

Terrain Data for Earthwork Quantities

L. L. FUNK, Photogrammetric Engineer, California Division of Highways

●IN DECEMBER 1957, an experimental project was undertaken by the California Division of Highways to determine the relative accuracy and costs of various methods of obtaining earthwork quantities. The project was initiated by the Design, Construction, and Photogrammetric Departments. The principal objective of the study was to provide data which would:

1. Assist Design and Construction in developing procedures to obtain acceptable pay quantities for earthwork with a minimum of engineering effort.
2. Furnish a guide to Design for selection of the most suitable method for obtaining terrain data and computing earthwork quantities on individual projects.

Test Site

The area selected for the test was a 3,000-ft section on new location between Stations 160+00 and 190+00 on Road III-But-21-B about two mi northeast of Oroville. As shown by the contour map in Figure 1, the terrain is rolling with fairly regular slopes ranging from 4 to 20 percent. The land is used for grazing. Portions of the area were covered by a relatively dense growth of grass up to a maximum of 3 in. in height. There was a minor amount of brush in two small creeks. A 2-ft contour map of the area had been previously obtained. Design was partially completed and the proposed centerline had been staked in the field.

Surveys

A total of nine surveys were made of the test site, which were designated as follows:

Field Surveys:

F1 - Precise

F2 - Made by commonly accepted methods

Photogrammetric cross-sections - Flying height 1,500 ft:

PS1 - Spot heights written on manuscript 1 in. = 50 ft

PS2 - Spot heights written on manuscript 1 in. = 50 ft

PS3 - Spot heights on punch cards - Model Scale 1 in. = 50 ft

PS4 - Spot heights written on manuscript 1 in. = 50 ft

Photogrammetric contour maps - Scale 1 in. = 50 ft, C.I. 2 ft:

CM1 - Flying height 1,500 ft

CM2 - Flying height 1,500 ft

CM3 - Flying height 2,100 ft

Field Surveys

The F1 survey was made by relatively precise methods for use as a yardstick in measuring the accuracy of other surveys. Cross-sections were taken with an engineer's level along lines at right angles to centerline. The right angles were turned with a transit. The maximum interval between cross-section lines was 25 ft. Density of points on individual cross-section lines was left to the judgment of the chief of party. In general, the resulting spacing did not exceed 50 ft with sufficient breaks in slope being read to insure the accuracy of earthwork quantities.

Field survey F2 was made by District III under instruction to use their conventional procedures for the type of terrain involved. This survey consisted of the following three steps:

1. Centerline profile read with an engineer's level.
2. Slope stakes set with a Rhodes Reducing Arc at 50-ft stations. Right angles were turned with a 90-deg prism. The maximum distance for Rhodes Arc readings was 100 ft.
3. Cross-sections taken at 50-ft stations plus nine additional cross-sections at designated breaks in the terrain.

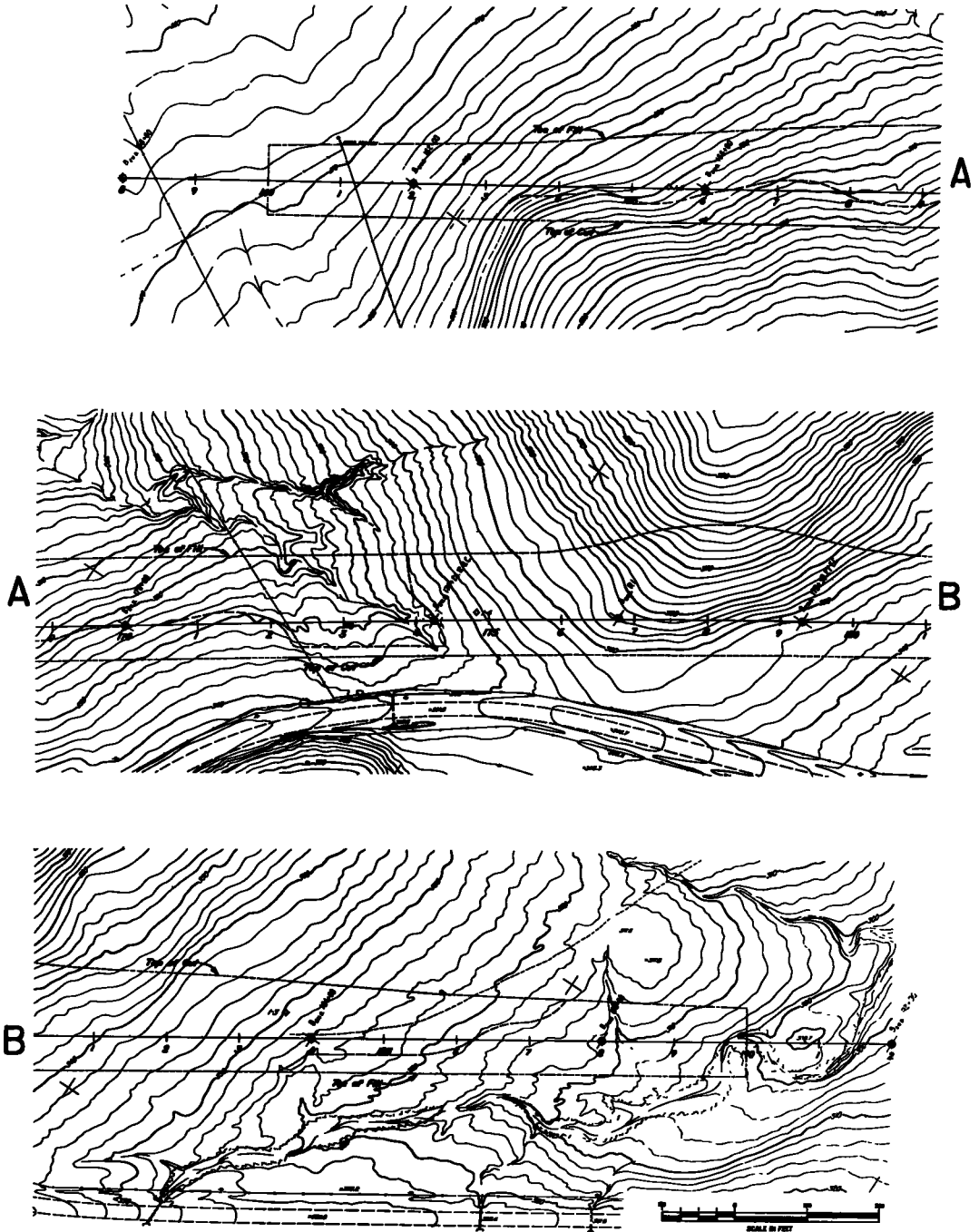


Figure 1.

The density of points on each cross-section line was again left to the judgment of the chief of party. The field notes indicate that it was slightly greater than for field survey F1. As will be shown later, the density of points in this type of terrain is of relatively little importance. From Station 160 to 163 and from Station 180+50 to 190, the cross-sections were taken with an engineer's level. From Station 163+50 to 180, the elevations were obtained by plus and minus differences from centerline using a Rhodes Reducing Arc.

The F1 survey was made subsequent to the F2 survey. In order to determine errors in reading with the Rhodes Arc, it included a reading on each slope stake set by the F2 survey. A reading was also made at the exact point designated by the F2 notes for the slope stake, to determine the difference in elevation caused by error in position.

Photogrammetric Surveys

A single flight of photography taken from a height of 1,500 ft with a Zeiss RMK/13 camera (nominal focal length of 6 in.) was used for all photogrammetric compilations, with the exception of CM3. Four stereomodels covered the length of the test area. Horizontal control consisted of three premarked points per model along centerline. Vertical control for each model consisted of one of the premarked points near the center plus four photo-identified wing points. This control was obtained by State forces.

Contour maps CM1 and CM2 and photogrammetric cross-sections PS1 and PS2 were compiled with a Kelsh plotter by professional mapping firms under contracts for plotter rental at an hourly rate. Elevations of points along the cross-section lines of PS1 and PS2 were written on the manuscripts.

PS3 consisted of a digital readout of the cross-section data using a Terrain Data Translator. This equipment was designated and manufactured by Benson-Lehner Corporation of Los Angeles to the requirements and specifications of Pafford and Associates, also of Los Angeles. It is adaptable for use in double projection type plotters or for taking digital data from a contour map. The data for PS3 were taken directly from the stereomodels in a Nistri Photomapper with the tracing table being guided along cross-section lines previously plotted on a manuscript. Output data consisted of IBM punch cards and a typed list of elevations and distances right and left of centerline for each cross-section.

PS4 consisted of readings of the slope stakes and centerline elevations only, and was done by Division of Highways operators usually engaged in map checking. One of the models was read in a Kelsh plotter and three in a Nistri Photomapper.

CM3 was a portion of a 14.8-mi mapping contract awarded in May 1956 at a contract price of approximately \$1,275 per mi. Photography for the portion included in the test site was taken on May 30, 1956, with a Wild RC 5A camera from a height of 2,100 ft. The specifications required a minimum of three horizontal and five vertical control points per model. Compilation of a 2-ft contour map at a scale of 1 in. = 50 ft was done in a Kelsh plotter modified to provide a ratio of 1 to 7 from photo scale to map scale.

VERTICAL ACCURACY OF SURVEYS

Comparative Accuracy

The accuracy of the various surveys in determining the elevation of discrete points is shown in Table 1. The points include slope stakes and centerline stations whose elevations were established by the F1 field survey. The first line of Table 1 shows the relative accuracy of two field surveys in reading 60 centerline stations with an engineer's level. The results show close agreement except for two blunders of 1.0 ft and one of 1.4 ft in the F2 survey.

Vertical errors due to the difference in the positions of the slope stakes, as set in the F2 survey, and their position, as recorded in the notes, are shown in the second line. Horizontal errors in position, due to poor right angles, amounted to as much as 10 ft in some cases. Horizontal errors in distances from centerline were very minor. Vertical errors in the Rhodes Arc readings of the F2 survey are shown in the third line. In some cases these tended to compensate the errors due to position.

TABLE 1
VERTICAL ACCURACY OF SURVEYS

Survey	No. of Points Measured	Probable Error 50 % Within (ft)	Specification Limit 90 % Within (ft)	Error Range (ft)	Standard Deviation ¹ (ft)	Arithmetic Mean (ft)
F2 - Centerline - Engineer's Level	60	± 0.1	± 0.2	-1.4 to +0.2	0.28	-0.02
F2 - Slope Stakes - Rhodes Arc						
Errors due to Position	118	0.0	± 0.4	-1.1 to +1.5		+0.03
Errors due to Reading	118	± 0.2	± 0.9	-2.4 to +1.6		-0.19
Combined Errors	118	± 0.2	± 0.9	-2.2 to +1.6	0.60	-0.14
PS1 - C/L and Slope Stakes	183	± 0.1	± 0.4	-0.5 to +0.6	0.19	+0.13
PS2 - Centerline	61	± 0.3	± 0.5	-0.1 to +0.7	0.19	+0.29
PS3 - C/L and Slope Stakes	183	± 0.2	± 0.4	-0.6 to +0.9	0.25	+0.09
PS4 - C/L and Slope Stakes	183	± 0.1	± 0.3	-0.5 to +0.7	0.20	+0.03
CM1 - C/L and Slope Stakes	183	± 0.2	± 0.4	-0.7 to +0.9	0.27	+0.05
CM2 - C/L and Slope Stakes	183	± 0.3	± 0.7	-0.6 to +1.2	0.34	+0.32
CM3 - C/L and Slope Stakes	183	± 0.4	± 1.1	-1.3 to +1.8	0.62	+0.24
CM3A - C/L and Slope Stakes (Portion from Sta. 164 to 190)	160	± 0.4	± 0.8	-1.3 to +1.4	0.52	+0.10

¹ Calculated on basis of deviations from the arithmetic mean.

The combined errors, shown in the fourth line, are the differences in elevations between the slope stakes, as determined by the F2 survey, and points at the described locations of the slope stakes as determined by the F1 survey. These errors should be compared to the various photogrammetric surveys in considering relative accuracy. The elevations of the slope stakes and centerline stations were determined by direct spot-height readings in the photogrammetric cross-section surveys and by interpolation between contours in the contour map surveys.

Five statistical measures of accuracy are shown for each of the surveys. Of these, the arithmetic mean has by far the greatest effect on the accuracy of earthwork quantities. It is computed by dividing the algebraic sum of the individual errors by the number of points tested. In effect, therefore, the arithmetic mean is the center of gravity of the entire group of errors. As such, it indicates the probable magnitude of the systematic errors.

Another valuable measure of accuracy is the standard deviation or mean square error. It is defined as the square root of the average of the squared deviations from the mean. The standard deviation is the best measure of the accuracy of individual points. In photogrammetric measurements, if systematic errors and blunders were eliminated, it would be a true measure of the magnitude of the random errors.

Four of the photogrammetric surveys (PS1, PS3, PS4, and CM1) were generally better than the F2 field survey by all five measures of accuracy. PS2 and CM2 were better than the F2 survey in all measures except the probable error and the arithmetic mean. The CM3 survey was slightly poorer than F2; however, if the portion from Station 160 to 163+50 is omitted from the CM3 survey, the resulting CM3A is better than F2 by all measures except the probable error. Accuracy of the CM3A portion is probably the most nearly representative of the general average of mapping being obtained under contract by the California Division of Highways.

It is generally considered that spot heights, such as photogrammetric cross-sections, read directly in a stereoplotter, have at least double the accuracy of points interpolated from a contour map. Results shown in Table 1 indicate lower standard deviations for the photogrammetric cross-sections on comparable surveys. There is no significant difference, however, in the arithmetic mean of points read in the cross-section surveys as compared to those from contour maps.

In considering the accuracy of the two methods of making photogrammetric measurements the type of terrain should not be overlooked. As slopes become flatter, the accuracy of points taken from contour map tends to decrease due to difficulties in interpolation. This is particularly true if the terrain is irregular.

Errors in Photogrammetric Measurements

Study and analysis of photogrammetric errors in numerous large-scale contour maps

have indicated the following characteristics which are pertinent to this study:

1. There are relatively few large individual blunders in photogrammetric measurements.
2. Systematic errors are not constant but tend to vary from model to model and even within individual stereomodels. Systematic errors in compilation, such as those caused by blunders in identification or values of photo control, tend to spread over a considerable area.
3. The arithmetic mean of the errors of a representative sample, such as a center-line profile, provides a good indication of the average of the systematic errors and blunders in the portion of the mapping from which the sample was taken.

These characteristics lead to the conclusion that for any selected small area, such as a single cross-section 2 or 3 hundred ft in length, systematic errors and blunders tend to remain fairly constant.

The standard deviation and arithmetic mean of the photogrammetric surveys in Table 1 show wide variations in accuracy between the surveys and even within the same survey. The latter is true of the CM3 survey where 16 out of 21 points in error by more than ± 1.0 ft occurred in the portion from Station 160 to 163+50. The arithmetic mean for this portion was +1.2 ft while for the remainder (CM3A) it was +0.10 ft.

A field profile had previously been run on a 4-mi section mapped under the same aerial survey contract as CM3. The arithmetic mean of 185 centerline points in this four miles was +0.01 ft, apparently indicating freedom from systematic errors. However, by dividing the profile into three sections, each having over 55 points, the arithmetic means of the individual sections were found to be +0.45 ft, -0.03 ft and -0.30 ft, respectively.

It has been previously noted that all of the photogrammetric surveys except CM3 utilized the same photography and control. All of the stereoplotter operators working on the various compilations were experienced, and reported that the photography and control were excellent. None of the operators were furnished any information concerning true elevations other than the five vertical control points per model. Under such conditions similar results might naturally be expected. Actually there was a wide variation, particularly in the arithmetic mean of the points read.

For some purposes, errors such as shown in Table 1 may be considered insignificant. However, it will be shown later than an arithmetic mean as small as ± 0.1 ft can cause serious discrepancies in earthwork quantities. Errors of this magnitude were not eliminated under the almost ideal conditions of photography and control prevailing on the test section. It therefore appears too much to expect that they will not occur under the more adverse conditions certain to be encountered in actual practice. This is particularly true when mapping specifications do not include a limitation on the arithmetic mean.

The importance of small systematic errors in the computation of earthwork quantities cannot be over-emphasized. It should be apparent that such errors cannot be found by casual inspection or by plotting comparative profiles. They can be detected, and their magnitude determined, only by calculation of the arithmetic mean of a sufficient number of points to form an adequate statistical base.

EARTHWORK QUANTITIES

Types and Uses

In California practice, three classes of earthwork quantities are used in the various stages of highway design and construction. They are:

1. Preliminary quantities for comparison of alternate lines and project report estimates. The accuracy requirements for this stage vary widely according to the demands of the individual project. Sources of data include: aerial photographs, USGS quadrangle maps, and photogrammetric reconnaissance maps ranging in scale from 1 in. = 400 ft with 20-ft contours to 1 in. = 200 ft with 5-ft or 10-ft contours.

2. Design quantities for positioning of the final line and for design of the grade line, slopes, etc. Projects are advertised for construction on the basis of the design quantities. They should have sufficient accuracy that troublesome revisions in alignment, grade line, or slopes will not be required during construction.

General practice is to obtain design quantities from a 1 in. = 50-ft photogrammetric map with 2-ft contours or, in flat terrain, with a grid of spot elevations. The latter may be in the form of photogrammetric cross-sections. Such sources have been generally satisfactory except for a few individual projects where errors in the photogrammetric mapping have resulted in serious imbalance in the quantities, causing difficulties and added cost during construction.

3. Pay quantities to be used as the basis for payment to the construction contractor. California specifications provide for payment on the basis of planned quantities plus authorized changes and unpreventable slides. Final cross-sections taken after construction are therefore seldom required. The accuracy of excavation quantities must be sufficient to insure equitable payment to the contractor. Embankment quantities on most projects are used only for balancing cut and fill, and as a guide to the payment of overhaul. Due to probable variations in estimated shrinkage factors, somewhat lower standards of accuracy could therefore be considered tolerable in embankment areas.

The usual practice is to obtain pay quantities by field cross-sections taken immediately prior to construction. This is generally during construction staking and after the project has been advertised for bids. The methods and standards of accuracy of the field cross-section survey are left to the judgment of the engineer in charge. One district has issued instructions to the effect that field cross-sections may be omitted in areas where sufficient checks of the photogrammetric maps indicate that resulting quantities will not vary more than one percent from those obtained by field surveys.

This study is primarily concerned with design quantities, pay quantities, and their efficient correlation.

Standards of Accuracy

Two facts must be immediately recognized in any study of earthwork quantities:

1. There are no rational, commonly accepted tolerance limits for their accuracy.
2. Any expression of earthwork quantities is approximate, as it involves measurements which can only approximate the actual terrain.

The difference in earthwork quantities as obtained from two surveys is commonly expressed as the difference in percent or the difference in cubic yards. The term "error" is seldom used as the engineer knows that both surveys are subject to error. He is frequently in doubt as to which is the more accurate. Differences of from 2 to 5 percent between photogrammetric quantities and those obtained from field surveys occasionally have been cited as evidence of the accuracy of photogrammetric surveys. Actual errors of this magnitude on large projects could result in completely unacceptable inequities in payment to the construction contractor.

Due to the difficulty and cost of obtaining a reliable yardstick for determining accuracy of earthwork quantities and the variation in requirements between projects, it is improbable that definite tolerance limits can be established. However, guides can be furnished which will assist the engineer in selecting the most suitable method of measuring earthwork quantities for a particular project or type of project. The probable accuracy and relative cost of various survey methods are the most important of these guides. Other factors which the engineer may consider are the unit cost of earthwork, the total quantity to be moved, and its relation to the total size of the construction project.

Sources of Error

In considering the premise that any measurement of earthwork is at best an approximation, the first step is to define the sources of errors. At least two such sources

are "built in" by many construction specifications including those of California. These are: computation by end area formula; and non-correction for the effect of curvature. In most cases these errors are relatively minor. In all cases they can be considered to result in equitable payment to the contractor as the method is specified and is presumably taken into consideration in bidding. It should be pointed out, however, that failure on the part of the designer to recognize the effect of curvature can, on some projects, result in an imbalance of quantities far exceeding that due to variations in shrinkage factors or errors in measurement.

Other sources of errors which must be given consideration are related to:

1. Computation of quantities.
2. Density of terrain measurements.
3. Accuracy of terrain measurements.

Errors in Computation

At present this is the least important of the sources of errors due to general use of high speed electronic computers to convert the basic measurements to cubic yards. It should be emphasized that machine computation is an exact method and the results are subject only to errors due to inadequate density of measurements and to errors in those measurements.

As the planimeter was, for many years, the standard method of measuring cross-sectional areas, computation by this method was included in the study. Cross-sections obtained by the F2 survey were plotted at a scale of 1 in. = 10 ft and the quantities determined by planimetered areas and by the Avol Rule. The latter is an instrument for determining earthwork quantities by cumulative measurements of equally spaced vertical ordinates. In this case the spacing was 5 ft.

Comparative quantities of excavation and embankment obtained by the three methods of computation are shown in Table 2. Results varied by a maximum of 0.6 percent.

Density of Terrain Measurements

In terrain similar to that of the test site, California practice is to take cross-sections at 50-ft intervals. The distance between points on each cross-section line generally does not exceed 50 ft. Cross-sections taken at 25-ft intervals in the F1 survey afforded an opportunity to determine the importance of density in this type of terrain.

The quantities resulting from 25-, 50-, and two different arrangements of 100-ft cross-section intervals are shown in Table 3. For the 100-ft intervals shown in Column 3, cross-sections were used at Stations 161, 162, 163, etc. For the results in Column 4 they were used at Stations 161+50, 162+50, 163+50, etc. The relatively minor errors in quantities due to 50- and 100-ft intervals indicate that intervals of 100 ft would be entirely satisfactory for design quantities in this type of terrain.

However, it should not be overlooked that design quantities are a source of information for roadbed notes and slope-stake data which are of considerable value in construction staking. While the designer may save time by using larger cross-section intervals on trial lines and grades, he should keep construction staking requirements in mind in preparing design quantities for the final line and grade. In general, therefore, slope-stake spacing rather than accuracy of quantities may govern the cross-section interval for the final line and grade.

Prediction of Errors in Photogrammetric Quantities

It has been previously mentioned that the arithmetic mean of a sample, such as a centerline profile, is an excellent guide to the average of the systematic errors

TABLE 2
COMPARISON OF EARTHWORK QUANTITIES
COMPUTED BY VARIOUS METHODS—F2 SURVEY

	Quantity Cu. Yd.	Error	
		Cu. Yd.	%
Excavation			
Machine Comp.	64,212		
Planimeter	64,364	+152	0.2
Avol Rule	63,840	-372	0.6
Embankment			
Machine Comp.	28,654		
Planimeter	28,774	+120	0.4
Avol Rule	28,612	-42	0.1

and blunders in the area covered by the sample. If this is true it should be possible to predict, within reasonable limits, the total error in approximating the terrain in each of the photogrammetric surveys. The results of such predictions are shown in Table 4.

The surface area of the test site, between slope stakes, was approximately 380,000 sq ft. Multiplying this area by the arithmetic mean of the centerline profile and dividing by 27 gave the predicted error in cubic yards. The actual errors are taken from Table 6. It will be noted that the maximum error in prediction for any of the surveys was only 0.6 percent of the total quantity involved. The accuracy of these predictions emphasizes the importance of the arithmetic mean as a guide to probable errors in photogrammetric quantities.

These results have several implications for the designer, map checker and the construction engineer. For the designer, it provides a method of determining the probable imbalance of excavation and embankment quantities between any desired limits as soon as a profile, on either the centerline or any base line between the slope stakes, is available. Such information if properly utilized will do much to eliminate major revisions during construction due to errors in the photogrammetric survey.

While a photogrammetric survey may cover an area 1,200 ft or more in width, in most cases the width between slope stakes will not exceed 200 to 300 ft. On many projects the location of the centerline is known within much smaller limits than the width of mapping would indicate. As accuracy required for earthwork quantities is confined to this relatively narrow band, it follows that the map checker should generally concentrate his efforts in this area of principal importance rather than attempt to test a remote corner of the mapping for compliance with specifications. By running test profiles generally parallel to the proposed centerline and calculating the arithmetic mean of the errors, the map checker can provide valuable information to the designer and to the construction engineer.

Comparison of Earthwork Quantities

Table 5 shows a comparison of excavation and embankment quantities obtained from two field surveys and six photogrammetric surveys. The same cross-section interval of 50 ft, plus nine additional cross-sections at breaks in the terrain, was used for all surveys. The resulting differences from the F1 survey are, for all practical purposes,

	(1) 25-ft Spacing	(2) 50-ft Spacing	(3) 100-ft Spacing	(4) 100-ft Spacing
Excavation:				
Quantity - cu. yd.	63,118	63,167	63,128	63,244
Error - cu. yd.		+49	+10	+126
Error - percent		0.1	0.0	0.2
Embankment:				
Quantity - cu. yd.	29,223	29,152	29,232	28,663
Error - cu. yd.		-71	+9	-560
Error - percent		0.2	0.0	1.9

TABLE 4

PREDICTED ERRORS IN TOTAL EARTHWORK QUANTITIES FROM PHOTOGRAMMETRIC SURVEYS

Survey	Arithmetic Mean of Centerline Profile (ft)	Predicted Error Cu yd	Actual Error Cu yd	Error in Prediction Cu yd	Error in Prediction as % of Total Quantity
PS1	+0.12	+1,690	+1,340	350	0.4
PS2	+0.29	+4,090	+3,822	268	0.3
PS3	+0.06	+ 845	+ 821	24	0.0
CM1	+0.04	+ 565	+ 262	303	0.3
CM2	+0.27	+3,800	+3,228	572	0.6
CM3	+0.18	+2,530	+2,247	283	0.3

due to errors in measurement of elevations. It is apparent that this is by far the most important of the various sources of errors in earthwork quantities.

Accuracy of excavation quantities of four of the photogrammetric surveys, PS1, PS3, CM1, and CM3, would be considered satisfactory for pay quantities by almost any standards. The F2 field survey and the PS2 and CM2 surveys would be generally satisfactory for design quantities. Some engineers might consider them satisfactory for pay quantities. In measurement of embankment quantities, however, only the F2, PS3, and CM1 surveys could be considered satisfactory.

For determination of probable error in balance between cut and fill, the total error as shown in Table 6 is undoubtedly the best measure. In this case all of the surveys showed plus errors in excavation and minus errors in embankment. Insofar as balance is concerned these errors are cumulative and the total error is their sum. In cases where both excavation and embankment errors have the same sign they would tend to compensate and the total error would be the difference.

Probably the best method of expressing the actual accuracy of the various surveys as related to earthwork quantities is the "Equivalent Vertical Error" also shown in Table 6. It was calculated by dividing the total error in cubic feet by the area between slope stakes in square feet. In effect, therefore, it is the mean vertical difference in each survey from the terrain as depicted by the F1 survey. The equivalent vertical error also could be determined by taking the arithmetic mean of the errors of a large number of equally spaced points over the entire area. The total error in volume could then be found by multiplying the equivalent vertical error by the area.

Comparison of the equivalent vertical error and the arithmetic mean of the centerline profile shows a very close relationship for each of the surveys. This relationship is

TABLE 5

COMPARISON OF EXCAVATION AND EMBANKMENT QUANTITIES FROM
FIELD AND PHOTOGRAMMETRIC SURVEYS

Survey	Excavation			Embankment		
	Quantity Cu. yd.	Error		Quantity Cu. yd.	Error	
		Cu. yd.	%		Cu. yd.	%
F1	63,167			29,152		
F2	64,212	+1,045	1.7	28,654	- 498	1.7
PS1	63,338	+ 171	0.3	27,983	-1,169	4.0
PS2	64,717	+1,550	2.5	26,880	-2,272	7.8
PS3	63,678	+ 511	0.8	28,842	- 310	1.1
CM1	63,187	+ 20	0.0	28,910	- 242	0.8
CM2	64,303	+1,136	1.8	27,060	-2,092	7.2
CM3	63,174	+ 7	0.0	26,912	-2,240	7.7

quite important as it indicates a means of correlating the accuracy of a survey, in measuring the elevation of discrete points, to the accuracy of earthwork quantities.

However, the values of the equivalent vertical error as shown in Table 6 are averages for the entire area, and are not applicable to individual cross-sections or to small portions of the mapping. This is clearly shown by the fact that, while total errors could be predicted very closely by the arithmetic mean of the centerline profile, the major errors in earthwork quantities occurred in embankment areas.

Lack of uniformity in the errors in individual surveys confirms previous

TABLE 6

COMPARISON OF TOTAL ERRORS IN QUANTITIES
FROM PHOTOGRAMMETRIC SURVEYS

Survey	Total Error Cu. Yd.	Total Error (%)	Equivalent Vertical Error (ft)	Arithmetic Mean of Centerline Profile (ft)
PS1	1,340	1.4	+0.10	+0.12
PS2	3,822	4.1	+0.27	+0.29
PS3	821	0.9	+0.06	+0.06
CM1	262	0.3	+0.02	+0.04
CM2	3,228	3.5	+0.23	+0.27
CM3	2,247	2.5	+0.16	+0.18

experience in comparing quantities from photogrammetric surveys with those from field surveys. It has been frequently found that errors in individual cuts and fills are far greater than the error for an entire project. Such variations are not the result of random accidental errors which are unpreventable. They can almost always be attributed to varying systematic errors and blunders. The fact that, in this particular case, the major errors occurred in embankment areas is not considered significant. It might be due to chance or to conditions peculiar to the test site. These conditions might be reversed on an adjacent project.

The comparisons shown in Tables 5 and 6 lead to the conclusion that the serious effect of relatively small systematic errors on earthwork quantities make the use of photogrammetric surveys for pay quantities questionable unless such surveys are adjusted or thoroughly checked in a satisfactory manner.

Adjustment of Quantities from Photogrammetric Surveys

The remarkably accurate results in predicting total errors shown in Table 4 indicate the possibility of reducing the errors in individual cross-sections by adjusting the terrain to a centerline elevation determined by a field survey. Several states have reported greatly increased accuracy in photogrammetric cross-sections by indexing on the field elevations at centerline while taking cross-sections from the stereomodel. Such a procedure requires determination of the final line and staking the line in the field prior to obtaining large-scale mapping. If similar results could be obtained by adjustment to a line staked in the field after the mapping was completed, it would provide much greater flexibility in design procedures.

As a test of this possibility, quantities from the six photogrammetric surveys were adjusted by raising or lowering the entire terrain at each cross-section by an amount equal to the error in the centerline elevation. The method of setting up the adjustments was to provide the tabulation section with a list showing difference in elevation at each centerline station between the F1 survey and the photogrammetric survey. New tabulations based on the adjusted terrain notes were provided by the tabulation section. On future projects it is anticipated that the difference in elevation at centerline will be computed by machine. In this case the only data to be furnished by the engineer will be the field and photogrammetric centerline elevations.

Errors in cubic yards and percent, both before and after adjustment, are shown for excavation quantities in Table 7 and for embankment quantities in Table 8. As a means of observing the localized effects of the adjustments, the quantities are shown in three segments for both excavation and embankment. Comparative quantities from the F2 field survey and the method of making the survey are also shown for each segment. No adjustment of the F2 quantities was possible as the Rhodes Arc elevations were based on centerline elevations, which had been corrected for obvious blunders.

The results show that in four of the six individual segments the adjusted quantities of all of the photogrammetric surveys were more nearly correct than quantities from the F2 field survey. Errors in both excavation and embankment totals by the F2 survey were greater than by any of the adjusted photogrammetric surveys. In only one case, embankment quantities by PS1, were the adjusted totals of either excavation or embankment quantities from photogrammetric surveys in error by more than one percent.

In several cases, where quantities were in error by relatively small amounts, the adjusted totals showed slightly greater errors than the original unadjusted quantities. This is to be expected and can be accepted if the centerline adjustments will (a) materially reduce large localized errors; and (b) result in quantities which are within tolerable limits.

The question of large localized errors is best illustrated by the portion of CM3 from Station 160 to 164. It has been previously noted that most of the errors of over 1.0 ft occurred in this section. As shown in Table 8 the centerline adjustment reduced the error in this portion of the survey from 1,378 cu yds to 95 cu yds and from 12.8 to 0.9 percent.

As to tolerable limits, adjustment of the six photogrammetric surveys of the test section resulted in quantities within limits generally considered tolerable for purposes

of payment. The adjusted quantities from all of the photogrammetric surveys were more nearly correct than those obtained from a field survey made by commonly accepted methods.

Comparative values for the previously discussed measures of total error and equivalent vertical error, both before and after adjustment, are shown in Table 9. In all cases, where the original quantities were in error by any appreciable amount, the total

TABLE 7
DETAILED COMPARISON OF EXCAVATION QUANTITIES AND ADJUSTMENTS

		F1	F2	PS1	PS2	PS3	CM1	CM2	CM3
Sta. 163+50 to 174	Quantity - cu. yd.	2065	2406	2152	2254	1956	2123	2438	2403
	Error - cu. yd.		+341	+87	+189	-109	+58	+373	+338
	Error - percent		16.5	4.2	9.1	5.3	2.8	18.1	16.4
	Adjustment - cu. yd.		(Rhodes	-116	-272	+42	-27	-305	-318
	Net Error - cu. yd.		Arc)	-29	-83	-67	+31	+68	+20
	Net Error - percent			1.4	4.0	3.2	1.5	3.3	1.0
	Relative accuracy		(7)	(2)	(6)	(4)	(3)	(5)	(1)
Sta. 174 to 180	Quantity - cu. yd.	45845	46570	45610	46807	48023	45571	46459	45673
	Error - cu. yd.		+725	-235	+962	+178	-274	+614	-172
	Error - percent		1.6	0.5	2.1	0.4	0.6	1.3	0.4
	Adjustment - cu. yd.		(Rhodes	+74	-679	+26	+326	-543	+481
	Net Error - cu. yd.		Arc)	-161	+283	+204	+52	+71	+309
	Net Error - percent			0.4	0.6	0.4	0.1	0.2	0.7
	Relative accuracy		(7)	(3)	(5)	(4)	(1)	(2)	(6)
Sta. 180 to 189	Quantity - cu. yd.	15257	15236	15576	15656	15699	15493	15406	15098
	Error - cu. yd.		-21	+319	+399	+442	+236	+149	-159
	Error - percent		0.1	2.1	2.6	2.9	1.5	1.0	1.0
	Adjustment - cu. yd.		(Engineer's	-494	-409	-446	-331	-144	+350
	Net Error - cu. yd.		Level)	-175	-10	-4	-95	+5	+191
	Net Error - percent			1.1	0.1	0.0	0.6	0.0	1.2
	Relative accuracy		(4)	(6)	(3)	(1)	(5)	(2)	(7)
Total excavation	Quantity - cu. yd.	63167	64212	63338	64717	63678	63187	64303	63174
	Error - cu. yd.		+1045	+171	+1550	+511	+20	+1136	+7
	Error - percent		1.7	0.3	2.5	0.8	0.0	1.8	0.0
	Adjustment - cu. yd.			-536	-1360	-378	-33	-992	+513
	Net Error - cu. yd.			-365	+190	+133	-13	+144	+520
	Net Error - percent			0.6	0.3	0.2	0.0	0.2	0.8
	Relative accuracy		(7)	(5)	(4)	(2)	(1)	(3)	(6)

TABLE 8
DETAILED COMPARISON OF EMBANKMENT QUANTITIES AND ADJUSTMENTS

		F1	F2	PS1	PS2	PS3	CM1	CM2	CM3
Sta. 160 to 164	Quantity - cu. yd.	10764	10644	10646	10202	10812	10619	10560	9386
	Error - cu. yd.		-120	-118	-562	+48	-145	-204	-1378
	Error - percent		1.1	1.1	5.2	0.4	1.3	1.9	12.8
	Adjustment - cu. yd.		(Engineer's	+73	+490	-7	+182	+244	+1473
	Net error - cu. yd.		Level)	-45	-72	+41	+37	+40	+95
	Net error - percent			0.4	0.8	0.4	0.3	0.4	0.9
	Relative accuracy		(7)	(4)	(5)	(3)	(1)	(2)	(6)
Sta. 164 to 174+50	Quantity - cu. yd.	15383	14978	14504	14085	15311	15335	14235	14640
	Error - cu. yd.		-405	-879	-1298	-72	-48	-1148	-743
	Error - percent		2.6	5.7	8.4	0.5	0.3	7.5	4.8
	Adjustment - cu. yd.		(Rhodes	+526	+1048	-7	+72	+1182	+517
	Net error - cu. yd.		Arc)	-353	-250	-79	+24	+34	-226
	Net error - percent			2.3	1.6	0.5	0.2	0.2	1.5
	Relative accuracy		(7)	(6)	(5)	(3)	(1)	(2)	(4)
Sta. 183 to 190	Quantity - cu. yd.	3005	3032	2833	2593	2719	2956	2265	2886
	Error - cu. yd.		+27	-172	-412	-286	-49	-740	-119
	Error - percent		0.9	5.7	13.7	9.5	1.6	24.6	4.0
	Adjustment - cu. yd.			+197	+468	+285	+237	+808	+221
	Net error - cu. yd.		(Engineer's	+25	+56	-1	+188	+68	+102
	Net error - percent		Level)	0.8	1.9	0.0	6.3	2.3	3.4
	Relative accuracy		(3)	(2)	(4)	(1)	(7)	(5)	(6)
Total embankment	Quantity - cu. yd.	29152	28654	27983	26880	28842	28910	27060	26912
	Error - cu. yd.		-498	-1169	-2272	-310	-242	-2092	-2240
	Error - percent		1.7	4.0	7.8	1.1	0.8	7.2	7.7
	Adjustment - cu. yd.			+796	+2006	+271	+491	+2233	+2210
	Net error - cu. yd.			-373	-266	-39	+249	+141	-30
	Net error - percent			1.3	0.9	0.1	0.9	0.5	0.1
	Relative accuracy		(7)	(6)	(5)	(2)	(4)	(3)	(1)

TABLE 9
EFFECT OF ADJUSTMENT ON TOTAL ERROR AND EQUIVALENT VERTICAL ERROR

Survey	Total Error in Cu. yd.		Total Error in %		Equivalent Vertical Error (ft)	
	Before Adjust.	After Adjust.	Before Adjust.	After Adjust.	Before Adjustment	After Adjustment
F2	1,543		1.7		+0.11	
PS1	1,340	8	1.4	0.0	+0.10	+0.00
PS2	3,822	456	4.1	0.5	+0.27	+0.03
PS3	821	172	0.9	0.2	+0.06	+0.01
CM1	262	262	0.3	0.3	+0.02	-0.02
CM2	3,228	3	3.5	0.0	+0.23	+0.00
CM3	2,247	550	2.5	0.6	+0.16	+0.04

error was considerably reduced by the centerline adjustment. The maximum resulting equivalent vertical error after adjustment was 0.04 ft shown by photogrammetric survey CM3.

The comparisons shown in Tables 6 and 9 indicate the value of the equivalent vertical error as a measure of the accuracy of earthwork quantities. It is the only type of measure which can be directly related to the accuracy of the survey in measuring the elevations of discrete points. Unlike the percent of error it is not affected by the volume of earthwork involved.

The results in Tables 7, 8, and 9 show little, if any, significant difference in accuracy, either before or after adjustment, between quantities from photogrammetric cross-sections and those obtained from photogrammetric contour maps. This is readily understandable when two factors are considered:

1. The added accuracy of photogrammetric cross-sections in reading the elevations of discrete points applies only to random errors.
2. Most of the large errors in earthwork quantities obtained from photogrammetric measurements are due to systematic errors.

Analysis of Adjustments

While the general effect of the centerline adjustment on earthwork quantities was very impressive, the results could be attributed largely to chance unless they tended to reduce the errors in individual cross-sections. As the cross-sections varied in width between slope stakes, from a minimum of 95 ft to a maximum of 178 ft, the errors in cross-sectional area could not be used as a basis for direct statistical comparison. The errors could, however, be reduced to a one-dimensional variable, the equivalent vertical error, by dividing the error in area of each cross-section by the width between slope stakes.

The equivalent vertical errors for each of the 70 cross-sections of the six photogrammetric surveys, both before and after adjustment, were calculated in this manner and arranged in frequency distributions. By comparing before and after frequency distributions for each survey it was apparent that the adjustments greatly improved the conformity to normal distribution, indicating much better statistical control.

Values of the standard deviation and arithmetic mean are shown in Table 10. The wide variation in accuracy of the various photogrammetric surveys has been previously noted. This is also shown by the variations in standard deviations of the equivalent vertical errors before adjustment, which range from 0.17 to 0.45 ft. The most significant fact shown by the analysis is that the centerline adjustments reduced this range to a minimum of 0.14 ft and a maximum of 0.23 ft. This clearly indicates the equalizing effect of the adjustment of the wide variations in accuracy of the various surveys.

As might be expected, the effect of the adjustment was to greatly reduce any large

TABLE 10
COMPARISON OF THE EQUIVALENT VERTICAL ERRORS OF
70 CROSS-SECTIONS

Survey	Standard Deviation		Arithmetic Mean		Standard Error of the Mean After Adjust.
	Before Adjustment	After Adjustment	Before Adjustment	After Adjustment	
	(ft)	(ft)	(ft)	(ft)	(ft)
PS1	0.17	0.14	+0.12	-0.01	0.02
PS2	0.20	0.17	+0.31	+0.01	0.02
PS3	0.19	0.15	+0.08	+0.02	0.02
CM1	0.17	0.19	+0.03	-0.04	0.02
CM2	0.31	0.22	+0.29	-0.01	0.03
CM3	0.45	0.23	+0.16	+0.02	0.03
CM3A ¹	0.33	0.23	+0.05	+0.02	0.03

¹ Portion from Stations 164 to 190 - 62 cross-sections.

errors in the arithmetic mean. Values of the arithmetic mean differ slightly from the equivalent vertical errors shown in Table 9 as the latter are, in effect, weighted averages of the individual cross-sections. Values of the standard error of the mean, after adjustment, shown in Table 10, indicate that the corresponding values of the arithmetic mean are, in all cases, within the limits of a normal distribution.

It will be noted that, after adjustment, the standard deviations of the photogrammetric cross-section surveys were slightly lower than those for the contour-map surveys. This indicates that quantities from photogrammetric cross-sections will tend to have slightly greater accuracy after adjustment than those from contour maps. For 70 cross-sections, however, the standard error of the mean is so small as to make the difference insignificant.

The primary purpose of the centerline adjustment of the photogrammetric measurements would be to reduce the errors in earthwork quantities. However, some consideration must be given to its effect on the accuracy of individual points, since the adjusted photogrammetric measurements may be used during construction staking. The measures of the 90 percent specification tolerance, standard deviation and arithmetic mean were used to determine the effects of the centerline adjustment on 122 slope stakes of the test section. Photogrammetric measurements of slope stake elevations were read directly in the stereoplotter for the PS1 and PS3 surveys. For the other surveys, elevations were determined by interpolation. Before and after adjustment values of the three measures for each of the surveys are shown in Table 11.

TABLE 11
EFFECT OF ADJUSTMENT ON POINT ACCURACY OF 122 SLOPE STAKES

Survey	90% Within		Standard Deviation		Arithmetic Mean	
	Before Adjust.	After Adjust.	Before Adjust.	After Adjust.	Before Adjust.	After Adjust.
	(ft)	(ft)	(ft)	(ft)	(ft)	(ft)
PS1	±0.4	±0.3	0.22	0.22	+0.13	+0.02
PS2	±0.7	±0.4	0.28	0.28	+0.36	+0.08
PS3	±0.4	±0.4	0.27	0.23	+0.11	+0.07
CM1	±0.5	±0.5	0.32	0.36	+0.06	+0.03
CM2	±0.8	±0.6	0.33	0.38	+0.35	+0.06
CM3	±1.1	±0.9	0.59	0.56	+0.29	+0.10

The general effects were to slightly improve the 90 percent limit, and to greatly reduce the arithmetic mean. The standard deviation was slightly greater after adjustment in two of the surveys, slightly less in two, and unchanged in the remaining two.

Probably the best explanation of the effect of the centerline adjustments can be found in the previously stated conclusion that "for any selected small area, such as a single cross-section two or three hundred feet in length, systematic errors and blunders tend to remain fairly constant." On this basis the centerline adjustment should tend to greatly reduce the systematic errors and blunders and to slightly increase the random errors. If this is correct, the effectiveness of the adjustment for a particular project would depend on the ratio of random errors to systematic errors and blunders. Unfortunately it is impossible to make a definite segregation of the types of errors in a photogrammetric survey.

Results from the surveys of the test section emphasize the importance of systematic errors and indicate that their effect can be minimized by the centerline adjustment.

COSTS

The over-all advantages of photogrammetric surveys over conventional field surveys have become generally recognized during the past few years. Therefore no attempt was made to expand the study to include complete costs of the two basic survey methods. However, the costs of certain phases of survey and design work are factors which must be considered in selecting a method for obtaining earthwork quantities. A record was therefore kept of the man-hours required for various operations in the phases of field surveys, stereocompilation and calculation of earthwork quantities. This information together with the calculated cost per mi for each operation is shown in Table 12.

Among the comparisons which can be made from Table 12 is the savings by use of machine computation over the former method of plotting cross-sections and planimetering the areas. Assuming that a contour map is the basic source of data, machine computation of earthwork quantities requires Items 9, 10, and 15 for a cost per mile of \$330. The planimeter method involves Items 12 and 13 for a total of \$570 per mi. The savings by use of machine computation in this particular case was \$240 per mile or 42 percent. Use of the Avol Rule instead of the planimeter would have saved \$125 per mi.

The cost of obtaining pay quantities by field cross-sections is the sum of Items 3 and 15, or \$725 per mi. Assuming that the design quantities had been taken from a preliminary line on the contour map, or that the interval was too great for pay quantities, the designer could have prepared new terrain notes (Item 10), obtained quantities (Item 15), and adjusted the quantities to the centerline profile (Item 15) for a total of \$305

TABLE 12
TIME AND COSTS¹

Item	Operation	Survey	Avg Width (ft)	Length (ft)	Man hours	Approx Cost per mi
1	Centerline Profile	F2		3,000	22	\$ 200
2	Cross-section - 25-ft interval	F1	140	3,000	160	1,480
3	Cross-section - 50-ft interval	F2	165	3,000	68	625
4	Set Slope Stakes	F2		3,000	50	460
5	Stereo Setup - per model	Avg except CM3		900	1.3	55
6	Stereo Compilation - Contours	Avg CM 1 and 2	500	3,400	10.4	120
7	Stereo Compilation - Cross-sections	Avg PS 1 and 2	250	3,400	6.9	80
8	Stereo Readout - Cross-sections	PS3	250	3,400	6.8	95
9	Prepare and Check Roadbed Notes	All		3,000	18	125
10	Prepare and Check Terrain Notes	Avg CM1, 2 and 3	170	3,000	15	105
11	Prepare and Check Terrain Notes	Avg PS1 and 2	190	3,000	8	55
12	Plot Cross-Sections and Templates	F2	165	3,000	44	310
13	Planimeter and Calculate Quantities	F2		3,000	37	260
14	Calculate Quantities - Avol Rule	F2		3,000	19	135
15	Key Punch and Machine Computation	Avg except PS3		3,000		100
16	Machine Computation	PS3		3,000		80

¹ Field survey costs include living expenses but no travel time.

Stereoplotter costs are based on Kelsh plotter rental of \$7.50 per hr including operator.

Stereoplotter costs plus Terrain Data Translator (PS3) are estimated at \$9.00 per hr.

per mi. For the test section, therefore, the saving by using adjusted photogrammetric quantities for payment in lieu of obtaining field cross-sections would have been \$420 per mi or 58 percent. It has been previously shown that adjusted quantities from all of the photogrammetric surveys of the test section were more nearly correct than those obtained from the F2 field survey.

A comparison of the relative cost of stereocompilation, preparing terrain notes and machine computation between a contour map (CM1), photogrammetric cross-sections written on a manuscript (PS1), and photogrammetric cross-sections taken directly from the stereomodel to punch cards by use of the Terrain Data Translator (PS3), is as follows:

CM1 - Items 6, 10, and 15	\$325 per mi
PS1 - Items 7 (Adjusted), 11, and 15	\$315 per mi
PS3 - Items 8 (Adjusted), and 16	\$270 per mi

The costs per mile of Items 7 and 8 as adjusted are twice the amounts shown in Table 10 to place stereocompilation costs on a uniform basis of 500-ft average width. The above comparison applies only to relative costs as a measure of savings and does not include items which are common to all of the methods such as photography, photo control, model set up and preparation of roadbed notes.

From the foregoing analysis the comparative savings which can be achieved in three of the steps involved in obtaining earthwork quantities can be summarized as follows:

1. Saving by machine computation as compared to plotting and planimentering cross-sections - \$240 per mi.
2. Saving by automation in taking digital data directly from the stereomodel to punch cards, as compared to use of a contour map - \$55 per mi.
3. Saving by adjusting photogrammetric quantities for payment in lieu of taking field cross-sections of the final line - \$420 per mi.

It should be noted that the savings under 1 are applicable to a number of trial lines as well as the final line while those under 2 and 3 generally apply only to the final line. Nevertheless the 58 percent saving in manpower by using adjusted photogrammetric quantities for payment indicate the value of developing a method for their use.

SELECTION OF METHODS

The two basic sources of photogrammetric terrain data are: contour maps; and cross-sections from spot heights read directly in the stereoplotter. Information concerning relative accuracy and costs developed by this study should assist the engineer in making a choice between the two sources.

The most frequently mentioned advantages of the cross-section method are the added accuracy and the saving in cost and manpower by taking digital terrain data from the stereomodel directly to punch cards or tape. This savings in cost for the test section amounted to \$55 per mi. The related saving in manpower, therefore, can be considered relatively minor when compared to the total engineering effort required to obtain earthwork quantities.

Surveys of the test section showed no significant differences in accuracy between quantities taken from a contour map and those from photogrammetric cross-sections. This is due to the fact that the added accuracy in reading spot heights applies only to random errors and has no effect on systematic errors. The latter have by far the most serious effect on the accuracy of earthwork quantities obtained from photogrammetric surveys. Except in comparatively flat terrain, the relative accuracy of the two methods does not appear to be an important factor.

The principal advantage of a contour map, as a source of terrain data, is the flexibility in procedure it provides. The designer can determine the approximate location of the final line from preliminary source data and obtain a large-scale contour map covering a band from 1,000 to 1,500 ft in width. The map can then be used to determine the exact position of the final line, and digital terrain data based on the final line can

be taken from it. Design work can proceed without the delay inherent in the photogrammetric cross-section methods due to staking the final line in the field, rephotographing the area or, as a minimum, resetting the models in the stereoplotter. These advantages appear to be the most important factors in making a choice between a contour map and photogrammetric cross-sections.

In flat terrain, where spot elevations are preferable to a contour map, consideration should be given to obtaining photogrammetric cross-sections based on a tentative centerline rather than an arbitrary grid of spot elevations. In such terrain the designer can frequently position the final line by the use of large-scale aerial photographs or other available data before obtaining a photogrammetric survey.

Development of photogrammetry and machine computation has provided the engineer with a wide variety of methods from which to choose in obtaining earthwork quantities. A method shown by this study to be relatively efficient and suitable for most terrain can be summarized as follows:

1. Obtain a photogrammetric contour map covering the previously selected route band. Except in very rough terrain a contour interval of two feet is preferable. Specifications for the mapping should place a limitation on the arithmetic mean of the points tested.
2. Develop earthwork quantities for trial lines by machine computation using the maximum cross-section interval consistent with the terrain. For terrain similar to that of the test section, intervals of 100 ft or more would be satisfactory. Similar spacing should be used in selecting points along the cross-section lines. Three-point, or even two-point, roadbed notes usually will be satisfactory for trial lines.
3. Develop design quantities from the contour map after the final line has been positioned on the map and calculated. The interval and stationing of cross-sections for the final line should be consistent with the terrain and with the requirements of slope staking and pay quantities. These intervals generally should not exceed 50 ft in rolling terrain and 100 ft in flat terrain.
4. Adjust the design quantities at such time as an accurate field profile is available. The time at which the field profile should be obtained depends on several factors. Among these are the physical characteristics of the project, the imminence of construction and the engineer's judgment as to the accuracy of the mapping.

CONCLUSIONS

Data developed by this study lead to the following conclusions:

1. The most important factor in the accuracy of earthwork quantities is the vertical accuracy of the survey measurements.
2. Photogrammetric surveys are subject to relatively small systematic errors. Such errors have a serious effect on the accuracy of earthwork quantities.
3. Use of photogrammetric surveys for pay quantities is questionable unless they are checked by statistical comparison with an accurate field profile.
4. The greatest saving in manpower in obtaining earthwork quantities which can be achieved under current California practice is development of a method of utilizing photogrammetric quantities for payment.
5. The most important fact developed by this study is that adjusted quantities from all of the photogrammetric surveys were within limits generally considered tolerable for pay quantities. They were more accurate than quantities obtained by a field survey made by commonly accepted methods.
6. The method of adjusting photogrammetric quantities by use of a centerline profile appears to have considerable potential value as a means of obtaining pay quantities with a minimum expenditure of manpower. Further tests of adjustments on projects scheduled for early construction are planned in the near future.

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