Frost Penetration: Relationship to Air Temperatures and Other Factors

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● THE FREEZING AND THAWING of soils and the detrimental effects on roads and pavements was one of the pertinent problems of highway engineers which led to the establishment of soil divisions in many of the state highway departments located in the frost zone of the United States. Early field investigations showed the important aspects of soil textures, available moisture, and temperature patterns and empirical procedures for coping with frost problems were developed. As this work continued, the need for fundamental studies in the mechanics of frost penetration was recognized and much work of this nature has been undertaken in recent years. Some of the studies were related to construction problems of the accelerated construction program in arctic and subarctic regions.

Publications during the past ten years, many in HRB reports, explain the information needed for frost depth calculations and the procedures to be used. This paper consists of two main divisions: first, a review of some of the basic considerations of frost penetration with reference to articles which explain methods of frost depth calculations; and second, the presentation of some field measurements which permit the study of one of the important assumptions used in such calculations.

The penetration of frost into a highway section is a heat-flow process; soil freezing occurs when the soil loses heat to the atmosphere above and the temperature of the soil drops below the freezing point. The temperature gradient which causes the upward flow of heat in the soil is produced by below freezing temperatures in the air above the soil; thus a good starting point for depth-of-freeze calculations is a record of air temperatures.

In order to see the relationship of some of the elements involved, one of the equations utilized for frost depth calculations may be considered. A very complete derivation and discussion of various equations for such calculations has been given by Jumikis (1). Methods of application of these to field situations have been given by Aldrich and Paynter (2), Aldrich (3) and Carlson (4). The equations which have been used in studies in Minnesota are the following:

$$h = \left(\frac{48k\,F\,C_f}{L}\right)^{1/2} \tag{1}$$

$$F_1 = \frac{L_1 h_1}{24}\,\frac{R_1}{2} \tag{2}$$

$$F_2 = \frac{L_2 h_2}{24}\left(R_1 + \frac{R_2}{2}\right) \tag{3}$$

$$F_n = \frac{L_n h_n}{24}\left(\Sigma R_{n-1} + \frac{R_n}{2}\right) \tag{4}$$

in which

- h = depth of freeze in a uniform soil;
- k = thermal conductivity of soil in Btu/sq ft/deg F/ft/hr;
- F = degree-days of freeze based on air temperatures, deg F;
- C_f = air-surface correction factor;
- L = volumetric latent heat of fusion in Btu/cu ft = 1.434 wd;
- w = moisture content of soil in percent;
- d = dry density of soil in lb/cu ft;

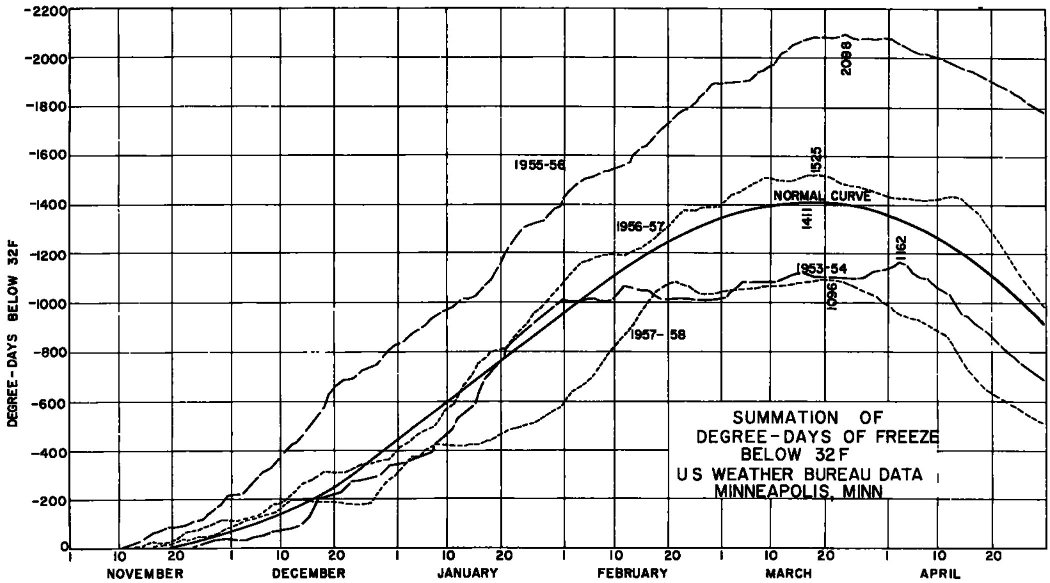


Figure 1.

F_1 = degree-days of freeze at surface required to freeze layer 1 in deg F;

L_1 = volumetric latent heat of fusion in Btu/cu ft for layer 1;

h_1 = thickness of layer 1 in ft; and

R_1 = thermal resistance of layer 1 = $\frac{h_1}{k_1}$.

Eq. 1 would be used for calculating the freeze in a soil profile with a uniform soil, that is, a one-layer system. The required data include that for the soil, k and L, and that from temperatures, F, plus the coefficient to change this to a surface temperature. This equation is one of the simplest which is in use; it is known as the Stefan Equation. Eqs. 2, 3, and 4 are adoptions of this to a layered system and are sometimes called the St. Paul Equations.

These equations are based on the simplifying assumptions that the heat represented by the latent heat of fusion of the soil mass constitutes the main heat quantity in the freezing process, and hence the only quantity which must be removed from the soil for it to freeze; that the average temperature below freezing during the freezing season can be used to establish the thermal gradient; and that the soil surface temperature is related to the air temperature by the coefficient C_f.

Values for the thermal conductivity of soils are usually obtained from published tables of charts. There have been various investigations on the determination of this coefficient. Van Rooyen and Winterkorn (5) have discussed many aspects of such determinations. Mickley (6), Makowski and Mochlinski (7), and Smith (8) have made different approaches to finding actual values, but the study covering the greatest variety of soils and soil conditions was that sponsored by the Corps of Engineers (9) and also reported by the Highway Research Board (10). This publication gives charts from which k-values can be selected for all moisture and density conditions apt to be encountered. Aldrich (3) has also presented this in convenient form.

The volumetric latent heat of fusion, L, is very simple to calculate. Since the latent heat of fusion of water is 143.4 Btu/lb, and the lb of water in a cubic foot of soil is $\frac{wd}{100}$, w being the moisture content expressed as a percentage of the dry weight and d the dry density in lb/cu ft, L = 1.434 wd.

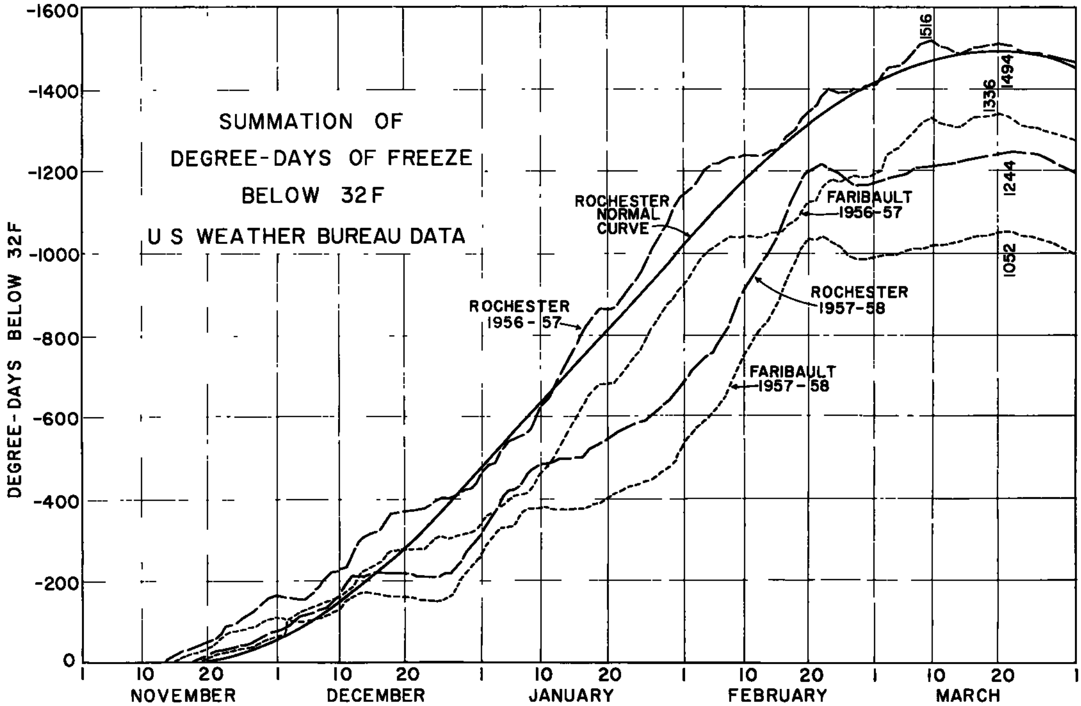


Figure 2.

Soils with high moisture contents will have high L values. This means that a large amount of heat must be extracted from them in order to change the water to ice. Although the soils with high moisture contents also have higher thermal conductivity co-

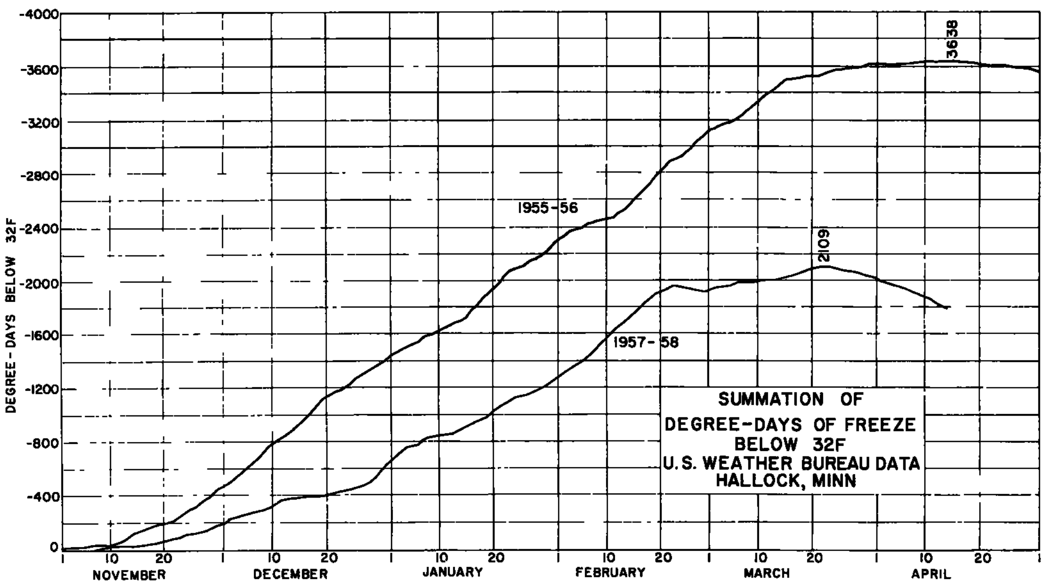


Figure 3.

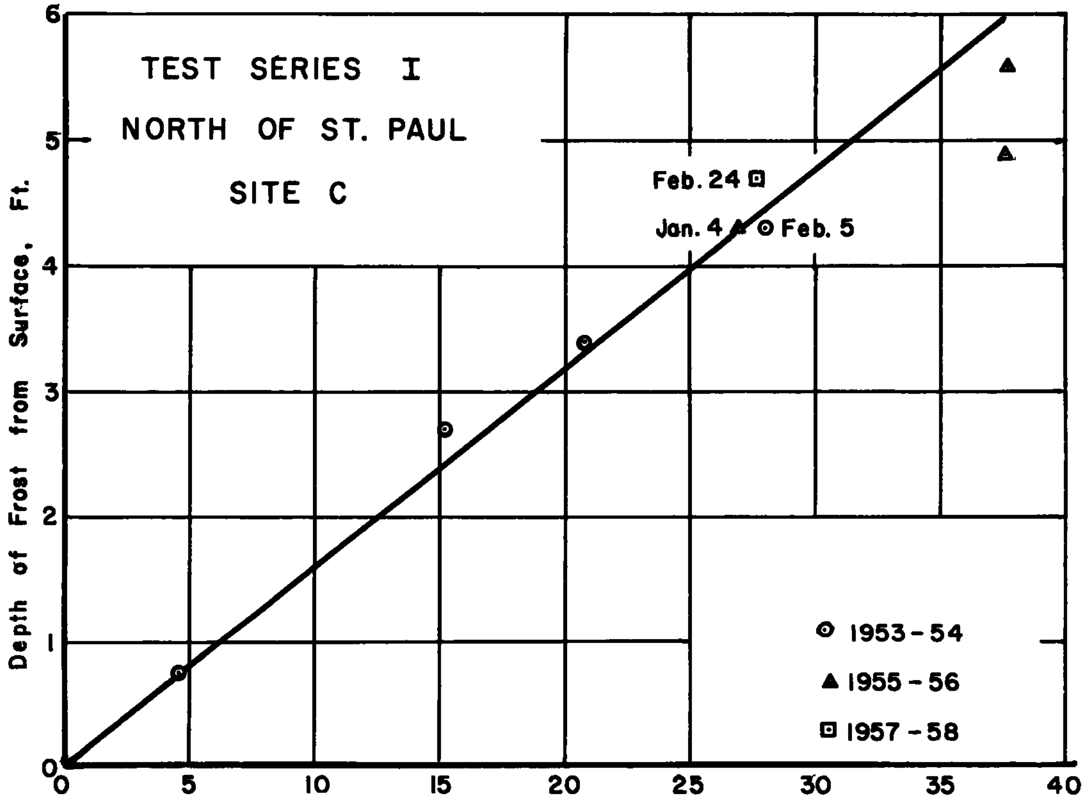


Figure 4.

efficients than drier soils, and hence would carry away the heat more readily, this effect is smaller than the effect of the high L value and thus frost will normally penetrate slower and to a lesser depth in wet soils than in dry ones.

Heat flows from the soil in the freezing process because there is a thermal gradient; that is, the ground surface is colder than the freezing front, or the level to which the frost has penetrated. Since air temperatures are usually available, these are utilized to approximate the gradient during the freezing period. The magnitude and duration of sub-freezing temperatures are measured in units of degree-days. For freezing calculations the degree-days, Fahrenheit, have usually been summed below 32 F. Thus a day with an average temperature of 19 F would give 32 - 19, or 13 degree-days. The magnitude of "cold" for any selected period is obtained by adding these values together for all days in that period. Examples of summations of degree-days of freeze for winter seasons are given in the latter part of this paper (see Figs. 1, 2, and 3).

It will be noted from Eq. 1 that the depth of freeze should vary with the square root of the degree-days of freeze. Also, one would obtain the same depth of freeze for a given number of degree-days (say 1,000) no matter whether this total was obtained in a long, slightly cold period, such as 100 days of 22 F temperatures, or for a short period of colder weather, such as 25 days of -8 F temperatures.

During freezing a pavement surface is usually warmer than the air above the pavement. Thus to transfer degree-days of cold based on air temperature readings to a pavement surface value the air values are multiplied by C_f, an air-surface correction factor, which has a value of less than unity. The Corps of Engineers (11) found that values of 0.6 or 0.7 were suitable for pavements in arctic or subarctic regions. John-

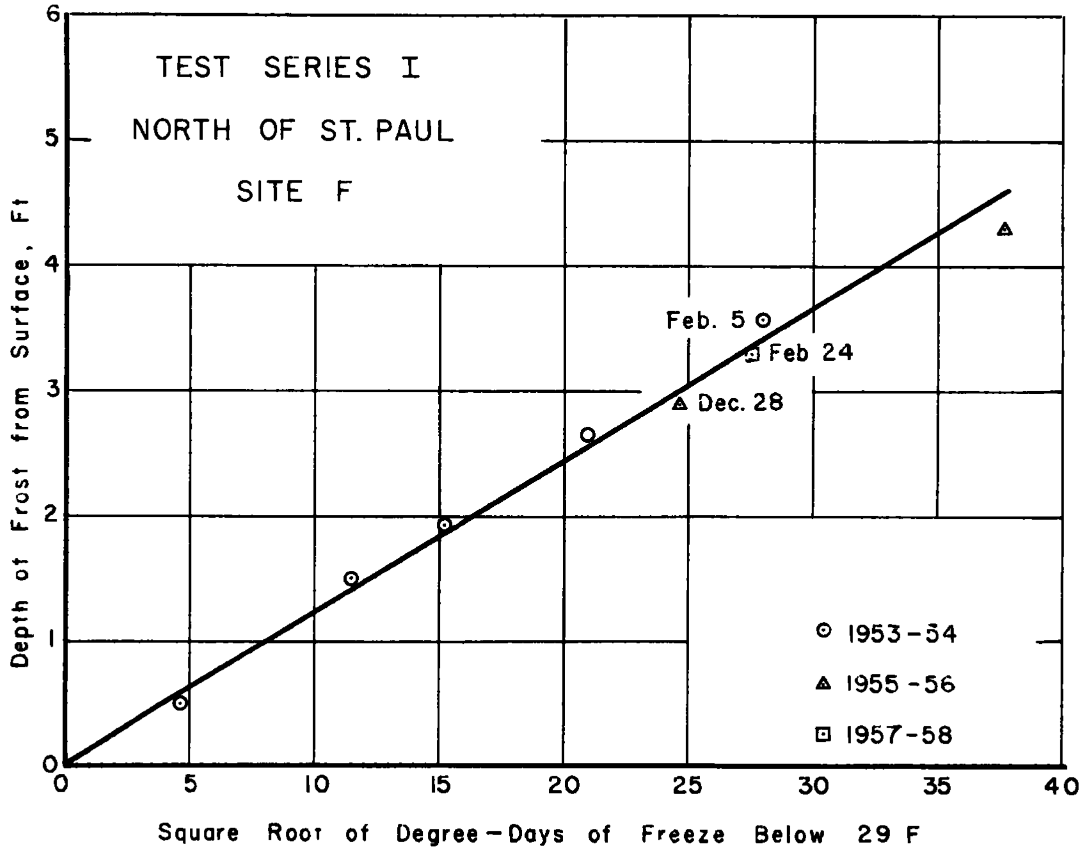


Figure 5.

son (12) found that 0.8 gave the best agreement between calculated and observed frost depths in a study in Minnesota and Braun (13) arrived at a value of 0.74 in another such study. Determination of reasonable values for this factor is one of the continuing needs in frost penetration studies.

Another way of handling this temperature difference between pavement surface and air would be to figure the degree-days of cold below a temperature other than 32 F and not use the C_f factor. Thus freezing below the pavement surface might be assumed to occur only when air temperatures are less than 29 F (or some other value less than 32 F). Degree-days of freeze would be represented by the summation below this temperature, again using the air temperature records.

Eqs. 2, 3, and 4 are merely an expansion of the reasoning followed in Eq. 1 to a layered system. Most highway sections would fall in this category, as they might have a pavement, a base course, and the underlying soil with possible variations in one or more of these parts.

All letters have the same meaning here as before; the subscripts refer to the layer number. In addition R is the thermal resistance of a layer and is equal to $\frac{h}{k}$, h being the thickness of the layer in ft, and k its thermal conductivity. ΣR_{n-1} is the resistance of all layers above layer n.

With the equations, one is able to calculate the degree-days of freeze at the surface to freeze each layer. The heat from any layer must flow through all layers above it, plus a part of the layer itself as it freezes. Since F_1, F_2, etc. are degree-days at the surface, they must be divided by C_f to obtain the equivalent number of degree-days of

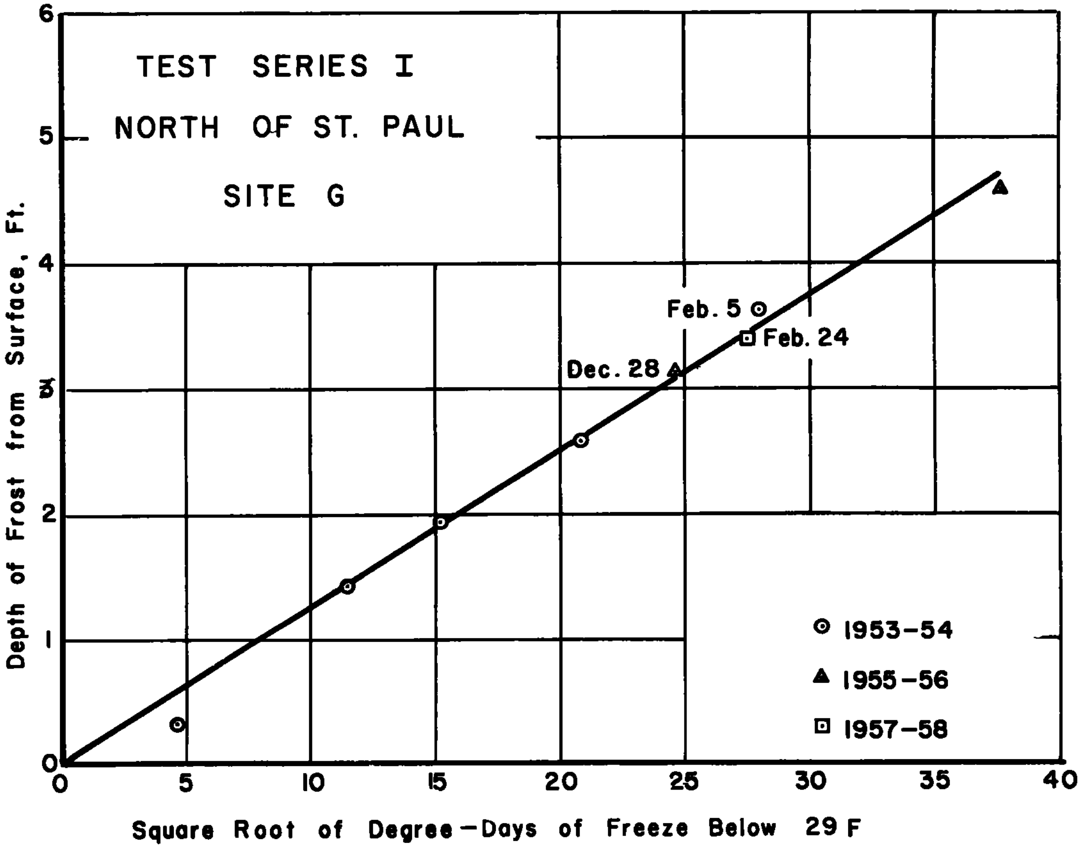


Figure 6.

freeze based on air temperatures. For a given winter season one may calculate the values of F_1 plus F_2 plus F_3, etc., change the total to degree-days air temperature, and find the depth for which this sum equals the total degree-days.

Further discussion of the layered system equations have been given by Carlsen and Kersten (14); Kersten and Johnson (12) have also given a detailed explanation and example.

The above equations have been found to be a useful tool for frost penetration studies in Minnesota. It is felt that they satisfy the important aspects of heat flow fundamentals and are simple enough to be applied without complications. One of the shortcomings in the approach is the neglect of those heat quantities not included in the latent heat of fusion, that is, that involved in cooling the soil mass to 32 F and also in dropping it to a lower temperature after freezing. Aldrich has studied this problem thoroughly and has presented a method of calculation which does take this factor into account (3). M. I. T. has made comparisons between results by this method and those by the St. Paul equations (15).

The remainder of this paper presents some actual field frost depth measurements and attempts to relate them to the air temperatures observed. As a somewhat new approach it has been attempted to ascertain if the summation of air temperatures below 29 F is a good measure of the effect of air temperature on frost penetration. If such is the case and if the soil conditions were reasonably uniform, it was reasoned that a plot of depth of freeze versus the square root of degree-days below the selected temperature would plot as a curve approximating a straight line, and that points for a given location should plot on the same curve for freezing seasons of different years, even though the winters were of different degrees of severity.

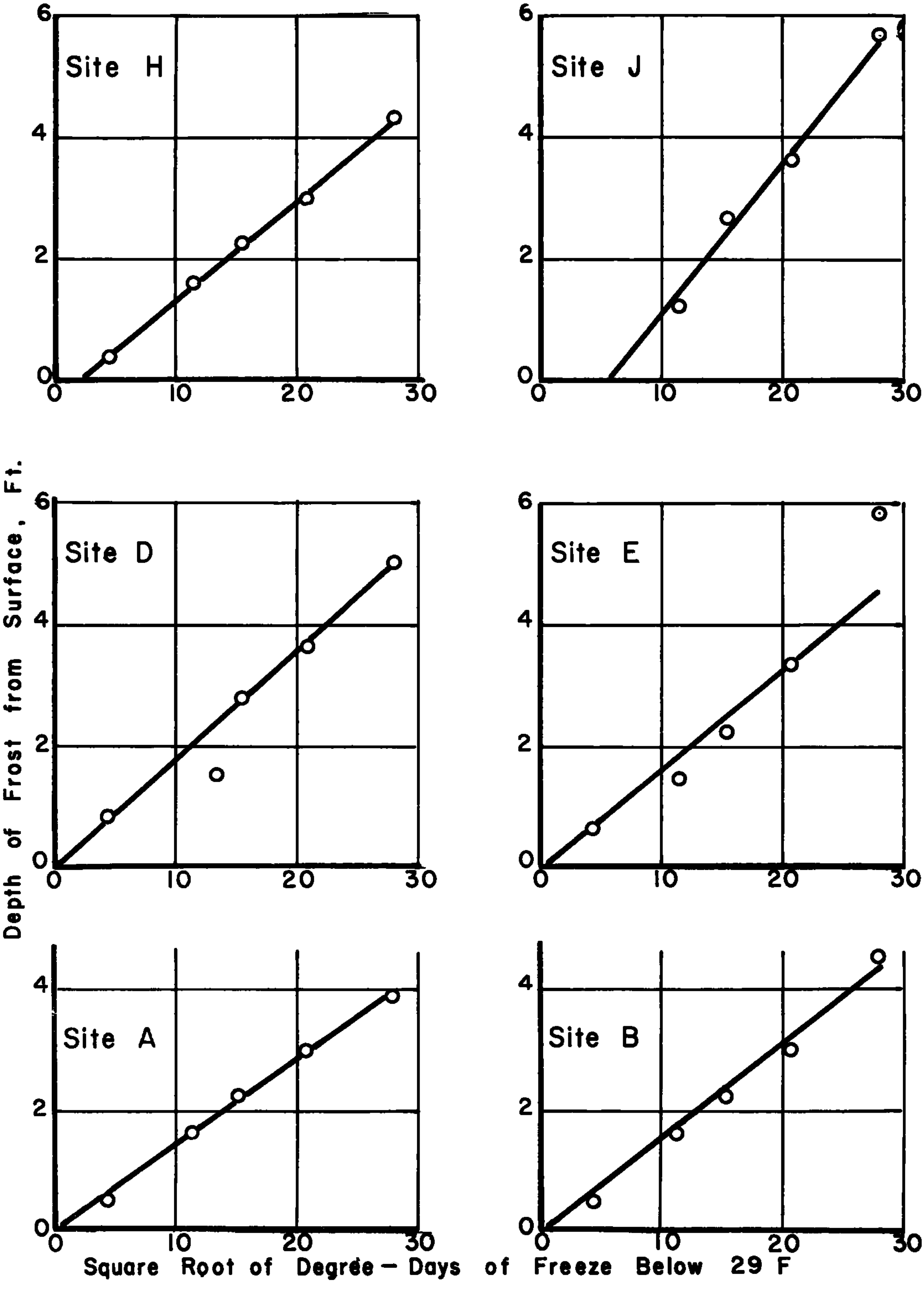


Figure 7. Test Series I, north of St. Paul, 1953-54.

There have been various programs of investigation at the University of Minnesota in which frost depths have been measured in different winters since 1953-54. On most of these the Minnesota Highway Department has cooperated; in one the Portland Cement Association has also rendered assistance. Most of these investigations have been parts of graduate studies of students of the University. By compiling the data from these various studies, some of which are current and continuing, the relationship

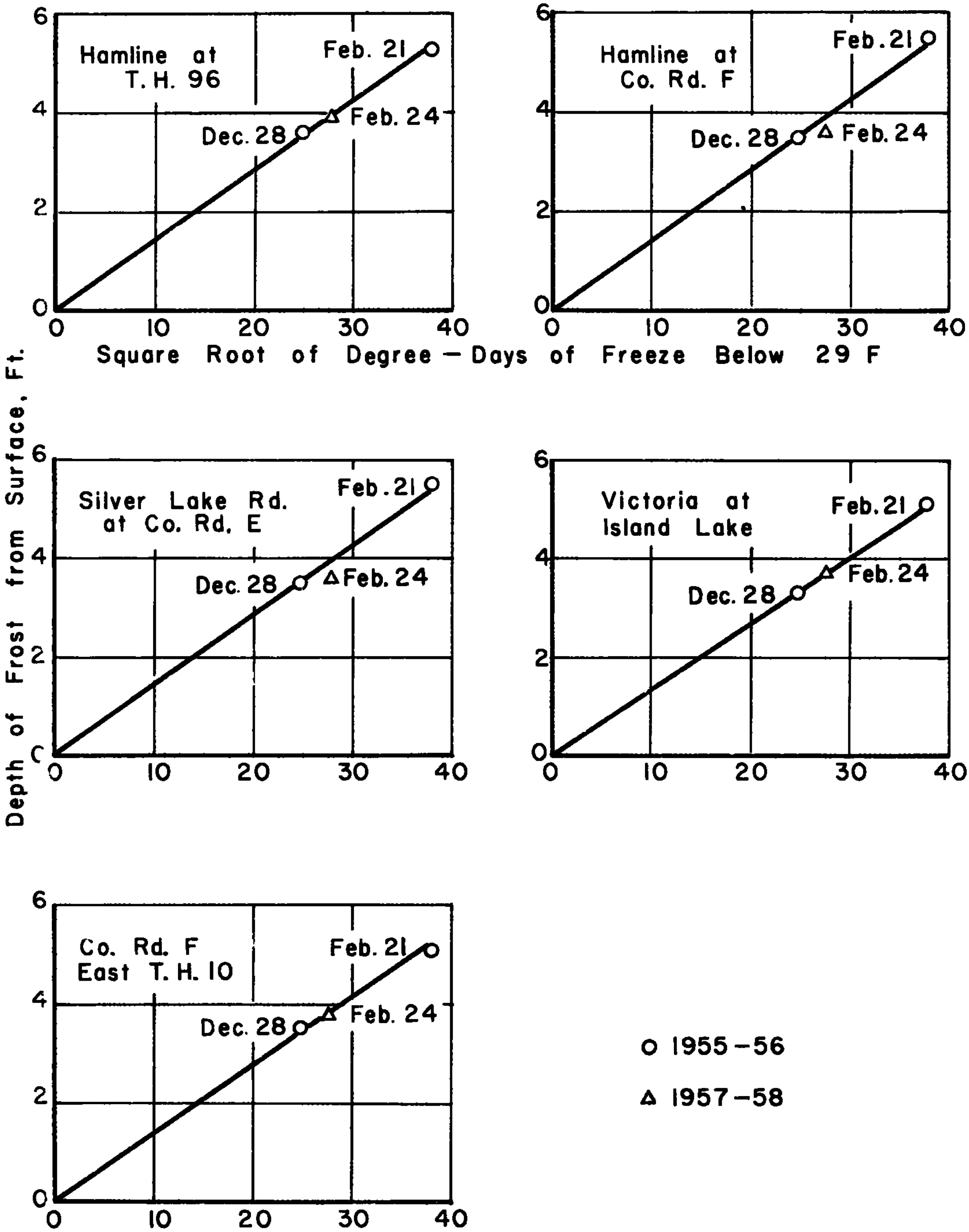


Figure 8. Test Series II, north of St. Paul.

between frost depth and air temperatures as represented by summation of degree-days can be observed.

Description of Test Locations

Five sets of data, each representing several test holes in one general location, were adaptable for this study. These will be described briefly.

Test Series I. In the winter of 1953-54 frost depths were measured at 9 locations in the area just north of St. Paul and northeast from Minneapolis. All points were in bituminous surfaced roads. The frost depths were determined by boring a hole and scraping the sides with a spoon, or "frost hook." Measurements were made five or six times.

Two sets of readings were taken at three of these points in 1955-56 and one set at the same three points in 1957-58.

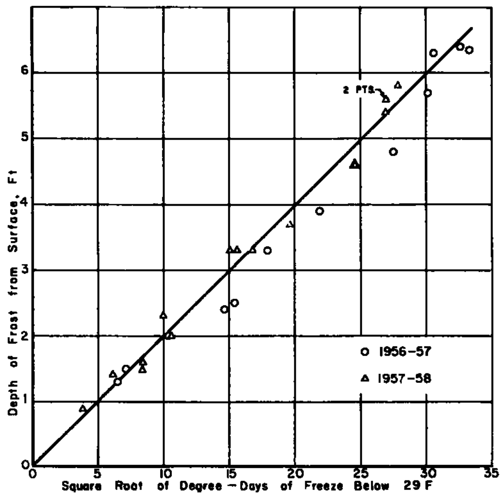


Figure 9. Test Series III; vicinity, Mpls. and St. Paul; Site 12A.

Test Series II. Five additional locations in the same general vicinity as Series I were selected and two measurements made in 1955-56 and another in 1957-58. These were also on bituminous surfaced roads and were made by auger.

Test Series III. As a part of a cooperative project by the Portland Cement Association, the Minnesota Highway Department, and the University of Minnesota, thermocouple assemblies were placed beneath concrete pavements in 6 locations near Minneapolis and St. Paul. These have been read at about weekly intervals during the winters of 1956-57 and 1957-58.

Test Series IV. In the winter of 1956-57; frost depth measurements were made at 10 locations in southeastern Minnesota (13). The frost depth was measured on four different dates during the freezing season. One set of measurements was made at 9 of these points in 1957-58. All points are on bituminous surfaced roads. The tests were made by auger.

Test Series V. To obtain information on frost depths in heavy clay soils in the coldest part of Minnesota, a single set of frost borings was made at 10 test points in the northwestern portion of the state in March 1956. These were repeated in late February 1958. All locations except one were on bituminous surfaced roads. They were located at various points between Ada and Hallock, Minnesota.

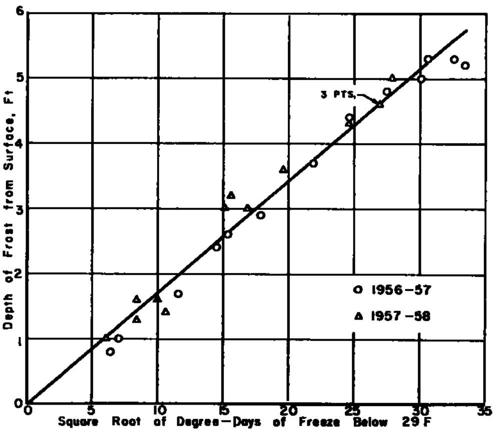


Figure 10. Test Series III; vicinity, Mpls. and St. Paul; Site 12B.

Temperature Data

Frost measurements at the various locations have been made during winters between 1953-54 and 1957-58. The manner used to portray the character of the air temperatures for these freezing seasons is a plot of the accumulation of degree-days below 32 F. Three such plots are shown.

Figure 1 is the curves for the Minneapolis area. These represent the climatic conditions for Test Series I, II, and III as described previously. The following items might be noted:

1. The year 1953-54 was about normal until February 1, but the month of

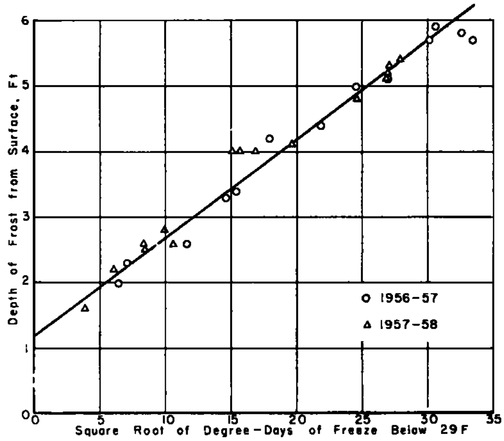


Figure 11. Test Series III; vicinity, Mpls. and St. Paul; Site 12C.

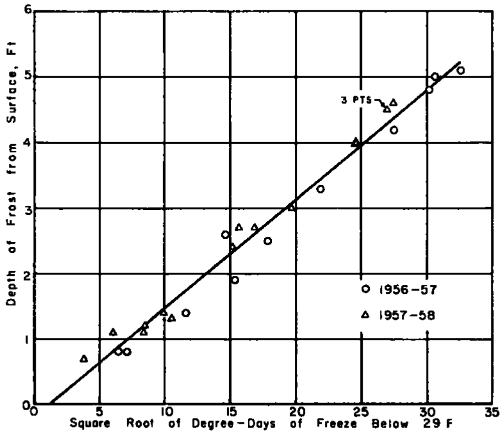


Figure 12. Test Series III; vicinity, Mpls. and St. Paul; Site 61A.

February was unseasonably warm, averaging about 32 F; March was only slightly colder. There was no measurable increase in frost depths after February 1.

2. The year 1955-56 was a steady, cold winter. Subfreezing temperatures started early and continued steadily with no midwinter thaws. Such cold winters have a frequency of perhaps once in ten years or more.

3. The year 1956-57 was very close to a normal year.

4. The year 1957-58 was distinctly a warm one. There were long mild periods in December and January. Some thawing occurred in the last week of February, although temperatures during the previous week had been sub-zero on several days. The freezing season was essentially ended at this time.

Figure 2 is a plot of the degree-days below 32 F at Rochester and Faribault, Minnesota. These data apply to the frost depths measured in southeastern Minnesota, Test Series IV. These two cities are only about 50 miles apart but it will be noted that Rochester had distinctly lower temperatures. Again 1956-57 was about a normal winter and 1957-58 was distinctly warmer than normal.

Figure 3 is a plot for one of the cities in the northwestern corner of Minnesota, Hallock. It may be noted that the degree-days of cold in this part of the state are about 50 percent greater than the other locations studied herein.

Discussion of Data

The various sets of data will be considered individually. Test Series I has been presented in part in a previous paper (12). The site descriptions and soil data will not be repeated here. The depth measurements made are not necessarily correct within 0.1 ft. There is a certain amount of judgment in selecting the depth at which resistance to the scraping changes; stony soils and light sandy soils with low moisture contents present difficult situations. Wet clay soils give the most definite indications. The points on the various graphs should therefore be considered to be the individual's best judgment of the frost line, and are probably correct within a quarter of a foot, but there may be a few points with more error than this.

Four or five depth measurements were made in 1953-54. The freezing season in that winter ended, for all practical purposes, at the beginning of February. At 3 of the test sites measurements have also been taken in 2 later winters. The data for these 3 points are shown in Figures 4, 5, and 6. The manner of plotting these curves as well as the others to be presented, is to express the temperature effect as the square root of the degree-days below 29 F. In previous studies with these and other data, there seemed to be some advantage to using such a scheme rather than using the degree-days below 32 F and multiplying by a coefficient, C_f, less than unity. However, these studies have not progressed to the point where one can definitely state that 29 F is the best base, although such an approach does have promise.

In the plots of depth of freeze versus the square root of degree-days below 29 F, it has been attempted to show the variation by a straight line.

Figure 4 is a plot of data for site C which had a fine sand soil with moisture contents between 4 and 7 percent. It is a soil condition for which it is somewhat difficult to recognize the exact frost depth by scraping. The plotted points give a fairly good straight-line relationship, with the exception of one of the Feb. 21, 1956 determinations. The data represent 3 distinctly different winters: 1953-54 was about normal until Feb. 1; 1955-56 was extremely cold; and 1957-58 was relatively mild (see Fig. 1). It is interesting to note that on Jan. 4, 1956, Feb. 5, 1954 and Feb. 24, 1958 at which time the degree-days of freeze were about the same, the depths of freeze were also approximately the same.

Figures 5 and 6 are for two locations which have very similar soils. The subgrade soils are silty clay loams with moisture contents in the middle 20's. The bituminous mat is relatively thin and the base course is somewhat mixed with the subgrade; the combined thickness is a half-foot or less. The frost line at these points was very sharp and easy to identify.

Both Figures 5 and 6 show very good straight-line plots, with the test points for all 3 years being essentially on a single line.

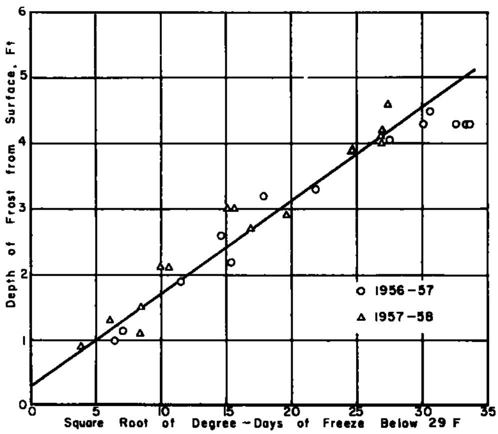


Figure 13. Test Series III; vicinity, Mpls. and St. Paul; Site 61B.

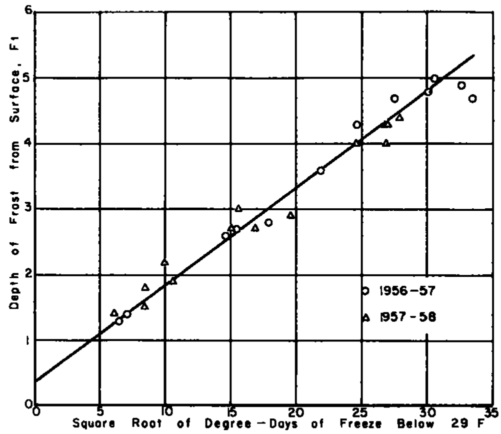


Figure 14. Test Series III; vicinity, Mpls. and St. Paul; Site 65A.

Tests were made at 6 other locations in 1953-54. No measurements have been made in subsequent years however. The plots for these 6 points are shown on a single plate, Figure 7. The frost depths vs square root of degree-days below 29 F are approximate straight-line relationships with the exception of Site E, where the final frost-depth measurement on February 5 seemed very high. At Sites H and J the straight line does not go through the origin, but is to the right on the plots. This would indicate that the freezing is effective only when the temperature is lower than some temperature colder than 29 F, or that a proportionately greater number of degree-days is required to freeze the upper layers of soil than the deeper layers, or perhaps some other unknown factor. Site J had a gravelly sand soil and it was difficult to identify the frost line because of the stones.

Considering all of the test points in this series, and particularly Sites F and G at which frost depth determinations are considered to be the most exact, it appears that the summation of air temperature degree-days below a value such as 29 F is a good guide to frost depth and that the relationship is represented by a straight-line graph between depth and the square root of degree-days. The frost depth would seem to be the same for a given number of degree-days regardless of whether this number was accumulated over a long period or over a shorter period of more intense cold.

The second group of tests, Test Series II, are from the same general area as Series

I, or just north of St. Paul. Frost-depth determinations were made on just 2 dates during 1955-56 which was the very cold winter and once near the end of the freezing season of 1957-58 which was quite mild. Although the number of measurements is small, they are considered valuable as comparisons of frost-depths for winters of a marked difference in magnitude of cold.

Tests were made at 5 points, all on secondary roads with bituminous surfacing and gravel bases usually about 0.7 ft thick. The soils in general were clay loam tills with moisture contents around 16 to 18 percent.

The plots of frost depth versus square root of degree-days below 29 F are shown in Figure 8. In all instances the points, though only three in number, indicate a straight line passing through or close to the origin. It is noted particularly that the frost depths on Feb. 24, 1958, at a time when the degree-days were only slightly more than on Dec. 28, 1955, check very closely the curves of the 1955-56 depths.

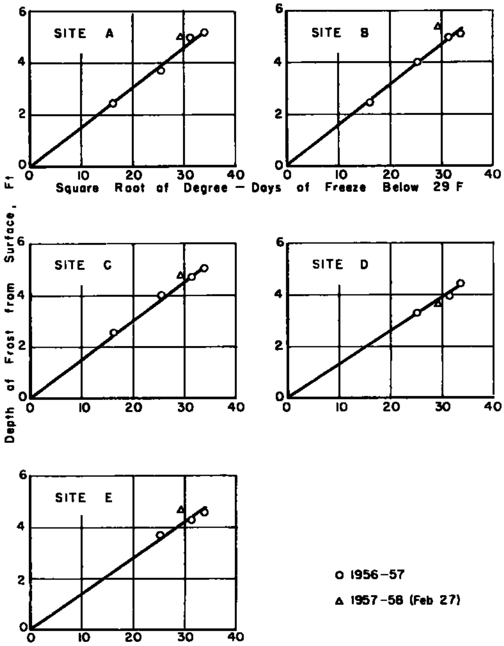


Figure 15. Test Series IV, Southeastern Minn. (near Rochester).

Test Series III differs from the other data being considered in two respects. First, they are measurements beneath portland cement concrete pavements; and second, the frost-depth determinations have been made by thermocouple readings. The soil profiles are also not as uniform as at most of the other locations. In this series the profiles consist of 0.75 ft of concrete pavement, a granular base course or lift of between 0.5 and 2.0 ft, and a fairly uniform subgrade soil. The subgrade soils vary in texture from clean sands to clay loam and silty clay loam. The 6 test locations are just outside of Minneapolis and St. Paul.

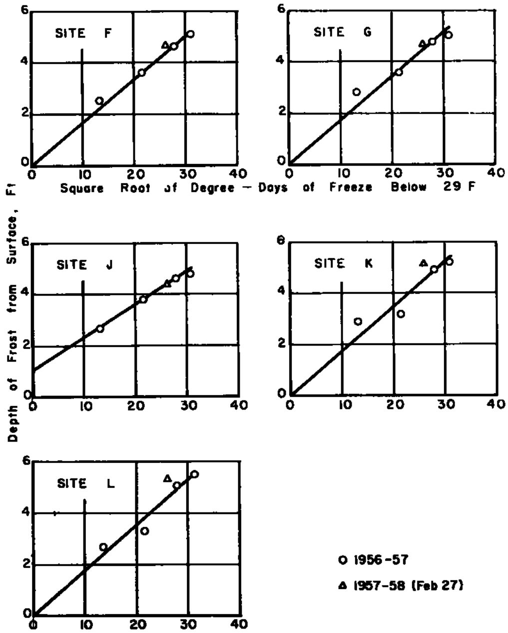


Figure 16. Test Series IV, Southeastern Minn. (near Faribault).

The details of the thermocouple test installation and the plotting of temperature profiles and determination of frost depth will not be presented here. In some instances the determination is judged to be quite exact—perhaps within 0.1 ft; in other temperature conditions, however, there is some personal judgment required, and possible variations of 0.3 or 0.4 ft may be encountered.

Readings have been taken for 2 winters, 1956-57 and 1957-58. The first of these 2 years was about a normal one; 1957-58 was warmer than normal, especially through January, with a cold February until Feb. 22, at which time the freezing season ended for all practical purposes (see Fig. 1).

Thermocouple readings were taken at about weekly intervals and hence there are a large number of points to show on the plots for the two winters. The results for the 6 locations are shown in Figures 9 to 14, inclusive.

Although there is some scattering of points on these plots, there still seems to be a straight-line relationship throughout the entire freezing season. At the end of the season when the degree-days are still showing some increase the depth of freeze does seem to remain about constant on most of the plots. In other words, at the end of the freezing period, factors other than air temperature must be having an influence, which was not the case in midseason. These might be variations in solar radiation effects, for example.

Again, the points for the 2 years seem to give the same general relationship. The specific dates for a given number of degree-days for the 2 years are as follows:

Degree-days below 29 F	Square Root of Degree-days	Date 1956-57	Date 1957-58
100	10	Dec. 9	Dec. 29
225	15	Dec. 30	Jan. 5
400	20	Jan. 10	Feb. 3
625	25	Jan. 22	Feb. 14
900	30	Feb. 3	Not reached

Thus, for example, a value of 20 on the square root scale of the figures represents Jan. 10 in 1957 and Feb. 3 in 1958; a reading of 25 represents Jan. 22 in 1957 and Feb. 14 in 1958. It will be noted that depths of freeze run about the same for the 2 years at these values.

Some of the curves in this series do not pass through the origin or close to it, particularly Location 12C (Fig. 11). This location has a clean sand subgrade which has a moisture content of only about 6 percent; this may or may not be significant, but it is the cleanest sand subgrade and the driest that has been worked with. Location 65A for which the curve is only about 0.4 ft from the origin is also a sand subgrade, but it is rather fine with around 8 to 20 percent passing the No. 200 sieve and a moisture content around 8 percent.

Test Series IV are tests made in the southeastern area of Minnesota near the cities of Rochester and Faribault. The locations are on state highways with bituminous surfaces. Tests were made on 4 different dates in 1956-57, which was a fairly normal winter, and on just one date at the end of the freezing season in 1957-58 which was very mild.

The results for 5 locations south of Rochester (most distant location about 25 miles southeast) are plotted on Figure 15. The subgrade soils at these locations were either clay loam tills or silty clay loams. It will be noted that the depth determinations made in 1956-57 plot reasonably close to a straight line passing through or close to the origin. The lines have been drawn on the basis of the 1956-57 data only. The single test in 1958 was made on Feb. 27. The degree-days of freeze on this date was 1164 below 32 F and 858 below 29 F. On Feb. 22 these values had been 1215 and 924, respectively, so there had been several days of warm weather just prior to the tests. In 1957 tests were made on Feb. 7 at which time the degree-days of freeze was 1225 below 32 F and 972 below 29 F. Thus there had been almost as much cold by that date (Feb. 7, 1957) as accumulated during the entire 1957-58 season (1244 degree-days below 32 F).

The frost depths for Feb. 27, 1958 plot above the curves of the 1956-57 data for all these locations except point D, and by an average of about 0.5 ft. This difference would be decreased a little if the points were plotted as having occurred on Feb. 22, or just at the end of the cold period and before the warm period.

The largest discrepancies are for Locations B and E where the 1958 depths are greater than any of those measured in 1956-57.

Tests in an area east and northeast of Faribault (most distant location about 22 miles) are shown in Figure 16. The subgrade soils represented are clay loam tills, silt loam, and clay. The plots of this figure are somewhat more irregular than those of Figure

15, but again it has been attempted to represent the trend of the 1956-57 points by a straight line. Locations K and L have similar clay soils (the locations are only 250 ft apart) and both seem to have a too-deep point for the first observation and a too-shallow point for the second one.

The single measurement made in the 1957-58 season (Feb. 27) again plots above the curve at all locations except one, and again by an average of almost 0.5 ft.

Test Series V is limited to just one frost-depth measurement near the end of the 1955-56 winter season (March 8) and one near the end of the 1957-58 season (Feb. 27) at 10 locations in northwestern Minnesota. The first of these seasons was an extremely cold one, the second an extremely mild one. The test locations were not referenced exactly at all points, and in some cases the frost-depth measurements may be a few hundred feet apart. The results should show general trends however.

The test points were all in bituminous surfaced roads. The extreme test locations in this series are about 110 miles apart. The temperature data are taken from the records of 3 towns or cities in this area, Ada, Crookston, and Hallock.

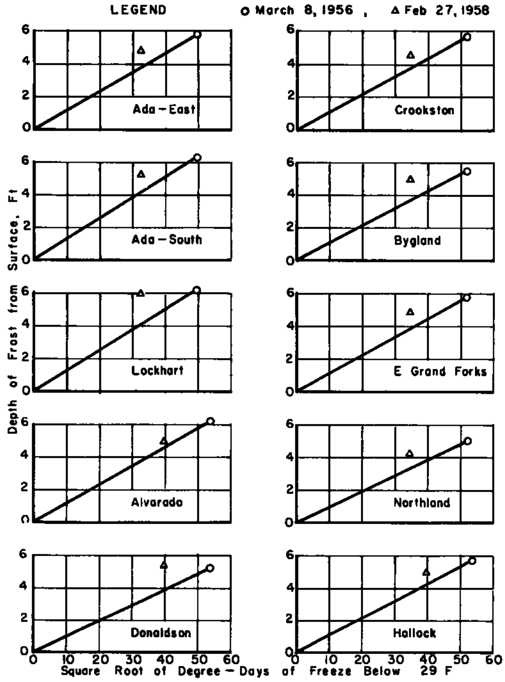


Figure 17. Test Series V, Northwestern Minnesota.

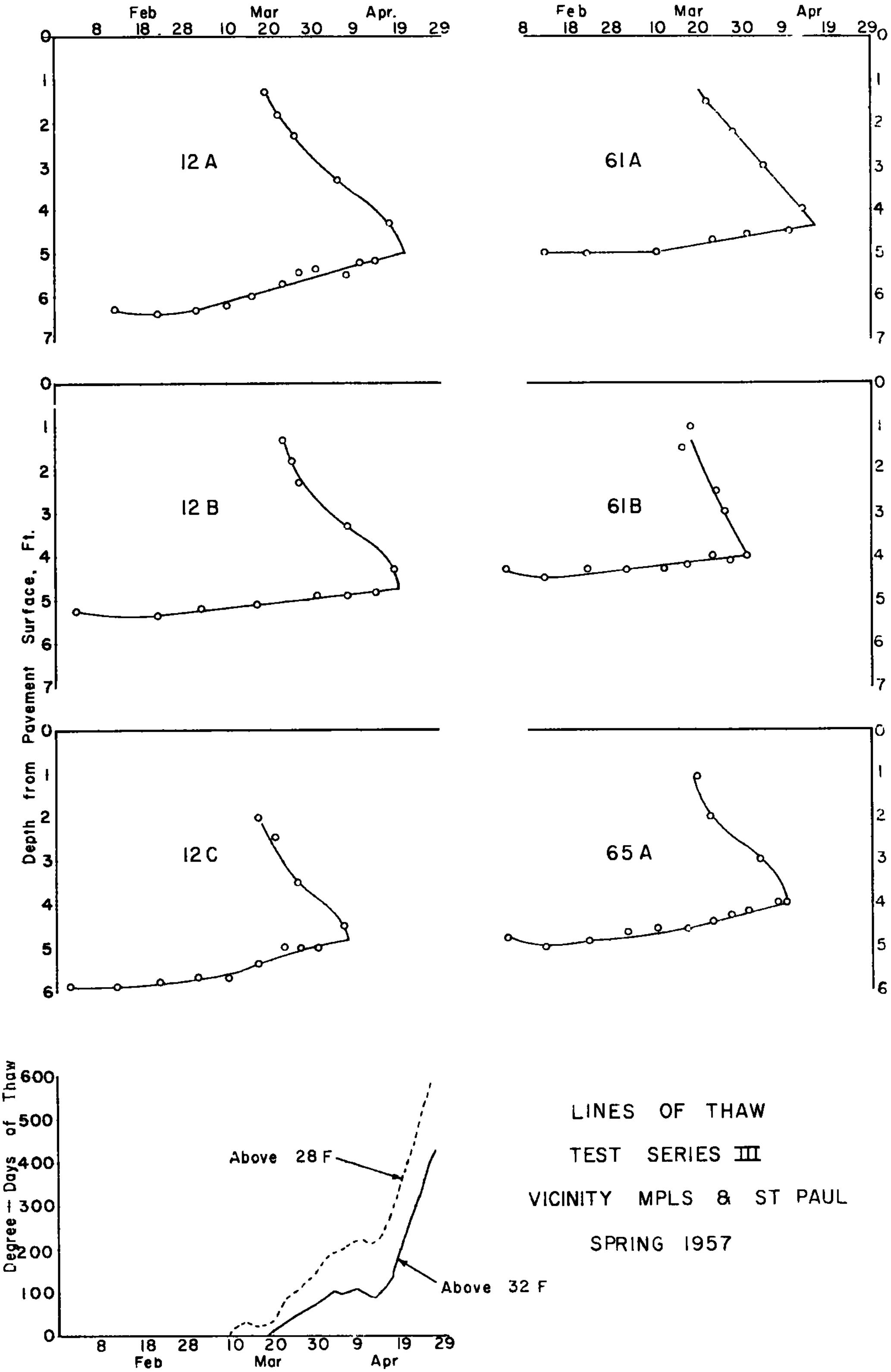
The test results are shown for all 10 points in Figure 17. The results seem to be quite different from those of the series previously presented.

The first obvious result is that the 1955-56 point, the 1957-58 point, and the origin are not on a common straight line. The 1955-56 tests (March 8, 1956) were taken by the author and Herbert Dale, District Soils Engineer of the Minnesota Highway Department. In Figure 17 straight lines have been drawn from the origin to these points. The 1957-58 depths (Feb. 27, 1958) were taken on the basis of mailed instructions by an Assistant District Soils Engineer who had not been on the previous year's measurements. This is not mentioned to question the accuracy of either set of data, but it is possible that a different judgment of what constituted the frost line may have been used. The author is prone to doubt this, however, since the frost line is usually quite definite in these soils. The 1958 readings were taken after 5 very warm days (average temperatures up to 49 F) and this may have caused some difficulties.

The Feb. 27, 1958 points all fall above the lines as drawn in Figure 17 and by an average amount of about 1 ft. This means that the frost line penetrated faster than one would anticipate from the 1955-56 experience. If straight lines were drawn through the 2 frost-depth determinations for each test location, the intercept at 0 degree-days would fall between 1 and 4 ft in a majority of the curves.

It is planned to make more calculations on these locations with the data on hand. Perhaps these will aid in explaining the seeming discrepancies. It would also be desirable to make more frost measurements in this region, with several depth determinations being made in one season.

To review the consideration of the 5 series of data, it would seem that 3 series, I, II and III, give good agreement between measured frost depths and an air temperature factor (degree-days below 29 F) for winters of different magnitudes of cold, and that a plot of depth versus the square root of this temperature factor approximates a straight line. On the basis of these data one would tend to accept the concept of the Stefan equa-



LINES OF THAW

TEST SERIES III

VICINITY MPLS & ST PAUL

SPRING 1957

Figure 18.

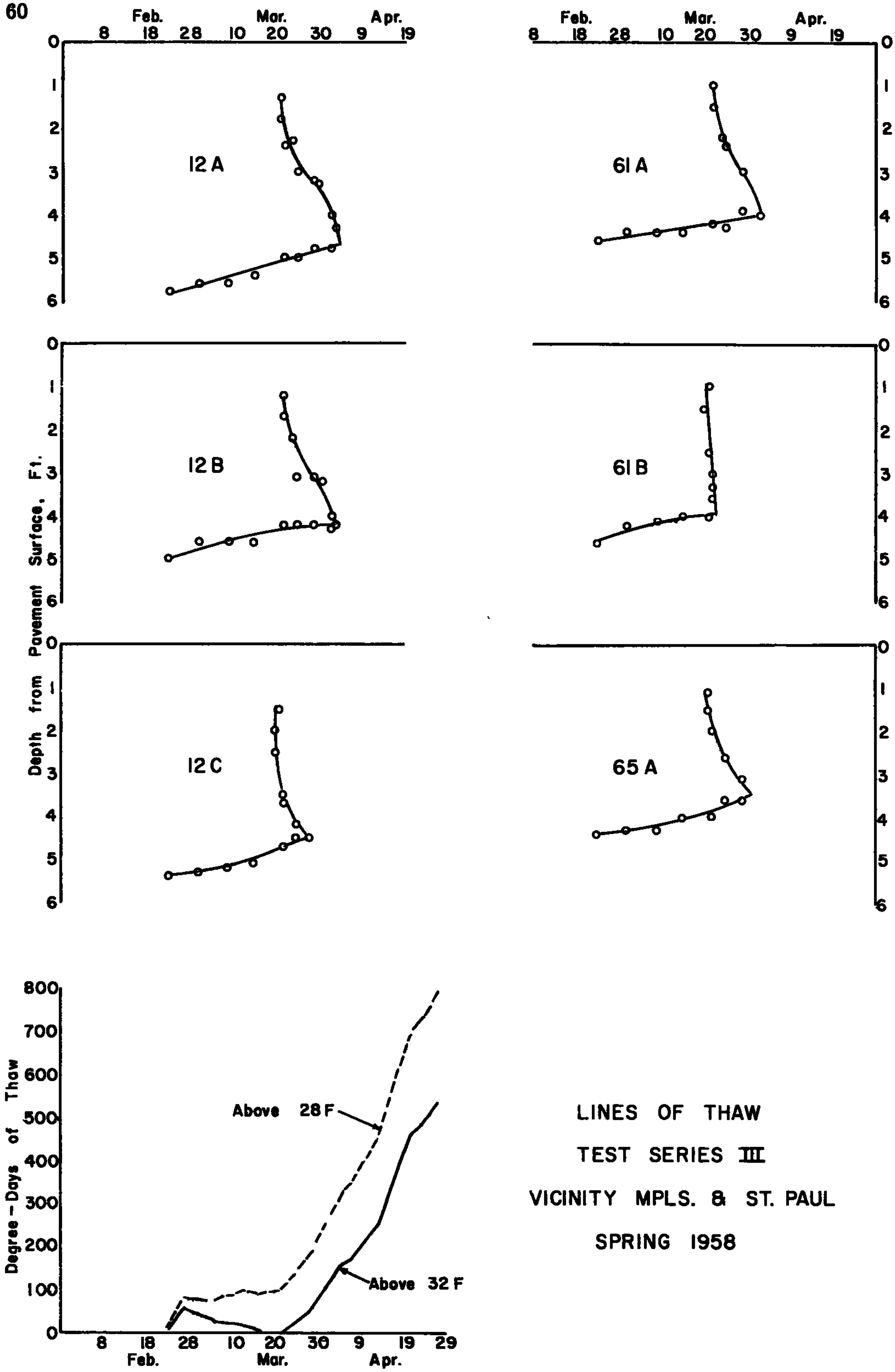

Feb.
Mar.
Apr.
8
18
28
10
20
30
9
19
12 A
61 A
12 B
61 B
12 C
65 A
Depth from Pavement Surface, Ft.
Degree - Days of Thaw
Above 28 F
Above 32 F
LINES OF THAW
TEST SERIES III
VICINITY MPLS. & ST. PAUL
SPRING 1958

Figure 19.

tion that depth does vary with the square root of a degree-day factor without mention of the time period required to accumulate this value.

Test series IV and V seem at some variance with this concept, however. In several measurements in these locations, frost depths at the end of a mild winter seemed to be about 0.5 to 1.0, plus or minus, deeper than would be estimated on a degree-day basis when compared with measured depths of a more normal winter.

It is entirely possible that other unmeasured factors caused these differences in Series IV and V. A rather important one is a possible variation in the moisture content of the subgrade. Complete moisture information was collected only in one year—1957-58 was a relatively dry winter and if this had a reflection in the subgrade moisture contents it could, and probably would, affect the frost depths. If this is true it would mean that one should take this factor into account along with the air temperature data in making an estimate of a frost depth on the basis of measurements during previous seasons. There is also the possibility that air temperatures at the field sites differed from those at the weather stations which were used.

Thawing of Subgrades

Nothing has been said thus far of the factors concerning and the calculation of the thaw of subgrade soils. The manner in which this thaw procedes would seem to have an important bearing on the loss of strength in the road structure and the possible "spring break-up." Since thawing is also a heat flow process, it would seem that it would be possible to write an equation for the process, substitute in it the necessary soils and air temperature data, and solve for the depths of thaw at selected times. Good checks were obtained by such procedures for some locations in Alaska (14). In climates such as that of Minnesota actual field measurements of thaw for making checks on such a procedure have been difficult to obtain, however. For one thing, the entire thawing is apt to occur in only a week or 10 days and it is easy to miss important measurements in many instances. Also, the temperatures in the frozen and thawed layers may be only a degree or two different, which means that accuracy is demanded in temperature readings.

There are only limited data in the studies in Minnesota to indicate how thaw does progress. The best of these are on the concrete pavement studies which were discussed in Test Series III. Figures 18 and 19 illustrate how these 6 sections thawed in the springs of 1957 and 1958. Although previous studies of thaw in the subarctic (14) had indicated that analytical calculations by a modified Stefan equation similar to freeze would give a good check for thaw, attempts to do the same for the Minnesota data have shown some complications.

While these studies are incomplete, some observations have been made which warrant further investigation. These include the following:

1. The amount of thaw from the bottom has varied from about 0.5 to 1.5 ft at the different locations. This thawing starts when the air temperatures start to average around 27 or 28 F. The thaw may be approximated by a calculation of upward heat flow using an average gradient during this period.
2. The thaw from the surface is quite rapid, exceeding that anticipated from previous observations. This thaw continues to deepen even on days with average air temperatures below 32 F once it has started.
3. Analytical calculations of thaw may have to utilize a large air-surface correction factor (such as 2.0) and also consider degree-days above some temperature less than 32 F, such as 28 or 29 F.

Conclusions

Studies of frost penetration including field observations, consideration of heat flow principles, and measurement of soil properties have led to the development of procedures by which calculations can be made for the depth and rate of ground freezing in pavement sections. References are available which explain such methods. It is felt that the methods and the required coefficients and other data are sufficiently correct so that reasonable answers can be obtained for most frost penetration problems of highway engineers. Continuing work will improve the knowledge of the coefficients and other items.

The calculation of rate of thaw is not nearly as advanced as the frost penetration procedures and further work is need on this phase of the problem.

The study of 5 series of frost measurements in Minnesota indicated in most instances that the freeze depths did vary in accordance with the concept of the Stefan equation in showing equal depths for a given magnitude of degree-days accumulated in varying periods of time for different winters. Since some measurements did not check this relationship, further study is desired.

The preliminary study of the Minnesota data also indicates that the summation of degree-days of freeze below some temperature less than 32 F, such as 29 F in this case, and the use of these values in the frost-depth equations, may have some merit.

ACKNOWLEDGMENTS

The author wishes to acknowledge the assistance in field work and analysis given by University of Minnesota graduate students, Rodney Johnson, Lawrence Stamstad and John Braun in Test Series I, III and IV, respectively.

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Discussion

FREDERICK J. SANGER, Special Assistant, Arctic Construction and Frost Effects Laboratory—The author does not say much about the modified Berggren equation beyond giving references to it (possibly to leave a topic for discussion). The experience of the Arctic Construction and Frost Effects Laboratory (Corps of Engineers, U. S. Army Engineer Division, New England) has been that this equation gives better results than others despite the limitations of any equation. All the variables in this equation are approximations; fortunately, errors usually cancel, but sometimes they do not.

ACFEL is attacking the frost penetration problem in two ways:

1. Taking the modified Berggren equation and trying to improve on its precision and make its use simpler; and
2. Approaching the problem de novo by the methods of micrometeorology, using analog computers to derive working tools with simplified techniques.

These remarks are decidedly of an interim nature but may be of general interest, and the work on the formula may have immediate value.

The modified Berggren formula, developed by Aldrich and Paynter (author's ref. 2) from the standard Neumann solution of the freezing problem is:

$$X = \lambda \sqrt{\frac{48knF}{L}}$$

in which

X = frost penetration in ft;
λ = a factor taking account of sensible heat;
n = surface correction factor = $\frac{\text{surface freezing index}}{\text{air freezing index}}$; and
k, F and L have already been defined.

From a statistical analysis of ACFEL field data, Aldrich and Paynter recommended a value of $n = 0.9$ for all bare paved surfaces; this value is being used, apparently with satisfactory results. The coefficient of thermal conductivity, k, has been studied extensively in the past year in an unsuccessful attempt to improve upon the author's curve of k, γ_d and w (author's ref. 9). Using Kersten's labor-

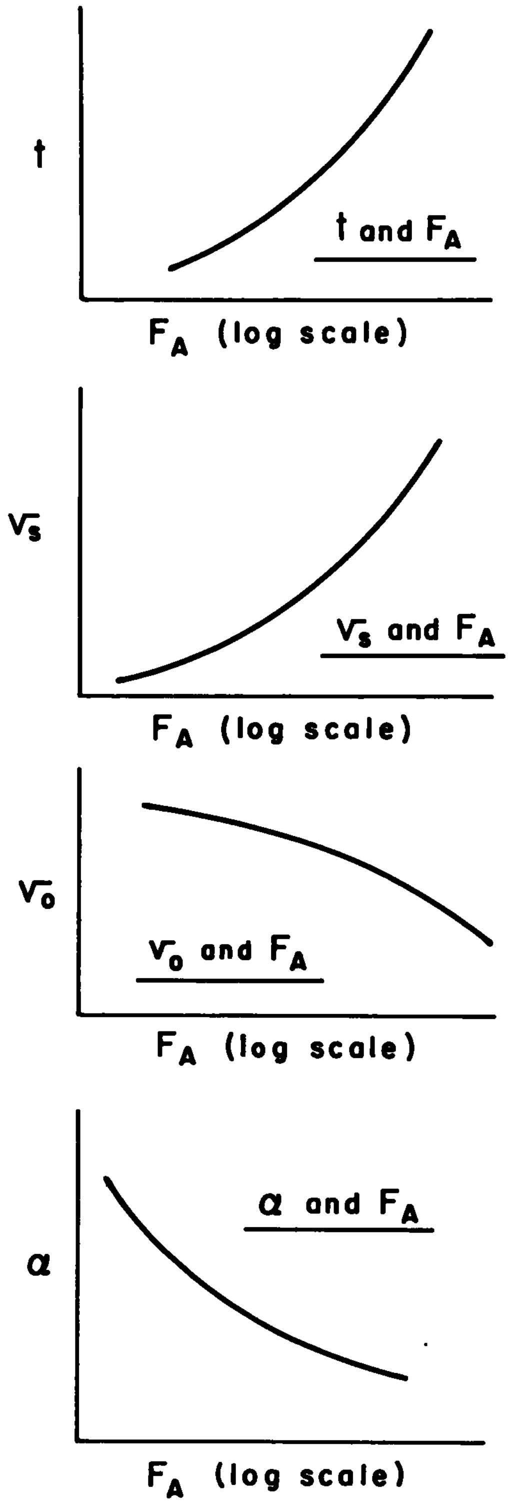

Figure 20.

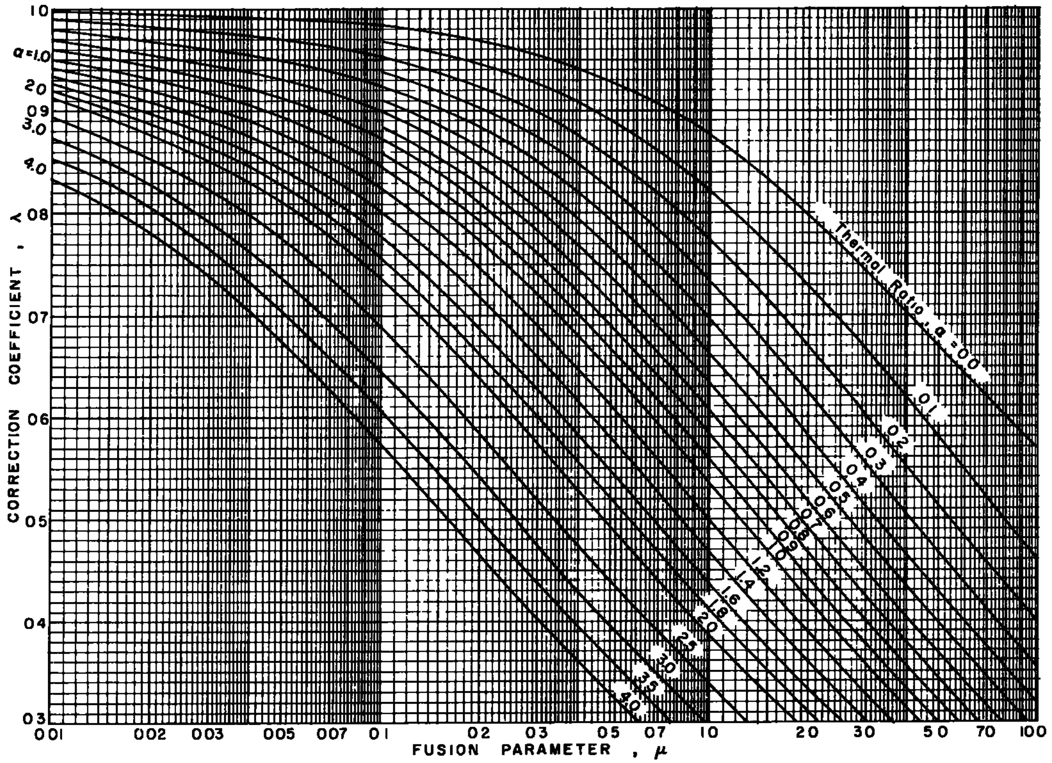

Figure 21. Correction coefficient λ and the modified Berggren formula.

atory data and soils descriptions, a theoretical formula suggested by Eucken (1) which assumes a non-continuous phase of solid particles in a continuous phase (air, water and ice were used in ACFEL studies) was used with mineralogical composition and k-values for individual minerals computed for the non-continuous phase. It soon became evident that quartz was the important mineral so that, finally, quartz content, dry unit weight, and moisture content were used for basic parameters. The results were no better than the existing curves, so ACFEL uses Kersten's values if the k-value of a particular soil has not been measured or cannot be estimated by comparison with other soils. For a job where soils are uniform and the profile is known for certain, laboratory tests are made, but that is a rare instance. The author's classification of soils into 2 groups—fine and coarse—is rather approximate but the mineralogical classification was no better; something else is needed to improve on the existing curves for k, possibly a classification based on the Unified Soil Classification System or on the Corps of Engineers Frost Effects Classification F1, F2, F3 and F4 soils (2).

The λ factor has been studied in ACFEL in some detail during the past year. As can be seen (author's ref. 2), λ is a function of μ and α; μ depends upon $\sqrt{s}$, the average surface temperature below 32 F, C, the volumetric sensible heat capacity of the soil and L, the volumetric latent heat of fusion; i. e., $\mu = \sqrt{s}\frac{C}{L}$ α is the ratio of $\sqrt{o}$, the mean annual temperature above 32 F, to $\sqrt{s}$ as just defined; $\alpha = \frac{\sqrt{o}}{\sqrt{s}}$. The original curves (author's ref. 2) did not cover a few soils encountered so ACFEL has recalculated the λ curves and has rearranged the parameters for convenience (Fig. 21). The λ factor was also studied statistically in an attempt at further simplification. A plot of air freezing index, F_A, against length of freezing season, t, for 16 locations in the northernmost states gave a good average curve showing the relationship between F_A and t.

(Actually Fairbanks, Alaska, and Thule, Greenland, fitted the curve excellently.)

The curve was very good above F_A = 200 degree-days approximately. Using F_A, t, and n = 0.9, the values of $\sqrt{s}$ were computed to give a curve of $\sqrt{s}$ and F_A. Values of $\sqrt{o}$ and F_A were then plotted to yield a curve of $\sqrt{o}$ and F_A. Then α was computed $(= \frac{\sqrt{o}}{\sqrt{s}})$ for a curve of α and F_A. (C and L are functions of dry unit weight and moisture content w, and $\frac{C}{L}$ merely of w.)

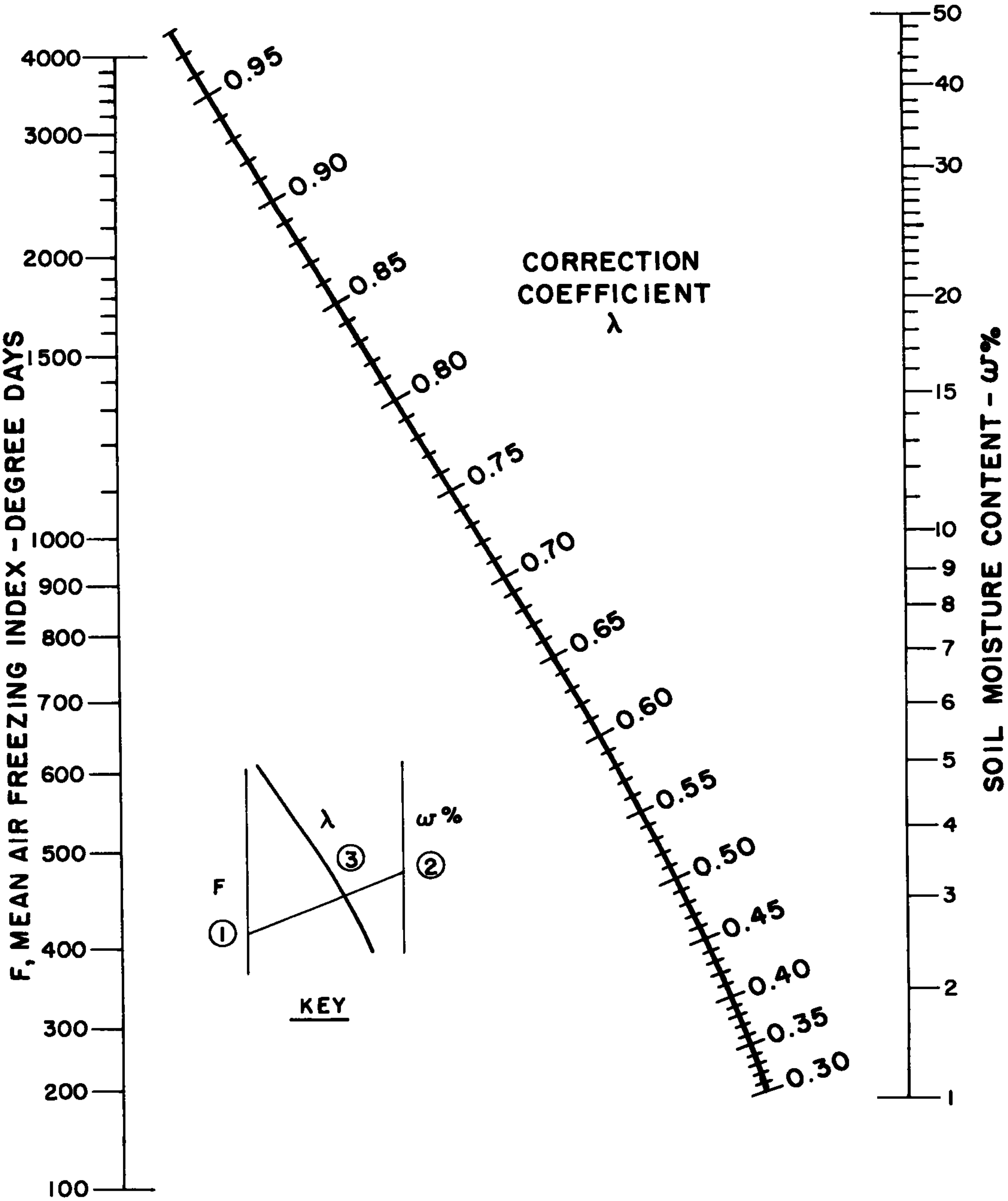

Figure 22. Modified Berggren Equation, F, w, λ.

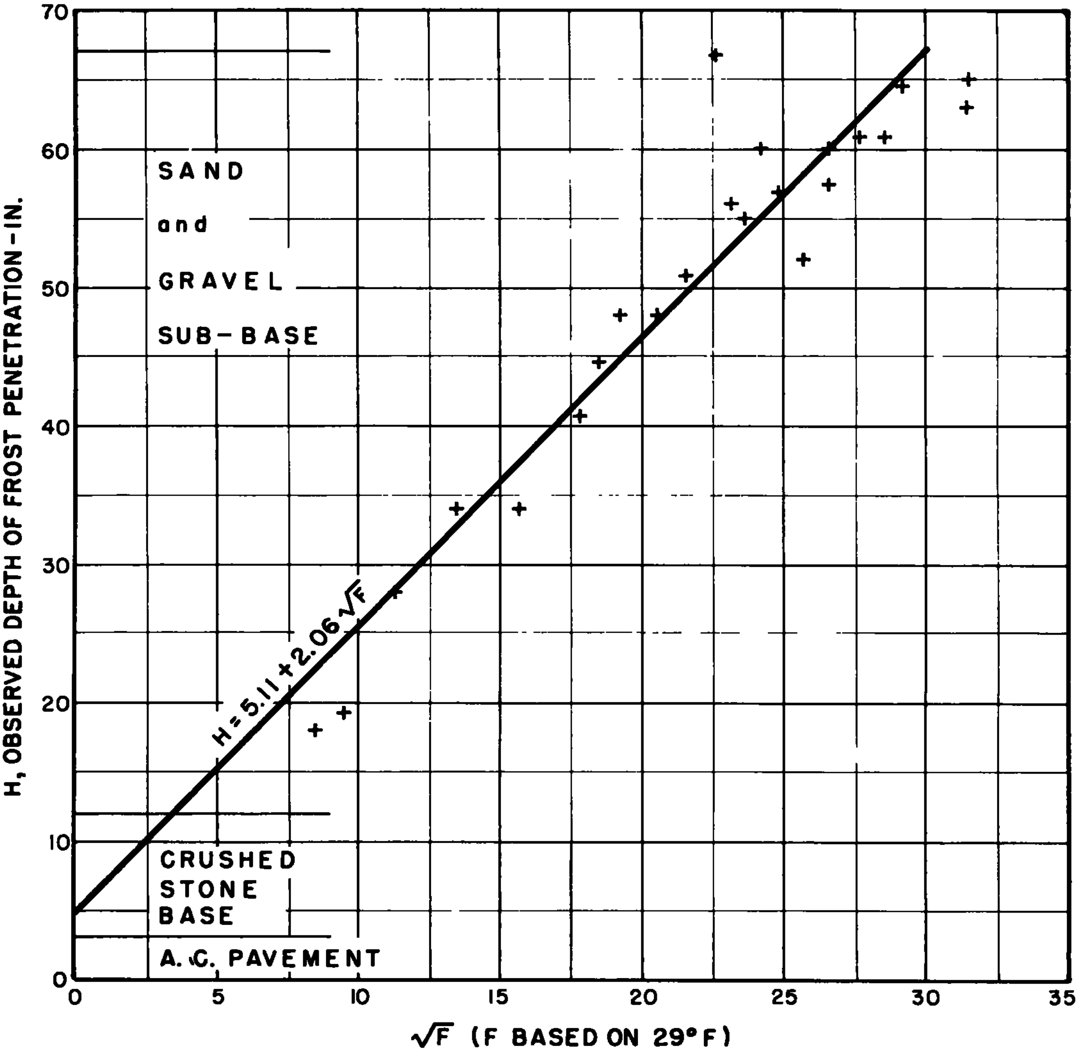

Figure 23. Observed depth of frost penetration and square root of freezing index based on 29 F.

Hence it was possible to compute $\mu = \sqrt{s}\frac{C}{L}$ on a basis of w, using values from the curve of $\sqrt{s}$ and F_A.

From the curves of Figure 21 values of λ were found and the result was the nomogram Figure 22 linking air freezing index, moisture content and λ. This diagram checks well despite its somewhat roundabout genesis. It is not recommended for use if λ can be computed more precisely, but quite often essential data are missing and then an approximate value is useful. The way that λ varies with freezing index and moisture content is very significant, even if the values of λ are not very precise. The diagram shows how large errors may arise from an assumption of a constant value of C_f and $\lambda = 1$, especially when w is small, then the sensible heat becomes as important as latent heat. Figure 22 is based on mean annual freezing and mean annual temperature; it becomes inaccurate for part freezing seasons.

The parameter, L, has been accepted for a long time as one of the easier to deal with. Actually, however, it may be responsible for many discrepancies between es-

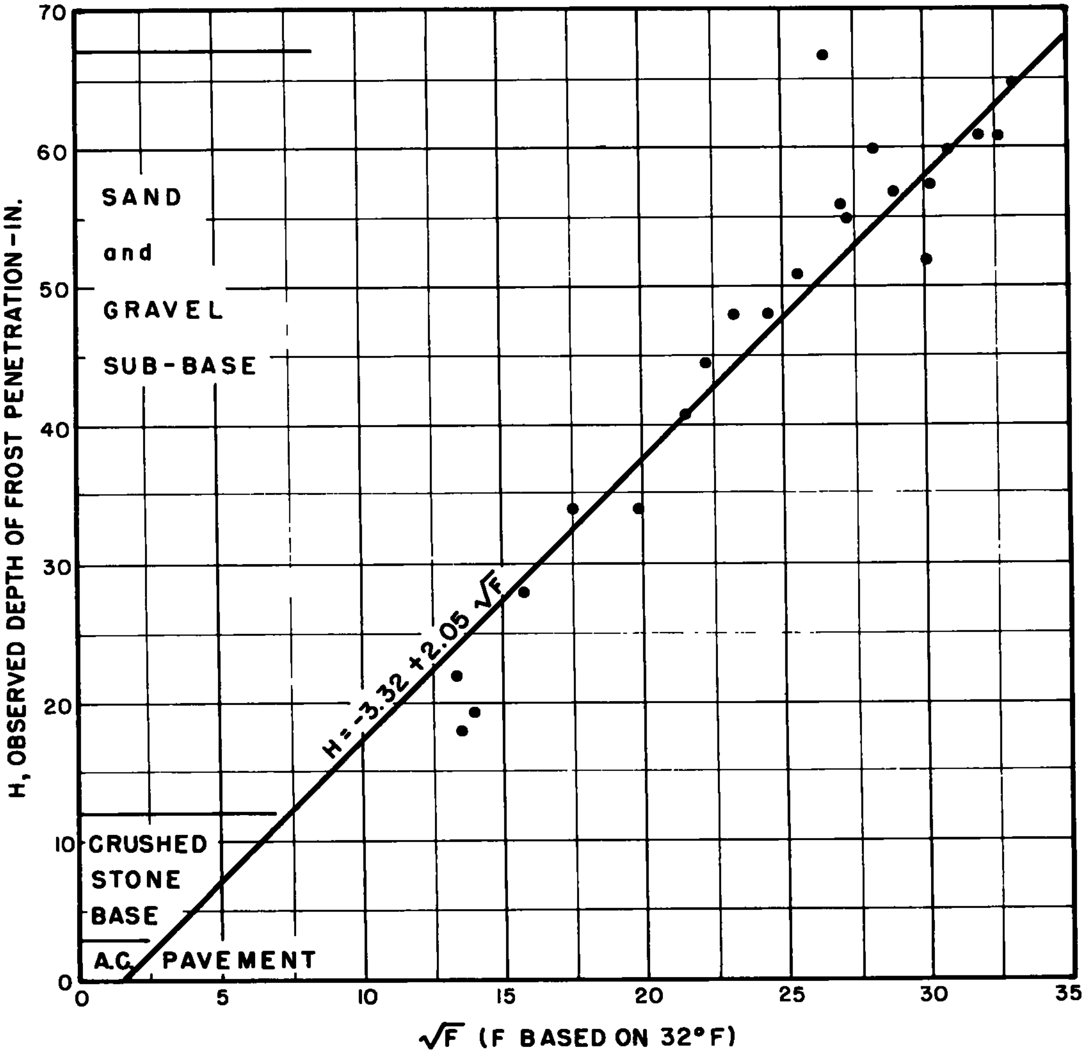

Figure 24. Observed depth of frost penetration and square root of freezing index based on 32 F.

timates and observations of frost penetration. It is computed on the assumption that all of the water in the soil freezes at ground temperatures, whereas it is known that this is far from the truth in fine-grained soils. The amount of unfrozen water varies with the soil, temperature, and time that the soil is exposed to freezing temperatures. Lovell (3) has recently written on the subject and Tsytovich (4), who gives further information on the phenomenon, shows that a clay may have about 25 percent of unfrozen water at 30 F, and about 16 percent at 20 F. Wintermeyer (5) summarizes the results obtained by many experimenters up to 1925. Taber, Bouyoucos, Winterkorn and Beskow (6) have all discussed this phenomenon.

The problem is still under study but the effect may explain why frost penetration in a clay is sometimes more than expected. It cannot explain discrepancies with gravels, however, because in clean sands and gravels essentially 100 percent of the water does freeze at ground temperatures. It looks as though the latent heat term should be studied seriously.

THE USE OF 29 F INSTEAD OF 32 F FOR THE DATUM IN COMPUTING DEGREE-DAYS OF FROST

The assumptions made in the development of frost penetration equations should be recalled when the various parameters in them are under discussion. The Stefan and modified Berggren formulas assume that the ground surface temperature suddenly drops below the freezing temperature of water and stays steady during the freezing season when the ground surface temperature suddenly rises again above the freezing temperature. The average temperature below freezing, $\sqrt{s}$, is multiplied by t, the duration of the freezing season in days, measures the energy abstracted from the soil. The Stefan equation assumes that all the energy goes into latent heat; the modified Berggren equation assumes that energy goes into sensible heat (in lowering ground temperatures) as well as into latent heat (in changing water to ice) and in that respect it is superior. The product of $\sqrt{s}\,t$ is given by the area under the temperature-time curve below the freezing temperature of water taken over the freezing season; this is called the surface freezing index F_s. Since surface temperatures are generally unknown air temperatures measured 4 to 6 ft above the surface which can be easily measured, or less reliably, estimated from observations taken at nearby places, are used with a correction factor C. In principle:

$$h = \sqrt{\frac{48kF_s}{L}} = C = \sqrt{\frac{48kF_A}{L}}$$

In the Stefan equation, C is usually associated with F_A, the air freezing index measured with 32 F as the freezing point of water:

$$h = \sqrt{\frac{48kF_A c_f}{L}} \qquad \text{(Stefan's equation)}$$

The c_f is found by equating observed frost penetration and values computed from the assumed formula.

It is clear that c_f has to cover many things, some quite unrelated to air ground temperatures, of which the sensible-heat effect is the most important.

In this paper the author takes several more steps to assume that "the soil surface temperature is related to the air temperature by the coefficient c_f" and applies it for part of a freezing season and apparently find discrepancies because he then starts to adjust F_A in a further assumption regarding the base temperature. In principle he changes his assumption of a constant ratio between air and surface temperatures to one of a constant difference between the temperatures. This may work well in a particular location; the more attractive way is to put the correction factor where it belongs—outside the root sign—and proceed to find it (if possible) rather than arbitrarily assuming a constant temperature difference. Actually the latter assumption may be a very good one if only freezing indexes are being considered, especially in the midwest where temperatures do drop quickly and stay low for the winter, then quickly rise again, but in general, temperatures do not fall and rise suddenly; at the end of the season there are fluctuations and the curve of temperature-time is far from rectangular, especially in New England.

Considering only freezing index again, the modified Berggren equation assumes that $\frac{\text{surface freezing index}}{\text{air freezing index}}$ is equal to 0.9 as a good statistical average based on a large quantity of data from ACFEL files. Recent analytical studies at M. I. T. under ACFEL contracts have shown that 0.9 is a good value to use north of latitude 45N but that it varies ±0.05 depending on solar radiation, wind and other factors. The writer thinks that the assumption of a constant temperature difference is at least as good as an assumption of constant temperature ratio but questions the temperature assumption as of general validity. By taking a whole freezing season the errors cancel to a considerable amount so that the assumption of a ratio between air and surface indexes for a whole season appeals to him as sound, whereas computations for a part season do not.

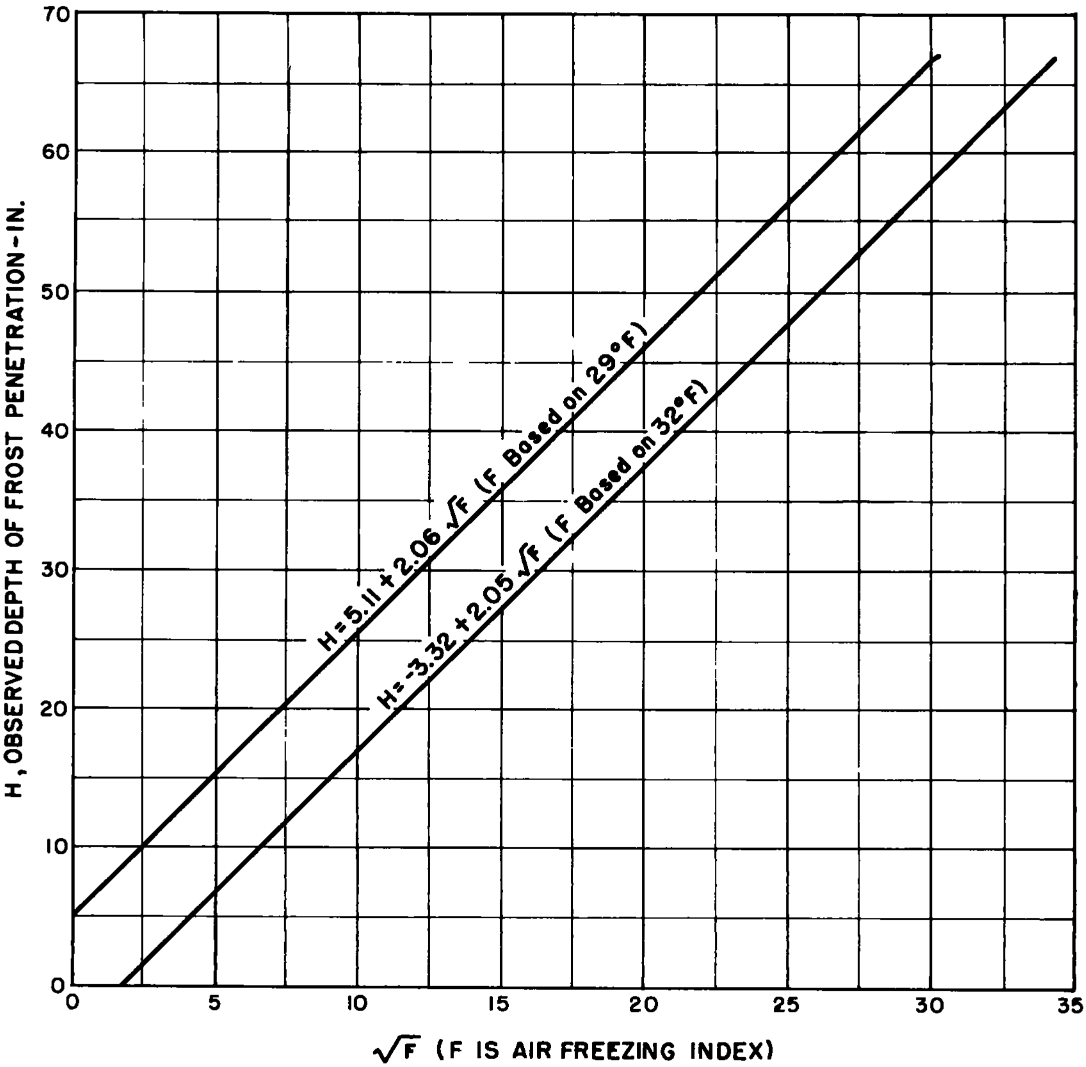

Figure 25. H, $\sqrt{F}$ curves from Figure 23 and 24 showing effect of freezing index based on 29 F and on 32 F.

In calculations for multi-layered systems there is no fallacy in computing degree-days for each layer but the result should not be taken to provide a plot of degree-days and time. The final penetration for the whole season is usually accurate enough using the modified Berggren equation and in a particular location, the simple Stefan equation (or an even simpler formula) might work well enough.

Author's Curves

A straight-line plot between H and $\sqrt{F}$ merely proves that within experimental limits H may be proportional to $\sqrt{F}$. Many straight lines (and curvilinear ones) can be drawn through experimental points; the "best" line is difficult to be sure of and the statistical analysis would be a big job. There are insufficient data available to use the author's curves and produce indexes based on 32 F but from ACFEL records of a location in northern Maine, plots have been made for a 29 F base and for a 32 F base. In each case a straight line has been drawn through the points using the method of least squares

(Figs. 23 and 24). Figure 25 shows both lines on a $\sqrt{F}$ base; which is better? If by means of the modified Berggren equation, X is plotted on a base of $\sqrt{F}$, the result is a curve of increasing slope (Fig. 26). It is probable that such a curve could be drawn through some of the author's points (again proving nothing).

The only real check on a new idea is computed depth versus measured depth at many locations, and under many conditions; but a formula of local application is still of value and should not be deprecated. The author has certainly covered a good range of soils in the Minneapolis area, and the use of 29 F with the Stefan formula may give results as good as, or better than, those obtainable using 32 F with that formula.

So much information on freezing indexes based on 32 F is now available that really strong evidence must be produced to justify a change from 32 F to 29 F for general usage.

The modified Berggren equation is not the final word in formulas; it shows discrepancies with granular soils of very low moisture content, giving results which are too big—on the safe side in design but uneconomical. The equation has other drawbacks also, especially in multi-layered systems. A constant value of surface temperature is assumed throughout a freezing season so that the formula should not be used for plotting frost penetrations against time throughout a freezing season although the result for a given full season is usually reliable. Nor at present can the equation be used in a system containing an interbedded insulating layer of zero moisture content.

No formula seems to work very well with granular soils of very low moisture content; perhaps the k values are the main reason for this.

At the present time (November 1958) methods based on micrometeorology are still not fully worked out although a promising start has been made at M. I. T. under contract with ACFEL and ACFEL is accumulating pertinent data for the evaluation of some quantities which so far have had to be estimated.

The writer considers that the best method of computing frost penetration is by a simple procedure and that the search for a very simple formula is on the wrong track. Numerical analysis with lumped elements, which is the basis of the analog solution (author's refs. 2, 15), is one possibility; the writing of Ingersoll (7) and Dusinberre (8) is recommended for the study of step-by-step procedures. All these approaches are fundamentally similar in using the first principles of heat flow. Structural designers use the moment distribution method without qualms—there is no reason why highway engineers should find the numerical treatment of heat transfer difficult; both techniques can give good results for specific problems where general solutions are very complicated or impossible. Accurate values of soil properties will always be a critical requirement whatever method is employed, and, of these, k is the most important.

In the meantime ACFEL is continuing the study of the modified Berggren equation to simplify its application still further.

THAWING OF SUBGRADES

A general formula for rate of thawing has not yet been found for seasonal frost areas but theoretical solutions to specific problems are obtainable by means of numerical procedures, either longhand or by analog computer (author's ref. 15). As the author points out, reliable measurements during the thaw period are extremely difficult to make; there is a considerable period of time when regions of the soil are at freezing temperature but part of the water is solid and part is liquid and the "depth of thaw" is meaningless. It is probable also that percolating water has an appreciable effect during the thaw period so that theoretical results may not be valid if this effect is ignored.

MICROMETEOROLOGICAL APPROACH

The direct method of studying the interaction between the air and the ground is a new approach which has shown promise but which still has a long way to go before it can be applied to engineering computations.

The weather data required are curves of air temperature and of windspeed observed at a standard height, cloud cover, sunshine duration, and vapor pressure (or relative

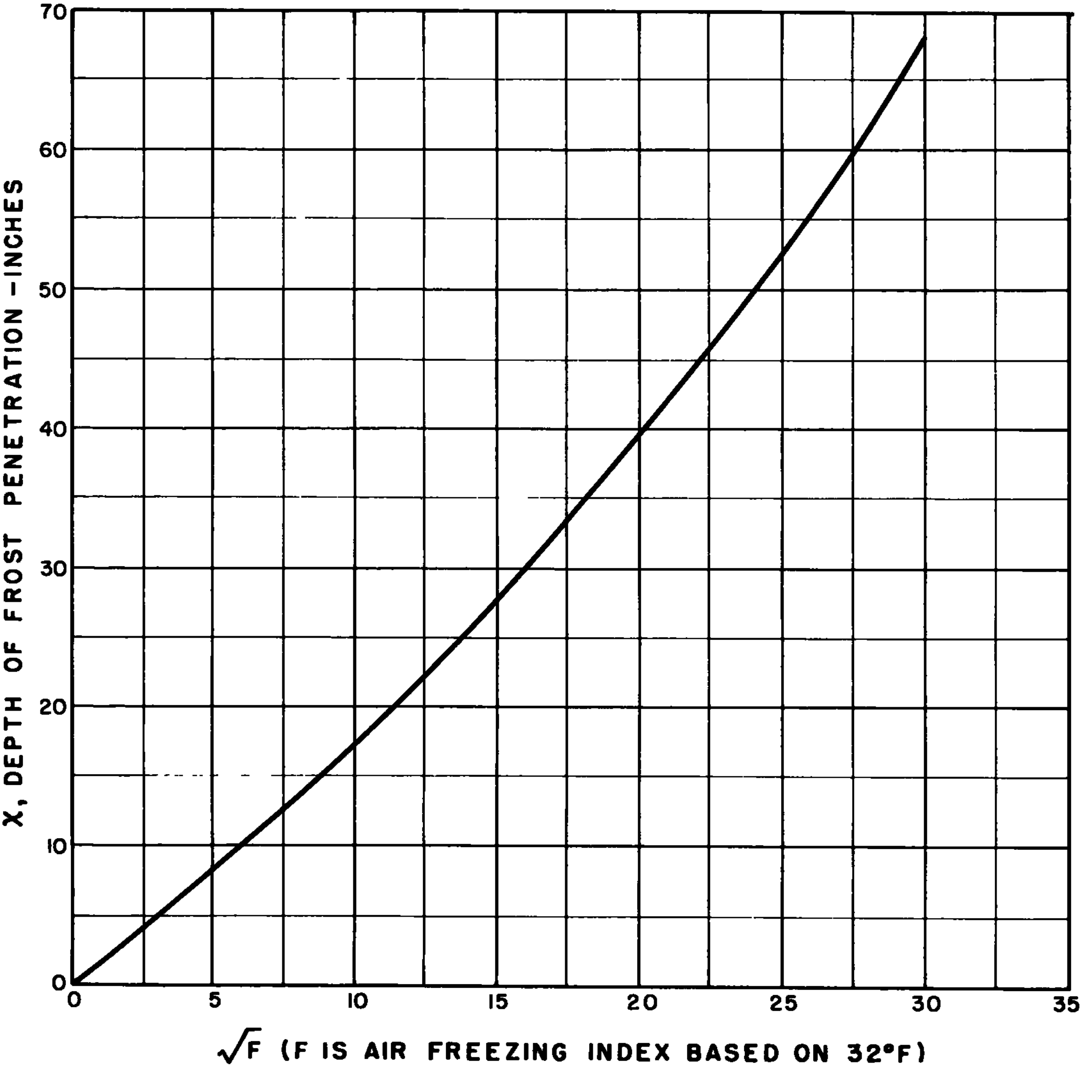

Figure 26. Modified Berggren formula, $X = 12\lambda\sqrt{\frac{48KnF}{L}}$. Typical curve of depth of frost penetration and square root of freezing index (γ_d = 140 pcf, w = 3.5%).

humidity) of the air. Ground data required are the nature of the srrounding surface and the soil profile with thermal properties of the soil. The latitude of the place must be known unless special measurements of heat radiation are made (not a usual procedure). If some of these requirements are not met, estimated values may cause serious errors.

The princple used, which is attractive, is to balance out the heat entering and leaving the surface of the ground; this enables the curve of air temperature to be modified to a curve of surface temperature from which the frost penetration can be computed either by a computer or by a numerical process from the known soil conditions. While attractive in principle, the method is quite difficult, combining the complexity of the weather with soil variability and the problem of freezing well known to "frost men."

The heat quantities are:

1. Solar Radiation. The sun radiates a known amount of energy received at the outside of the earth's atmosphere. This depends upon latitude and can be found in the

Smithsonian Tables. On its way through the atmosphere the flow is reduced by air, dust and clouds and at the surface some energy is reflected back, leaving a net amount of shortwave radiation, the main heat supply.

2. Longwave Radiation. If the earth's surface is at a different temperature from that of the air or of the clouds, longwave heat will be radiated between the "surfaces," the amount depending upon temperatures, surface emissivities and cloud cover. Estimation of this heat flow is somewhat uncertain at present. The surface temperature curve must be known for this heat computation and has to be assumed to start with. Its importance is evident from the low temperatures of calm, cloudless nights in winter when a lot of heat is being radiated away from the earth's surface.

3. Convective Heat Transfer at the Ground Surface. Air flowing over the surface leads to a heat exchange by forced convection at the surface. Windspeed over the surface is important and can be computed quite well from weather observations but surface roughness is not easy to evaluate because the wind is affected by a large area around the site.

4. Evaporation, Condensation, Sublimation, and Transpiration. Heat quantities involved in these phase changes of water substance and in plant metabolism are safely ignored in highway and airport computations, though very important in the micrometeorology of horticulture.

5. Conduction in the Soil. The net amount of heat left for conduction in the soil depends upon the balance of the other heat quantities and hence upon differences between large amounts—an unfortunate situation since small errors become enormously increased thereby.

The result of the heat balance computations is a curve of surface temperature and time from which the frost penetration can be found, preferably by means of a computer (the writer prefers the hydraulic analog type despite its drawbacks) but also by simple numerical procedures, or even, somewhat crudely, by formulas. In any event a computer is really necessary in the trial solutions for surface temperature; a complete curve has to be assumed and then checked, an extremely tedious process without a computer.

At present ACFEL has a theory and the computers. Field observations have been coming in but only for a year or two of complete micrometeorological data so it will be some time before the method will be satisfactory for engineering purposes. Most published material is for grassland, etc., during fair weather and that leaves a big gap. The analysis of a tremendous mass of weather data is a formidable task, but it is hoped that ultimately there will be simplified techniques for the direct solution of frost penetration problems from weather stations and soils data.

ACKNOWLEDGMENTS

Much of this discussion is based on data from the files of the Arctic Construction and Frost Effects Laboratory under the direction of Kenneth A. Linell. Peter A. Martus of the laboratory has been a valuable assistant to the writer in working up the data and making computations for the figures.

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HARL P. ALDRICH, Jr., Haley and Aldrich, Cambridge, Massachusetts—Situations arise where refined procedures for computing the depth of frost penetration are desired. The Stefan equation described by Kersten and even the modified Berggren formula discussed by Sanger may not fulfill requirements of a thorough study where an exact solution is required. The writer wishes to add, therefore, a few comments regarding analog, digital and hand numerical solutions, all of which have been used in recent years to solve specific problems or to verify approximate formulas.

These solution techniques are all based on a finite difference approximation of the differential equations governing the heat diffusion process. In simpler language the process involves dividing the soil profile in small "lumps" or layers of finite thickness and writing a series of simple mathematical equations for heat flow between midpoints of the lumps. The accuracy of the solution increases as the size of the lumps decreases.

Analogs

Two useful analogs have been used in recent years to solve a wide variety of complex practical problems, especially problems involving multi-layered soil profiles with surface temperatures varying with time. Both hydraulic and electronic analogs are in operation at the Arctic Construction and Frost Effects Laboratory of the New England Division, Corps of Engineers in Waltham, Mass.

Analogs follow from the formal correspondence between terms and equations in 3 important braches of physics, namely heat conduction, current conduction and laminar fluid flow, as given by Aldrich (1). These analogous relationships make it possible, for example, to model the heat flow frost problem with a simple fluid flow model containing a series of vertical standpipes connected by means of capillary tubes. Each standpipe represents a lump of soil while capillary tubes are scaled according to thermal conductivity and distance between lumps.

Design and operation of a hydraulic analog for one-dimensional frost problems is reported by M. I. T. (2). Scott (3) also describes the "computer" which was constructed at M. I. T. under a research contract with the New England Division, Corps of Engineers. Principal advantages of the hydraulic analog are initial cost, simplicity of programing and operation, and provision for a continual visual check on the solution. Principal disadvantage is the difficulty of maintaining a constant resistance in laminar flow capillaries.

An electronic analog computer for the frost problem was conceived in 1952 by Paynter of M. I. T. (4). During the following two years a computer containing 100 elements and capable of solving two-dimensional frost problems, was constructed and placed in operation at the ACFEL. The cost of the electronic analog is approximately ten times that of the hydraulic analog. Its principal advantage is the speed at which solutions can be obtained.

Numerical Solutions

Derivation of the simple mathematical equations which can be solved by hand using a desk calculator or slide rule or which may be solved by the IBM card programed computer, begin with the continuous Fourier one-dimensional conduction equation:

$$q = -k\frac{\delta v}{\delta x} \qquad (1)$$

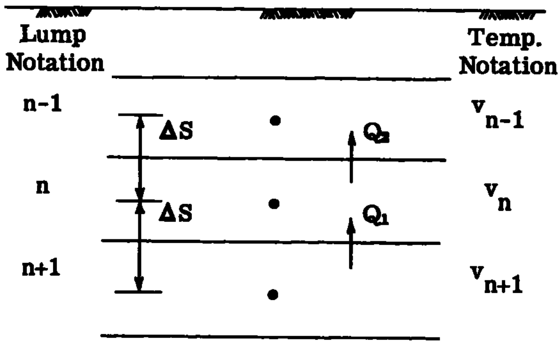
which is more familiar as:

$$Q = kiA \qquad (2)$$

in which

- v = temperature in deg F;
- Q = rate of heat flow in Btu/hr;
- k = thermal conductivity in Btu per (hr)(ft)(deg F);
- i = thermal gradient in deg F/ft; and
- A = area in sq ft, assumed 1.

The finite difference equation for one-dimensional heat flow in the absence of freezing and thawing is derived with the following notation:



where the following approximate equations may be written:

$$Q_1 = k\frac{v_{n+1} - v_n}{\Delta S}1$$

$$Q_2 = k\frac{v_n - v_{n-1}}{\Delta S}1$$

The net rate of heat flow into lump n is now:

$$Q_1 - Q_2 = k\frac{v_{n+1} + v_{n-1} - 2v_n}{\Delta S} \qquad (3)$$

The net rate of heat flow will be equal to the time rate of change of thermal energy u (Btu) of the layer:

$$Q_1 - Q_2 = \frac{\Delta u}{\Delta t}, \quad \text{where } \Delta u = C\Delta v_n(\Delta S \cdot 1 \cdot 1) \qquad (4)$$

$$= \frac{C\Delta v_n \Delta S}{\Delta t}$$

in which

- t = time in hrs; and
- C = volumetric heat in Btu per (cu ft)(deg F).

Equating Eqs. 3 and 4:

$$\Delta v_n = \frac{k}{C}\frac{\Delta t}{\Delta S^2}\left[v_{n+1} + v_{n-1} - 2v_n\right]$$

or

$$v_{n,k+1} - v_{n,k} = \beta \left[v_{n+1,k} + v_{n-1,k} - 2v_{n,k} \right] \quad (5)$$

in which

n-1, n, n+1 . . . = space notation;
k-1, k, k+1 . . . = time notation; and

$$\beta = \frac{k}{C} \frac{\Delta t}{\Delta S^2} \quad \text{(dimensionless)} \quad (6)$$

The developments presented above are extensively outline in the available literature. The introduction of latent heat from freezing or thawing complicates the basic solution somewhat. To the writer's knowledge, the numerical solution involving diffusion when latent heat effects are present was published first by Aldrich and Paynter (4).

For this development the heat storage relationship is considered for a cubic foot of soil containing water within at least a portion of its voids. A graph representing the idealized thermal energy versus temperature characteristics of the soil is shown in Figure 27.

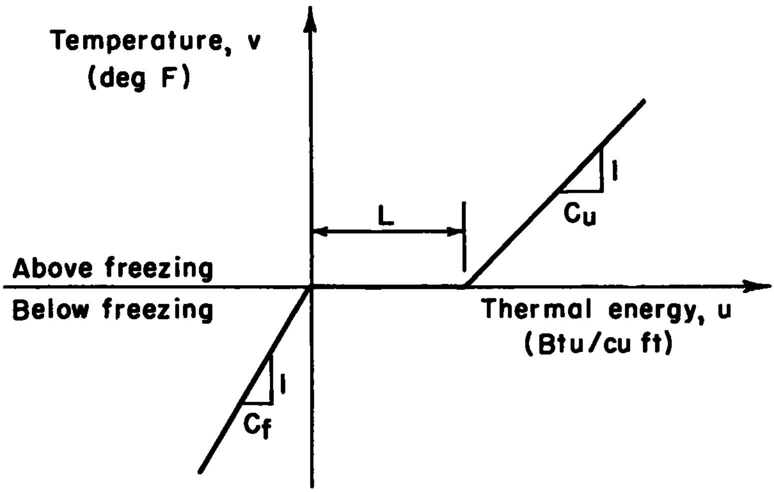


Figure 27.

in which

C_u, C_f = volumetric heat of unfrozen and frozen soil, respectively, expressed in Btu per (cu ft)(deg F); and
L = latent heat of soil moisture in Btu/cu ft.

Define a dimensionless quantity, e , which is a measure of the temperature, v, and is given by:

$$e = v\frac{C}{L} \quad (7)$$

Furthermore, define a dimensionless quantity, S, which is a measure of the thermal energy, u, and is related to e by the curve in Figure 28.

Since e is a function of v, Eq. 5 may be written:

$$e_{n,k+1} - e_{n,k} = \beta \left[e_{n+1,k} + e_{n-1,k} - 2e_{n,k} \right] \quad (8)$$

Finally, the following equations and conditions may be written:

$$S_{n,k+1} - S_{n,k} = \beta \left[e_{n+1,k} + e_{n-1,k} - 2e_{n,k} \right] \quad (9)$$

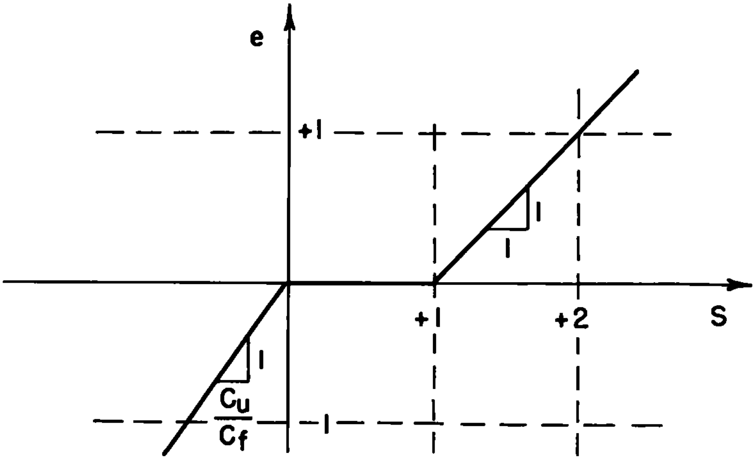


Figure 28.

where,

$e = S - 1$ for $S > 1$

$e = 0$ for $0 \leq S \leq 1$

$e = S\dfrac{C_f}{C_u}$ for $S < 0$

and where,

$\beta = a_u \dfrac{\Delta t}{\Delta S^2}$ for $S > 0.5$

$\beta = a_f \dfrac{\Delta t}{\Delta S^2}$ for $S \leq 0.5$

Numerical solutions based on the above set of normalized conditions have been made by Goldberg (5) for multi-layered soil profiles. As an illustration of the procedure and suggested form for tabulating computations, the following example is offered.

Consider a homogeneous stratum of soil semi-infinite in extent and initially at a uniform temperature of 62 F (30 F above the freezing point of soil moisture of v_o = 32 F). The temperature at the surface of the stratum is suddenly lowered to 22 F (10 F below freezing or v_s = 10 F). Assume thermal properties of soil as follows:

k = 1.0 Btu per (hr)(ft)(deg F);
C = 30 Btu per (cu ft)(deg F); and
L = 600 Btu/cu ft.

It is assumed here that k and C are the average for the frozen and unfrozen condition.

In terms of temperature then, the initial conditions and boundary conditions are:

depth ↓ n \ k (Time →)	0	1	2	3	
0	v_o 30 F	v_s 10 F	v_s 10 F	v_s 10 F	ΔS
1	v_o 30 F				ΔS
2	v_o 30 F				ΔS
3	v_o				
	Δt	Δt			

In terms of the dimensionless parameters e and S,

$$e = v\frac{C}{L}$$

Initially, then, throughout the depth,

$$e_o = v_o\frac{C}{L} = 30\frac{30}{600} = 1.50 \tag{a}$$

and at the surface when the temperature has been lowered to v_s:

$$e_s = -v_s\frac{C}{L} = -10\frac{30}{600} = -0.50 \tag{b}$$

Corresponding to condition (a), $S > 1$, therefore,

$$S_o = e_o + 1 = 2.50 \tag{c}$$

and corresponding to condition (b), from the thermal energy diagram it follows that $S < 0$, therefore,

$$S_s = e_s = -0.50 \tag{d}$$

since $C_f = C_u$ is assumed.

In terms of the dimensionless parameters the initial conditions and boundary conditions are:

n \ k	0		1		2	
Surface 0	1.50	2.50	-0.50	-0.50	-0.50	-0.50
1	1.50	2.50				
2	1.50	2.50				
3	1.50	2.50				

Notation:

n \ k	0	
0	$e_{n,k}$	$S_{n,k}$

For the numerical solution, select,

$$\beta = \tfrac{1}{4} = \frac{a\Delta t}{\Delta S^2} \tag{10}$$

Then from Eq. 9,

$$S_{n,k+1} = S_{n,k} + \tfrac{1}{4}(e_{n-1,k} - e_{n,k}) - \tfrac{1}{4}(e_{n,k} - e_{n+1,k}) \tag{11}$$

Finally, the numerical solution and a suggested form for recording computations is given on the following page.

n \ k	0		1		2		3		4		5
0	1.50	2.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50
	0	0	-2.00	-0.50	-1.50	-0.375	-1.25	-0.312			
1	1.50	2.50	1.50	2.50	1.00	2.00	0.75	1.75	0.594	1.594	
	0	0	0	0	-0.50	-0.125	-0.625	-0.156			
2	1.50	2.50	1.50	2.50	1.50	2.50	1.375	2.375	1.250	2.250	
	0	0	0	0	0	0	-0.125	-0.031			
	1.50	2.50	1.50	2.50	1.50	2.50	1.50	2.50	1.469	2.469	
							0	0			
							1.50	2.50	1.50	2.50	

ETC.

In the tabulation given above, the following notation is used:

	k		k+1	
n-1	$e_{n-1,k}$			
	$e_{n-1,k} - e_{n,k}$	$\frac{1}{4}(e_{n-1,k} - e_{n,k})$		
n	$e_{n,k}$	$S_{n,k}$	$e_{n,k+1}$	$S_{n,k+1}$
	$e_{n,k} - e_{n+1,k}$	$\frac{1}{4}(e_{n,k} - e_{n+1,k})$		
n+1	$e_{n+1,k}$			

The terms encircled are those appearing in Eq. 11. The real time and space relationship at any point in the solution is obtained from Eq. 10.

Numerical solutions have been programed on the IBM Card Programed Calculator or the M. I. T. Statistical Services Division and on the IBM—701 computer in New York City.

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R. W. J. PRYER, Soil Engineer, Quebec North Shore and Labrador Railway—Procedures which allow climatic data to be used more effectively for the prediction of ground freezing and thawing are of great practical value. The field measurements reported in this paper will be of general interest since they cover an extended period of time and are accompanied by adequate site descriptions.

A study of frost penetration records for a number of successive Canadian winters was reported by Legget and Crawford (1) in 1952. The results of this study prompted the authors to suggest that the freezing index concept might be modified to take into account the slope of the freezing index curve. The correlations obtained by Kersten are particularly interesting since his use of a cumulative degree-day total based upon 29 F tends to make such an allowance.

The author's review of the five series of data presented in the paper includes the statement, "On the basis of these data one would tend to accept the concept of the Stefan equation that depth does vary with the square root of a degree-day factor without mention of the time required to accumulate this value." However, since the term F in the Stefan equation is usually understood to be the summation of degree-days below 32 F, it should be noted that the substitution of 29 F as the base for the summation and the elimination of the surface correction factor C_f is equivalent to the use of F and a new correction factor, X. Thus:

$$h = C\sqrt{F_{29}} = C\sqrt{FX}$$

in which

F = summation of degree-days below 32 F; and
F_{29} = summation of degree-days below 29 F.

The value of X will be given by:

$$1 - \frac{3t}{F} + \frac{3t_1 - F_x}{F} \qquad \text{for values of } F > F_x$$

in which

F = summation of degree-days below 32 F;
F_x = summation of degree-days below 32 F when F_{29} = 0;
t = duration of the freezing perod from F = 0 (days); and
t_1 = interval between F = 0 and F_{29} = 0 (days).

The magnitude of the correction factor X depends, therefore, upon the ratio F/t or the average rate at which degree-days are accumulated during the freezing period. If t_1 is assumed to be small—a condition which is likely to exist when mean daily air temperatures drop rapidly at the start of the freezing season—the normal freezing index curve for Minneapolis (Fig. 1) indicates that X should increase from about 0.6 when F = 200 to about 0.8 when F = 1,200.

Experience in the Labrador Peninsula region of eastern Canada supports the author's observation that a given number of degree-days of cold can result in relatively deep frost penetration during a mild winter. In a locality where the normal freezing index is in the order of 5,000, importance is attached to the average mean daily air temperature during the 15-day period which precedes the onset of the freezing season. High temperatures during this period are usually associated with moderate rainfall, the melting of early snow cover and a marked increase in the moisture content of the soil. Measurements have been made in railroad subgrade sections in soils which are not susceptible to serious ice segregation and where snow plow operations produce fairly uniform snow cover conditions. The results suggest that when a mild winter is preceded by a two-week period during which the average mean daily air temperature is low (33 F), frost penetration may be more rapid than is the case when a severe winter follows a period of mild temperatures (44 F). This effect is most pronounced during the early part of the freezing season when estimates of frost penetration based upon degree-day totals and the measurements of previous years may be in error by as much as two or three feet.

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CLOSURE, Miles S. Kersten--The discussers have made valuable contributions to the subject of methods of calculating frost penetrations. The explanation of the use of of the modified Berggren equation by Sanger, and particularly the presentation of the chart for λ correlated with air freezing indexes, gives a promising procedure which can be strengthened by further correlation studies. The solution techniques described by Aldrich may likewise be useful where a "more exact" procedure is required. In all frost calculations, one must initially decide what degree of completeness is desired and then adopt either a simple, though approximate, method, or a more advanced one taking into account more of the influencing factors; in the latter case some knowledge of these factors is obviously required, and the accuracy of the results is dependent upon the correctness of these values as well as on the completeness of the method. Continued study of all of these methods, including the micrometeorological approach described by Sanger will lead to more dependable values and better results.

The experience of ACFEL has apparently indicated the poorest correlations between calculated and measured depths for sand soils. This corresponds to the observations in Minnesota.

The data presented in the paper were not intended to prove superiority of the use of degree-days below 29 F rather than 32 F. It merely suggested that such a system might have some merit, particularly for calculations in the initial parts of a freezing season, and that further studies might be profitable.

Pryer's discussion suggests another factor which merits consideration. This is the temperature and precipitation conditions in the 15-day period preceding the onset of the freezing season.