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Landslide and Foundation Investigations and Stability Analysis

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Contents

On the Methodology of Landslide Investigations in Soviet Russia

D. P. KRYNINE, Consulting Engineer, Berkeley, California

The present paper is based on Russian information concerning the landslide investigations done by special field stations located either in the regions with abundant slides or in the vicinity of a large slide in a state of slow motion in which final failure may or may not take place. Slides only, as distinct from falls and flows, are considered in this paper. Russian approaches to the slide classification are discussed first, after which cracks and fissures in the sliding body are considered in detail. Methods of measurement of the displacements at the surface and of those at a depth are described. The paper ends with the methodology of computing the balance of sliding masses at a given slope, the negative balance being generally an indicator of the tendency of the slide to stabilize.

● SLIDES ONLY as distinct from falls and flow are considered in this paper. The writer became interested in the information concerning the methods of slide study as used at the present time by Russian engineering geologists and engineers. Because these simple, but rather efficient methods are not well known outside of Russia, this paper has been prepared for information only. Basically it represents an outline of several chapters of a book written by a Russian woman engineering geologist (1) with information from other sources and some writer's comments.

FIELD SLIDE STATIONS

The slide research in Russia is done by field slide stations located at different parts of the country. The first station of this kind was organized in 1930 at the Koutchouk-Koy slide in the Crimea; and in the same year the first instructions for long-term observations were published. Methodology of making investigations of slides at field stations was afterwards gradually developed and discussed in the special press; and in 1934 the first All-Union slide conference took place.

Natural slides generally are not isolated phenomena but are spread over a territory characterized by certain geologic and geohydrologic conditions, and similarity in the development of the slopes. A slide is just a step in the process of denudation or gradual slope formation of a region. Therefore, a field slide station should be located within a certain region characterized by the abundance of slides. It may be also located in the neighborhood of a huge slide as in the example of the Crimean slide already quoted.

Stages of a Sliding Process

A field slide station has to classify local slides and establish the stages of the slide development in different slide types. Schematically speaking, if the sequence of slide stages in the slides of a given type is known, the present stage of the slide may be determined, for instance, from visual inspection and the next probable stage forecast. Slavianov (2) gives, for instance, the following example of stage sequence. A river starts to undermine a bank which becomes steeper, and in which fissures appear. A new factor takes place, namely penetration of water into the body of the slope through fissures with activation of the whole process. The next stage will be separation of the sliding mass and its translation. At the lower, flatter portions of the natural slope the sliding masses are arrested and the forepart of the slide ("tongue" in Russian terminology) enters the water. This is the stage of temporary equilibrium. In the natural state the ground water within the slope has outlets for discharge that may be covered or closed by sliding masses. Consequently the ground water may change its course and wet portions of the slope, heretofore dry. In this connection fissures and pools appear at the surface of the sliding mass. This stage is characterized by alternate periods of motion and rest. In connection with described phenomena, the relief of the locality gradually smooths out and the position of the sliding mass is stabilized on the slope. The sliding body is finally gradually covered up with soil coming from the inland or may be eroded and disappear.

Generally, the stage sequence observations should last a considerable time and encompass a large number of slides. Excessive automatization in the use of stage-sequence schemes should be avoided, however, and in each particular case the environment conditions properly appraised.

Activities of a Field Slide Station

In order to obtain a sufficient number of observations in the most economical way, simple sets of observations are done on a great number of objects spread through the region assigned to the station; and the more complicated the observations the fewer the number of individual objects of study. Finally, such complicated long-term observations as soil or water balances (discussed at the end of this paper) are done on a few objects only.

The region assigned to a station may be from several tenths of a mile to several hundred miles long (e.g., along a river canyon). On a portion of the region with especially intense slides (perhaps 12 to 30 mi long), regular visual observations are done from one to four times a year; additional observations are done after heavy displacements of existing slides; after new slides have started; after heavy rainstorms; after earthquakes; during construction, etc. On some individual slides where instrumental survey is done (e.g., systematic displacement measurement or ground water study), observations are done every two to five days, and sometimes every day.

More specifically, the following types of observation sections may be distinguished:

1. Sections containing slides at the stages when they threaten or may threaten the stability of existing or planned structures or facilities. The objective of the observations is the elaboration of measures tending to decrease the activity of the slide and stabilize it. The observations are short-termed and generally coincide with the period of preliminary or final engineering investigations for a given structure.
2. Sections on which the anti-sliding structures already exist and their effectiveness is studied. In the majority of cases the observa-

tions have to confirm complete stability of the section or show the rate of stabilization. The observations are rare, but may last decades.

3. Sections with typical slides in progress for obtaining information on the sliding law. Such sections may serve as field laboratories (for instance, by observing the stability of experimentally excavated slopes under variable conditions). These are usually long-term observations. The full-size experiments are combined with the work of the indoors laboratory where the experiments on smaller scale are done. In comparing the results the similitude laws may or may not be used. In the latter case the experiments are purely demonstrative, but may be sometimes helpful in the solution of some particular problems.

Mapping of the Region

The slide observations should be preceded by the mapping in of the whole region under the station jurisdiction using 1:25,000 to 1:100,000 scales. The contours of large slides are shown on such maps, and smaller slides are indicated by conventional signs.

The basic portion of the region under the constant observation by the station is usually mapped on a 1:5,000 to 1:10,000 scale, seldom larger. The mapping is supposed to (a) give the characteristics of each individual slide; (b) establish the dependence of the slide morphology on the petrography, genetics, location of the slope making formations, and on the plentifulness of the aquifers; (c) record the presence and intensity of other physico-geologic phenomena; (d) record all structures, particularly those designed for slide arresting purposes, and all artificial factors that may possibly affect the slope stability.

Individual slides are mapped on 1:500 to 1:2,000 scale, and very large ones on 1:5,000 scale. In this case geologic sections are shown. In this connection all available data should be utilized concerning the magnitude and direction of the displacements of the formations, their tippings and twists and other data obtained from the study of fissures and other deformations at the surface of the slide and from observations on reference points. Large-scale mapping is accompanied by boring and sampling of soils and rocks, and sometimes by the determination of the direction and velocity of the ground water flow, including pumping. Geophysical investigations are sometimes used. The boring logs should indicate not only the sequence of strata, but also their lithology, and technically important physical properties; presence, characteristics and orientation of cracks and fissures, sliding surfaces and slickensides (and scratches on them), also collapsed and smashed zones. Data from bore holes have to be checked against those from trenches and test pits. All basic bore holes must be sunk from 3 to 7 ft into the natural ground and one or two holes go to a deeper horizon whose immobility during the sliding process is above doubt. When the strata are horizontal, at least one bore hole on the slope must reach the elevation of the bottom of the deepest hole sunk through the sliding body itself.

Slope History

A purely geological problem of great importance is the history of a given slope considered as a whole. For a competent geologist, this is the basis of the understanding of the present day slides and possible prognosis of coming ones. Often the history of the slope is connected to the history of a water basin or a river canyon of which the slope is a part.

In addition to the history of the slope in remote times the station gathers all possible information concerning the present historical period (old maps, newspapers, questioning of the neighbors, etc.).

SLIDE TERMINOLOGY AND CLASSIFICATION

The Russian term for the slide is "ópolzen." The sliding body proper is the "body of the ópolzen." The visible vertical cliff-like scarp where the sliding body separates from the rest of the earth mass is "pull-off-wall." The sliding surface on which the sliding body reposes, if practically immobilized, is its "bed." The sliding body is limited by the right and left "flanks" or "sides," right and left being considered in the direction of sliding. The foremost portion of the slide is its "tongue" (not "toe").

A new slide changes the appearance of the slope. The relief changes, fissures and steps appear near the top of the slide, rock formation may become visible, there are new ground water outlets and swampy spots. If the displaced masses keep the new position for a certain time, the slope appearance changes again: the fissures are gradually filled up, their edges smoothen, and the newly exposed surfaces are covered with vegetation. This is the transformation of the slide into an "old slide." The duration of this transformation depends on the climate. It may be of the order of several hundreds of years in dry climate, whereas in the presence of considerable rainfall and rich vegetation two and even one year may suffice. It is necessary to distinguish between "old" and "ancient" slides. The latter are healed up slides (scars) formed in past geological times.

As may be concluded from Emel'ianova (1), the terms "new" or "modern," "old," and "ancient" slide are used by the stations' personnel. There are also slide classifications by Popov and Maslov, discussed later.

The slopes with old or ancient slides or slopes without slides at all may be similar to those on which new or modern slides are developed so far as the conditions of slope formation and existence are concerned. All such slopes deserve attentive study in order to clarify the reasons why under apparently identical conditions the slides may or may not develop; why the slides may be active or completely healed up.

Slide Classification

Popov's Slide Classification (3) is in reality an adaption of F. P. Savarensky's (3a) classification "somewhat developed and made preciser." Savarensky, a well-known Russian geologist, working mostly in engineering geology and seismology, was the first in Russia to introduce into the slide classification the time of manifestation of the slide and its state (stage). His classification, modified by Popov, is given in Table 1. On the basis of Table 1, Popov elaborated another table (Table 2) which in reality contains a small list of features on which a regional slide classification should be based and a small list of measures for the control of landslides. Table 2 is formulated in general terms and has no immediate practical value for a field engineer or geologist. It is not presented here.

Maslov's (4) slide classification considers four characteristics of the loss of slope stability, namely: (a) form, whether fall, slump with shear and rotation, shear at settlement, sliding, creep-displacement, creep, flows, plastic and viscous deformation, secular reworking of the

TABLE 1

CLASSIFICATION OF SLIDES ACCORDING TO AGE AND STAGE

Age	Stage	Characteristics	
		Of Age	Of Stage
Recent	Moving	With recent base level of erosion, and level of abrasion	Process tending to establishment of equilibrium
	Suspended		Action of cause temporarily balanced by some "security agent"
	Arrested		Cause temporarily eliminated
	Completed		Action of cause discontinued
Ancient	Exposed	With a different position of erosion and abrasion	Only soil and eluvium at the surface
	Buried		Slide covered with later deposits

slope; in total seven forms if fall and flow are not considered; (b) character of deformation, e.g., understanding under "sliding" a displacement along the planes of stratification, breaks, ancient movements, etc.; under "creep-displacement" almost horizontal displacement along a weak layer of cementing material between two strata caused by lateral pressure; (c) velocity of deformation expressed quantitatively only, e.g., "small and exceedingly small" in cm or mm per year; (d) natural environment (mostly geology of the site).

Cracks and Fissures

In the Russian originals discussed here no distinction is made between "crack" and "fissures." The general term used there is equivalent to English "fissure." In this paper the term "crack" is generally used; the term "fissure" is also used when real closed fissures are described.

Cracks and fissures at the slide surface are caused by stresses and displacements within the slide body. In the case of an elementary (simple) slide in clayey material there are tensile stresses and tensile cracks and fissures at the top of the slide; whereas at the tongue there may be bulging and, hence, compression cracks and fissuring. This is true in the case of non-sensitive clay. If the clay is sensitive, or if there is a sudden increase in the bed gradient close to the tongue, clay may move or flow down and spread on the terrain. There may be also fissures in the natural ground above the slide and below its tongue.

The portion of the simple slide between the upper (extended) and the lower (compressed) zones is a "displacement zone." If the curvature of the slide bed (shearing surface) is constant, e.g., as in the case of a perfect circular or plane shearing surface, the body of the slide is not stressed and no fissures are formed on it during the sliding process. If, in such a case, there are engineering structures built at the "displacement zone," they will be simply translated and may be tipped one way or the other according to their relative position with respect to the shear surface.

Figure 1 shows the crack classification by Ter-Stepanian (4) who subdivides all cracks on the sliding body into surface cracks and deep cracks or fissures. In their turn, the surface cracks constitute four large groups indicated on plan (Fig. 1, bottom).

Group I. Upper Cracks (Figs. 1a and 1b), open at top, more or less vertical, edges not smashed. Cracks (Fig. 1b) are in reality faults (or shear) surfaces. May be covered by the dry material falling from the upper "shoulder." Cracks (Fig. 1a) are tear (or tensile) cracks, not very long, dying out at the ends; both shoulders are at the same level.

Group II. Side Cracks (Figs. 1c, 1d, 1e and 1f), along sides of sliding body, right and left, considered along direction of movement. Each crack has two "shoulders," one movable on the sliding body, the other unmovable on the rest of the mass. At the beginning of the sliding process both shoulders have equal elevations but not so afterward, when the shoulders move relatively up and down, because of horizontal (and not vertical) displacements. However, at the end of the process there is a tendency for the movable shoulder to be lower than the unmovable at the top portions of the slide, and higher at the lower ones. This is explained by the erosional action at the cirques of the slide and the accumulative action of its tongue.

Figure 1. Types of cracks on a slide.

There may be four types of cracks along the sides of the sliding body. "Pushing" cracks (Fig. 1c) are formed when the direction of motion makes an acute angle with the edge of the sliding body, and both compression and shear stresses are acting. Basic pushing cracks (Fig. 1c) are accompanied by secondary curvilinear secondary crack probably caused by torsion. The presence of a moment in this case has been indicated in the United States by A. M. Ritchie (5, p. 55), and by the Russian investigators themselves (1, p. 40). "Squeezing" cracks (Fig. 1d) are of the same origin as cracks (Fig. 1c), only the acute angle is larger in this case. Also the secondary cracks are heavier; and there is a longitudinal roll of earth material squeezed up from below.

If the longitudinal sides of the sliding body are parallel to each other and to the general direction of sliding (Fig. 1e), the fissures separating the sliding body from the rest of the mass, are typical shear failure fissures covered with "lines" that probably are portions of the shear pattern and traces of wearing. In such cases slickensides may develop at the vertical or almost vertical sides of the sliding body. If the sliding body tends to widen in the direction of impending motion, "separation" cracks (Fig. 1f) are observed. Roughly such cracks approach a straight line, the shoulders are often torn and there are no "lines" and no signs of friction on them.

Group III. Central Cracks, are conventionally termed "compression cracks," though in their formation tensile stresses also participate. "Smashing cracks" (Fig. 1g) are in reality transverse closed fissures with one or more transverse earth rolls. The shoulders of the fissures are level. These cracks are formed at the place where the movement of the sliding body is decelerated by some obstacle, e.g., a heave in the bed of the slide.

"Opening" cracks are formed especially in the zone between the middle and the lower portion of the sliding body (Fig. 1h). These are transverse vertical cracks, formed by tensile stresses, e.g., in the case when the earth material accumulated in the lower portion of the slide, creeps over some obstacle (e.g., a heave in the bed) and breaks. The shoulders of these cracks are level.

Group IV. Lower Cracks. The fissures formed at the end of the tongue (Fig. 1i) are joints connecting the slide with the surrounding soil mass. These fissures are closed, their upper shoulder is high, sometimes tipped and even overturned; their lower shoulder is often hidden by the earth material.

This particular classification does not cover all kinds of cracks and fissures that may appear at the surface of a sliding body, e.g., dessication fissures; tectonic cracks, e.g., caused by faults; weathering fissures, etc. Besides cracks and fissures there may be other deformations of the sliding body such as folds, decrease and increase in thickness and other phenomena known as "slide tectonics."

The importance and general significance of cracks at the sliding body is well known (5, p. 54); here the methodology of crack study on the typical slides only as practiced by the Russian field slide stations, will be briefly discussed. In describing a crack in a report or a paper the following items are considered: (a) whether the crack is individual or belongs to a series; (b) shape in plan (straight, curved, wavy, broken, etc.), its length and its orientation and position on the sliding body; (c) width, max, min, average; (d) depth and state of the visible bottom; (e) walls of the crack (whether smooth, with "friction mirrors," scratches or "lines," or rough, notched, smashed; (f) state of the shoulder edges, whether sharp, fallen off, rounded; straight in plan torn, indented; (g) difference in level of the shoulders; (h) horizontal translations along the crack; (i) possible relation of the crack to geologic conditions, e.g., changes in the character of the crack when intersecting different rocks; (j) material filling the crack; (k) geohydrological significance of the crack; (l) possible causes of the crack. In certain cases pits and trenches are used for the detailed survey of a crack. Sometimes detailed instrumental mapping of cracks for individual sections of a large slide is advisable. An American example of such mapping should be recalled (7).

DEFORMATION OF STRUCTURES

A slide may occur under, or close to, an existing structure. In this case the structure may be heavily damaged or, under certain circumstances, may continue its service. Finally a structure (e.g., a road) may be constructed on a slowly progressing slide. In all these cases observations on the state of the structure are needed. The primary objective of these observations is to establish the relationship of the actual or possible damage of the structure with its position on the slide. It already has been suggested in this paper that the structures located at the central portion of an elementary slide are only slightly damaged, if at all. Obviously, the construction and depth of the foundation and the general condition of the structure prior to sliding are of importance.

Tipping and displacement of parts of structures are measured in three mutually perpendicular directions and shown schematically on the plans of the structure. Cracks and fissures are given special attention. Particularly, to establish whether a crack is widening or not, rectangular pieces of thin glass 1 to 3 cm wide are often fixed with gypsum or cement on the shoulders of the crack. Gypsum and cement overlays across the fissure are also used.

DEFORMATION OF VEGETATION

Plants may resist sliding or be entrained by the slide according to where their roots are fixed. In huge slides the whole root system is generally located within the sliding body. In a cylindrical slide an isolated vertical tree tips up slope; and moves forward, i.e., down slope, in the case of plastic flow. Young trees rotated by an early slide grow vertically afterwards, a growth which permits one to estimate how much time has transpired since the slide. It should be realized that bent trees are also found in windy regions.

DISPLACEMENTS OF SLIDING BODIES

Magnitude, direction and rate are to be recorded in the study of sliding body displacements. Regular surveying operations are performed on a system of "monuments" (Russian "reper") placed on and outside the slide. Besides determining whether or not there is a displacement of the sliding body, the boundaries of "active" slides or those of secondary slides (cirques) within huge old slides may be determined using such observations. It may also be found whether the sliding process is still progressing and, if so, what kind of movement is taking place: either displacement of the sliding body as a monolith; or differential displacement of its parts; or, finally, a plastic flow. It may be disclosed whether there is a lateral or an upward growth of the sliding body. Data for the stress distribution study within the sliding body may be collected. So far as the anti-sliding structures are concerned, data for the design of such structures may be collected, and when the structures themselves are constructed their efficiency checked.

Observations on "monuments" are referred to three mutually perpendicular axes. Displacements of a point along the axes and full displacement in space, velocities of displacement and increase or decrease of the true distance between the monuments are determined. Changes in the true distance between the monuments indicate tension or compression close to the slide's surface. Rotation of the sliding body about a horizontal or vertical axis is found from the angles of rotation of line AB connecting the two monuments A and B in the vertical or horizontal plane, respectively. In their turn, the values of these angles are computed from the coordinates of points A and B.

Monument observations may be used for determining the depth of the sliding body and the configuration of the sliding (shear) surface. The method proposed by Buckingham (8) is described by Emel'ianova (1, p. 59). During a displacement of the sliding body volume A_1 and ABB_1 (Fig. 2), may be assumed equal to the volume of soil passed through section BC. If the horizontal displacement of point B is Δ and assuming a uniform distribution of the velocities of the moving soil along vertical BC,

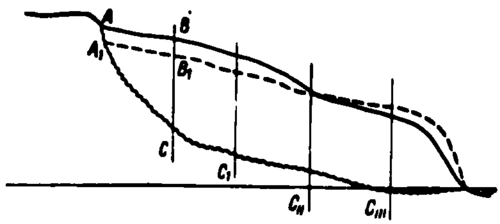

Figure 2. Approximate determination of the depth of a slide (8).

$$\text{depth BC} = \frac{\text{area } A_1ABB_1}{\Delta} \qquad (1)$$

If the sliding body is severely broken during the displacement, Eq. 1 may lead to considerable errors.

Location of Monuments. The monuments are located in longitudinal and transverse, mutually perpendicular rows. The longitudinal rows are usually parallel to the general direction of sliding. Some monuments should be placed beyond the perimeter of the sliding body proper. If there is a water basin at the foot of the slope, there should be monuments placed in water with at least one monument unmovable. Additional longitudinal monument rows are needed (a) on complex slides with secondary cirques (Figs. 3 and 4); (b) on frontal slides of considerable length along the slope; and (c) when besides translation also rotation may be expected. Figure 5 shows a case when monolithic sliding takes place along a plane dipping obliquely to the slope. It should be noted, however, that in many cases one good longitudinal monument row may suffice.

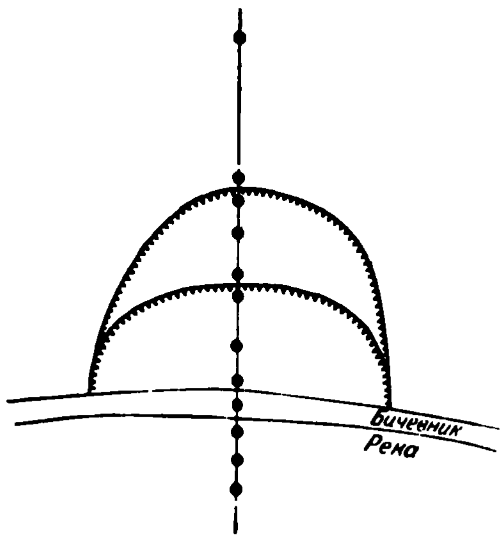

Figure 3. Location of monuments on a medium size two-step slide (schematic sketch).

Sometimes monuments are located in triangles as for a diminutive triangulation. Practice has shown, however, that the readings obtained from a sufficient number of mutually perpendicular monument rows are easier to analyze and are more illustrative than those obtained from triangular sets.

Location of monuments following a rectangular grid may be useful in

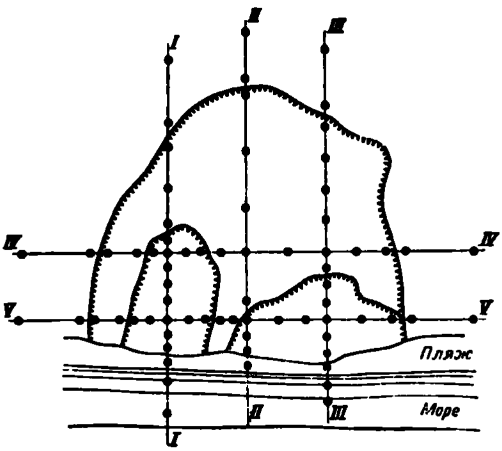

Figure 4. Location of monuments on a huge old slide with secondary cirques (schematic sketch).

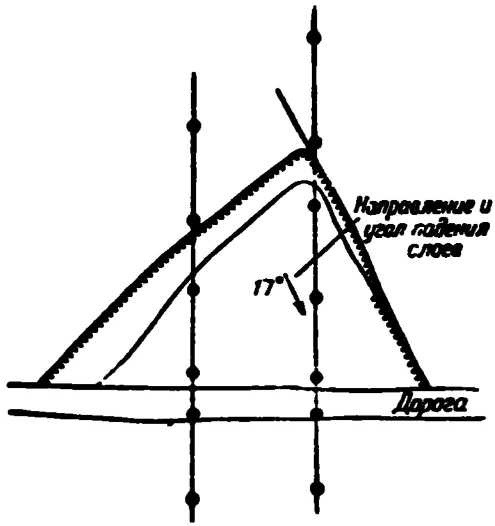

Figure 5. Location of monuments in the case of an oblique slide. Arrow shows direction and angle of dip of the sliding plane (schematic sketch).

the case of plastic flow and creep when the soil mass moves like a stream of viscous liquid.

Location of monument rows should be first planned on a large-scale topographic map according to available geomorphological information. The plot should be then corrected in the field by a geologist and an engineer together, according to the presence of cracks and shear steps, conditions of visibility, etc.

Monument Types. Monuments may be basic, for long-term observations, and working, for short duration use. Monuments of either type may be exposed or covered with earth. The monuments of the latter type are better so far as preservation of the monument is concerned but often are difficult to find in the field. It is advisable to make all monuments exposed and to cover them only if their safety is threatened.

The base of a monument should be placed below the freezing depth except for that case when the displacements of the mantle only are considered.

Figure 6 shows one covered monument (a) and two exposed. There are many types of working monuments; concrete is used in most of them for stability. A very common type is shown in Figure 7, left. In this case the monument of a metallic rod or pipe segment driven from the bottom of a shallow excavation filled afterwards with concrete. In the soils subject to swelling this type of monument is replaced by a concrete slab with a vertical rod placed at the bottom of a relatively deep pit and filled with compacted clay (Fig. 7, right).

Duration of Monument Observations. Practically always, observations on a slide, even in the areas served by field stations, start after the slide has already moved so that information on a very initial stage of sliding is missing. The sliding process develops non-uniformly and there may be periods of quiet between the displacements, lasting sometimes for years. Again, the history of some slides is limited to one displacement

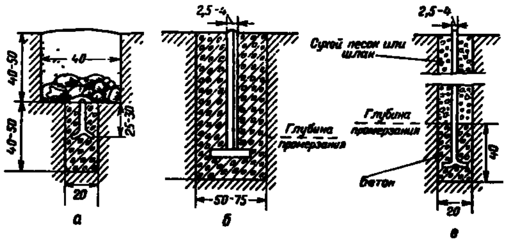

Figure 6. Types of basic monuments for long-term observation.

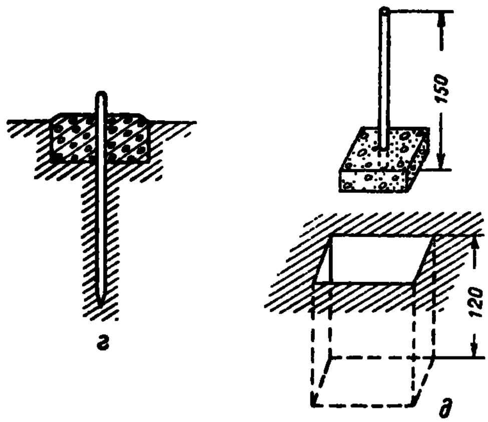

Figure 7. Typical monuments for short-term observations (the right one is used in frost-expansive soils).

only. Therefore, it is not necessary very often to perform full surveying operations, i.e., determination of the three coordinates x, y, z of all monuments. For instance, for intermediate observations of a slide, done 4 to 6 times per year, a systematic determination of one or two coordinates may suffice. The vertical coordinate z may be determined by simple leveling. The coordinate x may be found from the longitudinal monument rows, often by simple measurement of the distances between the monuments. The immobility of basic monuments should be checked once in awhile, however.

Measurement of any of the coordinates x, y, z should be done in dry weather or during a freezing spell. Obviously no coordinate measurement should be done during visible slide movement.

Simplified Observations. Regular surveying operations are expensive and require considerable time for their completion. Simplified observations between regular surveying operations are performed in order to investigate whether or not there are displacements during a certain time interval and what is the nature of these displacements. Temporary "marks" are placed or made at the surface of the sliding body for this purpose. Placing one mark at each side of a fissure and measuring distance between these marks may indicate the pressure or tension in the sliding body. Both of these marks may be located at the surface of the sliding body, or one on it, and the other outside, i.e., at the unmovable section of the earth surface. A great number of mark pairs may operate simultaneously at different sections of the slide. Marks may be located along the estimated direction of sliding and distances between them measured. To study a rapid sliding motion, a long graduated rod is fixed at the surface of the sliding body along the direction of sliding. Outside the edge of the slide, i.e., on the unmovable earth surface, a telescope tube is fixed perpendicular to the rod. Readings of the rod are then plotted against times of observation.

Obviously the marks should be simple, inexpensive and conveniently located. A pair of monuments from an existing longitudinal monument row located on both sides of a fissure or crack may work as marks. Boulders, trees, buildings and structures of all kinds may also be utilized for the purpose.

DISPLACEMENTS AT A DEPTH

Russian technical literature (9) advances the idea of existence of a "landslide nidus," or that region of the slope where the local concentration of tangential stresses occurs. The complete sliding cycle caused by these stresses consists of a phase of deep creep which may last for years, and a phase of shear in the form of quick displacement. An old slide may be healed up and become active again, and this may be repeated several times during the long geological history of the slope. Thus in one slide there may be traces of sliding surfaces at different levels. It is difficult to locate the shearing surface from a bore hole even in a recent slide; more correct data are obtained from pits and trenches. Russian field slide stations use deep monuments for the purpose.

Practically all methods of the field deep displacement-study are based on the consideration of deformation of a straight line (usually vertical) within the sliding body. When intersecting a sliding surface or a break the experimental straight line is displaced and the displacement of the sliding body equals the distance between the ends of the segments of the experimental vertical line. Within a displaced but monolithic block, the experimental line is still straight, but may be tipped. In the case of a plastic flow, the experimental line is curved and may be broken. Generally the shape of the curve in this case depends on the law of velocity changes along the vertical.

In one type of deep monuments their deformation is determined by carefully excavating the earth material around the monument which in this case may be used for one observation only. A simple type of such monuments consists of a bore hole filled in by wooden cylinders 4 to 10 in. in length, freely standing on each other and filling the bore hole. In another type the bore hole is filled with some loose material, e.g., gravel, sand, fine coal, etc. The color of this material obviously should differ from that of the slope materials. Very important in this case is the determination of the original deviation of the bore hole from the vertical which can be determined by placing into the hole a glass cylinder filled with appropriate liquid, e.g., acid affecting glass or some dye. In the latter case the walls of the glass cylinder should be lined inside with paper. The difference between the highest and the lowest level of liquid in the glass cylinder and its diameter furnish the data for computation of the slope of the bore hole at the level where the experimental glass cylinder is placed.

The excavation of a monument is costly and time consuming and is generally done when the monument is displaced horizontally 3 to 5 ft. Figure 8 represents a sketch of the wall of an excavation showing the displacement of a monument consisting of wooden cylinders. The monument is sheared, displaced vertically 120 cm and tipped 42 cm. Symbols on the sketch mean: (1) loose yellowish-brownish clay, slightly plastic; (2) grey sticky clays; (3) sandstone fragments and inclusions; (4) shearing (sliding) surface; (5) original position of monument.

A method permitting constant observations of the slide displacements consists in the excavation of special pits and trenches with non-rigid timber bracing. Bracing of a cylindrical pit (hole of large diameter) may consist of concrete rings 15 to 25 cm (6 to 10 in.) high, lying freely on each other. Horizontal displacements are well recorded by such installations but the vertical component of the sliding displacement cannot be correctly measured until the installation is completely sheared.

BALANCE OF SLIDING MASSES

A sliding mass may or may not vary in weight during the sliding process. Increase in weight is due to (a) active participation in the sliding process of the masses usually located at the top and at the sides of the slide; (b) deepening of the sliding body; (c) gradual arriving of earth material from the areas above the top of slide, because of falls (e.g., as with the Crimean slides), and because of erosion of these areas; (d) artificial additions to the sliding mass (e.g., because of the earthwork on railroads and highways located on the sliding body); (e) deposition of sediments on the slide's tongue and at the foot of the slope (e.g., in the case of formation of a sea or river terrace). Decrease in weight of the sliding body may take place because of (a) the slope's foot abrasion, erosion, flattening of slopes; (b) erosion of the slope with formation of ravines and gullies; (c) surficial soil removal (ablation); (d) artificial removal of earth masses in grading, building of cuts and excavations, removal of slides or heaves on the railroads and highways; (e) subsurface erosion. What is meant under the terms "balance of sliding masses" is the difference, positive or negative, of the increase and decrease of the weight of the masses constituting the sliding body.

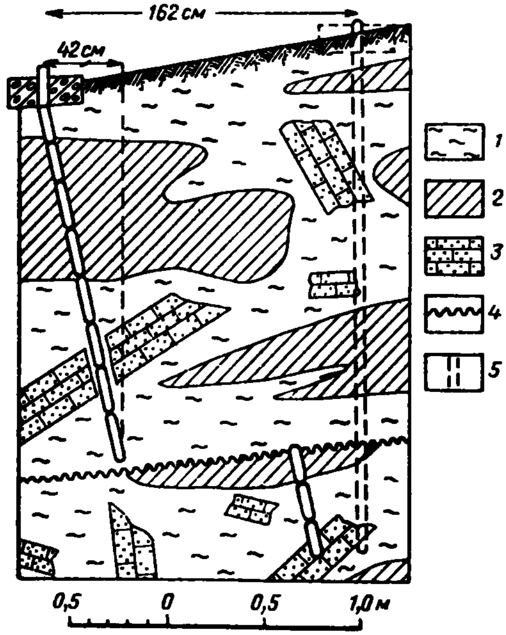

Figure 8. Idealized page from the field book of a slide observer showing translation and rotation of a deep monument.

Displaced soils are more or less saturated; and besides water may be displaced independently from the soil. Hence, strictly speaking, the balance of earth masses and the balance of water should be prepared independently, both of them being the components of the general balance of displaced masses. Hereafter, the water balance on the slope is not considered separately from the earth-mass balance.

With circular-cylindrical sliding surface the conditions of equilibrium of the sliding body depend on the position of its center of gravity. When the center of gravity is lowered, stability of the sliding body increases and vice-versa. Therefore, when considering a slide with circular-cylindrical sliding surface, it is necessary to consider separately the portions of the slide separated by a vertical plane passing through its center of gravity (determined at a certain initial time moment) and parallel to the strike of the slope. The position of this vertical plane in space is considered constant and the displaced masses are visualized as passing through that plane. In this way, the center of gravity of the slide is visualized as moving in space, and not within the earth mass. The negative balance of the upper portion of the sliding mass, and the positive balance of its lower portion mean the downward displacement of

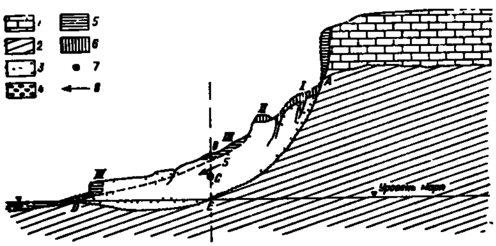

Figure 9. Balance of sliding masses: (1) limestone; (2) clay; (3) body of ópolzen; (4) beach gravel; (5) removed material; (6) added material; (7) center of gravity; (8) direction and length of displacement.

the center of gravity and increase in stability. The positive balance of the upper portion and the negative balance of the lower portion correspond to decrease in stability. If the sign of the two balances is the same, their absolute values are to be compared:

Example: The solid line AED in Figure 9 shows the position of the sliding body in 1947, with its center of gravity at C.

Before 1952 the following occurred (for symbols I, II, III, IV, see Fig. 9):

I. There was a limestone outfall, 340 m3 in volume.

II. Because of gradual settling, 180 m3 of imported gravel were placed on the existing highway.

III. As a result of the ravine and gullies, growth 170 m3 of earth were washed off.

IV. As a result of abrasion, the volume of the tongue decreased by 1,850 m3, whereas the volume and the regime of sediments in the shore portion of the sea did not change substantially. No suffosion in the slope was reported. Suffosion (term used by some Russian engineering geologists) is the internal erosion and removal of the eroded material at the base of a slope.

The average displacement of the sliding body for 5 yr, 1947-1952, as measured along a monument row passing through the center of gravity C was 95 cm horizontally and 24 cm vertically.

Assume the following unit weights of the materials: limestone, 2.35 tons per m3; gravel, 1.85 tons per m3; earth materials of the sliding body ravines and gullies, 1.75 tons per m3. Then:

1. Balance of the upper portion of the slide ABE.

The increase in weight consists of the weight of the limestone outfall (2.35 x 340 = 799 tons) and the weight of imported gravel (1.85 x 180 = 333 tons). Total increase = 799 + 333 = 1,132 tons.

The volume of the material lost by erosion in the ravines (170 m3) should be broken into two parts. Assume that 130 m3 have been eroded in the upper portion of the slide and 40 m3 in the lower. This gives a loss in weight in the upper portion of the slide equal to 1.75 x 130 = 227 tons.

Assume, furthermore, the average depth of the sliding body at 18 m and its front width (parallel to the strike of the slope) at 100 m, the volume of earth materials that passed through section BE is 0.95 x 18 x 100 = 1,710 m3 and its weight 1.75 x 1,710 = 2,992 tons. Hence, the total soil balance of the upper part of the slide ABE is + 1,132 - 227 - 2,992 = - 2,087 tons.

2. Balance of the lower portion of the slide BDE.

There is no addition of new material in this portion. The loss in weight equals the weight of the abraded material (1.75 x 1,850 = 3,237 tons) and that loss by abrasion in the ravines (1.75 x 40 = 70 tons), total 3,307 tons. The weight of the material passed through section BE (2,992 tons) is an increase in weight of the lower portion of the slide. Thus the total balance of the lower portion of the slide is:

$$- 3{,}307 + 2{,}992 = - 315 \text{ tons.}$$

Both the upper and the lower portion of the slide have negative soil balance. However, because the decrease in weight of the upper portion of the slide is larger than that in the lower portion, the center of gravity has moved down during the 5-yr interval which shows a tendency for stabilization of the slide. The balance of the slide as a whole is - 2,087 - 315 = - 2,402 tons, which means that the total weight of the sliding body is decreasing with time. Again, strictly speaking, the weight of the outfall (799 tons) should not be considered because this is a displacement of the material inside the slope. Under this assumption the negative balance of the slope as a whole would be - 2,402 - 799 = -3,201 tons and the average denudation intensity 3,201 ÷ 500 = 6.4 tons per lin ft of the slope per year.

OTHER ACTIVITIES OF FIELD STATIONS

Besides the continuous measurement of the displacements connected with the sliding process, as described, the field slide stations are conducting studies of (a) the sliding factors and (b) the effectiveness of the anti-sliding measures. Study of sliding factors in a given environment encompasses seismicity, meteorology, hydrology (including both surface and subsurface water, and water content regime of the slope), weathering, etc. An important sliding factor is the undermining of slopes by both stagnant and current water, and particularly by waves.

The anti-sliding measures studies are regulation of the run-off, drainage, mechanical stopping of sliding masses, modification of soil properties, stabilization of slopes, etc.

The Russian field slide stations belong to different "ministries" or Government repartitions. Most of the stations are under the jurisdiction of the Ministry of Geology, though some are in the Transportation Ministry, in the Ministry of Coal Industry (1), etc. The stations have to submit periodical reports and prepare recommendations when requested. Prognosis of sliding possibilities is also their responsibility.

CONCLUSIONS

The writer wishes to emphasize the purely informative character of this paper. No definite suggestions are made, but the writer believes that the idea of stationary (or rather regional) landslides observations deserves attention. Besides a thorough discussion of the subject matter—a discussion the writer thinks very desirable—it seems advisable to start a few field slide stations, not necessarily exactly of the Russian type, in zones affected by landslides that are produced as a natural step in the denudation process of a given region. Another proper location of such field stations would be next to river canyons in which several dams are planned. Quite a few facts became known when, after the construction of a reservoir and local collapse of its shores, expensive relocation of threatened highways and railroads running parallel to those canyons was necessary. States and counties, and in pertinent cases the railroad companies, should be interested in the idea.

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Soil and Foundation Investigations on the Patapsco Tunnel Project

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● THE PATAPSCO Tunnel Project constructed by the State Roads Commission of Maryland is the largest public works project ever undertaken in the state. It is comprised of a twin-tube tunnel under Baltimore Harbor between the industrial areas of Canton and Fairfield of Baltimore City, together with expressway approaches connecting the tunnel with Baltimore-Washington Boulevard and Baltimore-Washington Expressway on the west; with the Glen Burnie Bypass and Governor Ritchie Highway on the south; and with Pulaski Highway and the proposed Northeastern Expressway on the northeast. Figure 1 is a location map of the Patapsco Tunnel Project.

The entire roadway is 17.6 mi long, including 1.2 mi of tunnel, and 3.5 mi of structures. The approximate cost of the entire project is $127,000,000.

The purpose of this paper is to describe the procedures followed in the soil and foundation studies for this project from the civil engineering report phase through the design and construction phases, and, finally, into the maintenance and operations phase of the facility after it was opened to traffic. No attempt has been made to discuss the many interesting aspects of the tunnel investigations and related construction features. The tunnel portion of the project is worthy of an independent paper.

CIVIL ENGINEERING REPORT

In September 1953, the State Roads Commission of Maryland engaged J. E. Greiner Company to prepare a civil engineering report. One of the main reasons for the report was to isolate problem areas and to estimate the construction cost of the entire project. The report included a preliminary set of plans and specifications for the tunnel and its approaches and indicated the scope and general character of the work. Design criteria were also prepared for the purpose of establishing uniformity over the entire project. Because of its timing with regard to the construction seasons, it was considered advantageous in June 1954, to start the design work on the tunnel section. This enabled the design engineers to prepare a complete set of plans and specifications in time to start the tunnel construction prior to the construction of the approaches. This scheduling was necessary because the tunnel was contemplated to require 32 months for construction as compared to 27 months for construction of the approaches. On October 8, 1954, the civil engineering report was submitted to the State Roads Commission.

The first undertaking in the soil and foundation studies was the investigation of available sources of data. Soil Survey Reports of the U. S. Department of Agriculture, reports of the Maryland Department of Geology, Mines, and Water Resources, and publications of the U. S. Department of the Interior were used in developing the general soil and geological conditions of the area.

To acquaint the reader with the local soils and geology, a brief description of the general area is offered.

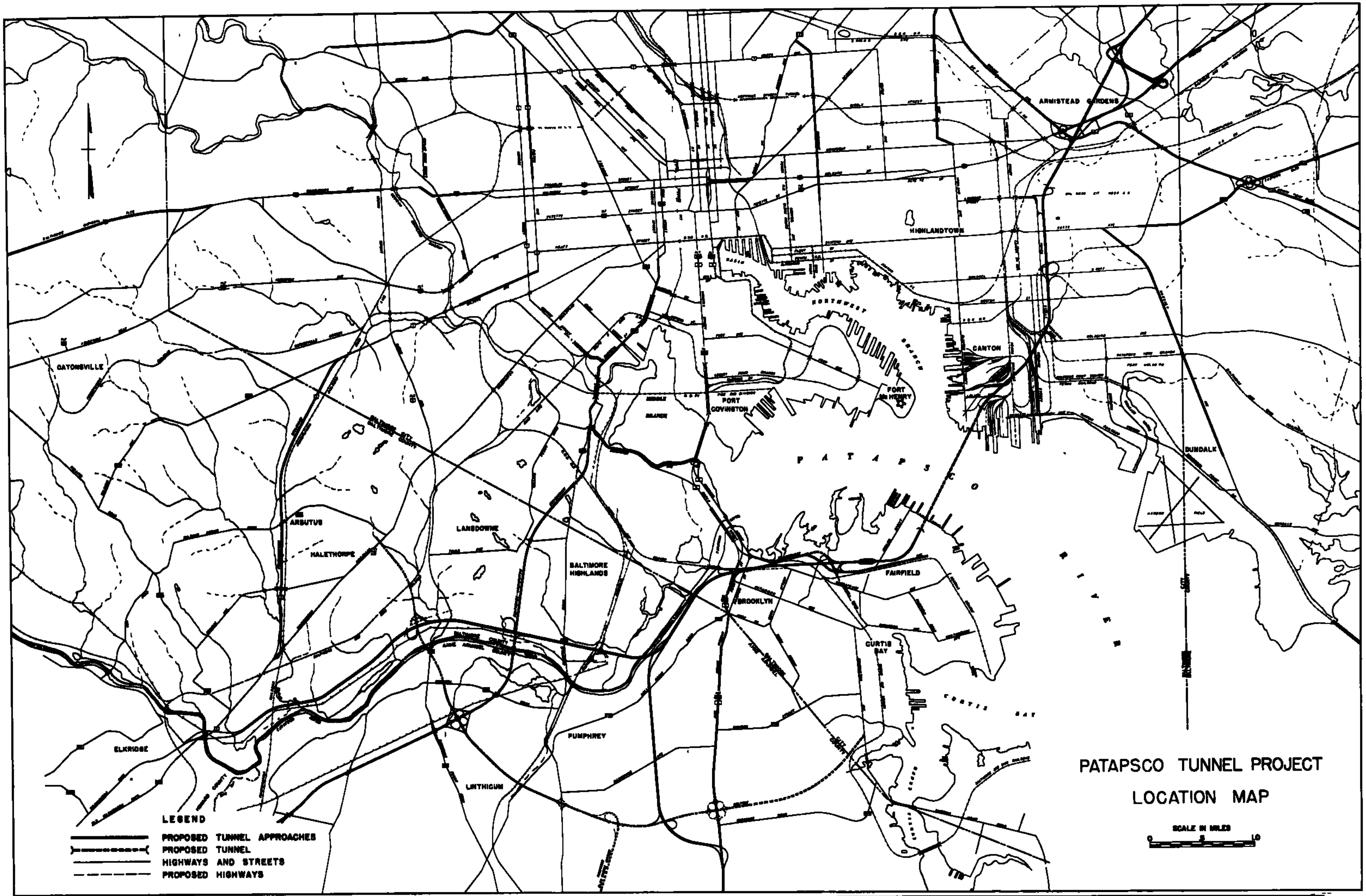


Figure 1. Location map.

The Patapsco Tunnel Project falls completely within the Coastal Plain. The predominant soil types are eroded sediments of an unconsolidated nature. Such topographic features as low hills, shallow valleys, flat plains, and numerous terraces are characteristic of this plain. Long, narrow peninsulas which extend in a southeasterly direction to the Chesapeake Bay are separated by bodies of water which have "drowned" existing valleys.

The surface soils most prevalent along the route represent formations of the Potomac Group of the Lower Cretaceous. Some recent sediments of the Quarternary are also evident.

The Potomac Group consists of three formations; namely, the Patuxent, the Arundel, and the Patapsco. These sediments rest upon crystalline rock, which, with the exception of the most westerly portion of the route near the Baltimore-Washington Boulevard, lies far below the limits of any foundation considerations.

The first deposits formed upon the rock were the coarse sediments of the Patuxent. After a period of erosive action, clays of the Arundel filled in old drainage depressions, and then increased in depth completely covering the Patuxent. This process occurred under swampy conditions as evidenced by the recovery of lignitic substances and segments of tree stumps during subsequent boring operations.

A general areal submergence of the Coastal Plain followed and resulted in the deposition by streams of sediments of a coarse nature over the Arundel. This formation is known as the Patapsco. A prolonged period of erosive action then took place.

The soils of recent age encountered in this area are gravel and clay deposits which were stream transported from the northwest during the periods of glacial action.

After all of the available information was compiled and analyzed, a field study was made, during which time the entire proposed alignment was covered on foot, observations made, and samples recovered for laboratory testing. Special care was exercised to assure that all areas of poor soil and foundation conditions, as well as areas of exceptionally good soils, were delineated.

Of the numerous areas of questionable soil and foundation conditions which were observed and described in the civil engineering report, five have been selected for discussion of their design treatments. Three of the five areas are discussed in more detail in the construction phase.

The civil engineering report noted that the valley of Herbert Run and the Kaiser fill represented two adjacent areas of concern. Though both areas were foundation problems, the soil conditions were quite dissimilar in character. In the valley, sediments of organic silty clays to depths exceeding 10 ft were encountered. The low lying terrain was very wet and swampy. Immediately to the east of this site was a loosely placed fill which, at the time of placement, was waste material from the construction of a nearby Kaiser Aluminum plant.

The tidal marsh west of the Patapsco River was recognized during the preparation of the report as an area deserving special foundation considerations. In addition to numerous probings which revealed organic deposits continuous over sands and gravels, a number of undisturbed samples in the organic deposits were recovered from contract borings. The organic soil was found to vary from 12 ft near the shoreline to 35 ft in the vicinity of one of the many tributaries located in the marsh. The results of consolidation tests performed on the undisturbed samples indicated that settlements ranging from 1 to 3 ft could be anticipated. Eighty-five percent of these movements would occur during the construction period.

The third area discussed is the miscellaneous fill west of Potee Street which extends for a considerable distance along the eastern bank of the Patapsco River. Because of its extent and variable character, contract borings of a limited nature were taken through this reach. Organic sediments and miscellaneous dumped materials were disclosed by the borings and found to vary from 15 to 35 ft.

Compressible soils north of the tunnel were revealed by borings taken in connection with the advanced tunnel design. Intermittent soft layers of organic deposits were evident to a depth of 80 ft.

The final area to be discussed is encountered in passing through the refinery properties of Standard Oil Company of New Jersey. Numerous miscellaneous fills of shallow extent were observed. One critical area in this vicinity was the miscellaneous fill adjacent to Tank 502.

DESIGN PHASE

After acceptance of the civil engineering report by the State Roads Commission and arrangements for financing completed, six design engineering firms were selected for the actual design work of the approach expressways. J. E. Greiner Company had already been retained by the Commission as General Consultant to act as its agent in the supervision of the work. During the progress of the design work, the Commission decided to extend the project at its northern terminus. Accordingly, a seventh form of engineers was selected to accomplish the additional work.

With numerous design firms performing their own soil and foundation studies, it was considered desirable to establish a set of standard specifications for the performance of all subsurface studies. These specifications were part of an over-all engineering specification outlining the functions to be performed by the design engineering firms. They assured that adequate soil and foundation studies would be performed for the proper design and treatment of the structural and roadway aspects of the particular design section.

A master soil plan and profile was prepared for each construction section by the design engineer for that section. These drawings served a number of different purposes. They presented a complete picture of the soil conditions to the contractor for his interpretation so that full benefit could be made of the various soils encountered. They also eventually became a complete record of all of the soil and foundation studies which were performed when supplemented with information obtained during the actual construction. Where the data was too voluminous for inclusion on the master set of drawings, an additional report was prepared of such information. In an effort to simplify these plans, only one typical boring for each structure site was included. The remaining structure borings were shown on the design plans for the particular structure. Also included on the master plan and profile sheets were test data used in the development of the design plans, such as compaction test data, unconfined compression test data, and consolidation test data. Figure 2 shows a typical master soil plan and profile sheet.

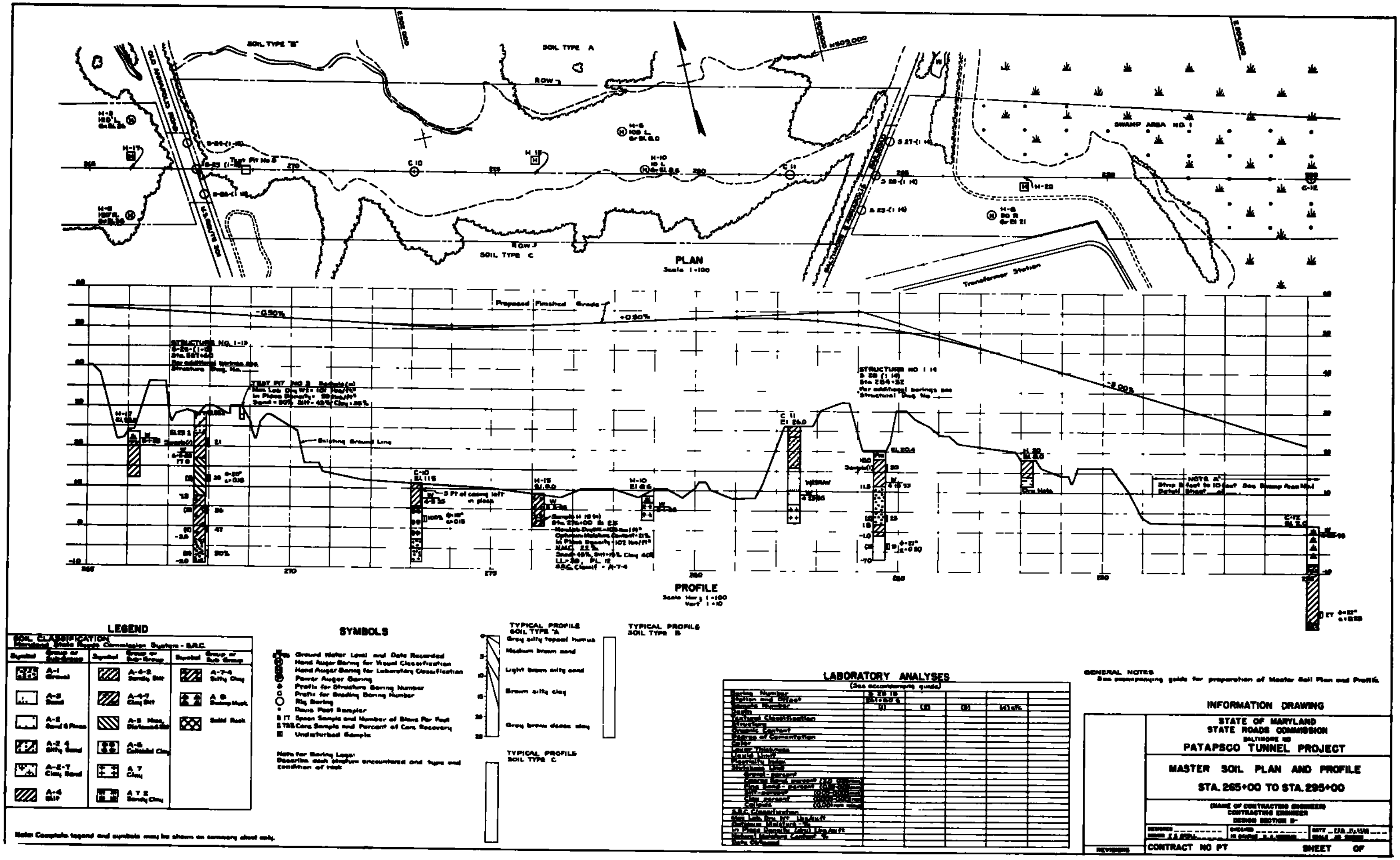


Figure 2. Master soil plan and profile.

The scales selected for the drawings were the same as those used in the preparation of the contract plans; namely, 1 in. = 10 ft vertically and 1 in. = 100 ft horizontally. The plans were kept simplified by only showing horizontal alignment, stationing along the centerline every 500 ft, locations of borings, and sufficient topographic features to permit proper orientation in the field. These studies also included interchange areas. The data obtained for inclusion on the master soil plan and profile sheets were developed from contract borings, hand and power auger borings, test pits, and special types of samples required in swampy areas.

Although the General Consultant provided general specifications for the performance of test borings, it was the responsibility of the design engineering firms to prepare the boring specifications and supervise the actual boring operations for their particular sections. The design engineers' own field personnel supplemented the contract boring data with whatever other necessary information was required to present a practical representation of the actual soil and foundation conditions.

Some of the general specifications relating to the soil and foundation studies are mentioned. Normally, one rig boring was taken at each abutment or pier location. Depending upon the structural design, borings were staggered in such a manner that the uniformity or lack thereof in the soil profile was established prior to increasing the number of borings required at a site. Contract borings were taken in cut sections when the cut was longer than 500 ft and the depth of cut exceeded 15 ft. Depending upon the length of the cut, borings were spaced at 300- to 600-ft intervals and taken to depths of at least 5 ft below profile grade line. Rig borings with undisturbed samples were also taken in areas of high embankments and questionable subsurface conditions. These samples were subjected to laboratory analyses necessary for the proper selection of the final design treatment.

Seven boring contracts were awarded by the Commission to obtain ordinary dry samples, core borings of a limited nature, and undisturbed samples for testing purposes. The aggregate amount of these contracts was $84,900, which represented 0.067 percent of the approximate project costs.

Other soil conditions were obtained from auger borings with particular attention given to transitional areas of cut to fill. These areas are quite frequently a source of pavement distress and deserved special review.

The design investigation for the valley of Herbert Runn and the Kaiser fill recognized two problems; namely, the undesirable soft organic soils in the valley and the presence of a loosely dumped, unconsolidated embankment to the east. Figure 3 is the soil profile of this area. The limits of the unsuitable material were determined by a grid pattern of 32 hand auger borings and probings which disclosed as much as 20 ft of soft organic clayey silt. Borings taken at the site of the structure over Herbert Run provided undisturbed samples for foundation studies in the valley area. These studies revealed that the weight of the 40-ft embankment across the valley would result in settlements of intolerable magnitudes requiring time of settlement far beyond the period of construction. It was decided, therefore, that the unsuitable soil be completely removed for the 800-ft valley crossing. the limits of removal of this organic soil were as shown on Figure 4, the standard section for removal of unsuitable material.

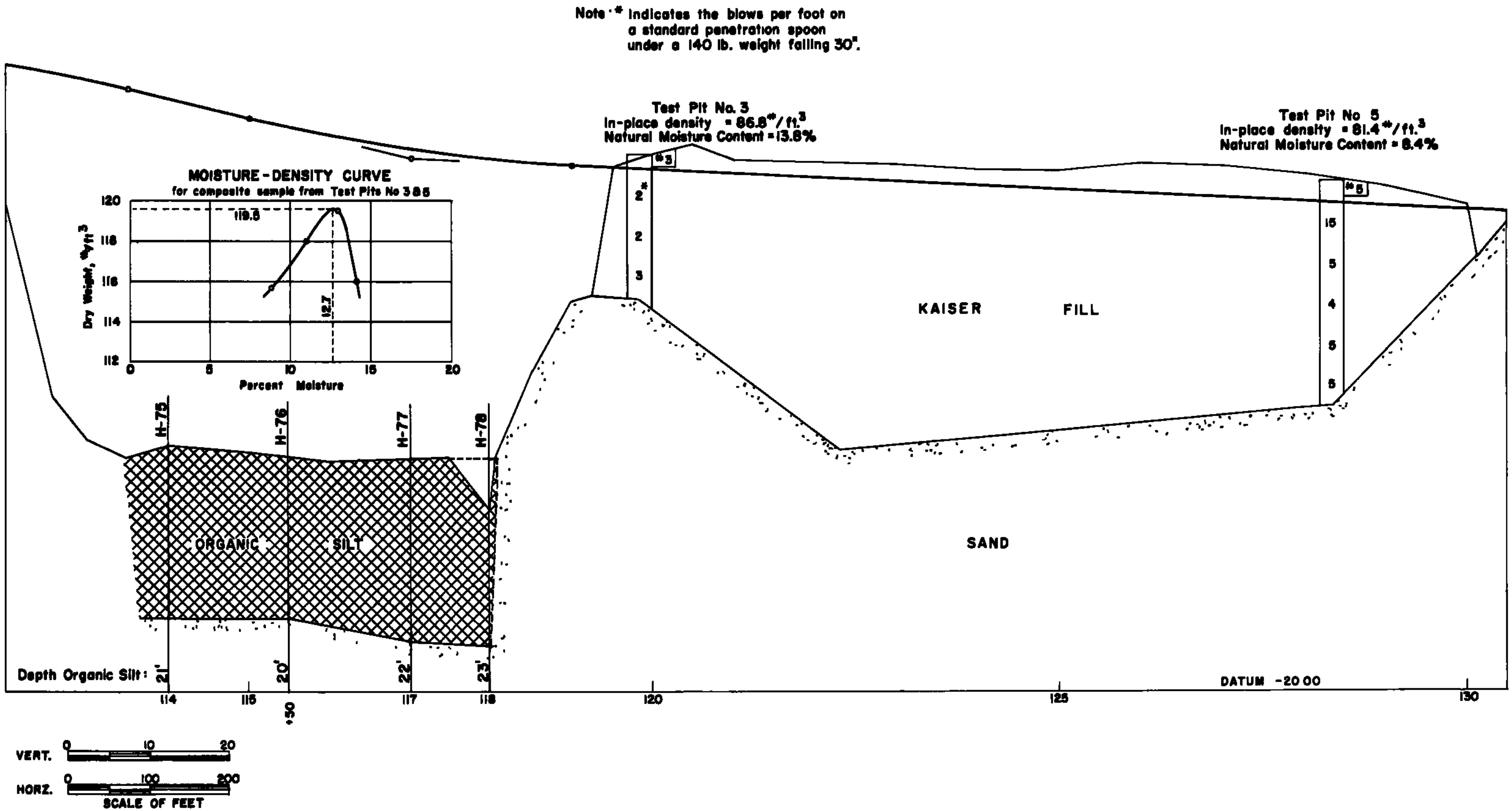
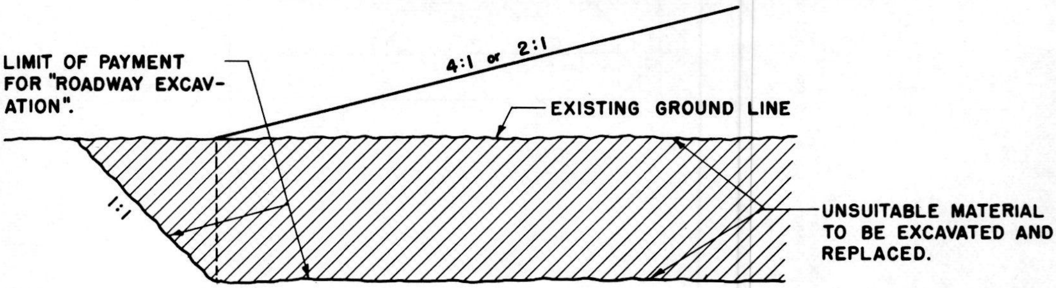


Figure 3. Soil profile—Herbert Run and Kaiser fill.



REMOVAL OF UNSUITABLE MATERIAL

Figure 4. Standard section for removal of unsuitable material.

The Kaiser fill immediately east of the valley was composed of 30 ft of granular materials, which had been placed by a bottom dumping operation. At the time of this operation, it had not been anticipated that the area would be utilized in the future for an expressway. Figure 5 is a view of the unconsolidated fill material.

Figure 5. Unconsolidated Kaiser fill.

Contract borings through this area, which extended for 1,000 ft, indicated that the very loose deposits had a resistance of 2 to 5 blows per foot on a standard penetration spoon driven by a 140-lb weight falling 30 in. This poor resistance was consistent through the entire 30-ft depth of the deposit. In-place density tests disclosed the average fill density to be 75 pcf. When compacted by the standard Proctor density test, the same soil had a dry unit weight of 120 pcf. The Highway Research Board Classification for this material was A-2-4.

Because the material was of very low density and would be subjected to frequent vibratory loads from the heavy volume of truck traffic, it was the opinion of the engineers that harmful differential settlements could occur. In the final design of this section, the removal of the Kaiser fill to a depth of 10 ft below profile grade line was considered adequate. Partial removal, rather than complete, realized substantial savings in the cost of this section due to the reduced earthwork quantities. Because the material was granular in character, it was used as underwater backfill in the valley to the west where the unsuitable organic soils were removed. After a surface preparation, the embankment up to profile grade line was constructed in accordance with the standard compaction requirements for fills.

The tidal marsh west of the Patapsco River was investigated by obtaining 15 contract borings and 22 hand auger borings and probings. It was found that unsuitable materials extended to a depth of 60 ft. These sediments were deepest toward the River and studies indicated a viaduct type structure to be most economical over these deep deposits.

Figure 2 shows the plan view of the western limit of the marsh area in the vicinity of Station 291. The presence of unsuitable organic silt along the centerline of the expressway is indicated on Figure 6. From this information, it was recognized that portions of the westbound lanes between Stations 287 and 291 would be underlain by varying depths of organic silt. The eastbound lanes were located on firm ground. To further complicate the differential conditions at this site, the westbound lanes required a fill of 40 ft in height to bring it to grade, whereas the eastbound lanes required only a 20-ft fill. This poor condition was recognized early in the route studies, but no change in location was made because it would have been necessary to relocate an electric substation, the property fences of which were already abutting the right-of-way boundaries. Economic studies of this situation favored a complete removal of the unsuitable materials and a backfill with clean granular soils below the water level and the use of satisfactory borrow materials above this elevation.

The miscellaneous fill west of Potee Street was found to follow the Patapsco River for a considerable extent. Investigations through this reach disclosed the presence of several different types of undesirable materials. The original surface soils in this area were organic silts which varied in depth from 8 to 25 ft. Beneath the silt layer were excellent deposits of sand and gravel. Because of its poor supporting quality, this land had never been improved for either commercial or residential purposes. At different times, it had been utilized as a municipal dumping area. Twelve borings taken for the Potee Street structure and the future Patapsco Avenue structure in addition to auger borings taken at the location of fills, furnished data to differentiate between the variously constituted fills; namely, garbage fills, trash fills, and cinder fills. This is shown in part on Figure 7. The most undesirable condition existed at the location of the trash fill where the combined thickness of trash and organic silt was 30 ft. The ground water level through these fills

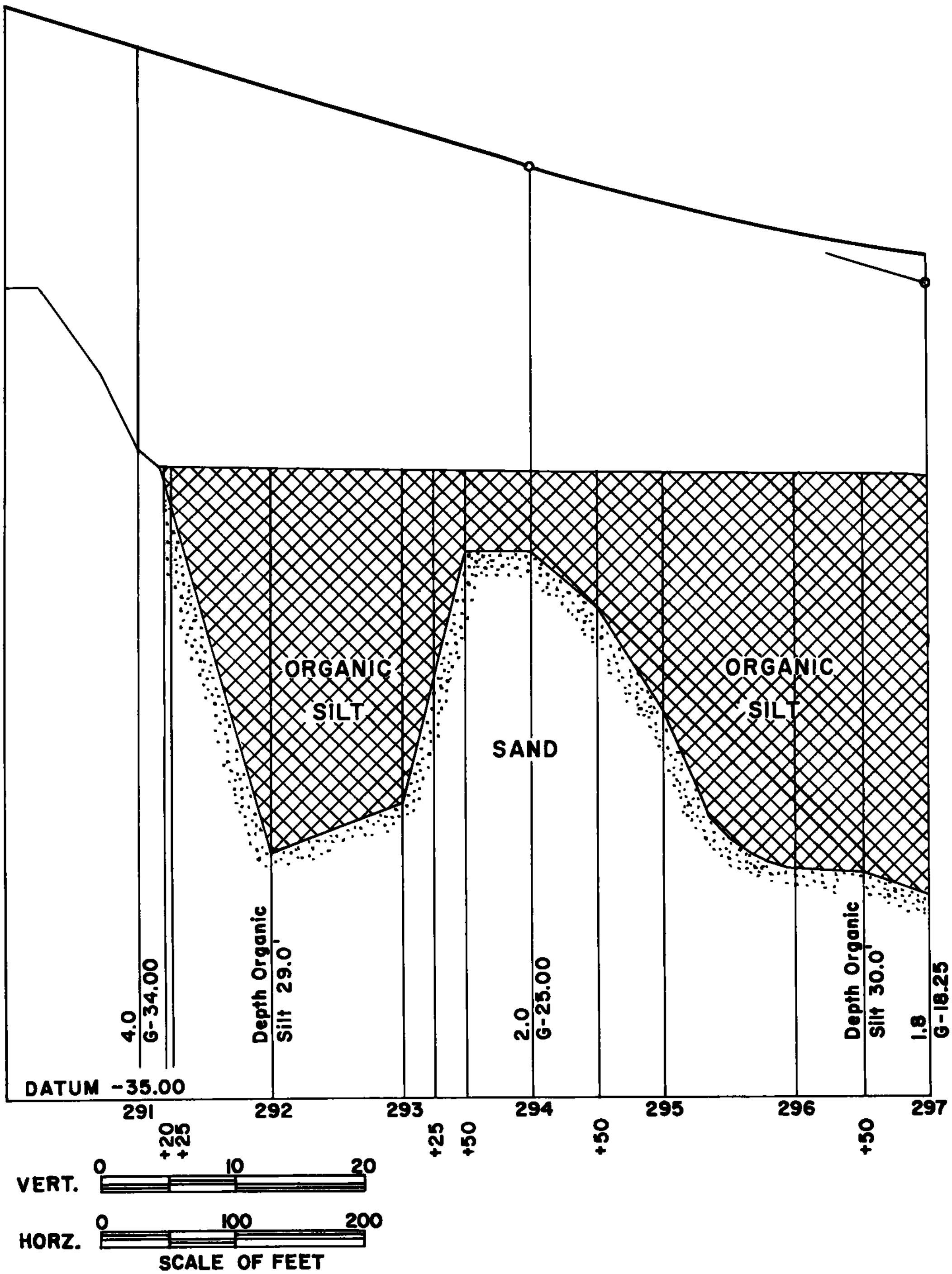


Figure 6. Soil profile—tidal marsh west of Patapsco River.

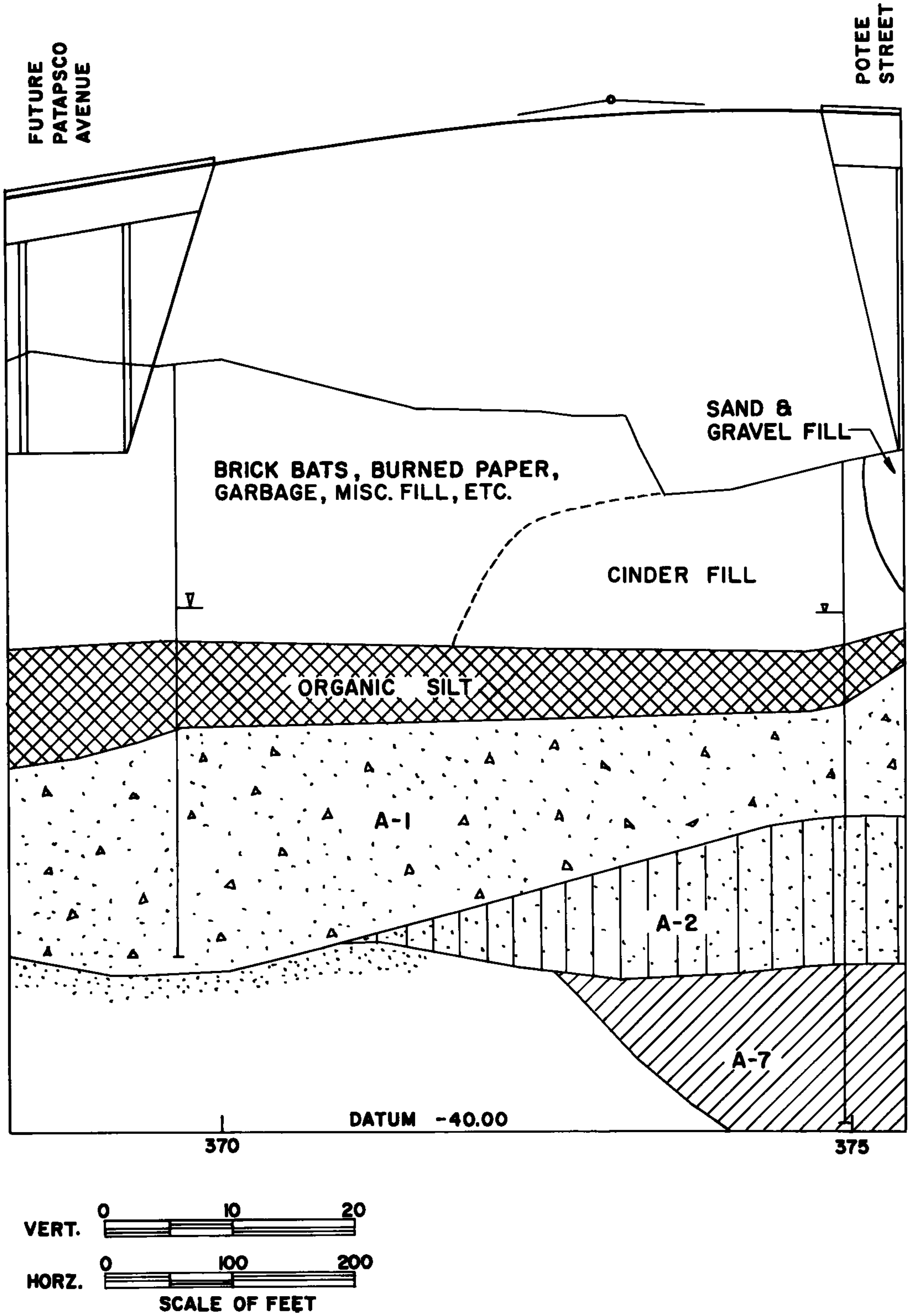


Figure 7. Soil profile—miscellaneous fill west of Potee Street.

was very close to the water level of the Patapsco River and, in many instances, was observed to fluctuate with tidal changes. This located the water level about 3 ft above the organic material.

The profile grade line through this area was approximately 50 ft above the sand and gravel stratum. This resulted in a fill of 30 ft in height. Because of the magnitude of this superimposed load, it was necessary to treat the various underlying fill materials in different manners. The garbage fill and trash fill were completely removed and granular materials used to backfill these areas. The section which was for the most part a cinder fill appeared capable of supporting the embankment and it was decided to surcharge this area rather than remove the cinders. This procedure resulted in a savings of $300,000. The design specified a 20-ft surcharge load which was to be placed by a carefully controlled field operation.

Though settlement computations were made in the underlying organic silt, it was the opinion of the engineers that the settlement in the cinder fill would be of more concern. Because of its heterogeneous character, settlement calculations were of little value in these deposits. However, the fact that the water table was at the lower extremity of the cinder material indicated the likelihood of very rapid deformations.

Another area requiring special foundation design and construction treatments occurred in the vicinity of the toll plaza which is located immediately west of the tunnel. Deep deposits of silt were encountered. The design engineers recommended the use of a surcharge loading to accelerate the consolidation of the unsuitable materials. Henry W. Janes, of Whitman, Requardt and Associates, thoroughly discusses this situation in "Deep Silt Consolidated by Surcharge Fill on the Patapsco Tunnel Approach," HRB Proceedings, Vol. 37, p. 538 (1958).

The compressible soils north of the tunnel are discussed briefly to emphasize the desire of designers to keep substructure and superstructure costs in economic balance. The roadway in this area ascends from the tunnel to the original ground surface and then to an elevated viaduct which crosses the railroad yards in Canton. Soft organic clays were the predominant soil throughout this portion of the alignment and were underlain by intermittent organic deposits to great depths. Design studies of the various foundation types indicated that retaining walls less than 9 ft in height could be founded on spread footings whereas higher walls required cast-in-place concrete pile foundations. Figure 8 indicates the open ramp north of the tunnel. The northernmost 400 ft of the walls on either side of the roadway are less than 9 ft in height.

A miscellaneous fill near the Esso tank farm was crossed as the route continued north from the tunnel. In addition to the presence of heterogeneous deposits of fill materials, other problems were encountered due to the maze of distribution lines which required relocation. These relocations were very critical because the change from one line to another had to be made during periods adjusted to satisfy the needs of the refinery. Water lines for fire fighting, crude oil distribution pipes, and safety dike reconstruction had to be coordinated into the over-all construction schedule.

A 20-ft depth of gravel, brick, and cinders was underlain by 8 ft of soft organic silt. The grade line through this area was 20 ft above the existing ground line. Examination of the fill samples indicated that the

Figure 8. Open ramp near north portal of tunnel.

consolidation of the organic silt was the main consideration. It was the opinion of the engineers that this compressible layer could be fully consolidated during the construction period.

CONSTRUCTION PHASE

The construction phase of the project became by far the most important. Situations arose which required immediate field decisions. Because judgment played an important part in these decisions, it was essential that the field personnel responsible for the soil and foundation aspects of the work be experienced and competent.

It must be appreciated that after construction was begun, the laboratory test data and office calculations were secondary in importance to actual field observations. The purpose of this data was fulfilled when used in designing foundations on questionable material. It then remained for field observations to govern final construction.

Many areas of interest were encountered during the construction phase; however, only three have been selected for further discussion in this paper.

First to be discussed is the tidal marsh west of the Patapsco River. This troublesome area was located in two design sections and, consequently, was constructed by two contractors.

The detailed studies favored the complete excavation of the unsuitable soft organic silty clay. At the western shoreline of the marsh only 3 ft of unsuitable soil was evident. This depth increased as the alignment progressed further into the swamp to an area where the maximum centerline thickness was 20 ft.

Initially, the unsuitable material was removed by a dragline in an underwater operation. The selected materials for backfill satisfied the standard specification for underwater backfills and consisted of either A-1, A-2-4, A-2-5, or A-3 soils. The material was placed by dumping it on the higher ground and then pushing it into the excavation by a bulldozer.

As discussed under the design treatment of this area, the soil conditions under the westbound lanes were much more critical than under the eastbound lanes.

During the progress of the work, a tension crack was observed along the edge of the embankment. Borings, which were taken to establish the cause for this visible indication of distress, revealed that there were areas not completely void of the unsuitable materials. A meeting was held between the design engineering firms concerned and the General Consultant to establish the course of action which should be taken to rectify this difficulty. It was the general opinion of the engineers that the probing method of assuring complete removal of unsuitable soils was defective under certain subsurface conditions and could prove misleading. To alleviate this situation, it was decided to remove the unsuitable material by dragline and attempt to lower the water table by pumping so that the actual soil conditions could be viewed during the removal operations.

Six days after the "excavation in-the-dry" operations were begun a hurricane passed through the Baltimore region and inundated the entire area as shown in Figure 9. It was necessary to use two 4-in. sump pumps on a 24-hr basis for $2\frac{1}{2}$ days to return the site to its original condition before the hurricane. Aside from the loss of this working time, no damage was evident.

Near the completion of the removal of the unsuitable soil, it was necessary to operate on an around-the-clock schedule. This continuous operation was necessary because the organic silts were unable to maintain stable slopes through the night and a great deal of time was lost each morning reestablishing stable working conditions. The lack of stability was the result of deepening silt deposits on an unfavorable cross slope away from the centerline of roadway. In addition, it was necessary to extend the standard limits for removal of unsuitable material (as shown in Figure 4) to provide a more stable condition.

It should be noted that the success of this operation of removal in-the-dry below the water level of the surrounding marsh is a direct function of the limited areas of operations. Small temporary dikes and generally two small sump pumps were sufficient to keep an area dry until the backfilling was completed. The experience here led to the conclusion that excavation of unsuitable material in-the-dry, if at all possible, is always preferable.

Figure 10 is a photographic view looking south at the completed embankment.

The area over the cinder fill portion of the miscellaneous fill in the vicinity of Potee Street was designed for the placement of a 20-ft surcharge load to accelerate the incalculable settlement. Because the

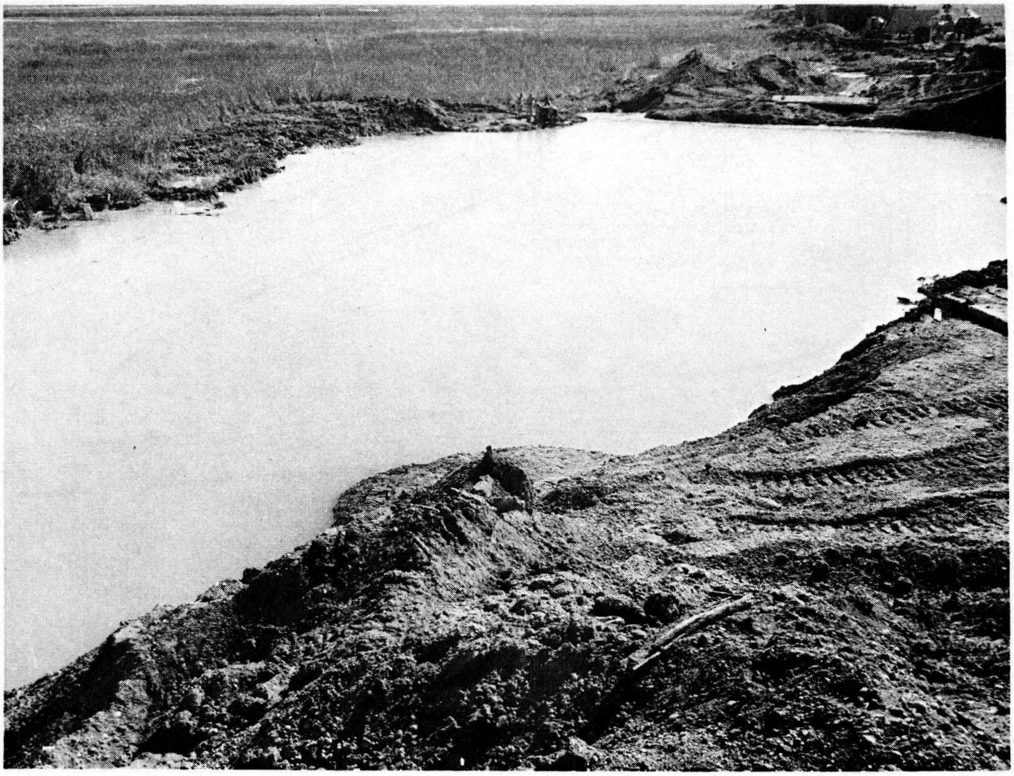


Figure 9. Station 287—area of unsuitable material.

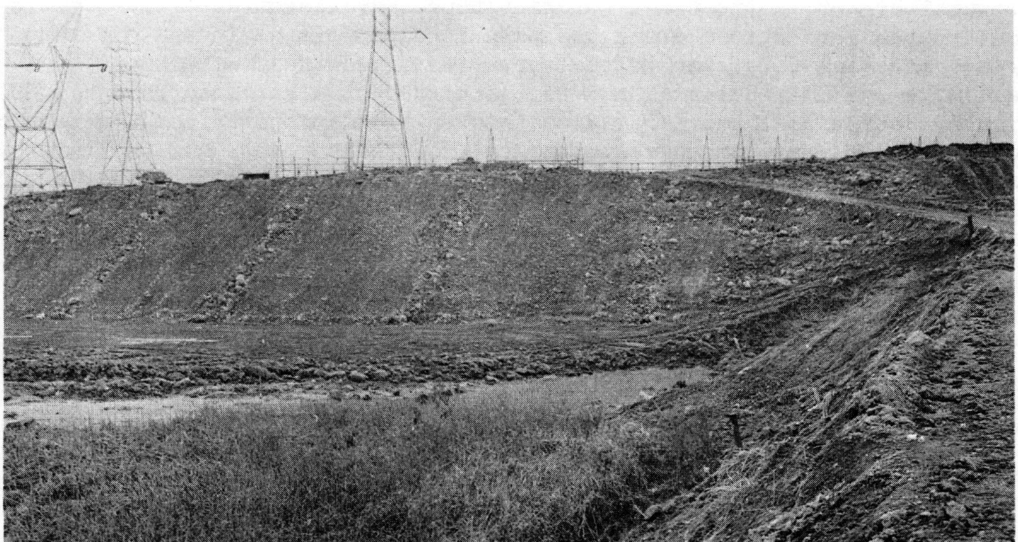


Figure 10. Station 287—completed embankment.

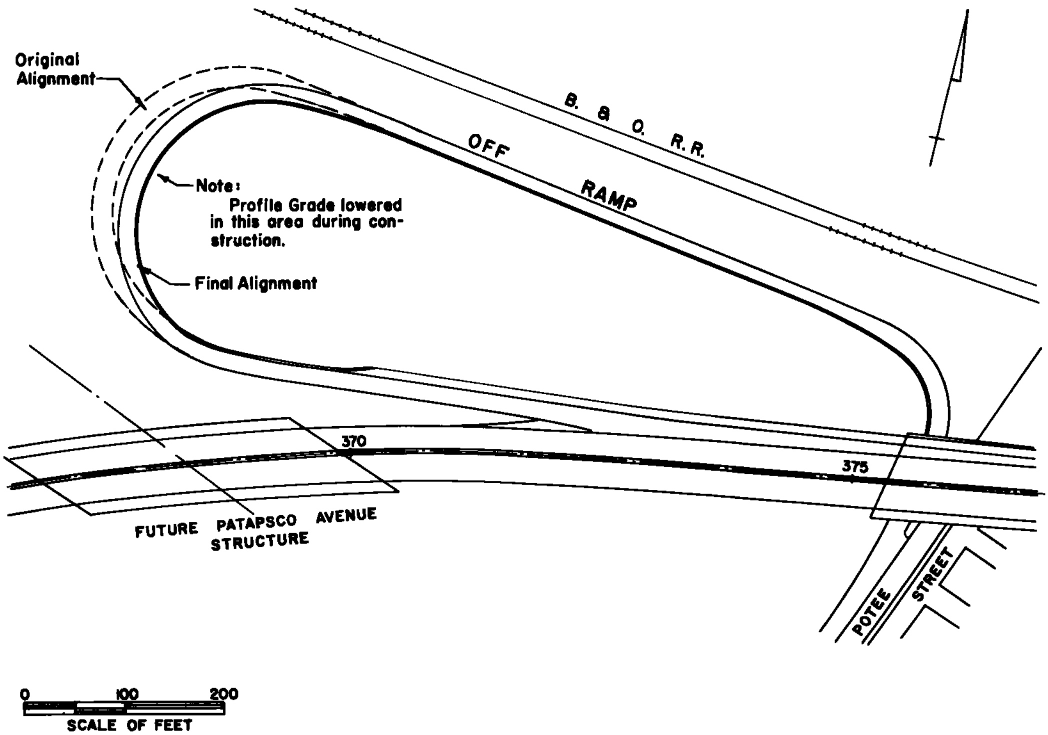


Figure 11. Station 375—main line and off-ramp plan views.

fill material overlying the relatively thin layer of organic silt was variable in character it was not the subject of settlement computations. Figure 11 shows the plan of the site.

In March 1956, construction was begun on the main line and off-ramp between the Potee Street and future Patapsco Avenue structures. Settlement plates were placed along the main line as well as in the off-ramp area. Abutting this ramp to the north was a two-track embankment of the Baltimore and Ohio Railroad and to the northwest a railroad bridge. Obviously detrimental settlements to this railroad property could not be tolerated. The new embankment, the railroad embankment, and the railroad bridge were in an area of questionable foundation subsoils, therefore, controls were established on one rail of the double track, the toe of slope of the railroad embankment, and the abutment and first pier of the railroad bridge so that settlements could be carefully observed and construction operations controlled to avoid difficulty.

Because the initial settlement readings were most important, two level runs a day were made. Shortly after construction was begun, rapid settlements were observed adjacent to the main line roadway embankment. Some heaving was also observed in the loop area. At this time the construction operations were halted and additional controls placed along the original ground surface on both sides of the main line fill. Thereafter, limited lifts of fill material were placed and their effect on the underlying compressible soils carefully observed prior to the placement of additional lifts.

Figure 12. Off-ramp slide failure.

The operations continued for the off-ramp according to plan until the elevation of the fill was within 2 ft of the profile grade line. At this time tension cracks were noted in the fill and a slide occurred toward the Patapsco River. The earth mass continued its movement for about 2 hr after which time it reached a stable condition. Figure 12 is a view of the failed embankment. Because of a forecasted rain storm, the General Consultant and the design engineer immediately decided to have the contractor grade the slide material to obtain proper drainage of the storm flow toward the river. The contractor was also requested to remove some of the undisturbed embankment to relieve the pressure on the underlying stratum which had sheared. This material was not wasted but utilized by the contractor in other areas which required borrow material.

After reviewing the original plan and profile for the off-ramp, it was the opinion of the engineers that a realignment of the ramp within the standards set forth in the design criteria of the project would allow a safe and economical rehabilitation of the area. This was done as shown in Figure 11. Certain advantages were gained from this design change. The horizontal realignment placed the roadway and shoulders on embankment not influenced by failure planes, and the lowering of the grade line made it possible to consider as surcharge, soils which initially were below grade.

Considering the fact that some heaving was experienced in the off-ramp loop area and the initial settlements were appreciable, it was de-

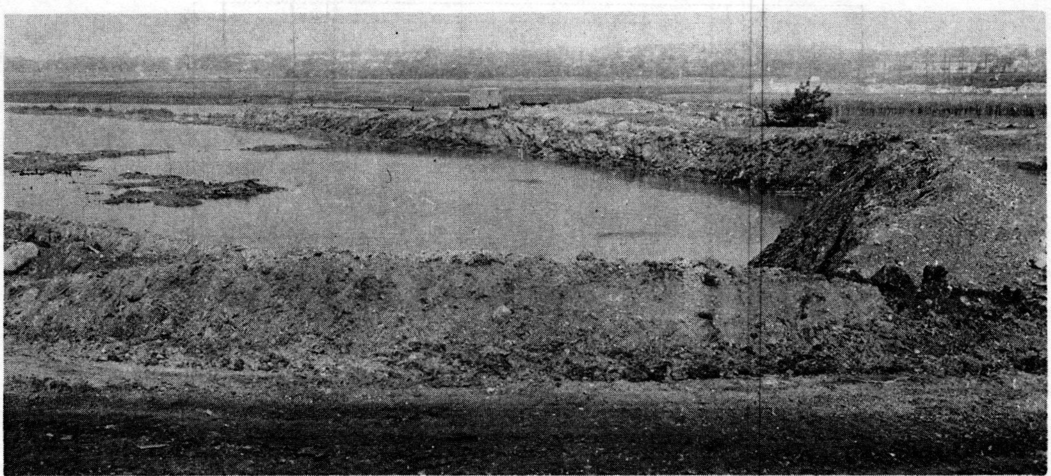


Figure 13. Station 360—area of unsuitable material.

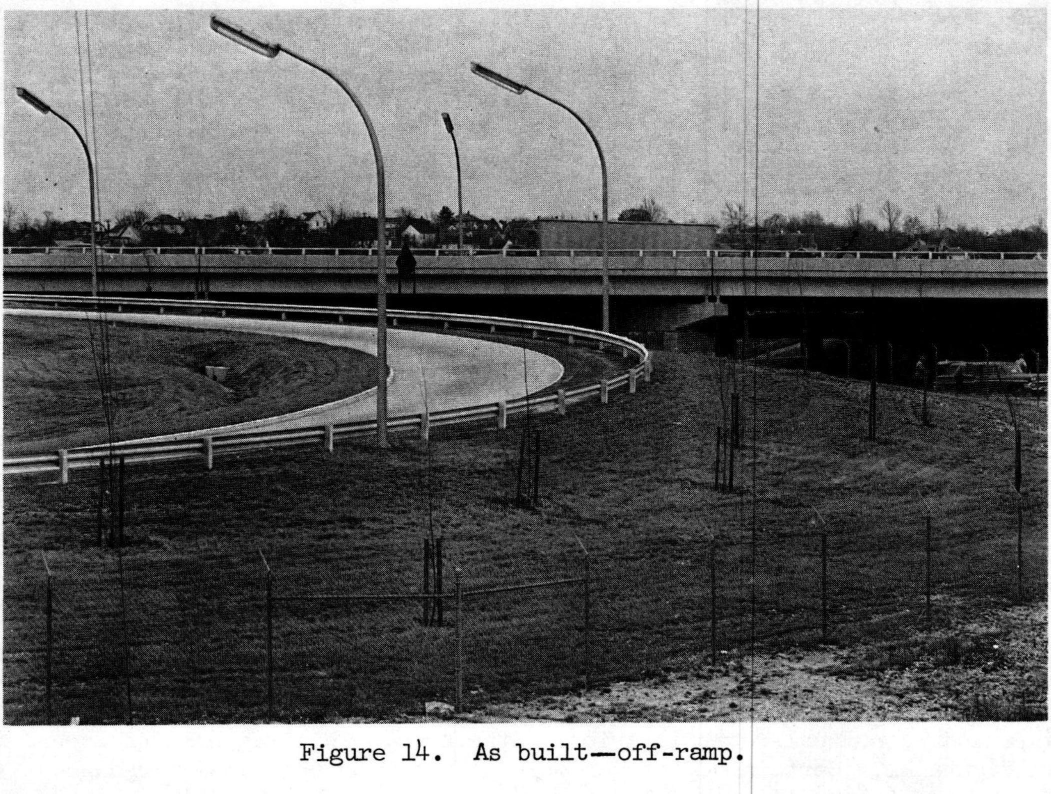


Figure 14. As built—off-ramp.

cided that a stabilizing berm in the loop would be beneficial. Its presence would permit faster placement of main line embankment by increasing the safety factor against a sliding failure through the soils underlying the loop. The materials used were obtained from an adjacent dredging operation as shown in Figure 13. Figure 14 shows the loop area as it appears at present.

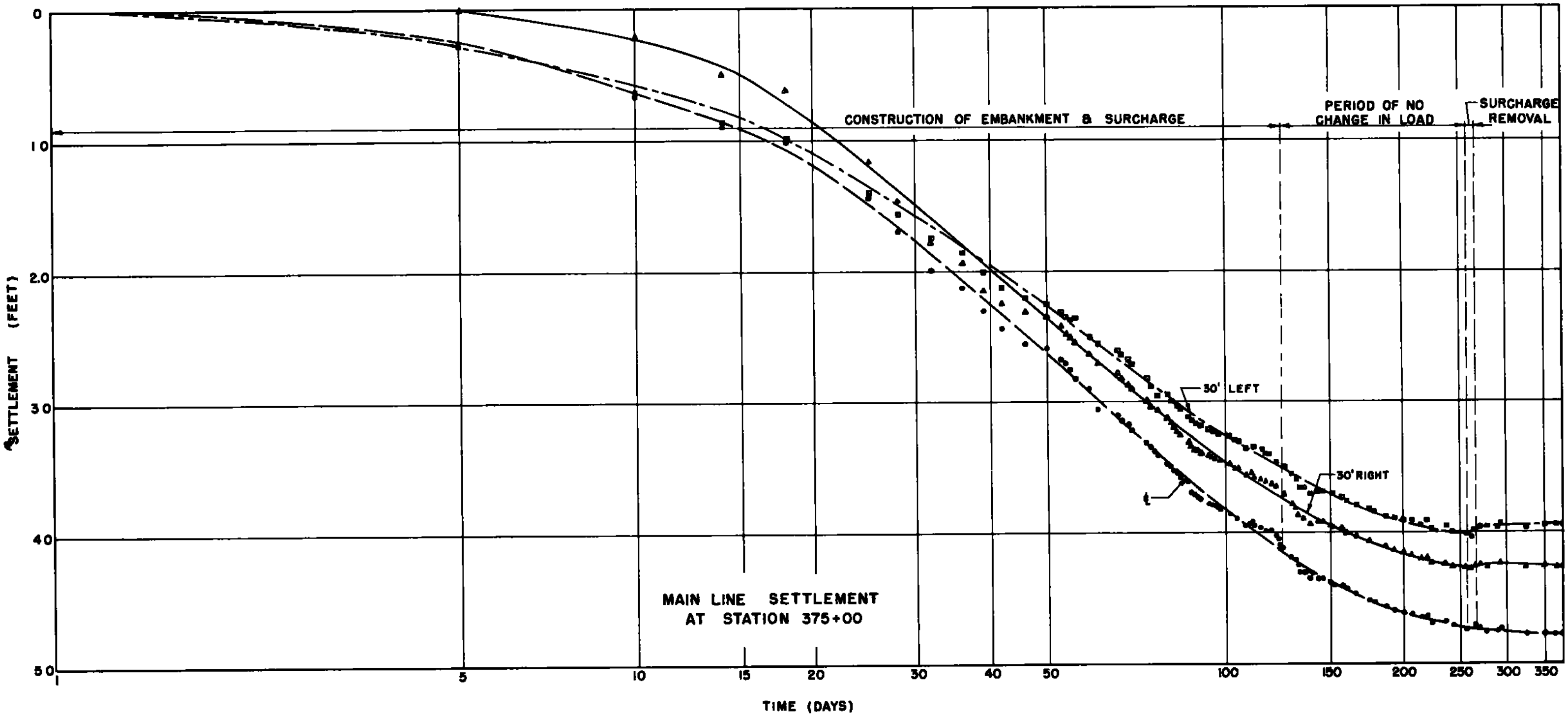


Figure 15. Station 325—main line—settlement curves.

Figure 16. Station 325—embankment and surcharge.

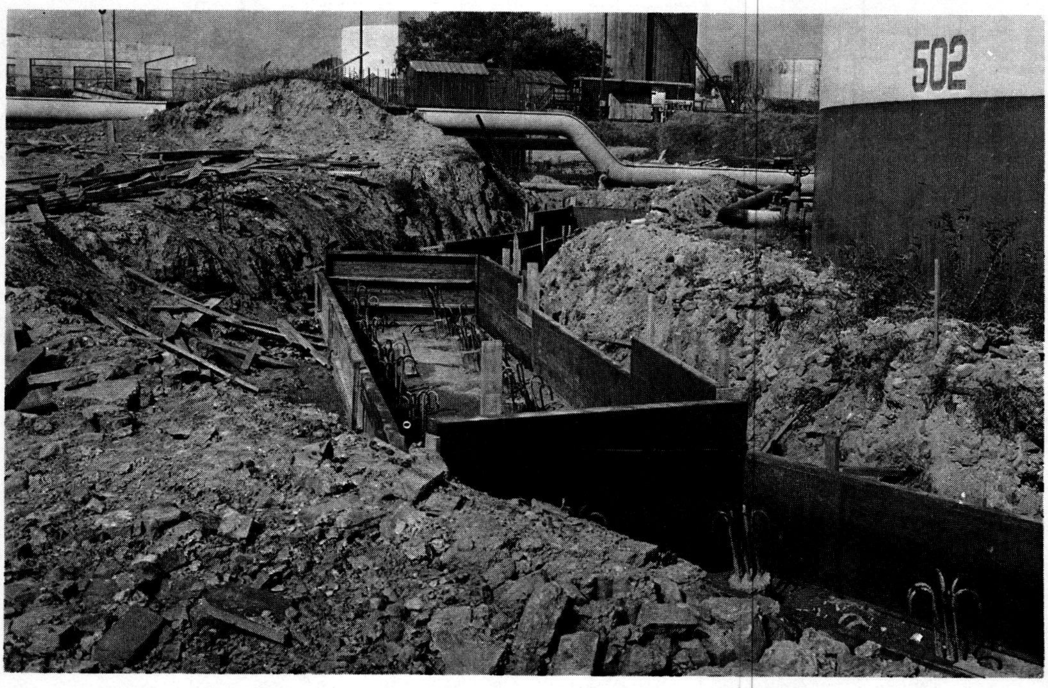


Figure 17. Retaining wall at Esso Tank 502.

The fill placement along the expressway was watched very carefully after the difficulties encountered in the area. It was decided on the basis of field observations that 13 ft of surcharge would be adequate to obtain the desired results rather than the 20 ft originally thought necessary. No further difficulties were experienced during or after the construction at this location. Figure 15 shows the curves obtained from the readings made on three settlement plates during the construction of the embankment and surcharge, the period of no change in surcharge, and the removal of surcharge. The embankment and surcharge are depicted in Figure 16. From field results obtained within the construction schedule, it was evident that rigid pavement could be used in conformance with the standard pavement section for the project.

The final subject to be discussed is a miscellaneous fill near the Esso tank farm. The problem here arose during the excavation for a retaining wall which was to contain the fill around Tank 502. This wall, which attained a maximum height of 11 ft, was designed for a spread-footing foundation, but the excavation for the footing revealed the presence of unsuitable fill material.

Cast-in-place concrete piles with an average depth of 25 ft were required to support the wall after this disclosure. It should be noted that Tank 502 had experienced some differential settlements in the past and the location of the retaining wall around this rank required, therefore, that all precautions be taken so as not to aggravate this situation. This is shown in Figure 17. An additional boring study of the site was made which disclosed that the southbound lanes passed over satisfactory soils while the northbound lanes were over unsuitable materials. The presence of bedsprings and brickbats combined with soft clays made the sampling operations quite difficult and it was decided that actual removal of these materials should be undertaken. With careful and continuous field supervision, the entire area of unsuitable soil which was found to be of limited extent, was removed by bulldozer and dragline. The removal of this material was most important because the fill over this area was approximately 20 ft in height and differential movements would have been quite serious.

MAINTENANCE AND OPERATIONS

The maintenance and operations phase of a large facility such as the Patapsco Tunnel Project normally does not gain full significance until a few years after its opening. Thereafter, the costs of maintaining and operating the project are expected to show an increasing trend.

In a strict sense, the problem mentioned in this section is incorrectly categorized because only recurring difficulties from wear are usually considered as maintenance and operations. However, this particular problem did occur after the roadway was opened to traffic.

During the fall and winter of 1957 an abnormal amount of rain and snow fell in a relatively short period of time. The unusually wet conditions caused a number of surface slides in cut sections and fill sections over 10 ft in height which were on a 1 on 2 side slope. The grass had not had sufficient time to establish its root system and consequently could not hold the 4-in. thick seeded topsoil layer to the side slopes. At one area located near the top of a cut, special drainage measures were necessary to control the flow of surface water.

At a number of other locations of distress, it was found that the topsoil was placed to greater depths than required by the plans and specifications. The general type of failure was the sliding of topsoil at the interface of topsoil and underlying fill. The basic problem, which has yet to be resolved, is a method whereby topsoil can be placed so as to become an integral part of the over-all fill.

SUMMARY

In October 1954, J. E. Greiner Company submitted a civil engineering report to the State Roads Commission of Maryland describing the Patapsco Tunnel Project with an estimate of project costs totaling $130,000,000. Construction was started in April 1955, and the tunnel and its approaches were opened to traffic in November 1957. The approximate project cost for this was $127,000,000.

J. E. Greiner Company of Baltimore, Maryland, was the General Consultant for the State Roads Commission and in this capacity was responsible for the preparation of the civil engineering report and the over-all supervision of the design and construction phases of the project. Eight design engineering firms were retained by the State Roads Commission to perform the actual design and field supervision of construction for the various sections of the work. A list of these design firms follows:

Design Section	Design Engineering Firm	Design Section	Design Engineering Firm
1	Green Associates, Inc. Baltimore, Md.	4	Singstad and Baillie New York, N. Y.
2A	Louis Berger & Associates Orange, N. J.	5	Joseph K. Knoerle & Associates, Inc. Baltimore, Md.
2B	Clarkson Engineering Co. Boston, Mass.	6	William H. MacFarland Binghamton, N. Y.
3	Whitman, Requardt, and Associates Baltimore, Md.	7	Rummel, Klepper and Kahl Baltimore, Md.

The design of the project in troublesome areas was based upon information available from borings, laboratory tests, and field tests. Certain areas were designated for elaborate field controls and careful field observations. However, all of the problems encountered during construction could not be anticipated at the time of design. The final treatment in certain areas was the result of field observations during the construction when some subsurface conditions became more clearly defined.

The importance of the revisions due to the field observations cannot be overemphasized as they affected the final costs and the construction schedule of the project. The results of these decisions stressed the importance and value of assigning competent field personnel to supervise the soils and foundation aspects of the work. Their presence assured a continuous reevaluation of the soil and foundation conditions as work proceeded.

Mathematical Expressions for the Circular Arc Method of Stability Analysis

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● THE DETERMINATION of the conditions under which earth slopes will be stable represents one of the most important applications of soil mechanics. A number of useful methods for investigating this problem involve the classical theories of Coulomb, Rankine, and Boussinesq. Other methods involve trial and error analysis for determining the most critical location of flat and curvilinear failure planes. This paper deals specifically with the cylindrical failure plane as applied to the general solution of the slope stability problem.

Analyses involving use of the cylindrical plane are based upon the two-dimensional case (1) in which the failure surface is represented by an arc of a circle (sometimes referred to as the Swedish Circle). The ratio of resisting moment to driving moment (taken about the center of the failure circle) or resisting force to driving force (taken along the failure arc) is used as a basis for describing the relative stability of the soil loading system for each specific failure surface. By successive trials, the location of the weakest plane can be established, and its corresponding moment (or force) ratio serves as an indication of its factor of safety. (This ratio is often used in various forms to represent the factor of safety of the earth slope against sliding. Although there is a great deal to be said about the definition of factor of safety (2), it is not the purpose of this paper to rationalize the point. It is noted, however, that the equations presented herein may be used in expressing any desired definition of factor of safety applied to the circular arc type of stability analysis.) Inasmuch as there are an infinite number of trial planes available for any problem (Fig. 1), the complete solution is often

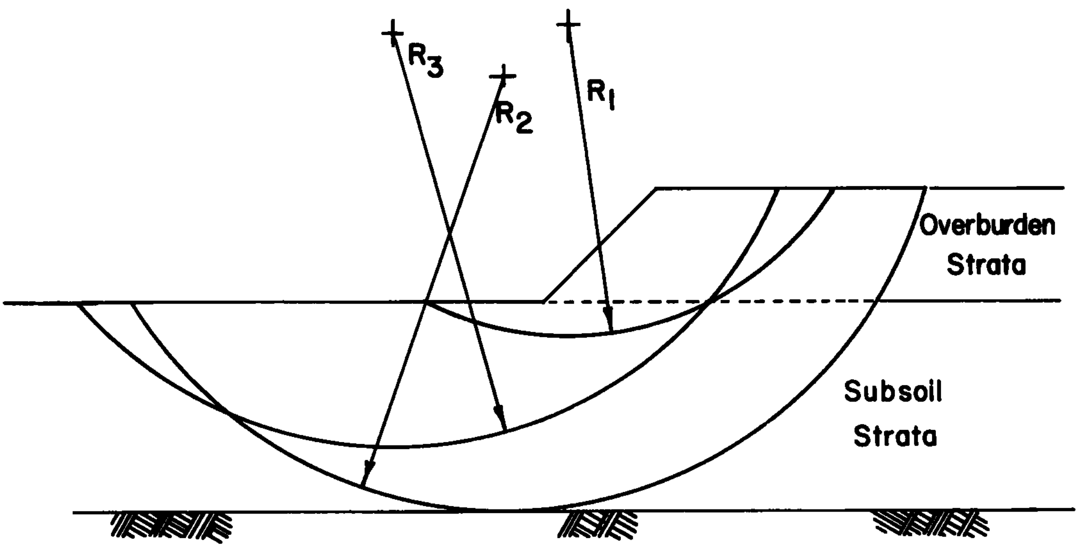


Figure 1. Typical circular arc failure surfaces.

tedious and time consuming. Where time limitations are imposed by design schedules, it is often impossible to properly locate the most critical failure plane. A number of attempts have been made to reduce the number of trials required (2) and, in the case of homogeneous soils, trial was eliminated (3). Unfortunately, the case of soil homogeneity is rarely if ever encountered, and to date the general solution for heterogeneous soil conditions defies simplification. The trial and error process is necessary for the present; however, the electronic computer now makes a rigorous approach economically feasible.

One of the first papers on the subject of the use of electronic computers for the solution of embankment stability was presented at the 37th Annual Meeting of the Highway Research Board (4). Other papers have appeared on the subject (5), particularly in England where the use of the Deuce electronic digital computer is widespread. These papers are technically excellent but are oftentimes limited in scope to suit the immediate needs of the users.

It is the purpose of this paper to present rigorous mathematical expressions which will permit direct application to computer programming as well as to organized manual computation for the determination of the weakest failure plane. Simplifying assumptions have been kept at a minimum to fully utilize the accuracy potential of the high-speed computer. Basic equations are presented for solution of the simple stability problem involving a constant earth slope of homogeneous material founded on a stratified subsoil. Special cases are also investigated, involving irregular or stratified slopes, the condition of toe failure, dam analysis, and related refinements demonstrating the flexibility of the derived expressions. In some instances, simplifying assumptions are made; however, it is left to the soils engineer to determine the suitability of the assumptions before attempting to use the equations for any specific problem. Specific examples of the use of these equations in the solution of a typical roadway embankment and an earth dam problem are presented in the Appendix. The Appendix also contains the basic forms from which the equations presented in the text are derived.

GENERAL DERIVATION

The method of investigating the stability of earth slopes, involving a multi-layer soil system, has generally been based upon the use of the method of slices. This involves the determination of forces developed by vertical segments of the soil system on corresponding incremental lengths of arc along the assumed failure plane. The method developed herein does not use the concept of slices but, instead, investigates the forces developed along the assumed failure plane due to the effect of each soil stratum. This approach to the problem results in completely rigorous mathematical expressions applicable to the general stability problem.

In deriving many of the equations presented, a coordinate system was used to relate the geometry of the soil system with that of the failure plane. The origin of this coordinate system is located at the toe of embankment slope (Fig. 2). To simplify application to electronic computer programming, the final form of the equations was often altered so that dimensions may be used as positive numbers without regard to coordinates. The term L (see "Glossary of Symbols"), which represents the horizontal distance from the toe of slope to the vertical axis through the center of the critical circle, is one of the few terms in the final equations that

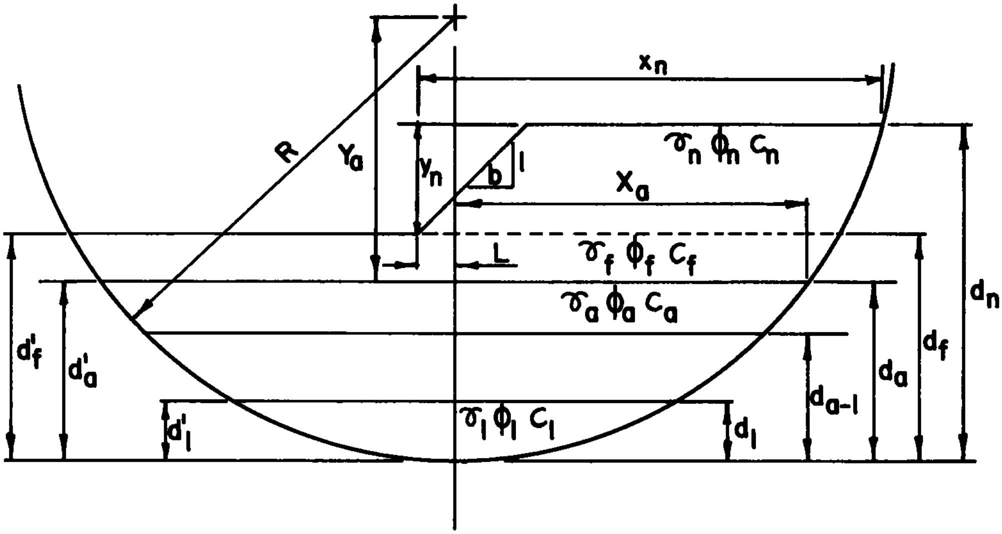


Figure 2. General earth slope problem.

are measured in accordance with the original coordinate system, and consideration must be given to its algebraic sign. When the term L is measured toward the embankment, its value is negative, whereas when it is measured away from the embankment, it is positive. When other exceptions are made, they are indicated in the text.

The basic solution of the slope stability problem, as presented herein, applies to the case of a uniformly sloping embankment of homogeneous material that is infinite in extent, situated on an unlimited number of layers of subsoil materials, as shown in Figure 2, for which the following simplifying assumptions are made:

1. The embankment is infinite in extent, has a uniform slope, and is homogeneous in nature.
2. The live loading, including its dynamic effects, may be represented by a surcharge applied to the embankment.
3. The subsoil can be represented as homogeneous horizontal strata, uniform in thickness.
4. The limits of the embankment slope, when projected downward onto the arc of the trial circle, fall within strata having the same angle of internal friction.
5. The stress distribution due to embankment loading is transmitted only vertically.

Where it is desired to avoid using these assumptions, supplementary mathematical expressions are presented in the section entitled "Special Cases." (Expressions to correct for the first and third assumptions as applied to embankments are found under the headings "Pore Pressure Distribution" and "Dam Analysis"; the second assumption may be eliminated in accordance with the explanation given under the heading "Concentrated Live and Dead Load." Assumption 4 can be eliminated by the method described under the heading "Shear Moments." A system for reducing the error incurred by assumption 5 is discussed under the heading "Shear Strength with Earth Slope Stress Distribution.")

Dead Load Moments

The dead load driving moment for a vertical embankment slope with the slope corresponding to the vertical axis (axis through center of trial circle) of the failure circle is expressed by Eq. 1, and the dead load resisting moment is expressed by Eq. 2.

Due to the existence of a sloping embankment, and the fact that the vertical axis of the failure circle may be located to either side of the toe of slope, a correction must be applied to both the driving and resisting moments. When the vertical axis falls outside of the embankment (L is positive or zero), the correction expressed by Eq. 3 must be subtracted from the driving moment. When the vertical axis passes through the embankment slope (L is negative), the value obtained from Eq. 4 is to be subtracted from the driving moment, and that obtained from Eq. 5 added to the resisting moment.

Shear Moments

In non-cohesive materials, there is general agreement that shear strength is adequately represented by the expression:

$$S = p \tan \phi$$

where p is the intergranular pressure (total pressure minus pore pressure) normal to the plane of failure, and ϕ is the angle of internal friction of the material. However, the manner of treatment of shear strength applied to cohesive materials is a point of controversy among engineers (7). For cohesive soils, the expression most often given is similar to that for cohesionless soils, except for the addition of the term C, representing the cohesion of the material. Thus, the shear strength of cohesive materials may be represented as follows:

$$S = C + p \tan \phi$$

This relationship indicates that the shear strength of any material can be thought of as being represented by the addition of a constant (C) and a variable ($p \tan \phi$). Separate equations are derived representing the resisting moment for each of these components and are referred to as moments due to cohesion and friction, respectively. The resisting moment due to cohesion is given in Eq. 6, and that due to friction is given in Eq. 7.

Inasmuch as the derivation of Eq. 7 includes a constant embankment load, the moment due to friction must be corrected for the effect of the sloping embankment by subtracting the value obtained in Eq. 8.

This correction is proper if assumption No. 4 is correct. Where the error involved in assumption No. 4 is not permissible, then a more exact solution can be made by using Eq. 8 incrementally, using values of X_L and L corresponding to the intersections of the various strata with the assumed failure plane projected onto the embankment slope line. The application of this refinement, it is believed, may be necessary in a limited number of cases.

Where the pore pressure effects are already included in the values of C and ϕ used in Eq. 6 and Eq. 7, or where no pore pressure exists, no further correction is necessary. However, where pore pressure has not otherwise been taken into account, a further correction in shear strength is necessary. Because of the method used in describing pore pressure, the subject of moments ascribed to those forces is handled under a separate heading.

DERIVATIONS—SPECIAL CASES

The foregoing presentation permits the solution of most stability problems involving simple slopes and loading configurations. However, in order to analyze conditions which cannot be handled by the basic equations the following special cases have been investigated: (a) concentrated live and dead load, (b) non-uniform slope, (c) stratified slope, (d) finite berm, (e) finite embankment, (f) dam analysis, (g) pore pressure distribution, (h) shear strength with stress distribution, and (i) toe failure investigation. Methods of handling these cases are developed and summarized in the following paragraphs. This list of special cases is not to be construed as being the only special cases possible but are presented to demonstrate that the derived expressions and the approach used in their derivations may be extended to include many special cases.

Concentrated Live and Dead Load

The incorporation of loads concentrated on an earth embankment may be desirable where heavy live loads are encountered and dynamic effects become important. This type of loading is assumed to effect the driving and resisting moments only, as their effects on shear strength and friction are assumed to be taken into account by distribution factors described in the text. When the value of moment V is positive, the result is used as a driving moment, and when negative, it is used as a resisting moment.

$$\text{Concentrated load moment} = V = P_L(1 + I)\ (L - E_L)$$

where P_L = Concentrated load;
E_L = Horizontal distance from toe of slope to load P_L, with its algebraic sign; and
I = Load increase factor due to dynamic effects.

$$(E_L + L)^2 \leq 2R(d_f+H) - (d_f+H)^2$$

Although the above limiting equation is specifically for loads at the top of the slope, a similar expression can be used for loads existing on the original ground surface, by using the term H as the height of the point of load application above the toe of slope.

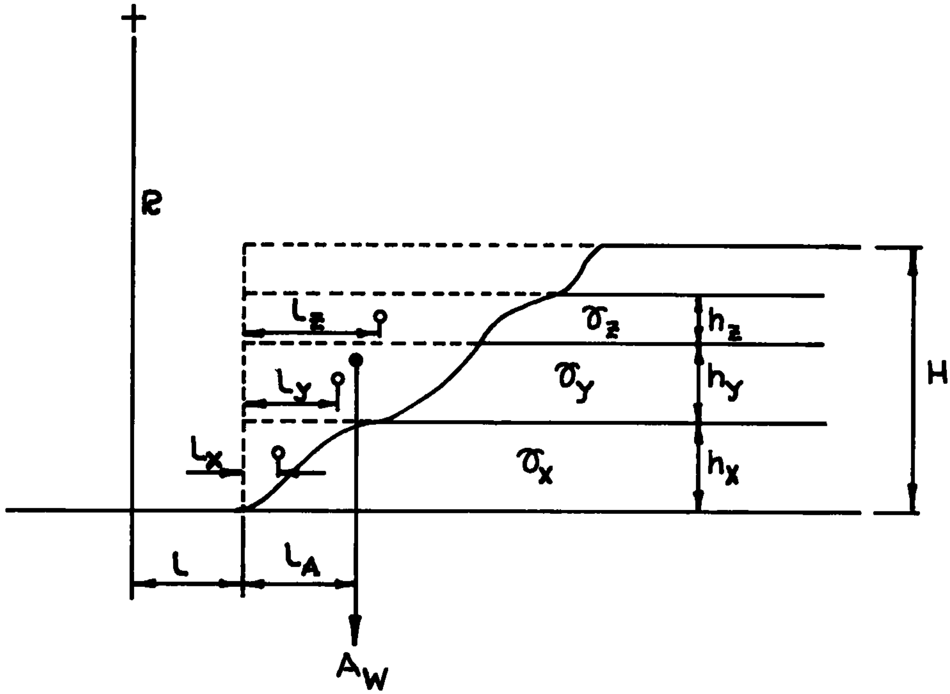
Non-Uniform Slope

The condition of non-uniformity in the embankment slope may be represented as a condition of a stratified embankment. Thus, Eq. 1 can be applied to determine the driving moments. The correction for the driving moments may be obtained by extending Eq. 3, using the proper values of X_L and L for each stratum, as shown in Figure 3A:

$$\text{Drive Correction} = \sum_{e=f+1}^{e=n} \frac{\gamma_n}{6b_e}\,(X_e{}^3 - L_e{}^3)$$

where the subscript e is used to denote embankment stratification. Where the vertical axis passes through the embankment, then the lower limit (f + 1) in the expression will be changed accordingly, and a correction to the resisting moment will be required. The identical expression to that given above will apply to the resisting moment correction except

3B



3A

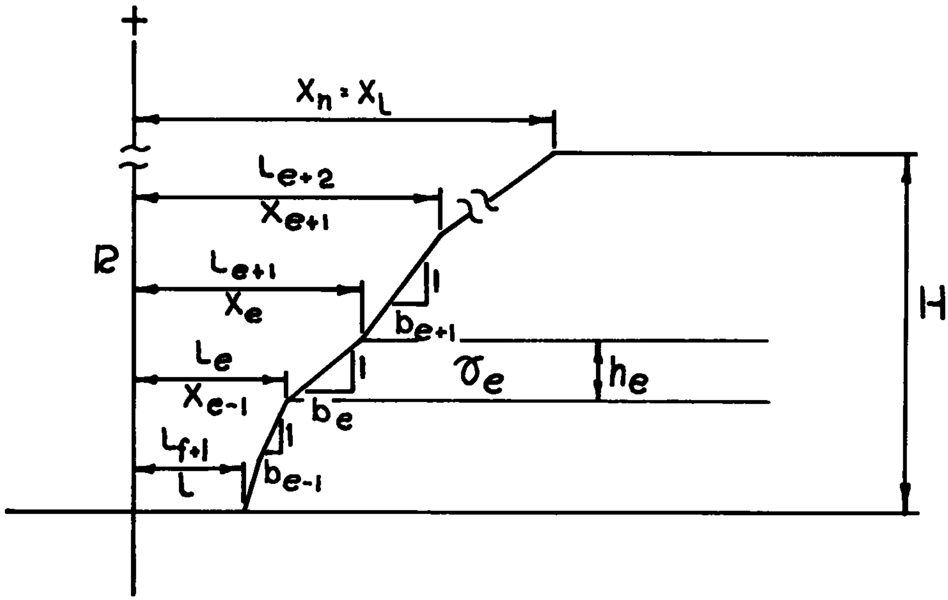


Figure 3. (A) Irregular earth slope, exact method; (B) Irregular earth slope, alternate method.

that the upper limit n will be reduced to correspond with the limiting stratum encountered. Further refinements may be required which may involve the use of Eq. 4 and Eq. 5; however, such detail will very rarely be required. (X_e and L_e take no sign.)

An alternate possibility for the correction of soil moments for a non-uniform slope may be obtained by Eq. 9, which equation is derived in accordance with Figure 3B. The derivation of this expression is based upon the determination of the weighted average soil characteristics of the area bounded by the embankment slope and the vertical axis. These characteristics are defined by Equations 9A, 9B and 9C, representing γ_A, A_W and L_A, respectively. The result obtained by means of these expressions represents the net driving moment correction due to the slope irregularity, and so is exact only for the condition where the vertical axis passes outside of the embankment. Where the vertical axis passes through the embankment, the correction to the driving moment obtained by Eq. 9 will be less than the actual, but this is offset by the fact that no correction is made to the resisting moment. When Eq. 9 is negative, its numerical value is added as a correction to the resisting moment. Although the use of Eq. 9 may slightly alter the position of the critical failure plane, its principal effect is in not presenting a true value of total soil driving and resisting moments when the vertical axis of the failure plane passes through the embankment.

The method of obtaining the shear moment under the general case applies, except that the number of strata is increased. The shear moment correction as given in Eq. 8 must be altered by providing a summation using appropriate values of X_e and L_e for the embankment stratification. It is noted that in most cases, the embankment slope can be approximated by an average value of b without causing an undue error in the shear moment, which will then permit using Eq. 8 directly for correction purposes without summation.

Stratified Slope

The discussion presented under the heading "Non-Uniform Slope" is directly applicable to the stratified slope condition, except that density is a variable with respect to the strata involved. Thus, the following equation is subtracted from the driving moments in accordance with Figure 3A:

$$\text{Drive Correction} = \sum_{e=f+1}^{e=n} \frac{\gamma_e}{6b_e}\left(X_e^3 - L_e^3\right)$$

The above equation may be applied to the determination of the resisting moment in the same manner as described for the case of non-uniform slope.

The alternate possibility for moment correction, using Eq. 9 as described for the non-uniform slope, holds here as well. However, in applying the alternate method to shear moment correction, the use of a weighted average density factor leads to an error which will vary with the range of density values involved; however, the error should generally be small. The error in shear correction is due to the fact that the weighted density average will vary with the value of L. Although refinements can be made to reduce the error, such as relating L and γ_A, it is not believed necessary that this be done except where extreme accuracy is needed. Where such accuracy is required, then the alternate method should not be used.

Finite Berm

Where a finite berm is used, Eq. 3 is directly applicable as a resisting moment, and Eq. 8 is applicable as a shear moment. These equations represent the effect of trapezoidal earth forms and are therefore applicable to a single or multiple finite berm configuration, where the failure arc does not cut through the berm. The alternate possibility described for determination of moments in the case of the non-uniform slope applies. In the case of the finite berm, a restriction is placed upon the radius of the failure plane, such that:

$$R \geq \frac{(d_f + h_B)^2 + (B - L)^2}{2(d_f + h_B)}$$

where B is the horizontal distance from the toe of slope to the top of berm, as shown in Figure 4.

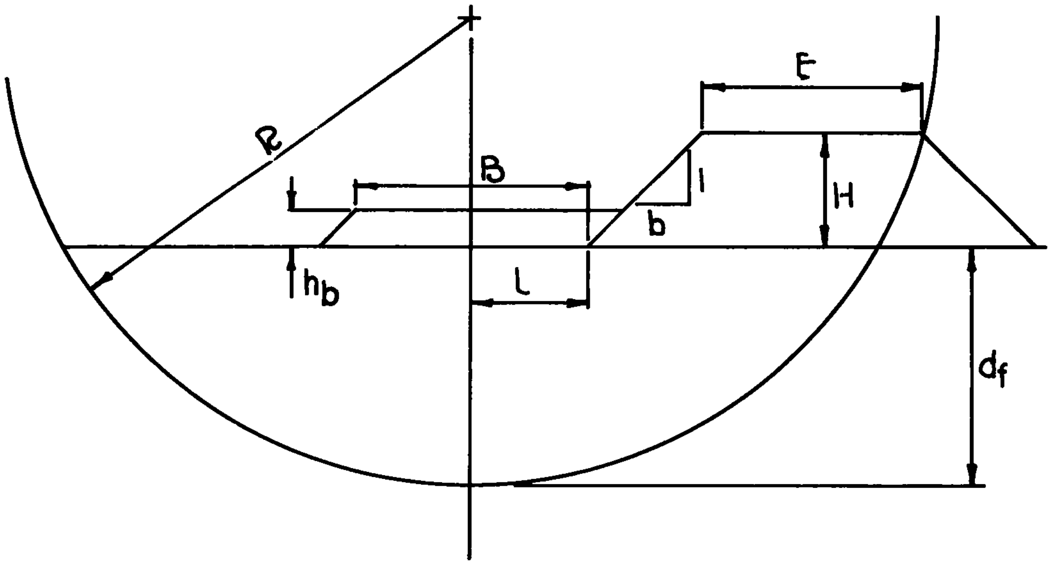


Figure 4. Finite embankment and berm.

Finite Embankment

The instance of the finite embankment is such that the following relationship holds, as shown in Figure 4:

$$R \leq \frac{(d_f + H)^2 + (L + bH + E)^2}{2(d_f + H)}$$

where E is the embankment width. This application is important in the investigation of the stability of roadway embankments along a plane transverse to its centerline.

The above equation is based upon the assumption that the failure circle does not intersect the far slope of the embankment. This assumption is valid for the vast majority of slope stability problems. Where the engineer prefers to provide a more detailed analysis by investigating circles which exceed the above limits for R, then the method of correction described in the Appendix under the heading "Investigation of Zoned Dam" may be used.

Dam Analysis

In order to apply these equations to the analysis of a dam, additional equations must be derived to express the geometry where the failure circle passes through the upstream face and also to account for the existence of a zoned (non-horizontally stratified) embankment.

Referring to Figure 5 for the variables, the distance above the toe of slope that the trial circle will pass through any interior slope is given by the following expressions (for a toe or deep circle):

$$h_a = \frac{-G_a + \sqrt{G_a^2 + (b_a^2 + 1)\,(2Rd_f - D_a^2 - d_f^2)}}{b_a^2 + 1}$$

where $G_a = b_a D_a - (R - d_f)$

and $D_a = L + m_a$

(The values of h_a can be determined in accordance with the coordinate system and therefore may be applied to subsurface stratification as well as earth slope stratification.)

The above equation is applied to determine the limiting points along the failure arc corresponding to the boundary lines for each zone.

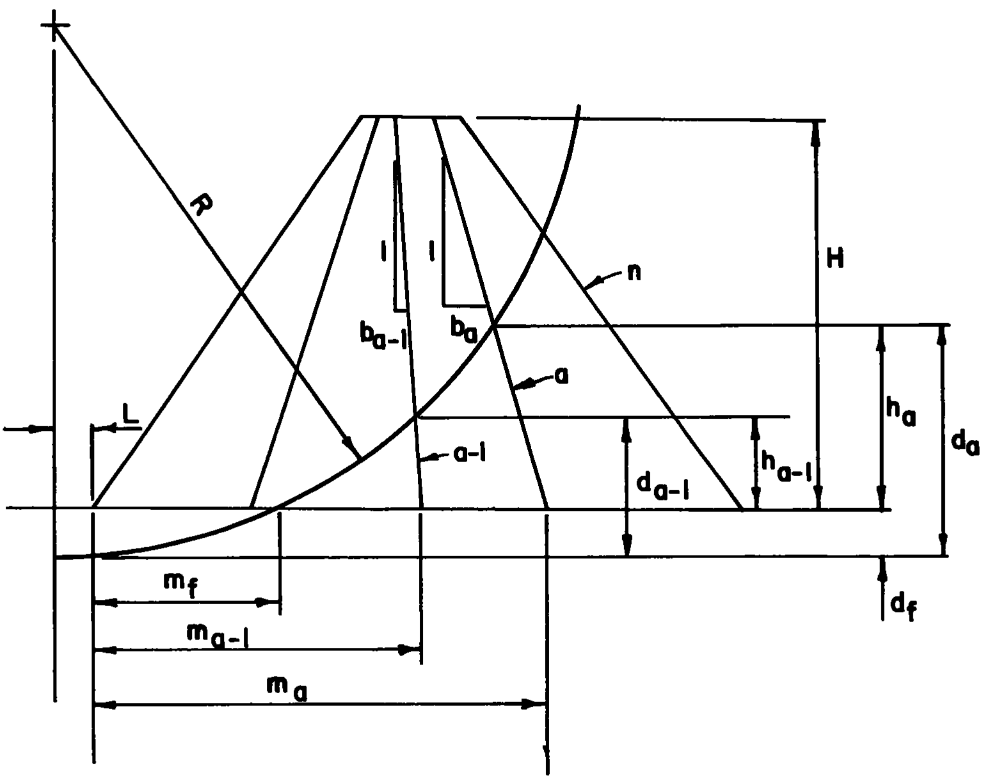


Figure 5. General zoned embankment problem.

In order to provide for the moment of an interior zone as a correction with respect to the original mass of the dam as a homogeneous material, the following equation is used to correct the driving moment obtained in using Eq. 1:

$$M_{Dc} = \frac{\gamma_d - \gamma_a}{6} \left[\frac{(D_a + b_a H)^3 - (D_a + b_a H_a)^3}{b_a} - \frac{(D_{a-1} + b_{a-1} H)^3 - (D_{a-1} + b_{a-1} h_{a-1})^3}{b_{a-1}} + (R - d_a)^3 - (R - d_{a-1})^3 + 3R^2 (d_a - d_{a-1}) \right]$$

where $d_a = b_a + d_f$.

A description of the application of the above equations to the solution of a specific problem is given in the Appendix.

Pore Pressure Distribution

It is not possible to express the distribution of pore pressure by mathematical symbols for application to all conditions, even where the case of soil homogeneity is assumed. However, an expression can be given based upon the assumption that pore pressure at any point can be described for a specific problem, if the position of that point is known with respect to the geometry of the system. Inasmuch as the failure plane is easily coordinated with respect to the toe of slope of the embankment, it follows that if tabular values of pore pressure are available and also coordinated with respect to the toe, that no rigorous mathematical expression is needed. Thus, if the pore pressure along the failure plane at a height d_a above the low point of the arc is known, and if this pressure is constant for a distance Δd equal to $(d_a - d_{a-1})$, then the smaller the value of Δd, the greater the accuracy in the analysis of the effect of pore pressure.

The problem thus resolves itself to the description of pore pressure at point d_a. From Figure 2 it is evident that the coordinate of any point can be expressed with respect to the toe of slope as $(d_a - d_f)$, $(x_a + L)$. The latter designation is consistent with the definition that x_a and L are negative when measured toward the embankment. As stated previously, the availability of a master tabulation of pore pressure related to the coordinate system employed would be the most accurate approach to the problem feasible at this time. However, for application to electronic computers where limited storage capacity is available, it is desirable to approximate the value of pore pressure by relating it to two independent factors, as given in the following relation:

$$U_a = Q_a + F_a f(h)$$

where f(h) represents a function of the overburden construction load. The values of Q_a and F_a are dependent only upon the geometry of the system, where the term F_a is used to express the effect of the earth slope and Q_a is a term encompassing all other factors affecting pore pressure. Assuming a uniform embankment height, the reduction in shear moment due to pore pressure is expressed by Eq. 10. The existence of a sloping embankment requires a reduction in the pore pressure. The latter correction requires that Eq. 11 be added to the shear moment. Inasmuch as Eq. 11 is related only to the earth slope, the Q factor is not involved in the expression.

In order to properly utilize Equations 10 and 11, it is necessary to relate the value of pore pressure with the geometry of the soil system under investigation. The values of Q_a and F_a are investigated for application to soils draining vertically, horizontally and to a case involving pressures in earth dams.

In the investigation of vertical draining subsoil, it is assumed that the value of Q is zero within the limits of the embankment. From the theory of consolidation, the pore pressure variation with depth is parabolic; and to demonstrate the case, it is assumed that the ground surface always permits free drainage. The value of F_a can then be expressed as follows:

$$F_a = M\left[1 - \left(\frac{md_a - (m-1)d_f}{d_f}\right)^2\right]$$

for $d_a \leq d_f$

where m is a constant varying from 1 to 2, depending upon whether the condition is one of single drainage, double drainage, or an intermediate drainage condition. The term M is a factor which represents the maximum pore pressure effect of a unit embankment load. (This equation may be altered to suit the condition where the upper stratum is not free draining, by substituting $d_f - d_a$ for d_a.) Pore pressure effects beyond the limits of the earth slope can be corrected by use of the Q-term.

Where only horizontal drainage is effective, the value of F_a would be constant with depth, and F_a would equal M. Although Q is considered as zero within the limits of the slope it may, nevertheless, be added when there is superimposed pressure through other means, such as ground water movement. Where both horizontal and vertical drainage occur, as well as water flow, the various terms may be combined to express the desired pore pressure distribution.

The conditions described above pertain to pore pressure below ground level; however, a necessary consideration is that of analyzing a ccmbination of pore pressure distribudion in subsoils as well as slopes such as earth dams. To demonstrate this application, it is assumed that the equipotential lines, as well as the phreatic line, can be described algebraically, as in Figure 6.

Taking the toe of the slope to be the origin, the general equation for the equipotential line can be expressed as:

$$y_e = f(x_e) + c_e$$

Using the approximation that the pore pressure between any vertical increment at a specific horizontal distance from the toe of slope is constant, a sufficiently large number of increments is arbitrarily established to assure desired accuracy. For any increment or hypothetical stratum located h_a above or below the toe of slope the horizontal distance from the toe to the intersection of this limit with the failure plane is:

$$x_a = L - \sqrt{R^2 - (R-d_f - h_a)^2}$$

(The sign in front of the radical is $\pm$ for $h_a \gtrless 0$).

Since y_e must equal h_a on the equipotential line and x_e must equal x_a, then c_e must be:

$$c_e = h_a - f(x_a)$$

Thus, the equation for the equipotential line passing through the failure plane at the height h_a must be:

$$y_e = f(x_e) + h_a - f(x_a)$$

If the equation of the phreatic line can be expressed as:

$$y_p = f(x_p) + c_p$$

then the point on the phreatic line h_p corresponding to h_a on the failure plane can be determined, and the pore pressure at height h_a on the failure plane will be:

$$Q_a = (h_p - h_a)\,\gamma_w$$

with Q_a assumed constant for that portion of the failure plane passing through the stratum. When Q_a is negative, then the phreatic line has been exceeded and the pore pressure is zero.

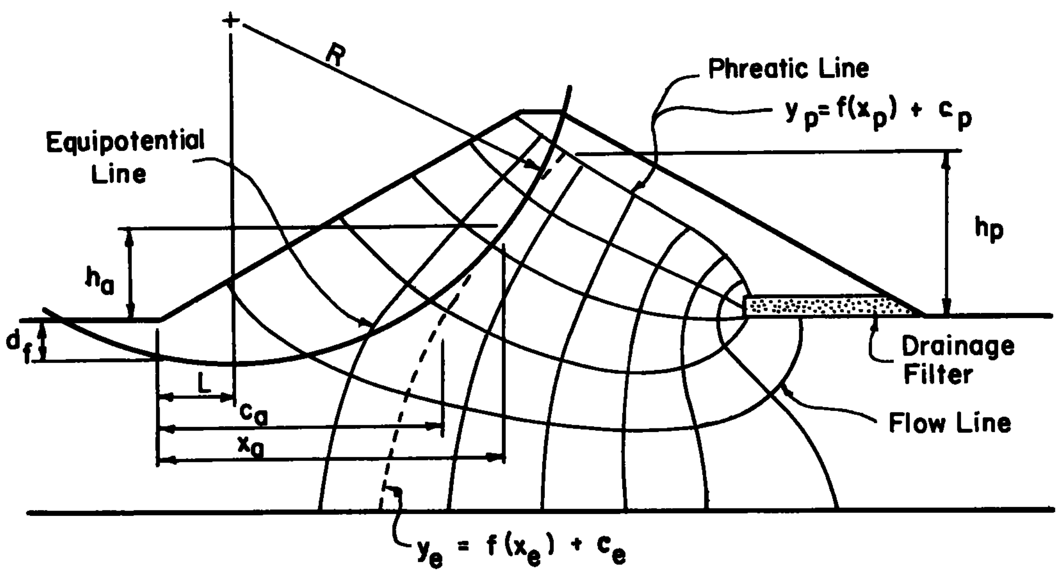


Figure 6. Typical flow net for earth dam.

The foregoing method may be applied to any system of equipotential and phreatic lines that can be expressed algebraically. Although most theoretical cases involve equipotential lines of the type shown in Figure 6, for demonstration purposes, an application of this method to the draw-down condition where the equipotential lines are vertical and the phreatic line follows the downstream face of the dam, will produce the following general expressions for toe failure investigations:

$$Q_a = \gamma_w \left(\frac{L}{b} - h_a - \frac{\sqrt{R^2 - (R-d_a)^2}}{b} \right)$$

where $L < -x_a < bH$

and

$$Q_a = \gamma_w\,(H - h_a)$$

where $bH + E > -x_a \geq bH$

For the case where the failure circle intercepts the downstream slope,

$$Q_a = \gamma_w \left[(H-h_a) + \frac{x_a + bH + E}{b_b} \right]$$

where $-x_a > bH + E$

b_b = downstream slope of dam
E = width of crest of dam

This method may be applied to many types of flow net conditions; however, it is emphasized that each case must be investigated separately to determine that the use of algebraic representation of the equipotential and phreatic lines will not result in undesirable error.

It is pointed out that the analysis of pore pressure may, in certain locations along the failure plane, produce a negative net frictional moment. However, this error is inherent to the procedure used in the method of slices and is compensated in part by an excess shear resistance effected along other portions of the arc by the neglect of stress distribution. In specific instances, it may be necessary to utilize stress distribution factors in the analysis. In such cases, where neutral stresses are large, the effect of such stresses acting on the vertical sides of hypothetical slices may be investigated analytically by using the pore pressure designation described herein.

Shear Strength with Earth Slope Stress Distribution

There may be instances where assumption 5 is undesirable. In such cases, it is necessary to take into account the effect of stress distribution due to the earth slope. The approximations recommended in this solution are similar to those discussed under the heading "Pore Pressure Distribution."

Although the total stress may be described as the summation of factors as in the case of pore pressure, unlike pore pressure the total stress at any point is not perpendicular to the plane of failure. It is therefore necessary to treat each stress value as two components:

$$P_v = Q_v + F_v(h)$$

$$P_h = Q_h + F_h(h)$$

where P_v and P_h represent the vertical and horizontal stress, respectively, at any point in the subsoil due to the earth slope. The procedure that may be followed is to determine the shear strength effects due to the subsoil loading by means of Eq. 7, taking the summation to the ground line (d_f) rather than to the top of slope (d_n).

The derivation of the equations for the effect of earth slope stress distribution on the shear strength along the failure plane are not given herein; however, the equation for the shear strength effect of the vertical component P_v is identical with that given in Eq. 7 applied so that P_v replaces the term $W_a - \gamma_a(R-d_a)$. The equation for the shear moment effect of the horizontal component P_h is:

$$M_{fh} = \sum_{a=1}^{a = f' \atop a \pm n} P_h \frac{\tan \phi_a}{2} (x_a^2 - x_{a-1}^2)$$

The qualifications applied to the use of this method are similar to those stated for pore pressure. Where the values of Q_v, F_v, Q_h, and F_h can be related algebraically with respect to the geometry of the soil system, then simplifications in the use of electronic computers may be possible. However, it is left to the engineer to determine whether or not the assumption of stress distribution is desirable as compared to the usual assumption that such distribution is entirely vertical.

Toe Failure Investigation

Where it is desired to investigate specifically for toe failure, as shown in Figure 7, the depth d_f to which the critical circle will pene-

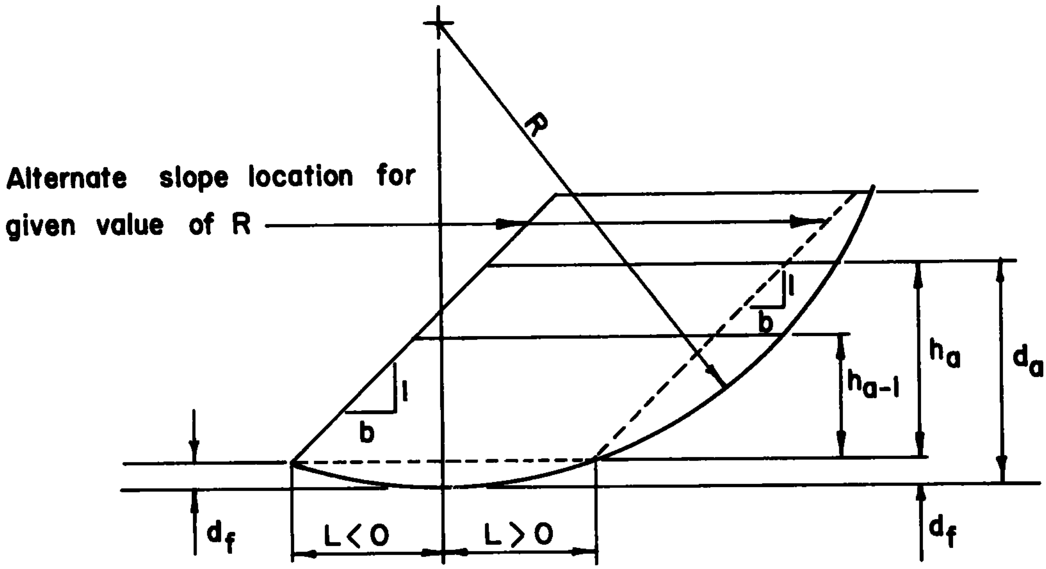


Figure 7. General toe failure problem.

trate the subsoil is not constant; therefore, the value of d for each stratum will vary with the location of the center of rotation. However, the height h of the top of each stratum above the toe of the slope is constant, as the stratification is assumed to be horizontal. To find d for any stratum, d_f must be added to h. The value of d_f for each circle is obtained as follows:

$$d_f = R - \sqrt{R^2 - L^2}$$

Thus, the value of d for any stratum becomes:

$$d_a = h_a + d_f$$

Using the appropriate values of d, Equations 1 through 11 may be used as previously described, applying the proper driving moment corrections.

LIMITATIONS

The mathematical expressions presented are limited by the assumptions made in their derivation. Where such assumptions are not permissible, they may be eliminated, or the associated inaccuracies reduced, by procedures outlined in the text. The assumption that is the most cumbersome to eliminate is that dealing with the horizontal stratification of the subsoil. However, suitable expressions may be obtained in a manner similar to that used for non-horizontal stratification in embankments.

The most difficult assumption to deal with is that concerning the representation of distributed pressure (excess hydrostatic as well as intergranular) where the theory can at best be presented as an approximation of the truth. Where such distribution can be expressed algebraically, the assumption that such pressures are constant within each stratum is a desirable postulate. If greater accuracy is desired, the number of strata may be increased to any practical limit. Due to the many complexities involved in theoretically analyzing pressure distribution, there is no certainty that the approach described herein is actually a limitation.

ACKNOWLEDGMENTS

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APPLICATIONS

The expressions presented herein are believed to be the most general possible for application to the solution of the problem of embankment stability by the circular arc method of analysis. These equations are now being programmed to permit the use of Bendix G-15D electronic computer in this work.

It is estimated that for a ten-layer system, under the most adverse conditions as concerns the number of variables, it would require no more than five minutes to investigate any one circle location for a program set up on the interpretation system. With a reduction in variables and number of layers, the machine time would be reduced proportionately. It is expected that those who are familiar with computer operations will be able to find many simplifying methods in applying the equations presented.

Charts and curves, based upon these equations, are now being prepared, using parametric values of the variables assuming a circular failure plane having a unit radius. These will permit rapid solution of the stability problem where electronic computers are not available.

These equations cannot be used to replace the judgment of the soils engineer, as such judgment is required in properly interpreting the nature of the problem, the physical characteristics of the subsoil and embankment materials involved, as well as to decide upon the applicability of the circle arc failure plane. Although these equations are presented for a specific type of failure plane, the method of analysis by investigating the effect of each stratum is applicable to any type of assumed failure surface.

GLOSSARY OF SYMBOLS

a = Stratum and zone designation subscript;
A_w = Weighted area for correction of driving moments;
b = Horizontal distance for unit rise of earth slope (positive when sloping upward and away from the toe of slope); as subscript, represents downstream face of dam;
B = Horizontal distance from toe of slope to top of berm; as subscript, refers to berm;
c = Constant; as subscript, denotes correction;
C = Cohesion;
d = Height of top of stratum above low point of failure circle on driving side of arc; as subscript, refers to dam;
D_a = $(L+M_a)$;
e = Embankment stratification subscript;
E = Embankment width (finite case), or crest width (dams);
E_L = Horizontal distance from toe of slope to concentrated load (with sign);
F = Load distribution factor due to earth slope;
f = Subscript denotes stratum at toe of slope, or friction;
$f(\ )$ = Denotes algebraic function;
G_a = $b_a D_a - (R - d_f)$;
H = Height of earth slope;
h = Stratum thickness, in earth slope; as subscript, refers to horizontal component;
I = Load increase factor due to dynamic effects;
L = Horizontal distance from toe of slope to vertical axis of trial circle;
L_e = Same as L, except measured from intersection of bottom of stratum e and earth slopeline;
$L_{A,x,y,z}$ = Moment arm, measured horizontally to toe of slope;
m = Constant denoting drainage condition in subsoil;
M = Maximum pore pressure effect of unit load;
M_D = Driving moment, dead and live load;
M_{Dc} = Driving moment correction;
M_f = Frictional moment;
M_p = Moment reduction due to pore pressure;
M_{pc} = Correction in moment reduction due to pore pressure;
M_R = Resisting moment, dead load;
M_{Rc} = Resisting moment correction;
M_S = Moment due to shear;
M_{Sc} = Shear moment correction;
n = Subscript designation for uppermost stratum;
p = Intergranular pressure; subscript, designating phreatic line;
P = Distributed pressure;
P_L = Concentrated load;
Q = Load distribution factor;
R = Radius of trial circle;
S = Shear strength in subsoil stratum;
U = Pore pressure;
v = As subscript, vertical component;
V = Concentrated load moment;
W_a = Summation of unit loads above a given stratum;
x = Horizontal distance from toe of slope in coordinate system;
X = Horizontal distance from vertical axis to intersection of top

of stratum with the failure arc or earth slope line;

X_L = Horizontal distance from vertical axis to top of slope;

y = Ordinate location of point within coordinate system;

Y = (R-d); density of stratum;

γA = Average density of total stratified slope;

ϕ = Angle of internal friction;

Δd The incremental vertical distance between strata; and

' = As superscript, represents dimensions taken with respect to strata to the side of the vertical axis away from the toe of slope.

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APPENDIX

Mathematical Expressions for the Circular Arc Method of Stability Analysis

INVESTIGATION OF FIVE-LAYER SOIL SYSTEM WITH BERM

To demonstrate the application of the equations presented it is assumed that it is desired to check the stability of the proposed embankment construction shown in Figure 8. An infinite embankment is assumed where

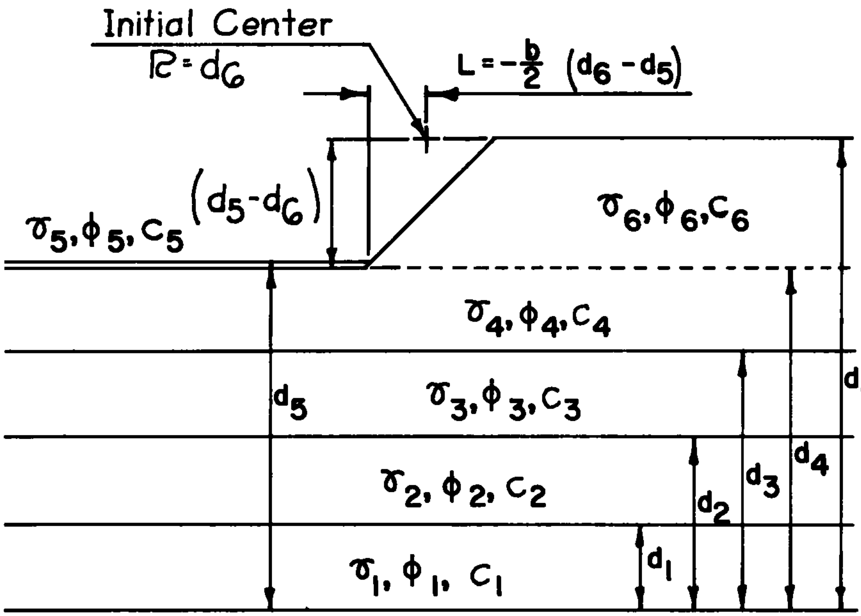


Figure 8. Typical 5-layer soil system stability problem.

the slope is 1/b, with four subsoil strata and one embankment stratum. Assuming that a berm may be required for stability purposes, a fictitious stratum is added, such that $d_4 = d_5$ initially. The expression for the driving moment M_D is obtained by means of Eq. 1. The driving moment needs to be corrected by subtracting M_{Dc}, using Eq. 3 or Eq. 4 whichever is applicable.

Due to the constant thickness of each strata, the resisting moment is equal to the driving moment less the driving moment of the embankment, such that:

$$M_R = M_D - \frac{\gamma_6}{2}\left[R^2 (d_6 - d_5) - \frac{(R - d_5)^3 - (R - d_6)^3}{3}\right]$$

This is corrected by adding M_{Rc} as given in Eq. 5, when L is negative.

Discounting the effect of the embankment, the shear moments on either side of the vertical axis of the trial circle are equal. Thus, using Eq. 6 and Eq. 7 to take care of moments on both sides of the vertical axis, the desired shear moment is obtained.

The values of W_1 through W_4 used in Eq. 7 are as follows:

$$W_1 = \gamma_2 h_2 + \gamma_3 h_3 + \gamma_4 h_4$$
$$W_2 = \gamma_3 h_3 + \gamma_4 h_4$$
$$W_3 = \gamma_4 h_4$$
$$W_4 = 0 \text{ (initially)}$$

To the above determined shear moment must be added the shear moment effect of the embankment, given by the following expression:

$$\frac{\gamma_6 (d_6 - d_5)}{2} \sum_{a=1}^{a=5} \tan \phi_a \left[X_a Y_a - X_{a-1} Y_{a-1} + R^2 \left(\tan^{-1} \frac{X_a}{Y_a} - \tan^{-1} \frac{X_{a-1}}{Y_{a-1}} \right) \right]$$

plus the values obtained from Eq. 6 and Eq. 7 for a = 6.

Due to the sloping embankment, Eq. 8 is used to obtain the shear moment correction M_{sc}, which is to be subtracted from the shear moment effect of the embankment.

Having obtained the necessary general equations and using a suitable definition for the factor of safety, a program can be set up for solution by electronic computer. Although a program is now being set up to investigate a more general case of soil stratification, its application to this would be approximately as described hereinafter.

Inasmuch as the problem is to investigate an infinite earth slope, the program will start at the minimum radius for the deepest stratum to be investigated:

$$R = d_6$$

having its center located at:

$$L = -\frac{b}{2} (d_6 - d_5)$$

(Equations are available that will permit a more desirable starting point; however, the use of such equations is left to the individual.) After this initial computation, the value of R will be varied in increments, and L maintained as a constant, and the factor of safety (ratio of resisting moments to driving moments) determined for each successive position. When a point is reached where the factor of safety at one location is greater than the preceding, the computer is returned to the previous location, R held constant and L is varied incrementally away from the slope, until the factor of safety increases, at which point the machine will revert back to the lower value. This cycle is repeated automatically, with the computer searching until the location of the minimum safety factor is determined.

Using the critical center so determined, the computer will automatically progress to the higher subsoil layers. The investigation will be continued until the factor of safety at a shallower stratum increases, at which point the machine reverts back to the lower depth.

Should the minimum factor of safety result in a value less than that desired, then d_5, which initially was set equal to d_4, is set to incrementally increase by any desired value Δh_B. The value of L will automatically be increased by the value b Δh_B, and the value of W_1 through W_4 is increased by the value $\gamma_5 h_B$.

Such trials can be investigated for the established critical failure plane and the berm height determined when the desired factor of safety is obtained. If necessary, the entire problem, including the berm, can be checked by returning the machine to any desired value of R and L.

Where desired, instead of investigating for a suitable berm height, the problem may be set up to determine the minimum slope for a specific factor of safety.

INVESTIGATION OF ZONED DAM

A typical zoned earth dam section on an impermeable base is shown in Figure 9. The required investigation is based on a toe circle analysis.

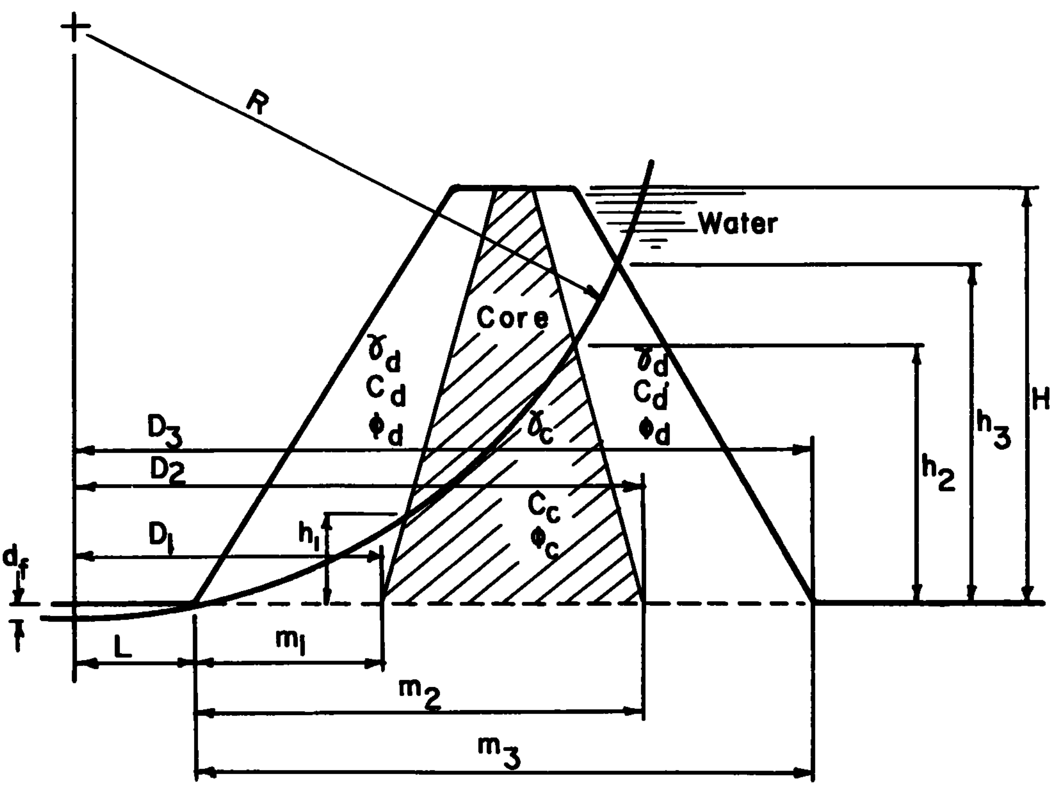


Figure 9. Typical zoned dam stability problem.

For simplicity, it is assumed that $\gamma_c = \gamma_d$ and that the trial circle intersects the upstream face of the dam, at a point h_3 located by the equation given under the heading "Derivations—Special Cases: Dam Analysis." The analysis proceeds as it would for a homogeneous embankment. To determine the driving moment, Eq. 1 and Eq. 3 are used from d_f to $(H + d_f)$. By repeating the use of these equations from h_3 to H a correction for the effect of water may be obtained, using $(\gamma_d - \gamma_w)$ in place of γ_a. No correction is needed for the interior zone as $\gamma_c = \gamma_d$. (When L is positive, no moment correction is made for subsoil moments.)

The frictional resistance is then determined by means of Eq. 7, assuming a constant friction angle ϕ_d for the entire length of arc and going in one step from d_f to $(H + d_f)$. The frictional correction for the downstream slope is made by means of Eq. 8, by going from L to $(L + bH)$. The frictional correction for the water on the upstream slope is made by going from $(h_3 + d_f)$ to $(H + d_f)$ in Eq. 7, and then from $(D_3 + b_3h_3)$ to $(D_3 + b_3H)$ in Eq. 8, with b_3 taking its algebraic sign.

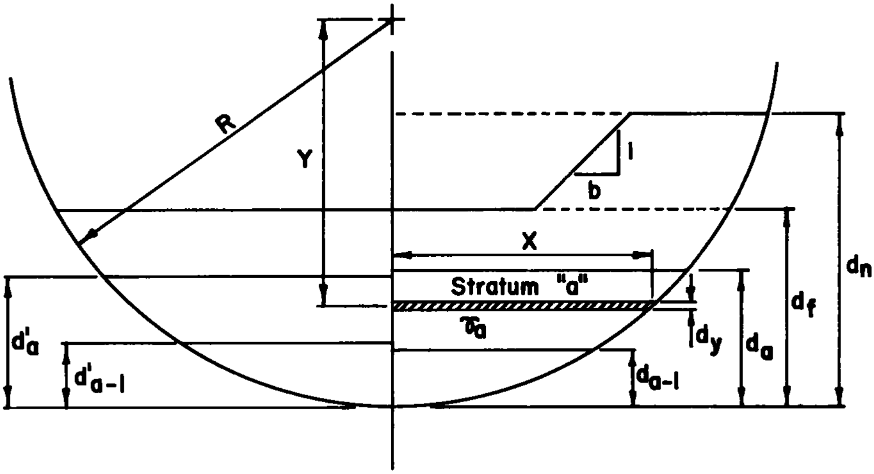
Knowing the values of h_1 and h_2 (equation for h_a, given under the heading "Derivations—Special Cases: Dam Analysis"), from Eq. 7 an approximate intergranular pressure correction is obtained by replacing $\gamma_a \tan \phi_a$ with $(\gamma_d \tan \phi_d - \gamma_c \tan \phi_c)$, where $W_a = (\gamma_d - \gamma_c)(H - h_2)$.

The shear resistance due to cohesion is obtained by means of Eq. 6, using the value of S_d from d_f to $(H + d_f)$ and then correcting for the upstream water by going from $(h_3 + d_f)$ to $(H + d_f)$. The correction for the central core is obtained by using $(S_c - S_d)$ in place of S_a in Eq. 6 and going from $(h_1 + d_f)$ to $(h_2 + d_f)$.

If h_3 were found to be greater than H, the circle arc would not intersect the dam backslope and the correction for water would be omitted from the analysis.

The effect of pore pressure is the remaining factor to be established, and this is done as in the case of the embankment analysis, by assuming as many horizontal layers as is desirable and applying Eq. 10 and Eq. 11.

The equations may be used in a similar manner for other configurations of dams and for deep-seated failures in dams as well.



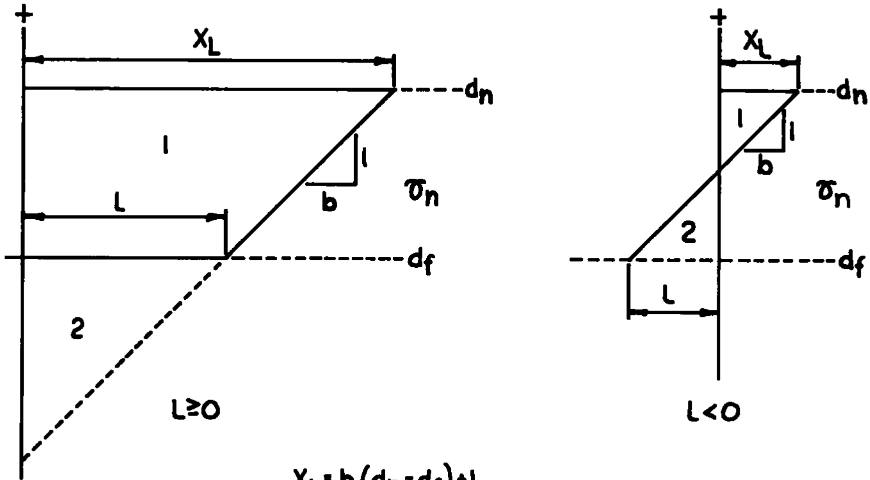
Equation of Arc $X^2 + Y^2 = R^2$

Moment of Stratum "a" about "O" = $\tau_a \int_{Y_a}^{Y_{a-1}} X \cdot \frac{X}{2} \cdot dy = M_a$

Eq. 1 $$M_D = \sum_{a=1}^{a=n} \frac{\tau_a}{2} \int_{R-d_a}^{R-d_{a-1}} \left(R^2 - Y^2\right) dy$$

Eq. 2 $$M_R = \sum_{a=1}^{a=f'} \frac{\tau_a}{2} \int_{R-d'_a}^{R-d'_{a-1}} \left(R^2 - Y^2\right) dy$$

Figure 10. Driving and resisting moments.



$$X_L = b(d_n - d_f) + L$$

$L \geq 0$

Subtract from Drive Moments

Eq. 3
$$\left.\begin{aligned} M_{1+2} &= \tau_n \frac{X_L}{b}\cdot\frac{X_L}{2}\cdot\frac{X_L}{3} = \frac{\tau_n X_L^3}{6b} \\ M_2 &= \tau_n \frac{L}{b}\cdot\frac{L}{2}\cdot\frac{L}{3} = \frac{\tau\, L^3}{6b} \end{aligned}\right\} \quad M_{Dc} = \frac{\tau_n}{6b}\left(X_L^3 - L^3\right) = M_1$$

$L < 0$

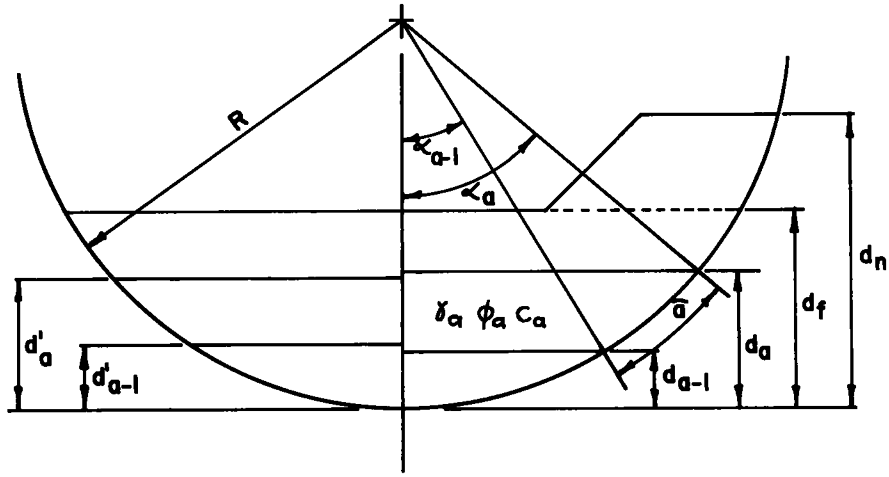
Subtract from Drive Moments

Eq 4
$$M_{Dc} = \tau_n \frac{X_L}{b}\cdot\frac{X_L}{2}\cdot\frac{X_L}{3} = \frac{\tau_n X_L^3}{6b}$$

Add to Resisting Moment

Eq. 5
$$M_{Rc} = -\tau_n \frac{L}{b}\cdot\frac{L}{2}\cdot\frac{L}{3} = -\frac{\tau_n L^3}{6b}$$

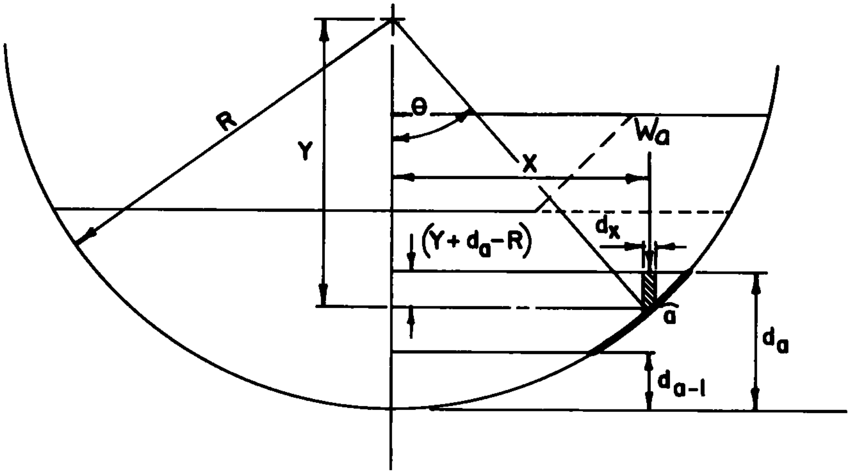
Figure 11. Correction in moment-drive and resistance.



Eq. 6
$$M_S = \sum_{a=1}^{a=n} R\,\hat{a}_a\,c_a + \sum_{a=1}^{a=f'} R\,\hat{a}_a\,c_a$$

$$\text{where } \hat{a}_a = R\left[\tan^{-1}\frac{\sqrt{2Rd_a - d_a^2}}{R - d_a} - \tan^{-1}\frac{\sqrt{2Rd_{a-1} - d_{a-1}^2}}{R - d_{a-1}}\right]$$

Figure 12. Shear moment due to cohesion.

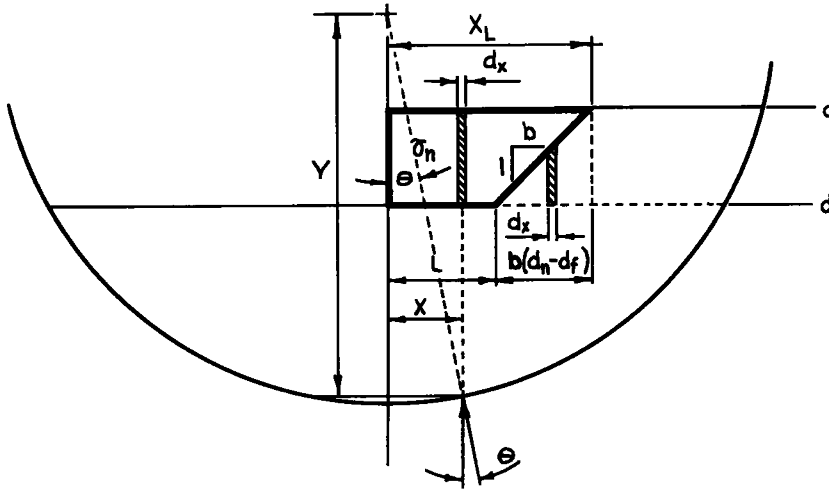


$$\text{Eq. 7} \qquad M_F = \sum_{a=1}^{\substack{a=f' \\ + \\ a=n}} \left[W_a - \gamma_a \left(R - d_a \right) \right] \tan \phi_a \int_{X_{a-1}}^{X_a} Y \, d_x$$

$$+ \sum_{a=1}^{\substack{a=f' \\ + \\ a=n}} \gamma_a \tan \phi_a \int Y^2 dx$$

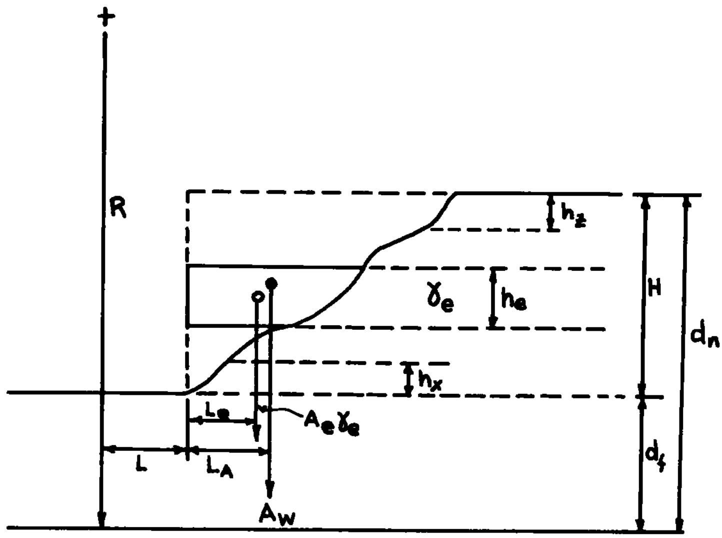
$$\text{where } W_a = \sum_{a=a}^{a=n} \gamma_{a+1} \left(d_{a+1} - d_a \right)$$

Figure 13. Shear moment due to friction.



Eq. 8 $$M_{fc} = \int_0^{X_L} \sigma_n (d_n - d_f)\sqrt{R^2 - X^2}\ \tan\phi\, d_x - \int_L^{X_L} \sigma_n \frac{(X-L)}{b}\sqrt{R^2 - X^2}\ \tan\phi\, d_x$$

Figure 14. Correction in shear moment due to friction.



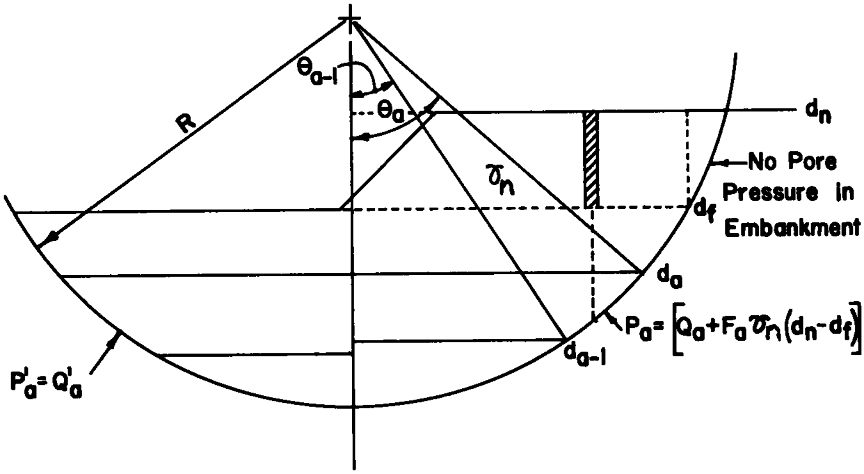
Eq. 9 $$M_{DC} = A_W(L+L_A) + \frac{1}{2}\gamma_A H L^2$$

Eq. 9A $$\gamma_A = \frac{\sum_{e=x}^{e=z} \gamma_e h_e}{\sum_{e=x}^{e=z} h_e}$$

Eq. 9B $$A_W = \sum_{e=x}^{e=z} \gamma_e A_e$$

Eq. 9C $$L_A = \frac{1}{A_W}\sum_{e=x}^{e=z} \gamma_e A_e L_e$$

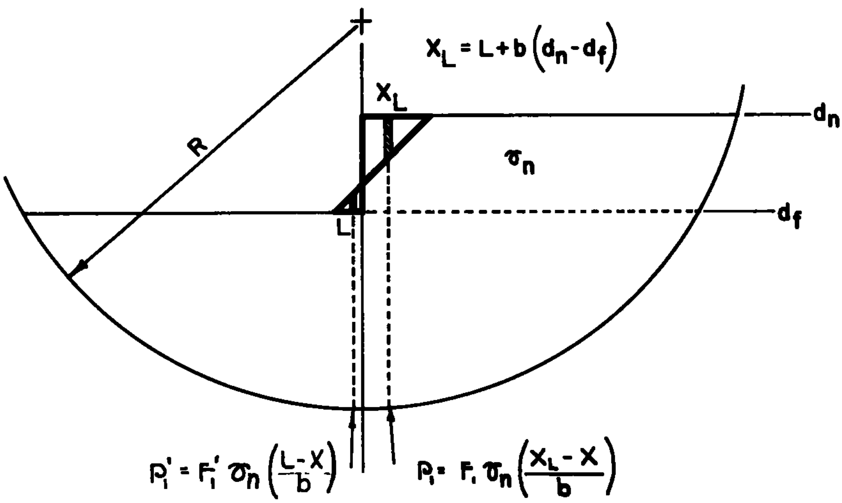
Figure 15. Driving moment correction—alternate method.



$$\text{Eq. 10} \qquad M_p = \int_{\theta_{a-1}}^{\theta_a} P_a R^2 \tan \Phi_a \, d\theta + \int_{\theta'_{a-1}}^{\theta'_a} P'_a R^2 \tan \Phi_a \, d\theta$$

$$\text{where } \theta_a = \tan^{-1} \frac{X_a}{Y_a}$$

Figure 16. Shear moment reduction due to pore pressure.



$$P'_1 = F'_1 \tau_n \left(\frac{L - X}{b}\right) \qquad P_1 = F_1 \tau_n \left(\frac{X_L - X}{b}\right)$$

$$\text{Eq. 11} \qquad M_{pc} = \int_0^{\theta} P_1 R^2 \tan \Phi_1 \, d\theta - \int_0^{\theta'} P'_1 R^2 \tan \Phi \, d\theta$$

Figure 17. Correction for shear moment reduction due to pore pressure.

Tabulation of Equations

Eq. 1 $$M_D = \sum_{a=1}^{a=n} \frac{\gamma_a}{6}\left[3R^2\left(d_a - d_{a-1}\right) + \left(R - d_a\right)^3 - \left(R - d_{a-1}\right)^3\right]$$

Eq 2 $$M_R = \sum_{a=1}^{a=f'} \frac{\gamma_a}{6}\left[3R^2\left(d'_a - d'_{a-1}\right) + \left(R - d'_a\right)^3 - \left(R - d'_{a-1}\right)^3\right]$$

Eq 3 $$M_{DC} = \frac{\gamma_n}{6b}\left(X_L^3 - L^3\right)$$ Eq 4 $$M_{DC} = \frac{\gamma_n}{6b} X_L^3$$

Eq. 5 $$M_{RC} = -\frac{\gamma_n}{6b} L^3$$

Eq 6 $$M_s = R^2 \sum_{a=1}^{a=f'+a=n} C_a\left(\tan^{-1}\frac{X_a}{Y_a} - \tan^{-1}\frac{X_{a-1}}{Y_{a-1}}\right)$$

Eq 7 $$M_f = \sum_{a=1}^{a=f'+a=n} \frac{\tan\phi_a}{2}\left[W_a - \gamma_a\left(R - d_a\right)\right]\left[X_aY_a - X_{a-1}Y_{a-1} + R^2\left(\tan^{-1}\frac{X_a}{Y_a} - \tan^{-1}\frac{X_{a-1}}{Y_{a-1}}\right)\right]$$
$$+ \sum_{a=1}^{a=f'+a=n} \gamma_a \tan\phi_a\left[R^2\left(X_a - X_{a-1}\right) - \frac{1}{3}\left(X_a^3 - X_{a-1}^3\right)\right]$$

Tabulation of Equations (Continued)

Eq. 8 $$M_{sc} = \frac{\gamma_n}{2b} \tan\phi_1 \left(X_L^2 \sqrt{R^2 - X_L^2} + X_L R^2 \tan^{-1} \sqrt{\frac{X_L^2}{R^2 - X_L^2}} \right) + \frac{\gamma_n}{3b} \tan\phi_1 \sqrt{(R^2 - X_L^2)^3}$$

$$- \frac{\gamma_n}{2b} \tan\phi_1 \left(l^2 \sqrt{R^2 - L^2} + R^2 \sqrt{L^2} \tan^{-1} \sqrt{\frac{L^2}{R^2 - L^2}} \right) - \frac{\gamma_n}{3b} \tan\phi_1 \sqrt{(R^2 - L^2)^3}$$

Eq. 9 $$M_{DC} = A_w \left(L + L_A \right) + \frac{1}{2} \gamma_A H L^2$$

Eq. 9A $$\gamma_A = \frac{\gamma_x h_x + \gamma_y h_y + \gamma_z h_z + \text{etc.}}{h_x + h_y + h_z + \text{etc.}}$$

Eq. 9B $$A_w = \gamma_x A_x + \gamma_y A_y + \gamma_z A_z + \text{etc.}$$

Eq. 9C $$L_A = \frac{\gamma_x A_x L_x + \gamma_y A_y L_y + \gamma_z A_z L_z + \text{etc}}{\gamma_x A_x + \gamma_y A_y + \gamma_z A_z + \text{etc.}}$$

Eq. 10 $$M_P = R^2 \sum_{a=1}^{\substack{a=f' \\ + \\ a=n}} P_a \tan\phi_a \left(\tan^{-1} \frac{X_a}{Y_a} - \tan^{-1} \frac{X_{a-1}}{Y_{a-1}} \right)$$

Eq. 11 $$M_{PC} = \frac{R^2 F_1 \gamma_n \tan\phi_1}{b} \left(X_L \tan^{-1} \sqrt{\frac{X_L^2}{R^2 - X_L^2}} + \sqrt{R^2 - X_L^2} - \sqrt{L^2} \tan^{-1} \sqrt{\frac{L^2}{R^2 - L^2}} - \sqrt{R^2 - L^2} \right)$$

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