# Automotive Test Track Design 

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- CONSIDERATION of automotive test track design before an audience of highway engineers is justified by the hope that some of the problems and possibly somewhat extreme treatments may be of help in the approach to highway problems.

An essential difference between the areas is that special types of construction and operation are easily possible on private road systems because of the opportunity to provide any driver training necessary and because operation can be supervised.

The special design problems of automobile test tracks may perhaps be approached best by consldering the objectives of automotive proving grounds and certain characteristics of the vehicles which are tested there.

The history of the establishment and the development of the General Motors Proving Ground, which was the first and is still the largest and most highly developed, will make these objectives clear.

## PROVING GROUND OBJECTIVES

The story is told that the concept of a proving ground crystallized from a series of tests conducted in 1923 when the 4 -wheel braking system was being developed for adoption by Buick. The General Technical Committee, with A. P. Sloan as chairman, had witnessed a test on an experimental 4 -wheel brake installation on a public road near Flint. After this demonstration the Committee suggested that certain design modifications be made, and it was agreed that the Committee would return and observe the performance of the modified system at a later date. When this time came and the Committee convened to run the test, they found that the County Road Commission had resurfaced the test strip, and it was obviously impossible to make a direct comparison with the earlier tests. That pointed out clearly the necessity for having a private road system where surfaces, gradients, and operating conditions could be controlled by the development groups, so that the test programs would be free from hazards to and interference from casual private transport and where the test conditions could be maintained at any standard desired.

Requirements of the site were convenience of access from the General Motors manufacturing plants at Detroit, Pontiac, Flint, and Lansing, and specific terrain characteristics; a tract of approximately 1,100 acres was located near Milford and purchased early in 1924.

In passing we may note that additional land has been purchased and the road system developed during the years until we now have 3,873 acres in use at the Proving Ground at Milford, 2, 280 acres in the Desert Proving Ground near Mesa, Arizona, and an Engineering Test Headquarters in Manitou Springs, Colorado, at the foot of Pikes Peak. The Desert Proving Ground was established because some portion of car development work must be conducted under climatic conditions prevailing in the south and southwest and in the mountains, and the Pikes Peak facility was established for mountain and high altitude testing. The road system on the Milford Proving Ground has grown to approximately 51.1 miles of several types of surface. The Desert Proving Ground has a 5mile circular track and a 1.2 -mile straightaway. Necessary garage, laboratory, and office bulldings on these Proving Grounds have more than $500,000 \mathrm{sq} \mathrm{ft}$ of floor area. We have accumulated more than $168,000,000$ test miles, and we operate at a current rate of about $12,000,000$ miles per year.

The primary objective in establishing the Proving Ground was to provide a place where the manufacturing divisions could carry on their development work free from interference and hazards of public highway travel and where privacy in development of new designs could be assured. General Motors divisions are autonomous, and each is responsible for design, development, manufacturing, and sale of its products. As a secondary objective, the Proving Ground Section was charged with the responsibility of conducting a comprehensive series of engineering tests on production cars of General


Figure 1. Rated horsepower.

Motors and competitors so that the management might know at all times exactly where General Motors products stand with respect to their competition, in the eyes of the customers. In the discharge of this function, the Proving Ground staff has become established as test experts and consultants, and instrument designers, and currently about one-third of the time of the engineering departments is being spent on special tests on division development programs.

Since division development work is the primary objective, a proving ground has to be reasonably close to the manufacturing operation, and consequently site location is restricted to a relatively small part of the country.

To make maximum use of such an area it must be as compact as possible, square or rectangular in shape, and in one piece. In view of property valuations and the nature of the established public road systems in areas reasonably close to industrial centers, the area of such a piece of property is necessarily limited and corresponding limitations are imposed on design.

## VEHICLE CHARACTERISTICS

The requirements of the road system on a proving ground are determined by the characteristics of the vehicles which are to use it.

First it must be recognized that the passenger car has developed and come into being to the number of more than $50,000,000$ units on the road today because it answers an urgent need in the mind and hearts of the American people for a means of swift, mobile, and inexpensive individual transportation. Because the passenger car provides, above


Figure 2. Maximum car speed. all, individual transportation, it has largely replaced mass transportation facilities, which may be as fast or faster and considerably cheaper but suffer the overwhelming disadvantage of being regimented. With our families in our cars we are as free as birds, and we drive about 500 billion miles each year. The automobile is creating a significant social revolution because of the char-


Figure 3. Speed - distance. acteristics of its behavior related to flexibility and mobility.

These characteristics have been summarized comprehensively elsewhere (1); for our purposes it will suffice to consider trends of power and speed, and the relations of distance and speed on full throttle acceleration, to characterize the proving ground road system needs of the current vehicle and to estimate reasonable future needs.

Figure 1 shows the trend of rated horsepower from 1930 to 1955, from which we
note that there has been a steady increase, greater in the current period, but always with an upward trend.

Associated with horsepower in the popular conception is the maximum speed which is shown in Figure 2. Here we note that in 1930 the average speed of representative American cars was approximately 65 mph , and in 1955 it was approximately 97 . The fastest car of the group developed about 73 mph in 1930 and about 108 mph in 1955 . Both the average and the highest speed curves show an increase year by year, and it may be expected that this increase will continue, probably with a decreasing slope. Note that we had $100-\mathrm{mph}$ production cars as early as 1941. We have noted several times that the values of maximum speed noted in Figure 2 are only the by-products of design compromises to achieve ever better traffic performance and fuel economy (1).

Figure 3 shows the speed-distance performance under full throttle acceleration on a level straightaway of a high-performing


Figure 5. Lateral acceleration vs radius.
speeds but because the satisfactory development of endurance capabilities, lubrication, and cooling systems requires extremely severe testing, testing under conditions far more exacting and demanding than will ever be used by the public. A proving ground intended for realistic development work on current automobiles should provide safe speed characteristics up to something like 110 to 120 mph ; from the trend of the maximum speed curve in Figure 2 it appears that the farsighted designer would allow himself a considerable margin for further increases in maximum speed.

It is my understanding that test tracks


Figure 4. Desert proving ground - lateral acceleration vs speed. 1955 car. Nearly one mile is required to attain 100 mph , and evidently a straightaway of two miles or more in length is required for any constant speed observations at 100 mph .

## AUTOMOTIVE TEST TRACE REQUIREMENTS

It seems clear, therefore, that a test track on an automobile proving ground must be so designed that it will permit development work up to the top speeds of current automobiles, not because manufacturer intends that the public will operate on those


Figure 6. General Motors commercial proving ground.


Figure 7. General Motors desert proving ground.
recently completed and now under construction have neutral speed capacities of from 140 to 150 mph .

The factor which makes design of automotive test tracks distinctive in highway engineering is that the area available is always very closely restricted because of land values and uses in the only locations where proving ground operation can be conducted most effectively. This means that the most careful refinements of design are required to extend the useful life of a facility to the maximum.

The physical laws related to speed, radius, and radial acelerations are applied in Figures 4 and 5. Figure 4 shows the radial acceleration in gravity units as a function of speed for a track with a radius of $4,200 \mathrm{ft}$; this refers to the General Motors Desert Proving Ground track at Mesa, Arizona. The same general second degree relationship holds for all values of radius except that the scale on the left side will be changed as the radius varies. The relationship between radial acceleration and the radius for an arbitrary value of speed is shown in Figure 5. Here the design speed was selected as 100 mph for purposes of illustration, and this general reciprocal relation holds for all values of speed. It is evident that the radial acceleration falls rapidly as the radius is increased up to $2,000 \mathrm{ft}$ or more and that there is further substantial decrease as the radius goes up to $5,000 \mathrm{ft}$. The operating and construction problems are very greatly simplified by the selection of a large radius.

Figure 6 shows the layout of the General Motors Proving Ground road system generally as it existed for 30 years; the test track is roughly triangular in shape because of the property line restrictions prevailing in 1924 and the terrain limitations. The three principal curves have radii of $1,042 \mathrm{ft}, 676 \mathrm{ft}$, and 661 ft , and the superelevation requirements for a uniform design speed change accordingly. On other sites with different terrain characteristics and property restrictions, test tracks have been constructed which are roughly oval in shape, with straight parallel sides and symmetrical circular curves at the ends. When the General Motors Desert Proving Ground site was selected in 1951 a sufficient area was provided to permit the construction of a circular track with a radius of approximately $4,200 \mathrm{ft}$ (Figure 7).

Since the Desert Proving Ground track occupies all the available space in a 2,280acre area, it seems that a practical maximum radius of the order from 3,500 to 5,000 ft will probably confront most designers of automotive test tracks. Note the location of the engineering test straightaway in Figure 7.

The design of the cross-section of an automotive test track differs from that of the usual highway superelevation in the very important respect that equilibrium of forces parallel to the pavement surface must be provided for a wide range of speeds. This is so because the tests conducted on such a track will range over every combination of speed and throttle opening from 5 or 10 mph up to continuous full-throttle maximum speed tests, or near full-throttle tests, in development work on such things as engine bearings and engine and axle lubricants. Simultaneously there will be numerous other test schedules requiring operation of a number of cars at any and all speeds from 10 to 20 mph up to the maximum. It is imperative that side forces be balanced to eliminate steering drag and undue tire wear. In other words, over a wide speed range, the design must provide operating conditions as close as possible to those on a straight road.

In view of this predominating requirement, the ceiling on any value of cross-section slope is removed; the superelevation slope is limited only by practical problems of construction. On the Milford track, slopes as high as 77 percent are used and operating
procedures are established accordingly. We do not carry loads of hay around such superelevated sections at low speeds, nor do we drive on them when they are covered with ice.

The equilibrium of forces parallel to the surface pavement is expressed by the fundamental equation of superelevated curves:

$$
\operatorname{Tan} \phi=\frac{\mathbf{V}^{2}}{\mathrm{gR}}
$$

where $\phi=$ angle of inclination of cross-


Figure 8. Variable speed super.
$V=$ speed in $\mathrm{ft} / \mathrm{sec}$
$\mathbf{R}=$ radius in feet
$\mathrm{g}=$ the acceleration due to gravity
This is derived by equating the components of centrifugal force and weight parallel to the pavement surface. In usual highway design, of course, the section is flat and $\phi$ is constant, but in an automotive test track or variable speed curve, $\phi$ varies continuously so that the higher speeds can be supported at the outer edge of the paved surface (Figure 8).

To provide a logical basis for the variation of the cross-section slope, it is convenient to select a cross-section of some simple mathematical form such as $y=F\left(x^{n}\right)$; here, of course, the value of $\tan \phi$ in the fundamental equation is the first derivative of the equation of the cross-section. This may be rewritten as

$$
y^{\prime}=V^{2} / g R
$$

The solution of this equation for $V$ gives

$$
\mathrm{V}=\sqrt{\mathrm{gRy}} \quad \text { (Figure 9) }
$$

Figure 9. Equilibrium of parallel forces.
Since $y^{\prime}$ is a function of $x$, it was clear that the variation of the equilibrium speed across the width of the section will depend upon the degree of $x$ in the equation of the crosssection. If the original section, for example, is an equation in $\mathrm{x}^{3}$, the equilibrium speed-width relationship will be linear. Other types of cross-section equations will give other types of relations as indicated in Figure 10.

Which of these relationships to select is always a problem to the designer; it could be approached more confidently if an estimate could be made of the relative proportion of high-speed traffic on the new test track.

For example, if the original equation is a quadratic and $10-\mathrm{mph}$ range speed lanes are laid out, the lanes will be packed together closely at the bottom of the pavement and spread more widely toward the outer edge. If, on the other hand, a 5th degree equation is used, the low-speed lanes toward the bottom of the cross-section will be spread rather widely and the high-speed lanes toward the top super will be packed in more densely. Thus the cross-section of the quadratic equation


Figure 10. Equilibrium speed.


Figure 11. Cross-section elevations.
has more room for high-speed lanes than for low, and the cross-section in the 5th degree equation has more room for the low-speed lanes than for high. Since no one can predict with certainty what the relative load will be at any time in the future, it is perhaps safer to select an intermediate value and use a cubic crosssection. Thus, if a speed range of from 30 to 120 mph is to be covered, the speed at the centerline would be halfway between, or 75 mph .

The total volume of earthwork under the cross-section and the shoulder changes importantly with the degree of the cross-section equation, and the construction problems become greater as the height at the outer edge increases. This is indicated in Figure 11, which shows the comparative cross-section elevations used in a design study of the Desert Proving Ground track, under the assumption that a design speed of 120 mph should be used. Note that the quadratic section is nearly 50 percent higher at the outer edge than that of the 5th degree equation. For curve designs using shorter radii with much higher total elevation, the differences are even greater, and the volume of earthwork required makes this consideration one of economic importance.

It may be noted, however, that the inner portions of the cross-sections computed from higher degree equations are very nearly flat, and there will usually be a serious question of whether it is really necessary to build the flat portion. The compromise in many design studies is to compute a relatively wide cross-section and use only the outer part of it, discarding several feet of the inside because of the low practical value of this portion. This compromise was reached on the Desert Proving Ground track; we computed a 4th degree cross-section 32 ft wide and used only the outer 24 ft . This gives the equilibrium speed-distance relation shown in Figure 12. At the inner edge the equilibrium speed is approximately 20 mph , while at the outer edge it is about 145 mph . The practical design speed, which is assumed to be at the middle of the outer traffic lane 3.5 ft from the outer edge, is 120 mph . This we considered would be adequate for safe operation on present vehicles and provide a considerable margin for possible increases in maximum speed during development work for the next several years. However the volume of test work exceeded expectations, and we are now reconstructing the track to increase both the surface width and the speed capacity. It will be noted, however, that any modification which discards the inner portion of cross-section results in an equilibrium speed-distance relationship which approaches the linear cubic relationship, and a cross-section elevation which approaches that of the cubic. It may be concluded, therefore, that any design using a cross-section equation of a degree higher than the third and eliminating a portion of the inner part of it will give a modified section which closely approximates the cubic, both with respect to equilibrium speed relations and cross-section elevations. The advantage of a high degree equation is that a considerable margin for higher speeds and safety is provided by a few feet of additional width at the outer edge.

There is no specific guide which can be given to the designer of the superelevated curves for automotive test tracks except that the maximum possible radius should be employed for ease of operation and simplicity and economy of construction, and that while the cross-section equation of the degree higher than the third will give no important construction economies


Figure 12. Circular speed loop cross-section and equilibrium speed.


Figure 13. Radial acceleration vs time observed in sharp turn.
unless the whole section is used, it will give a greater margin for safety and expansion for each foot of added width. If a flat inner lane will be used exten-
sively, significant economy is yielded by the high degree equation.

## TRANSITION SPIRALS

In all layouts where tangent sections are connected to horizontal curves it is imperative that a transition spiral of some sort be incorporated. We are indebted to R.L. McNeal, retired from the Proving Ground staff, for the adaptation of the mathematical form which will give a constant rate of change of radial acceleration through this transient section. This form appears in the literature as Cornu's Spiral. It is the same transition used by Joseph Barnett in his excellent "Transition Curves for Highways." Sample computations are reproduced in the Appendix of this paper.

Figure 13 shows the radial acceleration observed as a function of time by an automobile during the first few seconds of a sharp turn. This makes it clear that the car will follow a transition spiral as it enters a turn and suggest the desirability of building the road along the path the car will follow. This observation is introduced because some people question the need for transition spirals.

Figure 14 shows that the rate of change of radial acceleration, or curvature, is uniform, the distinguishing characteristic of this form.


Figure 14. Cornu's spiral - typical radial acceleration vs length.

Figure 15 is a list of the spiral formu-

Figure 15. Cornu's spiral formulas.

$$
\begin{aligned}
& K \text {-CONSTANT - } \sqrt{\frac{\pi v^{2}}{\alpha}} \text { (FEET) } \\
& \boldsymbol{Q}-\frac{\mathbf{v}}{\mathrm{RL}}-\frac{\mathbf{r}}{\mathrm{RI}} \\
& \text { L-LENGTH OF SPIRAL FROM ORIEIN TO ANY POINT-FEET } \\
& -\frac{L}{K} \\
& \text { 2- } \frac{u^{2}}{2} \text { ANGLE BETNEEN TANGENT LINE AT ORIGIN AND } \\
& \text { R- } \frac{1}{22} \text { radius at any point alonc spiral-feet } \\
& x-K \int_{0}^{u} \cos \left(\frac{\pi u^{2}}{2}\right) d u-K C(u) \\
& \mathbf{Y}-K \int_{0}^{u} \sin \left(\frac{\pi u^{2}}{2}\right) d u-K 5(u) \\
& \text { K-CONSTANT- } \sqrt{\frac{\pi v^{2}}{\alpha}} \text { (FEET) } \\
& \boldsymbol{Q}-\frac{\mathbf{v}}{\mathrm{RL}}-\frac{\mathbf{V}}{\mathrm{RI}} \\
& u-\frac{L}{K}
\end{aligned}
$$



Figure 16. Cornu's spiral - typical radius vs length.


PS
Figure 17. Typical spiral and locus of center of curvature.
las; while the spiral cannot be expressed very simply in elementary functions, the computations are but little more extensive than those of other spirals. Much more information is available; for example, the radius can be computed very simply for any station, which is essential to the design of the proper cross-section. In these formulas $a$ is the rate of change of radial acceleration and $\mathrm{u}^{2}$ is in right angles.

Of course, for public highway alignments, the tables in "Transition Curves" eliminate the need for use of the formulas; they are required for specialized applications discussed later.

Figure 16 shows how the radius changes along the length of the spiral.

Figure 17 shows the layout of a typical spiral of this form and the locus of the center of curvature.

Figure 18 defines Z, $x$, and $y$.
In the original adaptation, which is on file in the Engineering Societies' Library (2), the assumption was made that the radial acceleration should vary at a constant rate to provide the greatest ease of passing from the condition of zero centrifugal force to the maximum centrifugal force in the full circular curve. Of course, with the proper superelevation design in the spiral, the lateral forces are always balanced so, with constant speed, this derivation results in a form which gives a constant rate of change of curvature. A significant advantage of this form is that the length of radius and the direction of the tangent may be located precisely throughout the curve, and the coordinates of all the stations can be computed precisely for any length of spiral; the radius must be known at each station to develop precise cross-section design.

There are other types of spirals in the literature, developed first for use in the layout of railroad curves and subsequently adopted in one form or another with certain variations for highway use. These do not provide for a constant rate of change in curvature; in some cases approximations are made which do not permit them to be applied on long spirals turning through considerable angle.

There is no clear criterion of the value which should be selected for the constant rate of change of radial acceleration; in McNeal's derivation the assumption was made that a rate of $3 \mathrm{ft} / \mathrm{sec} / \mathrm{sec} / \mathrm{sec}$ should be the maximum, but this assumption has not been verified in practice. The current highway standard, according to Barnett, is 2 ft / $\mathrm{sec} / \mathrm{sec} / \mathrm{sec}$. For spirals into curves with a large radius it is our opinion, however, that selection of a conservative value will result in a spiral sufficiently long to permit reasonable rates of change of other elements of the transition; the designer can rest assured that the degree of permanence of his structure will depend on this value, and we are reasonably sure the value of $3 \mathrm{ft} / \mathrm{sec} / \mathrm{sec} / \mathrm{sec}$ is satisfactory for long radius curves on proving ground road systems. On short spirals leading into curves with radii in the neighborhood of 200 ft we have found a rate of from 6 to $8 \mathrm{ft} / \mathrm{sec} / \mathrm{sec} / \mathrm{sec}$ can be attained, but the operation is severe.


Figure 18.

It would be possible to compound the entering and leaving spirals and omit the portion of circular arc; this always requires a larger area which usually makes this design impractical, the feeling generally being that the maximum length of straight sections should be obtained.

## STRAIGHTAWAY TURNAROUNDS

In automotive development work, road elements other than a test track are required for certain types of engineering tests. Level straightaways of the maximum length possible are incorporated in most proving ground layouts, because they are essential to a comprehensive test program. These roads have possibly their maximum useful application in measurements of constant speed fuel economy. On these tests, it is essential that temperatures be stabilized, and the changes in temperature during the brief interruptions of turning around at the ends of the straightaway are significant factors in widening the so-called experimental error. It is, therefore, imperative that the test car be turned around at the end of each run in the shortest possible time and at as near the test speed as possible. This gives rise to numerous interesting and difficult design problems when we consider that fuel economy observations at speeds as high as 90 or 100 mph are important on present day automobiles. As a result the designer is expected to provide a high-speed turnaround in almost no space; on a turnaround, ideal from the standpoint of the test engineer, the slope and the total height of the superelevation would go up to colossal values and the transition spirals would have excessive rates of change of curvature.

## SPIRAL EASEMENT VERTICAL CURVE DESIGN

In cases such as this where the designer is cramped for room it will be found that the vertical curves leading the car up to the height of the fully superelevated structure in the high-speed lane are the critical factors in the design.

We find, for example, that the simplest type of easement from the level tangent section to the top of a superelevated curve structure would be a straight-sloped profile with simple vertical curves connecting the two horizontal levels on a structure such as shown in Figure 19. On high-speed, short-radius circular curves with short spirals, the radius of curvature in the vertical plane of both of these vertical curves must be short, and the centrifugal forces in the vertical plane are considerable. For example, as the car reaches the first concave vertical curve it will be pressed against the road surface with a force depending upon the speed of the car and the radius of the curve.

Figure 20 is used as an analogy of the case as the car climbs over the top of the second vertical curve. Figure 20 (left) illustrates an elementary problem in exterior ballistics; the projectile follows a parabolic path and climbs to a height determined by the velocity and initial angle of flight. The car at the right climbing a ramp of the same inclination at the same velocity would follow the same trajectory. If we constructed a vertical curve along the broken line, the weight of the car against the road surface would be zero, while if the ramp were continued indefinitely along the solid line, the road reaction would equal the car weight; on a vertical curve lying somewhere between, the reaction would be somewhere between zero and the car weight. The centrifugal lift means that the force in the vertical plane against the road surface is less than normal, and the gravity component down the slope which is used in the development of the fundamental equation of a superelevated curve is reduced proportionately. Therefore, the speed for balance of lateral forces parallel to the pavement surface will fall unless the superelevation slope is increased. This is a matter of considerable importance, because the critical point in operating on a high-speed turn near its capacity always occurs near the end of the transition section, and the effective operating speed of the turnaround system can be increased appreciably by careful design at this point.

We have developed a method whereby the designer can evaluate the vertical forces


Figure 20. Vertical curve analogy.
and a criterion which permits him to select proper rates of change of the vertical forces. We have applied McNeal's transition theory to the vertical curves, which means that the vertical disturbances of the vehicle path will occur at a constant rate. This derivation permits us to evaluate these disturbances by rather simple computation, so we can select the length of the vertical transition which will keep the rate of change in vertical forces within limits we may choose. Figure 21 shows the relative value of the vertical reactions as they occur during such a transition design using Cornu's Spirals.

Observations made at the Proving Ground indicate that the driver is sensitive to a lateral force unbalance of about $0.1 w$ producing a lateral, or radial, acceleration of 0.1 g . Figure 22 shows the effect of the vertical curve in terms of forces in the section on a car passing over the crest of a convex vertical curve leading from the transition section to the full super. The driver will be acutely aware and considerably disturbed as the component of the lift parallel to the plane of the pavement exceeds 0.1 w . The spiral in the vertical plane should therefore be selected to keep the lift component below this value or the design should account for it otherwise; this may mean that the slope of the superelevation should be increased to the point where the gravity component equals the sum of the parallel components of the lift force and the centrifugal force.

As a matter of practice in all our design studies on short-radius curves where this criterion had been employed, we have found that the length of the spiral is determined by the considerations of the vertical curve; when conservative or reasonable vertical curve rates are selected, the spiral length is such that rates in the horizontal spiral are small enough to be of no concern. It may be stated conclusively that in all highspeed, short-radius designs the considerations of the vertical curves are much more critical than those of the horizontal spiral.


Figure 22. Effect of vertical curve forces.

In cases where drainage conditions are


Figure 23. Desert proving ground turnaround loop - layout drawing and profiles.,


Figure 24.


Figure 25. North turnaround.
favorable, we have been able to depress the inner edge of the superelevated structure such that the climb in the high-speed lane is negligible; this design, we have found, tends to reduce the severity of operation to a considerable degree. The transition curves in the straightaway turnaround on the Desert Proving Ground were designed in this manner; the turnaround superelevation cross-section was rotated about the center of the high-speed lane so that the car traveling the high-speed lane remains on a level path. The edge profiles and layout are shown in Figure 23. The direction of travel is counterclockwise; the left edge grade is level of necessity to at least station $3+26$ where it leaves the straightaway line.

## Application to the North Turnaround

Some typical examples of proving ground design problems are found in the design of an extension and improvement of our North and South Engineering Test Straightaway. Figure 24 shows the general


NORTH TURNAROUND 200 FT RADIUS


Figure 26. North turnaround - 400 ft radius. Figure 27 . North turnaround -200 ft radius.


Figure 28. North turnaround - entrance spiral.
view of the area. With a distance of approximately three miles between the north and south property lines, sustained high speed operation will be possible over a considerable part of the length. To minimize hazards it seemed essential to divide the road, and after careful consideration, the layout and traffic control pattern shown in Figure 25 were selected. Traffic will flow in a clockwise direction to separate high-speed traffic as far as possible; proximity of the property line at the south end allowed space for a median of only 20 ft .

In developing the design of the north turnaround, the principal considerations were clearance with the west property line, the choice of design speeds and radii which would minimize the length of structure, and the most economical use of the total length available for straightaway purposes. Two values of design speed and radius were studied. The first of these, using $50-\mathrm{mph}$ design speed with suitable spirals, is shown in Figure 26. The unusual shape of this structure is caused by the proximity of the west property line. Here it will be noted that the total length of the curved structure is $4,265 \mathrm{ft}$, and the distance on the northbound lane from the end of the straight section to the extreme end is $1,702 \mathrm{ft}$, and the distance from the extreme end to the point where 85 mph will be obtained with a high-performing car is $1,958 \mathrm{ft}$.

Figure 27 shows corresponding values for the design speed of 35 mph using a 200-ft radius. Here again the limited clearance with the west property line made it necessary to move the extreme east portion of the curve slightly past the prolongation of the centerline of the straightaway. The total length of curved structure of this design is only $3,354 \mathrm{ft}$; the distance on the northbound lane from the


Figure 29. Speed vs distance - constant deceleration of $8 \mathrm{ft} / \mathrm{sec}^{2}$. end of the straight section to the extreme end is $1,584 \mathrm{ft}$; and the distance in the southbound lane from the extreme end to the $85-\mathrm{mph}$ point is $1,827 \mathrm{ft}$.

Thus, with the $35-\mathrm{mph}$ design speed, the total structure is shorter by more than 900 ft and both the north- and south-bound lanes are longer by more than 100 ft . It is possible that some greater economy might be gained at an even lower speed design, but practical operating considerations made it unnecessary to develop this farther.

The basic design decision of clockwise operation to keep high-speed traffic separated as far as possible meant that higher speed traffic would enter the curve on the left side of the road; to avoid crossing of traffic lanes, it is kept on the left side. Obviously the left lane requires a considerably greater superelevation slope than the right. Figure 28 shows a typical superelevation cross-section on the curve to the left in the northbound lane. Because the drainage problem in this area was expensive and serious, it did not seem feasible to build the whole cross-section with a uniform slope equal to that required by high-speed traffic in the left lane, so a split section with two dufferent planes is used. Since operation will be intermittent rather than continuous, some un-


Figure 30. North turnaround. balance of lateral forces can be tolerated and it is not necessary to use a curved cross-section.

An additional $12-\mathrm{ft}$ lane is added on the left side of the section to avoid serious erosion problems at the bottom of the slope. This will cover the whole length of the entering curve, and it is tapered at both ends (see Figure 30). Although it is intended primarily for erosion control,
it will be useful as a part of the operating surface, since drivers may straddle the intersection and gain some flexibility of neutral speeds.

The low and moderate speed lane on the right side has a relatively small slope,


Figure 31. North turnaround profile. and a $3-\mathrm{ft}$ wide beveled section is placed between these slopes to reduce the possibility of interference if the cars should pass from one lane to the other.

In the design of the entering spiral it was considered that the car would be operating at high speed approaching the turnaround, and a design speed at the beginning of the spiral was set arbitrarily at 100 mph with a value of $a=4$. It is assumed that a deceleration rate of $8 \mathrm{ft} / \mathrm{sec}^{2}$ can be used conservatively, and that the car would decelerate at this rate for a distance sufficient to bring it down to the design speed of 35 mph at the beginning of the curve to the right. These values have been used successfully in other recent designs. The relation of speed and distance at this rate of deceleration is shown on Figure 29.

These considerations led to a spiral with increasing curvature to the left 700 ft long followed by a 400 -ft spiral unwinding to straighten the car out. On these curves a value of $a=4$ at 100 mph was used, which means that most intense operation of the curve will occur at the beginning of the spiral, followed by decreasing intensity as speed is reduced. Superelevation slopes in the high-speed lane were computed at each station in accordance with fundamental equation and the operating speed developed in Figure 29.

We do not propose at this time to specify the speed range which will require use of the left lane; operation in this region will range from 20 to 100 mph , and what is "high speed" will depend on speed differences rather than on speed alone.

In Figure 27, the first spiral to the right leading to the short circular curve was designed on the basis of a uniform speed of 35 mph , and the rather considerable length is required primarily by the combination of vertical curve systems to reach the superelevation height without undue vertical reactions over the convex curve. It should be pointed out also that this is the only place where there is any safety margin for an operator who enters the system at too high a speed or experiences brake failure.

A section of circular arc approximately 240 ft long is used to turn the car far enough to enter a spiral out of the circular curve. The spiral out of the curve starts soon after the car starts to turn back toward the straightaway. Experience indicates that where drivers attempt to develop maximum speed after leaving a turnaround, they start to accelerate at about this position. Here we have designed the road to suit this practice. Failure to provide sufficiently for this practice is a deficiency in the design of other turnarounds in our road system.

This spiral is designed to permit full-throttle acceleration throughout its length of a current high-performing car, and it is estimated that a speed of 60 mph will be attained by the time the car reaches the end of the spiral.

Because the clearance with the west property line fence was limited and the edge of the curve system was forced across the line of the straightaway, the car is brought back to the straightaway line by means of a circular curve with a radius of $12,000 \mathrm{ft}$. The grade is brought down to the straightaway level at the beginning of this circular curve; with such a large radius it will function as a straight road.

Figure 30 shows the layout of the final design including the supplementary drainage fillets at the lower edge of the superelevated section and a stopping pad at the interior of the loop where repairs and adjustments will be made. The location of this pad was established at the point where even at high speed a car can stop safely and where the natural ground level is essentially at the grade of the inside edge of the paved surface.

Figure 31 shows the north turnaround profile.

## Application to South Turnaround

The design of the south turnaround is distinctive because advantage was made of terrain features. A complication arose from the fact that the west property line and the access to the straightaway slowly converge north to south so that there is a distance of


Figure 32. South turnaround.
only about 16 ft between the property line and the west edge of the new facility at the end of the former concrete pavement (Figure 32).

In order to take maximum advantage of the elevations just south of the former south turnaround, a study was made of the effect of elevation on acceleration and deceleration of cars. This relationship is shown in Figure 33. The mathematical relation is

$$
2 g h=\left(V_{1}{ }^{2}-V_{0}^{2}\right)
$$

where $h=$ change in elevation - feet
$\mathbf{V}_{\mathbf{1}}=$ terminal (or initial) velocity $-\mathrm{ft} / \mathrm{sec}$
$\mathbf{V}_{0}=$ initial (or terminal) velocity $-\mathrm{ft} / \mathrm{sec}$
$\mathrm{g}=$ acceleration due to gravity
It is obvious that elevations in the order of 60 or more feet would contribute materially to changes in vehicle speed, so that it should be possible to substitute change in elevation for length of structure and conserve the 3 -mile length. Figure 34 shows a layout used in design studies of the section including the turnaround loop in the old straightaway. The short turn at the right shows the old loop; the second one is a turnaround having minimum clearance with the then existing property. The chart in the lower part of this figure shows the relative elevations of the straightaway at point $A$, at point $B$ which was the south property line prior to 1955 , and the estimated elevation at the highest point on the private property between the proving Ground south property line and a public highway at point C. Note that on the Proving Ground property there is a maximum change of elevation of 60 ft and on the private property an estimated change of nearly 80 ft . It was apparent that procurement of this plece of property would permit taking advantage of the considerable increase in elevation as well as lengthening the com-


Figure 34. South turnaround.


Figure 33. Decrease in speed during hill climb.
plete structure more than 300 ft . Figure 33 suggests that the increase in elevation would give a large advantage at low speeds. This property was purchased. At the same time we decided to improve on nature and build a fill on top of the hill; design studies led to the decision that a fill of about 20 ft would be the best compromise. This put the outer edge of the structure at 100 ft above the straightaway grade, and it gives necessary clearance with the new property line and provides reasonable grades.

The next question to arise was the
choice of a design speed and radius of curvature. From Figure 33, showing the relation of height and velocity changes, it is evident that the effectiveness of any elevation is much greater at lower speeds and that the slope of the curves increases rapidly as the speed is reduced. It seemed evident that the $100-\mathrm{ft}$ elevation might be much more effective in contributing speed change at a very low starting speed at the top of the hill than at more conventional speeds. The layout of the vertical curve system would, it was felt, be much simpler at a low design speed.

A study was made of the speed-distance relationships at 0,25 , and 35 mph start-


Figure 35. South turnaround exit - speed attained from various starts. ing speeds, using the final design elevations (Figure 35); this slows clearly that the value of design speed selected would have little relation to the terminal speed at the foot of the hill. To provide some feeling of


Figure 36. South turnaround. progress, and to keep the vertical curve problems simpler, a design speed of 25 mph was selected arbitrarily.

The use of the $100-\mathrm{ft}$ elevation and the rather steep slopes leading up to it made it clear that we would have to provide a turnaround at a lower level for use in the wintertime and probably for use by large vehicles all year. The location and elevation selected made it possible to provide principal access to the new straightaway by means of an underpass (Figure 36). This location of the truck turnaround meant that the entering curve on the southbound lane would have to be started soon enough and thrown over rapidly enough to provide space for truck turning diameters. A second problem lay in the fact that the convergence of the straightaway and the property line made it necessary to throw the spiral leading from the turnaround arc back to the northbound lane over as rapidly as possible; the public highway along the west property line is being modernized and room is required to accommodate the difference in grade between the straightaway and the public highway. These requirements led to the consideration of compounding the horizontal spirals to increase the "throw" or deviation from the original tangent. Again it was considered that the design speed at the beginning of the first spiral would be 100 mph and moderate deceleration rates should be assumed.

If $a=4$ at 100 mph , any reduction in speed through the spiral will reduce the value of a rapidly and operation will become much more conservative. It seemed that we could make effective use of appropriate portions of successive spirals to


Figure 37. Comparison of simple and compounded spirals.


MAXIMUM $\frac{\mathrm{a}}{\mathrm{g}}$ ACCEPTABLE $=.3$
MAXIMUM $\frac{\mathrm{a}}{\mathrm{g}}$ FOR DESIGNING $=.2$
Figure 38. Maximum vertical reactions.
maintain a value of $a=4$ throughout the system and increase the curvature of the spiral much more rapidly. A technique was evolved to permit this; the results are demonstrated in Fugure 37.

Note that both spirals have a value of $a=4$ at $x=0$; the simple spiral, $K=$ 1,218, turns through an angle z of 38 deg - $40^{\prime}$ and reaches a radius of 611 ft , while in the same arc length the compounded spiral, where K varies in several steps from 1, 218 to 250, develops a rotation of the tangent to where $\mathrm{Z}=75 \mathrm{deg}$ $37^{\prime}$ and the radius decreases to 150 ft . With the reduction in speed, the value of a remains nearly 4 on the compounded spiral, and it falls rapidly on the simple spiral. The effectiveness of this system can be appreciated by the fact that the longer uncompounded spiral would have required a total rotation of about $21 / 2$ times to wind up at the terminal radius of the compounded spiral. The alternative to compounding would have been to start farther back and use up more straightaway.

The remainder of the horizontal design is analogous to that of the north turnaround, but modified in most details by speed differences caused by change in elevation and by property line clearance on the exit rather than the entering side (Figure 36). The deceleration to 25 mph can be accomplished easily by the help of the ascending 14 percent grade over the overpass.

The exit spiral is started as the car begins to turn toward the northbound lane, and the superelevation is based on full throttle acceleration of a current high-


Figure 39. South turnaround profile. performing car. A split-plane cross-section with an erosion control fillet at the bottom is used on both curves. As the design developed it became apparent that clearance distance from the west property line could be obtained by incorporating 600 ft of circular curve with a radius of $20,000 \mathrm{ft}$ between the end of the spiral and the straight section of the northbound lane. This will function


Figure 40. Effect of acceleration and drop on speed. as a straight section. Note that a speed of 85 mph is attained in 914 ft ; this contrasts with $1,827 \mathrm{ft}$ in the north turnaround.

The vertical curve system at the highest point of elevation was complicated because good design demanded that the accelerating exit spiral begin as the car begins to turn back toward the straightaway axis, and with the steep grade a short vertical curve was required to begin to make use of the drop at a low vehicle speed. This is the most critical point in design of the turnaround. It was necessary to design the convex vertical curve carefully to minimize the roller coaster effect, and no criteria had been established for guidance. A study was made of the proper values of vertical reactions by measuring tolerable and undesirable values on another part of the road system;
the consensus of opinion of the few observers was that a value of vertical acceleration of 0.3 g was about as high as would be acceptable. For design purposes, in this convex vertical curve system we used values of 0.2 g , recognizing that at times individual drivers might choose to exceed the design speed considerably. Cornu's spirals were used in this design. Figure 38 illustrates this concept.

Figure 39 shows the profile of the final design. It will be noted that a maximum grade of 14 percent is used on the entering side; this was required to provide sufficient clearance for the underpass access. A maximum grade of 12 percent is used on the downhill section. While the vertical scale is exaggerated, the vertical curve design problems are indicated clearly.

Figure 40 shows the expected distance-speed relationships of a current high-performing car. It shows the individual contributions to speed of the accelerating potential of the car and drops in elevation of 80 and 100 feet; after 600 ft a drop of 100 ft contributes more than the accelerating potential of a high-performing car starting at 35 mph .

As a summary, it may be said that automotive proving ground road system design uses the same principles as public highway design; the emphasis is different in that more severe operating conditions are both required and feasible - required because of area limitations and feasible because of the opportunity to tran drivers and supervise operations. This results in probably greater emphasis on detail design refinement to extend service life.

## REFERENCES

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2. R. L. McNeal, An Investigation of Some Problems in the Design of High Speed Tracks, GMPG No. 1. 1223 (Engineering Societies' Library).

## Appendix A

## Design of a Horizontal Transition Spiral

The problem was to develop a horizontal transition spiral to a circular arc having a radius of approximately 225 ft and a maximum slope in the superelevation cross-section of 30 percent; this would provide equilibrium of lateral forces parallel to the pavement at a speed of approximately 31.5 mph . The characteristics of the circular curve have already been determined by certain considerations which are not a part of this design problem.

The Cornu's Spiral formulas are:

$$
\begin{aligned}
& K=\text { constant }=\sqrt{\frac{\pi v^{3}}{a}} \text { (feet) } \\
& Q=\frac{v^{3}}{R L}=\frac{v^{2}}{R t} \\
& L=\text { Length of spiral from origin to any point }- \text { feet }
\end{aligned}
$$

$$
\begin{aligned}
& u=\frac{L}{K} \\
& Z=\frac{\pi u^{2}}{2} \quad \begin{array}{l}
\text { Angle between tangent line at origin and tangent line at any point on } \\
\text { spiral-radians }
\end{array}
\end{aligned}
$$

HORIZONTAL SPIRALS COMPUTED IN APPENDIX A


$$
\begin{aligned}
& R=\frac{L}{2 Z} \text { Radius at any point along spiral - feet } \\
& R=\frac{K}{\pi u} \\
& x=K \int_{0}^{u} \cos \left(\frac{\pi u^{2}}{2}\right) d u=K C(u) \\
& y=K \int_{0}^{u} \sin \left(\frac{\pi u^{2}}{2}\right) d u=K S(u) \\
& C(u)=u-.24674011 u^{5}+.02818550 u^{9}-.00160488 u^{13}+.00005407 u^{17}- \\
& S(u)=.52359878 u^{3}-.09228059 u^{7}+.00724478 u^{11}-.00031212 u^{15}+\ldots 1 \\
& u^{2}=\text { number of right angles }
\end{aligned}
$$

It was decided that a value of $\alpha=4$ was a proper criterion. Solution for $K$ gives:

$$
K=\sqrt{\frac{\pi V^{3}}{a}}=278.35
$$

From the formula

$$
R=\frac{K}{\pi u}, \text { for } u=.4, R=221.41^{\prime}
$$

which we accept as the terminal radius of the spiral and the radius of the circular arc.

$$
\begin{aligned}
& \text { Since } L=K u \\
& \qquad L=111.34, \text { th } \\
& \text { Then } C(u)=.39748 \\
& \qquad S(u)=.03336 \\
& x=K C(u)=110.64 \\
& y=K S(u)=9.29 \\
& \tan i=y / x=.08397 \\
& \qquad i=4^{\circ}-48^{\prime}
\end{aligned}
$$

$$
\mathrm{L}=111.34, \text { the total length of the spiral }
$$

where $i=$ deflection angle to end point on spiral, coordinates of which are (110.64, 9. 29)
Detailed computations including coordinates of each point at $10^{\prime}$ chords, deflection angles, radius, and required slope of the cross-section follows:
$R_{c}=221.41^{\prime} \quad u=\frac{L}{K}$
$L=111.34^{\prime}$
$K=278.35$
$V=31.5 \mathrm{mph}$
$Q=4.0 \mathrm{ft} / \mathrm{sec}^{3}$

| $\mathbf{L}$ | $\mathbf{u}$ | $\mathbf{u}^{2}$ | $\mathbf{u}^{3}$ | $\mathbf{u}^{5}$ | $\mathbf{u}^{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | .03593 | .0012907 | .0000464 | - | - |
| 20 | .07185 | .0051624 | .0003709 | .0000019 | - |
| 30 | .1078 | .0116165 | .0012520 | .000019 | .0000002 |
| 40 | .14370 | .0206497 | .0029674 | .0000613 | .0000013 |
| 50 | .17963 | .0322669 | .0057961 | .0001870 | .0000060 |
| 60 | .21556 | .0464661 | .0100162 | .0004654 | .0000216 |
| 70 | .25148 | .0632422 | .0159041 | .0010058 | .000636 |
| 80 | .28741 | .0826045 | .0237414 | .0019611 | .0001620 |
| 90 | .32333 | .1045423 | .0338017 | .0035337 | .0003694 |
| 100 | .35926 | .1290677 | .0463689 | .0059847 | .0007724 |
| 111.34 | .40000 | .16 | .064 | .0102400 | .0016384 |

$$
C(u)=u-.24674 u^{5}+\ldots \quad S(u)=.5236 u^{3}-.092281 u^{7}+\ldots
$$

| L | $\mathbf{u}$ | $.24674 u^{5}$ | C(u) | $x_{h}=K \mathbf{C}(u)$ | . $5236 u^{3}$ | . $092281 u^{7}$ | S(u) | $y_{h}=K S(u)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | . 03593 | - | . 03593 | 10.00 | . 00002 | - | . 00002 | . 01 |
| 20 | . 07185 | - | . 07185 | 20.00 | .00019 | - | . 00019 | . 05 |
| 30 | . 10778 | - | . 10778 | 30.00 | . 00066 | - | . 00066 | . 18 |
| 40 | . 14370 | . 00002 | . 14368 | 39.99 | .00155 | - | . 00155 | . 43 |
| 50 | . 17960 | . 00005 | . 17955 | 49.98 | . 00303 | - | . 00303 | . 84 |
| 60 | . 21556 | . 00011 | . 21545 | 59.97 | .00524 | - | . 00524 | 1.46 |
| 70 | . 25148 | . 00025 | . 25123 | 69.93 | . 00833 | . 00001 | . 00832 | 2.32 |
| 80 | . 28741 | . 00048 | . 28693 | 79.87 | . 01243 | . 00001 | . 01242 | 3.46 |
| 90 | . 32333 | . 00087 | . 32246 | 89.76 | . 01770 | . 00003 | . 01767 | 4.92 |
| 100 | . 35926 | . 00148 | . 35778 | 99.59 | . 02428 | . 00007 | . 02421 | 6.74 |
| 111.34 | . 4 | . 00253 | . 39747 | 110.64 | . 03351 | . 00015 | . 03336 | 9.29 |

$V=31.5 \mathrm{mph}=46.2 \mathrm{fps}$
$K=278.35$

$$
\begin{aligned}
& R=\frac{K}{\pi u}=\frac{88.601}{u} \\
& y^{\prime}=\frac{v^{2}}{g R}=\frac{66.37}{R}
\end{aligned}
$$

| L | $\mathbf{u}$ | ${ }^{\prime}$ | $y_{h}$ | $\tan i=y / x$ | 1 | $\mathbf{R}$ | $y^{\prime} \mathbf{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | . 03593 | 10.00 | . 01 | . 00100 | $0^{0}-031-26.9^{\prime \prime}$ | 2466.0 | . 0269 |
| 20 | . 07185 | 20.00 | . 05 | . 00250 | 0'-08'-35.2" | 1233.0 | . 0538 |
| 27.83 | . 1 | - | - | - | - - | 886.0 | . 0749 |
| 30 | . 10778 | 30.00 | . 18 | . 00600 | $0^{0}-20^{\prime}-37.2^{\prime \prime}$ | 822.1 | . 0807 |
| 40 | . 14370 | 39.99 | . 43 | . 01075 | $0^{0}-36^{\prime}-57.9^{\prime \prime}$ | 616.6 | . 1076 |
| 50 | . 17960 | 49.98 | . 84 | . 01681 | $0^{\circ}-57^{1-47.6 "}$ | 493.3 | . 1345 |
| 55.66 | . 2 | - | - | - | - | 443.0 | . 1498 |
| 60 | . 21556 | 59.97 | 1.46 | . 02435 | $1^{0}-23^{\prime}-41.4^{\prime \prime}$ | 411.0 | . 1615 |
| 70 | . 25148 | 69.93 | 2.32 | . 03318 | $1^{0}-54^{\prime}-02.1^{\prime \prime}$ | 352.3 | . 1884 |
| 80 | . 28741 | 79.87 | 3.46 | . 04332 | $2^{\circ}-28^{\prime}-49.7^{\prime \prime}$ | 308.3 | . 2153 |
| 83.49 | . 3 | - | - | - | - | 295.3 | . 2248 |
| 90 | . 32333 | 89.76 | 4.92 | . 05481 | $3^{0}-081-14.5^{\prime \prime}$ | 274.0 | . 2422 |
| 100 | . 35926 | 99.59 | 6.74 | . 06768 | $3^{0}-52{ }^{\prime}-18.61$ | 246.6 | . 2691 |
| 111.34 | . 4 | 110.64 | 9.29 | . 08397 | $4^{0}-48^{\prime}-00.0^{\prime \prime}$ | 221.5 | . 2996 |

## Chord Lengths of Horizontal Spiral

| L | $\Delta_{10} \mathrm{x}$ | $\Delta_{10^{\prime}}$ | $(\Delta x)^{2}$ | $(\Delta y)^{2}$ | $(\Delta x)^{2}+(\Delta y)^{2}$ | Chord $=\sqrt{(\Delta x)^{2}+(\Delta y)^{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10 | .01 | 100 | .0001 | 100.0001 |  |
| 20 | 10 | .04 | 100 | .0016 | 100.0016 | 10.00 |
| 30 | 10 | .13 | 100 | .0169 | 100.0169 | 10.00 |
| 40 | 9.99 | .25 | 99.80 | .0625 | 99.8625 | 9.99 |
| 50 | 9.99 | .41 | 99.80 | .17 | 99.97 | 10.00 |
| 60 | 9.99 | .62 | 99.80 | .38 | 100.18 | 10.01 |
| 70 | 9.96 | .86 | 99.20 | .74 | 99.94 | 10.00 |
| 80 | 9.94 | 1.14 | 98.80 | 1.30 | 100.10 | 10.00 |
| 90 | 9.89 | 1.46 | 97.81 | 2.13 | 99.94 | 10.00 |
| 100 | 9.83 | 1.82 | 96.63 | 3.31 | 99.94 | 10.00 |
| 111.34 | 11.05 | 2.55 | 122.10 | 6.50 | 128.60 | 11.34 |



$$
A=90^{\circ}-(Z-i)
$$

L $\quad u^{2} \quad Z=\frac{\pi}{-u^{2}}$

Z

$$
(Z-i)
$$

$$
A=90^{\circ}-(Z-1)
$$

| 10 | . 0012907 | . 002027 | $0^{\circ}-6^{\prime}-58.1{ }^{\prime \prime}$ | $0^{\circ}-031-26.9$ " | $0^{\circ}-03{ }^{\prime}-31.2{ }^{\prime \prime}$ | $89^{\circ}-56{ }^{\prime}-28.8{ }^{\prime \prime}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | . 0051624 | . 008109 | $0^{\circ}-27^{\prime-52.6 "}$ | 8'-35.2" | 19'-17.4" | $89^{\circ}-40^{\prime}-42.6{ }^{\prime \prime}$ |
| 30 | . 0116165 | . 018247 | $1_{0}^{0}-021-43.71$ | 20'-37.2" | 42'-06.5" | 89 ${ }^{\circ}-17{ }^{\prime}-53.5{ }^{\prime \prime}$ |
| 40 | . 0206497 | . 032437 | $1^{0}-51^{\prime}-30.51$ | 36'-57.9" | $1^{0}-14^{\prime}-32.61$ | 88 ${ }^{\circ}-45^{\prime}-27.41$ |
| 50 | . 0322669 | . 050685 | $2^{\circ}-54^{\prime}-14.5{ }^{\prime \prime}$ | 57'-47.6" | $1^{0}-56^{\prime}-26.9{ }^{\prime \prime}$ | $88^{\circ}-03{ }^{\prime}-33.11$ |
| 60 | . 0464661 | . 072989 | $4^{0}-10^{\prime}-54.9{ }^{\prime \prime}$ | $1^{0}-23$ '-41.4" | $2^{0}-47^{\prime}-13.5{ }^{\prime \prime}$ | $87^{\circ}-12{ }^{\prime}-46.5{ }^{\prime \prime}$ |
| 70 | . 0632422 | . 099341 | $5^{\circ}-41^{\prime}-30.5^{\prime \prime}$ | $1^{\circ}-54$ '-02.1" | $3^{\circ}-47^{1-28.4 "}$ | $86^{\circ}-12^{\prime}-31.6^{\prime \prime}$ |
| 80 | . 0826045 | . 129755 | $7^{\circ}-26^{\prime}-03.91$ | $2^{0}-28{ }^{\text {' }}$-49.7" | $4^{0}-57^{\prime}-14.2$ " | $85^{\circ}-02^{\prime}-45.8^{\prime \prime}$ |
| 90 | . 1045423 | . 164215 | $9^{\circ}-24^{\prime}-31.9$ " | $3^{\circ}-081-14.51$ | $6^{\circ}-16^{\prime}-17.4{ }^{\prime \prime}$ | $83^{\circ}-43^{\prime}-42.6{ }^{\prime \prime}$ |
| 100 | . 1290677 | . 202740 | $11^{0}-36^{\prime}-58.1{ }^{\prime \prime}$ | $3^{0}-52^{\prime}-18.6{ }^{\prime \prime}$ | $7^{\circ}-44^{\prime}-39.5{ }^{\prime \prime}$ | $82^{\circ}-15^{\prime}-20.5{ }^{\prime \prime}$ |
| 111.34 | 16 | . 251424 | $14^{\circ}-24$ | $4^{\circ}$ | $9^{\circ}-36{ }^{1-39.81}$ | $80^{\circ}-23^{\prime}-20.2$ " |

Figure A-1 shows spiral designs for various values of $K$ through a turn of 90 deg. A spiral can be calculated for any value of $K$ which depends on the velocity of car travel and the rate of increase of radial acceleration according to the following formula:

$$
\begin{aligned}
K=\sqrt{\frac{\pi V^{3}}{a}} \quad \text { where } V & =\mathrm{ft} / \mathrm{sec}^{3} \\
a & =\mathrm{ft} / \mathrm{sec}^{\mathrm{s}}
\end{aligned}
$$

The radius of curvature at any length along a spiral is shown in Figure A-2 for several different spiral designs denoted by K. The straight lines which radiate from the lower left corner and are denoted by u determine the angle, z , between the line tangent to the spiral at its origin and the line tangent to the spiral at any given point.


Figure A-1. Spiral designs to 90 deg.

For example, at $K-500$ and $R-400^{\prime}$, the length is $L-200^{\circ}$ and $u=0.4$.

$$
\begin{aligned}
\mathrm{Z} & =\frac{\pi u^{2}}{2} \\
& =1.5708(0.4)^{2} \\
& =.251328 \text { Radians }
\end{aligned}
$$

A spiral can be traveled over a wide range of speeds and Figure A-3 shows the effect of changes in speed on the rate of change in lateral acceleration for given spirals. From various studies, a value $a=4.0$ has been chosen as a maximum value desired at most speeds.

Figure A-4 is the specific case of spiral constants to be used at each speed to main$\operatorname{tain} a=4.0$, and Figure A-5 determines the length of spiral and terminal radius which may be used. The two straight-dashed lines across the family of curves show the points on each curve for cross-section slopes of 40 percent and 10 percent. Similar lines can be determined for any other slopes.


Figure A-2.


Figure A-3. At constant values of $K, V_{1 s}$ a function of $a$.

|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |

Figure A-4. Values of $K$ at various speeds for $a=4$.


Figure A-5. Length vs speed and radius at $\alpha=4$.

## Appendix $B$

## Design of Vertical Curves for Spiral Easement

The design problem was to develop the vertical curve detail for the horizontal spiral developed in Appendix A. Cornu's Spiral form was used to control the rate of change of vertical reactions and to provide a means of evaluating them conveniently.

The vertical system consisted of (1) a concave curve forming a transition between the horizontal tangent section and the ramp up to the top of the superelevated circular portion and (2) a convex curve between the ramp and the level at the top of the superelevated structure. Experience shows that it is convenient to divide each of these curves into two portions, and to develop Cornu's Spiral for the first portion of each curve; these spirals are then inverted to form an exit symmetrical with the entrance. In this way the radial forces on the car rise uniformly to a maximum of the midpoint of each curve and then decrease at the same uniform rate to the end, as illustrated in Figure 29 ; thus the vertical curves are designed in four phases, with the first and second and the third and fourth as counterparts. The first two may be identical in detail to the last two, except in direction of curvature, or they may differ; because of sensitivity of the car to decreased road reactions, where room is available, the convex curves should be longer than the concave. The convex curve may follow ammediately after the concave, or there may be a length of uniform gradient between, depending upon the length of the horizontal spiral and the height of the superelevation. The vertical transition system must coincide exactly with the horizontal transition.

Following are the detailed computations:
Construct vertical spiral along center of outside lane.

> | Vertical climb $=12 ' \times .05$ | $=.6^{\prime}$ |
| ---: | :--- |
| $6 ' \times .30$ | $=\underline{1.8^{\prime}}$ |
| $2.4^{\prime}$ |  |

$L=27.83=1 / 4$ length of horizontal spiral Let $\mathrm{y}=1.2^{\prime}$ for bottom two spirals
$\mathrm{y} \cong \mathrm{d} \cong \mathrm{I} \sin 2 \mathrm{Z}$
$1.2=27.83 \sin 2 Z$
$\sin 2 \mathrm{Z}=.04312$
$2 \mathrm{Z}=2^{\circ}-28^{\prime}-16.55^{\prime \prime}$
$Z=1^{\circ}-14^{\prime}-8.27^{\prime \prime}$
$R=\frac{L}{2 Z}=\frac{27.83}{.0431316}$
$R=645.23 \prime$
$a_{v}=\frac{v^{2}}{R}=\frac{2134.44}{645.23}=3.31 \mathrm{ft} / \mathrm{sec}^{2}$
$u^{2}=\frac{2 Z}{2}$

$u^{2}=\frac{.0431316}{3.1416}=.013729$$\quad$| $u^{3}=.001609$ |
| :--- |
| $u^{5}=.000022$ |
| $u^{2}=.0000003$ |

$$
\begin{aligned}
u & =.11717 \\
K & =\mathrm{L} / \mathrm{u}=27.83 / .11717=237.52
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{C}(u) & =u-.24674 u^{5}+\ldots \\
& =.11717-.000005+\ldots \\
& =.11717 \\
x & =\mathrm{K} C(u)=237.52(.11717) \\
& =27.83
\end{aligned}
$$

$$
\begin{aligned}
\sin 2 Z & =b / .20 \\
b & =.2(.04312) \\
& =.009 \\
\cos 2 Z & =c / 27.83 \\
c & =27.83(.99907) \\
& =27.81
\end{aligned}
$$

$$
\begin{aligned}
S(u) & =.5236 u^{3}-.092281 u^{7}+\ldots \\
& =.00084-\ldots \\
& =.00084 \\
y & =K S(u)=237.52(.00084) \\
& =.20
\end{aligned}
$$

$\sin 2 \mathrm{Z}=\mathrm{d} / 27.83$

$$
\mathrm{d}=27.83(.04312)
$$

$$
=1.20
$$

$\cos 2 Z=e / .2$

$$
\begin{aligned}
\mathrm{e} & =.2(.99907) \\
& =.2
\end{aligned}
$$

Total $\times$ for bottom spirals $=27.83+.01+27.81=55.65$
bottom and top spirals
111.30

Total y for bottom spirals $=.20+1.20-.20=1.20$
bottom and top spirals 2.40
$L=27.83^{\prime}$
$\mathbf{u}=\mathbf{I} / \mathbf{K}$
$K=237.52$
$\mathrm{V}=31.5 \mathrm{mph}$
Max. $\mathrm{a}=3.31 \mathrm{ft} / \mathrm{sec}^{2}$

$$
a=5.50 \mathrm{ft} / \mathrm{sec}^{3}
$$

| L | u | $u^{2}$ | $u^{3}$ | $u^{5}$ | $u^{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | . 02105 | . 0004431 | . 0000093 | - | - |
| 10 | . 04210 | . 0017724 | . 0000746 | - | - |
| 15 | . 06315 | . 0039879 | . 0002518 | . 0000010 | - |
| 20 | . 08420 | . 0070896 | . 0005969 | . 0000042 | - |
| 25 | . 10525 | . 0110776 | . 0011659 | . 0000129 | - |
| 27.83 | . 11717 | . 0137288 | . 0016086 | . 0000221 | - |

VERTICAL SPIRALS CALCULATED IN APPENDIX B


$$
C(u)=u-.24674 u^{5}+\ldots \quad S(u)=.5236 u^{3}-.092281 u^{7}+\ldots
$$

| L | $u$ | $.24674 u^{5}$ | $C(u)$ | $.5236 u^{3}=S(u)$ | $x_{v}=K C(u)$ | $y_{v}=K S(u)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | .02105 | - | .02105 | .00000 | 5.0 | .00 |
| 10 | .04210 | - | .04210 | .00004 | 10.0 | .010 |
| 15 | .06315 | - | .06315 | .00013 | 15.0 | .031 |
| 20 | .08420 | - | .08420 | .00031 | 20.0 | .074 |
| 25 | .10525 | - | .10525 | .00061 | 25.0 | .145 |
| 27.83 | .11717 | .00001 | .11716 | .00084 | 27.83 | .200 |

## Second of Four Phases

| $\mathrm{L}=27.83$ | $\mathrm{u}=\mathrm{L} / \mathrm{K}$ | $\sin 2 \mathrm{Z}=\sin 2^{0}-28^{\prime}-16.55^{\prime \prime}=.04312$ |
| :--- | :--- | :--- |
| $\mathrm{~K}=237.52$ | $\cos 2 \mathrm{Z}=\cos 2^{0}-28^{\prime}-16.55^{\prime \prime}=.99907$ |  |
| $\mathrm{~V}=31.5 \mathrm{mbh}$ |  |  |


| Horizontal Length | Length Along <br> Vertical Spiral$\quad u \quad u^{2}$ | $u^{3}$ | $u^{3}$ |
| :---: | :---: | :---: | :---: | :---: |


| $27.83+2.17=30$ | 25.66 | .10803 | .0116705 | .0012608 | .0000147 |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 20.66 | .08698 | .0075655 | .0006580 | .0000050 |  |
| 40 | 15.66 | .06593 | .0043468 | .0002866 | .0000012 |  |
| 45 | 10.66 | .04488 | .0020142 | .0000904 | .0000002 |  |
|  | 50 | 5.66 | .02383 | .0005679 | .0000135 | 0 |
|  | 55.65 | 0 | 0 | 0 | 0 | 0 |

$$
C(u)=u-.24674 u^{5}+\ldots \quad S(u)=.5236 u^{3}-.092281 u^{7}+\ldots
$$


$L \quad b=y \sin 2 Z \quad c=x \cos 2 Z \quad X=55.65-c-b d=x \sin 2 Z \quad e=y \cos 2 Z \quad Y=1.20-d+e$

| 30 | .007 | 25.64 | 30.00 | 1.106 | .157 | .251 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | .003 | 20.64 | 35.00 | .891 | .081 | .390 |
| 40 | .002 | 15.65 | 40.00 | .675 | .036 | .561 |
| 45 | .001 | 10.65 | 45.00 | .460 | .012 | .752 |
| 50 | .000 | 5.65 | 50.00 | .244 | .002 | .958 |
| 55.65 | .000 | 0 | 55.65 | 0 | 0 | 1.200 |

## Third of Four Phases


$L \quad b=y \sin 2 Z \quad c=x \cos 2 Z X=55.65+b+c d=x \sin 2 Z \quad e=y \cos 2 Z Y=1.20+d-e$

| 60 | .000 | 4.36 | 60.01 | .188 | 0 | 1.388 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 65 | .000 | 9.35 | 65.00 | .404 | .007 | 1.597 |
| 70 | .001 | 14.35 | 70.00 | .619 | .029 | 1.790 |
| 75 | .003 | 19.34 | 74.99 | .835 | .067 | 1.968 |
| 80 | .006 | 24.34 | 80.00 | 1.050 | .133 | 2.117 |
| 83.47 | .009 | 27.80 | 83.46 | 1.200 | .200 | 2.200 |

## Fourth of Four Phases

| $\begin{aligned} & \mathrm{L}=27.83 \\ & \mathrm{~K}=237.52 \\ & \mathrm{~V}=31.5 \mathrm{mph} \end{aligned}$ | $\mathbf{u}=\mathrm{T}_{1} / \mathbf{K}$ | $\begin{aligned} & C(u)=u-.24674 u^{5}+\ldots \\ & S(u)=.5236 u^{3}-.092281 u^{7}+\ldots \end{aligned}$ |  |  |  | $x=K C(u) \quad y=K S(u)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| Horizontal Length | Length Along Vertical Spiral | $u=C(u)$ | $u^{2}$ | $u^{3}$ | S(u) |  |  |
| $83.47+6.53=90$ | 21.30 | . 08968 | . 0080425 | . 0007213 | . 00038 | 21.30 | . 090 |
| 95 | 16.30 | . 06863 | . 0047101 | . 0003233 | . 00017 | 16.30 | . 040 |
| 100 | 11.30 | . 04757 | . 0022629 | . 0001076 | . 00006 | 11.30 | . 014 |
| 105 | 6.30 | . 02652 | . 0007033 | . 0000187 | . 00001 | 6.30 | . 002 |
| 111.30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |


| Horizontal Length | $\mathbf{X}=111.30-\mathbf{x}$ | $\mathbf{Y}=\mathbf{2 . 4 0}-\mathbf{y}$ |
| :---: | :---: | :---: |
|  |  |  |
| 90 | 90.00 | 2.310 |
| 95 | 95.00 | 2.360 |
| 100 | 100.00 | 2.386 |
| 105 | 105.00 | 2.398 |
| 111.30 | 111.30 | 2.40 |

## Appendix C

## Design of Superelevated Cross-Section in a Spiral Easement

The design problem was to develop the cross-section detail for a station in the apirail easement developed in Appendix A and B. For illustration the point at $L=70^{\prime}$ was selected; a cross-section of the form $y=f\left(x^{3}\right)$ was selected arbitrarily.

The conditions were that a cross-section was required having a slope in the middle of the outer traffic lane which would balance the component of centrifugal force in the

## VERTICAL CURVE FROM APPENDIX B



CROSS SECTION AT L $=70^{\prime}$, COMPUTED IN APPENDIX $C$

$18^{\prime}$

pavement plane as established in Appendix A and an elevation as established in Appendix B. The pavement width was selected arbitrarily as 24 ft ; this put the center of the outer lane at 18 ft from the inner edge of the pavement.

It was found that the origin of the cross-section equation meeting the conditions

$$
\begin{aligned}
@ x_{0}=18, \text { of: }: & y_{0} \\
y^{\prime} & =1.79 \mathrm{ft} \\
y^{\prime} & =.1884
\end{aligned}
$$

was 12.66 ft insıde the inner edge of the pavement; that is, the equation was:

$$
\begin{aligned}
y & =m\left(x_{0}+a\right)^{3}-m a^{3} \\
a & =12.66 \\
m & =.000066806
\end{aligned}
$$

In the design of cross-sections at successive stations along a spiral easement, the path providing equilibrium at design speed is always at a uniform distance from the pavement edge, but paths providing equilibrium at speeds below the design speed will not be at uniform distances from the edge; this occurs because section elevations are established by requirements of the vertical curve system. This means that a car following a traffic lane in the central or lower portion of the pavement will be subjected to a varying degree of lateral force unbalance; this condition is unavoidable, and it is unimportant at speeds below the design speed.

Computations of this design detail follow:

$$
\begin{aligned}
& y=m\left(x_{0}+a\right)^{3}-m a^{3} \\
& y^{\prime}=3 m\left(x_{0}+a\right)^{2}
\end{aligned}
$$

$$
\begin{array}{llll}
@ L=70 & y=1.79 & @ x_{0}=18 & \text { From Vertical Spiral } \\
& y^{\prime}=.1884 \quad @ x_{0}=18 & \text { From Horizontal Spiral } \\
& 1.79=m(18+a)^{3}-\mathrm{ma}^{3} \\
& .1884=3 \mathrm{~m}(18+\mathrm{a})^{2}
\end{array}
$$

$$
\begin{aligned}
& \frac{1.79}{.1884}=\frac{\mathrm{m}(18+a)^{3}-\mathrm{ma}^{3}}{3 \mathrm{~m}(18+\mathrm{a})^{2}} \\
& \dot{9.5011}=\frac{5832+972 a+54 a^{2}+a^{3}-a^{3}}{3(18+a)^{2}} \\
& 9.5011=\frac{18\left(324+54 a+3 a^{2}\right)}{3\left(324+36 a+a^{2}\right)} \\
& 9.5011=\frac{18\left(108+18 \mathrm{a}+\mathrm{a}^{2}\right)}{324+36 \mathrm{a}+\mathrm{a}^{2}} \\
& .5278\left(324+36 a+a^{2}\right)=108+18 a+a^{2} \\
& 171.0072+19.0008 a+.5278 a^{2}=108+18 a+a^{2} \\
& .4722 a^{2}-1.0008 a-63.0072=0 \\
& a=+1.0008 \pm \sqrt{1.00160064+119.00799936} \\
& \text {. } 9444 \\
& =+\frac{1.0008 \pm \sqrt{120.0096}}{.9444} \\
& =+\frac{1.0008 \pm 10.9549}{.9444} \\
& =\frac{11.9557}{.9444} \text { or } \frac{-9.9541}{.9444} \\
& =12.6596 \text { or } \mathbf{- 1 0 . 5 4 0 1}
\end{aligned}
$$

$\mathrm{a}=\mathbf{1 2 . 6 6}$

$$
\begin{aligned}
& .1884=3 \mathrm{~m}(18+12.66)^{2} \\
& \mathrm{~m}=.0628 \\
& (30.66)^{2} \\
& =.0628 \\
& 940.0356 \\
& =.000066806 \\
& y_{0}=.000066806\left(x_{0}+12.66\right)^{3}-.000066806(12.66)^{3} \\
& =.000066806\left(x_{0}+12.66\right)^{3}-.136 \\
& y^{\prime}=.000200418\left(\mathrm{x}_{\mathrm{o}}+12.66\right)^{2} \\
& \text { @ } x_{0}=18 \quad y_{0}=1.925-.136 \\
& y^{\prime}=.1884
\end{aligned}
$$

