Sign Brightness in Relation to Position, Distance, and Reflectorization

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There is need for quantitative comparison of the brightnesses of different sign materials in various situations on the highway. This paper describes a method for calculating the brightness of a reflective material, for a given distance and placement. The method is applied to investigate the effects of such factors as sign position with respect to the pavement, type of reflective material, type of headlamp, type of vehicle, and vertical and horizontal curves. Relationships of these factors to sign legibility and their implications for signing practice are discussed.

●FOR many years one of the principal means of communication between the highway engineer and the motorist has been the highway sign. Recently, with increased demands of traffic, highway signs have assumed a new importance, because they provide the best means of controlling operating speeds and directing traffic through the complicated interchanges that characterize modern highway systems. The message delivered by a sign, important enough during the daytime, becomes even more important at night when visibility is limited.

Only ten or fifteen years ago signs were difficult to read at night, but many of today's signs can be read almost as well after dark as in the daytime. The reflectorized sign has made a great contribution to the development of safe high-speed motor transportation. Today the purchaser of sign material can choose between several different types of reflectorized materials, which differ greatly not only in their cost but also in their brightness in given situations. Sometimes for the signing agency the availability of so many different types of materials can be a mixed blessing.

The Problem

Different demands are placed on a reflective sign material, depending upon the circumstances in which it is to be used. Obviously, the demands on a reflective material are quite different for an overhead sign on a high-speed expressway than for a confirmatory route marker on a country road. In choosing a reflective material for a given application, the engineer must rely on his experience and the advice of manufacturers. Sales representatives from different firms often give conflicting recommendations, and the factors affecting the engineer's choice are so complex that experience is difficult to apply. Wide differences in opinion are encountered regarding the suitability of a particular material for a particular application, and new materials, or recent modifications of old ones, are continually entering the market to confuse the engineer attempting to make a decision.

There is need for quantitative description of the performance of reflective materials on the highway. In addition to permitting a more intelligent choice between sign materials and more intelligent use of them after purchase, such a quantitative description would be a step toward quantitative design criteria applicable to signs for new situations. Van Lear (9) and Finch (2) have outlined procedures for photometric measurement of the reflective characteristics of materials. However, such measurements are of limited usefulness until they have been related to the conditions under which the sign is used on the highway.

The problem of the brightness of reflective materials might be stated as two questions: (a) "How bright should a sign be?" and (b) "How bright is a sign of a certain material in a given highway situation?" In regard to the first question, the basic relationships between brightness and legibility have been discussed in a previous paper (1). The present paper concerns itself primarily with the second question.

Purpose of Study

The purposes of this study were to contribute to the theory of brightness of highway signs in place by using certain photometric techniques, and to apply the theory to representative sign materials in representative highway situations.

METHOD

To determine the brightness, or luminance, of a sign in place on the highway, it is necessary to take into account the reflective characteristics of the sign material; the trigonometric relationships between the car, the sign, and the roadway; and the illumination reaching the sign from the headlamps.

Reflective Materials

The basic principles involved in a study of reflectorized materials (reflex reflectors or retrodirective reflectors) have been explained by Van Lear (9) and Finch (3). The reader is referred to these sources for a complete discussion. In addition, a brief explanation is given here of the principles touched on in this paper.

When light reaches a sign coated with ordinary pigmented paint, it is reflected more or less diffusely in all directions. Very little of the light is returned to the driver's eyes, and such a sign is difficult to read at night. The distinguishing feature of a reflectorized sign is that it concentrates a large proportion of the light into a beam which is directed back toward the source of the light. Since the reflectorized sign appears brighter to the driver, it can be read more easily at night.

Figure 1 shows how a reflectorized sign works. Part A shows the sign being illuminated by a headlamp beam. Some of the light from the source spreads out, goes past the sign, and is lost. Part B shows the sign reflecting light. Much of the reflectorized light is returned in a fairly narrow beam toward the source, but some is spread out in the return beam. The drawing is presented in two parts for simplicity, although illumination and reflection actually occur at the same time. To the person viewing the sign,

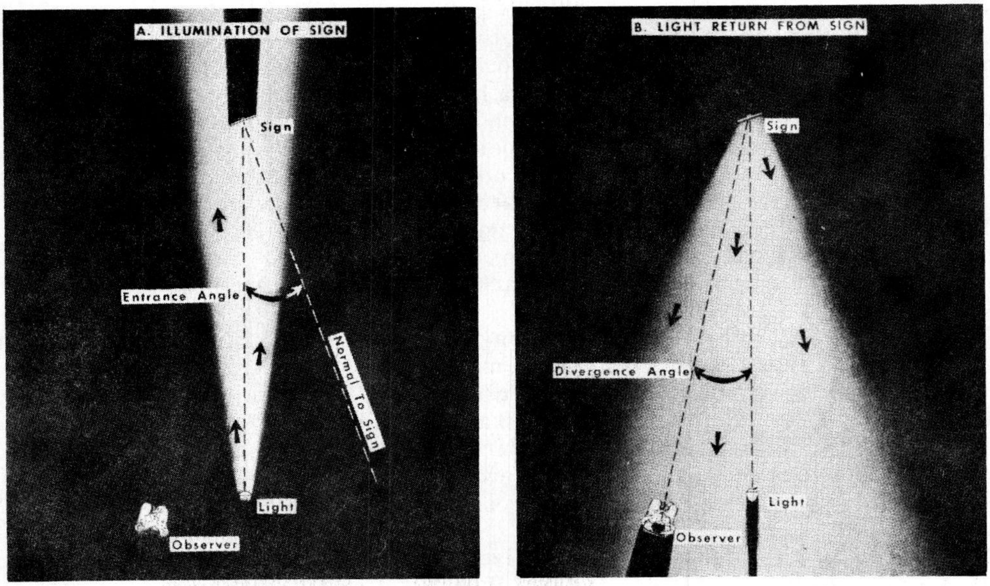

Figure 1. How a reflectorized sign works. Much of the light reaching the sign from the source is returned by the sign back toward the source.

it would appear brightest if his eyes could coincide with the line of the headlamp. As the eyes are moved farther away from the alignment of the light source, the sign appears less bright. The light-sign-eye angle is called the divergence angle. Figure 2 shows the distribution of light in the return beam from one type of sign material. The brightness of the sign is at the maximum when the divergence angle is zero, and it falls off as the eyes move away in any direction from the line of the light source.

Notice the entrance angle in Figure 1. Signs made of most reflectorized materials are brightest when facing the light squarely; that is, when the entrance angle is zero. As the sign is rotated so that the entrance angle increases, the brightness of the sign decreases. The dashed curve in Figure 2 shows the distribution of light in the return beam at a larger entrance angle.

"Luminance," often used to describe the measurement of the brightness of a sign, differs from "brightness" in that it has a specific meaning in terms of the physical measurement of light, whereas "brightness" is a more general term describing appearance to an observer (10).

Both the brightness and the luminance of a sign also depend on how much light it receives from the headlamp. The more light the sign receives from the headlamp, the more it will return to the eye. Figure 2 shows the luminance (in foot-Lamberts) when the sign receives 1 ft C of illumination. If the light received by the sign were 2 ft C, its luminance would be twice as much, and so on. The luminance of the sign when it received 1 ft C of illumination is termed "specific luminance," the unit of measurement used in this paper to describe the reflective characteristics of a material.

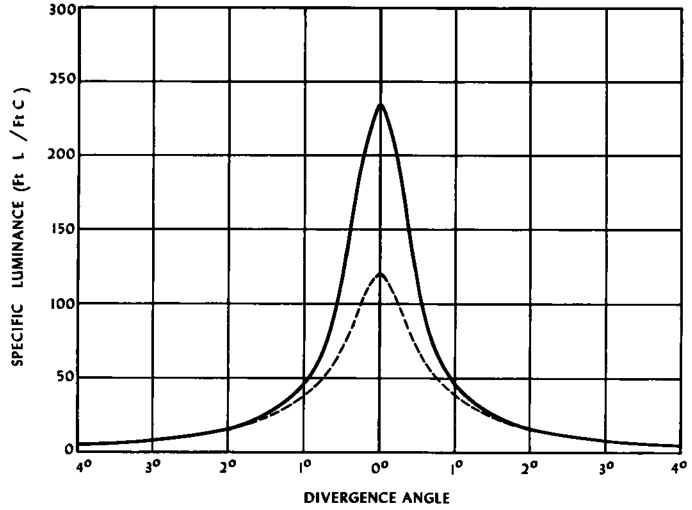

Figure 2. Light return from a reflectorized material. The solid line is for an entrance angle of 0 degrees, the dashed line for 30 degrees.

In plotting curves it is convenient to show luminance and specific luminance on a logarithmic scale. Equal units on a log scale of luminance appear as equal units of brightness to the eye (Fechner's law); perceptible differences in brightness are constant intervals on a log scale of luminance (Weber's law). The log scale was also found more meaningful for expressing relationships involving legibility (1). Figure 3 shows the data of Figure 2 plotted on a log scale of specific luminance. Since the curves for most reflective materials are symmetrical, the left half of the curve can be omitted. In the following section, curves of this type are shown for each type of material studied. Technical descriptions of the units of measurement and the method of measurement are given in Appendix A.

Trigonometric Relationships

To determine the light return from a sign in place on the highway, it is necessary to know the reflective characteristics of the material, which are a function of the entrance angle of the light reaching the sign from the car's headlamp, and the divergence angle between the headlamp and the driver's eyes. Calculation of these angles is not as simple as for the two-dimensional sketch (Fig. 1). Because the sign, the headlamps, and the driver's eyes are not at the same level, the problem must be solved in three dimensions instead of two. Figure 4 illustrates the divergence angle between the driver's eyes and each of his headlamps. It can be seen that the angles are different for each headlamp and that each angle is measured in a different inclined plane. For any given sign position, each of these angles will change continuously as the car approaches the sign.

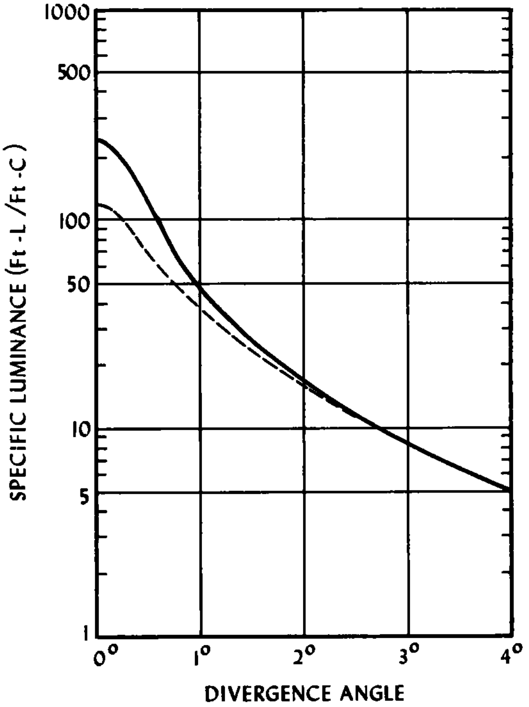

Figure 3. Data of Figure 2 plotted to a semi-log scale. Since the curves are symmetrical, left half is omitted.

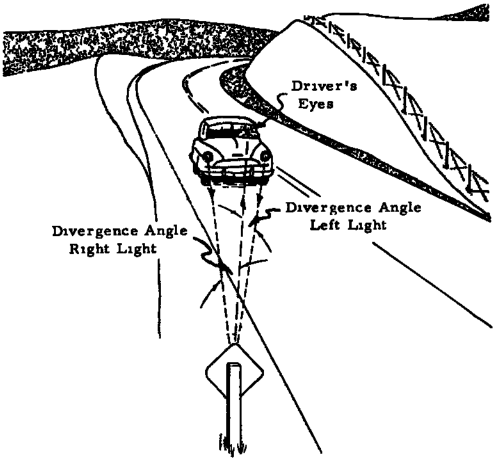

Figure 4. Divergence angles for right and left headlamps of a car approaching a sign.

Solutions were worked out for these angles for any sign position at any distance. Details of the solutions are given in Appendix B. To compute the angles, the dimensions of the car were necessary and measurements of typical late-model cars were made. The average values used in computations are shown in Figure B-1 in Appendix B.

Illumination from Headlamps

To compute the amount of light reaching the sign from the headlamp, it is necessary to know the distance to the sign, the distribution and intensity of light in the headlamp beam, and the position of the sign in the headlamp beam.

For the distribution of light in the headlamp beam, isocandle charts were obtained from headlamp manufacturers. One of these charts is reproduced (Fig. C-1) in Appendix C. It shows the intensity of light at any point in the headlamp beam. The method of determining the position of the sign in the headlamp beam also is explained in Appendix B. For any position in the headlamp beam, the intensity in candlepower can be read from the isocandle charts. The illumination reaching the sign (in foot-candles) is equal to the intensity in effective candlepower (as determined from an isocandle chart) divided by the square of the distance to the sign in feet.

Estimates of the illumination are no better than the degree to which these isocandle charts represent the distribution of light from the headlamps of cars. The isocandle charts used in computations were plotted from a typical headlamp. Although individual headlamps vary somewhat from one to another, the variations in voltages and headlamp aiming are more important. Roper (7) suggests that variations in voltage may introduce as much as 20 percent error in estimates of light output. He also suggests that variations in aiming are often as much as ½ degree, which could double or reduce to one-half the illumination of a point. As shown later, such variations are not as serious as they sound initially, because their effects are small when compared to the 20,000-to-1 range of sign brightness found on the highway. Nonetheless, these sources of error should be kept in mind when interpreting results.

Initial calculations of illumination were made from data for the sealed-beam lamps (No. 4030) produced as standard equipment until 1954. In a later section the effects of the new (No. 5040) headlamps, which have become standard equipment on cars produced since 1955, are discussed.

REFLECTIVE MATERIALS

Five types of reflective materials were chosen for study. These materials are now used in most reflectorized signs, except those using reflector buttons (which are not directly comparable to continuous-surface materials). Although letters made from buttons appear continuous to the eye when the sign is read at a distance, the relationships between the legibility and photometrics of buttons are essentially different from those of continuous-surface materials. Therefore, it would be misleading to include curves for reflector buttons with those of the continuous-surface materials. The method employed in this report can be used to compute the intensity of light of a reflector button in a manner comparable to the calculation of the luminance of continuous-surface materials (intensity in apparent candlepower is equal to illumination times specific intensity); however, no research to date has established the relationship between the legibility of reflector-button letters with a given intensity and that of continuous-surface letters (or background) of a given luminance. Until such research is done, it will not be possible to make a valid comparison of the two. Therefore, the present study confined itself to reflective materials with surfaces of uniform brightness. Each type of material was studied under a microscope. Figures 5 through 9 are photomicrographs of the five types and diagrams showing them in cross-section.

Beads on Paint

The least expensive type of material (Fig. 5) is built up directly on the aluminum or steel sign backing, and can be made in highway department sign shops. After the metal has been given a primer and an undercoat of paint, a special binder paint is applied. While the film of binder is still wet, a layer of tiny round glass beads is deposited on it. The beads are held in position by the paint. Figure 5 shows how this material works: a ray of light enters the bead and is reflected in the direction from which it came.

These beads are so small (about 0.004-in. diameter) that about 20 million are required to cover the surface of an average sign. Accurate control of paint film thickness and of the application of beads is required for a good-quality product. If the beads are not embedded firmly, they spall off in use and the sign will have poor durability. If they are embedded too deeply, paint will cover part of the bead surface (or even cover some beads completely), causing poor reflective performance. Signs made of this material range in quality from very good to very poor, depending upon the equipment used and the care and skill which goes into production of the material.

Beaded Sheeting

Except for its higher quality and higher cost, the beaded sheeting type (Fig. 6) is essentially like beads on paint. In mass production it is possible to achieve uniformly good control of the placement of beads in the binder. Although the beads are smaller than grains of salt, the microscope shows each bead to be firmly embedded in the binder, yet each bead has almost exactly one-half its surface exposed to reflect light. This material is supplied in rolls of sheeting, which are usually cut to size and applied to the user's sign blanks with an adhesive activated by heat or a special solvent.

Flat Sheeting

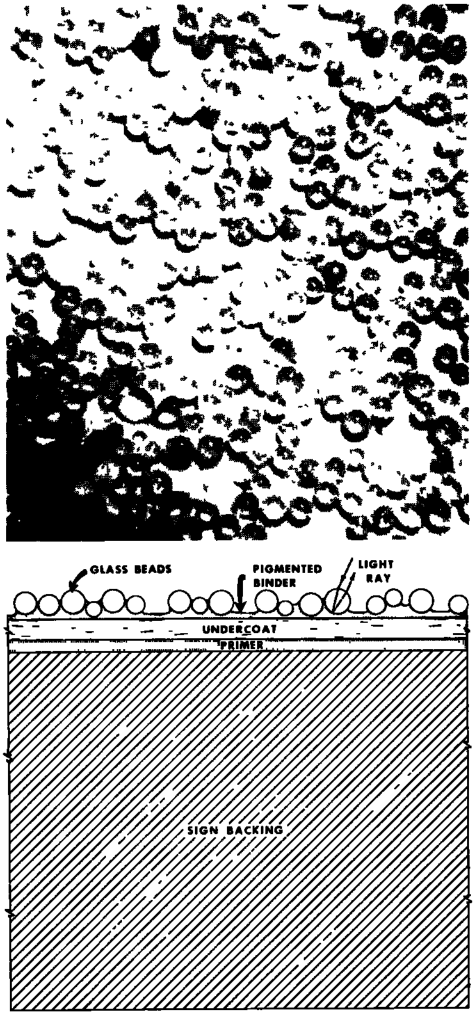

Figure 5. Photomicrograph and cross-section sketch of beads-on-paint type material (30 times actual size).

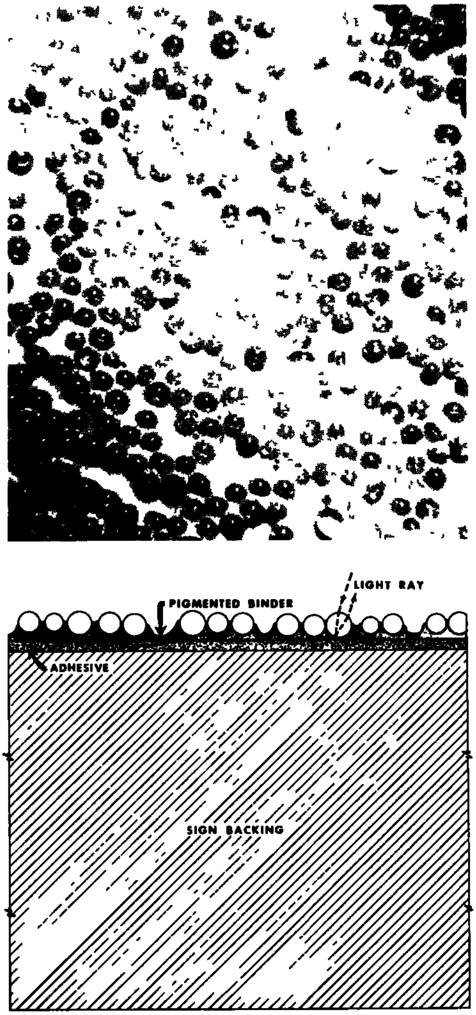

Figure 6. Photomicrograph and cross-section sketch of beaded sheeting type material (30 times actual size).

Flat sheeting (Fig. 7) is more complex and more efficient as a reflector, but more expensive than the two materials previously described. Smaller beads, with a higher index of refraction, are embedded in a low-index plastic and covered by a flat plastic surface. Behind the beads and their plastic matrix is a metal film reflector. Figure 7 shows a ray of light brought to focus on the metal film and returned in the direction from which it came. Flat sheeting is applied to the sign blank in the same manner as beaded sheeting.

Thin Lens-Mirror

The thin lens-mirror (Fig. 8) uses a different optical principle from the materials already described. Instead of gaining its reflective qualities from glass beads, it makes use of tiny lenses molded on the surface of a sheet of plastic. These lens focus the incoming light on the mirror-like back surface so that the light is reflected back toward its source. In cost and reflective efficiency, this type is comparable to flat sheeting. It usually is furnished in sheets to be applied to the user's sign blanks with an adhesive, although a new product furnished in rolls of sheeting has recently been made available.

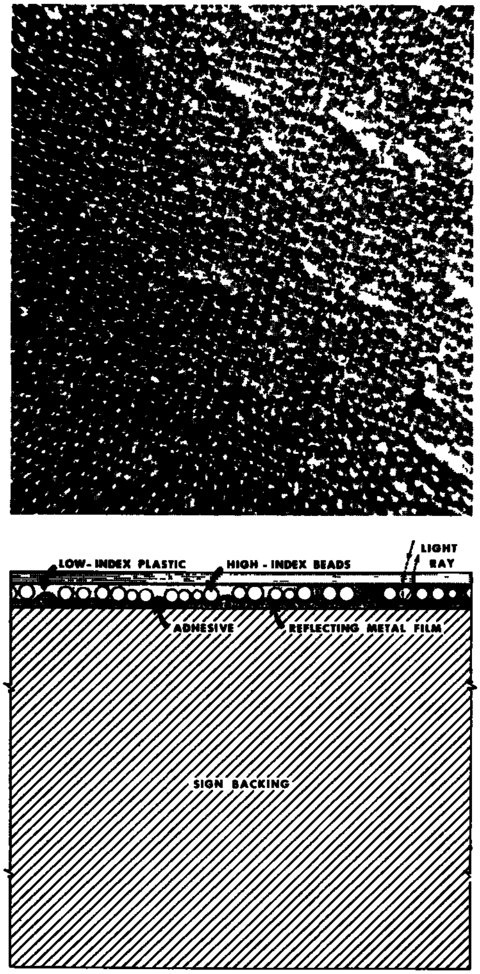

Figure 7. Photomicrograph and cross-section sketch of flat sheeting type material (30 times actual size).

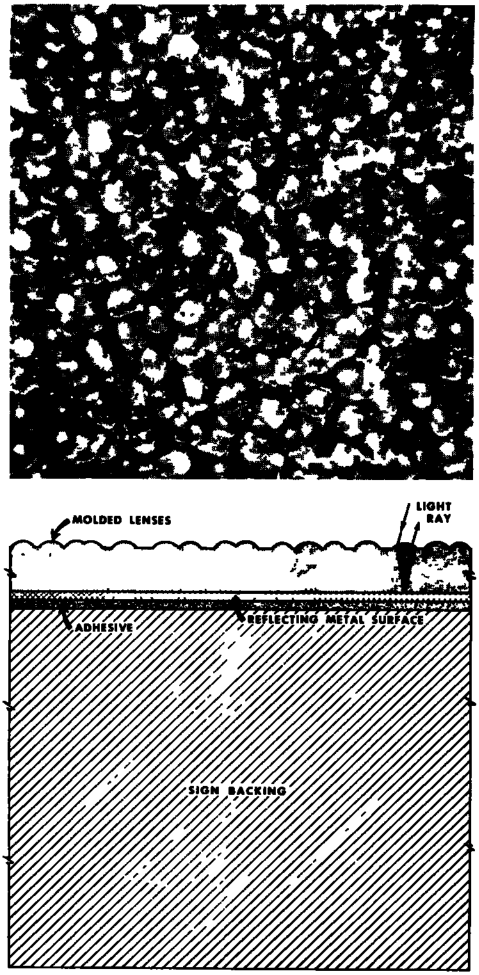

Figure 8. Photomicrograph and cross-section sketch of thin lens-mirror type material (30 times actual size).

Thick Lens-Mirror

The thick lens-mirror (Fig. 9) is similar to the preceding material except that the lenses are much larger and the plastic sheet is correspondingly thicker. It is commonly purchased in the form of cut-out letters or symbols, which are screwed or bolted to the face of the sign. The perfection with which these lenses are molded can be appreciated only when it is realized that each lens is about the size of the dot over an "i" on this page. Such perfection gives a very high reflective efficiency. This material is the most expensive of the types described.

Reflective Characteristics

In the laboratory, photometric measurements were made of specimens of each type of material using the method described in Appendix A. The results are shown in Figure 10, which shows the values of measurements on white or silver samples; curves for other colors would be essentially parallel. It should be emphasized that these measurements were made on only one or a few samples of each material and do not necessarily represent exactly the photometric characteristics of specific products available

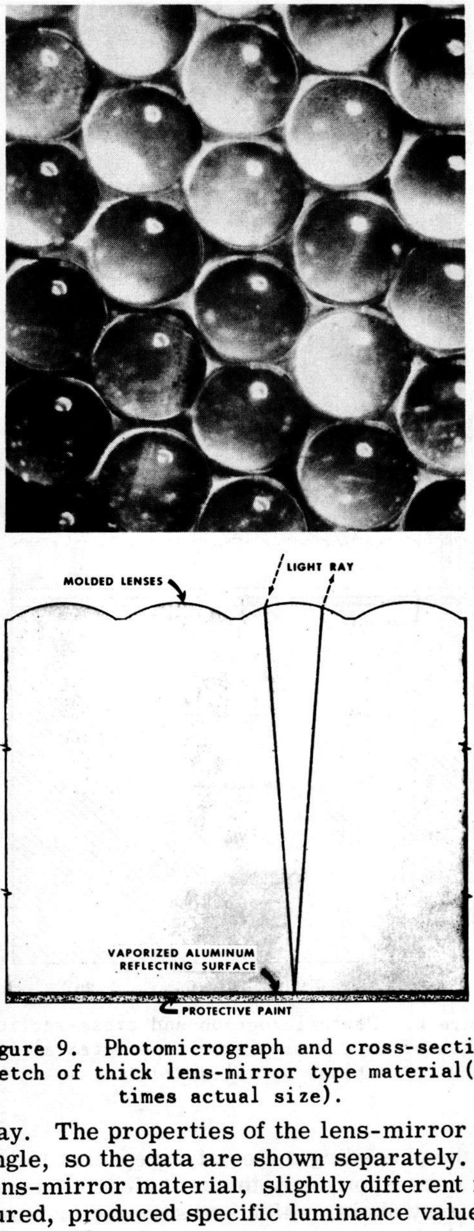

Figure 9. Photomicrograph and cross-section sketch of thick lens-mirror type material (30 times actual size).

on the market. Also, the measurements were made on new clean samples and do not take into account effects of age, dirt accumulation, or weather conditions. However, the range of photometric characteristics shown is representative of those found in different materials available today and constitutes a sound basis for study. This investigation was not intended to compare specific materials either favorably or unfavorably. Instead, an attempt was made to understand the characteristics of different types of continuous-surface material in their relationship to highway sign uses. A simple comparison of materials is invalidated as soon as the properties of one material change; instead of seeking information about a particular product, the reader is encouraged to think in terms of the basic relationships involved. A set of curves (Fig. 10) is shown for each of the materials described, with one curve for each entrance angle. Only one curve is shown for beads on paint, since the specific luminance of this type of material is almost the same at all entrance angles. The reflective properties of this type of material are quite variable, depending on the quality of workmanship, but the curve shown is judged to be the average to be expected from beads-on-paint signs produced in a highway department sign shop. (Since the data in Figure 10 were secured, significant modification has been made in beads-on-paint type material. A recent manufacturer's sample, apparently using higher index beads, produced a peak specific luminance almost twice as high as that shown.) The properties of the three materials shown on the left in Figure 10 are little affected by entrance angles commonly encountered on the highway. The properties of the lens-mirror types, however, are more affected by entrance angle, so the data are shown separately. It should be noted that a recent sample of thin lens-mirror material, slightly different in construction from samples previously secured, produced specific luminance values significantly higher than those shown in Figure 10.

In the past, curves of this type (in terms of specific intensity per square inch) have been used as a basis for specifications; however, until they are quantitatively related to the highway situation these curves have limited meaning.

RESULTS FOR A STRAIGHT LEVEL ROAD

The luminance or brightness of each material was calculated for distances from 40 to 2,000 ft, for typical sign positions on a straight level road, and for upper and lower headlamp beams. Because of space limitations it was not feasible to include the curves for all possible combinations of materials, headlamp beam, and sign position. A typical material is used to show the effect of sign position, and a typical position is chosen to show the effect of sign material. In the later section on vertical and horizontal curves,

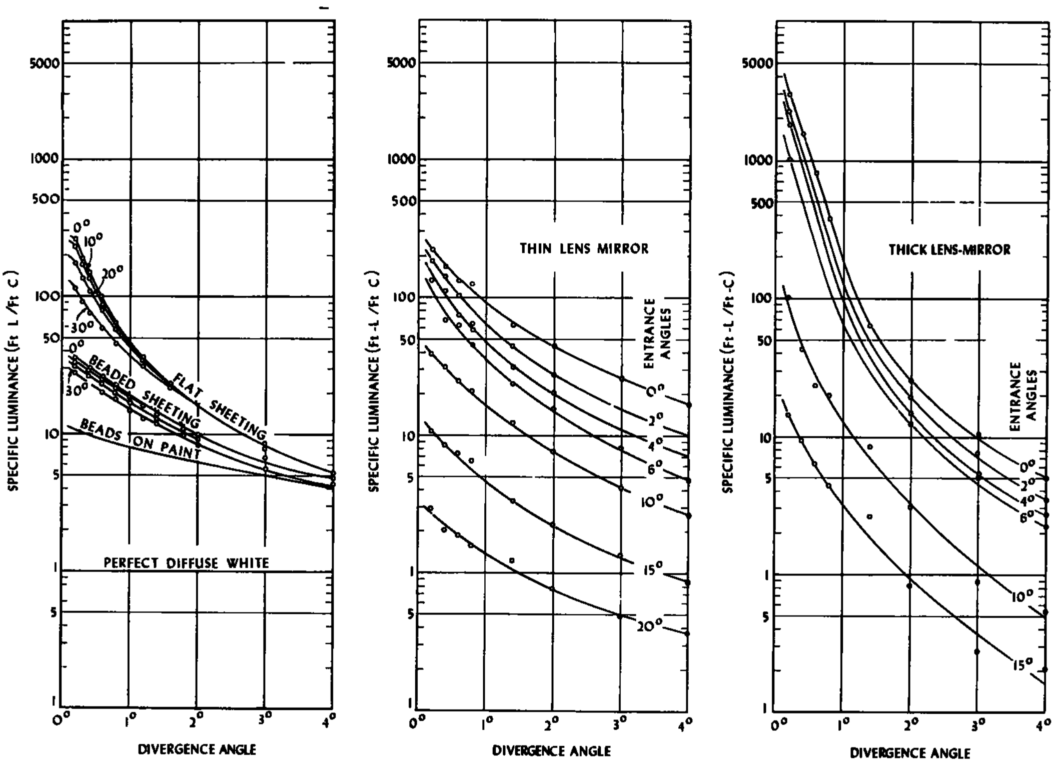

Figure 10. Laboratory photometric data for the five types of reflective materials studied in this paper.

the typical material and the typical sign position are used to show the effects of curvature. In each case where a type of material or a position other than the typical yielded different results, the fact is noted and discussed.

Because of the differences in the illumination provided by the left and right headlamps (see Appendix C) as well as in the trigonometric relationships for right and left headlamps (see Appendix B), it was necessary to make calculations for each headlamp separately. Figure C-3 shows the luminance of each headlamp separately; however, the total luminance seen by the driver is the sum of the luminances for the two headlamps. Therefore, all results shown in the text are based on the total luminance from both headlamps.

Assumptions

Calculations of luminance were based on the following assumptions:

1. All signs were considered to be mounted plumb and perpendicular to the roadway.
2. The car was positioned on the highway so that its right headlamp was 2 ft horizontally from the edge of the pavement.
3. The dimensions of the car were those of an average late-model car referred to earlier and shown in Appendix B, Figure B-1.
4. The car was equipped with standard GE No. 4030 headlamps in good condition with proper alignment and voltage.
5. Each type of sign material was new, clean, and dry, white or silver in color, and had photometric characteristics like the samples measured in the laboratory.

Sign Position

The position of the sign with respect to the pavement has an important bearing on its luminance to the driver. Six sign positions, covering the range of positions specified

TABLE 1

SIGN POSITIONS INVESTIGATED

Sign Position	Feet Above Pavement	Feet Over From Edge of Pavement	Common Application
1	5	6	Rural
2	5	10	Rural
3	8	2	Urban
4	8	6	Rural or Urban
5	8	10	Rural or Urban
6	16	-	Overhead

by the Manual on Uniform Traffic Control Devices (11), were chosen for investigation. These sign positions are shown in Table 1. Signs in any one of the six positions are likely to be encountered on rural highways. Position 3 is the most common in urban areas, although in such areas there is often another lane of traffic or a space for parked cars, so that sign positions 4 and 5 are, in fact, common in urban as well as rural areas. The overhead sign, position 6, is being used more frequently in both rural and urban areas.

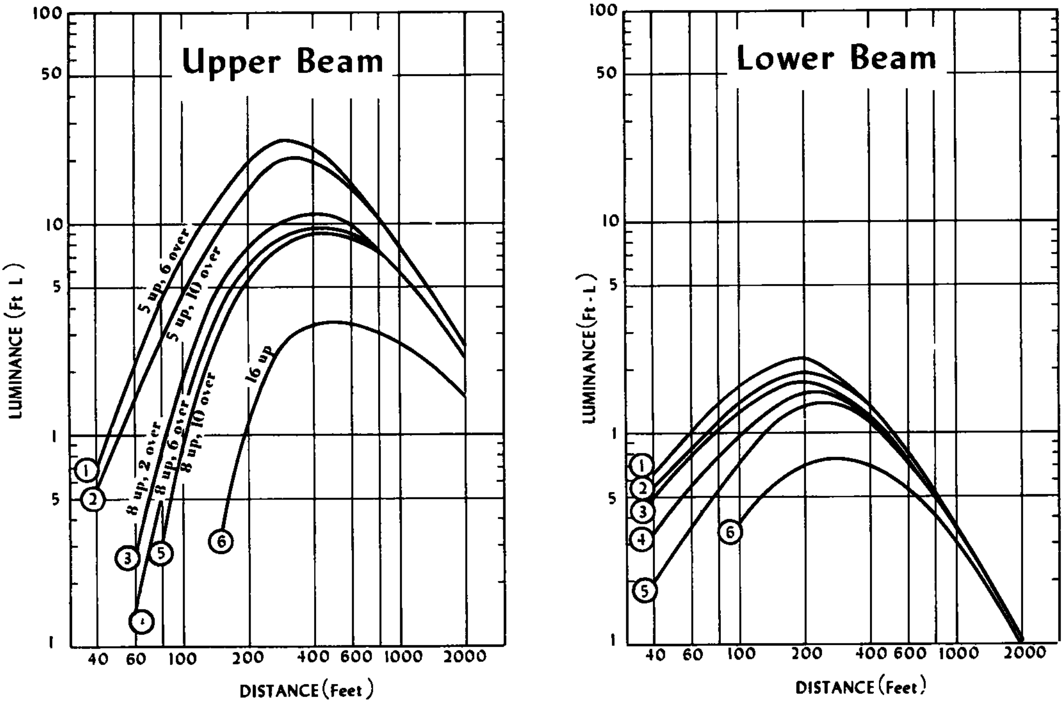

Figure 11. Effect of sign positions (see Table 1) on luminance for flat sheeting type material illuminated by a car equipped with No. 4030 headlamps approaching a sign on a straight level road.

Figure 11 shows the relationship between sign luminance and distance for various sign positions on a straight level road for flat sheeting. The shape of the curves is characteristic of most of the results. The luminance is relatively low at short distances, where divergence angles are large and the sign is out of the intense portion of the headlamp beam. The maximum luminance is reached at an intermediate distance and falls off gradually. When plotted on logarithmic paper, the luminance-distance relationship approaches a straight line at long distances.

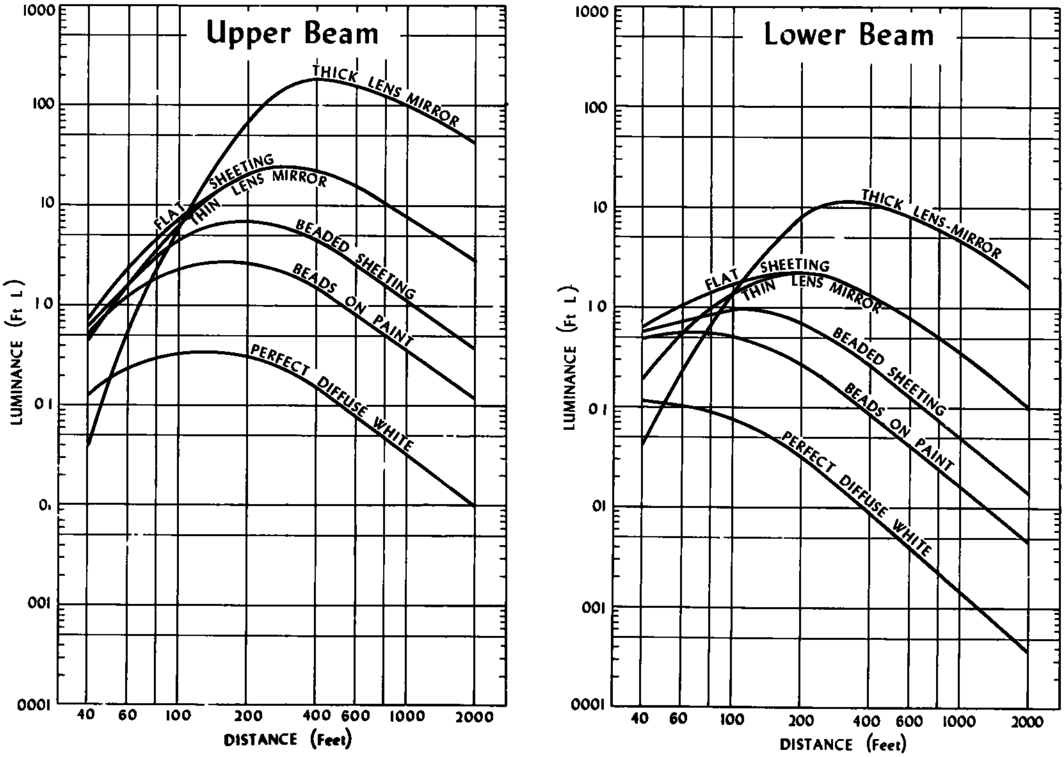

Figure 12. Effect of reflective materials on luminance for sign position No. 1 on a straight level road with illumination from No. 4030 headlamps.

Several characteristics of the curves should be noted. The curves show that height of signs has more effect on luminance than lateral distance from the pavement. The curves for signs placed 5 ft and 8 ft above the pavement are in distinct groups, with lateral distance having a significant effect only at quite short distances. The main variable causing this effect is the broad, flat shape of the headlamp beam. The overhead sign (position 6) yields a curve separate from the other two heights. The importance of overhead signs, which are placed at high-volume high-speed locations, coupled with the fact that the luminance of overhead signs is less than that of roadside signs, presents a problem worthy of special notice.

Headlamp Beams

In general, at short distances (roughly 200 ft) the luminance values for upper beams are about 10 times those for lower beams. At long distances, upper beams give 15 to 25 times the luminance for lower beams.

Reflective Materials

The luminances of the five types of reflective material, for sign position 1 (5 ft up and 6 ft from the edge of the pavement) are shown in Figure 12. The lowest curve, which shows the luminance of a perfect diffuser (a perfectly flat white paint), might be considered as a reference line representing performance of a sign without reflectorization. For all practical purposes it is also the curve for the illumination reaching the sign from both headlamps.

The curves for each material show how the luminance changes with distance to the sign. The curves for flat sheeting and thin lens-mirror were almost the same (for the samples tested) beyond 200 ft and are shown as the same curve beyond that distance. At near distances the luminance of thin lens-mirror is less because this type of material is more sensitive to changes in entrance angle than flat sheeting.

The thick lens-mirror type is very bright at its peak and maintains a high luminance even at 2,000 ft. At near distances, however, its luminance falls off rapidly. It should be noted that this low luminance at near distances is due not only to the entrance angle characteristics of the materials but also to the divergence angle characteristics. Any bright material concentrates its light into a beam of small divergence angle. There is, then, little light return at the large divergence angles encountered at near distances, and the high-brightness materials cannot give their best performance except at greater distances.

In most cases, low luminances at short distances are not important; once the sign becomes legible by virtue of its size as the car approaches, it will remain easily legible until the car is past the sign. However, it may be possible for materials of high brightness (that is, those which concentrate the returned light in a beam of narrow divergence angle) to become unreadable at near distances, especially if they are also sensitive to changes in entrance angle. Examination of the results so far indicates that one of the most important factors governing the choice of reflective material is the distance at which the sign must be read. Consider a sign which can be read with lower headlamp beams at 100 ft with letter size so small that the sign could not be read at far distances no matter what its luminance. Referring to Figure 12, at 100 ft little luminance is gained by use of a more expensive material. At 800 ft, however, flat sheeting or thin lens-mirror is required to give the same luminance as that produced by the cheapest material at 100 ft. At 2,000 ft only the brightest material has a luminance as great as that of the cheapest material at 100 ft. This indicates that for small signs with small lettering the material with the lowest long-range cost may be the best choice, whereas for a very large sign designed to be read at a great distance the most expensive material may be the best and only choice.

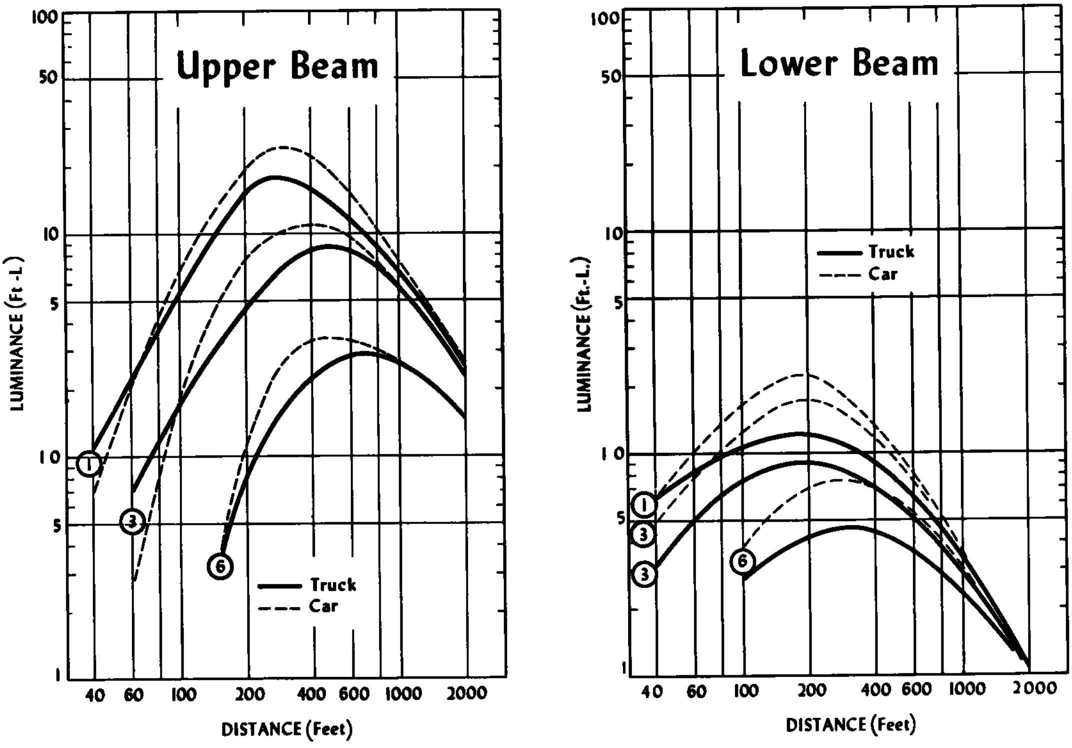

Figure 13. Truck-car comparison vs distance for various sign positions. Dashed lines repeat some data shown in Figure 11.

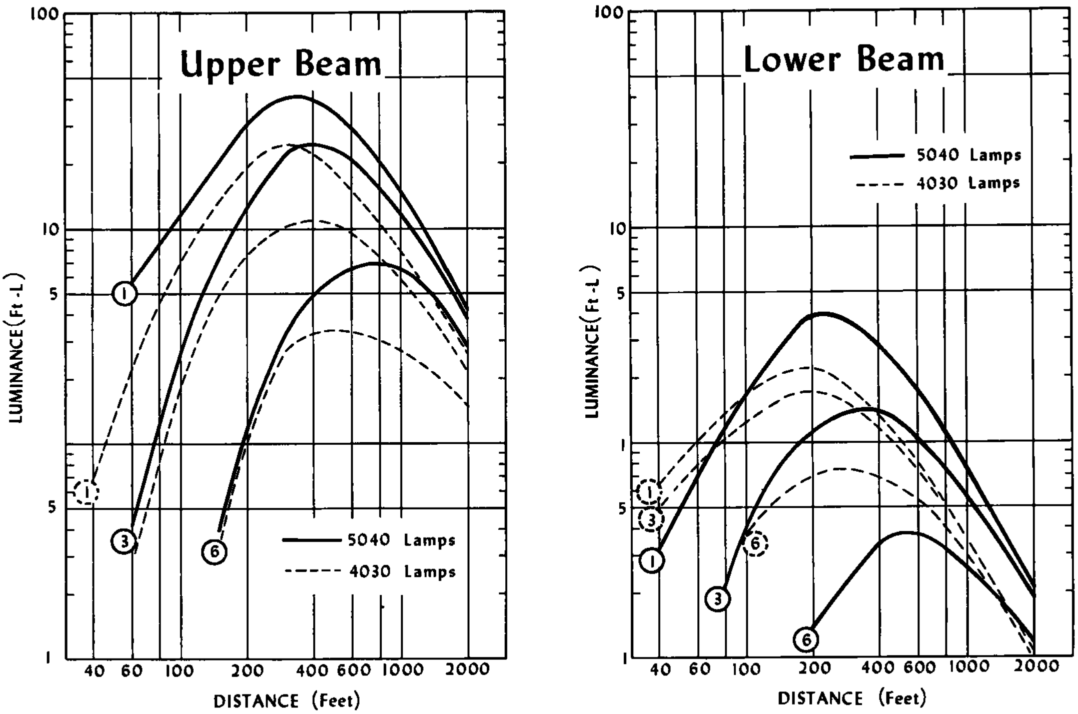

Figure 14. Comparison of luminance of No. 5040 and No. 4030 headlamps for various sign positions. Data shown by dashed lines also shown in Figure 11.

Truck-Car Comparison

Because of the different dimensions of a truck, the luminance of a sign will be somewhat different to the driver of a truck than to the driver of a car. The headlamps of a truck are higher above the pavement than car headlamps, and the driver's eyes are higher above the headlamps. Measurements were made on a sample of large trucks and the average dimensions used in calculations are given in Appendix B, Table 2. A wider variation in dimensions was encountered than in the sample of cars, so the average values used in truck calculations are somewhat less representative than those for the cars.

In Figure 13, the dashed lines are the results previously shown in Figure 11 for the average car and for flat sheeting at three sign positions. The solid lines show results for a large truck. Luminances as seen by the driver of a truck are generally lower, although the differences are not great. The factor accounting for the reduced luminance is the larger divergence angles. Although the illumination reaching the signs was slightly greater, because a truck's headlamps are mounted higher above the pavement, this effect was more than balanced by the increase in divergence angle. The reduction is greater for the brighter materials, since they concentrate the returned light into a beam of smaller divergence angle. However, even for such materials it is doubtful that the effect is of great importance. In general, the luminance of signs to the driver of a large truck will be slightly less, but this difference will be insignificant except for high-brightness materials at near distances.

New Type Headlamps

All previous data in this paper have used illumination values for the old type of sealed-beam headlamp, which was standard equipment on automobiles until 1955. Since 1955, the new No. 5040 lamp (50-w upper beam, 40-w lower beam) has replaced the old type No. 4030 (40-w upper beam, 30-w lower beam). The new headlamps have been described by Sherman (8). The upper beam of the new lamp differs from the old in having a higher intensity at the center of the beam and a sharper vertical cutoff. Two important changes have been made in the lower beam. With little change in the glare to opposing traffic, the new lower beam gives significantly increased visibility distances on the right-hand edge of the road. Also, addition of a filament cap over the lower-beam filament results in a large reduction in the stray light scattered upwards at near distances, giving considerably better visibility in fog, falling snow, or rain.

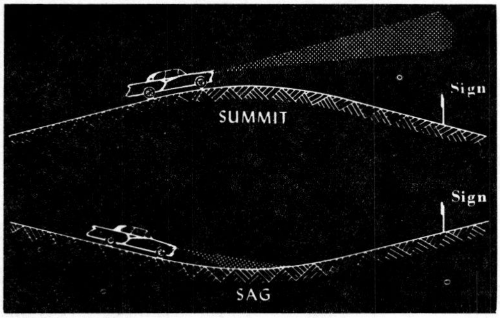

Figure 15. Relationship of main portion of headlamp beam to sign as car passes over vertical curves.

The effects of the No. 5040 headlamp on sign luminance were also investigated. The isocandle charts for 6-v No. 5040 lamps were used. Although the 12-v lamps have slightly lower rated candlepower, Roper (7) advises that the overvoltage found in vehicles using the 12-v system yields about the same candlepower distribution as for the 6-v lamps.

The dashed curves in Figure 14 are the data previously shown in Figure 11 for flat sheeting. Differences in luminance produced by the two headlamps would be similar for all other materials. The solid lines show results for the No. 5040 headlamp. The new upper beams give about the same luminance at near distances, but give greater luminance for all sign positions at far distances.

For the new lower beam, there is generally lower luminance at near distances and higher luminances at far distances. The overhead sign, position 6, deserves special attention. Except at far distances, there is a marked reduction in the light reaching an overhead sign from the new lower beam. For overhead signs already having rather low luminances, the problem is compounded by the reduction in light received from the lower beams of the new No. 5040 lamps.

RESULTS FOR VERTICAL AND HORIZONTAL CURVES

The data presented up to this point have been for a straight, level road, but vertical and horizontal curves have marked effects on sign luminance. Although it is possible to compute luminances for any combination of horizontal and vertical curvature, only a minimum of data is included to show the nature of the relationships and the relative magnitude of the effects. As in previous sections, results are shown for one type of material and one sign position, and important differences in relationships for other materials and positions are noted.

Vertical Curves

The most important effect of curvature is its influence on the position of the sign in the headlamp beams. The effect of vertical curves on the illumination reaching the sign is shown in Figure 15. As the car approaches the sign over a summit, the headlamp beams are aimed above the sign; for a sag, the beams of the approaching car are always aimed below the sign.

To arrive at a solution to the problem, some assumptions about length and symmetry of the curve, difference in grade, and sign position had to be made. Symmetrical curves 1,000 ft long were assumed, with algebraic differences in grade from 0 to 10 percent. This range of vertical curves is representative of those encountered on high-type major highways, but much more pronounced curves are found on secondary highways. Calculation of entrance and divergence angles and the position of the sign in the headlamp beam is somewhat more complicated than for a straight level road. The method is described in Appendix B. As before, the sign is assumed to be mounted plumb and perpendicular to the roadway.

Figure 16 shows the luminance of flat sheeting in the assumed sign position (5 ft up

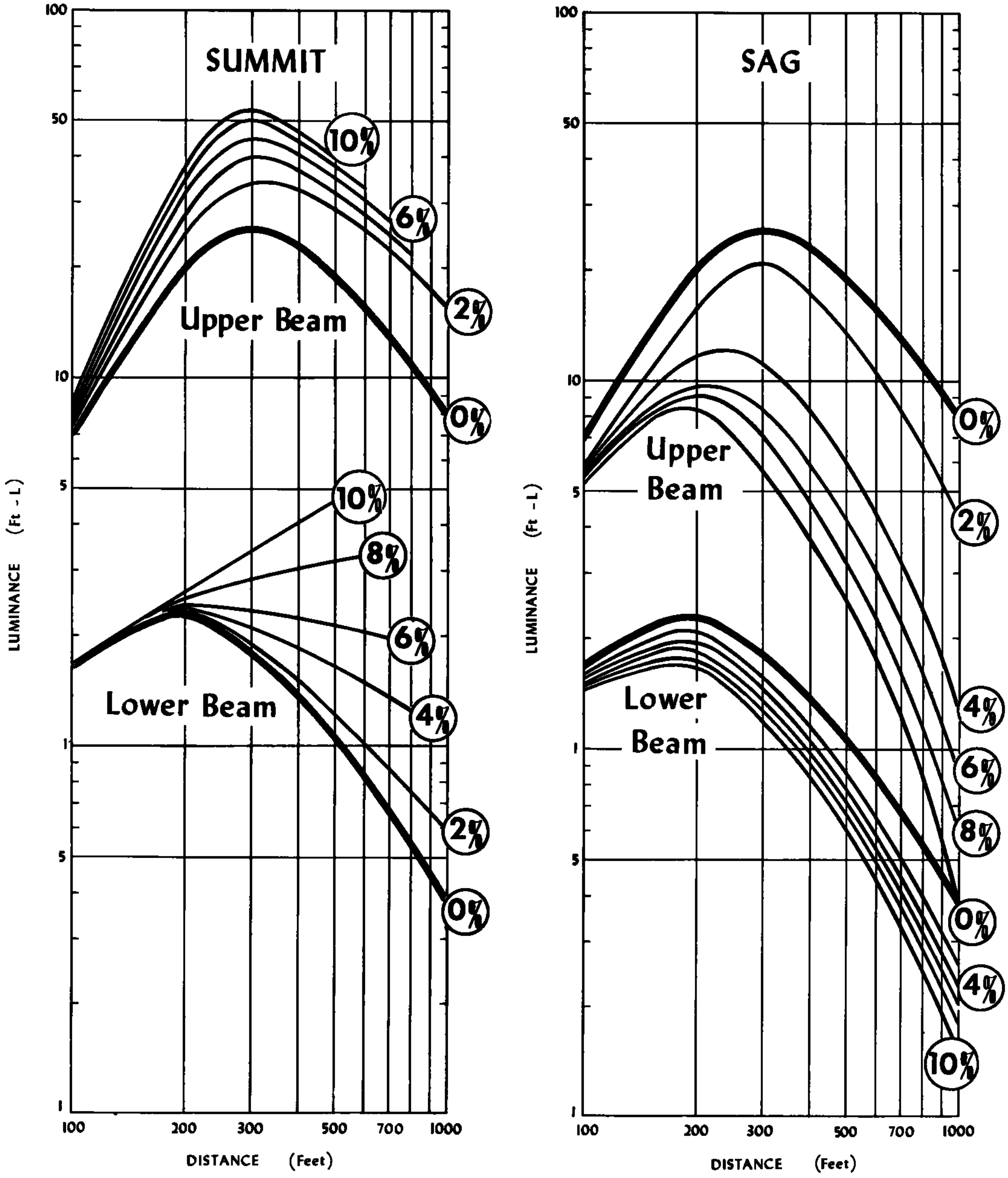

Figure 16. Luminance vs distance for various vertical curves. Bold lines show same data as in Figure 11 for sign position 1.

and 6 ft from the edge of the pavement and at the end of a 1,000-ft vertical curve) illuminated by No. 4030 headlamps. The heavy lines labeled 0 percent represent the data previously presented for a straight level road for position 1, and each of the other curves is labeled with the algebraic difference in grade. Since differences in luminance at near distances are small, only distances beyond 100 ft are shown. Each curve for a summit is cut off where the sign is shielded from the headlight beam by the crest of the hill.

Under the conditions assumed, luminances for a summit are always greater than for a level road, because the headlamp beam is aimed more directly at the sign. If a

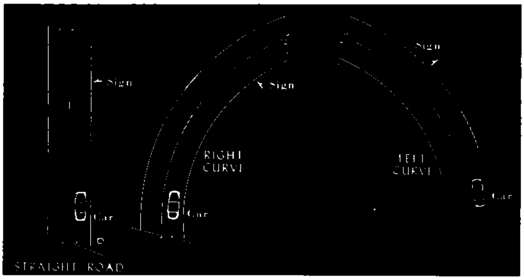

Figure 17. Relationship of main portion of right headlight beam with respect to sign as car proceeds around horizontal curves.

sign tends to be so bright on a level road that legibility is decreased (1) the problem would be accentuated on such a vertical curve. Conversely, luminances are always less for a sag. In certain applications on vertical curves, this reduction in luminance has a great effect on the distance at which a sign can be read. For example, an overhead sign would be still farther outside the intense portion of the headlamp beam, and would have still lower luminance than a roadside sign. The situation becomes more unfavorable when the lower beam of the No. 5040 headlamp is used. The resulting low luminance of any available reflective material suggests that artificial illumination may be necessary for overhead signs located beyond sags.

Luminance curves similar to Figure 16 for other types of sign material closely parallel those shown for flat sheeting. The characteristics of the material were not affected differently because vertical curves do not cause significant changes of entrance and divergence angles.

Horizontal Curves

The effect of horizontal curvature on the position of the sign in the headlamp beam is shown in Figure 17. A sign on the right shoulder of a right curve is always to one side of the centers of the headlamp beams. For a left curve, the center of the beam is at first to the right of the sign, but as the car approaches the center of the beam swings by the sign to the left.

The car was assumed to be approaching the sign on a continuous curve. Calculations were made for 0-, 2-, 4-, and 6-degree curves, both right and left. As before, the sign was assumed mounted plumb and perpendicular to the centerline of the roadway. No account was taken of superelevation. Methods of calculation are outlined in Appendix B.

Figure 18 shows the luminance-distance curves for the same material and position previously discussed (flat sheeting 5 ft up and 6 ft from the pavement edge. The curves for a straight level road are shown (0 degrees) as heavy lines for reference. The range of curve chosen (0 to 6 degrees) includes all curves usually found on high-type major highways, but curves considerably sharper are found on secondary highways. A right-of-way of 110 ft is assumed and the 4- and 6-degree curves terminate at the distance where the line of sight is interrupted by the fence line. The line of sight would be interrupted sooner in a cut, but would not be a limiting factor in a fill section.

Figure 18 shows that on right curves luminances are always less than on a straight road, while left curves have a peak luminance at near distances where the headlamps are pointed almost directly at the sign. The loss of luminance is greater for upper beams than for lower beams; although upper beam luminances are reduced to as low as one-tenth of their straight-road values, lower beam luminances do not become less than one-third of their straight-road values.

It should be remembered that these curves were prepared for flat sheeting only, which is not very sensitive to changes in entrance angle. Since divergence angles are not greatly affected by horizontal curvature, the main factor affecting the changes in luminance in Figure 18 is illumination.

Data for types of material other than flat sheeting were also computed. Because of the effects of entrance angle, the plotted results were not parallel to those shown for those types of material sensitive to changes in entrance angle. Therefore, it is necessary to consider the effects of entrance angle characteristics.

EFFECT OF SIGN ROTATION

As previously pointed out, different types of materials differ greatly in how much

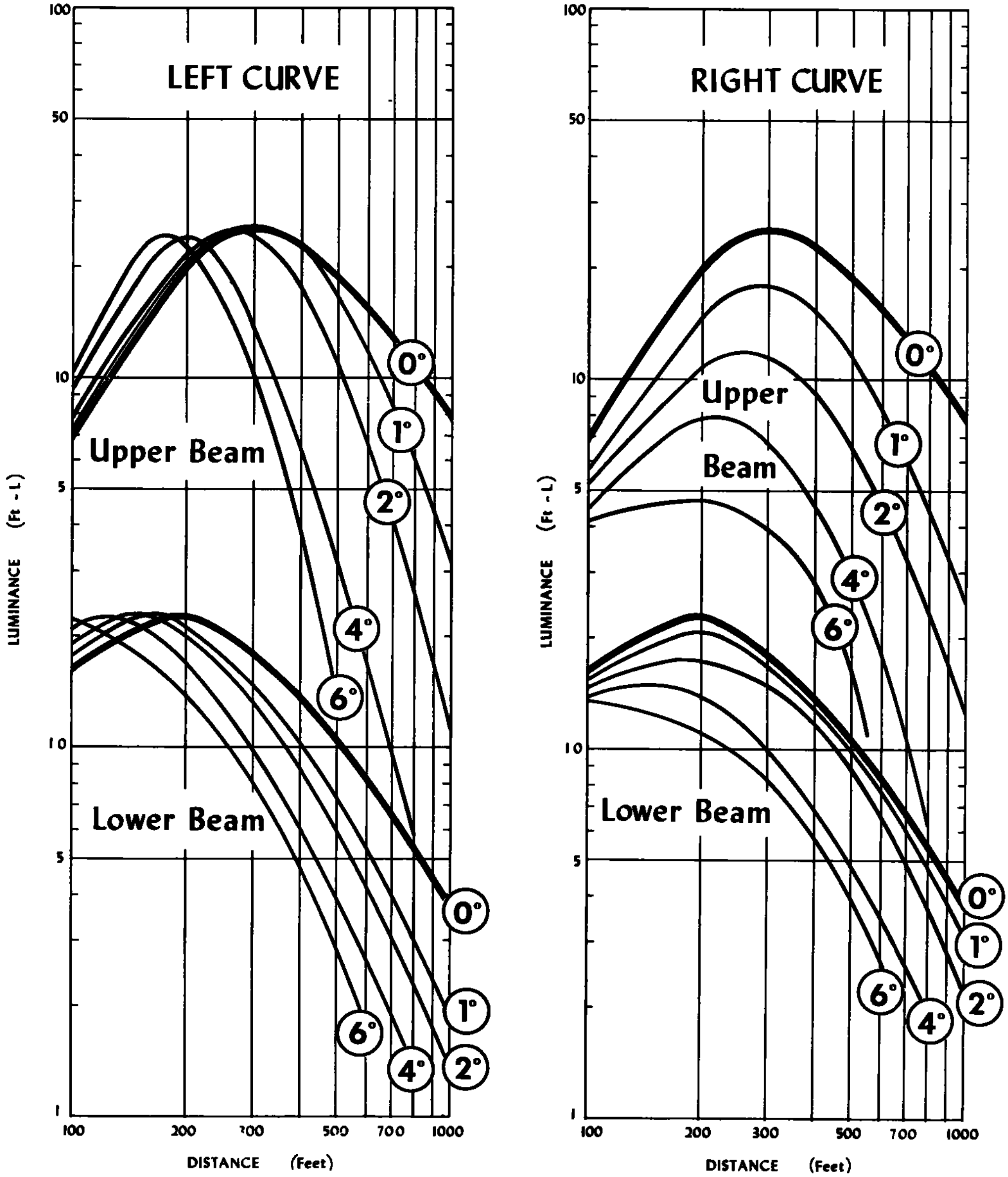

Figure 18. Luminance vs distance for various degrees of horizontal curvature. Bold lines show same data as in Figure 11 for sign position 1.

their brightness is affected by the entrance angle. Of the types included in this study, only the lens-mirror types were very sensitive to changes in entrance angle. In the following discussion, only the thin lens-mirror type is considered, since its brightness is roughly the same as the "typical" material, flat sheeting, except for its entrance angle characteristics.

When a sign is erected it can be rotated, within certain limits, to any desired angle with respect to the roadway. When painted signs were used, it was common practice to rotate them away from the road about five degrees to avoid specular reflection. This is still desirable for materials having high specular reflection, but it is certainly not desirable for materials sensitive to changes in entrance angle.

Rotation on a Straight Road

Figure 19 shows the effect of sign rotation on the luminance of thin lens-mirror type of material on a straight level road. The sign is assumed to be mounted as before, 5 ft above and 6 ft from the right edge of the pavement. The 0-degree curve is for no rotation, (sign face perpendicular to road). The clockwise rotation faces the sign toward the road and counterclockwise rotation faces the sign away from the road.

It can be seen (Fig. 19) that a 5 degree clockwise rotation does not affect luminance very much, but produces a slightly brighter sign at near distances and a slightly less bright sign beyond 250 ft. A 10 degree clockwise rotation brings about a substantial reduction in luminance at all but very near distances. As would be expected, counterclockwise rotation (facing the sign away from the road) causes even greater reductions.

To achieve maximum performance from a material sensitive to changes in entrance angle, care must be used in facing the sign properly when it is erected. To achieve maximum performance of these materials, orientation should be within ± 5 degrees; misaiming by more than 10 degrees will cause a serious loss of brightness. No reliable information is available regarding the ability of sign crews to aim signs with respect to the pavement, but with reasonable care competent workmen should be able to place a sign within 5 degrees of the the correct aiming on a straight road.

Rotation on Curves

As mentioned previously, the entrance angle conditions for both vertical curves and straight level road sections are quite similar; therefore, the problem of rotation on vertical curves is the same as for straight roads. Recognition of the effect of sign rotation on horizontal curves is important, however, in order to minimize the reduction

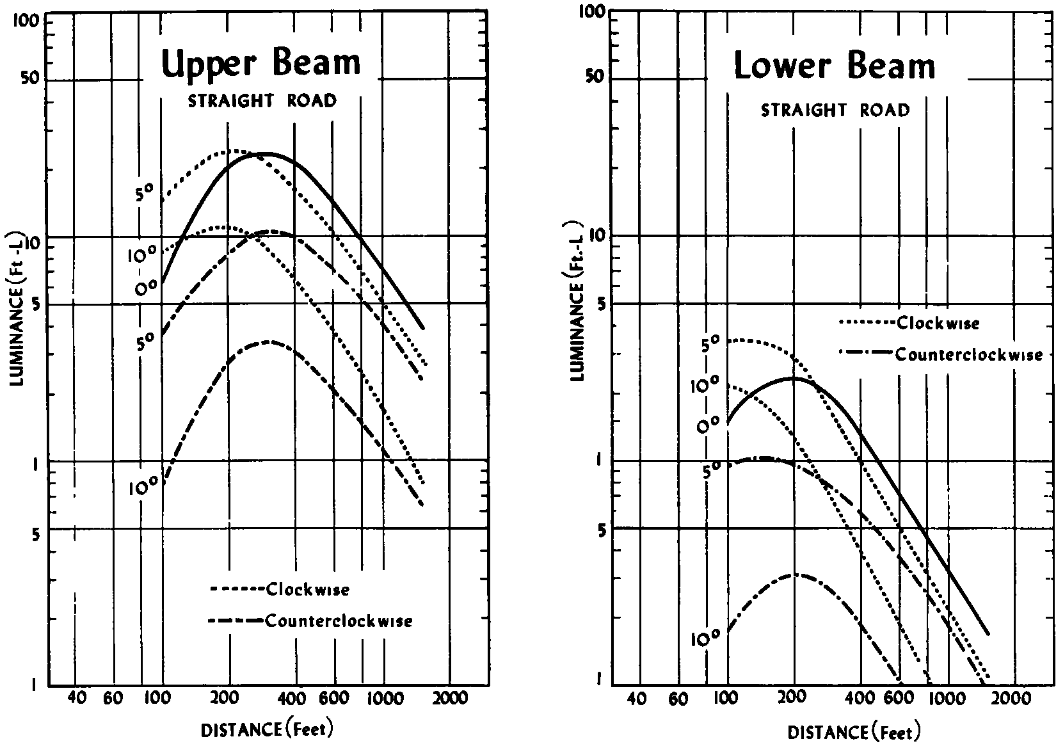

Figure 19. Luminance vs distance for various amounts of rotation of thin lens-mirror material. Roadway alinement is assumed flat and straight and sign is in position 1. Zero-degree curve is based on assumption that sign is mounted perpendicular to roadway.

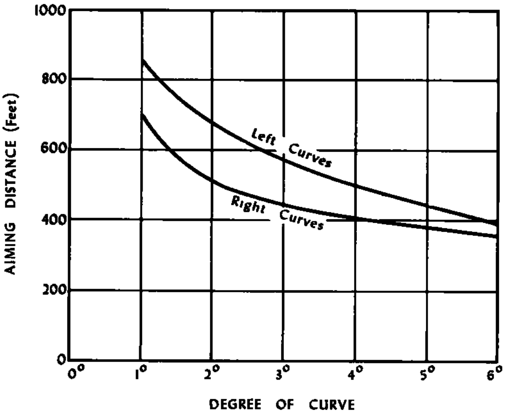

Figure 20. Aiming distance vs degree of horizontal curve for optimum performance of thin lens-mirror material.

of luminance resulting from large entrance angles.

It is possible to compute the luminance at any distance for any degree of rotation; however, the distance at which the sign is to be read is an important factor. Obviously it would be out of the question to orient a sign for luminance to a distance of 1,000 ft when its letter size at any luminance makes it illegible beyond 500 ft. However, it was of interest to see if it were possible to rotate the sign through a certain angle so that the effects of entrance angle characteristics would be small within the range of distance that the sign was visible around the curve. Luminances within this range of distances were plotted for several degrees of rotation, for right and left curves to values of 6 degrees of curve. The effects of entrance angle were found to be more critical for right curves than for left curves (for a sign mounted on the right-hand side of the road). On left curves, aiming the sign toward a point some distance back on the curve results in rotation toward the road, which increases brightness at near distances. For right curves, aiming toward a point some distance back on the curve rotates the sign away from the road. This causes reduced luminance at near distances.

Optimum rotations were estimated for thin lens-mirror material for both right and left curves. Figure 20 shows the optimum rotation in terms of aiming distance; that is, the distance from the sign to the point on the road toward which the sign should be aimed. For example, for optimum performance of thin lens-mirror material on a 4-degree right curve the sign should be aimed at a point about 400 ft back on the road. If the sign is aimed as indicated in Figure 20, the luminance will be approximately the same as if the material were not sensitive to changes in entrance angles and the detrimental effect of the curve on the luminance of the sign will be negated. For left curves no significant loss in luminance is encountered at any distance if the sign is aimed as suggested by Figure 20. For right curves the only significant decreases in luminance are found beyond about 600 ft on 4-degree curves and beyond about 500 ft on 6-degree curves. In no case, if the sign is so aimed, does curvature reduce the long-range luminance to that of the next bright material, beaded sheeting.

In practice, of course, it would be impractical to aim the sign exactly as specified in Figure 20; the purpose of the curves is to illustrate how the relationship between rotation and luminance could affect signing practice. In erecting signs of materials sensitive to changes in entrance angle, it is suggested that the sign be aimed toward the point where it should be read. However, for 5- or 6-degree right curves it is suggested that the sign be aimed at a point no farther than 400 or 500 ft down the road to avoid serious loss in luminance at near distances. If these recommendations are followed, there will be no serious loss of brightness on curves of less than 6 degrees if a material such as thin lens-mirror is used. There is need for further research on curves sharper than 6 degrees, and on other materials sensitive to changes in entrance angle.

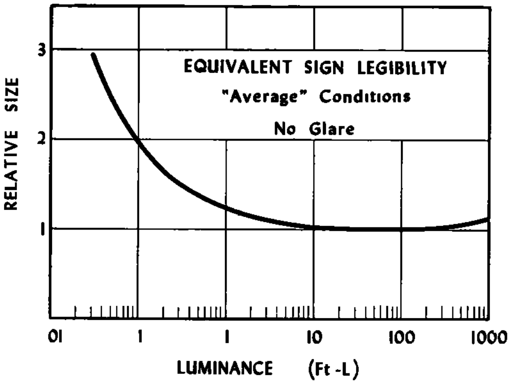

Figure 21. Effect of size on sign legibility under 'average" no-glare conditions.

RELATIONSHIP OF LEGIBILITY AND LUMINANCE

With all other factors constant, the brightness or luminance of a sign has an important bearing on the size of the sign required for legibility. Certain basic data on the relationships between brightness and legibility have been presented (1).

Figure 21 expresses an estimate of the relationship between sign luminance and the relative sign size required for equal legibility. The curve was derived from an extension of laboratory data (1).

Field validation of the laboratory legibility data has not yet been undertaken. Since the curve in Figure 21 is subject to change and refinement, it should not be used for design purposes. Furthermore, the curve is for "average" no-glare conditions, and does not show the important effects that some factors (such as stroke width, black-on-white vs. white-on-black) have on legibility. However, the basic relationship shown in Figure 21 illustrates a fundamental concept in reflectorized sign design. For example, if a given sign can just be read when its luminance is about 50 to 100 ftL, it would need to be about twice as large if its luminance were only 0. 1 ftL. The significance of this concept is shown by relating basic legibility data to those developed herein, as is done in the following examples.

Suppose that a comparison of sign designs is desired for a straight road application where traffic predominantly will be using lower beam. Assume that the sign is to be read at 500 ft. From Figure 12, the luminance of beads-on-paint type material is 0. 06 ftL; that of flat sheeting or thin lens-mirror types is 0. 9 ftL. From Figure 21, the corresponding relative size factors for equal legibility are 2. 4 and 1. 2, respectively. Therefore, at 500 ft any sign dimension of the less bright material would need to be 2. 4/1. 2 or 2. 0 times larger for equal legibility with the brighter materials. The required sign area is $(2.0)^2$; hence, for the less bright material to be of equal legibility about four times as much sign material would be needed. From an economic point of view, the average annual unit cost (per square foot) in service of the brighter materials could be as much as four times that of the beads on paint.

Consider another example under the same conditions except that the sign is to be read at a distance of 200 ft. The luminance of beads-on-paint type material is 0. 3 ftL; that of flat sheeting and thin lens-mirror types is 2. 0 ftL. These luminances have corresponding relative size factors of 1. 6 and 1. 1. Therefore, at 200 ft each dimension of a beads-on-paint sign would need to be 1. 6/1. 1, or about 1. 4 times larger than a sign of brighter material to be equal in legibility. The required sign area is $(1.4)^2$, or about twice as much. If the annual unit cost for the less bright material were less than one-half that of the brighter materials, it would be more desirable from the economic point of view.

Once again, it is pointed out that the legibility-brightness data referred to have not yet been developed sufficiently to permit application in design specifications, and the illustrations are given only to explain a basic concept of sign design.

SUMMARY AND CONCLUSIONS

A method has been described by which the luminance or brightness of a sign in place on the highway can be computed by taking into account (a) the position of the sign with respect to the pavement, (b) the distance between the headlamp and the sign, (c) vertical and horizontal curvature of the roadway, (d) laboratory photometric measurements of sign materials, and (e) the illumination reaching the sign from headlamps. Five common types of reflective materials were studied and their brightness values for representative highway situations were computed.

Although the results presented pertain to common highway sign applications, the same methodology could be applied to atypical vehicles (sports cars, buses), or other craft (airplanes, ships). It could also be applied to related problems (reflectorized pavement markings, advertising signs) and to a study of reflector buttons.

In general, highway signs have low luminance at near distances, maximum luminance at distances from 150 to 500 ft, and decreasing luminance at greater distances. Letter size is a factor of great importance in the selection of reflective material. For signs with small letter size, little is gained by using expensive high-brightness materials, because (a) small letters cannot be read at great distances no matter what their brightness, (b) adequate brightness is easily achieved using any material at near distances where illumination from headlamps is high, and (c) little increase in sign luminance is achieved through high-brightness materials because of the large divergence angles encountered at short distances. For large signs to be read at great distances, however, the more expensive materials may be more economical. Since little light reaches the sign from the headlamps, high-brightness materials are needed to achieve adequate luminance for good legibility.

Height of the sign is a more important factor than lateral placement. Overhead signs have luminances so much lower than roadside signs that they deserve special consideration.

For a sign mounted beyond a summit, sign luminance is always greater than on a level road. This would be the critical position for a material which might be too bright for good legibility. For a sign mounted beyond a sag, however, luminance is always less, and significantly so if the vertical curvature is abrupt or if the sign is to be read at a long distance. Vertical curvature can be a definite limitation on the placement of signs.

Horizontal curves cause marked reductions in sign luminance. Because signs on curves are outside the main portion of the headlamp beam, the illumination reaching the sign is reduced to as low as one-fourth of the straight-road value on a 6-degree curve. On horizontal curves a special problem arises from the use of reflective materials sensitive to changes in entrance angle. On a straight road, care should be taken to orient the sign within ± 5 degrees, or certainly within 10 degrees, of right angles to the road. Reasonably competent workmen should be able to orient the sign within these limits if given proper instructions. On vertical curves the problem of entrance angles is similar to that of a straight, level road, but on horizontal curves lack of care in aiming can result in a serious loss of brightness. With optimum orientation of the sign on horizontal curves, no serious loss of brightness need be encountered, at least for curves as sharp as 6 degrees.

Because luminance affects legibility, and since position and distance have great effects on luminance, it follows that position and distance should receive important consideration in the selection of the material used for particular sign applications. A fundamental concept of sign design necessary in order to make an objective comparison of sign materials for particular applications involves first equating of materials on the basis of legibility, and then taking into account average annual costs of satisfactory service.

It is important that field legibility studies take account of the luminance of the sign at the moment it is read. Unless this factor is considered, the validity of any generalizations concerning legibility will be open to question.

Further Research Needed

In addition to an extension of legibility studies, including field validation in which luminance is measured, further research is needed in several other areas relating to the problem of night performance of signs, as follows:

1. Study of the differences of headlamp illumination caused by such factors as misaim, voltage changes, sampling variation, age, dirt, and differences in manufacturing methods.
2. Quantitative measurement of the effects of age, rain, dew, fog, snow, and dirt on the reflective characteristics of various materials.
3. Study of the legibility of button-type letters and symbols, taking into consideration photometric factors.
4. Extension of the data presented in this paper to include other sign positions (such as turnpike locations). Further data are also needed on the effects of sign rotation and highway curvature on materials sensitive to entrance angle. After further precise data are developed, approximations permitting sufficient accuracy for estimating luminances for any material, position, etc., should be worked out for use in sign design.

ACKNOWLEDGMENTS

The work reported in this paper is part of the highway sign studies project jointly sponsored by the Virginia Department of Highways and the U. S. Bureau of Public Roads.

Grateful acknowledgment is made to the manufacturers of reflective materials for supplying several sets of samples; to D. M. Finch of the University of California for making photometric measurements on control samples; to W. C. Sinclair, B. B. Chamblin, H. T. Craft, H. E. Fielding, J. C. Bean, C. E. Giannini, and many others of the staff of the Virginia Council of Highway Investigation and Research for long hours spent in making measurements and tedious computations; to Miss Barbara Higgins for typing several drafts of the paper; to the members of the Council's Report Section for the illustrations, editing and reproduction of the report; and to T. E. Shelburne for his continuing patient support of the sign studies. The authors are also grateful to H. G. Larew, member of the University of Virginia Department of Civil Engineering staff, for his careful review of the manuscript and his many constructive suggestions.

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APPENDIX A

PHOTOMETRIC MEASUREMENTS OF MATERIALS

Specific Intensity

Previous studies of the photometric characteristics of reflective materials have used specific intensity as the unit of measurement (9, 3, 6). Specific intensity is defined as the apparent candlepower per unit of illumination of a specimen of material viewed as a point source. This unit of measurement is applicable to reflector buttons and to signs at such great distances that they appear as a point of light. At distances at which signs can be read, however, a sign cannot be considered a point source.

Specific Luminance

The legibility of a sign must be considered as a function of the luminance of the sign rather than of its intensity in apparent candlepower. The relationship between sign luminance and legibility has been outlined in a previous paper (1). The appropriate unit of measurement for this case is luminance per unit of illumination, measured in foot-Lamberts per foot-candle. A suitable name for this unit is specific luminance.

The relation between specific intensity and specific luminance is given by:

$$L = I\left(\frac{144\ \pi}{A \cos\ \phi}\right)$$

in which

L = specific luminance, in ft L per ft c;

I = specific intensity, in C per ft c;

$144\,\pi$ = constant for change of units

A = area of the specimen, in sq in.:

$\cos\ \phi = \cos\ (\theta\ +\ \Delta)$;

θ = entrance angle or angle of incidence; and

Δ = divergence angle.

In most practical cases ϕ is small, and specific luminance is approximately equal to 144 π times specific intensity per square inch.

The luminance factor of a body is defined as the ratio of its luminance to that of a perfect diffuser with the same conditions of illumination (10). Specific luminance is related to luminance factor by

$$L = \beta \cos\ \theta$$

in which

β = luminance factor; and

θ = entrance angle or angle of incidence.

Apparatus

The apparatus used for photometric measurements is shown schematically in Figure A1. The light source consisted of an automobile spotlight and a lens system to give an exit aperture of 1.2 in. The spotlight was operated at normal voltage and yielded a color temperature of about 2,900 deg K. A constant-voltage transformer was used to control light output. A corrected photocell (Weston) was used to measure the incident light on the specimen. A visual photometer with an objective lens (Luckiesh-Taylor Brightness Meter) was used to make direct measurements of the luminance of the surface of the specimen. The photometer reading in foot-Lamberts divided by photocell

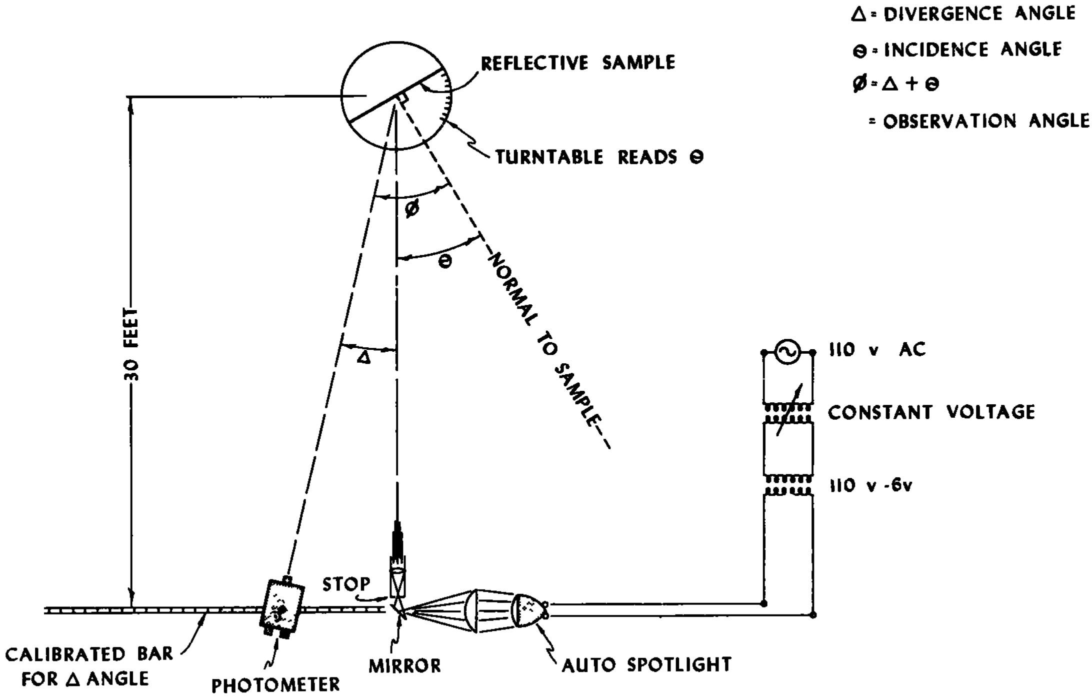


Figure A1. Schematic drawing of photometric apparatus used to measure reflective characteristics of materials.

reading in foot-candles yielded the specific luminance directly. (Careful cocalibration of photocell and photometer was essential for accurate results.) This method can be used only if the material has a surface of uniform brightness. Since only continuous-surface materials are considered in this report, the uses and limitations of this apparatus for testing reflector buttons are not discussed.

The apparatus is simple and relatively inexpensive. Other investigators (3, 6) have concluded that visual photometry does not yield accurate results; however, the authors' experience has shown that if the requirements for good visual photometry are met, accurate results can be obtained. The commonly used visual method requires the eye to match the point brilliance of the specimen with that of a comparison lamp. With the Luckiesh-Taylor instrument, however, the eye matches the luminance of an image of the specimen itself to the luminance of the photometer comparison field. This method gives smaller variation between readings and between observers. For a large number of measurements, the coefficient of variation between readings has not exceeded 2 percent for trained observers using a Luckiesh-Taylor meter. With the common visual method, substantial errors (and differences between observers) can be caused by the Purkinje effect when there is a difference in color between the specimen and the comparison lamp. In matching fields of brightness, however, errors due to the Purkinje effect can be avoided if image luminance exceeds 1 ft L (10). The apparatus used in this study more than meets this requirement.

Van Lear (9) and Finch (3) have pointed out the need for a large test distance, and suggested a distance of not less than 100 ft for testing reflector buttons. Use of a test distance as short as 30 ft introduces two sources of error. The first is the angular size of the light source and photometer apertures. The photometer used in this study had an aperture of about 6 minutes of arc, and the light source aperture was about 12 minutes of arc (comparable to a headlamp at about 150 ft). Apertures of these angles will introduce error of a moderate amount at small divergence angles (about the same amount of error as would be obtained with a 7-in. headlamp at 100 ft), but much less than some other methods in use (6, 4).

The second source of error lies in the failure of the inverse-square law to apply exactly at short test distances (9). The problem is analagous to the photometry of projectors. The minimum test distance is a function of the semi-divergence of the beam and the size of the individual reflector units. The reflector units of the materials considered in this report are many times smaller than those of reflector buttons: if 100 ft is adequate for reflector buttons, 30 ft is more than adequate for these materials.

Accuracy Check with Other Laboratories

For an evaluation of the accuracy of measurement, specimens from the same piece of material were sent to two other laboratories. It was found that their measurements compared favorably with those obtained in the authors' laboratory (see Figure A2). These laboratories were chosen because of their recognized standing and because they used two types of apparatus different from that of the authors. One laboratory (Electrical Testing Laboratories) used the standard visual method, wherein the point brilliance of the specimen at 100 ft was matched by a comparison lamp at the same distance. The second laboratory (University of California) used the photoelectric photometer described by Finch (3), and also used a test distance of 100 ft. All measurements were made at an angle of incidence of 10 degrees. The points plotted from the authors' data are the averages of five readings by one observer. A later check by another observer gave points which checked at least as well as those shown.

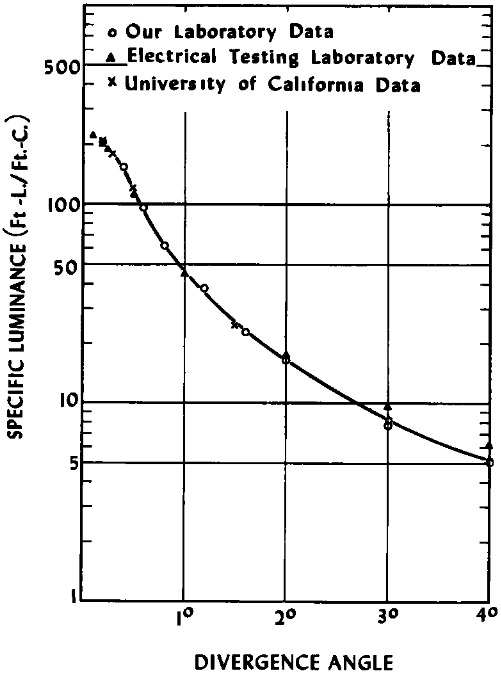


Figure A2. Comparison of photometric data from three laboratories.

It is felt that the accuracy is more than sufficient for the purposes of this paper. In the range of important divergence angles, there were no significant differences between the measurements by the three laboratories. At 3-degree and 4-degree divergence angles, the two visual methods differ significantly. Since the photoelectric apparatus did not give measurements at these divergence angles, no information could be obtained on which of the visual methods produced the correct results. However, since 3- and 4-degree divergence angles are encountered on the road only at distances less than 100 ft, the difference is of little practical significance.

APPENDIX B

TRIGONOMETRIC RELATIONSHIPS

To apply laboratory photometric measurements of reflective materials to practical highway situations, it was necessary to take into account the trigonometric relationships between the driver, the sign, and the headlamps of the car approaching the sign. Fundamental to the trigonometry of the problem were the dimensions of the car itself. Measurements were taken of 15 late model cars and averages were computed of all key dimensions. The "average" car dimensions used in this study are shown in Figure B1. The right headlamp was assumed to be 2.0 ft horizontally from the edge of the pavement in all computations.

Symbols Used

In the formulas developed, the following symbols were used:

S = headlamp-to-sign horizontal distance, measured parallel to roadway;

d = horizontal distance from the right headlamp to the sign, measured perpendicular to the roadway (see Fig. B1);

h = height of the sign above the road (see Fig. B1);

a = direct headlamp-to-sign distance, oblique in three dimensions;

b = direct eye-to-sign distance, oblique in three dimensions;

c = direct eye-to-headlamp distance, oblique in three dimensions;

k = headlamp-to-sign distance when headlamps are abreast of the sign, oblique in two dimensions;

Δ = divergence angle, the angle at the sign between the line from the headlamp to the sign and the line back to the eye, measured in the oblique plane which includes those three points; and

θ = entrance angle or angle of incidence, the angle at the sign between the line from the headlamp to sign and the line normal to the sign, measured in the oblique plane including that line and point.

Car on a Straight Level Road

By three-dimensional application of the Pythagorean theorem, using the parameters based on a car of the dimensions shown in Figure B1, the following relationships were derived:

$$a^2 = S^2 + d^2 + (h - 2.7)^2 \text{ for right light}$$

$$a^2 = S^2 + (d + 4.8)^2 + (h - 2.7)^2 \text{ for left light}$$

$$b^2 = (S + 7.0)^2 + (d + 3.7)^2 + (h - 4.3)^2$$

$$c^2 = (7.0)^2 + (3.7)^2 + (1.6)^2 = 65.25 \text{ for right light}$$

$$c^2 = (7.0)^2 + (1.1)^2 + (1.6)^2 = 52.77 \text{ for left light.}$$

Headlamp Angles

To use the isocandle diagrams for determining illumination (see Appendix C), it was necessary to know the horizontal and vertical angles from the headlight to the sign. These were computed as follows:

$$\tan H = \frac{d}{S} \text{ for right headlamp}$$

$$\tan H = \frac{(d + 4.8)}{S} \text{ for left headlamp}$$

$$\tan V = \frac{(h - 2.7)}{S}$$

Entrance Angles or Angles of Incidence

For a sign mounted plumb and perpendicular to the roadway, the entrance angle or angle of incidence is given by

$$\tan \theta = \frac{k}{S}$$

in which

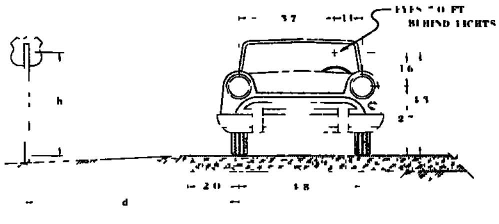
$$k^2 = d^2 + (h - 2.7)^2 \text{ for right light}$$

$$k^2 = (d + 4.8)^2 + (h - 2.7)^2 \text{ for left light.}$$

Divergence Angles

From the cosine law, the divergence angle is

$$\cos \Delta = \frac{a^2 + b^2 - c^2}{2\,ab}$$



(All dimensions in feet)

Figure B1. Dimensions of average car used in computations.

For accurate calculation of the cosines of the small angles involved, the calculation of the denominator of this equation required the extraction of square roots accurate to ten decimal places. However, when the distance S to the sign becomes very large, the fact that the eye is 7 ft behind the headlamps becomes unimportant, and the divergence angle approaches, as a limit:

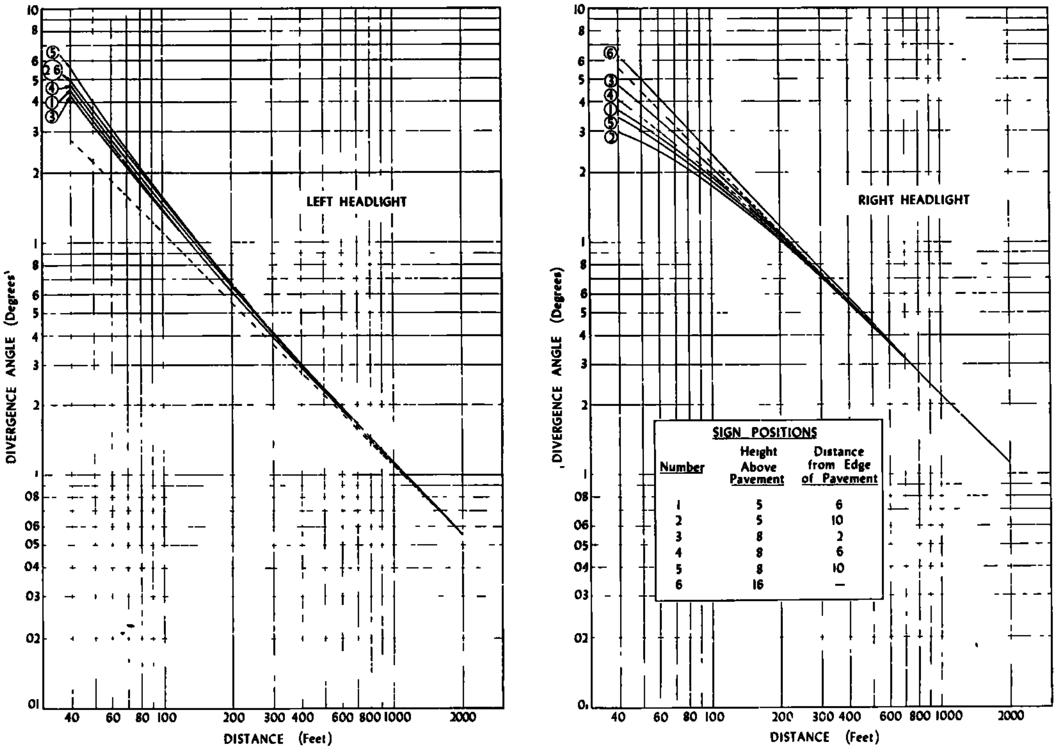
$$\tan \Delta = \frac{\sqrt{(1.1)^2 + (1.6)^2}}{S} = \frac{1.942}{S} \text{ for left light}$$

$$\tan \Delta = \frac{\sqrt{(3.7)^2 + (1.6)^2}}{S} = \frac{4.03}{S} \text{ for right light}$$

At such small angles the tangent and its angle are linearly related, so that

$$\Delta = \frac{111}{S} \text{ for left headlamp}$$

$$\Delta = \frac{231}{S} \text{ for right headlamp}$$



SIGN POSITIONS

Number	Height Above Pavement	Distance from Edge of Pavement
1	5	6
2	5	10
3	8	2
4	8	6
5	8	10
6	16	—

Figure B2. Divergence angles vs distance for left and right headlights of an average car for various sign positions.

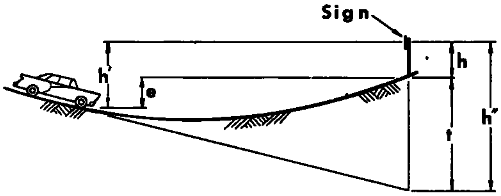


Figure B3. Geometry of vertical curves.

Figure B2 shows the computed values of the divergence angles plotted as a function of distance, for the six sign positions described in the text. The dashed lines show the long-distance approximations,

$$\frac{111}{S} \text{ and } \frac{231}{S}$$

Previous estimates of divergence angles by inexact formulas can be compared with these results. Finch (2) and Pocock and Rhodes (6) assumed a height of eye above headlamps of 21 in. Assuming the tangent of the divergence angle to be this separation divided by the distance to the sign, the divergence angle is approximately 100/S. Since height of eye above the headlamps is not the proper variable, these results were somewhat too low for the left headlamp, and less than one-half of the exact value for the right headlamp. Havens (4) used the cosine law for the special case where sign height and headlamp height are equal. Using somewhat different car dimensions, he obtained results for the right headlamp which agreed quite well with those in Figure B2 for sign position 1.

In future work approximations could be worked out, but should be evaluated in terms of the allowable error in the specific luminance for representative materials.

Truck on Straight Level Road

The trigonometric relationships for a truck approaching a sign are the same as for a car except that the parameters in the equations are different. Average values of measurements on a few late model trucks are given in Table 2 and compared with the values for the "average" car. Because of considerable variations in the dimensions of the few trucks measured, the truck values are not as reliable as those for cars.

Car on a Vertical Curve

Vertical curves were computed using the standard procedure for parabolic curves (5). The distance S was assumed as the horizontal distance. The geometry of vertical curves is illustrated in Figure B3. The difference in elevation, e, between the roadway at the sign location and the roadway at the car location was computed for each distance S. Divergence angles and incidence angles were calculated as for a straight road, except that h' was used in place of h, where

$$h' = (h + e) \text{ for sags}$$

and

$$h' = (h - e) \text{ for summits.}$$

As mentioned previously, headlamp angles are needed to use isocandle charts for determining illumination. The horizontal headlamp angle, H, for a vertical curve was the same as for a straight level road. To determine the vertical headlamp angle, V, it was necessary to project the tangent from the car to the sign, and compute the value of t in Figure B3. Headlamp angle V was computed in the same manner as for a straight level road except that h'' was used in place of h, where

$$h'' = (h + t) \text{ for sags}$$

and

$$h'' = (h - t) \text{ for summits.}$$

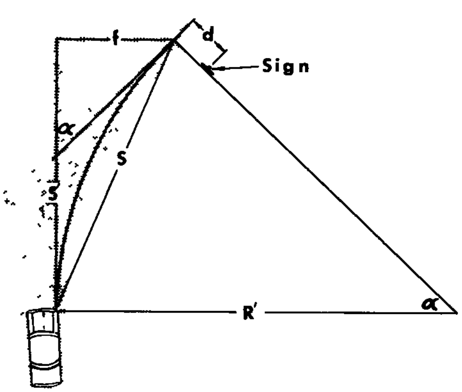


Figure B4. Geometry of horizontal curves.

Computed solutions for the vertical

curves were checked by large-scale plots of the vertical curves on profile paper.

TABLE 2

COMPARISON OF CAR AND TRUCK DIMENSIONS

(see Fig. B1)

Distances, ft	Car	Truck
Horizontal:		
Between headlamps	4.8	4.5
Eyes behind lamps	7.0	7.1
Right lamp from pavement edge	2.0	2.0
Line of sight from right lamp	3.7	3.3
Line of sight from left lamp	1.1	1.2
Vertical:		
Headlamp above pavement	2.7	3.4
Eyes above headlamp	1.6	3.5
Eyes above pavement	4.3	6.9

Car on Horizontal Curve

The geometry of horizontal curves is illustrated in Figure B4. The distance S was assumed as the chord distance on the path of the right headlamp. For a car and a sign both on a curve, the distance S corresponds to the distance S' on a straight road except for a greater lateral distance. The lateral distance to the sign is nearly equal to the sum of the distances f and d.

In highway engineering the degree of curve is defined as the central angle subtended by an arc of roadway 100 ft in length (5). For example, on a 3-degree curve the angle α would be 3 degrees when the arc distance at the centerline of the pavement was 100 ft. The radius of curvature can be found from

$$R = \frac{100 \text{ ft } (360^\circ)}{2 \pi D} = \frac{5729.58}{D}$$

where D is the degree of curve. On a 24-ft pavement with the right headlamp 2 ft from the edge of the pavement, the radius of curvature for the path of the right headlamp will be:

$$R' = (R + 10) \text{ for left curves}$$

$$R' = (R - 10) \text{ for right curves.}$$

Other necessary relationships are given by the following equations:

$$\alpha = 2 \sin^{-1}\left(\frac{S}{2 R'}\right)$$

$$S' = R' \sin \alpha$$

$$f = R' \text{ vers } \alpha$$

$$d' = (d + f) \text{ for right curves}$$

$$d' = (d - f) \text{ for left curves.}$$

Calculations of headlamp angles (for use on isocandle charts, see Appendix C) and divergence angles were made in the same manner as for a straight road except that the values S' and d' were used in place of S and d.

Calculations of entrance angles assumed that the sign was perpendicular to the roadway at the point of sign placement; in other words, that it is rotated through an angle α with respect to the car. For the sign position for which calculations were made (5 ft up and 6 ft from the edge of pavement) it was found that the vertical component of the entrance angle was negligible. In this case, the entrance angle is

$$\theta = (H + \alpha) \text{ for left curves}$$

$$\theta = (H - \alpha) \text{ for right curves.}$$

APPENDIX C

COMPUTATION OF SIGN LUMINANCE

For the calculated values of entrance and divergence angles for a given sign position and distance (see Appendix B), the specific luminance was obtained from the curves

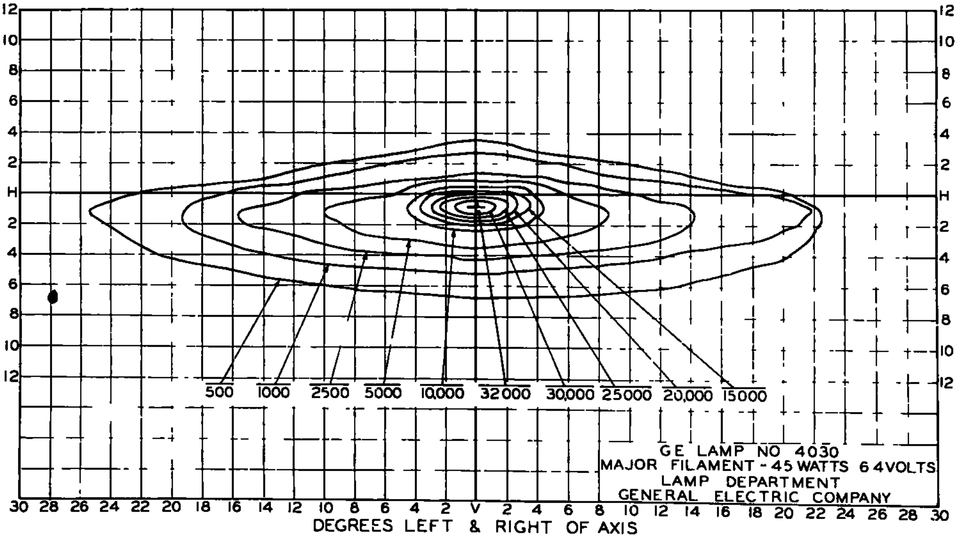
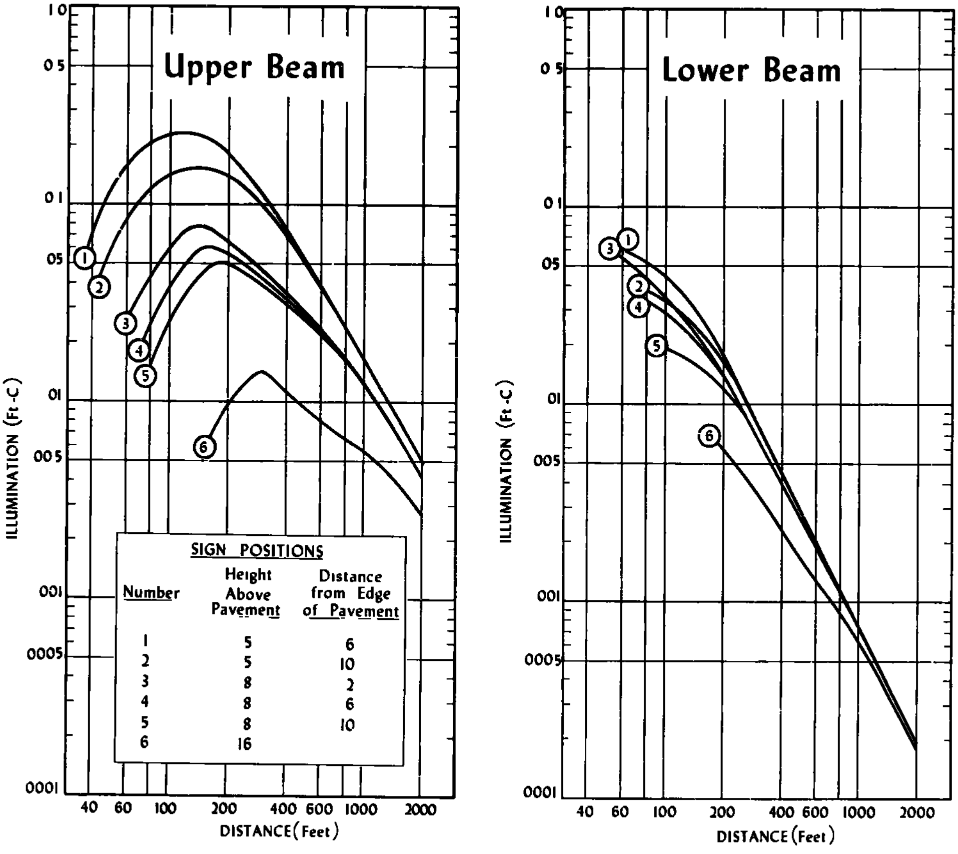


Figure C1. Example of isocandle chart showing headlamp candlepower distribution for upper beam of No. 4030 headlamp. Similar charts were used for No. 4030 lower beam and for No. 5040 upper and lower beam.



SIGN POSITIONS

Number	Height Above Pavement	Distance from Edge of Pavement
1	5	6
2	5	10
3	8	2
4	8	6
5	8	10
6	16	

Figure C2. Illumination vs distance for various sign positions. Data shown are for a No. 4030 right headlamp only, mounted in an average car.

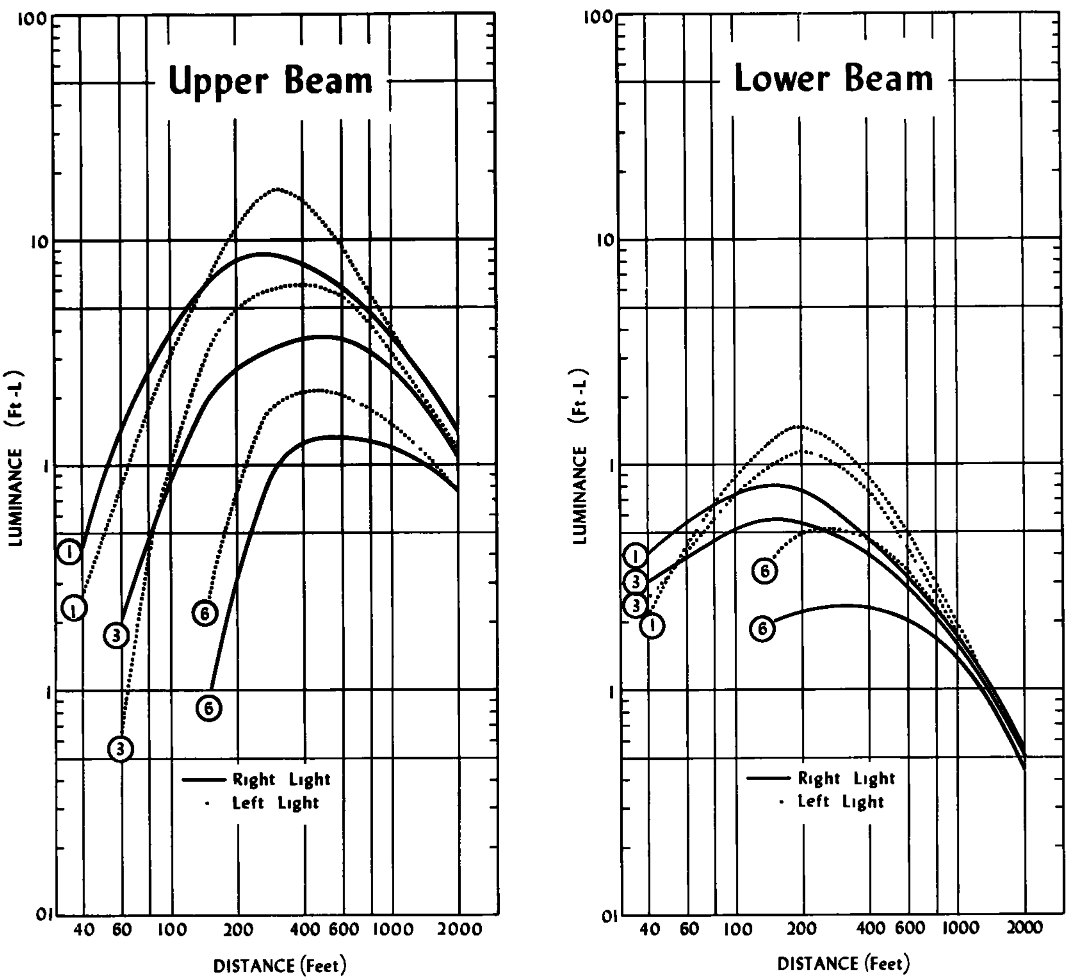


Figure C3. Luminance vs distance for left light and right light separately. Numbers on curves indicate sign positions.

based on photometric measurements of the reflective material (see Fig. 10). The luminance of the sign was then found by multiplying this value in specific luminance by the illumination reaching the sign from the headlamp.

The illumination was determined by the use of isocandle charts such as the one shown in Figure C1. These curves were supplied by the manufacturer as representing typical average production.

The headlamp angles H and V were computed using trigonometric relationships of the car approaching the sign as explained in Appendix B, and were used to find the intensity of candlepower from the isocandle curves. The illumination reaching the sign was:

$$E = \frac{\text{candlepower}}{(\text{distance})^2} = \frac{\text{C. P. for H and V}}{a^2}$$

Use of S^2 instead of a^2 (see symbols in Appendix B) would result in negligible error except at short distances.

Figure C2 shows curves of the illumination from the upper and lower beam of a car's right headlamp reaching signs in various positions on a straight level road. Similar illumination curves were plotted for the left headlamp, for the new No. 5040 headlamp, for trucks, and for vertical and horizontal curves.

Figure C3 shows the luminance for flat sheeting material, for left and right headlamps separately. In the computations it was necessary to treat each lamp separately because differences in illumination and differences in the geometry of divergence angles and entrance angles result in different luminances for each lamp. The total luminance of a sign as seen by a driver is the sum of the luminances for the two headlamps. Except for Figure C3, which illustrates the difference in luminances for each lamp, all of the luminance data in this paper are shown as the sum from both lamps.