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Report of Committee on Photogrammetry and Aerial Surveys

D. J. OLINGER, Chairman

●THE COMMITTEE on Photogrammetry and Aerial Surveys held its first meeting at Columbus, Ohio, in August 1956 and began its organization and planning for research activities under the chairmanship of Lowell E. Gregg.

In January 1957 the Committee successfully sponsored seven technical papers on the uses of aerial surveys and photogrammetry in highway location and design. These papers, together with numerous questions and answers developed in panel discussions, were published as a Symposium in Highway Research Board Bulletin 157.

Later in the spring of 1957 Mr. Gregg resigned chairmanship of the Committee and was replaced by the present chairman, D. J. Olinger, Chief Aerial Engineer, Wyoming State Highway Department.

The 37th Annual Meeting of the Board, in January 1958, was the occasion of another meeting of the Committee, at which time the Committee completed a formal statement of its purpose, aims and scope, as follows:

The purpose of this Committee is to encourage and aid highway departments and organizations in the adoption and use of photogrammetry and aerial surveys in each applicable phase of every stage of highway engineering.

In fulfillment of its purpose, the aims of the Committee are:

1. To perform and encourage research for development of new and improvement of old methods and procedures of utilizing photogrammetry and aerial surveys;
2. To point the way to adaptation of photogrammetric, aerial, and related methods and procedures from other fields of engineering to highway engineering;
3. To act as an evaluation and correlation center, and disseminate information and data obtained through research by the Committee and others;
4. To work for achievement of standardization in nomenclature and terminology; and
5. To encourage and participate in the preparation of reports and papers on utilization and accomplishments.

The interests and work of the Committee shall include local, regional, and national applications, developments, and accomplishments. In every stage of highway engineering, all phases of the utilization of, research in, and improvement of instruments, techniques, and procedures in photogrammetry and aerial surveys are included.

The Committee also decided on a further program of action, outlined as follows:

A subcommitte, comprised of C. L. Miller, Chairman, and L. L. Funk, R. D. Miles and M. D. Shelby, will study the problem of data presentation and requirements for map accuracy. They will function as a Committee on Requirements for Map Accuracy and Data Presentation.

A second subcommittee, on Utilization of Photogrammetry for Procurement of Rights-of-Way, comprised of H. G. Schlitt, M. D. Shelby, and L. L. Funk will report on this phase of photogrammetry as it is being applied within their own organizations at the next Annual Meeting.

All reports or papers prepared by Committee members (that is, (a) work reports, (b) directives issued within a committee member's organization for carrying out certain applications of photogrammetry and aerial surveys, (c) results of project work or research, and (d) any other subject within the purpose and scope of the Committee) are being sent all Committee members. Later it may be advisable to expand this service to include other organizations, or selected articles can be published and distributed by the Highway Research Board.

Adjustment of State Plane Coordinates

WILLIAM T. PRYOR, Chief of Aerial Surveys, Bureau of Public Roads

The system of plane coordinates established on sea level datum by the U.S. Coast and Geodetic Survey for each state is an excellent, unified method for coordinating and preserving all types of surveys, whether plotted to small or large scale. Most of the distances, however, from coordinates on maps controlled by and compiled on such a system, will not agree with distances on the ground unless scale corrections are applied. This effect becomes especially consequential wherever the maps are compiled at large scale.

To circumvent the need for correction of each map distance, as it is used for engineering, cadastral, or other purposes, it is easy to compute project coordinates by use of a combined scale and elevation adjustment factor applied to the system of state plane coordinates. When this is done before the mapping is accomplished, there is no need for scale correction because differences in coordinates and distances on the maps will agree with horizontal distances measured on the ground. Whenever state plane coordinates at sea level datum are required, they can be computed from the project coordinates by reverse use of the adjustment factor.

The principles of state plane coordinates and methods of their adjustment for large-scale engineering surveys are discussed.

●THE SURFACE of the earth is spheroidal in shape, which makes its representation in orthographic form on a plane extremely difficult. This is especially noticeable in large-scale mapping of extensive areas for engineering purposes. Although small segmental areas of the earth's surface do not curve perceptibly, the effects of curvature within them and their real or projected elevation above or below sea level must all be resolved to achieve a high degree of accuracy. The effects of curvature and elevation on the accuracy of projections of the earth's surface can be reconciled in several ways, or by some combination of ways.

In survey mapping at large scales for highways, the earth's surface is usually partitioned into small areas or short segmental strips whenever routes must be mapped in the preliminary survey stage for location and design purposes. Then certain types of adjustments are applied in making projections of the earth's surface onto a plane. The purpose of this paper is to discuss methods applicable to making such adjustments. Procedures are proposed for doing this in the beginning of surveying and mapping operations so that distances measured on the ground will agree, without adjustment, with distances determined from plane coordinates on the maps.

Within the national network of basic control surveys, it is unlikely that the precisely measured distance on the ground between any two stations would be exactly the same as the distance computed between the geodetic positions (latitude and longitude) or the plane coordinates (X and Y) of the stations. Such a difference in distance would not be an error. It is a difference which is exactly predictable because it is the result of elevation differences and distortions in map projections and the necessity for recording geodetic data of the control surveys on a common, national basis. Anyone who has been responsible for or involved in the planning or performance of surveys that are referenced to or are part of the national network is probably familiar with basic principles causing these differences, but a simplified presentation of them may refresh the memory.

For simplicity, large segments of the earth, as a country or a state, may be considered to have the surface shape of a sphere, although the earth is really a spheroid. The position of each survey station in the national network of contol surveys are geodetic coordinates on the sphere and the distances between the stations are distances along terrestrial arcs. For any given subtending angle, this distance would be proportional to the length of the radius of the arc. Thus, if a standard radius were not used, the distance between stations would be affected by their elevation. For example, the radius would be approximately one mile greater at Denver, Colorado, than at Miami, Florida. To consider the extreme in the United States, the radius would be almost three miles greater at Mt. Whitney than at Death Valley. Consequently, to provide a common standard, distances between stations in the national network must be recorded along an arc at some arbitrary but convenient elevation. The natural and most convenient elevation is the mean sea level spheroid, and the geodetic data of the national network of the control surveys are based on this surface which is one of the elements of the 1927 North American datum.

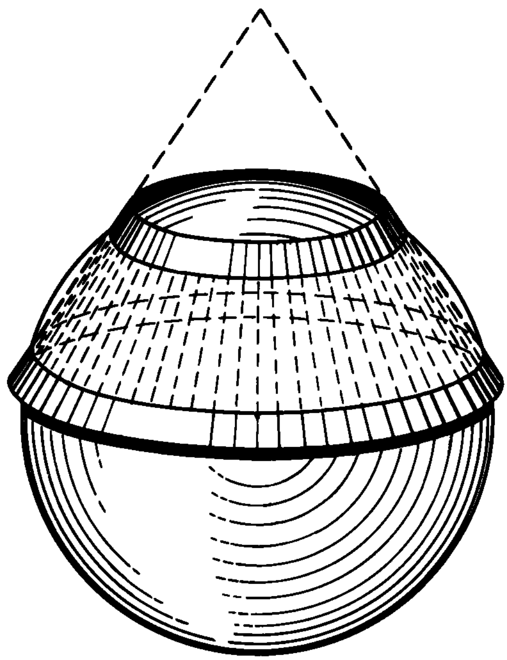


Figure 1. Lambert conformal projection (secant cone).

Geodetic surveying is a specialized field and to be of practical use in plane surveying for engineering purposes the geodetic position of stations on selected segments of the spheroid of the earth must be presented in a system of rectangular coordinates on a plane. Since any segment of the curved surface of the spheroid cannot be transformed to a plane without distortion some compromise had to be made. The solution accepted was to develop a system of rectangular coordinates on a plane related to a sphere by limiting the area of the curved surface to be represented on the plane so that the projectional distortions would be kept within acceptable limits. The designs made to accomplish this resulted in a system of state plane coordinates, by zones as necessary to limit distortions, on either a Lambert conformal (conic) or a transverse Mercator (cylindrical) map projection system, mathematically related to the curved surface of the earth (Figs. 1 and 2, respectively).

The first of the systems of state plane coordinates was established in 1933 by the Coast and Geodetic Survey in the United States Department of Commerce. This was done to fulfill the request made that year by a state highway engineer. Soon after, a system of plane coordinates was established for each state, positioned by the basic first- and second-order control survey network which extends from border to border and coast to coast of the United States. In these systems of coordinates, third-order control surveys have also been coordinated. In each of the systems of state plane coordinates, the Coast and Geodetic Survey has made available the X and Y coordinates of each control-survey monument and marker for which it has determined an adjusted geodetic position. By now, many of the states have adopted the applicable system of plane coordinates by legislative act.

Highway engineers will benefit in many ways by making fullest possible use of the system of state plane coordinates applicable where they must make surveys for highway location and design, for procurement of highway rights-of-way, and for highway

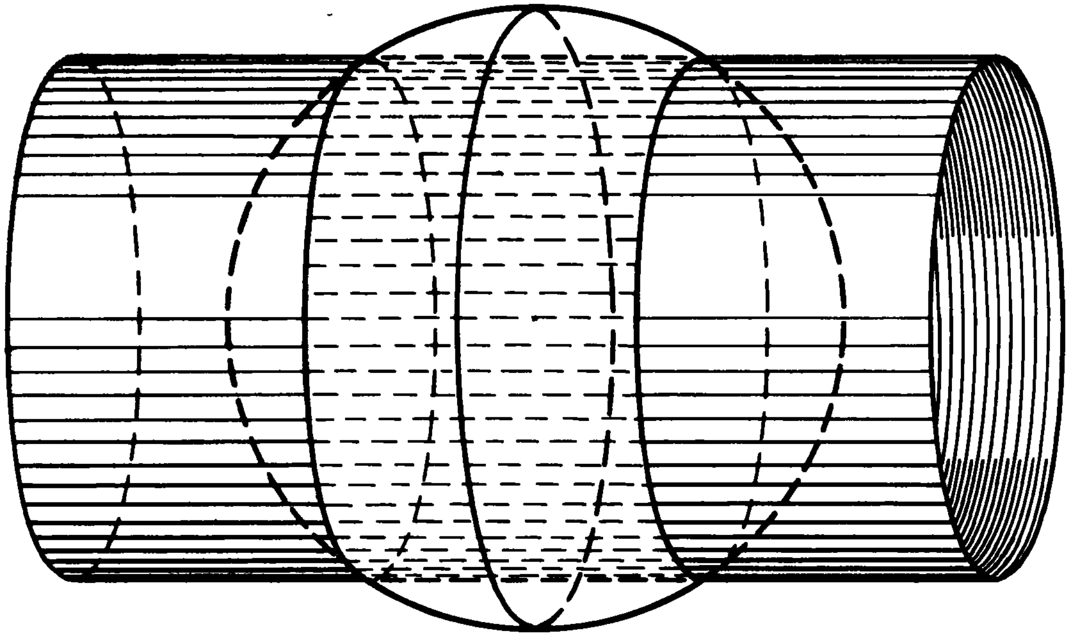


Figure 2. Transverse Mercator projection (secant cylinder).

construction. Furthermore, continuity in, and the preservation of all surveys made for highway engineering and cadastral purposes cannot be properly and fully attained until each survey is tied to and becomes an integral part of the state plane coordinate system established for the area of the state in which the survey is made. When this is done, highway surveys will attain continuity and uniformity, and the order of accuracy adequate for the detail and precision essential in both preliminary and location surveys for highways.

Since the plane coordinates of each state system were developed directly from the geodetic positions of station markers in the national network of horizontal control, the distance measured on the ground and the distance computed from plane coordinates of these stations will differ. The difference is caused by the combined effect of reduction to mean sea level of all stations physically above or below that datum and to the distortions occurring by projection of a curved surface onto a plane. These principles are portrayed in the accompanying illustrations for both the Lambert conformal and transverse Mercator projections. Actually, these projections (Figs. 1 and 2) are not perspective and cannot be displayed exactly graphically. Each projection is strictly a mathematical development, but the illustrations do indicate approximately what takes place.

In Figures 1 and 3 the sphere represents the sea level surface of the earth on which geodetic data are recorded and for the Lambert projection the cone represents the surface onto which points are projected from the curved surface. Whenever such a cone is cut along an element and a segment of the part intersecting the sphere is rolled out flat, it becomes the Lambert conformal projection. The cone cuts through the sphere on minor (small) circles of diameter T-U and V-W (Fig. 3) which is a cross-section (diagrammatically drawn, although not representative in scale) of the cone and sphere. Where the cone and sphere are coincident along T-U and V-W, two parallels of latitude are formed, known as standard parallels. These two parallels are the only lines along which geodetic and plane coordinate grid distances are equal. Thus, along these standard parallels, the scale factor between geodetic arc distances and distances on the Lambert conformal projection is one. Inside these parallels, the scale factor is less than one and grid distances on the plane of the projection are less than geodetic

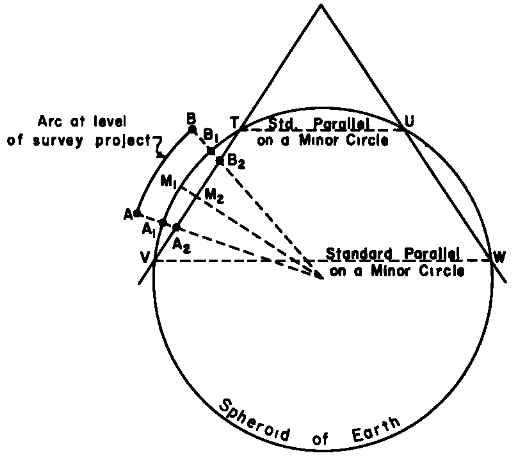


Figure 3. Lambert conformal projection; cross-section of intersection of cone and sphere.

distances on the arc; outside, the scale factor is greater than one and plane coordinate grid distances are greater than geodetic arc distances. A selected segment of the cone cut along its elements and rolled out flat to form the completed Lambert conformal projection, with a plane coordinate system superimposed thereon, would have the appearance illustrated in Figure 5. Scale factors of the plane coordinate grid on this projection are constant along each parallel of latitude, are variable along any line other than a parallel of latitude, and their greatest variation occurs along true North-South lines. At any one point, however, the scale is the same in all directions.

In Figure 3, the difference in distances is illustrated between points A and B, height A_1-A above the sphere, which are survey stations in the national network of basic ground control surveys. The distance that would actually be measured on the ground at the elevation of the survey project is arc A-B. The geodetic distance computed from the latitude and longitude of the stations is along the smaller arc A_1-B_1 at mean sea level. The grid distance computed from the state plane coordinates is the straight line distance A_2-B_2 on the projection surface.

The magnitude of the differences in distance, between A-B and A_1-B_1, between A_1-B_1 and A_2-B_2, and especially between A-B and A_2-B_2 may be so large as to seriously affect the apparent accuracy of a ground survey. For example, two U.S. Coast and Geodetic Survey triangulation stations in California, Sol and Eddy Gulch, may be substituted for A and B, and the appropriate values would be:

Plane coordinate grid distance	18,076.6 ft
Geodetic distance on arc of sphere	18,078.2 ft
Measured distance on ground at an elevation of 5,500 ft	18,082.8 ft

If the coordinate grid and measured distances were accepted at face value, a closure of 1:2,920 would be indicated. Such a closure is far below third order accuracy of 1:5,000 and does not greatly exceed fourth order accuracy of 1:2,500. Actually, however, when scale factors and elevation corrections are applied, it would be found that these distances are all correct at the datum and on the projection for which they were computed. Consequently a proper understanding and use of them is what is required.

The projection of points on the earth's spherical surface to a transverse Mercator projection is illustrated in Figure 4, in which there is shown diagrammatically the cross-section of an east-west cylinder intersecting the sphere on minor circles T-U and V-W. Points on the sphere representing the sea level surface of the earth, on which geodetic data are recorded, are projected to the surface of the cylinder. After a segment of the cylinder is cut along its elements and rolled out flat in a plane containing a line such as T-V (Fig. 4) it becomes the transverse Mercator (cylindrical) projection (Fig. 6) to which all distances were projected and on which a plane coordinate grid is superimposed. Scale factors of the plane coordinate grid on this projection are constant along the central meridian and all lines parallel to it, are variable along any other line, and their greatest variation occurs along lines perpendicular to the central meridian.

Only along the minor (small) circles T-U and V-W equidistant from the central meridian are the geodetic and grid distances equal and the scale factor is one. Between these two minor circles, grid distances on the projection plane are less than geodetic distances on the arc and the scale factor is less than one. Outside the two minor circles the grid distance is greater than the arc distance and the scale factor is greater than one. Again it should be noted that the scale is the same for all directions at a given point.

The difference in distances between points A and B at elevation of survey and points A_1 and B_1 on the sphere, and between points A and B and points A_2 and B_2 on the grid projection plane, are obvious in the illustration.

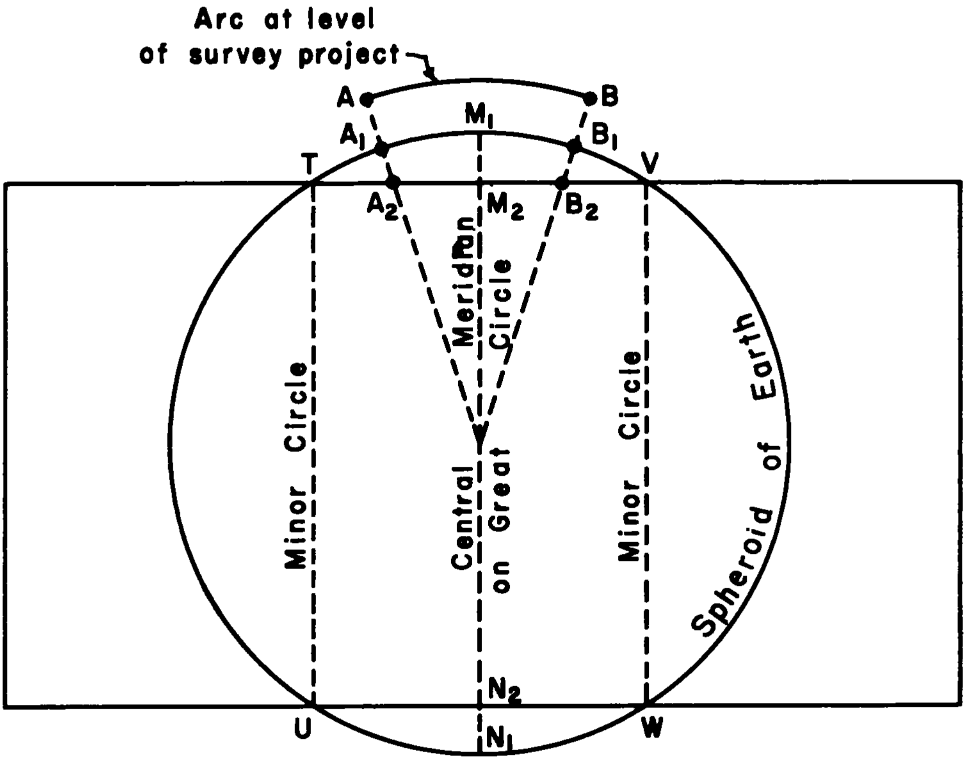


Figure 4. Transverse Mercator projection; cross-section of intersection of cylinder and sphere.

The Coast and Geodetic Survey has compiled and made available by segments scale factor tables for each of the established state plane coordinate systems. As previously demonstrated, these factors range from one, where the cone is coincident with parallels of latitude on the Lambert conformal projection and where the cylinder is coincident with parallel lines equidistant from the central meridian on the transverse Mercator projection, to less than one between such lines, and to greater than one outside of them. Moreover, these variations are caused by the fact that the rolled-out cone or cylinder forming the plane on which each plane coordinate system is projected coincides with the spheroid of the earth at only two parallels of latitude for the Lambert system and at only two north-south projection lines parallel to and equidistant from the central meridian for the transverse Mercator system. Other portions of the plane are either above or below the spheroidal surface. For the portions below the spheroid the distances measured horizontally on the ground are longer than distances projected onto the plane by either method. For those above the spheroid the effect is the opposite, and ground-measured distances are shorter than distances projected onto the plane.

TABLE 1

Distance the plane of projection is above or below the earth's spheroid (ft)	Number of times horizontal distance on the plane is larger or smaller than distance measured on spheroid. (These values are scale-correction factors expressed as a ratio of distance on ground)	Difference in distance on plane and spheroid expressed as an approximate fraction of any total distance
+2,400	1.000115	1:8,700
+2,200	1.000105	1:9,500
+2,000	1.000096	1:10,400
+1,800	1.000086	1:11,600
+1,600	1.000077	1:13,000
+1,400	1.000067	1:14,900
+1,200	1.000057	1:17,500
+1,000	1.000048	1:20,800
+ 800	1.000038	1:26,300
+ 600	1.000029	1:34,500
+ 400	1.000019	1:52,600
+ 200	1.000010	1:100,000
0	1.000000	--
- 200	0.999990	1:100,000
- 400	0.999981	1:52,600
- 600	0.999971	1:34,500
- 800	0.999962	1:26,300
-1,000	0.999952	1:20,800
-1,200	0.999943	1:17,500
-1,400	0.999933	1:14,900
-1,600	0.999923	1:13,000
-1,800	0.999914	1:11,600
-2,000	0.999904	1:10,400
-2,200	0.999895	1:9,500
-2,400	0.999885	1:8,700

Note: In practice these scale factors are taken from the state plane coordinate tables. They are listed for every minute of latitude in the Lambert projection, and for every 5,000 ft of x-distance from the central meridian in the transverse Mercator projection.

Table 1 contains in numerical form the effects, at 200-ft increments, of the plane of projection of state plane coordinate systems being above or below the earth's spheroid. A careful study of Table 1 will reveal why the various systems of state plane coordinates were established by zones so as to prevent differences between distances measured on the spheroid and distances determined from coordinates of the maps compiled at datum of the plane coordinate system from producing errors greater than approximately one part in 10,000. To achieve this, the zones of each state plane coordinate system were designed so that the height of the central parallel of latitude of each Lambert conformal system, and the central north-south meridian of each transverse Mercator system does not greatly exceed 2,000 ft above the plane on which the coordinate system is projected. Likewise, extensions of the plane beyond its intersection with the spheroid is limited to an altitude of about 2,000 ft above sea level. Actually, however, the scale factors were developed mathematically, not from consideration of the elevation of the plane above or below the spheroid.

From the foregoing, it is evident there are two causes of differences between distances measured horizontally on the ground and distances determined from coordinates on maps compiled on a state plane coordinate system. These are the variable distance that the plane of representation is below or above the spheroid of the earth, and the elevation that the survey project is above it or below it for the few places where the ground is lower than mean sea level. To cope with these conditions, the usual method of handling the associated problems is to reduce basic ground control survey data from values at elevation of survey to obtain the applicable coordinate data, geodetic or plane, as desired. Thus, arc distances measured horizontally by increments on the ground are combined with an elevation correction factor to determine an arc for this distance on the sea level spheroid; then a grid scale correction factor is applied to the spheroidal arc distance to determine the plane coordinate grid distance.

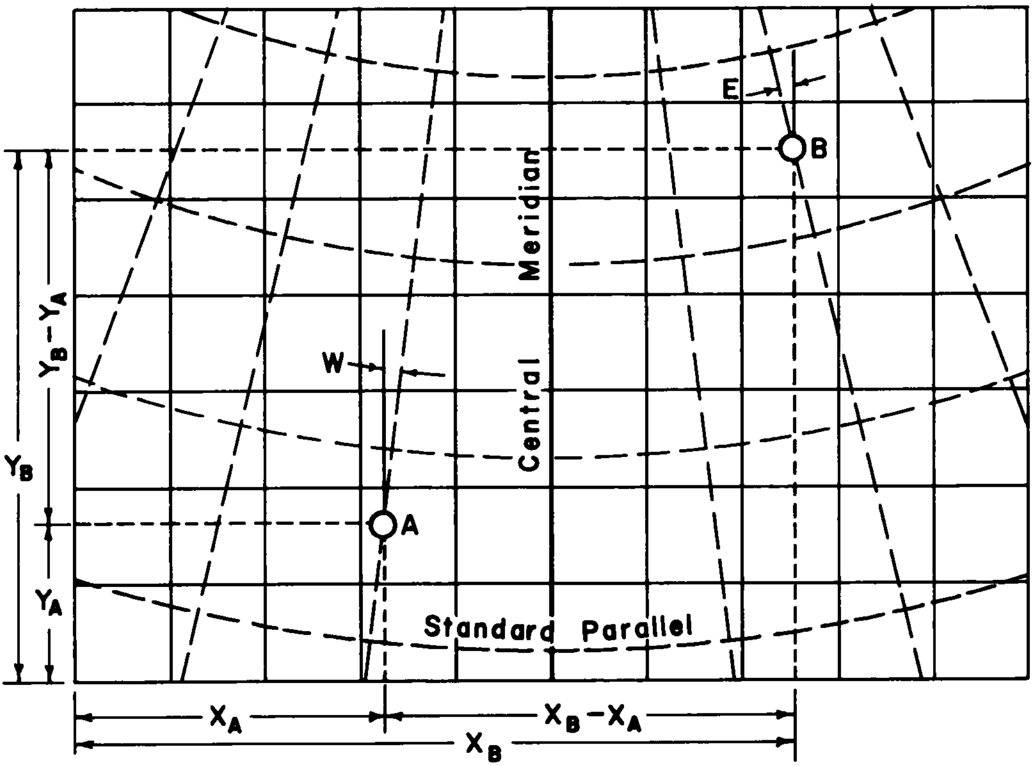


Figure 5. Plane coordinate grid on Lambert conformal projection (W and E are declinations of grid from true north at points A and B).

When the scale of a map is based upon plane coordinate grid control determined in such a manner, the ground and features on it are not delineated at constant scale, but at the variable scale occurring when projected onto a plane below or above the sea level spheroid illustrated. This does not create serious problems for users of small-scale maps, as 2,000 ft to one inch or smaller, because accuracy to the nearest foot or fraction of a foot is usually of no consequence in the reconnaissance surveys for which such maps are used. In compiling and using large-scale maps at a scale of 200 ft to one inch or larger for engineering design and cadastral purposes, however, the difference between map distances and ground distances caused by such variations in scale must be considered and appropriately resolved.

Practice has been to reduce all horizontally measured ground survey distances to the datum of the state plane coordinate system. Thus, where there is much elevation, distances measured on the ground must be adjustment-corrected in going from ground to map or from map to the ground. For example, if the alignment of a highway location at an elevation of several thousand feet is designed on maps compiled on datum of a state plane coordinate system, the designed curves, their radius and degree, tangent distances, distances along property and right-of-way lines—in fact every distance computed from the map for engineering or cadastral purposes—would have to be adjusted before the highway is staked on the ground in order to attain ground survey closures and assure proper positioning as designed.

Procedures are proposed for applying an essential adjustment to state plane coordinates so that map compilation datum is established at the average elevation of the survey rather than at the datum of the state system. This should be done in the beginning. Then distances measured on the ground will agree without need for correction-adjustment with distances determined from coordinates on the map. In this way, both convenience and savings in work are achieved by eliminating the need for adjustment of

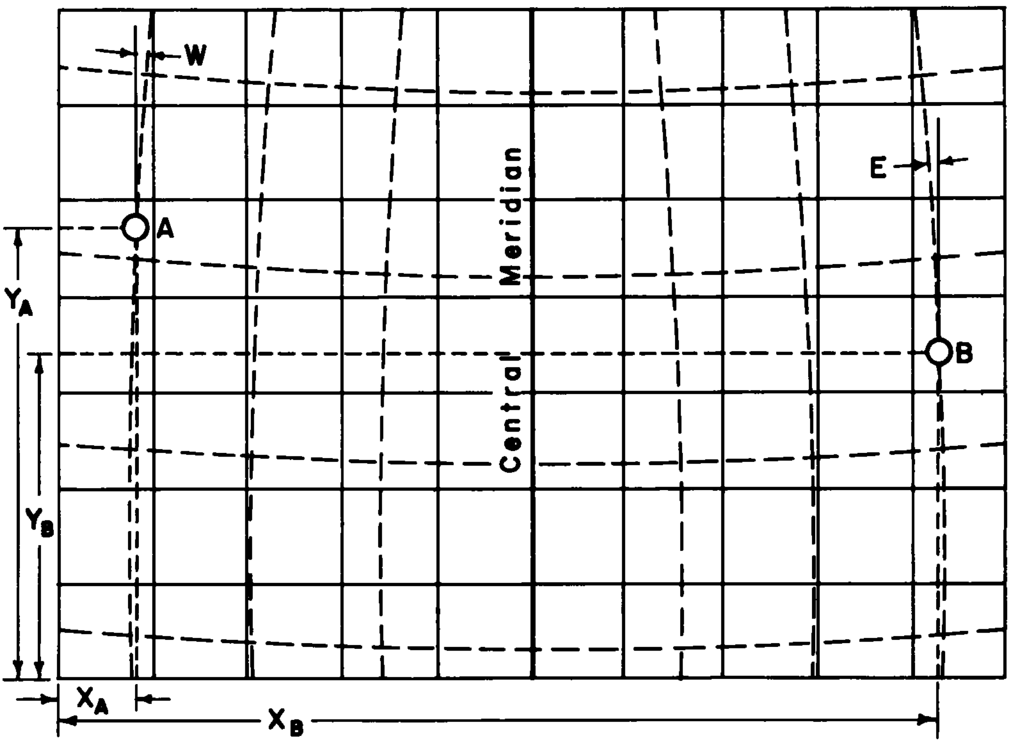


Figure 6. Plane coordinate grid on transverse Mercator projection.

each map distance to agree with its ground distance— the distance that was or will be measured depending upon its use sequence in highway engineering.

This method, contrary to some concepts, does not discard the state plane coordinate system. State plane coordinates of any point or feature on a map compiled in this survey system is readily determinable merely by the simple process of dividing coordinates of the point or feature in the system by the adjustment factor recorded on each map sheet.

Before considering procedures for doing this, analyze the numerical effects of distances measured in surveys made at an elevation above the sea level spheroid of the earth being reduced to the datum of a plane coordinate system. As illustrated in Figures 3 and 4, distance A-B of the survey is longer than its projection on the sphere. In Table 2, at 1,000-ft increments of elevation, the effect of elevation on horizontal distances, as compared to the unit distance 1.000000 along an arc of the earth at sea level datum is given.

Such differences cannot be ignored without serious consequences, especially when

TABLE 2

Elevation of survey above mean sea level (ft)	Number of times horizontal distance on ground at elevation listed is larger than horizontal distance at sea level datum (a multiplication factor)	Difference in distance expressed as an approximate fraction of any measured total distance.
0	1.000000	--
1,000	1.000048	1:20,800
2,000	1.000096	1:10,400
3,000	1.000143	1:7,000
4,000	1.000191	1:5,200
5,000	1.000239	1:4,200
6,000	1.000287	1:3,500
7,000	1.000335	1:3,000
8,000	1.000383	1:2,600
9,000	1.000430	1:2,300
10,000	1.000478	1:2,100
11,000	1.000526	1:1,900
12,000	1.000574	1:1,700
13,000	1.000622	1:1,600
14,000	1.000670	1:1,500
15,000	1.000717	1:1,400

their effects are additive to the effects of projecting map details onto any plane coordinate system established by either the Lambert or the transverse Mercator method, as given in Table 1. With this fact in mind, the next task is to devise a method which eliminates the need for any adjustment of distances other than the X and Y plane coordinates of the basic ground control survey stations. This must, of course, be done so that distances measured or to be measured on the ground have, without adjustment, the desired proportion to distances determined from the plane coordinates of planimetric and topographic features on the maps, and to the dimensions of designed highway alignment, structures, rights-of-way and so forth. There are three steps to achieving this, after the average elevation of the project survey has been ascertained.

First, select from Table 2 the multiplication factor applicable at the average elevation of survey project. This elevation should be a median not exceeding approximately 1,000 ft above the lowest and 1,000 ft below the highest point within the engineering survey area where large scale mapping (200 ft to one inch or larger) is required. Should much larger differences in elevation exist between the extreme high and low, consideration should be given to dividing the survey project into segments so as to prevent large discrepancies from occurring.

Second, select the appropriate scale-correction factor from the applicable state plane coordinate projection tables. In the tables prepared and made available for each state by the Coast and Geodetic Survey, this factor is expressed as a ratio of distance.

The scale-correction factor to use depends upon the latitude of the point or points to be adjusted in any Lambert designed system of plane coordinates, or upon the distance east or west of the central meridian of the point or points in any transverse Mercator designed system of plane coordinates. Actually, all scale-correction factors are similar in character to the numerical values in column two of Table 1, but which one to use for a particular survey point or geodetic station marker in the national network of control surveys must be obtained from the appropriate plane coordinate projection tables. The scale-correction factor which is median for surveys of small areas or short routes, or of selected segments of large areas or long routes extending across a major part or all of a state plane coordinate zone, will usually suffice.

Third, the combined-adjustment factor, which corrects for both elevation and scale variation, is computed by dividing the multiplication factor applicable from Table 2 by the scale-correction factor.

Once an applicable combined adjustment factor has been so determined, then the X and Y plane coordinates are computed, which would be applicable at the elevation of the survey project for each geodetic station marker to be used as basic control for the project. This is done by multiplying the state plane coordinates of each marker by the combined adjustment factor which is applicable.

Two geodetic station markers in Zone I of the Lambert conformal plane coordinate system in northern California may be considered, namely Sawtooth and Thompson Peak. Their geodetic coordinates are N 40° 58' 20.94" and W 123° 00' 04.22" and N 40° 56' 36.62" and W 122° 52' 18.17", respectively. Using their geodetic coordinates, other data pertinent to the markers were computed, such as their state plane coordinates in Zone I, the distance of 37,280.7 ft between them on the state plane coordinate grid, the geodetic distance of 37,284.5 ft on a great circle arc at sea level, and the arc distance of 37,293.4 ft on the ground at the survey project elevation of 5,000 ft.

The X and Y plane coordinates of Sawtooth and Thompson Peak station markers are 1,723,554.9 and 598,703.6 and 1,759,194.9 and 587,765.4, respectively. These station markers are about midway between the standard parallels of Zone I.

Although Sawtooth is actually at an elevation of 8915 and Thompson Peak at an elevation of 8383 ft, a combined scale- and elevation-adjustment factor must be ascertained which will apply at the average 5,000-ft elevation of the area of survey. This is because the survey area is in the canyon between the mountain tops on which the station markers are situated. For their average latitude of 40° 57.5', the scale factor in projection table of Zone I is 0.999897. Multiplication of distances on the spheroid by this factor gives distances on the state plane coordinate system, which is about 2150 ft below sea level in this area. Another way to visualize the significance of this correction is that each 100 ft on the spheroid is represented by 99.990 ft on maps compiled on state plane coordinate datum in this area. From Table 2 the multiplication factor applicable at the average elevation of 5,000 ft is 1.000239. Likewise, each 100 ft on the spheroid becomes 100.024 ft at the 5,000-ft elevation of the survey. The combined adjustment factor is 1.000342, which is 1.000239 divided by 0.999897. Thus, each 100 ft at datum of the state plane coordinate projection would measure 100.034 ft on the ground. In highway surveying and all construction stakeout work, such a difference would result in discrepancies of closure and in errors of positioning. Unfortunately these discrepancies and errors may cause engineers who are not familiar with the fact that the differences causing them are mathematically computable and adjustable to feel that the initial survey on state plane coordinate datum is full of errors.

The way in which to avoid the occurrence of such differences and to make distances determined from plane coordinates on the large-scale maps agree with distances measured on the ground at the level of the survey without affecting azimuth is to adjust the state plane coordinates of the geodetic station markers. Then use them in their adjusted position to control all surveying and mapping. In this example the new X and Y coordinates for Sawtooth are 1,724,144.4 and 598,908.3, and for Thompson Peak 1,759,796.5 and 587,966.4, which are merely the initial state plane coordinates multiplied by the combined adjustment factor of 1.000342. When desired in the future, X and Y coordinates of any points (the initial control, highway alignment points, property corners and right-of-way markers, and so forth) could be reduced for use at datum of the state plane coordinate system, as desired, by merely dividing their survey X and Y coordinates, respectively, by the same combined adjustment factor.

Plane coordinates for all surveys and mapping for highway engineering purposes can be easily utilized in the same manner. Thus, the systems of state plane coordinates are retained and used advantageously. In so doing, the need for resolving differences in distances is eliminated. Each designed alignment with its circular curves, transition spirals, and joining tangents can be staked on the ground without the nuisances of "apparent" errors in position, lengths, degrees of curvature, and the like.

In conclusion, it is urged that each and every highway engineer, organization, and department adopt and fully use these suggested methods. Then, and only then, will it be possible easily to attain accuracy, continuity, and permanency in surveys, and through these highly desirable benefits accrue savings in both time and money.

Discussion

L. G. SIMMONS, Coast and Geodetic Survey — It is evident from the paper on the Adjustment of State Plane Coordinates that the author has a good working knowledge of the nature of the projections forming the bases of the State Plane Coordinate Systems.

These systems were devised as a result of a request from a highway engineer about 25 years ago in order to take a practical advantage of the geodetic network throughout his state. Surveyors and engineers not engaged in geodetic work are unfamiliar with the type of computation required. The conversion of latitudes and longitudes of the triangulation stations to x and y rectangular coordinates puts the control network in a much more usable form.

Objections have frequently been raised by engineers throughout the country in regard to these state systems because of the fact that the actual ground lengths differ in some instances quite materially from the grid lengths, as determined by plane coordinates. There are two possible approaches in answer to this objection. One is to reduce each measured length for the scale of the grid and for its elevation above sea level and compute the coordinates which will be referred directly to the grid. Then, if any particular ground distance is needed to a high degree of accuracy for some special purpose, this can be determined by a reverse application of the sea level and grid scale factors. The other method proposed by Mr. Pryor is merely to change all the grid coordinates in a relatively small area by applying a combined elevation and scale factor and then proceed with the survey employing actual ground lengths in the computation.

For highway work, the second method appears to be quite practical. It minimizes the amount of computing necessary and results in a set of coordinates from which actual ground lengths can be determined immediately. Moreover, should it be desirable to obtain true grid coordinates after the detailed highway programs have been performed, these may be computed merely by applying the combined sea level and scale factors in reverse.

A note of caution should be injected here to avoid confusion in determining whether a set of coordinates applies strictly to the state grid or to a particular area within the grid at a certain elevation. Any list of coordinates, therefore, should carry a definite statement which will leave no doubt in the user's mind as to their nature.

WILLIAM T. PRYOR, Closure — Mr. Simmons discusses two vitally important points. The first is the fact that the distance between any two points, as determined from state plane coordinates, may not agree with the distance measured on the ground between the points with sufficient accuracy to satisfy engineering requirements, unless an adjustment is made. The purpose of the paper was to propose a method of making this adjustment in the state plane coordinates before mapping is undertaken. In this way the need for adjusting each measured distance on the ground, or each distance determined from coordinates on each map is precluded. Distances determined from maps compiled on an adjusted system of plane coordinates will agree within practicable limits of accuracy, without need for adjustment, with distances measured on the ground. Thus, a major obstacle to the adoption by highway engineers of the state plane coordinate systems is eliminated. Each engineer who makes use of the suggested method of adjusting state plane coordinates is not discarding the state plane coordinate system. He is actually using it in the most practicable way.

The second point, a note of caution by Mr. Simmons, is a good one. In order to avoid any confusion, each map sheet should contain a statement of the fact that the mapping coordinates were obtained by adjustment of the state plane coordinate system. In this way, the map user will be made immediately aware that the plane coordinates used apply only to the particular mapping project. Each map sheet should also contain the combined adjustment factor used to compensate distances for scale of the state plane coordinate system and the average elevation (datum) of the mapping project. Then whenever it is desired to convert the coordinates of any points or map features or engineering data to the datum of the state plane coordinate system the combined adjustment factor to use is readily available. The conversion can be accomplished by merely dividing their plane coordinates on the map by this factor.

Survey Security Through Photogrammetry

D. E. WINSOR, Design Engineer, U.S. Bureau of Public Roads, Denver

In view of the magnitude of the projected national highway program, it is becoming increasingly more important to exercise some sort of informational security on projected highway routing. In some instances it has been noted that conventional survey methods have generated speculation to the extent that the projected highway route was rendered economically prohibitive because of commercial encroachment. To insulate against such speculation, photogrammetry affords an ideal solution.

Even with the use of photogrammetry, ground control parties may alert the more persistent speculators to the fact that some sort of development is anticipated, even though this work does not offer much information as to specific location. To preclude this type of speculation, an engineer and one or two assistants can make a reconnaissance of the area to locate property corners, section corners, or existing survey markers, from which subdivisions have been made. On the day and the approximate hour of the flight to obtain low-level photography, two men familiar with the project can target the control corners for easy identification and remove the targets as soon as the flight has been accomplished.

From this low-level photography, accurate planimetric maps can be developed. From county records, descriptions of property tracts within the affected area are obtained. Through stereoscopic study of the photograph, the final highway routing can then be determined and projected upon the completed map. Likewise, the right-of-way requirements can be ascertained and computed in the manner and accuracy required by law.

To produce a comprehensive right-of-way map, the work manuscript is then reprojected on a controlled mosaic of convenient size against matching images. The centerline, right-of-way line, and property lines are outlined on this mosaic using white acetate tape in widths ranging from $\frac{1}{32}$ in. to $\frac{1}{8}$ in., as desired, for emphasis. This mosaic is then reproduced on a film autopositive. Inexpensive photoprints in any desired quantity are then available to the right-of-way negotiators. In this manner, speculation as to the projected development can be precluded, because security can be maintained up to the day that engineers accompany the negotiators to the field for acquisition of right-of-way.

● **TO PRECLUDE** speculation and controversy during reconnaissance surveys to determine highway routes, photogrammetry is an ideal tool. In view of the magnitude of the projected national highway program, it is becoming increasingly important to exercise some sort of informational security on projected highway routing. In some instances it has been noted that conventional survey methods have generated speculation to the extent that the projected highway route was rendered economically prohibitive because of commercial encroachment. To insulate against such speculation, proper and full use of photogrammetry in determining and comparing feasible route alternatives affords an ideal solution.

Even with the use of photogrammetry in making large-scale preliminary surveys of the best routes, ground control crews may alert the more persistent speculators to the fact that some sort of development is contemplated. Ground work performed by a field reconnaissance investigation group or by a survey party establishing control for making the preliminary survey by photogrammetric methods does not offer much information as to exact or specific location. However, work on the ground that

NAME AND ADDRESS	DATE	BOOK	PAGE	LEGAL DESCRIPTION OF PROPERTY
Mary Esther Fenton	7/8/47	840	347	Beginning at a point which bears South 24° 03' East 914.0 feet from the Northwest corner of Section 35, Township 5 North, Range 73 West of the 6th P. M.; and which bears North 67° 56' East 490.0 feet from a point on the section line South 04° 34' West 1023.25 feet of the Northwest corner of said Section 35, thence South 67° 56' West 38.3 feet, thence South 16° 39' East 958.8 feet to the center line of the Big Thompson River, thence along said River center line North 30° 01' East 42.2 feet, thence North 07° 27' West 322.0 feet, thence North 77° 12' East 264.4 feet, thence leaving said River North 16° 46' West 637.9 feet, thence South 72° 34' West 306.4 feet to point of beginning. (Warranty Deed) Excepting right of way conveyed to The Department of Highways State of Colorado November 3, 1955, recorded in Book 1006 at Page 525, Larimer County Records.
The Department of Highways, State of Colorado	11/3/55	1006	525	A parcel of land in the Northwest Quarter of Section 35, Township 5 North, Range 73 West of the 6th P. M. described as follows: Beginning at a point on the west property line from which point the Northwest corner of Section 35, Township 5 North, Range 73 West, bears North 21° 40' 30" West, a distance of 940.4 feet; 1. Thence, along the arc of a curve to the right, having a radius of 2,835.0 feet, a distance of 143.5 feet, the chord of which arc bears North 71° 31' East, a distance of 143.4 feet; 2. Thence, North 72° 58' East, a distance of 201.0 feet to a point on the east property line; 3. Thence, North 16° 46' West, a distance of 30.0 feet; 4. Thence, South 72° 58' West, a distance of 201.1 feet; 5. Thence, along the arc of a curve to the left, having a radius of 2,865.0 feet, a distance of 143.3 feet, the chord of which arc bears South 71° 32' West, a distance of 143.3 feet; 6. Thence, South 16° 46' East, a distance of 30.0 feet, more or less, to the point of beginning. (Special Warranty Deed). Above tract contains 0.237 acres, more or less, of which 0.179 acres are in the present road on November 1955.

Figure 1. Typical right-of-way description.

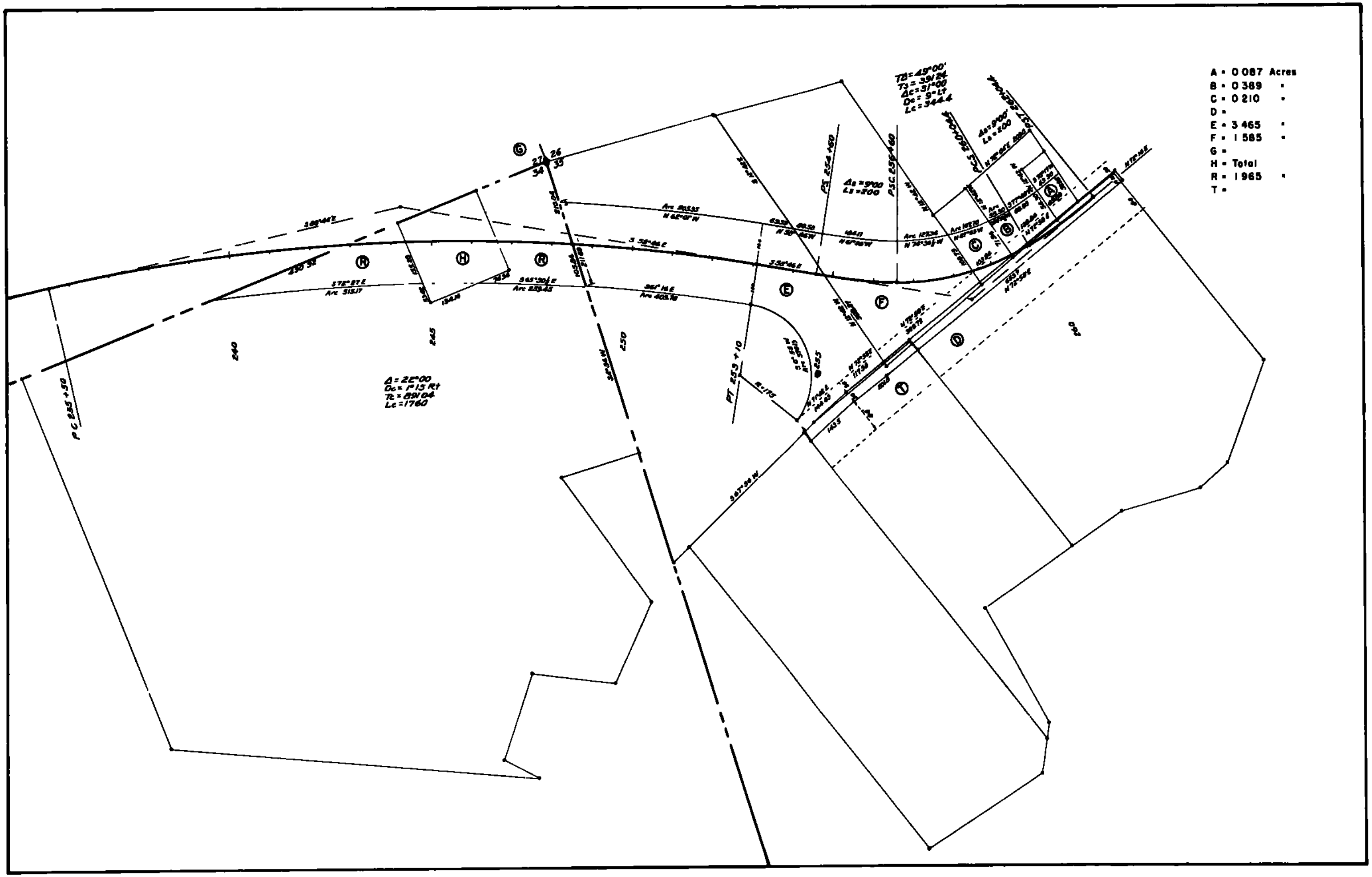


Figure 2. Typical right-of-way map.

would indicate development is contemplated in the area may create some speculation or cause premature ill will before the economics or feasibility of the routing have been determined and made public. To preclude this type of speculation and to avoid unwarranted concern by property owners and business establishments, routing can be refined to specific corridors without the exposure of the work performance by parties within the area. To accomplish this end, large-scale accurate planimetric and topographic maps can be compiled by photogrammetric methods using as control the pre-established land and subdivision corners.

The first step in this procedure is to obtain descriptions of the land tracts in the affected area from the county records. Using the key corner or corners from which these descriptions are based, the entire area is computed and plotted on a coordinate grid using bearings and distances of the recorded deeds.

Figure 1 is a copy of the recorded description of a tract within this area. Figure 2 represents the results of the first step. Small errors in closure of the original survey are apparent. Using the layout as a guide, a trip to the area is made to determine the possibility of photographic identification of some of the property ownership corners. It is essential that images of at least three corners be identified or targeted for each stereoscopic pair of the photographed strip. It may be found that in some instances key property ownership corners are difficult to identify even on large-scale photography. To overcome this handicap, arrangements are made to target these corners on the ground. About one hour before the area is photographed, portable targets are unobtrusively placed over the corners and removed as soon as the plane passes over.

Through stereoscopic study of the photographs, images of the corners or the photographic targets can be identified for horizontal control of the photographs. Factors affecting highway location on the selected route to be surveyed are determined at this time. Using hand-made radial or slotted templet methods or stereoscopic plotting equipment, all pertinent features can be brought into map position.

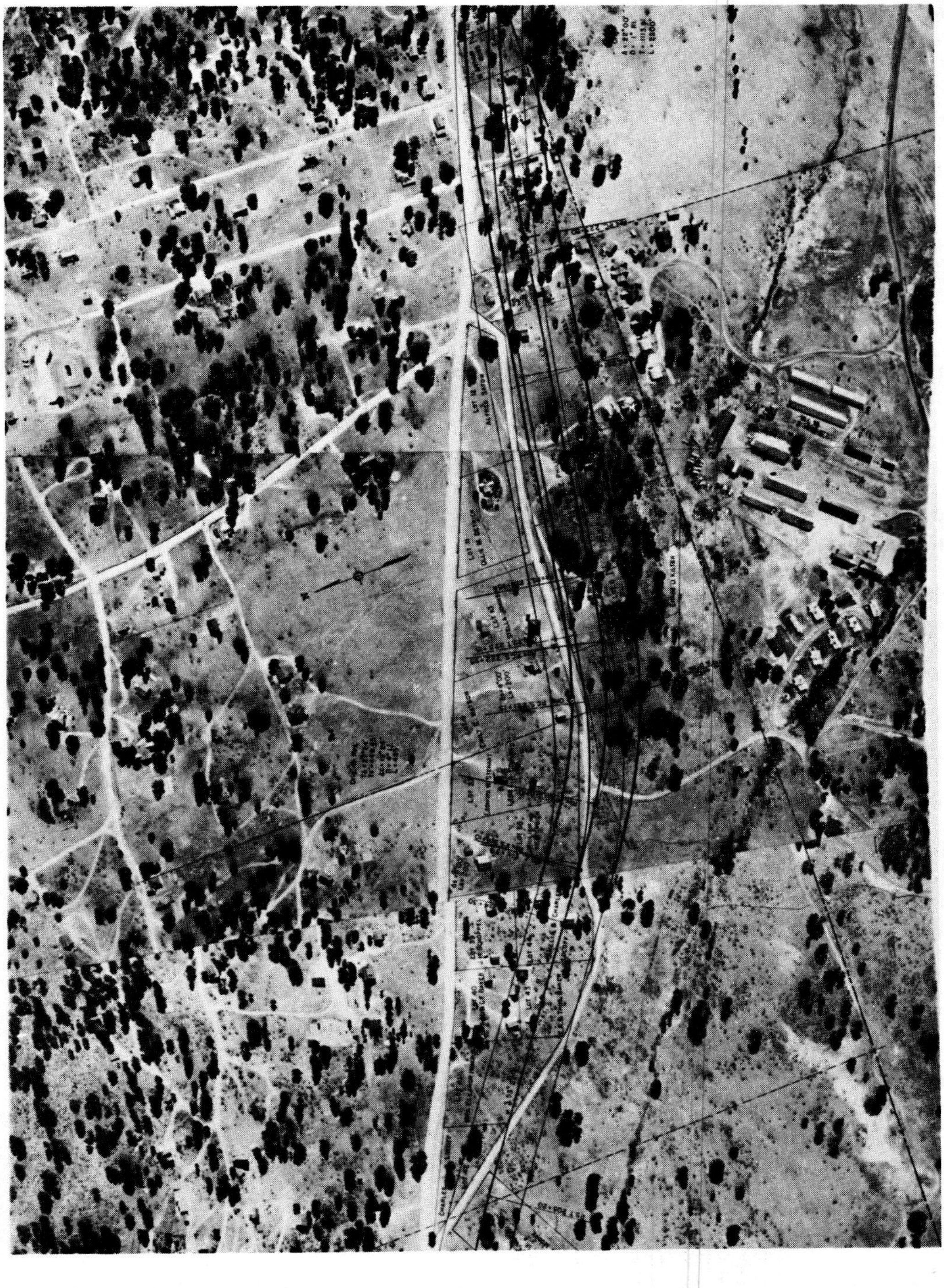
To further control the stereoscopic pairs (termed models) for mapping topographic features, a theodolite may be inconspicuously set up on an identifiable spot or object on the ground for which the image is seen on the photographs.

From this set up, vertical angles are read to four other identifiable objects appearing in the outer quadrants of the model. In orienting the photographs in a photogrammetric instrument for topographic mapping, the exact differences in elevation and horizontal distance are of central interest. With the use of photographic trigonometry, these differences are easily determined. Once the model is oriented in the vertical plane and properly adjusted to scale, the topographic features may be plotted to true elevation or to relative assumed elevations, depending upon the index available and used.

Figure 3 depicts a proposed intersection through highly desirable commercial property as developed by the photogrammetric methods described. On this project, the ownership descriptions also delineated right-of-way previously purchased for the connecting highway. This afforded a positive tie as to stations and bearings on this previously constructed project.

After the proposed centerline has been plotted on the planimetric map, the desired right-of-way areas may be accurately calculated and right-of-way descriptions prepared. To assemble a comprehensive map for use by negotiators, the property lines, centerline, and right-of-way are transferred to a controlled photographic mosaic using acetate tape or other means of delineating the project limits. The mosaic may then be rephotographed on autopositive film. Using this film, dry photographic prints of any quantity can be readily and cheaply processed. Both negotiators and property owners are enthusiastic over this method of presenting a true picture of the proposed development, and it has proved to be invaluable in appraisal and acquisition of right-of-way. Where it becomes necessary to resort to condemnation, this method of presentation will show the court the details of the area to be taken in a form that can be easily understood.

Figure 4 illustrates a portion of the final right-of-way map with parcels numbered



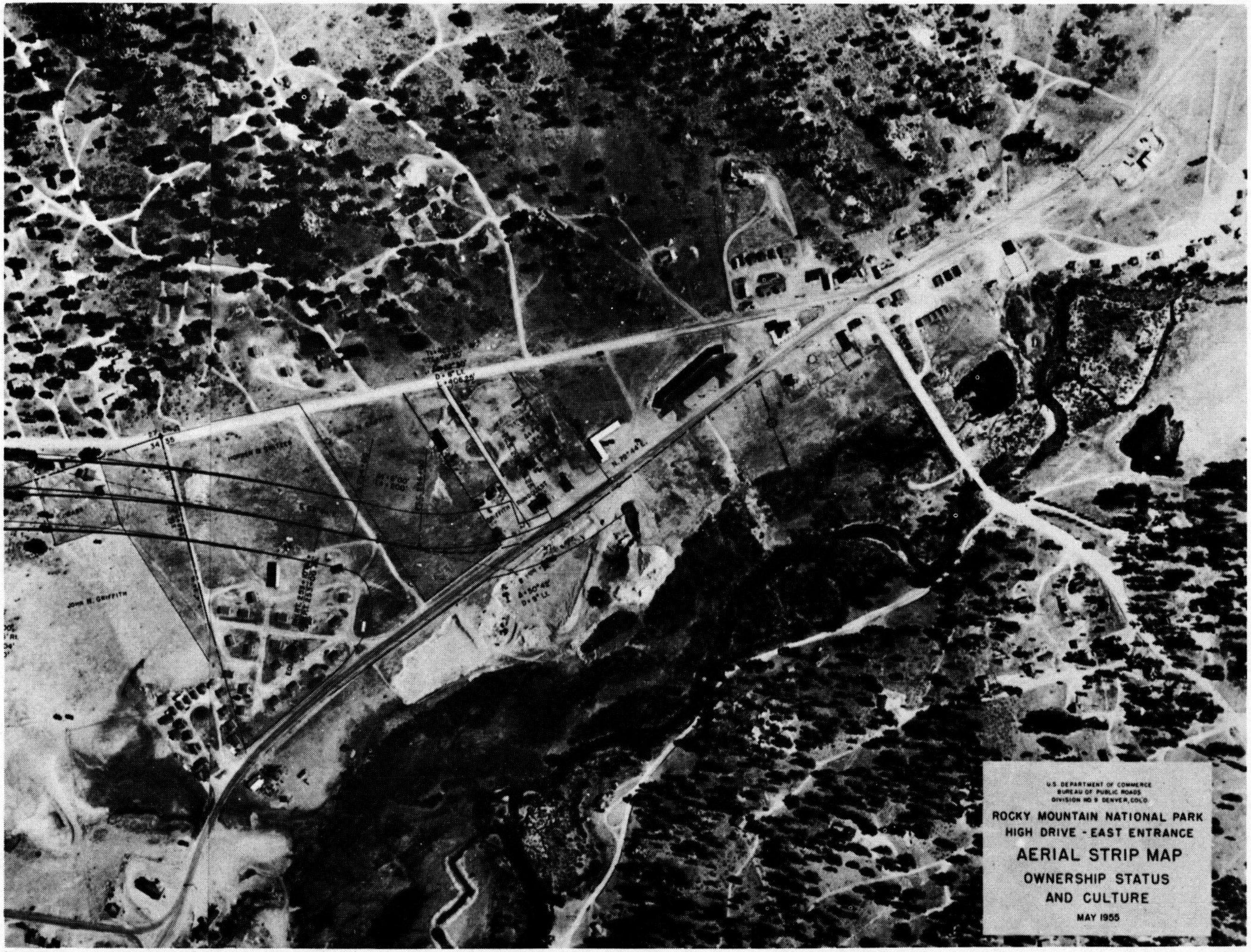


Figure 3. Aerial strip map; ownership status and culture.


1036
1025
CHARLES B. & AGNES M. CARIGNAN
27
26
34
35
BECK
GIMAR
JACK
DETRICK
1
12
N

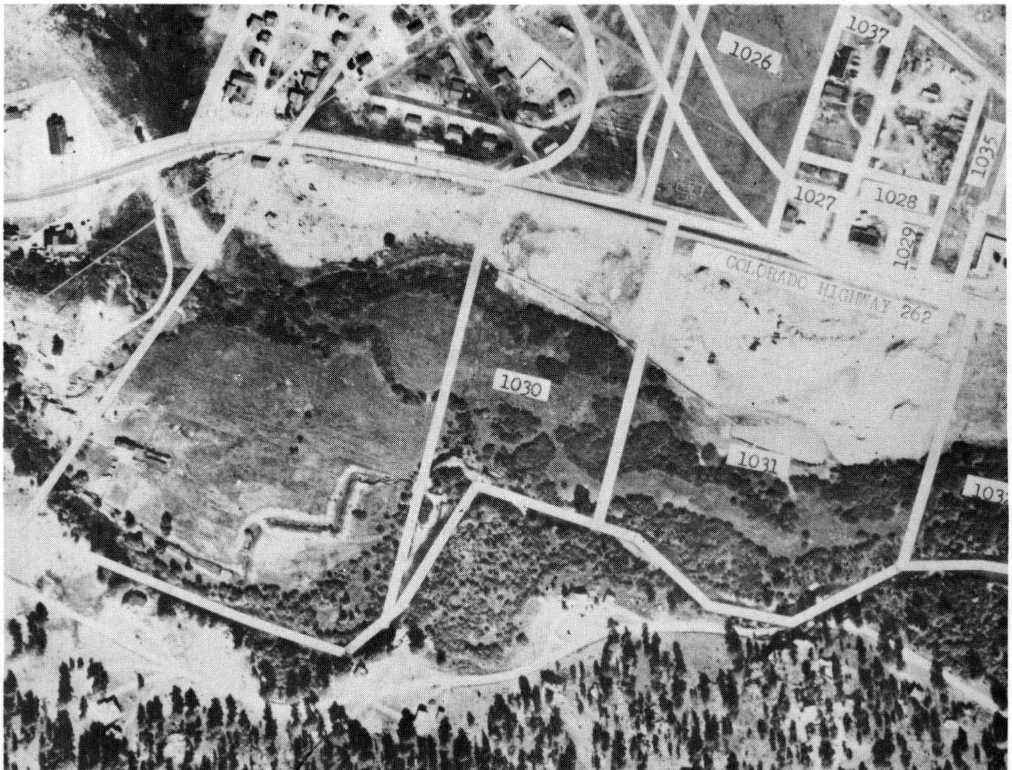


Figure 4. Aerial strip map; ownership status and culture.

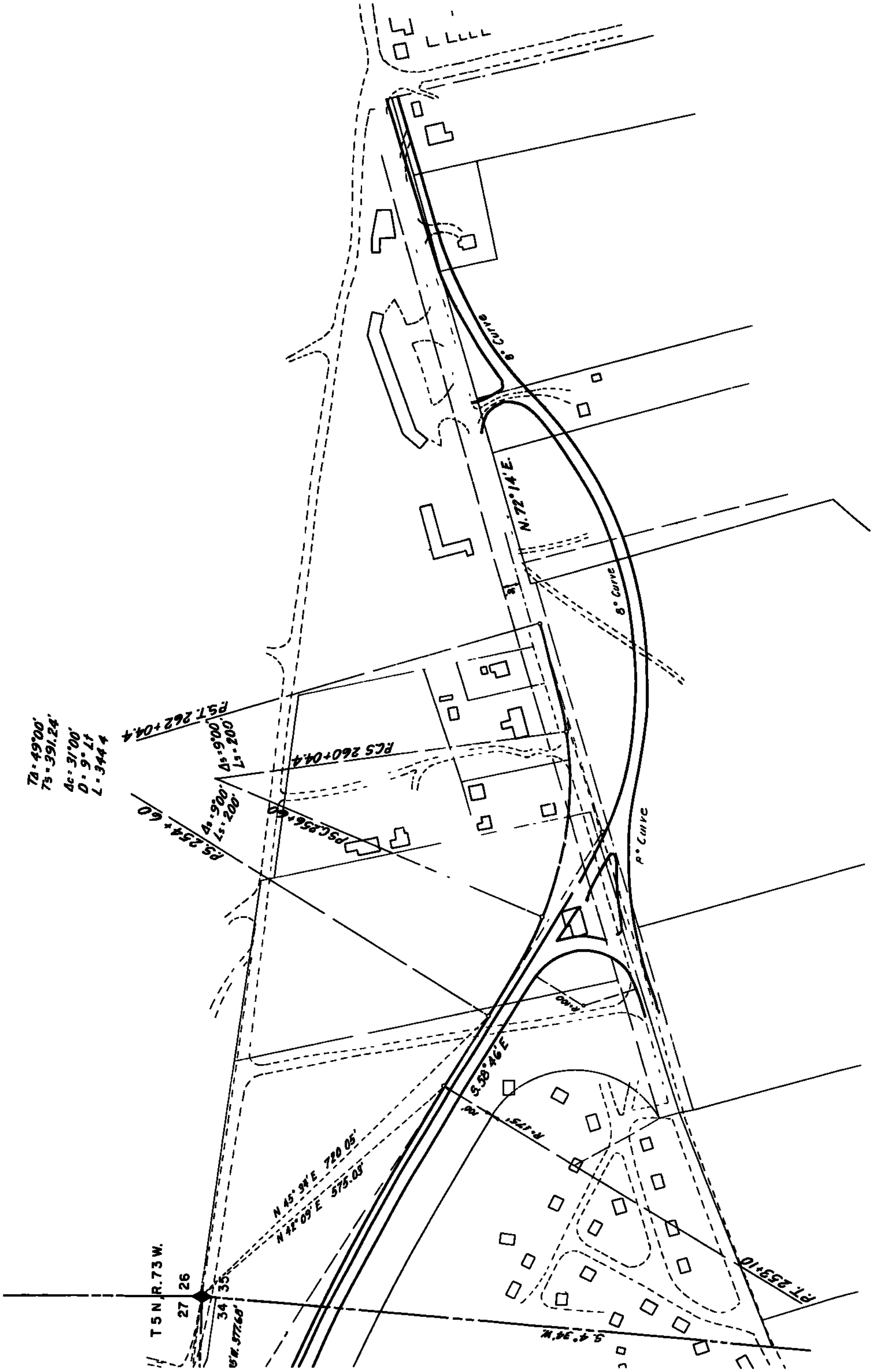

T 5 N. R. 73 W.
27
26
34
35
N. 72° 14' E.
S. 58° 46' E.
8° Curve
8° Curve
P.S.T. 262+04.4
P.C.S. 260+04.4
P.S.C. 256+60
P.S. 254+60
P.T. 253+10
Δs = 9°00'
Ls = 200'
Δs = 9°00'
Ls = 200'
Δc = 31°00'
L = 344.4
N 45° 34' E 720.05'
N 42° 09' E 575.05'
S. 4° 34' W.

Figure 5.

for easy reference as to ownership. Following this prescribed procedure, speculation can be precluded, because it is entirely possible to maintain security up to the day negotiation begins for procurement of rights-of-way.

Figure 5 illustrates the final layout for field staking. To reach this plateau, less than 8 hours of actual field work has been performed by three men.

Earthwork Data Procurement by Photogrammetric Methods

C. L. MILLER, Assistant Professor of Surveying, Director, Photogrammetry Laboratory, and
T. KAALSTAD, Research Assistant, Massachusetts Institute of Technology

Considerable interest has been shown in the use of photogrammetric methods for obtaining cross-sections and other forms of terrain data for the determination of earthwork quantities. This paper outlines some of the current and proposed methods and discusses the various factors involved. The relationship of the electronic computer and other instrumentation devices to the method of obtaining and processing the data is discussed. A summary is given of the current research activity by the M. I. T. Photogrammetry Laboratory related to earthwork data procurement.

●DURING THE PAST several years, there has been a tremendous increase in interest in photogrammetry as applied to highway engineering. This activity has been stimulated largely by the efforts of the Bureau of Public Roads to increase highway engineering productivity throughout the nation. Photogrammetry and electronic computers have received the principal emphasis in this program of encouraging highway organizations to take full advantage of the developments in modern technology.

The rapid rate of development of photogrammetric and electronic computer techniques is having considerable impact on highway engineering theory and practice. Acceptance and utilization of new methods ranges from enthusiasm to refusal. The variations in opinions and policies toward such subjects as the acceptance of photogrammetric data and the need for plotted cross-sections are well known. A review of the subject shows that there is good reason for such variations. The first part of this paper is devoted to a discussion of some of the basic factors involved in terrain data requirements for highway engineering which are at the root of many differences of opinion.

The second part of this paper is devoted to a discussion of some of the photogrammetric methods of obtaining earthwork data and the factors which must be considered. The balance of the paper is devoted to a discussion of some of the research developments of the Photogrammetry Laboratory of the M. I. T. Department of Civil Engineering which are being conducted under the sponsorship of the Massachusetts Department of Public Works in cooperation with the Bureau of Public Roads.

TERRAIN DATA FOR HIGHWAY ENGINEERING

Uses for Terrain Data

A representation of the surface of the terrain or topography of the area of interest is utilized as a source of data for each of the following aspects of highway engineering:

1. Analysis and selection of the location of the highway;
2. Computation of earthwork quantities;
3. Design of the highway and related features; and
4. Construction of the highway and related features.

The above are four separate and often distinct uses for terrain data and each use involves a different set of data requirements. The differing requirements are often overlooked in debating new methods and approaches. Heretofore, the same cross-sections adequately served for earthwork quantities, design and construction. However, it is now apparent that the differing requirements must be recognized in order to realize the greatest value from photogrammetry and electronic computers.

Form of Terrain Data Presentation

Terrain data may be presented in the following basic forms:

1. Graphical or analog form. Such data is usually in the form of continuous lines on a graphical plot. The contour map and the continuous line profile are examples of an analog form of presentation of terrain data.
2. Numerical or digital form. In contrast to the continuous line characteristic of analog data, digital data is composed of a number of discrete points with associated numbers. Tabulated cross-section data or a series of x, y, z coordinates are examples of digital data. Data recorded on punched cards or tape is in digital form.

For visualization and utilization by the human mind, data must usually be presented in analog form. The contour map and the plotted continuous line cross-section are the forms of terrain data presentation for the engineer. They also represent the form of data required when an analog computer such as a planimeter is to be used in conjunction with the data.

When the terrain data is to be utilized for numerical computational purposes such as input data for a desk calculator or electronic digital computer, it must be presented in a digital form acceptable by the computing machine. The distinction of analog form for human utilization and digital form for machine utilization is basic to a better understanding of data problems associated with new approaches to highway engineering. Heretofore, the same data has often adequately served for both the engineer and computational purposes. However, with the use of electronic computers, a distinction must be made between the forms of data presentation for the engineer and for the computer. The use of electronic computers by no means eliminates the need for an analog presentation for the benefit of the human engineer.

The fact that plotted cross-sections are not required for the computation of earthwork quantities does not necessarily mean that they can be eliminated from the scene. It does mean that the requirements for plotted cross-sections are now dictated by their use by the designer and constructor instead of the computational procedure for earthwork quantities. Recognition of this factor has been largely overlooked in the current debate over the need for plotted cross-sections. In conclusion, it may be stated that the form of terrain data presentation is a function of the intended use of the data.

Accuracy of Terrain Data

With regard to the accuracy of terrain data, it is important to recognize the following:

1. Terrain representation systems such as contour lines, spot elevations, profiles, and cross-sections are approximations to the true surface of the terrain in that they represent a sampling of the infinite number of lines or points which would be necessary to represent the complete surface.
2. A distinction should be made between point accuracy and statistical accuracy. By point accuracy is meant the absolute error (difference between represented surface and true surface) at a discrete point. By statistical accuracy is meant the mean error considering a large number of points representing an area of the surface. If the point errors are random in nature (equal chance for positive and negative errors, large number of small errors, small number of large errors), the mean error considering a large number of points will approach zero. Hence it is possible to have poor point accuracy and excellent statistical accuracy.

The accuracy of representation or degree of approximation is largely a function of the density of contour lines or points used to represent the terrain and the terrain slope characteristics and basically influence our choice of contour interval or point spacing. It is obvious that the intended use of the terrain representation will influence the degree of approximation which will be permitted. The intended use will also dictate whether point accuracy and/or statistical accuracy is needed. In earthwork volume computations, since a rather large number of points is being used, it is apparent that the requirement is often for point accuracy. In many design cases, the point accuracy requirement is such that the data may be presented in analog form such as a large scale plot of a cross-section. In some cases, the analog form must be supplemented by digital data at selected points such as recorded distances and elevations.

Sources of Terrain Data

For the purposes of representation, the terrain may be measured directly at full scale (field survey) or indirectly at reduced scale in terms of a model (photogrammetry). The data resulting from the field survey is digital in form. It may be retained and utilized in the digital form or it may be plotted and presented in analog form. The data resulting from measuring the photogrammetric model may be directly recorded in analog form or it may be recorded in digital form.

The graphical plot resulting from either source may be converted back to digital form but normally with some loss of accuracy over the original data in the case of the field survey and the possible direct digital data in the the case of the photogrammetric source. Essentially, it must be realized that data scaled from a map cannot be expected to have the same accuracy as the data used to compile the map. However, the map does serve as a very convenient medium for storing large quantities of terrain data for later recovery.

The sources of numerical terrain data may be summarized as follows:

1. Directly from the field survey recorded in digital form;
2. Indirectly from the field survey recorded in analog form;
3. Directly from the photogrammetric survey recorded in digital form; and
4. Indirectly from the photogrammetric survey recorded in analog form.

Source 1 is "capable" of furnishing the highest point accuracy. Since source 4 can usually furnish equivalent accuracy to source 2 at less cost, source 2 is of diminishing practical importance. Source 4 is "capable" of furnishing relatively good statistical accuracy for many purposes and sufficient point accuracy for some purposes. Source 3 is "capable" of furnishing better point and statistical accuracy than source 4 and better statistical accuracy than source 1 on the basis of economical comparison. The choice of data source is basically a function of intended use which in turn dictates form of presentation and accuracy required.

Terrain Data for Highway Location

For the study of a highway location problem by the engineer, the topographic map is of course the standard tool. A contour interval such as five feet usually furnishes a sufficient degree of terrain approximation and statistical accuracy, and the use of photogrammetry is well established. Possible new approaches to highway location utilizing electronic computers do not at the present time involve any change in the requirements and utilization of the location map by the engineer. The electronic computer will require its own set of terrain data when utilized to make numerical evaluations in the location phase in addition to the map as the engineer's form of data presentation.

Terrain Data for Earthwork Quantities

The basic requirements for terrain data for earthwork volume computations are as follows:

1. A degree of terrain approximation and statistical accuracy consistent with the earthwork accuracy requirement, and
2. A digital form of data presentation.

The earthwork accuracy requirement is a function of the intended use of the resulting volumes such as (a) preliminary quantities for use in comparing alternate routes and selecting the preliminary location, (b) preliminary quantities for estimating purposes, (c) final quantities for use in selecting the final alignment from possible variations from the preliminary location, (d) final quantities for estimating and bidding purposes, (e) final quantities for payment purposes. A data source and method of processing the data must be selected to be consistent with the intended use of the earthwork volumes.

For preliminary quantities, a 5- or 2-ft contour map will often be a suitable data source. For final quantities, a 2-ft contour map may suffice under certain conditions and direct digital data from the photogrammetric model will be adequate under most conditions. Occasionally direct digital data from a field survey will be necessary.

It is important to note that the requirement is for a large number of points with good statistical accuracy rather than a few points with good point accuracy. Statistical accuracy is only achieved if the point errors are random. A small systematic error can result in a large earthwork volume error.

The additional requirement that the data be in digital form adds an additional distinction between terrain data for earthwork quantities and the terrain data for other purposes.

Terrain Data for Design and Construction

The rather obvious but basic observation may be made that terrain data for design purposes is required wherever the engineer is called upon to exercise his design function. A better statement might be that the designer requires special terrain data wherever there is a departure from a standardized design in order to meet the special condition at hand. This means that a basic set of terrain data is required to delineate the sections of standard design and the sections of special design. A contour map will usually furnish sufficient data to delineate such sections and often will serve as an adequate source for the special terrain data for the special design. Drainage design and special slope treatment are examples of design problems for which the map might furnish adequate terrain data.

It is important to note that the map is a convenient and efficient source of data "on demand" whenever the designer needs special terrain data. Hence, it may be concluded that the large scale topographic map will continue to be an important tool of the designer and will not be eliminated by possible use of an electronic computer for quantity determination.

In some cases, the analog presentation of terrain data in the form of a map will not offer sufficient point accuracy for special design problems. In such cases, direct digital data from the photogrammetric model or field survey may be required. However, for visualization by the human engineer, such data will still be presented in analog form in addition to being available in digital form for design calculations.

The requirements for terrain data for design may be summarized as follows:

1. A representation of the terrain of the entire area of interest in analog form with sufficient degree of approximation and point accuracy to delineate sections of standard design and special design; and
2. A representation of the terrain in the sections of special design with sufficient point accuracy to achieve the special design and presented in analog form supplemented with digital data where necessary.

The requirements for terrain data for construction purposes may be simply stated as:

1. Sufficient information to convey the design, both standard and special, to the constructor; and
2. Sufficient information to furnish the constructor with a representation of the existing terrain.

Here again, since the concern is with the human use of the data, it must be presented in analog form supplemented with digital information when accuracy requirements exceed graphical scaling recovery abilities.

PHOTOGRAMMETRIC METHODS OF OBTAINING EARTHWORK DATA

Photogrammetric Map Data

The "stripping" of profile and cross-section data from a contour map is well known. Such methods have long been used for reconnaissance and preliminary estimates of earthwork and lately some agencies have used large scale and small contour interval photogrammetric maps for final quantities. Since the accuracy requirement for earthwork terrain data is statistical accuracy instead of point accuracy, the contour map is capable of furnishing adequate data if systematic errors have largely been eliminated in the preparation of the map. Decreasing the contour interval has the effect of increasing the sample and hence giving a better approximation of the true surface of the terrain. A very carefully prepared two-foot contour map will often be quite satisfactory as a source of terrain data for the computation of final earthwork quantities.

Unfortunately, the widely used tolerance type specification for vertical accuracy (90 percent of the contours correct within one half the contour interval) exercises no control over systematic errors in the map. Therefore, many maps that meet standard map specifications would not be acceptable sources of terrain data for earthwork computations. For example, considering a 2-ft contour interval, if the errors in map "A" are completely random, the expected average point error would be approximately 0.5 ft and the effective point error when 100 such points are used in a calculation would be 0.05 ft and would rapidly approach zero. Map "B" might have a systematic error of 0.9 ft at each point throughout the map, and the effective point error irrespective of how many points were used in the earthwork computation would still be 0.9 ft. Both maps would meet currently used specifications. Map "A" would yield very accurate earthwork quantities and map "B" would yield very erroneous results. Therefore, although the photogrammetric contour map is "capable" of furnishing adequate earthwork data, with currently used map specifications there is no assurance that the map data is adequate. The answer to this problem is to place a restriction on allowable systematic errors in contour maps to be used for earthwork quantities and to independently check each section of the map for compliance with the stated accuracy requirements.

Some of the advantages of the contour map as a source of terrain data for earthwork quantities include:

1. Obtaining cross-section data from the contour map is a very simple process which can be performed by subprofessional personnel and can be achieved with a simple scale as the only instrumentation requirement;
2. The preparation of contour maps is a well established practice with many available sources;
3. The contour map may be prepared in advance, without knowledge of the actual alignment or alignments which are to be considered; and
4. The terrain data is stored in a very convenient form.

Some of the disadvantages of the contour map as a source of terrain data for earthwork quantities are:

1. The accuracy and completeness of the terrain data may not be adequate in many cases, and
2. Taking the data from the map by manual methods is time consuming and subject to human errors and mistakes.

The first disadvantage may be overcome as previously discussed by proper control over the design and preparation of the map for its intended use. The second disadvantage may be partially overcome by adding some degree of automation to taking the data from the map. One of the current research projects of the M.I.T. Photogrammetry

Laboratory is the investigation and development of various instrumentation systems to facilitate the recovery of digital terrain data from contour maps.

Photogrammetric Model Data

A stationary measurement of the photogrammetric model with digital output (point coordinates) can be achieved with greater accuracy than a dynamic or continuous measurement with analog output (contours). Individually read and numerically recorded elevations of single points are usually considered to have twice the point accuracy of elevations of contour lines plotted from the same model. In addition, converting the data directly from the analog model into digital form eliminates the extra analog-to-digital conversion step of taking the data from the contour map. The advantages have led to considerable interest in taking cross-section data directly from the stereoplotter.

A word of caution is in order with regard to the increase in accuracy which may be gained. The increased point accuracy from reading spot elevations is primarily gained from a reduction in the operator's error in placing the "floating mark" in contact with the surface of the ground. This error is largely random except where it is a function of ground visibility. Therefore, reading spot elevations decreases the random error of observation and, using selected points, the possible systematic error due to ground cover or poor visibility. Decreasing the random error contributes little to increased accuracy of earthwork data. Of greater importance is the realization that reading opot elevations does nothing to decrease all other systematic errors in creating the steromodel. For example, an error in model orientation would influence a spot elevation to the same degree it influences a contour elevation.

The accuracy ot spot elevations has also been confused with the least count or smallest increment of the reading. Reading elevations (and distances) to the nearest tenth of a foot with the stereoplotter does not necessarily mean that they are accurate to the nearest tenth of a foot. The average error might well be one-half foot. Therefore, the accuracy of the measurements is not necessarily increased simply by amplifying the reading scales. Amplification is occasionally in order but the smallest reading increment is not to be confused with the accuracy of the reading.

The simplest approach, from the standpoint of instrumentation, for taking cross-section data with a stereoplotter would be to read and manually record spot elevations along graphically plotted section lines. The x and y digital values would then be obtained by scaling the graphical plot. The only instrumentation required would be any standard stereoplotter. The recorded data would be punched on computer input material as a separate and manual step. The simplicity of this approach is quite an advantage and it is already in fairly common use. The principal disadvantage is the manual measuring, recording, and punching steps which are time consuming and subject to human errors and mistakes.

Some interest has been shown in instrumentation for eliminating the manual scaling of the horizontal distances to the cross-section points. As a matter of convenience and increase in efficiency, such a component is justified. However, caution should be exercised in expecting greater accuracy to be realized. If more accurate measuring components have not been used to lay out the manuscript, plot the control, and orient the model to the control, higher order measurements within the model are not justified. Here, again, least reading should not be confused with accuracy. It should be noted that except for the case of terrain slope approaching 45 deg, it is not theoretically necessary to measure the cross-section distances with the same accuracy as the cross-section elevations. This is of course reflected in standard field survey practice.

Mention should be made of the problem of directional scanning. In conventional cross-sectioning practice for earthwork quantities, the offset or horizontal distances are measured at right angles to the centerline or baseline of the highway alignment. In the universal type of plotters, the instrument measuring axes are essentially fixed in direction and cannot be aligned at will in any desired direction. As a result, direct offset distances are not measured and observed with the plotter but rather the two xy components of the desired distance. In addition, the desired cross-section line is not directly observable by the plotter operator so an assistant is required to observe and direct the positioning of the floating mark on the cross-section line as plotted on the manuscript. Various solutions to this problem are being investigated by several groups including the instrument manufacturers. The same problem occurs when a three dimentional scanning unit is placed on a double projection plotter and related to the manuscript via the usual coordinatograph.

Automatic Output Instrumentation

In order to increase the operational efficiency of taking cross-section data directly with the stereoplotter, considerable interest has been shown in automatic instrumentation components for recording the output of the plotter directly on computer input material. The manual measuring, recording, and punching operations can be eliminated if desired by adding an automatic digital readout system to the stereoplotter. Such equipment is now available from several of the universal plotter manufacturers and a number of organizations are developing special systems for use on double projection stereoplotters. Since there are many ways in which the desired results can be accomplished, the various systems differ considerably in detail and technique although functionally they are quite similar. Although there is considerable commercial incentive to claim "the" solution to everyone's requirements and problems, there is still considerable room for new ideas and developments in this field. The future should reveal considerable improvement in flexibility, simplicity and reliability of operation, as well as a reduction in initial cost of such systems. Although the hardware associated with automatic output systems has attracted the most attention, the purpose and utilization of the resulting data is far more important and deserves more thorough study on the part of enthusiasts in this field.

Limitations of Photogrammetric Methods

With the wider acceptance and utilization of photogrammetric methods for obtaining highway terrain data, an understanding of the limitations as well as the applications of the approach is in order. There is great danger in misusing photogrammetric data. Although photogrammetry is capable of furnishing excellent results, unfortunately it is very easy to obtain very poor results. And poor results can be obtained much cheaper than good results.

A photogrammetric system involves a number of steps and instrumentation components each of which offer many sources of error. A complete listing of the sources of error would require many pages. A proper understanding and control over each of these many sources of error is required to obtain adequate results.

The selection and correct use of adequate instrumentation components is of course basic to obtaining desired accuracy. There is considerable variation in the accuracy capabilities of various cameras and stereoplotters, especially when they are not properly calibrated and operated. The dimensional instability of all materials used to temporarily store data while passing through the system is a common source of error. Inadequate or inaccurate ground control and the additional errors contributed by photogrammetric control extension often yield poor results. Model warpage due to careless or inconclusive orientation of the photogrammetric model in the stereoplotter is an extremely dangerous source of error and has accounted for many very poor results. A great deal depends on the skill and experience of the stereoplotter operator and of all the other human elements in the system. Finally, the knowledge and experience of the engineer responsible for selecting and designing the system to be used and responsible for the supervision of the operation of the system is of fundamental importance.

The relatively few thoroughly qualified photogrammetric engineers and skilled photogrammetric technicians is perhaps one of the greatest limitations of photogrammetry in the highway field at the present time. This will be the most serious obstacle to realizing the real potential value of photgrammetry.

Unfortunately, many users of photogrammetric data are responsible for the poor results they often obtain due to the premium placed on price in awarding photogrammetric contracts. Although organizational efficiency and the suitability of available equipment are important factors in cost results, low cost is easily obtained by compromising with accuracy or completeness of data. If price is to be the principal factor in contract award, an extensive independent checking program is essential to insure that adequate accuracy is delivered. A fair fee negotiated in a professional manner with a qualified and reputable organization is the best insurance that good results will be furnished.

A practical limitation of photogrammetry for obtaining earthwork data, one often encountered, is that of ground cover. Accurate and reliable measurements cannot be made when the surface of the terrain is obscured by vegetation or heavy shadows. Under some conditions this limitation can be quite restrictive and photogrammetric methods should not be attempted. However, the same conditions of heavy vegetation can seriously restrict field survey methods by placing a practical limit on the number of points it is economical to obtain. If there are frequent openings in the vegetation from an aerial viewpoint but not from a ground viewpoint, photogrammetry can feasibly obtain more data and thereby better results than a ground survey. However, solid and continuous ground cover even at low height is a limitation which must be respected.

Although speed is one of the principal advantages of photogrammetry over field surveys, the elapsed time between decision and final results can occasionally be in favor of field methods for conventional cross-sections. Although there is no question but what a plotter operator can take cross-sections faster than a field survey party, the plotter operation must await the procurement of suitable photography and ground control. If these phases have not been scheduled in advance, the elapsed calendar time might be in favor of field methods. This is particularly true under terrain and visibility conditions which do not handicap field operations. There are also cases in which it would be cheaper to take the cross-sections by field methods. The authors are by no means attempting to promote field methods but only to point out that photogrammetry cannot be looked upon as applicable to all conditions and cases, but should be used to advantage and not misused to disadvantage.

CONVENTIONAL APPROACHES TO EARTHWORK ANALYSIS

Limitations of Current Approaches

The principal limitation of the current approaches to determining earthwork is that the terrain cross-sections are taken with respect to the horizontal alignment. This presents two problems:

1. The horizontal alignment must be selected and established either on paper or on the ground before the terrain cross-section data can be obtained, and
2. A separate and new set of terrain cross-section data is required for each trial horizontal alignment or changed alignment except for minor lateral shifts.

Scheduling and Coordination Problems

The requirement that the alignment must be given presents a scheduling limitation. The engineering phase must proceed to the point of selecting the line or lines and then must cease, often for a considerable length of time while the cross-section data is being obtained. Even if the data is to be obtained by photogrammetry, considerable time is often required to arrange for a contract, await the proper season and obtain aerial photography, perform and deliver the work, inspect and check the data. If it is required that the line be staked on the ground before the cross-sections are taken, this imposes considerable additional time and scheduling problems in the middle of the engineering phase.

The above scheduling, coordination, and time problems could be reduced if it were possible to obtain all of the data necessary to represent the terrain for earthwork purposes prior to the step of actually selecting and staking the line.

Earthwork as a Location Factor

The second problem of requiring a separate set of cross-sections for each horizontal alignment places a limitation on the number of possible solutions which it is practical to consider. To evaluate ten trial lines or possible solutions would be essentially ten times more work than a single line. Therefore, it is common practice to reduce the number of trial lines to two or three by inspection or map study. Often the final line is selected without the numerical evaluation of any alternates. Earthwork (cuts and fills) are balanced by variations in the vertical alignment or grade line.

It can be said that the conventional approaches to highway location practice with respect to earthwork achieve an acceptable solution but not necessarily the optimum solution. It has heretofore been considered impractical to numerically evaluate a large number of trial solutions due to the engineering time and costs involved, despite the possible savings in the earthwork costs. Two different highway location engineers independently studying the same location problem with respect to earthwork will often arrive at quite different solutions to the problem although both solutions will be good and acceptable. The extent to which either solution approaches the optimum is really not known although each locator is confident that he has the "best" solution.

With respect to the desire for more thorough analysis of highway location with respect to earthwork, the argument is usually raised that earthwork seldom controls the location. It is pointed out that land use and right-of-way cost is often the controlling factor or that the earthwork volume is so small it is not considered a location factor. This is certainly true in many cases and more attention to earthwork is not advocated under such conditions. However, even in such cases, due to the time and cost involved, determining the earthwork even if it is a small amount warrants consideration of more efficient methods of obtaining the data and computing the quantities.

Although the complex and expensive urban highway projects are in the forefront of consideration by many highway engineers, rural locations are still very much in the picture. Depending on the terrain, earthwork may vary from a minor to a major role as a location factor. In many cases, there is ample justification for a more thorough numerical analysis of possible location solutions with respect to earthwork if such analysis can be made within practical reason. Even the most conservative estimates of the amount of money which will be spent on earthwork in accomplishing the national highway program indicated that a relatively small percentage saving in earthwork volume will account for hundreds of millions of dollars.

From the above, it may be concluded that any new approach to the earthwork problem should have the dual mission of:

1. Reducing the engineering time and costs involved in obtaining and processing earthwork data, and
2. Reducing the construction cost by permitting a more thorough analysis of possible location solutions.

Potential for New Approaches

The efficiency of photogrammetry for obtaining terrain data and the efficiency of the electronic digital computer for processing data is generally recognized by the highway engineering profession. These two tools are being used to varying degrees by most all highway organizations. However, the approach so far has been largely one of replacement. The stereoplotter is replacing the survey party for plotting maps and taking cross-sections. The electronic computer is replacing the planimeter and desk calculator for computing areas and volumes. However, the approach to the earthwork problem is largely the same as it has been for decades. The same type of cross-sections are still being taken except now with the stereoplotter. The same type of calculations are still being performed except now with the electronic computer.

It should be recognized that photogrammetry is a very efficient method of obtaining large volumes of terrain data and that the electronic digital computer is a very efficient method of processing large volumes of data. The efficiency of the combined systems offers an opportunity to deviate from the traditional approach to the earthwork problem which is predicated on the practical limitations of essentially manual methods.

Before photogrammetric methods were adopted for highway mapping, the highway location engineer was usually content with a topographic map of a relatively narrow band of several hundred feet in width. When the efficiency of photogrammetric mapping was recognized, the mapping requirement was changed to a band a mile in width in order to do a more thorough job of planning the location. It is expected and advocated that the same extension of thinking take place with respect to numerical analysis of location solutions.

DIGITAL TERRAIN MODEL APPROACH TO EARTHWORK ANALYSIS

The Digital Terrain Model

In order to accomplish the goals which have been presented above, the concept of a digital terrain model has been proposed by the senior author in several earlier papers and reports. By a digital terrain model is meant a statistical representation of the terrain with a system of discrete points with known xyz values. Digital representation of terrain is nothing new to the highway engineer. The map with relief shown by spot elevations and the conventional cross-section are two examples of digital representation by a system of discrete points. The proposed digital terrain model will differ from the above two examples by the following characteristics.

1. The representation is completely in numerical form, meaning that the location (x and y values) as well as the elevation of each point is directly available. (The map with spot elevations can appropriately be termed a digital map but the location of the points is given in analog form.)
2. The points are located in space with respect to a spatial or coordinate reference system which coincides with or is related to a ground recoverable reference system such as the state plane coordinate system. (Ordinary cross-section points are referenced to a given highway alignment.)
3. The storage medium for the digital terrain model is some form of electronic computer input material such as punched tape, punch cards, or magnetic tape.
4. The points are recorded on the computer input material in a sequence or system such that the computer can perform terrain analysis problems according to programmed instructions without the necessity of human interpretation of the terrain for the computer.
5. The digital terrain model in its stored form is usable for an infinite number of independent solutions to terrain analysis problem of the stored area with complete hor izontal and vertical freedom.

The fifth characteristic implies that the single set of stored terrain data could be used numerically to evaluate any chosen horizontal and vertical alignment for a proposed highway within the digitized zone. Such a facility is important to realizing the goals of reducing the amount of work required to consider a large number of trial solutions to a given section of highway. In addition it means that all of the terrain data could be obtained in advance of detailed consideration of the location of the highway alignment.

Digital Representation System

There are a number of possible systems of points to statistically represent the terrain surface. One would be the type of points used by the plane table or transit-stadia topographer where values are determined for terrain control points such as high points, low points, drain lines, and slope breaks. However, such a representation is random in appearance to the computer and the problem of storing, recovering and analyzing the terrain with such points would be quite difficult. The first degree of systematization could be added by taking points only along a system of parallel lines which shall be termed scan lines.

Assuming the scan lines are in the y direction, points may be taken at the following locations:

1. On equal increments of y: if the scan lines are a constant x increment apart, a rectangular grid system would result; if the y increment and x increment are constant and equal, square grid would result. Such a system would probably be suitable for flat or rolling terrain.
2. On equal increments of z: the horizontal density of the resulting points would vary with the terrain slope, with a higher density on the steeper slopes. Such a system would probably be suitable for terrain with a highly irregular surface with frequent slope changes.
3. On increments corresponding to a constant product of yz increments: for example if the constant was given as 20 units, a reading would be taken for such increment combinations of (y,z) as (20, 1), (10, 2), (40, ½), and (5, 4), etc. Here again the density would vary with the slope but such a system would give a better representation than (2) since horizontal as well as vertical changes are considered.
4. On terrain control points along the scan line such as high points, low points, and slope breaks: this system would correspond with the normal cross-section points taken in conventional practice. It would be suitable for all types of terrain and no doubt would be the most acceptable system to the highway engineer until other systems become better known.

The first three systems mentioned above have the advantage that a higher degree of automation could be achieved in the instrumentation for obtaining the data. For example, if the data is being taken with a stereoplotter, the scanning unit could be automatically moved across the model in accordance with the selected equal intervals, relieving the operator of all but one degree of freedom in operating the instrument. In fact, the scanning movement across the model could be continuous and automatic if means are provided for automatically recording the data.

With the above systems the data is taken and stored in a continuous systematic sequence. Therefore, it can be recovered in a relatively simple fashion providing the scan lines are spatially related to a known reference system.

Computer Analysis of the Digital Model

Some of the basic geometric problems which the electronic computer would be called upon to perform on the stored model according to programed intructions would include:

1. Surface elevation of the point corresponding to any given xy value;
2. Coordinates of the intersection of any given line with the surface of the model;
3. Profile values of the intersection of a vertical or random plane with the surface of the model;
4. Line of intersection and area of the quadrature defined by the intersection of a horizontal or random plane with the surface of the model;
5. Volume between the surface of the model and another given surface defined by a series of planes intersecting the model; and
6. Geometric problems such as the above but in which the lines and planes are not given but only their geometric limits and controls.

It is obvious that the surface of the digital terrain model, by definition, is discontinuous. By surface is therefore meant the envelope passing through all of the discrete points of the model. It is apparent that most of the geometric problems listed above involve "interpolated" z values determined by the computer. The accuracy of an interpolated value depends on the agreement between the assumed mathematical surface and the actual surface in the vicinity of the point and hence is a function of the density of the points and the actual terrain characteristics.

Computer Programs

Except for necessary research on the resulting degree of approximation for different types of terrain representation systems and the development and commercial availability of automatic instrumentation, the data procurement phase of the digital terrain model approach poses no real problems. However, the actual use of such an approach will await the availability of the necessary electronic computer programs to analyze the model for basic geometric problems outlined at the beginning of this section and for specific applied problems.

In general, the given lines and planes must be mathematically expressed in the same reference system as the terrain data or to a common reference system such as state plane coordinates. The mathematical relationships for the horizontal and vertical alignment and earthwork volumes for a highway based on simplified geometric controls have been derived and computer programed at the M. I. T. Photogrammetry Laboratory by S. Namyet and R. Laflamme. Related work is continuing on the development of other computer programs to analyze the digital terrain model.

General Applications

The characteristics of the digital terrain model and the interpolation considerations are such that it cannot be expected to yield "exact" values in the civil engineering design sense. It would not be practical to use a high enough density of points to yield interpolated elevations correct within a few tenths of a foot. However, there is a vast range of problems where such accuracy is not required, even in the design stage. In general, it can be said that whenever terrain data is required in graphical form, such as a plotted profile, the data could probably be taken from the digital model.

The major application of the digital terrain model will be to problems involving the determination of quantities such as areas and volumes. In such cases, a relatively large number of terrain points are used. Since the interpolation errors will tend to be random in nature, their effect will tend to compensate and approach zero for a large number of points.

Although this paper is primarily concerned with the digital terrain model for highway earthwork analysis, it should be noted that many of the same concepts and ideas are applicable to such problems as (a) line-of-sight and other terrain clearance problems related to location of communication systems, airport approach zones and military problems, and (b) areas and volumes of reservoirs, open pit mines, stock piles, and other excavation and storage projects. For the latter, the essential reference is to applying the traditional "borrow-pit" computation by a more efficient form of data procurement and processing.

Application to Highway Location

The basic steps involved in applying the digital terrain model approach to the numerical evaluation of the earthwork on many trial solutions may be summarized as follows:

1. Based on a route location study, the band of terrain to be digitized is selected.
2. A set of reference axes is selected for each long dimension of the band. These are designated the skew coordinate axes. The skew coordinate axes are mathematically related to the state plane coordinate system which serves as the basic spatial reference framework for relating all subsequent values to the physical ground.
3. Terrain data is obtained by taking cross-sections perpendicular to the x axis of the skew coordinate system. Such lines in the y direction are termed scan lines. The data may be taken from an available topographic map or may be taken directly with the stereoplotter with automatic digital output. In any event, the data is stored on computer input material in a systematic fashion so that it may be easily recovered.
4. The highway engineer selects as many trial alignments as he may desire to evaluate, furnishing limited input data to define each line horizontally.
5. A computer program furnishes the existing terrain profile data for each alignment.

6. The highway engineer selects one or more grade lines for each trial alignment and furnishes limited input data to define each trial vertically.
7. A computer program furnishes the earthwork data for each trial solution.
8. Based on the first runs, the highway engineer modifies the input data and reruns the apparent best solution until it is optimized.

Each of these basic steps will now be discussed in more detail.

Selection of the Band

The selection of the band to be digitized is basically a matter of delineating the limits within which the optimum location of the highway would appear to be, based on general highway location factors. In the case of a rural location, the location study goes through successive stages of narrowing these limits until they converge on a specific line.

The three basic stages of location which are usually recognized are (a) reconnaissance study (b) preliminary location, and (c) final location. It is apparent that the skew system can be applied to each of these different stages. Therefore, the particular stage being considered will be a basic factor in selecting the limits of the band to be digitized.

In conventional practice, a progressively larger scale, smaller contour interval topographic map is used in location studies as the project passes through the three stages. During each stage, the location analysis with respect to earthwork is primarily a matter of visual inspection of the maps. Two or perhaps three (rarely more) lines are plotted and numerically evaluated. The skew system would permit the numerical evaluation of a much larger number of trial lines but would not materially change the other aspects of location analysis.

Typical mapping requirements for the three basic location stages are:

Stage	Scale	C.I.	Band Width
Reconnaissance	1 in. = 2,000 ft	20 ft	20,000 ft
Preliminary	1 in. = 200 ft	5 ft	5,000 ft
Final	1 in. = 100 ft	2 ft	2,000 ft

Normally it will not be necessary to digitize a band as wide as the standard mapping limits. Visual analysis of the terrain will normally reduce the above limits by a factor of $\frac{1}{2}$ to $\frac{1}{4}$. Whereas in the mapping case, once a band has been selected, the total area is mapped irrespective of location obstacles, in the digital model case, data would be taken only where a highway location was feasible.

In urban areas, the location is usually dictated almost entirely by land use and traffic movement factors. In such locations, a digital model would offer little or no advantage.

Selection of Skew Coordinate Axes

There are no rigid controls on the selection of the skew coordinate axes. The only criteria is to minimize the skew of the scan lines by selecting an x axis paralleling the long dimension of the band. When there is a major change in the direction of the band (over 45 deg), it may be desirable to break the band into two or more sections and have a separate skew coordinate system for each in order to keep the skew within reason.

The reference line can be selected with the aid of a small map. Subsequent computation work and the plotting of the skew coordinate axes on subsequent larger scale maps is greatly facilitated if a reference line is selected which will pass through a number of standard state plane coordinate grid intersections. This would correspond to lines with a tangent of the bearing equal to 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc.

Once the bearing of the reference line and the state plane coordinates of one point on the line have been selected, it is explicitly fixed in space as an absolute data reference system.

Terrain Data Procurement

The two basic sources of terrain data are the topographic map and the stereomodel. In the case of taking data from an available map, the skew coordinate system (reference line and scan lines) is plotted on the map and data taken off manually or with the aid of automatic measuring and recording devices. In the case of the stereomodel, the skew coordinate system is plotted on the manuscript or the scanning system is oriented and indexed with the aid of numerical readout values. A scanning system and automatic digital readout system for double projection plotters which greatly facilitates the taking of terrain data for the digital terrain model approach has been developed at M. I. T. for the Massachusetts Department of Public Works. The full potential efficiency of the digital terrain model approach is only realized with such an automatic instrumentation system, but it can still be used to advantage when such instrumentation is not available.

One of many possible approaches to the use of the digital terrain model approach with regard to data procurement for a three phase application is as follows:

1. During the reconnaissance phase, if the topographic features governing the location do not clearly indicate the best general route, a band several miles or more in width can be digitized using an existing USGS topographic quadrangle map or a newly prepared reconnaissance scale map as the source of data. The digital terrain model would permit the investigation of a number of possible routes with an earthwork accuracy consistent with the reconnaissance phase.

2. Based on the reconnaissance phase results, a route is selected for more detailed attention in the preliminary location phase. Several alternates are possible in the use of the digital terrain model in the preliminary location phase:

a. A standard 1 in. = 200 ft, 5 ft contour interval map of a mile wide strip can be obtained. From a study of the map, a ½ mi or narrower band can be selected for digitalization using the map as the data source or for more accuracy by resetting and using the stereomodels used to compile the map.

b. If the band to be digitized can be selected directly from the reconnaissance study, the digital data can be otained from the stereomodels at the same time they are set for compiling the map. In this case, the mapping limits may exceed or be the same as the digital model limits.

3. The use of the digital terrain model in the final location phase will be primarily to refine the line selected in the preliminary location phase and to provide earthwork quantities for payment purposes. There may or may not be a major amount of shifting of the line in the final location phase, but the digital terrain model removes the problems associated with such shifting in allowing considerable freedom for optimizing the line from the standpoint of earthwork.

The digital data for final location studies would be obtained from the same stereomodels used to compile the large scale planimetric maps which serve as the design base maps. In this case it probably would not be necessary to contour the normal mapping limits but to compile small interval contours only in the vicinity of interchanges, bridge sites, and similar features. In addition to the digital data for obtaining earthwork, it probably would be desirable to obtain profile data for major drain lines and in other special design areas.

The above is only one approach to the data procurement problem. Many others are possible, depending on the extent to which the advantages of the digital terrain model are applicable.

Selection of Trial Lines and Data Processing

The highway engineer is solely responsible for the selection of the trial solutions to the location of the highway. The initial selections will be based primarily on judgment based on a study of the best available map, and a consideration of the many factors which influence the location of a highway.

For each trial horizontal alignment, the highway engineer must furnish the state plane coordinates of the origin, terminus, and each P.I. and the curve radius to be used at each P.I. This data, plus the terrain data previously obtained and already stored on computer input material, serves as the input for the first run through the computer. Based on a stored computer program, the electronic computer prints out the centerline geometrics (stations, P.C.'s, P.T.'s, etc.) and the existing ground elevati elevation of station points along the centerline. It would also be possible to obtain the profile data for any other profiles such as 50 ft to each side of the centerline or an average of the centerline and two or more offsets.

The existing terrain profile data would be automatically plotted by an xy point plotter or line plotter. Without such a plotter, it would be necessary to manually plot the profiles. This would place a practical limit on the number of trial lines it would be feasible to investigate.

With the plotted profiles, the highway engineer would select one or more trial grade lines for each trial horizontal alignment. The data required from the highway engineer for defining the vertical alignment would be the approximate station and elevation of each VPI, the origin, and the terminus, and the length of each vertical curve.

The final input data for which the highway engineer is responsible includes the cross-section design information. The highway engineer specifies the cross-section design criteria for the particular project, (widths and slopes of roadway elements, and side slopes for different conditions of cuts and fills). The basic design templet and design specifications are stored in the computer and the proper templet for each station determined by the computer according to stored instructions. Following the automatic design of the templet at each station, the computer determines the cross-sectional areas and earthwork volumes.

Point Classification

In the M.I.T. computer programs, allowance has been made to classify each terrain point. If the material at each point is classified by soil type (such as rock, peat and sand) the computer will accumulate the earthwork volumes of each type separately. Such classification information might be furnished by air photo interpretation. In the M.I.T. automatic digital output instrumentation system, the point classification is set on a decade dial and the recording performed automatically. The operator changes the setting when he passes from one classification zone to another. Many other such special features to extend the usefulness of the digital terrain model are being investigated.

The Digital Cost Model

The digital cost model approach to highway location evaluation is similar to the digital terrain model concept except that the "z" ordinate has the dimensions of dollars instead of feet. The "z" dimension represents the summation of the other location factors such as right-of-way cost, construction material costs, and other such factors in addition to earthwork which can be reduced to the dimensions of dollars per unit of highway. Traffic benefits would be reflected as a profit instead of a cost. Computer analysis of the digital cost model would have as its mission to determine the most economical solution to the highway location problem. Naturally such a system would have many practical limitations but could serve as a valuable tool and guide for the highway engineer. Its possibilities are more potential than real at the present time.

Research on the digital cost model approach was initiated at M.I.T. during the summer of 1956 and continues as an active subject of investigation. However, it is still in a very early stage of development and considerable work must be done before it reaches operational form even in an experimental sense. It is mentioned in this paper only to report that such research is underway.

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10-Foot Portable Subtense Bar

D. E. WINSOR, Design Engineer, U.S. Bureau of Public Roads, Region 9, Denver

The subtense bar has long been accepted theoretically as a base for triangulation. When used with precise survey instruments, very accurate results can be otained. In recognition of the potential of this device, Europeans have designed and used subtense bars for many years. Their subtense bars, at least those used commercially in this country, are limited to a 2-meter length.

It was reasoned that computations would be simplified and the range of operations would be proportionately increased if the bar length could be increased to 10 ft. To retain its value for field use, 10 ft was arbitrarily selected as the optimum for portability and wind resistance. A working model has answered all the requirements of portability and accuracy.

To hold the cost to a minimum, the basic parts of the instrument were constructed from obsolete surveying instruments. A graduated centering rod was incorporated so t'.e exact height of instrument could be easily determined. Although the primary purpose of the subtense bar is for horizontal traverse, it was reasoned that it would also be valuable for trigonometric leveling.

This subtense bar has been in almost constant use since it was developed in 1956, and the results have been highly satisfactory. It has been used on two 16-mile conventional ground surveys and in the prosecution of 100 linear miles of ground control work for mapping by photogrammetric methods. In establishment of preliminary traverse for highway location, the use of the bar in every instance has proved more accurate than the best chaining obtained from the field crews. The most encouraging development, however, is the elimination of gross errors which invariably develop in conventional chained traverse. The unexpected use in establishing supplemental vertical control for photogrammetry has even exceeded the original wishful thinking. The ease and accuracy with which these vertical controls can be established in rough terrain have saved many man-hours. The instrument has been accepted as standard equipment for the ground control crew, along with the theodolite, self-leveling level, etc.

● THE SUBTENSE bar has long been accepted theoretically as a base for short range triangulation. When used with precise instruments, very accurate results can be obtained. This principle of distance measurement consists of the precise measurement of the angle subtended by a bar or chord of known length. The bar is set up directly above the end point of the distance to be measured and in horizontal position.

Then: D = Distance
b = bar length
Δ = subtend angle

$$D = \frac{b}{2} \times \operatorname{ctg} \frac{\Delta}{2}$$

In recognition of the potential of this device, Europeans have designed and used subtense bars for several years. Their subtense bars, at least those available commercially in this country, are only 2 meters long. It was reasoned that computations would be simplified and the range of operations would be proportionally increased if the bar length could be increased to 10 ft. To retain its value for field use, 10 ft was arbitrarily selected as the optimum for portability and wind resistance. Predicated

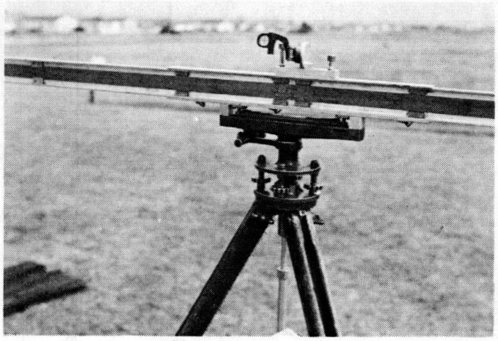

Figure 1. 10-ft subtense bar; leveling and sighting arrangement.

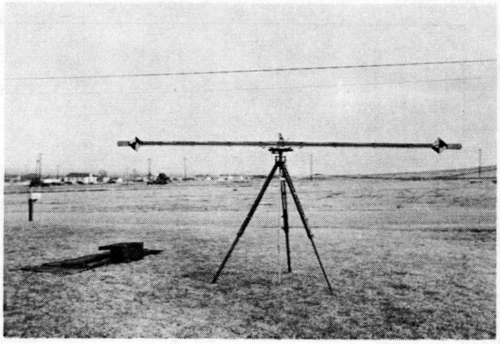

Figure 2. Bar in operating position.

by the acute need for a device to facilitate the making of precise surveys over rugged topography, the idea was translated into a working model. This model was constructed in May 1956, and has satisfactorily answered all the requirements of portability and accuracy.

To hold its cost to a minimum, the basic parts of the instrument were constructed from obsolete surveying instruments. A graduated centering rod was incorporated so that the exact height of instrument (H.I.) could easily be determined. Although the primary purpose of the subtense bar is for horizontal traverse, it was reasoned that it would also be valuable for leveling by trigometric methods. This subtense bar has been in almost constant use since it was developed in 1956, and the results have been highly satisfactory.

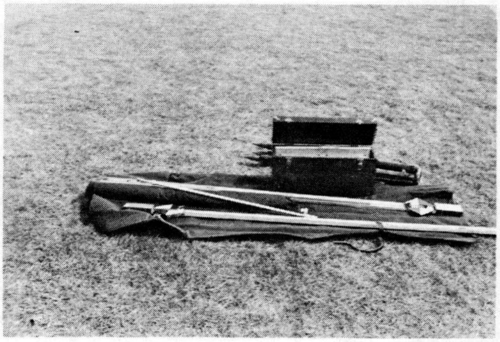

Figure 3. Bar undisturbed.

Measurements are affected by two types of errors, accidental and systematic. Accidental errors are the result of errors in reading the subtend angle. These errors are substantially eliminated by repetition of angle measurements. The bar is so constructed as to eliminate most of the systematic errors. To accomplish this, invar strips anchored at the center of the bar are held to correct distance under spring tension. Targets mounted on channels through which the invar floats have micrometric adjustment for exact setting. This device tends to eliminate any incorrect length of the base stemming from normal atmosphere fluctuation. The sensitive level bubble and accurate pointing device insure a horizontal bar perpendicular to the line measured. The vertical projection of the ground point to the elevation of the targets is accomplished by the use of a graduated centering rod equidistant from the two targets.

When side hill measurements are taken, unequal lateral refraction on the two targets may occur causing unfavorable results. This is difficult to eliminate because the observer does not know when the targets are affected. On this type of terrain, it is good practice to measure distances both forward and backward. The mean may be assumed to be more correct. Using a universal theodolite, which measures angles to the nearest second of arc, and limiting the range of measurements to 850 ft, third order accuracy in horizontal measurements has been achieved. In all instances, the traverse closures were more accurate than that attained by chaining in the plumb-bob and hand-level methods which are used in most highway survey work. Greater distances were accurately measured when atmosphere conditions were favorable. Some difficulty was encountered in pointing the instrument correctly to the targets

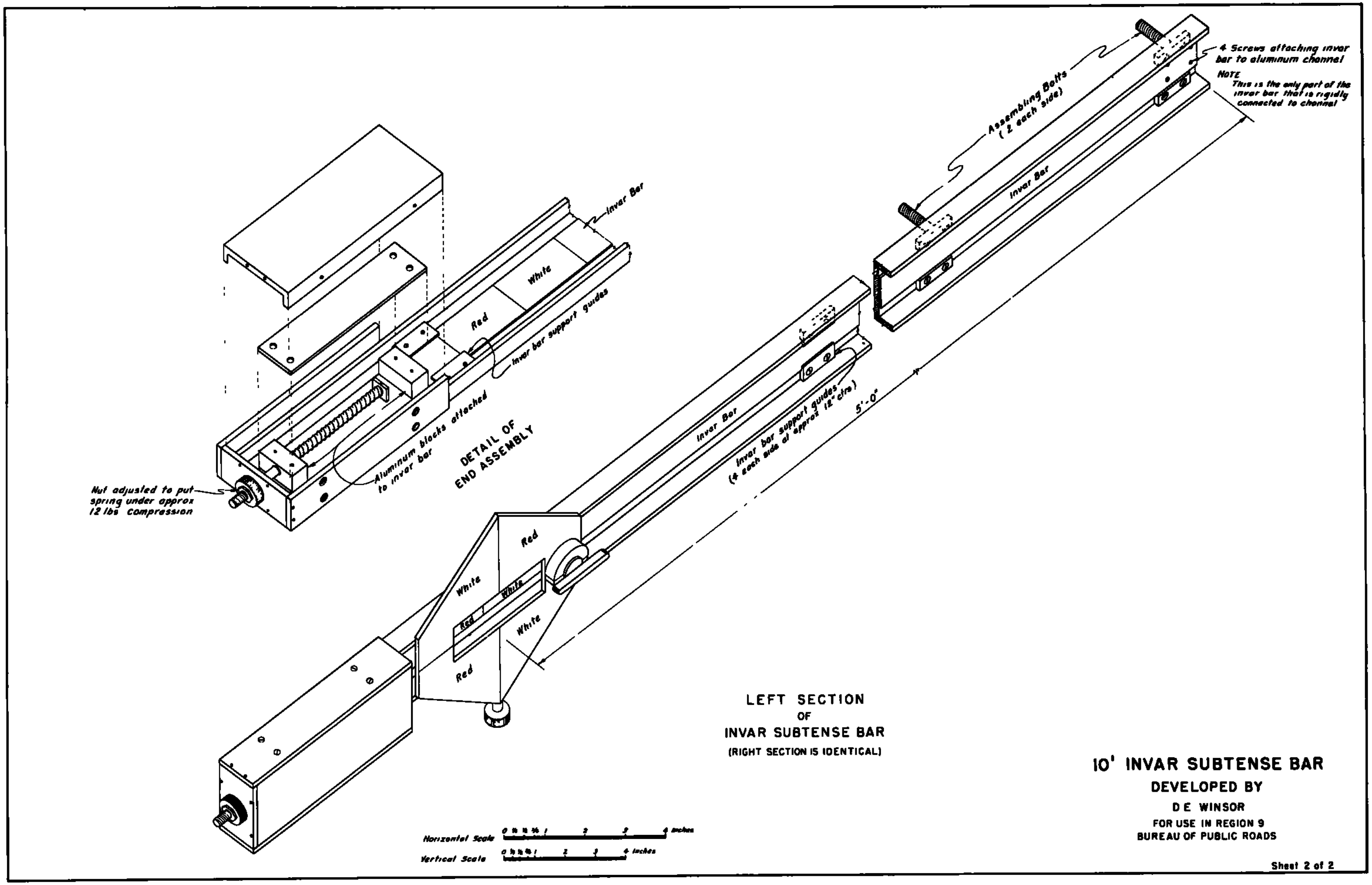

Figure 4. 10-ft invar subtense bar.

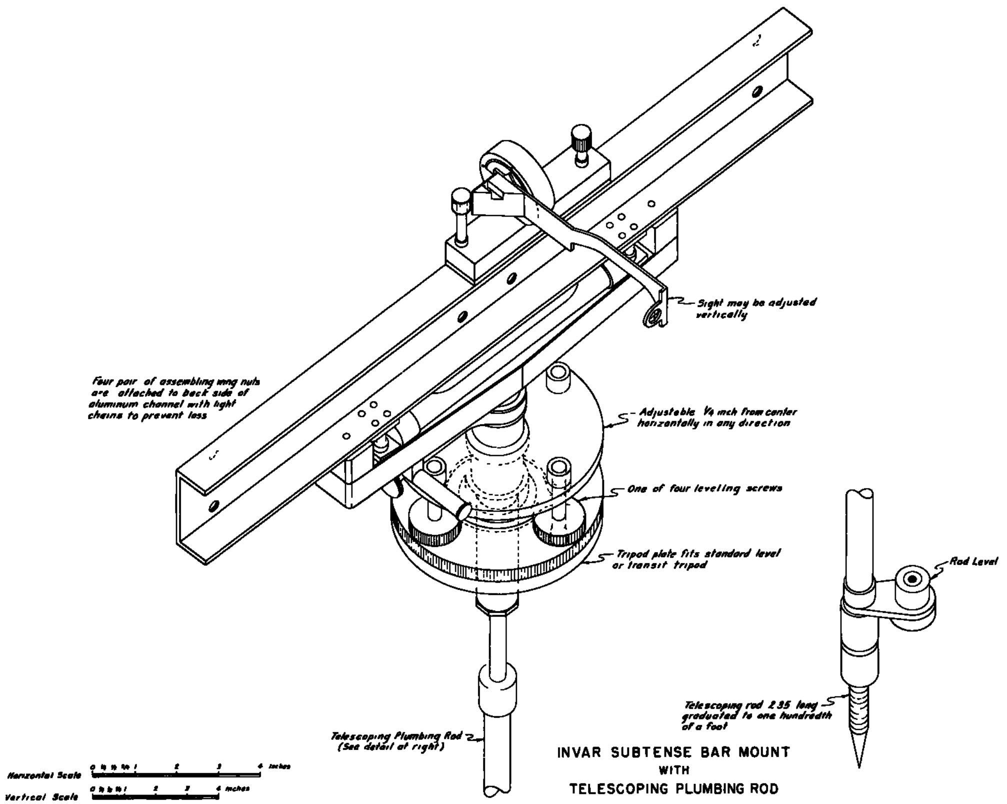

Figure 5. 10-ft invar subtense bar.

on the long courses. This may be helped by a re-design of the targets. Accurate trigonometric elevations can be achieved by limiting horizontal distance measurements to 1,000 ft and vertical angles of 15 deg maximum where practical.

In establishing vertical control throughout rugged topography for photogrammetric purposes, the subtense bar is an ideal tool. Using the leap frog method, or that normally employed in geometric leveling by rod and spirit level, only the H. I. of the subtense bar need be recorded. Level circuits of required accuracy for vertical control can be completed in one-fourth the time required by spirit level methods of running geometric levels. In summing up, the 10 ft subtense bar, when used where the topography is rugged, rough, and mountainous, is a time and labor saving device. Giving due consideration to the influence of errors, this instrument can provide a very adequate means for accurate measurement of horizontal distances and vertical elevations. It is a precision device and must be complemented with a precise angle-measuring instrument and a conscientious survey party.

Aerial Mapping in Areas of Heavy Ground Cover

IRWIN STERNBERG, District Location Engineer, Department of Highways, Tucson, Arizona[1]

This is a discussion of a study begun by the Washington State Highway Department for the purpose of investigating possible methods of securing large-scale aerial mapping of high accuracy in areas of heavy ground cover.

The paper covers work performed in the Fall of 1955 with the assistance of K. B. Wood and Associates of Portland, Ore., by means of mobile plotting equipment set up in the immediate vicinity of the area to be mapped and a small field crew equipped with portable radios in constant communication with the mapping unit so as to furnish necessary spot control as needed during the compiling operation.

● PRODUCTION of a large scale aerial contour map to an acceptable degree of accuracy is not a complex problem in an area where little or no ground cover is encountered. By use of modern precision equipment, proper "C" factor, competent operators, and with strict attention to weather conditions, it becomes a comparatively simple matter to obtain the accepted standard accuracy of a maximum one-half contour interval variation in 90 percent of the contours plotted.

However, with all methods in use at the present time it is necessary to be able to see at frequent intervals the actual image in the photographs, of the ground to be mapped in order to obtain a degree of accuracy agreeing with or comparable to the above.

Where there is heavy ground cover, such as is encountered in the heavy coniferous forests of the northwestern United States and in similar regions, the ground is completely obscured over large areas and assurance of attaining the above degree of accuracy becomes increasingly less as the density and area of the timber cover increases. The fact that these forests are predominantly evergreen minimizes any advantage gained by photographing the area when trees are bare of leaves.

Frequently such heavily timbered areas are precisely those where accurate, large-scale contour maps are desired. Consequently a number of methods have been proposed and tried, but so far few if any whereby a high degree of accuracy could be definitely assured.

The most common of these methods, by estimating the height of cover and making the proper correction during plotting operations, is at best only approximate. A degree of accuracy to approximately one-half the height of cover is all that can be expected. Any attempt to secure greater accuracy by this method usually does not materialize. Where trees are of heights reaching 100 ft and more and, particularly where these heights vary considerably, such accuracy is, of course, of little value for the purpose for which the maps are generally desired.

Attempts to secure a greater degree of accuracy by actual measurement of cover in the field and measurement by photogrammetric methods during the plotting operations have generally failed due to inability to secure accurate data in sufficient quantity to serve the purpose.

There has been considerable experimentation along various lines by some of the larger aerial survey organizations to solve this problem satisfactorily, such as by using extended stereoscopic coverage, utilizing sun angle with shadow images, making

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[1] Formerly, State Location Engineer, Department of Highways, Olympia, Washington

multiple flights, varying flight height and other methods all of which are efforts to more accurately measure height of cover or to increase the number of points where true ground images can be seen. Various degrees of success have been realized, but generally where heavy ground cover occurs weather conditions are poor, resulting in considerable delay in securing the necessary additional photography of a quality necessary to attain these results.

In the northwestern part of the United States, particularly in the coastal areas, many factors offset accuracy besides the timber problem. The principal one is the limitation of good flying weather. Heavy fogs along the coast obscure the terrain for long periods, rains are frequent and sometimes of long duration, low cloud formations are of frequent occurrence. In addition, precipitous canyons and high, sharp ridges cause extremely varied lighting conditions with accompanying heavy shadows even at noon.

The combined result of heavy timber and brush cover with adverse lighting conditions often results in photographs with less contrast than is found under more favorable conditions.

Two or more flights over the same area at different times of the day will frequently minimize heavy shadows in canyons. This method, however, requires the setting up of separate stereo-models and it also fails to a great extent where canyons run almost due east and west, which they most frequently do in the west coastal area.

It appears then that a logical approach to this specific problem is a method whereby stereo-compilation, combined with supplementary field control during and in conjunction with the actual compilation process, can be developed.

At the suggestion of Mr. K. B. Wood, president of K. B. Wood and Associates of Portland, Oregon, an experimental project was set up by the Washington Highway Department in the Fall of 1955 in which stereo-compilation would be brought into close coordination with field mapping. As Washington has no photogrammetric equipment of its own it was necessary to negotiate with Mr. Wood to perform the necessary work in connection with this experiment. Mr. Wood had done considerable work for Washington in the past and was himself concerned with the lack of accuracy obtained in this type of terrain with the usual methods of mapping. An area was desired which would impose problems of adverse condition both as to character of terrain and heavy forestation whereby the above methods could be fairly evaluated.

The site selected was immediately south of the Snoqualmie National Forest in Lewis County, approximately 12 mi west of Morton. Twenty-five percent of the area was covered with a 200 year old stand of virgin Douglas fir timber, 50 percent with a dense stand of alder in combination with an understory of heavy brush, and 25 percent was open fields. It was considered desirable to have part of the area in open fields for a direct comparison of accuracy. The area was cut transversely by a steep-sided canyon running almost due north and south and approximately 300 ft deep. In addition the Tilton River, a precipitous stream, about 100 ft wide, ran in an east-west direction through a canyon some 400 ft deep along the south edge of the strip to be mapped.

It can be seen therefore, that the area selected was a difficult one to map—perhaps too difficult as far as terrain was concerned—but the extent and variety of cover was ideal for the purpose in mind.

Secondary State Highway No. 1-K traverses the northern third of this section throughout its length and afforded easy access to the area. It was also a help in expediting primary control by transit traverse, which was considered desirable for this particular problem. The control was made at third order accuracy and was tied directly to the Washington Coordinate System which is based on the Lambert Grid. The strip, varying from 2,600 ft to 4,000 ft in width and $4\frac{1}{4}$ mi in length, was mapped at a scale of 1 in. to 200 ft with a 5 ft contour interval. The extent of the area covered was 2.54 sq mi or 1,622 acres.

The following procedure is taken directly from Mr. Wood's report which he submitted on completion of the project:

"In order to obtain the best possible coordination between stereo-compilation and field completion work, a three-projector multiplex machine was set up in a 24-ft steel trailer, equipped with 2-way radio so the multiplex operator could be in immediate touch with the field crew for spot checking areas on the photography where interpretation was difficult. In this manner also the compiler was able to operate as an office engineer for assembling and combining field sheets where ground mapping was accomplished. Consequently his time was fully utilized, even though the field work lagged behind the stereo-compilation operation on many occasions. Work was started in the Fall of 1955, and completed in January 1956. The weather during the course of the operation was typical of fall and winter west coast conditions, with a good deal of rain, some snow and some extremely cold weather.

Detailed procedures were as follows:

1. Aerial Photography. The area was flown in two separate strips at a scale of 1:6000, with a 6-in. precision mapping camera.

2. Ground Control. Ground control consisted of traverse run through the center of the area tied to primary traverse station 46, Washington 1912 on the west end, and tied for azimuth only by solar observation on the east end. Attempt was made to run the traverse not along existing roads, but through the center of the heavily timbered area, giving additional tie-ins for field mapping. Levels were run along the existing highway and closed in a loop back along the bottom of the Tilton River, giving good wing point control for levels and also tie-in points at numerous spots for field work. Levels were also run along the traverse, providing a vertical tie in the center of the project.

3. Aerial Triangulation. Prior to going to the field the project was set up on a multiplex long bar and the entire project was extended. Although scale points existed from field work on almost every model, it is usual procedure with this company to run through stereo-triangulation on multiplex in order to check the picture pointing and to set up additional scale controls on every model.

4. Compilation. On November 25, the portable multiplex unit and the compilation crews, consisting of four men, went to the field and a set-up for the portable unit was found where good communication was available to all parts of the area. The timber and ground cover conditions logically caused the work to fall in three categories:

a. Open fields and areas where 100 percent stereo-compilation was possible. (This work could have been done at the office plant in Portland.)

b. Areas where exact interpretation of the ground was extremely difficult. The usual procedure in these areas was to stereo compile the entire area, dropping good scale and vertical points on good photo image points on the ground for tying in field work. The field crew would then run strips between these points, correcting the topography where the stereo interpretation was in error. In other areas, field strips were run prior to the compilation and the photo compiler would use the field strips for orienting and adjusting his interpretation of the ground.

c. Areas of 175- to 200-ft timber. In these areas no stereo work whatsoever could be done except to drop points along the edges of the area for purposes of tying in the field mapping.

The field work was done by use of a staff compass line between either photo control points or ground control points. For distance, the chain was used in some cases and other cases where the distances between controls were not too far, a Wild range finder was used and the distances adjusted on tying into final control. An Abney level was used for purposes of carrying levels on the field strips. The accuracy of this type of field procedure was found to be within map accuracy wherever the distance between tie-ins, either to photogrammetric control or ground control, was not in excess of 1,500 ft.

Because of the unusually bad weather conditions, in most cases field sketching was not done. A set of field notes with spot elevations was carried in the field book and the topography plotted and interpreted from these field notes. There is no question but that the work could have been done more rapidly and possibly with slightly better accuracy, had field sketching been attempted. However, with continuous rain and snow, it would have been very difficult to maintain a sketch board in the field for a very long interval of time. The stereo-compilation work was completed on January 10, 1956.

5. Drafting. Drafting in accordance with standard State of Washington specifications was undertaken as soon as the compilation work was completed, and the work was delivered to the State of Washington on January 20, 1956. The cost of undertaking this work was, of course, excessive because of the fact that it was an experimental operation. Numberous procedures had to be developed, some of them retained and some of the rejected. In terms of man-hours, a total of 1,228 man-hours of work was expended on the job. This can be reduced to 289 man-hours per lineal mile, or 484 man-hours per square mile, for obtaining representative results. Due to the extremely rugged nature of the terrain and extreme brushiness of the project, strictly standard map accuracy was not obtained on the job. A total of 7 profiles were run by the State of Washington on the project, and the results of these profiles are as follows:

Accuracy results testing topography (based on 226 measured points on seven independent profiles):

Mean error	2.42 feet
Maximum error	9.00 feet
90.7% within	±5 feet
71.8% within	±2.5 feet

The interesting thing to note in regard to these profiles is that there is very little difference in accuracy between the relatively open semi-brushy areas which were done primarily with stereo-compilation, and the heavily timbered areas which were done primarily with fairly crude field methods. It is felt by this company and others with experience in the northwest timbered areas, that under conditions such as these, results better than this are almost impossible to obtain. It is also difficult, even in field work to ascertain in some places exactly where the ground is. Those who are familiar with west coast conditions realize that in walking on the ground one is often walking from two to five feet above the ground, on old windfalls and brush piles. A precise delineation of the mineral soil profile prior to a clearing operation, in many cases, is virtually impossible."

Cost

Cost of preparing maps on this one section ran approximately double what it would have cost using standard methods of computation. Somewhat cheaper costs could be expected on a larger project, and during a more favorable period of the year.

The actual cost of photography, preparing the maps, tying the primary control to the Lambert Grid, and all necessary supplementary field control, was $7,768.86 for the the 2.535 sq mi covered. To this should be added $1,650.00 spent by the state for test profiles, making a total of $9,418.86. The cost per square mile was $3,715.53 or $2,216.20 per lineal mile for a strip averaging 3,000 ft in width.

Projects over similar terrain mapped by regular aerial survey methods ran considerably less, but these were all to such a dubious degree of accuracy as to be worthless except for reconnaissance purposes. A fair estimate of performing this same work by usual aerial survey methods would be about $2,000 per square mile exclusive of the supplementary cost of checking. This is approximately one-half of the actual cost per mile of this experimental section. It should be remembered too that this was an isolated project of small size. Understandably such a project would cost considerably more per mile than one five or ten times as long.

Conclusions

Mr. Wood's conclusions as regards to possible accuracy that could be obtained over this particular area are concurred in by the writer. In other areas of less rugged terrain, or with a more uniform ground suface and with more refined methods of field work, a greater degree of accuracy might be attained.

It can be said that the primary objective of this survey, which was to obtain close coordination between stereo-compilation and field completion, was obtained. Many kinds of combinations in field and office work were easily possible to work out under these conditions. The use of two-way radio between the office and the field was not used as much as originally anticipated. Inasmuch as the compilation office was within a mile or two of the field operation, and the field crews checked in at noon for lunch at the compilation office, frequent contact was easily possible without the use of the radio. It was also found that the packing of a two-way radio in some of the brushy areas was sometimes excess baggage, although in other cases it was very desirable from a safety point of view.

Perhaps the most important principle learned in the course of this job was the need for the training of specialty men, both in compilation and in field work on this type of terrain. Those who are not familiar with northwest terrain and brush conditions cannot always understand this problem. Some engineers, with the proper training, can get along well with caulked shoes and conduct reasonably accurate engineering field operations. Others are simply not adapted to these conditions and are not suited to undertake this type of work. There are many places on this type of ground where the slopes are in excess of 75 percent, and the brush cover is such that a man cannot see more than ten feet in any direction. It is necessary also for the field crews to get over all of the ground in such areas, as there are many hidden draws and hidden springs, benches, cliffs, etc., which cannot be discerned from aerial photography, yet they are very critical from a design point of view.

The method shows promise and deserves further experimentation. It has proved to be one method whereby an aerial map of useable accuracy can be assured over difficult terrain and where heavy ground cover is encountered.

The area selected was extemely difficult to map under any condition and with any method. Without doubt, refinement of method, a more favorable period of the year and an area of greater extent would cut the average cost of mapping on this basis an appreciable amount.

Photographic Targets for Markers of Survey Control[1]

WILLIAM T. PRYOR, Chief of Aerial Surveys, Bureau of Public Roads

Benefits of accuracy in ground surveys to control aerial photographs in mapping by photogrammetric methods for the preliminary survey of highways are greatly enhanced when reliable and efficient methods are employed to utilize such control. Photographic targets of symmetrical shape, and suitable size and color, when properly placed over control points before aerial photographs are taken of the highway route band, not only increase the benefits of accurate ground control, but improve the efficiency and certainty with which the control can be used while the maps are compiled. Also, use of photographic targets implements as well as decreases the cost of staking the highway on the ground in the location survey stage. Similar advantages are gained by the availability of targets when supplemental control is bridged photogrammetrically.

Types of targets are discussed and illustrated. The principles of designing their size, shape, and spacing, and of placing them to prevent obscuration by tall trees and buildings are presented.

● AERIAL PHOTOGRAPHS are perspective views of the ground. They contain images of the ground and details of objects on it, as registered by the effect of light on chemicals which form a uniform coating on smooth base material as film, glass, or paper. The accuracy of maps made by use of aerial photographs reflects the precision capabilities of the photogrammetric system used and the ability of the mapper. Completeness and exactness are also dependent upon the type, quality, and amount of information the mapper has at hand, sees, and chooses to use. To achieve the utmost in accuracy during use of photogrammetrically compiled maps for highway engineering purposes, it is necessary to attain precise coordination of the position of basic control with its map and numerical position (as plane coordinates) and its physical location on the ground. This is true whether the basic control is established by ground surveys or by photogrammetric methods.

In mapping with precise photogrammetric instruments and in using maps produced thereby, a higher degree of accuracy may be attained if methods are devised and employed which promise certainty of identification and preciseness in coordination of positions on the ground with images on the aerial photographs and stereoscopic models formed therefrom for the mapping. Some procedures rely solely on the use of natural objects on the ground. Such objects often form faint and irregular-shaped images and patterns on the photographs, which lack point finiteness and are extremely difficult or impossible to use for horizontal and/or vertical control during the mapping operations. Often there are large areas where the photographic patterns lack well-defined images with point finiteness, because the patterns blend imperceptibly from image to image. Images with point finiteness, which may be visible on the photographs and could increase accuracy in the use of control, are often unidentifiable or inaccessible on the ground. There are many occasions when a lone small

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[1] This paper was not presented at the 37th Annual Meeting but was prepared and submitted for publication at the request of the Committee on Photogrammetry and Aerial Surveys.

bush in a forest or field can be positively identified on the photographs and used during the mapping operations. It is extremely difficult for the map user or surveyor to be sure he has identified and found the identical bush on the ground. In contrast, photographic targets placed on the ground remove confusion for the field survey parties when it is essential that they determine on the ground horizontal position and/or elevation of each point used or to be used in the stereomodels for mapping operations or for the measurement of profile and cross-sections. Moreover, natural and man-made features are both subject to change during the time lapse between photography for mapping by photogrammetric methods and actual use of the maps. Instances of this are where fence lines and field cultivation lines may change, where bushes and lone trees are cut down, and where a point on the bank of, or a rock in, a stream or lake becomes covered by a change in water level. Furthermore, many objects, such as trees, power line poles, and rocks in a stream, cannot be occupied by a surveying instrument when it becomes desirable to do so. All of these drawbacks will result in a loss of precision when the map user attempts to position-survey accurately on the ground from plane coordinates and elevations determined from the map. Consequently, it can be seen that it is better to place artificial markings of selected points on the ground to assure positive identification and utmost accuracy.

Geometrically designed photographic targets should be placed over points for which coordinate positions are to be bridged by photogrammetric means or are to be determined by ground survey methods. These coordinate positions are essential for control of stereomodels in the mapping operations and for points of origin and closure in subsequent surveys on the ground, such as testing the maps and staking (with its rights-of-way) the designed highway for construction. The target should be centered over the station marker, existing or set, in the ground at the point for which a coordinate position is known or is to be determined. Placement of the targets should be accomplished on the ground before the photography is taken. This procedure will aid in achieving efficiency, accuracy, and permanency, and save both time and money in the mapping; in using the maps; in alignment, structure, and right-of-way computations by use of plane coordinates; and in all stakeout work on the ground.

Close coordination between target placement and photography crews will assure best results. Lapse in time between their operations should not be so great that action of people, animals, or weather will destroy the targets' usefulness before the photography is completed. Also, very close coordination may be advantageous as a means of avoiding trespass claims and speculation. If the ground location of property corners, section corners, existing survey station markers, etc., were determined previously by reconnaissance, they could be targeted rapidly at the appropriate hour before the photographic flights are made. If necessary, the targets could be removed as soon as each photography mission is completed.

Materials for constructing the targets may be varied according to requirements for a particular project, taking land use, weather, and accessibility into consideration. In most cases, black or dark red muslin or black tar paper and white or yellow muslin would be suitable. Usually, black or dark red muslin is better than black tar paper for target centers where intense darkness is desirable. Black and white lime or similar materials may be better to use where targets constructed with muslin or tar paper might be destroyed by humans or animals before the photography is taken, especially in arid regions. Moreover, painting may be more advantageous where a target is placed on rock, or on a road. Plastics, plywood, masonite, cardboard, or similar materials (appropriately painted) will be more suitable for use under certain circumstances and conditions, as for example, where animals will destroy the muslin for its sizing or its other animal uses, and where the route to be photographed is in a very humid region.

Target dimensions must be predicated on the type of photogrammetric instruments that will be used, the extent to which bridging will be done photogrammetrically, and the scale of photography necessary to accomplish the required mapping. This is so, whether the mapping is to be topographic or planimetric, or whether profile and

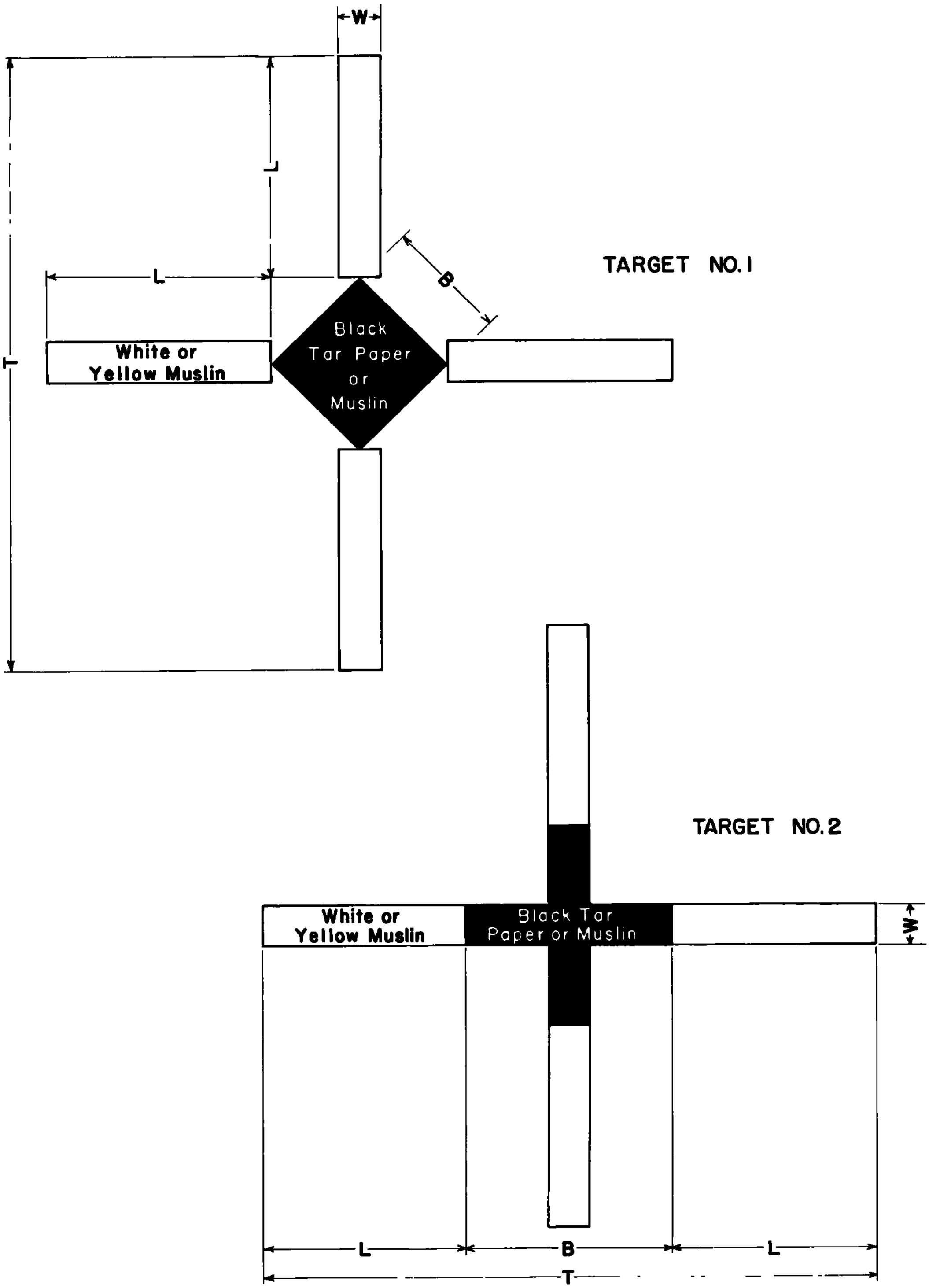
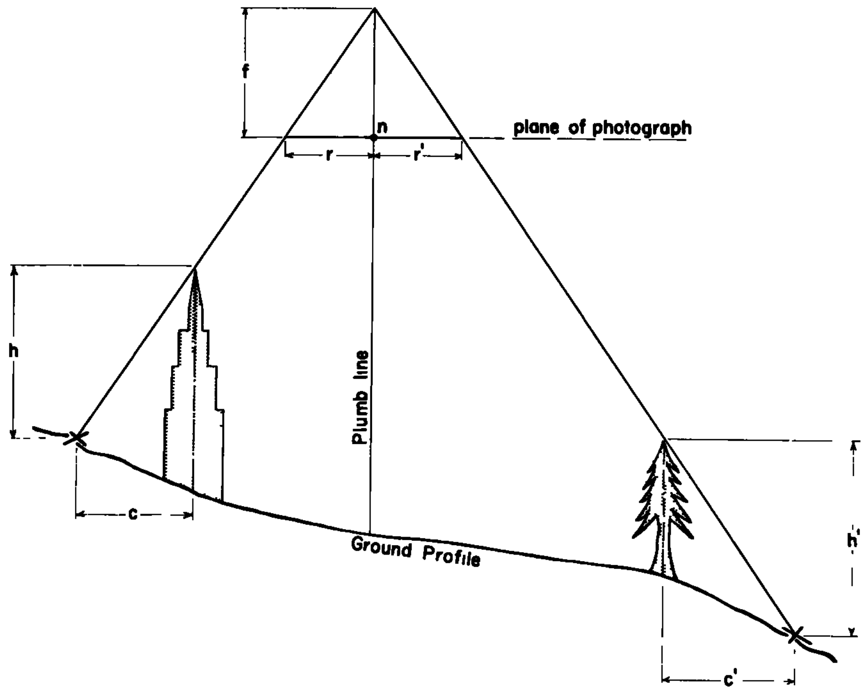


Figure 1. Sample photographic targets: these targets were designed for placement on the ground to produce a well defined symmetrical image on aerial vertical photographs. (Targets must be centered on station marker, hub, or other position markers in the ground.)



f = camera focal length

r & r' = radial distance on the photograph from the nadir point (n) to image of the top of the tall object.

h & h' = height of top of the tall object above the level of the target on the ground.

C & C' = horizontal distance on the ground obscured by "perspective layover" of image on photograph. This clearance from the object must be attained when the target is placed on the ground.

From Similar Triangles:

$\frac{C}{r} = \frac{h}{f}$ Thus: $C = \frac{rh}{f}$ and $C' = \frac{r'h'}{f}$

Figure 2. Clearance necessary to prevent obscuration of a target's image by "perspective layover" of a tall object.

cross-sections are to be measured photogrammetrically in conjunction with or independent of mapping. If the targets are to be used on markers of basic control points for bridging as well as for mapping, the dimensions must be designed so that the image of each target will be sharply defined as an easily identifiable image on the small scale bridging photographs and, at the same time, on the large scale mapping photographs. Moreover, each target must not be so large or irregular that its precise geometric center over the station marker cannot be determined when the photographs are projected for mapping by photogrammetric methods at the required scale.

The dimensions on the photograph of the image of a target may not always be proportional to its actual dimensions on the ground. This is due to the infringement of light emanating from light colored parts of the target or lighter tone areas surrounding it over the contiguous and darker portions of the target and adjacent dark ground. As a result of this infringement, the actual image size on a photograph of a dark object on the ground will be smaller than calculated and the image of a light object will be larger. The degree of photographic enlargement of objects light in color will depend upon lighting conditions, shape of the object, type and color tone of adjacent ground or objects, and the apparent image motion while the camera shutter was open.

Points to be considered include the effect of the infringement of white over black upon position accuracy, the target shape as seen on the aerial photographs, and the size of the target to be constructed on the ground. The visual appearance of the image of certain types of targets may bear little resemblance to their appearance on the ground, particularly on small scale photography. It is very important, therefore, that targets be designed and placed so that their images produced on photographs will permit precise determination of their geometric center over the marker on the ground. This image centering of the target on the marker must be achieved although infringement of light areas over dark areas has changed the over-all appearance and dimensions of the image of the target as compared to its dimensions on the ground.

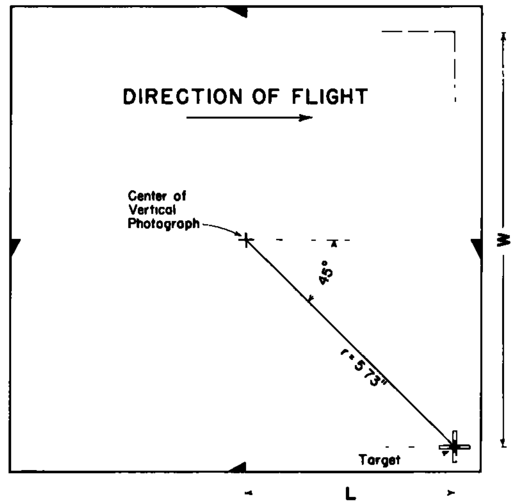


Figure 3. Extreme position of a photographic target on one photograph of a stereoscopic pair. Dimensions (L and W) of usable portion of one 9-in. x 9-in. photograph of a stereoscopic pair when minimum endlap is 55 percent. (L=45 percent x 9-in.=4.05-in.; W=90 percent x 9-in.=8.10-in.)

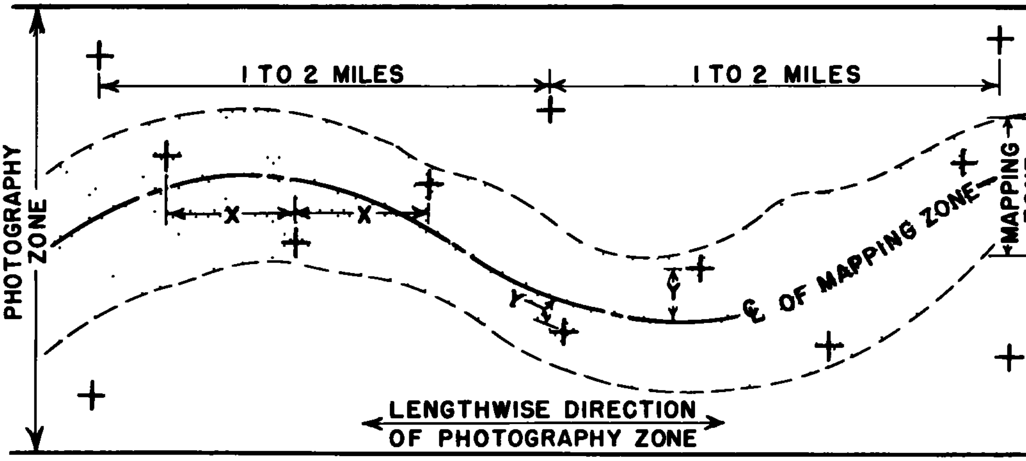
During photogrammetric bridging and mapping operations, accuracy of positioning of the image of any target will be dependent upon how well its center can be estimated visually; whether its shape appears symmetrical or irregular. In addition, white areas in the target's image will be "fuzzy" or "soft" in the stereoscopic model, and under such conditions it will be extremely difficult to determine precisely when the floating dot of the photogrammetric instrument is in contact with the ground. These limitations adversely affect the accuracy of the horizontal positioning to a small degree and the vertical accuracy to a large degree. If an area of dark color tone is provided at the geometric center of the target, both horizontal and vertical accuracy of photogrammetric plotting will be improved. Nevertheless, each target must contain white in contrast with the dark, the white being essential for ease and certainty in finding the target and the dark for greater accuracy. Under unusual conditions, however, an all black target in fully open areas where no shadows could occur and where the ground is all white or very light in color tone would be satisfactory.

The simplest type of target to meet all these conditions is a cross. Its separate dimensions should be appropriately proportioned. If the length of the arms of the cross is too short in relation to the width of the arms, the infringement of white over black will result in an irregular white image instead of a cross. Neither "T"- nor "L"-shaped targets should be used.

Figure 1 shows two cross-type targets which will produce images of suitable shape and relative dimensions if the following rules are applied:

Targets 1 and 2. Total length (T) in feet should be approximately one-fiftieth of the photography scale expressed in feet to one inch. Width of leg (W) in inches should be approximately one-sixtieth of the photography scale expressed in feet to one inch. The length of T should be within the limits of T (feet) equals (1.2 to 1.4) times W (inches).

Target 1. Length of leg (L) in feet should be approximately ten twenty-sevenths of the total length in feet. The dimension in feet of the black central square (B) should be not less than one-fifth nor larger than one-fourth the total length in feet.



+ = Targets within mapping zone for control of stereoscopic models. These should be placed where they will be visible from any possible camera position.

+ = Targets outside the mapping zone for correlation between separate stages of mapping. These must be placed in visible position within the photography zone.

X = Spacing along lengthwise direction of route photography zone. This spacing in feet should be two times the scale of photography expressed in feet to one inch. For example, if the photography scale is to be 500 feet to one inch, place targets at intervals of 1,000 feet.

Y = Spacing normal to centerline of mapping zone. This spacing should be random. Placement on alternate sides of the centerline of mapping zone is recommended.

Other targets should be placed, as desirable, for certainty in photogrammetrically positioning property corners or lines, ground control station markers (both basic and project), and other points essential for both mapping and engineering uses of the maps.

Figure 4. An example of the suggested placement of photographic targets.

Target 2. Length of white leg (L) and length of each part of the black cross (B) in feet should be one-third of the total length in feet.

The dimensions of each target to be constructed on the ground should be governed by the smallest scale photography on which an image of the target must be seen. These dimensions may be determined by considering the relationship of the scale of the map manuscripts to the mapping photography, and to the scale of bridging photography when it is to be used. The projection enlargement ratio of the scale of the stereoscopic model in each photogrammetric instrument to the scale of the aerial photographs must also be considered.

As a starting point, the scale of the map required is known. Using the Kelsh stereoplotter for six inch focal length photography, the projection enlargement ratio (1) is 5 or 7, according to type of instrument. The Kelsh stereoplotter for eight and one-quarter inch focal length photography has an enlargement ratio of 4 or 5, according to the type of instrument. The enlargement ratio is variable in first order instruments (2). In photogrammetric work for highway engineering purposes, a practical maximum enlargement ratio to employ in these instruments is 8. Other instruments have projection ratios of 2.4 and 3.4. Thus, the scale of the mapping photography will be $^{10}/_{24}$, $^{10}/_{34}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{1}/_{7}$, or $^{1}/_{8}$ the scale of the maps, depending on the photogrammetric instrument to be used.

To assure meeting accuracy requirements, the scale of the bridging stereomodels usually should be no smaller than one-half the scale of the mapping stereomodels. Accuracy may be acceptably sufficient at times, however, if the scale of the bridging models is one-third the scale of the mapping models. The bridging scale should

TABLE 1

TARGET DIMENSIONS ON GROUND

Photography	Scale (ft to 1 in.) Total Length (T), (ft)	Dimensions Leg Width (W), (in.)	Target 1 (center portion a black square) Black Square (B), (ft)	Target 1 White Leg Length (L), (ft)	Target 2 (center portion a black cross) Black Cross (B), (ft)	Target 2 White Leg Length (L), (ft)
250	5	4	1	2	2	2
500	12	9	2.5	4	4	4
750	15	12	3	5.5	5	5
1200	24	20	5	8.5	8	8
1500	30	25	6	11	10	10
2400	48	40	10	17	16	16
3000	60	50	12	21.5	20	20
4800	96	80	20	34	32	32
Site Mapping[1] 20 to 50	6 to 9	6	2	2 to 3	2 to 3	2 to 3

[1] Normally, control for site mapping would be obtained by methods other than by bridging photogrammetrically.

seldom, if ever, be as small as one-fourth the mapping scale.

As for mapping, the scale of the bridging photography will be $^{10}/_{24}$, $^{10}/_{34}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{1}/_{7}$, or $^{1}/_{8}$ the scale of the bridging model, according to instrument.

As an example, it may be assumed that topographic mapping at a scale of 100 ft to 1 in. is desired and that the instrument to be used is a Kelsh stereoplotter with an enlargement ratio of 5 using 6-in. photography. For the mapping, the required photography scale is 5 x 100, which is 500 ft to 1 in. For the photogrammetric bridging, it may be assumed that a first order type instrument is to be used, and that the scale of the bridging models is to be one-third the scale of the mapping models. In such a case, the scale of the bridging models will be 300 ft to 1 in. Using the practical maximum enlargement ratio for the first order instrument, the required bridging photography scale is 8 x 300, which is 2,400 ft to 1 in.

By applying the rules given previously, dimensions of the two suggested types of targets to produce a suitable image at the scale of 2,400 ft to 1 in. may be determined as follows:

<u>For Target 1 and Target 2</u>

$$\text{Total length (T)} = \frac{2,400}{50} = 48 \text{ feet}$$

$$\text{Width of leg (W)} = \frac{2,400}{60} = 40 \text{ inches}$$

$$\text{Proportions} = \frac{L\ (\text{ft})}{W\ (\text{in.})} = \frac{48}{40} = 1.20 \text{ feet per inch}$$

<u>For Target 1</u>

Black central square (B) = $^{1}/_{5}$ x 48 = 9.6, use 10 ft

Length of leg (L) = $^{10}/_{27}$ x 48 = 17.7, use 18 ft

<u>For Target 2</u>

Length of black cross (B) and Length of white leg (L) = $^{1}/_{3}$ x 48 = 16 ft

To produce a well-defined geometric image of the targets suggested, on photography at some commonly used scales, recommended on-the-ground dimensions for constructing the targets are listed in Table 1.

Applying the rules given and used in the examples and in compiling Table 1, image sizes in inches on the aerial photographs will be approximately within the limits given

TABLE 2

TARGET DIMENSIONS ON AERIAL PHOTOGRAPHS[1]

Total Length T		Leg Width W		Target 1 Black Square B		Target 1 White Leg L		Target 2 Black Cross B		Target 2 White Leg L	
min.	max.	min.	max.	min.	max.	min.	max.	min.	max.	min.	max.
1/50	1/12	1/750	1/190	1/250	1/62	1/150	1/37	1/150	1/37	1/150	1/37
.02	.08	.0013	.0052	.004	.016	.0067	.0267	.0067	.0267	.0067	.0267

[1] All dimensions are inches.

in Table 2, assuming a scale ratio of not more than four to one between the scales of mapping and bridging photography. Whenever the ratio between such scales is three or two to one, image size of targets should not be allowed to become smaller than the minimum in Table 2. The maximum size, however, should be either 3 or 2 times larger than the minimum, in the same proportion the scale of the mapping photography is larger than the scale of the bridging photograph. Moreover, targets for photography of scales other than listed in Table 1 should be designed similarly so that image sizes will be within the limits given in Table 2.

If each target placed on the ground is to serve the purpose for which it is designed, its image must be visible on photographs which might be taken from any possible position of the aerial camera. Photographic displacement of tall objects, however, obscures the ground between the object's base and its top. This is called "perspective layover." The ground area obscured in consequence of perspective layover depends on the scale of the photograph, slope of the ground, the height of the object, and its location on the photograph. Since the target's position on the photograph usually cannot be predicted before photography, it must be placed on the ground at a sufficient distance from tall objects, such as buildings, trees, walls, and monuments, so that their perspective layover cannot obscure the target's image. The distance from tall objects at which a target must be placed to preclude its obscuration by perspective layover may be determined by utilization of principles illustrated in Figure 2.

Experience has proven that, to assure visibility of each target, it should be anticipated that its image on vertical photographs somewhat "randomly taken," will possibly be at a maximum practical distance from any photograph's center. Assuming a minimum endlap of 55 percent on a 9- by 9-in. photograph, and that the image of the target may appear at the intersection of a line 45 deg to any line joining the fiducial marks and a line parallel to, and 0.45 in. from, the edge of the photograph, as illustrated in Figure 3, this anticipated maximum distance is 5.73 in. There are conditions, however, which may limit the usefulness of a target unless it lies at a distance of less than 5.73 in. from the center of a photograph. For example, when using Metrogon lens photography in a Kelsh stereoscopic plotter the cams do not correct for lens distortions beyond 5.0 in. from the principal point. On Aviogon lens photography the fiducial mark enclosures, which appear in the corners of the photographs, extend to within 5.65 in. of the center. By anticipating that each target may lie at an r-distance of 5.73 in. from any photograph's center, its placement on the ground will be where obscuration of the target by perspective layover of tall objects cannot occur.

Knowing the aerial camera focal length and applying the previously mentioned principles, clearance from perspective layover of tall objects can be attained by placing each target at the clearance distance, minimum (C) and desirable (C'), determinable by use of the equations in Table 3. In these equations, h is the height of the top of the object above level datum of the target.

For certainty in identification and use of vertical control, the elevation of targets placed within the mapping zone should be determined by spirit level or other accurate methods. The elevation of the corner points used to level each stereomodel for mapping

TABLE 3

HORIZONTAL CLEARANCE FOR TARGETS

Camera Focal Length (in.)	Horizontal Clearance Around Target Minimum	Desirable
6	C = 0.75 h	C' = 1.0 h
$8\frac{1}{4}$	C = 0.55 h	C' = 0.7 h
12	C = 0.40 h	C' = 0.5 h

is, depending on accuracy requirements, sometimes obtained by other means, such as photogrammetric bridging, or surveys on the ground by trigonometric leveling or barometric altimeter. The effects of small discrepancies in elevation at these points will be minimized within the mapping zone by the positive identification and utilization of the accurate elevation of targets within that zone. The plane coordinates of these targets, determined by photogrammetric bridging or ground control surveys, will be used as horizontal control for scaling stereomodels for the mapping and as plane coordinate points of origin for staking the designed highway alignment on the ground.

Spacing of these targets in the mapping zone should be governed by project requirements. Best results are achieved, however, when a minimum of two targets appear in each stereomodel. To be reasonably certain of fulfilling this requirement, target spacing, in feet, along the lengthwise direction of the route zone to be photographed for mapping by photogrammetric methods should be about two times the scale of the mapping photography expressed in feet to one inch. Target spacing normal to the centerline of the mapping zone should be random and targets should be placed where they will be visible from any camera position. Some of the targets should be placed over station markers which are not within the probable construction limits of the highway, in order to preclude their destruction during construction operations. Such targets should be placed close enough to the centerline of the mapping zone, however, for convenience in use on the ground when the highway is staked for construction. Placement of many of the targets alternately on one side and then the other side of the centerline of the mapping zone will be helpful in attaining greater accuracy. Wherever a P-line traverse, or a designed centerline has been staked on the ground before photography, however, targets should be placed advantageously over instrument points and other accurately positioned line stakes to attain the spacing interval required by the photography scale.

Where ground cover is tall and dense, the route zone may be photographed and mapped at suitable scale, an initial alignment determined, tentative grade lines and cross-sections measured, and clearing limits established. Once the best location, based on this initial design, is cleared and rephotographed, the surface of the ground can be accurately mapped for accomplishment of detailed design, measurement of grading and other quantities, and preparation of construction plans. To establish control for initial mapping of the route before it is cleared, for accurate mapping after the clearing is done, and for mapping, as desired, after construction to determine pay quantities, some targets should be placed outside the initial mapping zone, but still within the photography zone. Two such targets, which will not be disturbed during clearing or construction operations, should be placed at each end of the route zone, and others at intervals of about one or two miles along the route (Figure 4). Such targets will assure that positioning and coordination will be accurately accomplished in all three mapping operations without need for establishing basic control other than for the initial phase. Thus, systematic errors between the separately compiled maps may be kept to a minimum, if not entirely eliminated, and the optimum accuracy attained in each operation, regardless of which photogrammetric system is employed.

It may be of interest to note additional advantages in placement of photographic targets before photography is accomplished. The photography crew has positive visual guidance available on the ground, which adds certainty to their attaining exactly the photographic coverage required, without any likelihood of having to make several trial photographic missions. Targets enable field survey parties and the photogrammetric instrument operators to save time and thus decrease the cost of their work, because delays and errors resulting from misidentification of natural images is eliminated. These targets also enable photogrammetric instrument operators to determine whether there is image movement in the photography. This is accomplished by detecting where targets appear to "dig" or to "float" unnaturally. Each photogrammetric instrument operator can maintain consistently greater accuracy throughout his work by the elimination of doubt and the reduction of random errors, which would otherwise occur when photographic targets are not provided.

Ground and aerial photographs of various targets are included to illustrate good and bad types and proportions.

REFERENCES

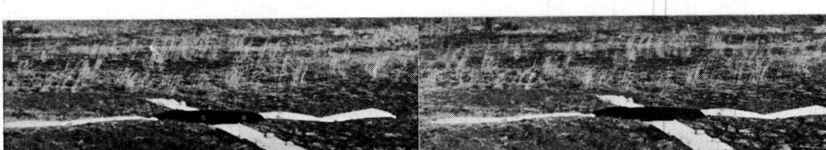
1. "Relationships in Contour Interval, Scales, and Instrument Usage," HRB Bulletin 157 (1957).

2. Manual of Photogrammetry, American Society of Photogrammetry, Second Edition (1952).

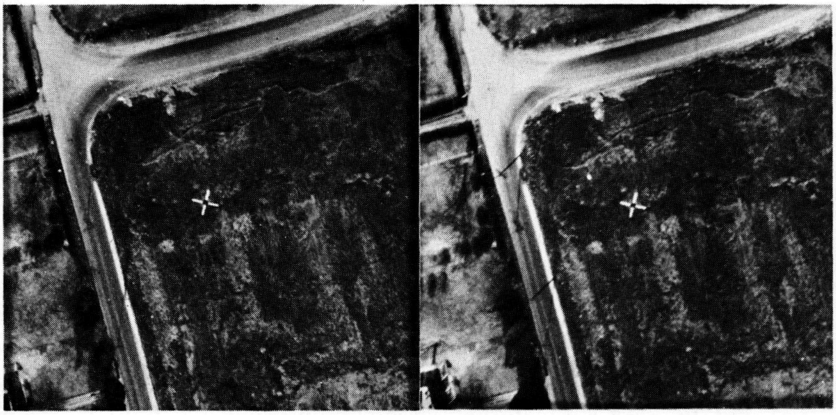
Appendix

Nineteen stereograms were selected to illustrate various types of targets used on several projects. Ground views are not of the same targets in the aerial views. Their similarities, however, are described. In the aerial views target images were small at contact scale, and for ease in their examination and certainty of reproduction herein, they were enlarged two diameters in preparation of the stereograms. The image of each target in the aerial stereograms is therefore twice as large as it actually appeared on the aerial photographs.

Stereograms 1 through 7 are of targets recommended as most suitable. Stereograms 8 through 14 are of targets which are usable and will serve well under special conditions. Stereograms 15 through 19 are of targets which contain unsatisfactory characteristics as noted.

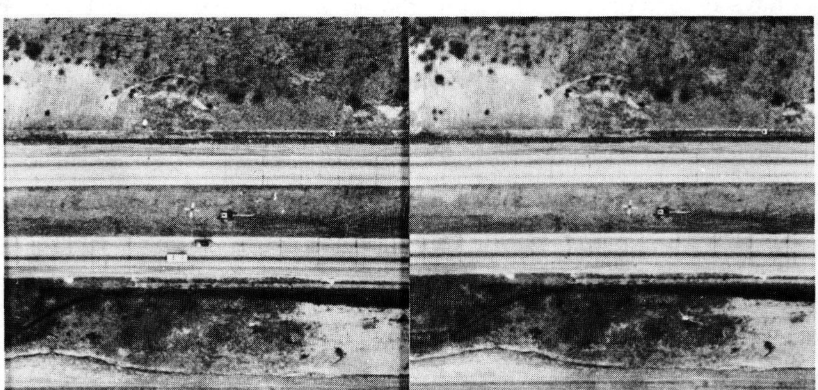


Stereogram 1. Ground view of cross-type target with central square. Material used is dark red and white muslin. This target is similar to Target No. 1 described in the paper. It is easily placed and removed as necessary, has reasonable durability, and its symmetry over the station marker remains in its image on the aerial photographs. Refer to illustration in Stereogram 2.



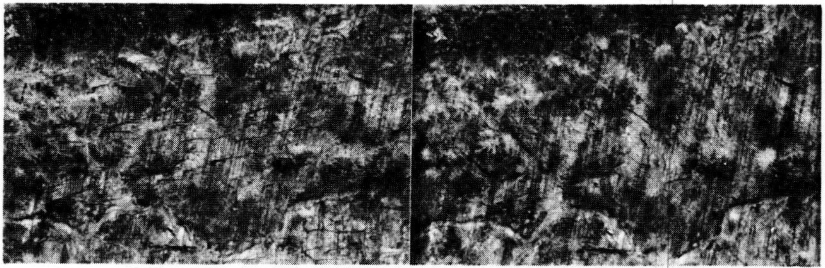
Stereogram 2. Aerial view of cross-type target, similar to the one in Stereogram 1. Bordering ground area has dark tone. Consequently there is considerable contrast between this area and the white legs, and little contrast between it and the dark red central square of the target. Target proportions are similar to those in Table 1 of the paper, but the dimensions are twice as large as necessary for the photography scale. (As seen here, however, the target appears four times as large as required for photogrammetric work because the photographs, as noted in the general explanation for all stereograms, are enlarged to twice their contact-print scale.)

Stereogram 3. Ground view of cross-type target. Materials used are dark red and white muslin. This target is similar to Target No. 2 described in the paper. Its other characteristics are similar to Target No. 1, as illustrated in Stereogram 1 and 2.

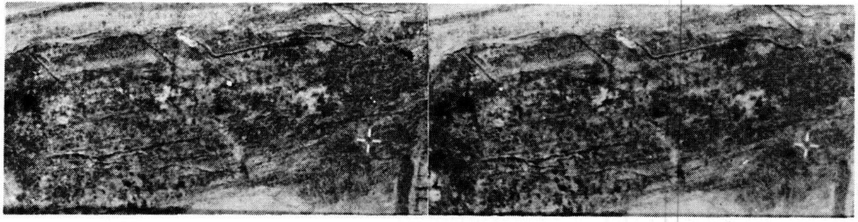


Stereogram 4. Aerial view of cross-type target similar to the one in Stereogram 3. This target was placed in the median strip of a 4-lane highway near a drainage inlet which appears on its right. A moving vehicle appears on the left photograph only. Grass area bordering the target is dark in color tone. Consequently the dark portions of the cross are only slightly darker than the grass. Target proportions and dimen-

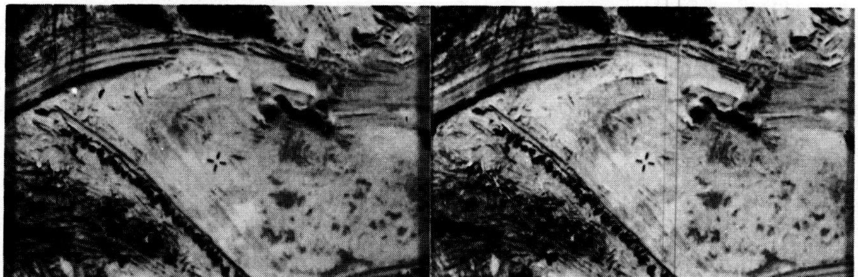
sions are similar to those in Table 1 of the paper. The natural overriding of white on dark tone details, however, causes images of the white legs to appear wider and longer proportionally than the black central portion of the cross.



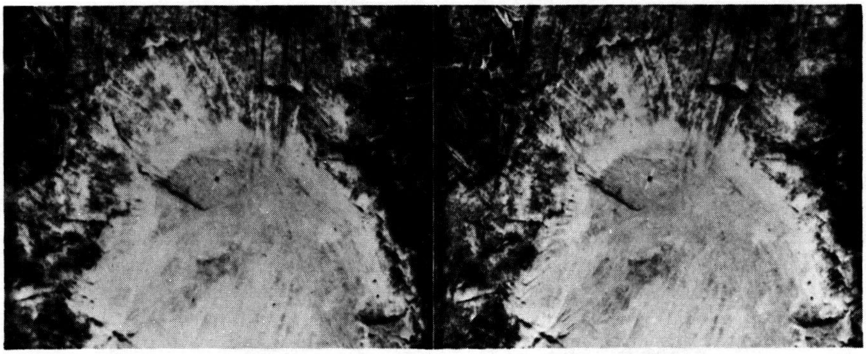
Stereogram 5. Aerial view of cross-type target, similar to Target No. 2 in the paper, placed in deciduous tree area without any clearing being done. Materials are black and white muslin. Magnification will reveal that, while the width of both black and white strips are the same on the ground, the white image is much wider than the black. Symmetry in the target's image is maintained, although this target was set 54 days before the photographs were taken. In the meantime there had been two snowfalls and enough rain to melt the snow.



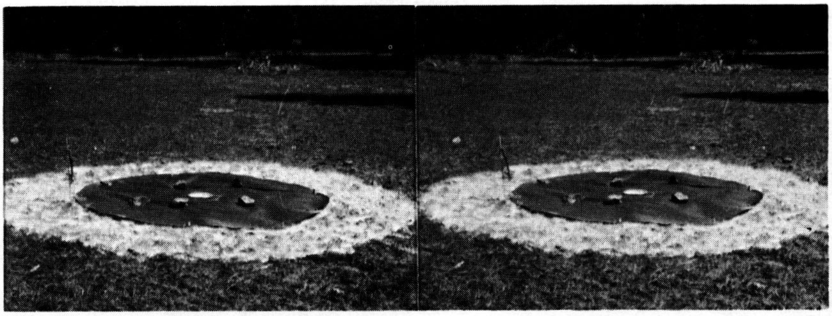
Stereogram 6. Aerial view of two cross-type targets, with white muslin legs. The small target at the left has a central black muslin square the same as Target No. 1, and the large target at the right has a central red muslin cross the same as Target No. 2 in the paper. For these photographs, the small target conforms in all respects with the dimensions in Table 1, and the other is twice as large. The small target was placed 54 days before photography, and the large one 5 days. Weather conditions for the small target were the same as for the target in Stereogram 5, whereas there were only 3 days of rainfall after the large target had been placed.



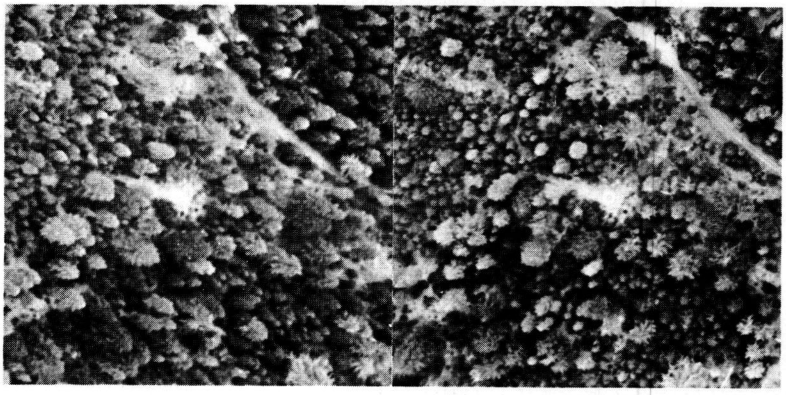
Stereogram 7. Aerial view of cross-type with legs of dark red muslin, and no central portion. Light color tone of ground makes dark legs easily identifiable, permits them to retain their symmetry, and eliminates need for central portion. Over-all ground dimensions of the target are the same as shown in Table 1, but width of dark red legs are twice as wide. On these aerial photographs, however, over-riding of light color tones over the dark red has reduced their black image width one half. A target of all dark legs should not be used wherever dark shadows might be cast by vegetation or buildings.



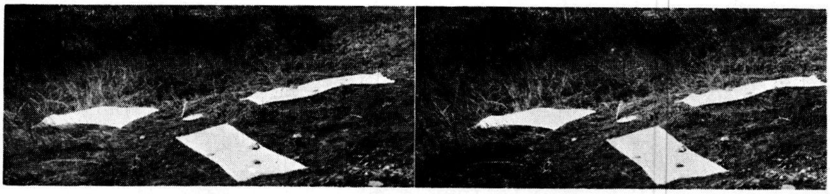
Stereogram 8. Aerial view of cross-type target in light color tone area. The legs are white muslin and the central portion is dark red muslin. On the ground dimensions vary slightly from those given in Table 1, the leg length being $1\frac{3}{4}$ longer and leg width $\frac{1}{8}$ narrower. Light color tone of area prevented the white legs from over-riding adjacent ground. Consequently they appear narrow and faint.



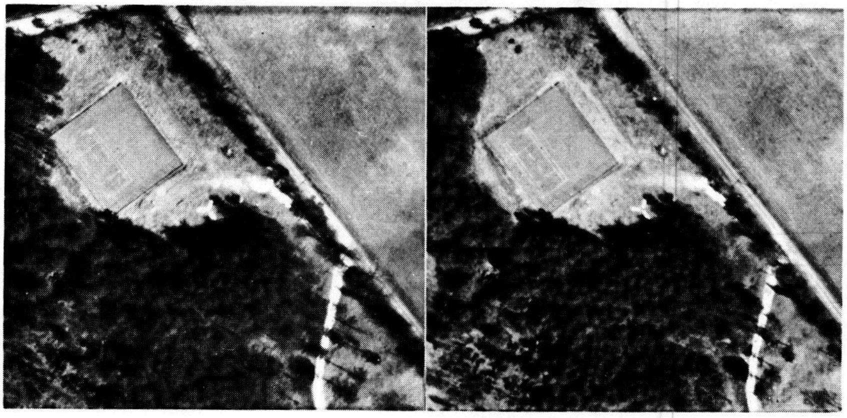
Stereogram 9. Ground view of bull's-eye type target. Station marker is centered by white paper pie plate over black tar paper bordered by white lime. This type of target is difficult to construct unless lime or similar material is used.



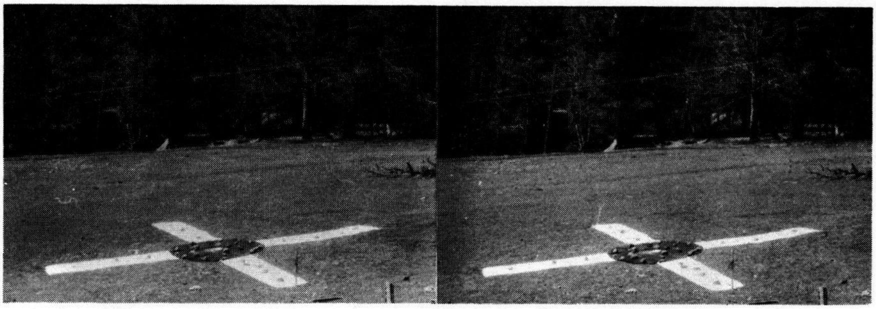
Stereogram 10. Aerial view of bull's-eye type target similar to the one in Stereogram 9. It should be noted that the white lime circle, although uniform in width on the ground, is not uniform in width on the aerial view. This is caused by the variable amount the white has over-ridden the adjacent areas. The central black area, however, is fairly symmetrical; thus centering on the station marker in the photogrammetric operations would be fairly precise.



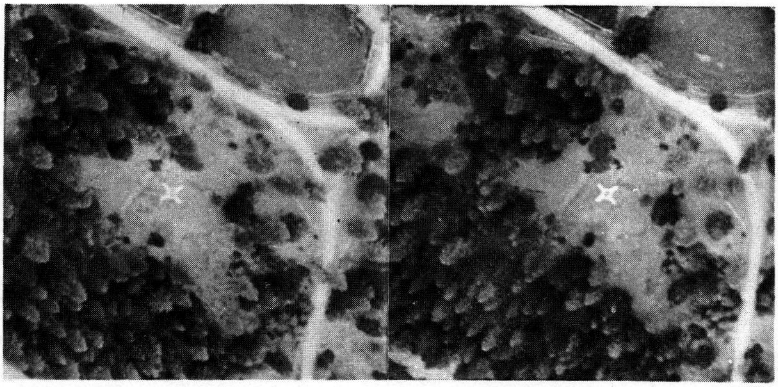
Stereogram 11. Ground view of three-legged target of white muslin. Station marker is centered by white paper pie plate over small triangle of black tar paper which has nearly the same color tone as adjacent ground. The guard stake leans toward the station marker.



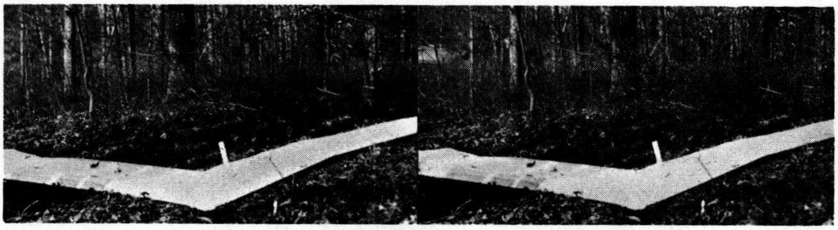
Stereogram 12. Aerial view of three-legged target similar to the one in Stereogram 11. It is difficult to place symmetrically each leg of this target with respect to the station marker. It is therefore not the best type of use, although it is easy to identify.



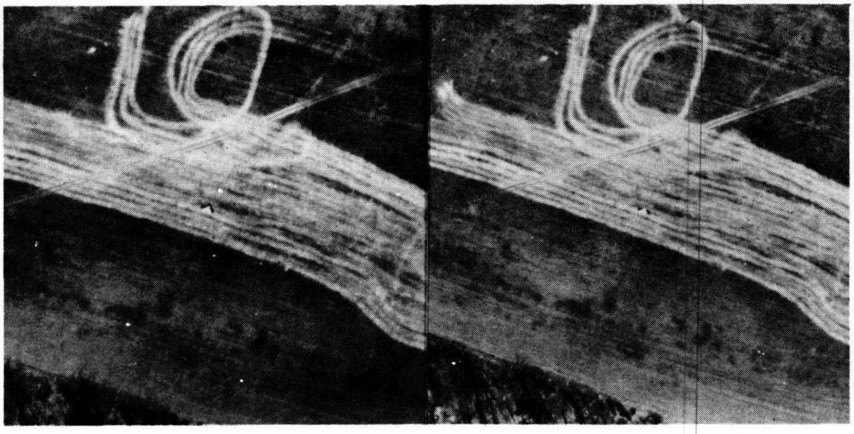
Stereogram 13. Ground view of cross-type target in which the white legs are too wide in proportion to their length and to the size of the central dark portion. Materials used are black tar paper, centered by a white paper pie plate over the station marker, and white muslin.



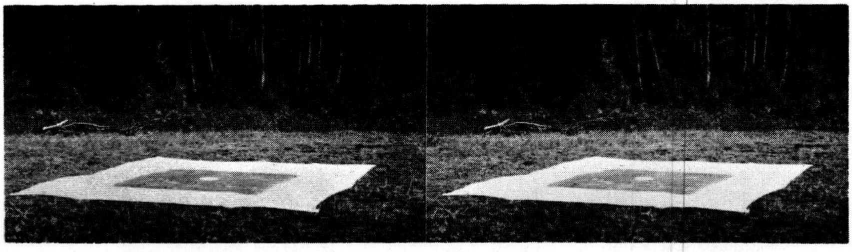
Stereogram 14. Aerial view of cross-type target similar to the one in Stereogram 13. The dominance of white over the adjacent ground and central dark portion of target has caused the image of this target on the aerial photographs to have the appearance that the central black portion was bordered by white when the target was set on the ground. This, of course, is not the case as can be seen in Stereogram 13. Excessive width of white legs in proportion to their length and to the central dark area is evident.



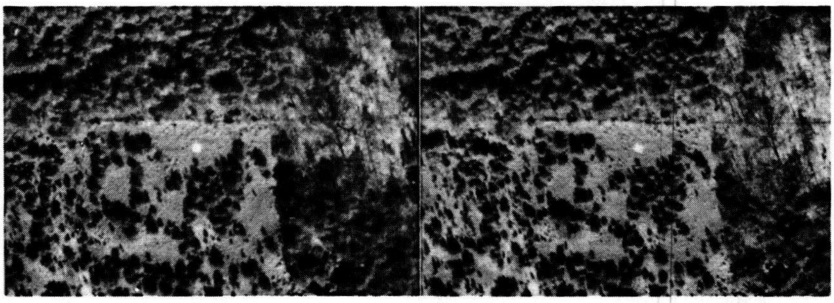
Stereogram 15. Portion of ground view of L-shaped target of white muslin, black omitted. The station marker is at the inside corner of the "L" toward which the guard stake is leaning. On a target of this shape, the over-riding of white on adjacent areas will displace the image corner from true position of the station marker on the ground.



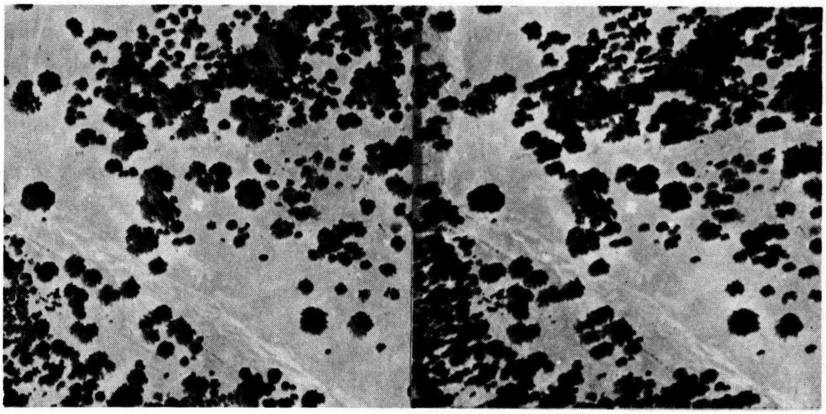
Stereogram 16. Aerial view of L-shaped target of white muslin on inside and black tar paper on outside. Because of light tone of adjacent ground, the tar paper was used in this area to attain sufficient contrast, but this use does not preclude the undesirable over-riding of white and resultant displacement of image at station marker.



Stereogram 17. Ground view of square target of black tar paper on inside, centered by paper pie plate over station marker and bordered by white muslin. Over-riding of white on other images decreases effectiveness of dark center area.



Stereogram 18. Aerial view of square shaped target, similar to the one in Stereogram 17, showing effect of white over-riding central dark portion to such an extent that the symmetry is decreased and center of the target on the station marker is not accurately discernable.



Stereogram 19. Aerial view of cross-type target constructed of white muslin and black tar paper. The white has completely over-ridden the black. This is because of improper proportions, white portions of the cross being too wide and short, and the black too small. The cross appearance has almost disappeared, there is no black to be seen. Consequently the stereoinstrument operator cannot be sure when the floating mark is on the station marker, which was at the center of the cross at the time the target was placed on the ground.

Discussion

W. S. HIGGINSON, Sloan and Associates, Pasadena — Targets for control points certainly remove a source of confusion from mapping projects. This is because they make possible the positive identification of surveyed points serving as control. Thus, they should be used wherever possible.

There will always be mapping projects, however, where it will not be feasible to place targets, and other instances where targets once placed will not be in place when the photography is taken. These situations should not prevent a thorough study of targets and their uses in order to establish a uniform practice in relation to their shape, use and distribution.

It should be realized at the outset that a stereophotogrammetric "bridge" is dependent on all points that are read in the traverse. Many of these points must be natural image points, since it would be impossible and impractical to attempt to locate the proper position of all pass points before the photographs are taken. This does not mean that since all bridge points or pass points cannot be ideal that it is not worth while to use targets on existing control or wherever control coordinates will be determined.

The design of targets may be based on personal opinion. As a result, targets may be of various shapes and sizes (crosses, L-shape, V-shape, T-shape, squares, circles, and other designs), any of which may be used to identify a particular point on the ground. Considerations leading to uniformity and the universality of these designs is well worth while. The diagram of targets as submitted by the author should produce excellent results. However, there are some features of these designs that could be eliminated and simplified without impairing their effectiveness. In this respect, a plain circle is suggested. The circle has no points, intersecting lines, or anything to scatter light rays, which usually results in exaggerated line dimensions or a diffused line between the dark and the light tone colors. The use of a circle eliminates all design problems except the diameter of the circle. The weight of the line could be heavy enough to make it visible in any photographic range; that is, for high altitude very wide lines and larger diameters would be used and for low-altitutde photography these dimensions could be reduced. The degree of accuracy with which one can set a pointer in the center of a circle depends on the diameter of that circle and this value can be determined easily before the photographic mission.

Random targets would not be an economical practice because "bridge points" or pass points must be selected within a rather limited area, and this area could never be determined previous to the photographic flying. A limited number of random targets would be of value in that it would provide a check of ground identification of points that were not targeted.

As Mr. Pryor has pointed out and clearly illustrated, contrasting tone values for markers and background are essential for positive identification within the accuracies desired in the large scale mapping. Information of this nature should be made use of in planning for all mapping projects.

It should also be pointed out that for almost any target, other than the circular type, it is necessary to cover the ground marker itself with the target material. This situation may not be too serious. It may be necessary, however, for the field engineers to examine, during their operations, the ground points targeted to serve as control for the mapping.

WILLIAM T. PRYOR, Closure — A few of Mr. Higginson's comments require amplification, inasmuch as the seeming conflict which he presents does not actually exist.

Mr. Higginson wholeheartedly accepts the fact that targets should be used wherever possible because of the certainty with which control points identified thereby can be utilized. He agrees that wherever natural image points are used simultaneously with targeted control, such control enables the photogrammetric instrument operator to localize errors or discrepancies which often occur. Whenever markers of control are not targeted, errors caused by incorrect identification of natural images tied to control are difficult, if not impossible, to reconcile; also, the essential reconciliation becomes costly in terms of the frustrations and time losses.

His suggestion that the design of targets may be based on personal opinion is acceptable to a certain extent—during the experimental use of targets. Personal opinion cannot govern, however, once the best shape and size for targets has been determined according to topographic and ground cover surroundings, and to photography and map scale requirements. Experiments indicate that overriding of white portions on dark portions of targets that are not symmetrical, as L-, V-, and T-shaped targets, invariably causes an image pattern displacement on the photographs with respect to the ground position of the point for which they were placed to identify and accurately mark. The circle- and cross-type targets, when symmetrical, do not contain such displacement. The center they establish, by their symmetry as images on the photographs, provide an accurate "pin point" for the actual position of the surveyed control points they mark on the ground; thus systematic errors resulting from undesirable displacements are avoided.

Experience has shown that it is easier to place the cross-type target than the concentric circle-type target. An exception of course would be the painting of an appropriate "bull's eye" on portable, durable materials such as metal, plywood, or masonite. An important fact to remember, however, is that materials used to construct targets should not have the tendency to cause "glare" from certain sun angles. This characteristic often photographically obliterates to ineffectiveness one of the images of a stereoscopic pair.

Mr. Higginson's comment regarding targets randomly placed seems contradictory, inasmuch as he mentioned in one sentence that their placement would not be an economical practice and in the next that they would be of value. The specific and random placement of targets on surveyed markers and on points for which control is to be bridged, as outlined in the paper, will achieve optimum results for the instrument operators and for highway engineers whose responsibility it is to stake the highway on the ground after it has been designed by use of the photogrammetrically compiled maps. It is believed that Mr. Higginson has overlooked the use benefits of targeted control, which are often greater after the mapping has been completed than while map compilation is in progress.

Mr. Higginson's thought that each target, except the circular type, must cover the marker is not in agreement with the author's experience. As necessary, any marker need not be covered by the target. Targets of any shape placed symmetrically over the marker can easily be center-opened, as desired, wherever the marker has been covered by the target. This can be done during the targeting operation or whenever the marker has to be occupied during ground survey work.

To complete or amplify some parts of the paper the following comments are offered:

Targets, as markers of survey control, can be as durable or perishable and removable as land use, climatic conditions, and engineering requirements demand. Targets constructed of cloth have provided satisfactory contrast and shape on photographs taken two months after placement of the targets, although they had been subjected to three heavy snowfalls which were melted away successively by heavy rains before weather was suitable for photography. In other cases, targets of painted plywood provided excellent photographic images in regions having a humid climate, where rainfall occurred periodically for six months or more before suitable photography was obtained.

In arid regions, lime or white cement and blackened lime or cement, distributed within the boundaries of a target templet, has been effective after rain, although it is not as resistant to the effects of weather as cloth, plywood, metal, and durable materials. The principal advantage of lime and cement is that animals do not destroy targets constructed of such materials to the same extent they will destroy targets constructed of wood, cloth, or plastic.

Photogrammetric Map Accuracy

L. L. FUNK, Photogrammetric Engineer, California Division of Highways

Highway engineers using large-scale photogrammetric mapping for detailed design work require certain standards of horizontal and vertical accuracy for both the control surveys and the mapping. Vertical accuracy of mapping, as represented by contours, is the most difficult to attain and is generally the greatest source of trouble.

The California Division of Highways is making a statistical analysis of the vertical accuracy of photogrammetric mapping obtained under contract. The information is derived from a comparison of field elevations with elevations interpolated from contour maps. Field elevations generally consist of a profile of the final line as staked on the ground. The data as developed for each project include the arithmetic mean, standard deviation, calculated C-factor, and a comparison of the error frequency distribution with the theoretical error or probability curve. The C-factor is calcuated from the theoretical contour interval which would comply with the 90 percent specification requirement as determined from the error frequency distribution.

This study is not a test of a particular type of plotting equipment under carefully controlled or ideal conditions. It is, rather, an evaluation by the map user of the accuracy of photogrammetric mapping obtained by contract under normal working conditions. For this reason the effect of the allowable horizontal shift is discussed but is not included in the results.

Analyses have been completed on several projects mapped for 2-ft contours at flying heights of from 1,500 to 2,100 ft. Results in most cases indicate close agreement between the error frequency distribution and the theoretical curve. The calculated standard deviations agree remarkably well with the 90 percent spread, indicating the validity of the statistical approach. In two cases values of the arithmetic mean indicate systematic errors in the mapping. The results show that C-factors of 1,000 or more should be used with extreme caution in planning for 2-ft contour mapping, particularly if the horizontal shift is not included in the specifications.

Study of the data has led to investigation of the more common types of map errors, their distribution, probable cause, and methods of prevention. The checking and investigation of photorammetric mapping by means of a Kelsh plotter have also developed data on spot height accuracy attainable under controlled conditions with this instrument.

● THE PAST few years have seen widespread acceptance of photogrammetry as a means of obtaining large-scale topographic maps for the design of major highway facilities. To be suitable for this purpose the maps must have sufficient horizontal and vertical accuracy that the facility, designed from the terrain as depicted by the maps, will fit the actual terrain when staked in the field.

The highway engineer planning to use such maps for computation of earthwork quantities is particularly interested in their vertical accuracy as represented by the contours. This type of accuracy is the most difficult to attain and is generally the greatest source of trouble. Attempts to obtain information concerning the probable accuracy of large-scale mapping, reveal the almost total lack of data on the subject.

Many engineers believe that photogrammetric mapping should be considered as a professional service with methods and equipment limitations left to the mapping contractor. The question of negotiation versus competitive bidding is outside the scope of this paper. The fact remains that a large volume of photogrammetric mapping for highway design is being obtained by competitive bids. The matter of price is also the controlling factor in many negotiated contracts.

At the present time photogrammetric mapping is a rapidly expanding, highly competitive field. There is a shortage of trained personnel particularly at the higher levels. New firms are entering the field, many of them without realizing the technical knowledge and experience required. These conditions frequently result in equipment ratios being stretched to the limit and field control reduced to a minimum in order to obtain work at a reasonable profit.

Many firms actually know very little about the accuracy of the maps they are producing. Acceptance by the contracting agency is frequently assumed to be proof of the specified accuracy. It is questionable whether satisfactory mapping can be assured under such circumstances without specifications which limit equipment ratios and require a definite amount of photo control. Development of factual data is needed so that highway engineers will know more about map accuracies actually being obtained and the specifications required to assure the desired accuracy.

The California Division of Highways is making a statistical analysis of the vertical accuracy of photogrammetric mapping obtained under contract. The information is derived from a comparison of field elevations with elevations interpolated from contour maps. The data as developed for each project include the arithmetic mean, standard deviation, calculated C-factor, and a comparison of the frequency distribution with the theoretical error or probability curve. The calculated C-factor is derived from the theoretical contour interval which would comply with the 90 percent specification requirement as determined from the error frequency distribution.

Data developed at the time the maps are checked for acceptance are generally insufficient to form an adequate statistical base. For this reason the analysis is generally not made until field elevations from a profile or slope stakes of the final line are available. The study is intended as an evaluation of the accuracy of photogrammetric mapping obtained by contract under normal working conditions. It is not a test of the absolute accuracy limits of a particular type of stereoplotter or photogrammetric system under carefully controlled conditions.

OBJECTIVES OF THE STUDY

Some of the broader aspects of the study include: adequacy of the present 90 percent within one-half contour interval specification; need for the horizontal displacement in determining contour accuracy; practical C-factor limitations for planning large-scale mapping for compilation in a Kelsh plotter; and the causes of map errors and possible methods of prevention.

Are Present Mapping Specifications Satisfactory?

National Map Accuracy Standards, which are the basis for most photogrammetric mapping specifications, require that: "Vertical accuracy, as applied to contour maps on all publication scales, shall be such that not more than 10 percent of the elevations tested shall be in error more than one-half the contour interval. In checking elevations taken from the map, the apparent vertical error may be decreased by assuming a horizontal displacement within the permissible horizontal error." For maps at scales of larger than 1:20,000 this permissible error is 0.033 in. It is generally assumed that error frequency distribution of photogrammetric mapping, within the limits of plus and minus one-half contour interval, follows the theoretical error or probability curve (1).

It is contended, however, by some writers (2, 3) that standard deviation, which is a measure of dispersion of the entire range of errors, would be a better method of specifying map accuracy. Information concerning the actual distribution of errors in mapping obtained under present specifications is needed to determine their adequacy.

Effect of the Horizontal Shift

The allowable horizontal shift of contours has the effect of lowering vertical accuracy as the steepness of slope increases. It is troublesome to apply, and the engineer using the map wants the same accuracy throughout. A recent memorandum of the U.S. Bureau of Public Roads states that the horizontal shift tolerance is not applicable to contours on large-scale topographic maps and should be omitted from the specifications. It is desirable to determine the effects on the accuracy and cost of photogrammetric mapping before eliminating the horizontal shift from the specifications.

Practical C-Factors for Large-Scale Mapping

The accuracy of photogrammetric mapping is closely related to the flying height. The relationship is frequently expressed by the term "C-factor" which is defined as the flying height divided by the contour interval. It should be understood that the C-factor is dependent not only on the type of stereoplotter but on many variables (5). However, with other conditions being equal, it is customary to consider that each type of stereoplotter has a certain C-factor. As over 90 percent of the design mapping for the California Division of Highways is compiled on a Kelsh plotter with a 6-in. focal length, it is the only instrument considered in this study.

There is little if any data available as to the C-factor attainable for contour intervals as small as 2 ft compiled in a Kelsh plotter under actual working conditions. Altenhofen (6) has stated that Geological Survey experience in small-scale mapping has indicated a C-factor range of from 850 to 1,000 for the Kelsh plotter. Struck (7) gives a value of 1,100 to be decreased 30 percent under poor conditions and increased 30 percent under favorable conditions. Based on tests of spot heights in a single model, Trorey (8) estimates a range of 1,000 to 1,500. Pennington (9) states that tests indicate a value of approximately 1,200. Pryor (10) has listed 1,200 as the value customarily employed.

It is believed that all of the above writers except Pryor had contour intervals of 5 ft or greater in mind. Harman (5) has stated that: "The C-factor of any plotting system will increase when the flight height decreases." This view is shared by most photogrammetrists. There is some reason to doubt that this opinion takes into account the effect that minor irregularities in ground surface and light growths of grass or weeds will have on the accuracy of 2-ft contours as compared to their effect on 20-, 10-, or even 5-ft intervals.

Present mapping specifications of the California Division of Highways do not directly specify a C-factor. However, the specified plotting ratio of 1 to 5 for a 6-in. Kelsh plotter has the effect of requiring a C-factor of 750 for 2-ft contours mapped at 1 in. = 50 ft. Many photogrammetric mapping organizations are optimistic about their ability to produce accurate maps at C-factors of 1,200 and even 1,500. The Kelsh Instrument Co. now makes a plotter with a ratio of 1 to 7 from photo scale to map scale. This instrument would allow a flying height of 2,100 ft for 1 in. = 50 ft mapping, with a resulting C-factor of 1,050 for 2-ft contours.

If greater flying heights can be used without sacrifice of accuracy it would result in fewer photographs with more width, less photo control, fewer models to compile and a lower resultant cost. Data on this subject are needed so that highway engineers will have a sound basis for planning large-scale mapping projects.

Types and Causes of Map Errors

Inaccuracies in any system of measurement can be classed as either random errors, systematic errors, or blunders. Random errors can be expected to follow error theory as to size, frequency and distribution. In photogrammetric mapping random errors,

TABLE 1

SUMMARY OF ANALYSIS OF TWELVE MAPPING PROJECTS

Figure	ASC NO.	Points Tested	Flying Height (ft)	Within ½ C.I (%)	Arithmetic Mean (ft)	Standard Deviation (ft)	Calculated C-Factor	90% Error Range (ft)	3.3 Std. Deviations
1	119 Loc. II	433	1,500	92.6	-0.09	0.58	800	1.9	1.9
2	90 Loc. III	472	1,500	88.8	-0.09	0.66	700	2.2	2.2
3	90 Loc. I	605	1,500	80.5	+0.62	0.65	600	2.0	2.1
4	150	224	1,500	88.1	-0.10	0.72	700	2.3	2.4
5	174	339	1,800	86.5	-0.40	0.62	800	1.9	2.0
6	169	185	2,100	76.5	+0.01	0.98	700	3.1	3.2
7	127	326	2,100	83.7	+0.19	0.79	800	2.4	2.6
8	159	528	1,500	83.0	-0.07	0.83	600	2.5	2.7
9	165	760	1,500	79.0	+0.09	0.98	500	2.9	3.2
10	146	356	1,500	91.5	-0.21	0.60	800	1.9	2.0
11	108	409	1,500	94.0	+0.13	0.52	950	1.6	1.7
12	172	484	1,500	91.5	0.00	0.61	750	1.9	2.0

together with small systematic errors which may be impossible to eliminate, determine the basic accuracy of the system. It follows that most of the larger and more troublesome inaccuracies are due to either large systematic errors or blunders.

Fortunately these two types of inaccuracies can generally be isolated and can frequently be traced to assignable causes. The investigation of types, distribution and causes of major inaccuracies is an important phase of this study.

METHODS AND RESULTS

Figures 1 to 12 illustrate the results of an analysis of twelve projects which were mapped at a scale of 1 in. = 50 ft with 2-ft contours A summary of the data is shown in Table 1. These projects represent the work of eight different mapping firms. Although no final conclusions can be drawn at this stage of the study, it is hoped that presentation of these data will stimulate thought and discussion on the subject of map accuracy.

Adequacy of Present Specifications

To test the conformity of the mapping to error theory the frequency distribution of errors has been compiled for each project. The standard deviation has been calculated on the basis of deviations from the arithmetic mean or average error. The horizontal shift, permitted by California specifications, has not been allowed in determining the size of errors. Points in areas of dashed contours where the ground was obscured by cover have not been used.

Frequency distributions have been plotted in cumulative form on arithmetic probability paper. They are plotted so that for any minus error the percent shows in error "more than" and for any plus error it shows "equal to or less than." For example, Figure 1 shows 6.7 percent of the points tested were in error by more than -1.0 ft and that 99.3 percent were in error by +1.0 ft or less. The difference between the two, or 92.6 percent is the percentage in error by not more than ±1.0 ft. The straight lines in Figures 1 to 12 represent the normal law of error distribution. This theoretical probability curve is plotted with 5 percent at -1.0 ft and 95 percent at +1.0 ft or, in other words, the 90 percent tolerance limits of mapping specifications. Various points on the theoretical curve are shown in detail in Figure 13.

Conformity of the mapping to error theory can be judged in several ways, one of which is by visual inspection of the curves. Another method is from the values of the arithmetic mean and the standard deviation. Statistically the value of the arithmetic mean should fall within certain limits dependent on the size of the sample (4). The range of sample sizes for the projects analyzed is from 185 points tested on the mapping shown in Figure 6 to 760 in Figure 9. For such sample sizes the arithmetic mean should be within the limits of ±0.1 ft. Seven of the twelve projects have an arithmetic mean within this range indicating the statistical validity of the approach. Where values are greater than ±0.1 ft the presence of blunders or systematic errors should be suspected. With the theoretical curve fixed by the 90 percent within one-half con-

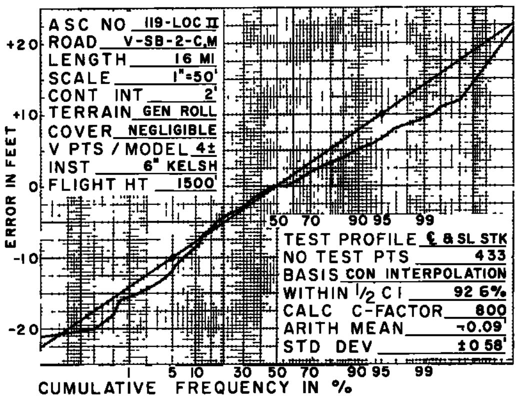


Figure 1.

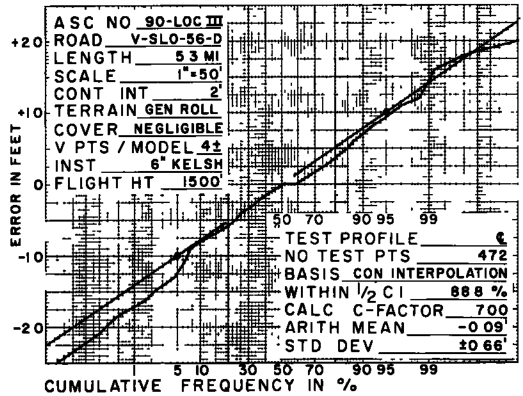


Figure 2.

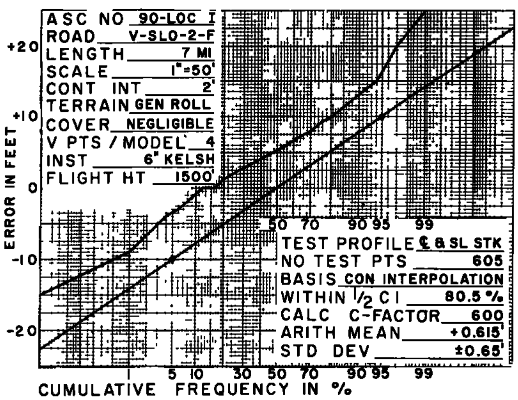


Figure 3.

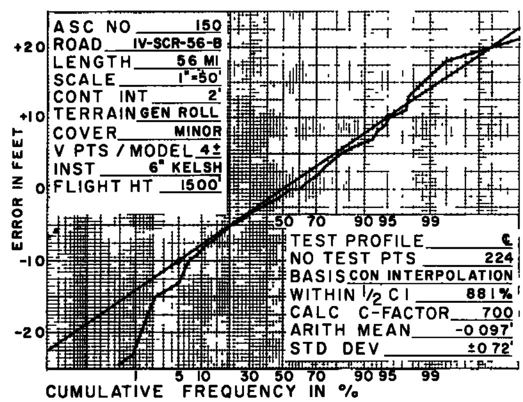


Figure 4.

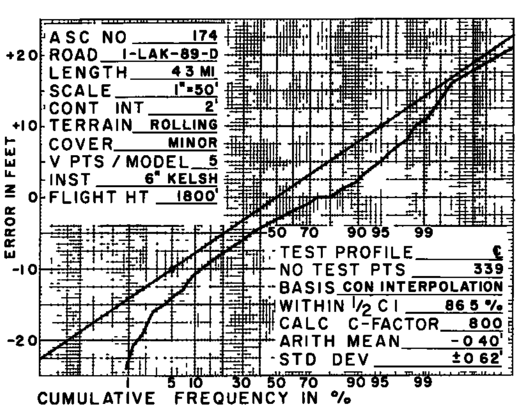


Figure 5.

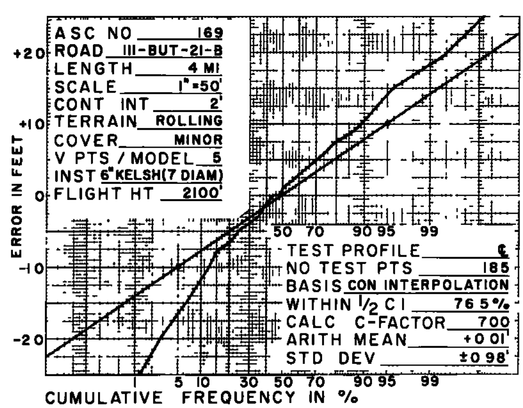


Figure 6.

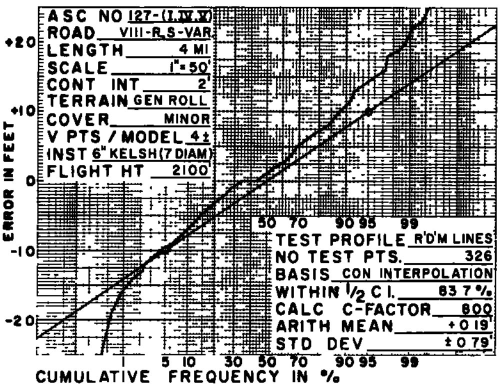


Figure 7.

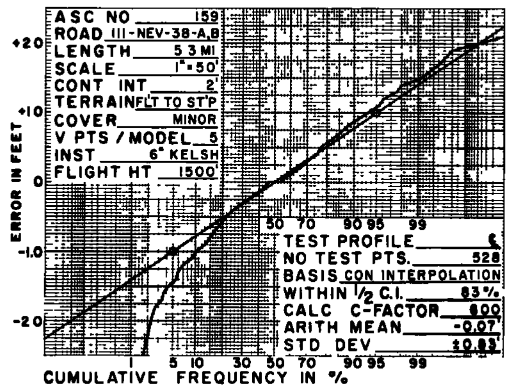


Figure 8.

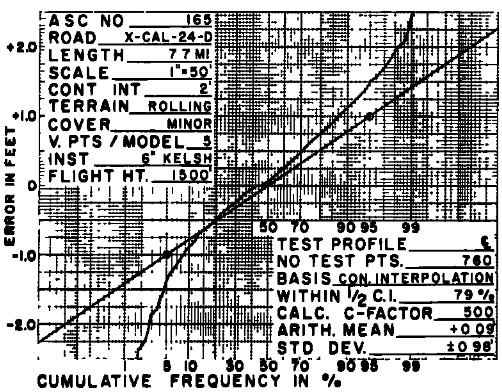


Figure 9.

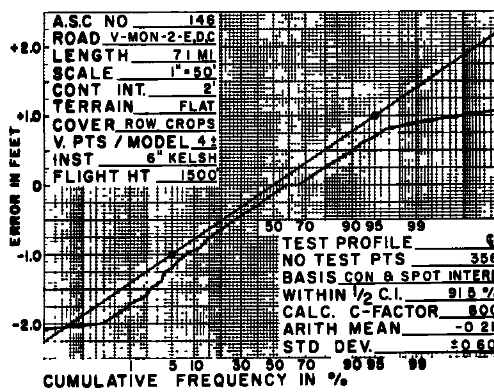


Figure 10.

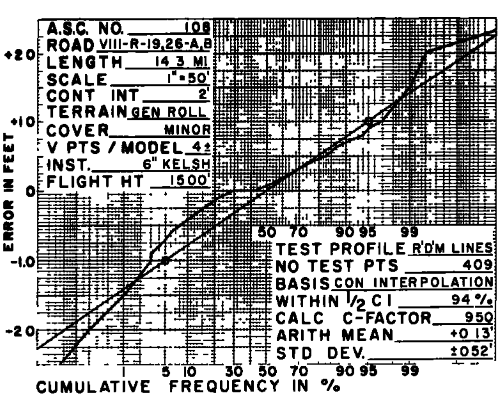


Figure 11.

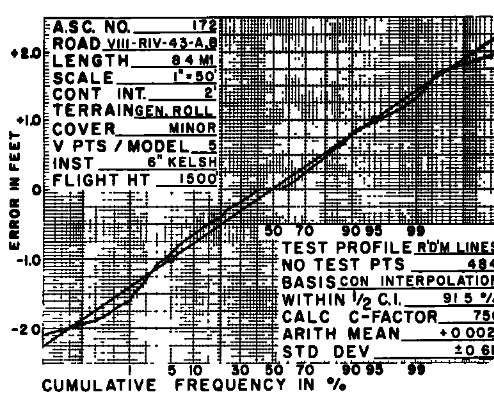


Figure 12.

tour specification, the value of the standard deviation should be 0.3 contour interval. Thus for 2-ft contour mapping it should be 0.6 ft.

Most of the projects show fairly good general conformity. There is a tendency (Figs. 6, 8, and 9) to deviate in the lower portions of the curve due to a disproportionate number of errors in excess of -1.0 ft. This is also shown by the relatively high values of the standard deviations. The mapping shown in Figure 3 follows error theory very closely but on a parallel curve due to a systematic error. The curves are approximately 0.60 ft apart measured vertically, which is the approximate amount of the arithmetic mean. The slope of the frequency distribution curve in Figure 6 is steeper than the theoretical curve for a 2-ft contour interval. It actually approximates the theoretical curve for 3-ft contour interval mapping very closely. This is also indicated by the high value of the standard deviation.

Compilation of the mapping shown in Figures 6 and 7 was done with a Kelsh plotter modified to enlarge seven diameters from photo scale to plotting scale. The flying height was 2,100 ft as compared to 1,500 ft on most of the other projects. California specifications no longer permit use of a plotting ratio of over 1 to 5 for Kelsh plotters.

The poorest conformity to error theory (Fig. 9) is due to the large number of errors in excess of ±2.0 ft. This is best shown by a comparison of the last two columns of Table 1. From error theory the value of the 90 percent point on the curve is 1.65 standard deviations (4). Therefore the 90 percent error range should theoretically equal 3.3 standard deviations. The actual error ranges and the values of 3.3 standard deviations are shown in the last two columns of Table 1. For example, on the frequency distribution curve of Figure 9, the 5 percent point is at -1.4 ft and the 95 percent point is at +1.5 ft or a 90 percent error range of 2.9 ft. This is a variation of 0.3 ft from the value of 3.3 standard deviations. All other projects show a variation of 0.2 ft or less between these values.

Analysis of these twelve projects shows general conformity to error theory and indicates that present specifications are adequate from the practical standpoint. There is, nevertheless, strong reason to question their basic soundness. To illustrate this statement a possible, although highly improbable, frequency distribution is shown on the lower portion of Figure 13. If a map with an error distribution such as this were delivered to a highway engineer he would be forced to accept it under present mapping specifications even though many would consider it entirely inadequate for the computation of earthwork quantities. Although such an extreme case as this might never occur, it cannot be considered good engineering practice to use a specification which could result in a product as unsatisfactory as this.

The standard deviation, by itself, would be equally unsatisfactory as a specification for map accuracy if the statistically correct method of computing it on the basis of deviations from the arithmetic mean were followed. This is illustrated by the values of the standard deviation for the mapping shown in Figure 3 and for the possible error distribution shown in Figure 13. The highway engineer is in a different position from most map users in that he is particularly concerned with the balance between plus and minus errors. It is probable that the most satisfactory specifications from this standpoint would embody both the arithmetic mean and the standard deviation.

Effect of the Horizontal Shift

From the fifth column of Table 1 it is apparent that only four of the twelve projects analyzed complied with the 90 percent within one-half contour interval tolerance. However, the percentages listed do not take into account the horizontal shift which is permissible under California specifications. To determine its effect, the allowable shift was applied to all points in error by more than ±1.0 ft on five of the projects. The effect on the one-half contour interval percentages was as follows:

Figure 5 increased from 86.5 to 93.4 percent
Figure 6 increased from 76.5 to 83.0 percent
Figure 7 increased from 83.7 to 87.7 percent
Figure 8 increased from 83.0 to 90.7 percent
Figure 9 increased from 79.0 to 87.3 percent

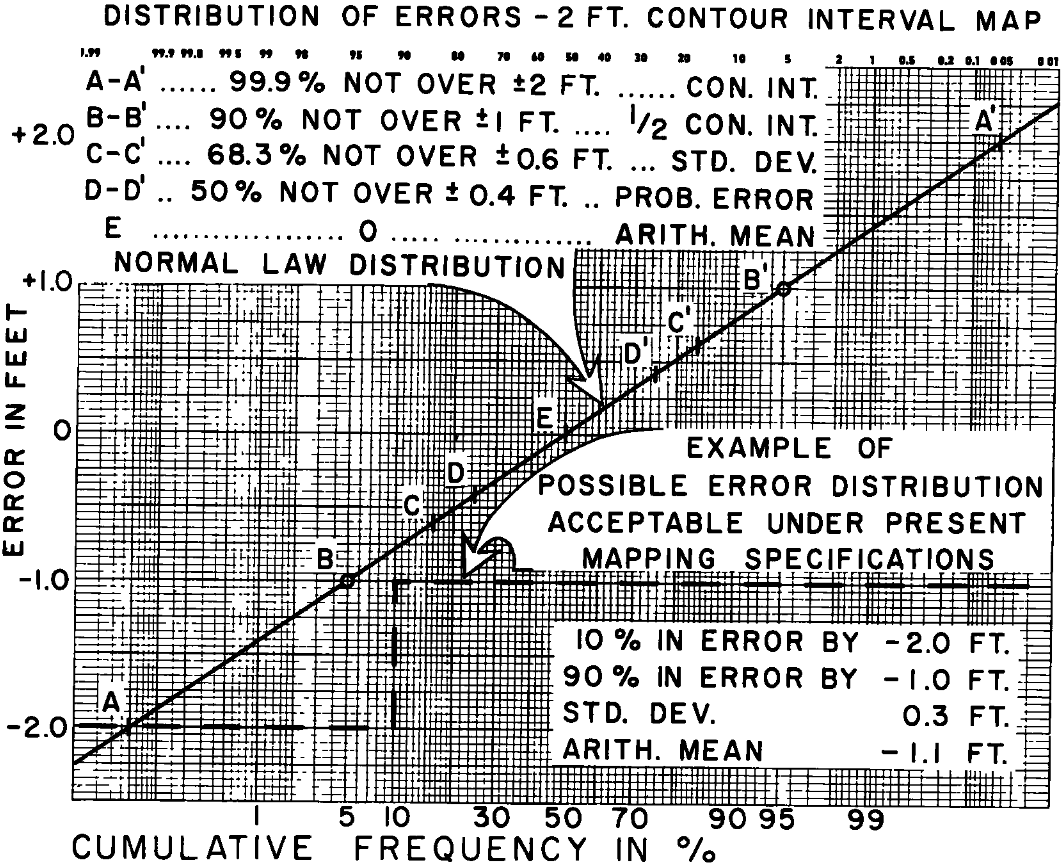


Figure 13.

From this comparison it is apparent that the effect of the horizontal shift in percentage of points within one-half contour interval ranged from 4 percent to 8.3 percent. As might be expected, the amounts varied with the steepness of the terrain. Thus the horizontal shift brought two of the projects within specification limits and would undoubtedly have done the same for the projects shown in Figures 2 and 4. With the effect of the horizontal shift taken into consideration, eight of the twelve projects thus complied with the 90 percent requirement and two more were over 87 percent which might be considered a tolerable variation.

The fact remains however, that many highway engineers want greater accuracy than is afforded by specifications which permit the horizontal shift. The principal causes of map errors in steep terrain as compared to those on flatter slopes are inaccuracies in drafting and generalization of contours by compilers and draftsmen. It is probable that the desired accuracies can be attained by using greater care in compiling and drafting, provided realistic C-factors are used in flight planning.

C-Factor Study

As a means of determining the C-factors actually being attained under working conditions the indicated C-factor has been calculated for each project. The method used is the same as that of the U.S. Geological Survey in their unpublished C-factor studies for small scale mapping. It consists of determining, from the frequency distribution curve, the contour interval which would have resulted in compliance with the 90 percent within one-half contour interval specification. The flying height is divided by this

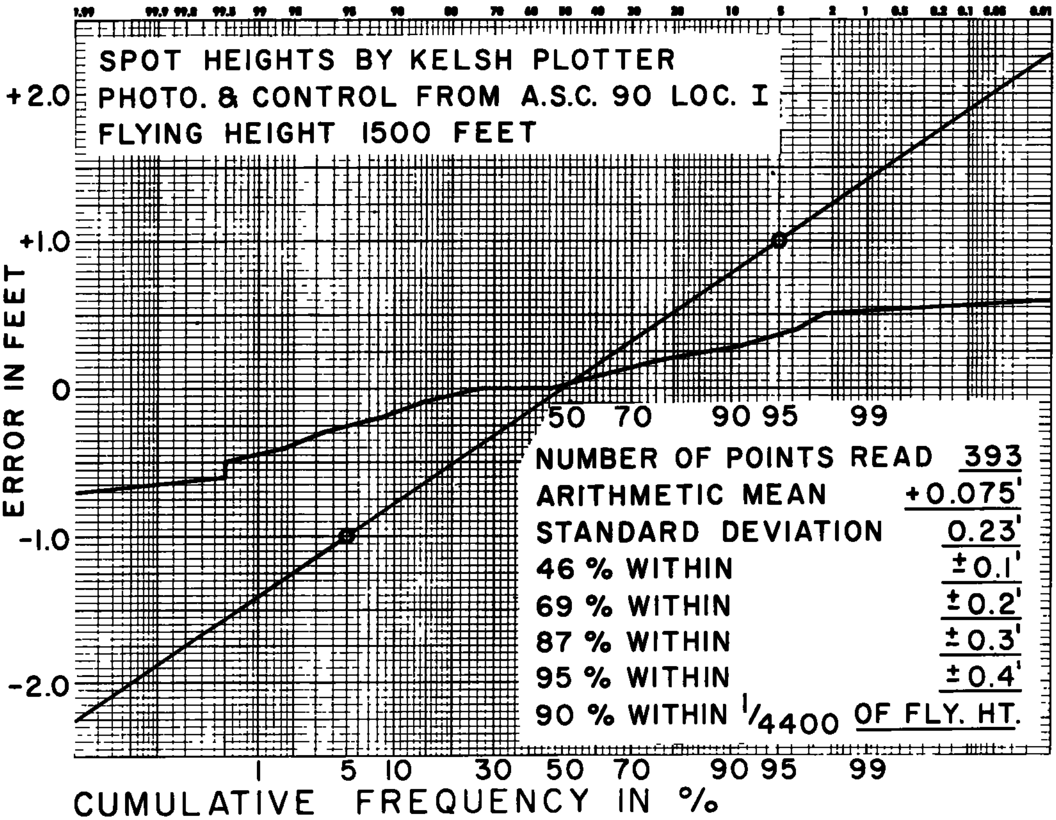


Figure 14.

contour interval to determine the calculated C-factor. The horizontal shift has not been applied to the individual points to determine errors for this study.

The resulting calculated C-factors as listed in Table 1 are considerably lower than those frequently used. As previously mentioned there is little if any published data to support the use of C-factors of 1,200 or 1,500 for mapping with a 2-ft contour interval to be compiled in a Kelsh plotter. Although values such as these can undoubtedly be attained in controlled tests or under ideal conditions, they do not provide any margin for systematic errors or blunders. Sound, conservative practice would indicate use of an operating cushion or factor of safety of at least 1.5 resulting in working C-factors of from 800 to 1,000. The data developed in this study confirm early experience of the California Division of Highways in contracting for large-scale mapping with no limitation on flying height.

Causes and Prevention of Map Errors

In discussing this phase of the study systematic errors will be considered first. Excellent examples of this type of errors are shown in Figures 3 and 5. The systematic error of approximately 0.6 ft on the project illustrated in Figure 3 caused an imbalance of approximately 90,000 cu yd in earthwork quantities. The errors were not disclosed in field checking the maps prior to acceptance, but were found at the time of running the centerline profile immediately prior to construction. The necessary changes in grade line made during construction were both troublesome and costly.

In studying the pattern of errors by comparing centerline and slope stake elevations with map elevations it was found that over one-half of the errors occurred in a 2-mi section. The pattern in individual stereomodels disclosed that errors were largest near the model centers. The general range was from + 1.0 ft to +1.5 ft near the model centers tapering down to +0.2 ft at the photo centers. Several models were set up in the Division of Highways Kelsh plotter and the elevations of 393 points, on which field elevations were available, were read. Diapositives furnished by the mapping contractor were used. Models were set up on the contractor's control without recourse to the additional field data available.

The results of the Kelsh plotter spot height readings as shown in Figure 14 were in close agreement with the field elevations with 95 percent being with ±0.4 ft. This indicated that both the photography and field control were satisfactory. In view of these facts and the intermittent pattern of the systematic errors it was concluded that they were due to malfunctioning of the distortion correction devices in the plotter at the time of compilation.

Present specifications of the California Division of Highways for mapping to be used in highway design require a minimum of three horizontal and five vertical field control points in each stereomodel, with the fifth vertical point located near the model center. Had this specification been in effect at the time the mapping in Figure 3 was undertaken the map errors would have been greatly reduced, if not entirely eliminated. The errors might have been found during field checking if it had been realized that model centers are areas of potential weakness in the mapping.

A less serious but more complex example of systematic errors, possibly combined with blunders, is shown in Figure 5. In this case the mapping complied with specifications after application of the horizontal shift. Three models were set up in the Division of Highways plotter. Spot height readings in these models failed to show consistent agreement with the field profile. Further investigation indicated that several causes contributed to the map errors. The flying height of 1,800 ft above average terrain required plotting ratios of over 1 to 6 in the lower areas. This resulted in reduced illumination and sharpness of focus and in magnification of any calibration errors. The mapping contractor also reported difficulties due to warping of the glass table top during compilation. A third source of possible error was the use of diapositives from distortion-free photography with the emulsion side up, even though the cams were disconnected. Future specifications will require compilation with the emulsion side down when distortion-free photography is used. In this case the centerline was staked in the field and a centerline profile run as soon as the projection was completed and the line calculated. The earthwork quantities can be readily corrected to a close approximation of their true value by raising or lowering the ground line of each cross-section to agree with the field centerline elevation.

In the analysis of blunders the control chart method (4) based on the number of defects per sample was used. Points in error by more than one-half contour interval were considered defects. Individual stereomodels were considered as samples. There is some doubt as to the statistical validity of this approach due to the small number of observations per sample. It is possible that the same results could be achieved by visual inspection. However, the grouping of defects by models is a sound procedure that will afford a clue to blunders in photo-control, photography, and compilation.

The control chart method was applied to errors of over ±1.0 ft on five projects. The following tabulation shows the percentage of total errors of over ± 1.0 ft which were outside the control chart limits and the percentage of the total number of models in which they occurred.

Figure 1:	81 percent of errors occurred in 10 percent of models.
Figure 5:	36 percent of errors occurred in 12 percent of models.
Figure 6:	21 percent of errors occurred in 6 percent of models.
Figure 8:	36 percent of errors occurred in 13 percent of models.
Figure 9:	40 percent of errors occurred in 17 percent of models.
Total:	40 percent of errors occurred in 13 percent of models.

The fact that on these five projects 40 percent of the errors outside the 90 percent specification tolerance occurred in only 13 percent of the models is strong evidence

TABLE 2

COMPARISON OF EARTHWORK QUANTITIES

Project	Contour Interval	Year Mapped	Total Exc. Quantity (Cu Yd)	Difference Between Field Survey and Map Quantities. Excavation (%)	Embankment (%)
I-Hum-1-G	5	1954	1,100,000	3[a]	3[a]
I-Men-15-A	5	1953	1,000,000	2[a]	c
II-Sha-3-C	5	1948	733,500	0.3	c
II-Sha-3-D, C	5	1948	1,750,000	0.8	4 4
III-But-21-B	2	1956	381,400	3.5	0.8
III-But-21-B	5	1954	1,849,200	1.1	1 4
IV-Mrn-1-C, D	5	1951	1,216,500	2.0	1.7
V-SLO-2-A	5	1951	764,400	0.1[b]	c
V-SB-2-J	2	1951	309,500	0.8	c
V-SLO-2-D, C	2	1952	1,013,000	1.9	0.03
V-SLO-2-F	2	1952	538,200	0.9	c
V-SB-2-M, L	2	1953	1,019,000	1.1	c
V-SLO-2-F	2	1955	784,000	2.9	7.6
V-SB-2-F	2	1953	349,200	5.4	3.8
V-SLO-2-PsRs-A	2	1951	935,600	0.9	3.7
V-SB-2-Q, G, F	2	1953	1,733,100	2.3	3.1
VI-Ker-140-D	5	1954	350,000	2.5	c
VII-LA-4-H, I, J	5	1947	1,150,000	1	c
VII-LA-4-LA, F	5	1947		1.5	c

[a] Contour map cross-sections were adjusted to conform to field centerline profile.
[b] Five-ft contours were supplemented by field profiles in level areas.
[c] Not available.

that blunders have a serious effect on the inherent accuracy of photogrammetric mapping. If blunders could be eliminated all measures of map accuracy including the calculated C-factors would be greatly improved.

The mapping shown in Figure 1 is an outstanding example of this. If the one poor model were disregarded the remainder of the mapping on this project would have had 98.2 percent of the points tested within one-half contour interval and a calculated C-factor of 1,100. Investigation of this model showed that it was difficult to get a satisfactory scale solution in the plotter due to sparsity of horizontal control. This was undoubtedly the major cause of error. Present specifications requiring three horizontal control points per model reduce the likelihood of this type of errors.

Two of the poorer models from the mapping shown in Figure 9 were also investigated. They contained 20 points in error by more than ±1.0 ft. Using the contractor's photography and control, the plotter operator read 87 percent of the points within ±0.6 ft of their field elevation and all but one within ±1.0 ft. This eliminated photography and control as a cause of the more serious errors and indicated that they occurred during compilation. In one of the models the errors appeared to be due to poor interpretation, by the compiler, of the height of a 1.5 ft to 3.0 ft growth of weeds or grass.

Checking Photogrammetric Mapping

The frequency of blunders as a major cause of map errors and the distribution of errors by models have an important bearing on map checking. They indicate that effective checking must include tests in every model. Unguided field checking is frequently a guessing game as to how much or what portion of the map to check. Experimental work by the California Division of Highways during the past few years has indicated that checking with a stereoplotter might be the most satisfactory solution. A Kelsh plotter was obtained in September 1956, and has been used for this purpose with results that have far exceeded expectations. It has proven to be the only feasible method of checking every model with a minimum expenditure of time and manpower.

By using the mapping contractor's diapositives, photo control and map manuscripts, an experienced operator can quickly determine how well the control fits and whether or not a satisfactory model setup can be made. This makes it possible to evaluate thoroughly the quality of the mapping. In many cases it is possible to analyze the underlying causes of substandard mapping such as misidentification, improper spacing or incorrect values of photo control, poor photography, poor plotter calibration or unsatisfactory compilation. This type of analysis and evaluation makes it possible to work with the mapping contractors to improve their techniques and will result in better quality of mapping on future projects.

In several instances plotter checking has disclosed serious errors which the contractor has corrected without the necessity for a field check. In other cases it has indicated areas of possible weakness for verification by field checking.

Special mention has been made of the investigation of errors in the mapping shown by Figures 1, 3, 5, and 9. Had a stereoplotter been available for map checking at the time these projects were completed the more serious errors would have been disclosed at once and the map sheets returned to the contractors for correction. The costs due to the difficulties encountered during construction, by reason of map errors, on the project illustrated by Figure 3 were alone more than enough to pay for a Kelsh plotter and a year's operation.

Earthwork Quantity Comparisons

No discussion of map accuracy would be complete without mention of earthwork quantity comparisons, as they are of particular interest to highway engineers. It is obvious that a comparison by percent of error in quantities, while of interest, is not a good test of actual map accuracy. Consider, for example, the frequency distribution of errors shown in the lower portion of Figure 13. If the arithmetic mean of -1.1 ft were applied to a 100-ft roadbed with 2:1 slopes and an average cut of 2.0 ft the resulting error would be 23,000± cu yd per mile, or approximately 56 percent of the quantity. If, however, the average cut was 10 ft, the error would be 30,000± cu yd per mile, but would only be approximately 13 percent of the quantity.

Table 2 shows quantities from photogrammetric mapping, as compared to quantities from field cross-sections. Only two of these projects are among the twelve analyzed for map accuracy. The mapping shown in Figure 3 is for the project shown on the 13th line of Table 2. The mapping in Figure 6 covered a portion of the project shown in the fifth line of Table 2. The total excavation quantity for the portion of the project in Figure 6 was 300,000 cu yd. The excavation differed by 9,000 cu yd, or 3 percent. Embankment quantities differed by only 90 cu yd. This close agreement, even though the mapping was relatively poor, was undoubtedly due to the low arithmetic mean of the errors.

Detailed breakdowns of quantity comparisons by individual cuts and fills were available for several of the projects listed. In most cases the discrepancies in individual cuts and fills were greater than for the entire project. This tends to confirm the evidence that a large proportion of the serious errors in photogrammetric mapping are due to blunders.

CONCLUSIONS

The most important phases of map accuracy discussed in this report are the C-factor study and the investigation of the types, distribution, and causes of map errors. Factual data concerning accuracies actually attained in photogrammetric mapping are needed to develop realistic C-factors for use in planning future mapping projects. Investigation of map errors should lead to methods of preventing them and result in increased map accuracy. Specifications defining methods to be used may be necessary to obtain the desired accuracy due to present conditions in the photogrammetric mapping industry.

As projects designed from photogrammetric mapping are staked for construction they afford a convenient, inexpensive source of information concerning map accuracy. Highway engineers should take the lead in the development of data on map accuracy as a means of increasing the usefulness of photogrammetric mapping.

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The writer wishes to acknowledge the contributions to this study made by Rex H. Fulton, Senior Highway Engineer and George P. Katibah, Supervising Photogrammetrist, of the California Division of Highways. Over a period of several years Mr. Fulton has been responsible for many of the improvements in California mapping specifications as well as for the use of the stereoplotter in map checking. Mr. Katibah has supplied the technical knowledge and supervision necessary for map accuracy investigation and plotter checking.

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Appendix

The basic information necessary for an analysis of map accuracy consists of field elevations and photogrammetric elevations of a number of points. For a valid analysis the measurements should be made in a uniform manner and under similar conditions. The analysis should include as many points as are available and should represent all portions of the mapping.

Minimum criteria for the selection of points require that the field elevations be determined by spirit level and that the ground is not completely obscured by brush and trees as would be evidenced by dashed contours.

Three types of comparative measurements are recognized and are generally tabulated and presented separately:

1. Points with field elevations available are plotted on the map and the photogrammetric elevations interpolated from the contours. The occasional points which fall directly on a contour are not segregated. This type of measurement is presented in Figures 1 to 9, inclusive, and Figures 11 and 12.
2. Points whose field elevations can be compared directly with photogrammetric spot height readings of the same point. The data in Figure 14 are based on this type of measurement.
3. Points with field elevations available are plotted on the map and the photogrammetric elevations interpolated from spot heights such as cross-sections. A portion of the data in Figure 10 was obtained in this manner. This portion was tabulated separately and compared with other data obtained by the first type of measurement. The two tabulations were in such close agreement that they were combined for presentation.

Data processing equipment is utilized in making the analysis. Information furnished the tabulating section for processing includes the identification, field elevation, and photogrammetric elevation of each point. The identification consists of the centerline station and, in the case of cross-sections, the distance right or left. Field elevations are generally supplied by the field notebook with the points to be used circled or checked.

The first step in data processing is to keypunch the identification, field elevation, and photogrammetric elevation of each point on a single card. In some cases where

TABLE 3

1	2	3	4
Error in feet	No. of Points	Cumulative Total	Cumulative (%)
+2.0	1	472	100
1.8	1	471	99.8
1.7	2	470	99.6
1.6	1	468	99.2
1.4	1	467	98.9
1.3	1	466	98.7
1.2	7	465	98.5
1.1	6	458	97.0
+1.0	7	452	95.8
0.9	11	445	94.3
0.8	10	434	92
0.7	11	424	90
0.6	13	413	87
0.5	17	400	85
0.4	20	383	81
0.3	21	363	77
0.2	30	342	72
+0.1	37	312	66
0.0	33	275	58
0.0		242	51
-0.1	33	209	44
0.2	35	174	37
0.3	24	150	32
0.4	27	123	26
0.5	15	108	23
0.6	20	88	19
0.7	22	66	14
0.8	19	47	10
0.9	10	37	7.8
-1.0	4	33	7.0
1.1	4	29	6.1
1.2	5	24	5.1
1.3	2	22	4.7
1.4	7	15	3.2
1.5	4	11	2.3
1.6	3	8	1.7
1.7	3	5	1.1
1.8	2	3	0.6
1.9	1	2	0.4
2.1	1	1	0.2
-2.6	1		0.0

5	6	7	8	9	10	11
	No of Points		Algebraic Σ $[(6)+(7)]x(5)$		Numerical Σ (6)+(7)	$n(e)^2$ $(10)x(5)^2$
Error in feet	+	-	+	-		
0.1	37	33	0.4		70	0.7
0.2	30	35		1.0	65	2.6
0.3	21	24		0.9	45	4.0
0.4	20	27		2.8	47	7.5
0.5	17	15	1.0		32	8.0
0.6	13	20		4.2	33	11.9
0.7	11	22		7.7	33	16.2
0.8	10	19		7.2	29	18.6
0.9	11	10	0.9		21	17.0
1.0	7	4	3.0		11	11.0
1.1	6	4	2.2		10	12.1
1.2	7	5	2.4		12	17.3
1.3	1	2		1.3	3	5.1
1.4	1	7		8.4	8	15.7
1.5		4		6.0	4	9.0
1.6	1	3		3.2	4	10.2
1.7	2	3		1.7	5	14.5
1.8	1	2		1.8	3	9.7
1.9		1		1.9	1	3.6
2.0	1		2.0		1	4.0
2.1		1		2.1	1	4.4
2.6		1		2.6	1	6.8
Total +119 -52.8					$\Sigma e^2 = 209.9$	

$$\text{Arithmetic Mean} = \frac{+11.9 - 52.8}{472} = \frac{-40.9}{472} = -0.09 \text{ ft}$$

$$\text{Std. Dev.} = \sqrt{\frac{209.9}{472} - (0.09)^2} = \sqrt{0.445 - (0.09)^2}$$

$$= \sqrt{0.437} = 0.66 \text{ ft}$$

Calc. C-factor

88.8% = ±1.0 ft
90.9% = ±1.1 ft
90.0% = ±1.06 ft

$$\frac{1500}{2 \times 1.06} = 708 \quad \text{Say } 700$$

either field or photogrammetric elevations have been previously used for earthwork computations they are gang punched into the card. The elevation difference, computed on IBM Machine Type 604, is punched into each card with a credit overpunch to indicate the direction of error. If the photogrammetric elevation is greater the error is considered plus, and if less, minus.

Two tabulations are produced by the data processing equipment:

1. A frequency distribution of calculated differences between field and photogrammetric elevations. The differences are expressed to the nearest 0.1 ft (for 2-ft contour mapping) and plus and minus differences are tabulated separately in ascending order. A sub-total of the number of points is furnished for each 0.1-ft group.
2. A list, in order by station, of all points having a difference of 1.0 ft or more.

Both tabulations show the identification, field elevation, photogrammetric elevation, and difference for each point.

The cards are sorted by elevation difference and the frequency distribution listing is produced on the IBM accounting machine Type 402. All points of the contour elevations whose difference is 1.0 ft. or more are interpolated. Points which have a difference of 3.0 ft or more are examined particular care for gross errors. If there appears to be any possibility of the field elevation being in error, the point is rejected. Also determine the effect of the horizontal shift on points whose difference is greater than 1.0 ft. and study the distribution of large errors by models.

The detailed method of making the analysis for the mapping is illustrated in Table 3. The error groups, to the nearest 0.1 ft, as shown by Table 1 are entered in Col. 1,

Table 3, in descending order from the largest plus error. The number of points in each error group is also taken from Table 1 and is entered in the Col. 2, Table 3. A cumulative total of these points is calculated and entered in Col. 3. It will be noted that the largest minus error of 2.6 ft is not entered opposite -2.6, but rather in the next error group, or opposite -2.1. This indicates that one point is in error by more than -2.1 ft. At the top of this column the total number of points, 472, is in error by +2.0 ft or less.

The cumulative total for each error group is converted to percent and entered in Col. 4. The cumulative percentages in Col. 4 provide an easy means of determining the percentage of the points within any desired error range. For example, 62 percent of the points (85 percent - 23 percent) are in error by not more than ±0.5 ft and 88.8 percent are in error by not more than ±1.0 ft. The percentages shown in Col. 4 are plotted on arithmetic probability paper to show the frequency distribution.

The error groups, without regard to sign, are entered in descending order in Col. 5. The number of plus points in each error group is entered in Col. 6 and the number of minus points in Col. 7. The algebraic sum of Col. 6 and 7 for each error group is multiplied by the size of the error and the result entered in either Col. 8 or Col. 9 depending on the sign. The algebraic sum of the totals of Col. 8 and 9 divided by the total number of points is the arithmetic mean or average error.

The numerical sum of Col. 6 and 7 for each error group is entered in Col. 10. Each numerical sum is then multiplied by the square of the error in feet and the result entered in Col. 11. If the total of Col. 11 were divided by the number of points and the square root extracted the result would be the standard deviation from zero error. However, to provide a true measure of dispersion the standard deviation must be calculated on the basis of deviations from the arithmetic mean. This can be readily done by dividing the total of Col. 11 by the number of points and subtracting the square of the arithmetic mean from the result. The square root of the resulting number gives the standard deviation from the arithmetic mean.

The calculated C-factor for contour mapping is determined by finding the error range in feet which includes a total of 90 percent of the points. Col. 3 is used to determine the 90 percent range. In this case ±1.0 ft includes 88.8 percent of the points (95.8 percent - 7.0 percent). Also ±1.1 ft includes 90.9 percent (97.0 percent - 6.1 percent). By interpolation ±1.06 ft includes 90 percent of the points. The flying height divided by twice this error range is the calculated C-factor. In other words it is the theoretical contour interval which would have complied with the 90 percent specification requirement. In the case of spot heights the fraction of the flying height which includes 90 percent of the points is calculated rather than the C-factor. This method of presentation is illustrated in Figure 14.

After a sufficient number of projects mapped at the same scale and under generally similar conditions have been analyzed, the calculated C-factors will be plotted as a cumulative frequency distribution. This will provide a means for predicting the probability of attaining any selected C-factor for mapping to be undertaken under comparable conditions. It should be a valuable guide in planning future mapping projects.

Statistical terminology and methods of computing the arithmetic mean and standard deviation have been based on A.S.T.M. Manual on Quality Control of Materials, Special Technical Publication 15-C. Particular reference is made to pages 13 and 14. Characteristics of the normal law distribution of errors are given in Table 1. The design of arithmetic probability paper is explained in the discussion of a paper by Allen Hazen published in Volume LXXVII of the Transactions of the American Society of Civil Engineers, December 1914, page 1666.

THE NATIONAL ACADEMY OF SCIENCES—NATIONAL RESEARCH COUNCIL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The ACADEMY itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the ACADEMY and the government, although the ACADEMY is not a governmental agency.

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