Indexes of Motor Vehicle Accident Likelihood

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Motor vehicle accident rates with the unit of risk expressed as mileage are useful for general safety promotional and educational purposes. Their usefulness for specific engineering and enforcement purposes is quite limited, however, as a result of the gross methods whereby mileage data are collected. Their meaningfulness also is reduced as the distance over which the rate is computed becomes smaller. In the limit, at a single point, the mileage-based statistics are completely meaningless. Another criticism of these statistics stems from the fact that the unit of risk is seldom identified with the site of the risk.

The single instance where site of risk is identified with unit of risk is in evaluations of the hazard of a particular stretch of road. It is demonstrated that even in a case as specific as this, a mileage-based index reduces in fact to volume-based indexes divided by a distance constant. This constant contributes nothing to the statistical behavior of the random risk factor, and, in fact, may obscure the true hazard of the road.

From these arguments it is concluded that motor vehicle accident rates should be expressed simply as volume-based indexes. These measures would relate the number of accidents occurring at a given point during some period of time to the volume of vehicles passing that point during the period. The point could be any stretch of road over which the volume of traffic is essentially constant. To increase the usefulness of the results, the points should be defined in operationally meaningful lengths.

●THE basic reason for maintaining and analyzing motor vehicle accident records is to quantify the hazard of motor vehicle operation and thereby provide direction to engineering, enforcement, and other personnel seeking to reduce the hazard. This hazard quantification is usually expressed as a rate relating the occurrence of an accidental event to some measure of risk. In general use today as the risk measure is vehicle mileage from which are derived well known indexes, such as fatalities per 100, 000, 000 vehicle-miles and collisions per 1, 000, 000 vehicle-miles. The messages implicit in such indexes are easily understood by the lay public, hence they are highly useful for the all-important public education and other promotional activities for obtaining public support for safety programs and improved facilities. However, fundamental difficulties arise when such indexes are utilized in more technical problems in engineering and enforcement, and the source of these difficulties essentially is the non-rigorous mileage measurement.

In this paper, the authors seek to analyze some of the factors which limit or possibly completely preclude the use of mileage as the risk measure in accident rates which are to provide bases for technical decisions. The analysis demonstrates that mileage, in its best form of measurement today for describing risk, is in effect a measurement of traffic volume. From this it follows that traffic volume itself can well be treated as the risk measure without any loss in rigor, and with the possibility of providing a much clearer quantification hazard.

MILEAGE AS THE UNIT OF RISK

The use of the vehicle-mile as the unit of risk in motor vehicle accident rates seemingly stems from the highly plausible concept that accident likelihood is measured by the ratio of the number of accidents and the amount of driving during the period in which those accidents occurred. It is well to note here that mileage is not the only measure of the amount of driving, nor is there any evidence, to the best knowledge of the authors, that it necessarily is the most meaningful basis for accident occurrence. For example, hours of driving without regard to distance covered also measures the amount of driving and could well correlate more closely with accidents than would mileage, particularly on long, open road trips, or nighttime versus daytime. The use of mileage, it would appear, further stems from the relative ease with which it can be plausibly estimated.[1] But in this paper, there is not so much concern with the validity of the rationale for the use of mileage as the risk measure as there is with the methods of measuring the mileage.

Perhaps the most general criticism of the mileage figure is that rarely, if ever, is it directly measured, but instead is usually estimated by indirect methods involving judicious manipulation of such variables as total gasoline consumption, car registrations, and average mileage per gallon. A reasonable estimate of total mileage can be obtained with these indirect methods if the region in question is sufficiently large, and the time period is sufficiently long. However, as the size of the exposure interval (whether expressed in distance or time) is increased in order to increase the precision of the mileage estimate, the value of the index for engineering and enforcement application is lessened. For example, an index over the entire United States is of little value for adequately identifying local problems, or, an annual index is of limited value for analyzing a seasonal problem. It can be stated almost axiomatically that mileage-based statistics do not usually identify the unit of risk with the site of the risk, and hence are of limited use for engineering and enforcement purposes, most of which require at least some degree of specificity.

Quite apart from the difficulties in measuring and localizing mileage so that the statistics will be useful for engineering and enforcement problems, there are other serious difficulties which relate to the distance over which the mileage is measured. If the distance is sufficiently long, there is no problem; as the distance decreases, however, the mileage-based statistics become less meaningful until in the limit, at a single point on the road, they are completely meaningless. Norden, Orlansky and Jacobs (2) recognize this and point out that mileage-based statistics would not be useful for study of accidents at toll gates. Other examples of the profession's recognition of this can be readily obtained.

AN ARGUMENT FOR TREATING VOLUME AS THE UNIT OF RISK

The argument for treating volume as the unit of risk in motor vehicle accident rates is developed in the following manner. First, it is postulated that the only application where mileage-based statistics come close to being rigorous is in highlighting the fact that a given stretch of road is unusually "safe" or "dangerous." It is then demonstrated that even for this, a relatively straightforward problem of mileage measurement, operational practice in making the measurement is such that the computed index is in effect a volume-based index divided by a constant. The constant, which turns out to be the length of the road in question, contributes nothing to the analysis of statistical fluctuation of the rate under the most ideal conditions, and in many cases can becloud the analysis. From this it is concluded that volume-based rates should be used.

Assume a thoroughfare, either rural or urban, that runs through two points, A and B. Parameters dealing with point A will be identified by the letter A; those dealing with B will carry the letter B; those dealing with the distance A to B will be identified by the

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[1] Motor vehicle accident rates expressed on a mileage basis have been dropping consistently over the years (1). It is of interest to speculate on the behavior of the rate had it been expressed on the basis of hours of driving, assuming that this risk measure could have been estimated. If it is assumed that higher average speeds have been attained over the years, then the amount of driving time could not have increased as rapidly as the distance covered. The possibility therefore exists that the time-based rate might be increasing. If this should happen to be the case, it could be said that motorists have been driving increasingly greater accident-free distances but their accident-free driving time has not been increasing as rapidly, and possibly might even be decreasing. This certainly would not be the pleasant picture implicit in the continually decreasing mileage-based rate.

letters AB. Thus, if N represents the number of cars and t represents some time interval, N_{tA} represents the number of cars passing point A during interval t, and $N_{(t)\ (AB)}$ represents the number of cars that pass A and then pass B during period t.

Also adopted is the convention that vehicles entering thoroughfare section A-B at point A will carry the subscript A without a bar, whereas those leaving at A will carry a bar over the subscript (for example, $\bar{A}$). The distance between A and B in the direction of A to B is represented by D_{AB}, which equals D_{BA}, the distance from B to A. Point E, any place between A and B, is the only entrance or exit other than A and B for cars on A-B.

Finally, the number of accidents occurring during period of time t will be represented by X_{tA}, the number at B will be X_{tB}; the number that occur anywhere between A and B, including those that occur at A and B, will be $X_{t(AB)}$. The subscript t is dropped for the present purpose, as all discussions are based on a single period. The nomenclature is summarized in Figure 1.

To simplify the discussion, let it be assumed that all vehicles move from A to B; that is, the thoroughfare is one way. The total vehicle mileage, M_{AB}, driven over AB is given by

$$M_{AB} = (N_A - N_{\bar{E}})\, D_{AB} + N_{\bar{E}}\, D_{AE} + N_E\, D_{EB} \tag{1}$$

which may be simplified, by substituting $(D_{AE} + D_{EB})$ for D_{AB}, to

$$M_{AB} = (N_A - N_{\bar{E}})\ (D_{AE} + D_{EB}) + N_{\bar{E}}\, D_{AE} + N_E D_{EB}$$

$$= N_A D_{AE} + N_A D_{EB} - N_{\bar{E}} D_{EB} - N_{\bar{E}}\, D_{EB} + N_{\bar{E}} D_{AE} + N_E\, D_{EB} \tag{2}$$

But the total number of vehicles, N_B, leaving at B, is

$$N_{\bar{B}} = N_A - N_{\bar{E}} + N_E \tag{3}$$

Therefore, substituting Eq. 3 in Eq. 2 gives

$$M_{AB} = N_A D_{AE} + N_{\bar{B}} D_{EB} \tag{4}$$

Thus, the accident rate on AB with vehicle mileage as the unit of risk becomes

$$R_{AB} = \frac{X_{AB}}{M_{AB}} = \frac{X_{AB}}{N_A D_{AE} + N_{\bar{B}}\, D_{EB}} \tag{5}$$

Eq. 5 facilitates demonstration of several points regarding an accident rate based on a distance unit of risk. First, and most obvious, the rate goes to infinity as the distance AB goes to zero. Thus, hazard analyses using such an index are meaningless at a single point such as an intersection or toll gate. Next, the probability structure underlying the rate R_{AB} as defined by Eq. 5 reduces to a volume-based rate divided by a constant, the distance over which the rate is computed. This will be demonstrated for two cases: first, where there are no entrances or exits between A and B; and, second, where there are entrances and exits.

CASE 1: No Intervening Entrances or Exits on the Road Section

Under the condition of no entrances or exits between A and B, all of the cars entering at A must leave at B,

$$N_A \equiv N_{AB} \equiv N_{\bar{B}} \tag{6}$$

Substituting this identity in Eq. 5 gives

$$R_{AB} = \frac{X_{AB}}{(N_A)\ (D_{AB})} \tag{7}$$

Considering the expected value, ξ, of R_{AB} in Eq.7, and noting that D_{AB} is a constant,

$$\xi\left[R_{AB}\right] = \frac{1}{D_{AB}}\ \xi\left[\frac{X_{AB}}{N_A}\right] \tag{8}$$

But the ratio of X_{AB} to N_A is simply the probability that a car on the road has an accident. Thus, the rate R_{AB} estimates the ratio of the pure risk, which is a statistical quantity, to the distance in question, which is simply a constant, or

$$\xi\left[R_{AB}\right] = \frac{\Pr\left\{\text{Acc on AB}\right\}}{D_{AB}} \tag{9}$$

CASE 2: Intermediate Entrances and Exits on Road Section

Consider the road section AB to have one entrance and exit at E between A and B, as shown in Figure 1. Because there are no entrances or exits between A and E, the separate analyses of sections AE and EB are both of the same form as that of section AB in Case 1. Thus,

$$R_{AE} = \frac{X_{AE}}{N_A D_{AE}} \quad \text{accidents per vehicle-mile} \tag{10}$$

$$R_{EB} = \frac{X_{EB}}{N_B D_{EB}} \quad \text{accidents per vehicle-mile} \tag{11}$$

$$\xi\left[R_{AE}\right] = \frac{\Pr\left\{\text{Acc on AE}\right\}}{D_{AE}} \tag{12}$$

$$\xi\left[R_{EB}\right] = \frac{\Pr\left\{\text{Acc on EB}\right\}}{D_{EB}} \tag{13}$$

The problem is to deal with the hazard over the section AB and not the individual hazards over the two sections AE and EB, these hazards being measured by Eqs. 12 and 13, respectively. It is plausible to consider that the hazard of the over-all section is some function of the individual hazards of the two sub-sections that comprise the whole. Inasmuch as the hazards are developed with vehicle-miles as the unit of risk, the over-all hazard may be considered as the weighted sum of the individual hazards, the weighting being according to the respective vehicle-mile totals for the two subsections.

$$\xi\left[R_{AB}\right] = \frac{(N_{AE}D_{AE})\ \xi\left[R_{AE}\right] + (N_{EB}\,D_{EB})\ \xi\left[R_{EB}\right]}{N_{AE}D_{AE} + N_{EB}D_{EB}} \tag{14}$$

or

$$\xi\left[R_{AB}\right] = \frac{(N_{AE})\Pr\left\{\text{Acc on AE}\right\} + N_{EB}\Pr\left\{\text{Acc on EB}\right\}}{N_{AE}D_{AE} + N_{EB}D_{EB}} \tag{15}$$

It can readily be seen that Eq. 14 is of the same form as Eq. 4, because $X_{AE} + X_{EB} = X_{AB}$. Therefore,

$$\frac{N_{AE}D_{AE}\left(\dfrac{X_{AE}}{N_{AE}D_{AE}}\right) + N_{EB}D_{EB}\left(\dfrac{X_{EB}}{N_{EB}D_{EB}}\right)}{N_{AE}D_{AE} + N_{EB}D_{EB}} = \frac{X_{AB}}{N_{AE}D_{AE} + N_{EB}D_{EB}} \tag{16}$$

Now interject into the argument the operational practice of either assuming that the volume over AE is the same as over EB, or obtaining ADT measurements on each of the two sections, averaging them, and applying the average to each section. Under these conditions Eq. 15 reduces to

$$\xi\left[R_{AB}\right] = \frac{\Pr\{\text{Acc on AE}\} + \Pr\{\text{Acc on EB}\}}{D_{AB}} \qquad (17)$$

Thus the hazard for the thoroughfare with the one exit reduces to the form with no exits; that is, Eq. 17 and Eq. 9 are similar, as

$$D_{AE} + D_{EB} = D_{AB}\,.$$

The argument can now be generalized to the case where there are n entrances or exits between A and B, with traffic volume assumed to be constant throughout. Thus, the section of road in question may be considered as being comprised of n + 1 sub-sections and

$$D_{AB} = \sum_{i=1}^{n+1} D_i \qquad (18)$$

where D_i is the length of the ith sub-section.

Let the risk associated with the ith sub-section be represented by Pr $\{i\}$. Then

$$\Pr\{i\} = \frac{X_i}{N_i} \qquad (19)$$

and

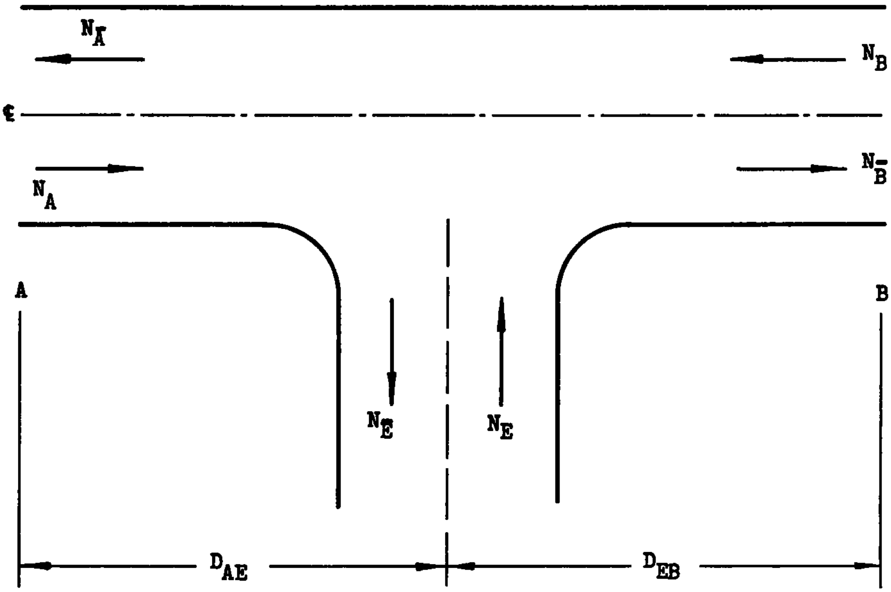
$$\xi\left[R_{AB}\right] = \frac{\sum_{i=1}^{n+1} \Pr\{i\}}{D_{AB}} \qquad (20)$$

Because distance does not enter into the evaluation of the risk of the ith sub-section, the distance D_i can be defined as a single point on the section AB, or any continuous length on AB. The sole requirement in this argument is that volume be constant over section i, and that all D_i add up to D_{AB}.

The discussion to this point has established that present methods of obtaining mileage-based accident rates are such that these rates identically are volume-based rates divided by a constant which is the distance over which the rate is computed. Furthermore, the volume-based rate (or pure risk factor) for the whole road is the sum of the individual risks of the separate sub-sections which together comprise the road. The next step is to examine what is lost by using a mileage-based index, bearing in mind that implicit in this usage are (a) an average risk over the entire road and (b) the length of the road.

It can readily be seen that change in the risk of section AB can come only from change in any of the risks of the sub-sections that comprise AB. If Pr $\{i\}$ is the same for all i over AB, or if it is uniformly changing over all i from one period to the next, there will not be any deleterious effects from the averaging process. If, on the other hand, the Pr $\{i\}$ is changing for some sub-sections and remaining constant for others, a situation that the engineering or enforcement agencies wish to be able to recognize, any arithmetic process of averaging must necessarily obscure the changing Pr $\{i\}$. This represents a loss of information, which in turn reduces the usefulness of the rate.

The length of the road, of course, contributes nothing to the analysis except where the risk of a road is to be compared with that of some other road of different length. (The case of comparisons between different roads of the same length is of no interest here.) This is the most widely practiced operational usage of mileage-based indexes and is basically valid. Difficulties arise, however, in the user's failure to differentiate between what may be called "unit risk" and "over-all risk." The expression "unit risk" pertains to the risk associated with one unit of length, whereas "over-all risk"



N_A: Number of cars passing point A and entering thoroughfare section AB.

$N_{\bar{A}}$: Number of cars passing point A and leaving thoroughfare section AB.

D_{AE}: Distance, point A to point E in direction A to E.

D_{EB}: Distance, point E to point B in direction E to B.

X_A: Number of accidents at point A.

Figure 1. Nomenclature, Schematic Diagram.

describes the risk of the total length. The mileage-based index is a statement of unit risk. Two roads having the same (mileage-based) accident rate, have the same unit risk. But if the roads are of unequal lengths, their over-all risks must be different. However, common usage of such rates rarely distinguishes between unit and over-all risk with careful qualifications such as ".... trip CD and AB have the same unit risk, but trip CD is more dangerous because it is longer....." It can accordingly be seen how usage of mileage-based statistics can be misleading even in this relatively simple application where distance is differing while the unit risk is unchanging. The confusion is greatly increased if both the distance and unit risk differ for the two roads being compared.

A somewhat more subtle aspect of this criticism of mileage-based rates is seen in a hypothetical example in which a tortuous road 10 miles long between two cities is replaced by a straight, improved road 5 miles long. Assume that the volume of traffic using the new facility is the same as it had been for the old road. Assume, further, that travellers between the two cities were involved in a total of 20 accidents per year, on the averag on the old road whereas this figure was 10 for the improvement. Clearly with the new road there was a reduction in the trip hazard by a factor of 50 percent. Expressing the hazard for the two roads as mileage-based rates, for the old road

$$\text{M.B.R.} = \frac{X}{(V)\ (L)} = \frac{20}{(V)\ (10)} = \frac{2}{V}$$ and for the new road

$$\text{M.B.R} = \frac{X}{(V)\ (L)} = \frac{10}{(V)\ \ (5)} = \frac{2}{V}$$

in which: M.B.R. = Mileage-based rate; X = Number of accidents; V = Volume of traffic; and L = Length of road. Thus, even with the obviously different over-all hazard, the mileage-based rates would be the same. The arithmetic of the computation is such that any decrease in trip hazard accompanying a reduction in trip distance must be obscured by the very reduction in distance causing the decrease in the hazard. Thus, it can be seen that comparing these rates has little meaning.

It is meaningful at this point to deviate from discussions of operational utility of mileage-based indexes in order to consider some of the long-range implications in their use for support purposes. The fact that an improvement results in a reduction in accident likelihood is certainly a strong argument for similar improvements. But it is reasonable to state that highway improvements, particularly in rural areas, will generally involve reductions in trip distance, as in the hypothetical example just given. By continuing to express the hazard as a mileage-based rate, engineers will soon find that it is increasingly difficult to show a reduction in the rate, because as trip distance is decreased while traffic volume remains constant, the mileage-based rate must go up, as is shown in the example, irrespective of whatever change in risk might have been achieved with the improvement. On the other hand, if trip distance is increasing, as it might be in a suburban development around large cities, the mileage-based rate would tend to present an unduly rosy picture of hazard.

RECOMMENDATION

It has been demonstrated that incorporating mileage into rate computations is not a particularly useful practice, either for immediate operational uses or for long-range support uses. Accordingly, it is recommended that the practice should be discontinued. When mileage is dropped from the analysis, what is left is simply the number of accidents divided by the volume of traffic generating the accident experience. This ratio is a pure risk measure, that is, the probability that a car using that section will be involved in an accident. It will have the binomial

$$\text{Risk on section AB} = \frac{\text{Number of Accidents on AB}}{\text{Volume of Traffic on AB}} \qquad (21)$$

distribution and, being small, can be treated as having the Poisson distribution. Extensive tables of both the binomial and Poisson distributions have been published. Also, control chart techniques such as those reported by Norden et al, (2), and Mathewson et al, (3) are directly applicable to the proposed risk measure (Eq. 21). Thus, the statistical analysis of this measure presents no major difficulty.

To use this risk measure in practice, the network of roads and streets would be divided into operational sections. The sole mathematical requirement of the section definition is that the traffic volume be essentially constant over the defined section. Thus, the measure could have been used in the hypothetical example and would have demonstrated the 50 percent hazard reduction. The important practical requirement is that those lengths of roads or road features be designated as sections-of-interest that in fact are of interest to operating personnel who seek to use the index.[2] For example, an intersection might be considered as a section-of-interest in a typical urban network; or, in a typical rural system, a stretch of road between two towns might be a section. Another practical requirement is that a section be so defined that reasonably accurate measurements of section traffic volume can be made. Still another practical criterion is that the definition assist accident investigators in identifying the accident with the section. This is exemplified by a system used in various parts of Europe wherein road markers are placed at fixed intervals along a road to assist travelers as well as to identify specific sections of the road.

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[2] The writers are indebted to Professor Harry Goode of the University of Michigan who first suggested to them that sections be so defined as to be of operational interest to persons who are to use the index in searching for and applying remedial measures to accident producing conditions on the streets and highways.

With the network somehow defined into operational sections, its risk pattern would simply be the frequency distribution of the risks of the defined sections. Then, by means of appropriate statistical tests on parameters of the distributions (such as the median, mean, interquartile range, etc.), it would be possible to compare the risk pattern of a network for one period with its pattern for another period. Similar comparisons could be made between patterns of different networks, provided the sections were defined the same way for the two networks.

One way of assuring the equivalence of sections is to define sections in standard lengths. An example of this system in practice is seen in California, where the Division of Highways considers its state-wide road system on the basis of coded quarter-mile sections. Another way of comparing two networks would be to compare operationally equivalent subsets of sections. Thus, the sub-set of sections comprised solely of intersections of one network might be compared with the sub-set of similar intersections from another network, or one sub-set of curves of a given superelevation could be compared with another set of similar curves. It is entirely possible that in time there would be developed risk standards of the form of Eq. 21 for all major operational or design features of a street and highway system. Risk analyses at specific locations would then only involve determining whether or not the deviation of its risk value from the standard was statistically and practically significant.

It should be noted that the recommended system is based solely on the two fundamental random variables, traffic volume and accident count, which are in fact the only data contributing anything to the statistical behavior of the mileage-based indexes. Thus the volume-based rate expresses all there is to be learned from the joint variation of these variables. Any attempt to reach additional inferences by interjecting normalizing constants or by performing arithmetic operations on these values can readily obscure whatever valid information might exist in the uncontaminated data.

SUMMARY

Motor vehicle accident rates with mileage as the risk measure are useful for general safety promotional and educational purposes. Their usefulness for specific engineering and enforcement purposes is quite limited, however, as a result of the gross methods whereby mileage data are collected. Their meaningfulness also is lessened as the distance over which the rate is computed becomes smaller, and in the limit (that is, at a single point) they are completely meaningless. Another weakness of these statistics stems from the fact that the unit of risk is seldom identified with the site of the risk.

The single instance where site of risk is identified with unit of risk is in evaluations of the hazard of a particular stretch of road. It is demonstrated that even in a case as specific as this, a mileage-based index reduces in fact to volume-based indexes divided by a distance constant. This constant contributes nothing to the statistical behavior of the random risk factor and, in fact, may obscure the true hazard of the road.

From these arguments, it is concluded that motor vehicle accident rates should be expressed simply as volume-based indexes that would relate the number of accidents occurring at a given "point" during some period of time to the volume of vehicles passing that "point" during the period. The "point" could be any stretch of road over which the volume of traffic is essentially constant. To increase the usefulness of the results, the "points" should be defined in operationally meaningful lengths.

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