Bond and Transflexural Anchorage Behavior of Welded Wire Fabric

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In this paper, mathematical theories are developed to predict the interactive bond-anchorage behavior of welded wire fabric when subject to a pull-out action, simulating the behavior of welded wire fabric in continuously reinforced concrete pavements. The theories are predicated on the concepts that the bond between the smooth longitudinal wires and the concrete is a function of minute movement between the steel and the concrete and that the anchorage of the transverse wires is achieved through the restrained flexural action of the transverse wires.

A summary of various general experimental techniques that other investigators have developed to measure bond is also presented as a background to the author's method, not hitherto used on small bars, in which electrical resistance strain gages are embedded in slots inside the bar. Based on this method, a description of the experiments on bond-anchorage behavior of welded wire fabric in concrete is presented. The tests are of a pull-out nature encompassing bond-anchorage behavior at first loading and after repeated cyclic loading.

A comparison of the theory and experiments shows general agreement.

●THE PRESENT investigation of bond-anchorage behavior of welded wire fabric in concrete had its origin in the study of continuously reinforced concrete pavements. In one of the author's previous papers (1) certain bond assumptions had to be made in the theory to predict the amount of crack opening and required wire steel. The only information available on this subject was some experimental work performed by Anderson (2). Although Anderson's findings are of value to the designer, the findings are lacking in certain needed areas for theoretical and research use. Through the cooperation of T. E. Shelburne of the Virginia Council of Highway Investigation and Research and Henry Aaron of the Wire Reinforcement Institute, facilities were made available to determine the basic information needed.

The first section of this work is theoretical; where equations are derived explaining the fundamental nature of the bond and anchorage action of a wire mesh in the region of a crack in a continuously reinforced pavement. The action at a crack is identical with pull-out action of a mesh embedded in a concrete slab, since at a crack the entire force is transmitted through the longitudinal wires. Exactly how this wire force is transmitted to the surrounding concrete is the essence of the problem at hand.

THEORY

For convenience of analysis only a single longitudinal wire is studied, as it is assumed that, at a crack, all the longitudinal wires act in the same manner. It is assumed that the steel and concrete remain elastic, that there is no creep effect, that the welds between the wires remain unbroken, that the stresses and strains in the mass concrete are very small, and that under certain conditions the adhesive bond between the steel and the concrete is negligibly small. In connection with the last assumption, elaboration is necessary.

First of all, bond is considered that restraining force acting like frictional shear only in the region of the surface of the longitudinal wire, as contradistinct to transflexural anchorage which is the restraint offered by the transverse wires. Although the effects are interactive, their basic behavior is different. Thus, for clarity of understanding as well as for other practical reasons, it will be initially assumed that no bond exists on the longitudinal wires. Once this transflexural anchorage behavior is explained, the bond behavior will then be separately analyzed. However, there are certain engineering bases for neglecting bond because of its unreliable and uncertain nature. Such things as the presence of grease or dirt on the wires, repeated loads causing agitation and erosion of the concrete around the wires, and reduction of wire diameter caused by Poisson's effect all tend to reduce bond. Only that form of bond created by bearing action (as in deformed bars) is reliable in all situations.

Figure 1 is a general plan view of the bar fabric in place in a cracked continuously reinforced concrete pavement. For information on the values of P_1 see (1).

Figure 2 is a skeleton view of one strand of wire fabric embedded in concrete with a crack formed at the right end. The force P in the wire is distributed along the length of the wire with decreasing value away from the crack. It is assumed that at the end of five transverse wire anchorages, this force P_6 is so small that it may be neglected.

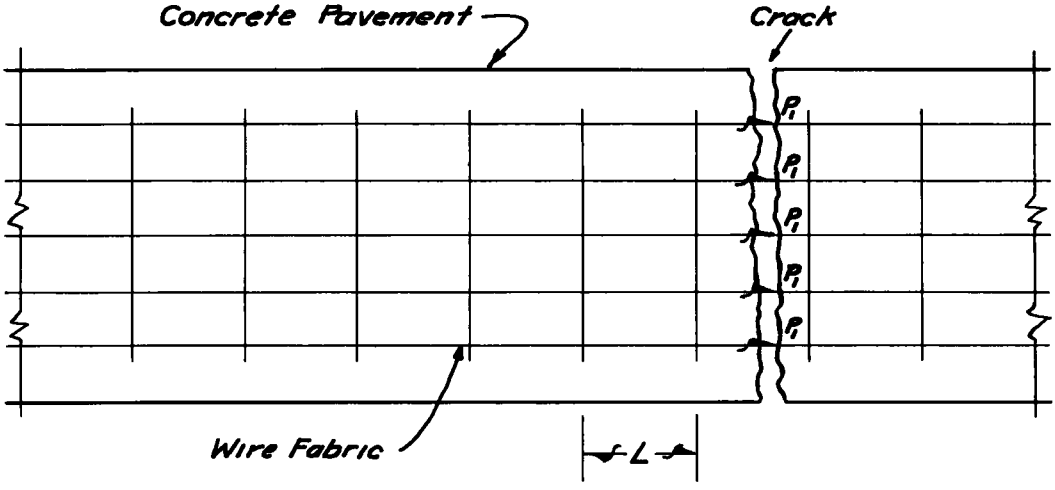

Figure 1. Plan view of fabric in pavement with crack.

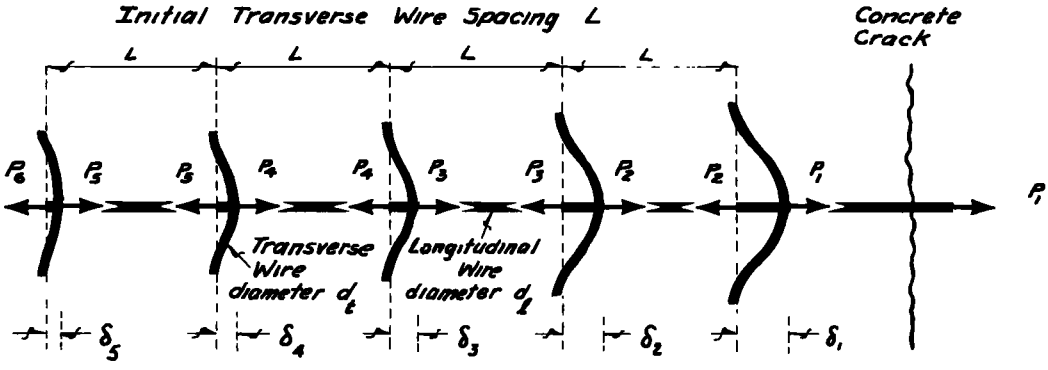

Figure 2. Longitudinal wire forces and transverse wire deflections.

The transverse wires act as flexural beams, restrained from bending by the compression of the surrounding concrete. (This condition is similar to that assumed by Friberg (14).) This situation is entirely analogous to the well known theory of beams on elastic foundations where the elastic support modulus is E_c (modulus of elasticity in compression of concrete) and the load is the unbalanced concentrated load across each joint (3). Interaction effect from adjacent longitudinal wires is negligibly small.

Thus the deflection δ, is

$$\delta_1 = \frac{(P_1 - P_2)\,\beta}{2\,E_c}$$

where

$$\beta = \sqrt[4]{\frac{E_c}{4\,E_s\,I_t}}$$

and E_s is the modulus of elasticity of the steel and I_t is the moment of inertia of the transverse wire $= \dfrac{d_t^4}{64}$

Likewise:

$$\delta_2 = \frac{(P_2 - P_3)\,\beta}{2\,E_c}$$

$$\delta_3 = \frac{(P_3 - P_4)\,\beta}{2\,E_c}$$

$$\delta_4 = \frac{(P_4 - P_5)\,\beta}{2\,E_c}$$

$$\delta_5 = \frac{(P_5)\,\beta}{2\,E_c}$$

However, the longitudinal wire stretches as it is subjected to load. From elementary theory

$$\delta_1 - \delta_2 = \frac{P_2\,L}{A_1\,E_s}$$

where A_1 is the area of the longitudinal steel $= \dfrac{\pi\,d_1^2}{4}$

Likewise:

$$\delta_2 - \delta_3 = \frac{P_3\,L}{A_1\,E_s}$$

$$\delta_3 - \delta_4 = \frac{P_4\,L}{A_1\,E_s}$$

$$\delta_4 - \delta_5 = \frac{P_5\,L}{A_1\,E_s}$$

Assuming P_1 is known, there are nine unknown quantities P_2, P_3, P_4, P_5, δ_1, δ_2, δ_3, δ_4, δ_5, and nine independent equations in P and δ as just presented. By standard elimination methods of simultaneous solutions, the following results for P may be obtained.

$$P_2 = \frac{(\gamma\phi^2 - 2\gamma^3\phi)}{(\gamma^4 + \phi^4 - 3\gamma^2\phi^2)} P_1$$

$$P_3 = \frac{(\gamma\phi^2 - \gamma^3)}{(\phi^3 - 2\phi\gamma^2)} P_2$$

$$P_4 = \frac{(\gamma\phi)}{(\phi^2 - \gamma^2)} P_3$$

$$P_5 = \frac{(\gamma)}{(\phi)} P_4$$

where

$$\gamma = \frac{3}{4E_c d_t} \sqrt[4]{\frac{E_c}{E_s}}$$

$$\phi = 2\gamma + \frac{\gamma L}{A_1 E_s}$$

To simplify these results it is observed that the fraction of longitudinal wire force R transmitted across each transverse wire is constant, and will be called the transfer ratio.

$$R = \frac{P_{m+1}}{P_m} \simeq \frac{\gamma\phi^3 - 2\gamma^3\phi}{\gamma^4 + \phi^4 - 3\gamma^2\phi^2} \simeq \frac{\gamma}{\phi} \qquad (1)$$

The simple ratio $\frac{\gamma}{\phi}$ is accurate to about 3 percent and the other longer expression in Eq. 1 is accurate to less than 1 percent.

Thus it is seen that the force in the longitudinal wire progressively diminishes away from the crack.

Example

$L = 10$ in.
$E_c = 3 \times 10^6$ psi
$E_s = 30 \times 10^6$ psi
Transverse wire size No. 1 ($d_t = 0.283$ in.)
Longitudinal wire size No. 5/0 ($d_l = 0.431$ in.)
$P_1 = 8150$ lb (limit value based on 56,000-psi stress)

Based on the preceding calculations, Figure 3 shows the distribution of force.

The force in the longitudinal wire diminishes very rapidly. Should bond exist, the longitudinal wire force would diminish even more rapidly, and the force in the longitudinal wire beyond the third transverse wire would essentially vanish as both Anderson's (2) and the author's tests show.

Bond

Since some force in the longitudinal wires is known to exist, strains must also exist. The steel must therefore move with respect to the concrete. The order of magnitude

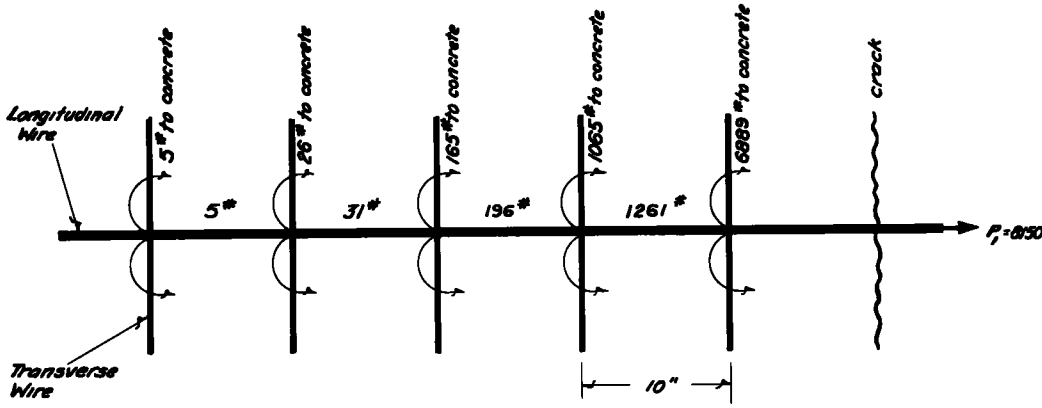

Figure 3. Distribution of wire forces.

of this movement is very small and cannot easily be measured. This movement may be due to actual slip or to a thin boundary layer of concrete adjacent to the steel being shear strained along with the steel. The latter condition implies true adhesive bond. However, either condition is covered by the proposed theory. Osterman (12) also found that this working hypothesis was very satisfactory for his studies of bond as used in a different application.

The first panel adjacent to the crack in Figure 2, is now assumed to have bond forces acting on the longitudinal wire. If the displacement of any element in the X direction as shown in Figure 4 is defined as u, then the unit surface bond stress is

$$\tau = K(u + \delta)$$

where K is an empirical constant dependent on such factors as concrete strength, roughness of wire, and moisture content of concrete.

In terms of the displacement the force in the wire at any position is

$$\frac{du}{dx} A_1 E_s$$

which from equilibrium must equal

$$P_1' + K\pi d_1 \int_0^x (u + \delta)\, dx$$

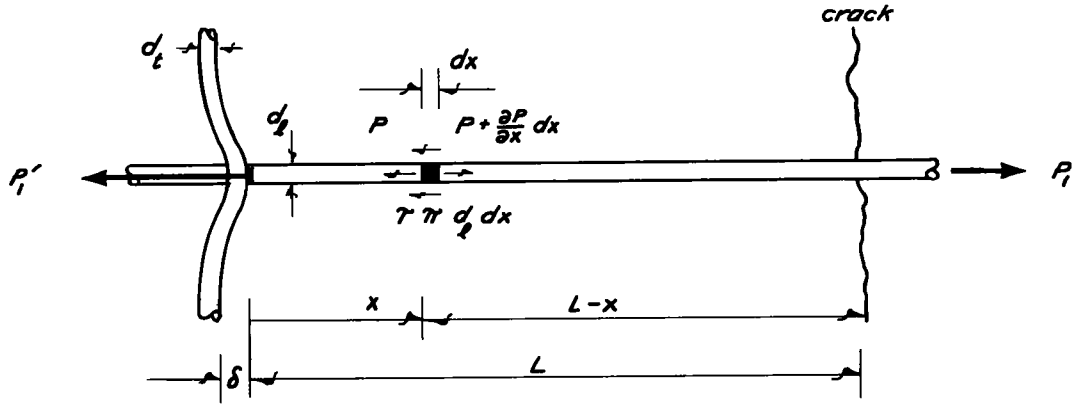

Figure 4. Bond forces on wire element.

Thus, the equation is formed as follows

$$\frac{du}{dx} A_1 E_s = P_1' + K\pi d_1 \int_0^x (u + \delta)\, dx \tag{2}$$

The solution to Eq. 2 which satisfies the boundary conditions is in the form of

$$u = Qe^{\alpha x} - Q \tag{3}$$

where α and Q are undetermined parameters. By substituting Eq. 3 into Eq. 2 and equating like terms, α and Q may be obtained as follows:

$$\alpha = \sqrt{\frac{K\pi d_1}{A_1 E_s}}$$

$$Q = \delta$$

Thus the solution to Eq. 2 is

$$u = \delta e^{x\sqrt{\frac{K\pi d_1}{A_1 E_s}}} - \delta \tag{4}$$

With the displacement u known, the force in the wire at any position may be obtained from the equilibrium equation below:

$$P(x) = P_1' + K\pi d_1 \int_0^x (u + \delta)\, dx$$

$$P(x) = P_1' + \frac{K\pi d_1 \delta}{\alpha} + \frac{K\pi d_1 \delta}{\alpha} e^{\alpha x}$$

Since

$$P_1 - P_1' = \int_0^l \tau \pi d_1 \, dx$$

the expression for P (x) may also be expressed in terms of P_1 as follows:

$$P(x) = P_1 - \frac{K\pi d_1 \delta}{\alpha} (e^{\alpha l} - e^{\alpha x}) \tag{5}$$

A typical plot of this equation is shown in Figure 5.

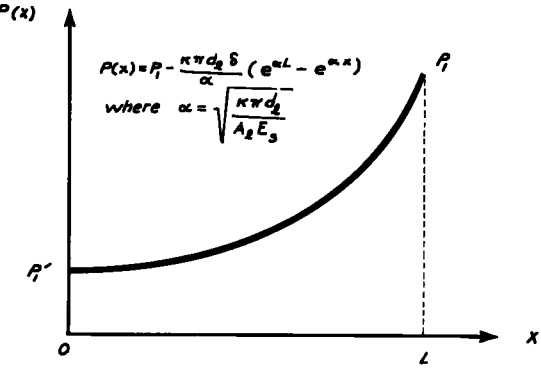

Figure 5. Wire force variation.

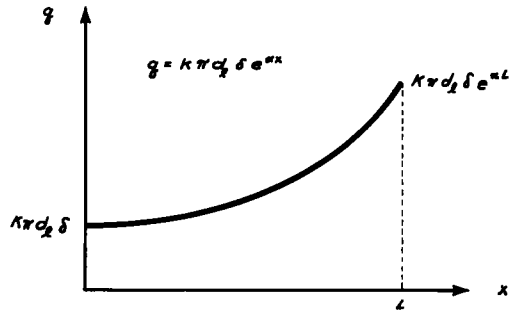

Figure 6. Bond force variation.

The bond force variation q may also be plotted from the equation

$$q = K\pi d_1 (u + \delta) = K\pi d_1 \delta e^{\alpha x} \tag{6}$$

as shown in Figure 6.

Both the wire force and bond force have the same general exponential shape. Experimental evidence to be cited later will show that this theory does agree with experiments.

EXPERIMENTAL METHODS FOR MEASURING BOND

A survey of existing literature on the subject of experimental methods for measuring the actual bond stress and its variation on plain and deformed bars reveals nine different techniques used by previous investigators. These nine methods will be discussed briefly here as background to the author's method. This survey and discussion of techniques may also be of benefit to other investigators faced with instrumenting for bond measurements.

Two simple methods (4) use extensometers to measure the strains in the steel at intervals. The steel bar may be made accessible for measurement in either of two ways. One method is to leave access holes in the concrete to the steel. The disadvantage here is that these holes may create stress irregularities as well as reduce the bond area. Also the gage length cannot practically be less than about 2 in., thereby resulting in loss of accuracy. Furthermore, if the concrete layer is thick, various extensions or levers must be attached to the extensometers to extend to the bars. The second alternative is to weld stubs on at intervals along the steel bar, protruding through access holes to the surface of the concrete. Here again the access holes must be carefully preformed such that the stubs will at all times be free of contact. Again accuracy is lost in that the stubs physically cannot be too close together. Loss of bond area and stress concentrations also play a detrimental role.

In another direction, much has been done with electrical resistance strain gages such as the SR-4 type. There are four techniques in this category. The first attempts were made by merely bonding the gages to the surface of the bar. Waterproofing and shielding of leads are important in this technique. Anderson (2) and McHenry and Walker (5) used this method. Some disadvantages of this method are: much bond area is lost (especially by the waterproofing application), wedging action of the protruding gage and the concrete takes place, and waterproofing is difficult.

To overcome some of these objections, Mains (6) split a reinforcing bar for its entire length and milled a small groove along the axis of the bar on the inner face. After attaching and waterproofing SR-4 gages on this inner slot, the two halves of the bar were spot welded back together. The leads of the gages all extended out the ends of the bar and no bond area was lost due to instrumentation. However, even this method is not without some drawbacks. Of course the major objection is in the trouble and expense of splitting and milling such bars; but in addition, such a hollow bar cannot be loaded to the same failure force as a normal bar, which means that information at normal failure loads cannot be obtained.

Bernander (7), using large diameter bars, inserted electrical resistance strain gages in preformed narrow slots along the bar. The slots extended only part way through the bars, and the gages were cemented to one side of these slots along the diameter line. The slots were filled with resin for moisture proofing. These slots were easy to form and waterproof and little bond area was lost. However, because the slots did not extend completely through the bar some bending or eccentric stresses were caused in the bar, even where direct tension was exerted at the ends.

Janney (8), using SR-4 gages, obtained bond information on very small diameter wire by a clever indirect method. The wires to be studied were cast in concrete prisms, with the ends of the wires extending beyond the prisms. SR-4 gages were then cemented to the exterior surface of the concrete along the axis of the prism in the direction of the wire. When the ends of the wire were pulled, the variation of the concrete strains could be measured and related to the wire strains. Certain weaknesses

are inherent in this method, such as the shear lag between the wire strains and the concrete surface strains, possible concrete cracking and the difficulty of relating non-linear concrete strains to steel stresses. However, for small diameter wires, no gaging could be attached to the wire itself, and so this method was used.

X-Ray techniques have been used in England to obtain bond stresses. Evans and Robinson (9) embed thin strips of platinum or lead in slots in the steel and X-Ray photograph the concrete and encased steel bars as load is applied to the steel. Images cast by the markers are thus recorded on film without disturbing the steel or concrete. By use of microscopes these filmed images may be studied and information on strains, stresses and bond may thus be obtained.

At the State Institute for Technical Research in Helsinki, a unique method was used for bond study (10). The steel stresses in the concrete were obtained without physically disturbing the steel or concrete in any way by a magneto-strictive method. This method is based on the principle that an alternating current passing through a steel bar creates a magnetic field, perpendicular to the longitudinal axis of the bar, which in conjunction with the resistance determines the electrical impedance of the steel. If the bar is subjected to tension while a constant current is applied, the impedance will be altered by magneto-striction. If the relation between the change in impedance and the stress in the steel is known (by pre-calibration), the average stress over a small given distance can be determined by measuring the voltage difference over this distance. Thus, in this way the bond variation may be obtained. The technique is not yet fully perfected as shrinkage, moisture and cracks influence the results.

Finally, to emphasize the variety of techniques used for bond studies, a ninth method of photoelasticity is mentioned. Beyer and Salakian (11) used a metal wire molded in photoelastic bakelite to determine bond stresses between the wire and the model, simulating reinforcing steel in concrete.

The method that served best for this particular study of bond of wire mesh was a variation of Bernander's method, in which the slot was extended all the way through the bar to eliminate the undesirable effects of unsymmetrical straining. Figure 7 shows how the slots were made and how the small SR-4 gages were attached. Minimum bond area was lost and successful waterproofing was easily accomplished by filling the slots with melted Petrosene wax. Extended water immersion tests showed that this method was quite satisfactory for the purpose intended. However, due to the removal of metal, the slotted wires had to be precalibrated to determine the relation

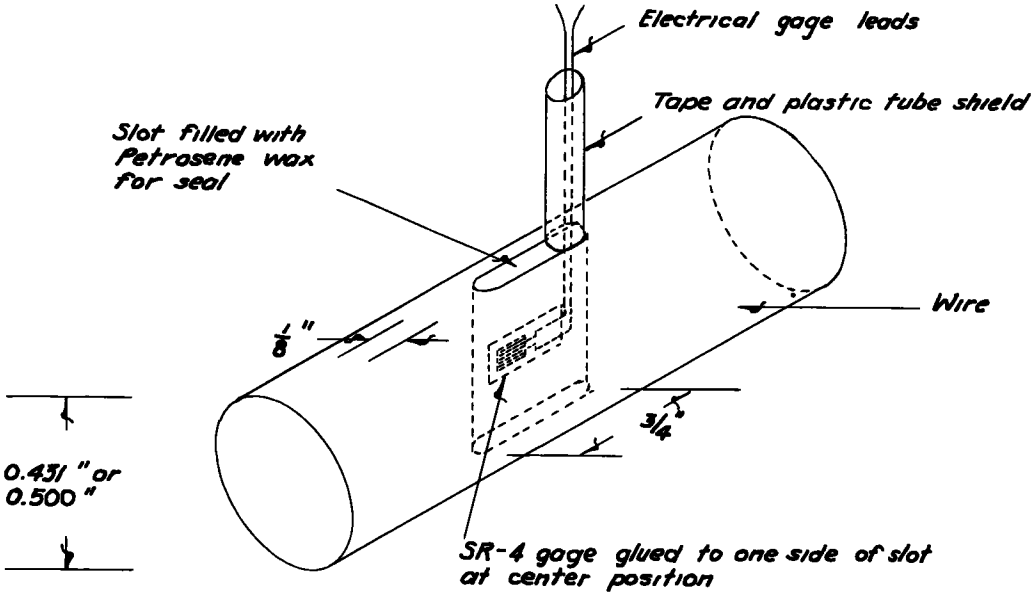

Figure 7. Attachment of gage to wire.

of the stress at the gage to the stress in the normal unslotted wire. This relation proved to be linear up to the yield stress of 62,000 psi.

TESTS

To determine the actual bond stress and its variation in welded wire mesh embedded in concrete, several tests were performed. Two mesh configurations were studied. These are shown in Figure 8.

Mesh A was embedded at mid-depth in a panel of concrete 16 in. by 4 in. by 42 in. and mesh B was embedded in a panel of concrete 16 in. by 4 in. by 62 in. Figure 9 shows these wire meshes in the forms prior to pouring. Note that only the center longitudinal wires are extended on which the pull out force is applied. Figure 8 also shows the locations of the SR-4 gages as positioned along the center longitudinal wires. At the time of pouring, test cylinders were made in order to obtain the concrete properties. The eye bolts shown are simply for the purpose of handling. After curing and hardening of the concrete for four weeks, the test panels were placed in a Universal Testing Machine and loaded (Fig. 10).

The center wire freely passed through the lower head and was gripped by the upper tension head. A bearing plate was provided to prevent crushing of the concrete. By this arrangement, a pull-out action was achieved.

Strain readings at each of the gage points were taken only at the first loading cycle

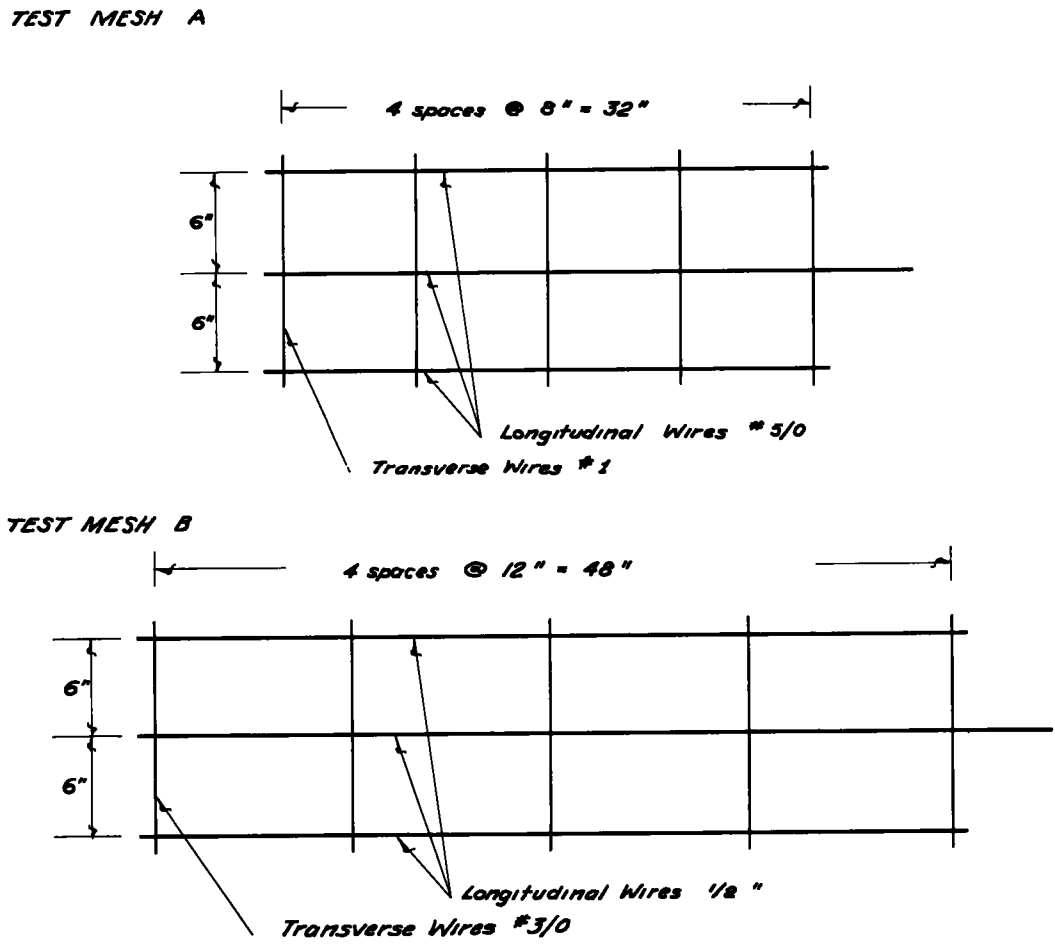

Figure 8. Test meshes.

and at the 60th loading cycle. Cyclic readings did not go beyond 60 cycles as the Universal Testing Machine used is awkward and time consuming to cycle. As the results of cyclic loading are of most importance, these results are plotted in Figures 11 and 12 for the A and B panels respectively based on an average center wire force of 1,000 lb. It is perhaps well to note that total loads on the wires were imposed to the extent of 80 percent of the yield stress load at the slot. At the range of load imposed, there

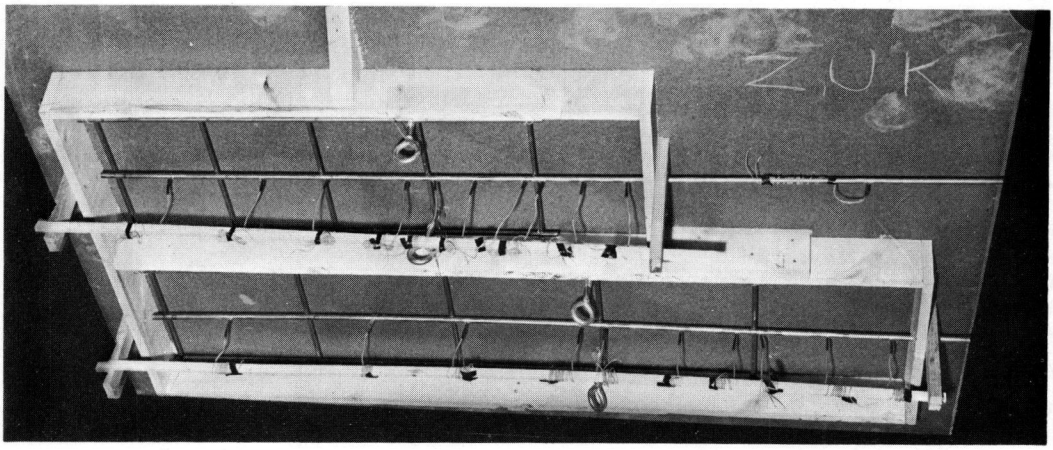

Figure 9. Test panels before pouring concrete.

did not seem to be any particular deviation from the same variation of wire force.

COMPARISON OF THEORY AND TESTS

The shape of the wire force curves between transverse wires compares well with the theory. That is, theoretical curves in Figures 5 and 6 correspond to experimental curves in Figures 11 and 12 between transverse wires. In addition, experimental curves found by Mains (6) in his Figure 7a of the pull-out force variation in a hooked plain bar also show good agreement with the theoretical curves. The anchorage action of a single hook is much the same as the transflexural anchorage of transverse wires as some movement also occurs at the anchorage such that the bar force and bond force do not vanish in this region.

Based on Eq. 5 and the data in the curves of Figures 11 and 12, the empirical bond value of K was determined as 49.8×10^4 lb per sq in. wire surface per in. slip. This value of K looks astronomically large, but the so-called slip is in the order of magnitude of less than one thousandth of an inch. Should large slips occur, entirely different bond-fric-

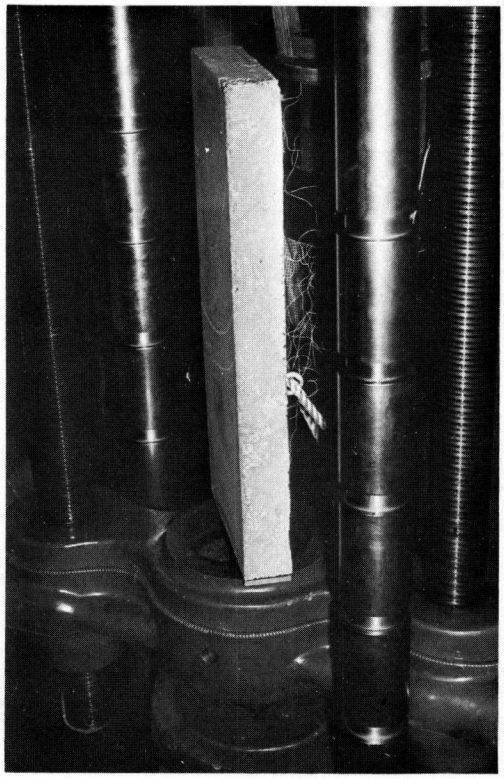

Figure 10. Panel in testing machine.

tion action would take place, invalidating the theory. Bond behavior becomes very irregular at large slip values as pointed out by Mains (6) and Mylrea (13).

Figures 11 and 12 show that no wire and bond forces exist beyond the second transverse wire. If such forces did exist, the strain gage readings were so small that they did not register. This information seems to verify Anderson's (2) observations on his own static tests that only the first two transverse wires are effective, even up to rupture of the steel. However, the full effects of cycling of stress many thousand times was not demonstrated in any of the tests, and the results should perhaps be viewed with this in mind. Such things as fatigue and agitation of the concrete in the region of the steel may well play an influential role in bond behavior under actual service conditions. It was noticed in these tests that the bond value of K was reduced from the first loading cycle to the 60th cycle, indicating a general trend in that direction. Future work using a special fatigue apparatus is thought to be necessary in this connection.

The computed values of the transfer ratio R of the wire force across the first transverse wire show agreement with theory regarding the effects of transverse wire size and spacing. Smaller values of $\frac{L}{A_1}$ mean less longitudinal wire stretching, and thin transverse wires mean less transflexural anchorage; resulting in more force being transmitted across the anchor points (larger values of R). Panel A has both a smaller $\frac{L}{A_1}$ ratio and smaller transverse wire size than panel B, resulting in a higher value of R (Figs. 11 and 12).

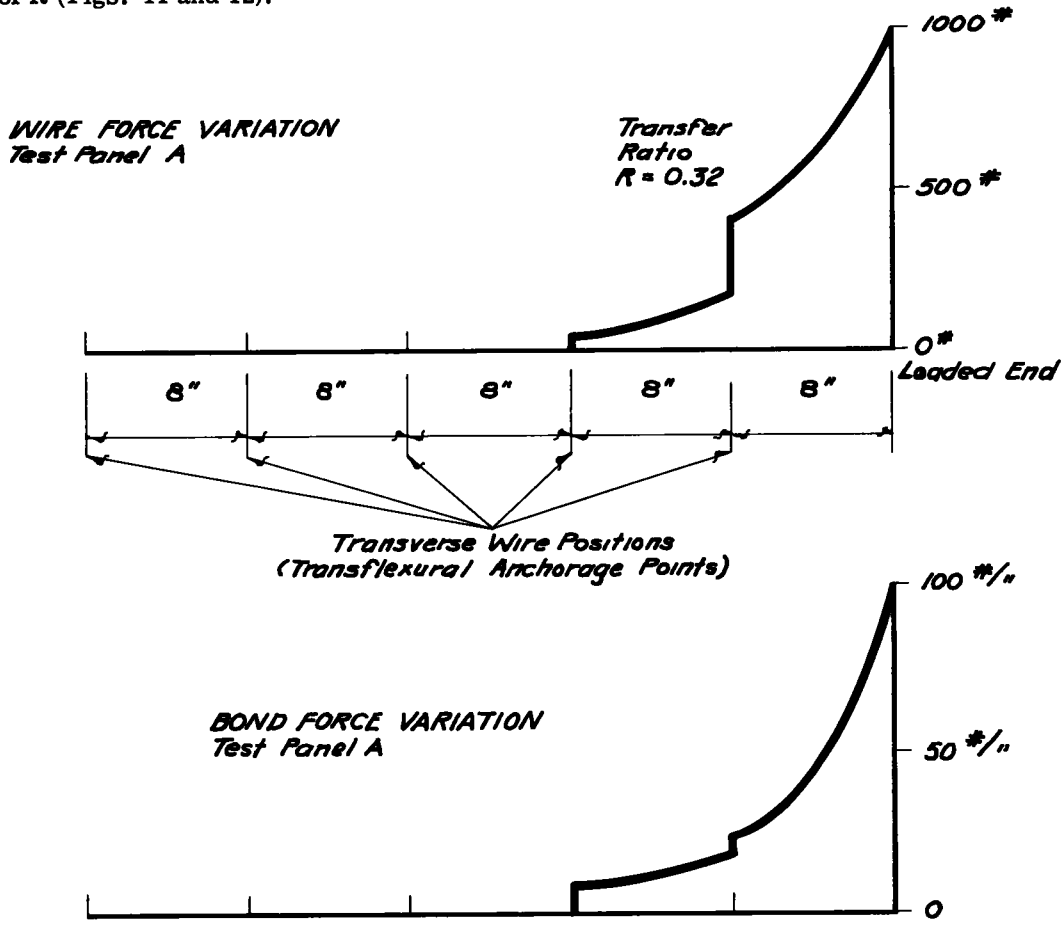

Figure 11. Wire and bond force variation—"A" panel after 60 cycles.

The theoretical value for the transfer ratio R for panels where both bond and transflexural anchorage exist may be obtained from the theory given at the beginning of this report. To do this, Eq. 5 is written for the first two spaces from the end as

$$P_1' = P_1 - \frac{K \pi d_1 \delta_1}{\alpha} e^{\alpha L}$$

and

$$P_2' = P_2 - \frac{K \pi d_1 \delta_2}{\alpha} e^{\alpha L}$$

Two transflexural beam equations are also needed, written as

$$\delta_1 = \frac{(P_1 - P_2)\beta}{2 E_c}$$

and

$$\delta_2 = \frac{P_2' \beta}{2 E_c}$$

In addition, a final equation of distortion, Eq. 4 for the second space is written as

$$\delta_1 = \delta_2 e^{\alpha L}$$

By solving these five equations simultaneously, the transfer ratio R for the combined

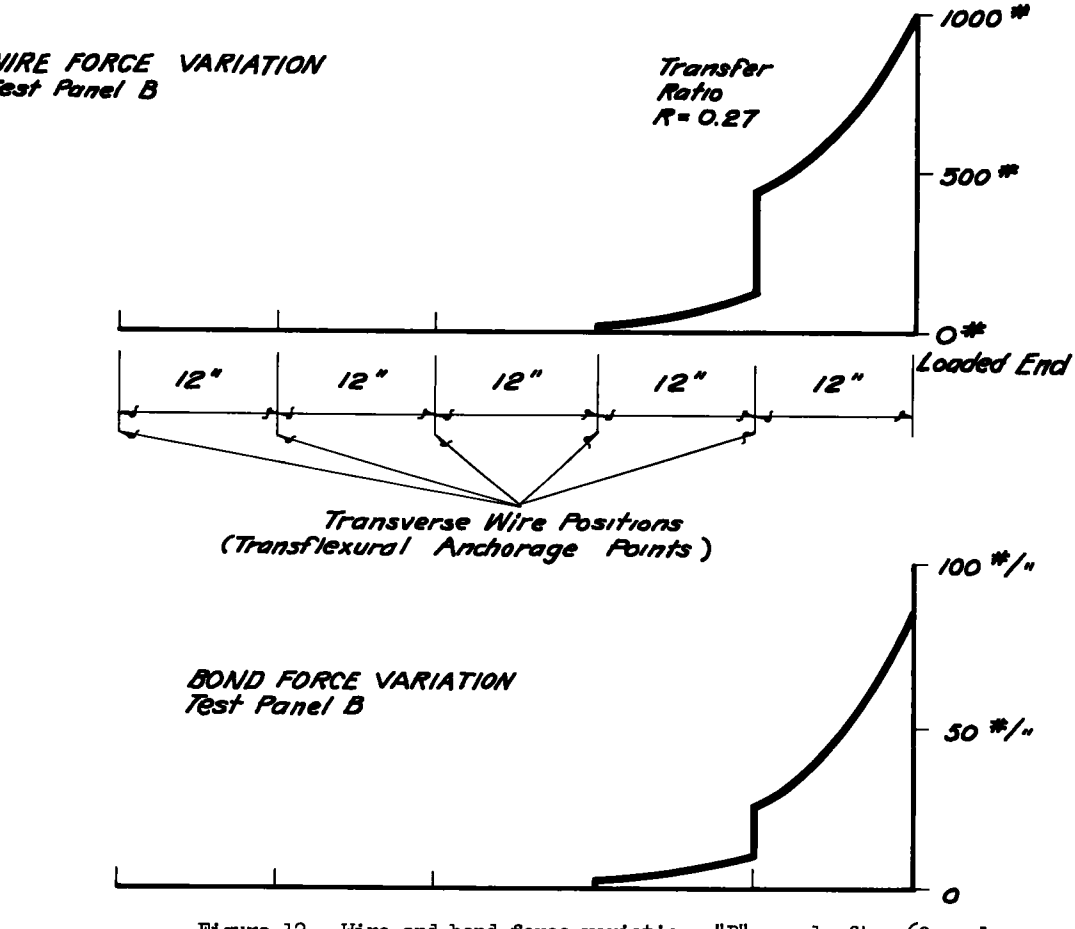

Figure 12. Wire and bond force variation— "B" panel after 60 cycles.

case of bond and anchorage is obtained as

$$R = \frac{N}{N + 2\alpha E_c e^{\alpha L}}$$

in which

$$N = 2\alpha E_c + K\pi d_1 \beta e^{\alpha L}$$

Using Eq. 7, R is 0.48 for panel A and 0.46 for panel B. The difference is in the correct direction but higher than experimental values indicate. The explanation for this probably lies in the adhesion of the transverse wires, which effect is not considered in the theory. Adhesion of the transverse wires would tend to reduce the transfer ratio.

CONCLUSIONS

The technique of using slotted holes employed in this set of tests seems to incorporate many advantages of simplicity, ease of fabricating, east of waterproofing, convenience when tested with standard equipment and good accuracy. Several disadvantages should also be noted. The slots must be milled exactly and the gages set exactly, otherwise the calibration of the stress concentration factor is difficult and unreliable. Special care must also be taken in bonding the gage to the wire under pressure. This is a little awkward in so narrow a slot (⅛ in. wide). An occasional gage in this set of tests did act erratically, due to poor adhesion of the gage. A final objection is that the force exerted on the wire had to be limited to about 60 percent of the rupture force on the normal bar to prevent yielding at the slot. This prevented the study of bond at normal rupture loads.

The theory and experiment showed good general agreement. This gives strong evidence to support the contention that bond of plain wires or welded wire fabric is a direct function of shear "slip" (at small values of slip) and that anchorage is primarily due to the restrained bending of the transverse wires. The theory and tests show that two distinct actions take place between the steel and the concrete to disperse the load in the longitudinal wires. The shearing boundary layer action between the longitudinal steel and the concrete is the one action called bond, and the restrained bending action of the transverse wire is the other action called transflexural anchorage. The theory developed shows the distinction and the relation between these two actions.

Further research is still necessary to study bond under repeated loads of many thousand cycles to determine the extent that bond action breaks down.

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14. ACI Committee 325, "Design Considerations for Concrete Pavement Reinforcement for Crack Control." ACI Journal, p. 361 (October 1956).

Discussion

J.L. WALING, School of Civil Engineering, Purdue University — Professor Zuk has undertaken an analysis of the distribution of the forces of anchorage of welded wire fabric in concrete. He is to be commended for attempting a solution to this difficult problem, complicated as it is by the uncertainty of the validity of the several assumptions which must be made in the theoretical analysis. The writer's questions and discussion will pertain primarily to these assumptions and will be considered not in the order of importance but in the order in which the material is presented in the paper by Professor Zuk.

The author states that "The action at a crack is identical with pull-out action of a mesh embedded in a concrete slab, since at a crack the entire force is transmitted through the longitudinal wires." This should be questioned since forces large enough to initiate a crack could also break the bond between the longitudinal wires of the fabric and concrete. Thus, the bond resistance available in the two cases may not be identical.

In stating the analogy of the action of the transverse wire to that of a beam on an elastic foundation, the author states that "...the elastic support modulus is E_c...." In the theory of beams on elastic foundations, the support modulus (usually denoted by K) is a measure of the support resistance in force per unit length of beam per unit deflection of the beam. It can be shown that

$$k = E_c \frac{w}{d}$$

in which

w = effective width of the beam support, and
d = effective depth of compressed supporting material.

It is not clear to the writer that the ratio w/d can be taken as unity. Nor is it clear that this ratio can be reliably approximated by theory alone.

The author stated the equation for unit surface bond stress as $\tau = K(u + \delta)$. However, if he means that $\tau = K$ times the movement of the longitudinal wire relative to the adjacent concrete, then the expression should be $\tau = Ku$, for the adjacent concrete is compressed by the transverse wire an amount equal to δ, at least in a region near $x = o$. Near $x = L$ the relationship of u to the bond shearing strain seems undetermined, since the author appears not to have considered concrete deformations near the crack.

If one assumes that the author is correct in the statement that $\tau = K(u + \delta)$ then the solution stated as Eq. 4 satisfies Eq. 2 only if $P_1' = K\pi d_1 \frac{\delta}{a}$. This seems to have escaped the attention of the author during the process of equating like terms after the substitution of Eq. 3 into Eq. 2.

If on the other hand one assumes that $\tau = Ku$, which would appear to be closer to reality over a greater length of the longitudinal wire, one can derive the following:

$$u = \frac{P_1' \alpha e^{\alpha x}}{K\pi d_1}$$

$$P(x) = P_1' e^{\alpha x}$$

and

$$q = P_1' \alpha e^{\alpha x}$$

where

$$\alpha = \sqrt{\frac{K\pi d_1}{A_1 E_s}} = \sqrt{\frac{4K}{d_1 E_s}}$$

Thus, it is seen that this assumption also yields exponential variations in wire force and bond force. It would be interesting to see a trial analysis of the experimental data made on the basis of these equations.

The method of attaching the SR-4 strain gages to the fabric wires warrants some additional attention. In test panel A, at the location of each gage, approximately 19 percent of the bond area and about 36 percent of the tensile area were lost. In test panel B the corresponding losses were about 16 and 31 percent, respectively. Thus the mechanical relationships governed by the area and perimeter of the longitudinal wires were somewhat distrubed in the length of the strain gage slots, even if the stress concentrations caused by the slots could be ignored. This is not to criticize the author for the use of the method, since no other method is accepted as appreciably better for the purpose.

The loads applied to the test panels (Figs. 11 and 12) seem unusually low. At the free end of the protruding wire at each panel a maximum load of 1,000 lb is indicated. This gives a maximum tensile stress of about 6,900 psi on the gross wire area in test panel A and about 5,100 psi in test panel B. Likewise, the corresponding maximum bond forces shown would give bond stresses of about 75 psi and 55 psi in the same panels, respectively. Because both of these stresses decrease rapidly along the wire toward the first gage slot in the concrete, it is difficult to imagine that stresses in magnitude of 80 percent of the yield strength of the wires were developed at a slot. This would indicate an extremely large stress concentration factor at a slot. In any case, the stresses imposed on gross areas of the wires in the experiments were quite small compared to stresses known to develop in continuously reinforced concrete pavement reinforcement. There is much evidence to indicate that maximum stresses which normally develop at cracks in such pavements cause bond slippage along the longitudinal wires of welded wire fabric, thus transferring the anchorage to the first transverse wire and longitudinal bond and transverse wires beyond.

By all means, the author should be encouraged in his commendable efforts to analyze the many difficult problems involved in the rational design of continuously reinforced pavements. He has made notable contributions to the present state of knowledge of these structural elements.

WILLIAM ZUK, Closure — The discussion of Professor Waling is very much appreciated. In particular, his refinement of the elastic support modulus and opinions on the shear slip are of interest. Perhaps the real fact of slip lies somewhere between the expression $\tau = K(u + \delta)$ and $\tau = Ku$. Instrumentation much more precise than that used in this study is necessary to confirm this.

The oversight error in Eq. 3 is also appreciated. The correct expression for Eq. 3 should have been $u = Q_1 e^{\alpha x} - Q_2$, resulting in Eq. 4 as

$$u = \frac{P_1' \alpha}{K\pi d_1} e^{\alpha x} - \delta$$

$P(x)$ would then be $P_1' e^{\alpha x}$ or $\frac{P_1}{e^{\alpha L}} e^{\alpha x}$ and "q" would be $P_1' \alpha e^{\alpha x}$

The equations thus correspond to Professor Waling's. It should also be mentioned that the correction of Eqs. 3 through 6 also change the equations on "Comparison of Theory and Tests." R is revised to read

$$\frac{K \pi d_1 \beta e^{2\alpha L}}{2 \alpha E_c + K \pi d_1 \beta e^{\alpha L}}$$

Perhaps as a result of simplification, there seems to be a misunderstanding in regard to the test load applied on the longitudinal wires. The actual peak loads imposed were of about 4 kips and 6 kips on panels A and B respectively. However, in plotting the curves (11) and (12) a unit value of 1 kip was used for comparison purposes. It is fully realized that at higher loads, more severe slipping would take place and a simple exponential expression for bond would no longer be true.