# A Method for Predicting Speeds Through Signalized Street Sections 

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#### Abstract

This paper presents the results of research work undertaken by the Chicago Area Transportation Study. It explains the development of a series of curves relating speed to the ratio of volume to capacity for signalized urban arterials.

The close similarity of the actual distribution of traffic to the Poisson distribution is shown for hourly volumes ranging from 300 to 2,100 vehicles. The Poisson distribution was used as a basis for generating vehicle arrival rates per minute for various hypothetical locations. Using these arrival rates in conjunction with known discharge rates, delays and stopped time were calculated and converted to speed for sections of known length. These speeds were plotted according to volume, resulting in a series of speed-volume curves, which were then converted to show the relationship of speed to the ratio of volume to capacity.

The need for and the application of these curves in urban traffic planning is also commented upon.


THE GOAL of the research reported in this paper was to develop a method of relating travel speed to volume, so that one could be used to predict the other. The principal reason for wanting to establish this relationship was to make the process of traffic assignment more realistic. Current assignment techniques make use of a time or speed estimate based on existing traffic conditions. These values are used as a basis for assigning traffic over a particular route. Often when the assignment is completed the assigned volume bears no relation to the initial assumption of time or speed. In fact, it is often the case that the traffic load assigned to an expressway could not be carried at any speed. This weakness in current assignment methods became more and more apparent as the work to develop an assignment technique progressed. Thus, if a relationship between speed and volume were established, it could be used in the assignment process to obtain more reasonable traffic estimates.

Another useful application of the relationship of speed to volume is in evaluating the efficiency and quality of service provided by a street system. For example, this kind of relationship provides data useful in determining how the entire street network would operate under loaded conditions, that is, after assignment of estimated future traffic volumes. Future bottlenecks could be detected immediately and, in addition, basic data could be provided to measure the quality of service for the entire street system.

Since defining this relationship seemed to provide a powerful planning tool, major emphasis was placed on this work. Several approaches were used to get at the problem. First, a carefully controlled study involving a moving vehicle (1) was used. In this study, route sections were selected and a car made runs each way recording the time between check points and observing the vehicles moving each way. Using the moving vehicle method (2), the volume of traffic in the section was estimated for each run. Unfortunately, this method did not produce the desired results so another approach was tried.

The second approach was based on the idea that speed on urban route sections was controlled largely by the ability to move through signalized intersections. Therefore, if the delay at intersections could be measured, the time to traverse a given route section, and thus the speed, could be determined. A further hypothesis was that if traffic were random on urban route sections, and thus described by a Poisson-type distribution, then the arrivals in any given time interval could be determined for any given hourly rate of flow. If this were true, and the output of the signal were known, then the
difference between the input and output could represent deiayed vehicles and, therefore, the effect of the signal on delay, and thus speed, could be determined for any hourly rate of flow. By setting up test examples, it would be possible to define an average relationship between speed and volume which could be used for any type of street.

This paper presents the results of the research following the hypothesis just described. The first part shows how the randomness of traffic was tested; the second part describes the estimations of average speed; the third part suggests some applications of this work; and, finally, a brief summary of the paper is given.

## RANDOMNESS OF TRAFFIC

## Previous Research

The success or failure of this research depended upon whether or not traffic on signalized urban streets was randomly distributed through time. Other researchers have concluded that traffic follows a random distribution on a 2-lane road (3). Greenshields, in a study made in Ohio in 1937 (4), found that "traffic may be considered purely random on a two lane road with volumes not exceeding four hundred vehicles per hour for the two lanes." In this same report a table is presented showing a theoretical distribution of traffic, as determined by Pois-


Figure 1. Cumulative distribution for hourly volume with mean arrival of 10 vehicles per minute. Arrivals stated in vehicles per minute and as a percent of twice the mean rate. son's law, compared with observed vehicle spacings on 2 and 4 lanes (4). The difference between theory and observations is insignificant.

## New Research

To determine if traffic is, in fact, random on signalized urban arterials, a series of field observations was made at 75 different locations. These arterials included 2-, 4-, and 6-lane surface streets and 8-lane expressways operating at various volumes and in 2 different cities-Chicago and Detroit.

The field study consisted of recording the traffic passing a point in one direction each minute for 90 min . A mid-block location was selected so there would be no interference from the signal ahead of the observation point. In addition, the width of pavement, the signal timing for signals on either side of the observation peint, and the number of travel lanes was recorded.

The data were summarized to show a cumulative frequency of arrival rates per minute. The cumulative frequency shows the percentage of time that $x$ or more vehicles arrive in a minute interval. In this case, the number passing the observation point each minute was termed arrivals (Fig. 1). In Figure 1 the x axis is identified in two ways, that is, by actual number of arrivals per minute, and as a percentage of the mean arrival rate. In the latter scale the mean arrival rate is equated to 50 percent. Thus, 100 percent on the scale equals twice the mean arrival rate. Figure 1
simply demonstrates the relationship of the mean arrival rate value to the scaled percentage values. Stating the arrivals as a percentage of the mean arrival rate gives a common base for plotting any distribution on the same chart regardless of its mean value.

Figure 2 shows a composite of cumulative distributions for mean arrival rate values ranging from 5 to 36 vehicles $/ \mathrm{min}$. The curves rotate clockwise around the 50 percent arrival rate as the mean value increases. It may also be noted that in the higher rates (above $17 \mathrm{veh} / \mathrm{min}$ ) there is no time when more than twice the mean number arrive. For example, at a volume of 1,200 vehicles per hour, or a mean of 20 vehicles per minute, it would be expected that 2 percent of the time more than 36 vehicles a minute would arrive-but never more than 38 vehicles would arrive in any one minute. This is a significant finding in itself.
Figures 7 through 12 show a comparison of observed values to values based on the Poisson distribution for the accumulated percentage of the time that $x$ or more vehicles arrive in any $1-\mathrm{min}$ interval. These figures are plotted for mean arrival rates ranging from 5 to 36 vehicles per minute. The observed values are averages of observations from 3 to 10 locations which have an average mean value as shown on the figure. This average mean value was used as the mean for the Poisson distribution plotted on the same chart.

These figures demonstrate a very similar distribution for observed and Poisson values. For any mean arrival rate, both observed and Poisson distributions have about the same range, and the shape and location of the accumulation curve is very similar.

The field data were summed to show the number of minutes any number ( $x$ ) of vehicles passed the point in 1 hr . Using the calculated mean value of each observed distribution, a theoretical distribution was


Figure 2. Cumulative distribution for arrival rates varying from 1 to 36.5 per minute. calculated showing the number of minutes containing $x$ number of vehicles. The $u$ sual chi-square test for goodness of fit was applied and the conclusion was that the observed data behaved in accordance with the Poisson law at the 5 percent significance level.

The randomness of arrivals for $60-\mathrm{sec}$ intervals has been demonstrated. The question naturally might arise as to whether the pattern of arrivals remains Poisson when observed at shorter time intervals. Gerlough (5) showed that the arrival pattern for intervals of 10 sec was nonrandom. However, his observations were that the arrivals in $30-$ sec groups could be considered random. His conclusions were based on a chi-square test using a 95 percent confidence level.

To test this conclusion, the authors made observations at $10-\mathrm{sec}$ intervals for 2 hr at one location. These $10-\mathrm{sec}$ observations were grouped to represent intervals of $20,30,40$, and 50 sec . The chisquare test was used to test arrival rates for each of these time groups against the Poisson distribution. The results indicated that arrivals during $10-$, $20-$, and $30-$ sec intervals are non-Poisson. However, those at 40 - and $50-$ sec intervals indicated a Poisson pattern.

Tests were also run for $70-$, $80-$, and $\mathbf{9 0 - s e c}$ groupings. These also proved to


Figure 3. Relationship of speed to volume-capacity ratio for 9 free speeds on a 2-lane section one-half mile long.
be Poisson. Thus, it was concluded that, in general, arrivals within the time span of the usual signal cycle, that is, 40 to 90 sec , are similar to a Poisson distribution.

The results of these field studies and their comparisons to theoretical Poisson distributions have been given as evidence that traffic, even on urban arterials under heavy pressure, can be considered to have random characteristics. The conclusion of the authors is that the distribution of traffic, through time, on urban arterials can be treated as being random and has a pattern similar to the Poisson distribution. Therefore, it became feasible to use the Poisson formula or tables to determine the range and distribution of arrival rates for known hourly traffic volumes.

## ESTIMATION OF AVERAGE SPEED

It has been shown that where the hourly one-directional volume on a signalized urban street is known, and hence the mean per-minute volume, the minute-by-minute arrivals can be predicted according to a Poisson table. If the minute-by-minute average maximum departure rate (essentially capacity) is known, or can be approximated, then it is a simple matter to predict the queue length at the signal at any minute during the hour. A table of queue lengths at each minute of the hour can be used to determine the number of partial or complete signal failures that occur.

A method of predicting the average speed of the stream, then, involves adding the accumulative stopped time, due to signal failures, at the signal or within the queue, to the known, or hypothesized accumulative travel time using best attainable legal speed, or so-called free speed, through the section at very low traffic volume. The following paragraphs discuss the estimation of accumulative stopped time, accumulative travel time and, finally, average speed.

## Maximum Accumulative Stopped Time

The maximum accumulative stopped time in a particular street section can be assumed to result when, (1) all entering vehicles within each signal cycle arrive at the signal, or at the end of the queue, instantaneously with the start of the red phase; and


Figure 4. Effect of route length on speed plots for free speed of 30 mph at volume= 600 vph .
(2) the number of vehicles which arrive within the first signal cycle of the observation hour is the largest predictable by the Poisson distribution of the total hourly volume entering the section; the number which arrive within the second signal cycle is the next largest; and so on through the hour. It can be assumed further, for simplicity, that there were no vehicles in the section prior to the first signal cycle.

Within the framework of these assumptions, there may be 3 types of stopped time resulting from the signal itself. Signal failures which, by definition, occur when arriving vehicles fail to discharge during the next following green phase, cause the greatest amount of stopped time ordinarily. Additional stopped time results from: (1) the assumption that all entering vehicles arrive at the beginning, and are stopped for the duration of the red phase; and (2) the fact that all vehicles are stopped for some portion of the discharging green phase. The latter types of stopped time may occur with or without accompanying signal failures. Eq. 1 expresses a method of solving for each type of stopped time separately.


Figure 5. Relationship of speed to volume-capacity ratio tor 2-, 4-, and 6-lane facilities.

$$
\begin{equation*}
S T_{\max }=(C) \sum_{c=1}^{n}\left(E_{c}+R_{c-1}-D_{c}\right)+(r)\left(\sum_{c=1}^{n} E_{c}\right)+\sum_{n=1}^{c}\left(\frac{D_{c}}{C a p}\right)\left(\frac{g}{2}\right)\left(D_{c}\right) \tag{1}
\end{equation*}
$$

in which

$$
\begin{aligned}
\mathrm{ST}_{\max } & =\text { maximum accumulative stopped time (hr); } \\
\mathrm{C} & =\text { total cycle length (hr); } \\
\mathbf{r} & =\text { total red phase }(\mathrm{hr}) ; \\
\mathrm{g} & =\text { total green phase }(\mathrm{hr}) ; \\
\mathrm{n} & =\text { number of cycles in the observation hour; } \\
\mathrm{E}_{\mathrm{c}} & =\text { number of vehicles entering the test section during cycle } \mathrm{c} ; \\
\mathrm{R}_{\mathrm{c}-1} & =\text { number of vehicles remaining in the test section from cycle } \mathrm{c}-1 \\
& \text { (the previous cycle); } \\
\mathrm{D}_{\mathrm{c}} & =\text { number of vehicles discharging from the test section during } \\
& \text { cycle c; and } \\
\text { Cap } & =\text { average maximum discharge rate per cycle. }
\end{aligned}
$$

The term (C) $\sum_{c=1}^{n}\left(E_{c}+R_{c-1}-D_{c}\right)$ accounts for the total stopped time resulting from signal failures. (C) is the total cycle length in hours. $\sum_{c=1}^{n}\left(E_{c}+R_{c-1}-D_{c}\right)$ is the summation for $n$ cycles of the number of vehicles entering the section each cycle ( $\mathrm{E}_{\mathrm{c}}$ ) plus the number of vehicles remaining from the previous cycle ( $\mathrm{R}_{\mathrm{c}-1}$ ) minus the number of vehicles discharging during the cycle ( $\mathrm{D}_{\mathrm{c}}$ ). The entire term might be written simply (C)( $\Sigma S F$ ) where ( $\Sigma S F$ ) is the total number of complete signal failures in the observation hour.


Figure 6. Final speed to volume-capacity ratio curves for free speeas as indicated.

The term (r) ( $\sum_{\mathbf{c}=1}^{\mathrm{n}} \mathrm{E}_{\mathrm{c}}$ ) accounts for the total stopped time resulting from the assumption that all entering vehicles arrive at the signal, or at the end of the queue, instantaneously with the start of the red phase. ( $r$ ) is the total red phase in hours. ( $\sum_{c=1}^{n} E_{c}$ ) is the total number of vehicles entering the section during the observation hour.

The term $\sum_{c=1}^{n}\left(\frac{D_{c}}{C a p}\right)\left(\frac{g}{2}\right)\left(D_{c}\right)$ accounts for the total stopped time resulting from the fact that all vehicles are stopped for some portion of the discharging green phase. When the number discharging equals the average maximum discharge rate ( $D_{c}=C a p$ ), it is assumed that all discharging vehicles depart midway of the green phase, or (1.00)( $\frac{\mathrm{g}}{2}$ ) $\left(D_{c}\right)$. But when the number discharging is less than the average maximum discharge rate ( $\mathrm{D}_{\mathrm{c}}<$ Cap), it is assumed that all discharging vehicles depart proportionately nearer the start of the green phase, or $\left(\frac{D_{c}}{C a p}\right)\left(\frac{g}{2}\right)\left(D_{c}\right)$.

Table 1 shows an example of the method of calculating these three terms for a hypothetical street section one mile long, where the total cycle length is 60 sec ( 0.017 hr ), the total red phase and the total green phase are each $30 \mathrm{sec}(0.0085 \mathrm{hr})$, the total entering volume is 1,794 vehicles, the total hourly maximum discharge rate is 1,920 vehicles or 32 vehicles per cycle, and the cyclical arrivals are arranged in descending order of their Poisson distribution. The latter arrangement can be demonstrated to produce the greatest number of signal failures for any given entering volume. Some experimentation with Table 1 will show clearly how the various terms are accumulated. It may be noted that repeated signal failures by any particular vehicles are automatically accounted for.

## Minimum Accumulative Stopped Time

The minimum accumulative stopped time in a particular street section can be assumed to result when, (1) all entering vehicles within each signal cycle arrive at the signal, or at the end of the queue, instantaneously with the start of the green phase, or as the last vehicle in the queue ahead clears, and (2) the number of vehicles which arrive during the first cycle of the observation hour is the largest predictable by the Poisson distribution of the total hourly volume; the number which arrive during the second cycle is the smallest predictable; the number which arrive during the third cycle is the second largest; the number which arrive during the fourth cycle is the second smallest; and so on through the hour. As before, it is assumed that there were no vehicles in the section prior to the first cycle.

Within the framework of these assumptions, there again may be three types of stopped time, although differing slightly from those described previously. Signal failures in this case may be one-half or


Figure 7. Comparison of cumulative frequency of actual arrivals per minute to Poisson derived arrivals per minute.

TABLE 1
DERIVATION OF ST $\max$ FOR A SIGNALIZED STREET SECTION ONE MILE LONG, WITH AN AVERAGE MAXIMUM DISCHARGE RATE OF 32 VEHICLES PER CYCLE, AN ENTERING VOLUME OF 794 VPH, AND A 60-SECOND CYCLE LENGTK WITH 50-50 TIMING

| Cycle Number <br> (c) | Arriving Vehicles Each Cycle $\left(E_{C}\right)$ | Discharging Vehicles Each Cycle (Avg. Max. Capacity = 32) (D) | Vehicles Stopped for Complete Cycle $\left(E_{c}+R_{c-1}-D_{c}\right)$ <br> $\left(R_{c}\right)$ | Proportion of Discharging Green Phase Stopped $\left(\frac{D_{c}}{C a p}\right)$ | One-half Green Phase in Hr, Times Proportion Stopped $\left(\frac{D_{c}}{\text { Cap }}\right)\left(\frac{g}{2}\right)$ | Total Stopped Time During Discharging Green Phase $\left(\frac{D_{c}}{C a p}\right)\left(\frac{g}{2}\right)\left(D_{c}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 42 | 32 | 10 | 1.00 | 0.0042 | 0. 1344 |
| 2 | 41 | 32 | 19 | 1.00 | 0.0042 | 0.1344 |
| 3 | 40 | 32 | 27 | 1.00 | 0.0042 | 0. 1344 |
| 4 | 39 | 32 | 34 | 1.00 | 0.0042 | 0. 1344 |
| 5 | 38 | 32 | 40 | 1.00 | 0.0042 | 0. 1344 |
| 6 | 37 | 32 | 45 | 1.00 | 0.0042 | 0.1344 |
| 7 | 37 | 32 | 50 | 1.00 | 0.0042 | 01344 |
| 8 | 36 | 32 | 54 | 1.00 | 0.0042 | 0. 1344 |
| 9 | 36 | 32 | 58 | 1.00 | 0.0042 | 0. 1344 |
| 10 | 35 | 32 | 61 | 1.00 | 0.0042 | 0. 1344 |
| 11 | 35 | 32 | 64 | 1.00 | 0.0042 | 0.1344 |
| 12 | 35 | 32 | 67 | 1.00 | 0.0042 | 01344 |
| 13 | 54 | 32 | 69 | 1.00 | 0.0042 | 0.1344 |
| 14 | 34 | 32 | 71 | 1.00 | 0.0042 | 0. 1344 |
| 15 | 34 | 32 | 73 | 1.00 | 0.0042 | 0.1344 |
| 16 | 33 | 32 | 74 | 1.00 | 0.0042 | 0. 1344 |
| 17 | 33 | 32 | 75 | 1.00 | 0.0042 | 0. 1344 |
| 18 | 33 | 32 | 76 | 1.00 | 0.0042 | 0. 1344 |
| 19 | 33 | 32 | 77 | 1.00 | 0.0042 | 0.1344 |
| 20 | 32 | 32 | 77 | 1.00 | 0.0042 | 0.1344 |
| 21 | 32 | 32 | 77 | 1.00 | 0.0042 | 0. 1344 |
| 22 | 32 | 32 | 77 | 1.00 | 0.0042 | 0. 1344 |
| 23 | 32 | 32 | 77 | 1.00 | 0.0042 | 0. 1344 |
| 24 | 31 | 32 | 76 | 1.00 | 0.0042 | 0. 1344 |
| 25 | 31 | 32 | 75 | 1.00 | 0.0042 | 0. 1344 |
| 26 | 31 | 32 | 74 | 1.00 | 0.0042 | 0. 1344 |
| 27 | 31 | 32 | 73 | 1.00 | 0.0042 | 0.1344 |
| 28 | 30 | 32 | 71 | 1.00 | 0.0042 | 0.1344 |
| 29 | 30 | 32 | 69 | 1.00 | 0.0042 | 0.1344 |
| 30 | 30 | 32 | 67 | 1.00 | 0.0042 | 0.1344 |
| 31 | 30 | 32 | 65 | 1.00 | 0.0042 | 0. 1344 |
| 32 | 29 | 32 | 62 | 1.00 | 0.0042 | 0.1344 |
| 33 | 29 | 32 | 59 | 1.00 | 0.0042 | 0.1344 |
| 34 | 29 | 32 | 56 | 1.00 | 0.0042 | 0.1344 |
| 35 | 29 | 32 | 53 | 1.00 | 0.0042 | 0. 1344 |
| 36 | 28 | 32 | 49 | 1.00 | 0.0042 | 0. 1344 |
| 37 | 28 | 32 | 45 | 1.00 | 0.0042 | 0.1344 |
| 38 | 28 | 32 | 41 | 1.00 | 0.0042 | 0. 1344 |
| 39 | 28 | 32 | 37 | 1.00 | 0.0042 | 0.1344 |
| 40 | 27 | 32 | 32 | 1.00 | 0.0042 | 0.1344 |
| 41 | 27 | 32 | 27 | 1.00 | 0.0042 | 0. 1344 |
| 42 | 27 | 32 | 22 | 1.00 | 0.0042 | 0.1344 |
| 43 | 27 | 32 | 17 | 1.00 | 0.0042 | 0.1344 |
| 44 | 26 | 32 | 11 | 1.00 | 0.0042 | 0.1344 |
| 45 | 26 | 32 | 5 | 1.00 | 0.0042 | 0. 1344 |
| 46 | 26 | 31 | 0 | 0.97 | 0.0041 | 0.1271 |
| 47 | 26 | 26 | 0 | 0.81 | 0.0034 | 0.0884 |
| 48 | 25 | 25 | 0 | 0.78 | 0.0033 | 0.0825 |
| 49 | 25 | 25 | 0 | 0.78 | 0.0033 | 0.0825 |
| 50 | 25 | 25 | 0 | 0.78 | 0.0033 | 0.0825 |
| 51 | 24 | 24 | 0 | 0.75 | 0.0032 | 00768 |
| 52 | 24 | 24 | 0 | 0.75 | 0.0032 | 0.0768 |
| 53 | 24 | 24 | 0 | 0.75 | 0.0032 | 0.0768 |
| 54 | 23 | 23 | 0 | 0.72 | 0.0030 | 0.0690 |
| 55 | 23 | 23 | 0 | 0.72 | 0.0030 | 0.0690 |
| 56 | 22 | 22 | 0 | 0.69 | 0.0029 | 0.0638 |
| 57 | 22 | 22 | 0 | 0.69 | 0.0029 | 0.0638 |
| 58 | 21 | 21 | 0 | 0.66 | 0.0028 | 0.0588 |
| 59 | 20 | 20 | 0 | 0.63 | 0.0026 | 0.0520 |
| 60 | 19 | 19 | 0 | 0.59 | 0.0025 | 0.0475 |
| Totals | 1,794 | 1, 794 | 2,438 |  |  | 7. 1653 |

$S T_{\max }=(C)\left(\sum_{c=1}^{n} E_{c}+R_{c-1}-D_{c}\right)+(r)\left(\sum_{c=1}^{n} E_{c}\right)+\sum_{c=1}^{n}\left(\frac{32}{C a p}\right)\left(\frac{\mathbf{g}_{2}^{2}}{2}\right)\left(R_{c}\right)$
$=(0.017)(2,438)+(0.0085)(1,794)+7.1653$
$=41.4460+15.2490+7.1653$
$=63.8603 \mathrm{hr}$
one and one-half signal failures (two of the three types). By assumption, there is no stopped time because of arrival during the red phase, but there is still stopped time during the discharging green phase. Eq. 2 accounts for each type separately.

$$
\begin{equation*}
S T_{\min }=(r) \sum_{c=1}^{n}\left(E_{c}+R_{c-1}-D_{c}\right)+(1.5 C) \sum_{c=1}^{n}\left(E_{c}+R_{c-1}-2 D_{c}\right)+\sum_{c=1}^{n}\left(\frac{R_{c}}{C a p}\right)\left(\frac{g}{2}\right)\left(R_{c}\right) \tag{2}
\end{equation*}
$$

Whether or not there is half or one and one-half failures depends upon the relation between ( $E_{c}+R_{c-1}-D_{c}$ ) and $D_{c}$. When the former is greater than the latter, the difference is the number of vehicles experiencing one and one-half signal failures. When the former is smaller than the latter, there are only half signal failures. Table 2 illustrates this.

During Cycle 1, 54 vehicles arrive at the start of the green phase and 32 vehicles discharge, leaving 22 vehicles stopped for a complete red phase (or a half signal failure). During Cycle 2, 26 additional vehicles arrive at the start of the green phase of which 10 can discharge after the 22 held over from Cycle 1. The remaining 16 are stopped for a complete red phase (or one-half signal failure). During Cycle 3, another 53 vehicles arrive of which 16 can discharge after the 16 held over from Cycle 2. There are then 37 vehicles remaining. Since only 32 can discharge during Cycle 4, the remaining 5 vehicles must be stopped for a complete red phase, plus a complete green phase, plus another complete red phase-altogether a signal failure and a half. The first two terms of Eq. 2 account for both types of stoppage.

The last term of Eq. 2 is changed slightly from Eq. 1. The basic difference results from the assumption that all vehicles arrive during the green phase. Those that are discharged immediately are assumed to suffer no stoppage. Only those that are held over from a previous cycle are assumed to suffer a stoppage during the discharging green phase. Thus, it is the remaining vehicles ( $\mathrm{R}_{\mathrm{c}}$ ) that must be considered in the last term, rather than the discharging vehicles ( $D_{c}$ ) as before.

Table 3 will make this distinction more clear. During Cycle 1,42 vehicles arrive and 32 discharge without breaking speed. The ten remaining discharge during Cycle 2, but they are stopped during some portion of the green phase. Thus, $\left(\frac{10}{32}\right)\left(\frac{\mathrm{g}}{2}\right)=0.0013$ each, and $(0.0013)(10)=$ 0.0130 . This type of stopped time is negligible when calculating $\mathrm{ST}_{\text {min }}$ but is important when finding $\mathrm{ST}_{\text {max }}$.

Why is it necessary to know both the minimum and maximum accumulative

TABLE 2

| Cycle | $E_{\mathbf{c}}$ | $\mathbf{D}_{\mathbf{c}}$ | $\mathbf{R}_{\mathbf{c}}$ | One-Half <br> Signal Fanlure | One and One-Half <br> Signal Failure |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 54 | 32 | 22 | 22 | 0 |
| 2 | 26 | 32 | 16 | 16 | 0 |
| 3 | 53 | 32 | 37 | 32 | 5 |
| 4 | 27 | 32 | 32 | 32 | 0 |



Figure 8 . Comparison of cumulative frequency of actual arrivals per minute to Poisson derived arrivals per minute.

TABLE 3
derivation of st min For a signalized street section one mile long, with an average MAXIMUM DISCHARGE RATE OF 32 VEHICLES PER CYCLE, AN ENTERING VOLUME OF 1794 VPH AND A 60-SECOND CYCLE LENGTH WITH 50-50 TIMING

| Cycle No. <br> (c) | Arriving Vehicles Each Cycle ( $\mathrm{E}_{\mathrm{c}}$ ) | Discharging Vehicles Each Cycle (Average <br> Max. Capacity $=32$ <br> Veh./Cycle) <br> (D) | $\begin{gathered} \text { Vehıcles } \\ \text { Stopped } \\ \text { for } \\ \text { Half Cycle } \\ \left(E_{c}+R_{c-1}-D_{c}\right) \\ \left(R_{c}\right) \end{gathered}$ | Vehicles <br> Stopped for <br> One and $A$ <br> Half Cycle $\left(E_{c}+R_{c-1}-2 D_{c}\right)$ | Proportion of Discharging Green Phase Stopped $\left(\frac{R_{c}}{C a p}\right)$ | One-Half Green Phase in Hr, Times Proportion Stopped $\left(\frac{R_{c}}{\text { Cap }}\right)\left(\frac{g}{2}\right)$ | Total Stopped Timeduring Discharging Green Phase $\left(\frac{R_{c}}{\text { Cap }}\right)\left(\frac{g_{2}}{2}\right)\left(R_{c}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 42 | 32 | 10 |  | 031 | 00013 | 00130 |
| 2 | 19 | 29 |  |  |  |  |  |
| 3 | 41 | 32 | 9 |  | 0.28 | 00012 | 00108 |
| 4 | 20 | 29 |  |  |  |  |  |
| 5 | 40 | 32 | 8 |  | 0.25 | 0.0011 | 00088 |
| 6 | 21 | 29 |  |  |  |  |  |
| 7 | 39 | 32 | 7 |  | 0.22 | 0. 0009 | 0.0063 |
| 8 | 22 | 29 |  |  |  |  |  |
| 9 | 38 | 32 | 6 |  | 0.19 | 00008 | 0.0048 |
| 10 | 22 | 28 |  |  |  |  |  |
| 11 | 37 | 32 | 5 |  | 016 | 00007 | 00035 |
| 12 | 23 | 28 |  |  |  |  |  |
| 13 | 37 | 32 | 5 |  | 016 | 00007 | 00035 |
| 14 | 23 | 28 |  |  |  |  |  |
| 15 | 36 | 32 | 4 |  | 0.13 | 0. 0005 | 00020 |
| 16 | 24 | 28 |  |  |  |  |  |
| 17 | 36 | 32 | 4 |  | 0.13 | 0. 0005 | 0.0020 |
| 18 | 24 | 28 |  |  |  |  |  |
| 19 | 35 | 32 | 3 |  | 0.09 | 0.0004 | 0.0012 |
| 20 | 24 | 27 |  |  |  |  |  |
| 21 | 35 | 32 | 3 |  | 0.09 | 0. 0004 | 0.0012 |
| 22 | 25 | 28 |  |  |  |  |  |
| 23 | 35 | 32 | 3 |  | 0.09 | 0. 0004 | 0.0012 |
| 24 | 25 | 28 |  |  |  |  |  |
| 25 | 34 | 32 | 2 |  | 0.06 | 0. 0003 | 00006 |
| 26 | 25 | 27 |  |  |  |  |  |
| 27 | 34 | 32 | 2 |  | 0.06 | 00003 | 0.0006 |
| 28 | 26 | 28 |  |  |  |  |  |
| 29 | 34 | 32 | 2 |  | 0.06 | 00003 | 00006 |
| 30 | 26 | 28 |  |  |  |  |  |
| 31 | 33 | 32 | 1 |  | 0.03 | 00001 | 0.0001 |
| 32 | 26 | 27 |  |  |  |  |  |
| 33 | 33 | 32 | 1 |  | 003 | 0.0001 | 0.0001 |
| 34 | 26 | 27 |  |  |  |  |  |
| 35 | 33 | 32 | 1 |  | 0.03 | 0. 0001 | 00001 |
| 36 | 27 | 28 |  |  |  |  |  |
| 37 | 33 | 32 | 1 |  | 0.03 | 0.0001 | 0.0001 |
| 38 | 27 | 28 |  |  |  |  |  |
| 39 | 32 | 32 |  |  |  |  |  |
| 40 | 27 | 27 |  |  |  |  |  |
| 41 | 32 | 32 |  |  |  |  |  |
| 42 | 27 | 27 |  |  |  |  |  |
| 43 | 32 | 32 |  |  |  |  |  |
| 44 | 28 | 28 |  |  |  |  |  |
| 45 | 32 | 32 |  |  |  |  |  |
| 46 | 28 | 28 |  |  |  |  |  |
| 47 | 31 | 31 |  |  |  |  |  |
| 48 | 28 | 28 |  |  |  |  |  |
| 49 | 31 | 31 |  |  |  |  |  |
| 50 | 28 | 28 |  |  |  |  |  |
| 51 | 31 | 31 |  |  |  |  |  |
| 52 | 29 | 29 |  |  |  |  |  |
| 53 | 31 | 31 |  |  |  |  |  |
| 54 | 29 | 29 |  |  |  |  |  |
| 55 | 30 | 30 |  |  |  |  |  |
| 56 | 29 | 29 |  |  |  |  |  |
| 57 | 30 | 30 |  |  |  |  |  |
| 58 | 29 | 29 |  |  |  |  |  |
| 59 | 30 | 30 |  |  |  |  |  |
| 60 | 30 | 30 |  |  |  |  |  |
| Totals | 1,794 | 1,794 | 77 |  |  |  | 0.0605 |

[^0]stopped time? Because it still is impossible to predict the arrangement of cyclical arrivals even when the frequency within the hour of various sized arrival increments is known from the Poisson distribution of the total hourly volume. The maximum and minimum accumulative stopped time values are simply the limiting conditions, assuming the most unlikely arrangements possible.

The problem has an element of chance. The actual stopped time at a signal, with all things equal might well differ from day to day, since identical cyclical arrangements of arrivals will occur only rarely. On the average, it was assumed that the actual stopped time will be halfway between minimum and maximum. That assumption is expressed by Eq. 3 which was used to compute the average speeds for the speed-volume/ capacity ratio curves.

$$
\overline{\mathrm{S}}=\frac{\mathrm{d}(\mathrm{~V})}{\mathrm{T}(\mathrm{~V})+\left(\mathrm{ST}_{\text {min }^{2}}+\mathrm{ST}_{\text {max }}{ }^{1 / 2}\right.}
$$

in which,
$\overline{\mathrm{S}}=$ average speed of all vehicles passing through the test section in an hour (mph);
$\mathrm{d}=$ test section length (miles);
$\mathrm{V}=$ volume per hour;
$\mathrm{T}=$ time to travel d; and others as defined previously.
It can be demonstrated, however, that as the volume entering a street section increases, the percentage of arrivals at the signal or at the end of the queue during the red phase increases. (It is obvious that at low volumes, drivers can and do adjust speeds in order to arrive during the green


Figure 9. Comparison of cumulative frequency of actual arrivals per minute to Poisson derived arrivals per minute.
phase as much as possible.) Some observations by CATS indicate that at signals with a 50-50 signal split the percentage of arrivals during the red phase may even rise above 50 percent as volume continues to increase. There is no apparent reason for the percentage to rise much above 50 percent, however.

The importance of the point is thisthat, if there is a demonstrable relationship between the percentage of arrivals during the red phase and a volume to capacity index (for comparability of different street sections having different discharge rates), the actual stopped time would tend away from the halfway point between minimum and maximum. The effect of this tendency upon the calculated average speed, however, should be slight.

## DERIVATION AND APPLICATION OF SPEED-VOLUME/CAPACITY RATIO CURVES

The application to actual route sections in the CATS arterial system required speed-volume curves in which speed could be solved for merely by the substitution of volume. Since the relation between volume and maximum discharge rate is implicit in this method of predicting average speeds, the speed-volume curves were
converted to speed-volume/capacity ratio curves. This conversion provided a common base for plotting the curves, that is, volume to capacity ratio, and made it possible to draw one curve for each speed range, yet account for route sections having different loads and different capacities. The following paragraphs discuss the derivation, simplification, and application of the required curves.

## Derivation

The family of curves was obtained by by a combination of manual and machine calculations. This process yielded speed points which, when plotted against the volume/capacity ratios corresponding, produced the necessary preliminary curves. It was decided that speed values from Eq. 3 would be calculated for 108 curves; that is, a different curve for route section lengths of $0.5,1.0,1.5$, and 2.0 miles; within this breakdown, a different curve for best attainable legal


Figure 11. Comparison of cumulative frequency of actual arrivals per minute to Poisson derived arrivals per minute.


Figure 10. Comparison of cumulative frequency of actual arrivals per minute to Poisson derived arrivals per minute.
speeds of $20,25,30,35,40,45,50,55$, and 60 mph ; and within this breakdown, a different curve for average maximum discharge rates of $600,1,320$, and $2,160 \mathrm{vph}$, assuming 50 percent green at the terminal signal. The latter capacity rates correspond to the capacity standards hypothesized by CATS for 2-, 4-, and 6-lane urban intersection approaches in intermediate type areas. These standards were the result of intersection, headway and starting reaction time surveys completed and analyzed by CATS.

Six speed values were calculated for each curve at volume to capacity ratios of $50,67,83,100,117$, and 138 percent. While more intermediate values could have been calculated at little additional cost, it was reasoned that 6 were sufficient to delineate accurate curves, since it was previously decided that the curves would have to be approximated later by straight lines. For purposes of illustration, the 9 curves for each best attainable legal speed through
a 4-lane section 1.0 mile long are plotted in Figure 3. The curves are eye-fitted.

## Simplification and Application

The application of 108 curves to matching CATS arterial street sections, however, presented a formidable machine problem. Simplification to a family of no more than 10-12 curves was highly desirable, and further analysis of the 108 curves proceeded with this in mind. Two important facts emerged. The first was that for a given section with all other things equal, increasing the section length from 0.5 to 2.0 miles shifted the speed points to the right, as shown in Figure 4. This was perfectly logical since vehicles might move at the best attainable legal speed up to the end of the queue, and the greater the section length, the greater the distance of unimpeded movement, generally.

The second important fact was, that all other things being equal, speed values for a 6 -lane section will be slightly higher than for a 4 -lane section, and the latter will be slightly higher than for a 2-lane section. This is demonstrated in Figure 5. The shift resulted from the fact that CATS capacity standards per unit of street width increase slightly as additional moving lanes are added. Preliminary releases of the revised "Highway Capacity Manual" show the same relationship.

These facts suggested that the 108 curves could be generalized by only 9 curves for which section length would be standardized at 0.5 mile and maximum discharge rate at 600 vph .

There were two reasons for choosing 0.5 mile as a standard-(1) the average section length of the 4,800 sections in the CATS system is roughly 0.5 mile; and (2) since deceleration and acceleration time losses are not accounted for by Eq. 3, the calculated speeds are overstated. Standardizing at 0.5 mile tends to compensate for this to some extent. Standardizing at 600 vph is an added compensation for the same bias.

The final family of eye-fitted curves, based on 9 free speeds, a section length of 0.5 mile, and an average maximum discharge rate of 600 vph , is shown in Figure 6. To facilitate machine application, the break point of all curves was taken at volume/capacity ratio $=60$ percent. The lowest speed allowable was taken as 5 mph at volume/capacity ratio $=110$ percent. They are reminiscent of the curves developed in a paper by Wardrop and Duff, titled "Factors Affecting Road Capacity." These are the curves to be used for the evaluation of the CATS arterial street assignment.

The reader may wonder how the volume can reach 110 percent of capacity. Obviously, the discharge of the signal cannot be greater than the maximum capacity. The volume referred to is the number of vehicles arriving at, and not necessarily discharging form, the intersection.

Briefly, the actual application might proceed as follows: during or after assignment, the volume assigned to any street is known, and may be comoared to the street capacity to obtain the volume/ capacity ratio. The appropriate curve, based upon the best attainable legal speed, is entered and the average speed, corres-


Figure 12. Comparison of cumulative frequency of actual arrivals per minute to Poisson derived arrivals per minute.
ponding to the assigned volume, determined. All this can and will be done mechanically.

## SUMMARY

This report has explained the development of a series of curves relating speed to traffic volume for urban signalized arterials. The resulting curves show the relationship of speed to the ratio of volume to capacity.

The similarity of the distribution of traffic on urban streets to the Poisson distribution was first demonstrated. Then the Poisson distribution was used to generate arrival rates for various hypothetical locations and to calculate travel delays and starting time as the volume of traffic varied. Next, the average speed was calculated by adding the accumulative stopped time to the time to travel through a section of known length at very low traffic volume. This was done for streets with varying discharge rates and for various approach volumes.

This resulted in the development of a series of speed-volume curves. Since the relation of volume and capacity is closely interwoven, these curves were converted to speed-volume to capcity ratio curves. This conv arsion also provided a common base, (that is, volume to capacity ratio) for expressing all the curves.

This is a crude approwh to the probiem of defining the relationship of speed to volume. Many simplifying assumptions were needed to develop the curves which are presented in this report. However, the authors feel that this is a good simplified first step to the development of a rational theory explaining traffic flow.

There are many obvious applications of the speed-volume relationship in the field of urban traffic planning. For example, after a traffic assignment is made, it would be possible to estimate the average operating speed of all streets under the assigned loads. Thus a measure of the quality of service, which would be afforded at some future date with new planned facilities, could be determined. Also, it is possible to determine what volume would be allowable to obtain a reasonable minimum speed on urban streets.

These relationships would provide an excellent basis for a time or speed feedback in an assignment problem. For example, using assumed speeds or times for route sections, a traffic assignment is made. After the assignment is completed, it would be possible to determine the speed and thus the times obtainable with the assigned volumes through the use of the speed-volume to capacity curves. These times could be fed back into the computer and used as a basis for a reassignment. The second assignment would redistribute the traffic and a series of these assignments should eventually result in a balanced load for the network. In this way it would be possible to simulate, with some realism, the manner in which traffic would distribute itself in a transportation network.

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[^0]:    $S T_{\min }=(r)\left(\sum_{c=1}^{n} E_{c}+R_{c-1}-D_{c}\right)+(1.5 C)\left(\sum_{c=1}^{n} E_{c}+R_{c-1}-2 D_{c}\right)+\sum_{c=1}^{n}\left(\frac{R_{c}}{C a p}\right)\left(\frac{g_{2}}{2}\right)\left(R_{c}\right)$
    $=(0.0085)(77)+(0.026)(0)+0.0605$
    $=0.6545+0.0605$
    $=0.7150 \mathrm{hr}$

