Influence of Soil Characteristics on Deformation of Embedded Flexible Pipe Culverts

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● BASED ON observations of flexible pipe culverts under earth fills, it is generally conceded that pipe failure may be defined as a deformation (1, p. 70; 3, p. 9-10). Specifically, failure is that deformation beyond which the culvert ceases to function satisfactorily. Under load the horizontal diameter increases and the vertical diameter decreases such that the culvert assumes an approximately elliptical shape with the major axis horizontal. In extreme cases the pipe either collapses or is in a state of incipient collapse at failure. Collapse can only occur after the horizontal diameter reaches its maximum value. Rational design requires that the increase in horizontal diameter should be held below the maximum or critical value. In some installations, it is conceivable that excessive deformation of the pipe might adversely affect the conveyance characteristics of the pipe or might allow the soil above the pipe to settle excessively. Under such conditions, failure could still be described in terms of a maximum allowable increase in horizontal pipe diameter. In this case, the allowable increase might be less than critical, but as for failure by collapse, it is important to predict the increase in horizontal diameter of the pipe as a basis for design. M. G. Spangler of Iowa State College has proposed a formula for doing this (3, p. 29). This formula, often referred to as the Iowa Formula, is as follows:

$$\Delta x = \frac{K W_c r^3}{EI + 0.061\,(er)r^3} \qquad (1)$$

where

Δx = increase in horizontal diameter of the pipe culvert (in.)
K = a parameter which is a function of the pipe bedding angle, α
(K = 0.083 for a bedding angle of 180)
W_c = vertical load per unit length of the pipe at the level of the top of the pipe (lb/in.)
r = mean radius of the pipe (in.)
E = modulus of elasticity of the material from which the pipe is constructed (psi)
I = moment of inertia of the cross-section of the pipe wall per unit length (in.^{4}/in.)

er = E' = a property of the soil which must be constant for any given soil according to Spangler's assumption in deriving the formula. (3, p. 28-29; 5, p. 45)

In this report er (or E') is referred to as the modulus of soil reaction.

The object of the investigation was to determine if the modulus of soil reaction is a function of any of the commonly recognized and easily measured soil properties. Clearly it would be uneconomical to determine values of this modulus from measurements on a test section or model for every culvert installation. Nevertheless, the modulus can be predicted for a specific installation by a test section or even by model analysis (6, p. 581), if the effort is justified. But more satisfactory is the use of model analysis to determine the relationship between the modulus and the more easily measured and more commonly recognized soil properties. This was the specific objective of the investigation.

Experimental data were collected from tests on a model soil-culvert system. A heavy box-like cell was constructed as shown in Figure 1. The ends of the cell were each lined with a rubber pressure diaphragm so that a simulated soil load could be applied by gas pressure. A particular soil sample was selected; then with the cell resting on one end and with the upper end removed, the soil was compacted in the cell under carefully controlled conditions. Since the cell was designed to hold exactly one cubic foot in volume it was an easy matter to determine the density of the compacted soil.

The upper end was then replaced and the cell was laid horizontally so that the bakelite window in the side of the cell was on top (Fig. 2). It was then possible to remove the bakelite window and force a model section of pipe into place in the cell. This operation was very sensitive in that slight misalignment caused considerable variation in the final results. To reduce misalignment a jig was constructed by means of which the model pipe section could be accurately positioned. Placement was accomplished by sucking soil from within the model pipe by a vacuum cleaner while the inside element of a jig bearing the model pipe section was forced downward until the pipe section was in place. The jig also provided for ejection of the model pipe section so that the jig itself could be withdrawn. This jig greatly improved the replicability of results. It was then possible to install a dial indicator gage mounted on ball bearings as shown in Figure 3. With the dial in place, the bakelite window was replaced, gas pressure lines were connected to both ends of the cell which were equipped with rubber diaphragms and the soil within the cell was subjected to gas pressure applied in increments (Fig. 4). With each increment of pressure a corresponding dial reading was taken and the results were plotted as a pressure-deflection curve.

In order to evaluate the modulus of soil reaction, the Iowa Formula may be worked backwards from known deflections and culvert loads to the corresponding value for the modulus. It is shown rewritten below in this proposed form.

$$E' = er = 1.36 \left(\frac{W_c}{\Delta x} \right) - 16.4 \left(\frac{EI}{r^3} \right) \tag{2}$$

(The value for K has been assumed to be 0.083)

For average design the load on the pipe W_c may be assumed to be directly proportional to the height of fill and the diameter of the pipe (2, p. 424); $W_c = C'DH\gamma_t$ where

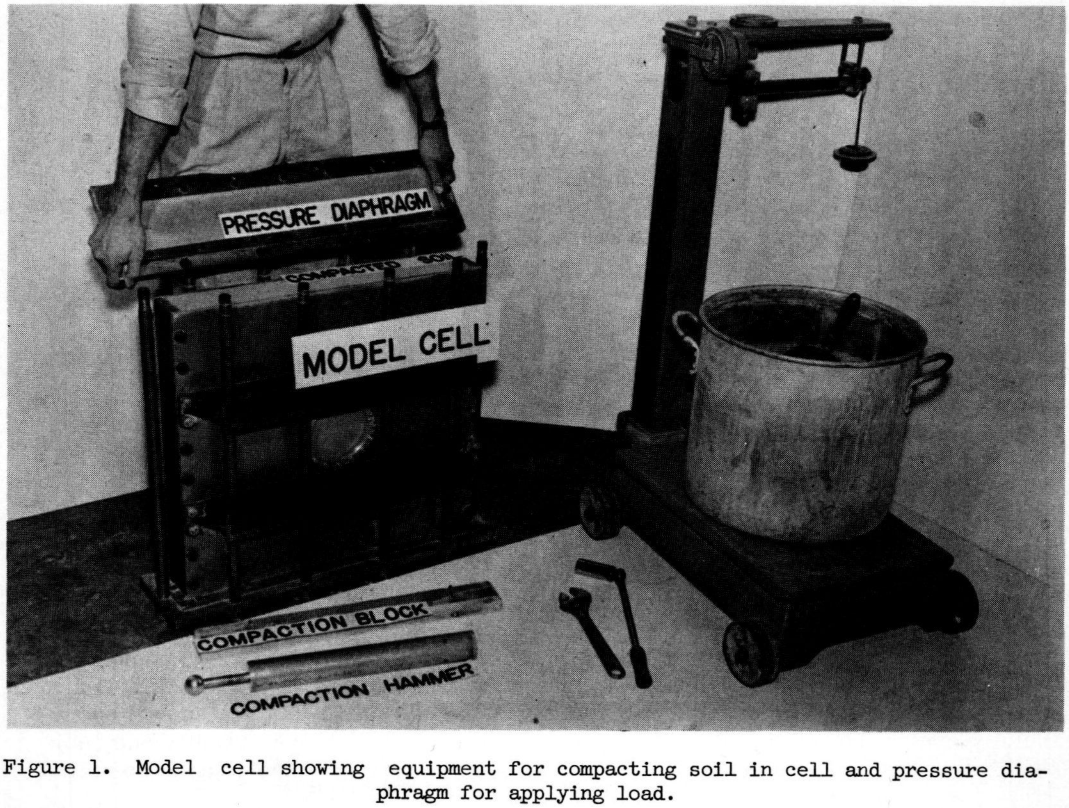

Figure 1. Model cell showing equipment for compacting soil in cell and pressure diaphragm for applying load.

C' = a constant, D = pipe diameter, H is the height of fill above the top of the pipe, and γ_t = a total unit weight of soil. In order to simulate these conditions on a model with the load supplied by an inflated membrane, an equivalent height of fill H is supplied by the pressure in the membrane or $H = \frac{p_v}{\gamma_t}$ where p_v = the vertical soil pressure at the level of the top of the pipe, or $p_v = pC_1$ where

p = diaphragm pressure

C_1 = the ratio of soil pressure at the level of the model pipe to the diaphragm pressure. C_1 is constant for a given soil in a given cell (see Fig. 5).

γ_t = total unit weight of soil.

Now by simply substituting these values in Eq. 2, the modulus becomes

$$E' = C\,D\left(\frac{p_v}{\Delta x}\right) - 131.2\left(\frac{EI}{D^3}\right)$$

where $p_v = \gamma_t H$ and C is the product of various constants.

For analysis on the model cell,

$$E' = CDC_1\left(\frac{p}{\Delta x}\right) - 131.2\left(\frac{EI}{D^3}\right) \qquad (3)$$

This is the Iowa Formula rewritten to evaluate the modulus in terms of pipe diameter D, pipe wall stiffness EI, and the ratio of soil pressure to horizontal deflection $\frac{p_v}{\Delta x}$. Spangler's assumption that E' is a soil constant must be true if $\frac{p}{\Delta x}$ is a constant for any

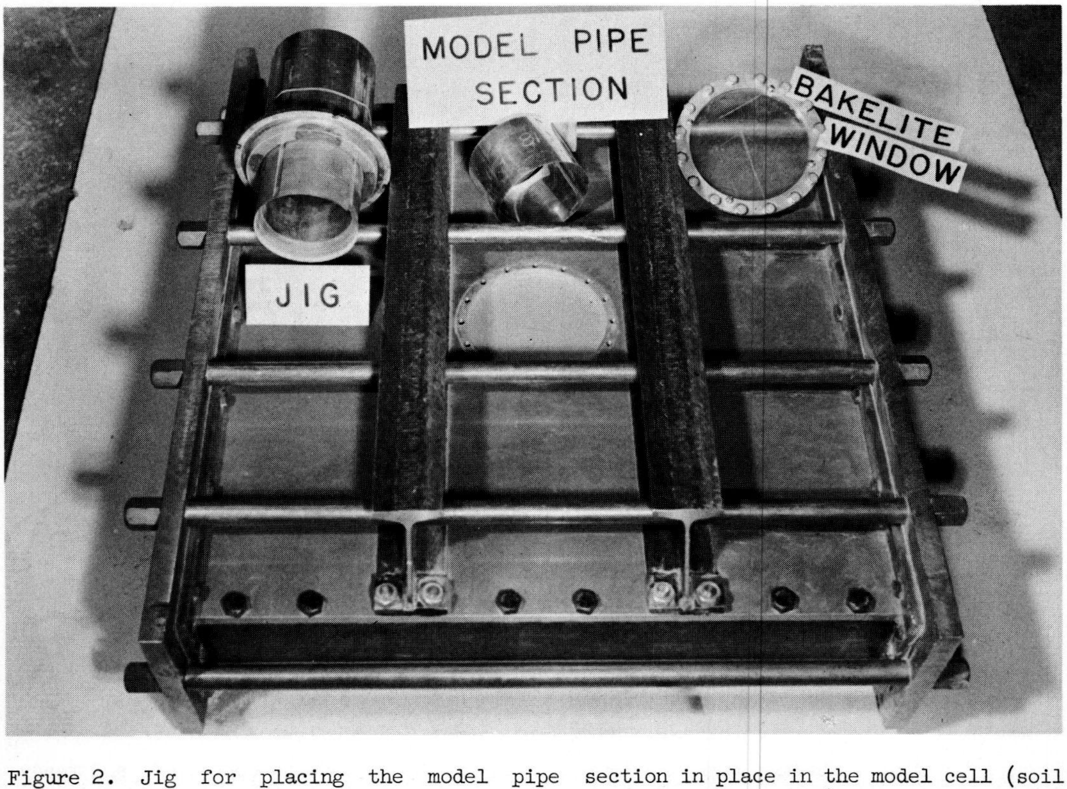

Figure 2. Jig for placing the model pipe section in place in the model cell (soil drawn from inside pipe section by a vacuum cleaner).

given pipe system. A cursory inspection of Figures 6 and 7 shows this to be the case.

When the pipe wall stiffness, EI, is varied, model tests show that the effect on E' is so small as to be negligible. This result is demonstrated by a typical test (see Figure 8) for which the following table applies:

TABLE 1

VALUES OF MODULUS OF SOIL REACTION, E', AS A FUNCTION OF CULVERT WALL STIFFNESS, EI FOR WHITE SILICON SAND

EI	E' (psi)		
	for C=1.36[a]	for C=0.95[a]	for C=0.88[a]
59 lb in.	2380	1780	1550
16.6 lb in.	2420	1810	1560
3.22 lb in.	2450	1810	1550

[a]C_1 = 0.18 in Eq. 3 for this particular model cell.

The table shows that E' is independent of pipe wall stiffness EI within the accuracy of the component measurements.

It may be concluded that the modulus of soil reaction for a flexible culvert is dependent on the soil only and may be assumed constant for any given soil. The dimension of E' lends further credence to this conclusion, for the dimension (lb in.$^{-2}$) is the same as the dimension for modulus of elasticity which is a property of the material only. If the modulus of soil reaction is similar to the modulus of elasticity, it should be possible to evaluate it by a simple stress-strain test such as the compression test.

It is interesting in Table 1 to note that even though E' is independent of EI, its

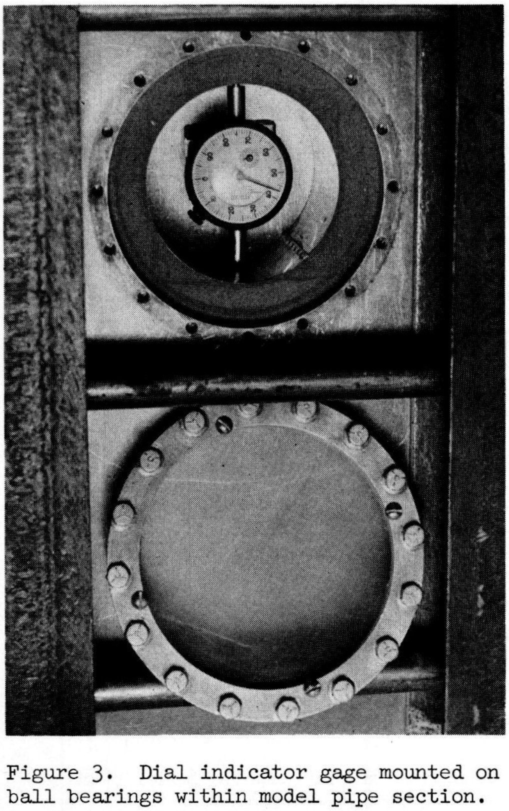

Figure 3. Dial indicator gage mounted on ball bearings within model pipe section.

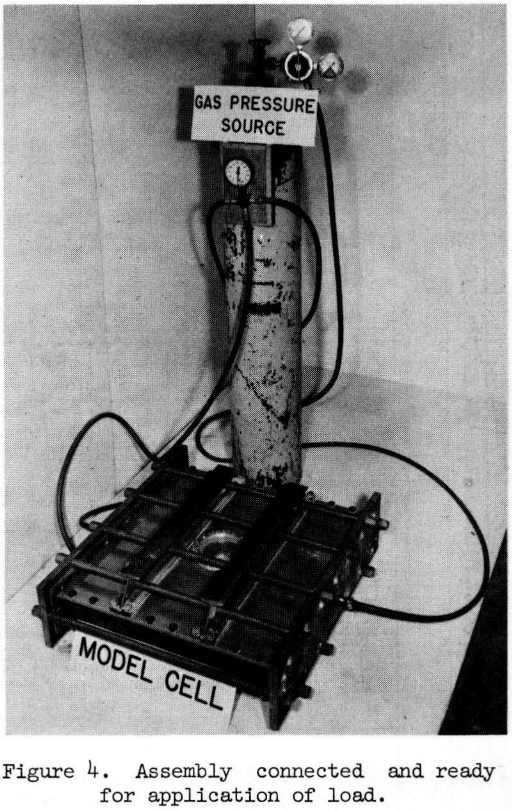

Figure 4. Assembly connected and ready for application of load.

absolute value is highly dependent on the constant C which in turn is dependent on a number of hard-to-evaluate factors such as the bedding angle which is unknown for a flexible pipe, the extent of projection condition or ditch condition, etc. For the design of most culverts under high fills, it appears advantageous to solve for both C and E' by

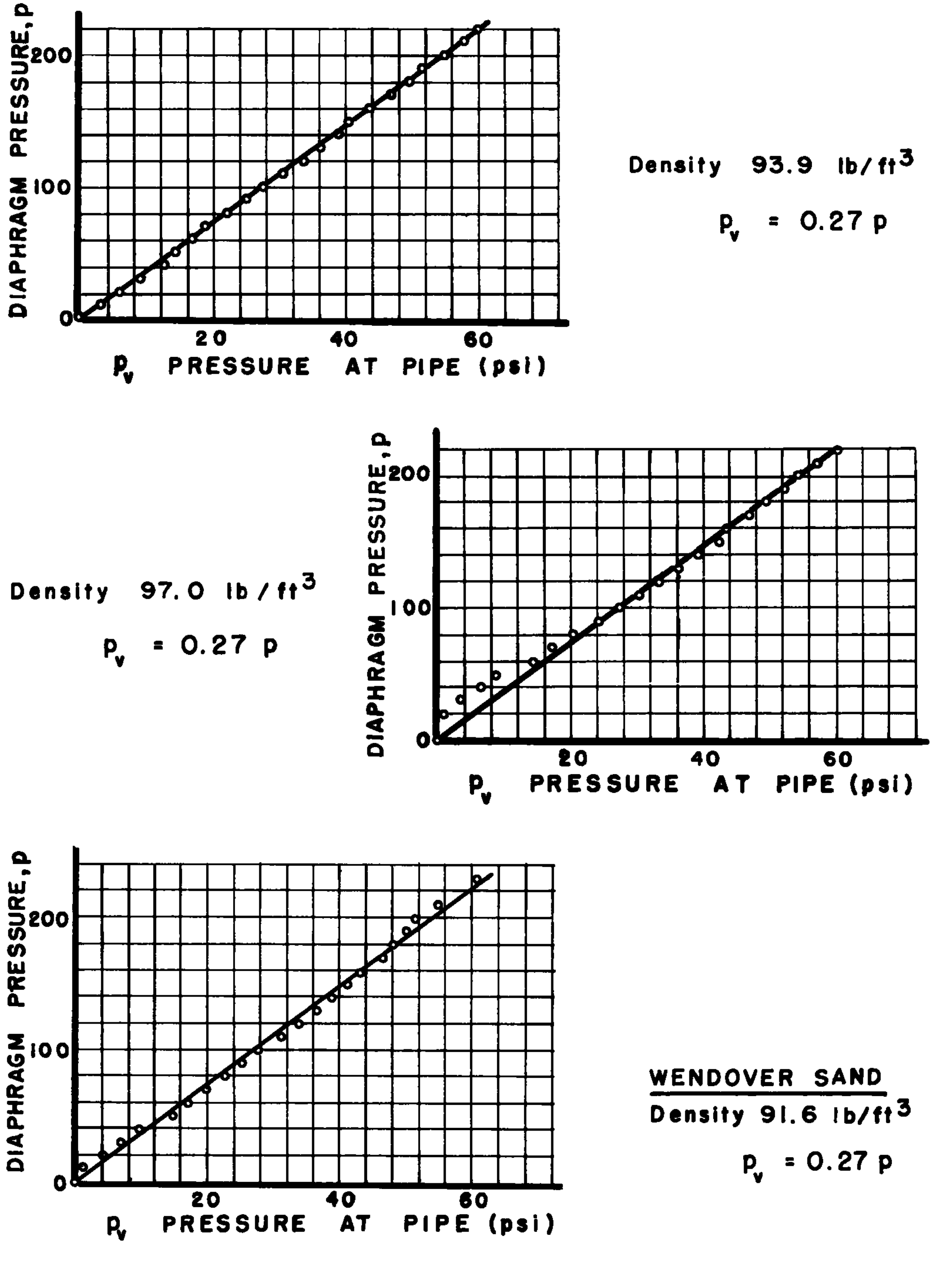

Figure 5. Diaphragm pressure required to develop soil pressure at the level of the top of the model pipe in Wendover sand at various densities.

plotting two or more model study curves varying EI (or D) as in Fig. 8, then by evaluating C and E' between any two curves by use of Eq. 3. If the resulting values of C and E' are used to predict culvert deflection the Iowa Formula becomes:

$$\Delta x = \left(\frac{C\, p_v}{E' + 131.2 \frac{EI}{D^3}} \right) D \qquad (4)$$

The soil constant problem has now been expanded to an evaluation of both C and E' for any given soil.

Evaluation of C

As is evident from Table 1, the constancy of E' is not sensitive to C. Even if a slightly incorrect value of C is used, the resulting calculated value of E' will still give good results in predicting Δx. A physical interpretation of this might include the following:

1. Flexible culvert will smooth out stress variations in the soil. For example, the bearing angle on the base will approach 180 deg as the culvert deforms regardless of the initial bearing angle.
2. The flexible culvert will tend to eliminate the difference between the settlement of the soil above and below it so that stresses tend to become symmetrical above and below the pipe. The model cell is designed on this assumption. Actually the assumption is not arbitrary because on the more carefully controlled projects, the flexible pipe is bedded on a good blanket of fill material.
3. The influence of pipe wall stiffness on the deflection is very small, therefore

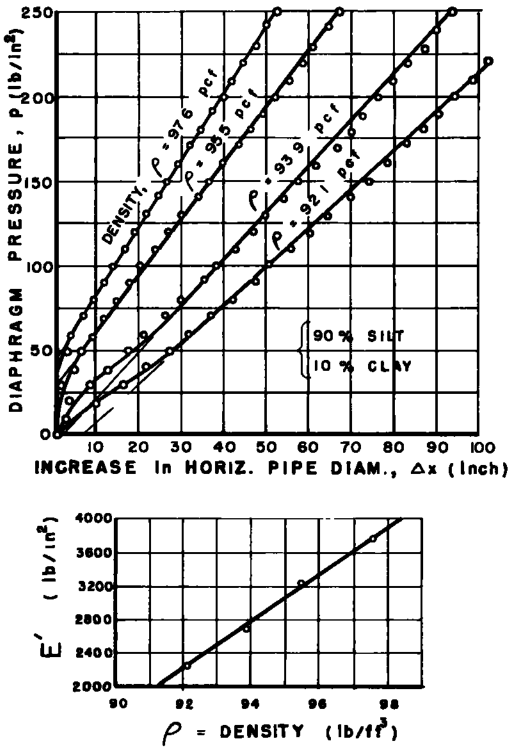

Figure 6. Pressure-deflection diagrams and E -density diagram for a mixture of 90 percent Bear River silt and 10 percent Trenton (Utah) clay.

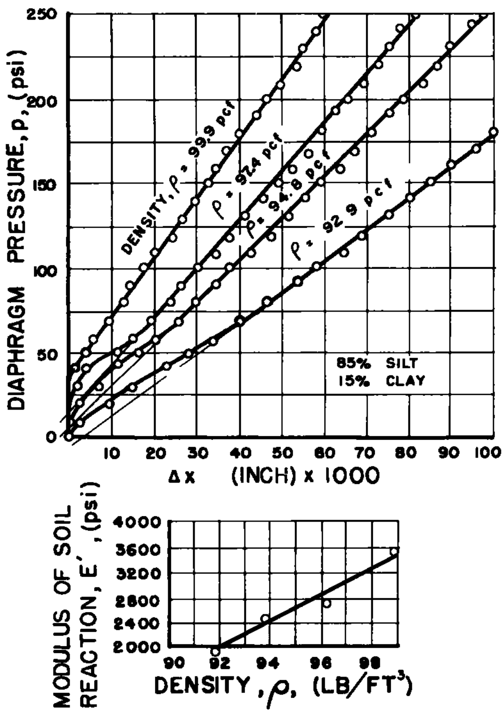

Figure 7. Pressure-deflection diagrams and E -density diagram for mixture of 85 percent Bear River silt and 15 percent Trenton (Utah) clay.

Δ x is determined largely by the ratio of C to E' (see Eq. 4) rather than the absolute value of either C or E'. For example, in Table 1, E' varies from about 1,500 to 2,500 psi while $\left(131.2 \frac{EI}{D^3}\right)$ varies from about 7 to 130 psi. Clearly Δx is more dependent on the ratio C: E' than the absolute values of C and E'.

It has been found that C may vary according to general soil types, but for a given type, it is relatively insensitive to other conditions such as compaction, stress, etc. For the sand analyzed in Table 1, either a value of C = 0.9 or C = 1 is good. A value of C = 1 has been assumed for the next paragraphs in which E' is investigated. The results appear to lose no accuracy by the assumption.

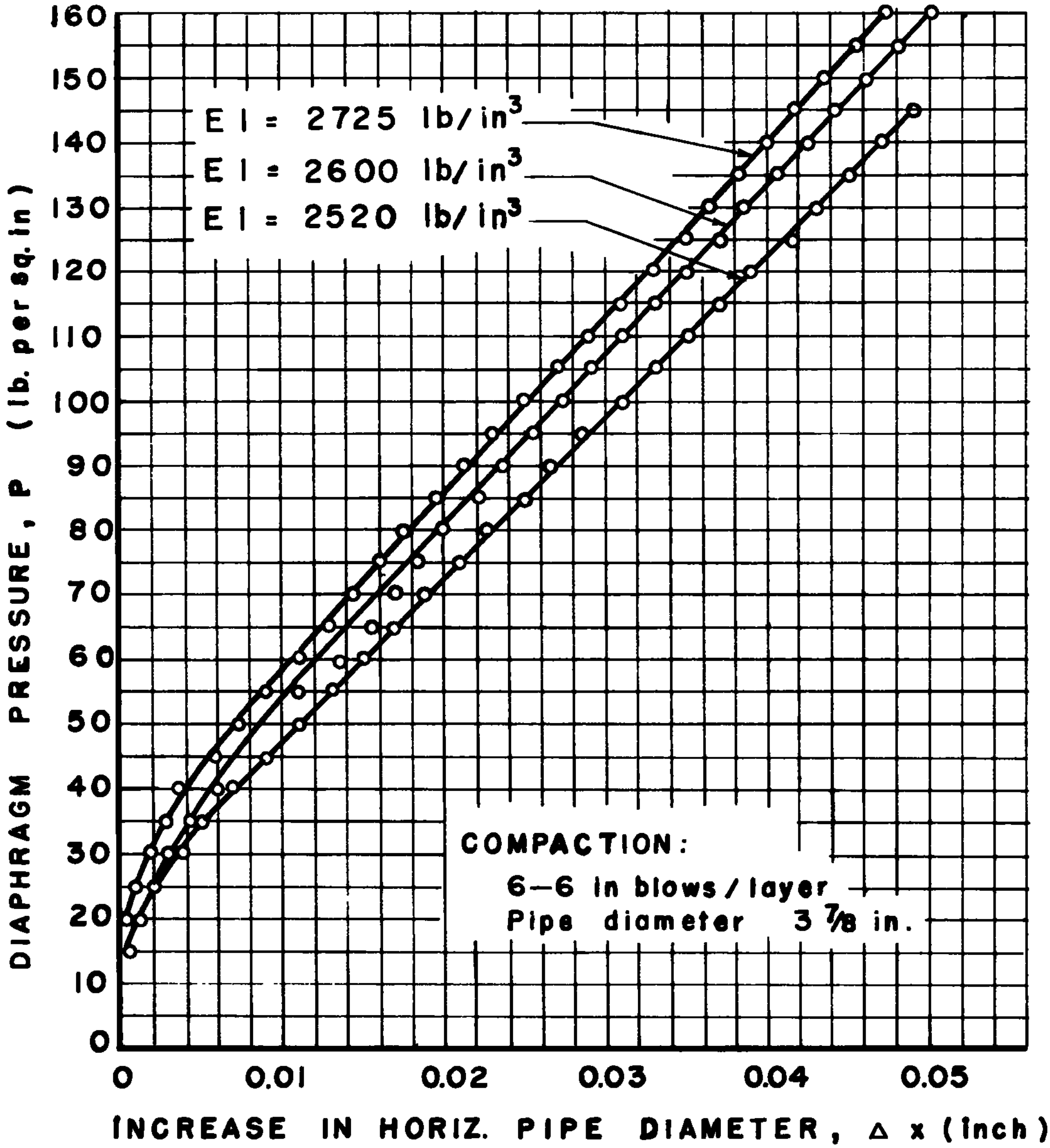

Figure 8. Pressure-deflection diagrams for white silica sand at varying values of the pipe wall stiffness factor, EI.

Evaluation of E'.

The modulus of passive resistance then remains the basic soil constant which must be determined. The most obvious conclusion from the pressure-deflection diagrams (Figs. 6 and 7) is that the soil density or degree of compaction is an important factor in determining E'. It appears in general that a linear relationship exists between soil density and E'. Some typical plots are shown in Figs. 6 to 9. Values of E' at low densities are the most likely to deviate from linearity, but this is due to void ratios in excess of critical and such low densities should possibly not be acceptable in culvert design. Critical void ratios will be discussed later.

Figure 10 shows how the slope of the E' density plot varies as a function of various soil types. In this case the soil types were synthesized by combining varying percentages of Bear River silt and Trenton (Utah) clay. E' was calculated by use of Eq. 3 with an assumed value of C = 1. A value of C_1 for the particular model cell was found to be 0.25 regardless of compaction and for all soil types tested in the series. This was determined by inserting an inflated innertube at the level of the top of the pipe, but without the pipe in place. The pressure in the innertube was measured as a function of load pressure in the membrane (see Fig. 5), then the ratio was calculated. The basic purpose of Figure 10 is to demonstrate how E' may be related to a compression modulus of soil as determined by simple compression tests.

Compression tests were run on silt-clay combinations. From compression tests it is common practice to plot the results as void ratio versus $\log_{10}$ of intergranular pressure. Such a curve usually plots as a straight line (4, p. 217). The slope is the compression modulus.

It should be noted that void ratio is an indication of the decrease in thickness of the sample, so it is an indication of strain in the sample. A compression curve for soil is effectively a stress-strain diagram with the stress plotted on a log scale so that the resulting plot will be a straight line. Just as the slope of the stress-strain diagram within the range of elasticity is the modulus of elasticity, so the slope of the compression diagram is in effect a compression modulus. That it is related to the modulus of soil reaction, E', for culverts is easily demonstrated. Figure 9 shows the modulus, E', as a function of soil density; therefore, it is necessary to plot the compression diagrams as a function of density likewise. This is possible since density is inversely proportional to void ratio and may be determined directly from it. Ordinarily the negative slope of the compression curve is computed and is designated as the compression index, C_c. This is defined as the rate of change of void ratio (or strain) with respect to the log of stress. Such an index is just the inverse of the type of quantity which one would define as a compression modulus. A plot of the compression modulus, $1/C_c$, as a function of soil type is shown in Figure 11. It is immediately apparent that the E'-density diagram and the $1/C_c$ diagram are related. As a matter of fact, the relationship is almost 1000 to 1 for the particular case shown; or

$$1000\left(\frac{1}{C_c}\right) = \frac{d\,(E')}{d_\rho}$$

By integrating the equation above

$E' = \left(\frac{1000}{C_c}\right)\rho$ + constant. When E' = 0, the constant $= \frac{1000}{C_c\,\rho_o}$ where ρ_o is the effective density of the soil at no soil pressure.

$$E' = \frac{1000}{C_c}\,(\rho - \rho_o)$$

The limits in the above equation are found from Figure 9. It would be very difficult to measure the density of the soil at no intergranular stress, but Figure 9 reveals that the value can be determined approximately from the intersection of the E' density lines with the E' = 0 line. It should be pointed out again that the low density values of E' are not actual since linearity ceases at low densities. For this particular diagram it is evident that ρ_0 varies within a rather narrow range of about 86 to 88 pcf depending on the soil. For any particular soil type a value of ρ_0 can be used as a constant.

So far this analysis has only been applied to sand, silt, and the combinations of silt and clay described above. There is a need to expand the analysis to include other soils to verify the theory. Also there is a need to evaluate the numerical factor relating E' to a soil compression modulus. The value of 1000 used here seems to be adequate for these tests, but there may be some variation in other soil types.

Figures 6 and 7 show a peculiarity that is common to most load-deflection diagrams. For the high density tests, the initial portion of the plot is concave down in such a fashion that the "y-intercept" for the straight line portion of the plot is a positive pressure. For the low-density tests, on the other hand, the initial portion of the plot is concave up and the "y-intercept" is a negative pressure. Tests show that this is true of the prototype as well as the model. The best explanation seems to be based on the concept of critical void ratio. If the soil is very loose, shearing strain or differential movement causes the soil grains to densify. But any densification tends to relieve the intergranular stresses. In order to maintain equilibrium, the stresses must be developed

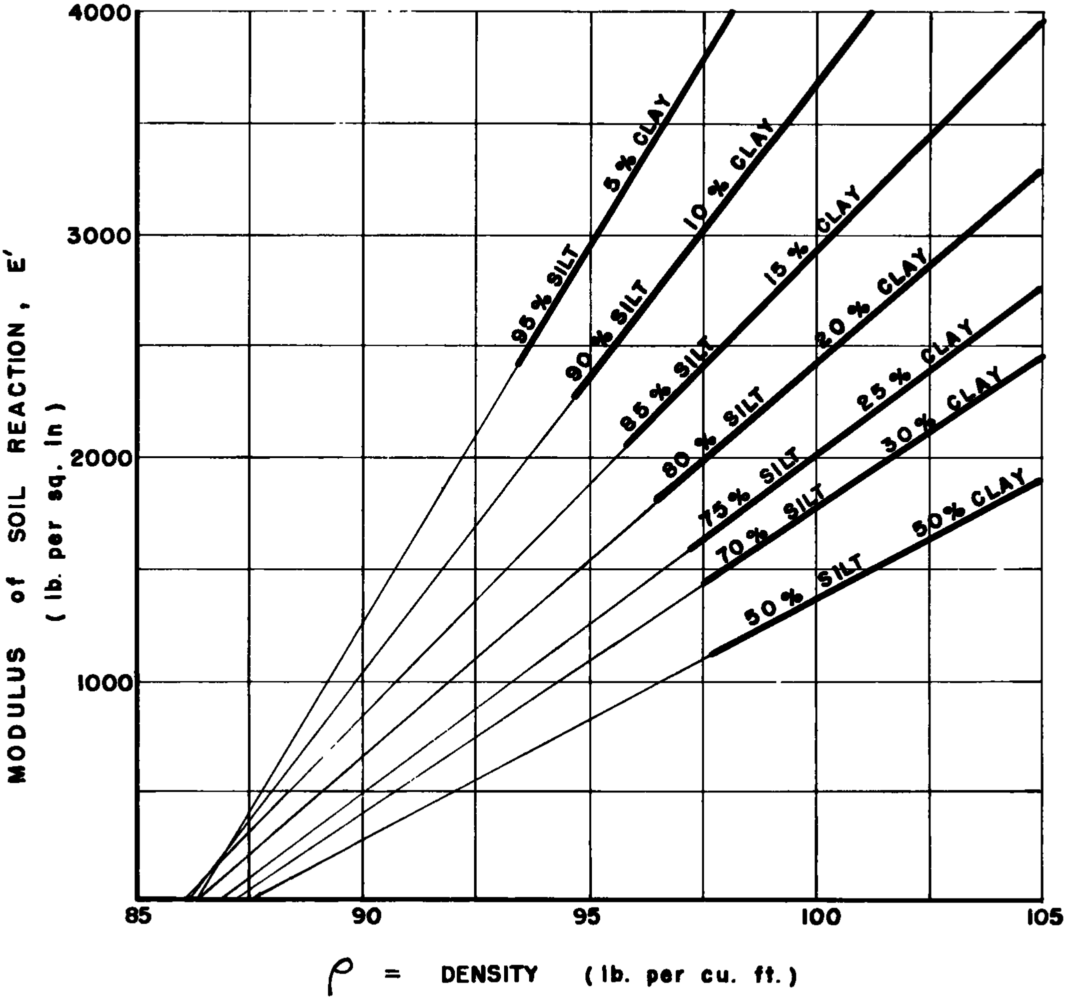

Figure 9. Plots of the modulus of soil reaction as a function of the soil density (so called E -density curves) for various combinations of Bear River silt and Trenton (Utah) clay.

by increased strains; so the relationship of strain to stress is greater while the soil is loose. On the other hand, if the soil is very dense, shearing strain causes the soil grains to roll over each other and to increase the volume of the soil mass. But such tendency to increase volume is resisted by the adjacent soil grains and excessive stresses are developed. For equilibrium these soil stresses must be relieved, but that can only be accomplished if the strains are relatively less. Of course, after the soil has generally shifted, the volume reaches an equilibrium point and the load deflection curve continues on as a straight line. The specific volume at which there is neither a tendency to increase nor to decrease during shear may be described in terms of critical void ratio. If the void ratio of the soil exceeds its critical value (that is, if the density is less than its critical) the actual deflection of the culvert will be greater than predicted. Since this is on the unsafe side in culvert analysis, a sure way to avoid this problem is to specify that the fill shall be of greater than critical density. With a density greater than critical, the reverse is true and the actual deflection of the culvert is less than predicted. This is on the safe side. If the project is of such magnitude that the deflection must be predicted more accurately, then the Iowa Formula should be modified to include a term which accounts for the "y-intercept" in the load-deflection curves.

The water content of soil was not a variable in the above considerations. Since the purpose of most culverts is to carry water, the soil adjacent to the culvert will probably be saturated part or all of the time. Work is yet to be done on the effect of water content, but it should be pointed out that most fills are placed dry or at optimum moisture content and most of the pipe deflection occurs during construction. For granular soil around the pipe, water does not alter the soil characteristics other than the vertical soil pressure, p_v which has no effect on E'. For a plastic soil the conditions of failure are entirely different. As the soil is saturated the pressures on the pipe approach hydrostatic pressures. In such event, the pipe would not continue to increase in horizontal diameter, but would tend to become circular again. For this case, some method of defining failure should be sought which is not based on deformation alone.

On the evidence that the modulus of passive resistance is a property of soil only, and that it can be readily related to the compression index for a given soil, it should be possible with a minimum of additional tests to arrive at the required relationships so that the deflection of a culvert can be predicted from a simple compression test in the laboratory.

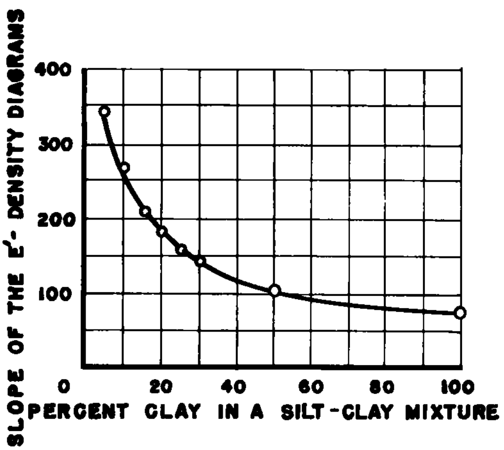

Figure 10. Slope of the E -density curve as a function of percent silt.

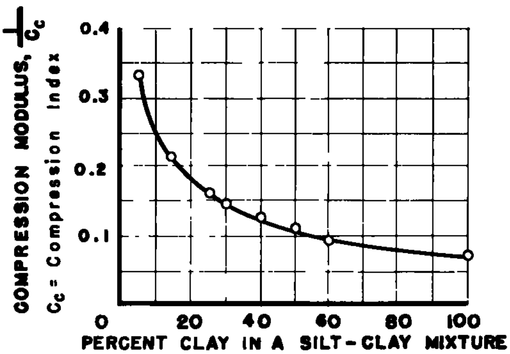

Figure 11. Compression modulus, $\frac{1}{C_c}$, as a function of the percent silt where C_c is the compression index of the soil as determined by a compression test.

Note: The shapes of the plots in Figs. 10 and 11 are so nearly the same as to show that the modulus of soil reaction, E', can be determined from the compression index of a soil.

REFERENCES

1. "Handbook of Drainage and Construction Products." Armco Drainage and Metal Products, Inc., Middletown, Ohio, (1955).
2. Spangler, M.G., "Soil Engineering." International Textbook Co., Scranton, Pa. (1951).
3. Spangler, M.G., "The Structural Design of Flexible Pipe Culverts." Iowa State College, Eng. Exper. Sta. Bull. 153 (1941).
4. Taylor, D.W., "Fundamentals of Soil Mechanics." John Wiley and Sons, New York.
5. Watkins, R.K., "Characteristics of the Modulus of Passive Resistance of Soil." Unpubl. Ph.D. Thesis, Iowa State College Library (1957).
6. Watkins, R.K., and Spangler, M.G., "Some Characteristics of the Modulus of Passive Resistance." HRB Proc., 37:576 (1958).

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