Wheel-Load-Stress Computations Related to Flexible Pavement Design

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● IT is recognized that no exact method for calculation of stresses beneath a wheel load exists; however, it is generally conceded that stresses existing below a uniformly loaded circular area are approximately the same as those existing beneath a pneumatic tire of similar contact area and pressure intensity. It has also been conceded by many that A. E. H. Love's solution of Boussinesq's equations of elasticity presents a desirable method of estimating stresses beneath a wheel load.

The Texas triaxial method of flexible pavement design was influenced a great deal by calculations involving the above mentioned theories. Although the Texas method has been developed largely for testing of materials taken from roads of known behavior, the question is often asked about what stress considerations are pertinent to the method in use. In an attempt to answer this question the writer has tried to present some specific calculations, based on certain assumptions, which will afford a comparison between their results and those of the Texas Highway Department method. This report represents only one approach to calculation of wheel load stresses out of many possible ones; therefore, it would not be unusual for the reader to prefer other assumptions and solutions to the problem.

The assumptions used are as follows:

1. The theory of elasticity involving:
   (a) A. E. H. Love's solution for stresses beneath a circular loaded area.
   (b) Warner H. Tuft's computation of influence values for Love's equations.
   (c) That the use of tangential stress omega-omega, $\widehat{ww}$, in a few cases where it is less than $\frac{1}{2}$ ($\widehat{zz} + \widehat{pp}$ - s) would not alter results of calculations for this problem sufficiently to justify their inclusion in this report.
   (d) Reduction of shearing stresses obtained from Love's solutions due to layers of pavements having higher moduli than subgrades. This is accomplished by multiplying Love's maximum shearing stresses by a factor F_S. F_S is the ratio of the maximum shear on the axis of the loaded area from F. H. Scrivner's solution[1] for "layer two" of the two layered system to the maximum shearing stress on the axis of loaded area from Love's solution. Mr. L. E. McCarty of the Texas Highway Department developed F_S as shown in Table 2 and Figure 4-A. It may be noted that F_S is a function of depth Z/a and ratio of moduli E_2/E_1. A set of empirical values of E_2/E_1 based upon experience were selected by the author and are shown in Table 3.
   (e) The addition of effects of surcharge weight equal to 1 psi. per foot of depth upon normal stresses for depths greater than 12 inches.
2. Poisson's Ratio μ = 0.5
3. A unit load of 100 psi. With exception of pavements located in zones where traffic makes extensive use of brakes for stopping, this unit pressure is thought to be high enough so as to eliminate the necessity of considering effects of added stresses due to acceleration or deceleration.
4. Wheel loads of 24,000 and 10,000 pounds were used in computations. Radii of loaded areas were 8.75 and 5.6 inches, respectively.

Calculation data are shown as follows:

1. Figure 1 shows geometrical dimensions used by Love in solution of the stress problem. Notations for Table 1 are also shown on Figure 1.
2. Table 1 lists Warner H. Tuft's computation of influence values for Love's equations.
3. Figures 2, 3, and 4 show the distribution of vertical stresses, radial or horizontal stresses and stress differences under a uniformly loaded circular area. The data

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[1] Some Numerical Solutions of Stresses in Two and Three Layered Systems by F. H. Scrivner, Volume 28, Proceedings of the Highway Research Board.

SOME GEOMETRICAL DIMENSIONS USED BY LOVE
IN THE CALCULATION OF STRESSES BENEATH
A UNIFORMLY LOADED CIRCULAR AREA

NOTATIONS

a = Radius of loaded area
p = Radial distance in cylindrical co-ordinates
z = Depth below surface
p/a = Radial distance expressed in terms of radius
z/a = Depth expressed in terms of radius
p = Applied uniform stress
$\widehat{zz}$ = Stress component acting in a vertical direction upon a horizontal plane
$\widehat{pp}$ = Stress component acting in a radial direction upon a plane perpindicular to the radius
S = Stress difference = $\sigma_I - \sigma_{III}$
μ = Poisson's ratio
σ_I = ½ $(\widehat{pp} + \widehat{zz} + S)$ Major principal stress
σ_{III} = ½ $(\widehat{pp} + \widehat{zz} - S)$ Minor principal stress*

*Tangential stress $\widehat{ww}$ not used. see text

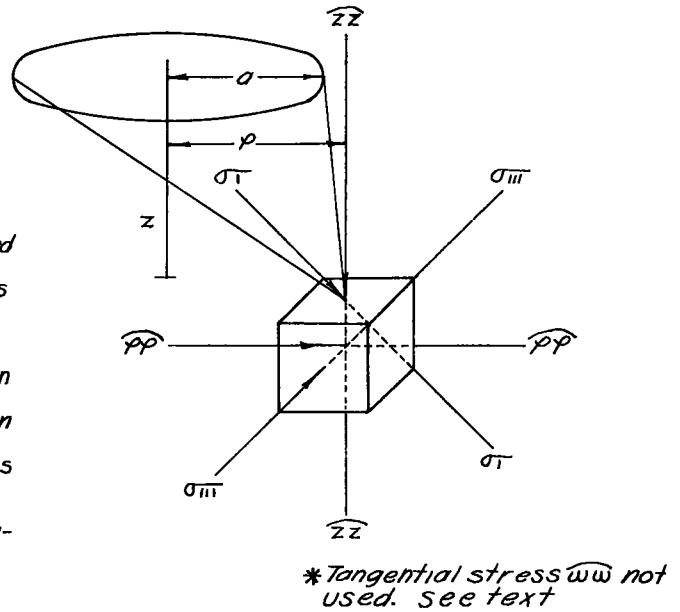Figure 1.

VERTICAL STRESS $\widehat{zz}$ CONTOURS WHEN p=100 P.S.I.
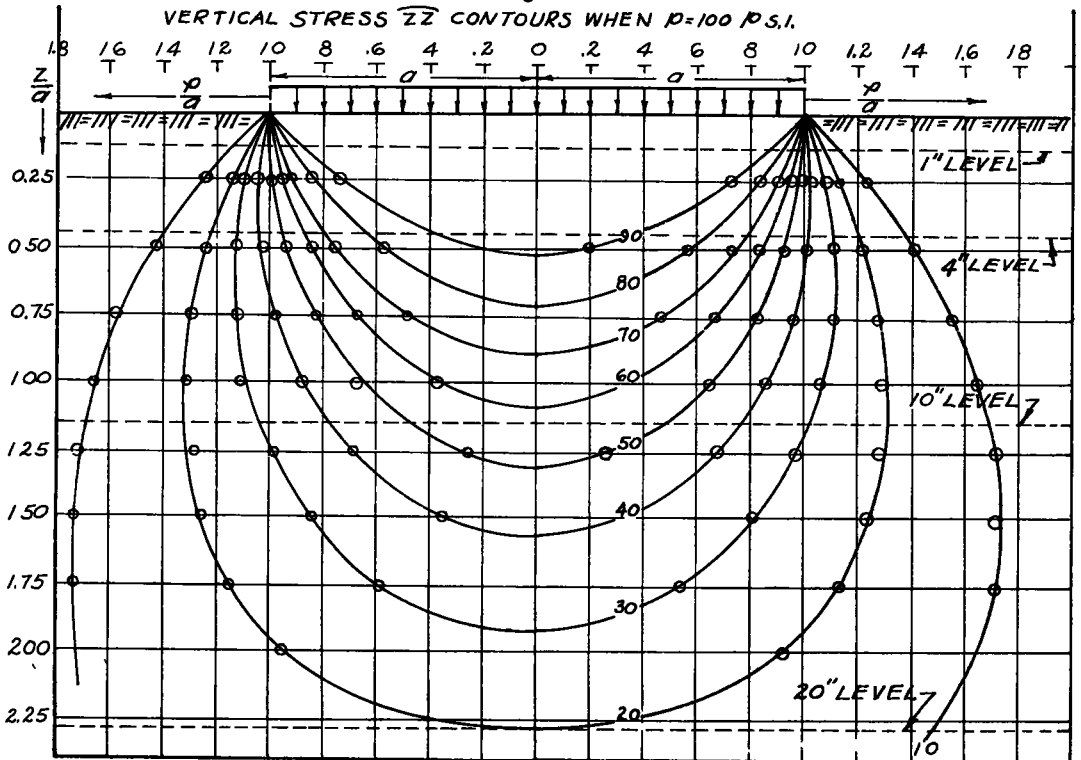
Figure 2.

TABLE 1

WARNER H TUFT'S COMPUTATION OF INFLUENCE VALUES FOR A E.H LOVE'S SOLUTION OF STRESSES EXISTING IN A SEMI-INFINITE ISOTROPIC SOLID (HAVING A POISSON RATIO OF 0.5) SUPPORTING A LOAD APPLIED UNIFORMLY OVER A CIRCULAR AREA

(Taken from 'Public Aids to Transportation Vol. IV")

$\frac{\rho}{a}$	$\frac{\overline{pp}}{\overline{p}}$	$\frac{\overline{zz}}{\overline{p}}$	$\frac{S}{\overline{p}}$
Z/a = 0.25			
0	6433	.9857	.3424
.0670	.6428	.9855	3429
.3131	6177	.9801	.3672
5670	5471	.9542	.4361
75	4580	8838	5322
8834	3970	.7243	.6080
1	.3849	.4596	6023
1.1166	3613	.2086	.4870
1.25	.2812	0737	3322
1.4330	1800	0211	.1961
1.6869	1015	0056	.1061
2 4178	0301	0005	0306
Z/a = 0 5			
0	.3739	.9106	.5367
.1340	3703	.9066	5395
2859	3567	.8913	5504
5	3249	.8396	.5754
.7668	2875	.6775	5968
1	2864	4175	5403
1 1820	2758	.2224	.4318
1 5	1962	.0604	.2448
1.7141	1430	0268	1657
2.0722	0848	0085	0924
2.8660	.0324	.0013	0337
Z/a = 0 75			
0	.2080	7840	5760
.1062	2079	.7803	5753
.3707	1999	.7354	5749
.5670	.1951	.6621	5662
.7990	.1989	.5241	5326
1	2087	.3745	.4722
1.2010	2094	.2353	3889
1 4330	.1886	1241	.2897
1 75	1419	0872	.1698
2.0711	1031	0639	0977
2.6084	.0561	0367	.0418
3.7990	.0196	0130	.0105
Z/a = 1 00			
0	.1161	6464	.5303
1609	1174	6378	.5273
4226	1205	5864	5136
6360	.1287	5103	.4880
9125	.1453	3773	4305
1.1763	.1563	2461	3306
1.4663	.1499	1363	2654
1 8391	1205	0595	1734
2 4281	0732	0173	0893
3 1445	0397	0050	.0444
4 7321	.0131	.0006	0137

$\frac{\rho}{a}$	$\frac{\overline{pp}}{\overline{p}}$	$\frac{\overline{zz}}{\overline{p}}$	$\frac{S}{\overline{p}}$
Z/a = 1 25			
0	0668	.5239	.4571
.1247	0686	.5197	4545
.4171	0752	.4774	.4388
.6651	0878	4100	.4094
.8906	.1017	.3322	.3706
1 1094	.1130	.2534	3245
1.4550	.1179	1479	.2466
1 8753	1027	0696	.1656
2.4897	0691	0233	0909
3.1651	.0422	0080	0499
4.4343	.0181	.0017	0196
Z/a = 1.5			
0	.0399	4240	.3840
.1340	.0417	.4203	.3815
.4540	0499	.3833	.3653
7355	.0637	.3240	.3357
1	0777	2562	2976
1.2645	0879	.1892	2540
1 5460	0914	.1292	2073
1.8660	.0866	0800	1567
2.2587	0736	0435	1143
2 7876	.0545	0197	.0732
3.5981	.0329	.0066	0393
4.2168	.0227	0032	0195
Z/a = 1 75			
0	.0249	.3455	3206
1840	0266	.3406	3179
5311	0361	.3058	3006
8469	0496	.2532	2719
1 1531	.0623	.1947	.2362
1.4689	0704	.1386	.1963
1.8160	0717	0901	1549
2 2254	0652	0523	.1143
2.75	.0522	0259	.0769
3 4993	0351	0102	0449
4 7529	.0180	0027	0206
Z/a = 2.0			
0	.0161	2845	.2863
2721	.0189	.2767	2640
6473	.0287	.2430	2461
1.3527	.0519	.1459	1859
1.7279	.0575	0996	.1502
2 1547	.0565	0612	1143
2.6782	.0488	0329	.0802
3 3835	0361	0145	0502
4.4641	.0215	0047	0261

$\frac{\rho}{a}$	$\frac{\overline{pp}}{\overline{p}}$	$\frac{\overline{zz}}{\overline{p}}$	$\frac{S}{\overline{p}}$
Z/a = 2.5			
0	.0075	.1996	.1921
.3301	.0098	1935	.1885
.7813	.0179	1681	.1742
1 2187	.0280	1327	.1524
1 6699	.0360	0955	.1264
2 1658	.0394	0621	.0988
2.7505	.0374	0357	0718
3 5	.0303	0174	0472
4.5703	.0204	0066	.0271
Z/a = 3.0			
0	0039	.1462	1423
.7375	.0098	.1300	.1327
1.2625	.0175	.1046	.1172
1 8038	.0245	0761	0976
2 3989	0282	.0497	.0762
3 5173	0254	0203	.0452
4.5753	0190	.0088	0276
Z/a = 4.00			
0	.0013	.0869	0856
.2947	.0023	0856	0840
1	.0056	.0761	.0789
1.7053	.0105	0564	0663
2.4559	.0148	.0463	0580
3 3094	0164	0355	.0452
4.3564	.0156	.0258	.0305
Z/a = 5.00			
0	.0006	.0571	0566
1	.0026	.0523	.0534
1.8816	0060	0410	0462
2 8198	0091	.0353	0417
4.5010	.0108	0241	0276
Z/a = 7 00			
0	.0002	.0299	.0297
1	.0010	.0283	0284
1 6124	.0017	.0245	0257
2 8756	.0038	.0236	0258
3.5478	.0047	.0224	0246
4.2641	.0051	.0210	.0230

for plotting of these figures were taken from Table 1 assuming an applied unit load of 100 psi.

4. Figures 5, 6, 7 and 8 show plottings of Mohr diagrams of stresses for the 1, 4, 10 and 20 inch levels, respectively, beneath a circular loaded area having a unit pressure of 100 psi. and a radius 8.75 in. The stress value of $\widehat{zz}$ for vertical, $\widehat{pp}$ for radial, and S/2 for maximum shear which were used in plotting of the Mohr diagrams were interpolated from Figures 2, 3 and 4. These values are tabulated in the upper right part of each Mohr diagram chart.

5. Figure 9 shows the stress envelopes from Figures 5, 6, 7 and 8 superimposed on the Texas triaxial classification chart. The lowest[2] classes of soil materials strong enough to pass above the calculated stress envelopes are shown on left side of chart.

The foregoing method of presenting stresses, i.e., by the use of Figures 2, 3 and 4, is far from the easiest but is used in this instance to present a picture of stress distribution. For those who are accustomed to using tables of influence values and who are

[2] Lowest graphically meaning weakest class.

TABLE 2
SHOWING CALCULATION OF MAXIMUM SHEAR STRESS RATIOS USED IN FIGURE 4-A

Two Layered System

E_2/E_1	.01		.05		0.1		0.3		0.5		1.0		Isotropic Medium
Z/a	S/2	F_s	S/2	F_s	S/2	F_s	S/2	F_s	S/2	F_s	S/2	F_s	S/2
1.00	.0180	.068	.0595	.224	.0935	.353	.1701	.642	.2119	.798	.2650	.999	.2651
1.25	.0131	.057	.0453	.200	.0726	.317	.1394	.610	.1775	.777	.2285	1.000	.2285
2.00	.0062	.046	.0227	.169	.0379	.282	.0768	.573	.1005	.749	.1341	1.000	.1341
2.50	.0042	.044	.0156	.163	.0263	.274	.0541	.563	.0714	.744	.0960	1.000	.0960
5.00	.0011	.039	.0043	.152	.0074	.261	.0155	.548	.0207	.731	.0283	1.000	.0283

not concerned with values for an exact depth will find it easier to plot directly from tables of influence values.

6. Figures 10 through 14 show plotting of Mohr diagram of stresses where values for vertical, radial and maximum shearing stresses were taken directly from Table 1 for depths of 1.4, 2.8, 7, 14 and 28 inch levels beneath a circular loaded area having a unit load of 100 psi. and a radius of 5.6 inches. The depth levels represent Z/a ratios of 0.25, 0.50, 1.25, 2.50, and 5.00 respectively, and influence values for stresses at such depths can be taken directly from Table 1. These data are used for plotting of the
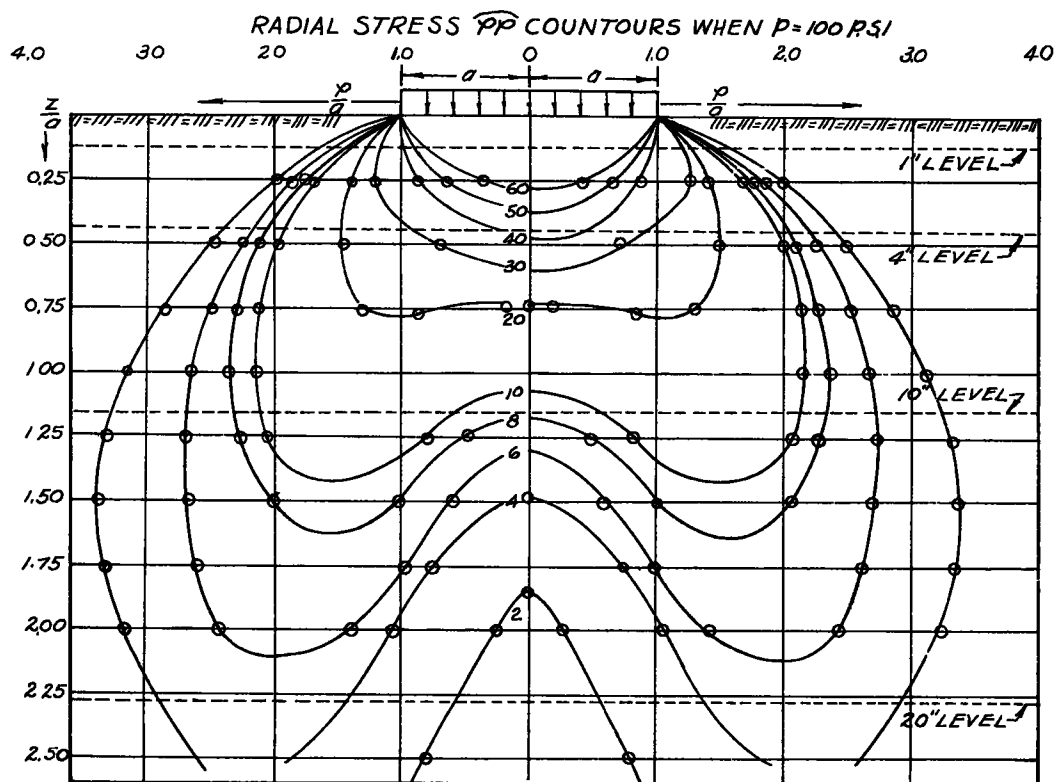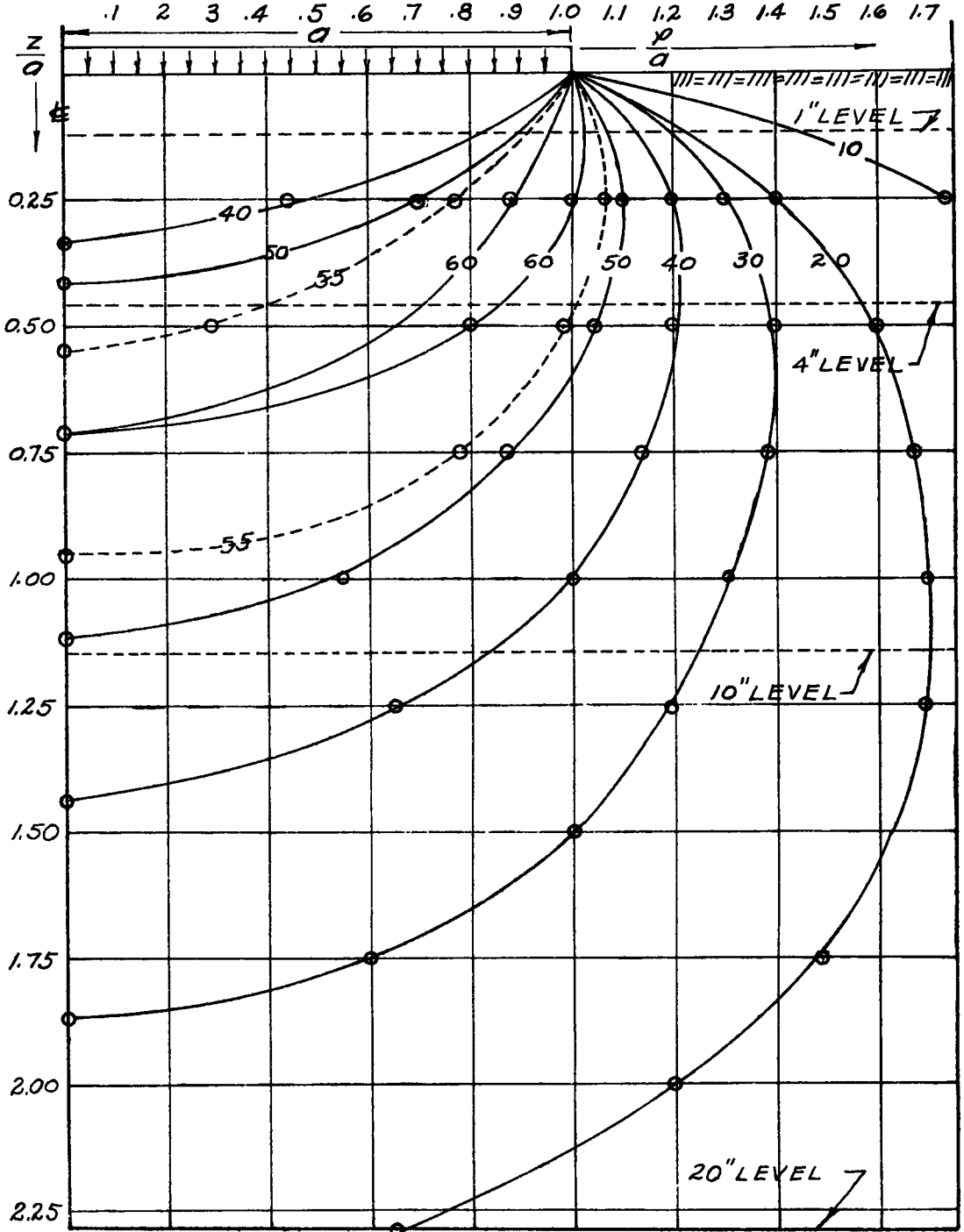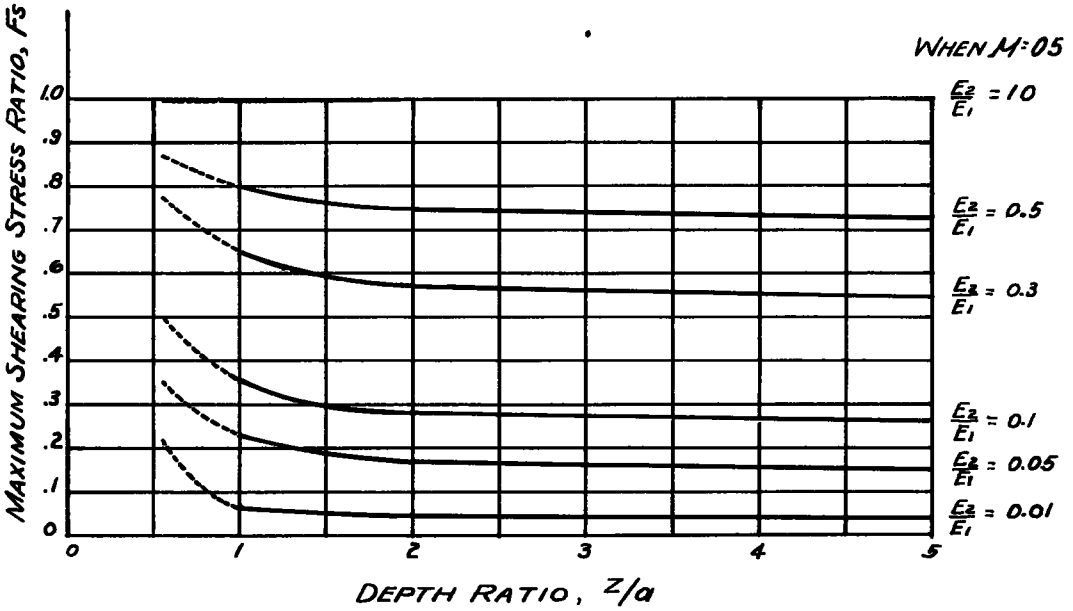

Figure 3.

Figure 4.

Figure 4-A. Shearing stress correction ratio, F_s, for converting Loves influence stresses to those for a two layered system.

ONE INCH LEVEL ρ = 100 psi., μ = 0.5, a = 8 75", F_s = 1 00

$\overline{zz}$	ρ/a	$\overline{\rho\rho}$	Z/a	Center $\overline{zz}/2 + \overline{\rho\rho}/2$	Radius $S/2$
96	℄	85	0.114	90	8
90	.88	50	"	70	25
80	.94	46	"	63	30
70	.96	44	"	57	31
60	.98	42	"	51	31
50	1.00	40	"	45	31
40	1 02	38	"	39	30
30	1.04	38	"	34	30
20	1 06	36	"	28	28
10	1 12	34	"	22	21

Figure 5.

FOUR INCH LEVEL, ρ = 100 psi., μ = 0.5, a = 8.75", F_S = 0.96

$\widehat{zz}$	ρ/a	$\widehat{pp}$	Z/a	Center $\dfrac{\widehat{zz}+\widehat{pp}}{2}$	Radius S/2	S/2x0.96
94	₵L	42	0 457	68	26	25
90	0.36	42	"	66	27.5	26 4
80	0 64	36	"	58	30	27 8
70	0.78	32	"	51	30	29.8
60	0.86	30	"	45	28.5	27 4
50	0.94	29	"	39 5	28	26.9
40	1.02	28	"	34	27	26
30	1.12	26	"	28	25	24
20	1.20	25	"	22.5	21	20.2
10	1.36	22	"	16	16	15 4

Figure 6.

TEN INCH LEVEL, ρ = 100 psi., μ = 0.5, a = 8.75", F_S · ¹.91

$\widehat{zz}$	ρ/a	$\widehat{pp}$	Z/a	Center $\dfrac{\widehat{zz}+\widehat{pp}}{2}$	S/₂	S/2x0.91
57	₵L	9	1.142	33	₂5	22.7
50	0.43	10	"	30	23	20.9
40	0.76	11	"	25.5	21	₁9.1
30	1.00	12	"	21	18	16.4
20	1.28	12	"	16	15	13.6
10	1.68	12	"	11	11	10.0
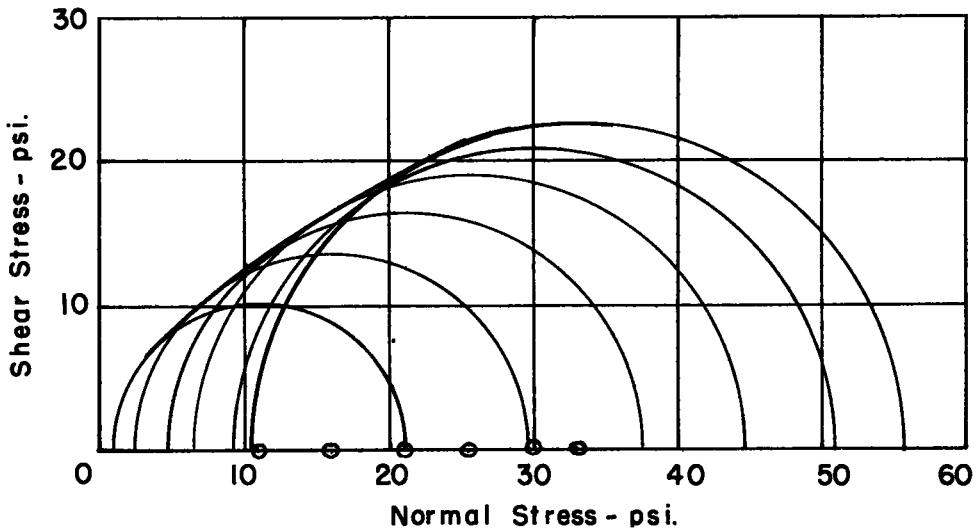
Figure 7.

TWENTY INCH LEVEL, $\rho = 100$ psi., $\mu = 0.5$, $a = 8.75''$, $F_s = 0.82$

$\widehat{zz}$	ρ/a	$\widehat{\rho\rho}$	Z/a	Center $\dfrac{\widehat{zz} + \widehat{\rho\rho} + 0.7*}{2}$	$S/2$	Radius $S/2 \times 0.82$
20	$\underset{L}{C}$	2	2.28	11.7	10.5	8.6
15	1.115	3	"	9.7	8	6.6
10	1.500	4	"	7.7	7	5.8
5	2.350	5	"	5.7	5	4.1

* For all depths greater than 12 inches surcharge weight in psi. =
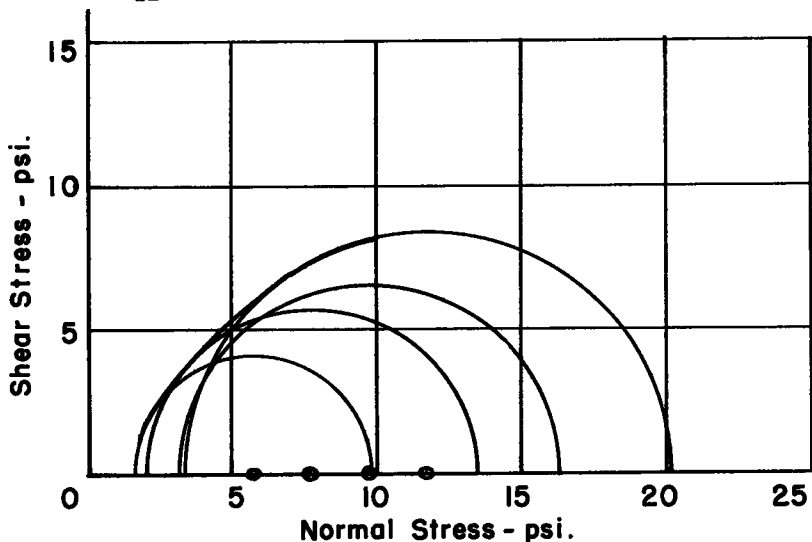$\dfrac{\text{depth level inches - 12 x 1}}{12}$

Figure 8.

TABLE 3

Depth Divided by Radius of Loaded Area Z/a	Approximate Average Value of Moduli[a] E_2/E_1	Shearing Stress Ratio Factor F_s
0.25	1.0	1.00
0.50	0.95	0.96
0.75	0.90	0.95
1.00	0.85	0.93
1.25	0.80	0.90
1.50	0.75	0.89
1.75	0.70	0.87
2.00	0.65	0.84
2.50	0.6	0.81
3.00	0.5	0.76
4.00	0.4	0.67
5.00	0.35	0.58

[a] These are empirical values based upon experience.

Mohr diagrams and are shown in the upper right hand portion of these charts.

7. Figure 15 shows stress envelopes from Figures 10 through 14 superimposed on the Texas classification chart. The lowest* class of soil strong enough to pass above calculated stress envelopes are also shown.

8. Table 4 shows depth levels at which stress computations were made, the weakest triaxial classifications not subject to overstressing, and the depths required for long-life roads built on such soil materials. These depths are taken from the Texas wheel load analysis chart, Figure 16. This chart has been worked out on the following basis:

(a) Depths are known for a given wheel load.

(b) Tire pressures remain constant when wheel loads vary and the magnitude

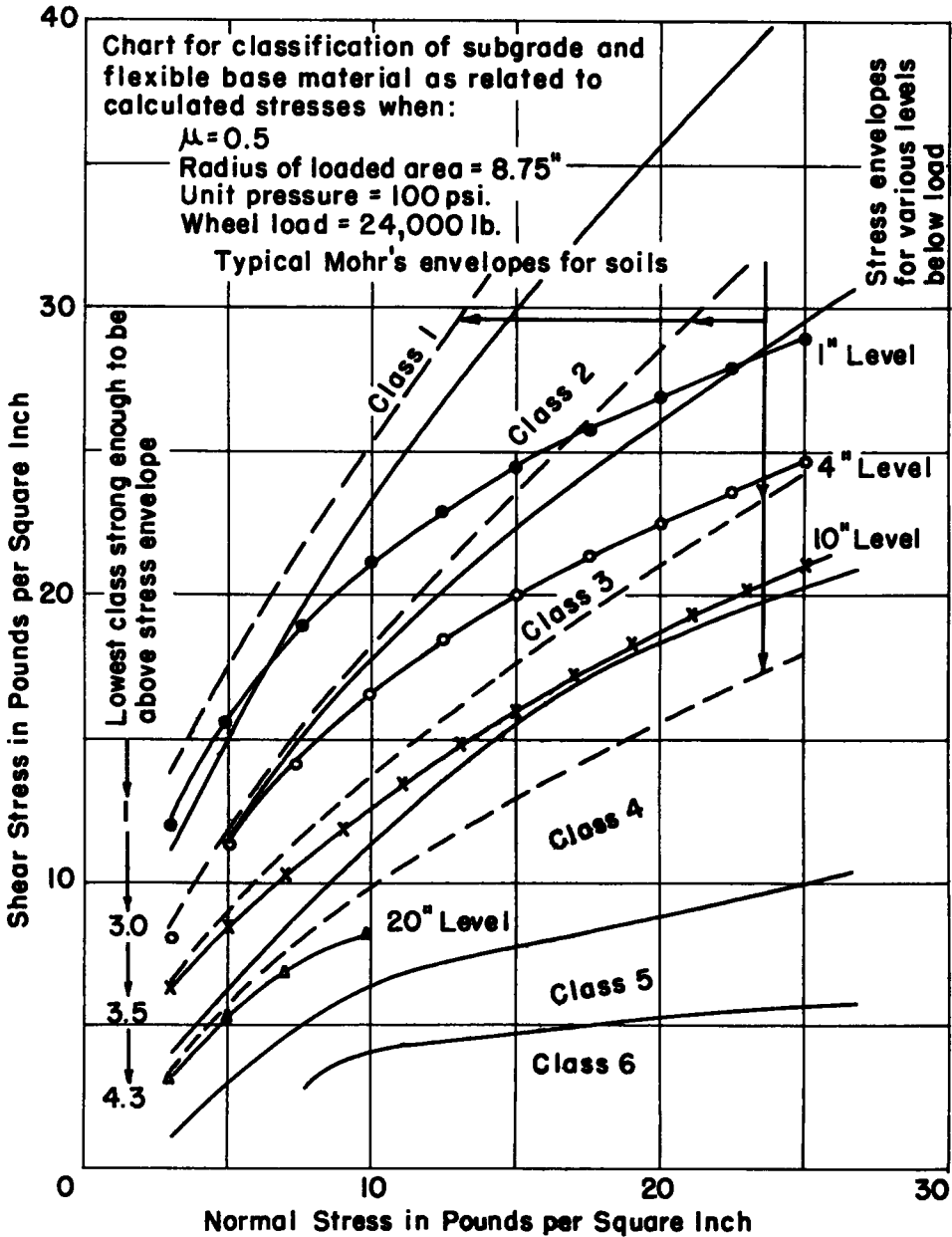*Lowest graphically meaning weakest class.

Figure 9.

of radii of contact areas will change accordingly.

Then: Depth for desired design wheel load = Known depth for given wheel load

$$\sqrt{\frac{\text{wheel load desired for design}}{\text{wheel load for which depths are known}}}$$

A comparison of levels shown in the last and third from last column of Table 4 indicates that the Texas triaxial method requires approximately the same depths as those required by the theoretical concepts presented herewith.

1. 4 INCH LEVEL ρ = 100 psi., μ = 0.5, a = 5. 6", F_s = 1.00

$\widehat{zz}$	ρ/a	$\widehat{\rho\rho}$	Z/a	Center $\dfrac{\widehat{zz}+\widehat{\rho\rho}}{2}$	Radius S/2
98. 6	0	64. 3	0.25	81. 5	17. 1
72. 4	0. 8834	39. 7	"	56. 1	30 4
46. 0	1. 000	38. 5	"	42. 3	30 1
20. 9	1. 1166	36. 1	"	28. 5	24. 4
7. 4	1. 2500	28. 1	"	17. 8	16. 6
2. 1	1 4330	18. 0	"	10. 1	9. 8
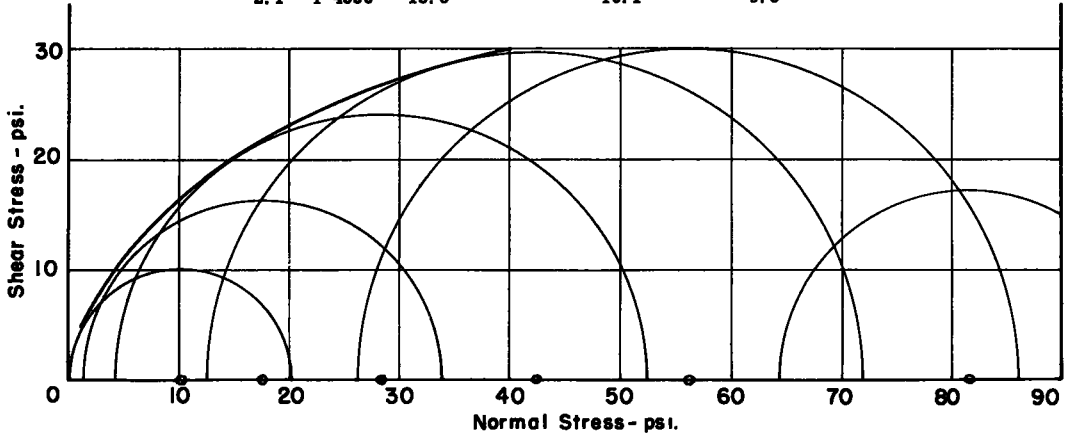
Figure 10.

2. 8 INCH LEVEL ρ = 100 psi., μ = 0.5, a = 5. 6", F_s = 0. 96

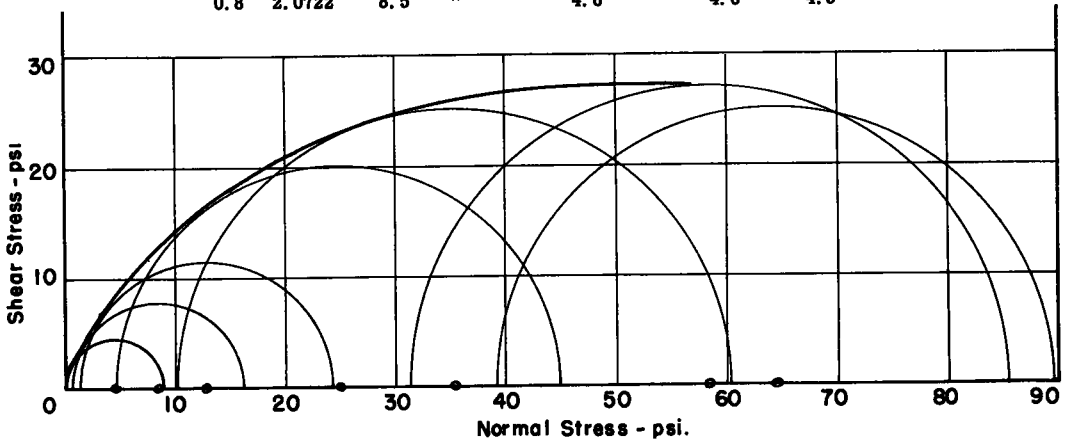| $\widehat{zz}$	ρ/a	$\widehat{\rho\rho}$	Z/a	Center $\dfrac{\widehat{zz}+\widehat{\rho\rho}}{2}$	S/2	Radius S/2 x 0. 96
91. 0	0	37. 4	0. 5	64. 2	26. 9	25. 8
84 0	0. 500	32. 5	"	58. 3	28 8	27. 6
41. 8	1 000	28 6	"	35. 2	27. 0	25. 9
22. 2	1. 1820	27. 6	"	24. 9	21 6	20 7
6. 0	1. 500	19. 6	"	12. 8	12. 3	11. 8
2 7	1. 7141	14. 3	"	8. 5	8. 3	8. 0
0. 8	2. 0722	8. 5	"	4. 6	4. 6	4. 3

Figure 11.

TABLE 4

WHEEL LOAD DATA
UNIT PRESSURE = 100 PSI

Load in Lbs.	Radius (Inches)	Z/a Ratio of Depth to Radius	Depth Level at which Calculations were made (Inches)	Strength Class of Material Required to Prevent Overstress	Depth Required for Long-Life Roads (Inches)
24,000	8.75	0.114	1	1	1
"	"	0.457	4	3.0	5.3
"	"	1.14	10	3.3	9.0
"	"	2.28	20	4.3	20.5
10,000	5.6	0.25	1.4	2.7	2.5
"	"	0.50	2.8	3.3	5.5
"	"	1.25	7.0	3.7	9.0
"	"	2.50	14.0	4.0	11.5
"	"	5.0	28.0	6.4	28.0

7 0 INCH LEVEL, ρ = 100 psi , μ = 0.5, a = 5.6", F_s = 0.90

$\widehat{zz}$	ρ/a	$\widehat{\rho\rho}$	Z/a	Center $\dfrac{\widehat{zz} + \widehat{\rho\rho}}{2}$	S/₂	Radius S/₂ x 0.90
47.8	0.4171	7.5	1.25	27.7	22.0	19.8
41.0	0.6651	8.8	"	24.9	20.5	18.4
25.3	1.1094	11.3	"	18.3	16.2	14.6
14.7	1.4550	11.8	"	13.3	12 3	11 1
7.0	1.8753	10 3	"	8 6	8.3	7 5
2 3	2.4897	6.9	"	4.6	4.5	4.1
0.8	3 1651	4.2	"	2.5	2.5	2 2
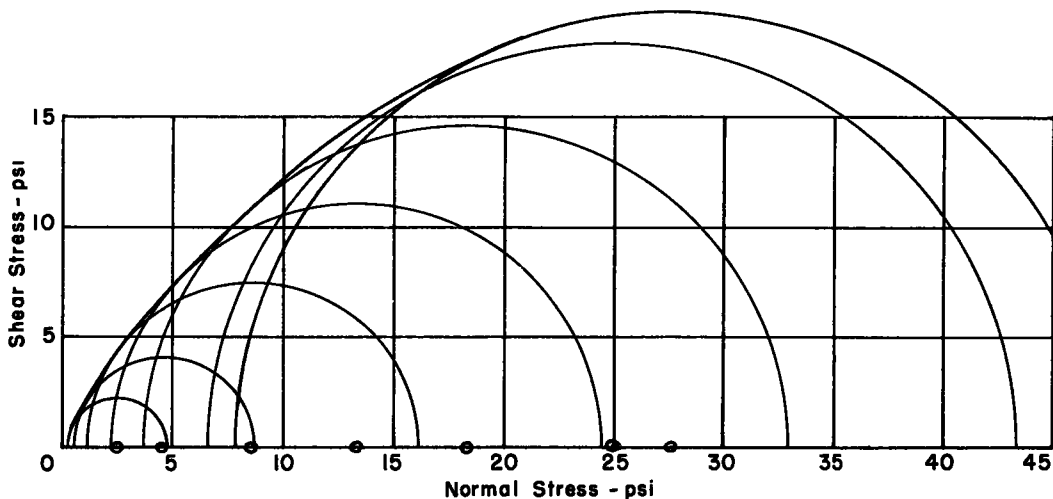
Figure 12.

14 INCH LEVEL, $\rho = 100$ psi., $\mu = 0.5$, $a = 5.6''$, $F_s = 0.81$

$\widehat{zz}$	ρ/a	$\widehat{\rho\rho}$	Z/a	Center $\dfrac{\widehat{zz} + \widehat{\rho\rho}}{2}$	$S/2$	Radius $S/2 \times 0.81$
20	0	0.8	2.5	10.4	9.6	7.8
16.8	0.7813	1.8	"	9.3	8.7	7.0
13.3	1.2187	2.8	"	8.0	7.6	6.2
6.2	2.1658	4.0	"	5.1	5.0	4.1
3.6	2.7505	3.8	"	3.7	3.6	2.9
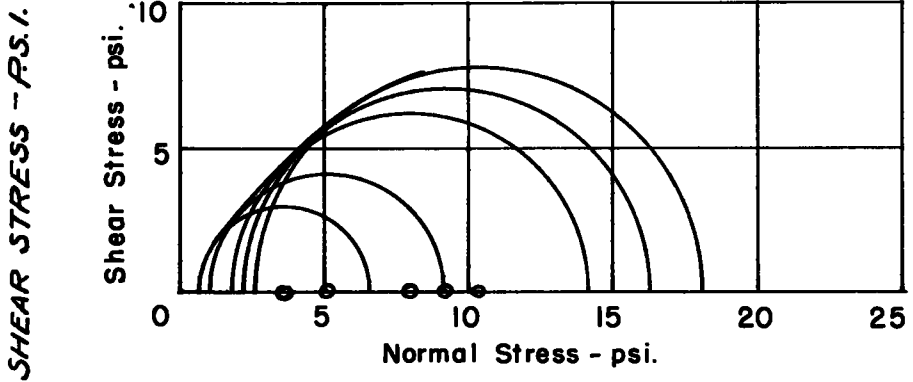
Figure 13.

28 INCH LEVEL, $\rho = 100$ psi., $\mu = 0.5$, $a = 5.6''$, $F_s = 0.58$

$\widehat{zz}$	ρ/a	$\widehat{\rho\rho}$	Z/a	Center $*\dfrac{\widehat{zz} + \widehat{\rho\rho} + 1.3}{2}$	$S/2$	Radius $S/2 \times 0.58$
5.7	0	0.1	5.0	4.2	2.8	1.6
5.2	1.000	0.3	"	4.1	2.7	1.6
4.1	1.8816	0.6	"	3.7	2.3	1.3
3.5	2.8196	0.9	"	3.5	2.1	1.3
2.4	4.5010	1.1	"	3.1	1.8	1.0

*For all depths greater than 12 inches, surcharge weight in psi. = $\dfrac{\text{depth level inches} - 12 \times 1}{12}$
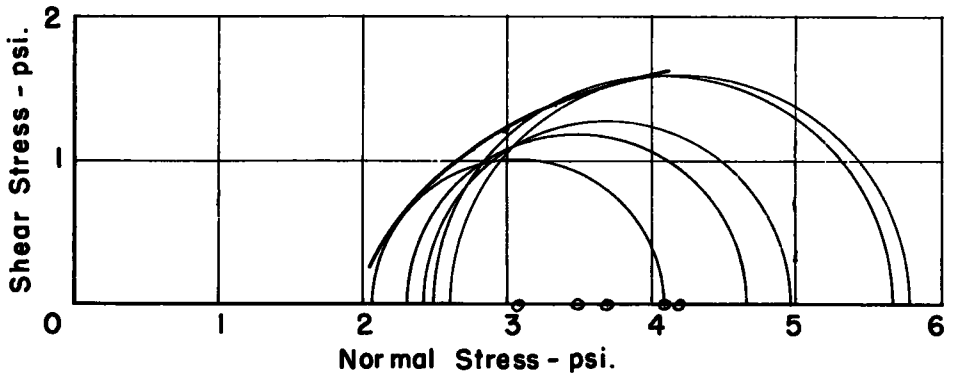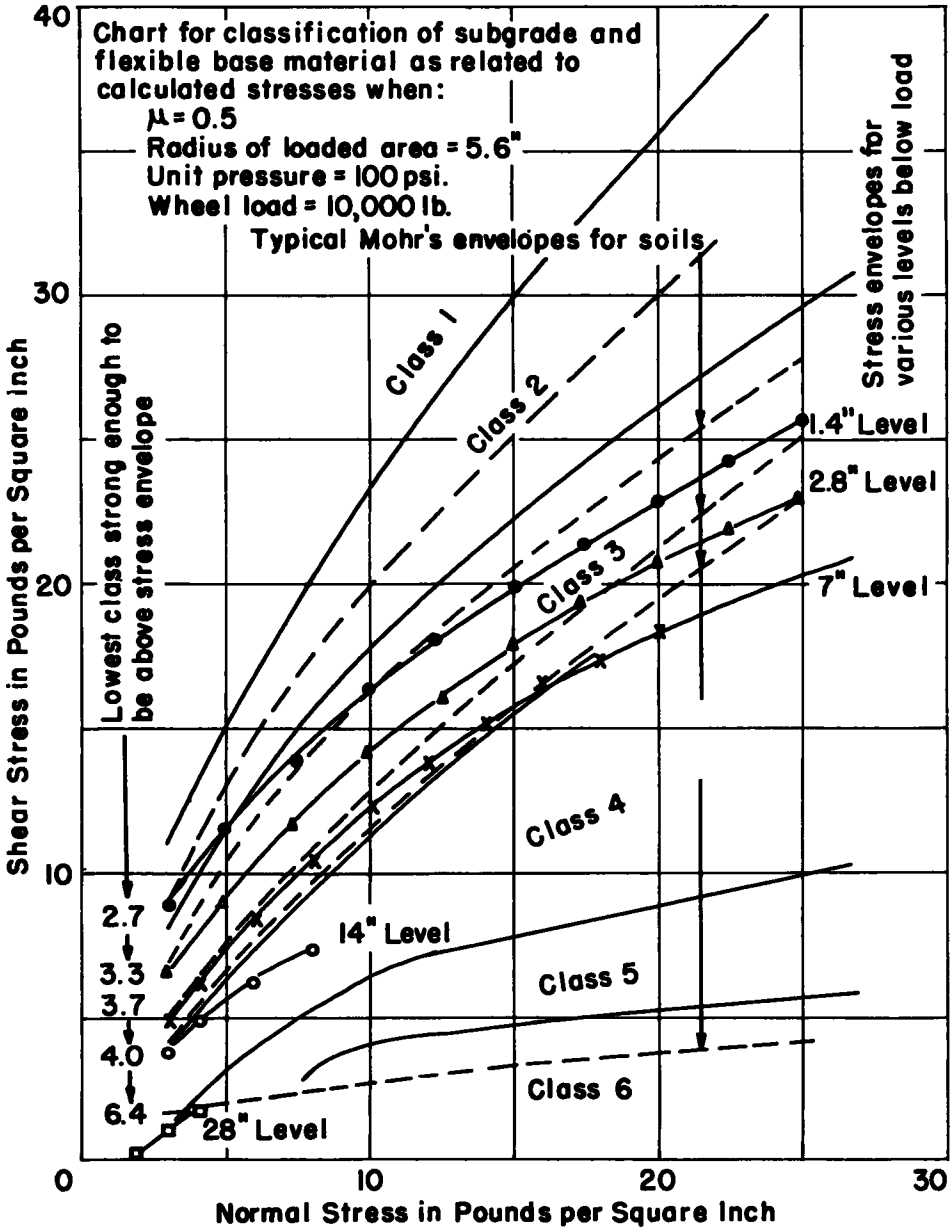

Figure 14.

Chart for classification of subgrade and flexible base material as related to calculated stresses when:
μ = 0.5
Radius of loaded area = 5.6"
Unit pressure = 100 psi.
Wheel load = 10,000 lb.

Typical Mohr's envelopes for soils

Stress envelopes for various levels below load

Class 1
Class 2
Class 3
Class 4
Class 5
Class 6

1.4" Level
2.8" Level
7" Level
14" Level
28" Level

Lowest class strong enough to be above stress envelope

2.7
3.3
3.7
4.0
6.4

Shear Stress in Pounds per Square Inch

Normal Stress in Pounds per Square Inch

Figure 15.

Wheel Load in Thousands of Pounds*
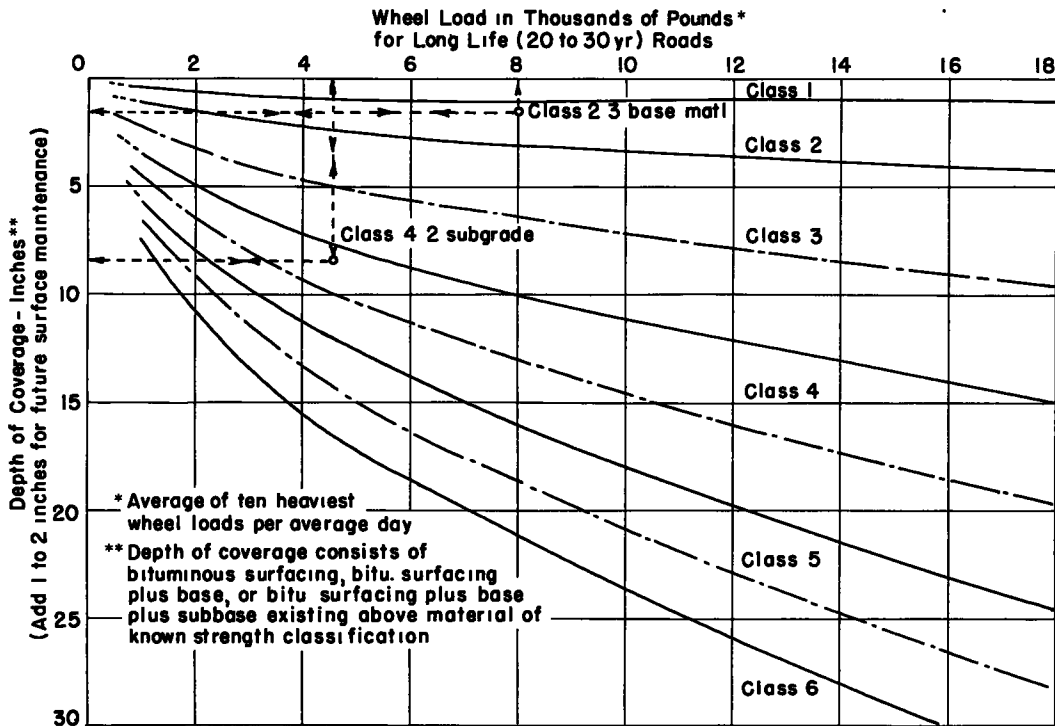for Long Life (20 to 30 yr) Roads

Figure 16.

Discussion

R. G. AHLVIN, Chief, Reports and Special Projects Section, Waterways Experiment Station, Vicksburg, Miss.—McDowell is to be commended for his treatment of this very difficult problem. The step from empirical to rational design methods for flexible pavements is a very large one, and McDowell has, with this paper, narrowed it somewhat.

A few minor comments appear pertinent, after which reference to an apparently remarkable correlation between the Texas Highway Department design method and the Corps of Engineers CBR design method may be of interest.

In Item 1(c) under the assumptions, the tangential stress, $\widehat{ww}$, is recognized as being the minor principal stress in a few cases. However, when Poisson's ratio is taken as 0.5 as it is here, the tangential stress is everywhere the intermediate principal stress. The assumption listed as Item 1(c) is therefore not needed.

It should perhaps be noted here that values of Poisson's ratio other than 0.5 can lead to critical maximum shear stresses (or principal stress differences) up to about 20 percent larger than those computed using the 0.5 value.

With reference to the center column of Table 3, it is not clear why the ratio of moduli, E_2/E_1, varies with depth. It is perhaps also notable that realistic ratios of moduli of two pavement layers, E_2/E_1, could be as low as about 0.2 at fairly small depths.

In Figure 4, principal stress difference contours are presented. These are the same as maximum shear stress contours $\left(\tau_{max} = \dfrac{\sigma_I - \sigma_{III}}{2}\right)$ except that their values are twice as large. In our stress-distribution work at the Waterways Experiment Station we have had occasion to develop very careful and, we believe, quite accurate maximum shear stress contours. A copy of these for a Poisson's ratio of 0.5 is shown as Figure A. The contours presented are only slightly different than those used by McDowell but they include smaller intervals between contours in the critical zone.

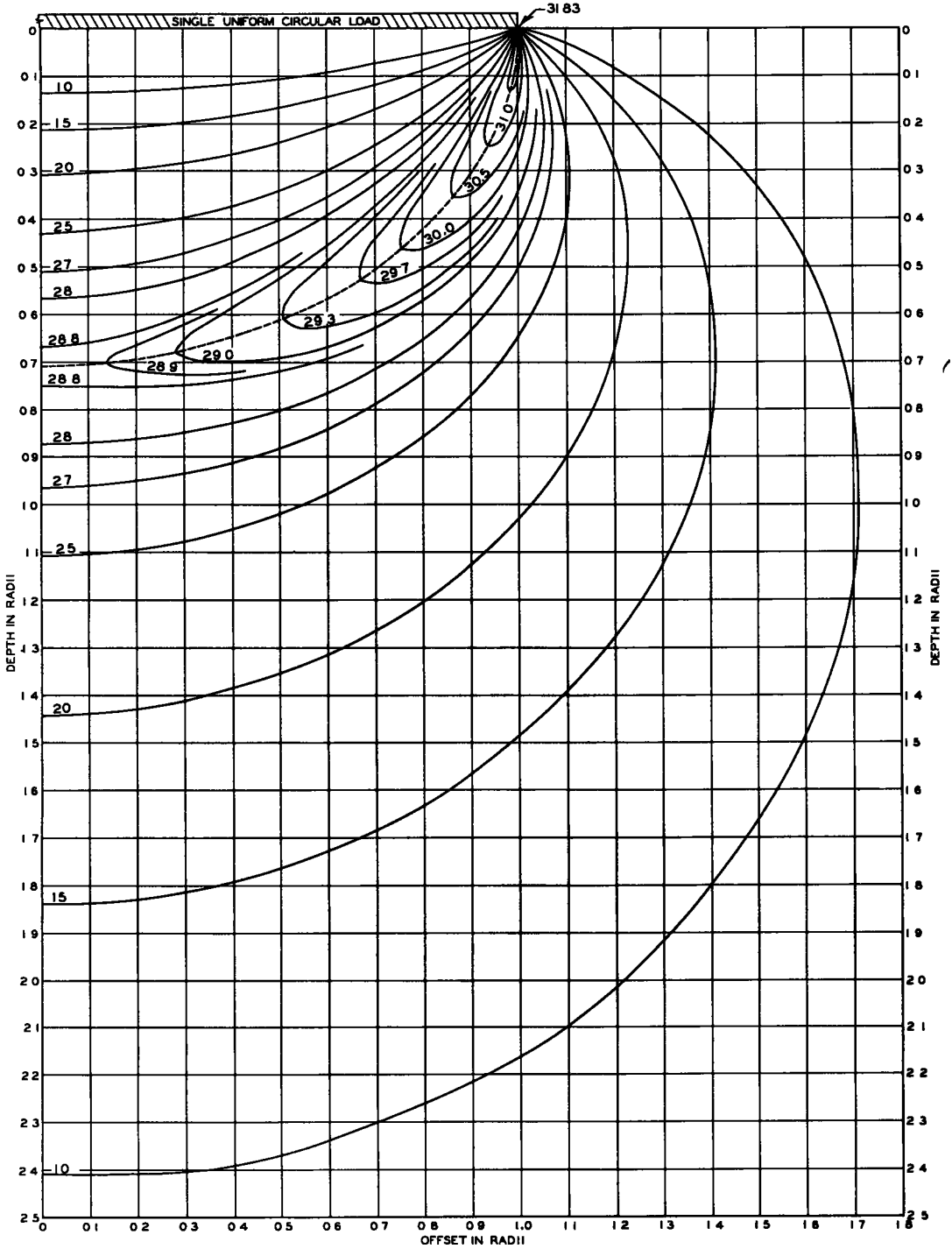The relations expressed by the Texas Highway Department Flexible Pavement

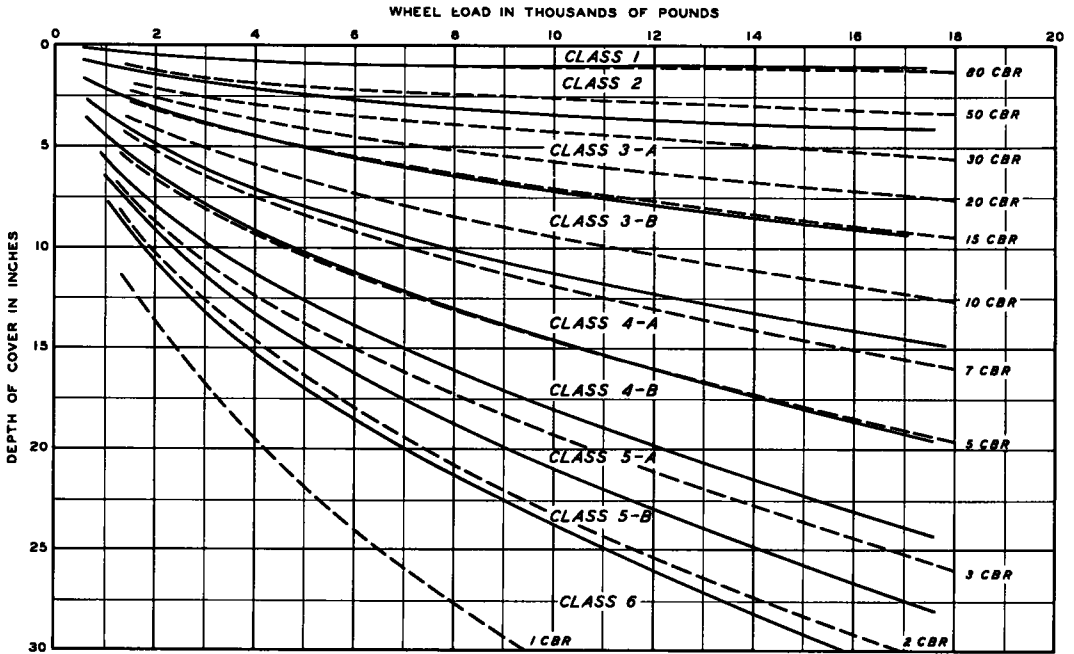Figure A.

WHEEL LOAD IN THOUSANDS OF POUNDS

Figure B.

Analysis Chart appear to be similar in form to those of the Corps of Engineers CBR curves. Figure B shows a plot of both sets of curves for direct comparison. As can be readily seen, a remarkable parallelism exists between curves indicating a near linear relation between the Texas Highway Department class and CBR. The following table shows this relation. I have taken the liberty of adding A and B to classes 3, 4, and 5, which are divided into two parts in Figure 16 of McDowell's paper.

Texas Highway Department Class	Corps of Engineers CBR Range [a]
1	80 and above
2	38 – 80
3-A	15 – 38
3-B	7.7 – 15
4-A	5.0 – 7.7
4-B	3.4 – 5.0
5-A	2.5 – 3.4
5-B	1.9 – 2.5
6	1.9 and below

[a] The CBR relations used were those for single wheels having 100-psi. tire pressure and for capacity operation (5,000 or more coverages of aircraft traffic).

Discussion

STUART M. FERGUS, Asphalt Research Laboratory, Standard Oil Company, Cleveland Ohio—McDowell presents a method for computing subgrade stresses and comparing them with those obtained in laboratory tests of triaxial specimens. The method is developed mainly from the theory of elasticity and involves the development of a Mohr envelope for each particular wheel load. The theoretical values of the normal and shearing stresses used in plotting each Mohr diagram are obtained first by assuming that the subgrade is a semi-infinite, homogeneous, isotropic, elastic body in the usual sense and then reducing the stresses by a factor F_s. This factor is the ratio of the

maximum shearing stress in a two-layered body to that in a homogeneous body taken at a point beneath the center of the loaded circular area.

At any given depth the value of F_S depends on the ratio (E_2/E_1) of the moduli of elasticity of the two layers. The Mohr envelopes developed in this way are then plotted on the Texas Triaxial Classification Chart which presumably consists of Mohr envelopes developed from laboratory triaxial tests of soils of the several strength classifications used by the Texas Highway Department. The final step in McDowell's presentation is the relating of the wheel load stress computations to the soil strengths and the introduction of a table (Table 4) showing the depths for which the theoretical computations were made (4th column) are approximately equal to the combined thickness of pavement and base course (6th column) required by the Texas Design Curves. Computations are presented for wheel loads of 10,000 and 24,000 lb. with a surface contact pressure of 100 psi.

In a paper of this kind, dealing as it does with terms and notations with which the average engineer-reader may not be too familiar, the writer is always torn between the one extreme of explaining too little and the other of explaining what may be obvious. In the end, if the paper is to serve adequately its purpose of exposition, it is probably best to err on the side of too much rather than too little detail. With this in mind, it is suggested that the paper be amplified as follows:

1. An explanation of the source of the Mohr envelopes for the several soil strength classes of the Texas Classification Chart.

2. Some discussion of the laboratory triaxial test as presently used by the Texas Highway Department together with mention of any procedures or techniques which may be unusual.

3. Identification in some way of the terms such as "weakest class, strong enough, overstressing, etc." used in describing the strength characteristics of the subgrade, with the location, direction, and magnitude of the stresses or stress envelopes considered to be critical.

4. For most American engineers who plot the Mohr stress circle, Love's notation will be unfamiliar. A brief statement in the following form might be helpful.

$$\frac{\widehat{zz} + \widehat{\epsilon\epsilon}}{2} = \frac{\sigma z + \sigma \epsilon}{} = \frac{\sigma 1 + \sigma 3}{}$$

$$\frac{S}{2} = \frac{\sigma 1 - \sigma 3}{2}$$

With the exception of the factor F_S, the assumption set forth in paragraphs 1 (a) to 1 (e) pertaining to the theory of elasticity are those usually made in treating the suggrade as an elastic body. It might be argued that the ratio F_S should not be applied to stresses at points away from the z-axis since a stronger top layer does not reduce all stresses uniformly. However, this seems a minor point. In this connection it might be remarked that a uniform reduction in all stresses would result if the surface contact pressure were reduced but that this would be possible only if the total load were reduced. With respect to paragraph 1 (c), a recent report[1] has shown that for the special case $\mu = 0.5$, $\widehat{ww}$ is the intermediate principal stress at all points beneath a uniform circular load.

The bases given in paragraph 8 for the development of the Texas Wheel Load Analysis Chart are also those generally accepted by pavement engineers. The proportionality of the design thickness to the square root of the wheel load is inherent also in the California Bearing Ratio method of design. It is of interest to point out in this connection that the design thicknesses given for the various wheel loads by the Texas Flexible Pavement Analysis Chart are practically identical to those required by the CBR method if the following relations are assumed:

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[1] See, "Theoretical Stresses Induced by Uniform Circular Loads," TM-3 323, September, 1953. U.S. Waterways Experiment Station, Vicksburg, Mississippi.

Texas Highway Dept. Class	Calif. Bearing Ratio percent
1	100
2	80
3	40
4	8
5	3. 5
6	2

A wide difference of opinion, however, can be expected on the question of the relations assumed to exist between the pattern of stress distribution in a subgrade and that in a laboratory triaxial specimen. To give one example: in the ordinary triaxial tests the lateral stress is usually held constant while the vertical stress is increased, but in a real subgrade both the lateral and the vertical stresses vary constantly between a small initial value and a maximum as the wheel load is moved on and off. It may also be pointed out that the modulus of elasticity (some writers prefer stress-strain ratio) is not a constant for soils but varies inversely with the stress. It would, therefore, seem likely that the ratio E_2/E_1 is also not a constant. One result of this variability is that computations of the strain values based on summing up or on superposition of stresses are not valid. In other words, the strain produced by a stress of 50 psi. is more than 10 times that produced by a stress of 5 psi.

All of these remarks may quite correctly be considered as quibbles which do not help to produce a satisfactory theory of flexible pavement design. McDowell's paper at the very least has merit in that it is an effort toward that goal. This reviewer, however, feels it necessary to point out that while all the present methods of design make use in one way or another of the theory of elasticity, no method has thus far been submitted which is entirely free from empiricism.

CHESTER McDOWELL, Closure—We are indebted to Ahlvin for his pertinent comments. He states that assumption item 1(c) is not needed and for all practical purposes Ahlvin is correct. However, since it is not obvious from Love's equations that 1(c) is not needed and since Tuft's tables of influence values indicate tensile values in one or two cases, I am not sure that the assumption can be deleted. Evidently, Ahlvin has proof of this point, and deletion of the assumption probably is in order. Ahlvin certainly is justified in questioning E_2/E_1 ratios shown by center column in Table 3 of the report. I can see that I failed to offer any explanation of why the values were selected. There certainly are many E_2/E_1 ratios other than those shown in Table 3. They may vary from 0. 03 for soil-cement bases on subgrade soil to values far above one, the latter being cases where flexible bases are placed in rock cuts or over old concrete pavements. In these cases depth problems are not usually highly significant; but the selection of high quality base courses that are capable of resisting reflected stresses is important.

In cases where smaller E_2/E_1 ratios exist at shallower depths than those shown, it seems that shear stresses for similar depths and loadings should be less than those we would obtain by use of the values selected for use. For instance, shear stresses in subgrade under soil-cement or concrete are generally accepted to be lower than those in subgrade under flexible base.

It is believed that troubles arise when low E_2/E_1 ratios are applied because of fatigue of "slab effects". In cases where "slab effects" are attained and deflections are so low that fatigue is unlikely, some reduction of overall depths are indicated due to existence of low E_2/E_1 ratios, but we are not sure how this can be accounted for properly. Therefore, it should be stated that the E_2/E_1 ratios shown in Table 3 are preferred because shear stresses obtained by their use will be higher than those obtained by employment of lower ratios at similar depths. The decrease in ratio with depth is in the order of what we consider typical sections which utilize soil layers economically. See Table A for an example.

It is believed that the values in the table are on the safe side because shear stress

calculations obtained by their use will be higher than if lower E_2/E_1 ratios are employed at similar depths.

<div align="center">TABLE A</div>

Type of Material	Depth in Terms of Z/a	Average of E_2/E_1 Ratio	Shearing Stress Factor F_S
Base $E = 10,000$	0 to 1	8500/10,000 = 0.85	0.93 at Z/a = 1
Good Subbase $E = 8,500$	1 to 2	6000/9250 = 0.65	0.84 at Z/a = 2
Fair Subbase $E = 6,000$	2 to 3	3800/7625 = 0.50	0.76 at Z/a = 3
Select Soil $E = 3,800$	3 to 5	2000/5713 = 0.35	0.58 at Z/a = 5
Subgrade $E = 2000$			

It is interesting to note the correlation Ahlvin shows between CBR values and our strength classifications. Although I have never attempted or seen such a correlation, it is not surprising. It should be pointed out that data for the relations shown by Ahlvin were taken from similar theories of load distribution, but that actual laboratory test results do not correlate so well. For instance, many of our strength Class 3 materials would have CBR values of 100 plus and this is one of the main reasons that we never did adopt the CBR method. We are just as interested in knowing what to build our bases out of as we are to know how thick they should be. So far as we are concerned, mold restraint is too critical in case of the CBR test when used to test base materials.

Fergus's comments certainly are of assistance in amplifying and explaining parts of the report which are not clear to the reader. He suggests that four items be amplified. It should be explained that during the author's presentation of a paper (see report in HRB Research Report 16B) covering Items 1, 2 and 3 that members of the HRB Flexible Pavement Committee requested that the report be amplified by submitting comments on the type of stress analysis considered pertinent to the Texas triaxial method of flexible pavement design. It is not surprising that this report within itself appears to be rather incomplete; however, if read in conjunction with reports given in Research Report 16-B, Bulletin 93 and Volume 26 of the Proceedings of the Highway Research Board, it will be much clearer to the reader.

Fergus correctly points out that the basis for development of the Texas Wheel Load Chart is the same as that used by the CBR method, but it is not safe to assume that his tabulated correlation between CBR and strength class exists when actual tests are made. In fact some class 3 materials containing aggregate have CBR values of 100 plus. Strength class 3 materials containing small amounts to no aggregate, such as fine sand-clays, may have CBR values as low as 15. Although certain theoretical aspects of the problem correlate, it does not necessarily follow that laboratory test results show any such correlation between CBR and Texas strength classes.

It is gratifying to note that Fergus is in agreement with Ahlvin relative to the unimportance of assumption 1(c).

Fergus points out the difficulties encountered in comparing modulus of elasticity from triaxial tests to that which develops under wheel loads. It is admitted in this instance that we are dealing with very elusive or variable sets of values and we do not contend that the two sets of modulae are necessarily similar. Although such E values may be dissimilar, it does not follow that the E_2/E_1 ratios derived from triaxial tests

are necessarily different from those produced by wheel loads because in all cases each E value used has been obtained by the same procedure.

It is admitted that the Texas Triaxial Method of Flexible Pavement Design has many empirical portions in it without which the method would not function. The parts covering testing technique of the Texas method are perhaps of more value than the theoretical parts such as are being discussed here. It is doubtful if any process or method has ever been developed from theory to practical application without some empirical steps being taken. In order to utilize methods of pavement design, it seems desirable that they contain well balanced portions of theoretical and empirical steps. Considerable differences of opinion will arise as to how these portions shall be balanced.

We are indebted to Ahlvin and Fergus for their constructive and informative discussions. They will be of assistance as developments are made in our pavement design methods.