Factors Affecting Vertical Loads on Underground Ducts Due to Arching

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A theory of earth pressure on underground conduits is presented. Expressions for the general case of an $s = c + \sigma \tan\phi$ material have been derived. Expressions relating to $s = \sigma \tan\phi$ and $s = c$ soil types appear as special cases of the general case.

It is shown that the pressure on top of both covered-up and mined-in conduits is governed by the same mathematical relations. However, the values of the physical factors appearing in the theoretical expressions depend on the geometry and nature of installation, the physical properties, and the initial state of the materials, as well as on the construction methods and workmanship employed.

Curves for the evaluation of the load on top of covered-up conduits installed under an $s = \sigma \tan\phi$ material have been constructed. Under certain conditions the same curves can also be used for the general case of an $s = c + \sigma \tan\phi$ material.

The load on covered-up conduits becomes a minimum if the conduit side supporting material is thoroughly compacted, the ditch directly above the conduit is made as high as economically feasible, and the ditch is filled with a compressible, loose material.

● THIS paper was intended originally to be the theoretical part of a report on a three-year research project directed by the North Carolina State Highway and Public Works Commission. The project involved the study of the performance of a 66-in. flexible, metal-pipe culvert installed under a 170-ft. earth embankment that was constructed by end-dumping.

Existing earth pressure theories on underground conduits are applicable to low or medium height embankments consisting of perfectly granular material. Because of the unusual fill height and the construction methods employed in this project it was considered desirable to review and extend these theories, and revise them if necessary, in order to make them applicable to the above conditions.

In the process of extending these theories it was noticed that the mathematical expressions that govern the loading action of a fill placed on top of a conduit also govern the loading action of a natural earth deposit on a conduit that has been installed by a tunneling process. The geometrical similarity existing among various types of conduits covered by an earth fill and a conduit installed by a mining process is shown in Figure 1.

All underground conduits are either covered with an earth embankment after they have been assembled in place or are mined-in through a natural earth deposit. Therefore, an earth pressure theory that is applicable to these two main categories is generally applicable to all types of underground conduits.

Because of these considerations the general theoretical treatment is presented here as a separate study. The experimental part of the same project appears as a separate report by the North Carolina State Highway and Public Works Commission (Costes and Proudley, 1955). In the latter report appropriate mathematical expressions were derived from the general theory to make a speculative analysis of the earth pressure existing on top of the particular culvert under study.

Definitions

In this paper, an underground conduit is defined as a hollow prismatic structure that is installed with its longitudinal axis substantially horizontal under either a man-made earthen embankment or a natural earthen deposit.

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Underground conduits can be used for a multiplicity of purposes; they can be used as aquaducts, drainage structures, sewers, viaducts, runways for conductors or cables, gas mains, etc.

If a conduit is installed first, and then an earth embankment is constucted above it, the conduit is defined as a "covered-up conduit." If the conduit is installed through a natural earthen deposit by means of a mining process, the conduit is defined as a "mined-in conduit."

If judged according to their relative stiffness, underground conduits may be classified as "rigid conduits" or as "flexible conduits." The demarcation line between these two classes is not defined clearly.

Problems Relating to Underground Conduit Design

When designing an underground conduit, the engineer faces a variety of problems whose relative influence on the final design of the conduit depends on the purpose for

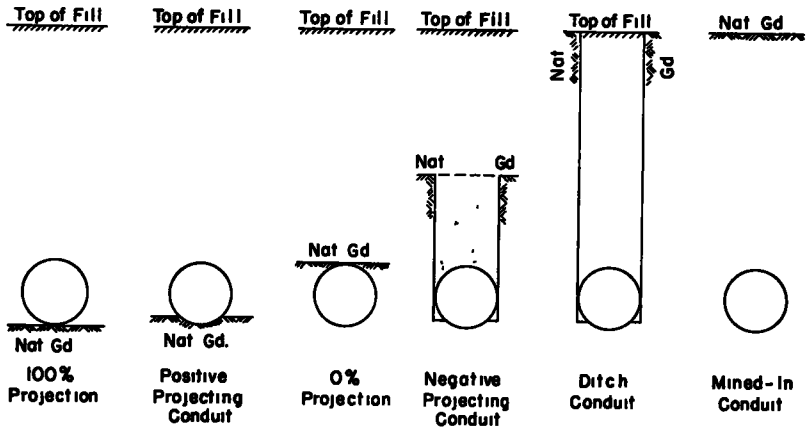

Figure 1. Geometrical relationship among underground conduits.

which the conduit is installed, the desired life expectancy of the conduit, and the size of the earth mass that the conduit will sustain. Some of these problems relate to: (1) durability; (2) hydraulic factors in case the conduit is installed as an aquaduct; (3) traffic considerations in case the conduit is installed as a viaduct; (4) adequate space in case a close inspection of the conduit is desired; and (5) structural capacity.

If the conduit is treated from the structural point of view the designer is mainly concerned with: (1) choosing the right conduit material and employing the right construction methods in order that the load on the conduit will be a minimum; (2) providing for adequate side support so that the conduit will not fail by excessive lateral bulging; (3) selecting the proper bedding material and deciding on the proper camber so that the conduit will not go out of alignment as the foundation settles; and (4) designing properly the thickness and the structural connections of the conduit so that it will withstand the internal stresses that are generated in its structure by the external pressures, namely, the top load, the lateral pressures exerted by the side supporting material, and the bottom reaction from the bedding material.

If the earth mass above the conduit is not too high, then, in addition to the dead load due to the earth mass, the influence of live loads that may exist on the surface of the mass must be considered also.[2] If the earth mass, however, is sufficiently high and pressure waves due to live loads are dissipated before reaching the conduit, the main load on the conduit will be due to the pressure of the earth that it sustains.

Scope of Paper

The purpose of this paper is to (1) present a general, uniplanar, theoretical study of

[2] For such treatment see References, Spangler and Hennessy (1946).

the factors influencing the pressure that is developed on top of both covered-up and mined-in conduits due to the earth mass alone; (2) apply the general study to special cases; (3) examine the physical meaning of the derived mathematical expressions; (4) draw conclusions in connection with the implications that certain installations may have on the conduit load; (5) construct curves from which the load on top of a conduit can be obtained for as many cases as possible; (6) suggest the main principles that should guide the engineer's judgment when designing an underground conduit; and (7) make recommendations relating to future research efforts in connection with this field of engineering.

This study makes no differentiation between "rigid" or "flexible" conduits. The shape of the conduit is also considered not to be a variable.

The mathematical treatment deals with the pressures acting in the plane perpendicular to the longitudinal axis of the conduit. The study of the development of earth pressures above the conduit and in the direction parallel to its longitudinal axis is beyond the scope of this paper.

REVIEW OF PREVIOUS RELATED STUDIES

All earth pressure theories relating to underground conduits have been based on one of the most universal phenomena encountered in soils, both in the laboratory and in the field, the so-called "arching effect." The arching effect as defined by Terzaghi (1943a) is a transfer of pressure from a yielding mass of soil onto adjoining relatively stationary parts. This pressure transfer takes place through a mobilization of the shearing resistance of the material which tends to oppose the relative movement within the soil mass.

Most of the existing theories on arching deal with the pressure of dry sand on yielding horizontal strips. Terzaghi (1943a) divides these theories into three groups:

1. In the first group only the conditions for the equilibrium of the sand immediately above the loaded strip have been considered. No attempt has been made to investigate whether or not the results of the computations have been compatible with the conditions for the equilibrium of the sand at a greater distance from the strip.
2. The theories of the second group have been based on the unjustified assumption that the entire mass of sand located above the yielding strip is in a state of plastic equilibrium.
3. In the third group the assumption has been made that the vertical sections through the outer edges of the yielding strip represent surfaces of sliding and that the pressure on the yielding strip is equal to the difference between the weight of the sand located above the strip and the full frictional resistance along the vertical sections.

No attempt will be made in this paper to describe each one of the above groups in any further detail.[3]

As far as studies of pressures on underground conduits are concerned, one may go as far back as the year 1882 when Forchheimer (1882) studied the development of earth pressures on the roof of a tunnel. This study was related to the studies by Janssen and Airy on the development of pressures observed in bins and grain elevators (Janssen, 1895), (Ketchum, 1913). As a matter of reference, the term, "bin effect," may be found in place of the term, "arching effect," in some publications.

Dean Anson Marston, Professor M. G. Spangler, and their associates of Iowa State College, deserve great credit for advancing the knowledge of loads developed on underground conduits. Under their direction, an extensive program of research, starting in 1908, has been carried out. Their main aim was to develop a rational method for determining the loads on covered-up conduits. The result of their work has been the "Marston Theory of Loads on Underground Conduits." This theory has been applied extensively in this country in the design of covered-up conduits. The theory is applicable

[3] For a comprehensive summary of each theory, see K. Terzaghi, Theoretical Soil Mechanics (New York: John Wiley & Sons, 1943), pp. 69-74. Detailed information on the same subject may be obtained from the following References: Engesser (1882), Kötter (1899), Janssen (1895), Koenen (1896), Bierbaumer (1913), Caquot (1934), Terzaghi (1936), Völlmy (1937) and Ohde (1938).

mainly to embankments constructed of perfectly granular materials (Marston, 1913, 1930), (Spangler, 1950a, 1950b).

In addition to the work conducted by Marston in the Iowa Engineering Experiment Station, several other extensive studies concerning earth pressures on underground conduits have been carried out both in the United States and in other countries.

These studies include the following:

1. Experiments were conducted at the University of North Carolina in 1927 in which the top vertical pressure, radial pressures, and the decrease in the conduit vertical diameter were measured in pipes of various diameters and materials, installed as positive projecting conduits (Braune, Cain and Janda, 1929).
2. Pressure tests were conducted on corrugated metal, concrete, and cast iron pipe culverts by the American Railway Engineering Association at Farina, Illinois, during the period 1923-1926 (Area, 1928).
3. During the construction of liner-plate and shield tunnels installed in the Chicago, Illinois, subway, an extensive research on earth pressures developed in mined-in conduits due to plastic clay, as well as on the deformations of the conduit structures, was conducted and reported by Terzaghi (1942-1943) and Peck (1943).
4. Similar tests on earth pressure on tunnels installed in plastic clay were conducted and reported by Housel (1943) in Detroit, Michigan.
5. Strain gage and load cell pressure measurements, as well as data from deformations and settlements, were obtained by the Alabama State Highway Department and Armco engineers from corrugated metal culvert pipe installations under 137 ft. of embankment (Timmers, 1953).
6. Similar tests were conducted by the North Carolina State Highway and Public Works Commission on a Multi-Plate culvert pipe installed under 170 feet of embankment. An attempt to develop a technique to measure directly the earth pressures exerted on the culvert under study is discussed also (Costes and Proudley, 1955).
7. In the laboratories of the Zurich Technical University, Switzerland, Völlmy (1936, 1937) conducted a series of tests on sand located above a yielding support to prove his assumption that the potential sliding surfaces are oblique planes.
8. Experiments on pipe models by using centrifuges to generate forces similar to these acting on the pipes in ditch conduit installations were conducted in the Moscow Municipal Academy (Pokrowski, 1937).[4]
9. A series of articles on culvert pipe analysis has been published in France by the Hungarian engineer Bela (1937), and by Guerrin (1938).
10. Information of culvert pipe analysis may also be found in the catalogues and publications of pipe manufacturers.[5]

THEORETICAL STUDY

Method of Analysis

The theoretical concepts and the resulting relations of this paper are presented as follows: (1) the basic assumptions are stated and discussed; (2) the fundamental differential equation describing the loading action of an earth mass on top of an underground conduit is derived; (3) the general load equation for an $s = c + \sigma \tan\phi$ material is derived; (4) Case I is defined and discussed; (5) Case II is defined and discussed; (6) factor $u = (2K_e \tan\phi_e)H_e/B_d$ is evaluated and discussed for Case II existing in covered-up and mined-in conduits; (7) the analysis of the general case is applied to an $s = \sigma \tan\phi$ material; (8) the analysis of the general case is applied to an $s = c$ material; and (9) families of curves are constructed for which the load by an $s = \sigma \tan\phi$ material on a covered-up conduit can be obtained. Conditions are stated under which the same curves can be used for the evaluation of the conduit load when the loading agent is an $s = c + \sigma \tan\phi$ material.

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[4] For a brief summary of the findings and conclusions of the experiments mentioned in items (7) and (8), see D. P. Krynine, "Design of Pipe Lines from Standpoint of Soil Mechanics," Proceedings of the Highway Research Board, XX (1940), 726-727.

[5] see References.

Analysis

Statement of Assumptions. The following basic assumptions are made in the evaluation of the theoretical relations governing the loading action of masses on top of underground conduits:

1. The loading agent is an ideal, homogeneous, isotropic material whose shearing resistance, s, per unit of area can be represented by the empirical equation: $s = c + \sigma \tan\phi$ where σ is a force per unit area, normal on a section through a mass. The symbol c represents the cohesion, which is equal to the shearing resistance per unit area if $\sigma = 0$. The symbol ϕ represents the angle of internal friction of the material.
2. Because of the fact that the foundation, which supports the material directly above the conduit, does not yield the same amount as the foundation, which supports the material adjacent to the middle mass, the former subsides more or less than the adjacent material depending upon the relative yielding of their respective supports. The relative subsidence takes place along vertical plane surfaces extending from the top of the conduit to some horizontal plane above the conduit designated as, "plane of equal settlement." Above the plane of equal settlement no relative subsidence takes place and all parts of the fill material settle the same amount due to the consolidation of the fill. Henceforth, the mass directly above the conduit will be referred to as the "interior prism." During the subsidence of the interior prism, horizontal layers remain horizontal.
3. The side supporting material has not been compressed excessively so as to cause the structure to fail by excessive horizontal bulging.
4. The internal stresses generated in the conduit structure on account of the external pressures have not exceeded the critical buckling load of the structure.
5. The unit weight of the material, γ, is constant throughout the fill height.
6. The angle of internal friction of the material, ϕ, is constant along the potential sliding planes.
7. The cohesion of the material, c, is constant along the potential sliding planes.
8. The ratio of the horizontal principal stress component within an element of the fill material to the vertical principal stress acting on the same element, K_e, is constant along the potential sliding planes. The ratio may be called, therefore, "hydrostatic pressure ratio."

Discussion of Assumptions. Every stress theory is based on the assumption that the material subject to stress is either homogeneous and isotropic or that the departure from these ideal conditions can be described by simple equations. If the material is also assumed strictly to follow Hooke's law, then the term "homogeneity" denotes identical elastic properties at every point of the material in identical directions whereas the term "isotropy" involves identical elastic properties throughout the material and in every direction at any point of it. When the material under study is soil not subject to stratification, then both assumptions may be understood to have a statistical average value.

Assumption 2 that the potential surfaces of sliding are vertical planes, is unlikely to occur in the actual case and it is made only to simplify the mathematical computations. Actually, as Terzaghi (1943a) points out, the real surfaces of sliding are curved and at the top of the fill their spacing is considerably greater than the width of the conduit. From this, it follows that along the assumed vertical, potential sliding surfaces the internal friction of the material will never be fully mobilized and, thus, plastic equilibrium conditions are not realized. The error due to ignoring this fact is on the unsafe side.

Also, during the relative subsidence of the material above the conduit, horizontal layers within the interior prism do not remain horizontal, but they become either concave or convex curved surfaces depending on whether or not the interior prism subsides more or less than the adjacent masses. Therefore, the surfaces of equal, normal pressure are not plane but are curved, like arches.

The existence of the "plane of equal settlement" was discovered on purely mathematical grounds by Marston (1922). The actual existence of such a plane has been demonstrated by laboratory models, and by measurements of the settlements of the soil both over and adjacent to some experimental conduits (Spangler, 1950a, 1950b).

Assumptions 3 and 4 must be fulfilled in order that the analysis made in this paper

has a meaning. The problems of insuring adequate side support to the conduit as well as designing the conduit structure to withstand the internal stresses that are generated due to the external pressures are beyond the scope of this paper.

Assumption 5 requires that an overall average value of the unit weight of the material be used. Actually, everything else remaining constant the unit weight of the material will vary with the fill height with higher values at the bottom of the fill. The method of fill construction and the water content are major factors influencing γ.

Assumptions 6 and 7 pertain to the values of the angle of internal friction and cohesion that are actually mobilized along the potential sliding planes. Because of the reasoning applied in discussing Assumption 2 , both values will generally be smaller than the laboratory values of ϕ and c exhibited by a series of tests from samples of the same material. Therefore, in the subsequent theoretical treatment of the problem the values of ϕ and c that are used will denote the amount of both properties that are actually mobilized. They will be designated as ϕ_e and c_e respectively. These values depend not only on the nature of the soil and its initial state, but also on the rate of stress application, the permeability of the material, the deformation characteristics, and the size of the mass.

The last assumption, that the ratio of the horizontal principal stress to the vertical principal stress acting on an element within the mass of the material is constant along the potential sliding planes, is at great variance with reality. Everything else remaining constant this ratio depends on the nature, initial state, and strain characteristics of the material.

If the material is a solid block, then the ratio is equal to zero. If the material behaves like a liquid then the ratio is equal to one.

For a semiinfinite, sedimentary deposit of cohesionless material, it has been found experimentally that this ratio varies between 0.45 and 0.55 depending on the geologic history of the deposit and it is approximately the same for every point of the mass. In this particular case, the ratio is called the coefficient of earth pressure at rest, or coefficient of natural earth pressure and it is denoted by K_o. The range of values of K_o for clays in their natural state is not yet known.

If a homogeneous, semiinfinite mass bounded by a horizontal plane and extending to infinity downward and in every horizontal direction is given an opportunity for lateral expansion to a very great depth, z, in such a manner that the lateral strain remains constant with depth, then the mass passes from an initial state of elastic equilibrium to an active state of plastic equilibrium. In this condition the internal resistance of the material is fully mobilized and conditions of incipient shear failure exist along two sets of surfaces of sliding that are symmetrical to each other with respect to a vertical axis and inclined at an angle of $45^\circ - \phi/2$ with the vertical. Under such conditions the lateral intensity of pressure decreases to the smallest value compatible with equilibrium. Such a condition is called an active earth pressure condition. The value of the lateral earth pressure is designated σ_A and the ratio K is equal to

$$K_A = \tan^2 (45^\circ - \phi/2) - \frac{2c}{\gamma z} \tan(45^\circ - \phi/2) \tag{1}$$

for an $s = c + \sigma \tan\phi$ material.

For an $s = \sigma \tan\phi$ material

$$K_A = \tan^2 (45^\circ - \phi/2). \tag{2}$$

For a perfectly cohesive material; that is, for an s = c material

$$K_A = 1 - \frac{2c}{\gamma z}. \tag{3}$$

If the same semiinfinite mass is compressed laterally to a great depth, z, in such a manner that the lateral compressive strain remains constant, then the mass reaches a passive state of plastic equilibrium. In this state the internal resistance of the material is fully mobilized and conditions for incipient shear failure exist along two sets of sliding surfaces, symmetrical to each other with respect to a vertical axis and inclined at an angle equal to $45^\circ + \phi/2$ with the vertical. Under such conditions the lateral intensity of pressure increases to the largest value compatible with equilibrium. Such a condition

is called a state of passive earth pressure. The corresponding lateral pressure is designated σ_p and the ratio K is equal to

$$K_p = \tan^2 (45^\circ + \phi/2) + \frac{2c}{\gamma z} \tan (45^\circ + \phi/2) \tag{4}$$

for an $s = c + \sigma \tan\phi$ material.

For a cohesionless material; that is, for an $s = \sigma \tan\phi$ material

$$K_p = \tan^2 (45^\circ + \phi/2). \tag{5}$$

For a perfectly cohesive material; that is, for an $s = c$ material

$$K_p = 1 + \frac{2c}{\gamma z} \cdot \tag{6}$$

In the actual case, the lateral expansion or compression which cohesive soils must

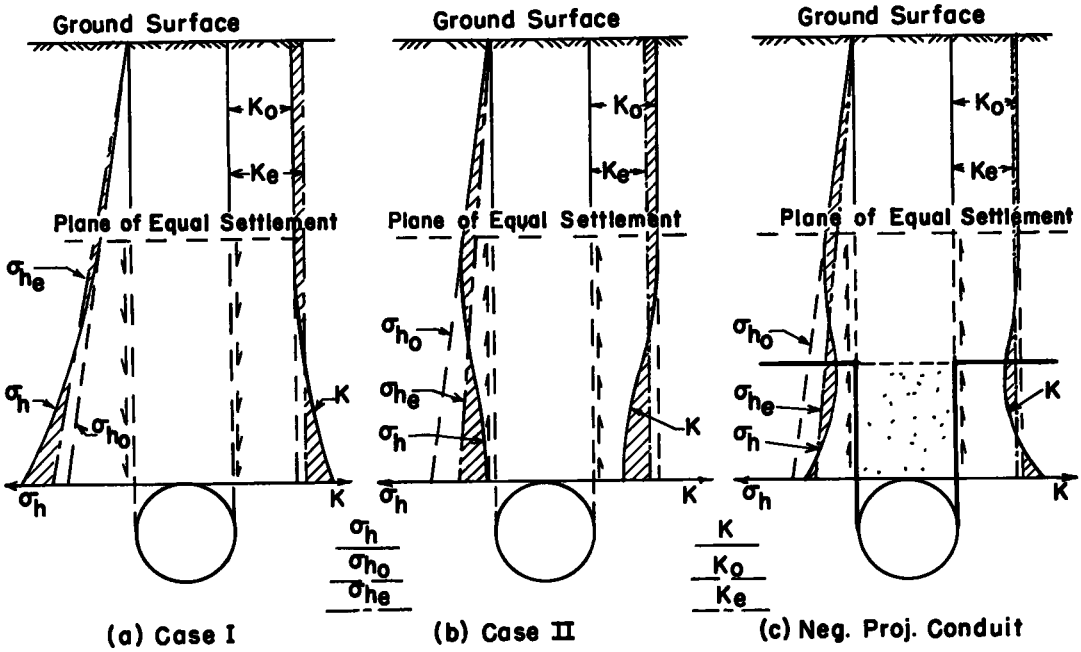

Figure 2. Variation of σ_h and K with fill height.

undergo in order that they reach active or passive states of plastic equilibrium is much greater than any allowable movement within the engineering structures which they bear contact with and, therefore, the ratio K will always lie between the limiting values K_A and K_p.

With cohesionless soils such as dry clean sand, a very small lateral stretching is sufficient to insure active state conditions, whereas a considerable compressive movement must precede passive state conditions.

However, even if the least trace of moisture is present in a cohesionless mass, the material will exhibit a property known as "apparent cohesion" and it will behave like a cohesive material (Terzaghi 1943a). Since in engineering practice water is almost always present in a soil mass, even a granular mass must be stretched laterally a considerable amount before an active state of plastic equilibrium is reached and before K assumes the limiting value K_A.

From Equations 1 and 3 it can be seen that for cohesive materials in an active state of plastic equilibrium K depends mathematically on the fill height and for small values of the fill height it may assume even negative values.

The above discussion on the ratio K was made in reference to constant strain conditions for various materials. If the lateral strain within the mass varies with depth then

K must be expected to vary also.

In the case of an underground conduit, as the middle prism slides along the vertical planes, the lateral strain along these planes may be visualized to vary as follows: Along the vertical extensions of the sliding planes from the top of the embankment to the plane of equal settlement, the lateral strain within the mass is zero because the settlement is uniform at all parts of the mass. Therefore, K may be expected to be constant within this region. If the conduit is mined in a sedimentary deposit of granular material the value of K will be K_o. If the conduit is installed under a man-made granular embankment the value of K will be K_s. K_s will be dependent on the nature and condition of the material, the methods of compaction, the degree of compaction and the height of the fill.

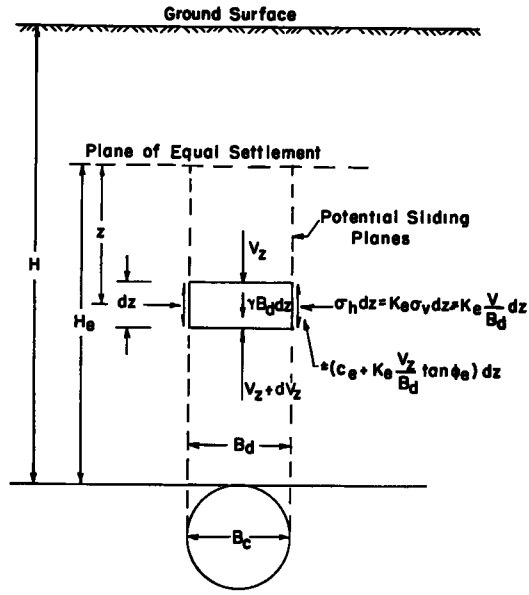

Figure 3. Force diagram for an underground conduit.

In the region between the plane of equal settlement two cases may develop: (1) the adjacent mass may settle more then the interior prism and (2) the interior prism may settle more than the adjacent mass.

In the first case the lateral strain changes from zero at the plane of equal settlement and becomes compressive gradually increasing to a maximum at the top of the conduit. Accordingly, K should be expected to increase from the value K_o or K_s at the plane of equal settlement to a maximum value in the vicinity of the top of the conduit (Figure 2a).

In the second case the lateral strain changes from zero at the plane of equal settlement and becomes tensile gradually increasing to a maximum at the yielding support of the conduit. Accordingly, K should be expected to decrease from the values K_o or K_s at the plane of equal settlement to a smaller value approaching K_A in the vicinity of the yielding support of the middle prism (Figure 2b).

Since the object of the subsequent mathematical treatment is to develop a relation for the load on top of the conduit upon which the integrated influence of K is reflected, the diagram of the variation of K with fill height may be substituted with an equivalent diagram in which K is constant and has a value equal to the mean abscissa, K_e of the diagrams of Figure 2. Thus, the mathematical computations will be simplified appreciably without altering the resulting load expression. Adequate experimentation will give values of K_e for various types of installations and earthen materials.

In Figure 2 the lateral principal stress diagrams σ_{h_o}, σ_h, and σ_{h_e}, corresponding to $K = K_o$, $K = K$, and $K = K_e$ respectively, are also shown for the two cases. From these diagrams it can be seen that the ordinates of the equivalent hydrostatic stress diagram, σ_{h_e}, are larger or smaller in magnitude than the ordinates of the lateral stress at rest diagram, σ_{h_o}, depending on whether the interior prism subsides less or more than the adjacent mass.

Differential Equation Describing the Loading Action of an $s = c + \sigma \tan\phi$ Material on Top of Underground Conduits

Let Figure 3 represent the installation conditions and the force diagram for an underground conduit of external diameter B_c installed under an embankment composed of an $s = c + \sigma \tan\phi$ material.

Let:

H = height of embankment measured from the top of the conduit, ft.

H_e = height of the potential sliding planes from the top of the conduit to the plane of equal settlement. Henceforth, this height will be referred to as "height of arching," ft.

z = distance from the plane of equal settlement down to any horizontal plane, ft.

B_c = width of conduit, ft.

B_d = effective width of the interior prism, ft.

γ = unit weight of the material on top of the conduit, pcf.

ϕ_e = portion of the angle of internal friction of the material that is mobilized along the potential sliding planes.

c_e = portion of the cohesion of the material that is mobilized along the potential sliding planes, psf.

σ_v = vertical principal stress acting on an element of the material along the sliding planes at a distance z from the plane of equal settlement, psf.

σ_h = horizontal principal stress acting on an element of the material along the sliding planes at a distance z below the plane of equal settlement, psf.

$K_e = \frac{\sigma_h}{\sigma_v}$ = equivalent hydrostatic pressure ratio along the sliding planes.

$V_z = \sigma_v B_d$ = resultant vertical pressure acting on a horizontal layer in the interior prism at a distance z from the plane of equal settlement, lb. per lin. ft. of length.

W_c = vertical load on top of the conduit due to overburden material, lb. per lin. ft. of length.

$W = \gamma B_d H$ = weight of the earth column on top of the conduit, lb. per lin. ft. of length.

The weight of the thin slice of the interior prism with a thickness dz at a depth z below the plane of equal settlement is $\gamma B_d dz$ per unit of length perpendicular to the plane of the drawing. The slice is acted upon by the forces indicated in the figure. The condition that the sum of the vertical components that act on the slice must equal to zero can be expressed by the equation

$$\gamma B_d dz + V_z - dV_z - V_z \pm 2\left(c_e + K_e \frac{V_z}{B_d} \tan\phi_e\right) dz = 0, \tag{7}$$

or

$$-\frac{dV_z}{dz} \pm 2K_e \frac{V_z}{B_d} \tan\phi_e \pm 2c_e + \gamma B_d = 0. \tag{8}$$

Equation 8 is the fundamental differential equation describing the conditions of equilibrium during the loading action of an $s = c + \sigma \tan\phi$ material acting on top of an underground conduit. The plus or minus signs represent the case in which the interior prism subsides less or more than the adjacent masses respectively.

Evaluation of the General Load Expression for an $s = c + \sigma \tan\phi$ Material

Equation 8 is a linear differential equation of first order.

Integrating and considering the limits

$$V = (H - H_e)\gamma B_d \quad \text{for } z = 0$$

$$V = V_z \quad \text{for } z = z$$

one obtains after rearranging terms

$$V_z = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{\pm(2K_e \tan\phi_e)\frac{z}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H - H_e}{B_d}\right) \pm \left(1 \pm \frac{2c_e}{\gamma B_d}\right)\right] \mp \left(1 \pm \frac{2c_e}{\gamma B_d}\right) \right\}. \tag{9}$$

When

$$z = H_e \qquad V_z = W_c.$$

Substituting in Equation 9 one obtains

$$W_c = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{\pm(2K_e \tan\phi_e)\frac{H_e}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H - H_e}{B_d}\right) \pm \left(1 \pm \frac{2c_e}{\gamma B_d}\right)\right] \mp \left(1 \pm \frac{2c_e}{\gamma B_d}\right) \right\}. \tag{10}$$

Equation 10 is the general load expression for an $s = c + \sigma \tan\phi$ material. The plus or minus signs represent respectively the cases in which the interior prism subsides

less or more than the adjacent masses.

Equation 10 may be written also

$$W_c = \gamma B_d (B_d/2K_e \tan\phi_e) C, \tag{11}$$

where

$$C = e^{\pm (2K_e \tan\phi_e)\frac{H_e}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H-H_e}{B_d}\right) \pm \left(1 \pm \frac{2c_e}{\gamma B_d}\right)\right] \mp \left(1 \pm \frac{2c_e}{\gamma B_d}\right). \tag{12}$$

Henceforth, factor C will be called the "load factor."

Letting

$$(B_d/2K_e \tan\phi_e) C = H_{eff} \tag{13}$$

and substituting in Equation 11 one obtains

$$W_c = \gamma B_d H_{eff}. \tag{14}$$

Factor H_{eff} may be thought of as an effective height along which no relative subsidence occurs between the material directly above the conduit and the adjacent material. In such case neither mass would tend to brace itself against the adjacent one, no sliding surfaces would tend to form, and the load on top of the conduit per unit length would be equal to the full weight of the column of the material directly above it.

By inspection of Equation 12 , and since:

$$C = \left[(2K_e \tan\phi_e)/B_d \right] H = C_o,$$

$$H_{eff} = H,$$

and

$$W_c = \gamma B_d H = W, \tag{15}$$

when

$$H_e = 0,$$

it can be seen that if the interior prism subsides less than the adjacent masses, in which case the shearing resistance of the material mobilized along the sliding planes have the same direction and sense as the weight of any thin slice within the interior prism, the positive signs are used in Equations 7 through 12,

$$C_p = \text{Load factor with positive signs} > C_o ,$$

$$H_{eff} > H,$$

and

$$W_c > W.$$

Similarly, if the interior prism subsides more than the adjacent masses, in which case the shearing resistance of the material mobilized along the sliding planes has the same direction but opposite sense than the weight of any thin slice within the interior prism, the negative signs are used in Equations 7 through 12,

$$C_n = \text{load factor with negative signs} < C_o,$$

$$H_{eff} < H,$$

and

$$W_c < W.$$

In the subsequent analysis the above two cases will be studied separately. However, every engineer dealing with underground conduits should direct all his efforts toward creating the proper environmental conditions during the construction of such structures in order that conditions corresponding to the second case will be realized.

Case I. The Interior Prism Subsides Less Than the Adjacent Masses

This case may develop as a result of the following two environmental conditions in the construction of a conduit.

1. In the case of a covered-up conduit, the conduit is installed by means of the so-called "positive projection" method (Spangler 1946). According to this method the conduit is installed with its top projecting some distance above the natural ground surface. Then, the fill material is placed around and on top of the conduit. No special effort is made to compact the side material to a higher degree of compaction than the rest of the fill material (Figure 4).

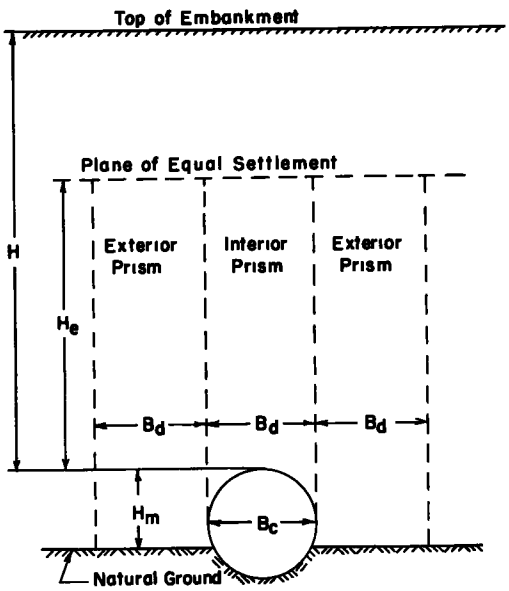

Figure 4. Installation diagram for a positive-projecting conduit.

Assuming that the natural ground surface settles by the same amount everywhere, let us compare the vertical deformation of the interior prism with the deformation of the two adjacent masses extending from the natural ground surface to the plane of equal settlement and having a width equal to the width of the interior prism which in this case is equal to the width of the conduit. Henceforth, these two masses will be called "exterior prisms."

All three prisms are loaded with the same overburden weight equal to $(H-H_e)\gamma B_d$. Therefore, any relative differential deformation existing among them would be a function of the weight of each prism which, accordingly, is a function of its height as well as the characteristics of the material. Hence, if no contact existed among these prisms and each one were allowed to deform freely, the summation of deformations from the bottom upward would normally be at a greater rate in the high prisms than in the lower ones.

Since the two exterior prisms are higher than the interior prism by an amount H_m, and since the material within this region is compacted by the same amount as any other part of the fill, the exterior prisms will tend to settle at a greater rate than the interior prism. However, in the actual case all three prisms are in contact with each other and, consequently, the exterior prisms transfer part of their vertical pressures to the interior prism. The result is that, because of this stress transfer, the rate of summation of vertical deformations will be reduced in the exterior prisms and increased in the interior prism. The total summation of deformations in the interior prism will approach that in the exterior prisms, and the height at which the deformations become equal is the height of equal settlement (Marston 1922).

2. In the case of a mined-in conduit, the conduit is installed in a bed of a very soft compressible material, and the conduit is too rigid to "give in" under the influence of the top vertical load. Under such conditions it is conceivable that the material adjacent to the conduit will have the tendency to settle more than the material on top of it. Therefore, as in the case of "positive projecting conduits," the exterior prisms will tend to brace themselves against the interior prism and in doing so they will transfer part of their vertical pressures on to the interior prism.

From the above discussion and for reasons which were discussed, one should use the positive signs in the general load expression when the conditions insuring the existence of Case I have been realized.

Hence, Equation 10 becomes

$$W_c = \frac{\gamma B^2_d}{2K_e\tan\phi_e}\left\{e^{+(2K_e\tan\phi_e)\frac{H_e}{B_d}}\left[(2K_e\tan\phi_e)\left(\frac{H-H_e}{B_d}\right) + \left(1+\frac{2c_e}{\gamma B_d}\right)\right] - \left(1+\frac{2c_e}{\gamma B_d}\right)\right\}, \quad (16)$$

from which

$$C_p = e^{+(2K_e\tan\phi_e)\left(\frac{H_e}{B_d}\right)}\left[(2K_e\tan\phi_e)\left(\frac{H-H_e}{B_d}\right) + \left(1+\frac{2c_e}{\gamma B_d}\right)\right] - \left(1+\frac{2c_e}{\gamma B_d}\right). \quad (17)$$

A quick inspection of Equations 16 and 17 will show that in this case the shearing resistance of the material on top of the conduit works against the engineer; the more resistant to shear the material is and the larger the portion of its shear components that is mobilized along the sliding planes, the greater will be the load on top of the conduit.

Furthermore, from Figure 2b, it was shown that in Case I the equivalent hydrostatic pressure ratio K_e will generally be larger in magnitude than the coefficients of earth pressure at rest, K_o, or K_s. By inspecting Equations 16 and 17 again, one can also see

that the larger the value of K_e the larger will be the load.

Now let us examine what other serious implications Case I might have on the load expression.

Equations 16 and 17 contain the ascending exponential function e^u, where $u = (2K_e \tan\phi_e) H_e/B_d \geqslant 0$, multiplied by a positive sum. The ascending exponential function is equal to 1 for u = 0, and increases very rapidly with increasing values of u. For example:

$$\text{if } u = 1,\ e^u \approx 2.7;\ \text{if } u = 2,\ e^u \approx 7.4;$$
$$\text{if } u = 4,\ e^u \approx 54;\ \text{if } u = 8,\ e^u \approx 2980,\ \text{etc.}$$

Therefore, if the over-all height of the material on top of the conduit is in the region of 100 ft. or more, which with modern construction equipment has come within the realm of engineering endeavor, the load on top of the conduit, W_c, will be many times greater than the weight of the column of the material, W. Consequently, even if the side-supporting material is able to mobilize sufficient reactive pressure to equalize the top pressure before the structure bulges out excessively, the ring stresses that are generated in the conduit structure will exceed the critical buckling load of the conduit and the results will be catastrophic.

To illustrate the above, let

$$H = 100 \text{ ft.}$$
$$B_d = 5.0 \text{ ft.}$$
$$K_e = 1.0$$
$$\phi_e = 10^0$$
$$c_e = 200 \text{ psf.}$$
$$\gamma = 120 \text{ pcf.}$$

The weight of the column of the material above the conduit is, therefore, $W = \gamma B_d H$ = 60,000 lb. per lin. ft.

Substituting the above data in Equations 16 and 17 and solving for $H_e = 0$, H_e = 10 ft., H_e = 20 ft., and H_e = 50 ft. one obtains respectively:

H_e	C_p	H_{eff}	H_{eff}/H	W_c/W
0	7.1	100	1.0	1.0
10 ft.	14.6	207 ft.	2.1	2.1
20 ft.	28.3	401 ft.	4.0	4.0
50 ft.	176	2495 ft.	25.0	25.0

In other words, if the height of arching is one-half the fill height, the load on a 5.0 ft. diameter conduit due to a 100 ft. fill will be almost twenty-five times the weight of the column of the material on top of it; i.e., W_c = 1,320,000 lb. per lin. ft. No conceivable factor of safety employed in the design of the conduit will provide for such a possibility and stay within reasonable economical limits.

From the above, one may conclude that conditions for Case I are very undesirable from the engineering standpoint and, therefore, every effort should be made to avoid them in the field.

If a conduit is installed by the "positive projection" method, the material immediately adjacent to the conduit should be thoroughly compacted to a much higher degree than the remainder of the fill material. If such a procedure is followed, the stiffness of the mass within the height H_m will be much greater than that of the material within the rest of the exterior prism. Consequently, the effective height of the exterior prism will be decreased to a value approaching the height of the interior prism. Furthermore, if the conduit is sufficiently flexible, the support furnished by the stiffened mass to the shortened exterior prism, will yield much less than the support under the interior prism. Therefore, the reverse action will take place; the interior prism will tend to brace itself against the exterior prisms thereby reducing the load on top of the conduit.

In the case of a mined-in conduit within a bed of soft compressible material, if the conduit is made sufficiently flexible so as to adjust its shape to any external differential pressure, then, even if the top load is originally greater in magnitude than the weight of the column of the material, a subsequent change in the conduit shape will result in a redistribution of the external pressures. Further changes in the conduit shape will result in further redistribution of the external pressures and this process will continue until

all differential moments that are generated within the conduit structure are eliminated and only axial ring stresses will exist. Hence, if the conduit is designed to withstand these stresses, no failure will occur and the conduit will function satisfactorily.

Case II. The Interior Prism Subsides More Than the Adjacent Masses

This case will be discussed in detail, because it is most likely to occur in the field. It may be present even in positive projecting conduits, provided their side supporting material has been compacted very thoroughly. The engineer should always be able to visualize the action which takes place in this case and to know what to expect in terms of load ranges from various construction methods and materials.

The existence of Case II is insured by the following construction methods and conditions:

1. Covered-up conduits are installed by the following three methods:

(a) The Ditch Conduit Method. According to this method (Spangler, 1946) the conduit is placed in a ditch not wider than two or three times its outside width and it is covered up with backfill material that is in a relatively loose condition as compared to the natural ground in which the ditch is dug. (Figure 5a).

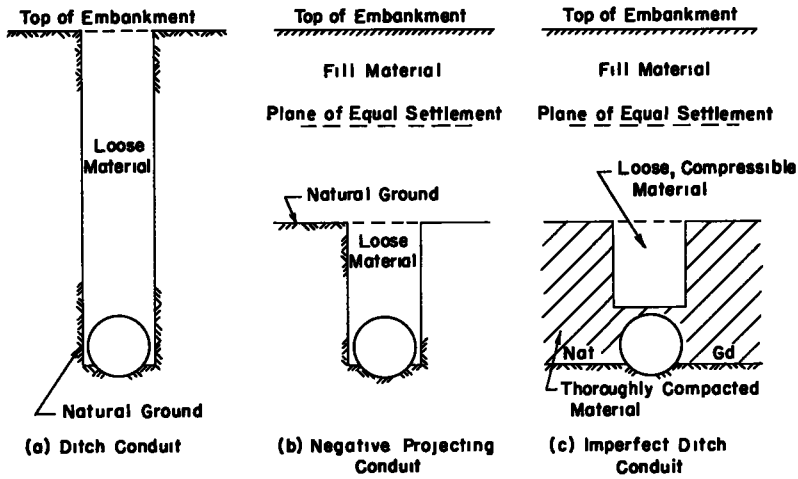

Figure 5. Covered-up conduits.

In a ditch conduit the potential sliding planes will be the walls of the ditch. The backfill material has the tendency to settle downward. In doing so it tends to brace itself against the sides of the ditch transferring part of its weight onto the natural ground. Thus, the load on top of the conduit is reduced by an equal amount.

(b) The Negative Projecting Conduit Method. Conduits falling within this category are placed in shallow ditches of such depths that the top of the conduit is below the adjacent natural ground surface that is covered by an embankment as shown in Figure 5b (Spangler, 1946).

(c) The Imperfect Ditch Conduit Method. In this method of construction the conduit is originally installed as a positive projecting conduit (Spangler, 1946). The soil on both sides and above the conduit for some distance above its top is thoroughly compacted. Then a ditch is dug in this compacted fill by removing the prism of material directly over the conduit. The ditch is refilled with very loose compressible material, after which the embankment is compacted above it (Figure 5c).

In the last two cases the potential sliding planes are assumed to be the vertical extensions of the sides of the ditch on top of the conduit. These planes will extend as far as the plane of equal settlement. In both cases the material on top of the ditch will subside more than the adjacent masses. The loose material in the ditch furnishes a support that yields much more than the adjacent natural ground in the case of a negative projecting conduit or more than the very well compacted material in the case of an imperfect ditch conduit.

2. In the case of a mined-in conduit that is flexible enough so that its roof will give in sufficiently to act as a yielding support to the material above, three cases (Terzaghi, 1942-1943, 1943a, 1943b) are of interest:

(a) The conduit is installed through cohesive material and its lower part is located within an exceptionally stiff layer of clay between soft layers (Figure 6a). The sliding planes will extend through the edges of the bottom of the conduit (Terzaghi, 1942-1943).

(b) If the cohesive material on both sides of the conduit is not exceptionally stiff (Figure 6b), the width of the interior prism is approximately $B_d = B_c + 2H_m$ (Terzaghi, 1942-1943).

(c) The conduit is installed through cohesionless granular material (Figure 6c). In this case, because of the yield of the timbering and the imperfection of the joints on the sides of the conduit, the granular material adjoining these sides subsides to the same extent as the subsiding material on top of the conduit on account of the yield of its roof. This lateral yield may cause the granular mass to come to an active state of plastic equilibrium. In such case the boundaries of the zone of subsidence will rise at the bottom of the conduit at an angle $45^\circ - \phi/2$ with respect to the vertical and gradually the boundaries will become vertical at the plane of equal settlement. The width of the interior prism will, therefore, be equal to:

$$B_c + 2H_1 \tan(45^\circ - \phi/2) = B_d \text{ on top of the conduit}$$

and

$$B'_d \text{ at the plane of equal settlement where } B'_d > B_d.$$

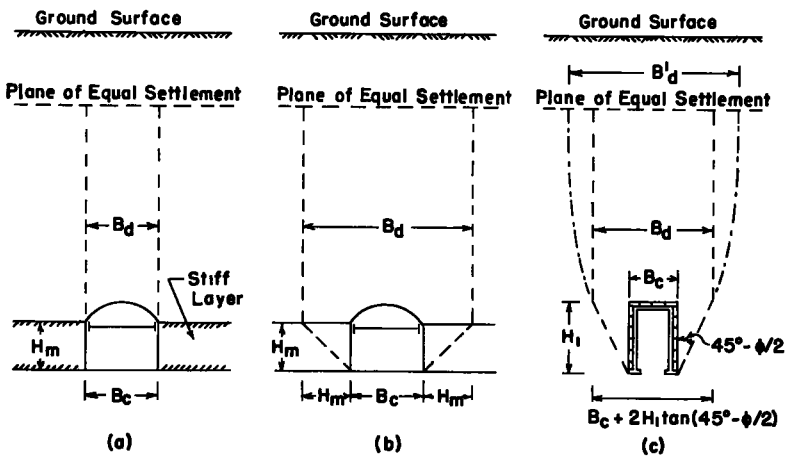

Figure 6. Mined-in conduits.

In order that the mathematical computations be simplified, it is assumed that the effective width of the interior prism is equal to B_d throughout the height from the top of the conduit to the plane of equal settlement (Terzaghi, 1943a).

From the above discussion and for reasons that were discussed previously one should use the negative signs in the general load expression when the conditions insuring the existence of Case II have been realized.

Hence, Equation 10 becomes

$$W_c = \frac{\gamma B^2_d}{2K_e \tan\phi_e} \left\{ e^{-(2K_e \tan\phi_e)\frac{H_e}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H - H_e}{B_d}\right) - \left(1 - \frac{2c_e}{\gamma B_d}\right) \right] + \left(1 - \frac{2c_e}{\gamma B_d}\right) \right\}, \quad (18)$$

from which

$$C_n = e^{-(2K_e \tan\phi_e)\frac{H_e}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H - H_e}{B_d}\right) - \left(1 - \frac{2c_e}{\gamma B_d}\right) \right] + \left(1 - \frac{2c_e}{\gamma B_d}\right). \quad (19)$$

A quick inspection of Equations 18 and 19 will show that the shearing resistance of the material on top of the conduit works to the engineer's advantage. The more resistant

to shear the material is and the larger the portion of its shear components that is mobilized along the sliding planes, the lower will be the load on top of the conduit.

The above discussion may be expressed in mathematical form as follows:

$$\lim_{\phi_e \to 90^0} W_c = 0, \tag{20}$$

$$\lim_{c_e \to \infty} W_c = -\infty . \tag{21}$$

Equation 21 has mathematical meaning only. Physically, it may mean that for a certain installation, if the material is able to mobilize a sufficient amount of cohesion and if the deformation characteristics within the mass are such that such an amount is mobilized along the sliding planes, the load on top of the conduit will be a minimum approaching zero.

Let us see now what other implications some other conditions may bring on the load expression.

Equations 18 and 19 contain the descending exponential function e^{-u} where $u = (2K_e \tan\phi_e)\, H_e/B_d \geqslant 0$. This function is equal to 1 for $u = 0$ and decreases very rapidly with increasing positive values of u, and approaches zero. For example:

if $u = 1$, $e^{-u} \approx 0.3679$; if $u = 2$, $e^{-u} \approx 0.1353$;
if $u = 4$, $e^{-u} \approx 0.0183$; and, if $u = 8$, $e^{-u} \approx 0.003$, etc.

From the above it can be seen that if $u \gg 1$ the first part of Equation 19 will become negligible and

$$C_n \approx 1 - \left[2c_e/\gamma B_d \right], \tag{22}$$

from which

$$W_c \approx \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left(1 - \frac{2c_e}{\gamma B_d}\right). \tag{23}$$

Hence, if the material is potentially able to mobilize along the sliding planes an amount of cohesion equal to $c_e = \frac{\gamma B_d}{2}$ the load on top of the conduit will be:

$$W_c \approx 0. \tag{24}$$

The above expression is at variance with reality because the general load expression was evaluated on the assumption that the normal stresses in the interior prism are the same everywhere on a horizontal layer. Actually, the surfaces of equal normal stresses will be curved like arches. If the conduit has a flat roof, then the region within the surface of zero pressure and the roof of the conduit will be in a state of tension. Consequently, the material within this planoconvex region will have the tendency to drop out of the roof. As Terzaghi points out, "in order to prevent such an accident, an unsupported roof in a tunnel through cohesive earth should always be given the shape of an arch."[6]

In the case of either a covered-up or a mined-in conduit whose top is curved, such as in the case of circular, eliptical, or oval shaped conduits, Equation 24 may describe conditions very close to reality if the proper deformation conditions are insured within the mass and if the material is able to mobilize a sufficient amount of cohesion along the sliding planes.

From the above discussion, it was shown that if the factor $u = (2K_e \tan\phi_e)\, H_e/B_d$ is made sufficiently large, the load factor C_n and, accordingly, the load W_c will become minimum on top of the conduit. Therefore, an understanding of the behavior of the factor u for various physical conditions is considered to be an indispensable guide in directing the engineer's judgment when dealing with underground conduit design.

In the following chapter, a study of the factors governing the behavior of u will be made for covered-up as well as for mined-in conduits.

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[6] K. Terzaghi, Theoretical Soil Mechanics (New York: John Wiley & Sons, 1943), p. 199.

Evaluation of the General Expression Governing the Behavior of Factor $u = (2K_e \tan\phi_e) H_e/B_d$ for Case II

1. Evaluation of u for Covered-Up Conduits. In this treatment a negative projecting conduit represents the general case. An imperfect ditch conduit as well as a ditch conduit can be deduced as special cases.

Let Figure 7 represent a negative ditch conduit installation in which the previous notation is employed with the addition of the following:

H_d = height of ditch above the top of the conduit, ft.

$H' = H - H_d$ = height of fill above the top of the compacted material, ft.

$H'_e = H_e - H_d$ = height of the plane of equal settlement above the surface of the compacted material, ft.

s_f = settlement of the conduit foundation, ft.

d_c = shortening of the vertical dimension of the conduit, ft.

s_d = compression of the loose material in the ditch within the distance H_d, ft.

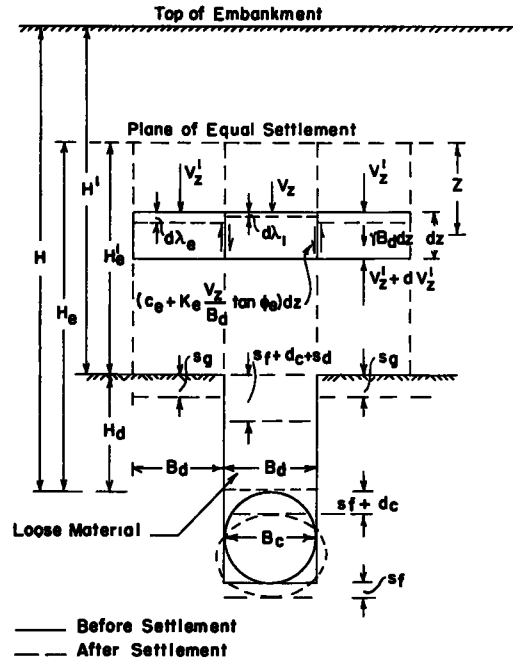

Figure 7. Force diagram for a covered-up conduit.

$s_f + d_c + s_d$ = settlement of the surface of the loose material, ft.

s_g = settlement of the surface of the compacted material, ft.

r_{sd} = settlement ratio = $[s_g - (s_d + d_c + s_f)]/s_d$

V'_z = resultant vertical pressure acting on a horizontal layer of width B_d in the exterior prism at a distance z from the plane of equal settlement, lb. per lin. ft. of length.

λ'_i = compression of the interior prism between the surface of the compacted material and the plane of equal settlement due to the vertical pressure within the fill height H', ft.

λ'_e = compression of the exterior prisms between the surface of the compacted material and the plane of equal settlement due to the vertical pressure within the fill height H', ft.

E_F = modulus of deformation of all fill material except the loose mass in the ditch within the distance H_d, lb. per ft. per ft.

E_L = modulus of deformation of the loose mass in the ditch within the distance H_d, lb. per ft. per ft.

$a' = E_L/E_F$.

The following assumptions must be made in addition to the previously stated basic assumptions:

(a) The average behavior of both the compacted and the loose fill materials is such that these materials may be considered to obey Hooke's law when subjected to compression. Their respective moduli E_F and E_L, therefore, are assumed to be constant within any region of the fill.

(b) The settlement ratio r_{sd} is considered to be constant throughout the life of the conduit.

(c) The internal friction of the fill materials distributes the infinitely small decrements of pressure from shear into the interior prism below the plane of equal settlement in such a manner that the effect on settlement is substantially the same as for uniform vertical pressure (Spangler, 1950a).

(d) The internal friction in the fill materials distributes the infinitely small increments of pressure from shear onto each of the exterior prisms below the plane of equal

settlement in such a manner that the effect on settlement is substantially the same as though the pressure were distributed uniformly over a width of prism equal to the width of the interior prism, B_d (Spangler, 1950a).

(e) With the exception of the moduli of deformation both the compacted and the loose masses exhibit the same physical properties.

Assumptions (a) through (e) are made in order that the subsequent mathematical treatment will be simplified. Their variance with reality depends upon the nature of the materials used, the method of construction, and the magnitude of the quantities involved. The engineer's judgment, based on previous experience, will determine how large the involved error is and what allowances should be made in each individual case.

To evaluate factor u one must consider the deformation characteristics of the interior and exterior prisms.

The over-all settlement of the interior prism at the plane of equal settlement must equal the over-all settlement of the exterior prism at the same plane.

Hence

$$\lambda_i' + s_d + d_c + s_f = \lambda_e' + s_g \tag{25}$$

or

$$\lambda_i' = \lambda_e' + s_g - (s_d + d_c + s_f). \tag{26}$$

Since

$$r_{sd} = \left[s_g - (s_d + d_c + s_g)\right] / s_d,$$

Equation 26 may be written

$$\lambda_i' = \lambda_e' + r_{sd} s_d. \tag{27}$$

Since the material within the interior and exterior prisms is assumed to obey Hooke's law, the vertical compression of a thin horizontal slice of the interior prism with thickness dz at a depth z below the plane of equal settlement must equal

$$d\lambda_i = (V_z/B_d E_F)\, dz. \tag{28}$$

Similarly, the vertical compression of a thin horizontal slice of the exterior prism with thickness dz at a depth z below the plane of equal settlement must equal

$$d\lambda_e = (V_z'/B_d E_F)\, dz. \tag{29}$$

Substituting in Equation 28 the value of V_z from Equation 9, in which the negative signs have been employed, and integrating between the limits

$$\lambda_i = 0 \quad \text{for} \quad z = 0$$

$$\lambda_i = \lambda_i' \quad \text{for} \quad z = H_e'$$

one obtains after rearranging terms

$$\lambda_i' = \frac{\gamma B_d^2}{E_F} \frac{1}{(2K_e \tan\phi_e)^2} \left\{ e^{-(2K_e \tan\phi_e)\frac{H_e'}{B_d}} \left[\left(1 - \frac{2c_e}{\gamma B_d}\right) - 2K_e \tan\phi_e \left(\frac{H' - H_e'}{B_d}\right) \right] + \right.$$
$$\left. \left[2K_e \tan\phi_e \left(\frac{H' - H_e'}{B_d}\right) + \left(1 - \frac{2c_e}{\gamma B_d}\right)\left((2K_e \tan\phi_e)\left(\frac{H_e'}{B_d}\right) - 1 \right) \right] \right\}. \tag{30}$$

To evaluate V_z' the conditions of static equilibrium are considered for a thin slice of the exterior prism with a thickness dz at a depth z below the plane of equal settlement (Figure 7). The conditions that the sum of the vertical forces that act on the slice must equal zero can be expressed by the equation

$$\gamma B_d dz + \left(c_e + K_e \frac{V_z}{B_d} \tan\phi_e\right) dz + V_z' - V_z' - dV_z' = 0, \tag{31}$$

or

$$dV_z' = \left(\gamma B_d + c_e + K_e \frac{V_z}{B_d} \tan\phi_e\right) dz. \tag{32}$$

Substituting in Equation 32 the value V_z from Equation 9 and integrating between the limits

$$V_z' = (H - H_e)\, \gamma B_d \quad \text{for} \quad z = 0$$

$$V_z' = V_z' \quad \text{for} \quad z = z$$

or, since $H - H_e = H' - H'_e$, between the limits

$$V'_z = (H' - H'_e)\,\gamma B_d \quad \text{for } z = 0$$

$$V'_z = V'_z \quad \text{for } z = z$$

one obtains after rearranging terms

$$V'_z = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left\{\frac{3}{2}\cdot(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d} + \frac{z}{B_d}\right)\right.$$

$$\left. - \frac{1}{2}\left[e^{-(2K_e\tan\phi_e)\frac{z}{B_d}}\left\{(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d}\right) - \left(1 - \frac{2c_e}{\gamma B_d}\right)\right\} + \left(1 - \frac{2c_e}{\gamma B_d}\right)\right]\right\},$$

or

$$V'_z = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left[\frac{3}{2}(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d} + \frac{z}{B_d}\right)\right] - \frac{1}{2}V_z. \tag{33}$$

Substituting in Equation 29 the value V_z from Equation 33 one obtains

$$d\lambda_e = \frac{1}{B_d E_F}\cdot\frac{\gamma B_d^2}{2K_e\tan\phi_e}\left[\frac{3}{2}(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d} + \frac{z}{B_d}\right)\right]dz - \frac{1}{2}\frac{V_z}{B_d E_F}\,dz,$$

or, from Equation 28,

$$d\lambda_e = \frac{1}{B_d E_F}\cdot\frac{\gamma B_d^2}{2K_e\tan\phi_e}\left[\frac{3}{2}(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d} + \frac{z}{B_d}\right)\right]dz - \frac{1}{2}d\lambda_i. \tag{34}$$

Integrating between the limits

$$\lambda_e = 0 \quad \lambda_i = 0 \quad \text{for } z = 0$$

$$\lambda_e = \lambda'_e \quad \lambda_i = \lambda'_i \quad \text{for } z = H'_e$$

one obtains after rearranging terms (35)

$$\lambda'_e = \frac{\gamma B_d^2}{E_F}\,\frac{1}{(2K_e\tan\phi_e)^2}\left\{\frac{3}{2}\left[(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d}\right) + \frac{1}{2}(2K_e\tan\phi_e)\frac{H'_e}{B_d}\right]\right\}(2K_e\tan\phi_e)\frac{H'_e}{B_d} - \frac{1}{2}\lambda'_i.$$

Since the loose material in the ditch is considered to obey Hooke's law, the vertical compression of the prism within the distance H_d due to the vertical pressure $V_z = H'_e$ on top of the ditch is

$$s_d = \frac{V_z = H'_e}{B_d E_L}\cdot H_d. \tag{36}$$

Substituting $z = H'_e$ in Equation 9 and since $H - H_e = H' - H'_e$ one obtains

$$V_{z = H'_e} = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left\{e^{-(2K_e\tan\phi_e)\frac{H'_e}{B_d}}\left[(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d}\right) - \left(1 - \frac{2c_e}{\gamma B_d}\right)\right] + \left(1 - \frac{2c_e}{\gamma B_d}\right)\right\}. \tag{37}$$

Hence, Equation 36 becomes

$$s_d = \frac{\gamma B_d^2}{E_L B_d}\,\frac{H_d}{2K_e\tan\phi_e}\left\{e^{-(2K_e\tan\phi_e)\frac{H'_e}{B_d}}\left[(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d}\right) - \left(1 - \frac{2c_e}{\gamma B_d}\right)\right] + \left(1 - \frac{2c_e}{\gamma B_d}\right)\right\},$$

or

$$s_d = \frac{\gamma B_d^2}{a' E_F}\,\frac{1}{(2K_e\tan\phi_e)^2}\cdot(2K_e\tan\phi_e)\frac{H_d}{B_d}\left\{e^{-(2K_e\tan\phi_e)\frac{H'_e}{B_d}}\left[(2K_e\tan\phi_e)\left(\frac{H' - H'_e}{B_d}\right) - \left(1 - \frac{2c_e}{\gamma B_d}\right)\right]\right.$$

$$\left. + \left(1 - \frac{2c_e}{\gamma B_d}\right)\right\}. \tag{38}$$

Substituting the values of λ'_i, λ'_e, and s_d from Equations 30, 35, and 38 in Equation 27 and letting

$$v' = (2K_e\tan\phi_e)\frac{H'}{B_d}, \tag{39}$$

$$u' = (2K_e\tan\phi_e)\frac{H'_e}{B_d}, \tag{40}$$

$$w' = (2K_e\tan\phi_e)\frac{H_d}{B_d}, \tag{41}$$

one obtains after collecting terms

$$v' = \frac{\left(\frac{3}{4}u'^2 - \frac{3c_e}{\gamma B_d}u'\right) - \left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)\left(1 - \frac{2c_e}{\gamma B_d}\right) + \left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)\left(u' + 1 - \frac{2c_e}{\gamma B_d}\right)e^{-u'}}{\left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)e^{-u'} + \frac{3}{2}(u' - 1)} \quad . \tag{42}$$

Equation 42 governs the behavior of u' for a given installation and material. Since $H_e = H_d + H'_e$, it follows that $u = u' + w'$. Therefore, Equation 42 governs the behavior of factor u as well. All other quantities are independent variables in Equation 42.

Factor u' can be obtained from the above equation implicitly. This, however, would be a cumbersome and time consuming operation for design purposes. Since v' is a single valued function of u', one may solve Equation 39 for v' and construct curves from which u' can be obtained in a reverse manner for a given installation and material.

An inspection of Equation 42 will show that if the denominator

$$\left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)e^{-u'} + \frac{3}{2}(u' - 1)$$

approaches zero, v' increases without limit.

The physical significance of the above is that for a given material and conduit width, if the fill is made very high, factor u' and, accordingly, the height of arching, H_e, does not depend on the cohesion and the unit weight of the material.

Hence, no matter what the values of cohesion or the unit weight of the material are, for infinitely high fills, the height of arching is governed by the equation

$$\left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)e^{-u'} + \frac{3}{2}(u' - 1) = 0. \tag{43}$$

It should be noted that in Equation 42 u' can be larger in magnitude than v' for certain conditions. However, physically, u' is limited in the region $0 \leq u' \leq v'$ because the height of arching, H_e, can vary only in the region $H_d \leq H_e \leq H$.

If u' is mathematically larger than v', the plane of equal settlement becomes imaginary. In such case, a trough-like depression appears at the surface of the embankment directly above the conduit.

If u' is mathematically smaller than v', then the arching effect does not extend along the whole fill height. Consequently, the plane of equal settlement will be below the top of the embankment, and no settlement will be noticeable at the surface.

The above discussion holds for both imperfect ditch and negative projecting conduits because no differentiation was made between the stiffness of the thoroughly compacted material and the stiffness of the natural ground in the above theoretical treatment.

In the case of a ditch conduit:

$$H_d = H_e = H. \tag{44}$$

Substituting Equation 44 in Equation 18 one obtains as the load expression for a ditch conduit and an $s = c + \sigma \tan\phi$ material

$$W_c = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left\{e^{-(2K_e\tan\phi_e)\,H/B_d}\left[-\left(1 - \frac{2c_e}{\gamma B_d}\right)\right] + \left(1 - \frac{2c_e}{\gamma B_d}\right)\right\},$$

or

$$W_c = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left\{\left(1 - \frac{2c_e}{\gamma B_d}\right)\left(1 - e^{-(2K_e\tan\phi_e)\,H/B_d}\right)\right\}. \tag{45}$$

If $H \gg 1$

$$W_c \approx \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left(1 - \frac{2c_e}{\gamma B_d}\right), \tag{46}$$

which is identical to Equation 23.

Letting $(2K_e\tan\phi_e)\,H/B_d = v$, and substituting in Equation 45 one obtains

$$W_c = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\left\{\left(1 - \frac{2c_e}{\gamma B_d}\right)\left(1 - e^{-v}\right)\right\}, \tag{47}$$

from which

$$C_n = (1 - \frac{2c_e}{\gamma B_d})(1 - e^{-v}) . \tag{48}$$

The method of utilizing dimensionless factors reduces the number of independent variables in any problem and facilitates the mathematical computations considerably. Therefore, in the subsequent analysis their use will be extensive.

Since $u = w' + u'$ and $H - H_e = H' - H'_e$ in imperfect ditch and negative projecting conduits, by substituting the dimensionless factors of Equations 39, 40, and 41 in Equations 18 and 19 one obtains, respectively

$$W_c = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{-w'} e^{-u'} \left[(v' - u') - (1 - \frac{2c_e}{\gamma B_d}) \right] + (1 - \frac{2c_e}{\gamma B_d}) \right\} \tag{49}$$

and

$$C_n = e^{-w'} e^{-u'} \left[(v' - u') - (1 - \frac{2c_e}{\gamma B_d}) \right] + (1 - \frac{2c_e}{\gamma B_d}) . \tag{50}$$

As has been discussed previously, if either of the two exponents w' and u' in Equation 50 are large enough, C_n will approach the value $1 - (2c_e/\gamma B_d)$.

Factor u' is governed by Equation 42 in which many independent variables must be determined in order that this factor can be evaluated.

Factor w', however, is an independent variable in Equation 42 and depends only on the properties of the material, the width of the conduit, and the height of the ditch on top of the conduit. Therefore, for a given material and width of conduit, if the height of ditch is made large enough so that $w' \gg 1$ then the load on top of the conduit will be

$$W_c \approx \frac{\gamma B_d^2}{2K_e \tan\phi_e} (1 - \frac{2c_e}{\gamma B_d}) , \tag{51}$$

which is identical to Equations 23 and 46.

Again, if the cohesion of the material that is mobilized along the sliding planes is equal to $c_e = \frac{\gamma B_d}{2}$ theoretically there should be no load on top of the conduit.

Equation 42 may be written also (52)

$$(\frac{3}{2} + \frac{r_{sd} w'}{\alpha'})(1 - e^{-u'}) - \frac{3}{2} u' = \frac{\gamma B_d}{2} \left\{ \frac{(\frac{3}{2} + \frac{r_{sd} w'}{\alpha'}) \left[1 + (v' - u' - 1) e^{-u'} \right] - \frac{3}{4} u'^2 + \frac{3}{2} v' (u' - 1)}{c_e} \right\} .$$

If c_e is allowed to increase without limit, the left hand member of Equation 52 will approach zero. Hence in the limit one obtains

$$(\frac{3}{2} + \frac{r_{sd} w'}{\alpha'})(1 - e^{-u'}) - \frac{3}{2} u' = 0. \tag{53}$$

Equation 53 may also be written

$$e^{-u'} = 1 - \frac{\frac{3}{2} u'}{(\frac{3}{2} + \frac{r_{sd} w'}{\alpha'})} . \tag{54}$$

From Equation 53 or 54 it can be seen that for all real values of the parameter $r_{sd} w'/\alpha'$, the only solution of Equation 54 is $u' = 0$.

One may conclude, therefore, that

$$\lim_{c_e \to \infty} u' = 0 \tag{55}$$

In a similar manner it can be shown that

$$\lim_{\left|\frac{r_{sd} w'}{\alpha'}\right| \to \infty} u' = \infty , \tag{56}$$

and

$$\lim_{\frac{r_{sd} w'}{\alpha'} \to 0} u' = 0 \tag{57}$$

The physical significance of Equations 55, 56, and 57 is as follows:

(a) The higher the amount of cohesion that is mobilized along the sliding planes, the lower will be the height of arching. However, in such case the quantity $1 - (2c_e/\gamma B_d)$ becomes the predominant factor in the general load equation. Therefore, although in the same equation the descending exponential e^{-u} will become maximum for an infinite amount of cohesion, the load on top of the conduit, as it has been discussed in the previous section will vanish.

(b). For a given installation and material, the height of arching H_e varies directly with the settlement ratio r_{sd}, the height of the ditch on top of the conduit, H_d, and the relative stiffness between the compacted fill material and the loose material in the ditch, which is expressed by the ratio $1/\alpha'$. Therefore, the larger the above quantities are, the higher will be the height of arching and, consequently, the lower will be the load on the conduit.

From the above discussion it can be seen that if w' is made large enough, not only will the exponential $e^{-w'}$ decrease, but the exponential $e^{-u'}$ will also decrease.

As it was pointed out previously, from a physical standpoint, u' cannot be larger than v' even if the quantity $r_{sd}w'/\alpha'$ increases without limit. Therefore, for a given installation and material, u' is bounded by the condition u' = v'.

Substituting the above in Equation 42, one obtains for a given installation and materials the maximum fill height for which the material on top of the conduit will brace itself against the adjacent mass along the whole fill height in a similar manner as in a ditch conduit.

Hence, for u' = v'

$$v' = \frac{\left(\frac{3}{4}v'^2 - \frac{3c_e}{\gamma B_d}v'\right) - \left(\frac{3}{2} + \frac{r_{sd}w'}{\alpha'}\right)\left(1 - \frac{2c_e}{\gamma B_d}\right) + \left(\frac{3}{2} + \frac{r_{sd}w'}{\alpha'}\right)\left(v' + 1 - \frac{2c_e}{\gamma B_d}\right)e^{-v'}}{\left(\frac{3}{2} + \frac{r_{sd}w'}{\alpha'}\right)e^{-v'} + \frac{3}{2}(v' - 1)},$$

or, after collecting terms

$$e^{-v'} = \frac{\frac{3}{4}v'^2 - \frac{3}{2}v'\left(1 - \frac{2c_e}{\gamma B_d}\right)}{\left(1 - \frac{2c_e}{\gamma B_d}\right)\left(\frac{3}{2} + \frac{r_{sd}w'}{\alpha'}\right)} + 1 \qquad (58)$$

from which v' may be obtained by successive trials.

(c) If the ditch material, the conduit, and the conduit foundation have an over-all stiffness that is equal to the stiffness of the adjacent masses, the middle prism will settle the same amount as these masses. Consequently, there will be no arching effect.

Since the material in the ditch behaves like the adjacent masses, no distinction can be made between the two materials; consequently, w' = 0 and u' = u = 0.

Substituting the above in Equation 18 and since v' - u' = v - u and $v = (2K_e\tan\phi_e)H/B_d$ one obtains

$$W_c = \gamma B_d H = W \quad \text{when} \quad r_{sd}w'/\alpha' = 0, \qquad (59)$$

which is identical to Equation 15.

Evaluation of u for Mined-In Conduits

In this treatment case (a) of Figure 6 will be considered to be the general case. Cases (b) and (c) can be treated in a similar manner if the quantities involved in the expressions derived for case (a) are modified accordingly.

Let Figure 8 represent a mined-in conduit installed through cohesive material with its lower part located within an exceptionally stiff layer of clay between soft layers. The same notation is employed as in previous sections with the addition of the following:

H_m = thickness of the stiff layer on either side of the conduit, ft.

λ_i = compression of the interior prism between the top of the conduit and the plane of equal settlement, ft.

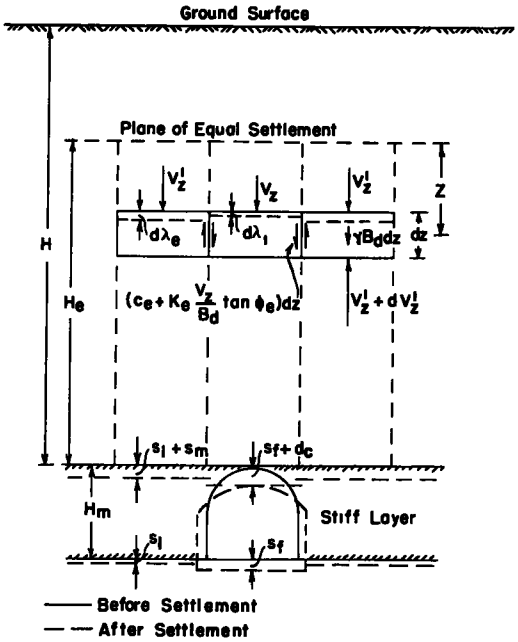

Figure 8. Force diagram for a mined-in conduit.

λ_e = compression of the exterior prism between the top of the conduit and the plane of equal settlement, ft.

s_m = compression of the stiff mass on either side of the conduit within the distance H_m, ft.

s_l = settlement of the foundation supporting the stiff layer, ft.

$s_m + s_l$ = settlement of the mass supporting the exterior prisms, ft.

$d_c + s_f$ = settlement of the mass supporting the interior prism, ft.

r_{sm} = settlement ratio = $[(s_m + s_l) - (d_c + s_f)]/s_m$.

E_f = modulus of deformation of all other material on top of the conduit except the stiff layer on its sides, lb. per ft. per ft.

E_m = modulus of deformation of the stiff mass within the distance H_m, lb. per ft. per ft.

$\alpha = E_m/E_f$.

The assumptions made in the case of covered-up conduits are modified in order that the subsequent analysis can be made. Thus:

(a) The average behavior of the material surrounding the conduit is such that it may be considered to obey Hooke's law when subjected to compression. Thus, the moduli E_f and E_m are assumed constant within any region occupied by their respective materials.

(b) The settlement ratio r_{sm} is considered constant throughout the life of the conduit.

Assumptions (c), (d), and (e) are the same as in the case of covered-up conduits. Assumption (e), however, should be modified to include the stiff mass on the sides of the conduit instead of the loose mass within the ditch on top of a covered-up conduit.

In addition to the above:

(f) In setting up the expression for s_m, the friction between the sides of the conduit and the stiff layer is neglected to simplify the mathematical computations (Spangler, 1950b).

As in the previous case, for the evaluation of u one considers the relative deformation of the interior and exterior prisms. The over-all settlement of the interior prism at the plane of equal settlement must equal the over-all settlement of the exterior prism at the same plane. Hence

$$\lambda_i + d_c + s_f = \lambda_e + s_m + s_l$$

or

$$\lambda_i = \lambda_e + s_m + s_l - (d_c + s_f). \tag{60}$$

Since

$$r_{sm} = [(s_m + s_l) - (d_c + s_f)]/s_m,$$

Equation 60 may be written

$$\lambda_i = \lambda_e + r_{sm} \cdot s_m. \tag{61}$$

To evaluate λ_i one substitutes in Equation 28 the value of V_z from Equation 9 employing the negative signs, and integrates between the limits

$$\lambda_i = 0 \quad \text{for} \quad z = 0,$$

$$\lambda_i = \lambda_i \quad \text{for} \quad z = H_e.$$

Rearranging terms and letting

$$(2K_e \tan\phi_e)\, H/B_d = v, \tag{62}$$

$$(2K_e \tan\phi_e)\, H_e/B_d = u, \tag{63}$$

$$(2K_e \tan\phi_e)\, H_m/B_d = w, \tag{64}$$

one obtains

$$\lambda_i = \frac{\gamma B_d^2}{E_f} \frac{1}{(2K_e \tan\phi)^2} \left\{ e^{-u} \left[(1 - \frac{2c_e}{\gamma B_d}) - (v - u) \right] + \left[(1 - \frac{2c_e}{\gamma B_d})(u - 1) + (v - u) \right] \right\}. \tag{65}$$

Similarly, by substituting Equation 33 in Equation 29 and integrating between the limits

$$\lambda_e = 0 \qquad \lambda_i = 0 \qquad \text{for} \qquad z = 0$$

$$\lambda_e = \lambda_e \qquad \lambda_i = \lambda_i \qquad \text{for} \qquad z = H_e$$

one obtains in terms of the dimensionless factors v and u defined from Equations 63 and 64,

$$\lambda_e = \frac{\gamma B_d^2}{E_f} \frac{1}{(2K_e \tan\phi_e)^2} \left\{ \frac{3}{2} (v - \frac{1}{2} u)\, u \right\} - \frac{1}{2} \lambda_i . \tag{66}$$

Since the stiff mass within the distance H_m is considered to obey Hooke's law, the vertical compression of the prism of width B_d and height H_m, due to the vertical pressure $V'_{z = H_e}$ on top of the stiff layer, is

$$s_m = (V'_{z = H_e}/B_d E_m)\, H_m . \tag{67}$$

Substituting $z = H_e$ in Equation 33 and since $H' - H'_e = H - H_e$, one obtains in terms of the dimensionless factors v and u

$$V'_z = H_e = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ \frac{3}{2} v - \frac{1}{2} e^{-u} \left\{ \left[(v - u) - (1 - \frac{2c_e}{\gamma B_d}) \right] + (1 - \frac{2c_e}{\gamma B_d}) \right\} \right\}. \tag{68}$$

Substituting in Equation 67 the value for $V'_{z = H_e}$ from Equation 68 and, since $E_m = \alpha E_f$ and $2K_e \tan\phi_e \frac{H_m}{B_d} = w$ one obtains

$$s_m = \frac{\gamma B_d^2}{E_f} \frac{1}{(2K_e \tan\phi)^2} \frac{w}{\alpha} \left\{ \frac{3}{2} v - \frac{1}{2} \left\{ e^{-u} \left[(v - u) - (1 - \frac{2c_e}{\gamma B_d}) \right] + (1 - \frac{2c_e}{\gamma B_d}) \right\} \right\}. \tag{69}$$

Substituting in Equation 61 the values λ_i, λ_e, and s_m from Equations 65, 66, and 69, respectively, one obtains after rearranging terms

$$v = \frac{(\frac{3}{2} u^2 - \frac{6c_e}{\gamma B_d} u) - (3 - \frac{r_{sm} w}{\alpha})(1 - \frac{2c_e}{\gamma B_d}) + (3 - \frac{r_{sm} w}{\alpha})(u + 1 - \frac{2c_e}{\gamma B_d})\, e^{-u}}{(3 - \frac{r_{sm} w}{\alpha})\, e^{-u} + 3(u - 1 + \frac{r_{sm} w}{\alpha})} . \tag{70}$$

Equation 70 governs the behavior of u for a given installation and material in the case of a mined-in conduit. The same equation may be used in the case of positive projecting conduits if their side supporting material has been thoroughly compacted.

As in the case of covered-up conduits, u may be obtained from curves that have been constructed by solving Equation 63 for v.

It can be seen for this case that again, for infinitely high fills u does not depend on the cohesion and the unit weight of the material but is governed by the equation

$$(3 - \frac{r_{sm} w}{\alpha})\, e^{-u} + 3(u - 1 + \frac{r_{sm} w}{\alpha}) = 0. \tag{71}$$

By following the same method of approach applied to covered-up conduits, it can be shown also that

$$\lim_{c_e \to \infty} u = 0 \tag{72}$$

$$\lim_{\left| \frac{r_{sm} w}{\alpha} \right| \to \infty} u = \infty \tag{73}$$

$$\lim_{\frac{r_{sm} w}{\alpha} \to 0} u = 0 \tag{74}$$

Again, physically, u is bounded by the condition u = v.

Substituting u = v in Equation 70 one obtains

$$v = \frac{(\frac{3}{2}v^2 - \frac{6c_e}{\gamma B_d}v) - (3 - \frac{r_{sm}w}{a})(1 - \frac{2c_e}{\gamma B_d}) + (3 - \frac{r_{sm}w}{a})(v + 1 - \frac{2c_e}{\gamma B_d})\, e^{-v}}{(3 - \frac{r_{sm}w}{a})\, e^{-v} + 3(v - 1 + \frac{r_{sm}w}{a})},$$

or

$$e^{-v} = \frac{\frac{3}{2}v^2 - 3v\left[(1 - \frac{2c_e}{\gamma B_d}) - \frac{r_{sm}w}{a}\right]}{(1 - \frac{2c_e}{\gamma B_d})(3 - \frac{r_{sm}w}{a})} + 1. \tag{75}$$

From Equation 75 one may obtain for an $s = c + \sigma \tan\phi$ material, the maximum height of the mass on top of a mined-in conduit for which the arching effect will extend as far as the ground surface thereby causing a trough-like depression to appear at the surface directly above the conduit.

From Relation 74 and Equation 18 one may see that, as in the case of a covered-up conduit, if the over-all stiffness of the body furnishing support to the middle prism equals the stiffness of the mass supporting the exterior prisms above a mined-in conduit, there will be no relative settlement between the interior and exterior prisms and, consequently, there will be no arching effect. Consequently u = 0 and the load on top of the conduit will be

$$W_c = \gamma B_d H = W \quad \text{when} \quad r_{sm}w/a = 0, \tag{76}$$

which is identical to Equations 59 and 15.

In both cases of covered-up and mined-in conduits the corresponding settlement ratios r_{sd} and r_{sm} are empirical quantities and must be determined by direct measurement. Since in either case the interior prism subsides a greater amount than the exterior prism, both quantities are negative.

A positive settlement ratio would indicate that the reverse action has taken place in the relative subsidence of the masses on the top of the conduit. Under such circumstances, conditions corresponding to Case I would be present, which, as it has been discussed previously, is very undesirable because of its detrimental influence on the conduit. Therefore, every effort must be made in the design and construction of an underground conduit in order that the settlement ratio of the masses above it remains negative at all times.

Both settlement ratios were defined originally by Dean Anson Marston (1922) and Professor Spangler (1950b) of Iowa State College in their theoretical treatment of covered-up positive and negative projecting conduits installed in a granular material. To avoid confusion, the writer has adopted the same definitions in his treatment of the general case. However, he believes that if both ratios had a common denominator, say d_c, which would always be a positive quantity, then the two cases could have been united into one general treatment. Furthermore, if both ratios were defined in such a manner that they would be positive quantities, the mathematical treatment and the resulting expressions for all cases would have been much less complicated.

The employment of the shortening of the vertical dimension of the conduit, d_c, as a denominator in the expressions for settlement ratios would also tie in the height of arching and, consequently, the load expression, with the stiffness of the conduit and the distribution of external pressure on its sides and bottom. Consequently, the resulting load expression would have been also a function of the support which the side material can furnish to the conduit, as well as of the stiffness of the conduit. Such treatment, however, is beyond the scope of this paper.

The Analysis of the General Case as Applied to an $s = \sigma \tan\phi$ Material

The theoretical relations describing the loading action of a perfectly cohesionless material on top of underground conduits can be deduced from the expressions derived for the general case in which the loading agent is an $s = c + \sigma \tan\phi$ material, by taking the limits of these expressions when c_e is allowed to approach zero. Thus:

1. From Equation 10 the general load expression for an $s = \sigma \tan\phi$ material will

become

$$\lim_{c_e \to 0} W_c = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{\pm (2K_e \tan\phi_e) \frac{H_e}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H - H_e}{B_d}\right) \pm 1 \right] \mp 1 \right\}, \tag{77}$$

from which

$$C = e^{\pm (2K_e \tan\phi_e) \frac{H_e}{B_d}} \left[(2K_e \tan\phi_e)\left(\frac{H - H_e}{B_d}\right) \pm 1 \right] \mp 1 .\ ^{7} \tag{78}$$

In terms of the dimensionless factors v and u, the above equations may be written respectively

$$\underset{c_e = 0}{W_c} = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{\pm u} \left[(v - u) \pm 1 \right] \mp 1 \right\} \tag{79}$$

and

$$C = e^{\pm u} \left[(v - u) \pm 1 \right] \mp 1. \tag{80}$$

In the event that the interior prism subsides less than the exterior prisms, which was defined previously as Case I, the positive signs should be used in Equations 77 through 80. Hence, in terms of the dimensionless factors v and u, one obtains

$$\underset{c_e = 0}{W_c} = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{+u} (v - u + 1) - 1 \right\} \tag{81}$$

and

$$C_p = e^{+u} (v - u + 1) - 1 . \tag{82}$$

From the above equations one may conclude that Case I will be just as detrimental to an underground conduit installed under an $s = \sigma \tan\phi$ material.

In the event that the interior prism subsides more than the exterior prisms, which case was previously defined as Case II, the negative signs should be used in Equations 79 and 80. Hence, in terms of the dimensionless factors v and u, one obtains, respectively

$$\underset{c_e = 0}{W_c} = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left[e^{-u} (v - u - 1) + 1 \right] \tag{83}$$

and

$$C_n = e^{-u} (v - u - 1) + 1 . \tag{84}$$

If $u \gg 1$, e^{-u} will become negligible. Hence;

$$C_n \approx 1$$

and

$$\underset{c_e = 0}{W_c} \approx \frac{\gamma B_d^2}{2K_e \tan\phi_e} . \tag{85}$$

Equation 85 is identical to the expression derived by Terzaghi for the pressure on top of deep tunnels through dry sand, i. e., for Case c of Figure 6.[8]

For a ditch conduit, $H_d = H_e = H$. Hence, $u = v$ and Equation 83 becomes

$$\underset{c_e = 0}{W_c} = \frac{\gamma B_d^2}{2K_e \tan\phi_e} (1 - e^{-v}) . \tag{86}$$

As in the case of an $s = c + \sigma \tan\phi$ material, if the height of the fill is large enough so that $v = (2K_e \tan\phi_e) H/B_d \gg 1$, e^{-v} becomes negligible and the load on top of the conduit approaches the value

$$\underset{c_e = 0}{W_c} \approx \frac{\gamma B_d^2}{2K_e \tan\phi_e}$$

given by Equation 85.

For either a negative or an imperfect ditch conduit, since $u = w' + u'$ and $H - H_e = H' - H'_e$

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[7] Equations 77 and 78 are identical to Equations 11 and 12 obtained by Spangler. See References, Spangler (1950b), p. 24.

[8] Terzaghi, op. cit., p. 196.

Equations 83 and 84 may be written, respectively, as

$$\underset{c_e=0}{W_c} = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \left\{ e^{-w'} e^{-u'} (v' - u' - 1) + 1 \right\} \tag{87}$$

and

$$C_n = e^{-w'} e^{-u'} (v' - u' - 1) + 1 . \tag{88}$$

Because of the descending exponential functions $e^{-w'}$ and $e^{-u'}$, if either w' or u' is large enough, C_n will approach one and the load on top of the conduit will be given by Equation 85.

Since $w' = (2K_e \tan\phi_e) H_d/B_d$, it follows that for a given material and width of ditch, if the height of ditch is made large enough so that $w' \gg 1$, the conduit load will be

$$\underset{c_e=0}{W_c} \approx \frac{\gamma B_d^2}{2K_e \tan\phi_e} .$$

2. The equation governing the behavior of u' for a covered-up conduit may be obtained by taking the limit of Equation 42 when c_e is allowed to approach zero. Thus

$$\lim_{c_e \to 0} v' = \frac{\frac{3}{4} u'^2 - (\frac{3}{2} + \frac{r_{sd} w'}{a'}) + (\frac{3}{2} + \frac{r_{sd} w'}{a'})(u' + 1) e^{-u'}}{(\frac{3}{2} + \frac{r_{sd} w'}{a'}) e^{-u'} + \frac{3}{2}(u' - 1)} . \tag{89}$$

Similarly, the equation governing the behavior of u in the case of a mined-in conduit may be obtained by taking the limit of Equation 70 when c_e is allowed to approach zero. Thus

$$\lim_{c_e \to 0} v = \frac{\frac{3}{2} u^2 - (3 - \frac{r_{sm} w}{a}) + (3 - \frac{r_{sm} w}{a})(u + 1) e^{-u}}{(3 - \frac{r_{sm} w}{a}) e^{-u} + 3(u - 1 + \frac{r_{sm} w}{a})} .^{9} \tag{90}$$

By inspection it can be seen that if the denominator of the right hand member of the above equations approaches zero, factors v' and v will increase without limit.

Hence, for a given material, if the mass on top of a conduit is infinitely high, u' and u will be governed by the equations

$$(\frac{3}{2} + \frac{r_{sd} w'}{a'}) e^{-u'} + \frac{3}{2}(u' - 1) = 0 \tag{91}$$

and

$$(3 - \frac{r_{sm} w}{a}) e^{-u} + 3(u - 1 + \frac{r_{sm} w}{a}) = 0 \tag{92}$$

respectively.

Equations 91 and 92 are respectively identical with Equations 43 and 71 which had been derived for infinitely high masses consisting of $s = c + \sigma \tan\phi$ material. One may conclude, therefore, that for very high earth masses on top of either covered-up or mined-in conduits, the influence of the cohesion of the material on the height of arching is negligible and an $s = c + \sigma \tan\phi$ material will behave like a perfectly granular material. Since the general load equation depends primarily on the height of arching it follows that for very high masses consisting of $s = c + \sigma \tan\phi$ material, the conduit load may be computed as for an $s = \sigma \tan\phi$ material. The error due to neglecting the cohesion, besides being on the safe side, will be negligible.

What constitutes a very high earth mass will depend not only on the height, H, but on the factors $2K_e \tan\phi_e$ and B_d as well, because $v = (2K_e \tan\phi_e) H/B_d$.

3. As in the general case of an $s = c + \sigma \tan\phi$ material the following relations can be established for an $s = \sigma \tan\phi$ material:

(a)

$$\lim_{\substack{c_e=0 \\ \phi_e \to 90^\circ}} W_c = 0. \tag{93}$$

[9] Equations 89 and 90 are respectively identical to Equations 18 (Spangler, 1950a, p. 158), and 18 (Spangler, 1950b, p. 28).

(b) In the case of a covered-up conduit:

$$\lim u' = \infty, \quad \left|\frac{r_{sd}w'}{a'}\right| \to \infty \tag{94}$$

$$\lim u' = 0. \quad \frac{r_{sd}w'}{a'} \to 0 \tag{95}$$

(c) In the case of a mined-in conduit:

$$\lim u = \infty, \quad \left|\frac{r_{sm}w}{a}\right| \to \infty \tag{96}$$

$$\lim u = 0. \quad \frac{r_{sm}w}{a} \to 0 \tag{97}$$

As in the general case u' and u are physically bounded by the conditions u' = v', and u = v respectively.

The maximum height of a mass of a perfectly granular material above an underground conduit for which the arching effect will extend as far as the surface of the mass causing a trough-like depression to appear on the surface may be obtained by substituting u' = v' and u = v in Equations 89 and 90 respectively.

Thus, for a covered-up conduit one obtains after rearranging terms

$$e^{-v'} = \frac{\frac{3}{4}v'^2 - \frac{3}{2}v'}{\left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)} + 1. \tag{98}$$

For a mined-in conduit

$$e^{-v} = \frac{\frac{3}{2}v^2 + 3v\left[\frac{r_{sm}w}{a} - 1\right]}{3 - \frac{r_{sm}w}{a}} + 1. \tag{99}$$

From relations 95 and 97 and by applying the same reasoning as in the general case it can be shown also that

$$\lim_{c_e = 0} W_c = \gamma B_d H = W, \quad \frac{r_{sd}w'}{a'} \to 0 \tag{100}$$

$$\lim_{c_e = 0} W_c = \gamma B_d H = W. \quad \frac{r_{sm}w}{a} \to 0 \tag{101}$$

The Analysis of the General Case as Applied to an s = c Material

As in the previous section, the theoretical relations describing the loading action of a perfectly cohesive material on top of underground conduits may be deduced from the general case by taking the limits of the expressions derived from an $s = c + \sigma \tan\phi$ material when ϕ_e is allowed to approach zero. Thus:

1. From Equation 10, the general load expression for an s = c material will become equal to

$$\lim_{\phi_e \to 0} W_c = \gamma B_d^2 \left\{ \left(\frac{H - H_e}{B_d}\right) + \left(1 \pm \frac{2c_e}{\gamma B_d}\right)\frac{H_e}{B_d} \right\}, \tag{102}$$

from which

$$C = \left(\frac{H - He}{B_d}\right) + \left(1 \pm \frac{2c_e}{\gamma B_d}\right)\frac{H_e}{B_d} \quad . \tag{103}$$

Let

$$H/B_d = v_o, \qquad H'/B_d = v'_o,$$
$$He/B_d = u_o, \qquad H'_e/B_d = u'_o,$$
$$H_m/B_d = w_o, \qquad H_d/B_d = w'_o,$$

Substituting in Equations 102 and 103 one obtains

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left\{(v_o - u_o) + \left(1 \pm \frac{2c_e}{\gamma B_d}\right) u_o\right\}, \tag{104}$$

and

$$C = (v_o - u_o) + \left(1 \pm \frac{2c_e}{\gamma B_d}\right) u_o \tag{105}$$

respectively.

In the above equations the positive signs should be used in the event that the interior prism subsides less than the exterior prisms, and the negative signs in case the reverse action takes place.

Hence, if Case I obtains

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left\{(v_o - u_o) + \left(1 + \frac{2c_e}{\gamma B_d}\right) u_o\right\} \tag{106}$$

and

$$C_p = \left\{(v_o - u_o) + \left(1 + \frac{2c_e}{\gamma B_d}\right) u_o\right\} . \tag{107}$$

If Case II obtains

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left\{(v_o - u_o) + \left(1 - \frac{2c_e}{\gamma B_d}\right) u_o\right\} \tag{108}$$

and

$$C_n = \left\{(v_o - u_o) + \left(1 - \frac{2c_e}{\gamma B_d}\right) u_o\right\} . \tag{109}$$

From the above equations, it can be seen that the conditions for Case I are just as undesirable for an $s = c$ material as for an $s = c + \sigma \tan\phi$ or for an $s = \sigma \tan\phi$ material.

For a ditch conduit, since $H_d = H_e = H$,

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left(1 - \frac{2c_e}{\gamma B_d}\right)\frac{H}{B_d} , \tag{110}$$

or

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left(1 - \frac{2c_e}{\gamma B_d}\right) v_o . \tag{111}$$

Hence, if $c_e = \frac{\gamma B_d}{2}$,

$$W_c = 0. \tag{112}$$

Equation 112 is similar to Equation 24.

For a negative or an imperfect ditch conduit, since $H_e = H_d + H'e$ and $H - He = H' - H'_e$, Equation 102 may be written

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left\{\left(\frac{H' - H'_e}{B_d}\right) + \left(1 - \frac{2c_e}{\gamma B_d}\right)\left(\frac{H'_e + H'_d}{B_d}\right)\right\} , \tag{113}$$

or

$$\underset{\phi_e = 0}{W_c} = \gamma B_d^2 \left\{(v'_o - u'_o) + \left(1 - \frac{2c_e}{\gamma B_d}\right)(u'_o + w'_o)\right\} . \tag{114}$$

in terms of the dimensionless factors v'_o, u'_o, and w'_o.

2. The equation governing the behavior of u'_o in the case of a covered-up conduit may be obtained from the general case as follows:

Equation 42 may be written also

$$\frac{H'}{B_d} = v'_o = \frac{1}{2K_e \tan\phi_e} \cdot \frac{\left(\frac{3}{4}u'^2 - \frac{3c_e}{\gamma B_d}u'\right) - \left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)\left(1 - \frac{2c_e}{\gamma B_d}\right) + \left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)\left(u' + 1 - \frac{2c_e}{\gamma B_d}\right)e^{-u'}}{\left(\frac{3}{2} + \frac{r_{sd}w'}{a'}\right)e^{-u'} + \frac{3}{2}(u' - 1)} . \tag{115}$$

Applying L'Hospital's rule twice on Equation 115 and letting ϕ_e approach zero one obtains

$$\lim_{\phi_e \to 0} v'_o = -\frac{2c_e}{\gamma B_d} \cdot \frac{\frac{3}{4}u'^2_o - \frac{r_{sd}w'_o}{a'}u'_o}{\frac{r_{sd}w'_o}{a'}} \tag{116}$$

from which

$$u'_o = \frac{2}{3}\frac{r_{sd}w'_o}{a'}\left\{1 \pm \sqrt{1 - \frac{3v'_o}{\frac{r_{sd}w'_o}{a'} \cdot \frac{2c_e}{\gamma B_d}}}\right\} . \tag{117}$$

Similarly the equation governing the behavior of u_o in the case of a mined-in conduit if Equation 70 is written in the form

$$\frac{H}{B_d} = v_o = \frac{1}{2K_e \tan\phi_e} \cdot \frac{\left(\frac{3}{2}u^2 - \frac{6c_e}{\gamma B_d}u\right) - \left(3 - \frac{r_{sm}w}{a}\right)\left(1 - \frac{2c_e}{\gamma B_d}\right) + \left(3 - \frac{r_{sm}w}{a}\right)\left(u + 1 - \frac{2c_e}{\gamma B_d}\right)e^{-u}}{\left(3 - \frac{r_{sm}w}{a}\right)e^{-u} + 3\left(u - 1 + \frac{r_{sm}w}{a}\right)} . \tag{118}$$

If L'Hospital's rule is applied twice on Equation 118 and ϕ_e is allowed to approach zero one obtains

$$\lim_{\phi_e \to 0} v_o = -\frac{2c_e}{\gamma B_d} \cdot \frac{\frac{3}{4}u_o^2 + \frac{1}{2}\frac{r_{sm}w_o}{a}u_o}{\frac{r_{sm}w_o}{a}} \tag{119}$$

from which

$$u_o = \frac{1}{3}\frac{r_{sm}w_o}{a}\left\{-1 \pm \sqrt{1 - \frac{12v_o}{\frac{r_{sm}w_o}{a} \cdot \frac{2c_e}{\gamma B_d}}}\right\} . \tag{120}$$

By inspection of Equations 117 and 120 and by noting that r_{sd} and r_{sm} are negative quantities the physical meaning of these equations can be interpreted if written as follows:

$$u'_o = \frac{2}{3}\frac{r_{sd}w'_o}{a'}\left\{1 - \sqrt{1 - \frac{3v'_o}{\frac{r_{sd}w'_o}{a'}\frac{2c_e}{\gamma B_d}}}\right\} \tag{121}$$

for all values of $\frac{3v'_o}{\frac{r_{sd}w'_o}{a'}\frac{2c_e}{\gamma B_d}}$.

$$u_o = -\frac{1}{3}\frac{r_{sm}w_o}{a}\left\{1 + \sqrt{1 - \frac{12\,v_o}{\frac{r_{sm}w_o}{a} \cdot \frac{2c_e}{\gamma B_d}}}\right\} \tag{122}$$

for $\left|\frac{12v_o}{\frac{r_{sm}w_o}{a}\frac{2c_e}{\gamma B_d}}\right| \neq 0$,

and

$$u_o = 0 \tag{123}$$

for

$$\frac{12v_o}{\frac{r_{sm}w_o}{a}\frac{2c_e}{\gamma B_d}} = 0.$$

3. From the above equations the following relations can be established for an s = c material:

(a) $\lim\limits_{\substack{\phi_e = 0 \\ c_e \to \infty}} W_c = -\infty$. (124)

(b) In the case of a covered-up conduit:

$$\lim_{c_e \to \infty} u'_o = 0, \tag{125}$$

$$\lim_{\left|\frac{r_{sd} w'_o}{a'}\right| \to \infty} u'_o = \infty, \tag{126}$$

$$\lim_{\frac{r_{sd} w'_o}{a'} \to 0} u'_o = 0. \tag{127}$$

(c) In the case of a mined-in conduit:

$$\lim_{c_e \to \infty} u_o = 0, \tag{128}$$

$$\lim_{\left|\frac{r_{sm} w_o}{a}\right| \to \infty} u_o = \infty, \tag{129}$$

$$\lim_{\frac{r_{sm} w_o}{a} \to 0} u_o = 0. \tag{130}$$

As in the cases of $s = c + \sigma \tan\phi$ material and $s = \sigma \tan\phi$ material, u'_o and u_o are bounded by the conditions $u'_o = v'_o$ and $u_o = v_o$, respectively.

The maximum height of a mass of s = c material above an underground conduit for which the arching effect will extend as far as the surface of the mass causing a trough-like depression to appear on the surface may be obtained by substituting $u'_o = v'_o$ and $u_o = v_o$ in Equations 116 and 119, respectively.

Thus for a covered-up conduit one obtains after rearranging terms:

$$v'_{o_{max}} = -\frac{2}{3}\frac{\gamma B_d}{c_e}\frac{r_{sd} w'_o}{a}\left(1 - \frac{2c_e}{\gamma B_d}\right). \tag{131}$$

If

$$c_e = \frac{\gamma B_d}{2}, \qquad v'_{o_{max}} = 0. \tag{132}$$

For a mined-in conduit:

$$v_{o_{max}} = -\frac{2}{3}\frac{\gamma B_d}{c_e}\frac{r_{sm} w_o}{a}\left(1 + \frac{c_e}{\gamma B_d}\right). \tag{133}$$

From Relations 127 and 130 and by applying the same reasoning as in the general case, it can be shown also that

$$\lim_{\substack{\phi_e = 0 \\ \frac{r_{sd} w'_o}{a'} \to 0}} W_c = \gamma B_d H = W, \tag{134}$$

$$\lim_{\substack{\phi_e = 0 \\ \frac{r_{sm} w_o}{a} \to 0}} W_c = \gamma B_d H = W. \tag{135}$$

From Equations 102, 117, and 120, it can be seen that the load on top of underground conduits installed under a purely cohesive material is independent of the pressure ratio K.

Construction of Load Curves for a Covered-Up Conduit Installed Under an $s = \sigma \tan\phi$ Material

The purpose of the construction of the following families of curves is to facilitate the computation of load on top of covered-up conduits if the loading agent is an $s = \sigma \tan\phi$ material. Curves to estimate the earth load on top of mined-in conduits can be constructed in a similar manner.

The presentation of these curves will follow the same sequence as the order in which they would be used by a designer to make a load estimate for a given installation and material.

1. Factor $K\tan\phi$ has been plotted against the angle of internal friction ϕ for various values of the equivalent hydrostatic pressure ratio $K = K_e$. (Table A and Figure A in the Appendix).

The upper boundary of this family of curves is the curve $K_p\tan\phi$ which is obtained if K assumes the upper limiting value, $K_p = \tan^2 (45^0 + \phi/2)$ for a passive state of plastic equilibrium.

The lower boundary of the same family of curves is the curve $K_A\tan\phi$ which is obtained if K assumes the lower limiting value, $K_A = \tan^2(45^0 - \phi/2)$ for an active state of plastic equilibrium.

K_e can be obtained by constructing an equivalent pressure ratio diagram similar to Figure 1a. The value ϕ_e may be considered to be a fraction of the maximum value of the angle of internal friction of the material. This fraction will depend on the desired factor of safety for the particular project.

2. By solving Equation 89 for v', curves were prepared showing the relation v' versus u' for various values of $r_{sd}w'/\alpha'$. From these curves, which are not presented in this paper, u' was plotted against $r_{sd}w'/\alpha'$ for various values of v'. (Table B and Figure B in the Appendix).

The upper boundary of this family of curves is the curve for which u' = v' and it is obtained by solving Equation 98.

The lower boundary of the same family of curves is the curve for which $v' = \infty$, and it is obtained by solving Equations 43 or 91.

From the given data of the project and the value $K_e\tan\phi_e$ obtained in Step 1, the values $v' = (2K_e\tan\phi_e)\, H'/B_d$ and $w' = (2K_e\tan\phi_e)\, H_d/B_d$ can be computed.

From available records of previous installations, the settlement ratio r_{sd} as well as the stiffness ratio $\alpha' = E_L/E_F$ can be estimated for a given installation and material. Hence, the quantity $r_{sd}\, w'/\alpha'$ can be computed. Accordingly, u' can be obtained from the above family of curves.

By substituting the obtained values of v', u', w' and $2K_e\tan\phi_e$, in Equation 87, the load W_c can be computed.

From the above curves it can be seen that for a given finite value of $r_{sd}w'/\alpha'$, as v' increases, u' decreases from a maximum value $u' = v'_{max}$ to a lower limiting finite value $u'_{v'} = \infty$. For values of v' less than v'_{max}, u' is larger than v' in magnitude and, therefore, it becomes imaginary from a physical standpoint.

From the above, the nature and extent of arching for a given material and ditch width can be visualized as follows:

If the yield of the loose mass in the ditch induces a constant relative movement within the fill material above the top of the ditch, the shearing resistance of the material will be mobilized along the whole fill height and will oppose this movement. This action is called the "arching effect." During this action a visible, trough-like depression will exist on the surface of the fill directly above the conduit.

If the fill height exceeds a maximum value H_{max}, the arching effect will extend upward to the surface below the top of the fill which is called, "the plane of equal settlement." Above this plane no relative movement exists within the soil mass, therefore, no depression will appear on the surface of the fill directly above the conduit.

If the fill height increases without limit, the height of arching will approach a lower limiting finite value.

3. To facilitate the computational part of the above described procedure, two other families of curves have been plotted as follows:

The load factor C_n may be written also:

$$C_n = 1 + e^{-w'} C_m \tag{136}$$

where

$$C_m = e^{-u'}(v' - u' - 1). \tag{137}$$

By substituting in Equation 137 the values for v' and u' from the family of curves presented in Step 2, C_m has been plotted versus v' for different values of $r_{sd}w'/a'$ (Table C and Figures C and C1 in the Appendix).

The above family of curves makes possible the evaluation of the conduit load W_c without computing u' first.

The upper boundary of these curves is the curve for which $r_{sd}w'/a' = 0$. From Relation 95 and the discussion involved in evaluating the general expression governing the behavior of factor u, it was shown that under such conditions, $u' = 0$ and $w' = 0$. Hence, $v' = v$ and the upper boundary will be the curve

$$C_m = v' - 1 = v - 1. \tag{138}$$

Since u' is physically bounded by the condition $u' = v'$ the above curves will be bounded by the same condition.

Letting $u' = v'$ in Equation 137 one obtains

$$C_m = -e^{-v'} \tag{139}$$

which, as it can be seen from Equation 86 is the corresponding C_m factor for a ditch conduit. Hence, the locus of the lower points of the above curves, is the C_m curve obtained for a ditch conduit.

Since Equations 137 and 139 have obviously the same derivative with respect to v' at the point $v' = u'$, it follows that the ditch conduit C_m curve is tangent to each one of the curves of the above family. Hence, at their respective $v' = u'$ points, each one of these curves merges with the ditch conduit curve.

From the above it follows that the process of arching as visualized in Step 3 is also mathematically continuous.

It can be shown also that for a fixed value of $r_{sd}w'/a'$

$$\lim_{v' \to \infty} C_m = \infty, \tag{140}$$

whereas

$$\lim_{\left|\frac{r_{sd}w'}{a'}\right| \to \infty \atop v' \to \infty} C_m = 0. \tag{141}$$

4. By solving Equation 136, C_n has been plotted versus C_m for different values of w' (Table D and Figure D).

The upper boundary of this family of curves is the curve obtained for $w' = 0$. As it was previously shown, under such conditions

$$\frac{r_{sd}w'}{a'} = 0 \qquad \text{and} \qquad u' = 0.$$

Therefore,

$$C_m = v - 1 \qquad \text{and} \qquad C_n = v.$$

Since

$$v = (2K_e \tan\phi_e)\, H/B_d,$$

$$W_c = \frac{\gamma B_d^2}{2K_e \tan\phi_e} \cdot (2K_e \tan\phi_e)\, H/B_d = \gamma B_d H = W.$$

The lower boundary of the same family of curves is the curve for which $w' = \infty$. Under such conditions the second member of Equation 136 vanishes, C_m becomes equal to one, and the load assumes the value

$$W_c = \frac{\gamma B_d^2}{2K_e \tan\phi_e}.$$

From the last family of curves one can see that the load on top of the conduit is greatly influenced by the factor $w' = (2K_e\tan\phi_e)\ H_d/B_d$.

Recapitulating, one can compute the load on top of covered-up underground conduits installed under a cohesionless material as follows:

Given the material and the dimensions H, H_d, and B_d:

(a) Estimate K_e, ϕ_e, γ, r_{sd}, and α'.

(b) From Table A or Figure A obtain the value $2K_e\tan\phi_e$.

(c) Compute factors, $v' = (2K_e\tan\phi_e)\ H'/B_d$, $w' = (2K_e\tan\phi_e)\ H_d/B_d$, and $r_{sd}w'/\alpha'$.

(d) From Table C or Figures C or C1 determine the value of C_m corresponding to v' and $r_{sd}w'/\alpha'$ computed in Step c.

(e) From Table D or Figure D determine the value of C_n corresponding to the values of C_m obtained in Step d, and w' computed in Step c.

(f) Substitute the value of C_n and $2K_e\tan\phi_e$ in the equation $W_c = \frac{\gamma B_d^2}{2K_e\tan\phi_e}\ C_n$ and compute the load.

Although the above procedure utilizes three different charts for the load evaluation instead of one utilized in other publications, it has the over-all advantage over the latter in that the same charts can be used for any kind of covered-up conduit installation. Load factor charts in other publications can be used for only one value of the quantities $K\tan\phi$, and H_d/B_d.

In previous sections it was shown that if factor $v' = (2K_e\tan\phi_e)\ H'/B_d$ increases without limit, $u' = (2K_e\tan\phi_e)\ H'_e/B_d$ is governed by Equations 43 or 91,

$$\left(\frac{3}{2} + \frac{r_{sd}w'}{\alpha'}\right) e^{-u'} + \frac{3}{2}(u' - 1) = 0,$$

regardless of whether the material is an $s = c + \sigma\tan\phi$ type or an $s = \sigma\tan\phi$ material. Hence, if factor $2K_e\tan\phi_e$ is made large enough by proper construction methods and if the ratio H/B_d of the fill height to the width of the ditch is also large enough, the use of Equation 43 instead of Equation 42, for determining factor u' for an $s = c + \sigma\tan\phi$ material, will not result in a serious error.

Under the above conditions, one may solve the load Equation 49 for an $s = c + \sigma\tan\phi$ material, by substituting the value of u' obtained from Equation 43.

It can also be shown by numerical examples that under the same conditions, for the values of cohesion c_e up to $c_e = \gamma B_d/2$, an $s = c + \sigma\tan\phi$ material may be assumed to be cohesionless and the error, besides being on the safe side, will not be appreciable enough to affect economy. Therefore, in such cases the above constructed charts may be used also for evaluating the conduit load for an $s = c + \sigma\tan\phi$ material.

If the construction of a high ditch with very loose material on top of the conduit is economically feasible, then factor $w' = (2K_e\tan\phi_e)H_d/B_d$ will be large enough and, consequently, the exponential function e^{-u} will approach zero and the first part of the load factor C_n will become negligible. Hence, under these conditions the load becomes independent of factor u' and it may be obtained either from Equation 23 or from Equation 85. However, if the construction conditions are such that w' is not large enough to make the first part of the load factor C_n negligible, the load must be computed by means of the appropriate equations.

SUMMARY AND CONCLUSIONS

I. From the construction point of view, underground conduits may be classified into two main categories:

A. Covered-Up Conduits.
Conduits belonging in this category are installed under artificial earth embankments that are constructed after the conduits have been assembled in place.

B. Mined-In Conduits.
Conduits of this category are installed by a mining process through natural earthen deposits.

II. Mathematical relations have been derived, describing the loading action on top of an underground conduit of a material whose shearing resistance can be represented by the general Coulomb equation $s = c + \sigma\tan\phi$.

III. Theoretical relations governing the loading action of a perfectly cohesionless or a perfectly cohesive material have been obtained by taking the limits of the expressions derived for the general case when c or ϕ are allowed to approach zero, respectively.

IV. From the mathematical standpoint the earth load on top of either covered-up or mined-in conduits can be evaluated by means of the same general expression

$$W_c = \gamma B_d H_{eff}$$

where:

W_c = vertical load on top of conduit, lb. per lin. ft.
γ = unit weight of the material, pcf.
B_d = effective width of the earth column above the conduit, ft.
H_{eff} = effective height of the earth column above the conduit, ft.

V. The effective height H_{eff} is a fictitious quantity and is a measure of the arching effect that takes place within the earth mass above the conduit. Generally:

A. If the mass directly above the conduit subsides less than the adjacent masses, the effective height, H_{eff}, is greater in magnitude than the actual height of the mass, H. Accordingly, the conduit load will be greater than the weight of the earth column directly above the conduit. This possibility has been defined as Case I.

B. If the reverse action takes place, H_{eff} is smaller in magnitude than H, and, consequently, the conduit load will be less than the weight of the earth column above the conduit. This possibility has been defined as Case II.

C. If no relative movement takes place within the earth mass above the conduit, H_{eff} will equal H and, consequantly, the weight on top of the conduit will be equal to the weight of the earth column directly above it.

VI. Case I has been shown to have detrimental effects on underground conduits regardless of the type of overlying soil. Therefore, this condition should always be avoided by proper methods of design and construction.

Case II with possibility C as a limiting condition is very advantageous and, consequently, the conditions for this case must always be sought by proper methods of design and construction. The theoretical analysis presented in this paper has dealt mainly with the factors influencing the conduit load when the conditions insuring the existence of Case II have been realized.

VII. From the analysis for Case II one may conclude that the effective height is the product of the effective width of the column of earth directly above the conduit multiplied by a function of dimensionless factors that depend:

A. Directly:

1. On the geometry of the installation.
2. On the initial state and physical properties of the loading agent.
3. On the height of arching, H_e.

B. Indirectly:

1. On the relative movement that takes place within the soil mass directly above the conduit.
2. On the relative stiffness between the body supporting the soil prism directly above the conduit defined as the "interior prism," and the body supporting the soil prisms adjacent to the interior prism, defined as "exterior prisms."
3. On the construction methods and workmanship employed.

VIII. Elaborating on Item VII, the following may be deduced from the mathematical analysis for Case II:

A. The higher the factor $v = (2K_e \tan\phi_e)\, H/B_d$, the higher will be the effective height.

B. The higher the factors $2K_e \tan\phi_e$, $2c_e/\gamma B_d$ and $u = (2K_e \tan\phi_e)\, H_e/B_d$, the lower will be the effective height.

C. For given values of v and $2c_e/\gamma B_d$, factor u varies directly with:

1. Factor $r_{sd}w'/a' = r_{sd}(E_F/E_L)(2K_e\tan\phi_e)\ H_d/B_d$ for covered-up conduits.

2. Factor $r_{sm}w/a = r_{sm}(E_f/E_m)(2K_e\tan\phi_e)\ H_m/B_d$ for mined-in conduits, or positive projecting conduits with their side supporting material compacted very thoroughly.

Factors $r_{sd}w'/a'$ or $r_{sm}w/a$ are measures of the relative yielding and the relative stiffness between the supports of the interior and the exterior prisms in covered-up or mined-in conduits respectively. Therefore, the larger in magnitude of these quantities, the higher will be the factor u, or, for a given installation, the higher will be the height of arching, H_e.

D. For fixed values of $2c_e/\gamma B_d$, and given values of $r_{sd}w'/a'$ or $r_{sm}w/a$, if v' or v are allowed to increase without limit, u' or u decrease respectively from maximum values $u' = v'_{max}$ or $u = v_{max}$ to limiting finite values $u' = u'_{v' = \infty}$ or $u = u_{v = \infty}$, that are independent of the cohesion c_e.

For values of v' or v smaller in magnitude than v'_{max} or v_{max}, factors u' or u are respectively greater in magnitude than v' or v and, therefore, they do not have a physical meaning dimensionwise.

E. For given values of v' and $r_{sd}w'/a'$, or v and $r_{sm}w/a$, if $2c_e/\gamma B_d$ is allowed to increase without limit, u' or u will approach zero. Under the same conditions the effective height will approach the value $-\infty$. However, physically, the above height can approach only the value zero. Consequently, the load on top of the conduit will vanish.

IX. From a knowledge of the behavior of the physical factors that are involved in the aforementioned mathematical analysis, the following conclusions may be drawn relative to the development of earth pressure on top of underground conduits:

A. The unit weight γ, the angle of internal friction ϕ and the cohesion c of the loading agent are understood to have a statistical average value meaning. Local deviations from this value depend:

1. In the case of covered-up conduits on the type of earthen material, the method of fill construction, and the water content of the fill material.
2. In the case of mined-in conduits on the geologic history, the initial state, and the water content of the overlying natural earth deposit.

B. Inasmuch as the potential sliding surfaces are not vertical planes but are in reality curved surfaces whose spacing is considerably greater at the top of the mass than it is at the top of the conduit, the shearing resistance of the soil is only partially mobilized along the assumed vertical planes in order to oppose any relative movement within the soil mass above the conduit. Consequently:

1. Only a portion of the maximum value of the angle of internal friction of the soil is mobilized along the vertical planes.
2. Only a portion of the maximum value of the cohesion of the material is mobilized along the vertical planes.
3. The earth pressure ratio, K, will never assume the extreme values K_A and K_P that are realized respectively for active and passive states of plastic equilibrium, but it will vary between these two limiting values.

To simplify the theoretical treatment of the problem, the above factors are assumed constant along the vertical planes and equal to ϕ_e, c_e, and K_e, respectively.

C. The values of ϕ_e, c_e, and K_e depend on the type, the initial state, the permeability, and the strain characteristics of the soil as well as on the size of the mass and the rate of application of stress to it.

From the foregoing it may be concluded that for a given installation and soil type, the construction methods and the workmanship employed, as well as time are major factors influencing ϕ_e, c_e, and K_e.

D. The strain characteristics of the material as well as the stress application mentioned in Item C, depend on the settlement ratio defined as r_{sd} for covered-up conduits, and r_{sm} for mined-in conduits. This ratio, which is a measure of the relative yield between the support of the interior prism and the support of the exterior prisms, depends on time and varies directly with the relative stiffness between supports.

E. For a given installation and soil type, the greater the magnitude of the settlement ratio, the higher will be the height of arching, H_e.

Physically, H_e can be only as high as the height of the mass above the conduit, H. If numerically, H_e is greater in magnitude than H, a trough-like depression will appear at the top of the mass directly above the conduit. This indicates that the soil mass along the sliding planes is being subjected to a greater amount of strain.

If the above conditions are realized, the effective values ϕ_e and c_e that are mobilized along the vertical planes will approach the limiting values of ϕ and c and K will approach the limiting value K_A at the top of the mass.

F. The maximum fill height up to which the surface of the fill may be expected to settle directly above an installed conduit can be obtained from Equation 58.

G. Everything else remaining the same, as the fill height increases the height of arching decreases and a greater portion of the mass acts directly on the conduit.

H. For very high fills the influence of the cohesion of the material becomes negligible and the material acts as if it were perfectly granular.

X. From the discussion of Item IX one may conclude that:

A. In a Covered-Up Conduit:

1. The better the gradation of the fill material and the more uniformly it is compacted, the greater will be the values, γ, ϕ, and c.
2. The greater the relative stiffness between the support of the interior prism and the support of the exterior prisms, the greater will be the relative movement within the soil above the conduit. Consequently, the greater will be the amount of ϕ and c that is mobilized along the vertical planes. If such conditions are realized, a greater portion of the load will be sustained by the shearing resistance of the material; hence, the pressure on top of the conduit will be reduced.
3. Since in the discussion of Item 2 it was indicated that the governing factor in the development of pressure on top of the conduit is the relative stiffness between the supports of the interior and exterior prisms and not the individual stiffness of each constituent part of these supports, it follows that the more rigid the conduit is:
   a. The stiffer should be the side supporting material.
   b. The looser and the more compressible should be the material in the ditch directly above the conduit.
   c. The more yielding should be the foundation.
4. The higher the ditch and the more compressible the material in it, the higher will be the equivalent earth pressure ratio K_e.

Since the shortening of the vertical diameter of the conduit is very small as compared with the compression of the material in the ditch, it follows that as this material is compacted due to the weight of the fill it subjects the side masses to compression. Consequently, K increases gradually from the minimum value it attains at the top of the ditch, to a maximum value at the top of the conduit. Thus, the value of the equivalent hydrostatic pressure ratio, K_e is increased also (Figure 2c).

5. From the mathematical analysis and the discussion of Item 4 it follows that the higher the ditch above the conduit and the more compressible the material in it, the greater will be the factor $w' = (2K_e \tan\phi_e)\, H_d/B_d$ and, consequently, the lower will be the conduit load.
6. The effective width B_d may be considered to be equal to the mean width of the

ditch above the conduit.

7. The settlement ratio r_{sd} can be determined by measuring directly the subsidence of the parts constituting the supports of the interior and exterior prisms.

8. It may be concluded that:

a. The side supporting material of a covered-up conduit should consist of thoroughly compacted, well- graded, granular material.

b. The ditch material should be of a compressible type and it should be placed in such a way that it will be in the loosest possible state.

c. The ditch should be made as high as economically feasible. In order that a sheeting and bracing operation be avoided during the construction of a high ditch, the following method may be adopted:

The heavy equipment, which is compacting the material adjacent to the ditch, may follow a course perpendicular to the longitudinal axis of the conduit. Each pass should end at lines pre-staked along the conduit axis and on both sides of the ditch. When the ditch sides become sufficiently high, the compressible material may be end-dumped from the heavy equipment into the ditch.

By the above method the ditch can be filled up with compressible material at the same time it is constructed. Therefore, it can reach any desirable height with its sides remaining vertical.

d. If Items a, b and c are fulfilled and should other considerations indicate that they will be more economical to use than flexible conduits, rigid conduits may also be installed safely under high fills.

B. In a Mined-In Conduit:

1. The values ϕ and c of a natural earth deposit will generally be greater than the same values of an identical material that has been remolded and used as a fill material on top of a covered-up conduit.

2. The greater the relative stiffness between the supports of the interior and exterior prisms, the greater will be the corresponding settlement ratio; accordingly, the greater will be the height of arching and the portions of ϕ and c that are mobilized along the sliding planes. Consequently, the load will be lower.

The softer the layer of the soil adjacent to the conduit, the more flexible should the conduit be made so that it will adjust its shape and thereby minimize the development of nonuniform external pressures.

3. The effective width of the earth column on top of the conduit, B_d, depends on the relative stiffness between the material adjacent to the conduit and the remainder of the mass above it.

4. The load on top of mined-in conduits that are installed under a deep natural earth deposit consisting of as $s = c + \sigma \tan\phi$ material becomes independent of the cohesion and may be evaluated by means of Equation 85.

5. The maximum height of the mass up to which the surface of the mass should be expected to settle assuming a trough-like shape directly above an installed conduit can be obtained from Equation 75.

6. In mined-in conduits the settlement ratio r_{sm} can be determined only indirectly. In positive projecting conduits, however, it can be obtained by direct measurements.

C. Generally:

A good knowledge of the physical properties of the soils that are involved in a project is always necessary. Therefore, a good soil exploration of the site where the conduit is to be installed is imperative.

The factor of safety can be applied to the values ϕ_e, and c_e that are considered to be the developed fractions of the maximum values of the angle of internal friction and cohesion of the material.

The value K_e will vary with the method of installation and with the soil type. Tentatively, it is suggested that for small depths of overburden its value be chosen between 0.5 and K_A; for high depths its value be chosen between 0.5 and 1.0

The relative stiffness between the supports of the interior and exterior prisms should be made as high as possible.

XII. Families of curves have been constructed from which the conduit load due to an $s = \sigma \tan\phi$ material can be computed. Under certain conditions these curves can be used to evaluate the conduit load due to an $s = c + \sigma \tan\phi$ soil type.

XIII. In this investigation consideration has been given to pressures that act in the plane of a right, cross-section of the conduit with no allowance for variations caused by arching along the longitudinal axis of the conduit nor to tangential forces that act along this axis.

Since an underground conduit has the tendency to settle more in the middle than at its ends, the earth mass, which is above its middle portion, will tend to brace itself against the end masses thereby increasing the normal pressure and the longitudinal strains at the ends and decreasing the pressure at the middle. The effects of such arching action will be especially significant in the case of long conduits installed under high fills.

RECOMMENDATIONS FOR FUTURE STUDY

In order to obtain a better understanding of earth pressures on underground conduits, the author believes that future efforts should be concerned mainly with the development of techniques by means of which the earth pressure exerted around the circumference of an underground conduit can be measured directly. From such information one will be able to gather substantial information to:

I. Evaluate the values ϕ_e, c_e, and K_e for given installation conditions.

II. Determine the magnitude of the lateral pressure that a precompacted side supporting material is potentially able to mobilize per unit of lateral bulging of the conduit.

III. Determine by rational means the type, the size, and the degree of precompaction of the side supporting material in order that a given conduit may not bulge excessively.

IV. Develop a theory expressing the vertical load on top of the conduit as a function of the lateral pressures exerted by the side supporting material as well as the bottom reaction of the bedding material.

V. Understand the arching effect on the earth mass above and along the conduit axis.

VI. Obtain information on settlement ratios especially for mined-in conduits.

VII. Permit the use of a substantially smaller factor of safety and thereby achieve a more economical design.

In conjunction with the above discussion the reader is referred to a report prepared by the research department of the North Carolina State Highway and Public Works Commission (Costes and Proudley, 1955) in which an attempt to develop a technique for the direct measurement of the lateral earth pressures acting on a flexible culvert pipe installed under a high fill is outlined.

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Appendix

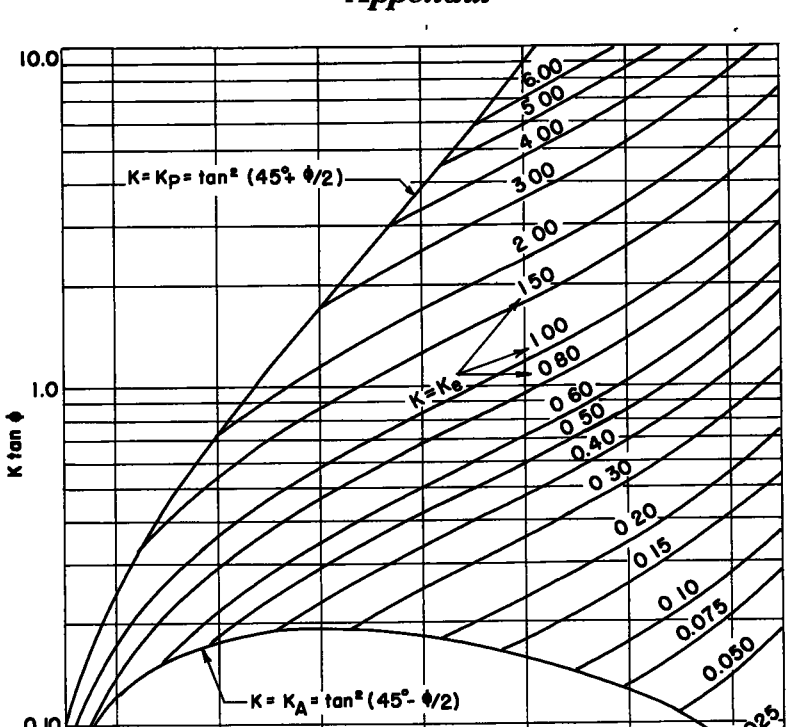

Figure A.

TABLE A

VALUES OF Ktanϕ FOR GIVEN VALUES OF K AND ϕ

K	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°
K_A	0 073	0 125	0 158	0 178	0 189	0 192	0 190	0. 182	0 171	0. 159	0 141	0 125	0. 105	0. 085	0 063
0 025															0. 093
0 050													0 107	0. 137	0. 187
0 075												0. 130	0. 161	0. 206	0. 280
0 100											0. 143	0 173	0 215	0 275	0. 373
0 150										0 179	0. 214	0. 260	0 322	0. 412	0 560
0. 200									0 200	0. 238	0 286	0 346	0. 429	0 550	0 746
0 300							0 210	0 252	0 300	0. 358	0. 428	0 520	0 644	0 824	1. 120
0 400						0 231	0 280	0 336	0 400	0 477	0. 571	0. 693	0. 858	1 099	1 493
0 500				0 182	0. 233	0 289	0 350	0. 420	0 500	0. 596	0 714	0. 866	1 073	1 374	1 866
0 600			0 161	0 218	0 280	0 346	0 420	0. 503	0 600	0. 715	0. 857	1. 039	1 287	1 649	2. 239
0 800		0 142	0 214	0 291	0. 373	0 462	0 560	0 671	0 800	0 954	1 142	1 386	1 716	2 198	2 986
1 000	0 087	0 177	0 268	0 364	0 466	0 577	0 700	0 839	1 000	1 192	1 428	1 732	2 145	2 748	3 732
1 500			0 402	0 546	0 699	0 866	1 050	1. 259	1 500	1 788	2 142	2 598	3 218	4 122	5 598
2 000				0 728	0 932	1 154	1 400	1. 678	2 000	2 384	2 856	3 464	4 290	5 496	7 464
3 000						1 731	2 100	2 517	3 000	3 576	4 284	5 196	6 435	8 244	11 196[a]
4 000								3 356	4 000	4 768	5 712	6 928	8 580	10 992[a]	14 928[a]
5 000									5 000	5 960	7 140	8. 660	10 725[a]	13 740[a]	18 660[a]
6 000										7. 152	8 568	10. 392[a]	12 870[a]	16. 482[a]	22. 392[a]
K_P	0 104	0 252	0 455	0 742	1 147	1 731	2 583	3. 860	5 827	8 995	14. 369[a]	24 123[a]	43 649[a]	88 378[a]	215 333[a]

[a] Values of Ktanϕ beyond plotting range of Figure A

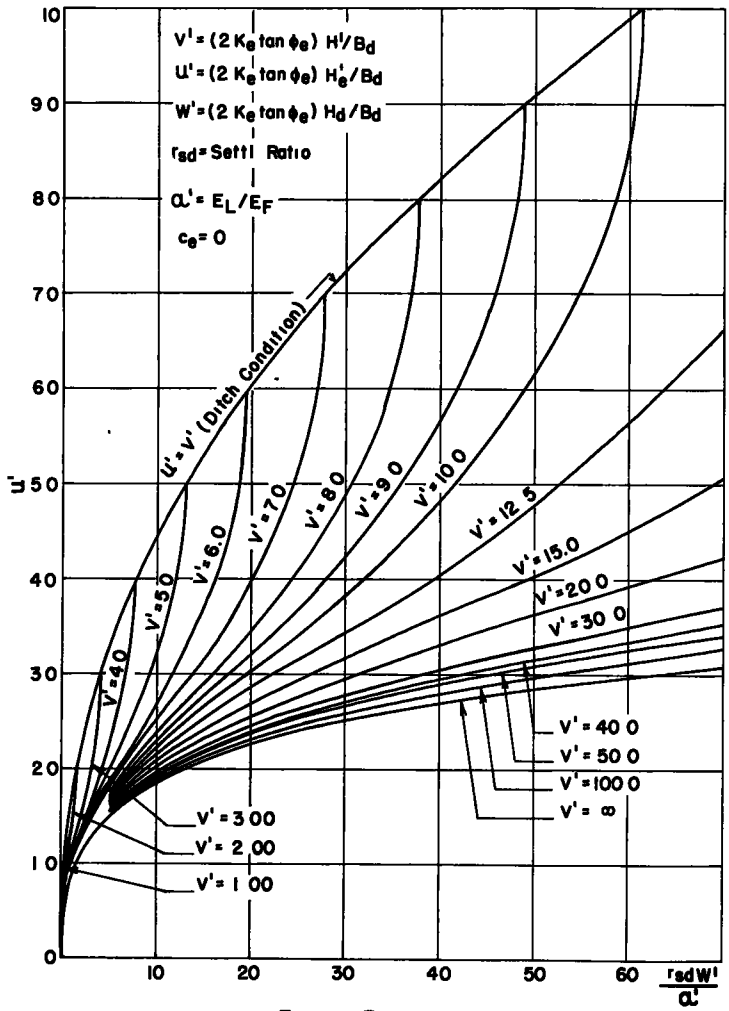

Figure B.

TABLE B

VALUES OF u' FOR GIVEN VALUES OF v' AND $r_{sd}w'/\alpha'$

v'	0.0	-0.5	-1.0	-1.5	-2.0	-3.0	-4.0	-5.0	-7.5	-10	-20	-30	-40	-50	-60	-70
							$\frac{r_{sd}w'}{\alpha'}$									
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
v' = u'	0.000	1.234	1.680	2.000	2.260	2.693	3.043	3.343	3.975	4.495	6.058	7.243	8.200	9.100	9.888	10.595
1.0	0.000	-[a]	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2.0	0.000	0.794	1.210	2.000	-	-	-	-	-	-	-	-	-	-	-	-
3.0	0.000	0.732	1.025	1.270	1.495	2.000	-	-	-	-	-	-	-	-	-	-
4.0	0.000	0.708	0.970	1.170	1.347	1.658	1.960	2.265	3.625	-	-	-	-	-	-	-
5.0	0.000	0.695	0.941	1.125	1.281	1.546	1.775	1.990	2.575	3.19	-	-	-	-	-	-
6.0	0.000	0.687	0.925	1.100	1.245	1.483	1.688	1.875	2.280	2.69	-	-	-	-	-	-
7.0	0.000	0.682	0.915	1.085	1.222	1.445	1.637	1.799	2.163	2.49	3.95	-	-	-	-	-
8.0	0.000	0.677	0.907	1.070	1.205	1.420	1.598	1.745	2.077	2.37	3.45	4.90	-	-	-	-
9.0	0.000	0.674	0.900	1.063	1.193	1.401	1.572	1.720	2.040	2.29	3.21	4.21	5.70	-	-	-
10.0	0.000	0.672	0.897	1.056	1.185	1.389	1.550	1.680	1.978	2.23	3.04	3.85	4.80	6.19	8.50	-
12.5	0.000	0.668	0.889	1.044	1.168	1.362	1.518	1.649	1.920	2.18	2.81	3.40	3.95	4.62	5.38	6.22
15.0	0.000	0.666	0.883	1.035	1.158	1.348	1.493	1.625	1.875	2.08	2.69	3.18	3.62	4.04	4.51	5.07
20.0	0.000	0.663	0.877	1.026	1.145	1.327	1.475	1.589	1.831	2.03	2.56	2.96	3.30	3.62	3.93	4.24
30.0	0.000	0.660	0.870	1.016	1.133	1.309	1.456	1.565	1.778	1.97	2.45	2.78	3.05	3.29	3.51	3.71
40.0	0.000	0.658	0.867	1.012	1.128	1.301	1.441	1.545	1.765	1.94	2.40	2.71	2.95	3.16	3.34	3.51
50.0	0.000	0.657	0.866	1.010	1.124	1.298	1.437	1.535	1.750	1.93	2.37	2.66	2.90	3.09	3.26	3.41
100.0	0.000	0.655	0.861	1.005	1.117	1.283	1.419	1.525	1.728	1.92	2.32	2.59	2.79	2.96	3.11	3.25
∞	0.000	0.653	0.859	1.000	1.110	1.278	1.408	1.514	1.718	1.872	2.271	2.524	2.701	2.845	2.980	3.065

[a] For these values of v' and $r_{sd}w'/\alpha'$, u' is equal to v' physically

TABLE C

VALUES OF C_m FOR GIVEN VALUES OF v' AND $r_{sd}w'/a'$

$\frac{r_{sd}w'}{a'}$ \ v'	0.0	1.0	2.0	3.0	4.0	5.0	6 0	7.0	8.0	9.0	10
0.00	-1.000	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7 000	8.000	9.000
0.01	-1.000	-0.110	0.799	1.700	2.580	3.480	4.390	5.280	6.14	7.04	7.90
0.05	-1.000	-0.200	0.586	1.370	2.180	2.950	3.720	4.500	5.30	6.11	6.90
0.10	-1.000	-0.265	0.464	1.175	1.910	2.620	3.340	4.050	4.77	5.51	6.25
0.20	-1.000	-0.325	0.314	0.959	1.600	2.230	2.850	3.490	4.13	4.77	5.40
0.40	-1.000	-0.368[a]	0.155	0.704	1.230	1.800	2.350	2.900	3.45	4.00	4.48
0.60	-1.000	-0.368	0.054	0.548	1.020	1.550	2.040	2.520	3.02	3.52	4.00
0.80	-1.000	-0.368	-0.010	0.440	0.880	1.350	1.800	2.250	2.68	3.13	3.58
1.00	-1.000	-0.368	-0.059	0.352	0.777	1.190	1.610	2.020	2.43	2.85	3.26
1.50	-1.000	-0.368	-0.132	0.211	0.572	0.932	1.300	1.680	2.04	2.40	2.75
2.00	-1.000	-0.368	-0.135[a]	0.119	0.440	0.760	1.070	1.400	1.76	2.05	2.40
3.00	-1.000	-0.368	-0.135	0.000	0.262	0.530	0.800	1.080	1.37	1.62	1.90
4.00	-1.000	-0.368	-0.135	-0.049	0.150	0.380	0.617	0.858	1.13	1.35	1.59
5.00	-1.000	-0.368	-0.135	-0.050[a]	0.078	0.280	0.488	0.701	0.93	1.13	1.35
6.00	-1.000	-0.368	-0.135	-0.050	0.028	0.201	0.386	0.580	0.80	0.98	1.17
8.00	-1.000	-0.368	-0.135	-0.050	-0.017	0.100	0.249	0.410	0.59	0.75	0.92
10.00	-1.000	-0.368	-0.135	-0.050	-0.018[a]	0.035	0.155	0.293	0.44	0.58	0.73
15.00	-1.000	-0.368	-0.135	-0.050	-0.018	-0.007[a]	0.034	0.123	0.23	0.33	0.45
20.00	-1.000	-0.368	-0.135	-0.050	-0.018	-0.007	-0.003[a]	0.035	0.11	0.20	0.29
∞	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

[a] For values of v' and $r_{sd}w'/a'$ below marked horizontal lines, u' is equal to v' physically and $C_m = -e^{-v'}$ (Ditch Condition).

$\frac{r_{sd}w'}{a'}$ \ v'	15	20	30	40	50	60	70	80	90	100	∞
0.00	14.000	19.000	29.000	39.000	49.000	59.000	69.000	79.000	89.000	99.000	∞
0.01	12.4	16.8	25.8	34.7	43.8	52.5	61.0	70.8	80.0	88.5	∞
0.05	10.8	14.7	22.6	30.5	38.5	46.3	54.2	62.1	70.5	78.0	∞
0.10	9.85	13.4	20.5	27.9	35.2	42.6	49.8	57.0	64.0	71.5	∞
0.20	8.62	11.8	18.2	24.6	31.0	37.5	44.0	50.2	57.0	63.0	∞
0.40	7.35	10.1	15.6	21.0	26.5	32.1	37.5	43.1	48.9	54.0	∞
0.60	6.48	8.95	13.8	18.8	23.8	28.7	33.6	38.7	43.8	48.6	∞
0.80	5.87	8.10	12.7	17.3	21.9	26.5	31.0	35.5	40.3	44.8	∞
1.00	5.40	7.50	11.7	16.0	20.2	24.5	28.7	33.0	37.3	41.5	∞
1.50	4.59	6.40	10.1	13.8	17.5	21.1	24.8	28.4	32.0	35.9	∞
2.00	4.04	5.66	8.98	12.3	15.6	18.8	22.1	25.3	28.7	32.0	∞
3.00	3.28	4.65	7.48	10.2	13.0	15.7	18.5	21.4	24.2	27.0	∞
4.00	2.78	4.00	6.45	8.90	11.3	13.8	16.2	18.7	21.2	23.5	∞
5.00	2.46	3.56	5.75	8.00	10.2	12.4	14.6	16.8	19.0	21.2	∞
6.00	2.18	3.17	5.20	7.20	9.20	11.2	13.3	15.3	17.4	19.4	∞
8.00	1.75	2.61	4.35	6.10	7.82	9.59	11.3	13.1	14.8	16.5	∞
10.00	1.50	2.24	3.75	4.09	6.85	8.39	9.90	11.5	13.0	14.5	∞
15.00	1.02	1.64	2.86	4.04	5.30	6.50	7.75	9.00	10.2	11.5	∞
20.00	0.77	1.27	2.28	3.30	4.30	5.35	6.40	7.43	8.50	9.55	∞
∞	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0

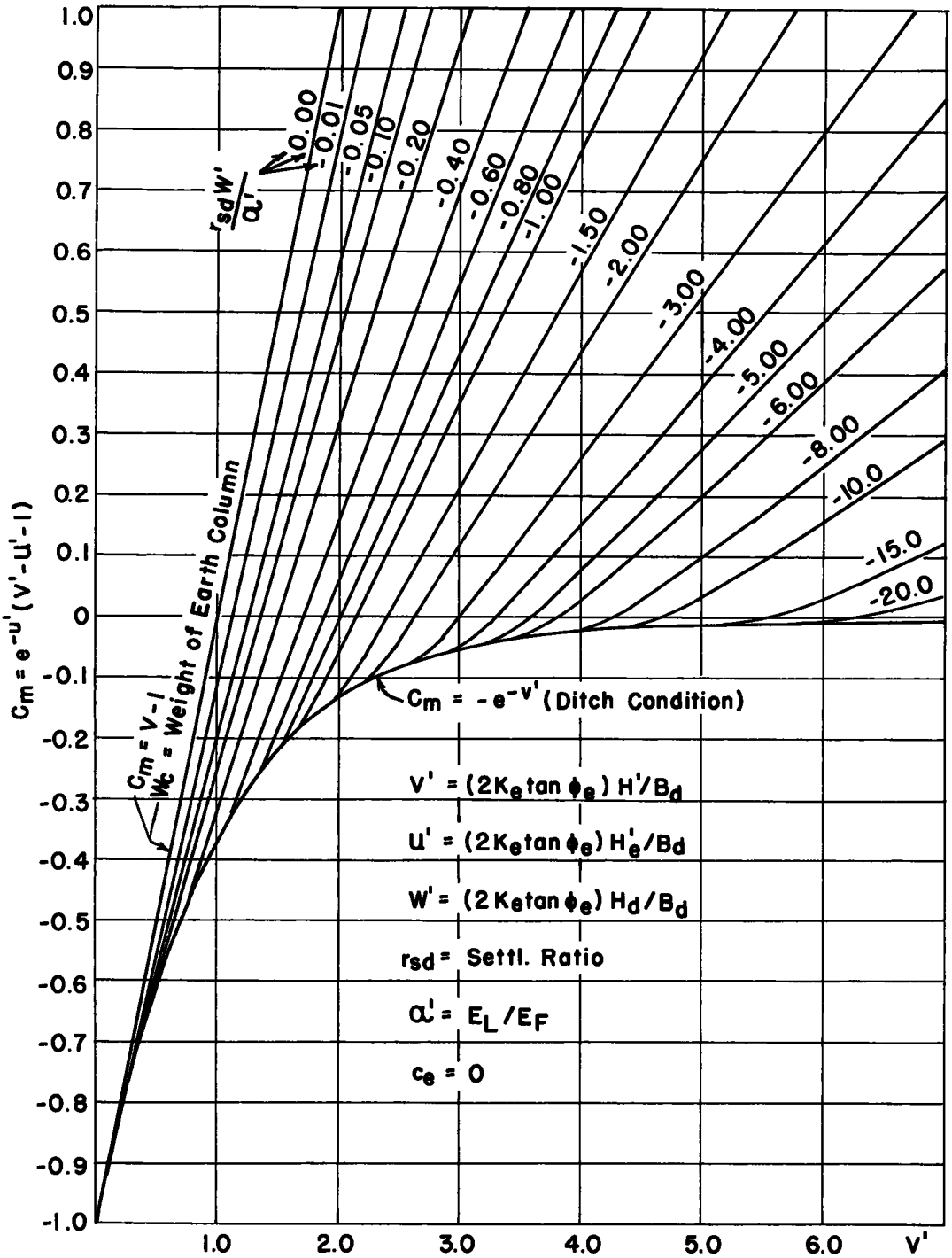
$C_m = e^{-u'}(v'-u'-1)$
$\frac{r_{sd}W'}{\alpha'}$
0.00
-0.01
-0.05
-0.10
-0.20
-0.40
-0.60
-0.80
-1.00
-1.50
-2.00
-3.00
-4.00
-5.00
-6.00
-8.00
-10.0
-15.0
-20.0
$C_m = V-1$
W_c = Weight of Earth Column
$C_m = -e^{-v'}$ (Ditch Condition)
$V' = (2K_e \tan\phi_e)\, H'/B_d$
$u' = (2K_e \tan\phi_e)\, H'_e/B_d$
$W' = (2K_e \tan\phi_e)\, H_d/B_d$
r_{sd} = Settl. Ratio
$\alpha' = E_L/E_F$
$c_e = 0$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
1.0
2.0
3.0
4.0
5.0
6.0
V'

Figure C.

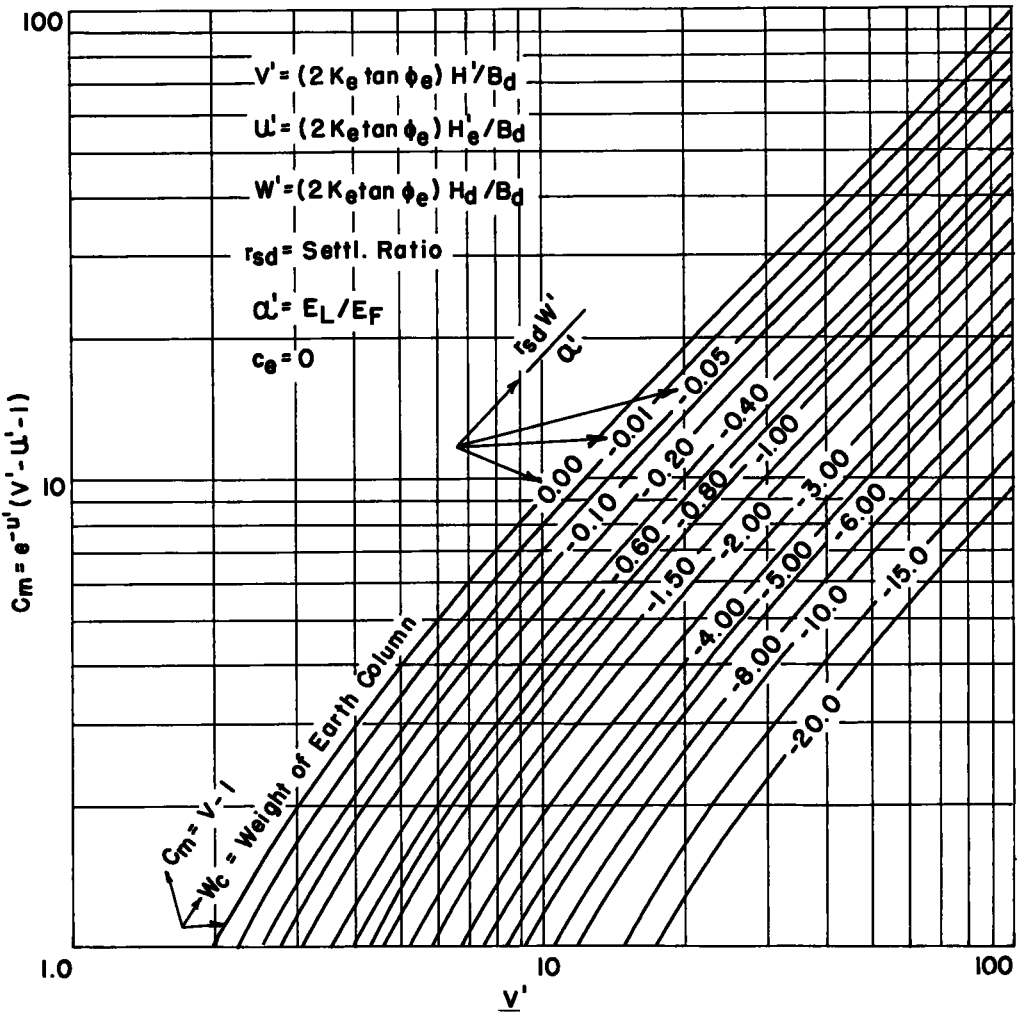

Figure C1.

TABLE D

VALUES OF C_n FOR GIVEN VALUES OF C_m AND w'

C_m	0.0	0.5	1.0	1.5	2.0	2.5	w' 3.0	3.5	4.0	4.5	5.0	6.0	7.0	8.0	∞
0.1	1.10	1.06	1.04	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.20	1.12	1.07	1.04	1.03	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.40	1.24	1.11	1.09	1.05	1.03	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.50	1.30	1.18	1.11	1.07	1.04	1.02	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00
0.6	1.60	1.36	1.22	1.13	1.08	1.05	1.03	1.02	1.01	1.01	1.00	1.00	1.00	1.00	1.00
0.8	1.80	1.49	1.29	1.18	1.11	1.07	1.04	1.02	1.01	1.01	1.01	1.00	1.00	1.00	1.00
1.0	2.00	1.61	1.37	1.22	1.14	1.08	1.05	1.03	1.02	1.01	1.01	1.00	1.00	1.00	1.00
2.0	3.00	2.21	1.74	1.45	1.27	1.16	1.11	1.06	1.04	1.02	1.01	1.00	1.00	1.00	1.00
4.0	5.00	3.43	2.47	1.89	1.54	1.33	1.20	1.12	1.07	1.04	1.03	1.01	1.00	1.00	1.00
5.0	6.00	4.03	2.84	2.12	1.68	1.41	1.25	1.15	1.09	1.06	1.03	1.01	1.00	1.00	1.00
6.0	7.00	4.64	3.21	2.34	1.81	1.49	1.30	1.18	1.11	1.07	1.04	1.01	1.00	1.00	1.00
8.0	9.00	5.85	3.94	2.79	2.08	1.66	1.40	1.24	1.15	1.09	1.05	1.02	1.00	1.00	1.00
10.0	11.00	7.07	4.68	3.23	2.35	1.82	1.50	1.30	1.18	1.11	1.07	1.02	1.01	1.00	1.00
20.0	21.00	13.13	8.36	5.46	3.71	2.64	2.00	1.60	1.37	1.22	1.13	1.05	1.02	1.01	1.00
40.0	41.00	25.26	15.72	9.93	6.41	4.28	2.99	2.21	1.73	1.44	1.27	1.10	1.04	1.01	1.00
50.0	51.00	31.33	19.39	12.16	7.77	5.11	3.49	2.51	1.92	1.56	1.34	1.12	1.05	1.02	1.00
60.0	61.00	37.39	23.07	14.39	9.12	5.93	3.99	2.81	2.10	1.67	1.40	1.15	1.05	1.02	1.00
80.0	81.00	49.52	30.43	18.85	11.83	7.57	4.98	3.42	2.47	1.89	1.54	1.20	1.07	1.03	1.00
100.0	101.00	61.65	37.79	23.31	14.53	9.21	5.98	4.02	2.83	2.11	1.67	1.25	1.09	1.03	1.00
∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	1.00

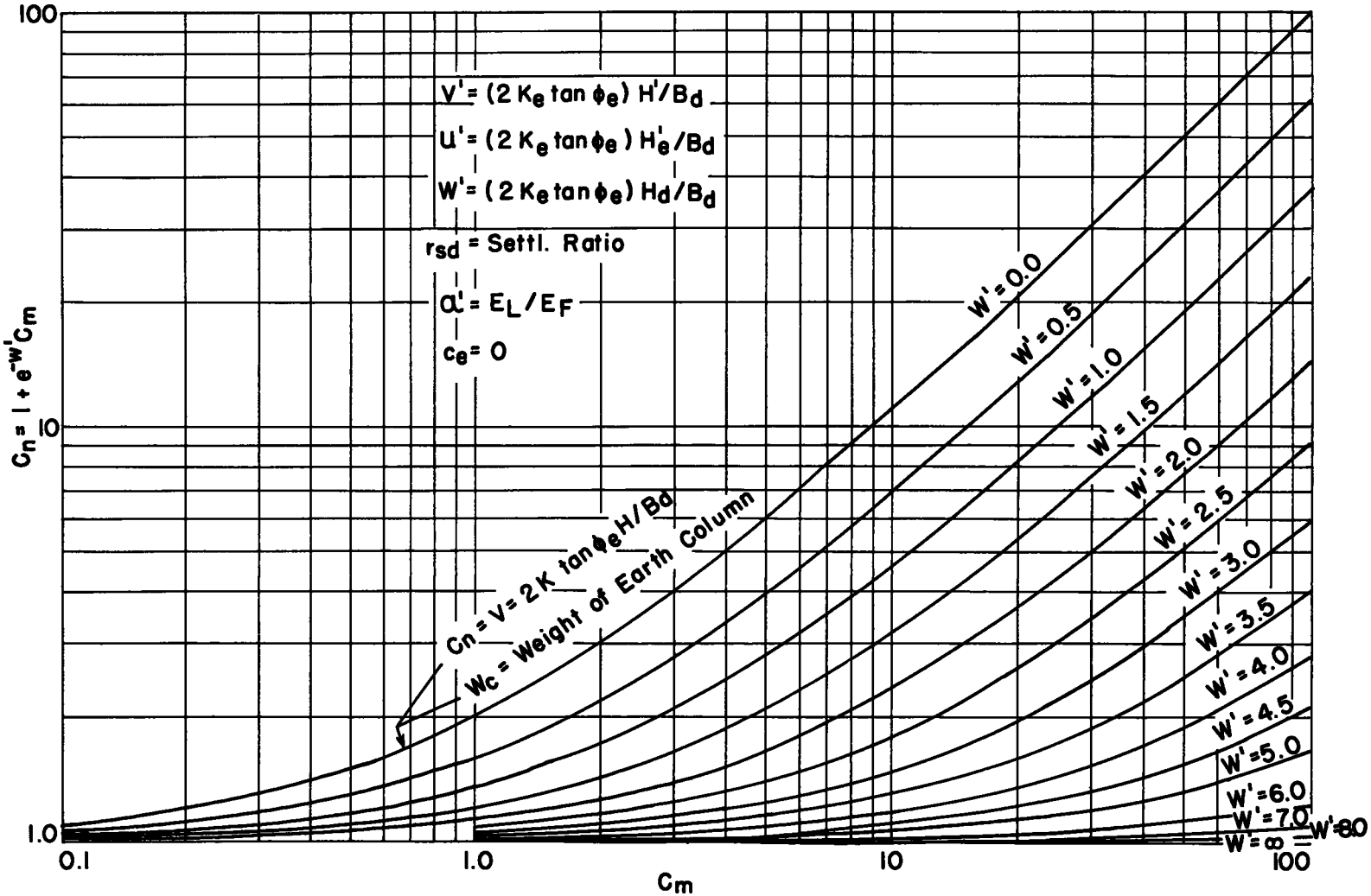
V' = (2 Ke tan φe) H'/Bd
U' = (2 Ke tan φe) H'e/Bd
W' = (2 Ke tan φe) Hd/Bd
rsd = Settl. Ratio
α' = EL/EF
ce = 0
Cn = V = 2K tan φe H/Bd
Wc = Weight of Earth Column
W' = 0.0
W' = 0.5
W' = 1.0
W' = 1.5
W' = 2.0
W' = 2.5
W' = 3.0
W' = 3.5
W' = 4.0
W' = 4.5
W' = 5.0
W' = 6.0
W' = 7.0
W' = ∞ = W' = 80
Cn = 1 + e^-W' Cm
Cm
100
10
1.0
0.1
1.0
10
100

Figure D.