HIGHWAY RESEARCH BOARD

Bulletin 228

Photogrammetry: Developments and Applications 1959

National Academy of Sciences—

National Research Council

publication 688

HIGHWAY RESEARCH BOARD

Officers and Members of the Executive Committee

1959

OFFICERS

HARMER E. DAVIS, *Chairman* PYKE JOHNSON, *First Vice Chairman*

W. A. BUGGE, *Second Vice Chairman*

FRED BURGGRAF, *Director* ELMER M. WARD, *Assistant Director*

Executive Committee

BERTRAM D. TALLAMY, *Federal Highway Administrator, Bureau of Public Roads* (ex officio)

A. E. JOHNSON, *Executive Secretary, American Association of State Highway Officials* (ex officio)

LOUIS JORDAN, *Executive Secretary, Division of Engineering and Industrial Research, National Research Council* (ex officio)

C. H. SCHOLER, *Applied Mechanics Department, Kansas State College (ex officio, Past Chairman 1958)*

REX M. WHITTON, *Chief Engineer, Missouri State Highway Department* (ex officio, Past Chairman 1957)

R. R. BARTLESMEYER, *Chief Highway Engineer, Illinois Division of Highways*

J. E. BUCHANAN, *President, The Asphalt Institute*

W. A. BUGGE, *Director of Highways, Washington State Highway Commission*

MASON A. BUTCHER, *Director of Public Works, Montgomery County, Md.*

C. D. CURTISS, *Special Assistant to the Executive Vice President, American Road Builders Association*

HARMER E. DAVIS, *Director, Institute of Transportation and Traffic Engineering, University of California*

DUKE W. DUNBAR, *Attorney General of Colorado*

FRANCIS V. DU PONT, *Consulting Engineer, Cambridge, Md.*

H. S. FAIRBANK, *Consultant, Baltimore, Md.*

PYKE JOHNSON, *Consultant, Automotive Safety Foundation*

G. DONALD KENNEDY, *President, Portland Cement Association*

BURTON W. MARSH, *Director, Traffic Engineering and Safety Department, American Automobile Association*

GLENN C. RICHARDS, *Commissioner, Detroit Department of Public Works*

WILBUR S. SMITH, *Wilbur Smith and Associates, New Haven, Conn.*

K. B. WOODS, *Head, School of Civil Engineering, and Director, Joint Highway Research Project, Purdue University*

Editorial Staff

FRED BURGGRAF ELMER M. WARD HERBERT P. ORLAND

2101 Constitution Avenue Washington 25, D. C.

N.R.C. # HIGHWAY RESEARCH BOARD

Bulletin 228

Photogrammetry: Developments and Applications 1959

Presented at the
38th ANNUAL MEETING
January 5-9, 1959

1959
Washington, D. C.

$1.20

Department of Design

T. E. Shelburne, Chairman
Director of Research,
Virginia Department of Highways

COMMITTEE ON PHOTOGRAMMETRY AND AERIAL SURVEYS

D. J. Olinger, Chairman
Aerial Engineer, Wyoming State Highway Department

William T. Pryor, Secretary
Chief, Photogrammetry and Aerial Surveys
Bureau of Public Roads

Fred B. Bales, Virginia Department of Highways, Richmond
I. W. Brown, Photronics Engineer, Mississippi State Highway Department, Jackson
E. M. Every, Assistant Engineer in Charge of Photogrammetry and Location, Wisconsin Highway Commission, Madison
W. S. Foy, L. E. Gregg and Associates, Lexington, Kentucky
L. L. Funk, Photogrammetric Engineer, California Division of Highways, Sacramento
W. S. Higginson, Sloan and Associates, Pasadena, California
Fred W. Hurd, Director, Bureau of Highway Traffic, New Haven, Conn.
David S. Johnson, Engineer of Planning, Connecticut State Highway Department, Hartford
John E. Meyer, Route Location Engineer, Michigan State Highway Department, Lansing
Robert D. Miles, Research Engineer and Assistant Professor of Highway Engineering, Purdue University, Lafayette, Indiana
Charles L. Miller, Director, Photogrammetry Laboratory, Massachusetts Institute of Technology, Cambridge
E. S. Preston, Director, Ohio Department of Highways, Columbus
Arthur C. Quinnell, Location Engineer, Interstate Division, Montana Highway Commission, Helena
Henry G. Schlitt, Deputy State Engineer, Nebraska Department of Roads, Lincoln
M. D. Shelby, Supervising Research Engineer, Texas State Highway Department, Austin
Preston C. Smith, Highway Research Engineer, Bureau of Public Roads
Irwin Sternberg, District Location Engineer, Arizona Highway Department, Tucson
Clyde T. Sullivan, Division of Engineering, U. S. Department of Agriculture
L. W. Verner, State Highway Location Engineer, State Highway Department of Georgia, Atlanta
Dwight E. Winsor, Bureau of Public Roads, Denver, Colorado
Marshall S. Wright, Jr., Chief Engineer, Jack Ammann Photogrammetric Engineers, Inc., San Antonio, Texas

Contents

North Dakota's Use of Aerial Inventory for County General Highway Maps

E. T. BOWEN, Road Inventory Manager, North Dakota Highway Planning Survey, C. J. CRAWFORD, Highway Planning Survey Engineer, North Dakota Highway Department and J. B. KEMP, District Engineer, North Dakota Division, U. S. Bureau of Public Roads

One of the principal activities of the Statewide Highway Planning Survey since its beginning in the middle thirties has been inventory and mapping. Probably the principal use of inventory data has been for the preparation of county general highway maps. North Dakota prepares these maps (see Fig. 1) in three colors. A number of methods for collecting the inventory data have been developed by the several states.

The North Dakota Highway Planning Survey has instituted an aerial method of collecting a vast majority of the field data. To the best of knowledge, North Dakota was the first state to employ the aerial inventory method. It has been in operation for the past three years and has produced better inventory data at less cost and in less time than the conventional ground inventory methods. During the past three years all 53 counties in the State have been inventoried. These 53 counties cover 70, 183 square miles and include about 115, 000 miles of roads and streets.

The following subjects will be discussed:

1. The aerial inventory method developed and used in North Dakota and the supporting ground crew activities.
2. The equipment and personnel employed for this operation.
3. Comparative cost data.
4. Proposed future uses of the aerial method.

SYNOPSIS OF INVENTORY OPERATIONS

●THE INVENTORY operations in North Dakota involve essentially a combination of:

1. An office preparation of a work map "loaded" with road and cultural data obtained from aerial photographs.
2. A verification or a revision of the work map based on field observation from the air.
3. The estimating and classifying of certain information (e.g. surface widths and drainage structure sizes and types) from the air.
4. Obtaining structural data, inventory data in incorporated places and certain horizontal control information by a ground crew.

Office Preparation of Work Map

A print of the previous county general highway map at a scale of one mile to one in. serves as a base for the work map. Information such as road identification numbers, map segment numbers and other data which serve to orient the air crew are added to the base map. Data are added to or deleted from the base map by a review of the aerial photographs. The work maps then are cut into segments of convenient size for ease of handling and manipulation in aircraft.

Work Map Check-Off

The work map with the data available from the aerial photographs is then taken in the aircraft and each road is covered from the air. The roads in North Dakota generally are on the N-S or E-W section lines in the rectangular land grids. The road and cultural data that are observed to be as indicated on the work map are so identified by a check mark on the map. Cultural or other features not shown on the work map but found to be in existence are added by color code to the work map. Features shown incorrectly are corrected and features shown but not existing are crossed out.

Aerial Classification of Surface Widths and Minor Structures

Experience in estimating surface widths from the air has indicated that a satisfactory

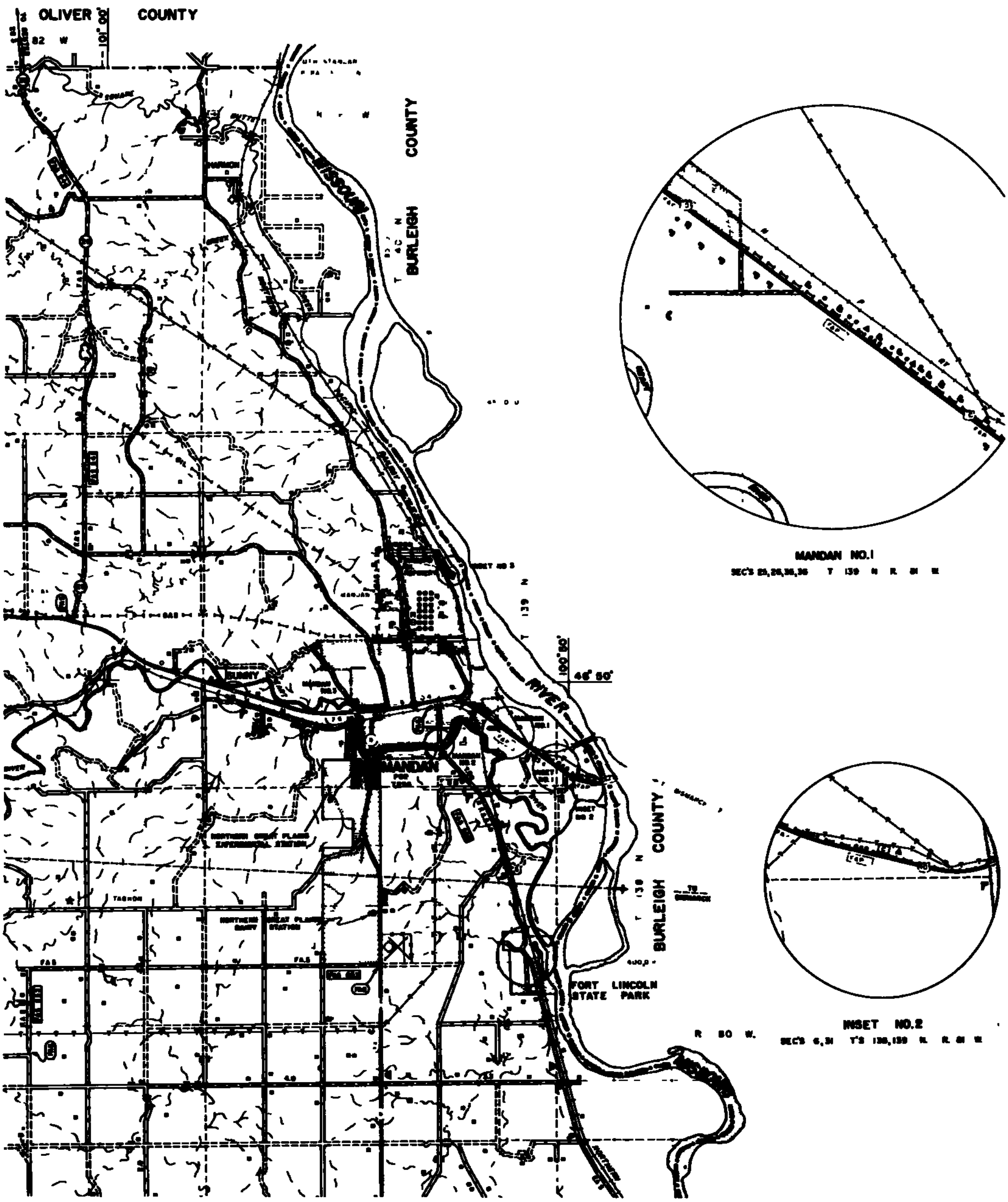


Figure 1.

grouping into surface width classes can be made. Three classes, under 20 ft, 20 to 26 ft, and over 26 ft, are used for the 2-lane roads. Widths from construction plans are available for most all multi-lane roads.

Earth and gravel-surfaced roads under 20 ft are characterized by a single pair of tracks (Fig. 2). The middle class (20 to 26 ft) is characterized by three tracks (Fig. 3) while the wide widths (over 26 ft) in general have two pairs of tracks or four clearly defined wheel tracks (Fig. 4).

There are very few dustless surfaced roads off the Federal-Aid systems in North Dakota. Widths from construction plans are available in the State Highway Department for practically all dustless surfaces.

Minor structures, 10 to 20 ft inclusive are classified according to size and type from the air. Experienced air crew personnel can estimate these types and sizes with a high degree of accuracy.

On the basis of test runs, it was found that experienced air crew personnel can estimate surface width and structure size and type information to better than 95 percent accuracy. This is comparable to ground measurement accuracy for other than dustless surface types.

Ground Crew Activities

The ground crew activities are coordinated with the air crew activities, and they are in effect subservient to the air activities. The aerial inventory is made first; all the data possible of collection are gathered and the balance is left for the ground crew.

In areas where there is inadequate coverage of triangulation stations, the ground crew takes the aerial photographs into the field, locates enough section corners to give adequate horizontal control data for mapping, and pin points the section corner locations on the aerial photographs. It should be noted in passing that the collection of this horizontal control information by the ground crew is a one-time operation. It has been completed following statewide coverage available since 1955.

For structures over 20-ft clear span, the ground crew collects width, length, waterway opening, type, and related data. The gathering of a vast majority of this information has been completed. Future re-inventory operation need be concerned only with the structures built, replaced or

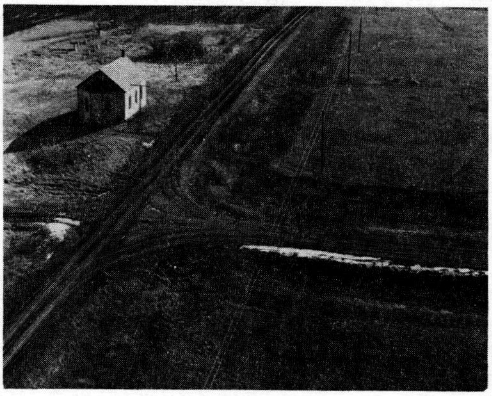


Figure 2.

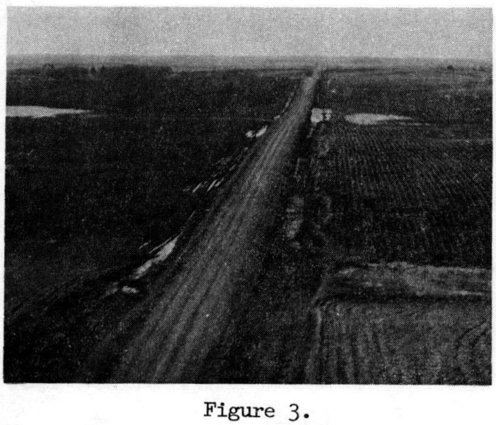


Figure 3.

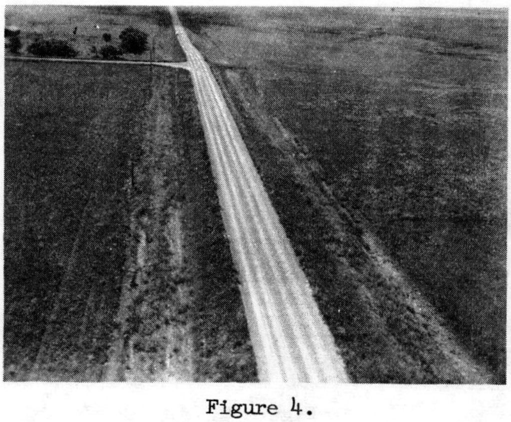


Figure 4.

destroyed since the previous inventory. Such structures can be identified readily by the air crew. Many structures are built on one of the Federal-Aid systems and plans are available for them.

The air crew has inventoried the unincorporated compacts working from photographic copy enlargements of the aerial photographs by the check-off method in the less congested compacts. In a few of the larger unincorporated compacts, especially those over 500 population, some assistance from the ground crew has been required.

AERIAL EQUIPMENT, PERSONNEL AND OPERATION

The aerial inventory is accomplished in a Cessna 170B four-place all-metal aircraft equipped with all-weather instruments including a directional gyro compass (Figs. 5 and 6). The aircraft is leased from a privately owned corporation exclusively for the inventory operation. The rental rate is \$12.00 per hour without pilot; there is no minimum guarantee. The purchase price new of an airplane of this general type so equipped would be in the order of \$10,000 to \$15,000.

The State has full control of all operations and maintenance. The State orders whatever repairs or maintenance is deemed necessary, whether it be a new motor or a small screw, and the corporation pays the bill.

There is strict adherence to the CAA safety regulations both in maintenance and in flight. The corporation carries insurance covering the airplane, and liability and property damage covering all State or Federal personnel that may be in the craft. No other commercial activities are covered by the insurance.

The air crew consists of two, a pilot and recorder. The State employs a pilot on an annual salary, which includes an increment for flight activities. He is assigned office duties during the winter months and when he is not in the field. There are a large number of individuals holding commercial pilot licenses which qualify them for this type of work. The Road Inventory Manager, is a licensed commercial pilot and forms the nucleus of a standby crew. The recorder is one of the regular draftsmen in the Inventory and Mapping Section. Several draftsmen have been trained for recorder duties.

The flight time required by the aircraft to reach 75 percent of the counties in North Dakota from the State Capitol in Bismarck does not exceed one hour (Fig. 7). The air activities have headquartered out of Bismarck to save subsistence and quarters allowances and to be able to use the air crew in the office during inclement weather. Further, up to about one-half hour of flight time each way can be used profitably for paper work, keeping records of operations, costs, arranging maps, and so forth.

In working the outlying counties the State collects inventory data in intermediate counties enroute both ways. By the time the outlying counties are completed, a good portion of the aerial inventory in the intermediate counties has been completed. By proper planning in scheduling the counties for inventory in any given year, little time

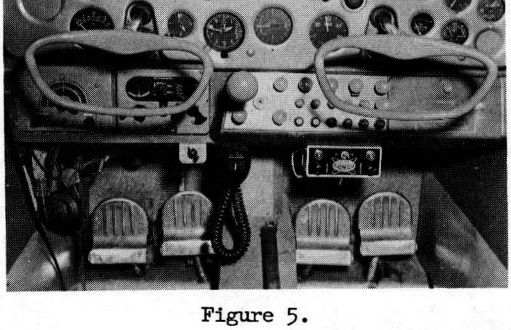


Figure 5.

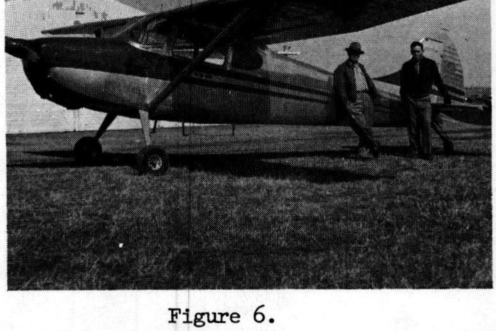


Figure 6.

is lost in "dead heading." In rough terrain, such as the badlands, high level reconnaissance at 2,000 to 5,000 ft above the ground is necessary for orientation.

After completion of the high level reconnaissance, the flight altitude is reduced to 500 to 1,000 ft above the ground for detailed road information.

In flat or gently rolling terrain the high level reconnaissance is employed only in the more congested areas. In rural areas the section lines ordinarily are well defined and there is a road or trail on most of them. Orientation presents no special problem in such areas and the inventory data can be collected by low level flights. These flights generally are at an altitude of 200 to 500 ft above the ground. The air speed is about 80 mph at the low level.

These low level flights require CAA and North Dakota low flight waivers. At the low flight altitudes county wide runs are made covering one section line at a time. Depending on the wind direction on a given day, either N-S or E-W section lines are flown.

Occasional circling and reruns are necessary if features are not clearly identifiable on the first run. This may be necessary to ascertain whether a dwelling unit in a grove of trees is occupied or vacant. Further, at the low altitude clusters of houses sometimes go by too fast relative to the observer. If the culture is quite dense, it may be necessary to climb to a higher altitude to get a good view.

COMPARATIVE COST DATA

As shown in Table 1 and Figure 8, the average county in North Dakota contains 1,324 square miles. During 1955, the last year the ground inventory method was used the inventory operations in an average county cost $6,103.64 or $4.61 per square mile. The cost in previous years approximated this amount.

In 1956, the first year of the aerial inventory, this average cost was $2,118.40 per county or $1.60 per square mile. In 1957 this unit cost was reduced to $1,880.08 per average county or $1.42 per square mile while in 1958 it was further reduced to $1,681.48 per average county or $1.27 per square mile.

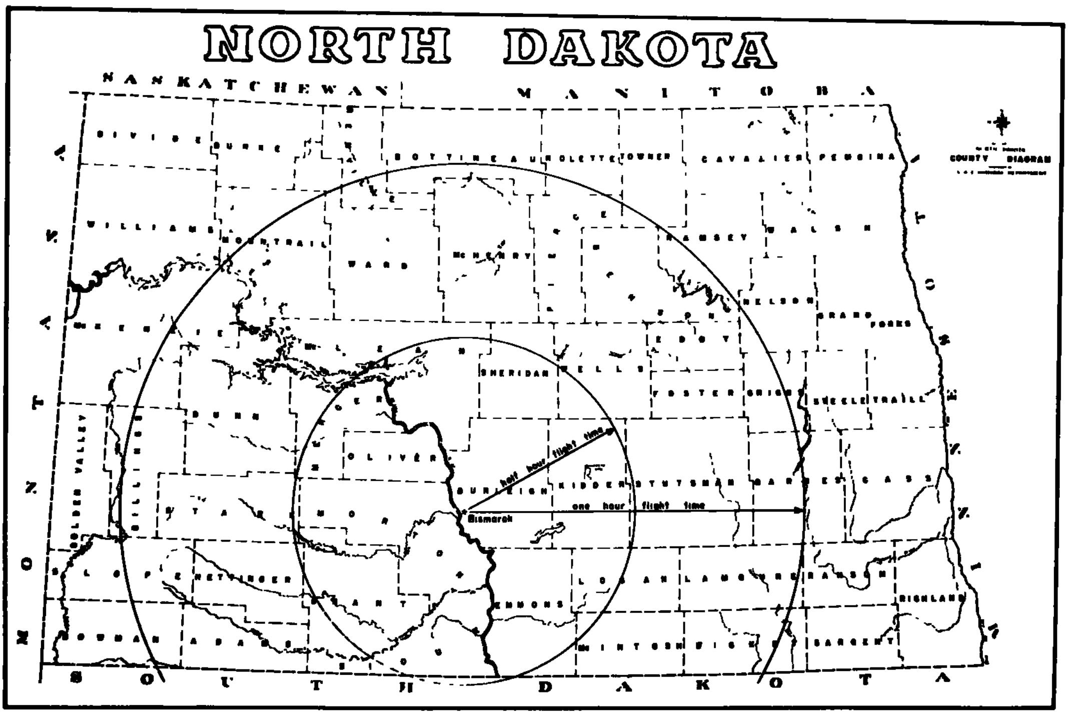


Figure 7.

TABLE I,
COMPARATIVE AVERAGE PER COUNTY AND PER SQUARE MILE INVENTORY COSTS, 1955 TO 1958, BASED ON AN AVERAGE OF 1324 SQUARE MILES PER COUNTY.

YEAR	PER COUNTY			PER SQUARE MILE			UNIT COST DATA ADJUSTED TO 1955 COSTS, 1955 = 100	
	AERIAL INVENTORY	GROUND INVENTORY	TOTAL	AERIAL INVENTORY	GROUND INVENTORY	TOTAL	PER COUNTY	PER SQ MILE
	$	$	$	$	$	$	$	$
1955	——	6103.64	6103 64	——	4 61	4 61	6103 64	4 61
1956	1165.12	953 28	2118.40	0 88	0 72	1 60	1991 30	1 50
1957	966 52	913.56	1880 08	0 73	0 69	1 42	1748.47	1.32
1958	807.64	873 84	1681 48	0 61	0 66	1 27	1563.78	1.18

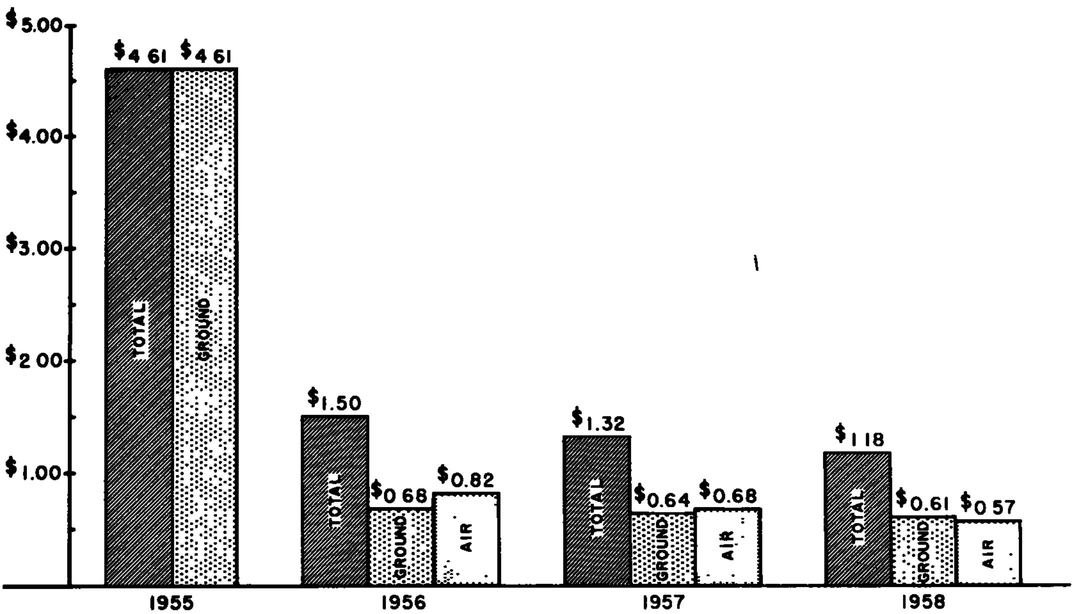


Figure 8.

It is the opinion of the State that the field inventory costs will be reduced further during subsequent reinventory operation.

FUTURE PLANS

It is planned that the reinventory operations will be continued on a 3-year cycle basis. The 3-year cycle is considered a practical one for efficient inventory and mapping operations, and the extensive use made of the maps makes this 3-year period desirable.

Every effort is being made to transfer ground crew activities to the air crew. In the past the ground crew has inventoried all incorporated places primarily to obtain official corporate limit information. New methods of obtaining reliable corporate limit information by other than a direct visit are under consideration. Also under consideration is the purchase of a camera and appropriate enlarging, developing, and reproduction equipment to permit direct photographing of areas having relatively heavy cultural development. It now appears that within a short time all incorporated places under 500 population, which represents about 70 percent of all incorporated places in North Dakota, will be inventoried by aerial inventory methods without assistance from a ground field crew.

It is expected that before long the ground crew activities will be limited to:

1. City street information in cities over 500 population.
2. The measurement and typing of structures.

SUMMARY AND CONCLUSIONS

1. North Dakota is, in general a prairie state. Its terrain varies from flat to rolling with some rough badlands areas. This type of terrain and the sparsely settled areas are ideal for aerial inventory operations. It is believed that the aerial methods would work well in a large portion of the U.S.

2. The aerial inventory methods in North Dakota have produced better inventory data in much less time and at much less cost than the conventional methods. The average inventory costs by the aerial methods are approximately 25 percent of the conventional method costs.

3. In North Dakota where there is a snow cover during three to five months of most years, a 3-year inventory cycle permits an optimum balance between the inventory and mapping operations for a permanent crew. Under this method of operation each county general highway map is current every three years.

4. Three years experience with the aerial inventory methods have convinced us of its merits.

North Dakota only has scratched the surface in the use of aerial methods for inventory. Further experience undoubtedly will produce many refinements and improvements.

A Photogrammetric Approach to Highway Route Location and Reconnaissance

ARTHUR C. QUINNELL, Location Engineer, Montana State Highway Commission

● AS HAS BEEN the case in many western states, Montana's roadways grew up along easily traversable routes which usually meant river valleys. As a result, most major highways are located within the confines of rugged terrain, along rivers and through the higher-valued farming lands. Major population concentrations are also located along these routes as well as most major industry. Due to these factors and many others, the most important of which are economic barriers, great care must be taken by the highway engineer in selecting a location for a new highway. Very thorough and fairly detailed studies must be made, and, due to the greatly increased pace in highway programming and construction, these studies must be accomplished in the shortest period of time.

Available data in the form of PMA and Forest Service photography at scales of 1:20,000 and 1:63,000, U.S. Geological Survey quadrangle maps, and planimetric maps made by photographic projection were used previously as guides to highway reconnaissance. These data are invaluable if utilized correctly. However, they are very limited in scope since no accurate comparisons of alternate routes can be made. In very rugged terrain, where a minor horizontal or vertical projection might result in a double construction cost, these data are not accurate enough and do not provide an acceptable base for comparing alternate routes.

The general problem, keeping in mind the aforementioned factors, would be to utilize all available information, and, in addition, make the most economic use of various additional photogrammetric processes and equipment, which will result in a speedy, accurate and economical method of highway reconnaissance and location, as well as provide an accurate base by which alternate routes may be compared.

If utilized properly, U. S. Geological photography and mapping and planimetric maps, together with traffic data and a good understanding of the objectives of a highway with respect to land use, topography, economy and development typical to the respective locality, a reliable and economical method of making reconnaissance of broad areas for route possibilities is established. Necessarily, other problems, such as recency of photography and mapping and possible future changes in types of development, are taken into consideration. However, it has been found that, due to various circumstances, typical only to Montana, these other problems are very minor in all but a few instances.

By thoroughly reviewing the foregoing factors and information relative to many route possibilities, it is entirely possible to delete all but a few alternates and, in many instances, only one location is evident.

Once several route alternatives have been chosen, the inevitable assignment of the highway engineer to compare and choose the best route arises. Along with this assignment, the questions arise as to the method to be used, the economic aspects of the method chosen, the rapidity of the method and the accuracy. The first three of these considerations may be grouped and discussed as one item, since they tend to be contingent upon each other. The method utilized should assure the utmost in economy, and, due to the rapidly accelerated highway program, should be very speedy. As has been found throughout the highway engineering profession, the fastest, most economic methods of ascertaining data required in comparing route alternates is encompassed in photogrammetry. The methods of utilizing photogrammetry in highway reconnaissance and location differ widely throughout the country. However, the following two methods are generally used:

1. Strip Photography, at a scale such as 1:12,000, is acquired of the alternate routes. By measuring parallax, using a parallax bar or other measuring device, fairly accurate differences in elevation may be computed and utilized in planning gradients

and profile. In this manner the routes may be compared as to physical features as well as approximate over-all costs. After a route has been chosen, the final route may be set by topographic maps made photogrammetrically or by field survey

2. Strip photography at an appropriate scale is acquired of the alternate routes. Topographic maps are prepared from this photography for use in route location, laying grades, computing earthwork quantities for comparisons and arriving at a final location.

TABLE 1

Item	Photogrammetric Method	Field Survey Method
Length	4.06 mi	4.07 mi
Excavation	1,148,667 cu yd	1,144,505 cu yd
Embankment	997,194 cu yd	1,028,505 cu yd
Time of completion	36 man days	189 man days
Vertical accuracy	± 5 ft	*
Maximum gradient	+ 1.8 percent	+ 1 86 percent

* Use as basis of comparison unless tabulated.

There are both advantages and disadvantages to both the above methods. The first method may be speedy and very economical. However, when applied in some of the very rugged terrain typical to more than one-half of Montana, it was found that in many cases where the alternates to be compared were located very close together, the method was not accurate enough. In one case in particular, a horizontal shift of 100 ft entailed approximately three-quarter million cubic yards of additional rock excavation in one cut alone.

In the second method it was found that the required accuracy was attainable; but it cost too much and the time consumed was considerably more than could be allowed in many instances.

If a method could be devised that would utilize the good features of both methods generally outlined above and do away with the bad features, it would be more applicable, subject to governing conditions perhaps typical only to Montana. The desire was to utilize a method or reconnaissance of area and determination of route possibilities, as well as accurately comparing alternate routes and selecting the best one. The following

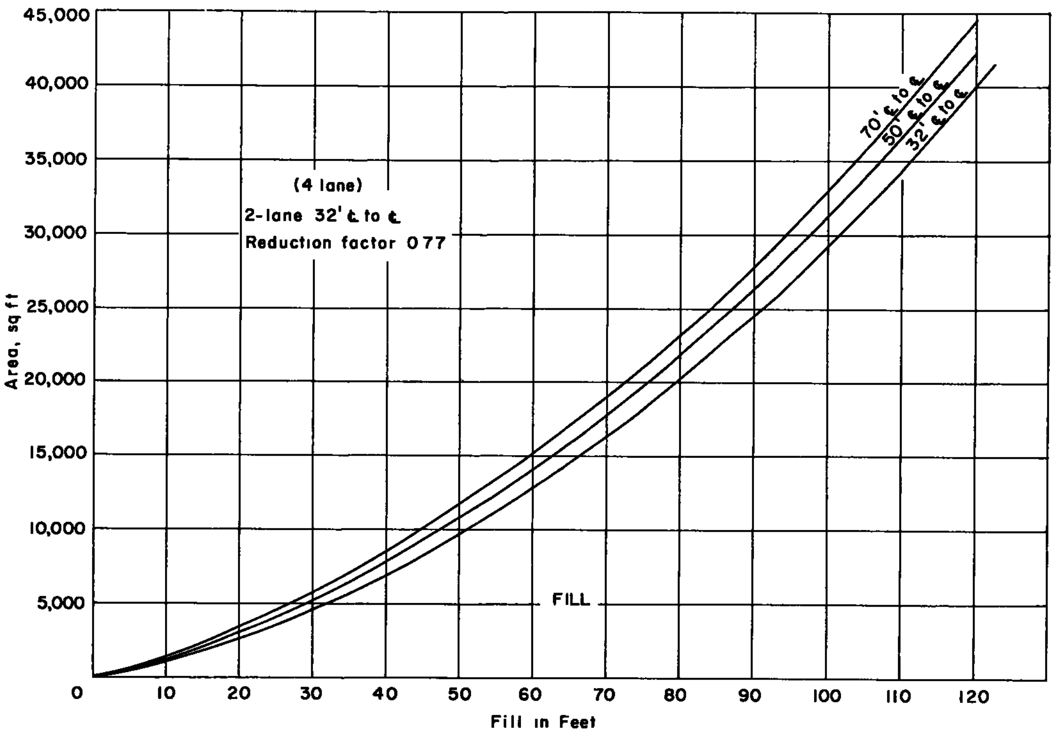


Figure 1.

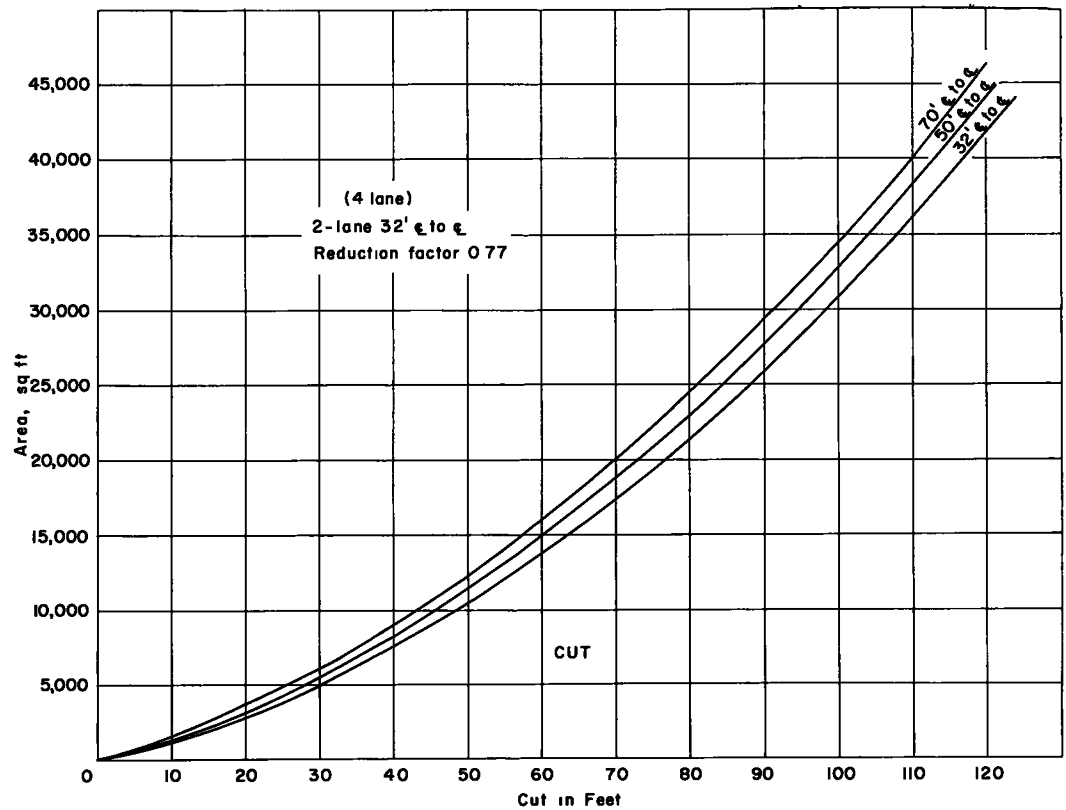


Figure 2.

outline is descriptive of the method now being used in Montana:

Step 1. Reconnaissance of area and determination of route possibilities utilizing:

(a) U. S. Geological Survey and Forest Service Photography at 1:20, 000 to 1:63, 000 scale.

(b) U. S. Geological Survey Quadrangle maps.

(c) Planimetric maps.

Step 2. Photographing route possibilities selected in Step 1 at a scale of 1:12, 000.

Step 3. Compare route possibilities by mirror stereoscope and parallax methods.

Step 4. Compare final alternates by actual cost estimates:

(a) Of entire project where required.

(b) Of problem sections of project where a high degree of accuracy is required (as is the case in most instances). This is accomplished by means of a small economical contact-print type stereoplotter, acquiring ground control from highway construction plans, U. S. Geological Survey maps, and previously mapped areas.

As a case in point, an Interstate Project located in central Montana, is illustrative of this method.

The project is 10. 26 miles in length, lies generally along the Yellowstone River, traverses land from highly productive irrigated types to rugged mountainous terrain and included four major structures.

Seven route possibilities were established from mosaics made of PMA photography at a scale of 1:20, 000. Through judicious use of U. S. Geological Survey quadrangle maps and knowledge of the land use and development, four of these possibilities were selected as alternates. After photographing at a scale of 1:12,000, reviewing stereoscopically and obtaining gradients by measuring parallax, two of the alternate routes remained equal as far as could be ascertained. Preliminary cost estimates were, for all practical purposes, equal. It would appear that either of these two routes could be chosen, designed and constructed to accomplish the same result. However, some questions remained, and, to answer them, a more accurate and detailed study was required. The crux of the situation lay in a 4-mi section located approximately in the center of the project. Accurate excavation quantities were required, since all excavation was classified as varying from 50 to 100 percent rock. Since the acquisition of cross-sections is too time-consuming a process for these studies, a method was devised utilizing an accurate ground profile and grade line which is considerably faster. The ground profile was plotted utilizing a contact-print type stereoplotter. The accuracy attainable is best illustrated by the comparison, tabulated in Table 1, between the photogrammetric and field methods of the final location. By choosing a centerline cut or fill from the profile and grade line and interpreting cross-sectional area from graphs such as those in Figures 1 and 2, an accurate and speedy means of obtaining earthwork quantities is possible.

As well as being useful in preparing more accurate data for cost comparisons, the contact-print type stereoplotter has been used successfully in preparing design topographical mapping over limited areas. The final location selected by this method resulted in an approximate savings of $230,000.00. This savings was not evident until application of this more accurate method of acquiring data for cost comparisons.

Photogrammetry in Highway Planning

DAVID S. JOHNSON, Assistant Chief (Engineer of Planning), Connecticut State Highway Department

Photogrammetry has two main fields of usefulness in highway planning—preliminary engineering and public relations.

In preliminary engineering, which in Connecticut is a function of planning, photogrammetry is used for determining and weighing alternate lines, and for transmitting the recommended schematic layout for processing through the survey and design stages.

Furthermore, it is used as a base for portraying proposed highway improvements at the public hearings required by federal and state statutes.

● ODDLY enough, the subject of this paper, "Photogrammetry in Highway Planning," makes more sense to the layman than to the professional. That is because most professionals in this field place a different meaning on the word "planning" than does the layman, and the layman's usage is the correct one. The term "planning", as generally used today, came into the highway engineer's lexicon through the State-Wide Highway Planning Surveys initiated by the U.S. Bureau of Public Roads in the late thirties as cooperative ventures with the several states. The planning of these surveys was the gathering of data, which provides the frame of reference for planning, but it was no more planning in itself than the site, which provides a frame of reference for a building, is the building itself.

The activities commonly and erroneously thought of as planning are, in Connecticut, carried on by the Records and Statistics Section which for many years was under the jurisdiction of the Bureau of Business Administration. These activities include gathering the data for recording the physical configuration, or structural arrangement of all road sections on the state system; type and width of pavement and of shoulders, sight distances, alignment, gradient, superelevation, etc. Also forming part of the inventory are data relating to bridge clearances and waterway areas, railroad-highway intersections, whether at grade or separated, school bus and mail routes, etc. Records and Statistics also keeps so-called road life records which are merely the historical data relating to the various road sections. Loadometer and other data-gathering surveys are made by this section which, in the main, has service rather than creative functions. There does not seem to be much need, or even any great area of usefulness, for photogrammetry in the accumulation of statistics.

A highway is an area reality, not a lineal abstraction. It is the recognition of this fact and its translation into the answers for "what, where and how" that constitutes planning. One of the most important aids, from the initial general "look-see" at the area of concern to the completion of the final schematic scaled layout for the survey and design of a particular project, is photogrammetry.

Connecticut is participating, on a continuing basis, with the U.S. Geological Survey in the cost of preparing the USGS quadrangles for the state. Formerly these were based on plane table mapping but in more recent years on aerial photogrammetry. About half of Connecticut's quadrangles are photogrammetric. With the passage of time, and the requirements for revision on a 5-year cycle, the entire state will be covered by maps produced photogrammetrically. These USGS quadrangles are the "work horses" of planning.

The major planning studies currently being made by the Connecticut Highway Department deal with expressway projects. A typical planning study begins with the examination of the area of concern on USGS mapping. This office examination is, of course, supplemented by field investigations. An expressway study is not treated as a lineal problem between two points, but rather, as an area service problem. It is possible to delimit traffic drainage sheds and arrive at likely locations for roadside traffic interview stations, thus programming the various traffic studies largely on the basis of USGS mapping.

This USGS mapping also is useful as both a source and a mapping base for land use information, a necessary factor, along with traffic data, in highway planning.

After, or along with, the compilation of the traffic and land use data relating to a highway project, various alternate locations are studied on USGS mapping to determine feasible routes and the limits of the area in which the new expressway will be located. The desirable scale for the photogrammetric coverage (usually 200 ft to the inch with 5-ft contours) is decided upon and the map sheets for photogrammetry are laid out on USGS mapping and form the basis for dealing with the photogrammetric contractor.

When the preliminary photogrammetric map sheets are received the project engineer proceeds to transfer to them the various alternates, first laid out on the USGS mapping. The better mapping reveals the undesirability, or impossibility, of some part, or all, of various lines and the possibility, or the necessity, of adjusting and refining the others. Also, he is usually able to add new alternates for study to replace those he has been forced to discard.

Eventually there result from two to five or more alternates on photogrammetry. These alternates are developed to fully engineered and scaled schematic layouts on the photogrammetry. Centerlines are shifted around until the most acceptable profile appears; the possibility of separated roadways in likely areas is investigated and decision reached, conformity with alignment and gradient criteria can now be assured; interchanges are engineered to scale and the treatment of intersected roads shown. It is now possible to make comparable cost estimates for both right-or-way and construction for the several alternates which, used in conjunction with the traffic operations data assembled by the project engineer, permit the computation of benefit-cost ratios.

After the several alternates are developed they go through channels for review and decision. The atlas containing the several alternate layouts for Interstate 91, between New Haven and Meriden, had 32 full photogrammetric map sheets. Figure 1 shows a section of the index map for this atlas and the wide area of necessary coverage. The tendency would have been to skimp, had old line methods of mapping been employed.

Figure 2 is an interchange developed on 200-scale photogrammetry for the intersection of Connecticut 8 with Interstate 84 in Waterbury. If the mapping had been obtained by old line methods, it is highly unlikely that the coverage would have been wide enough for a satisfactory solution. The problems were:

1. Full interchange had to be provided between two expressways and between the expressways and the local street system.
2. The traffic volumes were very heavy.
3. The heavily industrialized area to the east of the Naugatuck River had to be spared.
4. The Naugatuck River itself; and
5. The area available to the west of the river was steeply sloping.

The highway planner is working in three dimensions, as is clearly evident, and contoured photogrammetry most conveniently provides the base he needs.

The photogrammetry surcharged with the carefully engineered schematic layouts for the various alternates then serves as a basis for discussion of the several alternates: (a) with the several levels of decision within the department, (b) with other affected agencies of the state, (c) with local public and/or semi-public agencies, and (d) with the Bureau of Public Roads. After study of the several alternates, not only from the strictly engineering and cost standpoints but also from the standpoint of effect on the traversed communities and on local area planning objectives, decision is reached as to the general line location.

The chosen alternate is carefully refined to provide as complete a guide as possible for the ground survey and design at 40 scale. The survey is usually the transit and tape variety, although it seems probable that photogrammetry will claim an increasing share of this field. Figure 3 is a typical planning map completed by the Planning Division and forwarded for survey and design. It is a section of sheet No. 1 of 3 for the relocation of route Connecticut 12 in Killingly. The centerline for the expressway is shown by a single heavy line. Where separated roadways are specified by planning

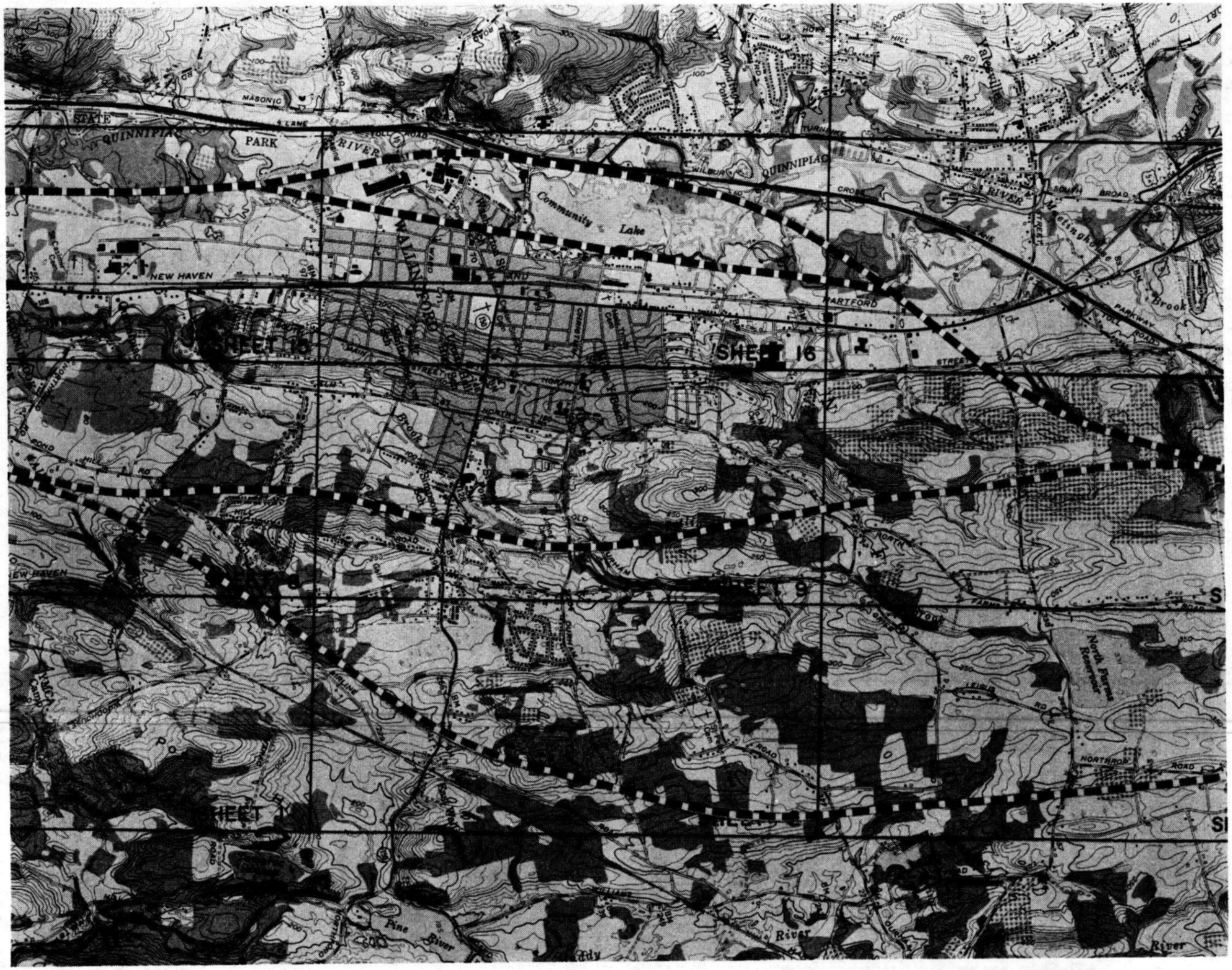

STATE
QUINNIPIAC
PARK
RIVER
MASONIC
Community
Lake
NEW HAVEN
HARTFORD
SHEET 15
SHEET 16
SHEET 9
PARKWAY
Brook
North Farms Reservoir
NORTHROP
ROAD
NEW HAVEN
River

Figure 1.

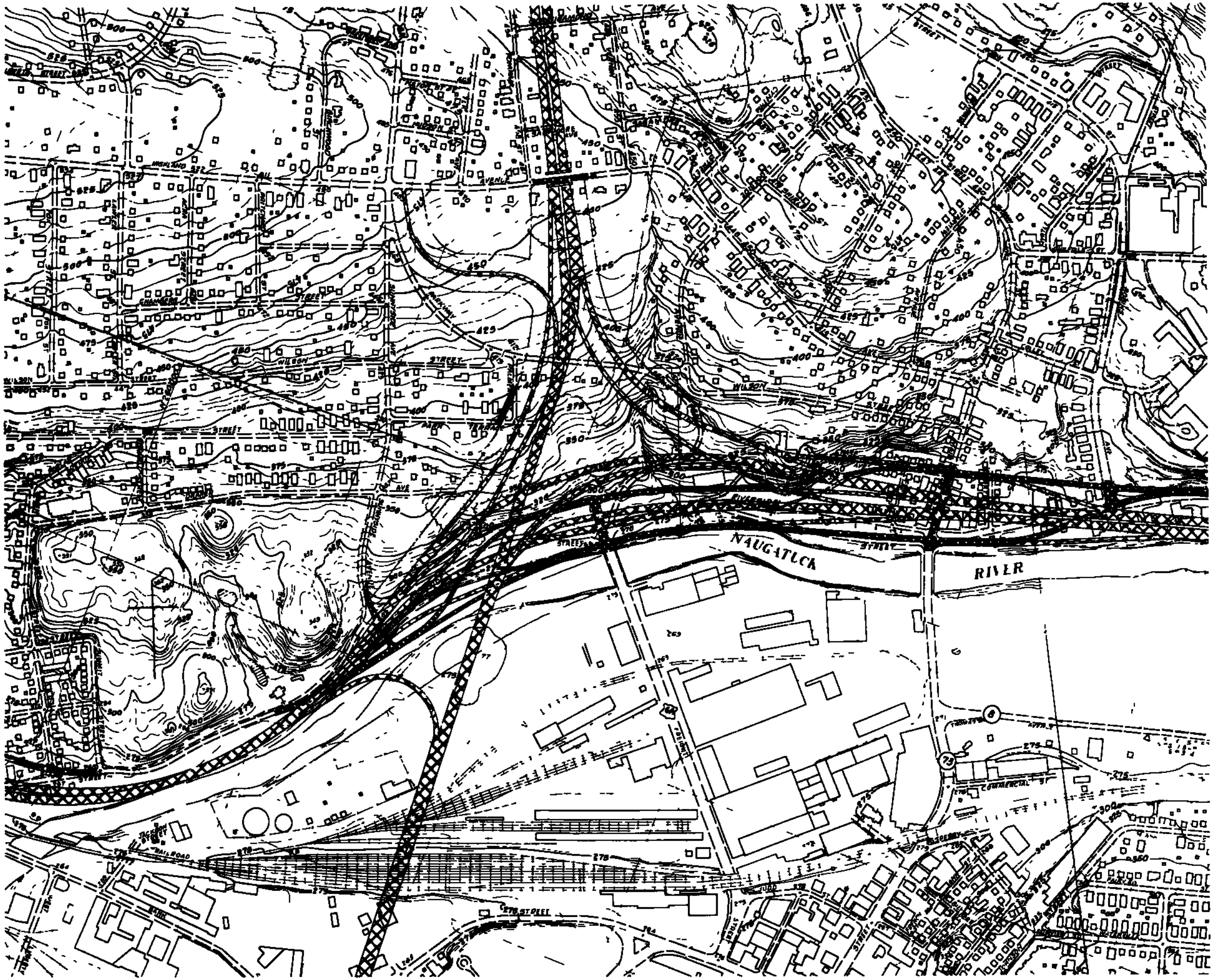


Figure 2.

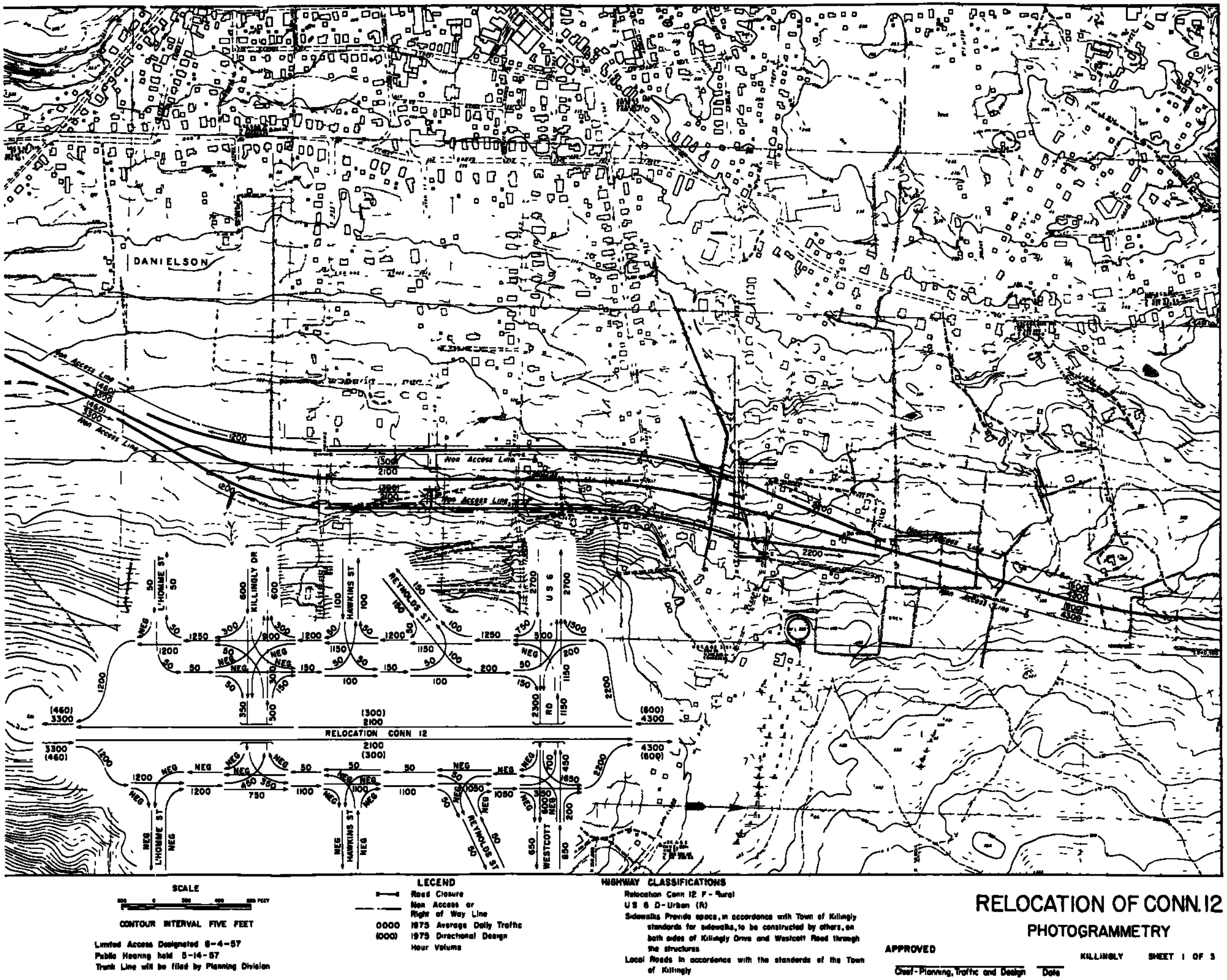

DANIELSON
Non Access Line
RELOCATION CONN 12
L'HOMME ST
KILLINGLY DR
HAWKINS ST
REYNOLDS ST
US 6
WESTCOTT RD
SCALE
CONTOUR INTERVAL FIVE FEET
Limited Access Designated 6-4-57
Public Hearing held 5-14-57
Trunk Line will be filed by Planning Division
LEGEND
Road Closure
Non Access or Right of Way Line
0000 1975 Average Daily Traffic
(000) 1975 Directional Design Hour Volume
HIGHWAY CLASSIFICATIONS
Relocation Conn 12 F - Rural
U S 6 D - Urban (R)
Sidewalks Provide space, in accordance with Town of Killingly standards for sidewalks, to be constructed by others, on both sides of Killingly Drive and Westcott Road through the structures
Local Roads in accordance with the standards of the Town of Killingly
RELOCATION OF CONN. 12
PHOTOGRAMMETRY
APPROVED
Chief - Planning, Traffic and Design
Date
KILLINGLY
SHEET 1 OF 3

Figure 3.

the centerlines for both roadways are shown. Interchange ramp centerlines are shown, and the positioning of the bridge rails indicates whether planning intends an intersecting road to overpass or underpass the expressway. Frontage roads, road closures, cul-de-sacs, and non-access lines are also shown. Traffic volumes for the design target year, both as ADT's and DDHV's, where necessary, are shown both on the line portrayal and on separate traffic diagrams on those portions of the sheet where the base coverage is expendable. In the 3-in. margin at the bottom of the sheet are notes advising of the status of the limited-access designation, trunkline filing and the public hearing. Also noted, are the highway classifications applicable to all of the roads involved in the proposed improvement on this map sheet. Other information may be included, such as sidewalk calls, commitments, and special notes. Also, each map sheet is approved by the Chief of Planning, Traffic and Design. The intent is to give the Surveys Section all the information necessary to make the ground survey and to give the Design Division all the information necessary for agreements with local bodies and to prepare the contract drawings. The planning maps do not, of course, include any drainage information or directives.

The trunkline filing maps, necessary for legal reasons, are filed with the local municipal clerks, after completion of the planning stage, on photogrammetry used as the base mapping.

Photogrammetry in the field of planning is particularly useful in public relations. Of all phases of highway engineering, planning is most affected by, and responsive to, public opinion. Both state and federal statutes require the holding of public hearings for all but the routine type of highway improvement. These hearings are generally held in each town traversed by the proposed improvement. Although, in rare cases, one hearing may serve more than one town, in no case are county lines crossed.

A few weeks before the date of the public hearing a set of the planning maps is sent to the municipal officials showing the proposed highway improvement through that town, for public display in the town or city hall. The right-of-way requirements, between the non-access lines, are colored with a yellow wash on this set of prints. As the map base of these planning layouts is photogrammetric, townspeople inspecting the display can see if they are affected. Also, the relationship of the proposal to the existing street system is readily apparent, even to the layman.

Figure 4 shows a photogrammetric-based display map such as used at public hearings. For some hearings various types of data charts may also be used to bring out pertinent points, but generally these map sheets are the only displays, and in all cases, they are the "heart" of the hearing. A series of such map sheets, from one end of the included area to the other, is mounted on easels and the departmental spokesman describes, on the basis of these maps, the entire improvement and answers all questions from the floor. After the formal part of the hearing people come up to the display maps and search out their homes or businesses and ask informal questions with reference to the map to indicate their area of concern. Figure 4 shows a section of Interstate 84 at the Danbury-Bethel town line. Proposed new construction was colored red, as were also those sections of the local system which will be rebuilt. Existing roads, not to be rebuilt, were shown in black. Existing US 6 has been split into two one-way frontage roads on either side of Interstate 84 and the addition of ramps has produced a basket-type interchange.

Figure 5 shows another type of portrayal of hearings information on a photogrammetric map base. This is the same project as in Figure 3. For the purpose of presentation at the hearing that same sheet was rendered in color to make the improvement clear to the audience. Yellow wash was used to delineate the areas required in the right-of-way for the expressway; red wash, the right-of-way requirements of related improvements that will be constructed by the state and deeded to the town, such as the frontage roads to be constructed initially on either side of the expressway between L'Homme Street and Westcott Road. A brown wash was used to show the relationship of, and adjustment in, streets of the existing road network in the area. The severance and preservation of local streets in the vicinity of the frontage roads tells the audience, and the hearings official in his presentation to the audience, the effect of the expressway on this neighborhood. Local landmarks, such as the standpipe north of Stearns Street, were also colored to aid in orientation.


FOUNDATION
FOUNDATION
435
450
427
425
375
435
415
MT VIEW
TERRACE
RD
400
385
700
3550
4650
7000
1100
2350
3350
425
450
365
355
1100
3900
350
375
550
3950
340
1350
3300
342
325
315
350
365
420,000
375
418,000
300
390
700
330
326
479

Figure 4.

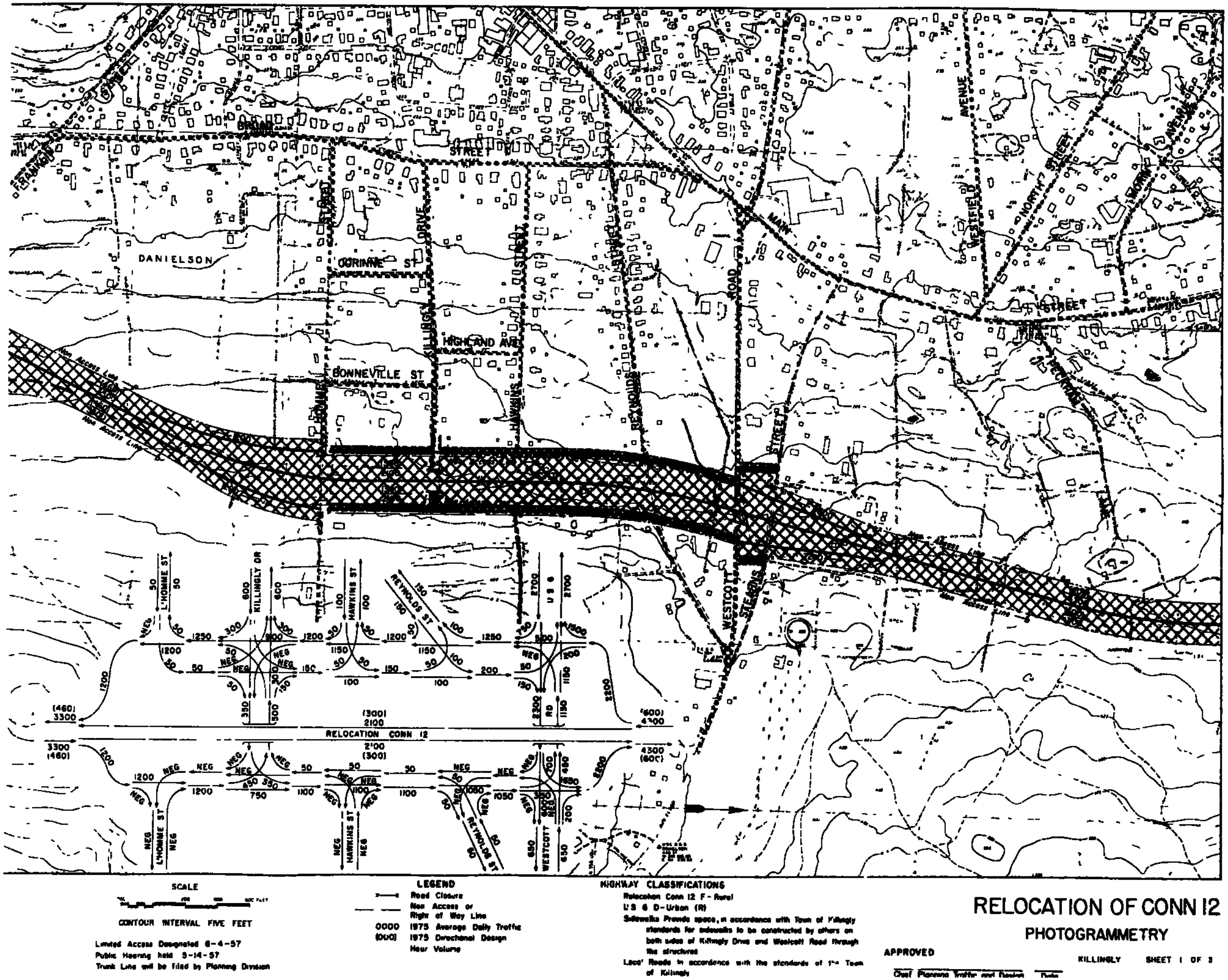

DANIELSON
BROAD STREET
CORINNE ST
HIGHLAND AVE
BONNEVILLE ST
KILLINGLY DRIVE
HAWKINS ST
REYNOLDS ST
WESTCOTT ROAD
STEARNS STREET
MAIN STREET
WESTFIELD AVENUE
NORTH STREET
RELOCATION CONN 12
L'HOMME ST
KILLINGLY DR
US 6
SCALE
CONTOUR INTERVAL FIVE FEET
Limited Access Designated 6-4-57
Public Hearing held 5-14-57
Trunk Line will be filed by Planning Division
LEGEND
Road Closure
Non Access or Right of Way Line
0000 1975 Average Daily Traffic
(000) 1975 Directional Design Hour Volume
HIGHWAY CLASSIFICATIONS
Relocation Conn 12 F - Rural
US 6 D-Urban (R)
Sidewalks Provide space, in accordance with Town of Killingly standards for sidewalks to be constructed by others on both sides of Killingly Drive and Westcott Road through the structures
Local Roads in accordance with the standards of the Town of Killingly
RELOCATION OF CONN 12
PHOTOGRAMMETRY
APPROVED
KILLINGLY SHEET 1 OF 3

Figure 5.

With such a presentation, it is evident that highway planning has now matured to an independent discipline in which purely technical considerations form only a part of the pertinent criteria. It is not a coincidence that this maturing of highway planning as a discipline coincides so closely in time with the use of photogrammetry as a planning tool. It is even doubtful that this maturity would have come about without the aid of a quick, low-manpower, low-cost method of mapping coverage, such as that provided by photogrammetry.

Adjustment of Photogrammetric Surveys

L. L. FUNK, Photogrammetric Engineer, California Division of Highways

PHOTOGRAMMETRIC surveys are subject to systematic errors, variable in size and difficult to eliminate. Such errors, if they fail to compensate, can cause serious discrepancies in earthwork quantities even though the surveys may comply with mapping specifications and with National Map Accuracy Standards. A previously reported experimental project (4) by the California Division of Highways indicated that the accuracy of earthwork quantities could be greatly improved by adjusting photogrammetric surveys to an accurate field profile.

To test this method under actual field conditions three sections of photogrammetric mapping, totaling 10.7 mi in length, from three construction projects were selected for study. Conditions on all of the projects were ideal for photogrammetric mapping with ground cover being almost negligible in each case. Data concerning the accuracies of the mapping, as measured by field profiles, are shown in Figures 1, 3, and 5. The method used in analyzing map accuracy has been discussed in a previous article (3).

In each case earthwork quantities for design and advertising the construction contracts had been obtained by taking terrain cross-section notes from the 2-ft interval contour maps. The terrain notes and corresponding roadbed notes were then processed by electronic computers. Field cross-sections for determining pay quantities of roadway excavation had been taken as the projects were slope staked for construction.

The field cross-sections were taken either with an engineer's level or by reading vertical angles with a transit. Right angles were determined with a 90 deg prism for the cross-sections taken with a level. A few individual points on the project on IX-Mno-23H were read with a hand level. While no definite statement can be made as to the absolute accuracy of the field surveys on these projects, it is believed they are slightly less accurate than the F1 survey of the experimental section (4) but somewhat better than the accuracy of the F2 survey.

EXCAVATION AND EMBANKMENT QUANTITIES

In developing comparisons of quantities the same stations were used for cross-sections from both field and photogrammetric surveys. In general, the cross-section interval was 50 ft. Results were screened for large, obvious blunders in the area of individual cross-sections. Adjustments of the photogrammetric surveys were made by raising or lowering the entire terrain at each cross-section by an amount equal to the difference in elevation from the field survey at centerline.

For purposes of comparison the three projects were divided into 10 segments each approximately 1 mi in length. Differences between field and photogrammetric survey quantities, both before and after adjustment, are shown in Table 1. The differences in excavation quantities, before adjustment, for the 10 segments ranged from 0.3 percent to 5.4 percent with an average of 2.5 percent. After adjustment the differences ranged from 0.0 percent to 1.8 percent with an average of 0.5 percent. Corresponding differences for embankment quantities were 0.9 to 9.7 percent with an average of 3.1 percent before adjustment and 0.1 to 1.8 percent with an average of 0.6 percent after adjustment.

Difficulties have been previously encountered on several projects where large localized errors in photogrammetric surveys caused serious imbalance in earthework quantities. These occured even though the projects as a whole balanced fairly well. Comparisons were therefore developed to determine the effect of the adjustments on 14 individual cuts and fills which showed serious differences between field and photogrammetric survey quantities. These comparisons are shown in Table 2. It will be noted that the differences, before adjustment, ranged from 0.7 percent to 10.2 percent with an average of 5.2 percent. Adjustment of the photogrammetric surveys to a field profile reduced these differences to a range of from 0.0 to 1.3 percent with an average of

TABLE 1

COMPARISON OF EARTHWORK QUANTITIES FROM FIELD AND PHOTOGRAMMETRIC SURVEYS

Project Sta. to Sta.	Excavation: Field Survey Quantity (cu yd)	Excavation: Photogrammetric Survey Difference (cu yd) Before Adjust.	Excavation: Photogrammetric Survey Difference (cu yd) After Adjust.	Excavation: Photogrammetric Survey Difference (%) Before Adjust.	Excavation: Photogrammetric Survey Difference (%) After Adjust.	Embankment: Field Survey Quantity (cu yd)	Embankment: Photogrammetric Survey Difference (cu yd) Before Adjust.	Embankment: Photogrammetric Survey Difference (cu yd) After Adjust.	Embankment: Photogrammetric Survey Difference (%) Before Adjust	Embankment: Photogrammetric Survey Difference (%) After Adjust.
ASC 188-IX Mno 23H										
283 to 340	50,121	+2,016	+ 887	4 0	1 8	44,039	-4,398	-145	9.7	0 3
340 to 390	87,661	-2,638	+ 369	3.0	0.4	67,894	-3,209	+481	4 7	0 7
390 to 440	15,658	- 667	+ 9	4.3	0.1	27,960	+1,671	+192	6.0	0.7
440 to 491	81,024	+4,123	+1,145	5.1	1.4	86,657	+2,281	-585	2.6	0.7
Total	234,464	+2,834	+2,410	1 2	1.0	226,550	-3,655	- 57	1.6	0.0
ASC 135-V SLO 33B										
92 to 150	83,535	+ 316	- 5	0.4	0.0	130,356	-2,175	-866	1.7	0.7
150 to 208	136,321	+7,306	- 237	5.4	0 2	23,120	- 447	+411	1.9	1 8
Total	219,856	+7,622	- 242	3.5	0.1	153,476	-2,622	-455	1.7	0.3
ASC 192-VI Ker 58D										
220 to 280	303,664	+2,744	+2,060	0.9	0 7	356,359	-5,006	+535	1.4	0.2
280 to 330	356,445	+1,230	+ 145	0.3	0.0	397,553	+3,500	+867	0.9	0.2
330 to 390	586,841	+4,326	+4,073	0.7	0.7	1,251,489	+14,371	+851	1.1	0.1
390 to 460	977,477	+10,690	-1,183	1.1	0.1	752,711	-7,172	+2,042	1.0	0.3
Total	2,224,427	+18,990	+5,095	0.9	0.2	2,758,112	+5,693	+4,295	0 2	0.2

0.5 percent. The wide variation in the arithmetic mean of the centerline profile in these 14 cuts and fills illustrates the variability of systematic errors in the mapping.

COMPARISONS OF TOTAL DIFFERENCES

The effects of adjustment on the total differences and the equivalent vertical differences for the ten 1-mi segments are shown in Table 3. The total difference is the difference in cubic yards between the terrain as depicted by the contour maps and the terrain as developed by the field survey. For a project designed for balanced cut and fill the total difference would, therefore, represent the imbalance caused by errors in the

TABLE 2

EFFECT OF ADJUSTMENT ON LARGE ERRORS IN INDIVIDUAL CUTS AND FILLS

Project Sta. to Sta.	Arithmetic Mean of Centerline Profile (ft)	Field Survey Quantity (cu yd)	Photogrammetric Survey Difference: Cubic Yards Before Adjust.	Cubic Yards After Adjust.	Percent Before Adjust.	Percent After Adjust.
ASC 188-IX Mno 23H						
363 to 378	-0.75	62,891 exc.	4,497	584	7.1	0.9
447 to 460	+0.31	80,648 exc.	4,106	1,017	5.1	1.3
287 to 320	+0.53	36,855 emb.	3,771	125	10.2	0.3
377 to 385	+1.45	46,682 emb.	3,899	382	8.3	0.8
459 to 491	-0.45	48,792 emb.	3,285	651	6.7	1.3
ASC 135-V SLO 33B						
180 to 200	+0.74	120,022 exc.	7,712	171	6.4	0.1
133 to 155	+0 20	113,554 emb.	1,538	55	1.4	0.0
ASC 192-VI Ker 58D						
391 to 416	-0.22	723,816 exc.	5,014	297	0.7	0.0
422 to 430	+0.91	114,805 exc.	6,091	3	5.3	0.0
439 to 449	+1.79	138,373 exc.	10,402	835	7.5	0.6
220 to 230	+1.03	48,012 emb.	4,625	234	9.6	0.5
275 to 288	-0.35	481,148 emb.	4,629	1,016	1 0	0.2
330 to 356	-0.52	657,624 emb	12,789	3,680	1.9	0.6
446 to 460	+1.28	502,201 emb.	8,402	337	1.7	0.1

TABLE 3

RELATION OF MAP ACCURACY TO DIFFERENCES IN EARTHWORK QUANTITIES

Project Sta. to Sta.	Centerline Profile: No of Points	Centerline Profile: Within ½ C. I. (%)	Centerline Profile: Arithmetic Mean (ft)	Centerline Profile: Standard Deviation (ft)	Total Difference (cu yd): Before Adjust.	Total Difference (cu yd): After Adjust.	Equivalent Vertical Dif. (ft): Before Adjust.	Equivalent Vertical Dif. (ft): After Adjust.
ASC 188-IX Mno 23H								
(1) 283 to 340	113	89	+0 30	0. 61	+6, 414	+1, 032	+0. 35	+0. 06
(2) 340 to 390	100	80	+0. 04	0 90	+ 571	- 112	+0 03	-0. 01
(3) 390 to 440	100	93	-0. 15	0. 61	-2, 338	- 183	-0 16	-0. 01
(4) 440 to 491	103	88	-0. 12	0. 68	+1, 842	+1, 730	+0. 09	+0. 09
Total	416	88	+0. 03	0 75	+6, 489	+2, 467	+0. 09	+0. 04
(4A) 440 to 460	40	90	+0. 38		+5, 115	+ 937	+0. 52	+0 10
(4B) 460 to 491	63	87	-0. 43		-3, 273	+ 793	-0. 32	+0. 08
ASC 135-V SLO 33B								
(5) 92 to 150	110	90	+0 05	0 61	+2, 491	+ 861	+0. 11	+0. 04
(6) 150 to 208	110	86	+0. 26	0. 67	+7, 753	- 648	+0. 30	-0. 03
Total	220	88	+0 16	0. 64	+10, 244	+ 213	+0. 21	0. 00
ASC 192-VI Ker 58D								
(7) 220 to 280	118	79	+0. 28	0. 83	+7, 750	+1, 525	+0. 19	+0. 04
(8) 280 to 330	99	86	-0. 05	0. 88	-2, 270	- 722	-0. 07	-0. 02
(9) 330 to 390	123	58	-0. 46	1. 45	-10, 045	+3, 222	-0. 21	+0. 07
(10) 390 to 460	146	62	+0. 44	1. 08	+17, 862	-3, 225	+0. 32	-0 06
Total	486	70	+0. 07	1. 13	+13, 297	+ 800	+0. 08	0. 00

photogrammetric survey. The equivalent vertical difference was calculated by dividing the total difference in cubic feet by the area between the slope stakes in square feet. In effect, it is the mean vertical difference between the average of the terrain as represented by the contour map and the average of the terrain from the field survey. It is, therefore, a one-dimensional variable which is directly related to the difference in earthwork quantities.

It will be noted in Table 3 that the equivalent vertical differences of the ten 1-mi segments ranged from -0.21 to +0.35 ft before adjustment of the photogrammetric surveys. The average (without regard to sign) for these 10 segments was 0.18 ft. The adjustment reduced the equivalent vertical differences to a range of -0.06 to +0.09 ft with an average of 0.04 ft.

To further study the effect of adjustment on imbalance of quantities the total differences were calculated in 1,000-ft

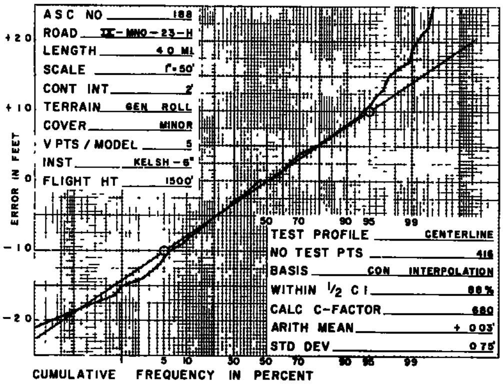


Figure 1

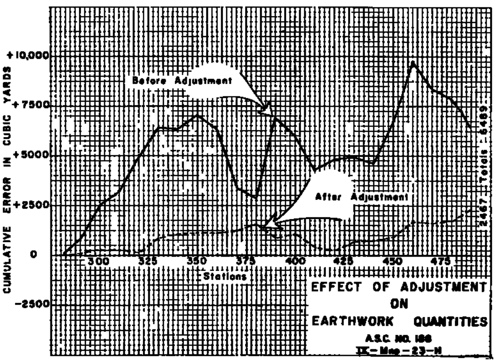


Figure 2

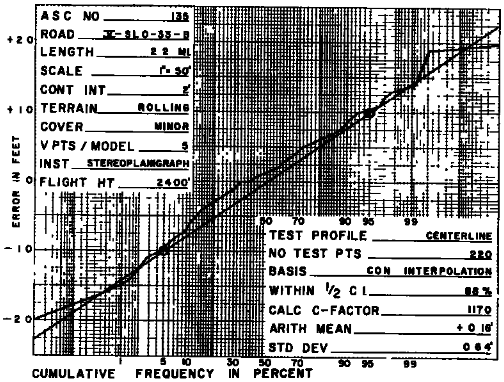


Figure 3

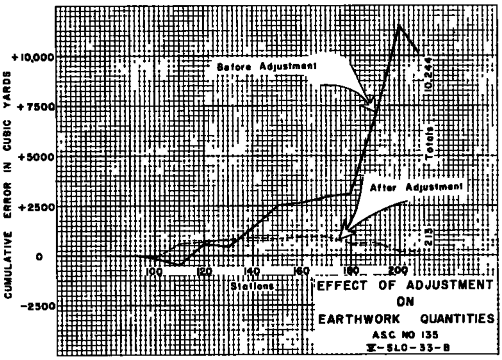


Figure 4

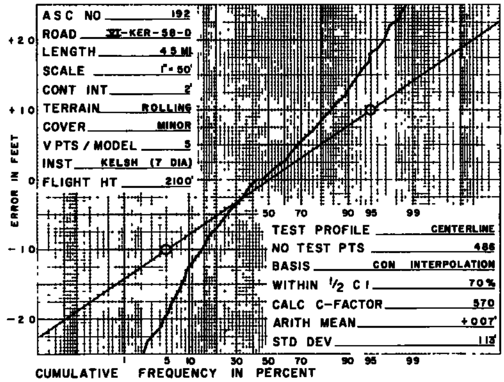


Figure 5

segments for the three projects. Cumulative total errors (differences) before and after adjustment were then plotted as ordinates with centerline stations as abscissae. The resulting curves in Figures 2, 4, and 6 show the imbalance in quantities caused by errors in the photogrammetric surveys. They also illustrate the dampening effect of adjustment of the photogrammetric surveys on errors in earthwork quantities. The only evidence of serious discrepancies in the after adjustment curves is between Stations 330 and 370 on the VI-Ker-58-D project shown in Figure 6. These discrepancies were probably caused by large individual blunders in either the field or photogrammetric surveys.

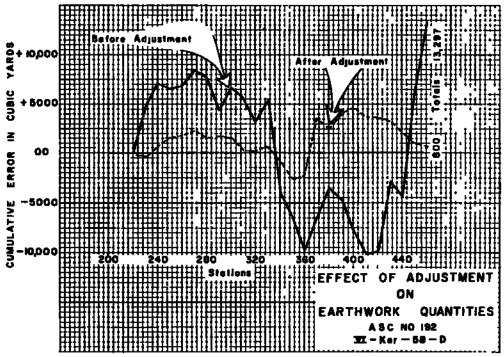


Figure 6

RELATION OF MAP ACCURACY TO EARTHWORK QUANTITIES

The National Map Accuracy Standards are the basis for most photogrammetric mapping specifications. For vertical accuracy the requirement is, in effect, that 90 percent of the points shall be within one-half contour interval of their true elevation. One of the objectives of this study was to determine if this or any specification for map accuracy can be directly related to the resulting accuracy of earthwork quantities.

As previously noted the equivalent vertical difference is a one-dimensional measure of the accuracy of earthwork quantities. The equivalent vertical differences before adjustment and the percentage of points on the centerline profile within one-half contour interval are both shown in Table 3. A comparison for the various segments of the mapping shows little, if any, relation between these two values. For example, the portion of the project on IX-Mno-23-H from Stations 340 to 390 has the lowest percentage of points within one-half contour on this project (80 percent) and also has the lowest equivalent vertical difference (+0.03 ft). Similarly, of the three projects, the one on VI-Ker-58-D has by far the lowest percentage of points within one-half contour interval (70 percent) and also has the lowest equivalent vertical difference (+0.08 ft) as compared to +0.09 ft and +0.21 ft for the other two projects. The mapping on V-SLO-33-B was very good by conventional map accuracy standards, having 88 percent of the points tested within one-half contour interval and a standard deviation of 0.64 ft, and yet the equivalent vertical difference before adjustment of +0.21 ft is the highest of the three projects.

The lack of relationship between National Map Accuracy Standards and accuracy of

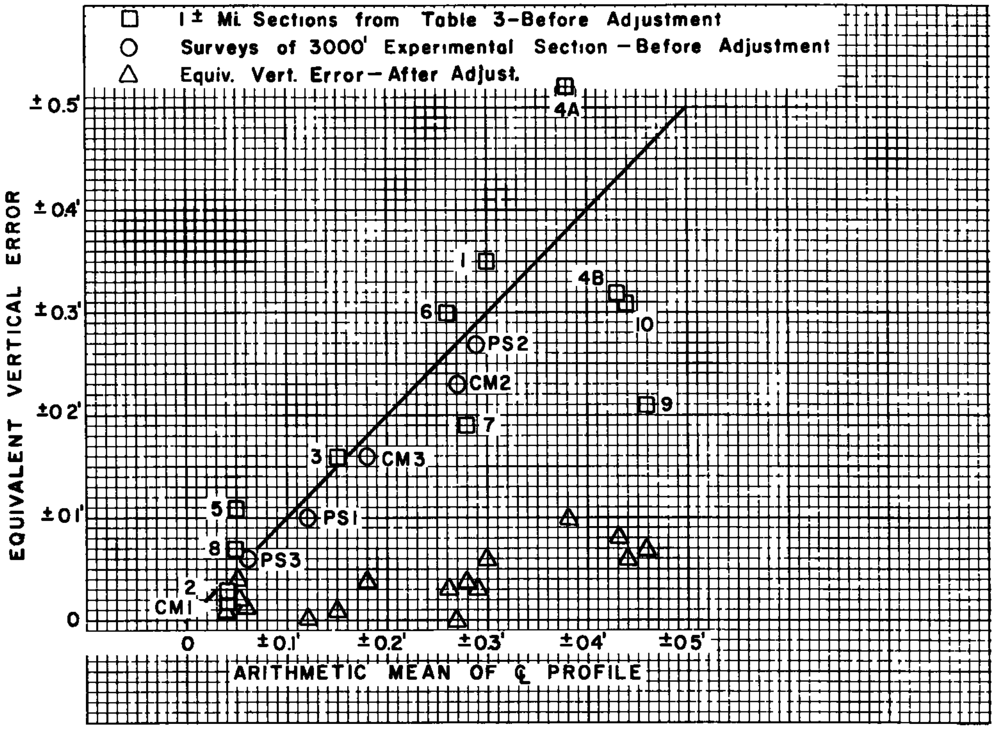


Figure 7

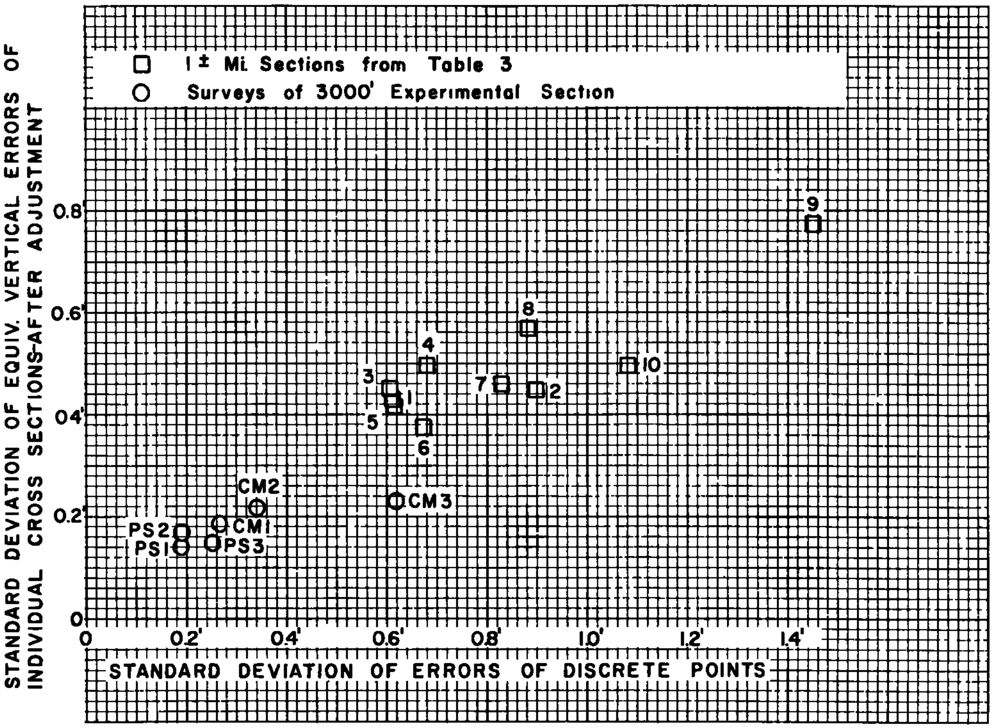


Figure 8

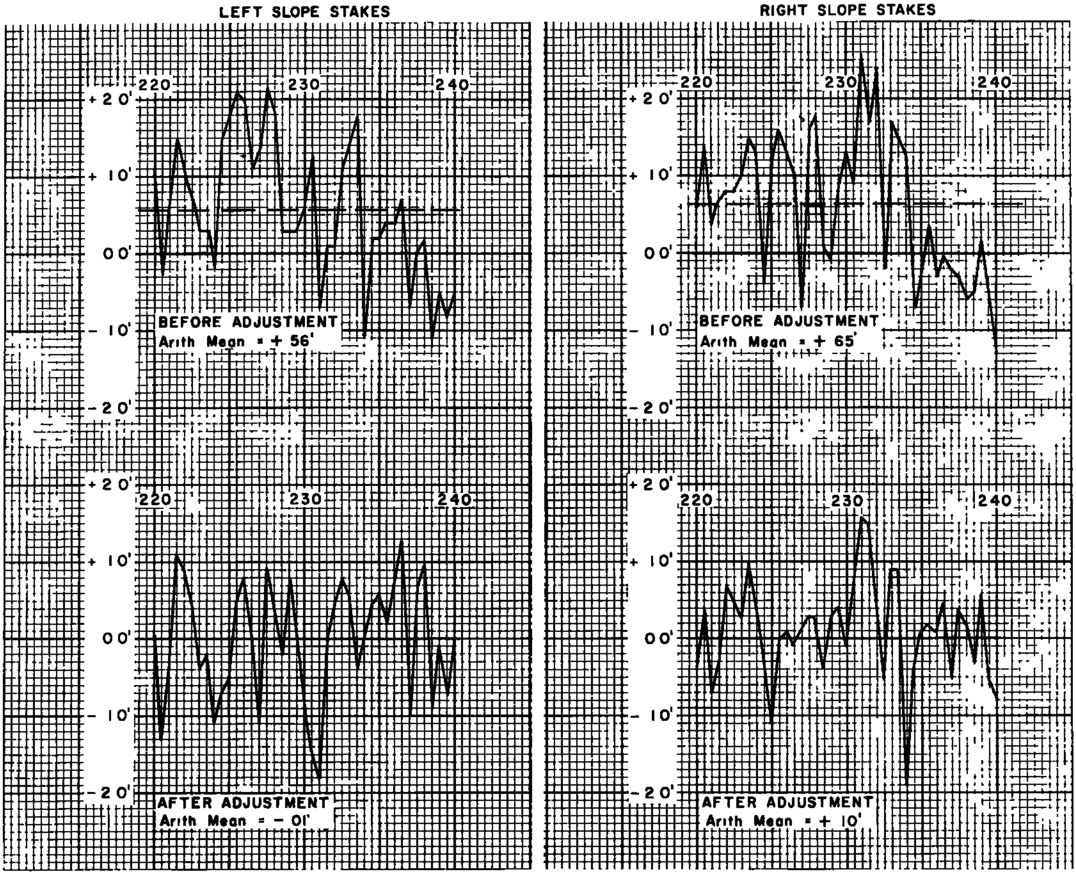


Figure 9. VI-Ker-58-D, effect of adjustment on slope stakes.

earthwork quantities is due, of course, to the serious effect of relatively small systematic errors on earthwork quantities as compared to the relatively minor effect of much larger random errors. This has been previously pointed out by Miller in connection with photogrammetric measurements for earthwork quantity determination (1).

In a previous article (4) the close relationship between the arithmetic mean of a centerline profile and the accuracy of earthwork quantities was noted. In comparing the arithmetic mean of the field profiles with the equivalent vertical differences before adjustment for the 1-mi segments, as shown in Table 3, a similarly close relationship is apparent for 7 of the 10 segments. The exceptions are shown in Lines 4, 9, and 10. For the segments shown in Lines 9 and 10 the less direct relationship is probably due partially to the large variation in width between slope stakes in the rough terrain and partially to blunders in the field and photogrammetric surveys. For the section from Station 440 to Station 491 of IX-Mno-23-H, shown in Line 4, the arithmetic mean of the centerline profile is -0.12 ft, as compared to an equivalent vertical difference of +0.09 ft. As shown by Lines 4A and 4B this apparent discrepancy is caused by averaging 2 segments having widely different systematic errors.

The relationships between the arithmetic mean of the centerline profiles and the equivalent vertical errors (differences) for the various segments of Table 3 and for the six photogrammetric surveys of the experimental section (4) are shown graphically in Figure 7. These data indicate that a field profile will furnish an excellent guide to the probable accuracy of earthwork quantities. They lead to the conclusion that mapping specifications should include a limitation on the arithmetic mean of points tested if the mapping is to be used as a source of terrain data for earthwork quantities.

If adjustment of photogrammetric surveys to a field profile tends to greatly reduce systematic errors the remaining discrepancies in earthwork quantities should be largely due to random errors. In this case the accuracy of earthwork quantities after adjustment should be proportional to the standard deviation of the random errors and inversely proportional to the number of points tested or the number of cross-sections. To determine whether any such relationship could be developed, the equivalent vertical differences for each individual cross-section of the three projects were calculated. The standard deviations of these individual equivalent vertical differences were then computed for each of the ten 1-mi segments. In only three of the ten cases were the equivalent vertical differences after adjustment, as shown in the last column of Table 3, greater than the standard error of the mean for the number of cross-sections involved. In all cases they were well within the limits of a normal distribution.

The standard deviations of the equivalent vertical errors (differences) for the 1-mi segments are shown as ordinates in Figure 8 with the standard deviations of the centerline profile from Table 3 plotted as abscissae. Corresponding values are shown for the 6 photogrammetric surveys of the experimental section (4). The resulting pattern gives strong indication of a straight-line relationship between the two values. If this is verified by further research it will provide a means of estimating the accuracy of earthwork quantities from adjusted photogrammetric surveys in terms of probability.

Further evidence of the effect of adjustment on systematic errors is illustrated by Figure 9. The graphs show the errors in the left and right slope stake points before and after adjustment for the section from Station 220 to Station 240 on the VI-Ker-58-D project. The adjustment reduced the arithmetic mean of the left slope stakes from +0.56 to -0.01 ft and of the right slope stakes from +0.65 to +0.10 ft. The reduction in the equivalent vertical difference for this 2,000-ft section was from +0.58 to +0.06 ft. It will be noted that there is no appreciable change in the magnitude of the random errors.

CONCLUSION

The results of this study have generally confirmed those developed by the previously reported 3,000-ft experimental section. The slightly greater differences between field survey quantities and photogrammetric survey quantities both before and after adjustment were anticipated. They can be attributed partially to the fact that the photogrammetric mapping was obtained under actual working conditions. More important, however, is the probability of less accuracy of the field surveys which were used as a yardstick. For this reason the term "difference" rather than "error" has been used in most instances in this report.

The most important conclusion which can be drawn from the study is that adjustment of photogrammetric surveys by means of accurate field profile will:

1. Materially reduce large localized errors in earthwork quantities; and
2. Result in over-all quantities which are within limits generally considered tolerable for purposes of payment.

REFERENCES

1. Miller, C. L., "Digital Terrain Model Approach to Highway Earthwork Analysis." M. I. T. Photogrammetry Laboratory, Publication III (August 1957).
2. Miller, C. L., and Kaalstad, T. H., "Earthwork Data Procurement by Photogrammetric Methods." M. I. T. Photogrammetry Laboratory, Publication 115; also HRB Bull. 199 (1958).
3. Funk, L. L., "Photogrammetric Map Accuracy." Photogrammetric Engineering (June 1958); also HRB Bull. 199 (1958).
4. Funk, L. L. "Terrain Data for Earthwork Quantities." HRB Bull. 228 (1959).

Relationship of Topographic Relief, Flight Height, and Minimum and Maximum Overlap

WILLIAM T. PRYOR, Chief of Aerial Surveys Branch, Bureau of Public Roads

The effects of topographic relief on overlap in aerial stereoscopic photography become acute when flight height must be sufficiently low for taking aerial photographs suitable for large scale mapping by photogrammetric methods for highways. While these same effects are present in small scale photography used to compile small scale maps, their consequences are not acute because the large flight height permits a greater relief height. For the double projection photogrammetric instruments commonly used, the ratio of relief height to flight height (h/H) varies from 0.21 to 0.36.

Principles governing the design of endlap (overlap in line of flight) and sidelap (overlap of one strip of photographs on another) are presented. Considerations that must be made when determining the minimum flight height that can be utilized according to the relief height existing in the area to be photographed and mapped at large scale with small contour interval are outlined, and their effects on the maximum scale attainable are pointed out. Whenever large scale mapping for highway surveys is to be undertaken by precise photogrammetric methods, the specific relationship between relief height in the area to be mapped and the photography flight height must be fully considered. Graphs are provided to serve as aids in ascertaining limiting conditions.

●STEREOSCOPIC photographic coverage of the ground is the cardinal requirement for mapping by stereophotogrammetric methods. As the aircraft moves the aerial camera forward along its line of photographic flight, this coverage is attained by photographing ground detail from separate camera stations. Separation of the camera stations is such that part of the area covered by each successively taken photograph is common to an area covered on the preceding photograph.

The area of overlap in photographic coverage along the flight line is called forward lap or endlap. The absolute minimum in endlap to obtain stereoscopic coverage by vertical photography is 50 percent of the flight line dimension of each photograph. In practice, an endlap greater than 50 percent is necessary for choosing pass points between successive stereoscopic models and for attaining continuity in mapping from model to model. These pass points serve in somewhat the same manner as backsights and foresights in running traverses and in spirit leveling by ground survey methods.

If several parallel strips of vertical photography are required for coverage of an area, they must have a common area of overlap called sidelap. In this way, image points common (conjugate) to photographs in adjacent strips are available for selection to serve as pass points so that continuity can be attained in mapping from one set of stereoscopic models to the other sets which are immediately adjacent in the separate flight lines of photography.

For efficiency in photogrammetric utilization of vertical photography, the maximum endlap should not exceed the percent needed to provide full stereoscopic coverage of the ground plus a small area of common stereoscopic coverage from one stereoscopic model to another. In addition, such percent cannot be allowed to become greater than the percent admissible by the photogrammetric instruments. That which follows is a presentation of principles which should be understood and applied in specifying endlap and sidelap, according to the topographic relief encountered and aircraft flight height required within the area to be photographed for aerial surveys and mapping by photogrammetric methods.

If the ground area photographed were flat and the photographic mission performed with perfection, the overlap of the photographs would consistently agree with the ideally designed value. In actuality, however, ground areas contain relief and no photographic crew performs perfectly at all times. Consequently, within each specific area, overlap attained in the photography varies in line of flight for endlap from one successive stereoscopic pair to another, and for sidelap between the adjacent strips of photographs.

EFFECTS OF RELIEF

Topographic relief causes radial displacement of the photographic images of ground points. For any given flight height, this displacement is proportional to the height of the point above or below the datum plane and to the radial distance between the nadir (plumb) point and the displaced point. High points are displaced outward from the nadir point and low points are displaced inward toward this point. Thus, a high point near the edge of an area to be photographed could be displaced so far perspectively as to not appear on the photographic format.

Perspective displacement of high relief can cause a gap in the stereoscopic coverage—an area that could not be mapped (a) in line of flight, (b) along the edge of a single strip of photographs, and (c) between the adjacent parallel strips. Situations causing the gaps must be avoided by proper design of photography endlap and sidelap limits, flight height, and flight lines. To accomplish this by increasing the amount of overlap (both endlap and sidelap) increases the number of photographs necessary to cover an area stereoscopically. Then the cost of bridging or mapping is increased proportionately. An increase in endlap results in a shorter airbase. The accuracy of the mapping is unduly lowered whenever unnecessary shortening of the airbase decreases the precision with which relief can be perceived and measured within the stereoscopic model. Actually, the relationship of relief height to flight height is a primary consideration in coping with such problems.

EFFECTS OF TILT

The effect of tilt is not accounted for in compilation of the tables, and in preparation of the figures and graphs. The consequences, however, and the numerical effects of tilt on endlap and sidelap are subsequently explained.

Sidelap and endlap will be decreased on the portion of each aerial negative tilted above the plane of the vertical and will be increased on the portion tilted below that plane. Whenever tilt does not exceed five degrees, the decrease per degree of tilt is approximately 1.8 percent and 2.0 percent, respectively, and the increase is 1.9 percent and 2.1 percent, respectively, on photographs taken with 6-in. and 8.25-in. focal length aerial cameras. For practical purposes, the increase and decrease in endlap and sidelap can be considered as two percent per degree of tilt.

The Reference Guide Outline, Specifications for Aerial Surveys and Mapping by Photogrammetric Methods for Highways—1958, stipulates that tilt in any one photograph shall not exceed three degrees, and the average tilt shall not exceed one degree for the entire project. Whenever tilt is kept within these specification limits, only the few photographs which have tilt exceeding two degrees would cause sufficient change in overlap as to reduce endlap to less than 51 percent.

Accordingly, when the minimum endlap on vertical photography is 55 percent, adjacent photographs with tilt exceeding two and one-half deg will have their endlap reduced to about 50 percent on one side and increased to about 60 percent on the other side. Thus, to avoid resultant gaps in stereoscopic coverage caused by tilt, tilt, must be less than two degrees, or the minimum endlap limit of 55 percent on vertical photography should be changed to 57 percent if tilt of three degrees is permitted, 59 percent for four degrees, and 61 percent for five degrees.

Axiomatically, the effective width of stereoscopic coverage on a single strip is decreased about two percent per degree of tilt occurring on the x-axis, the line of flight. Endlap in line of flight is similarly decreased on one edge and increased on the other by tilt occurring on the y-axis, the axis normal to the line of flight. Tilt occurring on other axes will have combination effects of less than two percent per degree of tilt on endlap and on width of stereoscopic coverage.

The analyses subsequently presented are for tilt-free vertical photographs—practical applications of which will not be so adversely affected as to be nullified when tilt does not exceed the reasonable minimum. The alternative is to maintain minimum endlap on vertical photography greater than 55 percent to prevent endlap becoming less than usable on tilted photography. This practice, because tilt cannot be eliminated, decreases the efficiency of mapping by photogrammetric methods. Sidelap will be affected in a similar manner, and also the continuity of photographic coverage along the edge of a single strip, such as in route photography.

PRINCIPLES

For double projection, photogrammetric instruments like the Multiplex, Balplex, Kelsh, and Photocartograph (called Photomapper in the U.S.), there is a limit to which the airbase can be shortened by increasing the endlap to satisfy relief-height to flight-height relationship requirements. Whenever this limit is exceeded, a stereoscopic model cannot be produced because projectors of the instrument will touch before the desired stereomodel scale is attained. The maximum allowable endlap will vary with the double projection instrument used and the map-scale to photography-scale projection ratio. The allowable endlap limits in percent determined by the projector positions of such instruments are listed in the final column of Table 1.

Optical train instruments are capable of using pairs of photographs containing larger percentages of endlap than can be utilized in double projection instruments. As circumstances permit, however, and unless only two photographs are available when excessive overlap occurs, the second photograph of each three is omitted. Thus, photographs numbered one, three, five, and so forth of each flight line are used when feasible.

Another and more critical factor, which limits the amount the airbase can be shortened by increasing the endlap to satisfy the requirements of h/H (relief height divided by flight height), is the range in vertical measurement of photogrammetric instruments. The fourth column of Table 1 lists the vertical measurement range of the various double projection instruments. This range is set by the projection zone in which the stereoscopic model is sharp enough to be measured with ease and consistency. Whenever differences in elevation of relief within a model are so large as to encompass all or most of this range, then such differences, called relief height, must be appropriately considered in relation to the flight height above the points of lowest elevation, or both endlap and sidelap requirements may not be met.

In column 5 of Table 1, h/H equals the vertical measurement range of the instrument in inches divided by the maximum projection distance in inches. This maximum

TABLE 1

INSTRUMENT LIMITATIONS TO MAXIMUM ALLOWABLE ENDLAP

Double Projection Photogrammetric Instrument	Projection Ratio[1]	Photography Focal Length (in.)	Vertical Measurement Range[2] (in.)	h/H Ratio	Maximum Endlap Governed by Vertical Measurement Range[3] (%)	Maximum Endlap Governed by Projector Position (%)
Multiplex	2.4:1	6	6.7	0.36	71	74
Balplex (525)	3.4:1	6	7.0	0.28	67	70
Kelsh stereoplotter	4:1	8.25	9.9	0.25	66	71
Kelsh stereoplotter	5:1	8.25	9.9	0.21	64	77
Kelsh stereoplotter	5:1	6	9.0	0.25	66	77
Balplex (760)	5:1	6	9.0	0.25	66	79
Photocartograph	5:1	6	9.0	0.25	66	73
Kelsh stereoplotter	7:1	6	11.0	0.22	65	83
Photocartograph	7:1	6	11.0	0.22	65	80

[1] Number of times stereoscopic model scale is larger at an optimum projection distance than the scale of vertical photography.
[2] Depth of focus of the projection lenses of the instrument in projecting a visually sharp stereoscopic model.
[3] For each instrument, this is the maximum endlap allowable at the point of lowest relief appearing on one edge of the stereoscopic overlap when the point of highest relief is 5 percent of the length of the photograph from the opposite edge of such overlap. This condition results in a minimum endlap of 55 percent at the level of the point of highest relief.

projection distance is the projection ratio of the photogrammetric instrument times the focal length of the aerial camera plus approximately 60 percent of the vertical measurement range of the instrument, and the minimum projection distance is the maximum projection distance minus its vertical measurement range. To compute the percent of endlap in column 6 of Table 1, the h/H ratio in column 5 is used in the equation for maximum endlap, $E_1 = E_2 + 50 + (50-E_2)\, h/H$, which is developed later. The percents in the same column are also equal to 100 minus the quantity of 45 times the minimum projection distance divided by the maximum projection distance.

The final column of Table 1 lists the maximum endlap, as governed by the position of the projectors in double projection instruments. Since the preceding column contains smaller percents of endlap, the vertical measurement range of each instrument limits the maximum allowable endlap in the photography for mapping with double projection instruments.

Endlap limits of 55 to 65 percent with an average of 57 percent have been specified for aerial vertical photography. It will be shown later, in development of the relationship of minimum and maximum endlap, that the 55 to 65 percent limits will accommodate a ratio of relief height to flight height of only 2/9. These limits are easily complied with for small scale photography where the flight height is relatively high. For example, photography taken from a flight height of 20,000 ft and containing the 55 to 65 percent endlap at points of highest and lowest relief, respectively, would accommodate a maximum relief of 4,444 ft. These 55 to 65 percent limits, however, are difficult and sometimes almost impossible to adhere to under certain relationships of relief height and low flight heights.

When the end product required is maps of large scale for engineering purposes, and

TABLE 2

MAP SCALE CONTROLLING USE OF PHOTOGRAMMETRIC INSTRUMENTS

Photogrammetric Instrument	Ratio of Map Scale to Photog. Scale	Map Scale (ft to 1 in.)	Photog. Scale (ft to 1 in.)	Flight Height (ft)	Maximum Relief[1] (ft)	Feasible Contour Interval[2] (ft)	Resultant C-factor[3]
Multiplex	2.4:1	20	48	288*	64	0.5	576
		40	96	576*	128	1	576
		50	120	720*	160	2	360
		80	192	1152	256	2	576
		100	240	1440	320	2.5	576
		200	480	2880	640	5	576
Balplex (525)	3.4:1	20	68	408*	91	0.5	816
		40	136	816*	181	1	816
		50	170	1020	227	1.5	680
		80	272	1632	363	2	816
		100	340	2040	453	2.5	816
		200	680	4080	907	5	816
Balplex (760)	5:1	20	100	600*	133	0.5	1200
Kelsh stereoscopic		40	200	1200	267	1	1200
plotter and Nistri		50	250	1500	333	2	750
Photocartograph		80	400	2400	533	2	1200
		100	500	3000	667	2.5	1200
		200	1000	6000	1333	5	1200
Kelsh stereoscopic	7:1	20	140	840*	187	1	840
plotter and Nistri		40	280	1680	373	2	840
Photocartograph		50	350	2100	467	2	1050
		80	560	3360	747	2.5	1344
		100	700	4200	933	4	1050
		200	1400	8400	1867	10	840
Optical Train:							
Wild Autograph, A-7;	8:1	20	160	960*	213	1	960
Zeiss Stereoplani-		40	320	1920	427	2	960
graph, C-8; Nistri		50	400	2400	533	2	1200
Photostereograph,		80	640	3840	853	4	1210
B-2; and Galileo-		100	800	4800	1067	5	960
Santoni Stereocartograph		200	1600	9600	2133	10	960

* Under usual conditions these flight heights are lower than practicable.

[1] Should endlap be larger than 65 percent for points of lowest relief, the maximum admissible by some of the instruments, the maximum relief measurable would be slightly larger than listed in this column.

[2] As a practical unit, the contour interval is one-half or nearest full foot only.

[3] Resultant C-factors must not be construed as an accuracy measurement of the photogrammetric instrument. In most cases map compilation scale governs, therefore, nearly all resultant C-factors are less than those commonly used (Table 3) and, whenever this occurs, the accuracy in contour compilation should be improved.

the map compilation is to be done by photogrammetric methods at the scale specified for the finished maps, the flight height must be relatively low. As a result, if compliance with 55 and 65 percent endlap limits were to be held to, with the resulting relief height to flight height ratio of 2/9, the maximum relief that could be accommodated for various photogrammetric instruments, photography scales, and flight heights, when 6-in. focal length photography is to be used and the map compilation scales are as listed, would be as given in Table 2. Columns 7 and 8 of Table 2 also list the feasible contour interval obtainable and the resultant C-factor when the map scale is allowed to control use of the photogrammetric instrument.

If the contour interval desired is small, the C-factor often applied in photogrammetric instrument operation may cause the contour interval to control the flight height. The maximum relief that can be accommodated by the 55 and 65 percent limits, when the contour interval controls, is given in Table 3. As an example, if a Kelsh stereoscopic plotter using 6-in. focal length photography is to be used to compile a topographic map with a contour interval of 1 ft, and the projection ratio of map scale to photography scale is 7 to 1, a C-factor of 1,300 might be used for this instrument. Using this C-factor as an indicator of the capability of the instrument, it is assumed that contours at the 1-ft interval may be delineated by use of photography taken from a flight height of 1,300 ft. The maximum relief which may be accommodated at this 1,300-ft flight height with a maximum endlap of 65 percent is 2/9 of 1,300, or 289 ft. The photography scale expressed in terms of feet per inch is equal to the flight height in feet divided by the focal length of the aerial camera in inches (1,300 divided by 6), which is 217 ft to 1 in. The desirable resultant compilation scale on the map manuscript is nearly seven times larger than the photography scale, or 30 ft to 1 in. Should map compilation at a scale of 30 ft to 1 in. be required for topographic mapping with the the same instrument and a contour interval of two feet, photography would have to be

TABLE 3

CONTOUR INTERVAL CONTROLLING USE OF PHOTOGRAMMETRIC INSTRUMENTS

Photogrammetric Instrument	Ratio of Map Scale to Photog. Scale	C-factor Commonly Used	Contour Interval (ft)	Flight Height (ft)	Maximum Relief[1] (ft)	Resultant Compilation Scale on Map Manuscript from Stereomodel (ft to 1 in.)
Multiplex	2.4:1	600	0.5	300*	67	20
			1.0	600*	133	40
			2.0	1200	267	80
			5.0	3000	667	200
			10.0	6000	1333	400
Balplex (525)	3.4:1	1000	0.5	500*	111	15
			1.0	1000	222	30
			2.0	2000	444	60
			5.0	5000	1111	150
			10.0	10,000	2222	300
Balplex (760) Kelsh stereoscopic plotter, and Nistri Photocartograph	5:1	1200	0.5	600*	133	20
			1.0	1200	267	40
			2.0	2400	533	80
			5.0	6000	1333	200
			10.0	12,000	2667	400
Kelsh stereoscopic plotter, and Nistri Photocartograph	7:1	1300	0.5	650*	144	15
			1.0	1300	289	30
			2.0	2600	578	60
			5.0	6500	1444	150
			10.0	13,000	2889	300
Optical Train: Wild Autograph, A-7 Zeiss Stereoplanigraph, C-8; Nistri Photostereograph, B-2; and Galileo-Santoni Stereocartograph	8:1[2]	1500	0.5	750*	167	15
			1.0	1500	333	30
			2.0	3000	667	60
			5.0	7500	1667	150
			10.0	15,000	3333	300

* Under usual conditions these flight heights are lower than practicable; also the 0.5-ft contour interval is smaller than practicable unless there is little or no ground cover and height of relief is very small within the area to be photographed and mapped by stereophotogrammetric methods.

[1] Should endlap be larger than 65 percent for points of lowest relief (that is, equal to the maximum admissible by most of the instruments) the maximum relief measurable would be slightly larger than listed in this column.

[2] By changing ratios on the coordinatograph of the optical train instruments, map as desirable, can be compiled at scales smaller than eight times the photography scale, as six, five, and so forth.

taken to the same scale from the flight height of 1,300 ft. The resultant C-factor would be 650, and, consequently, it should be especially easy, wherever the ground can be seen from the air, to achieve the desired accuracy in contour delineation.

Examination of flight heights involved (column 5) and maximum relief (column 6) that can be accommodated, when endlap is limited between 55 and 65 percent, indicates that flexible limits are desirable. This is especially true when large scale photography for large scale topographic mapping with a small contour interval is required. An increase in maximum endlap limits would permit an increase in maximum relief measurable within a stereoscopic model. The maximum endlap limits, however, cannot exceed the endlap acceptable to the particular photogrammetric instrument that will be used. Maximum endlap admissible by one instrument is 71 percent, 67 percent for another, and 66 to 64 percent for the remaining commonly used double projection instruments (Table 1, column 6). The minimum endlap is fixed by stereo-requirements.

In column 7 of Table 3 is listed the resultant compilation scale on the map manuscript for 5 different contour intervals (column 4) and for various photogrammetric instruments. In some cases these are not standard map scales. Manuscripts at such scales would generally be photographically reduced to the nearest smaller standard scale for preparation of the finished maps.

Factors affecting endlap were examined and an expression was developed to correlate the relationship between minimum and maximum endlap and relief height and flight height. In deriving the equations subsequently presented, only vertical photographs without crab or tilt were considered. Six variable factors were involved: minimum endlap, maximum endlap, flight height, relief height, and the limiting position on each photograph of the point of highest relief and the point of lowest relief.

Since the position of points of highest and lowest relief cannot be predetermined, they are assumed to be at the position where the perspective geometry of the photographs will cause maximum radial displacement. The position of the point of highest relief is defined, therefore, as lying somewhere on a line normal to the flight line and passing through the principal point of one of the photographs of the stereoscopic pair. (Referring to Figure 1, n_1 to a_1 is the line on which the point of highest relief appears in this space geometry illustration.) The position of the point of lowest relief is defined as lying somewhere on the extreme opposite edge of the same photograph, the edge in the stereoscopic overlap that is approximately parallel to the line on which the point of highest relief causing minimum endlap actually lies. For simplification, the point of lowest relief is assumed to be in the datum plane, as represented by point G_1.

To expand the problem to include sidelap, the same variable factors are involved. In addition, the image of principal points of the adjacent photograph do not normally appear in the sidelap area. Thus, the position of the point of highest relief, fixed arbitrarily for definition purposes, is defined as lying on a line parallel to and at a minimum sidelap distance from the edge of each of the adjacent sidelapping photographs; therefore, this line lies midway within the sidelap area. The position of the point of lowest relief lies on the near edge of each sidelapping photograph, and is assumed to be at the datum plane for the particular photograph on which sidelap is being measured.

EQUATIONS

Equations expressing the relationships of minimum and maximum endlap and sidelap are derived by use of the following terms, which are illustrated in Figures 1 to 3:

E_1 is the maximum endlap at the datum plane. The distance E_1 is measured from a point lying in the datum plane at the edge of one photograph to the conjugate image on the same photograph of a point which lies in the datum plane at the edge of the photograph which is adjacent in line of flight. E_1 is expressed as a percent of the dimension of the photograph in line of flight.

E_2 is the minimum endlap distance that the point of highest relief affecting endlap is from the edge of the photograph. The distance E_2 is measured from the edge of the photograph to the image of the point of highest relief. E_2 is expressed as a percent of the dimension of the photograph in line of flight.

S_1 is the maximum sidelap at the datum plane. It is the distance from the edge of the photograph to the conjugate image on the same photograph of a point at the datum plane appearing at the edge of the photograph which is in the adjacent line of flight. S_1 is expressed as a percent of the dimension of the photograph normal to the line of flight.

S_2 is the minimum sidelap distance that the point of highest relief is from the edge of the photograph. This distance is equal on photographs in adjacent flight lines whenever the minimum sidelap requirements are met on both photographs. S_2 is expressed as a percent of the dimension of the photograph normal to the line of flight.

h is the height above the datum plane of the point of highest relief which affects endlap or sidelap.

H is the aircraft flight height above the datum plane from which the stereoscopic pair of photographs being considered were, or will be, taken. Two intermediate values used in deriving the relationships are:

r is the projection of the radial distance between the principal point and the image of

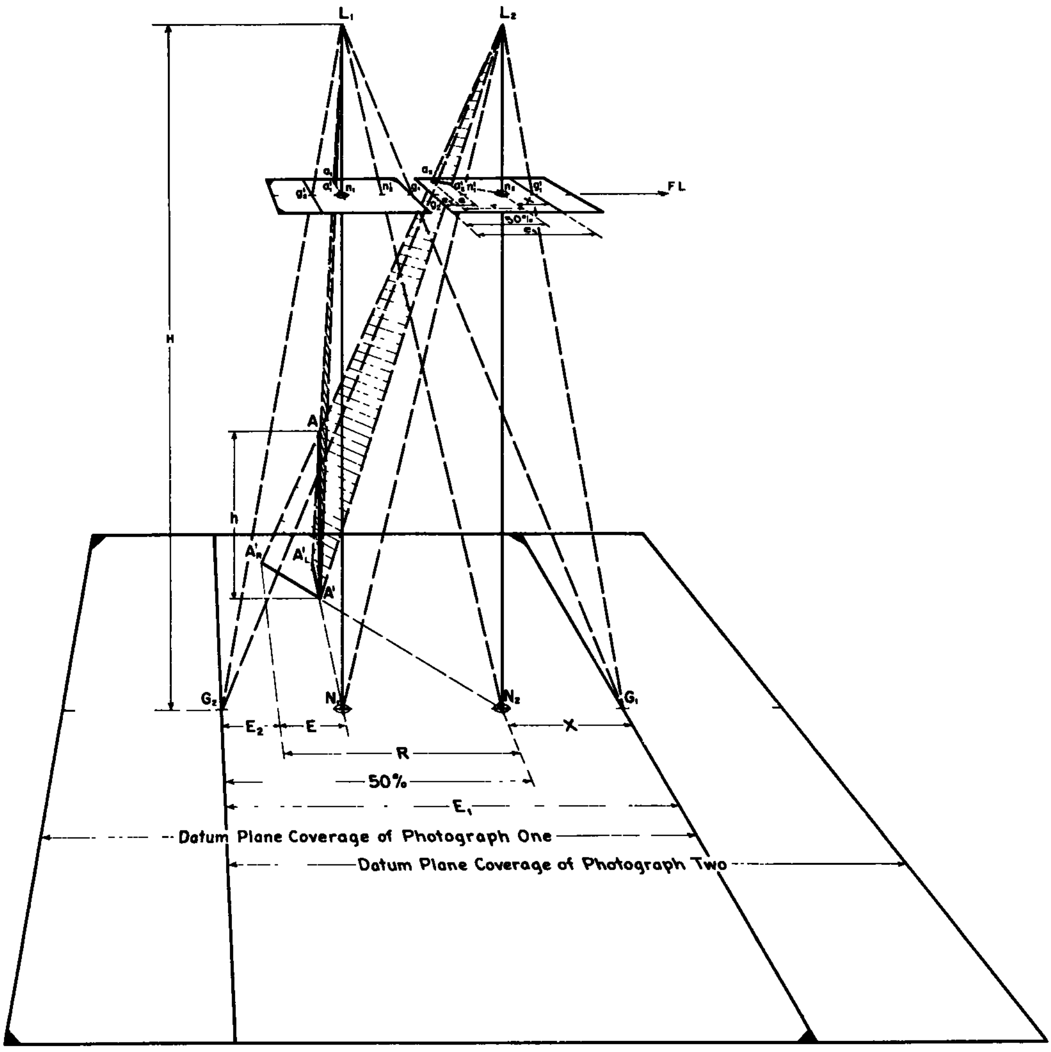


Figure 1. Space geometry of pair of aerial vertical photographs adjacent in line of flight to show endlap (E_1) at datum plane and endlap (E_2) at point of highest relief.

$$E_1 = E_2+50+(50-E_2)\,\frac{h}{H}$$

the point of highest relief on to the plane of endlap or sidelap measurement. In Figure 1 this projection is made orthographically on to a line parallel to the flight line for endlap. For sidelap, it is made on to a line normal to the flight line.

e is the projection of the radial displacement of the point of highest relief on to the line of endlap or sidelap measurement. The separate projections for sidelap and endlap are made in the same manner as for r.

With these terms defined, and with the position of the points of highest and lowest relief fixed, as stated previously, examination of Figures 1, 2, and 3 results in the following relationships:

By similar triangles, e, e_1, e_2, s_1, s_2, etc., on the photographs, are analogous, respectively, to E, E_1, E_2, S_1, S_2, etc., in the datum plane. The capitalized representation, as shown for the datum plane conditions, are subsequently used in all equations and charts.

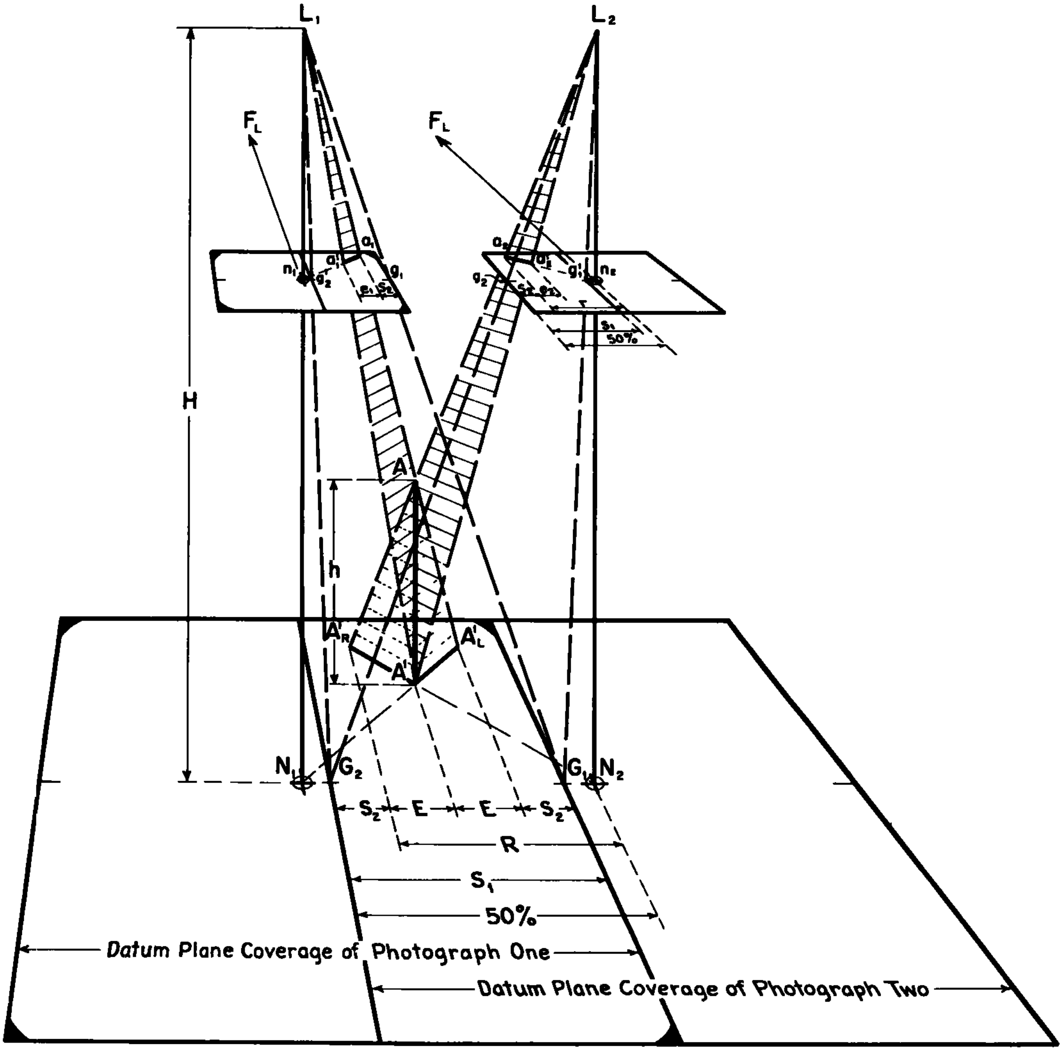


Figure 2. Space geometry of aerial vertical photographs in adjacent flight lines, at optimum spacing, to show sidelap (S_1) at datum plane and sidelap (S_2) at point of highest relief.

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H}$$

Endlap (Figure 1)

$E_1 = 50 + X$ By similar triangles, $X = E_2 + E$

Therefore:

$E_1 = 50 + E_2 + E$ Also by similar triangles, $\frac{e}{r} = \frac{E}{R} = \frac{h}{H}$

and R in any case $= 50 - E_2$

Therefore:

$$E = R\frac{h}{H} = (50 - E_2)\frac{h}{H} \text{ and } E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$

This expression may be rearranged thus:

$$\frac{h}{H} = \frac{E_1 - E_2 - 50}{50 - E_2}$$

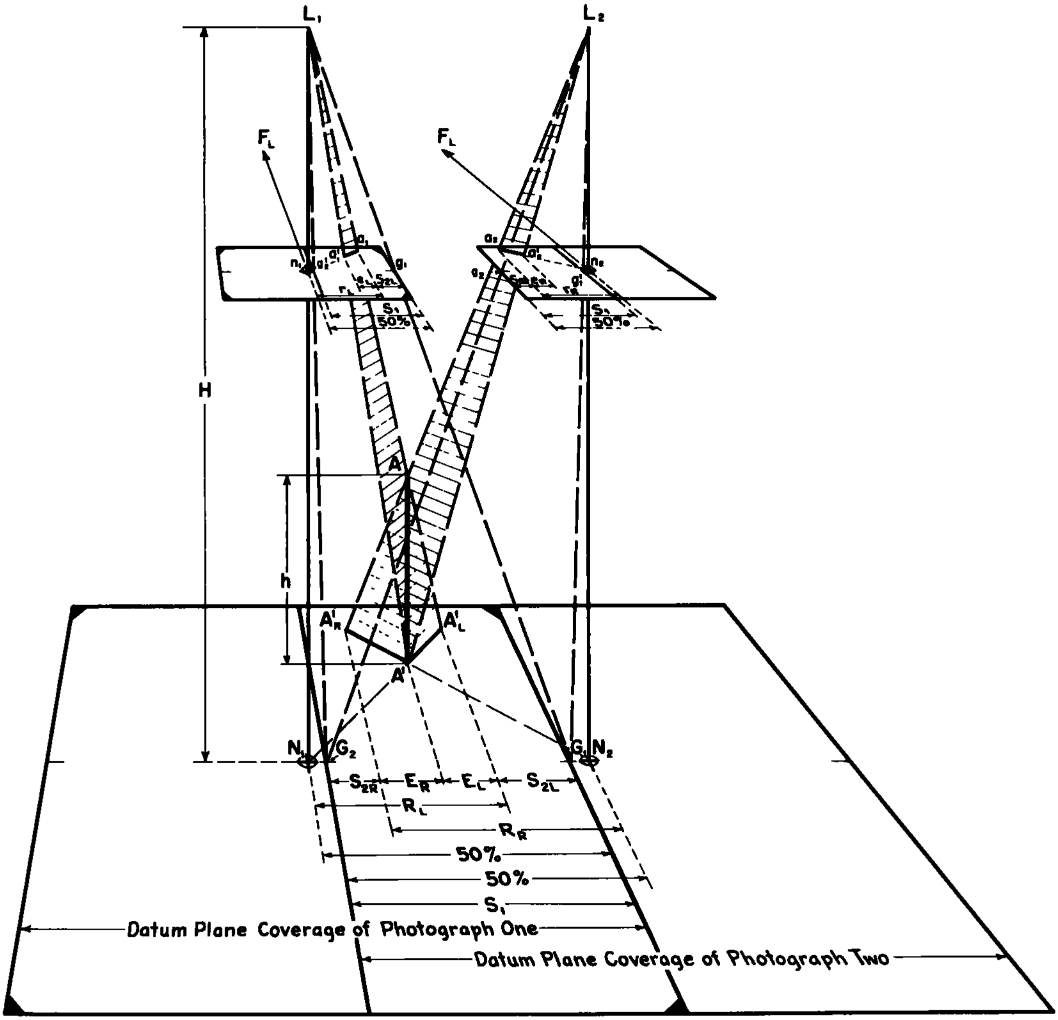


Figure 3. Space geometry of aerial vertical photographs in adjacent flight lines, not at optimum spacing, to show sidelap (S_1) at datum plane and sidelaps (S_{2R} and S_{2L}) at point of highest relief.

$$S_1 = S_{2R} + S_{2L} + (100 - S_{2R} - S_{2L})\frac{h}{H}$$

If the 55 percent and 65 percent limits are substituted:

$$\frac{h}{H} = \frac{65 - 5 - 50}{50 - 5} = 2/9$$

This value (2/9) is the relief height-flight height ratio used in compiling Tables 1 and 2.

Sidelap (Figure 2)

In this case, the flight lines are at optimum spacing, and minimum sidelap (S_2) is obtained on both adjacent photographs.

$$S_1 = S_2 + E + E + S_2 = 2S_2 + 2E$$

by similar triangles,

$$\frac{e_2}{r} = \frac{E}{R} = \frac{h}{H}$$

and R in any case = $50 - S_2$

Therefore:

$$E = (50 - S_2)\frac{h}{H}$$

and

$$S_1 = 2S_2 + 2\,(50 - S_2)\frac{h}{H}$$

In the case of Figure 3, the flight lines are not at optimum spacing, and, as a result, S_{2R} and S_{2L} and E_R and E_L are not equal on adjacent photographs.

$$S_1 = S_{2R} + S_{2L} + E_R + E_L$$

Again by similar triangles,

$$\frac{E_L}{R_L} = \frac{h}{H} = \frac{E_R}{R_R}$$

and

$$R_L = 50 - S_{2L} \text{ and } R_R = 50 - S_{2R}$$

Therefore:

$$E_L = (50 - S_{2L})\frac{h}{H} \text{ and } E_R = (50 - S_{2R})\frac{h}{H}$$

and

$$S_1 = S_{2R} + S_{2L} + (100 - S_{2R} - S_{2L})\frac{h}{H}$$

Since S_2 for this case is not equal on adjacent photographs, the position of the point of highest relief does not lie on the previously defined line. Figure 3 and its equation are presented as an example of noncritical conditions, and graphs have not been prepared from this equation.

On most of the vertical photographs taken for any one project, intermediate endlap and sidelap values will usually occur because the extreme conditions will seldom exist on more than a few of the vertical photographs. But the anticipated extreme must be used in planning survey projects, establishing flight lines at specific places and for the entire area of survey, and in administering specifications. The positions considered, therefore, are for the points of highest and lowest relief where their perspective displacement on the photographs has the greatest effect on overlap (endlap or sidelap). Then, if the point of highest relief is at a minimum 5 percent of the lengthwise dimension of a particular photograph from its back edge, the maximum endlap will be measurable from the back edge to the image on this photograph which is conjugate to the image of the point of lowest relief appearing on the leading edge of the preceding photograph. Conversely, if the point of highest relief is at a minimum 5 percent from the forward edge, the maximum endlap will be measurable from that edge to the image on this photograph which is conjugate to the image of the point of lowest relief appearing on the back edge of the succeeding photograph. Sidelap is measurable in a similar manner.

Moreover, such occurrences will also affect the width of stereoscopic coverage on a single strip of photographs by decreasing it in proportion to the height above datum of points on the ground which appear as images along edges of the strip. The decrease on one side is expressed by this equation:

$$S_3 = \frac{11.11\ r\ h}{H}$$

in which S_3 is the percent of decrease in width of ground coverage caused by relief, r is the distance in inches from the center of the photograph to the image point of highest relief appearing on its edge, h is the relief height of the ground point above datum plane, and H is the flight height above datum.

Flight height and relief height must be in the same units of measure. If r is assumed to be 4. 5 in. for the usual 9- by 9-in. vertical photographs, the equation for S_3 becomes:

$$S_3 = 50\,\frac{h}{H}$$

Thus, all single strips of photographs are decreased by relief on the edges of the strips in their effective width of stereoscopic coverage. This condition must be fully accounted for in designing photography flight lines.

GRAPHS

Five graphs have been prepared from the equations for endlap and sidelap. These graphs show the relationships of minimum and maximum endlap and/or sidelap, flight height, and height above datum of point of highest relief. Graphs 1 and 2 are, respectively, endlap and sidelap graphs for flight heights to 40, 000 ft. Graph 3 is applicable to determination of either endlap or sidelap for flight heights to 24, 000 ft. Graph 4, similar to Graph 3, is for determination of either endlap or sidelap for flight heights to 9, 000 ft. In effect, Graph 4 is simply an enlargement of the lower portion of Graph 3. Graph 5 is for the determination of either endlap or sidelap for the single flight height of 3, 000 ft.

It should be noted that in each case in using these graphs, the value of H is the aircraft flight height above the datum plane, and the datum plane is assumed to pass through the point of lowest elevation governing maximum endlap in stereoscopic pairs of the vertical photographs. The flight height to consider in attaining a particular map scale, however, is the optimum flight height, the flight height measured from the aircraft to the elevation point which corresponds to the point of optimum projection in the stereoscopic model rather than the flight height above the defined datum plane. The point of optimum projection lies above the datum plane a distance equivalent to about 60 percent of the relief height. Thus, the optimum flight height is equal to the aircraft flight height above the datum plane minus 60 percent of the relief height. Examples illustrating uses of these graphs follow:

GRAPH 1

With E_2 specified, and given values for any two of the three variables H, h, or E_1, the third value may be determined from Graph 1 for flight heights up to 40, 000 ft.

Example No. 1

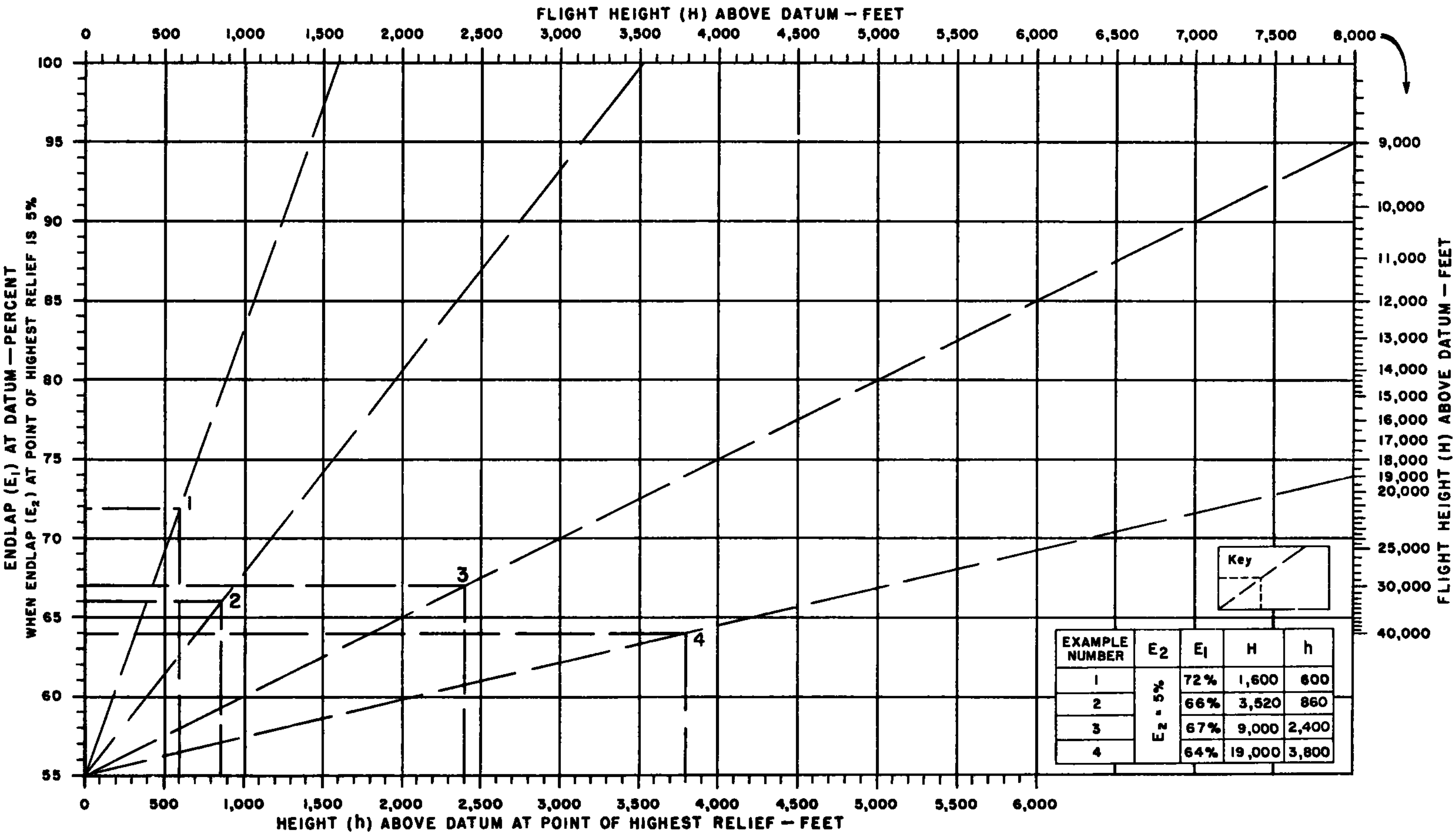
To determine:	E_1 at datum
Given:	H = 1, 600 ft
	h = 600 ft
	E_2 = 5 percent

1. Construct a sloping line from the point of minimum endlap, 55 percent, to 1, 600 ft on the abscissa for flight height (H).

2. From 600 ft on the abscissa for relief height (h), construct a vertical line to intersect the first line.

RELATION OF PERCENTAGE OF ENDLAP (E_1) AT DATUM, AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM, AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF, WHEN PERCENTAGE OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF IS 5%

$$E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$

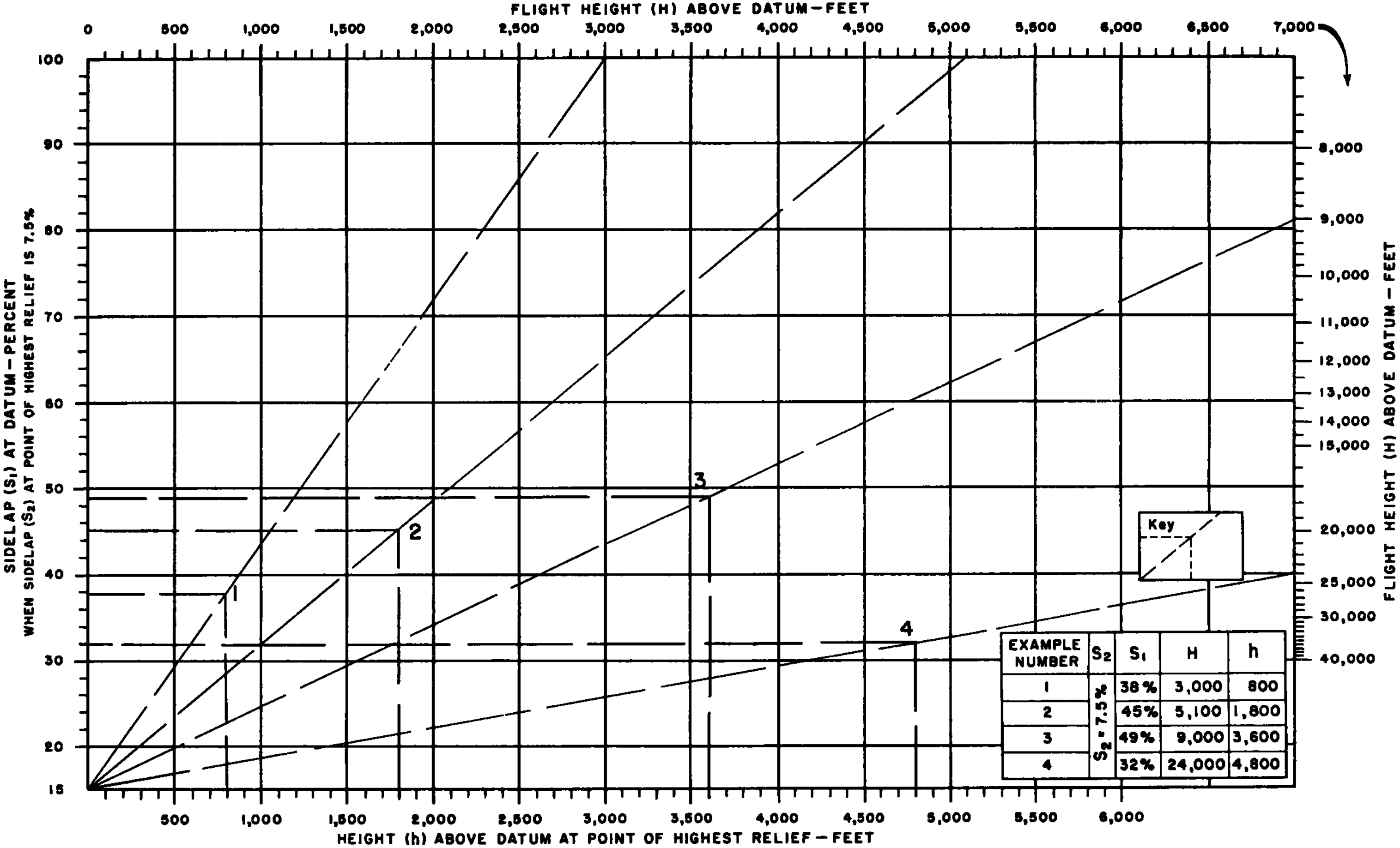


EXAMPLE NUMBER	E_2	E_1	H	h
1	E_2 = 5%	72%	1,600	600
2		66%	3,520	860
3		67%	9,000	2,400
4		64%	19,000	3,800

Graph 1

RELATION OF PERCENTAGE OF SIDELAP (S_1) AT DATUM, AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM, AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF WHEN PERCENTAGE OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF IS 7.5%

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H}$$

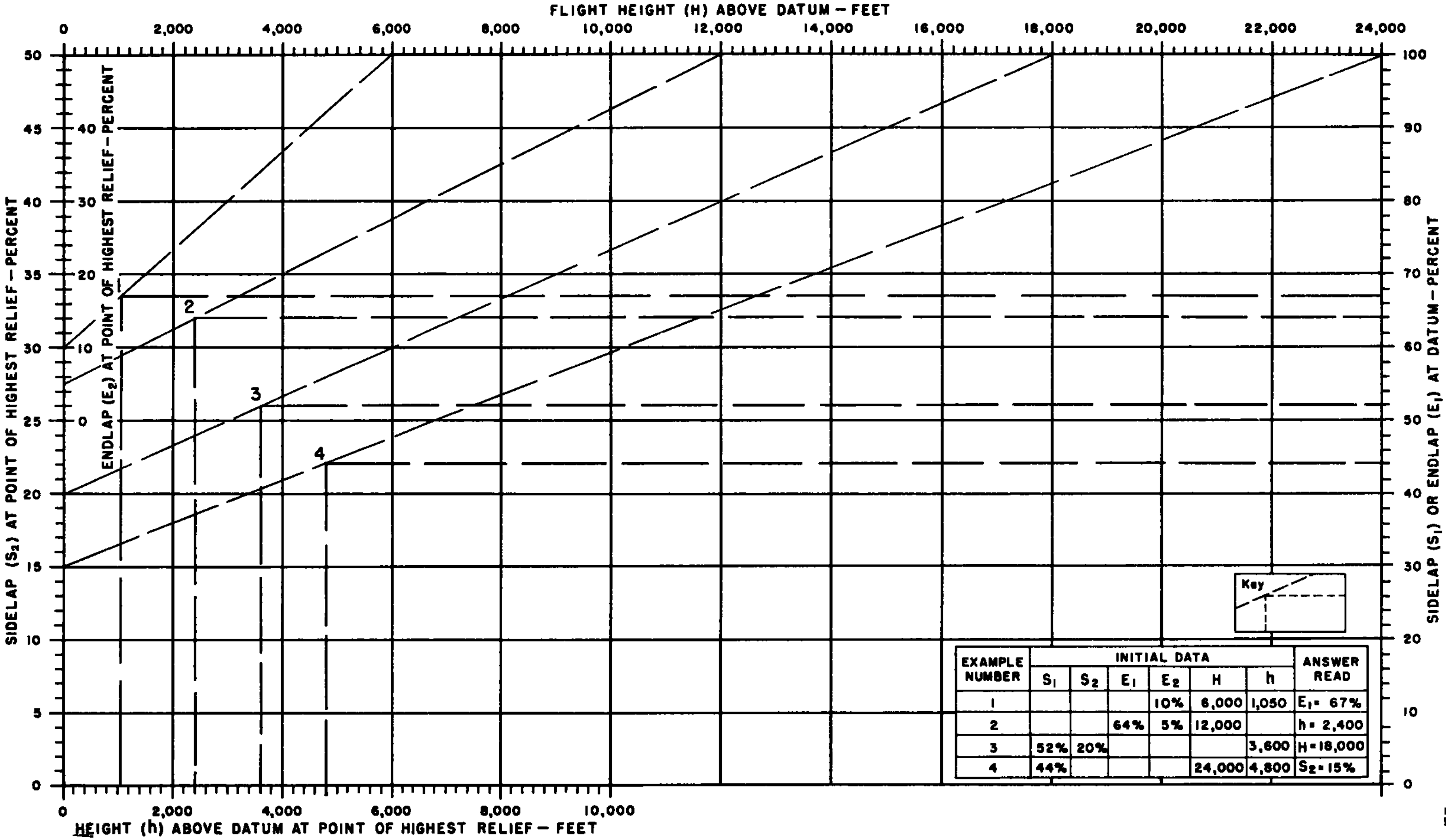


EXAMPLE NUMBER	S_2	S_1	H	h
1	S_2 = 7.5%	38%	3,000	800
2		45%	5,100	1,800
3		49%	9,000	3,600
4		32%	24,000	4,800

Graph 2

RELATION OF PERCENTAGE OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF SIDELAP (S_1) AT DATUM, OR OF PERCENTAGE OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF ENDLAP (E_1) AT DATUM, AND AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF
(FLIGHT HEIGHT TO 24,000 FEET)

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H} \quad \text{AND} \quad E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$



EXAMPLE NUMBER	INITIAL DATA						ANSWER READ
	S_1	S_2	E_1	E_2	H	h	
1				10%	6,000	1,050	E_1 = 67%
2			64%	5%	12,000		h = 2,400
3	52%	20%				3,600	H = 18,000
4	44%				24,000	4,800	S_2 = 15%

Graph 3

3. From the point of intersection of lines one and two, construct a horizontal line to the endlap ordinate, and read the endlap in percent. E_1 = 72 percent.

Resultant endlap of 72 percent in this example, and in example 1 on Graph 4, is unrealistic for double projection instruments because, according to Table 1, none of these instruments is capable of handling an endlap of 72 percent. An optical train instrument, however, could utilize photographs with such an endlap.

Example No. 2

A better approach to solving the endlap problem is given in Example 2 on Graph 1. First consider the type of photogrammetric instrument; also the scale at which the map compilation is desired. Should the instrument for which endlap and sidelap and photography flight lines are to be designed be a Kelsh stereoscopic plotter using 6-in. focal length photography and a 5:1 projection ratio, the optimum flight height would be 3,000 ft for map compilation at a scale of 100 ft to 1 in. This 3,000 ft is a product of the map scale of 100 ft to 1 in., the projection ratio of 5, and the photography focal length of 6 in. On all graphs, the optimum flight height, H_o, plus 60 percent of the relief height equals the flight height, H. When the maximum permissible E_1 at datum is 66 percent, the minimum E_2 is to be not less than 5 percent, and the optimum flight height required for the map compilation scale desired is 3,000 ft, proceed as follows to determine the maximum h which can be accommodated and the actual flight height, H, that will be required above the datum passing through the point of lowest relief. From Table 1, select the 66 percent maximum for E_1. Utilize a minimum endlap of 55 percent at point of highest relief, which results in an E_2 of 5 percent. In reducing the equation for E_1 such values result in an equation, in this case, wherein $H = 4.1h$. Also, from preceding data $H = 0.6h + H_o$. Therefore, by substitution of 3,000 ft for H_o, and 4.1h for H, the value of h is determined to be 860 ft. Consequently, Example 2 on Graph 1, reduces to:

To determine: H above datum

Given: E_1 = 66 percent
E_2 = 5 percent
h = 860 ft

1. Construct a line parallel to the abscissa of the graph from the endlap ordinate of 66 percent.
2. Construct a line parallel to the ordinate of the graph from the relief height abscissa of 860 ft.
3. From the point of intersection of the ordinate and abscissa lines of this graph, at the minimum endlap of 55 percent for point of highest relief, construct a sloping line to pass through the point of intersection of the two lines constructed in steps one and two. Extend this line to the H abscissa. This intersection marks an H of 3, 520 ft, the answer. The practical H to use in this case would be 3, 500 ft.

Continuing further with this example, by considering only a single strip of aerial photographs, the walls of a canyon 860 ft high would decrease the width of photographic coverage 24.4 percent. This is twice the decrease on one side, as computable by use of the equation for S_3 which is the decrease in percent of width of ground coverage by perspective displacement of relief.

GRAPH 2

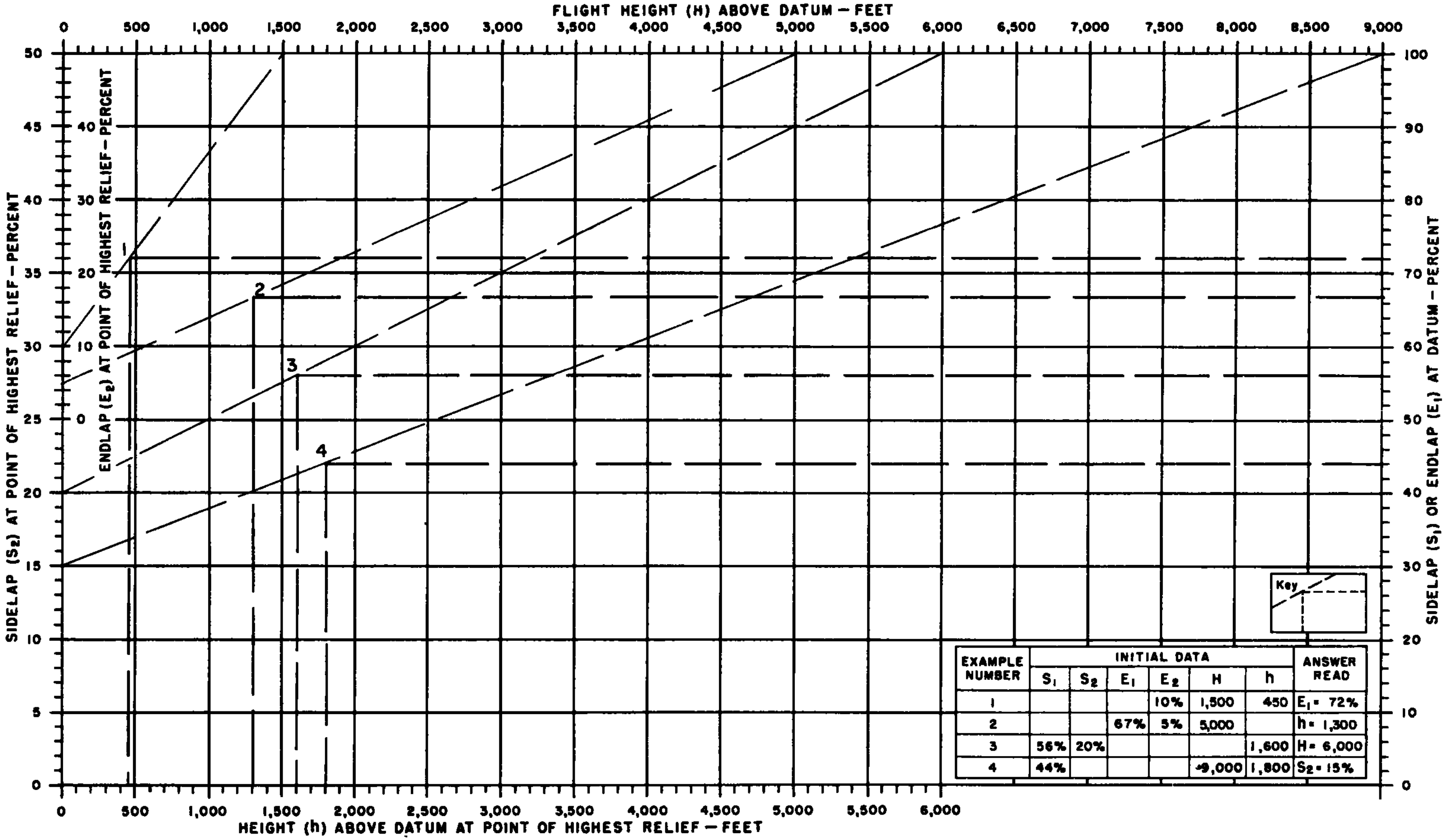
With S_2 specified, and given values of any two of the three variables H, h, or S_1, the third value may be determined from Graph 2 for flight heights up to 40, 000 ft.

Example No. 1

To determine: S_1 at datum

RELATION OF PERCENTAGE OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF SIDELAP (S_1) AT DATUM, OR OF PERCENTAGE OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF TO PERCENTAGE OF ENDLAP (E_1) AT DATUM, AND AIRCRAFT FLIGHT HEIGHT (H) ABOVE DATUM AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF
(FLIGHT HEIGHT TO 9,000 FEET)

$$S_1 = 2S_2 + 2(50 - S_2)\frac{h}{H} \quad \text{AND} \quad E_1 = E_2 + 50 + (50 - E_2)\frac{h}{H}$$



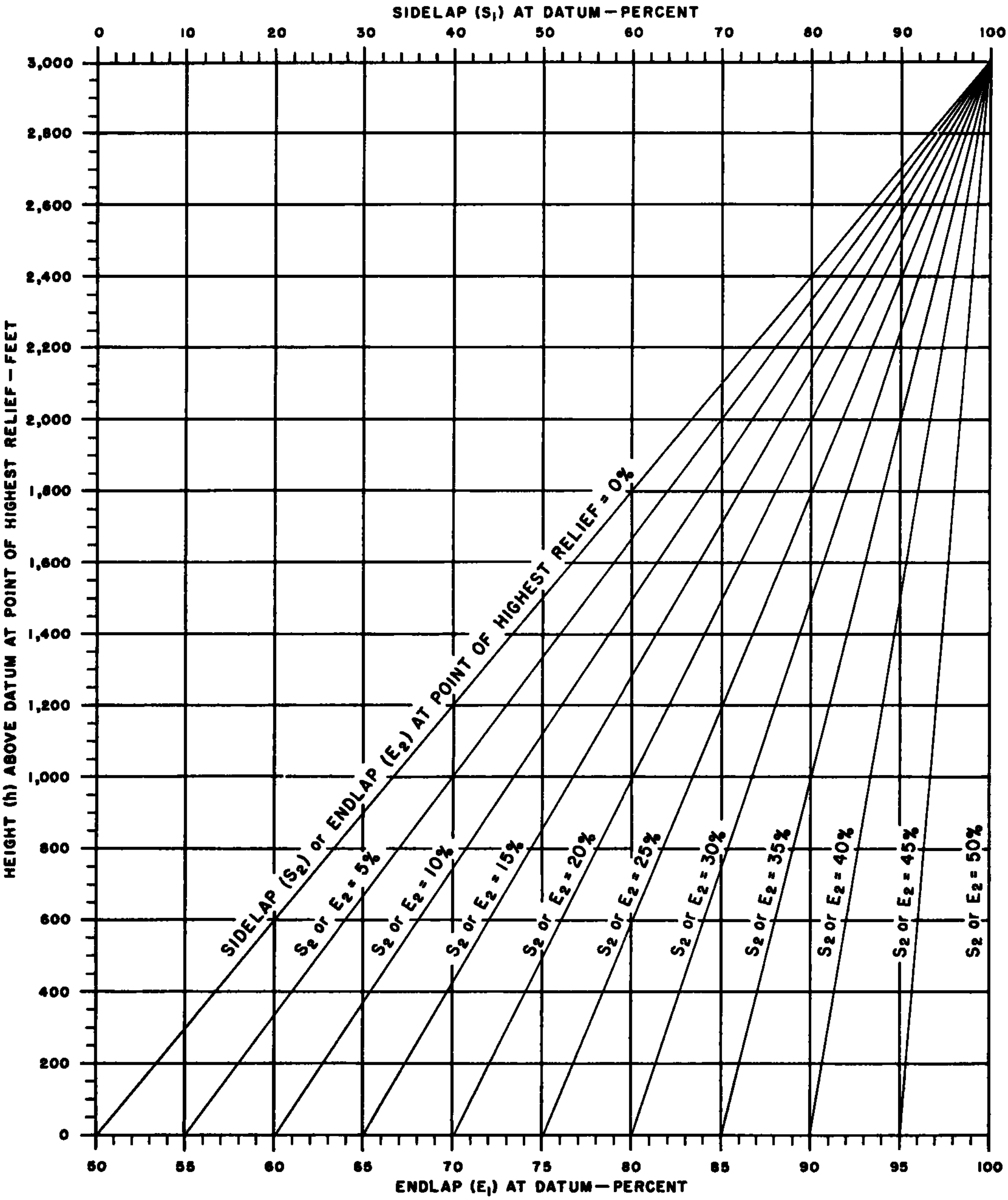
EXAMPLE NUMBER	INITIAL DATA						ANSWER READ
	S_1	S_2	E_1	E_2	H	h	
1				10%	1,500	450	E_1 = 72%
2			67%	5%	5,000		h = 1,300
3	56%	20%				1,600	H = 6,000
4	44%				9,000	1,800	S_2 = 15%

Graph 4

Given: $H = 3,000$ ft
$h = 800$ ft
$S_2 = 7.5$ percent

1. Construct a sloping line from the point of twice the minimum sidelap distance

RELATION OF VARIOUS MINIMUM PERCENTAGES OF SIDELAP (S_2) AT POINT OF HIGHEST RELIEF AND PERCENTAGE OF SIDELAP (S_1) AT DATUM, OR VARIOUS MINIMUM PERCENTAGES OF ENDLAP (E_2) AT POINT OF HIGHEST RELIEF AND PERCENTAGE OF ENDLAP (E_1) AT DATUM, AND HEIGHT (h) ABOVE DATUM OF POINT OF HIGHEST RELIEF, WHEN AIRCRAFT FLIGHT HEIGHT ABOVE DATUM IS 3,000 FEET



Graph 5

(in this case $2S_2$ = 15 percent) to 3,000 ft on the abscissa for flight height (H).

2. From 800 ft on the abscissa for relief height (h), construct a vertical line to intersect the sloping line previously drawn.

3. From the point of intersection of the first and second lines, construct a horizontal line to the sidelap (S_1) ordinate, and read the sidelap in percent. S_1 = 38 percent.

GRAPH 3

Graph 3 is a combination sidelap-endlap graph for flight heights to 24,000 ft. It presents the relationship of the aircraft flight height (H) above datum, height (h) of the point of highest relief, and the percentage of sidelap (S_2) or endlap (E_2) at point of highest relief, and the percentage of sidelap (S_1) or endlap (E_1) at the datum.

Given any three of the four variables, S_1, S_2, H, and h, or E_1, E_2, H, and h, the fourth may be determined from this graph.

<u>Example No. 1 on Graph 3 for Endlap</u>

To determine: E_1 at datum

Given: H = 6,000 ft
h = 1,050 ft
E_2 = 10 percent

1. Construct a sloping line from 10 percent on the ordinate for endlap (E_2) to 6,000 ft on the abscissa for flight height (H).

2. From 1,050 ft on the abscissa for relief height (h), construct a vertical line to intersect the sloping line.

3. From the point of intersection of lines one and two, construct a horizontal line to the ordinate for endlap (E_1), and read the endlap in percent. E_1 = 67 percent. (This endlap is excessive for all but two of the double projection instruments listed in Table 1, the Multiplex and Balplex (525). It is also usable in the optical train instruments.)

<u>Example No. 2 on Graph 3 for Sidelap</u>

To determine: Flight height (H)

Given: h = 3,600 ft
S_1 = 52 percent
S_2 = 20 percent

1. From 3,600 ft on the abscissa for relief height (h), construct a vertical line.

2. From 52 percent on the ordinate for sidelap (S_1) at the datum, construct a horizontal line to intersect the vertical line from the abscissa for relief height (h).

3. Construct a line from 20 percent on the ordinate for sidelap (S_2) through the point of intersection of lines one and two to the abscissa for flight height (H), and read the flight height in ft. H = 18,000 ft.

GRAPH 4

Graph 4 is a combination sidelap-endlap graph for flight heights to 9,000 ft. It is constructed and used in the same manner as Graph 3, and is in effect, an enlargement of the lower portion of that graph.

GRAPH 5

Graph 5 is constructed for the special case of an aircraft flight height of 3,000 ft above datum. It presents the relationships of various percentages of sidelap (S_2) at point of highest relief and percentages of sidelap (S_1) at datum, or various percentages of endlap (E_2) at point of highest relief and percentages of endlap (E_1) at datum, and height (h) of point of highest relief.

Example No. 1 using Graph 5 for Endlap

To determine: E_1 at datum

Given: E_2 = 5 percent
h = 800 ft

Draw a horizontal line from 800 ft on the ordinate for relief height (h) to intersect with the sloping line labeled E_2 = 5 percent. From this point of intersection, construct a line vertically downward to the abscissa for endlap (E_1) at datum. In this case E_1 = 67 percent. (A flight height larger than 3,000 ft would be necessary to achieve an endlap at datum of less than 67 percent, as required by most double projection instruments.)

Example No. 2 using Graph 5 for Sidelap

To determine: S_1 at datum

Given: S_2 = 5 percent
h = 800 ft

Draw a horizontal line from 800 ft on the ordinate for relief height (h) to intersect with the sloping line labeled S_2 = 5 percent. From this point of intersection, construct a line vertically upward to the abscissa for sidelap (S_1) at datum. In this case S_1 = 34 percent.

CONCLUSION

In this paper, an attempt has not been made to achieve an exhaustive analysis of the interrelationships of relief, flight height, tilt, and overlap. Proof of the significance of their effects on utilization of photogrammetric methods of mapping at large scales for highway engineering purposes was undertaken. In actuality, certain combinations of relief heights and flight heights place a limit on how close to the ground an aircraft can be flown on photography missions for such mapping. Utilization of the principles presented and graphs prepared will enable highway engineers to ascertain the largest scale and smallest contour interval that are practicable for a particular relief height within the route band or area of survey. Whenever the principles are fully applied, it will always be possible to attain optimum overlap.

Discussion

W. S. HIGGINSON, Sloan and Associates, Pasadena, California — Study of Pryor's paper has given me the idea that the question of C-factor would have more meaning if the elements that really assign a numerical value to it are considered. While the C-factor used for each photogrammetric instrument is mostly empirical, there are a number of elements that affect it.

Planning flight lines for photographic missions has been the subject of study for many years. There is no method of planning photography yet where a planner can apply a particular procedure and use specific equations and produce a satisfactory aerial photography plan for all projects. In any planning it is necessary to decide a few important factors from the best available information before a usable photography plan can be evolved. These factors are minimum ground elevation, critical ground elevations (greatest difference in elevation that may occur in a single stereomodel), photography scale, and focal length of the camera to be used. If these factors are applied to a particular pattern a suitable plan can be produced for an aerial photography project. Such planning is based on the theory that overlaps, either end or side, are to be constant quantities rather than variable. To consider endlap variable, changing it from any value other than the ideal value, will change the accuracy of plotting map detail in the same manner as altering the C-factor, because the base-height ratio (b/H) is one of the elements that affect the C-factor.

This practice, of course, admits that it is impossible to compile a map at a specified scale and contour interval for areas of extreme vertical relief, or in cases where h/H is greater than 2/9 which would require other than 55 percent minimum and 65 percent maximum endlap limits as stated in the paper. The planner can favor a particular project by a slight change in some of the arbitrary values he has fixed for average or critical elevations; in fact, both of these values could be adjusted to change the plan to a considerable extent. It might be suggested that special additional photography should be planned to apply, at the optimum elevation, over the limited areas of critical or extremely high ground. The additional photography over these limited areas could provide adequate quality of mapping photography for the entire project area.

Most aerial photography mapping plans are based on the empirical equation: H = contour interval times "C", and "C" is the C-factor applicable to a particular instrument. If this "C" is separated into $c_1 + c_2 + c_3 + c_4 + c_5$, one of the small c elements could change with a minimum decrease in the C-factor and a maximum change in the ground area coverage per photograph that will result in the required map scale and contour interval. The five c-factor elements designated as small c are considered to be scale, B/H ratio, projection distance, quality of the photography, and ground cover. A proper evaluation of each of these elements is necessary before any plans could be evolved. Since this approach is the real basis of planning aerial photography for most mapping projects, it is of the utmost importance.

WILLIAM T. PRYOR, Closure — In his discussion, Higginson stresses the importance of considering several factors which have an effect on efficiency in the photogrammetric use of aerial photography. Attaining the greatest possible effeceency is always desirable. It was not intended, however, to include all facets of aerial photography flight planning in this paper, particularly those which are especially applicable to photography for small scale mapping.

The purposes of the paper were to present the effects of relief on selecting map scales and contour intervals, and in determining the endlap and sidelap limits controlled thereby according to the various types of photogrammetric instruments used in large scale topographic mapping for highway engineering purposes within the U.S.

Unless the effects of relief height to flight height are considered, the consequences are insufficient endlap which, in turn, results in inability to map the desired areas. Obviously, the base-height ratio (B/H) would be a maximum wherever the minimum admissible endlap is attained. But, to attain maximum efficiency, the B/H ratio would have to change when the ratio of relief height to flight height (h/H) changes within the admissible limits. Should a constant, but minimum, B/H ratio be maintained to assure the attainment of full stereoscopic coverage, regardless of the h/H ratio, then inefficiency will result. But, if the principles presented are properly applied, the B/H ratio will decrease in proportion to the increase in h/H ratio. Conversely, the B/H ratio can be increased as the h/H ratio decreases.

It should be emphasized, of course, that the foregoing statements regarding influence of the h/H ratio on the B/H ratio are specifically applicable, when aerial photographs are taken from a low flight height for large scale topographic mapping where the relief is such that the h/H ratio approaches, or tends to exceed, the specific values in column 5 of Table 1. Literally, relief in these considerations is the difference in elevation and the height of trees and buildings within the successive stereomodels. When the h/H influences are ignored, either intentionally or by oversight, the consequences are inefficiency. Particularly, photography lacking sufficient overlap and proper scale for accomplishing the mapping required, will result. Moreover, to ignore these influences can also result in specifying a map scale which is larger, and a contour interval which is smaller, than can be attained photogrammetrically because the h/H ratio, whether arising from topographic relief or object heights, or both, will not admit taking photography of sufficient scale to accomplish the mapping.

Whenever the contour interval or the ratio of photography scale to map scale must be considered in relation to the h/H ratio, the resultant C-factor (refer to column 8 in Table 2) may be much less than the C-factor commonly used (refer to column 3 in Table 3). Therefore, map scale and the h/H ratio are primarily the governing factors in much of the large scale topographic mapping required by highway engineers. (In consideration of the foregoing, it should be remembered that the ratio of photography scale to map scale is actually the projection ratio for double projection photogrammetric instruments, and that the practical limit of the ratio of photography scale to map scale for the optical train instruments is recognized as being 8:1.)

It was not the purpose in this paper to focus attention on the empirical C-factors. These factors were used merely as a means to stress the importance of the h/H ratio and the degree to which it controls what can be done photogrammetrically in large scale, small contour interval, topographic mapping. Only when the h/H ratio is insignificantly small can the C-factor become fully significant and the B/H ratio be kept reasonably uniform. Consequently, when the relief height is large and variable from one stereoscopic pair to another, sufficient stereoscopic overlap cannot be achieved unless the B/H ratio is varied inversely as the h/H ratio changes from one stereoscopic pair to another. To achieve this greatest possible efficiency, photographic crews must be alert and effective in applying the principles outlined.

Terrain Data for Earthwork Quantities

L. L. FUNK, Photogrammetric Engineer, California Division of Highways

●IN DECEMBER 1957, an experimental project was undertaken by the California Division of Highways to determine the relative accuracy and costs of various methods of obtaining earthwork quantities. The project was initiated by the Design, Construction, and Photogrammetric Departments. The principal objective of the study was to provide data which would:

1. Assist Design and Construction in developing procedures to obtain acceptable pay quantities for earthwork with a minimum of engineering effort.
2. Furnish a guide to Design for selection of the most suitable method for obtaining terrain data and computing earthwork quantities on individual projects.

Test Site

The area selected for the test was a 3,000-ft section on new location between Stations 160+00 and 190+00 on Road III-But-21-B about two mi northeast of Oroville. As shown by the contour map in Figure 1, the terrain is rolling with fairly regular slopes ranging from 4 to 20 percent. The land is used for grazing. Portions of the area were covered by a relatively dense growth of grass up to a maximum of 3 in. in height. There was a minor amount of brush in two small creeks. A 2-ft contour map of the area had been previously obtained. Design was partially completed and the proposed centerline had been staked in the field.

Surveys

A total of nine surveys were made of the test site, which were designated as follows:

Field Surveys:

F1 - Precise
F2 - Made by commonly accepted methods

Photogrammetric cross-sections - Flying height 1,500 ft:

PS1 - Spot heights written on manuscript 1 in. = 50 ft
PS2 - Spot heights written on manuscript 1 in. = 50 ft
PS3 - Spot heights on punch cards - Model Scale 1 in. = 50 ft
PS4 - Spot heights written on manuscript 1 in. = 50 ft

Photogrammetric contour maps - Scale 1 in. = 50 ft, C.I. 2 ft:

CM1 - Flying height 1,500 ft
CM2 - Flying height 1,500 ft
CM3 - Flying height 2,100 ft

Field Surveys

The F1 survey was made by relatively precise methods for use as a yardstick in measuring the accuracy of other surveys. Cross-sections were taken with an engineer's level along lines at right angles to centerline. The right angles were turned with a transit. The maximum interval between cross-section lines was 25 ft. Density of points on individual cross-section lines was left to the judgment of the chief of party. In general, the resulting spacing did not exceed 50 ft with sufficient breaks in slope being read to insure the accuracy of earthwork quantities.

Field survey F2 was made by District III under instruction to use their conventional procedures for the type of terrain involved. This survey consisted of the following three steps:

1. Centerline profile read with an engineer's level.

2. Slope stakes set with a Rhodes Reducing Arc at 50-ft stations. Right angles were turned with a 90-deg prism. The maximum distance for Rhodes Arc readings was 100 ft.

3. Cross-sections taken at 50-ft stations plus nine additional cross-sections at designated breaks in the terrain.

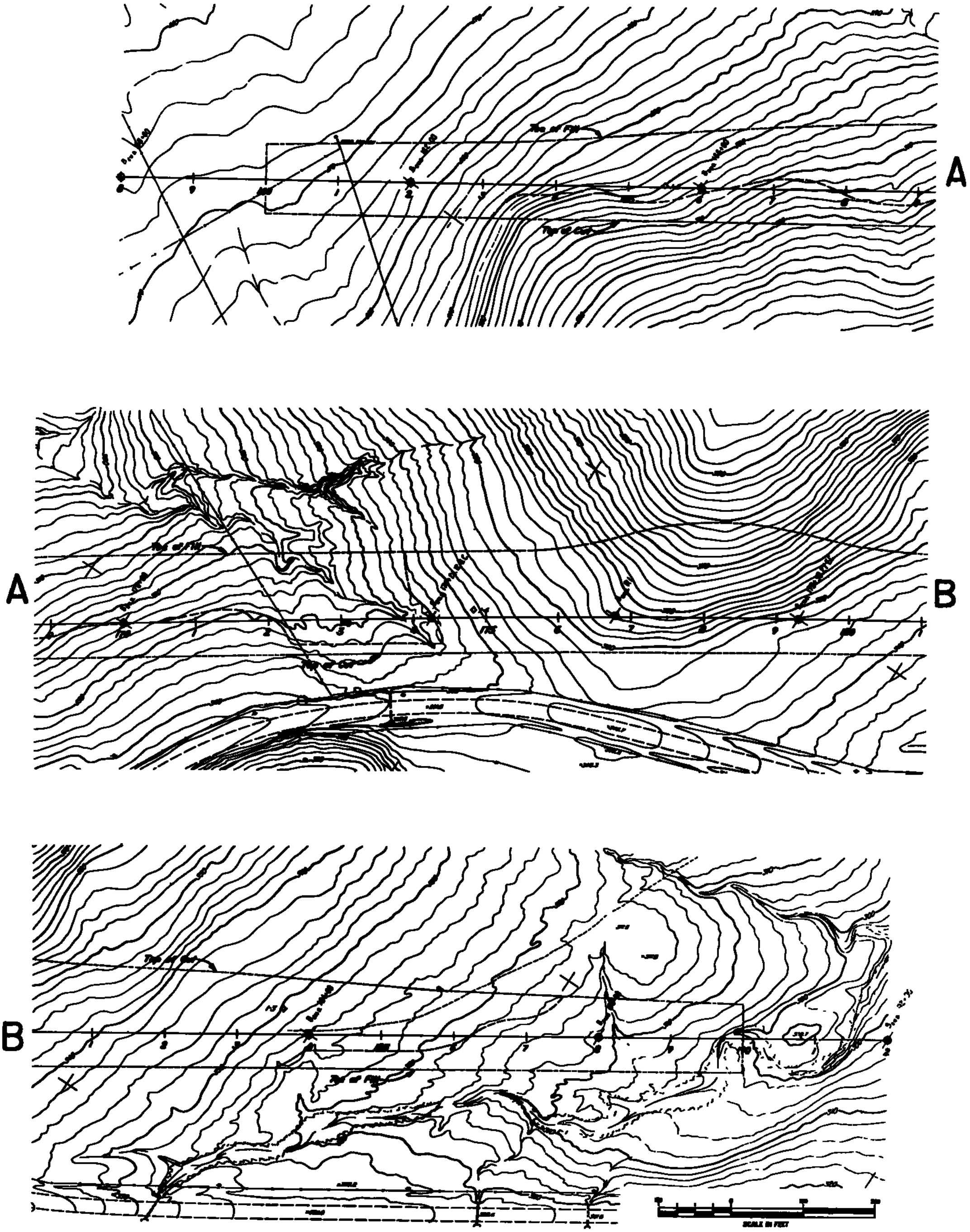


Figure 1.

The density of points on each cross-section line was again left to the judgment of the chief of party. The field notes indicate that it was slightly greater than for field survey F1. As will be shown later, the density of points in this type of terrain is of relatively little importance. From Station 160 to 163 and from Station 180+50 to 190, the cross-sections were taken with an engineer's level. From Station 163+50 to 180, the elevations were obtained by plus and minus differences from centerline using a Rhodes Reducing Arc.

The F1 survey was made subsequent to the F2 survey. In order to determine errors in reading with the Rhodes Arc, it included a reading on each slope stake set by the F2 survey. A reading was also made at the exact point designated by the F2 notes for the slope stake, to determine the difference in elevation caused by error in position.

Photogrammetric Surveys

A single flight of photography taken from a height of 1, 500 ft with a Zeiss RMK/13 camera (nominal focal length of 6 in.) was used for all photogrammetric compilations, with the exception of CM3. Four stereomodels covered the length of the test area. Horizontal control consisted of three premarked points per model along centerline. Vertical control for each model consisted of one of the premarked points near the center plus four photo-identified wing points. This control was obtained by State forces.

Contour maps CM1 and CM2 and photogrammetric cross-sections PS1 and PS2 were compiled with a Kelsh plotter by professional mapping firms under contracts for plotter rental at an hourly rate. Elevations of points along the cross-section lines of PS1 and PS2 were written on the manuscripts.

PS3 consisted of a digital readout of the cross-section data using a Terrain Data Translator. This equipment was designated and manufactured by Benson-Lehner Corporation of Los Angeles to the requirements and specifications of Pafford and Associates, also of Los Angeles. It is adaptable for use in double projection type plotters or for taking digital data from a contour map. The data for PS3 were taken directly from the stereomodels in a Nistri Photomapper with the tracing table being guided along cross-section lines previously plotted on a manuscript. Output data consisted of IBM punch cards and a typed list of elevations and distances right and left of centerline for each cross-section.

PS4 consisted of readings of the slope stakes and centerline elevations only, and was done by Division of Highways operators usually engaged in map checking. One of the models was read in a Kelsh plotter and three in a Nistri Photomapper.

CM3 was a portion of a 14. 8-mi mapping contract awarded in May 1956 at a contract price of approximately $1, 275 per mi. Photography for the portion included in the test site was taken on May 30, 1956, with a Wild RC 5A camera from a height of 2, 100 ft. The specifications required a minimum of three horizontal and five vertical control points per model. Compilation of a 2-ft contour map at a scale of 1 in. = 50 ft was done in a Kelsh plotter modified to provide a ratio of 1 to 7 from photo scale to map scale.

VERTICAL ACCURACY OF SURVEYS

Comparative Accuracy

The accuracy of the various surveys in determining the elevation of discrete points is shown in Table 1. The points include slope stakes and centerline stations whose elevations were established by the F1 field survey. The first line of Table 1 shows the relative accuracy of two field surveys in reading 60 centerline stations with an engineer's level. The results show close agreement except for two blunders of 1. 0 ft and one of 1. 4 ft in the F2 survey.

Vertical errors due to the difference in the positions of the slope stakes, as set in the F2 survey, and their position, as recorded in the notes, are shown in the second line. Horizontal errors in position, due to poor right angles, amounted to as much as 10 ft in some cases. Horizontal errors in distances from centerline were very minor. Vertical errors in the Rhodes Arc readings of the F2 survey are shown in the third line. In some cases these tended to compensate the errors due to position.

TABLE 1

VERTICAL ACCURACY OF SURVEYS

Survey	No. of Points Measured	Probable Error 50 % Within (ft)	Specification Limit 90 % Within (ft)	Error Range (ft)	Standard Deviation[1] (ft)	Arithmetic Mean (ft)
F2 - Centerline - Engineer's Level	60	±0.1	±0.2	-1.4 to +0.2	0.28	-0.02
F2 - Slope Stakes - Rhodes Arc						
Errors due to Position	118	0.0	±0.4	-1.1 to +1.5		+0.03
Errors due to Reading	118	±0.2	±0.9	-2.4 to +1.6		-0.19
Combined Errors	118	±0.2	±0.9	-2.2 to +1.6	0.60	-0.14
PS1 - C/L and Slope Stakes	183	±0.1	±0.4	-0.5 to +0.6	0.19	+0.13
PS2 - Centerline	61	±0.3	±0.5	-0.1 to +0.7	0.19	+0.29
PS3 - C/L and Slope Stakes	183	±0.2	±0.4	-0.6 to +0.9	0.25	+0.09
PS4 - C/L and Slope Stakes	183	±0.1	±0.3	-0.5 to +0.7	0.20	+0.03
CM1 - C/L and Slope Stakes	183	±0.2	±0.4	-0.7 to +0.9	0.27	+0.05
CM2 - C/L and Slope Stakes	183	±0.3	±0.7	-0.6 to +1.2	0.34	+0.32
CM3 - C/L and Slope Stakes	183	±0.4	±1.1	-1.3 to +1.8	0.62	+0.24
CM3A - C/L and Slope Stakes (Portion from Sta. 164 to 190)	160	±0.4	±0.8	-1.3 to +1.4	0.52	+0.10

[1] Calculated on basis of deviations from the arithmetic mean.

The combined errors, shown in the fourth line, are the differences in elevations between the slope stakes, as determined by the F2 survey, and points at the described locations of the slope stakes as determined by the F1 survey. These errors should be compared to the various photogrammetric surveys in considering relative accuracy. The elevations of the slope stakes and centerline stations were determined by direct spot-height readings in the photogrammetric cross-section surveys and by interpolation between contours in the contour map surveys.

Five statistical measures of accuracy are shown for each of the surveys. Of these, the arithmetic mean has by far the greatest effect on the accuracy of earthwork quantities. It is computed by dividing the algebraic sum of the individual errors by the number of points tested. In effect, therefore, the arithmetic mean is the center of gravity of the entire group of errors. As such, it indicates the probable magnitude of the systematic errors.

Another valuable measure of accuracy is the standard deviation or mean square error. It is defined as the square root of the average of the squared deviations from the mean. The standard deviation is the best measure of the accuracy of individual points. In photogrammetric measurements, if systematic errors and blunders were eliminated, it would be a true measure of the magnitude of the random errors.

Four of the photogrammetric surveys (PS1, PS3, PS4, and CM1) were generally better than the F2 field survey by all five measured of accuracy. PS2 and CM2 were better than the F2 survey in all measures except the probable error and the arithmetic mean. The CM3 survey was slightly poorer than F2; however, if the portion from Station 160 to 163+50 is omitted from the CM3 survey, the resulting CM3A is better than F2 by all measures except the probable error. Accuracy of the CM3A portion is probably the most nearly representative of the general average of mapping being obtained under contract by the California Division of Highways.

It is generally considered that spot heights, such as photogrammetric cross-sections, read directly in a stereoplotter, have at least double the accuracy of points interpolated from a contour map. Results shown in Table 1 indicate lower standard deviations for the photogrammetric cross-sections on comparable surveys. There is no significant difference, however, in the arithmetic mean of points read in the cross-section surveys as compared to those from contour maps.

In considering the accuracy of the two methods of making photogrammetric measurements the type of terrain should not be overlooked. As slopes become flatter, the accuracy of points taken from contour map tends to decrease due to difficulties in interpolation. This is particularly true if the terrain is irregular.

Errors in Photogrammetric Measurements

Study and analysis of photogrammetric errors in numerous large-scale contour maps have indicated the following characteristics which are pertinent to this study:

1. There are relatively few large individual blunders in photogrammetric measurements.
2. Systematic errors are not constant but tend to vary from model to model and even within individual stereomodels. Systematic errors in compilation, such as those caused by blunders in identification or values of photo control, tend to spread over a considerable area.
3. The arithmetic mean of the errors of a representative sample, such as a centerline profile, provides a good indication of the average of the systematic errors and blunders in the portion of the mapping from which the sample was taken.

These characteristics lead to the conclusion that for any selected small area, such as a single cross-section 2 or 3 hundred ft in length, systematic errors and blunders tend to remain fairly constant.

The standard deviation and arithmetic mean of the photogrammetric surveys in Table 1 show wide variations in accuracy between the surveys and even within the same survey. The latter is true of the CM3 survey where 16 out of 21 points in error by more than ± 1.0 ft occurred in the portion from Station 160 to 163+50. The arithmetic mean for this portion was + 1.2 ft while for the remainder (CM3A) it was +0.10 ft.

A field profile had previously been run on a 4-mi section mapped under the same aerial survey contract as CM3. The arithmetic mean of 185 centerline points in this four miles was +0.01 ft, apparently indicating freedom from systematic errors. However, by dividing the profile into three sections, each having over 55 points, the arithmetic means of the individual sections were found to be +0.45 ft, -0.03 ft and -0.30 ft, respectively.

It has been previously noted that all of the photogrammetric surveys except CM3 utilized the same photography and control. All of the stereoplotter operators working on the various compilations were experienced, and reported that the photography and control were excellent. None of the operators were furnished any information concerning true elevations other than the five vertical control points per model. Under such conditions similar results might naturally be expected. Actually there was a wide variation, particularly in the arithmetic mean of the points read.

For some purposes, errors such as shown in Table 1 may be considered insignificant. However, it will be shown later than an arithmetic mean as small as ±0.1 ft can cause serious discrepancies in earthwork quantities. Errors of this magnitude were not eliminated under the almost ideal conditions of photography and control prevailing on the test section. It therefore appears too much to expect that they will not occur under the more adverse conditions certain to be encountered in actual practice. This is particularly true when mapping specifications do not include a limitation on the arithmetic mean.

The importance of small systematic errors in the computation of earthwork quantities cannot be over-emphasized. It should be apparent that such errors cannot be found by casual inspection or by plotting comparative profiles. They can be detected, and their magnitude determined, only by calculation of the arithmetic mean of a sufficient number of points to form an adequate statistical base.

EARTHWORK QUANTITIES

Types and Uses

In California practice, three classes of earthwork quantities are used in the various stages of highway design and construction. They are:

1. Preliminary quantities for comparison of alternate lines and project report estimates. The accuracy requirements for this stage vary widely according to the demands of the individual project. Sources of data include: aerial photographs, USGS quadrangle maps, and photogrammetric reconnaissance maps ranging in scale from 1 in. = 400 ft with 20-ft contours to 1 in. = 200 ft with 5-ft or 10-ft contours.

2. Design quantities for positioning of the final line and for design of the grade line, slopes, etc. Projects are advertised for construction on the basis of the design quantities. They should have sufficient accuracy that troublesome revisions in alignment, grade line, or slopes will not be required during construction.

General practice is to obtain design quantities from a 1 in. = 50-ft photogrammetric map with 2-ft contours or, in flat terrain, with a grid of spot elevations. The latter may be in the form of photogrammetric cross-sections. Such sources have been generally satisfactory except for a few individual projects where errors in the photogrammetric mapping have resulted in serious imbalance in the quantities, causing difficulties and added cost during construction.

3. Pay quantities to be used as the basis for payment to the construction contractor. California specifications provide for payment on the basis of planned quantities plus authorized changes and unpreventable slides. Final cross-sections taken after construction are therefore seldom required. The accuracy of excavation quantities must be sufficient to insure equitable payment to the contractor. Embankment quantities on most projects are used only for balancing cut and fill, and as a guide to the payment of overhaul. Due to probable variations in estimated shrinkage factors, somewhat lower standards of accuracy could therefore be considered tolerable in embankment areas.

The usual practice is to obtain pay quantities by field cross-sections taken immediately prior to construction. This is generally during construction staking and after the project has been advertised for bids. The methods and standards of accuracy of the field cross-section survey are left to the judgment of the engineer in charge. One district has issued instructions to the effect that field cross-sections may be omitted in areas where sufficient checks of the photogrammetric maps indicate that resulting quantities will not vary more than one percent from those obtained by field surveys.

This study is primarily concerned with design quantities, pay quantities, and their efficient correlation.

Standards of Accuracy

Two facts must be immediately recognized in any study of earthwork quantities:

1. There are no rational, commonly accepted tolerance limits for their accuracy.
2. Any expression of earthwork quantities is approximate, as it involves measurements which can only approximate the actual terrain.

The difference in earthwork quantities as obtained from two surveys is commonly expressed as the difference in percent or the difference in cubic yards. The term "error" is seldom used as the engineer knows that both surveys are subject to error. He is frequently in doubt as to which is the more accurate. Differences of from 2 to 5 percent between photogrammetric quantities and those obtained from field surveys occasionally have been cited as evidence of the accuracy of photogrammetric surveys. Actual errors of this magnitude on large projects could result in completely unacceptable inequities in payment to the construction contractor.

Due to the difficulty and cost of obtaining a reliable yardstick for determining accuracy of earthwork quantities and the variation in requirements between projects, it is improbable that definite tolerance limits can be established. However, guides can be furnished which will assist the engineer in selecting the most suitable method of measuring earthwork quantities for a particular project or type of project. The probable accuracy and relative cost of various survey methods are the most important of these guides. Other factors which the engineer may consider are the unit cost of earthwork, the total quantity to be moved, and its relation to the total size of the construction project.

Sources of Error

In considering the premise that any measurement of earthwork is at best an approximation, the first step is to define the sources of errors. At least two such sources are "built in" by many construction specifications including those of California. These are: computation by end area formula; and non-correction for the effect of curvature. In most cases these errors are relatively minor. In all cases they can be considered to result in equitable payment to the contractor as the method is specified and is presumably taken into consideration in bidding. It should be pointed out, however, that failure on the part of the designer to recognize the effect of curvature can, on some projects, result in an imbalance of quantities far exceeding that due to variations in shrinkage factors or errors in measurement.

Other sources of errors which must be given consideration are related to:

1. Computation of quantities.
2. Density of terrain measurements.
3. Accuracy of terrain measurements.

Errors in Computation

At present this is the least important of the sources of errors due to general use of high speed electronic computers to convert the basic measurements to cubic yards. It should be emphasized that machine computation is an exact method and the results are subject only to errors due to inadequate density of measurements and to errors in those measurements.

As the planimeter was, for many years, the standard method of measuring cross-sectional areas, computation by this method was included in the study. Cross-sections obtained by the F2 survey were plotted at a scale of 1 in. = 10 ft and the quantities determined by planimetered areas and by the Avol Rule. The latter is an instrument for determining earthwork quantities by cumulative measurements of equally spaced vertical ordinates. In this case the spacing was 5 ft.

Comparative quantities of excavation and embankment obtained by the three methods of computation are shown in Table 2. Results varied by a maximum of 0.6 percent.

Density of Terrain Measurements

In terrain similar to that of the test site, California practice is to take cross-sections at 50-ft intervals. The distance between points on each cross-section line generally does not exceed 50 ft. Cross-sections taken at 25-ft intervals in the F1 survey afforded an opportunity to determine the importance of density in this type of terrain.

The quantities resulting from 25-, 50-, and two different arrangements of 100-ft cross-section intervals are shown in Table 3. For the 100-ft intervals shown in Column 3, cross-sections were used at Stations 161, 162, 163, etc. For the results in Column 4 they were used at Stations 161+50, 162+50, 163+50, etc. The relatively minor errors in quantities due to 50- and 100-ft intervals indicate that intervals of 100 ft would be entirely satisfactory for design quantities in this type of terrain.

However, it should not be overlooked that design quantities are a source of information for roadbed notes and slope-stake data which are of considerable value in construction staking. While the designer may save time by using larger cross-section intervals on trial lines and grades, he should keep construction staking requirements in mind in preparing design quantities for the final line and grade. In general, therefore, slope-stake spacing rather than accuracy of quantities may govern the cross-section interval for the final line and grade.

TABLE 2

COMPARISON OF EARTHWORK QUANTITIES COMPUTED BY VARIOUS METHODS—F2 SURVEY

	Quantity Cu. Yd.	Error Cu. Yd.	Error %
Excavation			
Machine Comp.	64,212		
Planimeter	64,364	+152	0.2
Avol Rule	63,840	-372	0.6
Embankment			
Machine Comp.	28,654		
Planimeter	28,774	+120	0.4
Avol Rule	28,612	- 42	0.1

Prediction of Errors in Photogrammetric Quantities

It has been previously mentioned that the arithmetic mean of a sample, such as a centerline profile, is an excellent guide to the average of the systematic errors and blunders in the area covered by the sample. If this is true it should be possible to predict, within reasonable limits, the total error in approximating the terrain in each of the photogrammetric surveys. The results of such predictions are shown in Table 4.

TABLE 3

COMPARISON OF EARTHWORK QUANTITIES FOR VARIOUS CROSS-SECTION INTERVALS—F1 SURVEY

	(1) 25-ft Spacing	(2) 50-ft Spacing	(3) 100-ft Spacing	(4) 100-ft Spacing
Excavation:				
Quantity - cu. yd.	63, 118	63, 167	63, 128	63, 244
Error - cu. yd.		+49	+10	+126
Error - percent		0. 1	0. 0	0 2
Embankment:				
Quantity - cu. yd.	29, 223	29, 152	29, 232	28, 663
Error - cu. yd.		-71	+9	-560
Error - percent		0. 2	0. 0	1 9

The surface area of the test site, between slope stakes, was approximately 380, 000 sq ft. Multiplying this area by the arithmetic mean of the centerline profile and dividing by 27 gave the predicted error in cubic yards. The actual errors are taken from Table 6. It will be noted that the maximum error in prediction for any of the surveys was only 0. 6 percent of the total quantity involved. The accuracy of these predictions emphasizes the importance of the arithmetic mean as a guide to probable errors in photogrammetric quantities.

These results have several implications for the designer, map checker and the construction engineer. For the designer, it provides a method of determining the probable imbalance of excavation and embankment quantities between any desired limits as soon as a profile, on either the centerline or any base line between the slope stakes, is available. Such information if properly utilized will do much to eliminate major revisions during construction due to errors in the photogrammetric survey.

While a photogrammetric survey may cover an area 1, 200 ft or more in width, in most cases the width between slope stakes will not exceed 200 to 300 ft. On many projects the location of the centerline is known within much smaller limits than the width of mapping would indicate. As accuracy required for earthwork quantities is confined to this relatively narrow band, it follows that the map checker should generally concentrate his efforts in this area of principal importance rather than attempt to test a remote corner of the mapping for compliance with specifications. By running test profiles generally parallel to the proposed centerline and calculating the arithmetic mean of the errors, the map checker can provide valuable information to the designer and to the construction engineer.

Comparison of Earthwork Quantities

Table 5 shows a comparison of excavation and embankment quantities obtained from two field surveys and six photogrammetric surveys. The same cross-section interval of 50 ft, plus nine additional cross-sections at breaks in the terrain, was used for all surveys. The resulting differences from the F1 survey are, for all practical purposes,

TABLE 4

PREDICTED ERRORS IN TOTAL EARTHWORK QUANTITIES FROM PHOTOGRAMMETRIC SURVEYS

Survey	Arithmetic Mean of Centerline Profile (ft)	Predicted Error Cu yd	Actual Error Cu yd	Error in Prediction Cu yd	Error in Prediction as % of Total Quantity
PS1	+0. 12	+1, 690	+1, 340	350	0. 4
PS2	+0. 29	+4, 090	+3, 822	268	0. 3
PS3	+0. 06	+ 845	+ 821	24	0. 0
CM1	+0. 04	+ 565	+ 262	303	0. 3
CM2	+0. 27	+3, 800	+3, 228	572	0. 6
CM3	+0. 18	+2, 530	+2, 247	283	0. 3

due to errors in measurement of elevations. It is apparent that this is by far the most important of the various sources of errors in earthwork quantities.

Accuracy of excavation quantities of four of the photogrammetric surveys, PS1, PS3, CM1, and CM3, would be considered satisfactory for pay quantities by almost any standards. The F2 field survey and the PS2 and CM2 surveys would be generally satisfactory for design quantities. Some engineers might consider them satisfactory for pay quantities. In measurement of embankment quantities, however, only the F2, PS3, and CM1 surveys could be considered satisfactory.

For determination of probable error in balance between cut and fill, the total error as shown in Table 6 is undoubtedly the best measure. In this case all of the surveys showed plus errors in excavation and minus errors in embankment. Insofar as balance is concerned these errors are cumulative and the total error is their sum. In cases where both excavation and embankment errors have the same sign they would tend to compensate and the total error would be the difference.

Probably the best method of expressing the actual accuracy of the various surveys as related to earthwork quantities is the "Equivalent Vertical Error" also shown in Table 6. It was calculated by dividing the total error in cubic feet by the area between slope stakes in square feet. In effect, therefore, it is the mean vertical difference in each survey from the terrain as depicted by the F1 survey. The equivalent vertical error also could be determined by taking the arithmetic mean of the errors of a large number of equally spaced points over the entire area. The total error in volume could then be found by multiplying the equivalent vertical error by the area.

Comparison of the equivalent vertical error and the arithmetic mean of the centerline profile shows a very close relationship for each of the surveys. This relationship is quite important as it indicates a means of correlating the accuracy of a survey, in measuring the elevation of discrete points, to the accuracy of earthwork quantities. However, the values of the equivalent vertical error as shown in Table 6 are averages for the entire area, and are not applicable to individual cross-sections or to small portions of the mapping. This is clearly shown by the fact that, while total errors could be predicted very closely by the arithmetic mean of the centerline profile, the major errors in earthwork quantities occurred in embankment areas.

Lack of uniformity in the errors in individual surveys confirms previous

TABLE 5

COMPARISON OF EXCAVATION AND EMBANKMENT QUANTITIES FROM FIELD AND PHOTOGRAMMETRIC SURVEYS

	Excavation			Embankment		
	Quantity	Error		Quantity	Error	
Survey	Cu. yd.	Cu. yd.	%	Cu. yd.	Cu. yd.	%
F1	63,167			29,152		
F2	64,212	+1,045	1.7	28,654	- 498	1.7
PS1	63,338	+ 171	0.3	27,983	-1,169	4.0
PS2	64,717	+1,550	2.5	26,880	-2,272	7.8
PS3	63,678	+ 511	0.8	28,842	- 310	1.1
CM1	63,187	+ 20	0.0	28,910	- 242	0.8
CM2	64,303	+1,136	1.8	27,060	-2,092	7.2
CM3	63,174	+ 7	0.0	26,912	-2,240	7.7

TABLE 6

COMPARISON OF TOTAL ERRORS IN QUANTITIES FROM PHOTOGRAMMETRIC SURVEYS

Survey	Total Error Cu. Yd.	Total Error (%)	Equivalent Vertical Error (ft)	Arithmetic Mean of Centerline Profile (ft)
PS1	1,340	1.4	+0.10	+0.12
PS2	3,822	4.1	+0.27	+0.29
PS3	821	0.9	+0.06	+0.06
CM1	262	0.3	+0.02	+0.04
CM2	3,228	3.5	+0.23	+0.27
CM3	2,247	2.5	+0.16	+0.18

experience in comparing quantities from photogrammetric surveys with those from field surveys. It has been frequently found that errors in individual cuts and fills are far greater than the error for an entire project. Such variations are not the result of random accidental errors which are unpreventable. They can almost always be attributed to varying systematic errors and blunders. The fact that, in this particular case, the major errors occurred in embankment areas is not considered significant. It might be due to chance or to conditions peculiar to the test site. These conditions might be reversed on an adjacent project.

The comparisons shown in Tables 5 and 6 lead to the conclusion that the serious effect of relatively small systematic errors on earthwork quantities make the use of photogrammetric surveys for pay quantities questionable unless such surveys are adjusted or thoroughly checked in a satisfactory manner.

Adjustment of Quantities from Photogrammetric Surveys

The remarkably accurate results in predicting total errors shown in Table 4 indicate the possibility of reducing the errors in individual cross-sections by adjusting the terrain to a centerline elevation determined by a field survey. Several states have reported greatly increased accuracy in photogrammetric cross-sections by indexing on the field elevations at centerline while taking cross-sections from the stereomodel. Such a procedure requires determination of the final line and staking the line in the field prior to obtaining large-scale mapping. If similar results could be obtained by adjustment to a line staked in the field after the mapping was completed, it would provide much greater flexibility in design procedures.

As a test of this possibility, quantities from the six photogrammetric surveys were adjusted by raising or lowering the entire terrain at each cross-section by an amount equal to the error in the centerline elevation. The method of setting up the adjustments was to provide the tabulation section with a list showing difference in elevation at each centerline station between the F1 survey and the photogrammetric survey. New tabulations based on the adjusted terrain notes were provided by the tabulation section. On future projects it is anticipated that the difference in elevation at centerline will be computed by machine. In this case the only data to be furnished by the engineer will be the field and photogrammetric centerline elevations.

Errors in cubic yards and percent, both before and after adjustment, are shown for excavation quantities in Table 7 and for embankment quantities in Table 8. As a means of observing the localized effects of the adjustments, the quantities are shown in three segments for both excavation and embankment. Comparative quantities from the F2 field survey and the method of making the survey are also shown for each segment. No adjustment of the F2 quantities was possible as the Rhodes Arc elevations were based on centerline elevations, which had been corrected for obvious blunders.

The results show that in four of the six individual segments the adjusted quantities of all of the photogrammetric surveys were more nearly correct than quantities from the F2 field survey. Errors in both excavation and embankment totals by the F2 survey were greater than by any of the adjusted photogrammetric surveys. In only one case, embankment quantities by PS1, were the adjusted totals of either excavation or embankment quantities from photogrammetric surveys in error by more than one percent.

In several cases, where quantities were in error by relatively small amounts, the adjusted totals showed slightly greater errors than the original unadjusted quantities. This is to be expected and can be accepted if the centerline adjustments will (a) materially reduce large localized errors; and (b) result in quantities which are within tolerable limits.

The question of large localized errors is best illustrated by the portion of CM3 from Station 160 to 164. It has been previously noted that most of the errors of over 1.0 ft occurred in this section. As shown in Table 8 the centerline adjustment reduced the error in this portion of the survey from 1,378 cu yds to 95 cu yds and from 12.8 to 0.9 percent.

As to tolerable limits, adjustment of the six photogrammetric surveys of the test section resulted in quantities within limits generally considered tolerable for purposes of payment. The adjusted quantities from all of the photogrammetric surveys were more nearly correct than those obtained from a field survey made by commonly accepted methods.

Comparative values for the previously discussed measures of total error and equivalent vertical error, both before and after adjustment, are shown in Table 9. In all cases, where the original quantities were in error by any appreciable amount, the total

TABLE 7

DETAILED COMPARISON OF EXCAVATION QUANTITIES AND ADJUSTMENTS

		F1	F2	PS1	PS2	PS3	CM1	CM2	CM3
Sta. 163+50 to 174	Quantity - cu. yd.	2065	2406	2152	2254	1956	2123	2438	2403
	Error - cu. yd.		+341	+87	+189	-109	+58	+373	+338
	Error - percent		16.5	4.2	9.1	5.3	2.8	18.1	16.4
	Adjustment - cu. yd.		(Rhodes	-116	-272	+42	-27	-305	-318
	Net Error - cu. yd.		Arc)	-29	-83	-67	+31	+68	+20
	Net Error - percent			1.4	4.0	3.2	1.5	3.3	1.0
	Relative accuracy		(7)	(2)	(6)	(4)	(3)	(5)	(1)
Sta. 174 to 180	Quantity - cu. yd.	45845	46570	45610	46807	46023	45571	46459	45673
	Error - cu. yd.		+725	-235	+962	+178	-274	+614	-172
	Error - percent		1.6	0.5	2.1	0.4	0.6	1.3	0.4
	Adjustment - cu. yd.		(Rhodes	+74	-679	+26	+326	-543	+481
	Net Error - cu. yd.		Arc)	-161	+283	+204	+52	+71	+309
	Net Error - percent			0.4	0.6	0.4	0.1	0.2	0.7
	Relative accuracy		(7)	(3)	(5)	(4)	(1)	(2)	(6)
Sta. 180 to 189	Quantity - cu. yd.	15257	15236	15576	15656	15699	15493	15406	15098
	Error - cu. yd.		-21	+319	+399	+442	+236	+149	-159
	Error - percent		0.1	2.1	2.6	2.9	1.5	1.0	1.0
	Adjustment - cu. yd.		(Engineer's	-494	-409	-446	-331	-144	+350
	Net Error - cu. yd.		Level)	-175	-10	-4	-95	+5	+191
	Net Error - percent			1.1	0.1	0.0	0.6	0.0	1.2
	Relative accuracy		(4)	(6)	(3)	(1)	(5)	(2)	(7)
Total excavation	Quantity - cu. yd.	63167	64212	63338	64717	63678	63187	64303	63174
	Error - cu. yd.		+1045	+171	+1550	+511	+20	+1136	+7
	Error - percent		1.7	0.3	2.5	0.8	0.0	1.8	0.0
	Adjustment - cu. yd.			-536	-1360	-378	-33	-992	+513
	Net Error - cu. yd.			-365	+190	+133	-13	+144	+520
	Net Error - percent			0.6	0.3	0.2	0.0	0.2	0.8
	Relative accuracy		(7)	(5)	(4)	(2)	(1)	(3)	(6)

TABLE 8

DETAILED COMPARISON OF EMBANKMENT QUANTITIES AND ADJUSTMENTS

		F1	F2	PS1	PS2	PS3	CM1	CM2	CM3
Sta. 160 to 164	Quantity - cu. yd.	10764	10644	10646	10202	10812	10619	10560	9386
	Error - cu. yd.		-120	-118	-562	+48	-145	-204	-1378
	Error - percent		1.1	1.1	5.2	0.4	1.3	1.9	12.8
	Adjustment - cu. yd.		(Engineer's	+73	+490	-7	+182	+244	+1473
	Net error - cu. yd.		Level)	-45	-72	+41	+37	+40	+95
	Net error - percent			0.4	0.8	0.4	0.3	0.4	0.9
	Relative accuracy		(7)	(4)	(5)	(3)	(1)	(2)	(6)
Sta. 164 to 174+50	Quantity - cu. yd.	15383	14978	14504	14085	15311	15335	14235	14640
	Error - cu. yd.		-405	-879	-1298	-72	-48	-1148	-743
	Error - percent		2.6	5.7	8.4	0.5	0.3	7.5	4.8
	Adjustment - cu. yd.		(Rhodes	+526	+1048	-7	+72	+1182	+517
	Net error - cu. yd.		Arc)	-353	-250	-79	+24	+34	-226
	Net error - percent			2.3	1.6	0.5	0.2	0.2	1.5
	Relative accuracy		(7)	(6)	(5)	(3)	(1)	(2)	(4)
Sta. 183 to 190	Quantity - cu. yd.	3005	3032	2833	2593	2719	2956	2265	2886
	Error - cu. yd.		+27	-172	-412	-286	-49	-740	-119
	Error - percent		0.9	5.7	13.7	9.5	1.6	24.6	4.0
	Adjustment - cu. yd.			+197	+468	+285	+237	+808	+221
	Net error - cu. yd.		(Engineer's	+25	+56	-1	+188	+68	+102
	Net error - percent		Level)	0.8	1.9	0.0	6.3	2.3	3.4
	Relative accuracy		(3)	(2)	(4)	(1)	(7)	(5)	(6)
Total embankment	Quantity - cu. yd.	29152	28654	27983	26880	28842	28910	27060	26912
	Error - cu. yd.		-498	-1169	-2272	-310	-242	-2092	-2240
	Error - percent		1.7	4.0	7.8	1.1	0.8	7.2	7.7
	Adjustment - cu. yd.			+796	+2006	+271	+491	+2233	+2210
	Net error - cu. yd.			-373	-266	-39	+249	+141	-30
	Net error - percent			1.3	0.9	0.1	0.9	0.5	0.1
	Relative accuracy		(7)	(6)	(5)	(2)	(4)	(3)	(1)

TABLE 9

EFFECT OF ADJUSTMENT ON TOTAL ERROR AND EQUIVALENT VERTICAL ERROR

	Total Error in Cu. yd.		Total Error in %		Equivalent Vertical Error (ft)	
Survey	Before Adjust.	After Adjust.	Before Adjust.	After Adjust.	Before Adjustment	After Adjustment
F2	1,543		1.7		+0.11	
PS1	1,340	8	1.4	0.0	+0.10	+0.00
PS2	3,822	456	4.1	0.5	+0.27	+0.03
PS3	821	172	0.9	0.2	+0.06	+0.01
CM1	262	262	0.3	0.3	+0.02	-0.02
CM2	3,228	3	3.5	0.0	+0.23	+0.00
CM3	2,247	550	2.5	0.6	+0.16	+0.04

error was considerably reduced by the centerline adjustment. The maximum resulting equivalent vertical error after adjustment was 0.04 ft shown by photogrammetric survey CM3.

The comparisons shown in Tables 6 and 9 indicate the value of the equivalent vertical error as a measure of the accuracy of earthwork quantities. It is the only type of measure which can be directly related to the accuracy of the survey in measuring the elevations of discrete points. Unlike the percent of error it is not affected by the volume of earthwork involved.

The results in Tables 7, 8, and 9 show little, if any, significant difference in accuracy, either before or after adjustment, between quantities from photogrammetric cross-sections and those obtained from photogrammetric contour maps. This is readily understandable when two factors are considered:

1. The added accuracy of photogrammetric cross-sections in reading the elevations of discrete points applies only to random errors.
2. Most of the large errors in earthwork quantities obtained from photogrammetric measurements are due to systematic errors.

Analysis of Adjustments

While the general effect of the centerline adjustment on earthwork quantities was very impressive, the results could be attributed largely to chance unless they tended to reduce the errors in individual cross-sections. As the cross-sections varied in width between slope stakes, from a minimum of 95 ft to a maximum of 178 ft, the errors in cross-sectional area could not be used as a basis for direct statistical comparison. The errors could, however, be reduced to a one-dimensional variable, the equivalent vertical error, by dividing the error in area of each cross-section by the width between slope stakes.

The equivalent vertical errors for each of the 70 cross-sections of the six photogrammetric surveys, both before and after adjustment, were calculated in this manner and arranged in frequency distributions. By comparing before and after frequency distributions for each survey it was apparent that the adjustments greatly improved the conformity to normal distribution, indicating much better statistical control.

Values of the standard deviation and arithmetic mean are shown in Table 10. The wide variation in accuracy of the various photogrammetric surveys has been previously noted. This is also shown by the variations in standard deviations of the equivalent vertical errors before adjustment, which range from 0.17 to 0.45 ft. The most significant fact shown by the analysis is that the centerline adjustments reduced this range to a minimum of 0.14 ft and a maximum of 0.23 ft. This clearly indicates the equalizing effect of the adjustment of the wide variations in accuracy of the various surveys.

As might be expected, the effect of the adjustment was to greatly reduce any large

TABLE 10

COMPARISON OF THE EQUIVALENT VERTICAL ERRORS OF 70 CROSS-SECTIONS

Survey	Standard Deviation Before Adjustment (ft)	Standard Deviation After Adjustment (ft)	Arithmetic Mean Before Adjustment (ft)	Arithmetic Mean After Adjustment (ft)	Standard Error of the Mean After Adjust. (ft)
PS1	0.17	0.14	+0.12	-0.01	0.02
PS2	0.20	0.17	+0.31	+0.01	0.02
PS3	0.19	0.15	+0.08	+0.02	0.02
CM1	0.17	0.19	+0.03	-0.04	0.02
CM2	0.31	0.22	+0.29	-0.01	0.03
CM3	0.45	0.23	+0.16	+0.02	0.03
CM3A[1]	0.33	0.23	+0.05	+0.02	0.03

[1] Portion from Stations 164 to 190 - 62 cross-sections.

errors in the arithmetic mean. Values of the arithmetic mean differ slightly from the equivalent vertical errors shown in Table 9 as the latter are, in effect, weighted averages of the individual cross-sections. Values of the standard error of the mean, after adjustment, shown in Table 10, indicate that the corresponding values of the arithmetic mean are, in all cases, within the limits of a normal distribution.

It will be noted that, after adjustment, the standard deviations of the photogrammetric cross-section surveys were slightly lower than those for the contour-map surveys. This indicates that quantities from photogrammetric cross-sections will tend to have slightly greater accuracy after adjustment than those from contour maps. For 70 cross-sections, however, the standard error of the mean is so small as to make the difference insignificant.

The primary purpose of the centerline adjustment of the photogrammetric measurements would be to reduce the errors in earthwork quantities. However, some consideration must be given to its effect on the accuracy of individual points, since the adjusted photogrammetric measurements may be used during construction staking. The measures of the 90 percent specification tolerance, standard deviation and arithmetic mean were used to determine the effects of the centerline adjustment on 122 slope stakes of the test section. Photogrammetric measurements of slope stake elevations were read directly in the stereoplotter for the PS1 and PS3 surveys. For the other surveys, elevations were determined by interpolation. Before and after adjustment values of the three measures for each of the surveys are shown in Table 11.

TABLE 11

EFFECT OF ADJUSTMENT ON POINT ACCURACY OF 122 SLOPE STAKES

Survey	90% Within Before Adjust. (ft)	90% Within After Adjust. (ft)	Standard Deviation Before Adjust. (ft)	Standard Deviation After Adjust. (ft)	Arithmetic Mean Before Adjust. (ft)	Arithmetic Mean After Adjust. (ft)
PS1	±0.4	±0.3	0.22	0.22	+0.13	+0.02
PS2	±0.7	±0.4	0.28	0.28	+0.36	+0.08
PS3	±0.4	±0.4	0.27	0.23	+0.11	+0.07
CM1	±0.5	±0.5	0.32	0.36	+0.06	+0.03
CM2	±0.8	±0.6	0.33	0.38	+0.35	+0.06
CM3	±1.1	±0.9	0.59	0.56	+0.29	+0.10

The general effects were to slightly improve the 90 percent limit, and to greatly reduce the arithmetic mean. The standard deviation was slightly greater after adjustment in two of the surveys, slightly less in two, and unchanged in the remaining two.

Probably the best explanation of the effect of the centerline adjustments can be found in the previously stated conclusion that "for any selected small area, such as a single cross-section two or three hundred feet in length, systematic errors and blunders tend to remain fairly constant." On this basis the centerline adjustment should tend to greatly reduce the systematic errors and blunders and to slightly increase the random errors. If this is correct, the effectiveness of the adjustment for a particular project would depend on the ratio of random errors to systematic errors and blunders. Unfortunately it is impossible to make a definite segregation of the types of errors in a photogrammetric survey.

Results from the surveys of the test section emphasize the importance of systematic errors and indicate that their effect can be minimized by the centerline adjustment.

COSTS

The over-all advantages of photogrammetric surveys over conventional field surveys have become generally recognized during the past few years. Therefore no attempt was made to expand the study to include complete costs of the two basic survey methods. However, the costs of certain phases of survey and design work are factors which must be considered in selecting a method for obtaining earthwork quantities. A record was therefore kept of the man-hours required for various operations in the phases of field surveys, stereocompilation and calculation of earthwork quantities. This information together with the calculated cost per mi for each operation is shown in Table 12.

Among the comparisons which can be made from Table 12 is the savings by use of machine computation over the former method of plotting cross-sections and planimetering the areas. Assuming that a contour map is the basic source of data, machine computation of earthwork quantities requires Items 9, 10, and 15 for a cost per mile of $330. The planimeter method involves Items 12 and 13 for a total of $570 per mi. The savings by use of machine computation in this particular case was $240 per mile or 42 percent. Use of the Avol Rule instead of the planimeter would have saved $125 per mi.

The cost of obtaining pay quantities by field cross-sections is the sum of Items 3 and 15, or $725 per mi. Assuming that the design quantities had been taken from a preliminary line on the contour map, or that the interval was too great for pay quantities, the designer could have prepared new terrain notes (Item 10), obtained quantities (Item 15), and adjusted the quantities to the centerline profile (Item 15) for a total of $305

TABLE 12

TIME AND COSTS[1]

Item	Operation	Survey	Avg Width (ft)	Length (ft)	Man hours	Approx Cost per mi
1	Centerline Profile	F2		3,000	22	$ 200
2	Cross-section - 25-ft interval	F1	140	3,000	160	1,480
3	Cross-section - 50-ft interval	F2	165	3,000	68	625
4	Set Slope Stakes	F2		3,000	50	460
5	Stereo Setup - per model	Avg except CM3		900	1.3	55
6	Stereo Compilation - Contours	Avg CM 1 and 2	500	3,400	10.4	120
7	Stereo Compilation - Cross-sections	Avg PS 1 and 2	250	3,400	6.9	80
8	Stereo Readout - Cross-sections	PS3	250	3,400	6.8	95
9	Prepare and Check Roadbed Notes	All		3,000	18	125
10	Prepare and Check Terrain Notes	Avg CM1, 2 and 3	170	3,000	15	105
11	Prepare and Check Terrain Notes	Avg PS1 and 2	190	3,000	8	55
12	Plot Cross-Sections and Templates	F2	165	3,000	44	310
13	Planimeter and Calculate Quantities	F2		3,000	37	260
14	Calculate Quantities - Avol Rule	F2		3,000	19	135
15	Key Punch and Machine Computation	Avg except PS3		3,000		100
16	Machine Computation	PS3		3,000		80

[1] Field survey costs include living expenses but no travel time.
Stereoplotter costs are based on Kelsh plotter rental of $7.50 per hr including operator.
Stereoplotter costs plus Terrain Data Translator (PS3) are estimated at $9.00 per hr.

per mi. For the test section, therefore, the saving by using adjusted photogrammetric quantities for payment in lieu of obtaining field cross-sections would have been $420 per mi or 58 percent. It has been previously shown that adjusted quantities from all of the photogrammetric surveys of the test section were more nearly correct than those obtained from the F2 field survey.

A comparison of the relative cost of stereocompilation, preparing terrain notes and machine computation between a contour map (CM1), photogrammetric cross-sections written on a manuscript (PS1), and photogrammetric cross-sections taken directly from the stereomodel to punch cards by use of the Terrain Data Translator (PS3), is as follows:

CM1 - Items 6, 10, and 15	$325 per mi
PS1 - Items 7 (Adjusted), 11, and 15	$315 per mi
PS3 - Items 8 (Adjusted), and 16	$270 per mi

The costs per mile of Items 7 and 8 as adjusted are twice the amounts shown in Table 10 to place stereocompilation costs on a uniform basis of 500-ft average width. The above comparison applies only to relative costs as a measure of savings and does not include items which are common to all of the methods such as photography, photo control, model set up and preparation of roadbed notes.

From the foregoing analysis the comparative savings which can be achieved in three of the steps involved in obtaining earthwork quantities can be summarized as follows:

1. Saving by machine computation as compared to plotting and planimetering cross-sections - $240 per mi.
2. Saving by automation in taking digital data directly from the stereomodel to punch cards, as compared to use of a contour map - $55 per mi.
3. Saving by adjusting photogrammetric quantities for payment in lieu of taking field cross-sections of the final line - $420 per mi.

It should be noted that the savings under 1 are applicable to a number of trial lines as well as the final line while those under 2 and 3 generally apply only to the final line. Nevertheless the 58 percent saving in manpower by using adjusted photogrammetric quantities for payment indicate the value of developing a method for their use.

SELECTION OF METHODS

The two basic sources of photogrammetric terrain data are: contour maps; and cross-sections from spot heights read directly in the stereoplotter. Information concerning relative accuracy and costs developed by this study should assist the engineer in making a choice between the two sources.

The most frequently mentioned advantages of the cross-section method are the added accuracy and the saving in cost and manpower by taking digital terrain data from the stereomodel directly to punch cards or tape. This savings in cost for the test section amounted to $55 per mi. The related saving in manpower, therefore, can be considered relatively minor when compared to the total engineering effort required to obtain earthwork quantities.

Surveys of the test section showed no significant differences in accuracy between quantities taken from a contour map and those from photogrammetric cross-sections. This is due to the fact that the added accuracy in reading spot heights applies only to random errors and has no effect on systematic errors. The latter have by far the most serious effect on the accuracy of earthwork quantities obtained from photogrammetric surveys. Except in comparatively flat terrain, the relative accuracy of the two methods does not appear to be an important factor.

The principal advantage of a contour map, as a source of terrain data, is the flexibility in procedure it provides. The designer can determine the approximate location of the final line from preliminary source data and obtain a large-scale contour map covering a band from 1,000 to 1,500 ft in width. The map can then be used to determine the exact position of the final line, and digital terrain data based on the final line can be taken from it. Design work can proceed without the delay inherent in the photogrammetric cross-section methods due to staking the final line in the field, rephotographing the area or, as a minimum, resetting the models in the stereoplotter. These advantages appear to be the most important factors in making a choice between a contour map and photogrammetric cross-sections.

In flat terrain, where spot elevations are preferable to a contour map, consideration should be given to obtaining photogrammetric cross-sections based on a tentative centerline rather than an arbitrary grid of spot elevations. In such terrain the designer can frequently position the final line by the use of large-scale aerial photographs or other available data before obtaining a photogrammetric survey.

Development of photogrammetry and machine computation has provided the engineer with a wide variety of methods from which to choose in obtaining earthwork quantities. A method shown by this study to be relatively efficient and suitable for most terrain can be summarized as follows:

1. Obtain a photogrammetric contour map covering the previously selected route band. Except in very rough terrain a contour interval of two feet is preferable. Specifications for the mapping should place a limitation on the arithmetic mean of the points tested.
2. Develop earthwork quantities for trial lines by machine computation using the maximum cross-section interval consistent with the terrain. For terrain similar to that of the test section, intervals of 100 ft or more would be satisfactory. Similar spacing should be used in selecting points along the cross-section lines. Three-point, or even two-point, roadbed notes usually will be satisfactory for trial lines.
3. Develop design quantities from the contour map after the final line has been positioned on the map and calculated. The interval and stationing of cross-sections for the final line should be consistent with the terrain and with the requirements of slope staking and pay quantities. These intervals generally should not exceed 50 ft in rolling terrain and 100 ft in flat terrain.
4. Adjust the design quantities at such time as an accurate field profile is available. The time at which the field profile should be obtained depends on several factors. Among these are the physical characteristics of the project, the imminence of construction and the engineer's judgment as to the accuracy of the mapping.

CONCLUSIONS

Data developed by this study lead to the following conclusions:

1. The most important factor in the accuracy of earthwork quantities is the vertical accuracy of the survey measurements.
2. Photogrammetric surveys are subject to relatively small systematic errors. Such errors have a serious effect on the accuracy of earthwork quantities.
3. Use of photogrammetric surveys for pay quantities is questionable unless they are checked by statistical comparison with an accurate field profile.
4. The greatest saving in manpower in obtaining earthwork quantities which can be achieved under current California practice is development of a method of utilizing photogrammetric quantities for payment.
5. The most important fact developed by this study is that adjusted quantities from all of the photogrammetric surveys were within limits generally considered tolerable for pay quantities. They were more accurate than quantities obtained by a field survey made by commonly accepted methods.
6. The method of adjusting photogrammetric quantities by use of a centerline profile appears to have considerable potential value as a means of obtaining pay quantities with a minimum expenditure of manpower. Further tests of adjustments on projects scheduled for early construction are planned in the near future.

REFERENCES

1. Miller, C. L., "Digital Terrain Model Approach to Highway Earthwork Analysis."

M. I. T. Photogrammetry Laboratory, Publication 111, August (1957).
2. Miller, C. L. and Kaalstad, T. H., "Earthwork Data Procurement by Photogrammetric Methods." M. I. T. Photogrammetry Laboratory, Publication 115, HRB Bull. 199.
3. Funk, L. L., "Photogrammetric Map Accuracy." Photogrammetric Engineering, June (1958). HRB Bull. 199.
4. ASTM Manual on Quality Control of Materials, Special Publication 15-C.

General Discussion

Questions Asked of and Answered by L. L. FUNK, California Division of Highways

Question: What type of ground survey was made (1st, 2nd, or 3rd order) in measuring cross-sections for the test project?

Answer: The primary horizontal and vertical control surveys were of second order accuracy. A transit was used for turning right angles for the cross-section lines, a steel tape was used for distances out from centerline and elevations were read to .01 ft with an engineer's level. (The person who asked this question remarked that, on field surveys made in his State for measuring cross-sections, they had been found to be in error as much as 3 ft in elevation.)

Question: Was the precision of the ground survey sufficient to give a reliable comparison of all topographic mapping done by photogrammetric methods?

Answer: Yes.

Question: Was the basic vertical control for orientation of stereoscopic models the same for each mapping of the test project?

Answer: The same vertical control was used for the five compilations designated as PS1, PS2, PS3, CM1 and CM2. No difficulties in setting up the stereomodels were reported by any of the operators.

Question: Did all contractors use the same photographic glass plate transparencies in accomplishment of the five separate mappings of this test project?

Answer: Three sets of diapositives were made. Comparisons of these diapositives with a flash plate both at the time they were made and subsequently do not show any measurable dimensional changes.

Question: Were specific 0.0 points determined between the excavation and embankment sections—
(a) On side hill and
(b) At changes from all cut to all fill, and vice-versa?

Answer: No. As cross-sections for each of the surveys were taken at the same stations, omission of zero cut and fill points would have practically no effect on the relative quantities from the various surveys.

Question: Do you consider that adjustment of photogrammetric surveys to a field profile would be practical on projects where excavation costs amounted to as much as $4 or $5 per cu yd?

Answer: This would depend largely on the type of terrain. Such high costs would usually be associated with rock excavation in rugged terrain. If ground cover is not excessive a good photogrammetric survey in irregular terrain should be more accurate than a conventional field survey. Particularly if a line of horizontal and vertical photographic control has been established close to the proposed centerline. Adjustment of the photogrammetric survey to an accurate field profile should effectively eliminate any systematic errors which might occur.

Question: You mentioned that spot elevations measured photogrammetrically were not more accurate than elevations interpolated from the contours of the topographic maps. Is this equally true for all types of photography, the flat (nearly level), the rolling, and the rugged (mountainous)?

Answer: The statement regarding relative accuracy of spot heights and elevations interpolated from contours of the topographic maps was a conclusion drawn from the test section, which was in rolling terrain. In flatter areas the accuracy of interpolation from contours would tend to decrease and the relative accuracy of spot heights would increase. In rugged, irregular terrain the reverse would probably be true unless an extremely high density of spot heights was used.

Question: Is California convinced, in consequence of the tests on which you based your report, that grading quantities computed during design would be sufficiently accurate for contract payment, provided adjustments are made for the difference between the ground elevation at the point where the L-line, as staked on the ground, intersects the cross-section as compared with the elevation interpolated from contours of the map at each identical point? It is realized of course that adjustments would also be made for unavoidable overbreak and slides, and for authorized changes.

Answer: The results on the three test projects were very favorable and have aroused considerable interest. Actual use of the method will undoubtedly develop gradually over a period of several years. Some engineers will continue to take field cross-sections for payment, particularly on projects where the accuracy of the photogrammetric surveys is questionable. Evaluation of the results in such cases will provide additional information as to the magnitude of discrepancies which can be corrected by adjustment to a field profile. This may lead to establishment of acceptable tolerances for the measurement of earthwork quantities.

Question: What has been the attitude of the contractors with regard to specifying greater accuracy, particularly in requiring that the mean of all points tested shall not exceed a specific value in order that systematic errors might be reduced to a practical minimum?

Answer: The California Division of Highways started using specifications which limit the mean vertical error of each map sheet about six months ago. There has been no apparent increase in the contract cost of mapping as a result. This specification has made the mapping contractors realize the importance of correcting the causes of small systematic errors. We believe it will result in considerable improvement in future mapping work. We consider that it is still in the experimental stage insofar as results on any specific project are concerned and also in the limiting values which can be reasonably specified.

Question: Were the tests made by the State to ascertain how well the photogrammetric firms had used the aerial photography and compiled the topographic maps made with the same photography as the contractors used?

Answer: Yes. This is an essential part of our regular procedure in checking the quality of the contractors' use of photogrammetric instruments. By using the same diapositives and the same photographic control used by the mapping contractor our operators can quickly evaluate the quality of the mapping.

Question Asked of and Answered by W. T. PRYOR, Bureau of Public Roads

Question: Are the charts you prepared for determining overlap (endlap and sidelap) applicable to only one focal length aerial camera, such as 6 inches?

Answer: The charts are independent of focal length. Minimum and maximum endlap and sidelap can be determined therefrom, as governed by flight height and relief height above datum. In each case, the datum is considered as passing through the point of lowest elevation affecting compliance with overlap requirements in each of the stereoscopic pairs of aerial vertical photographs to be taken for mapping by use of photogrammetric instruments.

HRB:OR-268

THE NATIONAL ACADEMY OF SCIENCES—NATIONAL RESEARCH COUNCIL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The ACADEMY itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the ACADEMY and the government, although the ACADEMY is not a governmental agency.

The NATIONAL RESEARCH COUNCIL was established by the ACADEMY in 1916, at the request of President Wilson, to enable scientists generally to associate their efforts with those of the limited membership of the ACADEMY in service to the nation, to society, and to science at home and abroad. Members of the NATIONAL RESEARCH COUNCIL receive their appointments from the president of the ACADEMY. They include representatives nominated by the major scientific and technical societies, representatives of the federal government, and a number of members at large. In addition, several thousand scientists and engineers take part in the activities of the research council through membership on its various boards and committees.

Receiving funds from both public and private sources, by contribution, grant, or contract, the ACADEMY and its RESEARCH COUNCIL thus work to stimulate research and its applications, to survey the broad possibilities of science, to promote effective utilization of the scientific and technical resources of the country, to serve the government, and to further the general interests of science.

The HIGHWAY RESEARCH BOARD was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the NATIONAL RESEARCH COUNCIL. The BOARD is a cooperative organization of the highway technologists of America operating under the auspices of the ACADEMY–COUNCIL and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the BOARD are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.