Use of Backwater in Designing Bridge Waterways

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How is the length of a bridge over a stream determined? This question has many answers since bridge engineers, responsible for such decisions, have had to rely principally on personal observation and experience for the answers. In short, no generally accepted method for bridge waterway design has existed. A comparison of the small number of bridge failures to the total number of bridges throughout the country attest to the commendable job bridge designers have performed with the limited design tools available. Their record is most certainly impressive.

What proportion of existing bridges are under-designed and what proportion are over-designed from the standpoint of length and clearance? With many new bridges scheduled to be constructed under the accelerated highway program, the above question deserves serious thought from the standpoint of safety and economy. Under-designed bridges usually speak for themselves, given sufficient time. In the case of over-design, no reliable standards exist at the present time by which these structures can be judged impartially.

CURRENT RESEARCH

●AS A STEP aimed at placing bridge waterway design on a sounder footing, a cooperative research project was initiated in 1954 by the Bureau of Public Roads at Colorado State University. The project has been active since that time. To date the investigations have centered on the determination of backwater produced by bridges (1, 2), scour at bridge abutments, scour around piers, and methods for alleviating such scour. Two other research projects at the University of Iowa, sponsored by the Iowa State Highway Commission and the Bureau of Public Roads, have also contributed much needed information on scour at bridge piers (3) and scour at bridge abutments (4).

Bridge waterway problems are diversified and complex, which accounts to some extent for the limited headway made in understanding and resolving this phase of design in the past. Because of the many variables involved, hydraulic models were called upon to serve as the principal research tool in all the work mentioned above. It is possible with models to hold a certain number of variables constant while investigating the effect of others; then by systematically rotating the combination of variables in the test program, holding some constant and allowing others to vary, to isolate the part that certain principal variables play in the final result. In addition to aiding in a better understanding of the theory and mechanics involved, the models are indispensable since experimental coefficients are required, which can be obtained in no other way.

The waterway problem is much too extensive for even a condensed treatment here; the context of this paper will thus be confined principally to a discussion of the bridge backwater phase. It contains a brief account of the problem, the research results, the design information derived therefrom, and the application of bridge backwater to waterway design.

EXPERIMENTAL BACKWATER STUDIES

A comprehensive record of the experimental data, test procedures, and analysis of results on bridge backwater appears in a report issued by Colorado State University (1). For those interested only in the design application, a booklet titled "Computation of Backwater Caused by Bridges" is available (2). The latter contains design charts, an explanation of design procedures, and five practical examples. Since the above information is available in printed form, it will be necessary to draw from it only sufficiently to understand the contents of this paper.

The manner in which flow is contracted in passing through a channel constricted by bridge embankments is illustrated in Figure 1. The flow bounded by each pair of

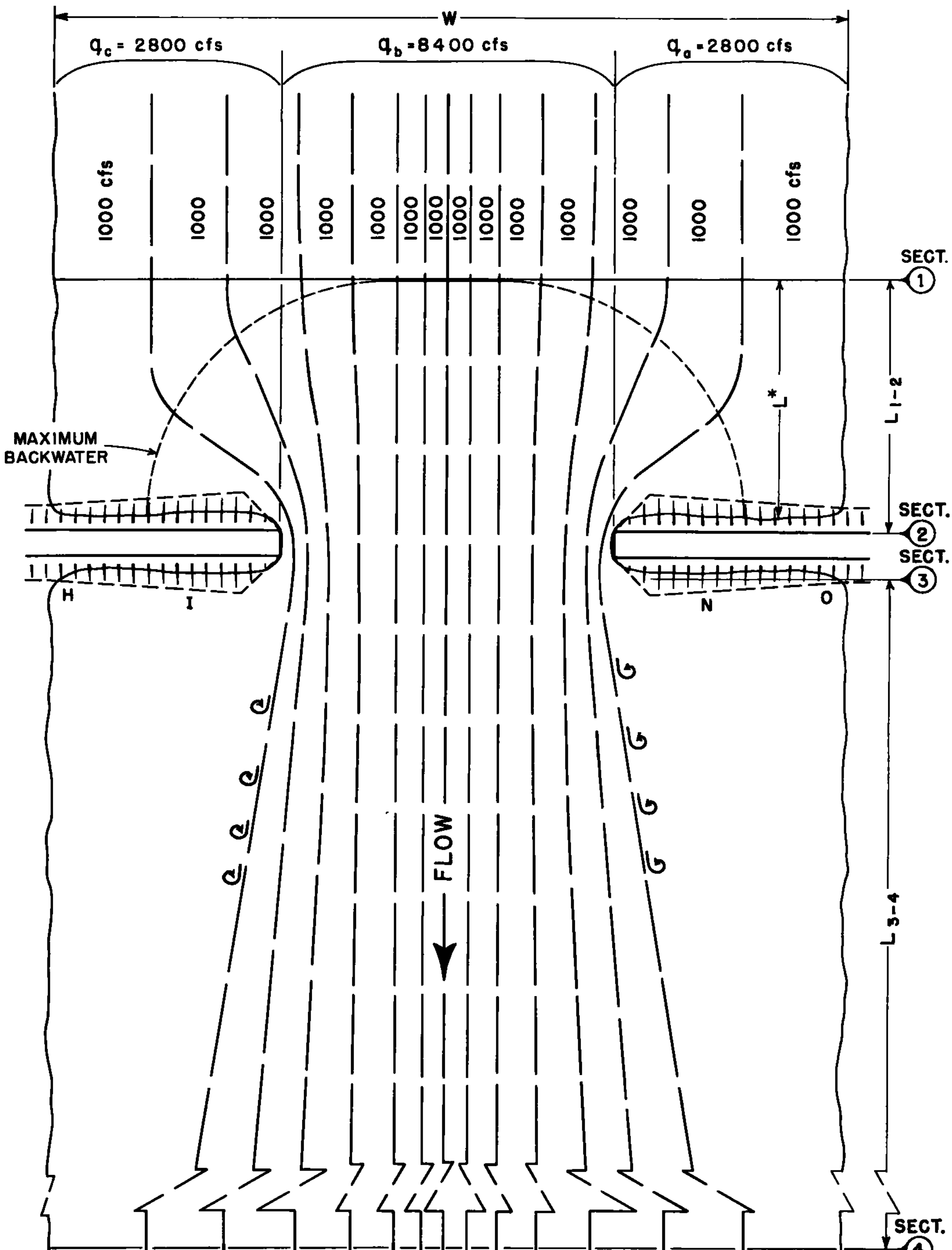

Figure 1. Flow lines— typical normal crossing.

streamlines represents 1,000 cfs. Note that channel constriction appears to produce very little alteration in the shape of the streamlines near the center of the channel, while a marked change is evident near the abutments where flow from the flood plains enter the constriction. As the discontinuity is greatest in this region, it is not difficult to visualize that areas adjacent to the abutments are most vulnerable to attack by scour during floods. Upon leaving the constriction the flow, which is now concentrated in the central portion of the channel, expands at an angle of 5 to 7 deg on a side until normal conditions are again re-established downstream, which may involve a considerable reach of the river.

Constricting the flow of a stream, of course, produces a loss of energy, the greater portion of this occurring in the re-expansion process downstream from the constriction. This loss of energy is reflected in a rise in both the water surface and the energy gradient upstream from the bridge, as demonstrated by a profile of this same crossing taken along the centerline of the stream (Fig. 2). The normal stage or water surface existing for a given flood, prior to construction of the bridge, is represented by a straight dash line labeled "normal stage." The water surface for the same flood, with constricting bridge embankments, is denoted by the solid line labeled "water surface on centerline." The water surface is now above normal stage at section 1, passes through normal stage in the vicinity of section 2, reaches minimum depth near section 3, and returns to normal stage a considerable distance downstream at section 4 where the original regime of the river has not been disturbed. The energy at section 4 is thus the same with or without the bridge. The energy at section 1, on the other hand, must increase to provide head to overcome the loss introduced by the constriction. The major portion of this energy is reflected in the backwater, which is the rise in water surface at section 1 (denoted by the symbol h_1^*) on Figure 2.

Note that the drop in water surface measured across the roadway embankment is not the backwater as is so often presupposed to be the case. The water surface as indicated in the central part of the channel at section 3, which is essentially the water surface along the downstream side of the embankments, is invariably lower than normal stage, so the difference in level across the embankments Δh, is always larger than the backwater h_1^*.

It was found that the backwater to be expected at a bridge for a given discharge is dependent on a number of factors, the more prominent of which are:

1. The degree of constriction of the channel;
2. The number, size, shape and orientation of piers in the constriction;
3. Eccentricity of the bridge with the low-water channel or flood plain;
4. The angle or skew of the bridge with the stream;

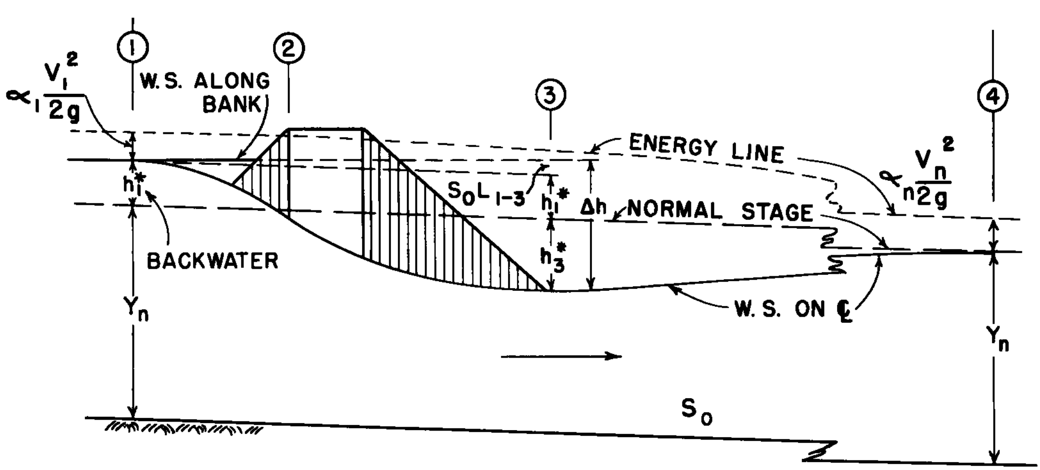

Figure 2. Profile on centerline of stream.

5. The type and slope of bridge abutments (important only for the shorter bridges);
6. The amount of scour experienced in the constriction; and
7. Whether the crossing consists of a single bridge or two or more parallel bridges on a divided highway.

Contrary to expectations, the width of the abutment or fill had no significant effect on the backwater.

It should be quite evident at this point that a backwater study can have but limited value without a reliable stage-discharge curve for the bridge site. Also a knowledge of the flood frequency and magnitude are required to intelligently determine the design discharge for a bridge and the necessary clearance (2).

In spite of the number of principal variables enumerated above, the backwater expression and the procedure for computing backwater, as developed from the experimental studies, are very much down to earth. A man with some training in hydraulics should have no particular difficulty in mastering this phase of waterway design.

An abbreviated form of the expression for computing bridge backwater is given here:

$$h_1^* = K^* \frac{V_{n2}^2}{2g} + (\ldots. \qquad (1)$$

in which K^* consists of a combination of experimental backwater coefficients multiplied by a velocity head. The coefficient K^* varies with the seven geometric factors mentioned above while the velocity is computed with respect to the average water cross-section under the bridge relative to normal stage. The remainder of the expression, which has been omitted for the sake of simplicity, consists of the change in kinetic energy between sections 1 and 4 (Fig. 2) produced by alteration of the stream by the bridge. In the majority of cases, this factor represents a small portion of the total backwater, but this is not always the case. Guides are provided whereby the importance of this factor can be readily recognized and omitted from the computations where permissible (2).

To give a general idea of the manner in which Eq. 1 operates, the backwater coefficient for a symmetrical normal stream crossing, having wing-wall abutments, but without piers or other complicating features, would be obtained directly from Figure 3. The coefficient K_b (known as the base curve value) varies with the degree of constriction of the channel M, and the type of abutment. The parameter M is the ratio of the quantity of flow which can pass through the constriction unimpeded to the total discharge of the river. For M = 1, there is no constriction of the stream, and the coefficient is zero. As the degree of constriction increases, M becomes smaller and the coefficient K_b increases in value. To illustrate, the contraction ratio for the condition shown on Figure 1 would be

$$M = \frac{8,400}{14,000} = 0.60.$$

Should piers, eccentricity, or skew be involved, the effect of these factors are accounted for by adding incremental coefficients to the value obtained from the base curve (Fig. 3), thus the over-all coefficient

$$K^* = K_{b\,(\text{base})} + \Delta K_{p\,(\text{piers})} + \Delta K_{e\,(\text{eccentricity})} + \Delta K_{s\,(\text{skew})}.$$

The value of the incremental coefficients for the effect of piers, eccentricity, skew, etc., are obtained from charts prepared for that purpose. For a detailed description of the procedure and the charts see (2). A general idea as to the magnitude of the individual coefficients can be gleaned from an inspection of the first eight columns of Table 2.

RELIABILITY OF MODEL RESULTS

A tremendous difference can exist between a model and a field structure insofar as bridges are concerned. Because of the model limitations, it was imperative that some means be devised to verify or disprove the validity of the experimental information. This was accomplished by applying the computational procedure, developed from the model studies, to existing bridges on which the Geological Survey had furnished field measurements obtained during floods. Reliable measurements on bridge backwater are extremely difficult to make in the field, but the drop in water surface across embankments Δh, is readily measurable (Fig. 2). Model results showed a very definite relation to exist between the drop in water surface across the embankments Δh, and the backwater h_1^*, so model computations and prototype measurements are compared on the basis of Δh. A comparison of measured and computed values for several bridges varying from 20 to 340 ft in length is presented in Table 1. Columns 2 through 6 give the bridge length, flood discharge, average velocity under the bridge, the contraction ratio, and the computed backwater, respectively. The computed and measured values of Δh are shown in Columns 7 and 8, respectively, while the percentage difference in each case is shown in Column 9. The differences range from -8.5 to +13 percent, the deviation being positive in six instances and negative for six; the average deviation is +2 percent. The deviation in the majority of the cases is well within the error of field measurement. The experimental error of the model experiments is estimated as comparable to the average deviation. Thus, the comparison affords satisfactory verification to date. Field measurements on longer bridges are needed but these have not been forthcoming as yet.

APPLICATION OF BACKWATER TO DESIGN

Now that it is possible to compute bridge backwater with a fair degree of confidence, to what practical purpose can this information be used in design?

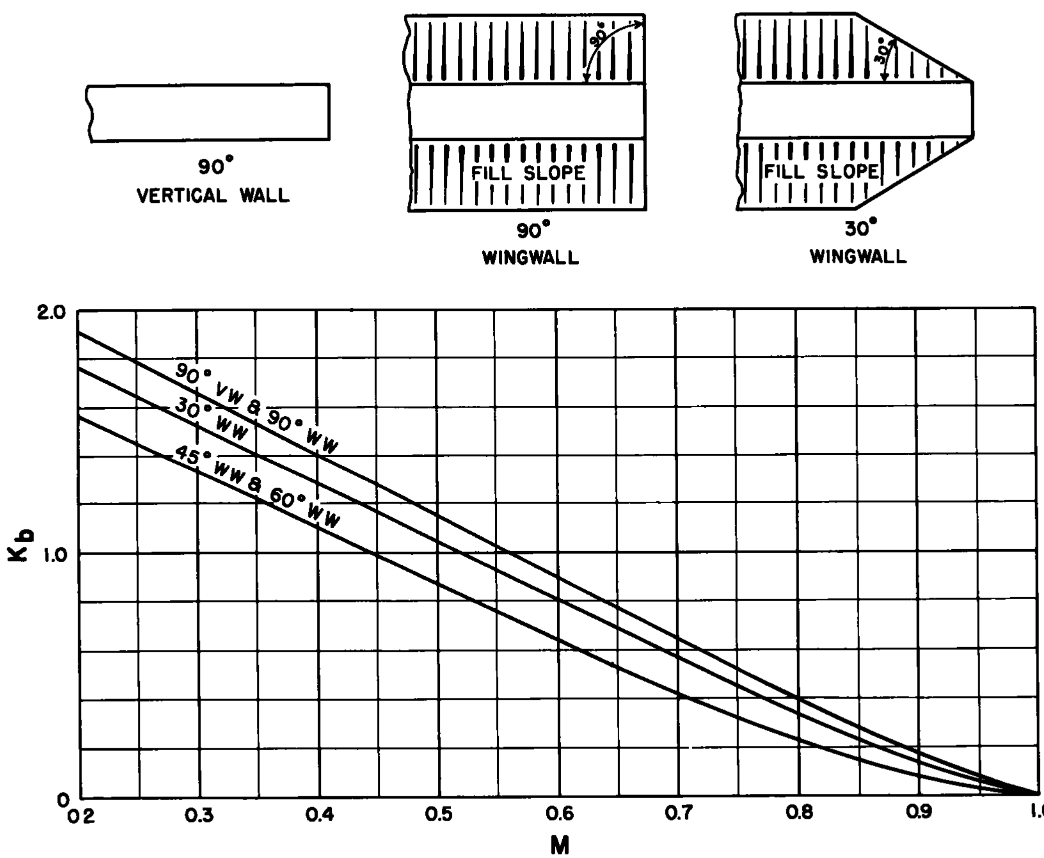

Figure 3. Backwater coefficient K_b for wing-wall abutments (base curve).

1. It makes it possible to proportion bridges to operate during flood flows at a limited specified backwater.
2. It offers a fair means of settling claims involved in backwater damage suits instigated by upstream property owners.
3. It makes it possible to understand and compute the hydraulics involved in cases where approach roadways can be overtopped during infrequent floods.
4. It provides a large share of the necessary hydraulic information for a proposed economic analysis to determine the optimum design discharge and the most economical length of bridge.

In the case of item 2, no reliable method has existed for computing backwater produced by bridges. Backwater based on field measurements made by the novice were also justifiably questionable. Thus, damage suits of this nature have resulted in indefinite delays or settlements have been made on considerations other than fact.

TABLE 1

COMPARISON OF COMPUTED Δh VALUES WITH FIELD MEASUREMENTS

Bridge No.	Length (ft)	Discharge (cfs)	Velocity Under Bridge (fps)	Contraction Ratio, M	Computed Backwater, h_1^* (ft)	Drop Across Embankments, Δh (ft) Computed	Measured	% Diff. Δh
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	20	1,370	9.1	0.57	1.07	1.90	1.99	- 4.5
2	84	4,340	6.8	0.85	0.21	0.65	0.70	- 7.2
3	220	27,500	7.5	0.90	0.28	0.76	0.83	- 8.5
4	83	5,240	8.6	0.60	1.03	1.81	1.60	+13
5	72	12,000	10.2	0.83	0.57	1.94	1.95	- 0.5
6	58	3,400	7.1	0.82	0.18	0.61	0.55	+10.9
7a	44	2,620	7.8	0.66	0.63	1.23	1.15	+ 6.9
7b	44	1,450	5.4	0.70	0.30	0.66	0.69	- 4.4
8	112	9,640	9.0	0.33	1.80	2.53	2.24	+12.9
9	340	70,000	10.5	0.90	0.77	2.57	2.70	- 5.0
10	68	7,230	Deck girder immersed			1.53	1.48	+ 3.4
11	120	2,600	Deck girder immersed			1.70	1.61	+ 5.6

TABLE 2

COMPARISON OF LENGTH AND COST OF SKEW BRIDGES WITH NORMAL CROSSINGS

Bridge	Skew Angle (deg)	Contraction Ratio, M	Backwater Coefficients Base K_b	Piers ΔK_p	Ecc. ΔK_e	Skew ΔK_s	Total K^*	$\frac{V_{n2}^2}{2g}$ (ft)	Backwater h_1^* (ft)	Projected Bridge Length	$\frac{L_s \cos \phi}{L_n}$	$\frac{L_s}{L_n}$	$\frac{C_s}{C_n}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
A	0	0.90	0.12	0.09	0.07	0	0.28	1.70	0.76	340	1.0	1.0	1.0
	30	0.90	0 12	0.09	0.07	-0.02	0.26	1.84	0.76	335	0.98	1.14	1.22
	45	0.90	0.12	0.09	0.07	-0.03	0.25	1.89	0.76	330	0.97	1.37	1.54
B	0	0.67	0.48	0.04	0.15	0	0.67	1.22	0.89	2,000	1.0	1.0	1.0
	30	0.665	0.50	0.05	0.15	-0.05	0.65	1.26	0.89	1,925	0.96	1.12	1.18
	45	0.66	0.51	0.06	0.15	-0.08	0.64	1.30	0.89	1,900	0.95	1.34	1.50
C	0	0.64	0.55	0.19		0	0.74	2.66	2.18	87	1.0	1.0	1.0
	30	0.635	0.55	0.19		-0.06	0.68	2.84	2.18	84	0.96	1.12	1.22
	45	0.63	0.56	0.19		-0.09	0.66	2.90	2.18	80	0.92	1.30	1.50
D	0	0.62	0.60	0.03	0.16	0	0.79	1.65	1.41	1,100	1.0	1.0	1.0
	30	0.62	0.60	0.04	0.16	-0.07	0.73	1.79	1.41	1,025	0.93	1.08	1.15
	45	0.61	0.62	0.05	0.16	-0.11	0.72	1.82	1.41	1,010	0.92	1.30	1.45
E	0	0.53	0.92	0.08		0	1.00	1.11	1.19	630	1.0	1.0	1.0
	30	0.52	0.96	0.09		-0.19	0.86	1.27	1.19	600	0.96	1.10	1.16
	45	0.51	1.00	0.12		-0.37	0.75	1.46	1.19	575	0.91	1.29	1.42
F	0	0.46	1.13	0.15	0.04	0	1.32	0.67	0.93	1,075	1.0	1.0	1.0
	30	0.43	1.16	0 16	0.04	-0.26	1.10	0.82	0.93	990	0.92	1.06	1.08
	45	0.42	1.21	0.19	0.04	-0.49	0.95	0.90	0.93	925	0.86	1.22	1.31
G	0	0.46	1.06	0.06		0	1.12	0.90	1.05	75	1.0	1.0	1.0
	30	0.44	1.08	0.08		-0.25	0.91	1.09	1.05	69	0.92	1.06	1.12
	45	0.42	1.14	0.09		-0.48	0.75	1.35	1.05	64	0.86	1.20	1.34

The attainment of a sound method of procedure for determination of the optimum design discharge and the most economical length of bridge (item 4) constitutes the ultimate goal in the present research program.

APPLICATION OF BACKWATER TO LENGTH OF SKEW CROSSING

A practical application to which the bridge backwater information may be used to advantage can be demonstrated in comparing the length and cost of skew bridges with the length and cost of equivalent normal crossings, on the basis of backwater. The procedure consists of first choosing an existing normal stream crossing and computing the backwater which the bridge will produce for a given flood condition; then, holding stream conditions the same, computing the length of equivalent skew bridges which have the same effective waterway, or in other words, produce the same backwater. This course of computation was followed through for seven existing crossings and the results are given in Table 2. The normal length of these bridges varied from 75 to 2,000 ft and included both wing-wall and spill-through type abutments. The faces of the abutments under the bridge were oriented parallel with the flow, as shown in the sketch in Figure 4. This is the most efficient skew abutment shape. Types with faces at an angle to the flow require more length of bridge.

The ordinate in Figure 4 is the ratio of skew length to normal length of crossing in percent, which is plotted with respect to the skew angle as abscissa and the contraction ratio M as a third variable. In the case of M = 1.0 (no constriction of the stream) the skew length is simply $L_n/\cos \phi$. With constriction of the stream, the ratio L_s/L_n reduces with the value of M.

What is occurring can be better understood by referring to Figure 5. On this chart the ordinate is the ratio of the projected skew length to the normal length (see sketch) while the other two parameters remained unchanged. For M = 1.0, no constriction, the ordinate is 1.0 for all angles of skew. With constriction of the stream, the projected skew length, required to produce the same amount of backwater, is shorted than the normal bridge length. This characteristic is to be expected but the actual relationship has been until now entirely a matter of conjecture. The curves in Figures 4 and 5 offer actual values which may prove useful in design.

A plot relating the cost ratio in percent of skew to normal crossings for the same

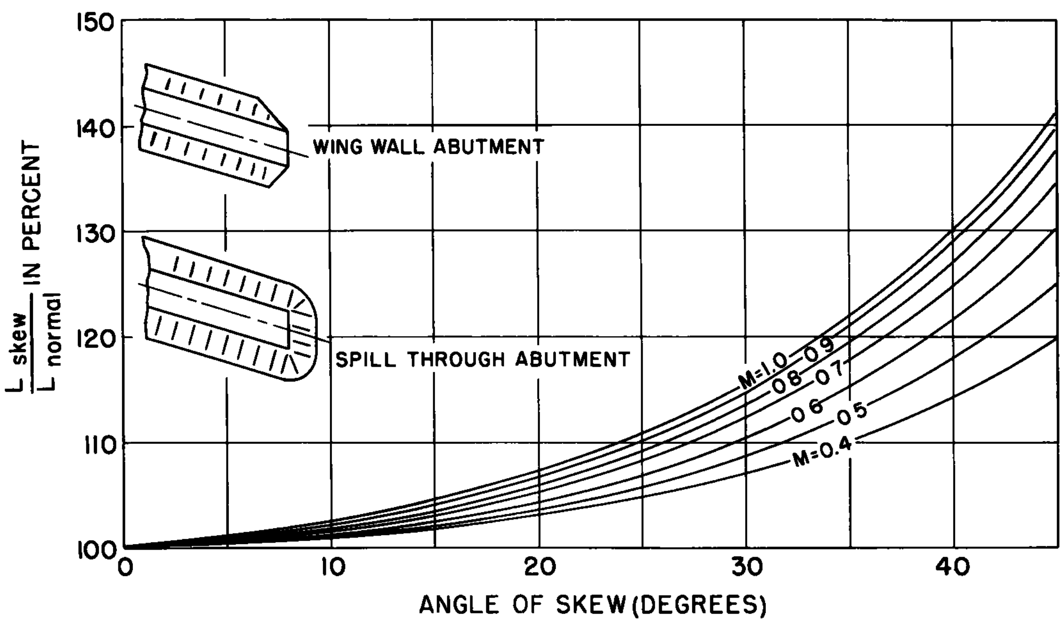

Figure 4. Length ratio of skew to normal crossing for equivalent backwater.

bridges, is presented in Figure 6. Again the parameters are the same except for the ordinate. The consistency is not of the same order found in the length curves since it was necessary to adjust span lengths and provide additional piers for the skew bridges.

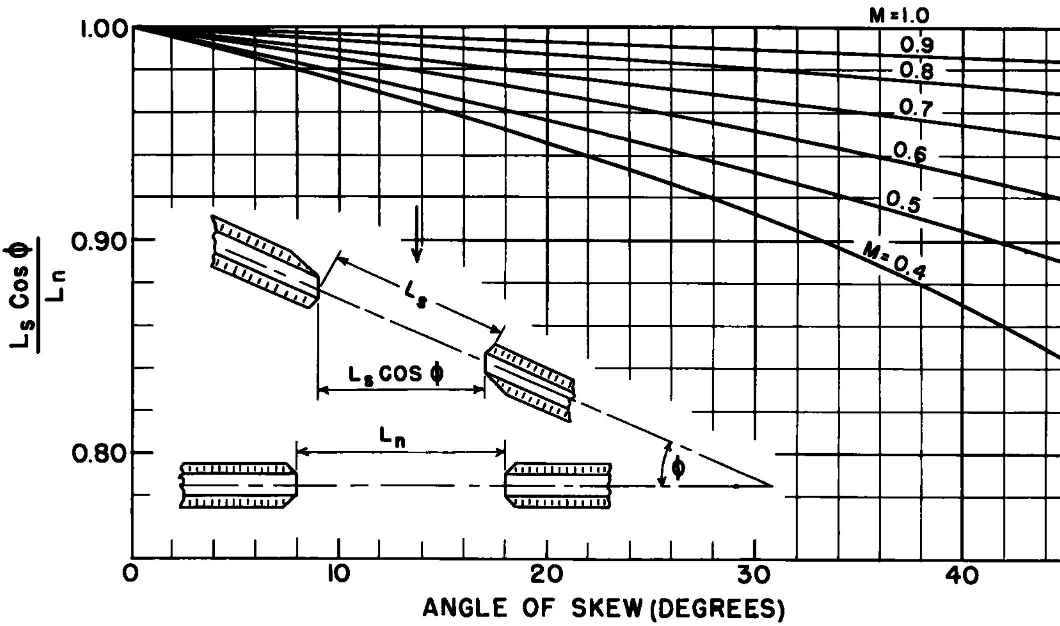

Figure 5. Ratio of projected skew length to normal bridge length for equivalent backwater.

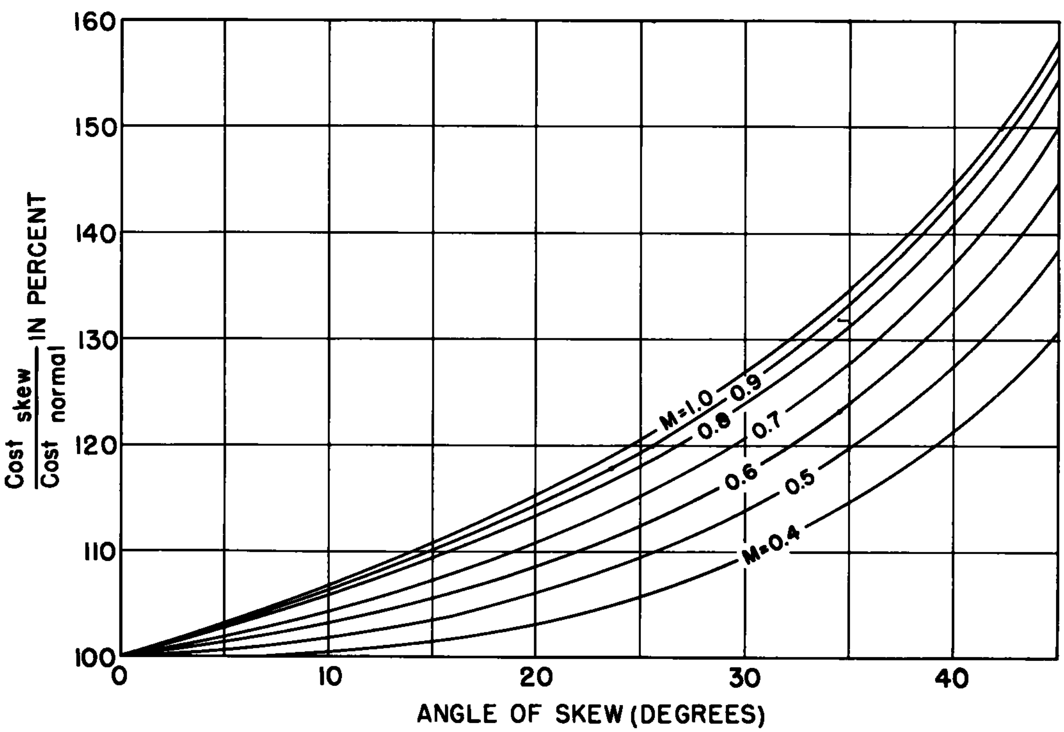

Figure 6. Cost ratio of skew to normal crossing for equivalent backwater.

Also the higher unit cost of skew construction and the increased length of embankments were considered. The cost was affected to a greater extent than the length by these factors. The criterion for determining span lengths for the skew crossing consisted of balancing the cost of superstructure against the cost of piers on an equal basis. The increase in cost of superstructure per square foot was assumed at 5 percent for the 30-deg skew and 10 percent for the 45-deg skew.

For the purpose of comparison, the length varies from 107 to 115 percent or normal for the 30-deg skew while the cost varies from 110 to 127 percent for the same range of contraction ratios. In the case of the 45-deg skew, the length varies from 120 to 141 percent of normal compared with a cost variation of 130 to 158 percent for the same values of M.

As can be observed, skew angles up to approximately 20 deg produce only a small increase in both length and cost over the normal crossing. As the angle increases above this value, the curves steepen and the length and cost rise rapidly. In this same connection, it was observed in the course of the model studies that the hydraulic flow problems encountered with skew crossings, for angles up to approximately 20 deg, were little different than for normal crossings. As the angle exceeds this value, the flow and scour problems increase in complexity.

LIMITATION OF BACKWATER AND ACCOMMODATION OF SUPER FLOODS

Another application in which the backwater design information can be used to advantage is for the case where approach roadways can be depressed to protect the bridge during floods of extreme proportions. Although it is seldom economically feasible to construct a bridge sufficiently long to accommodate the super type of flood, it is possible in many cases to design for a 35- or 50-yr flood but make provision to pass flows of much greater magnitude with little or no damage to the bridge proper and, at the same time, keep the backwater within specified limits. The most effective way to present this case is by an actual illustration.

The stream at a proposed crossing has a low-water channel about 700 ft across while in flood the stream may be a mile wide. Records show that within the past 50 yr, two floods approximating frequencies of 100 yr have occurred on this stream, the last one destroying a bridge at the site. This is on a state route carrying a fairly heavy volume of traffic which will increase with time. A considerable amount of residential and business development, occupying portions of the flood plain, have sprung up within the last decade. It is therefore important from the traffic viewpoint that the bridge proper not fail or be out of service for an extended length of time during its expected life; and from the standpoint of life and property damage, it is desirable that the bridge backwater be limited to a definite figure for all flows. For the purpose of illustration, the bridge will be reconstructed to satisfy the above requirements and limit the backwater to 0.5 ft for any discharge likely to occur during the life of the bridge.

There is a choice here of designing a long bridge to take the full flow of the river for, say, a 100-yr frequency flood, keeping the embankments above high water at all times, or the alternative of choosing a shorter bridge and using the ½ to ¾ mile of roadway transversing the flood plain as a spillway during high water. In either case the superstructure will be located above extreme high water at all times.

The case where the embankments are located above high water and the bridge is required to accommodate the entire flow will be first investigated. The chart on Figure 7 shows the backwater relative to length of bridge and discharge for this type of operation. In addition, scales have been superimposed showing flow recurrence interval at top and cost of bridge at right. Were there no restriction on backwater, a bridge 1,500 ft long, producing 1.5 ft of backwater, might be a reasonable choice. But with backwater limited to 0.5 ft, it can be observed that the bridge should be 2,250 ft long for a 50-yr flood or 2,600 ft long for a 100-yr flood. From the scale on the right, the cost involved in reducing the backwater from 1.5 to 0.5 ft approximates $400,000 in this case or about 40 percent of the initial cost. This comparison demonstrates how limitation of backwater can increase the initial cost.

How can limitation of backwater be accomplished less painfully? The alternative, the depressed roadway, will now be examined. Figure 8 demonstrates a very extreme case; the bridge has been shortened to 800 ft with approximately 3,500 ft of depressed roadway. The lower broken line labeled "normal stage" represents the stage-discharge

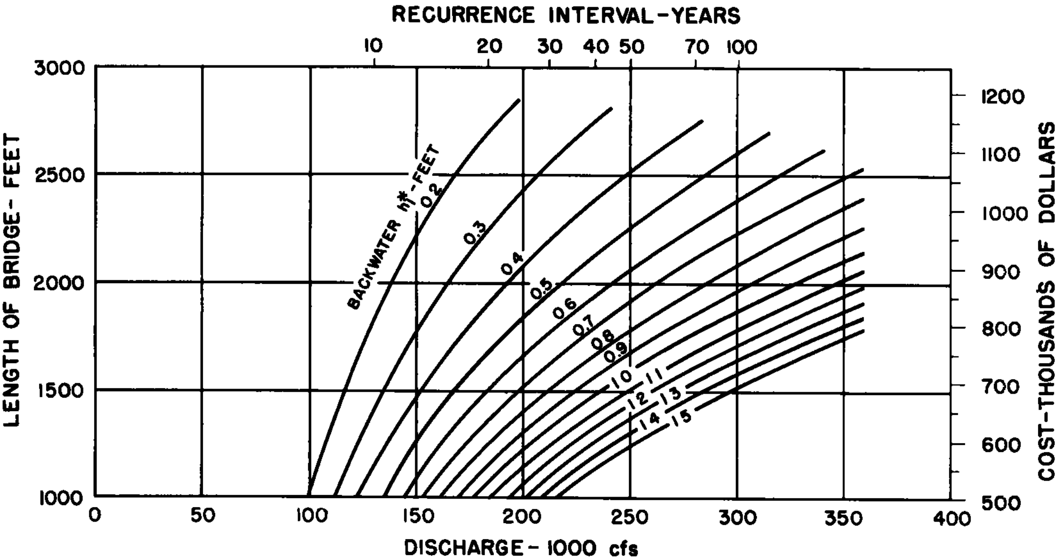

Figure 7. Variation of backwater with length of bridge and discharge.

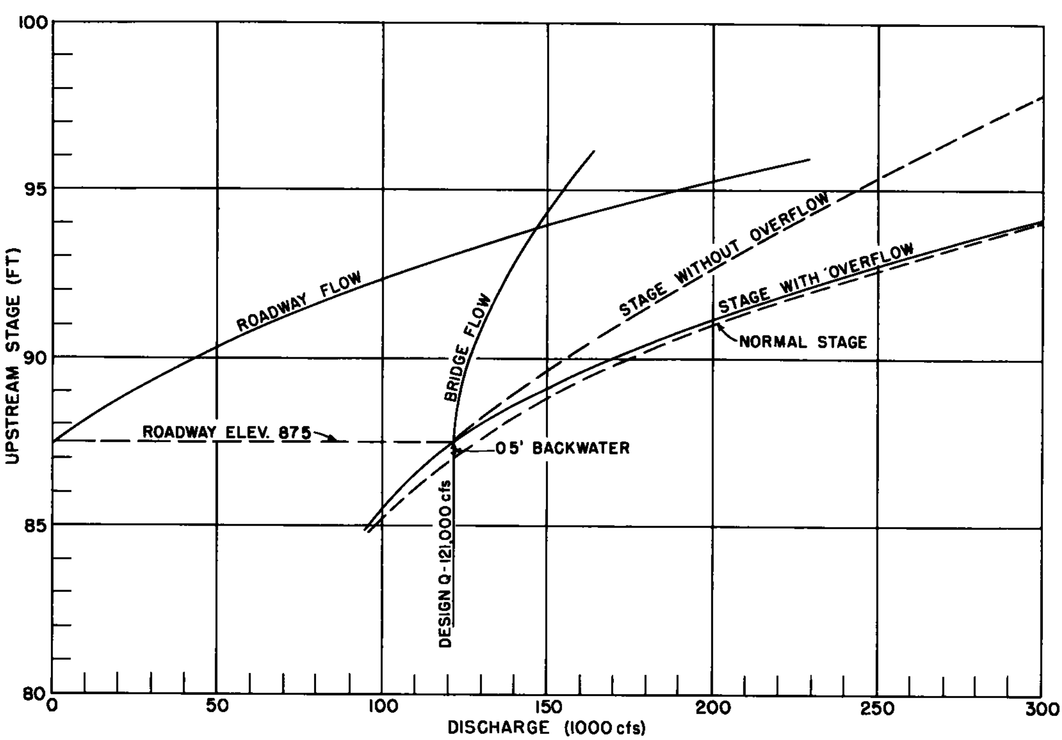

Figure 8. Operation with depressed roadway and 800-ft bridge—backwater limited to 0.5 ft.

curve for the river prior to construction of the bridge. The upper dotted line labeled "stage without overflow" represents the stage discharge to be expected upstream from the 800-ft bridge without overflow. The difference between the two curves represents the backwater. Note that for a discharge of 250,000 cfs (50-yr flood) the backwater is 2.5 ft and for 300,000 cfs (100-yr frequency flood) the backwater approximates 4 ft. The limitation of 0.5 ft for backwater is reached at a discharge of 121,000 cfs. If the approach embankment is placed at elevation 87.5 so water will spill over the roadway for flows greater than 121,000 cfs, the backwater will decrease with further increase in discharge, falling off to about 0.1 ft for a discharge of 300,000 cfs. The backwater with overflow is represented by the difference between the lines labeled "normal stage" and "stage with overflow." The flow under the bridge and flow over the roadway are indicated by the lines so labeled. Note that as the roadway overflows, the discharge under the bridge now increases slowly with upstream stage, while flow over the roadway increases rapidly with stage. At stage 93.8, the roadway is carrying as much flow as the bridge.

As the roadway is elevated, the backwater and flow characteristics remain similar to those shown on Figure 8 but the bridge length must be increased if the backwater is to be limited to 0.5 ft for upstream stage level with the new roadway. Figure 9 demon-

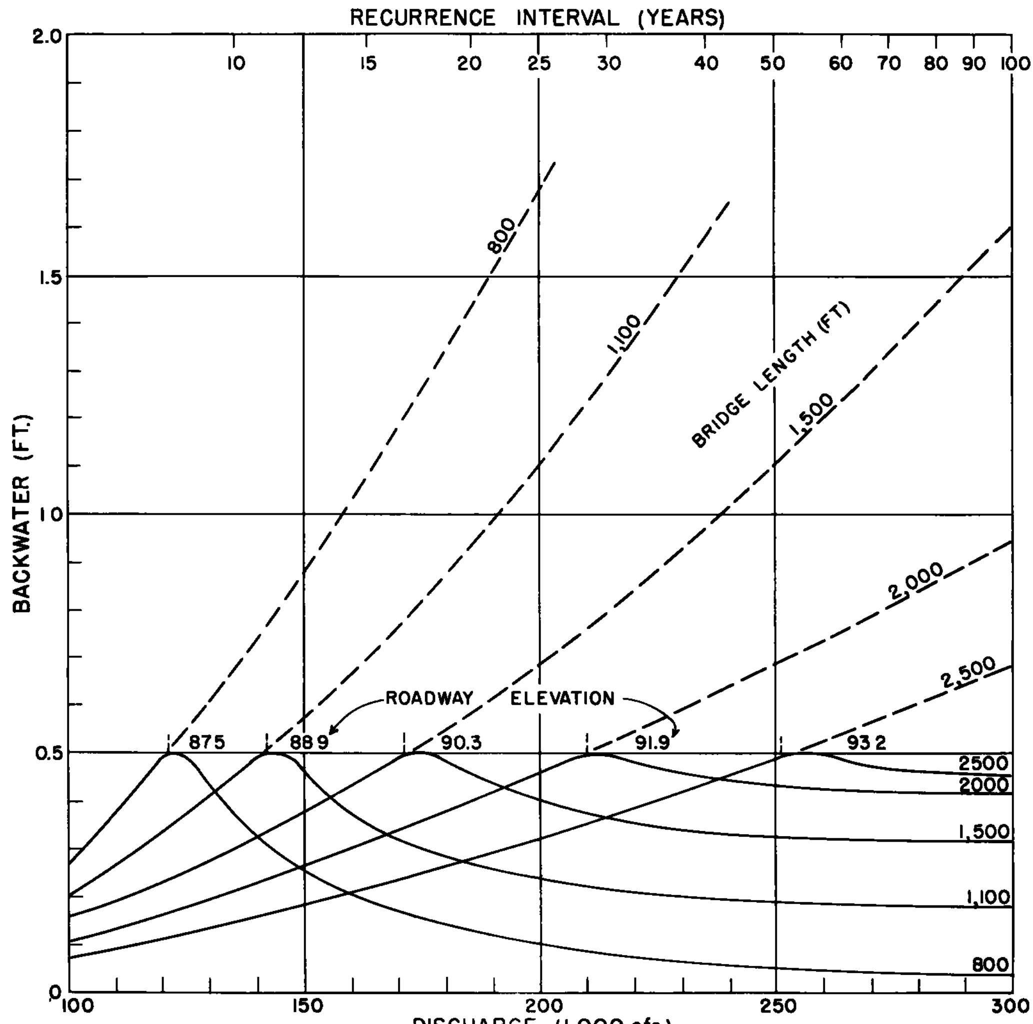

Figure 9. Operation with depressed roadway for several lengths of bridge—backwater limited to 0.5 ft.

strates how the backwater varies with roadway elevation and length of bridge. The dash lines denote the backwater which could be expected for several bridge lengths were flow over the road not permitted. The solid lines demonstrate how flow over the roadway limits the backwater to a maximum of 0.5 ft regardless of the discharge.

The depressed roadway not only serves to hold the backwater within limits but offers a means of accommodating the superflood without undue overloading of the bridge proper. It is true that the higher the approaches and the shorter the length of embankment, the longer the bridge must be for a given amount of backwater; nevertheless, it is usually possible to set embankments for the 50-yr flood stage and still retain the safety valve feature.

CONCLUSIONS

The above illustrations represent only a few ways in which the recently acquired bridge backwater information can be applied to waterway design. Gaps still remain which eventually will be plugged as reliable field data become available.

An equally important, or second phase, involving the hydraulics of waterways is the reasonable prediction of maximum scour depths at abutments and piers. Some information is already available for streams with alluvial beds (3, 4) and additional information will be forthcoming.

It will be found that the hydraulic analysis offers many variations of supposedly equally good waterway proportions. How, then, is an unbiased choice to be made? It is believed that this can best be accomplished through development of a generally acceptable type of economic analysis, which at the present does not exist, taking into account all tangible and certain intangible costs which may be incurred by the highway agencies in building and maintaining a bridge and by the highway users who travel over the bridge. In this way, a fair monetary value can be assigned to each design, whereby comparisons can be made on a basis familiar to all parties concerned. Determination of the fundamental concepts on which such an economic analysis should be founded constitutes the third major phase of this research program, which is now under consideration.

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