L_1 h_1^2}{48 k_1} = \frac{L_1 h_1}{24} \cdot \frac{R_1}{2}$$

in which F_1 = degree-days of freeze at surface required to freeze layer 1 in degrees Fahrenheit.

L_1 = volumetric latent heat of fusion in Btu. per cu. ft.
= 1.434 wd
w = moisture content of soil in percent
d = dry density of soil in pounds per cubic foot
h_1 = thickness of layer in feet
k_1 = thermal conductivity in Btu. per square foot per degree F. per foot per hour.
R_1 = thermal resistance $= \frac{h_1}{k_1}$

For layer 2

$$F_2 = \frac{L_2 h_2}{24} \left(R_1 + \frac{R_2}{2}\right)$$

For layer n

$$F_n = \frac{L_n h_n}{24} \left(R_1 + R_2 + \ldots\ldots + R_{n-1} + \frac{R_n}{2}\right)$$

$$= \frac{L_n h_n}{24} \left(\Sigma R_{n-1} + \frac{R_n}{2}\right)$$

in which ΣR_{n-1} = the summation of the thermal resistances of all layers above layer n.

To utilize these equations it is necessary to know certain characteristics of each layer, namely, moisture content, density, and thermal conductivity. Moisture contents and densities can be measured in the field at the test location. In the field studies reported herein, extensive moisture-content data were collected. Only a few density tests were made, but it is thought that reasonably accurate values could be estimated from textural information. Values of the coefficient of thermal conductivity have been selected from diagrams based on extensive laboratory studies previously reported (3, 4). Experience thus far has indicated that these values are reasonably correct.

The other item of information required for utilization of the equations is temperature data. Average daily temperatures during the freezing season were obtained by use of a maximum-minimum thermometer. Readings were recorded at 6 P.M. each day and

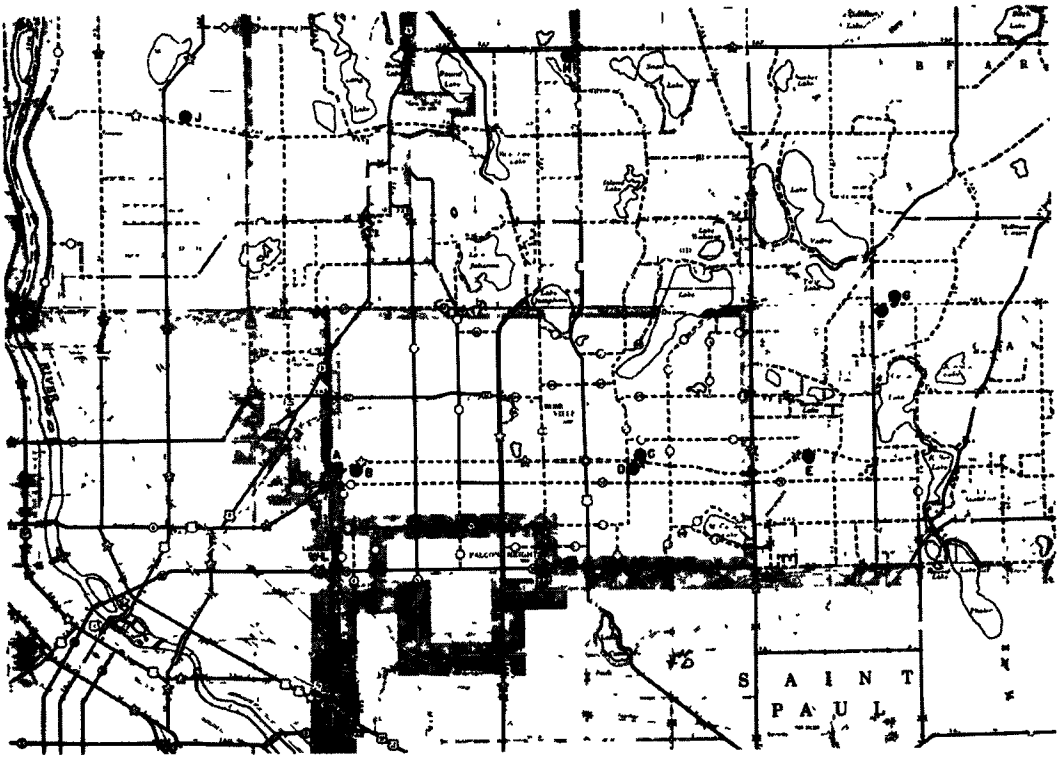

Figure 1. Map showing site location.

the two values averaged for calculation of the degree-days. Also, average daily temperatures were obtained from the U. S. Weather Bureau at Minneapolis.

The degree-days of freeze at the pavement surface, the term F in the equation, was taken as the degree-days of freeze based on air temperatures multiplied by an air-surface correction factor, C_f. The studies in Alaska had indicated a value of $C_f = 0.6$ to be correct. Therefore this same value was initially assumed for this investigation. However, other values of C_f were also considered in this study to attempt to obtain the best correlation between observed and calculated frost depths.

SITES SELECTED FOR STUDY

The field sites selected for the study are shown in Figure 1. The area is just north of St. Paul, Minnesota; all nine locations are within an area of about 50 square miles. All locations are points on roads with bituminous surfaces. It was attempted to select areas with a variety of textures of subgrade soils. The detailed information for each test point is given in the following paragraphs and in Figures 2 to 10, inclusive.

Soil textures were determined by visual and manual manipulation with numerous checks by hydrometer analysis; moisture content tests were made for every 0.5 foot; most of the densities are estimates based on the texture of the soils and some laboratory compaction tests at natural field moisture contents. Moisture contents for all test sites are given in Table 1 for ready comparison.

Site A

The location of Site A is on Minnesota Trunk Highway 36 just outside the corporate limits of Minneapolis (see Figure 1). The area surrounding this location is relatively level, having little cover with the exception of scattered brush to the south of the pavement. The road itself is a tangent section about 1/2 mile long, connecting relatively

TABLE 1
MOISTURE CONTENTS - FIELD TEST SITES

Depth in Feet[a]	Moisture Content, Percent of Dry Weight Site								
	A	B	C	D	E	F	G	H	J
0 - 0.5	3.2	2.0	3.3	7.9	8.1	17.6	19.1	6.9	6.6
0.5 - 1.0	14.5	3.7	6.7	6.2	7.2	17.4	27.3	8.2	4.8
1.0 - 1.5	13.5	6.3	5.1	7.1	10.3	20.2	23.9	10.3	4.8
1.5 - 2.0	15.0	11.7	5.4	5.1	9.4	28.6	24.1	13.3	5.8
2.0 - 2.5	13.0	11.8	5.8	6.6	8.7	25.1	24.3	15.7	6.1
2.5 - 3.0	14.0	10.4	5.4	7.3	8.8	24.7	22.8	15.5	5.9
3.0 - 3.5	14.7	9.5	5.1	7.9	8.7	25.3	23.8	18.3	6.7
3.5 - 4.0	14.7	10.2	5.0	6.9	8.3	24.3	25.7	19.8	6.0
4.0 - 4.5	20.6	9.1	4.5	5.4	9.0	25.7	28.0	19.2	5.7
4.5 - 5.0	19.8	9.4	4.5	4.8	9.4	25.7	30.7	17.6	7.5
5.0 - 5.5	22.2	12.3	4.2	5.0	9.4	25.9	32.3	16.8	8.3
5.5 - 6.0	22.0	13.5	4.6	4.7	9.9	24.6	34.7	18.8	10.1
6.0 - 6.5	19.8	12.4	9.2	4.4	10.0	23.1	31.1	17.6	9.8
6.5 - 7.0	19.0	8.2	9.1	4.4	9.9	24.7	29.4	17.2	6.8
7.0 - 7.5	15.5	8.3	7.5	5.5	10.3	24.9	29.3	17.5	6.9
7.5 - 8.0	16.2	9.5	6.1	10.4	10.3	26.3	26.1	17.7	7.4

[a] All depths measured from top of bituminous surface.

flat reversed curves. The cross-section is level, with moderate ditches; the entire area offers little cover or wind protection.

The soil encountered in this location is described in the Soil Survey of Ramsey County, United States Department of Agriculture, as Miami loam. Hydrometer analysis of the soil gave a loam texture (United States Bureau of Chemistry and Soil Classification) for the entire depth of the hole. The particle size of 0.02 mm. is sometimes considered as being significant in the frost-susceptibility of a soil. About 33 percent of this soil is smaller than 0.02 mm.

Site B

Site B is just 0.1 mile east of Site A. Cover conditions are similar, the road being in an exposed location. The soils at this site, however, differ from those at Site A. The subgrade is a mixture of sandy loams of both gray and red drift origin. On the soil map this location is on the apparent border of Miami and Gloucester soils.

Sites C and D

Sites C and D can be considered together, since their locations are close to one another and the soils are the same. This section of the road and the area adjacent to the sites are relatively level; there is no brush or trees within several hundred feet. Pasture and cultivated areas abut the roadway both north and south. The natural moisture content of the soil in borings made October 11, 1953, showed 6 to 9 percent at Site C and 5 to 8 percent at Site D. The soil at both locations was a relatively clean fine sand. The area is mapped as the Merrimac series.

Site E

Site E has more protection from wind and less exposure to sun than most of the other sites. The highway at this location is wooded on both sides (see Figure 6); the site was chosen in order to have a soil of the Gloucester fine sandy loam texture. There soils are derived from glacial red drift; the stony nature of this material proved to be an

TABLE 2
MEASURED DEPTHS OF FROST PENETRATION

Site	Depth of Frost Penetration, Inches[a]							
	1953					1954		
	Dec. 11	Dec. 18	Dec. 19	Dec. 23	Dec. 31	Jan. 15	Feb. 5	Mar. 9
A	7	20			27	36	47	76[b]
B	6	19			27	36	55	90[c]
C	9	12[b]			31	40.5	51.5	32[b]
D	10			19	33.5	43.5	60.5	30[b]
E	8		18		27	40.5	70	37[d]
F	6	18			23	32	43	40
G	4	17			23	31	43.5	41
H	5		19		27	36	52	42[b]
J	0[b]		15		32	43.5	68	52[e]

[a] All depths measured from top of bituminous surface.
[b] Reliability of data in doubt.
[c] Thawed depth approximately 2 to 2.5 feet.
[d] Thawed depth approximately 2 feet.
[e] Thawed depth approximately 1.5 feet.

undesirable feature. In the actual boring operation the presence of stones, cobbles, etc., made boring difficult and determination of depth of freeze was not readily made since the granular nature of the soil did not result in a positive line of demarcation between the frost zone and nonfrozen soil. This made the value of Site E as a test location somewhat dubious.

Sites F and G

Sites F and G will also be considered together, since the soils are the same and the two are thought of as verifications of each other. The location is on a county road with a narrow (20-foot) road-mix bituminous surface. The surrounding terrain is level and cultivated fields abut north and south. The top soil is black with high organic content. A relatively poor natural drainage accounts in part for the high moisture content of 23 to 30 percent. The texture of the soil is sandy clay loam. The soil survey map shows the area as the Clyde series. A mechanical analysis of this material indicates about 22 percent of clay and 60 percent of silt. These two locations represent the heaviest soils which could be found in the general test area. The entire study would have benefited could a location having a heavy clay soil been found.

Site H

The highway in the area of Site H is level with open space both north and south affording no wind or sun protection. The subgrade soil is a sandy loam containing about 13 percent of clay and 27 percent of silt. The variations in moisture content are attributed to some differences in the amount of silt and clay at different depths.

Site J

Site J is a departure from the cover and wind exposure conditions of the other sites as it is in a deep cut section of highway with trees bordering to the north and south. The soil survey map of Anoda county shows this soil to be a fine sandy loam of the Miami series. The hydrometer analysis indicated the soil to be a gravelly sand near the surface and a sandy loam till derived from red glacial drift at deeper depths. The good grading and gravel content of the soils resulted in the highest densities, 110 to 118 pcf., of the soils tested.

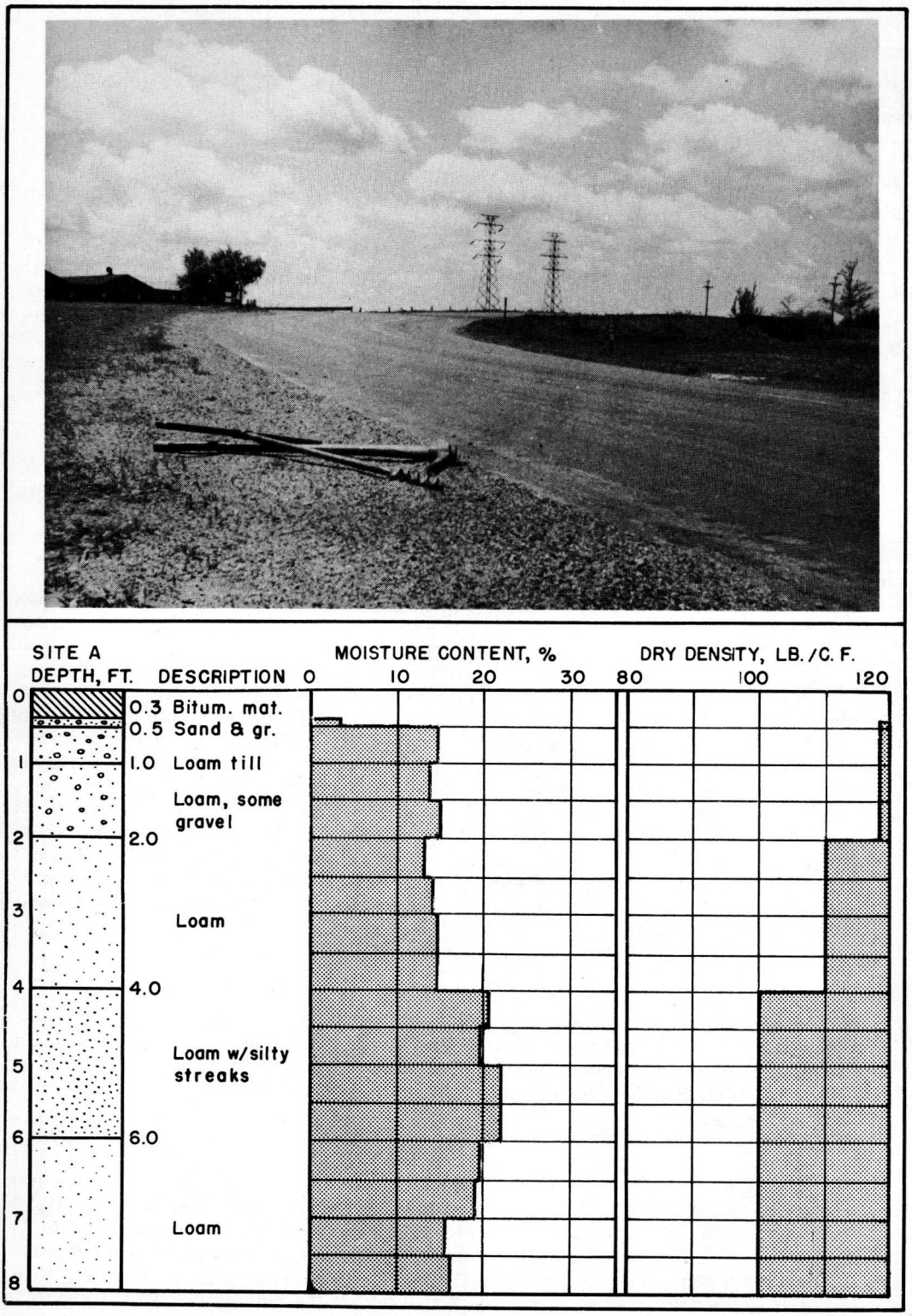
SITE A
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C. F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
0.5 Sand & gr.
1.0 Loam till
Loam, some gravel
2.0
Loam
4.0
Loam w/silty streaks
6.0
Loam

Figure 2.

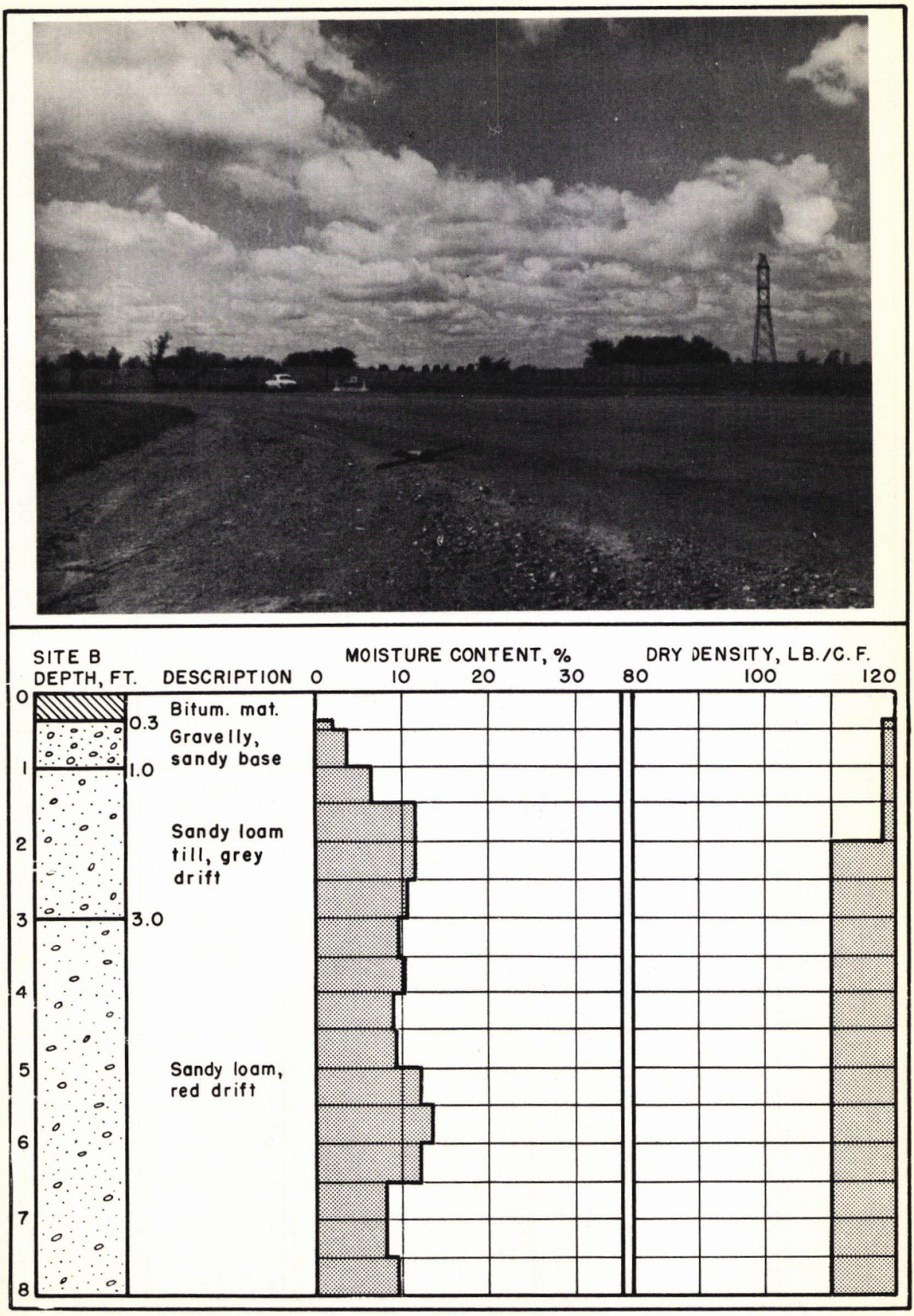
SITE B
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3
1.0
3.0
Bitum. mat.
Gravelly, sandy base
Sandy loam till, grey drift
Sandy loam, red drift

Figure 3.

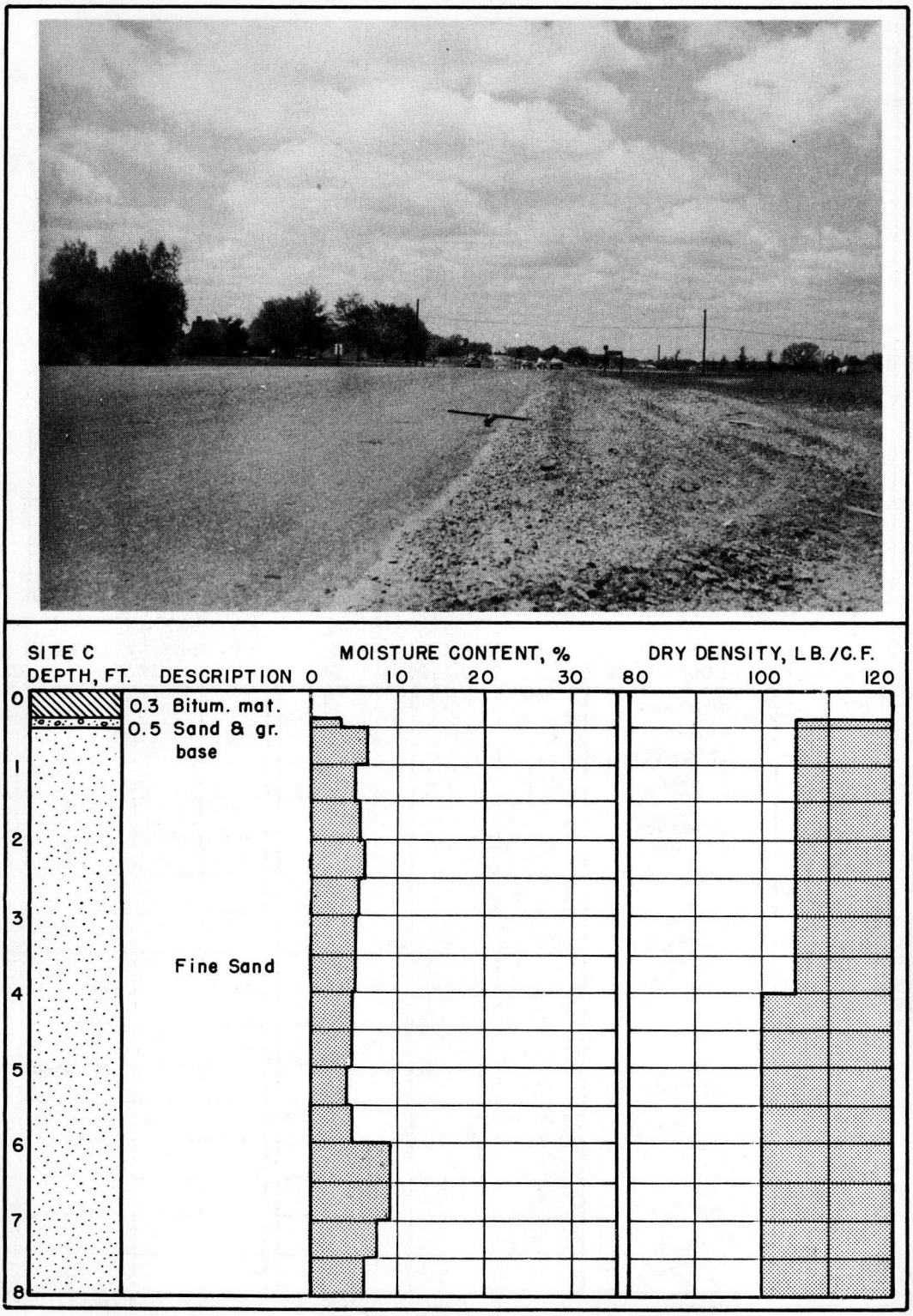
SITE C
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
0.5 Sand & gr. base
Fine Sand

Figure 4.

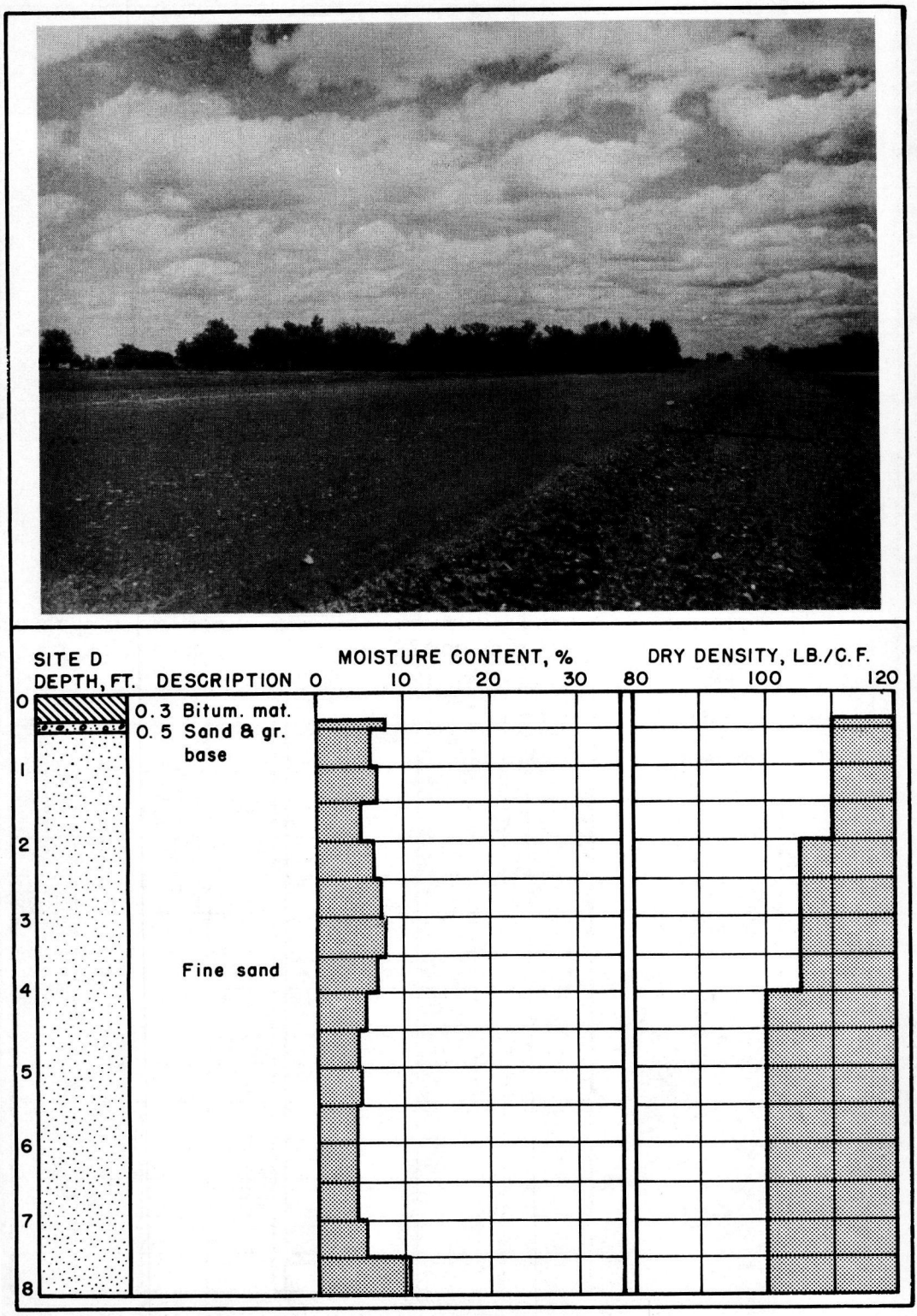
SITE D
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
0.5 Sand & gr. base
Fine sand

Figure 5.

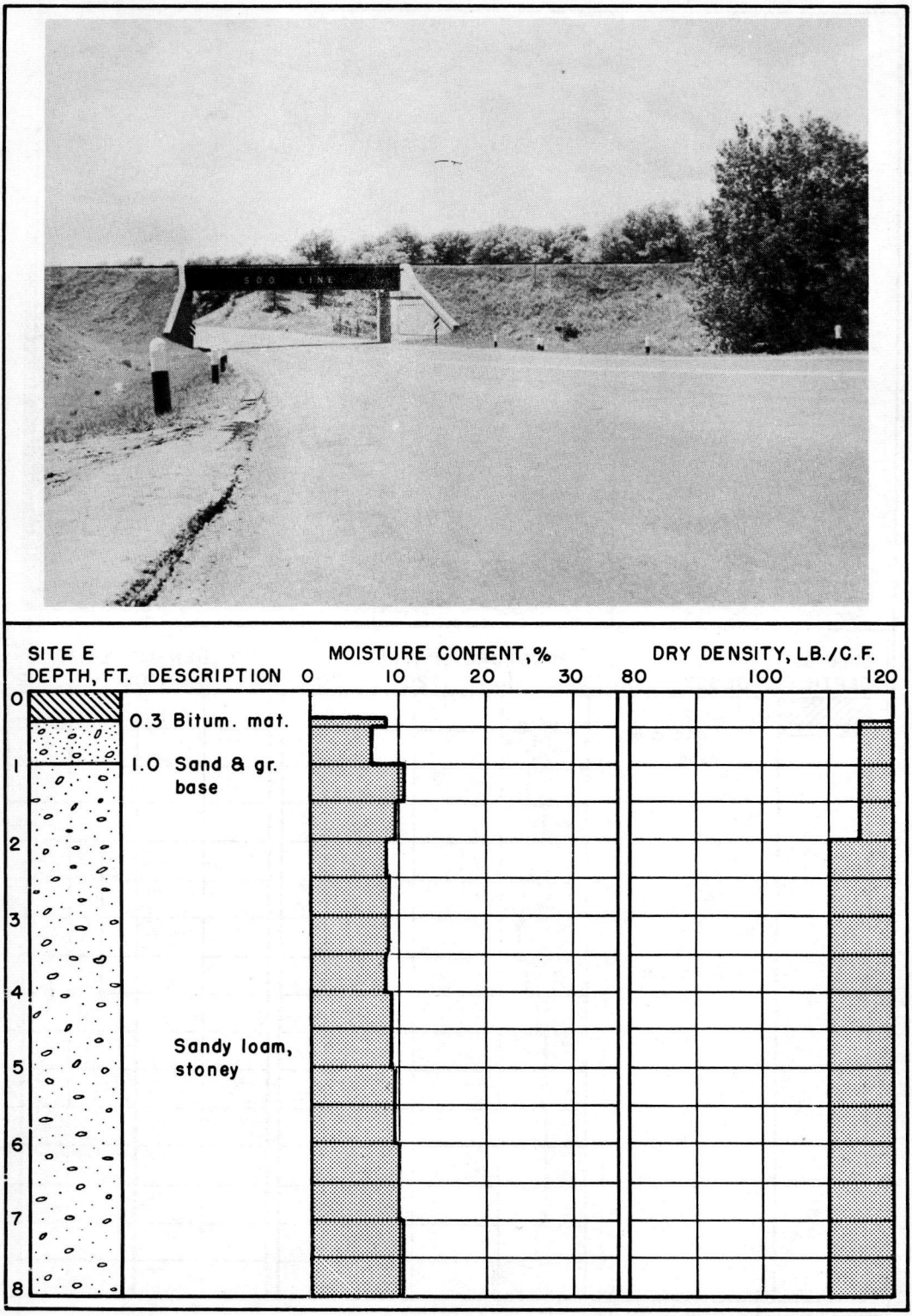
SITE E
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
1.0 Sand & gr. base
Sandy loam, stoney

Figure 6.

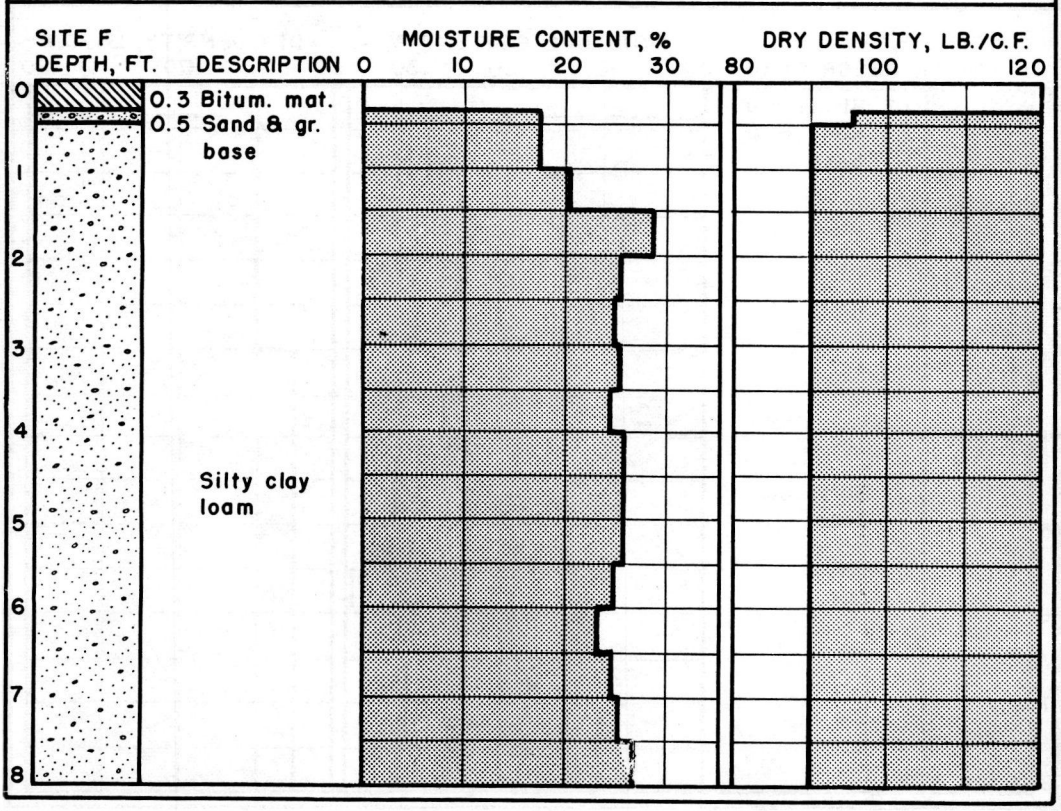
SITE F
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
0.5 Sand & gr. base
Silty clay loam

Figure 7.

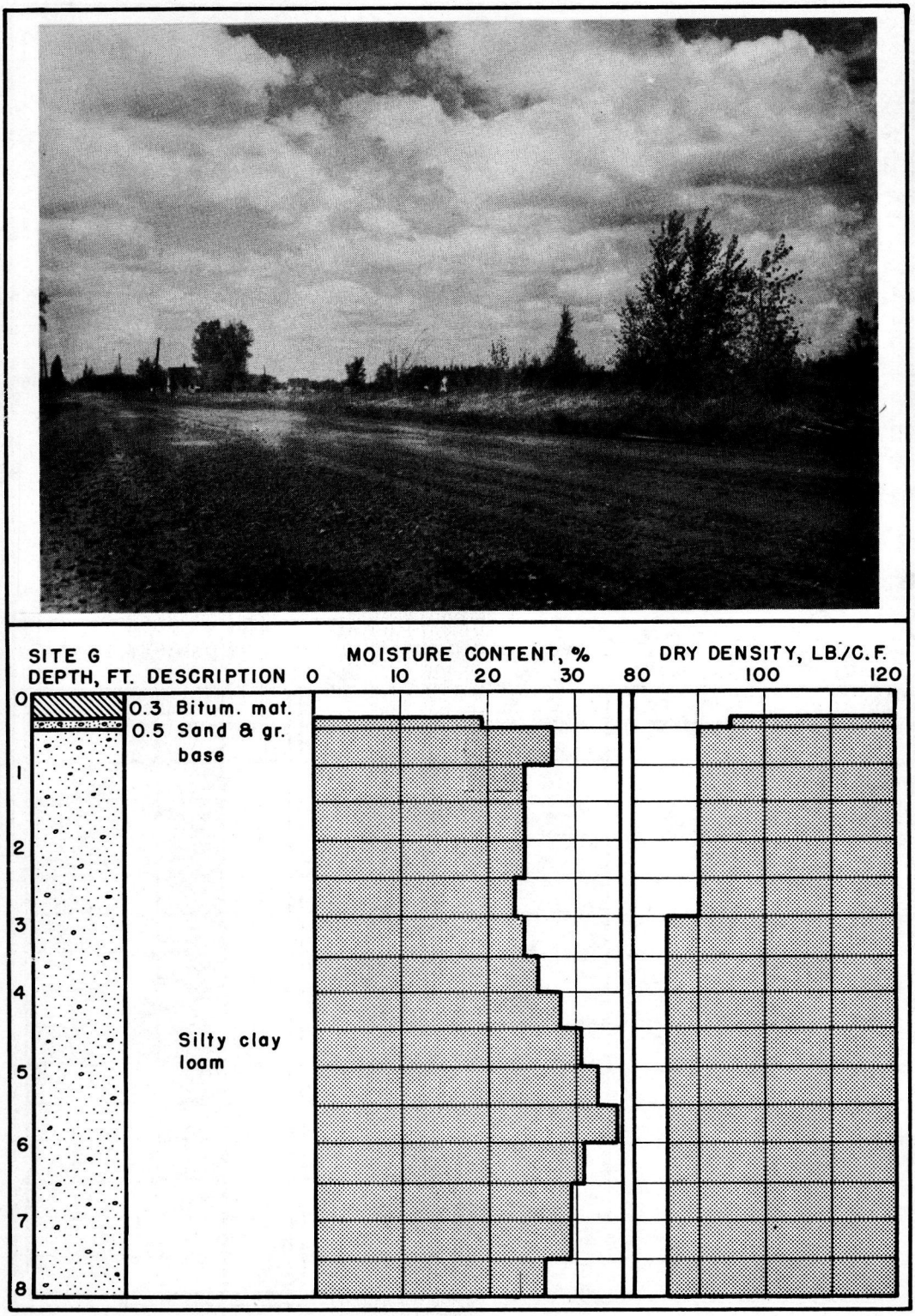
SITE G
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
0.5 Sand & gr. base
Silty clay loam

Figure 8.

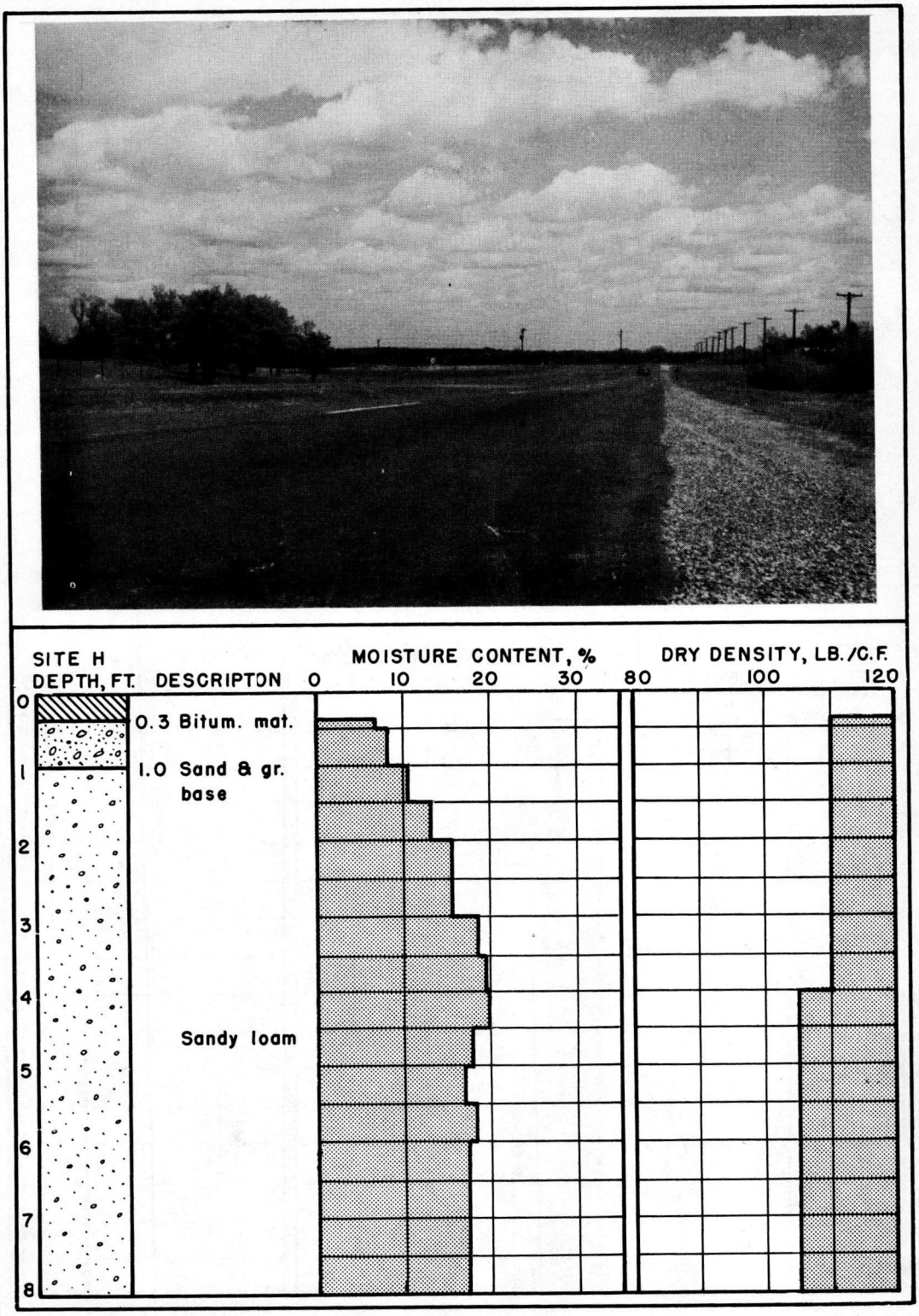
SITE H
DEPTH, FT.
DESCRIPTON
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
1.0 Sand & gr. base
Sandy loam

Figure 9.

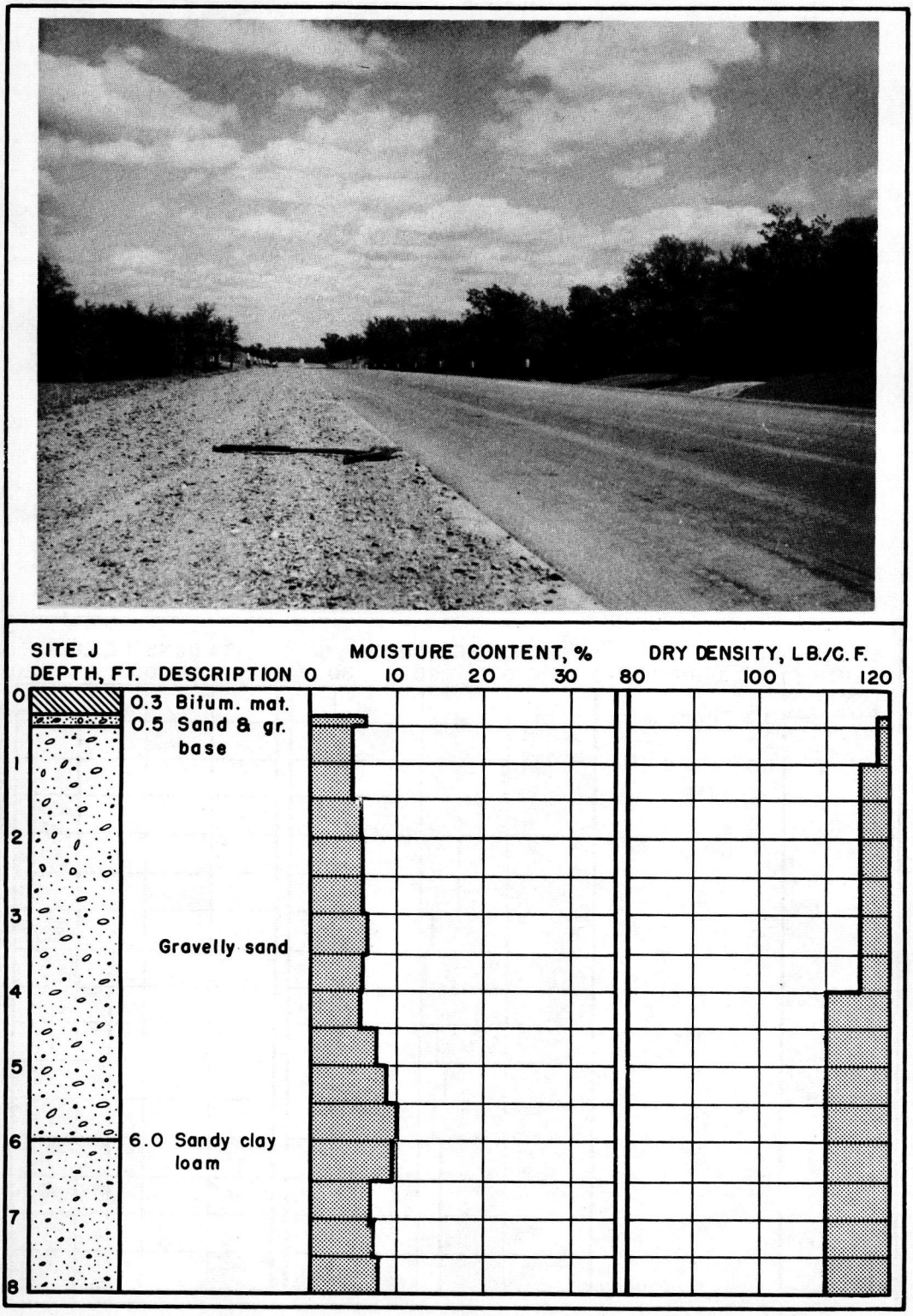
SITE J
DEPTH, FT.
DESCRIPTION
MOISTURE CONTENT, %
0
10
20
30
DRY DENSITY, LB./C.F.
80
100
120
0
1
2
3
4
5
6
7
8
0.3 Bitum. mat.
0.5 Sand & gr. base
Gravelly sand
6.0 Sandy clay loam

Figure 10.

CLIMATIC DATA

The location of the station at which temperatures were measured was about 2½ miles southwest of Site A (Figure 1). This was the residence of the investigator and was selected for that reason. A maximum-minimum recording thermometer was mounted in a location away from sun and wind yet representative of the actual air temperature. From November 1, 1953, to April 30, 1954, daily maximum and minimum temperatures were recorded at 6 P.M., from which the observed average daily temperature was determined. The location of the U.S. Weather Bureau Station was near the Twin City Metropolitan Airport, approximately 10 miles south of the location of the other recording station. Average daily temperatures at this station were based on the maximum and minimum values for a midnight to midnight period.

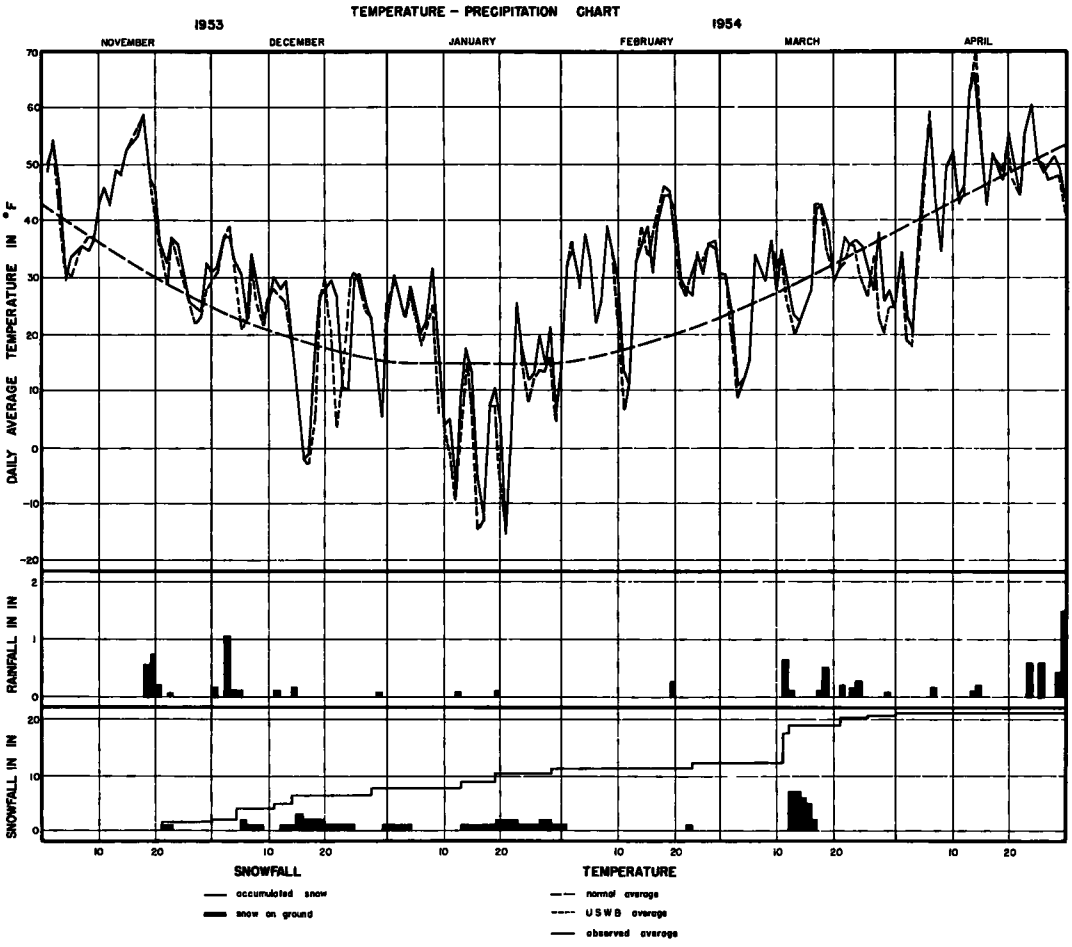

Figure 11.

It is believed that the temperatures at the airfield and frost study sites approximate each other rather closely.

The freezing season in Minneapolis according to records of the U.S. Weather Bureau normally begins about November 17 and ends about March 17, a period of four months. During this time wide fluctuations of temperature occur with highs of 60 F. to 70 F. recorded at the beginning and end of the season and lows of -20 F. during the middle of the season. The mean temperature of this 4-month period is 20 F.

Temperature and precipitation data are shown in Figure 11. The winter of 1953-54 differed from a normal winter in two ways: (1) a very-warm February and (2) a small

amount of snow. Total snowfall was only 23.3 inches as compared to an average fall of 42 inches. The light snow had little or no effect on the frost study, since road surfaces are always kept clean. The abnormally warm February, however, cut off the study at an early date, since there was no increase in frost depths after the first part of February.

The measurement of cold for frost-penetration calculations is by degree-days. A degree-day is defined as each degree in any one day that the average daily air temperature varies from 32 F. The difference between the average daily temperature and 32 F. equals the degree-days for that day. The degree-days are minus when the average daily temperature is below 32 F. and plus when above. For temperate climates, where hourly fluctuations of temperature are common, a better system would be the degree-hour, which would be a more-accurate measurement of cold. This is apparent, since the average daily temperature is the basis for calculation of degree-days. Hence, a sudden rise or fall in temperature, especially near the recording time, would reflect an incorrect amount of cold, the actual average being several degrees higher or lower. The use of the degree-hour is considerably more laborious, however, and makes its value a practical question. A variation of this idea will be shown later when differences arise from time of recording.

Figure 12 is a plot of degree-days of freeze for a normal winter and for the winter of

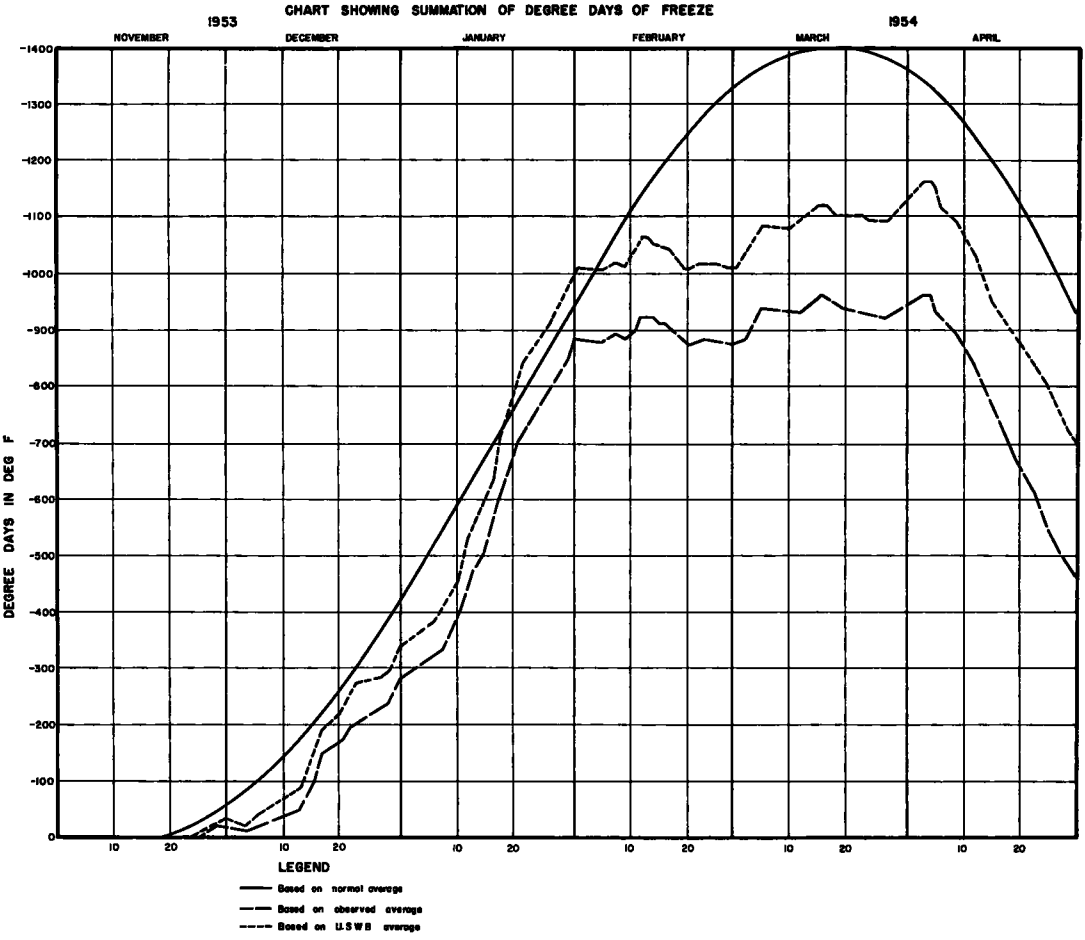

Figure 12.

1953 -54 using both the temperature data near the test site (observed) and the U.S. Weather Bureau values. The differences in values for these two curves is believed to be caused by two factors which influence the daily average temperature.

The first of these factors is that of location of the recording stations, the station for

observed data being a more-sheltered urban area of Minneapolis, while that of the Weather Bureau is in an exposed suburban area with less protection from climatic variations. This fact, it is believed, could account for the somewhat lower daily average temperature and a higher accumulation of cold.

The other (and perhaps most-important) factor is that of time of recording. The time of recording for the observed data was 6 P.M. while the USWB averages are computed on a 12 M. to 12 M. basis. Using the hourly temperature recordings from the official USWB climatological data sheets, the maximum and minimum daily temperatures for a 6 P.M.-to-6 P.M. period were compiled. The averages of these temperatures were computed and from these a new total degree-days of freeze was figured for the entire freezing period.

The following results were obtained:

	Total Degree-days of Freeze		
	U.S.W.B. 12 M to 12 M	U.S.W.B. 6 PM to 6 PM	Observed 6 PM to 6 PM
February 13, 1954	-1064	-982	-923
April 3, 1954	-1162	-1021.5	-965.5

The result of the change to 6 P.M. recording time is that the total degree-days using USWB data are 140 days closer to the total of observed and only 56 days greater (April 3 totals). This difference in totals is considered reasonable, being within 6 percent of the observed degree-days total. A difference of 5 percent is deemed allowable for location difference.

Closer agreement between the calculated and observed depths of frost penetration could be shown were these USWB (6 P.M.-to-6 P.M.) data used instead of the 12 M.-to-12 M.data However, the concept is a new and unproven one, and therefore the official data were used.

It should be mentioned here that although it was at first intended to include a comparison of actual and calculated depths of thaw for the same locations, the unseasonal weather in February and early March made such a study impossible. The value of such a comparison is unquestionable, its application to the fields of highway and airport engineering being perhaps greater than that of frost penetration.

FIELD OBSERVATIONS OF FREEZE

At the present time two methods of field determination of depth of freeze are in general use. These are the thermocouple and the auger, either hand or powered. Each method offers advantages and disadvantages over the other.

The hand method using the simple helical auger or the "Iwan" posthole auger is the easiest and most practical where only a few borings are to be made. This method is adapted for recovery of unfrozen shallow depth samples for laboratory analysis, since the drill hole is kept clean and actual depths of samples can be closely determined. This method is not very satisfactory for depth of freeze determination. The use of this method for frozen soil borings appears to be limited to about a foot. For this study the hand auger, helical type, was used only for determination of the depth of freeze at the onset of the cold season. The frost lines is readily found, however, from the break-through point.

The variety of power augers and boring rigs is wide and varies from hand supported, electric powered, small-diameter helical augers to large, truck-mounted, large-diameter augers and percussion drilling rigs. After the frost line had reached depths exceeding a foot, a jeep-mounted power auger supplied through the cooperation of the Minnesota Department of Highways was used. This unit had a bore 3 inches in diameter and used a continuous helical screw auger.

In practice the actual point of break through the frozen to unfrozen soil could not be closely determined during augering. Instead, a sampling spoon having a hooked lip was scraped along the walls of the hole until the spoon hooked the frozen earth. The depth was then measured to the lip of the spoon. The only disadvantage of this method was that in

particularly granular soils (Sites E and J) the pebbles and rocks held firmly to the walls, so the spoon catching one of them gave the impression of being at the frost line. No difficulty was experienced in soils such as those found at Sites C, D, F, and G as the frost line at these locations was sharp and definite and no pebbles were present in the soil.

Conceivably a stoney clay, silt, or loam could occur which ordinarily would have a texture permitting easy frost-depth determination in the homogeneous state; however, due to the stoney texture some ambiguity could occur in frost determination. Manipulation by hand of scrapings from the sides of the bore was of some assistance in locating the frost line and, also, frozen soil often having the appearance of containing ice or ice crystals as compared to unfrozen soil having a moist appearance.

After stabilization of the freezing season in December, the borings were continued throughout the winter at intervals of two to three weeks.

The depths of freeze as determined are tabulated in Table 2. Depth of freeze versus time plots are shown on Figures 13 to 21, inclusive.

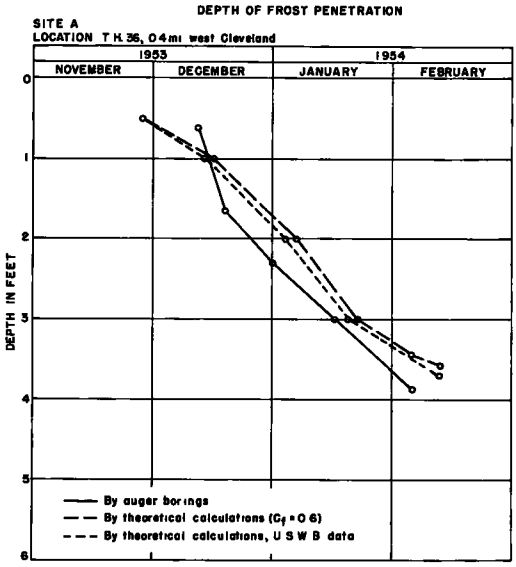

Figure 13.

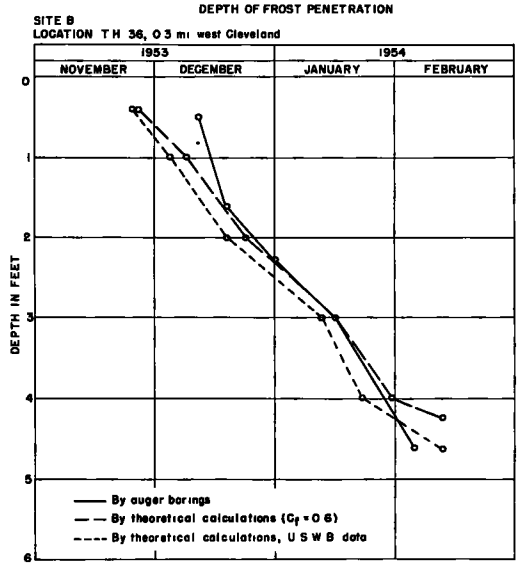

Figure 14.

CALCULATED DEPTHS OF FREEZE

The equation for calculation of depth of freeze in a layered soil has already been given as

$$F_n = \frac{L_n h_n}{24} \qquad (\Sigma R_{n-1} + \frac{R_n}{2})$$

This equation was applied and depths of freeze computed. The maximum freezing index of -923 degree-days, reached on February 12, 1954, was used for all sites.

Using Site A as an example, the calculations of freeze are shown in Table 3. To calculate the degree-days to freeze the sand and gravel base, the following computation is made:

$$F = \frac{L\,h}{24} \quad (\Sigma R_{n-1} + \frac{R}{2})$$

$$= \frac{540\,(.2)}{24} \quad (.38 + \frac{.30}{2})$$

$$= 5$$

The volumetric latent heat of fusion of the sand and gravel is 540, 0.2 is the thickness of this layer, 0.38 the thermal resistance of the overlaying bituminous mat, and 0.30 the thermal resistance of the sand and gravel itself. The average resistance for the layer

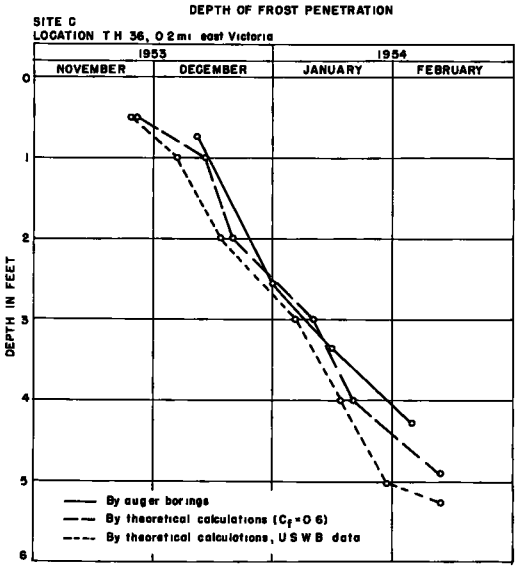

Figure 15.

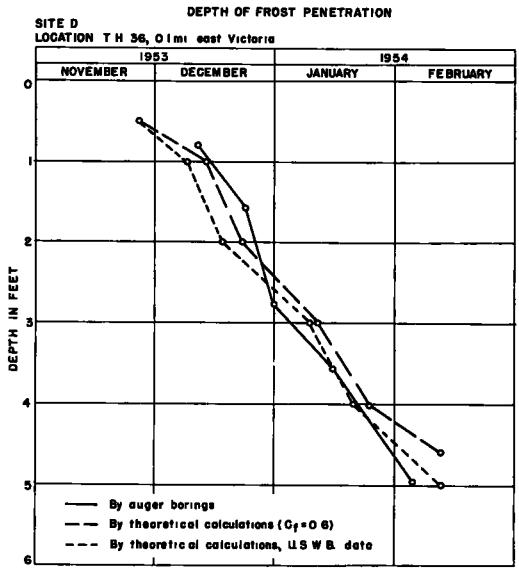

Figure 16.

during its becoming frozen is, then, 30/2. The value of 5 degree-days is a surface value. Since the relation between the surface values and air indices is given by the surface correction factor, this can be transposed into air degree-days by dividing by the value of C_f. Table 3 shows air values for C_f values of 0.6, 0.8, and 1.0.

For calculating the total number of degree-days to freeze 0.5 feet additional depth, the next calculation is made, utilizing the same equation as above, but using the L value for the loam till, a thickness of 0.5 feet, the combined result of the bituminous mat and sand and gravel base for ΣR and the average R of the 0.5 feet of loam till. The ΣF column totals the F values for all layers to that depth.

It will be noted that the maximum index of -923 degree-days produced a depth of freeze between 3 and 4 feet on February 12. In computing the depth of freeze in the final layer, it is necessary to determine the thickness of soil which can be frozen by the available degree-days of the freezing index. Since 717 degree-days are required to freeze a depth of 3 feet, (for C_f = 0.6).

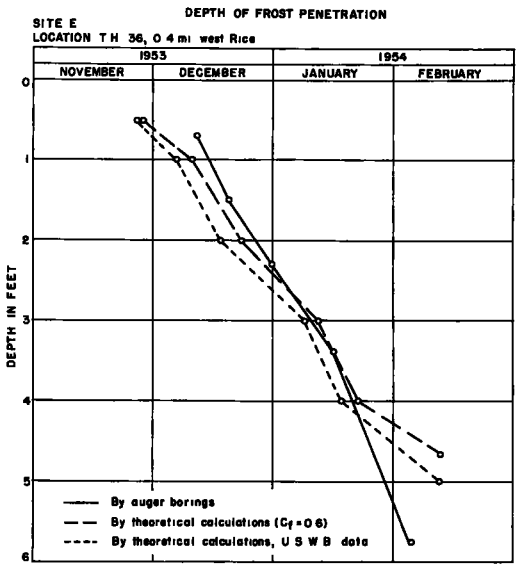

Figure 17.

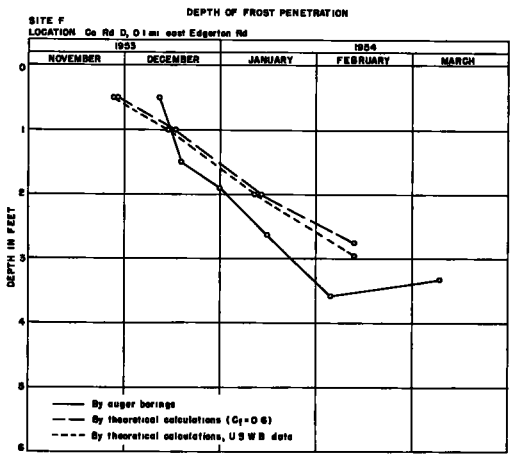

Figure 18.

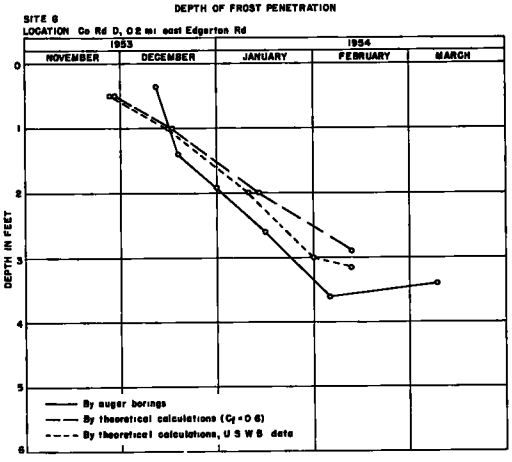

Figure 19.

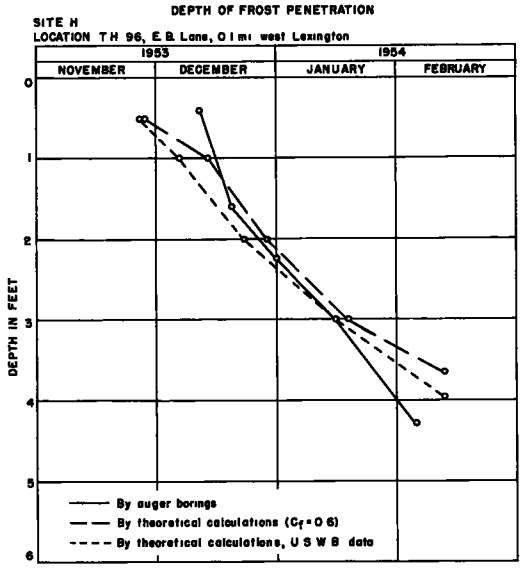

Figure 20.

$$923 - 717 = 206 \text{ degree-days.}$$

available to freeze below 3 feet.

Converting this to surface freezing index by application of the surface correction factor, C_f, gives

$$206\ (0.6) = 124 \text{ degree-days.}$$

Applying the formula again:

$$F_n = \frac{L_n h_n}{24} \left(\Sigma R_{n-1} + \frac{R_n}{2}\right)$$

$$124 = \frac{2320\ (h)}{24} \left(2.96 + \frac{h}{2(1)}\right)$$

or: $2.96\ h + .5\ h^2 = 1.28$

$$h = .40 \text{ feet.}$$

The total depth of freeze on February 12 is then 3 + 0.40, or 3.40 feet.

The same procedure was followed for all other sites in calculating the depth of freeze in the final layer. In addition, computations using the USWB freezing index of -1,064 degree-days were made and plotted on the same time-depth charts.

After the degree-days required to freeze to the various depths have been calculated, the date on which this number was reached can be taken from the curves, such as those of Figure 12. The calculated time-depth of freeze curves are then drawn. Those calculated using an air-surface correction factor of 0.6 are shown in Figures 13 to 21 inclusive.

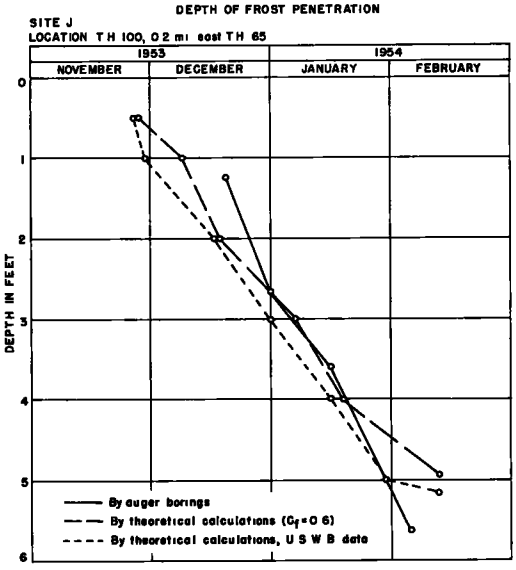

Figure 21.

EFFECT OF VARIATION IN DENSITY

Inasmuch as densities were taken at only

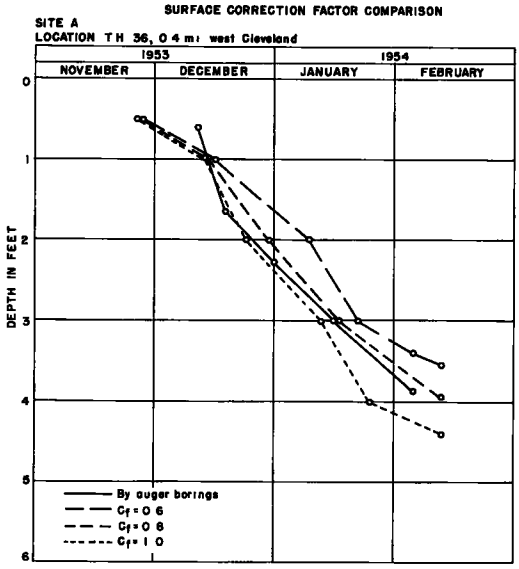

Figure 22.

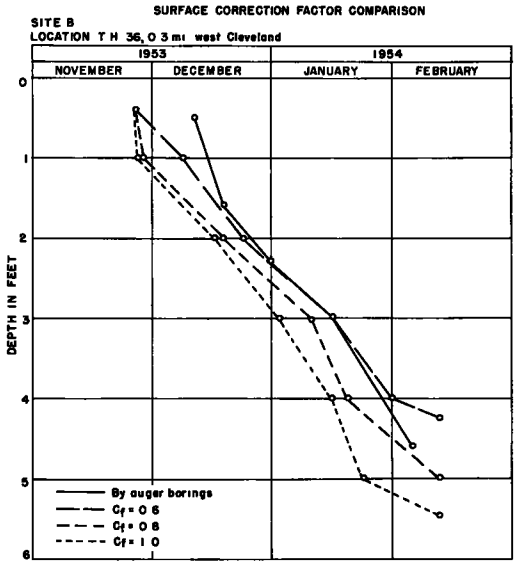

Figure 23.

two sites, A and G, to a depth of 2.5 feet and inasmuch as this depth represents only a portion of the total frozen depth, it can be said for all practical purposes that the densities as used are assumptions which are, nevertheless, believed to be indicative of the actual field conditions.

In recognizing that they are assumptions, it is also necessary to recognize that they may be in error, some perhaps by as much as 10 pcf. plus or minus. Since the values for the other variables as used are based upon test or previously determined results, the densities used are the most significant source of error.

In the Stefan equation used here, L, the volumetric latent heat of fusion is given as

$$L = 1.434\ wd$$

where: w = moisture content in percent of dry weight
and d = dry density in pounds per cubic foot.

It is possible to determine statistically the exact effect that deviations from the assumed correct value for both moisture content and density would have upon L and hence also the

TABLE 3

CALCULATION OF FREEZING

Site A

Depth	Material	Thick. h	Density δ	w %	L	k	R	ΣR	$\Sigma R + \frac{R}{2}$	F	ΣF	ΣF/0 6	ΣF/0.8
0-0.3	Bit. Mat.	3				18	.38						
0.3-0.5	Sand and Gr Base	2	118	3.2	540	67	30	.38	53	5	5	8	6
0.5-1.0	Loam till, some gr.	.5	118	14 5	2490	1.17	.43	.68	.90	47	52	87	65
1 0-2 0	Loam till	1	118	14 5	2490	1.17	85	1.11	1.54	159	211	352	264
2 0-3.0	Loam	1	110	13 5	2130	1 0	1.0	1.96	2.46	219	430	717	538
3.0-4 0	Loam	1	110	14.7	2320	1.0	1.0	2 96	3.46	334	764	1270	955
4.0-5.0	Loam w/silty streaks	1	100	20 4	2920	1 0	1.0	3 96	4.46	543	1307	2180	1635
5 0-6.0	Loam w/silty streaks	1	100	22 0	3150	1.1	.9	4 96	5.41	710	2017	3360	2520
6.0-7.0	Loam	1	100	19.4	2780	1.0	1.0	5 86	6 36	735	2752	4580	3440

Units are as follows·
Thickness, h, in feet
Density, δ, in lb. per cu. ft (dry density)
Moisture content, w, in percent of dry weight
Volumetric latent heat of fusion, L, in BTU per cu. ft.
Thermal conductivity, k, in BTU per sq ft. per degr F. per ft. per hour
Thermal resistance, R = h/k
Degree-days of freeze, F, degree-days Fahrenheit

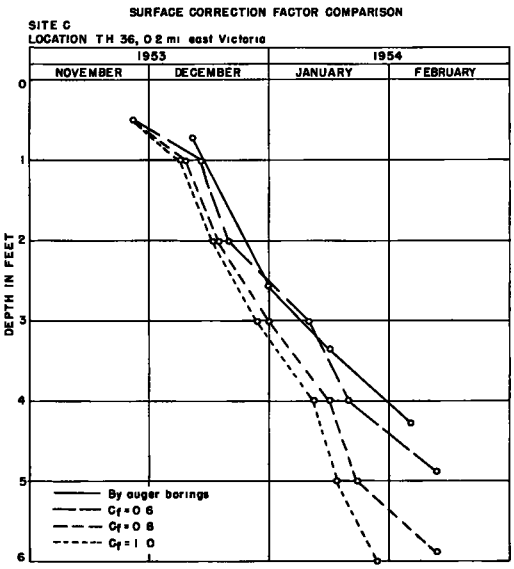

Figure 24.

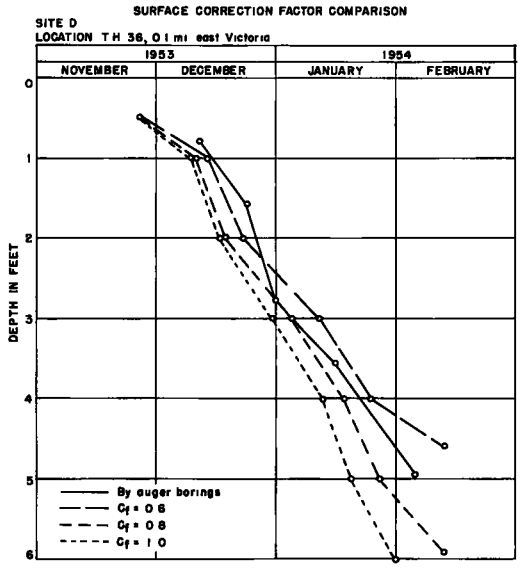

Figure 25.

effect upon the depth of freeze. The moisture contents in this study are the result of actual test and are thought to be accurate within a percentage point or two; variations in density from assumed values are more probable, and hence only these will be considered.

For this analysis the broad assumption was made that the densities could have been in error by as much as 10 pcf. plus or minus. Calculations were made for the soils at Site A representing the sandy loams, C representing the fine sands, and G representing the heavier silty clay loams. In the calculations for each site, the densities were both increased and decreased by a value of 10 pcf. and the depth of freeze calculated for the February 12 date when the air freezing index was -923 degree-days. The resultant depths are shown in Table 4.

It can be seen that a maximum variation of 0.3 to 0.4 feet in calculated depth of freeze would result for a variation of 20 pcf. (Plus or minus 10 pcf. from actual assumed values).

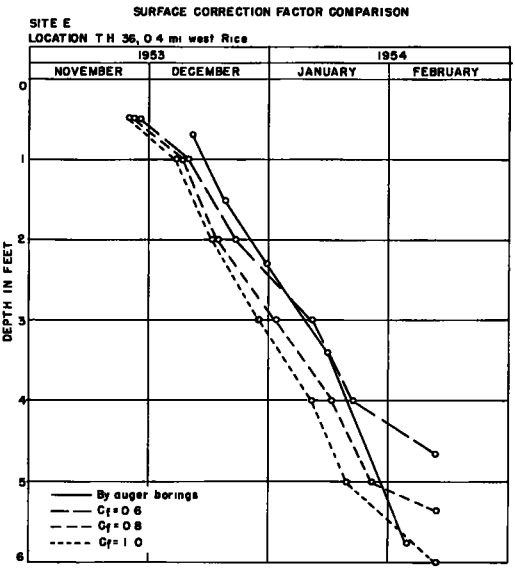

Figure 26.

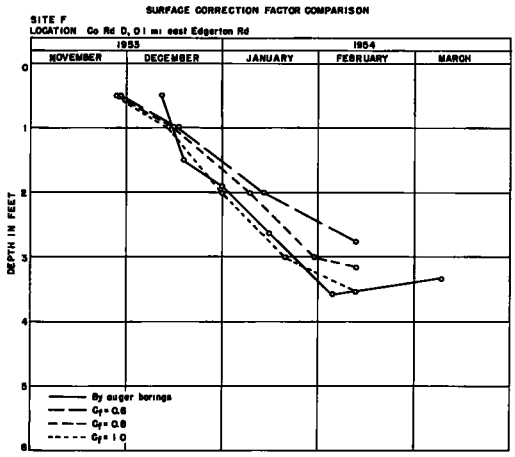

Figure 27.

COMPARISON OF CALCULATED AND OBSERVED DEPTHS

One of the important purposes of this study was to compare the actual and calculated depths of freeze and, also, to study the effect of the surface correction factor on calculated depths of freeze. The results of the former are shown by Figures 13 to 21 and of the latter by Figures 22 to 30.

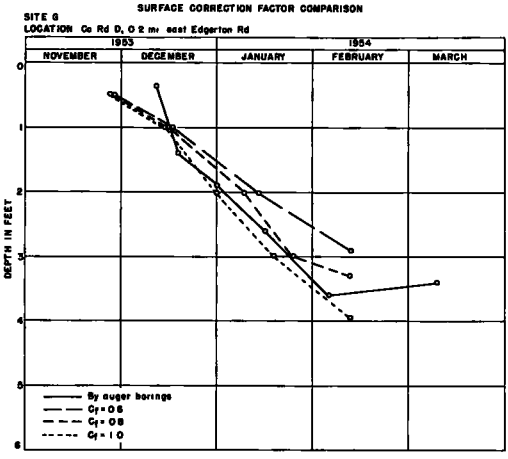

Figure 28.

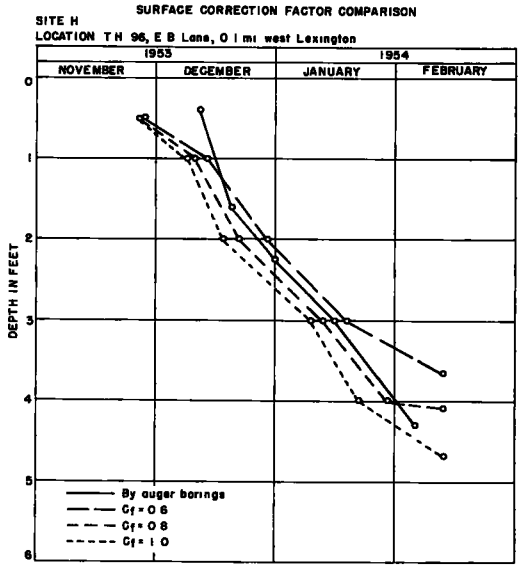

Figure 29.

Inspection of Figures 13 to 21 shows that the predicted depth of frost penetration is less than actual depth, and that the use of U.S. Weather Bureau temperature data gives closer correlation between them than does use of observed temperature data. The difference between the two calculations is not especially significant. It will be noted that in each case the curves are closely parallel, differing only in magnitude.

Using the USWB curves, the differences in observed and calculated depths of freeze on February 5, 1954, are shown in Table 5. In all cases except at Site C, the actual depth was greater than the predicted depth. This leads one to conclude that Site C may have been in error regarding the final depth. This conclusion may actually be correct, since some doubt existed at the time the borings were made.

The ambiguity regarding the depth stemmed from the fact that the soil at Sites C and D was a cohesionless sand with a low moisture content; this made determination of the depth

TABLE 4

CALCULATION OF DEPTH OF FREEZE FOR ASSUMED VARIATIONS IN DENSITY

Site	Assumed Density pcf.	Calculated Depth of Freeze, Feb. 12, 1954 feet
	10 #/c.f. greater	3.3
A	As in Figure 2	3.5
	10 #/c.f. less	3.6
	10 #/c.f. greater	4.7
C	As in Figure 4	4.9
	10 #/c.f. less	5.1
	10 #/c.f. greater	2.7
G	As in Figure 8	2.9
	10 #/c.f. less	3.1

TABLE 5

DIFFERENCE BETWEEN ACTUAL AND CALCULATED DEPTHS OF FREEZE, FEBRUARY 5, 1954.

AIR-SURFACE CORRECTION FACTOR = 0.6

Site	Soil	Measured Depth of Freeze, ft.	Difference, Measured and Calculated Depths, ft.	
			Observed Temps.	U.S W.B. Temps.
A	Loam	3.9	-0.45	-0.40
B	Sandy loam	4.6	-0.50	-0.20
C	Fine sand	4.3	+0.35	+0.85
D	Fine sand	5.0	-0.65	-0.25
E[a]	Sandy loam	5.8	-1.35	-1.10
F	Silty clay loam	3.6	-1.00	-0.85
G	Silty clay loam	3.6	-0.90	-0.50
H	Sandy loam	4.3	-0.85	-0.60
J	Gravelly sand	5.7	-1.00	-0.55

[a]Results of this site in doubt.
Note: A plus sign indicates calculated depth is greater than observed.

of the frost front difficult. Therefore in the case of Site C, the borings were repeated on February 6 in order to correct what seemed to be an obvious error, with an inconclusive result.

The differences shown in Table 5 represent those which would result from the original assumptions for the theoretical equation, i.e., an air-surface correction factor of 0.6. With the exception of the dubious frost depth determinations at Site E, it will be noted that the error in such predictions did not exceed 1.0 foot for actual frost depths which varied between 3.6 and 5.8 feet. This is considered a fairly good correlation.

TABLE 6

DIFFERENCE BETWEEN ACTUAL AND CALCULATED DEPTHS OF FREEZE

U.S.W.B. TEMPERATURE DATA

AIR-SURFACE CORRECTION FACTOR = 0.8

Site	Soil	Measured Depth of Freeze, ft.		Difference, Measured and Calculated Depths, ft.	
		January 15	February 5	January 15	February 5
A	Loam	3.0	3.9	-0.2	-0.2
B	Sandy loam	3.0	4.6	+0.6	+0.2
C	Fine sand	3.4	4.3	+0.6	+1.3
D	Fine sand	3.6	5.0	+0.3	+0.6
E	Sand loam	3.4	5.8	+0.5	-0.6
F	Silty clay loam	2.7	3.6	-0.3	-0.5
G	Silty clay loam	2.6	3.6	-0.2	-0.4
H	Sandy loam	3.0	4.3	+0.2	-0.2
J	Gravelly sand	3.6	5.7	+0.5	-0.1

Note: A plus sign indicates calculated depth is greater than observed.

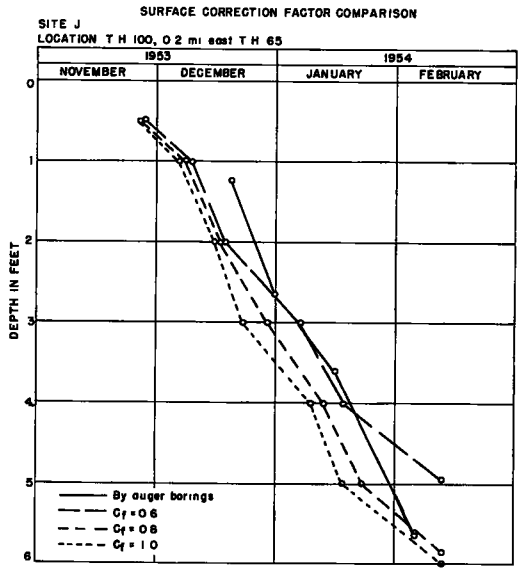

Figure 30.

At the start of the investigation it was realized that some air-surface correction factor other than 0.6 might result in a better check. As inspection of the curves of Figures 22 to 30 indicates that a value of C_f = 0.8 does give better results. Table 6 shows the differences between observed and calculated depths using C_f = 0.8 and the USWB temperature data for two different dates.

Inspection of the table shows an average difference of only about ½ foot with some calculated values being greater than the observed and others being less.

It is of interest to note that the relationship of texture and moisture content with frost penetration was the same as that shown by theoretical calculations. All other conditions being the same, the least frost penetration should occur in a fine-textured soil with a high moisture content. Sites F and G with the silty clay loam soils and moisture contents in the middle 20's had the smallest depths of freeze, about 43 inches. The greatest frost penetrations should occur for sandy soils at low moisture contents. Inspection of Table 1 shows the Sites C, D, and J had the lowest moisture contents, averaging around 5 or 6 percent.

Results show that these sites did have reasonable deep frost penetrations although Sites B, E, and H had greater depths than Site C (some doubt on accuracy of measurements at Site E has previously been mentioned). The soils with moisture contents between those of the dry sands and the silty clay loams such as the loam at Site A (15 percent) and the sandy loam at Site H (10 to 19 percent) showed intermediate depths of freeze. It is not felt that the accuracy of the frost depth determinations was sufficient to permit making more than a generalized comparison of frost depths for different textures of soils such as has been done above.

CONCLUSIONS

The study involved the collection of factual data on the soils at nine selected sites, temperature data during the freezing season, and depths of freeze at intervals. The latter were determined by means of auger borings. Utilizing the soil and temperature data, calculations of anticipated frost depths were made by a modified form of the Stefan equation. The following conclusions result from this investigation:

1. The modified Stefan equation is a useful tool for calculating frost depths. The information required for its use includes texture and moisture contents of the soil profile, density determination by tests or estimates, and average daily air temperatures during the freezing season.

2. The use of an air-surface correction factor of 0.8 gave the best agreement between the calculated depths of freeze and the observed depths. Differences in calculated and observed depths averaged about ½ foot when this factor was used.

3. Frost penetration is greatest in those soils with low moisture contents and least in those with high moisture contents. These results are obtained with the theoretical equation and were also found in the field observations.

4. For calculating the freezing index from the average of maximum and minimum 24-hour period temperatures, the time selected for readings is significant. Since midnight-to-midnight periods are commonly used, it is suggested that this be taken as standard. Further study is suggested to determine variations when another hour is selected for readings.

5. This relatively short project has indicated the desirability of other studies. It would be valuable to make measurements during a more-severe winter, with greater resultant

frost depths. The inclusion of more silt and clay soils would be helpful. Further study of the air-surface correction factor is needed. The study could be extended to include depth of thaw, since these calculations can also be made by the Stefan equation. It should also be of interest to make similar investigations in other states or areas with different climatic conditions.

References

1. Carlson, Harry and Kersten, M. S., "Calculation of Depth of Freezing and Thawing Under Pavements," Soil Temperature and Ground Freezing, Highway Research Board, Bulletin No. 71, pp. 81-98 (1953).

2. Carlson, Harry, "Calculation of Depth of Thaw in Frozen Ground," Frost Action in Soils, A Symposium, Highway Research Board Special Report No. 2, pp. 192-223 (1952)

3. Kersten, M.S., Thermal Properties of Soils (1949) Bulletin No. 28, Eng. Exp. Stat Univ. of Minn.

4. Kersten, M.S., "Thermal Properties of Soils," Frost Action in Soils, A Symposium Highway Research Board Special Report No. 2, ppl 161-166 (1952).