Dynamic Load Analysis and Design of Highway Bridges

CHARLES F. SCHEFFEY, Assistant Professor of Civil Engineering
University of California

Live loads on highway bridges produce three types of dynamic effects of interest to the designer: (1) those due to the speed at which the load rolls smoothly across the span, (2) those due to shock effects of deck irregularities or obstructions, and (3) those involving a resonance effect due to repetition of load at or near the natural frequency of the structure. This paper first discusses a rational approach to the determination of the effects of the smoothly rolling loads. The approach is to solve the general nonlinear partial differential equation which results from consideration of the essential factors involved by a step-by-step process, in which it is reduced to a linear ordinary equation with constant coefficient for short intervals of time. The constant coefficients for this equation are obtained by a quantitative analysis of the manner in which the coefficients of the basic equation vary. The application of the analysis to structures with simple and continuous spans is discussed, and the results are compared to field tests. A brief analysis of shock loads and of resonance effects produced by certain axle spacings, spacing of vehicles, and deflections in the floor system is given. The paper concludes with a discussion of this analysis method to the design problem and suggests the lines along which the author believes specifications' clauses whould be developed to permit more-rational design for dynamic effects.

● THEORETICAL methods for investigation of the effects of dynamic loadings on highway bridges have two important objectives: (1) the explanation or prediction of these effects on existing structures and (2) the formulation of proper allowances for these effects on structures being designed. Much excellent work has been and is being done on the first of these objectives, which perhaps we should call the fundamental investigation. Great refinement in the analysis is necessary in order that the investigator may be confident that he has the proper explanation for the observed behavior in all of its detail. However, it is the opinion of the author that the state of knowledge of this phase has now progressed far enough that a cautious approach to the second objective can be made by the use of some approximate methods of dynamic load analysis.

This paper will analyse the influence of various dynamic effects of live load on typical modern highway bridges and their significance from a standpoint of design. The effect of smoothly rolling loads will be considered first, then the effects of vehicle springing, impact of wheels passing over obstructions, and dynamic effects induced by varying stiffness of the floor system. The relative importance of each of these effects for various spans of structure will be developed, and then the significance of the results in terms of design will be discussed.

ANALYSIS FOR SMOOTHLY ROLLING LOADS WITHOUT SPRINGING

The fundamental problem which must be solved in any dynamic load analysis is the solution of the differential equation of motion for a rolling mass moving at uniform velocity across a structure which in its simplest form consists of a simply supported beam of uniform mass and moment of inertia (Figure 1). The differential equation in which the effect of the mass of the load as well as the mass of the beam and the damping of the structure is included (but in which only the first mode of vibration of the structure is considered) is:

$$EI \frac{\partial^4 y}{\partial x^4} + 4\pi n_b m \frac{\partial y}{\partial t} + M \frac{\partial^2 y}{\partial t^2} = \frac{2}{\ell} \left[Mg - M\alpha \right] \sin 2\pi nt \sin \frac{\pi x}{\ell}$$

In which the notation, borrowed from C. E. Inglis (1) has the following meaning.

EI = Elastic constant of the beam
m = mass per unit length
$4\pi n_b m$ = damping constant, in which n_b has the dimensions of a frequency
g = acceleration of gravity
α = vertical acceleration of the load due to movement of the beam, taken positive when downwards

$$n = \frac{v}{2L} = \frac{\text{speed of vehicle}}{2 \times \text{length of beam}}$$

By assuming a solution of the form:

$$y = f(t) \sin \frac{\pi x}{L}$$

We can, by substitution of an appropriate expression for α as a function of f(t), derive an ordinary differential equation of the form:

$$\frac{d^2 f(t)}{dt^2} + 2\,\Omega \frac{df(t)}{dt} + 4\,\pi^2 N_o^2\, f(t) = \frac{Mg}{\beta} \sin 2\,\pi\, nt$$

The coefficients of this equation are not constants but are a function of the position of the load on the span, and consequently a function of time, as follows:

$$\Omega = \frac{2\,\pi\,(n_b + n\frac{M}{M_G} \sin 4\,\pi\, nt)}{(1 + 2\frac{M}{M_G} \sin^2 2\,\pi\, nt)}$$ The "equivalent damping"

$$N_o = n_o \sqrt{\frac{M_G}{M_G + 2M \sin^2 2\pi\, nt}}$$ The "equivalent natural frequency"

where n_o = unloaded natural frequency

and $\beta = 1 + 2\,\frac{M}{M_G} \sin^2 2\,\pi\, nt$ The "equivalent" total mass factor

An exact solution of this equation can be obtained by the use of infinite series, but fortunately there is no need to do this for the purposes at hand.

Let us consider the actual amount by which these coefficients vary for the range of modern highway bridges. To to this we need to have some idea of the typical dynamic parameters of these structures. Figure 2 shows the typical variation of the weight per foot, the centerline moment of inertia, and the dead load to live load ratio for highway bridges for spans from 20 to 300 feet. These curves were developed by preliminary designs for two lane structures with H20-S16 loadings, according to usual AASHO specifications. The effective moment of inertia was based on an allowance for composite action of roadway slab where appropriate (2). The corresponding parameters for railway bridges taken from Inglis's work are also shown to point out the fact that, although the investigations of the dynamics of railway bridges are fruitful sources of insight into the highway-bridge problem, they must be carefully interpreted due to the great difference in the dead load to live load ratio.

The resulting natural unloaded and loaded frequencies are shown in Figure 3, which also shows an estimated damping factor. The frequencies shown by these curves are merely indicative of average values for each type of bridge. However, it will be noted that the abrupt discontinuities in the curves at which a change in type occurs do not cause changes of frequency of more than about 20 percent. Of further significance is the fact that the loaded frequencies do not differ appreciably from the unloaded values.

The estimated damping factor ρ , the ratio of successive residual deflections, is quite frankly an educated guess, since little data is available on this problem. It was arrived at by a study of the variation of damping in railway bridges reported by Inglis, taking particular note of the fact that much of the high damping for short span railway bridges was found to result from the friction of the track on the ballast, a factor completely lacking in highway bridges. A few spot checks were made on this curve with

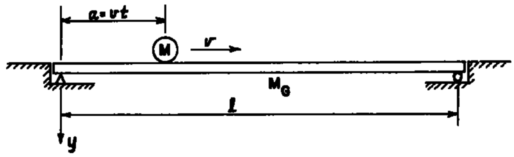

Figure 1. Smoothly rolling mass on a beam of uniform cross-section.

results which indicate it is of the correct order of magnitude.

Returning now to our equation of motion for the case of the smoothly rolling load, we will use these estimated dynamic characteristics to evaluate the range of variation of the equivalent damping Ω, the equivalent natural frequency N_0, and the equivalent mass factor, β.

Figure 4 indicates the results of studies of a 20-foot span, such a short span being most sensitive to these variations. It will be noted that the effect of the second term of the equivalent damping factor is to increase the damping as the load moves toward the center of the span, absorbing energy by participating in larger oscillations, and then decreases the damping as it leaves the span by feeding energy back into the structure. It will also be noted that the effect on the equivalent natural frequency for this span is quite large, reducing it by some 30 percent when the load is at the middle of the span.

These coefficients do not vary over nearly so wide a range when the span length increases, as is shown by Figure 5. The equivalent damping not only decreases in absolute value but also the range of variation decreases rapidly up to spans of about 100 feet, after which both the absolute value and range are quite small. The variation curve

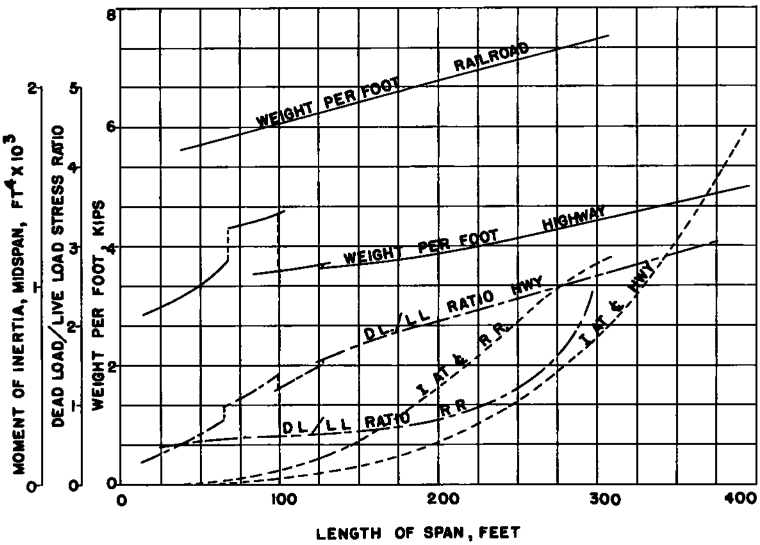

Figure 2. Design characteristics of steel highway and railway bridges.

for the equivalent natural frequency also reaches a rather stable value for spans above 100 feet. The discontinuities in this curve are due to taking successively larger live loads to represent the rolling mass M as the span increases.

Having established that the variation of the coefficients is important primarily for spans under 100 feet, we may now obtain an approximate solution for our equation for even a quite short span by considering the coefficients as constants for very short intervals of time.

Under these conditions, the solution of our equation within any such time interval is like the familiar solution for a one degree of freedom system with damping.

$$y = \left[e^{-\Omega t}(A \sin 2\pi N_0 t + B \cos 2\pi N_0 t) + C \cos 2\pi nt + D \sin 2\pi nt\right] \sin \frac{\pi x}{L}$$

Which may be rewritten, after ignoring a slight phase shift and modification of one term by damping, which can be shown to be negligible for even high speeds and short spans, and by introducing proper boundary conditions, as:

$$y = \delta_{St.}\left[\frac{1}{1-(\frac{n}{n_o})^2}\right]\left[\sin 2\pi nt - e^{-\Omega t}(\frac{n}{n_o}) \sin 2\pi N_o t\right] \sin \frac{\pi x}{L}$$

where $\delta_{St.}$ = Static deflection at a given position for the same position of the load.

The physical interpretation of this expression can be best studied by a reference to Figure 6, which depicts the results of applying this analysis to a 20-foot span with a single axle moving at 60 mph. In this figure we have plotted the centerline deflection against time. The shape of this curve for a slowly moving "crawl" load is shown by the dotted line.

The effect of the first term of the expression above is to produce a slight increase in the amplitude of the maximum deflection, in the amount of $\frac{1}{1-(\frac{n}{n_o})^2}$ of the static deflection, resulting in the dashed line curve. The effect of the second term is to add a free oscillation to the system to meet the starting condition of zero velocity, producing the solid line curve. The frequency and amplitude of this oscillation vary in accordance with the variations of Ω and N_o discussed above. The results in this case were computed by correction of these values every one half cycle, making ten steps of calculations to trace the behavior all the way across the span. These same oscillations are isolated in the lower part of the figure to show the effects of these variations.

Some conclusions as to the importance of smoothly rolling load effects can be drawn by a study of this figure plus Table 1. Since the effect of the equivalent damping during the period the load is approaching the center is always to reduce the free oscillations, the assumptions of constant damping is conservative. The effect of the variable frequency has little significance with regard to amplification of static deflections, since a small change of velocity can shift the peaks of the oscillations so that a maximum downward oscillation occurs as the load is near the center of the span. It is only the envelope of this oscillation which is important. The dynamic increments under these conditions are dependent on the aforementioned amplification factor and upon:

$$y = \delta_{st}(\frac{n}{n_o})\, e^{-2\pi\frac{n_b}{4n}}$$

These quantities are tabulated in Table 1 and it can be seen that the smoothly rolling effects of a single axle load are quite small for all except short spans.

TABLE 1

MAXIMUM DYNAMIC INCREMENTS FOR SINGLE AXLE SMOOTHLY ROLLING LOADS

Span ft.	$\frac{n}{n_o}$	$e^{-\frac{\pi}{2}\frac{n_b}{n}}$	Dynamic Increments Free Oscillations	Forced Component %	Totals
20	0.101	0.690	6.9	1.02	7.9
50	0.0937	0.755	7.1	0.8	7.9
100	0.0809	0.800	6.5	0.6	7.1
200	0.0652	0.774	5.0	0.4	5.4
300	0.0553	0.668	3.7	0.3	4.0

EFFECTS OF VEHICLE SPRINGING

The previous discussion of the dynamic effects due to a smoothly rolling load gives results which are in some respects more severe and in other respects less severe than when the effects of vehicle springing are included. Spring mounting of the load has three important effects on the response of the structure: (1) if the load enters the span with its springs in the equilibrium position, it may be set into motion by the vibrations of the structure, absorbing some of the energy; (2) if the load enters the span with its springs in any position other than that of equilibrium, the potential energy thus available will be partly transmitted to the structure, adding to the effects of the smoothly rolling load; (3) the presence of the spring-borne load changes the frequency of the system and provides a second point of energy dissipation if heavy damping is present in the spring system.

Before discussion of how these effects are produced, it is necessary to select some typical vehicle characteristics for study and to establish the range of variation of these characteristics which are of importance.

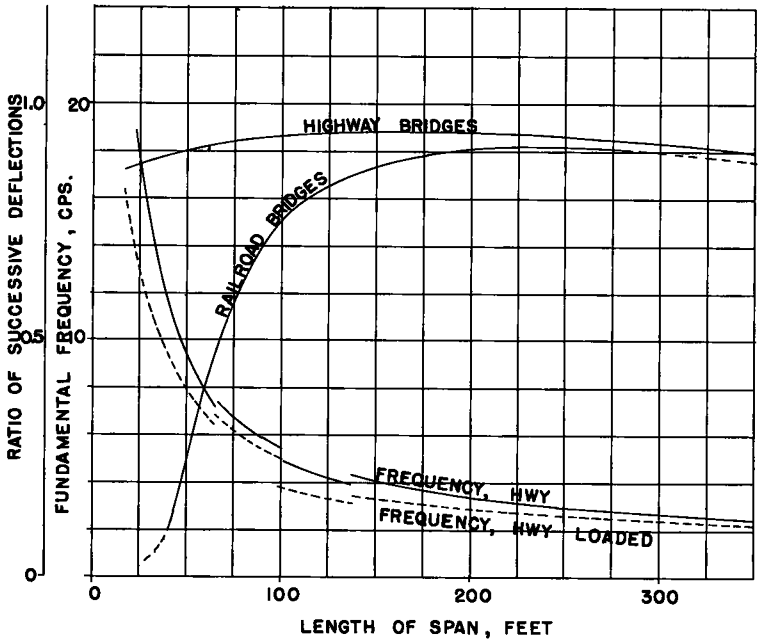

Figure 3. Dynamic characteristics of highway bridges.

Since we are ultimately concerned in this study with dynamic effects significant from a design standpoint, we will confine our interest to those types of vehicles which can produce maximum live load stresses; namely, the heavy truck and truck-trailer combinations. We shall exclude vehicles of extreme axle weights or special equipment which would travel under permit and, hence, supposedly under controlled conditions. Information on this problem is scattered and incomplete and the best that can be offered is the following estimate for axles corresponding to an H20-S16 loading:

Sprung weight per wheel	12,000 lb.
Unsprung weight per wheel	4,000 lb.
Spring constant per wheel	2,000 lb. per inch
Spring damping factor	0.7 (Ratio of residual oscillations, equivalent viscous)
Force to initiate spring action	1,600 lb. per wheel
Natural frequency of spring mass on springs	1.28 cps.

Since the tires of the vehicle also have spring characteristics which may be involved, it is well to indicate average values for these (for the same H20-S16 axle):

Tire spring constant	21,400 lb. per in.

Tire damping constant	0.6 (greatly variable)
Natural frequency of unsprung mass on tires	7.20 cps.
Natural frequency of total mass on tires	3.60 cps.

It must be emphasized that considerable range for all of these factors must be anticipated, particularly when such matters as the possibility of resonance with the structure are considered.

Let us now consider the beneficial effects of vehicle springing for the case where the load comes on to the span in the equilibrium condition. In this event, the vehicle will act somewhat as a dynamic vibration absorber. Inglis has shown, in his work on railway bridges, that a good approximation of this effect can be obtained by mounting this spring-borne mass at the center of the bridge in a stationary position. For the effect of springing of the vehicle upon the oscillations induced in highway bridges, we can make a similar approximation by considering a simple two degrees of freedom system in which one mass is the mass of the girder plus the unsprung weight and the other mass is that of the sprung weight.

Now assume that the structure has begun to oscillate in its own fundamental frequency due to a rolling load entering the span. Initially the springs of the vehicle will remain locked and the load will follow the structure since at the end of the bridge the amplitudes of the fundamental mode vibrations are small and produce small accelerative forces. At any position of the load along the span, maximum value of these forces may be expressed approximately as:

$$\text{Force} = M \left(\frac{\partial^2 y}{\partial t^2} \right)_{max.} = Me^{-2\pi n_b t} \, 4\pi^2 n_o^2 \left(\frac{n}{n_o}\right) \sin \frac{\pi x}{L} \left[\delta st\right] \quad \text{or,}$$

neglecting damping,

$$\text{Force} = M \left[4\, \delta_{st}\, \pi^2 n_o^2 \left(\frac{n}{n_o}\right) \right] \sin \frac{\pi x}{L}$$

That is, the accelerations to which the load is subject are the maximum accelerations of the structure at its centerline multiplied by the factor $\sin \frac{\pi x}{L}$. Spring action will not be initiated until this product equals the force necessary to overcome friction.

Based on the average dynamic characteristics of structures and vehicles presented above, the significance of this will be found by inspection of Table 2. The results of this calculation for the assumed spring initiation force of 1,600 lb. shows that the free oscillations of the spans are far too mild to bring the springing into play except for the short spans under about 40 feet. Even if a low value of the force is taken, say 240 lb. or 2 percent of the sprung mass, the effect is still nonexistent except for spans under 100 feet.

For these short spans where spring action is initiated, the system may be idealized as shown in Figure 7. The symbols used in this figure are those common in dynamic theory, except for the little link "F", which represents the frictional force in the springs which keeps them locked until the acceleration forces are large enough to initiate spring movement. This is a rather incomplete analogy of the Coulomb type friction damping, but will serve its purpose for our present problem.

Initially, both masses will oscillate together as a single degree of freedom system. At some instant of time "a", the link F is broken, after which the system behaves as a two degree of freedom system. The displacements and velocities of the two masses at the instant "a" determine what happens during this second phase. The general solution for the two degrees of freedom system without damping in terms of the generalized coordinates q_1 and q_2 is:

$$q_1 = A_1 \cos(p_1 t + \alpha_1) + B_1 \cos(p_2 t + \alpha_2)$$

$$q_2 = A_2 \cos(p_1 t + \alpha_1) + B_2 \cos(p_2 t + \alpha_2)$$

where p_1 and p_2 are the two real positive solutions of the frequency equation derived from Lagrange conditions,

$$M_G M_p^4 - \left[(M + M_G) K_s + MK_G\right] p^2 + K_s K_G = 0$$

and the constants A_1, A_2, B_1, B_2 are determined from the boundary condition with respect to time. For example, if we consider a span of 50 feet and the effects of a single 32,000 lb. axle, of which 24,000 is sprung weight, we have:

$$M = \frac{24000}{g} = 747 \text{ slugs} \qquad M_G = \frac{16700 + 8000}{g} = 5430 \text{ slugs}$$

$$K_s = 2(12)(2000) = 4.8 \times 10^4 \text{ lb./ft.} \qquad K_G = 5.58 \times 10^6 \text{ lb./ft.}$$

By substitution of these values in the frequency equation we find that:

$$p_1^2 = 65 \quad p_1 = 8.04 \qquad f_1 = \frac{p_1}{2\pi} = 1.28 \text{ cycles per second}$$

$$p_2^2 = 1037 \quad p_2 = 32.1 \qquad f_2 = \frac{p_2}{2\pi} = 5.12 \text{ cycles per second}$$

TABLE 2

POSITION OF LOAD IN SPAN TO INITIATE SPRING ACTION

Based on a Sprung Mass of 12 Kips per wheel

Span	Static Deflection 64^k Load	Natural Frequency n_o	Centerline Maximum Accelerations	Maximum Centerline Force/Wheel	Position as Fraction of Length to Initiate Spring Action F=1,600 lb.	F=240 lb.
ft.	ft.	cps.	ft/sec^2	lb.		
20	0.0120	21.75	22.40	8,340	0.061	0.0092
50	0.0115	9.38	3.78	1,410	No action	0.055
100	0.0123	5.43	1.17	436	"	0.186
150	0.0119	4.01	0.54	201	"	No action
200	0.0119	3.37	0.37	138	"	"
250	0.0117	2.96	0.27	101	"	"
300	0.0113	2.66	0.16	60	"	"

Let us now assume that the spring frictional forces are released as the structure reaches the peak of a downward oscillation at the instant of time "a". This will impose boundary conditions with respect to time on the two degree of freedom system as follows, introducing a new variable t' which is the time after instant "a". When t' = 0; q_1 = amplitude of the free oscillation of the structure, D

$$D = \delta_{1\,\text{Kip}} \,(\text{Load}) \left(\frac{n}{n_o}\right) = 0.000179\,(32)\,(0.094) = 5.38 \times 10^{-4} \text{ ft.}$$

$$q_2 = 0$$

$$\dot{q}_1 = \dot{q}_2 = 0$$

The resulting motion is governed by the following equations, and is presented graphically in Figure 7.

$$q_1 = D\left[-0.012\cos(8.04t') + 1.012\cos(32.1\,t')\right]$$

$$q_2 = D\left[-1.077\cos(8.04t') + 1.077\cos(32.1\,t')\right]$$

The oscillations of both the vehicle and the structure now consist of two superimposed sinusoidal vibrations, the periods of which correspond closely to the natural periods of structure and vehicle. It can be seen that if no damping is present in the vehicle springing, no beneficial effect is produced on the oscillations of the structure. The more-complete treatment of the problem, using equivalent viscous damping to approximate the Coulomb friction effects, yields equations nearly identical to those above, except that each term is modified by a time decay function and a small phase angle is introduced as follows:

$$q_1 = D\left[-0.012\,e^{-0.425t'}\cos(8.04t'-\alpha_1) + 1.012\,e^{-0.645t}\cos(32.1t'-\alpha_2)\right]$$

$$q_2 = D\left[-1.077\,e^{-0.425t'}\cos(8.04t'-\alpha_1) + 1.077\,e^{-0.645t}\cos(32.1t'-\alpha_2)\right]$$

where $\alpha_1 = 3.03$ deg.
$\alpha_2 = 1.15$ deg.

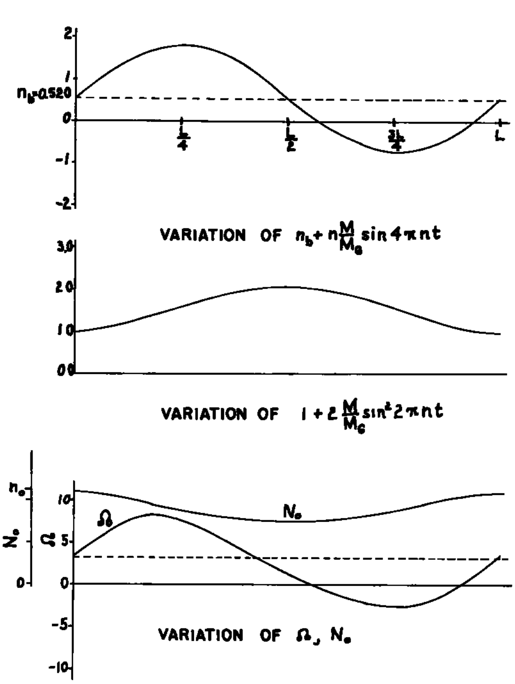

Figure 4. Variation of equivalent damping and frequency of 20-ft. span with 60 mph. load.

The results of these effects upon the centerline deflection of the structure are depicted in Figure 8. As can be seen, the number of oscillations which take place while the deflection is near a maximum is small for these short spans in which spring action can be initiated, and consequently, the reduction of amplitude of the oscillations with time due to damping is of little importance.

If we now consider what takes place when the springs of the vehicle are compressed or the vehicle is oscillating as the load enters the span, we will find an entirely different situation. As regards the length of span in which this effect will be important, we must consider spans for which the natural frequency of the structure will be close to the natural frequency of the load on its springs, since this will permit a resonance condition loading to large amplitudes.

Suppose that we take the same single axle load of 32,000 lb. as before and select a span as near the same natural frequency as that of the sprung mass as possible. Now, for the average vehicle and bridge characteristics assumed in this discussion, the longest spans considered do not have natural frequencies below 2 cps., hence, complete resonance is not possible. If we presume that the vehicle frequency may go as high as 1.5 cps. and that a structure of 200-foot span may have a frequency of as low as 2 cps. we will be about as close to resonance as we can get.

Let us further assume that the cause of the initial oscillation of the load, whether it be due to deck roughness, or to some type of self excited vehicle oscillation, shall continue to supply the energy necessary to keep the vehicle oscillating at constant amplitude. Under these conditions we have a problem almost identical to that of the hammer blow effect of locomotives on railway bridges. The one difference is that the magnitude and frequency of the pulsating force is independent of the speed of the vehicle. This leads to a peculiar paradox: The dynamic increment of deflection is greatest for a moderate speed, since this allows the pulsating force to apply more increments of energy to the structure.

The differential equation which governs the motion of the span under the influence of the primary component of the oscillating force is:

$$EI\frac{\partial^4 y}{\partial x^4} + 4\pi n_b m\frac{\partial y}{\partial t} + m\frac{\partial^2 y}{\partial t^2} = \frac{2}{L}\left[\sin 2\pi n t \sin\frac{\pi x}{L}\right]\sin 2\pi N t$$

where the notation is that used in the analysis of the smoothly rolling load except that:

N = Frequency of oscillation of sprung load.

The solution for this equation has been developed by Inglis and is of the form:

$$y = \frac{D_p}{2}\left[\frac{\cos\left\{2\pi(N-n)t - \Phi\right\} - e^{-2\pi n_b t}\left\{G\sin 2\pi n_o t + I\cos 2\pi n_o t\right\}}{\left\{\left[1-\left(\frac{N-n}{n_o}\right)^2\right]^2 + \left[\frac{2n_b(N-n)}{n_o^2}\right]^2\right\}^{1/2}}\right]\sin\frac{\pi x}{L}$$

$$-\frac{D_p}{2}\left[\frac{\cos\left\{2\pi(N+n)t - \Psi\right\} - e^{-2\pi n_b t}\left\{H\sin 2\pi n_o t + J\cos 2\pi n_o t\right\}}{\left\{\left[1-\left(\frac{N+n}{n_o}\right)^2\right]^2 + \left[\frac{2r_b(N+n)}{n_o^2}\right]^2\right\}^{1/2}}\right]\sin\frac{\pi x}{L}$$

In which:

D_p = static deflection due to a load equal to pulsating force P applied at centerline.

$$\Phi = \mathrm{Tan}^{-1}\frac{2n_b(N-n)}{n_o^2-(N-n)^2}$$

$$\Psi = \mathrm{Tan}^{-1}\frac{2n_b(N+n)}{n_o^2-(N+n)^2}$$

The coefficients G, H, I and J of the free oscillations to satisfy initial conditions are:

$$G = \frac{N-n}{n_o}\sin\Phi + \frac{n_b}{n_o}\cos\Phi \qquad I = \cos\Phi$$

$$H = \frac{N+n}{n_o}\sin\Psi + \frac{n_b}{n_o}\cos\Psi \qquad J = \cos\Psi$$

A calculation of the constants and coefficients for spans of 20, 50, 100, 200, and 300 feet indicates: (1) the second term of the denominator is always negligible with comparison to the first; (2) the coefficients G and H are always small; and (3) the coefficients I and J are almost exactly equal to one since the phase angles Φ and Ψ have maximum values of about 5 deg. A summary of these results is given in Table 3. Because of these facts, a simplified solution of high accuracy can be written for the centerline deflection as:

$$y_{\mathbb{C}} = \frac{D_p}{2}\left[C_1\cos 2\pi(N-n)t - C_2\cos 2\pi(N+n)t - e^{-2\pi n_b t}(K\sin 2\pi n_o t + L\cos 2\pi n_o t)\right]$$

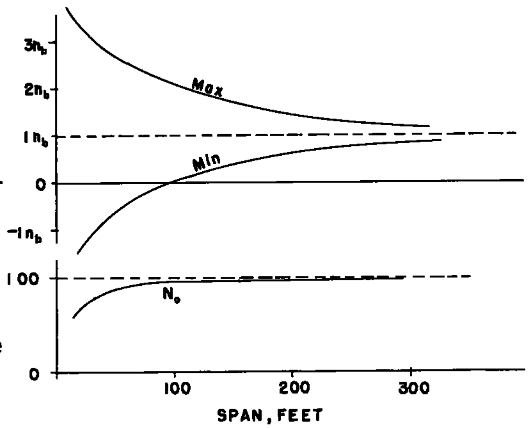

Figure 5. Range of variation of equivalent damping and equivalent frequency for spans 20 ft. to 300 ft.

In which: $C_1 = \frac{1}{1-\left(\frac{N-n}{n_c}\right)^2}$ $C_2 = \frac{1}{1-\left(\frac{N+n}{n_o}\right)^2}$

$K = C_1G - C_2H \qquad L = C_1 - C_2$

The values of these new constants and the frequencies and periods of the oscillations induced for the same series of spans are given in Table 4. The result of the superposition of these oscillations is shown graphically in Figure 8. In this figure, only the first few cycles of oscillations are shown for the free vibrations of the structure, since these are rapidly damped into insignificance with respect to the forced oscillations. It is best to look first at the effects on the 300-foot span, since here the repetitive "beating" pattern of oscillation has had time to fully develop. Under these conditions, the maximum dynamic affect of the pulsating load is produced by the superposition of the two main oscillations in phase, so that the amplitude is

$$y = \frac{1}{2}(C_1 + C_2)\, D_p$$

The value of this amplitude in terms of D_p is also given in Table 4. A better appreciation for the significance of this amplitude can be obtained by assuming that the pulsating force is some reasonable percentage of the total spring load on the axle, say 25 percent. The ratio of the dynamic increment to the total deflection for this assumption is given in the last column of Table 4. Thus, it is seen that the effect of a single oscillating load can be taken at its static equivalent for short spans but must be markedly

TABLE 3

FACTORS FOR CALCULATION OF EFFECTS OF OSCILLATING LOADS ON VARIOUS SPANS

Span	Structure		Moving Load		$1-\left(\frac{N-n}{n_o}\right)^2$	$\frac{2n_b\,(N-n)}{n_o^2}$	$1-\left(\frac{N+n}{n_o}\right)^2$	$\frac{2n_b\,(N+n)}{n_o^2}$
	n_o	n_b	N	n(40 mph)				
ft.								
20	15.00	0.520	1.5	1.465	1.000	0.0000	0.961	0.0137
50	7.05	0.157	1.5	0.586	0.983	0.0008	0.912	0.0018
100	4.79	0.063	1.5	0.293	0.938	0.0014	0.859	0.0021
200	3.09	0.036	1.5	0.147	0.807	0.0033	0.714	0.0040
300	2.55	0.038	1.5	0.098	0.696	0.0064	0.605	0.0073
200 Sp.	2.00	0.036	1.5	0.147	0.538	0.0122	0.320	0.0148

Span	Φ	Ψ	$\frac{n_b}{n_o}$	G	H	I	J
ft.	deg.	deg.					
20	0.00	0.82	0.0347	0.0347	0.0375	1.0000	0.9999
50	0.33	0.84	0.0223	0.0299	0.0265	1.0000	0.9999
100	0.40	0.65	0.0131	0.0149	0.0174	1.0000	0.9999
200	0.71	0.98	0.0117	0.0172	0.0209	0.9999	0.9998
300	1.38	1.75	0.0149	0.0282	0.0340	0.9997	0.9995
200 Sp.	2.58	5.28	0.0180	0.0486	0.0942	0.9990	0.9958

TABLE 4

AMPLITUDES AND FREQUENCIES DUE TO OSCILLATING LOAD

Span	C_1	C_2	(N-n)	(N+n)	K	L	$\frac{1}{2}(C_1+C_2)$	Percent δ st.
ft.								
20	1.000	1.040	0.035	2.965	-0.0043	-0.040	1.020	25.6
50	1.017	1.097	0.914	2.086	0.0013	-0.080	1.057	26.4
100	1.065	1.165	1.207	1.793	-0.0044	-0.100	1.125	28.1
200	1.240	1.401	1.353	1.647	-0.0080	-0.161	1.320	33.0
300	1.438	1.652	1.402	1.598	-0.0156	-0.214	1.545	38.6
200	1.858	3.125	1.353	1.647	-0.2002	-1.267	2.492	62.3

increased for longer spans. Moreover, present designs of highway bridges of spans of about 300 feet are getting quite close to the condition of resonance for oscillations of heavy vehicles, which would accent this effect greatly, unless some type of heavy damping is incorporated into the structure.

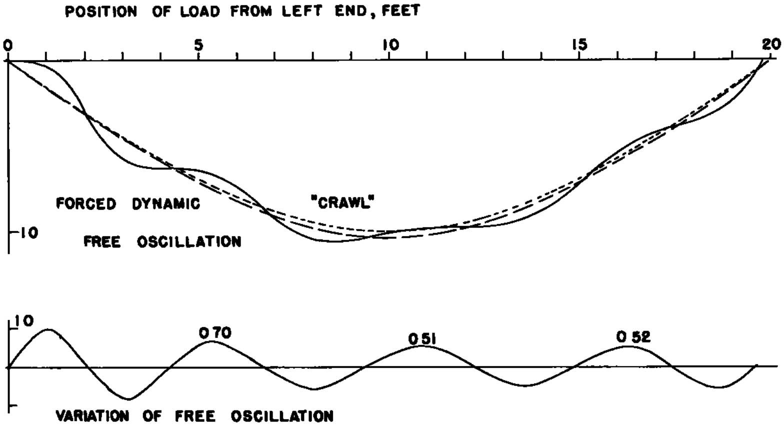

Figure 6. Response of a 20 ft. span to single axle at 60 mph.

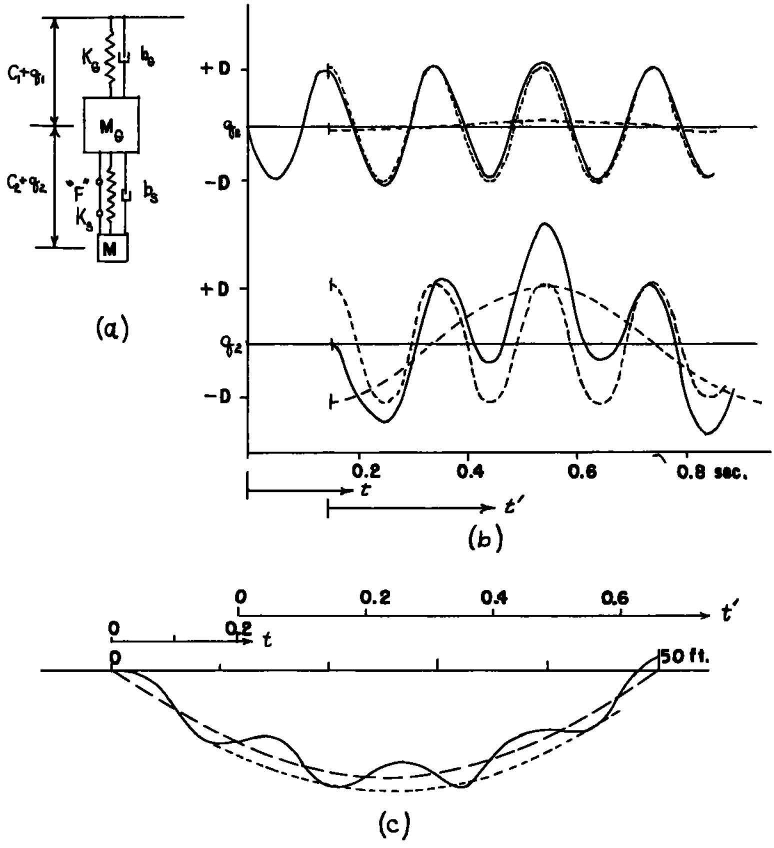

Figure 7. Effect of vehicle springing upon oscillation of the structure.

In closing this section it would be well to mention a study which was made of the possibility of induced oscillations of effective contact force between load and deck due to differential deflections in the deck system for various positions of the load. It was found that it was quite possible to obtain a pulsating force of about 3 percent of the live load with a frequency dependent on the vehicle speed, hence the possibility of resonance exists. The dynamic increments of deflections produced under a condition of resonance with the springing effect of the vehicle included is about 30 percent of the total axle load.

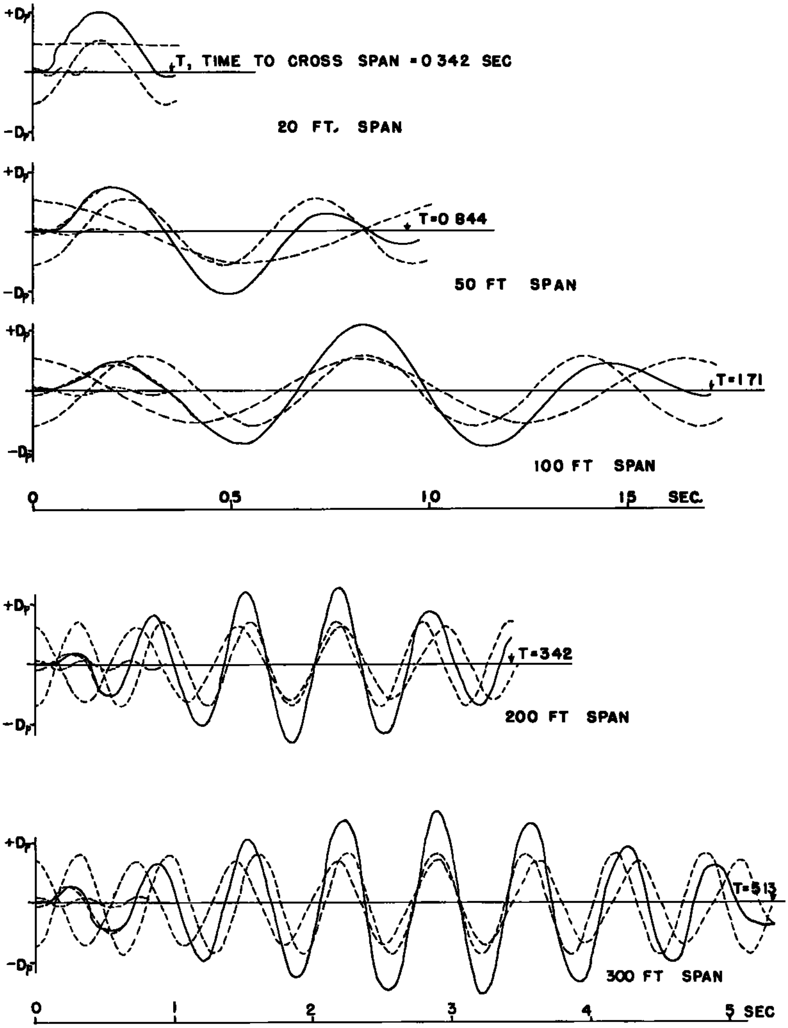

Figure 8. Oscillations produced in spans by oscillating load.

SHOCK EFFECTS OF DECK ROUGHNESS AND OBSTRUCTIONS

The presence of small irregularities in the running surface will have as their chief effect the production of vehicle oscillation and its maintenance as discussed above. Sharp defects or obstructions will have a more direct and somewhat different effect. A study was made of this effect by assuming the simplified system shown in Figure 9. Here the tire of the vehicle is replaced by a massless roller and an elastic spring, upon which rests the unsprung mass of the vehicle. The sprung mass of the vehicle is assumed to follow a horizontal path during the short interval that the wheel is passing over the obstruction, hence the force of the spring acting down on the unsprung mass can be represented as shown. The sinusoidal bump represents an idealized path of the hub of the wheel in passing slowly over a 1-by-2-inch board.

The analysis of this system has shown that the contact force between the wheel and the surface becomes negative at the rather moderate speed of 15 mph., that is, the

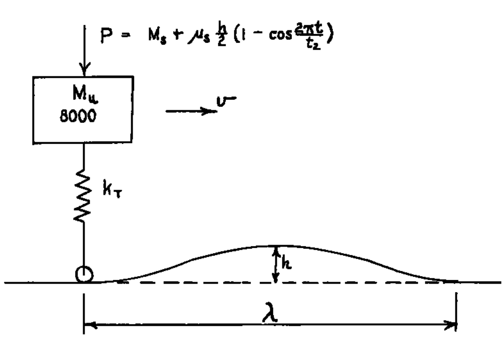

Figure 9. Simplified system representing vehicle passing over obstruction.

the tire leaves the surface and follows a trajectory in space. This is the case of most interest, since the shock on the structure which occurs as the wheel returns to the surface is much greater than the initial shock of contact with the bump.

What happens in this case is illustrated qualitatively by Figure 10. The effect of the initial shock is ignored, since it is rather small compared to what follows. As the wheel leaves the surface, the structure is without any load; hence it begins to return to a no-load position at its own natural frequency of vibration. If the wheel is out of contact with the surface for a time just equal

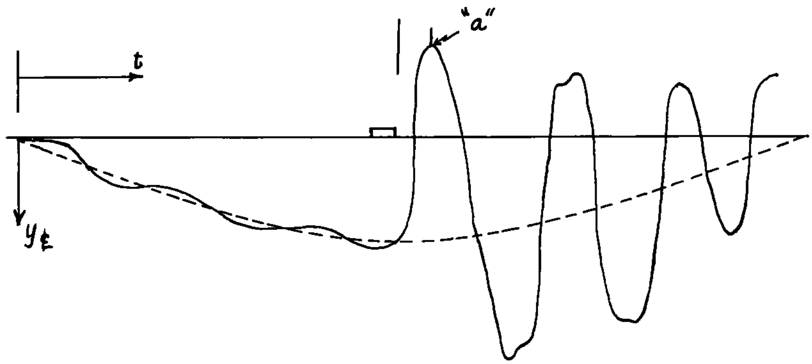

Figure 10. Qualitative behavior of structure as wheel passes over obstruction.

to half of a natural period for the structure, the maximum effect will be produced. This is possible for short spans, where this time is of the order of from 0. 03 to 0. 10 sec. and the vehicle would have moved only 1. 60 to 6. 00 feet. In this event, the structure would have just reached the maximum upward deflection represented by point "a" on the figure. At this instant we have the sudden application of the load, which is augmented by the downward velocity of the unsprung mass.

Under these conditions it would be possible to get the theoretical 100 percent impact of the classic analysis of a suddenly applied load, followed by oscillations of the structure and vehicle. Actual tests of a structure in which these conditions were approached gave an increment of about 60 percent.

It will be noted that the shock effects of single axles will be greatly reduced for spans beyond about 100 feet, since incomplete recovery of the structure to its no load position will have taken place. The natural frequency of heavy tires is of the order of 7 cps., giving a half period for the tire of about 0. 07 sec., and the half period for the oscillation of the unsprung mass against the vehicle springs is about 0. 195 sec., which would be the upper bound of the length of time the tire would not be in contact with the surface.

SIGNIFICANCE OF THE VARIOUS DYNAMIC EFFECTS IN DESIGN

The designer is faced with two problems in making proper allowance for the dynamic effects of live loads. The first of these is the establishment of a suitable allowance in terms of percentage of the static affect of the live load to provide a proper stress margin. The second one, not yet always recognized, is to prevent resonance effects which are dangerous to the structure, or are psychologically disturbing to the users.

In the previous portions of this discussion, we have considered separately the various dynamic effects which can be produced by single-axle live loads. We must now relate this information to the first design problem, namely, the stress problem. We shall begin by recapitulation of the magnitude of these separate effects for spans of various lengths, as presented graphically in Figure 11.

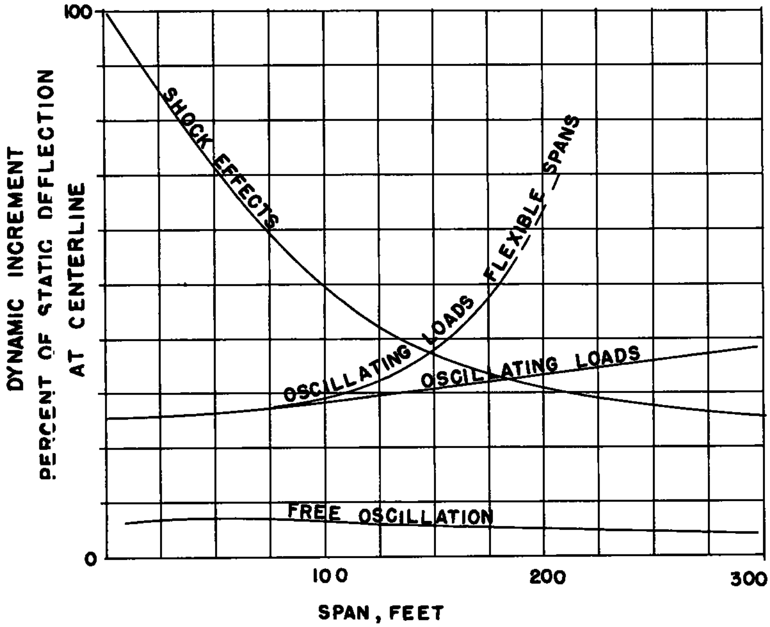

Figure 11. Variation of single-axle effects with span length.

The "forced" oscillation due to the smoothly rolling load is so small as to be negligible for all spans. The free oscillation due to the smoothly rolling load decreases with span length due both to frequency relationship conditions and due to the fact that in the longer spans the damping will have had time to be effective. The effects of an oscillating single axle load become more and more pronounced as the frequency of the span approaches the frequency of the vehicle, a tendency which exists for either longer spans or more flexible structures.

Obstructions and sharp deck irregularities can produce large effects in short spans. The curve presented for this effect is intended to indicate maximum possible resulting deflections, because it will be later developed that its significance is not nearly as great as would appear.

Obviously one should not combine the percentages of each effect to get the total allowance or equivalent additional static load. Rather, the allowance must be based upon the following considerations: (1) the ratio of the maximum stress produced by a single axle load to the total design live load stress; (2) the probability of the simultaneous occurrence of effects, and the relative phase of the separate oscillations; (3) the possible beneficial effects of the presence of live loads other than the single axle being considered; and (4) the relationship of stress in the member to centerline deflection of the structure.

The first of these factors is perhaps the most important of all, as will be seen by reference to Table 5. The rapid decrease of the percentage of total live load furnished by any one axle for spans in excess of 100 feet greatly reduces the importance of the effects of oscillating loads and shock effects for these spans.

The matter of the probability of simultaneous occurrence of the various dynamics effects or of the superposition of the effects of a number of axles in phase is a most difficult problem to treat quantitatively on the basis of presently available data. It is related to the more general problem of the scientific treatment of safety factors as discussed by A. M. Freudenthal (3) and others. At present, the best which the writer can offer is based on estimated probability for each type of effect and the combination of

TABLE 5

RATIOS OF SINGLE AXLE TO FULL LIVE LOAD FOR VARIOUS SPANS TWO LANE H20-S16 BRIDGES

Span	Composition of Assumed Full Live Load	Full Live Load Moment	Single Axle Moment	Ratio
ft.		ft. Kip	ft. Kip	
20	One 32 Kip axle per lane	320	160	0.500
50	One H20-S16 vehicle per lane	1,040	400	0.389
75	One H20-S16 vehicle per lane	1,940	600	0.309
100	Two H20-S16 vehicles per lane	2,690	800	0.298
150	640 lb. per ft. plus 18 Kip per lane	4,950	1,200	0.242
200	" " " " " " " "	8,200	1,600	0.195
250	" " " " " " " "	12,250	2,000	0.163
300	" " " " " " " "	17,100	2,400	0.140

these probabilities by the elementary algebraic methods.

To begin with we must recognize that the probability of ever getting full design live loads on a span decreases rapidly as the span length increases. Anyone who has had the experience of trying to find enough heavy vehicles to apply a full design live load to a structure will be convinced of this. Therefore, we should not be overly conservative with the manner in which we pile on dynamic effects, since we are probably already dealing with small probabilities.

If we assume that we have all truck traffic and that vehicles equivalent to the H20-S16 vehicle account for about 5 percent of this traffic, then the probability of full live load on both lanes of spans of various lengths is as given in Table 6. These probabilities are large enough to make certain that the structure will receive its full load at some time during its life. The value of this listing will come only in connection with combining these probabilities with those due to dynamic effects.

TABLE 6

PROBABILITY OF FULL LIVE LOAD ON VARIOUS SPAN LENGTHS

Span	Probability p, in n Loadings
ft.	
20	0.0250
50	0.0125
100	0.0062
150	0.0042
200	0.0031
250	0.0025
300	0.0021

Here we must assume an order of magnitude of probability of reaching full design stress which is considered proper. Based on the use of allowable stresses in the AASHO specifications, a probability of not greater than 1×10^{-4} would appear to be generously adequate. This is admittedly the weakest point in our analysis, since we are mixing the concept of equal probabilities of failure with those of allowable stress. We must also assume some reasonable probabilities for superposition of individual axle effects.

Suppose that: (1) 0.25 is the probability that the free oscillations introduced in a structure by two successive axles will be in phase; (2) 0.25 is the probability that one axle will be oscillating with a pulsating force of 25 percent of static load; (3) 0.25 is the probability that the oscillations produced by two successive oscillating axles will be in phase; and (4) 0.01 is the probability that a fully loaded axle will strike an obstruction in each of two lanes at the same time.

Further assume that only one obstruction of importance exists in each lane and that the probability of its being within the midspan one fifth of the structure is 0.2. Based on these assumptions, we will make allowances at full effect for any combinations which give combined probabilities of greater than 1×10^{-4} and reject those which give less than this probability. For instance, suppose we wish to determine for various spans the number of axles which should be assumed to induce free vibrations of the

TABLE 7

VARIATION OF EFFECTIVE DYNAMIC INCREMENT OF CENTERLINE DEFLECTION FOR VARIOUS SPANS

Span	Free Oscillations Axles	Per Axle	Eff.	Shock Effects Axles	Per Axle	Eff.	Vehicle Oscillation Axles	Per Axle	Eff.	Combined Eff.	AASHO "Impact"
ft.	No.	%	%	No.	%	%	No.	%	%	%	%
20	1	6.9	3.4	1	100	50.0	1	25.6	12.8	53.4	30.0
50	2	7.1	5.5	1	70	26.9	2	26.4	21.0	29.6	28.5
100	3	6.5	5.8	1	50	14.9	2	25.0	14.9	16.6	22.2
150	3	5.8	4.2	1	25	6.0	2	24.6	11.6	13.0	18.2
200	3	5.0	2.9	1	25	4.9	2	22.0	8.5	9.5	15.4
250	3	4.3	2.1	1	25	4.1	2	21.6	7.1	7.8	13.3
300	3	3.7	1.5	1	25	3.5	2	19.3	5.4	5.9	11.8

structure which are in phase. The probability of getting any five axles so superimposed when full live load is on a 300 foot span is:

$$p = (0.0021)\,(0.25)^4 = 8.4 \times 10^{-6}$$

which would not be used. Three axles would give a probability of 1.32×10^{-4} which is within our limit. The increment of live load moment, based on full live load moment would be:

$$3(3.7)\,(0.140) = 1.55 \text{ percent}$$

For the superposition of oscillating load effects, the probability of four axles being in phase on this same span is:

$$p = (0.0021)\,(0.25)\,(0.25)^3 = 8.4 \times 10^{-6}$$

Thus, for this effect we must take only two axles, for which the probability is 1.32×10^{-4} and the dynamic increment is:

$$2(38.6)\,(0.140) = 10.8 \text{ percent}$$

For the possible shock effects we have the following probability that a fully loaded axle will strike an obstruction at the same instant in each lane near the center of the bridge when full live load is on the structure and that the resulting oscillations will be in phase:

$$p = (0.0021)\,(0.01)\,(0.2)\,(0.25) = 1.05 \times 10^{-6}$$

which is rejected.

Using only one axle, the resulting shock effect is:

$$1(0.25 \text{ percent})\,(0.140) = 3.5 \text{ percent}$$

It is also possible to combine these effects in such a way that the combined probability is greater than 1×10^{-4}, as long as any effects combined are not physically incompatible, as is the case with combining oscillating loads with obstruction effects where only two axles can be near the center of the span at one time. The following combinations are proposed: (1) free oscillations for one axle plus obstruction effect for one axle and (2) free oscillations for one axle plus forced oscillations for two axles.

Before indicating how this approach affects the establishment of live load dynamic allowances, let us consider the third problem mentioned above, namely, the beneficial effects of having the structure loaded. These are of particular importance in considering the effects of vehicle oscillations. An analysis made earlier indicated that the springing of vehicles had little benefit in reducing the forced oscillations of the structure. However, for those oscillations induced by bouncing vehicles, the presence of the vehicles on the span of nearly the same frequency would provide an excellent dynamic vibration absorber. This would only be true for spans beyond about 75 feet, because for shorter spans the induced vibrations have frequencies much higher than those of the oscillating vehicle. It will be assumed in what follows that this absorption of energy is roughly proportioned to the number of vehicles which are on span other than the one supplying the energy, with percentage reduction as follows: 10 percent at 75 feet, 25 percent at 150 feet, and 50 percent at 300 feet.

The result of applying these limiting probabilities and other modifications to the single-axle live load effects shown previously in Figure 11 is given in Table 7. Here the percentage allowance based on total live load for each effect and for certain combinations is given. The currently used AASHO impact allowance formula is also shown for comparison.

The allowances computed by the above methods are based on the assumption that the increase in stress in any member is proportional to the increase of centerline deflection. This is a good approximation for simple spans composed of rolled shapes or plate girders of moderate length, in which the centerline bending moment controls the design. The assumption is also good for quarter point moment. The assumption is not nearly so good for shearing forces at the end of the structure as the length of the span increases. It is also questionable in its application to the members of the web system of a truss, in which the live load would be placed only on a portion of the influence line but for which any dynamic forces due to inertia of the span would be distributed throughout the span length in sinusoidal fashion. For instance, the oscillation of a structure due to a shock effect at the center of the span produces practically no shear at this point. The use of the concept of loaded length for such a member in determining the allowance for dynamic effects is also questionable, since the response of the member is primarily a function of the frequency of the supporting element of which it is a part, namely, the main truss.

In conclusion, it is suggested that the rational treatment of dynamic stress allowances for highway bridges should involve the following steps, some of which can be greatly aided by proper specification clauses and by prepared tables and charts for making estimates: (1) estimation of the fundamental frequency of vibration of the structure, taking into account the type of material, structural system and span length; (2) selection of the reasonable combinations of dynamic effects which are probable with full live load, considering span length, class of load and number of lanes; (3) computation of the probable percent increase of centerline deflection due to dynamic effects; (4) relating dynamic stress increase in each member to increase in centerline deflection; and (5) checking structure after completion of design for resonance and near resonance effects.

ACKNOWLEDGMENTS

The material of this paper is drawn from three sources: (1) a thesis written by the author in 1951 for the Division of Civil Engineering, University of California; (2) the research project on the San Leandro Creek Bridge conducted by the Institute of Transportation and Traffic Engineering of the University of California; and (3) the data collected by the Committee on Deflection Limitations of Bridges of the American Society of Civil Engineers, of which G. S. Vincent is chairman. The author gratefully acknowledges the encouragement and assistance of the many investigators with whom he has been associated in this work.

References

1. C. E. Inglis, "A Mathematical Treatise on Vibrations in Railway Bridges," Cambridge University Press (1934).
2. L. E. Vandegriff, "Vibration Studies of Continuous Span Bridges," Bul. 119, Engr. Exp. Station, Ohio State University.
3. A. M. Freudenthal, "The Safety of Structures," Trans. ASCE, V. 112, p. 125 (1947).