Relationship of Topographic Relief, Flight Height, and Minimum and Maximum Overlap

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The effects of topographic relief on overlap in aerial stereoscopic photography become acute when flight height must be sufficiently low for taking aerial photographs suitable for large scale mapping by photogrammetric methods for highways. While these same effects are present in small scale photography used to compile small scale maps, their consequences are not acute because the large flight height permits a greater relief height. For the double projection photogrammetric instruments commonly used, the ratio of relief height to flight height (h/H) varies from 0.21 to 0.36.

Principles governing the design of endlap (overlap in line of flight) and sidelap (overlap of one strip of photographs on another) are presented. Considerations that must be made when determining the minimum flight height that can be utilized according to the relief height existing in the area to be photographed and mapped at large scale with small contour interval are outlined, and their effects on the maximum scale attainable are pointed out. Whenever large scale mapping for highway surveys is to be undertaken by precise photogrammetric methods, the specific relationship between relief height in the area to be mapped and the photography flight height must be fully considered. Graphs are provided to serve as aids in ascertaining limiting conditions.

● STEREOSCOPIC photographic coverage of the ground is the cardinal requirement for mapping by stereophotogrammetric methods. As the aircraft moves the aerial camera forward along its line of photographic flight, this coverage is attained by photographing ground detail from separate camera stations. Separation of the camera stations is such that part of the area covered by each successively taken photograph is common to an area covered on the preceding photograph.

The area of overlap in photographic coverage along the flight line is called forward lap or endlap. The absolute minimum in endlap to obtain stereoscopic coverage by vertical photography is 50 percent of the flight line dimension of each photograph. In practice, an endlap greater than 50 percent is necessary for choosing pass points between successive stereoscopic models and for attaining continuity in mapping from model to model. These pass points serve in somewhat the same manner as backsights and foresights in running traverses and in spirit leveling by ground survey methods.

If several parallel strips of vertical photography are required for coverage of an area, they must have a common area of overlap called sidelap. In this way, image points common (conjugate) to photographs in adjacent strips are available for selection to serve as pass points so that continuity can be attained in mapping from one set of stereoscopic models to the other sets which are immediately adjacent in the separate flight lines of photography.

For efficiency in photogrammetric utilization of vertical photography, the maximum endlap should not exceed the percent needed to provide full stereoscopic coverage of the ground plus a small area of common stereoscopic coverage from one stereoscopic model to another. In addition, such percent cannot be allowed to become greater than the percent admissible by the photogrammetric instruments. That which follows is a presentation of principles which should be understood and applied in specifying endlap and sidelap, according to the topographic relief encountered and aircraft flight height required within the area to be photographed for aerial surveys and mapping by photogrammetric methods.

If the ground area photographed were flat and the photographic mission performed with perfection, the overlap of the photographs would consistently agree with the ideally designed value. In actuality, however, ground areas contain relief and no photographic crew performs perfectly at all times. Consequently, within each specific area, overlap attained in the photography varies in line of flight for endlap from one successive stereoscopic pair to another, and for sidelap between the adjacent strips of photographs.

EFFECTS OF RELIEF

Topographic relief causes radial displacement of the photographic images of ground points. For any given flight height, this displacement is proportional to the height of the point above or below the datum plane and to the radial distance between the nadir (plumb) point and the displaced point. High points are displaced outward from the nadir point and low points are displaced inward toward this point. Thus, a high point near the edge of an area to be photographed could be displaced so far perspectively as to not appear on the photographic format.

Perspective displacement of high relief can cause a gap in the stereoscopic coverage—an area that could not be mapped (a) in line of flight, (b) along the edge of a single strip of photographs, and (c) between the adjacent parallel strips. Situations causing the gaps must be avoided by proper design of photography endlap and sidelap limits, flight height, and flight lines. To accomplish this by increasing the amount of overlap (both endlap and sidelap) increases the number of photographs necessary to cover an area stereoscopically. Then the cost of bridging or mapping is increased proportionately. An increase in endlap results in a shorter airbase. The accuracy of the mapping is unduly lowered whenever unnecessary shortening of the airbase decreases the precision with which relief can be perceived and measured within the stereoscopic model. Actually, the relationship of relief height to flight height is a primary consideration in coping with such problems.

EFFECTS OF TILT

The effect of tilt is not accounted for in compilation of the tables, and in preparation of the figures and graphs. The consequences, however, and the numerical effects of tilt on endlap and sidelap are subsequently explained.

Sidelap and endlap will be decreased on the portion of each aerial negative tilted above the plane of the vertical and will be increased on the portion tilted below that plane. Whenever tilt does not exceed five degrees, the decrease per degree of tilt is approximately 1.8 percent and 2.0 percent, respectively, and the increase is 1.9 percent and 2.1 percent, respectively, on photographs taken with 6-in. and 8.25-in. focal length aerial cameras. For practical purposes, the increase and decrease in endlap and sidelap can be considered as two percent per degree of tilt.

The Reference Guide Outline, Specifications for Aerial Surveys and Mapping by Photogrammetric Methods for Highways—1958, stipulates that tilt in any one photograph shall not exceed three degrees, and the average tilt shall not exceed one degree for the entire project. Whenever tilt is kept within these specification limits, only the few photographs which have tilt exceeding two degrees would cause sufficient change in overlap as to reduce endlap to less than 51 percent.

Accordingly, when the minimum endlap on vertical photography is 55 percent, adjacent photographs with tilt exceeding two and one-half deg will have their endlap reduced to about 50 percent on one side and increased to about 60 percent on the other side. Thus, to avoid resultant gaps in stereoscopic coverage caused by tilt, tilt, must be less than two degrees, or the minimum endlap limit of 55 percent on vertical photography should be changed to 57 percent if tilt of three degrees is permitted, 59 percent for four degrees, and 61 percent for five degrees.

Axiomatically, the effective width of stereoscopic coverage on a single strip is decreased about two percent per degree of tilt occurring on the x-axis, the line of flight. Endlap in line of flight is similarly decreased on one edge and increased on the other by tilt occurring on the y-axis, the axis normal to the line of flight. Tilt occurring on other axes will have combination effects of less than two percent per degree of tilt on endlap and on width of stereoscopic coverage.

The analyses subsequently presented are for tilt-free vertical photographs—practical applications of which will not be so adversely affected as to be nullified when tilt does not exceed the reasonable minimum. The alternative is to maintain minimum endlap on vertical photography greater than 55 percent to prevent endlap becoming less than usable on tilted photography. This practice, because tilt cannot be eliminated, decreases the efficiency of mapping by photogrammetric methods. Sidelap will be affected in a similar manner, and also the continuity of photographic coverage along the edge of a single strip, such as in route photography.

PRINCIPLES

For double projection, photogrammetric instruments like the Multiplex, Balplex, Kelsh, and Photocartograph (called Photomapper in the U.S.), there is a limit to which the airbase can be shortened by increasing the endlap to satisfy relief-height to flight-height relationship requirements. Whenever this limit is exceeded, a stereoscopic model cannot be produced because projectors of the instrument will touch before the desired stereomodel scale is attained. The maximum allowable endlap will vary with the double projection instrument used and the map-scale to photography-scale projection ratio. The allowable endlap limits in percent determined by the projector positions of such instruments are listed in the final column of Table 1.

Optical train instruments are capable of using pairs of photographs containing larger percentages of endlap than can be utilized in double projection instruments. As circumstances permit, however, and unless only two photographs are available when excessive overlap occurs, the second photograph of each three is omitted. Thus, photographs numbered one, three, five, and so forth of each flight line are used when feasible.

Another and more critical factor, which limits the amount the airbase can be shortened by increasing the endlap to satisfy the requirements of h/H (relief height divided by flight height), is the range in vertical measurement of photogrammetric instruments. The fourth column of Table 1 lists the vertical measurement range of the various double projection instruments. This range is set by the projection zone in which the stereoscopic model is sharp enough to be measured with ease and consistency. Whenever differences in elevation of relief within a model are so large as to encompass all or most of this range, then such differences, called relief height, must be appropriately considered in relation to the flight height above the points of lowest elevation, or both endlap and sidelap requirements may not be met.

In column 5 of Table 1, h/H equals the vertical measurement range of the instrument in inches divided by the maximum projection distance in inches. This maximum

TABLE 1

INSTRUMENT LIMITATIONS TO MAXIMUM ALLOWABLE ENDLAP

Double Projection Photogrammetric Instrument	Projection Ratio[1]	Photography Focal Length (in.)	Vertical Measurement Range[2] (in.)	h/H Ratio	Maximum Endlap Governed by Vertical Measurement Range[3] (%)	Maximum Endlap Governed by Projector Position (%)
Multiplex	2.4:1	6	6.7	0.36	71	74
Balplex (525)	3.4:1	6	7.0	0.28	67	70
Kelsh stereoplotter	4:1	8.25	9.9	0.25	66	71
Kelsh stereoplotter	5:1	8.25	9.9	0.21	64	77
Kelsh stereoplotter	5:1	6	9.0	0.25	66	77
Balplex (760)	5:1	6	9.0	0.25	66	79
Photocartograph	5:1	6	9.0	0.25	66	73
Kelsh stereoplotter	7:1	6	11.0	0.22	65	83
Photocartograph	7:1	6	11.0	0.22	65	80

[1] Number of times stereoscopic model scale is larger at an optimum projection distance than the scale of vertical photography.
[2] Depth of focus of the projection lenses of the instrument in projecting a visually sharp stereoscopic model.
[3] For each instrument, this is the maximum endlap allowable at the point of lowest relief appearing on one edge of the stereoscopic overlap when the point of highest relief is 5 percent of the length of the photograph from the opposite edge of such overlap. This condition results in a minimum endlap of 55 percent at the level of the point of highest relief.

projection distance is the projection ratio of the photogrammetric instrument times the focal length of the aerial camera plus approximately 60 percent of the vertical measurement range of the instrument, and the minimum projection distance is the maximum projection distance minus its vertical measurement range. To compute the percent of endlap in column 6 of Table 1, the h/H ratio in column 5 is used in the equation for maximum endlap, $E_1 = E_2 + 50 + (50 - E_2)\ h/H$, which is developed later. The percents in the same column are also equal to 100 minus the quantity of 45 times the minimum projection distance divided by the maximum projection distance.

The final column of Table 1 lists the maximum endlap, as governed by the position of the projectors in double projection instruments. Since the preceding column contains smaller percents of endlap, the vertical measurement range of each instrument limits the maximum allowable endlap in the photography for mapping with double projection instruments.

Endlap limits of 55 to 65 percent with an average of 57 percent have been specified for aerial vertical photography. It will be shown later, in development of the relationship of minimum and maximum endlap, that the 55 to 65 percent limits will accommodate a ratio of relief height to flight height of only 2/9. These limits are easily complied with for small scale photography where the flight height is relatively high. For example, photography taken from a flight height of 20,000 ft and containing the 55 to 65 percent endlap at points of highest and lowest relief, respectively, would accommodate a maximum relief of 4,444 ft. These 55 to 65 percent limits, however, are difficult and sometimes almost impossible to adhere to under certain relationships of relief height and low flight heights.

When the end product required is maps of large scale for engineering purposes, and

TABLE 2

MAP SCALE CONTROLLING USE OF PHOTOGRAMMETRIC INSTRUMENTS

Photogrammetric Instrument	Ratio of Map Scale to Photog. Scale	Map Scale (ft to 1 in.)	Photog. Scale (ft to 1 in.)	Flight Height (ft)	Maximum Relief[1] (ft)	Feasible Contour Interval[2] (ft)	Resultant C-factor[3]
Multiplex	2.4:1	20	48	288*	64	0.5	576
		40	96	576*	128	1	576
		50	120	720*	160	2	360
		80	192	1152	256	2	576
		100	240	1440	320	2.5	576
		200	480	2880	640	5	576
Balplex (525)	3.4:1	20	68	408*	91	0.5	816
		40	136	816*	181	1	816
		50	170	1020	227	1.5	680
		80	272	1632	363	2	816
		100	340	2040	453	2.5	816
		200	680	4080	907	5	816
Balplex (760)	5:1	20	100	600*	133	0.5	1200
Kelsh stereoscopic		40	200	1200	267	1	1200
plotter and Nistri		50	250	1500	333	2	750
Photocartograph		80	400	2400	533	2	1200
		100	500	3000	667	2.5	1200
		200	1000	6000	1333	5	1200
Kelsh stereoscopic	7:1	20	140	840*	187	1	840
plotter and Nistri		40	280	1680	373	2	840
Photocartograph		50	350	2100	467	2	1050
		80	560	3360	747	2.5	1344
		100	700	4200	933	4	1050
		200	1400	8400	1867	10	840
Optical Train:							
Wild Autograph, A-7;	8:1	20	160	960*	213	1	960
Zeiss Stereoplani-		40	320	1920	427	2	960
graph, C-8; Nistri		50	400	2400	533	2	1200
Photostereograph,		80	640	3840	853	4	1210
B-2; and Galileo-		100	800	4800	1067	5	960
Santoni Stereocartograph		200	1600	9600	2133	10	960

* Under usual conditions these flight heights are lower than practicable.

[1] Should endlap be larger than 65 percent for points of lowest relief, the maximum admissible by some of the instruments, the maximum relief measurable would be slightly larger than listed in this column.

[2] As a practical unit, the contour interval is one-half or nearest full foot only.

[3] Resultant C-factors must not be construed as an accuracy measurement of the photogrammetric instrument. In most cases map compilation scale governs, therefore, nearly all resultant C-factors are less than those commonly used (Table 3) and, whenever this occurs, the accuracy in contour compilation should be improved.

the map compilation is to be done by photogrammetric methods at the scale specified for the finished maps, the flight height must be relatively low. As a result, if compliance with 55 and 65 percent endlap limits were to be held to, with the resulting relief height to flight height ratio of 2/9, the maximum relief that could be accommodated for various photogrammetric instruments, photography scales, and flight heights, when 6-in. focal length photography is to be used and the map compilation scales are as listed, would be as given in Table 2. Columns 7 and 8 of Table 2 also list the feasible contour interval obtainable and the resultant C-factor when the map scale is allowed to control use of the photogrammetric instrument.

If the contour interval desired is small, the C-factor often applied in photogrammetric instrument operation may cause the contour interval to control the flight height. The maximum relief that can be accommodated by the 55 and 65 percent limits, when the contour interval controls, is given in Table 3. As an example, if a Kelsh stereoscopic plotter using 6-in. focal length photography is to be used to compile a topographic map with a contour interval of 1 ft, and the projection ratio of map scale to photography scale is 7 to 1, a C-factor of 1,300 might be used for this instrument. Using this C-factor as an indicator of the capability of the instrument, it is assumed that contours at the 1-ft interval may be delineated by use of photography taken from a flight height of 1,300 ft. The maximum relief which may be accommodated at this 1,300-ft flight height with a maximum endlap of 65 percent is 2/9 of 1,300, or 289 ft. The photography scale expressed in terms of feet per inch is equal to the flight height in feet divided by the focal length of the aerial camera in inches (1,300 divided by 6), which is 217 ft to 1 in. The desirable resultant compilation scale on the map manuscript is nearly seven times larger than the photography scale, or 30 ft to 1 in. Should map compilation at a scale of 30 ft to 1 in. be required for topographic mapping with the the same instrument and a contour interval of two feet, photography would have to be

TABLE 3

CONTOUR INTERVAL CONTROLLING USE OF PHOTOGRAMMETRIC INSTRUMENTS

Photogrammetric Instrument	Ratio of Map Scale to Photog. Scale	C-factor Commonly Used	Contour Interval (ft)	Flight Height (ft)	Maximum Relief[1] (ft)	Resultant Compilation Scale on Map Manuscript from Stereomodel (ft to 1 in.)
Multiplex	2.4:1	600	0.5	300*	67	20
			1.0	600*	133	40
			2.0	1200	267	80
			5.0	3000	667	200
			10.0	6000	1333	400
Balplex (525)	3.4:1	1000	0.5	500*	111	15
			1.0	1000	222	30
			2.0	2000	444	60
			5.0	5000	1111	150
			10.0	10,000	2222	300
Balplex (760)	5:1	1200	0.5	600*	133	20
Kelsh stereoscopic			1.0	1200	267	40
plotter, and Nistri			2.0	2400	533	80
Photocartograph			5.0	6000	1333	200
			10.0	12,000	2667	400
Kelsh stereoscopic	7:1	1300	0.5	650*	144	15
plotter, and Nistri			1.0	1300	289	30
Photocartograph			2.0	2600	578	60
			5.0	6500	1444	150
			10.0	13,000	2889	300
Optical Train:						
Wild Autograph, A-7	8:1[2]	1500	0.5	750*	167	15
Zeiss Stereoplani-			1.0	1500	333	30
graph, C-8; Nistri			2.0	3000	667	60
Photostereograph, B-2;			5.0	7500	1667	150
and Galileo-Santoni			10.0	15,000	3333	300
Stereocartograph						

* Under usual conditions these flight heights are lower than practicable; also the 0.5-ft contour interval is smaller than practicable unless there is little or no ground cover and height of relief is very small within the area to be photographed and mapped by stereophotogrammetric methods.

[1] Should endlap be larger than 65 percent for points of lowest relief (that is, equal to the maximum admissible by most of the instruments) the maximum relief measurable would be slightly larger than listed in this column.

[2] By changing ratios on the coordinatograph of the optical train instruments, map as desirable, can be compiled at scales smaller than eight times the photography scale, as six, five, and so forth.

taken to the same scale from the flight height of 1,300 ft. The resultant C-factor would be 650, and, consequently, it should be especially easy, wherever the ground can be seen from the air, to achieve the desired accuracy in contour delineation.

Examination of flight heights involved (column 5) and maximum relief (column 6) that can be accommodated, when endlap is limited between 55 and 65 percent, indicates that flexible limits are desirable. This is especially true when large scale photography for large scale topographic mapping with a small contour interval is required. An increase in maximum endlap limits would permit an increase in maximum relief measurable within a stereoscopic model. The maximum endlap limits, however, cannot exceed the endlap acceptable to the particular photogrammetric instrument that will be used. Maximum endlap admissible by one instrument is 71 percent, 67 percent for another, and 66 to 64 percent for the remaining commonly used double projection instruments (Table 1, column 6). The minimum endlap is fixed by stereo-requirements.

In column 7 of Table 3 is listed the resultant compilation scale on the map manuscript for 5 different contour intervals (column 4) and for various photogrammetric instruments. In some cases these are not standard map scales. Manuscripts at such scales would generally be photographically reduced to the nearest smaller standard scale for preparation of the finished maps.

Factors affecting endlap were examined and an expression was developed to correlate the relationship between minimum and maximum endlap and relief height and flight height. In deriving the equations subsequently presented, only vertical photographs without crab or tilt were considered. Six variable factors were involved: minimum endlap, maximum endlap, flight height, relief height, and the limiting position on each photograph of the point of highest relief and the point of lowest relief.

Since the position of points of highest and lowest relief cannot be predetermined, they are assumed to be at the position where the perspective geometry of the photographs will cause maximum radial displacement. The position of the point of highest relief is defined, therefore, as lying somewhere on a line normal to the flight line and passing through the principal point of one of the photographs of the stereoscopic pair. (Referring to Figure 1, n_1 to a_1 is the line on which the point of highest relief appears in this space geometry illustration.) The position of the point of lowest relief is defined as lying somewhere on the extreme opposite edge of the same photograph, the edge in the stereoscopic overlap that is approximately parallel to the line on which the point of highest relief causing minimum endlap actually lies. For simplification, the point of lowest relief is assumed to be in the datum plane, as represented by point G_1.

To expand the problem to include sidelap, the same variable factors are involved. In addition, the image of principal points of the adjacent photograph do not normally appear in the sidelap area. Thus, the position of the point of highest relief, fixed arbitrarily for definition purposes, is defined as lying on a line parallel to and at a minimum sidelap distance from the edge of each of the adjacent sidelapping photographs; therefore, this line lies midway within the sidelap area. The position of the point of lowest relief lies on the near edge of each sidelapping photograph, and is assumed to be at the datum plane for the particular photograph on which sidelap is being measured.

EQUATIONS

Equations expressing the relationships of minimum and maximum endlap and sidelap are derived by use of the following terms, which are illustrated in Figures 1 to 3:

E_1 is the maximum endlap at the datum plane. The distance E_1 is measured from a point lying in the datum plane at the edge of one photograph to the conjugate image on the same photograph of a point which lies in the datum plane at the edge of the photograph which is adjacent in line of flight. E_1 is expressed as a percent of the dimension of the photograph in line of flight.

E_2 is the minimum endlap distance that the point of highest relief affecting endlap is from the edge of the photograph. The distance E_2 is measured from the edge of the photograph to the image of the point of highest relief. E_2 is expressed as a percent of the dimension of the photograph in line of flight.

S_1 is the maximum sidelap at the datum plane. It is the distance from the edge of the photograph to the conjugate image on the same photograph of a point at the datum plane appearing at the edge of the photograph which is in the adjacent line of flight. S_1 is expressed as a percent of the dimension of the photograph normal to the line of flight.

S_2 is the minimum sidelap distance that the point of highest relief is from the edge of the photograph. This distance is equal on photographs in adjacent flight lines whenever the minimum sidelap requirements are met on both photographs. S_2 is expressed as a percent of the dimension of the photograph normal to the line of flight.

h is the height above the datum plane of the point of highest relief which affects endlap or sidelap.

H is the aircraft flight height above the datum plane from which the stereoscopic pair of photographs being considered were, or will be, taken. Two intermediate values used in deriving the relationships are:

r is the projection of the radial distance between the principal point and the image of

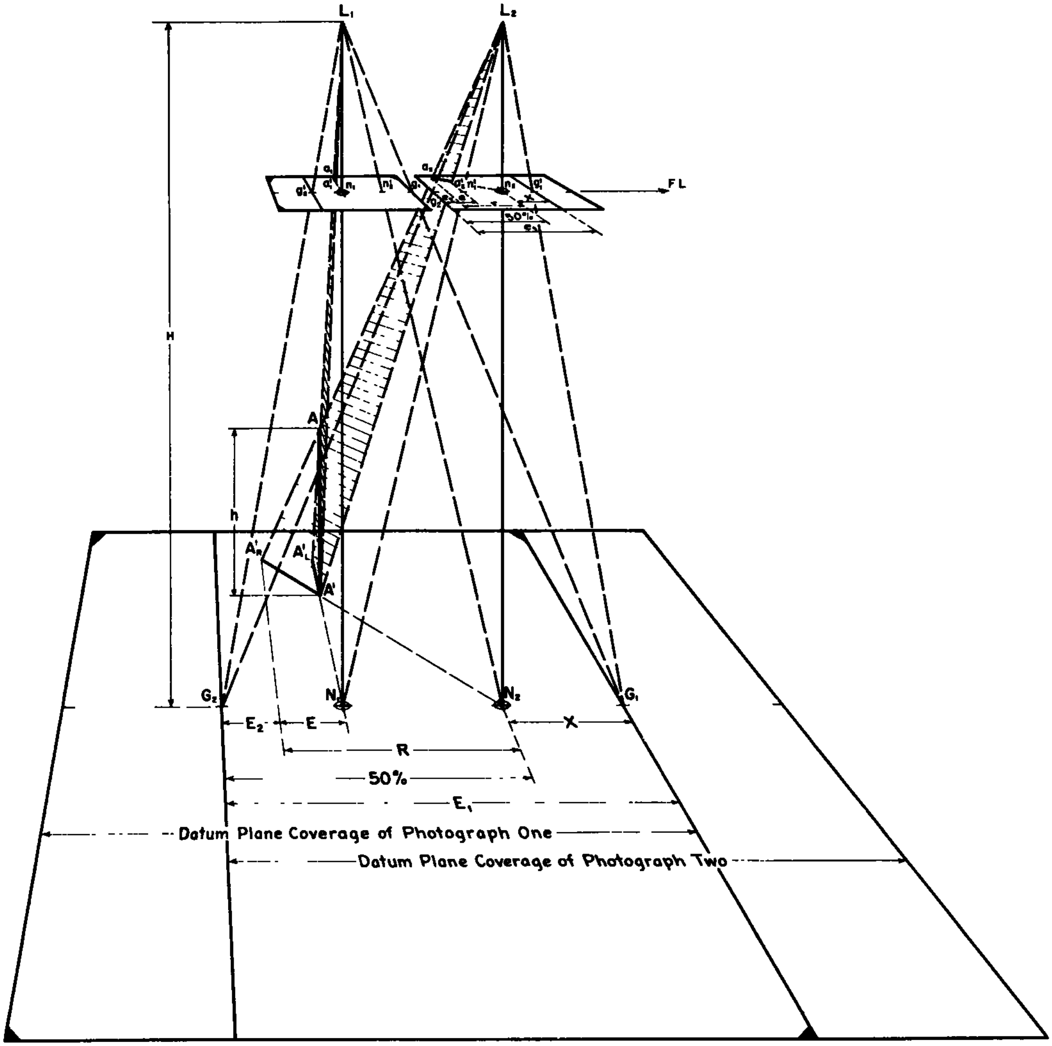


Figure 1. Space geometry of pair of aerial vertical photographs adjacent in line of flight to show endlap (E_1) at datum plane and endlap (E_2) at point of highest relief.

$$E_1 = E_2+50+(50-E_2)\ \frac{h}{H}$$

the point of highest relief on to the plane of endlap or sidelap measurement. In Figure 1 this projection is made orthographically on to a line parallel to the flight line for endlap. For sidelap, it is made on to a line normal to the flight line.

e is the projection of the radial displacement of the point of highest relief on to the line of endlap or sidelap measurement. The separate projections for sidelap and endlap are made in the same manner as for r.

With these terms defined, and with the position of the points of highest and lowest relief fixed, as stated previously, examination of Figures 1, 2, and 3 results in the following relationships:

By similar triangles, e, e_1, e_2, s_1, s_2, etc., on the photographs, are analogous, respectively, to E, E_1, E_2, S_1, S_2, etc., in the datum plane. The capitalized representation, as shown for the datum plane conditions, are subsequently used in all equations and charts.

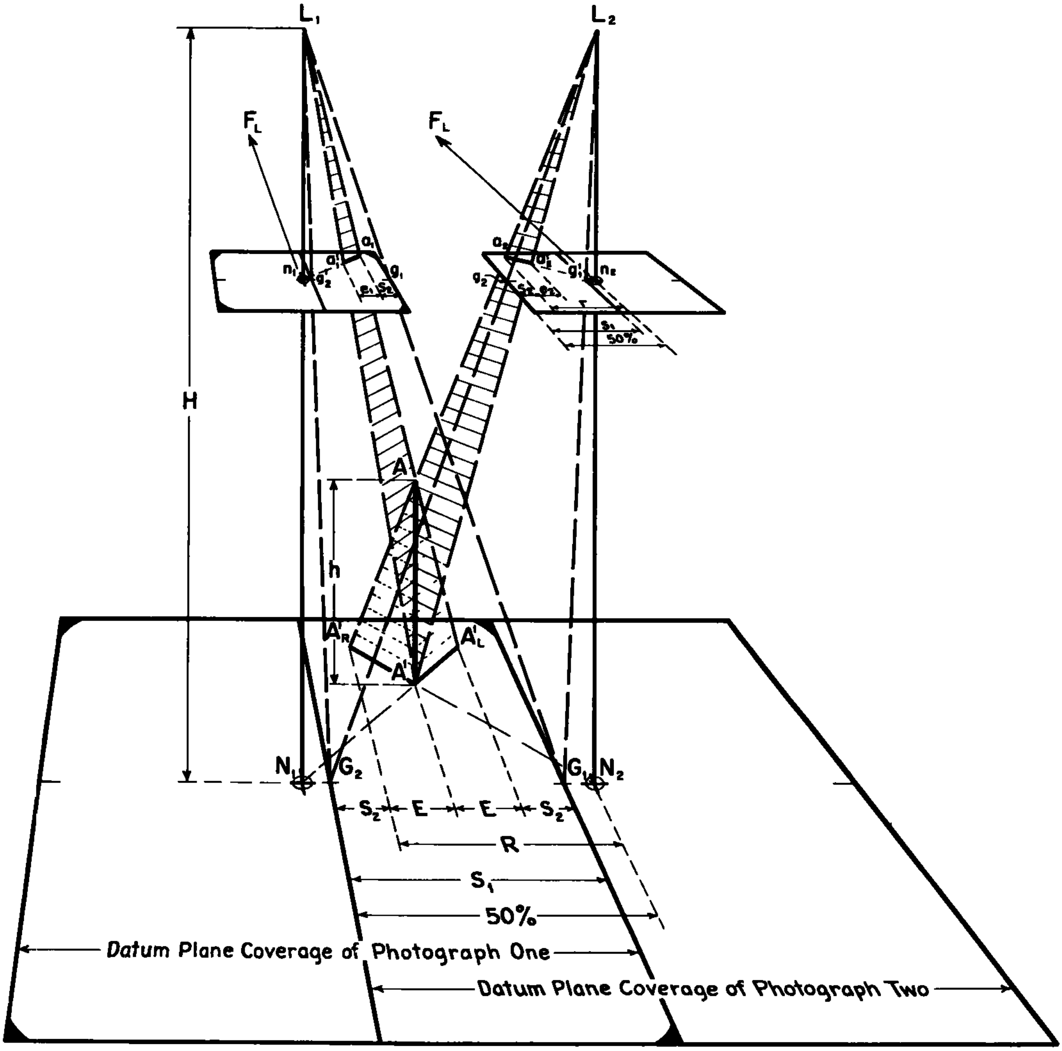


Figure 2. Space geometry of aerial vertical photographs in adjacent flight lines, at optimum spacing, to show sidelap (S_1) at datum plane and sidelap (S_2) at point of highest relief.

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H}$$

<u>Endlap (Figure 1)</u>

$E_1 = 50 + X$ By similar triangles, $X = E_2 + E$

Therefore:

$E_1 = 50 + E_2 + E$ Also by similar triangles, $\frac{e}{r} = \frac{E}{R} = \frac{h}{H}$

and R in any case $= 50 - E_2$

Therefore:

$$E = R\frac{h}{H} = (50 - E_2)\frac{h}{H} \text{ and } E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$

This expression may be rearranged thus:

$$\frac{h}{H} = \frac{E_1 - E_2 - 50}{50 - E_2}$$

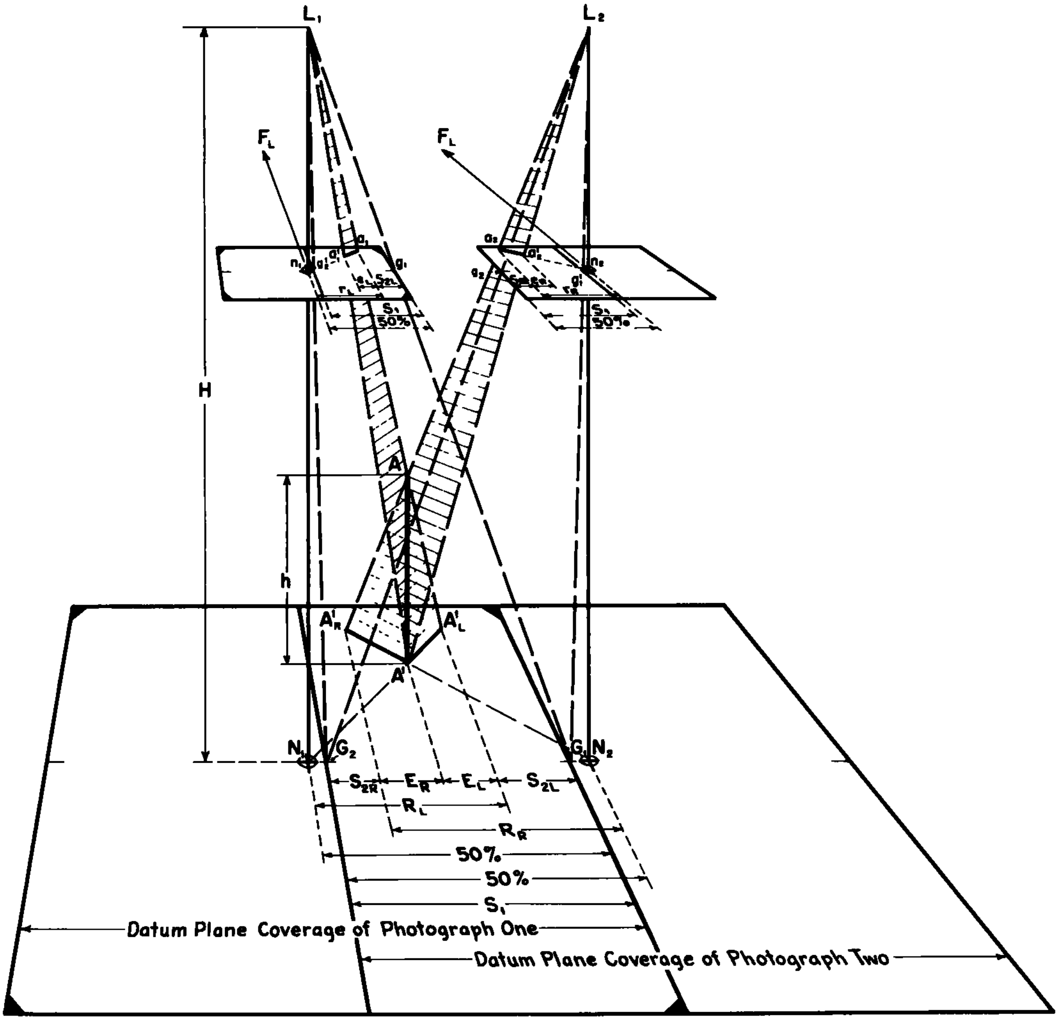


Figure 3. Space geometry of aerial vertical photographs in adjacent flight lines, not at optimum spacing, to show sidelap (S_1) at datum plane and sidelaps (S_{2R} and S_{2L}) at point of highest relief.

$$S_1 = S_{2R} + S_{2L} + (100 - S_{2R} - S_{2L})\frac{h}{H}$$

If the 55 percent and 65 percent limits are substituted:

$$\frac{h}{H} = \frac{65 - 5 - 50}{50 - 5} = 2/9$$

This value (2/9) is the relief height-flight height ratio used in compiling Tables 1 and 2.

Sidelap (Figure 2)

In this case, the flight lines are at optimum spacing, and minimum sidelap (S_2) is obtained on both adjacent photographs.

$$S_1 = S_2 + E + E + S_2 = 2S_2 + 2E$$

by similar triangles,

$$\frac{e_2}{r} = \frac{E}{R} = \frac{h}{H}$$

and R in any case = 50 - S_2

Therefore:

$$E = (50 - S_2)\frac{h}{H}$$

and

$$S_1 = 2S_2 + 2\,(50 - S_2)\frac{h}{H}$$

In the case of Figure 3, the flight lines are not at optimum spacing, and, as a result, S_{2R} and S_{2L} and E_R and E_L are not equal on adjacent photographs.

$$S_1 = S_{2R} + S_{2L} + E_R + E_L$$

Again by similar triangles,

$$\frac{E_L}{R_L} = \frac{h}{H} = \frac{E_R}{R_R}$$

and

$$R_L = 50 - S_{2L} \text{ and } R_R = 50 - S_{2R}$$

Therefore:

$$E_L = (50 - S_{2L})\frac{h}{H} \text{ and } E_R = (50 - S_{2R})\frac{h}{H}$$

and

$$S_1 = S_{2R} + S_{2L} + (100 - S_{2R} - S_{2L})\frac{h}{H}$$

Since S_2 for this case is not equal on adjacent photographs, the position of the point of highest relief does not lie on the previously defined line. Figure 3 and its equation are presented as an example of noncritical conditions, and graphs have not been prepared from this equation.

On most of the vertical photographs taken for any one project, intermediate endlap and sidelap values will usually occur because the extreme conditions will seldom exist on more than a few of the vertical photographs. But the anticipated extreme must be used in planning survey projects, establishing flight lines at specific places and for the entire area of survey, and in administering specifications. The positions considered, therefore, are for the points of highest and lowest relief where their perspective displacement on the photographs has the greatest effect on overlap (endlap or sidelap). Then, if the point of highest relief is at a minimum 5 percent of the lengthwise dimension of a particular photograph from its back edge, the maximum endlap will be measurable from the back edge to the image on this photograph which is conjugate to the image of the point of lowest relief appearing on the leading edge of the preceding photograph. Conversely, if the point of highest relief is at a minimum 5 percent from the forward edge, the maximum endlap will be measurable from that edge to the image on this photograph which is conjugate to the image of the point of lowest relief appearing on the back edge of the succeeding photograph. Sidelap is measurable in a similar manner.

Moreover, such occurrences will also affect the width of stereoscopic coverage on a single strip of photographs by decreasing it in proportion to the height above datum of points on the ground which appear as images along edges of the strip. The decrease on one side is expressed by this equation:

$$S_3 = \frac{11.11\ r\ h}{H}$$

in which S_3 is the percent of decrease in width of ground coverage caused by relief, r is the distance in inches from the center of the photograph to the image point of highest relief appearing on its edge, h is the relief height of the ground point above datum plane, and H is the flight height above datum.

Flight height and relief height must be in the same units of measure. If r is assumed to be 4. 5 in. for the usual 9- by 9-in. vertical photographs, the equation for S_3 becomes:

$$S_3 = 50\,\frac{h}{H}$$

Thus, all single strips of photographs are decreased by relief on the edges of the strips in their effective width of stereoscopic coverage. This condition must be fully accounted for in designing photography flight lines.

GRAPHS

Five graphs have been prepared from the equations for endlap and sidelap. These graphs show the relationships of minimum and maximum endlap and/or sidelap, flight height, and height above datum of point of highest relief. Graphs 1 and 2 are, respectively, endlap and sidelap graphs for flight heights to 40, 000 ft. Graph 3 is applicable to determination of either endlap or sidelap for flight heights to 24, 000 ft. Graph 4, similar to Graph 3, is for determination of either endlap or sidelap for flight heights to 9, 000 ft. In effect, Graph 4 is simply an enlargement of the lower portion of Graph 3. Graph 5 is for the determination of either endlap or sidelap for the single flight height of 3, 000 ft.

It should be noted that in each case in using these graphs, the value of H is the aircraft flight height above the datum plane, and the datum plane is assumed to pass through the point of lowest elevation governing maximum endlap in stereoscopic pairs of the vertical photographs. The flight height to consider in attaining a particular map scale, however, is the optimum flight height, the flight height measured from the aircraft to the elevation point which corresponds to the point of optimum projection in the stereoscopic model rather than the flight height above the defined datum plane. The point of optimum projection lies above the datum plane a distance equivalent to about 60 percent of the relief height. Thus, the optimum flight height is equal to the aircraft flight height above the datum plane minus 60 percent of the relief height. Examples illustrating uses of these graphs follow:

GRAPH 1

With E_2 specified, and given values for any two of the three variables H, h, or E_1, the third value may be determined from Graph 1 for flight heights up to 40, 000 ft.

Example No. 1

To determine: E_1 at datum

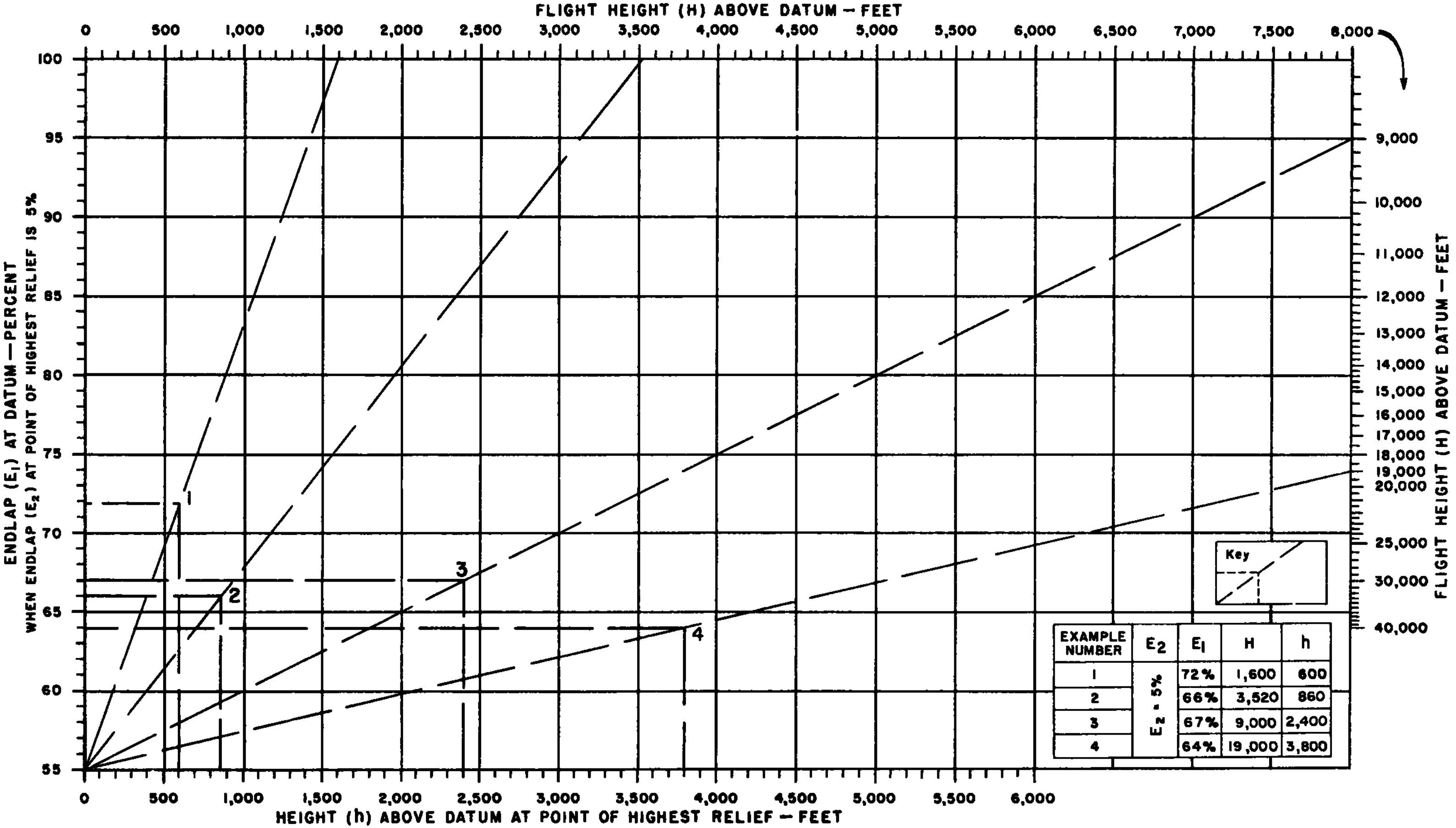
Given: H = 1, 600 ft
h = 600 ft
E_2 = 5 percent

1. Construct a sloping line from the point of minimum endlap, 55 percent, to 1, 600 ft on the abscissa for flight height (H).

2. From 600 ft on the abscissa for relief height (h), construct a vertical line to intersect the first line.

RELATION OF PERCENTAGE OF ENDLAP (E_1) AT DATUM, AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM, AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF, WHEN PERCENTAGE OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF IS 5%

$$E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$

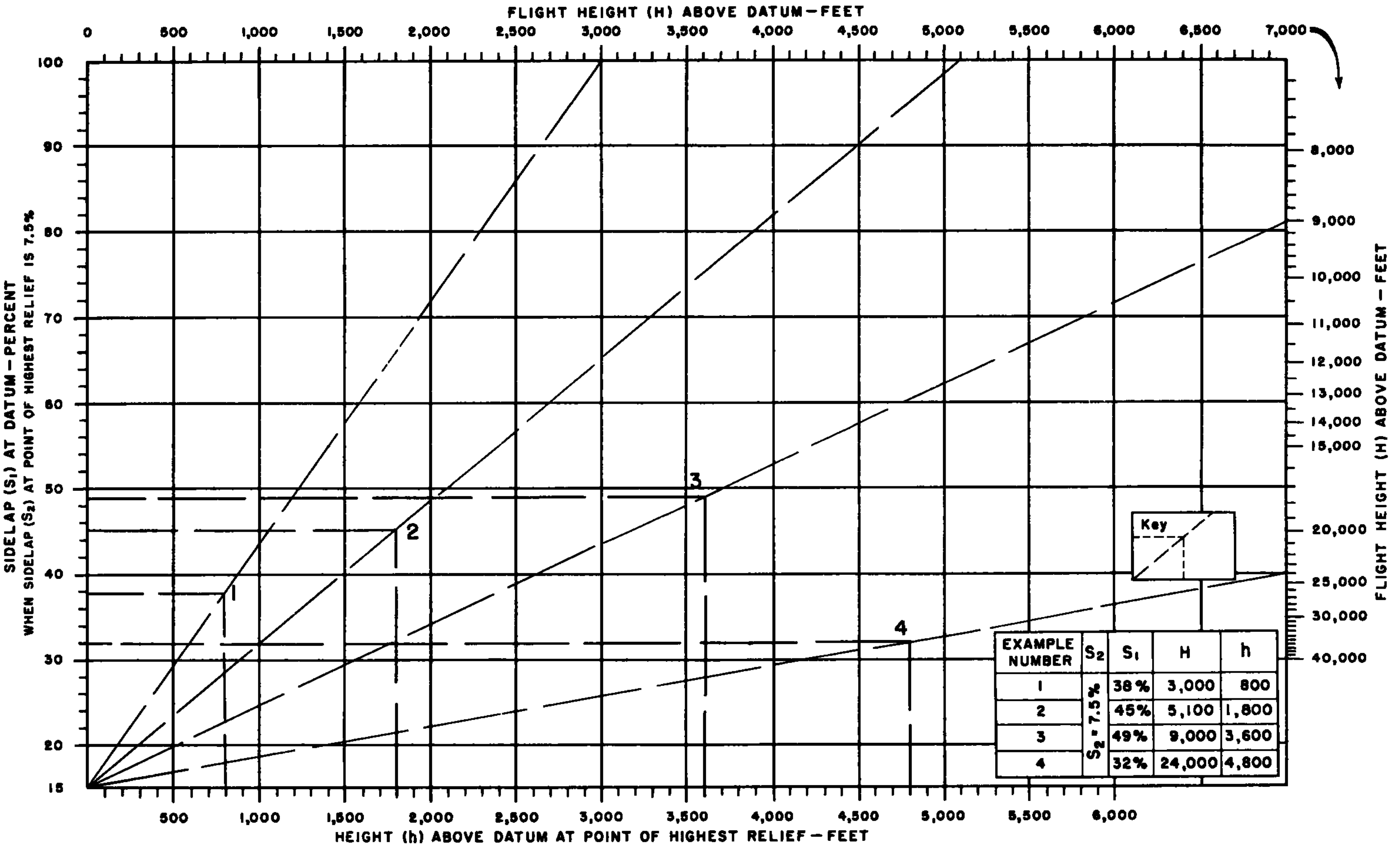


EXAMPLE NUMBER	E_2	E_1	H	h
1	E_2 = 5%	72%	1,600	600
2		66%	3,520	860
3		67%	9,000	2,400
4		64%	19,000	3,800

Graph 1

RELATION OF PERCENTAGE OF SIDELAP (S_1) AT DATUM, AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM, AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF WHEN PERCENTAGE OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF IS 7.5%

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H}$$

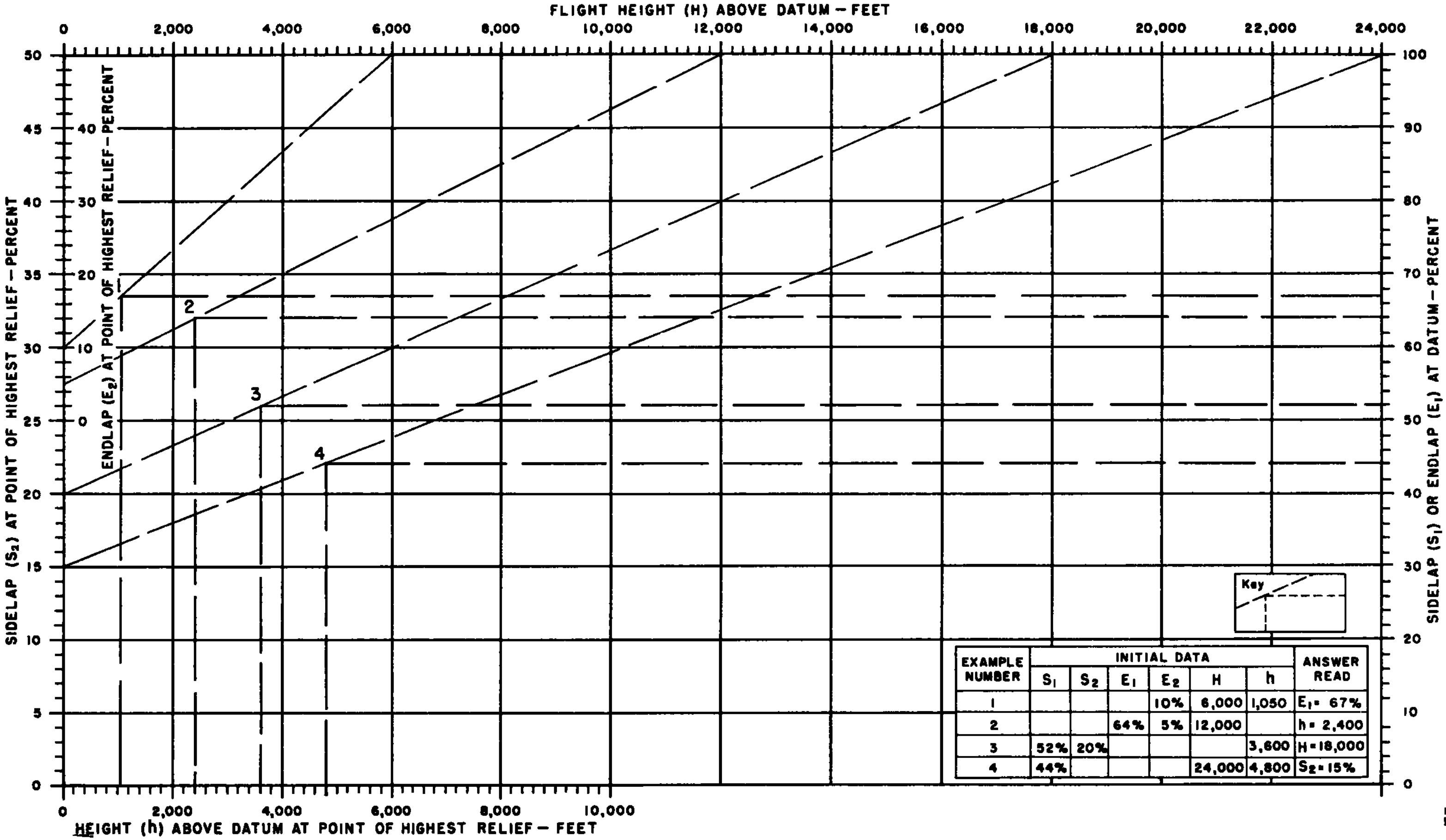


EXAMPLE NUMBER	S_2	S_1	H	h
1	$S_2 = 7.5\%$	38%	3,000	800
2		45%	5,100	1,800
3		49%	9,000	3,600
4		32%	24,000	4,800

Graph 2

RELATION OF PERCENTAGE OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF SIDELAP (S_1) AT DATUM, OR OF PERCENTAGE OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF ENDLAP (E_1) AT DATUM, AND AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF
(FLIGHT HEIGHT TO 24,000 FEET)

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H} \quad \text{AND} \quad E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$



EXAMPLE NUMBER	INITIAL DATA						ANSWER READ
	S_1	S_2	E_1	E_2	H	h	
1				10%	6,000	1,050	E_1 = 67%
2			64%	5%	12,000		h = 2,400
3	52%	20%				3,600	H = 18,000
4	44%				24,000	4,800	S_2 = 15%

Graph 3

3. From the point of intersection of lines one and two, construct a horizontal line to the endlap ordinate, and read the endlap in percent. E_1 = 72 percent.

Resultant endlap of 72 percent in this example, and in example 1 on Graph 4, is unrealistic for double projection instruments because, according to Table 1, none of these instruments is capable of handling an endlap of 72 percent. An optical train instrument, however, could utilize photographs with such an endlap.

Example No. 2

A better approach to solving the endlap problem is given in Example 2 on Graph 1. First consider the type of photogrammetric instrument; also the scale at which the map compilation is desired. Should the instrument for which endlap and sidelap and photography flight lines are to be designed be a Kelsh stereoscopic plotter using 6-in. focal length photography and a 5:1 projection ratio, the optimum flight height would be 3, 000 ft for map compilation at a scale of 100 ft to 1 in. This 3, 000 ft is a product of the map scale of 100 ft to 1 in., the projection ratio of 5, and the photography focal length of 6 in. On all graphs, the optimum flight height, H_o, plus 60 percent of the relief height equals the flight height, H. When the maximum permissible E_1 at datum is 66 percent, the minimum E_2 is to be not less than 5 percent, and the optimum flight height required for the map compilation scale desired is 3, 000 ft, proceed as follows to determine the maximum h which can be accommodated and the actual flight height, H, that will be required above the datum passing through the point of lowest relief. From Table 1, select the 66 percent maximum for E_1. Utilize a minimum endlap of 55 percent at point of highest relief, which results in an E_2 of 5 percent. In reducing the equation for E_1 such values result in an equation, in this case, wherein H = 4. 1h. Also, from preceding data $H = 0.6h + H_o$. Therefore, by substitution of 3, 000 ft for H_o, and 4. 1h for H, the value of h is determined to be 860 ft. Consequently, Example 2 on Graph 1, reduces to:

To determine: H above datum

Given: E_1 = 66 percent
E_2 = 5 percent
h = 860 ft

1. Construct a line parallel to the abscissa of the graph from the endlap ordinate of 66 percent.
2. Construct a line parallel to the ordinate of the graph from the relief height abscissa of 860 ft.
3. From the point of intersection of the ordinate and abscissa lines of this graph, at the minimum endlap of 55 percent for point of highest relief, construct a sloping line to pass through the point of intersection of the two lines constructed in steps one and two. Extend this line to the H abscissa. This intersection marks an H of 3, 520 ft, the answer. The practical H to use in this case would be 3, 500 ft.

Continuing further with this example, by considering only a single strip of aerial photographs, the walls of a canyon 860 ft high would decrease the width of photographic coverage 24. 4 percent. This is twice the decrease on one side, as computable by use of the equation for S_3 which is the decrease in percent of width of ground coverage by perspective displacement of relief.

GRAPH 2

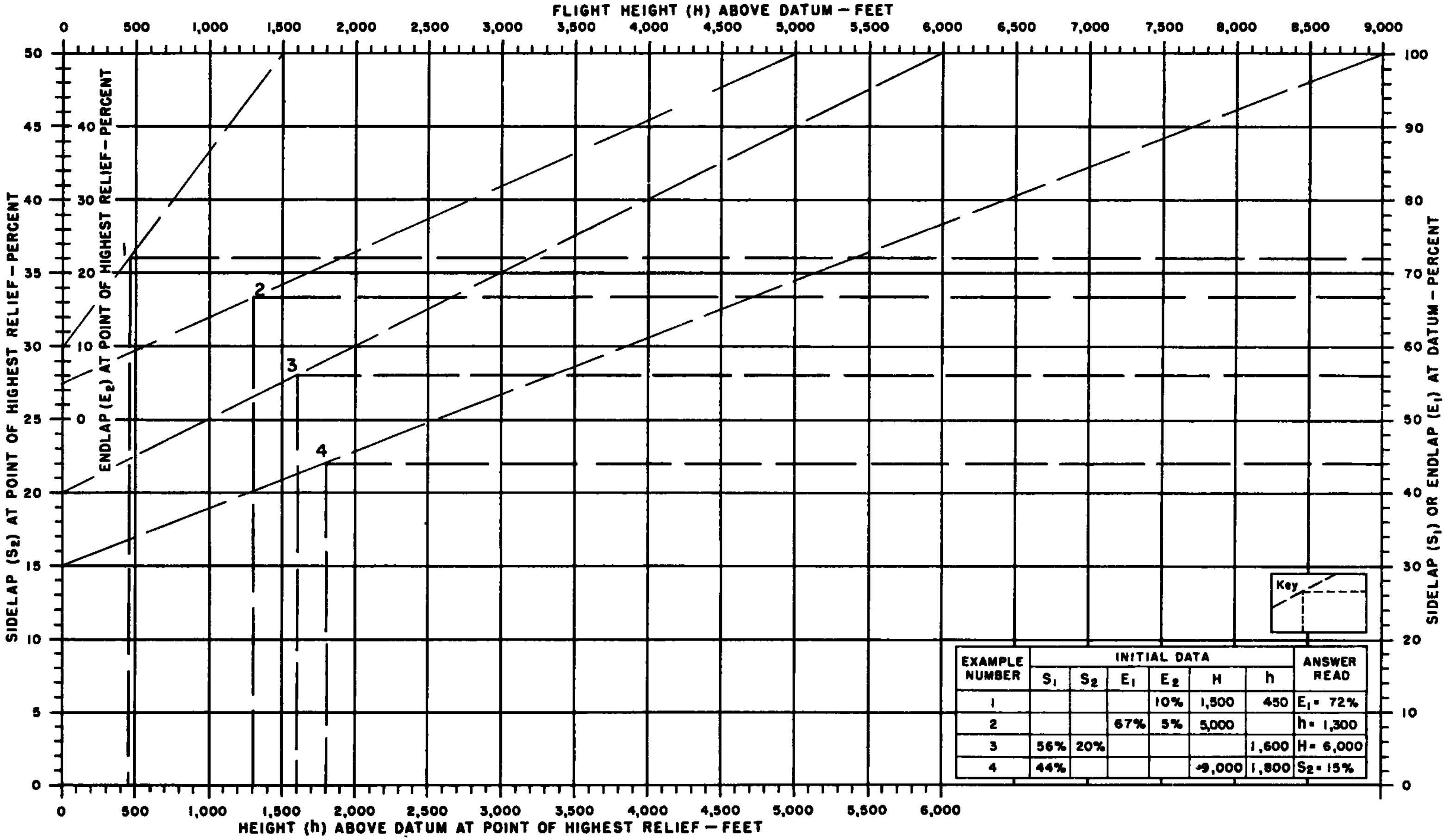
With S_2 specified, and given values of any two of the three variables H, h, or S_1, the third value may be determined from Graph 2 for flight heights up to 40, 000 ft.

Example No. 1

To determine: S_1 at datum

RELATION OF PERCENTAGE OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF SIDELAP (S_1) AT DATUM, OR OF PERCENTAGE OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF ENDLAP (E_1) AT DATUM, AND AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF (FLIGHT HEIGHT TO 9,000 FEET)

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H} \quad \text{AND} \quad E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$



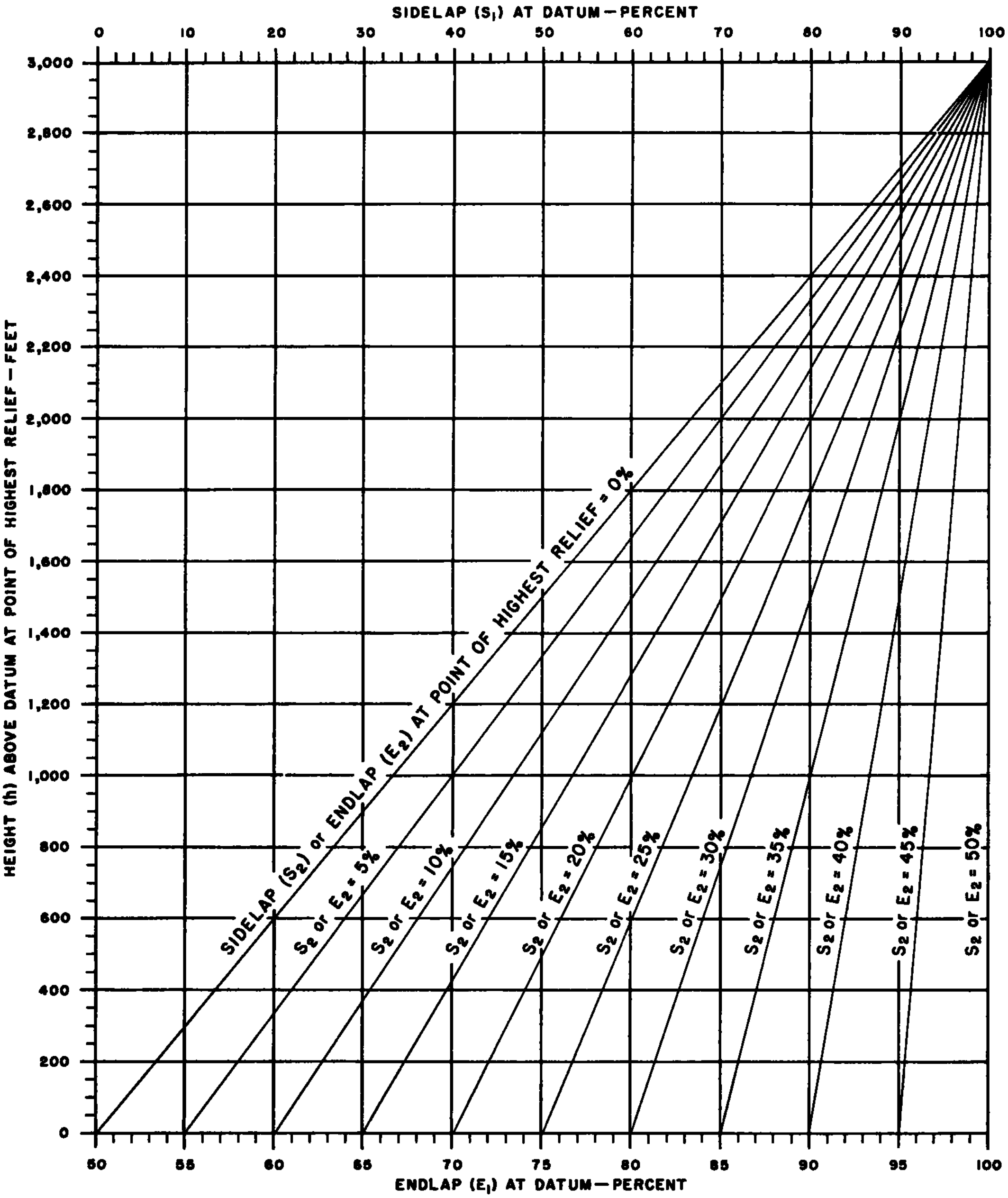
EXAMPLE NUMBER	INITIAL DATA						ANSWER READ
	S_1	S_2	E_1	E_2	H	h	
1				10%	1,500	450	E_1 = 72%
2			67%	5%	5,000		h = 1,300
3	56%	20%				1,600	H = 6,000
4	44%				9,000	1,800	S_2 = 15%

Graph 4

Given: H = 3, 000 ft
h = 800 ft
S_2 = 7. 5 percent

1. Construct a sloping line from the point of twice the minimum sidelap distance

RELATION OF VARIOUS MINIMUM PERCENTAGES OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF AND PERCENTAGE OF SIDELAP (S_1) AT DATUM, OR VARIOUS MINIMUM PERCENTAGES OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF AND PERCENTAGE OF ENDLAP (E_1) AT DATUM, AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF, WHEN AIRCRAFT FLIGHT HEIGHT ABOVE DATUM IS 3,000 FEET



Graph 5

(in this case $2S_2$ = 15 percent) to 3,000 ft on the abscissa for flight height (H).

2. From 800 ft on the abscissa for relief height (h), construct a vertical line to intersect the sloping line previously drawn.

3. From the point of intersection of the first and second lines, construct a horizontal line to the sidelap (S_1) ordinate, and read the sidelap in percent. S_1 = 38 percent.

GRAPH 3

Graph 3 is a combination sidelap-endlap graph for flight heights to 24,000 ft. It presents the relationship of the aircraft flight height (H) above datum, height (h) of the point of highest relief, and the percentage of sidelap (S_2) or endlap (E_2) at point of highest relief, and the percentage of sidelap (S_1) or endlap (E_1) at the datum.

Given any three of the four variables, S_1, S_2, H, and h, or E_1, E_2, H, and h, the fourth may be determined from this graph.

<u>Example No. 1 on Graph 3 for Endlap</u>

To determine: E_1 at datum

Given: H = 6,000 ft
h = 1,050 ft
E_2 = 10 percent

1. Construct a sloping line from 10 percent on the ordinate for endlap (E_2) to 6,000 ft on the abscissa for flight height (H).

2. From 1,050 ft on the abscissa for relief height (h), construct a vertical line to intersect the sloping line.

3. From the point of intersection of lines one and two, construct a horizontal line to the ordinate for endlap (E_1), and read the endlap in percent. E_1 = 67 percent. (This endlap is excessive for all but two of the double projection instruments listed in Table 1, the Multiplex and Balplex (525). It is also usable in the optical train instruments.)

<u>Example No. 2 on Graph 3 for Sidelap</u>

To determine: Flight height (H)

Given: h = 3,600 ft
S_1 = 52 percent
S_2 = 20 percent

1. From 3,600 ft on the abscissa for relief height (h), construct a vertical line.

2. From 52 percent on the ordinate for sidelap (S_1) at the datum, construct a horizontal line to intersect the vertical line from the abscissa for relief height (h).

3. Construct a line from 20 percent on the ordinate for sidelap (S_2) through the point of intersection of lines one and two to the abscissa for flight height (H), and read the flight height in ft. H = 18,000 ft.

GRAPH 4

Graph 4 is a combination sidelap-endlap graph for flight heights to 9,000 ft. It is constructed and used in the same manner as Graph 3, and is in effect, an enlargement of the lower portion of that graph.

GRAPH 5

Graph 5 is constructed for the special case of an aircraft flight height of 3,000 ft above datum. It presents the relationships of various percentages of sidelap (S_2) at point of highest relief and percentages of sidelap (S_1) at datum, or various percentages of endlap (E_2) at point of highest relief and percentages of endlap (E_1) at datum, and height (h) of point of highest relief.

Example No. 1 using Graph 5 for Endlap

To determine: E_1 at datum

Given: E_2 = 5 percent
h = 800 ft

Draw a horizontal line from 800 ft on the ordinate for relief height (h) to intersect with the sloping line labeled E_2 = 5 percent. From this point of intersection, construct a line vertically downward to the abscissa for endlap (E_1) at datum. In this case E_1 = 67 percent. (A flight height larger than 3, 000 ft would be necessary to achieve an endlap at datum of less than 67 percent, as required by most double projection instruments.)

Example No. 2 using Graph 5 for Sidelap

To determine: S_1 at datum

Given: S_2 = 5 percent
h = 800 ft

Draw a horizontal line from 800 ft on the ordinate for relief height (h) to intersect with the sloping line labeled S_2 = 5 percent. From this point of intersection, construct a line vertically upward to the abscissa for sidelap (S_1) at datum. In this case S_1 = 34 percent.

CONCLUSION

In this paper, an attempt has not been made to achieve an exhaustive analysis of the interrelationships of relief, flight height, tilt, and overlap. Proof of the significance of their effects on utilization of photogrammetric methods of mapping at large scales for highway engineering purposes was undertaken. In actuality, certain combinations of relief heights and flight heights place a limit on how close to the ground an aircraft can be flown on photography missions for such mapping. Utilization of the principles presented and graphs prepared will enable highway engineers to ascertain the largest scale and smallest contour interval that are practicable for a particular relief height within the route band or area of survey. Whenever the principles are fully applied, it will always be possible to attain optimum overlap.

Discussion

W. S. HIGGINSON, Sloan and Associates, Pasadena, California — Study of Pryor's paper has given me the idea that the question of C-factor would have more meaning if the elements that really assign a numerical value to it are considered. While the C-factor used for each photogrammetric instrument is mostly empirical, there are a number of elements that affect it.

Planning flight lines for photographic missions has been the subject of study for many years. There is no method of planning photography yet where a planner can apply a particular procedure and use specific equations and produce a satisfactory aerial photography plan for all projects. In any planning it is necessary to decide a few important factors from the best available information before a usable photography plan can be evolved. These factors are minimum ground elevation, critical ground elevations (greatest difference in elevation that may occur in a single stereomodel), photography scale, and focal length of the camera to be used. If these factors are applied to a particular pattern a suitable plan can be produced for an aerial photography project. Such planning is based on the theory that overlaps, either end or side, are to be constant quantities rather than variable. To consider endlap variable, changing it from any value other than the ideal value, will change the accuracy of plotting map detail in the same manner as altering the C-factor, because the base-height ratio (b/H) is one of the elements that affect the C-factor.

This practice, of course, admits that it is impossible to compile a map at a specified scale and contour interval for areas of extreme vertical relief, or in cases where h/H is greater than 2/9 which would require other than 55 percent minimum and 65 percent maximum endlap limits as stated in the paper. The planner can favor a particular project by a slight change in some of the arbitrary values he has fixed for average or critical elevations; in fact, both of these values could be adjusted to change the plan to a considerable extent. It might be suggested that special additional photography should be planned to apply, at the optimum elevation, over the limited areas of critical or extremely high ground. The additional photography over these limited areas could provide adequate quality of mapping photography for the entire project area.

Most aerial photography mapping plans are based on the empirical equation: H = contour interval times "C", and "C" is the C-factor applicable to a particular instrument. If this "C" is separated into $c_1 + c_2 + c_3 + c_4 + c_5$, one of the small c elements could change with a minimum decrease in the C-factor and a maximum change in the ground area coverage per photograph that will result in the required map scale and contour interval. The five c-factor elements designated as small c are considered to be scale, B/H ratio, projection distance, quality of the photography, and ground cover. A proper evaluation of each of these elements is necessary before any plans could be evolved. Since this approach is the real basis of planning aerial photography for most mapping projects, it is of the utmost importance.

WILLIAM T. PRYOR, Closure — In his discussion, Higginson stresses the importance of considering several factors which have an effect on efficiency in the photogrammetric use of aerial photography. Attaining the greatest possible effeciency is always desirable. It was not intended, however, to include all facets of aerial photography flight planning in this paper, particularly those which are especially applicable to photography for small scale mapping.

The purposes of the paper were to present the effects of relief on selecting map scales and contour intervals, and in determining the endlap and sidelap limits controlled thereby according to the various types of photogrammetric instruments used in large scale topographic mapping for highway engineering purposes within the U. S.

Unless the effects of relief height to flight height are considered, the consequences are insufficient endlap which, in turn, results in inability to map the desired areas. Obviously, the base-height ratio (B/H) would be a maximum wherever the minimum admissible endlap is attained. But, to attain maximum efficiency, the B/H ratio would have to change when the ratio of relief height to flight height (h/H) changes within the admissible limits. Should a constant, but minimum, B/H ratio be maintained to assure the attainment of full stereoscopic coverage, regardless of the h/H ratio, then inefficiency will result. But, if the principles presented are properly applied, the B/H ratio will decrease in proportion to the increase in h/H ratio. Conversely, the B/H ratio can be increased as the h/H ratio decreases.

It should be emphasized, of course, that the foregoing statements regarding influence of the h/H ratio on the B/H ratio are specifically applicable, when aerial photographs are taken from a low flight height for large scale topographic mapping where the relief is such that the h/H ratio approaches, or tends to exceed, the specific values in column 5 of Table 1. Literally, relief in these considerations is the difference in elevation and the height of trees and buildings within the successive stereomodels. When the h/H influences are ignored, either intentionally or by oversight, the consequences are inefficiency. Particularly, photography lacking sufficient overlap and proper scale for accomplishing the mapping required, will result. Moreover, to ignore these influences can also result in specifying a map scale which is larger, and a contour interval which is smaller, than can be attained photogrammetrically because the h/H ratio, whether arising from topographic relief or object heights, or both, will not admit taking photography of sufficient scale to accomplish the mapping.

Whenever the contour interval or the ratio of photography scale to map scale must be considered in relation to the h/H ratio, the resultant C-factor (refer to column 8 in Table 2) may be much less than the C-factor commonly used (refer to column 3 in Table 3). Therefore, map scale and the h/H ratio are primarily the governing factors in much of the large scale topographic mapping required by highway engineers. (In consideration of the foregoing, it should be remembered that the ratio of photography scale to map scale is actually the projection ratio for double projection photogrammetric instruments, and that the practical limit of the ratio of photography scale to map scale for the optical train instruments is recognized as being 8:1.)

It was not the purpose in this paper to focus attention on the empirical C-factors. These factors were used merely as a means to stress the importance of the h/H ratio and the degree to which it controls what can be done photogrammetrically in large scale, small contour interval, topographic mapping. Only when the h/H ratio is insignificantly small can the C-factor become fully significant and the B/H ratio be kept reasonably uniform. Consequently, when the relief height is large and variable from one stereoscopic pair to another, sufficient stereoscopic overlap cannot be achieved unless the B/H ratio is varied inversely as the h/H ratio changes from one stereoscopic pair to another. To achieve this greatest possible efficiency, photographic crews must be alert and effective in applying the principles outlined.