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Field Applications of Soil Consolidation: Time-Dependent Loading and Varying Permeability

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Field Applications of Soil Consolidation: Time-Dependent Loading and Varying Permeability

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Field Applications of Soil Consolidation Under Time-Dependent Loading and Varying Permeability

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In 1958 an extension to the Terzaghi theory of consolidation of fine-grained soil was presented (1). This new theory established the basis for considering variable loading during the consolidation process as an external condition. In addition the theory included mathematical procedures for analyzing the permeability variation of the soil during the process of consolidation.

The present paper presents computed results of the extended consolidation theory, to enable soil engineers to use these theories in practice.

Certain specific problems of consolidation are covered in this study, as follows:

1. One-dimensional consolidation of a doubly drained layer of clay. The soil mass is loaded with a construction loading that is linear in time until the end of construction and is constant thereafter. Tables and charts are presented for the value of the excess pore pressure within the clay mass as a function of time and space position of the piezometer installation.
2. Radial flow to a sand-drain for the time-independent loading but variable permeability. Tables and charts are presented for the exact solution to the equal strain, radial sand-drain problem. In this problem, the drain is considered to be a perfect drain, with no smear zone. The tabulations presented are for the average excess pore pressure as a function of time.
3. The effect of variable load for equal-strain sand-drains under radial flow is presented in the form of tables and charts of average excess pore pressure for conditions of peripheral smear in the case of constant permeability.
4. The approximate theory of consolidation is presented for sand-drains for equal strain in the case of construction loading and variable permeability. This case presents tabular and charted values for the average excess pore pressure where no smear zone is considered.

In addition to these specific problems, general procedures are presented for the interpolation of the tabular and charted values, so that accurate pore pressure predictions can be made for any piezometer position and any time.

THEORY OF CONSOLIDATION

Theory

A generalized theory of consolidation considering a time-dependent loading and variable permeability has been presented in a previous paper (1).

The working conditions upon which the theory is based are as follows:

1. The soil mass is completely saturated with an incompressible fluid, and is made up of incompressible small particles.
2. Darcy's law of permeability is instantaneously valid. The coefficient of permeability, as measured along a velocity path is a scalar point function of time and space.

$$\vec{v} = k(\nabla h) \tag{1}$$

in which

$\vec{v}$ = vector velocity function;
k = coefficient of permeability; and
h = total fluid head.

3. The change in volume with imposed pressure is linear and small as compared to the original volume.

The resulting differential equation for consolidation in which there is an internal pressure generation is

$$\nabla k \cdot \nabla u + k\nabla^2 u + Q\gamma_w = m\gamma_w \frac{\delta u}{\delta t} \tag{2}$$

in which

u = excess pore pressure;
Q = rate of head generation;
γ_w = unit weight of water; and
m = modulus of volume change.

Computational Theorem

There are many cases of practical interest when the permeability of a soil mass can be regarded as constant throughout the process of consolidation. If the rate of surface loading (and thus the rate of pore pressure development) is further restricted to space functions only, Eq. 2 reduces to

$$C\nabla^2 u + R(P) = \frac{\delta u}{\delta t} \tag{3}$$

in which

R = rate of imposition of surface load; and
C = coefficient of consolidation.

As shown in Figure 1, Eq. 3 refers to the behavior of the excess pore pressure, u, as a function of time, t, and the generalized space variable, P. To completely specify a solution to this equation a set of conditions on the boundary P' must be designated, together with a set of initial conditions. In general these conditions will be

$$\frac{\delta^n u}{\delta P^n}(P', t) = F(P', t) \qquad 0 \leq t \leq \infty \tag{4a}$$

$$u(P, 0) = G(P) \qquad 0 \leq P \leq P' \tag{4b}$$

Eq. 4a specifies the nth space derivative of excess pore pressure on the boundary P'. If the consolidating mass were bounded by free-draining soils such as sands, the pore pressure (for example, the zero derivative of u) would vanish. For impermeable boundaries the first derivative of the pore pressure, at the boundary, would vanish. Eqs. 4a and 4b are set up to consider any general boundary and initial conditions that may apply.

The computational theorem used here is based on a similar theorem proposed by Awberry (2). This theorem breaks up the solution to Eq. 3 into two separate solutions, as follows:

$$u(P, t) = u_1(P) + u_2(P, t) \tag{5}$$

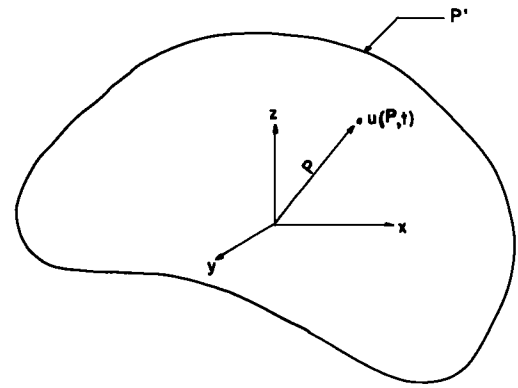

Figure 1. Generalized consolidating porous mass.

In addition to Eq. 5, it can be further specified that $u_1(P)$ will be the solution to the following Kelvin type of differential equation with arbitrary boundary conditions:

$$C\nabla^2 u_1 + R(P) = 0 \tag{6a}$$

$$\frac{\delta^n u_1}{\delta P^n}(P') = H(P') \tag{6b}$$

The boundary function, H, in Eq. 6b is completely arbitrary and can be chosen in such a way as to fit the ease of the computer.

Given Eqs. 5, 6a, and 6b, a substitution in Eq. 3 will result in the necessary differential equation which $u_2(P, t)$ must satisfy. This is

$$C\nabla^2 u_2 = \frac{\delta u_2}{\delta t} \tag{7}$$

By substituting the prescribed boundary and initial conditions (Eqs. 4a and 4b) in Eq. 5, along with the arbitrary boundary condition (Eq. 6b), the necessary boundary and initial conditions associated with $u_2(P, t)$ become

$$\frac{\delta^n u_2}{\delta P^n}(P', t) = F(P', t) - H(P') \qquad 0 \le t \le \infty \tag{8a}$$

$$u_2(P, 0) = G(P) - u_1(P) \qquad 0 \le P \le P' \tag{8b}$$

Thus, a transient consolidation problem with a forcing function has been broken down into a linear combination of a steady-state problem and an unforced transient problem. This separation is particularly convenient because there are many computed solutions to Eq. 7 both in the soil mechanics literature and the literature on heat transfer (3).

ONE-DIMENSIONAL CONSOLIDATION

General Equations

Classically, the one-dimensional consolidation case considers a clay stratum bounded by layers of sand, as shown in Figure 2. For true one-dimensional compression to take place the imposed surface loading must be infinite in lateral extent, as would occur for a blanket fill. In terms of the coordinate system and loading shown in Figure 2, the governing differential equation and boundary conditions for time-dependent loading and constant permeability are

$$C\frac{\delta^2 u}{\delta z^2} + R(z, t) = \frac{\delta u}{\delta t} \tag{9a}$$

$$u(0, t) = 0 \qquad 0 \le t \le \infty \tag{9b}$$

$$u(2H, 0) = 0 \qquad 0 \le t \le \infty \tag{9c}$$

$$u(z, 0) = \sigma(z) \qquad 0 \le z \le 2H \tag{9d}$$

The general solution to Eqs. 9a, 9b, 9c, and 9d is

$$u(z, t) = \frac{1}{H}\sum_{n=1}^{\infty} \operatorname{Sin}\frac{n\pi}{2H} z \left\{\int_0^{t'} \left[\int_0^{2H} R(z, t) \operatorname{Sin}\frac{n\pi}{2H} z\, dz\right] e^{-(Cn^2\pi^2/4H^2)(t-\tau)}\, d\tau\right\}$$

$$+ \frac{1}{H}\sum_{n=1}^{\infty} \left[\int_0^{2H} \sigma(z) \operatorname{Sin}\frac{n\pi}{2H} z\, dz\right] \operatorname{Sin}\frac{n\pi}{2H} z\, e^{-(Cn^2\pi^2/4H^2)t} \tag{10}$$

The solution as presented in Eq. 10 is readily computable, once the functions R and σ are known. It is not, however, in the form for efficient computation. To develop the most efficient computational scheme the previously developed computational theorem will be used.

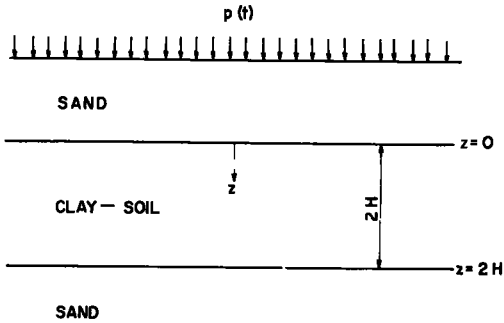

Figure 2. Double-drainage clay layer; one dimension.

Constant Load

The classical theory of consolidation, as formulated by Terzaghi (4), is most widely applied for a case of constant initial excess pore pressure. The formulation of this problem is as follows:

$$C\frac{\delta^2 u}{\delta z^2} = \frac{\delta u}{\delta t} \qquad (11a)$$

$$u(0,\ t) = 0 \qquad 0 \leq t \leq \infty \qquad (11b)$$

$$u(2H,\ t) = 0 \qquad 0 \leq t \leq \infty \qquad (11c)$$

$$u(z,\ 0) = u_0 \qquad 0 \leq z \leq 2H \qquad (11d)$$

The solution to the differential equation and boundary and initial conditions, as expressed by Eqs. 11a, 11b, 11c, and 11d is

$$u(z,\ T) = \frac{4\ u_0}{\pi} \sum_{n\ =\ 1;\ 3,\ 5,}^{\infty} \frac{1}{n} \operatorname{Sin}\frac{n\pi}{2}\ \frac{z}{H}\ e^{-(n^2\pi^2/4)T} \qquad (12a)$$

$$T = Ct/H^2 \qquad (12b)$$

in which T is the time factor.

The constant-load case as expressed in Eq. 12a is a special case of the variable-load problem involved in later computation. As a result, detailed theoretical point pore pressure values were developed as a matter of course. These computations were performed on an IBM 650 computer. Values of (u/u_0) for various values of the dimen-

TABLE 1

POINT PORE PRESSURE RATIOS (u/u_0) FOR CONSTANT LOAD AND CONSTANT PERMEABILITY; ONE-DIMENSIONAL CONSOLIDATION

T \ z/H	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.001	0.73645	0.97465	0.99999	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.0015	0.63869	0.93211	0.99974	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.002	0.57080	0.88615	0.99843	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.003	0.48139	0.80329	0.99018	0.99989	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.004	0.42384	0.73645	0.97465	0.99920	0.99999	1.0	1.0	1.0	1.0	1.0	1.0
0.005	0.38292	0.68269	0.95450	0.99730	0.99994	1.0	1.0	1.0	1.0	1.0	1.0
0.006	0.35192	0.63869	0.93211	0.99383	0.99974	0.99999	1.0	1.0	1.0	1.0	1.0
0.007	0.32740	0.60198	0.90903	0.98877	0.99928	0.99998	1.0	1.0	1.0	1.0	1.0
0.008	0.30737	0.57080	0.88615	0.98229	0.99843	0.99992	1.0	1.0	1.0	1.0	1.0
0.009	0.29061	0.54394	0.86396	0.97465	0.99713	0.99981	0.99999	1.0	1.0	1.0	1.0
0.01	0.27633	0.52050	0.84270	0.96611	0.99532	0.99959	0.99998	1.0	1.0	1.0	1.0
0.015	0.22717	0.43630	0.75179	0.91674	0.97908	0.99611	0.99947	0.99995	1.0	1.0	1.0
0.02	0.19741	0.38292	0.68269	0 86639	0.95450	0.98758	0.99730	0.99953	0.99994	0.99999	1.0
0.03	0.16174	0.31691	0.58578	0.77933	0.89753	0.95877	0.98569	0.99573	0.99891	0.99975	0.99991
0.04	0.14032	0.27633	0.52050	0.71116	0.84270	0.92290	0.96610	0.98667	0.99530	0.99844	0.99919
0.05	0.12563	0.24817	0.47291	0.65722	0.79410	0.88615	0.94221	0.97310	0.98844	0.99507	0.99687
0.06	0.11477	0.22717	0.43630	0.61352	0.75178	0.85107	0.91668	0.95651	0.97855	0.98913	0.99222
0.07	0.10631	0.21073	0.40702	0.57732	0.71493	0.81849	0.89101	0.93812	0.96615	0.98056	0.98495
0.08	0.09948	0.19741	0.38292	0.54672	0.68263	0.78852	0.86592	0.91873	0.95180	0.96959	0.97516
0.09	0.09381	0.18633	0.36263	0.52044	0.65406	0.76100	0.84173	0.89886	0.93598	0.95658	0.96316
0.1	0.08901	0.17692	0.34522	0.49752	0.62856	0.73565	0.81854	0.87882	0.91907	0.94192	0.94931
0.15	0.07255	0.14447	0.28404	0.41423	0.53132	0.63252	0.71609	0.78114	0.82741	0.85504	0.86422
0.2	0.06215	0.12387	0.24425	0.35783	0.46165	0.55318	0.63040	0.69181	0.73633	0.76329	0.77231
0.3	0.04778	0.09526	0.18812	0.27627	0.35751	0.42984	0.49152	0.54106	0.57730	0.59939	0.60680
0.4	0.03725	0.07426	0.14669	0.21550	0.27899	0.33560	0.38393	0.42281	0.45129	0.46865	0.47449
0.5	0.02909	0.05801	0.11458	0.16834	0.21795	0.26219	0.29997	0.33037	0.35263	0.36621	0.37078
0.6	0.02273	0.04532	0.08953	0.13153	0.17029	0.20486	0.23438	0.25813	0.27553	0.28614	0.28971
0.7	0.01776	0.03541	0.06995	0.10277	0.13305	0.16006	0.18313	0.20169	0 21528	0.22358	0.22636
0.8	0.01388	0.02767	0.05465	0.08030	0.10396	0.12506	0.14309	0.15759	0.16821	0.17469	0.17687
0.9	0.01084	0.02162	0.04270	0.06274	0.08123	0.09772	0.11180	0.12313	0.13143	0.13649	0.13819
1.0	0.00847	0.01689	0.03337	0.04902	0.06347	0.07635	0.08736	0.09621	0.10269	0.10665	0.10798
1.5	0.00247	0.00492	0.00972	0.01428	0.01848	0.02223	0.02544	0.02802	0.02991	0.03106	0.03144
2.0	0.00072	0.00143	0.00283	0.00416	0.00538	0.00647	0.00741	0.00816	0.00871	0.00904	0.00916
3.0	0.00006	0.00012	0.00024	0.00035	0.00046	0.00055	0.00063	0.00069	0.00074	0.00077	0.00078
4.0	0.00001	0.00001	0.00002	0.00003	0.00004	0.00005	0.00005	0.00006	0.00006	0.00007	0.00007
5.0	0	0	0	0	0	0	0	0	0.00001	0.00001	0.00001
6.0	0	0	0	0	0	0	0	0	0	0	0

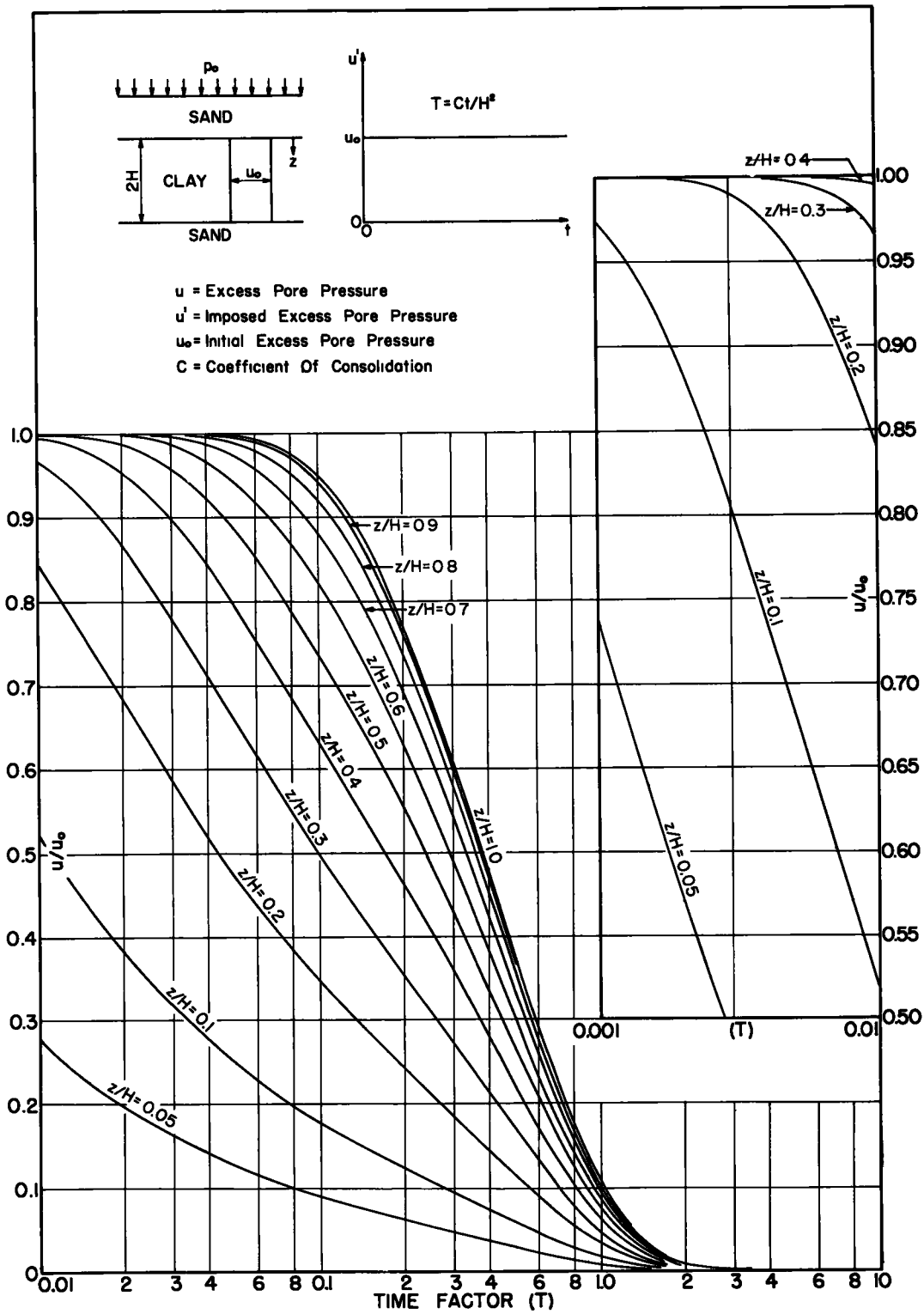

Figure 3. Point pore pressures for one-dimensional consolidation and constant load (depth parameter).

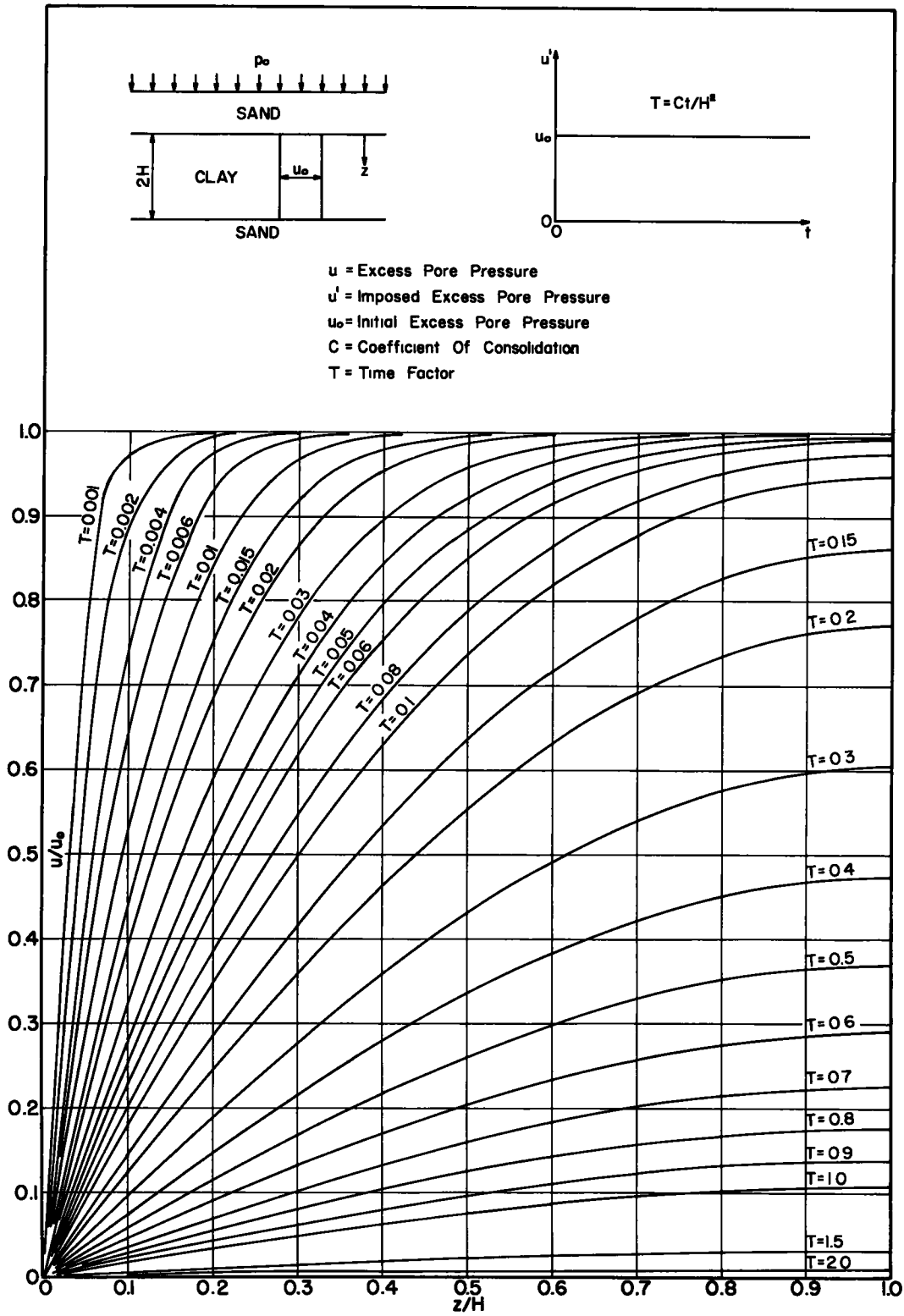

Figure 4. Point pore pressures for one-dimensional consolidation and constant load (time factor parameter).

sionless depth ratio , z/H, and the time factor, T, are presented in Table 1. The point pore pressure values are presented as continuous curves in Figures 3 and 4. In Figure 3, T is carried as the independent variable and z/H as a parameter. In Figure 4, T is carried as a parameter, with z/H as the independent variable.

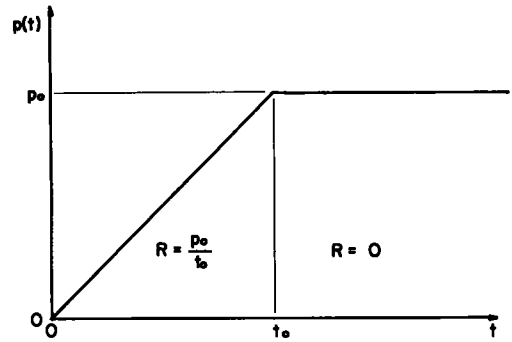

Figure 5. Construction loading.

Linear Loading

A linear loading often can be used to approximate the surface load build-up during construction. Such a construction loading will be approximated as shown in Figure 5. In this type of loading the surface load will be built up at a constant rate, in time t_o to a uniform load of p_o. Subsequent to time t_o the loading will remain constant. Thus, the rate of loading will be

$$R = p_o/t_o \tag{13}$$

In the development of numerical values for this type of loading, certain additional working conditions on the theory are postulated, as follows:

1. The permeability of the soil mass is considered to be constant at all times.
2. The imposed pore pressure, u', is a constant with respect to the thickness of the clay-soil stratum.
3. The rate of imposition of the surface load is equal to the rate of imposition of excess pore pressure, culminating at the end of construction in a total imposed excess pore pressure of magnitude u_o. This condition can be formulated as follows:

$$R = u_o/t_o \tag{14}$$

As a result of all the working conditions postulated, the consolidation problem has the following formulation, applicable during construction:

$$C\frac{\delta^2 u}{\delta z^2} + \frac{u_o}{t_o} = \frac{\delta u}{\delta t} \tag{15a}$$

$$u(0, t) = 0 \qquad 0 \leq t \leq t_o \tag{15b}$$

$$u(2H, t) = 0 \qquad 0 \leq t \leq t_o \tag{15c}$$

$$u(z, 0) = 0 \qquad 0 \leq z \leq 2H \tag{15d}$$

Although Eq. 15a can be solved directly, the previously offered computational theorem will aid in presenting the results in a form more readily useful to computation, with the solution to the boundary value problem being broken down into the sum of two solutions, as follows:

$$u(z, t) = u_1(z, t) + u_2(z) \tag{16}$$

On the basis of Eq. 16, the boundary value problem during construction becomes

$$C\frac{\delta^2 u_1}{\delta z^2} = \frac{\delta u_1}{\delta t} \tag{17a}$$

$$u_1(0, t) = 0 \qquad 0 \leq t \leq t_o \tag{17b}$$

$$u_1(2H, t) = 0 \qquad 0 \leq t \leq t_o \tag{17c}$$

$$u_1(z, 0) = -u_2(z) \qquad 0 \leq z \leq 2H \tag{17d}$$

$$C\frac{\delta^2 u_2}{\delta z^2} + \frac{u_o}{t_o} = 0 \tag{17e}$$

$$u_2(0) = 0 \qquad 0 \leq t \leq t_o \tag{17f}$$

$$u_2(2H) = 0 \qquad 0 \le t \le t_0 \tag{17g}$$

The resulting solution is

$$u(z, t) = \frac{u_0 H^2}{C t_0} \left\{ -\frac{16}{\pi^3} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^3} \operatorname{Sin} \frac{n\pi}{2} \frac{z}{H} e^{-(Cn^2\pi^2/4H^2)t} + \frac{z}{H} - \frac{1}{2}\left(\frac{z}{H}\right)^2 \right\}$$

$$0 \le z \le 2H \qquad 0 \le t \le t_0 \tag{18}$$

The boundary value problem for post-construction consolidation follows the usual Terzaghi theory, where the initial condition is the excess pore pressure developed at the end of construction.

$$C\frac{\delta^2 u}{\delta z^2} = \frac{\delta u}{\delta t} \tag{19a}$$

$$u(0, t) = 0 \qquad t_0 \le t \le \infty \tag{19b}$$

$$u(2H, t) = 0 \qquad t_0 \le t \le \infty \tag{19c}$$

$$u(z, t_0) = \frac{u_0 H^2}{C t_0} \left\{ \frac{z}{H} - \frac{1}{2}\left(\frac{z}{H}\right)^2 - \frac{16}{\pi^3} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^3} \operatorname{Sin} \frac{n\pi}{2} \frac{z}{H} e^{-(Cn^2\pi^2/4H^2)t_0} \right\}$$

$$0 \le z \le 2H \tag{19d}$$

The solution to the post-construction consolidation problem is:

$$u(z, t) = \frac{16\, u_0 H^2}{C t_0 \pi^3} \left\{ \sum_{n=1,3,5,}^{\infty} \frac{1}{n^3} \operatorname{Sin} \frac{n\pi}{2} \frac{z}{H} e^{-(Cn^2\pi^2/4H^2)(t - t_0)} - \sum_{n=1,3,5,}^{\infty} \frac{1}{n^3} \operatorname{Sin} \frac{n\pi}{2} \frac{z}{H} e^{-(Cn^2\pi^2/4H^2)t} \right\}$$

$$0 \le z \le 2H \qquad t_0 \le t \le \infty \tag{20}$$

By introducing the concept of a time factor, T, and an end time factor, T_0, the problem can be put on a dimensionless basis:

$$T_0 = Ct_0/H^2 \tag{21}$$

For computational purposes, two new functions are introduced, as follows:

$$\frac{u_z}{u_0}(z) = \frac{z}{H} - \frac{1}{2}\left(\frac{z}{H}\right)^2 \tag{22a}$$

$$\frac{u_p}{u_0}(z, T) = \frac{16}{\pi^3} \sum_{n=1,3,5,}^{\infty} \frac{1}{n^3} \operatorname{Sin} \frac{n\pi}{2} \frac{z}{H} e^{-(n^2\pi^2/4)T} \tag{22b}$$

The solution of point pore pressure for the consolidation under time-dependent loading then becomes:

$$\frac{u}{u_0}\left(\frac{z}{H}, T\right) = \frac{1}{T_0}\left[\frac{u_z}{u_0}\left(\frac{z}{H}\right) - \frac{u_p}{u_0}\left(\frac{z}{H}, T\right)\right] \qquad 0 \le T \le T_0 \tag{23a}$$

$$\frac{u}{u_0}\left(\frac{z}{H}, T\right) = \frac{1}{T_0}\left[\frac{u_p}{u_0}\left(\frac{z}{H}, T - T_0\right) - \frac{u_p}{u_0}\left(\frac{z}{H}, T\right)\right] \qquad T_0 \le T \le \infty \tag{23b}$$

The computations for u_p/u_0 are presented in Figures 6 and 7. In Figure 6 the depth ratio, z/H, is held as a parameter while the time factor, T, is the independent vari-

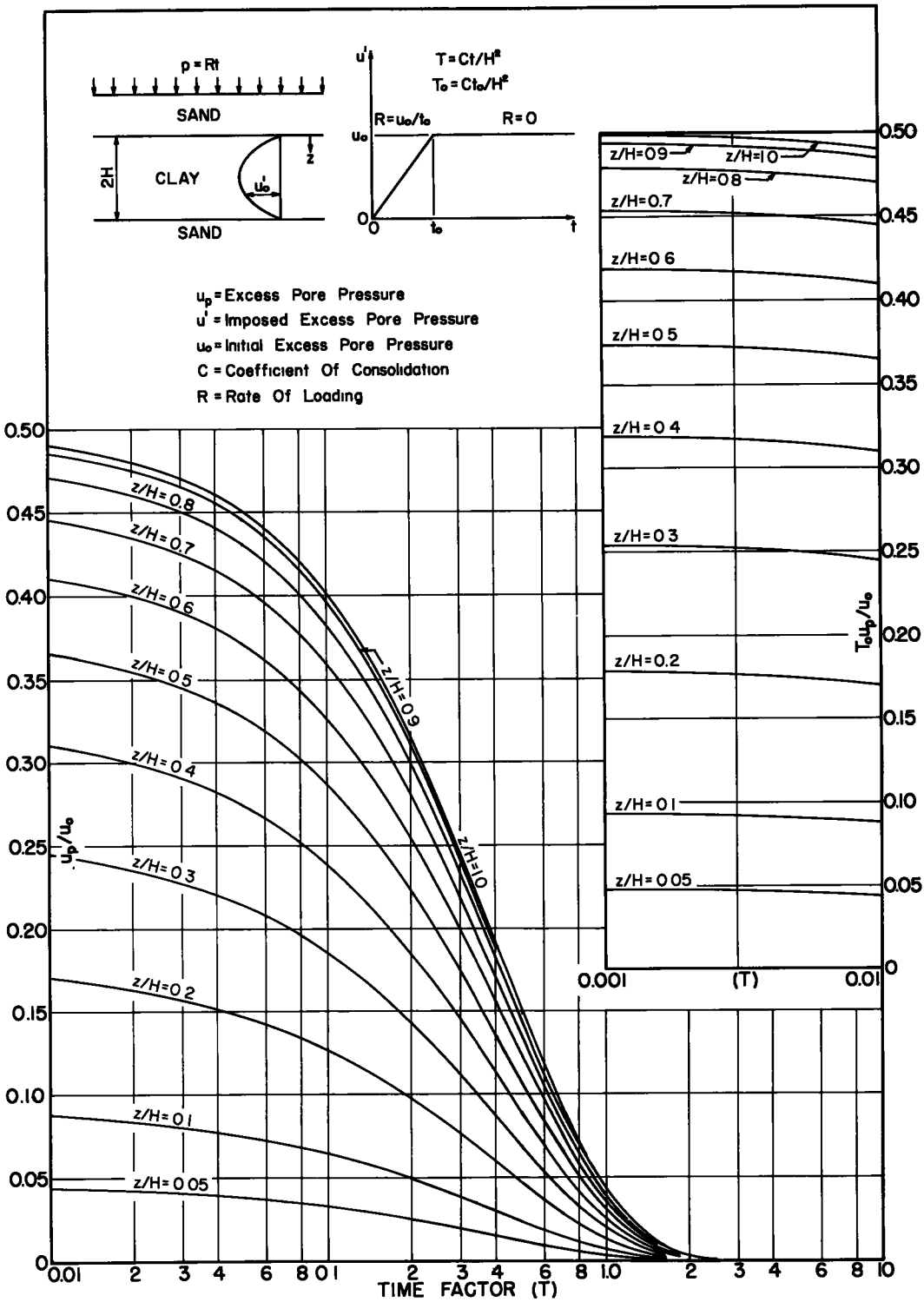

Figure 6. Point pore pressures for one-dimensional consolidation and construction loading (depth parameter).

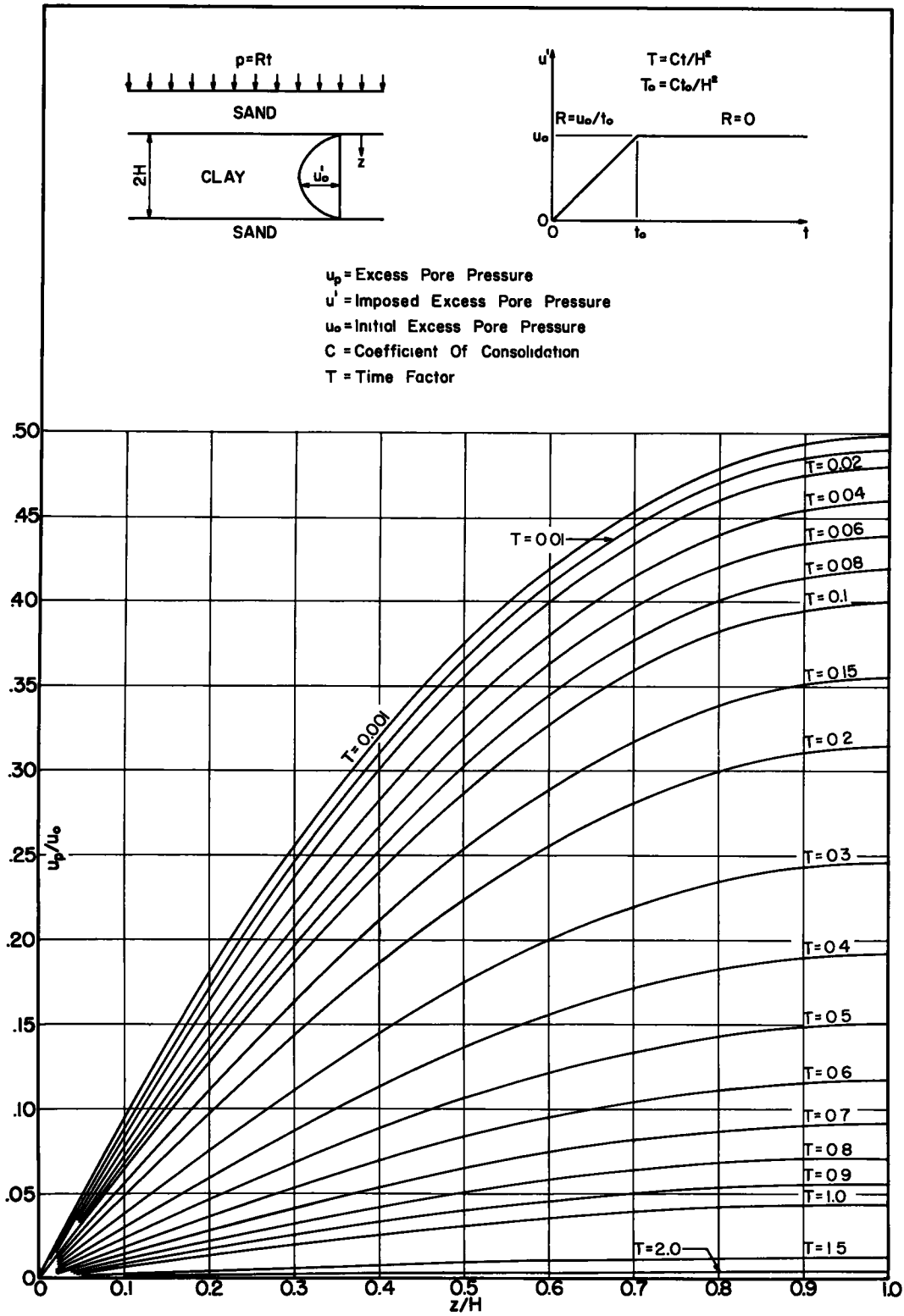

Figure 7. Point pore pressures for one-dimensional consolidation and construction loading (time factor parameter).

TABLE 2

POINT PORE PRESSURE RATIOS (u_p/u_o) FOR LINEAR LOADING AND CONSTANT PERMEABILITY, ONE-DIMENSIONAL CONSOLIDATION

T \ z/H	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.001	0.04787	0.09401	0.17900	0.25400	0.31900	0.37400	0.41900	0.45400	0.47900	0.49400	0.49900
0.0015	0.04752	0.09353	0.17850	0.25350	0.31850	0.37350	0.41850	0.45350	0.47850	0.49350	0.49850
0.002	0.04722	0.09307	0.17800	0.25300	0.31800	0.37300	0.41800	0.45300	0.47800	0.49300	0.49800
0.003	0.04670	0.09223	0.17701	0.25200	0.31700	0.37200	0.41700	0.45200	0.47700	0.49200	0.49700
0.004	0.04625	0.09146	0.17602	0.25100	0.31600	0.37100	0.41600	0.45100	0.47600	0.49100	0.49600
0.005	0.04585	0.09075	0.17506	0.25000	0.31500	0.37000	0.41500	0.45000	0.47500	0.49000	0.49500
0.006	0.04548	0.09009	0.17411	0.24901	0.31400	0.36900	0.41400	0.44900	0.47400	0.48900	0.49400
0.007	0.04514	0.08947	0.17319	0.24801	0.31300	0.36800	0.41300	0.44800	0.47300	0.48800	0.49300
0.008	0.04482	0.08889	0.17230	0.24703	0.31200	0.36700	0.41200	0.44700	0.47200	0.48700	0.49200
0.009	0.04452	0.08833	0.17142	0.24605	0.31100	0.36600	0.41100	0.44600	0.47100	0.48600	0.49100
0.01	0.04424	0.08780	0.17057	0.24508	0.31001	0.36500	0.41000	0.44500	0.47000	0.48500	0.49000
0.015	0.04299	0.08542	0.16659	0.24037	0.30507	0.36001	0.40500	0.44000	0.46500	0.48000	0.48500
0.02	0.04194	0.08339	0.16301	0.23591	0.30023	0.35505	0.40001	0.43500	0.46000	0.47500	0.48000
0.03	0.04016	0.07992	0.15671	0.22770	0.29097	0.34531	0.39009	0.42502	0.45000	0.46500	0.47000
0.04	0.03866	0.07697	0.15119	0.22026	0.28227	0 33590	0.38032	0.41510	0.44003	0.45501	0.46000
0.05	0.03733	0.07435	0.14624	0.21343	0.27409	0.32685	0.37078	0.40530	0.43011	0.44504	0.45002
0.06	0.03613	0.07098	0.14170	0.20709	0.26637	0.31817	0.36148	0.39565	0.42027	0.43512	0.44007
0.07	0.03503	0.06979	0.13749	0.20114	0.25904	0.30982	0.35244	0.38618	0.41055	0.42527	0.43019
0.08	0.03400	0.06775	0.13354	0.19552	0.25205	0.30179	0.34366	0.37689	0.40096	0.41551	0.42038
0.09	0.03304	0.06584	0.12982	0.19019	0.24537	0.29404	0.33512	0.36781	0.39152	0.40588	0.41069
0.1	0.03212	0.06402	0.12628	0.18510	0.23896	0.28656	0.32682	0.35892	0.38224	0.39639	0.40113
0.15	0.02812	0.05606	0.11069	0.16248	0.21013	0.25249	0.28854	0.31744	0.33855	0.35140	0.35571
0.2	0.02477	0.04938	0.09753	0.14324	0.18539	0.22292	0.25494	0.28066	0.29948	0.31095	0.31481
0.3	0.01932	0.03852	0.07608	0.11177	0.14471	0.17407	0.19915	0.21932	0.23409	0.24310	0.24612
0.4	0.01509	0.03009	0.05943	0.08732	0.11305	0.13600	0.15560	0.17136	0.18291	0.18996	0.19232
0.5	0.01179	0.02351	0.04644	0.06822	0.08833	0.10626	0.12157	0.13389	0.14292	0.14842	0.15027
0.6	0.00921	0.01837	0.03628	0.05331	0.06901	0.08302	0.09499	0.10462	0.11167	0.11597	0.11741
0.7	0.00720	0.01435	0.02835	0.04165	0.05392	0.06487	0.07422	0.08174	0.08725	0.09061	0.09174
0.8	0.00562	0.01121	0.02215	0.03254	0.04213	0.05069	0.05799	0.06387	0.06817	0.07080	0.07168
0.9	0.00439	0.00876	0.01731	0.02453	0.03292	0.03960	0.04531	0.04990	0.05327	0.05532	0.05601
1.0	0.00343	0.00685	0.01352	0.01987	0.02572	0.03094	0.03540	0.03899	0.04162	0.04322	0.04376
1.5	0.00100	0.00199	0.00394	0.00579	0.00749	0.00901	0.01031	0.01135	0.01212	0.01259	0.01274
2.0	0.00029	0.00058	0.00115	0.00168	0.00218	0.00262	0.00300	0.00331	0.00353	0.00367	0.00371
3.0	0.00002	0.00005	0.00010	0.00014	0.00018	0.00022	0.00025	0.00028	0.00030	0.00031	0.00031
4.0	0	0	0.00001	0.00001	0.00002	0.00002	0.00002	0.00002	0.00003	0.00003	0.00003
5.0	0	0	0	0	0	0	0	0	0	0	0

able. In Figures 7, the depth ratio is the independent variable, while the time factor is held as a parameter. The presentation of the data in these two fashions will enable cross-interpolation for values which were not computed. Complete values of the computation for u_p/u_o are presented in Table 2.

The theoretical data presented in Table 2 can be used as a check against field piezometer readings. Because piezometers cannot always be located in positions where theoretical calculations have been made, it may become necessary to interpolate between calculated points. The simplest interpolation which will achieve a high order of accuracy is the Gregory-Newton difference interpolation scheme (5). Such schemes exist using either forware or backward differences, depending on the range of data available.

The usefulness of the computation scheme presented in Eqs. 23a and 23b can be demonstrated by a simple example. Consider that it is desired to compute the excess pore pressure-time factor relation when the end time factor is 0.1 and the piezometer is located at the mid-point in the clay stratum. The computation is outlined in Table 3. The values listed in Col. 1 are the time factors that are desired in the computation. Col. 2 presents

TABLE 3

SAMPLE COMPUTATION OF TIME-PORE PRESSURE RELATION

$T_o = 0.1$ $z/H = 1.0$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
T	u_z/u_o	$\frac{u_p}{u_o}(T)$	$T-T_o$	$\frac{u_p}{u_o}(T-T_o)$	$T_o u/u_o$	u/u_o
0.01	0.5	0.49000	-	-	0.01000	0.1000
0.015	0.5	0.48500	-	-	0.01500	0.1500
0.02	0.5	0.48000	-	-	0.02000	0.2000
0.03	0.5	0.47000	-	-	0.03000	0.3000
0.04	0.5	0.46000	-	-	0.04000	0.4000
0.05	0.5	0.45002	-	-	0.04998	0.4998
0.06	0.5	0.44007	-	-	0.05993	0.5993
0.07	0.5	0.43019	-	-	0.06981	0.6981
0.08	0.5	0.42038	-	-	0.07962	0.7962
0.09	0.5	0.41069	-	-	0.08931	0.8931
0.1	0.5	0.40113	-	-	0.09887	0.9887
0.15	-	0.35571	0.05	0.45002	0.09431	0.9431
0.2	-	0.31481	0.1	0.40113	0.08632	0.8632
0.3	-	0.24612	0.2	0.31481	0.06869	0.6869
0.4	-	0.19232	0.3	0.24612	0.05380	0.5380
0.5	-	0.15027	0.4	0.19232	0.04205	0.4205
0.6	-	0.11741	0.5	0.15027	0.03286	0.3286
0.7	-	0.09174	0.6	0.11741	0.02567	0.2567
0.8	-	0.07168	0.7	0.09174	0.02006	0.2006
0.9	-	0.05601	0.8	0.07168	0.01567	0.1567
1.0	-	0.04376	0.9	0.05601	0.01225	0.1225
1.5	-	0.01274	1.4	0.01631	0.00357	0.0357
2.0	-	0.00371	1.9	0.00475	0.00104	0.0104
3.0	-	0.00031	2.9	0.00041	0.00010	0.0010

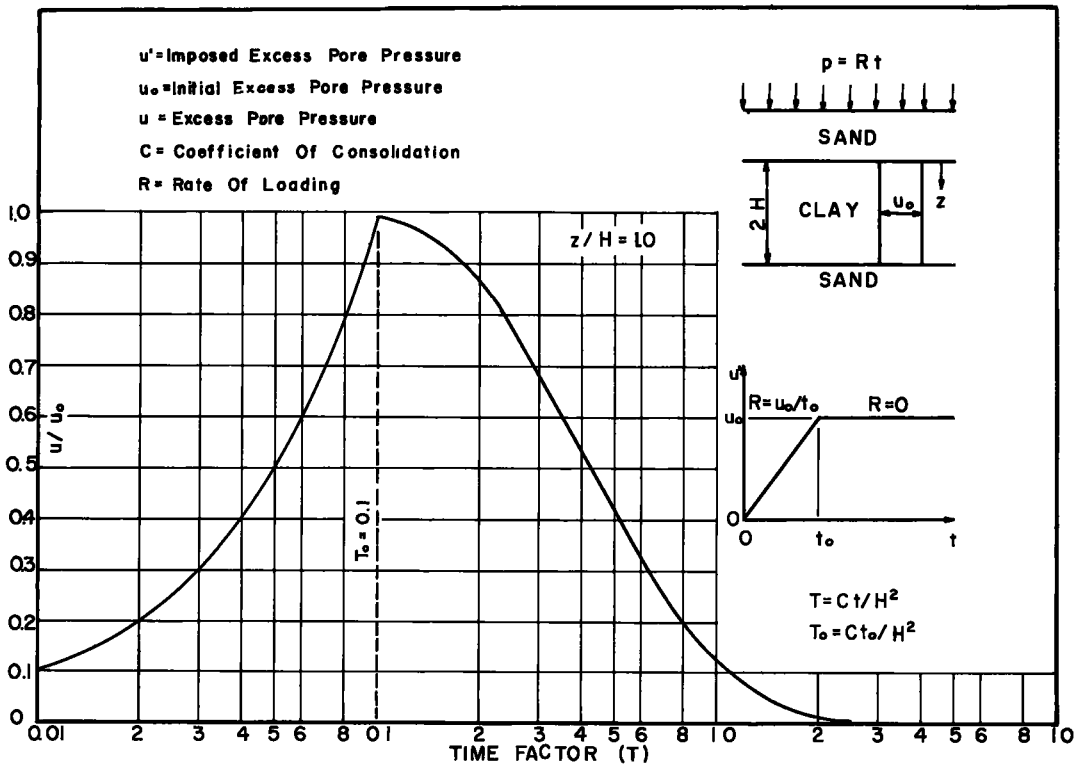

Figure 8. Point pore pressure example.

the value of u_z as computed from Eq. 22a. Col. 3 lists the values of u_p at the various time factors. These values are either read from Figures 6 and/or 7 or taken from Table 2. For this example they were taken from Table 2. For values of T less than T_0, Col. 4 and 5 are not used. Col. 6 is the difference between Cols. 2 and 3 in accordance with Eq. 23a. The excess pore pressure ratio (Col. 7) is finally determined by dividing the values in Col. 6 by T_0. To determine the pore pressure subsequent to construction, the argument $T - T_0$ must be determined, as in Col. 4. The value of u_p for this argument is presented in Col. 5. In accordance with Eq. 23b, the values in Col. 6 for values of T greater than T_0 are the difference between Col. 5 and Col. 3. Col. 7 again computes the excess pore pressure ratio. The results of this example are plotted in Figure 8.

EQUAL-STRAIN SAND-DRAINS

The theory of equal-strain sand-drains was developed originally by Barron (6). In this work several solutions were computed for sand drains placed in a triangular pattern, as shown in Figure 9. If the drains are placed in a triangular pattern the zone of influence becomes hexagonal and can be approximated by a circle. The basic sand-drain theory for triangular patterns considers that the drain itself acts as a pressure-free "hole" surrounded by an infinite cylinder with the cylinder boundary being flux-free. In some instances the remolding effects (smear) of installing the drain can be included in the analysis.

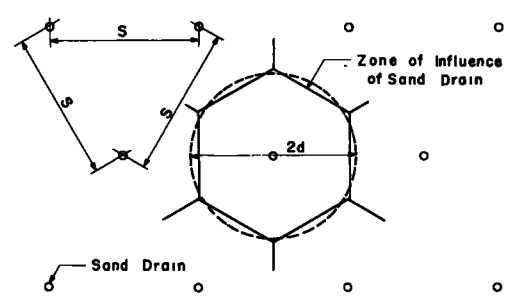

Figure 9. Sand-drain pattern.

A dimensioned section of the drain is shown in Figure 10.

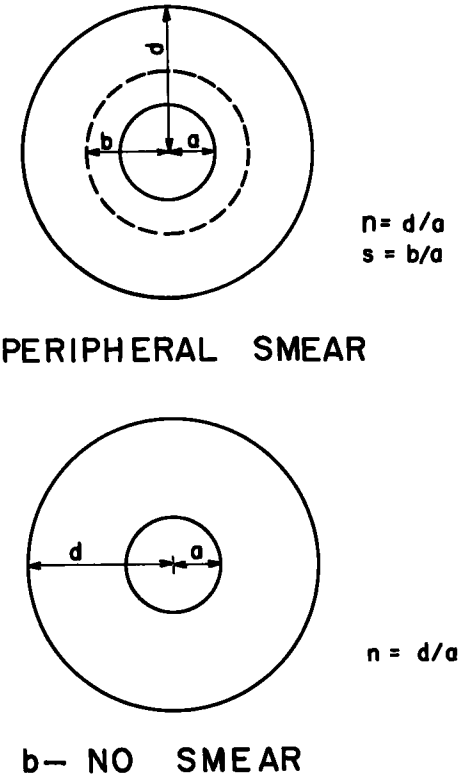

Figure 10. Sand-drain section.

Constant Load, Variable Permeability

An exact solution to the equal-strain sand-drain theory has been developed (1) for situations in which the following conditions apply:

1. The surface load activating the sand-drain action is time-independent.
2. The drain is perfect and the soil surrounding the drain is completely undisturbed (that is, no smear).
3. The soil permeability is time-dependent according to

$$\bar{k}(t) = \alpha\,\bar{u}(t) + k_f \tag{24}$$

in which

$\bar{k}$ = average coefficient of permeability;
k_f = coefficient of permeability at the end of consolidation;
α = coefficient of variation; and
$\bar{u}$ = average excess pore pressure.

The solution of the consolidation problem under the foregoing conditions and when the soil is consolidating under an imposed average excess pore pressure, $\bar{u}_o$, is

$$\frac{\bar{u}}{\bar{u}_o}(T_f) = \frac{\bar{w}_r}{\bar{w}_r + \mu(1 - \bar{w}_r)} \tag{25a}$$

$$\bar{w}_r = \frac{\bar{u}_r}{\bar{u}_o} = e^{-8T_f/F(n)} \tag{25b}$$

$$\mu = \bar{k}_o/k_f \tag{25c}$$

$$F(n) = \frac{n^2}{n^2 - 1}\ln(n) - \frac{3n^2 - 1}{4n^2} \tag{25d}$$

$$T_f = C_f t/4d^2 \tag{25e}$$

In Eq. 25, $\bar{k}_o$ refers to the initial average coefficient of permeability, and C_f is the coefficient of consolidation based on the final permeability. As a result of this definition of C_f, the time factor, T_f, is also defined.

It may be interesting to note that this exact solution for excess pore pressure with variable permeability is expressible in terms of $\bar{u}_r$, the solution for excess pore pressure when the permeability is constant, and μ, the ratio of end point permeabilities. Thus, the computations for Eq. 25a can take advantage of the available wealth of computed data on sand-drains.

Figures 11, 12, 13, 14 and 15 are charts in which various parameters of Eqs. 25a through 25e are computed. These charts are arranged for relative ease of interpolation. Figures 14 and 15 plot identical entities, except that Figure 15 enables the computation at early times during consolidation.

In addition to the values in Figures 11 through 15, computed values of F are presented in Table 4.

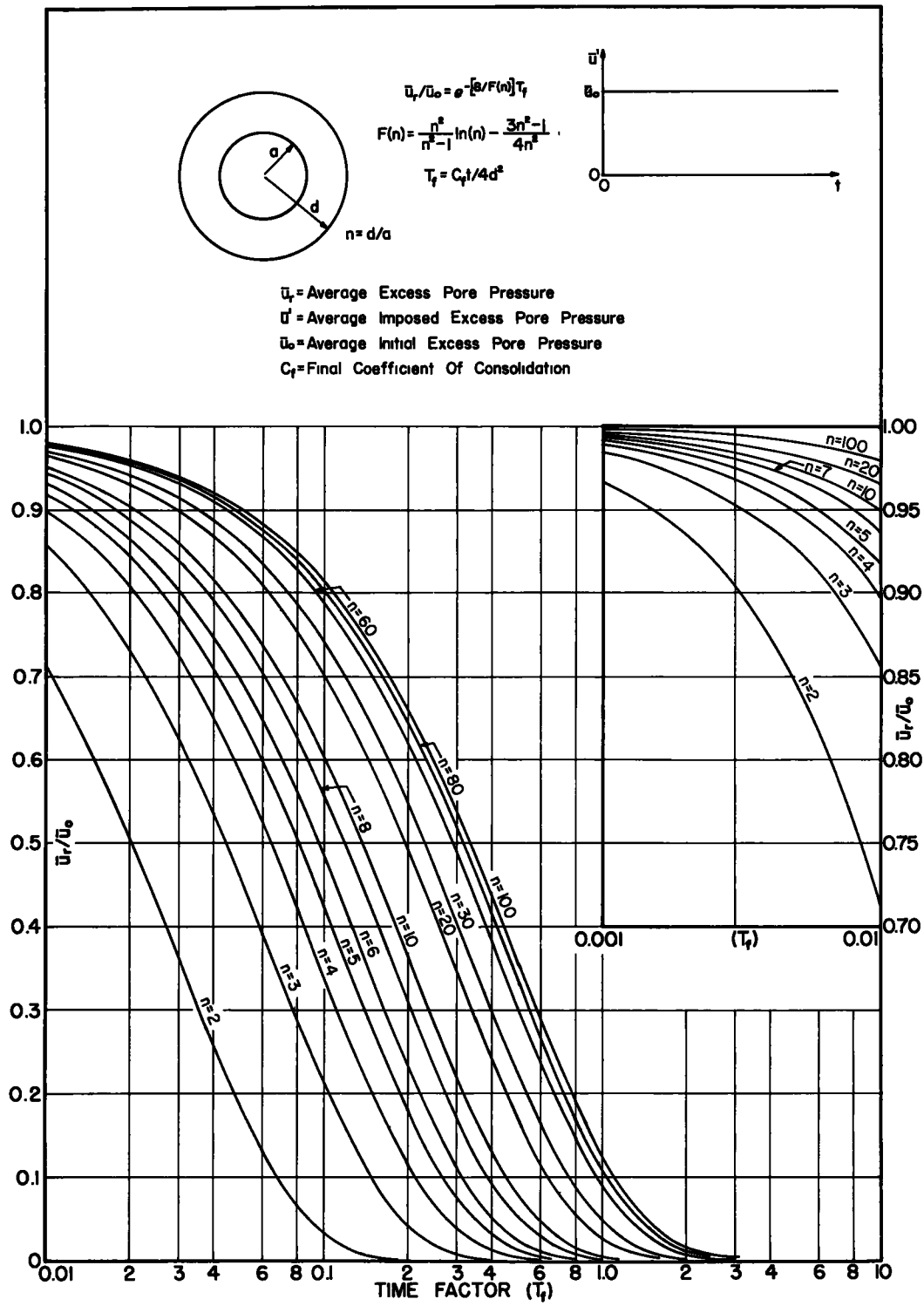

Figure 11. Average pore pressure for equal-strain sand-drains, for no smear and constant load (drain ratio parameter).

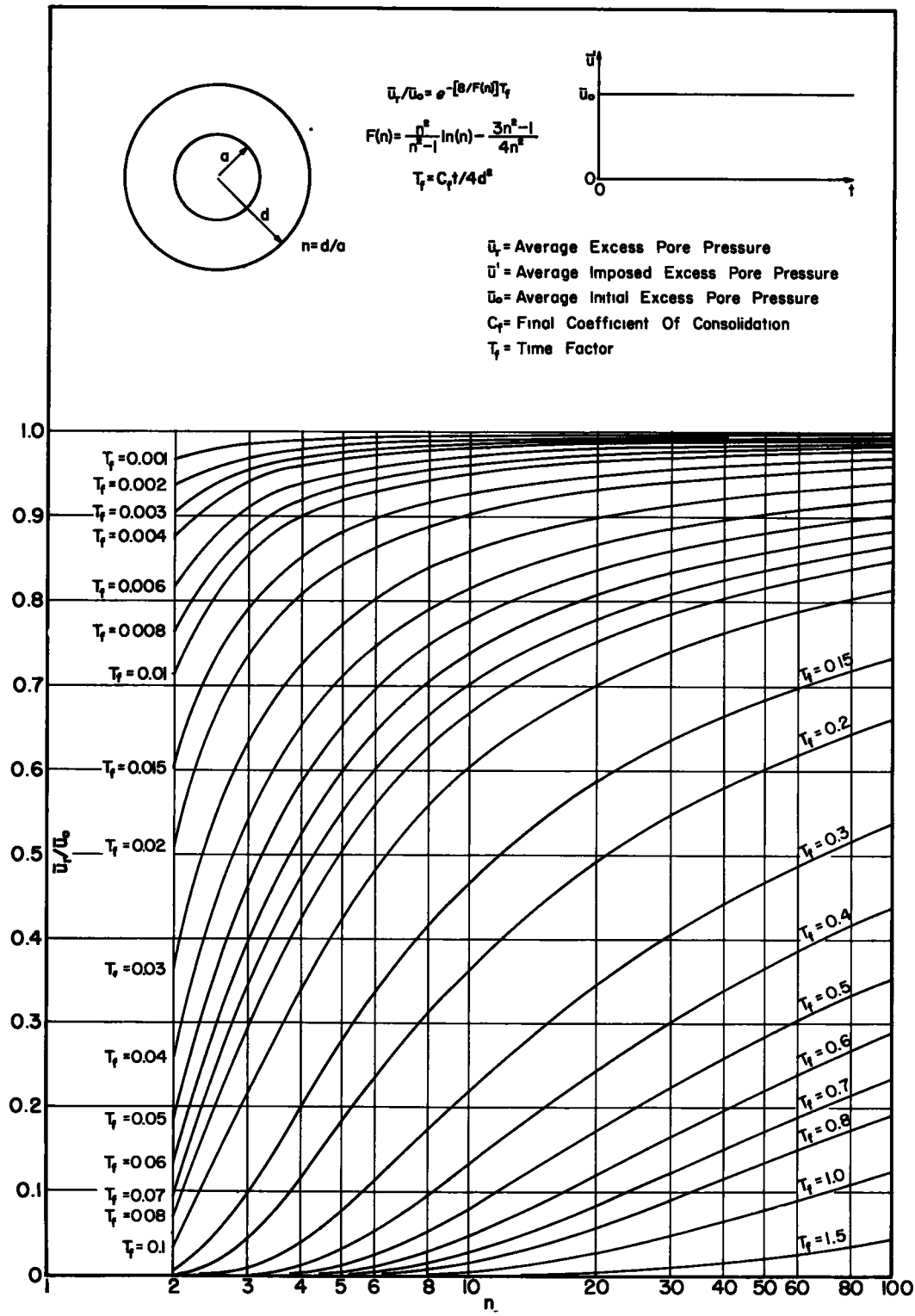

Figure 12. Average pore pressures for equal-strain sand-drains, for no smear and constant load (time factor parameter).

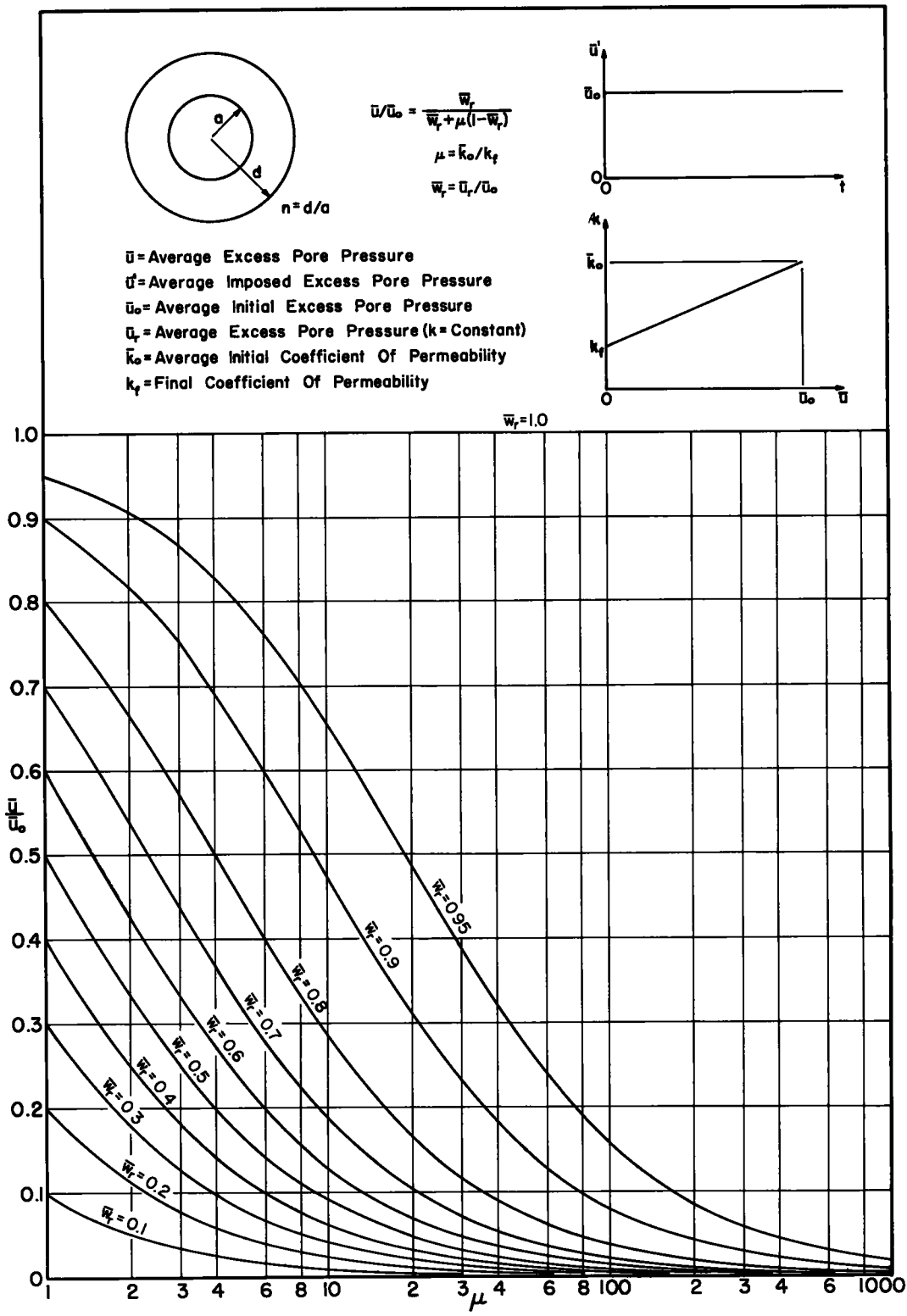

Figure 13. Average pore pressures for equal-strain sand-drains under constant load and variable permeability with no smear (constant permeability pore pressure parameter).

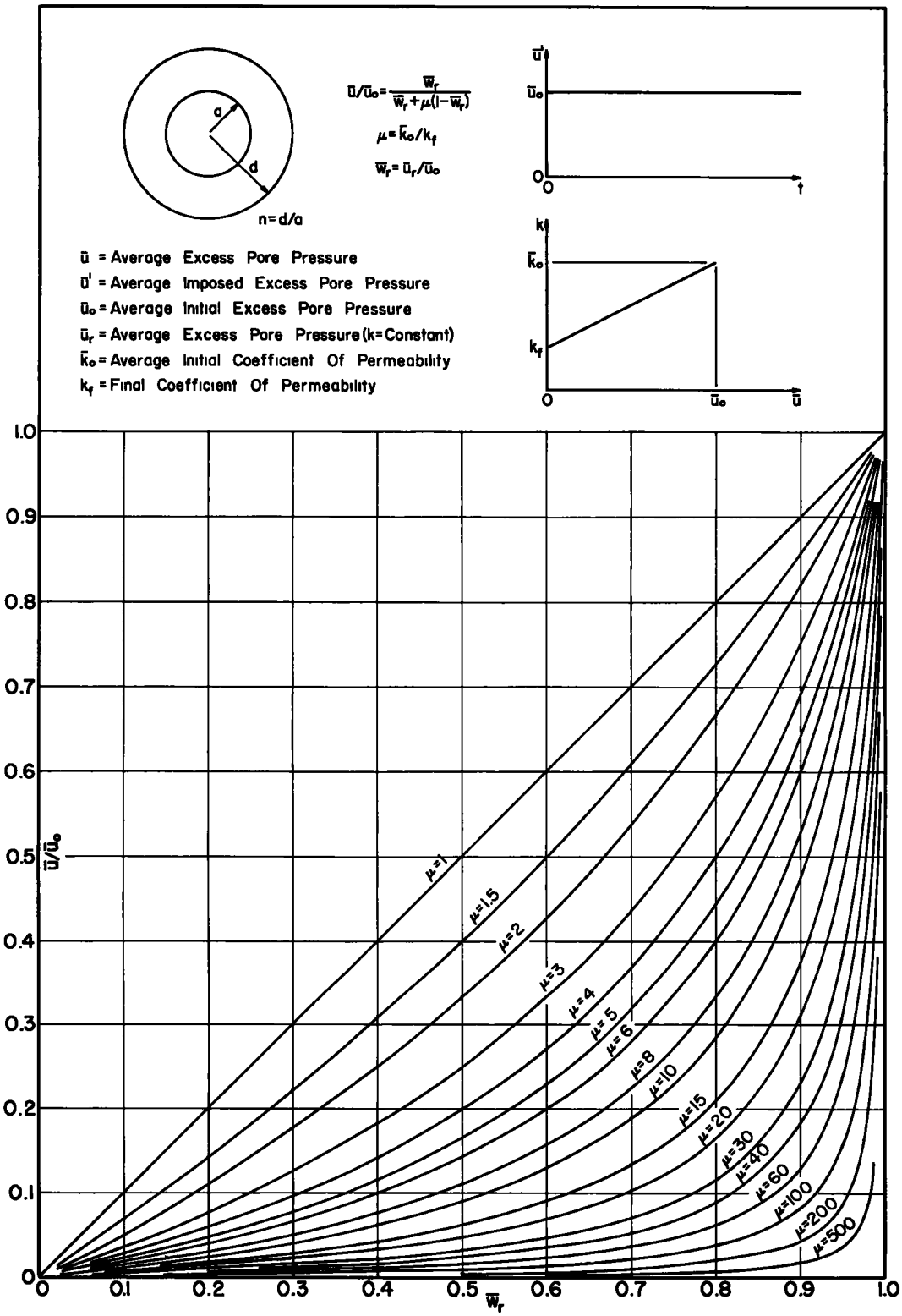

Figure 14. Average pore pressures for equal-strain sand-drains under constant load and variable permeability with no smear (permeability ratio parameter).

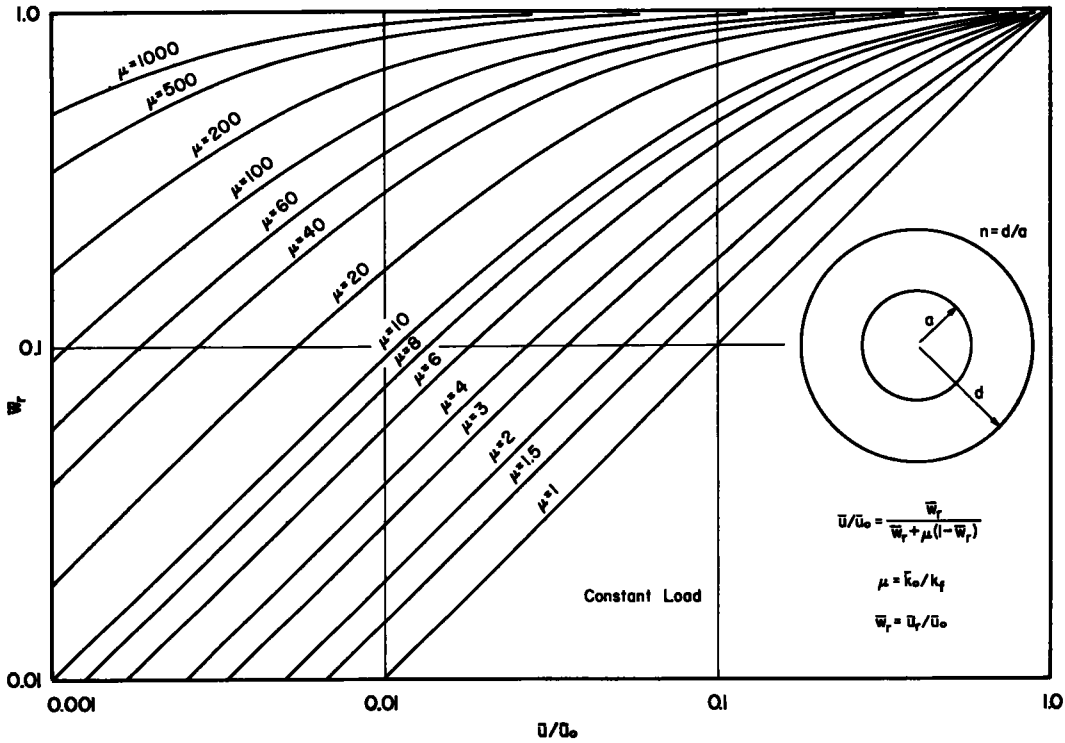

Figure 15. Expanded-scale relation of average pore pressures for equal-strain sand-drains under constant load and variable permeability with no smear (permeability ratio parameter).

Constant Permeability, Construction Loading

The exact equal-strain sand-drain solution can be achieved for a sand-drain with smear and for a construction loading. To achieve a closed form solution, the permeability of the soil was held to a constant during the process of consolidation. The construction loading was considered to be a linear build-up of average imposed excess pore pressure; that is,

$$R = \bar{u}_o/t_o \tag{26}$$

in which $\bar{u}_o$ is the average imposed excess pore pressure at the end of construction, t_o. Essentially the rate, R, is the rate of build-up of the surcharge loading.

The excess pore pressures, both during and after construction, are in accordance with the following:

$$\frac{\bar{u}}{\bar{u}_o} = \frac{1}{8T_o}\left[F(n, s) + \frac{G(n, s)}{\chi}\right]\left[1 - e^{-8T_h/F(n, s)}\right] \qquad T_h \leq T_o \tag{27a}$$

$$\frac{\bar{u}}{\bar{u}_o} = \frac{1}{8T_o}\left[F(n, s) + \frac{G(n, s)}{\chi}\right]\left[1 - e^{-8T_o/F(n, s)}\right] e^{-8(T_h - T_o)/F(n, s)} \qquad T_h \geq T_o \tag{27b}$$

$$F_1(n, s) = \frac{n^2}{n^2 - s^2}\ln\left(\frac{n}{s}\right) + \frac{s^2 - 3n^2}{4n^2} \tag{27c}$$

$$F_2(n, s) = \frac{n^2 - s^2}{n^2}\ln(s) \tag{27d}$$

$$G(n, s) = \frac{1 - s^2(1 - 2\ln s)}{2n^2} \tag{27e}$$

$$F(n, s) = F_1(n, s) + \theta F_2(n, s) \qquad (27f)$$

$$\theta = k_r/k_r'' \qquad (27g)$$

$$\chi = C_h''/C_h \qquad (27h)$$

in which

$F_1(n, s)$ = geometric drain parameter;

$F_2(n, s)$ = geometric drain parameter;

$G(n, s)$ = geometric drain parameter;

k_r = radial coefficient of permeability (undisturbed);

k_r'' = radial coefficient of permeability (remolded);

C_h = radial coefficient of consolidation (undisturbed); and

C_h'' = radial coefficient of consolidation (remolded).

TABLE 4

GEOMETRIC PARAMETERS FOR NO-SMEAR EQUAL-STRAIN SAND-DRAINS UNDER CONSTANT LOAD

n	F(n)	8/F(n)
2	0.23670	33.79859
3	0.51372	15.57279
4	0.74434	10.74779
5	0.93650	8.54247
6	1.09990	7.27341
7	1.24155	6.44355
8	1.36635	5.85499
9	1.47778	5.41354
10	1.57834	5.06861
20	2.25387	3.54946
30	2.65526	3.01289
40	2.94134	2.71985
50	3.16369	2.52869
60	3.34555	2.39123
70	3.49941	2.28610
80	3.63275	2.20219
90	3.75040	2.13311
100	3.85566	2.07487

Figures 16 through 21 present charts which will enable the computation of the geometric parameters F_1, F_2, and G. These parameters also are presented in Table 5.

As an example of the type of time-average pore pressure calculations that can be made, the following problem is presented:

$$n = 10, \; s = 2, \; \theta = 5, \; \chi = 2, \; T_0 = 0.2.$$

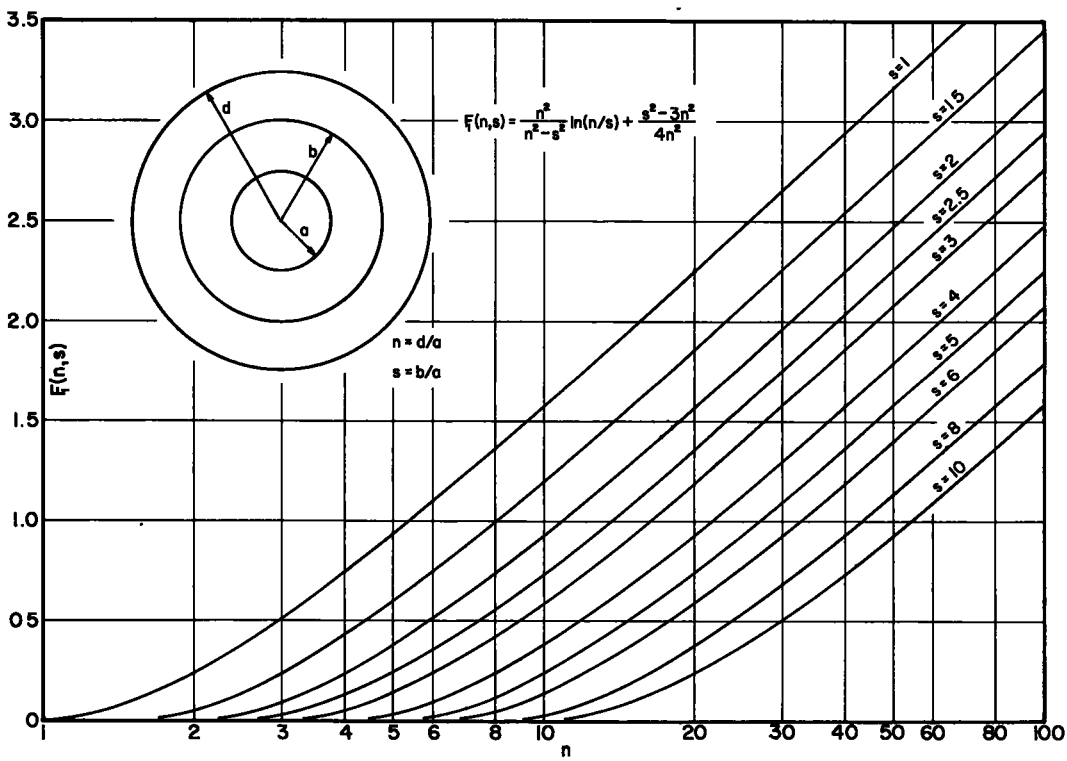

Figure 16. Geometric smear factor F_1 (n, s) for equal-strain sand-drains (smear ratio parameter).

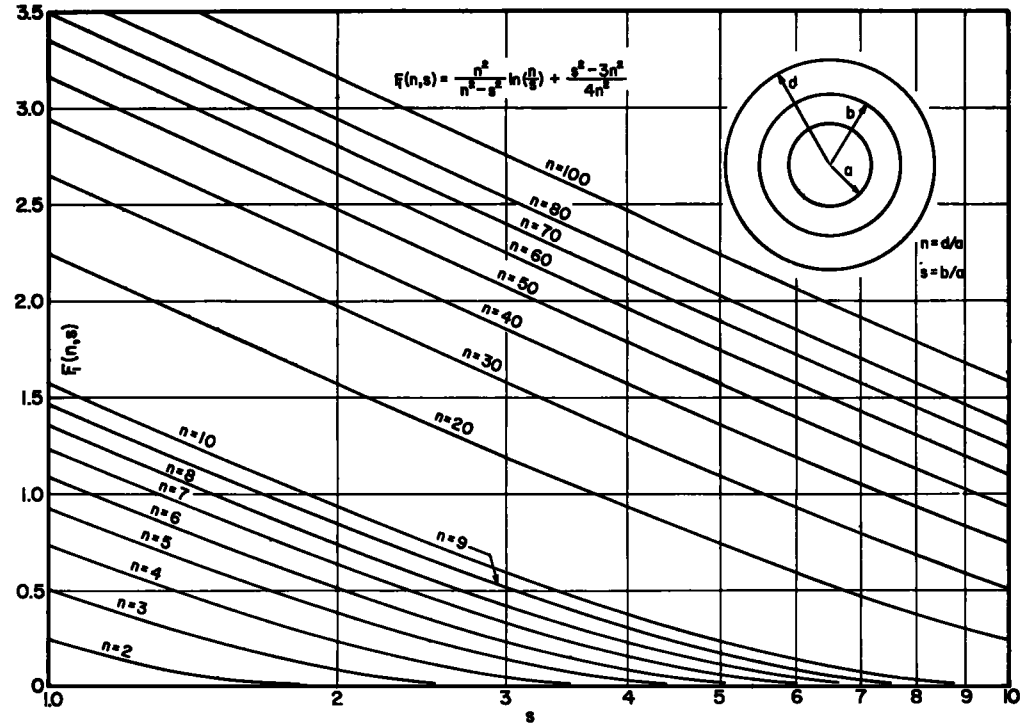

Figure 17. Geometric smear factor F_1 (n, s) for equal-strain sand-drains (drain ratio parameter).

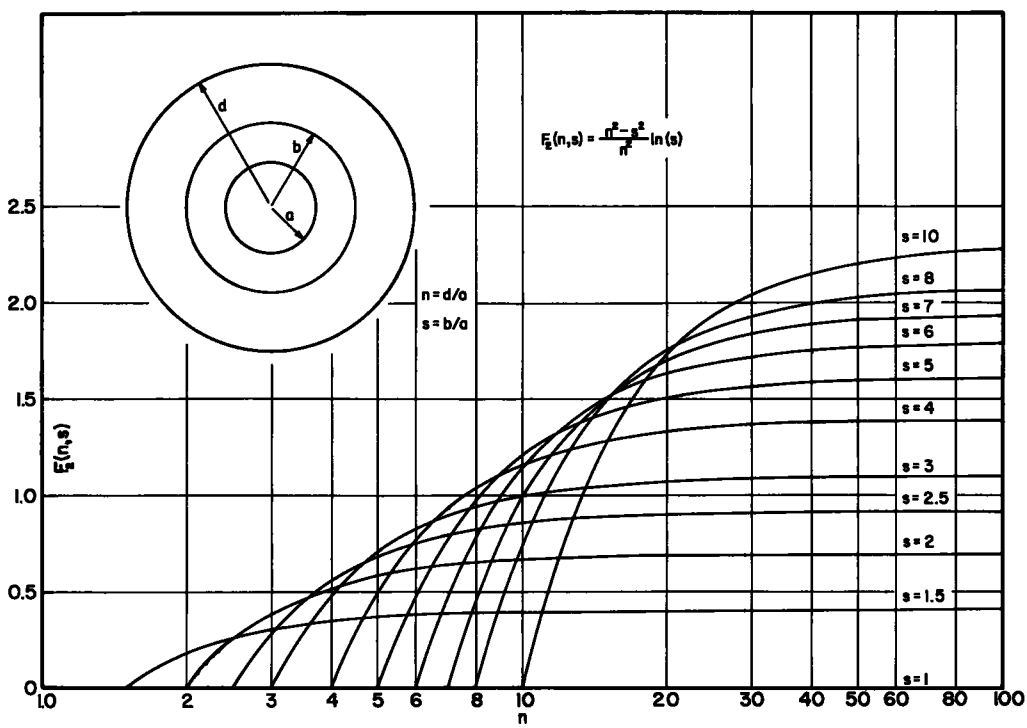

Figure 18. Geometric smear factor F_2 (n, s) for equal-strain sand-drains (smear ratio parameter).

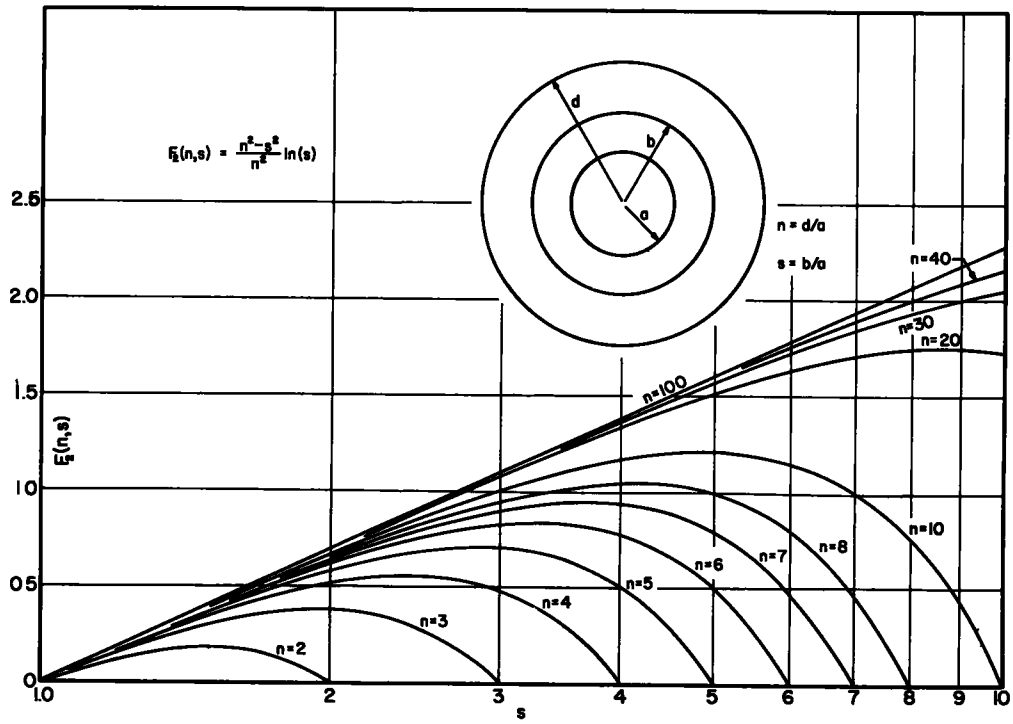

Figure 19. Geometric smear factor F_2 (n, s) for equal-strain sand-drains (drain ratio parameter).

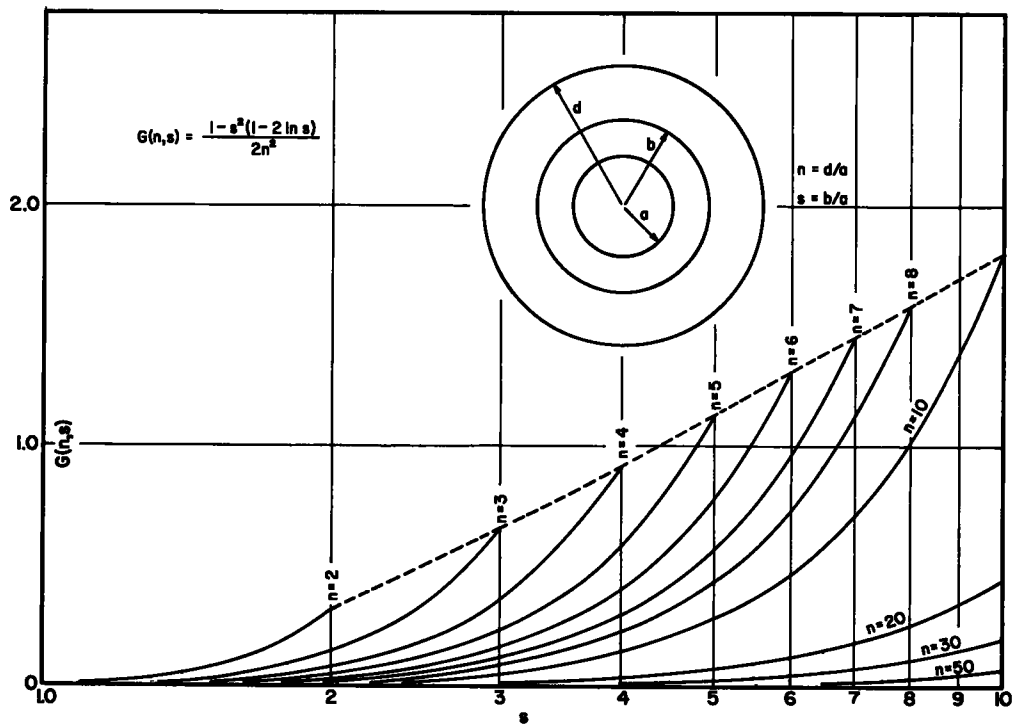

Figure 20. Geometric smear factor G (n, s) for equal-strain sand-drains (drain ratio parameter).

TABLE 5

GEOMETRIC SMEAR FACTOR FOR EQUAL-STRAIN SAND-DRAINS WITH CONSTANT PERMEABILITY; CONSTRUCTION LOADING

s \ n	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100
									a. Smear Factor $F_1(n, s)$									
1.0	0.23670	0.51372	0.74434	0.93650	1.09990	1.24155	1.36635	1.47778	1.57834	2.25387	2.65526	2.94134	3.16369	3.34555	3.49941	3.63275	3.75040	3.85566
1.1	0.18274	0.44275	0.66551	0.85324	1.01385	1.15363	1.27709	1.38752	1.48734	2.15998	2.56067	2.84648	3.06868	3.25046	3.40427	3.53757	3.65519	3.76044
1.2	0.13816	0.38082	0.59555	0.77874	0.93650	1.07433	1.19641	1.30582	1.40484	2.07448	2.47443	2.75994	2.98199	3.16369	3.31744	3.45071	3.56830	3.67352
1.3	0.10157	0.32652	0.53307	0.71164	0.86646	1.00231	1.12296	1.23131	1.32952	1.99602	2.39521	2.68040	2.90230	3.08390	3.23759	3.37082	3.48838	3.59358
1.4	0.07186	0.27877	0.47700	0.65086	0.80269	0.93650	1.05569	1.16294	1.26032	1.92358	2.32196	2.60683	2.82855	3.01006	3.16369	3.29687	3.41441	3.51958
1.5	0.04818	0.23670	0.42648	0.59555	0.74434	0.87606	0.99376	1.09990	1.19641	1.85633	2.25387	2.53839	2.75994	2.94134	3.09491	3.22805	3.34555	3.45071
1.6	0.02984	0.19960	0.38082	0.54502	0.69072	0.82033	0.93650	1.04149	1.13713	1.79360	2.19027	2.47443	2.69580	2.87710	3.03060	3.16369	3.28116	3.38629
1.7	0.01628	0.16692	0.33945	0.49872	0.64128	0.76873	0.88335	0.98717	1.08191	1.73485	2.13062	2.41442	2.63560	2.81678	2.97021	3.10325	3.22069	3.32579
1.8	0.00703	0.13816	0.30189	0.45617	0.59555	0.72081	0.83385	0.93650	1.03032	1.67963	2.07448	2.35789	2.57887	2.75994	2.91330	3.04629	3.16369	3.26877
1.9	0.00170	0.11295	0.26775	0.41698	0.55314	0.67619	0.78763	0.88907	0.98195	1.62757	2.02146	2.30448	2.52526	2.70621	2.85948	2.99242	3.10979	3.21484
2.0	0	0.09094	0.23670	0.38082	0.51372	0.63453	0.74434	0.84457	0.93650	1.57834	1.97125	2.25387	2.47443	2.65526	2.80846	2.94134	3.05867	3.16369
2.5	0	0.02030	0.11894	0.23670	0.35279	0.46204	0.56345	0.65733	0.74434	1.36635	1.75042	2.03444	2.25387	2.43401	2.58678	2.71937	2.83648	2.94134
3.0	0	0	0.04818	0.13816	0.23670	0.33386	0.42648	0.51372	0.59555	1.19641	1.57834	1.85633	2.07448	2.25387	2.40614	2.53839	2.65526	2.75994
3.5	0	0	0.01114	0.07186	0.15207	0.23670	0.32022	0.40056	0.47700	1.05569	1.43148	1.70683	1.92358	2.10213	2.25387	2.38574	2.50234	2.60683
4.0	0	0	0	0.02984	0.09095	0.16258	0.23670	0.30993	0.38082	0.93650	1.30582	1.57834	1.79360	1.97125	2.12239	2.25387	2.37017	2.47443
4.5	0	0	0	0.00703	0.04818	0.10635	0.17078	0.23670	0.30189	0.83385	1.19641	1.46597	1.67963	1.85633	2.00684	2.13788	2.25387	2.35789
5.0	0	0	0	0	0.02030	0.06452	0.11894	0.17735	0.23670	0.74434	1.09990	1.36635	1.57834	1.75402	1.90387	2.03444	2.15009	2.25387
5.5	0	0	0	0	0.00484	0.03457	0.07868	0.12939	0.18274	0.66551	1.01385	1.27709	1.48734	1.66195	1.81109	1.94117	2.05647	2.15998
6.0	0	0	0	0	0	0.01470	0.04818	0.09095	0.13816	0.59555	0.93650	1.19641	1.40484	1.57834	1.72676	1.85633	1.97125	2.07448
6.5	0	0	0	0	0	0.00353	0.02602	0.06064	0.10157	0.53307	0.86646	1.12296	1.32952	1.50187	1.64952	1.77856	1.89309	1.99602
7.0	0	0	0	0	0	0	0.01114	0.03737	0.07186	0.47700	0.80269	1.05569	1.26032	1.43148	1.57834	1.70683	1.82096	1.92358
7.5	0	0	0	0	0	0	0.00268	0.02030	0.04818	0.42648	0.74434	0.99376	1.19641	1.36635	1.51240	1.64031	1.75402	1.85633
8.0	0	0	0	0	0	0	0	0.00873	0.02984	0.38082	0.69072	0.93650	1.13713	1.30582	1.45102	1.57834	1.69162	1.79360
8.5	0	0	0	0	0	0	0	0.00212	0.01628	0.33945	0.64128	0.88335	1.08192	1.24932	1.39367	1.52038	1.63321	1.73485
9.0	0	0	0	0	0	0	0	0	0.00703	0.30189	0.59555	0.83385	1.03032	1.19641	1.33988	1.46597	1.57834	1.67963
9.5	0	0	0	0	0	0	0	0	0.00171	0.26775	0.55314	0.78763	0.98195	1.11467	1.28928	1.41474	1.52664	1.62757
									b. Smear Factor $F_2(n, s)$									
1.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	0.06648	0.08250	0.08810	0.09070	0.09211	0.09296	0.09351	0.09389	0.09416	0.09502	0.09518	0.09524	0.09526	0.09528	0.09529	0.09529	0.09530	0.09530
1.2	0.11669	0.15315	0.16591	0.17182	0.17503	0.17696	0.17822	0.17908	0.17970	0.18167	0.18203	0.18216	0.18222	0.18225	0.18227	0.18228	0.18229	0.18230
1.3	0.15152	0.21310	0.23465	0.24463	0.25005	0.25332	0.25544	0.25689	0.25793	0.26126	0.26187	0.26209	0.26219	0.26224	0.26227	0.26229	0.26231	0.26232
1.4	0.17160	0.26320	0.29525	0.31009	0.31815	0.32301	0.32617	0.32833	0.32988	0.33482	0.33574	0.33606	0.33621	0.33629	0.33634	0.33637	0.33639	0.33641
1.5	0.17739	0.30410	0.34845	0.36897	0.38012	0.38685	0.39121	0.39420	0.39634	0.40318	0.40445	0.40489	0.40510	0.40521	0.40528	0.40532	0.40535	0.40537
1.6	0.16920	0.33631	0.39480	0.42188	0.43658	0.44545	0.45120	0.45515	0.45797	0.46700	0.46867	0.46925	0.46952	0.46967	0.46976	0.46982	0.46986	0.46988
1.7	0.14725	0.36024	0.43479	0.46929	0.48803	0.49933	0.50667	0.51170	0.51529	0.52679	0.52892	0.52967	0.53001	0.53020	0.53032	0.53039	0.53044	0.53047
1.8	0.11168	0.37618	0.46876	0.51161	0.53489	0.54892	0.55803	0.56428	0.56874	0.58303	0.58567	0.58660	0.58702	0.58727	0.58740	0.58749	0.58755	0.58760
1.9	0.06258	0.38440	0.49704	0.54917	0.57749	0.59457	0.60565	0.61325	0.61868	0.63606	0.63928	0.64041	0.64093	0.64121	0.64138	0.64149	0.64157	0.64162
2.0	0	0.38508	0.51986	0.58224	0.61613	0.63656	0.64983	0.65892	0.66542	0.68622	0.69007	0.69141	0.69204	0.69238	0.69258	0.69271	0.69280	0.69287
2.5	0	0.27998	0.55836	0.68722	0.75721	0.79942	0.82681	0.84559	0.85902	0.90197	0.90993	0.91271	0.91400	0.91470	0.91512	0.91540	0.91558	0.91572
3.0	0	0	0.48064	0.70311	0.82396	0.89683	0.94412	0.97654	0.99974	1.07389	1.08763	1.09243	1.09466	1.09587	1.09659	1.09707	1.09739	1.09762
3.5	0	0	0.29362	0.63891	0.82648	0.93957	1.01298	1.06330	1.09930	1.21440	1.23571	1.24317	1.24662	1.24850	1.24963	1.25037	1.25087	1.25123
4.0	0	0	0	0.49907	0.77016	0.93363	1.03972	1.11246	1.16449	1.33084	1.36164	1.37243	1.37742	1.38013	1.38177	1.38283	1.38356	1.38408
4.5	0	0	0	0.28577	0.65803	0.88249	1.02818	1.12806	1.19950	1.42793	1.47024	1.48504	1.49189	1.49562	1.49786	1.49932	1.50032	1.50103
5.0	0	0	0	0	0.49177	0.78830	0.98075	1.11270	1.20708	1.50885	1.56473	1.58429	1.59334	1.59826	1.60123	1.60315	1.60447	1.60541
5.5	0	0	0	0	0.27229	0.65233	0.89899	1.06810	1.18906	1.57583	1.64745	1.67252	1.68412	1.69042	1.69422	1.69669	1.69838	1.69959
6.0	0	0	0	0	0	0.47536	0.78389	0.99542	1.14673	1.63050	1.72009	1.75144	1.76596	1.77384	1.77860	1.78168	1.78380	1.78531
6.5	0	0	0	0	0	0.25785	0.63612	0.89546	1.08097	1.67409	1.78393	1.82237	1.84017	1.84983	1.85566	1.85945	1.86204	1.86389
7.0	0	0	0	0	0	0	0.45607	0.76875	0.99241	1.70754	1.83997	1.88632	1.90777	1.91942	1.92645	1.93101	1.93414	1.93638
7.5	0	0	0	0	0	0	0.24399	0.61566	0.88152	1.73156	1.88898	1.94407	1.96957	1.98342	1.99177	1.99719	2.00091	2.00357
8.0	0	0	0	0	0	0	0	0.43643	0.74860	1.74673	1.93157	1.99626	2.02621	2.04247	2.05228	2.05865	2.06301	2.06613
8.5	0	0	0	0	0	0	0	0.00212	0.59387	1.75352	1.96827	2.04343	2.07822	2.09712	2.10851	2.11591	2.12098	2.12460
9.0	0	0	0	0	0	0	0	0	0.41747	1.75229	1.99047	2.09599	2.13603	2.14779	2.16090	2.16942	2.17525	2.17943

1.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.1	0.00258	0.00115	0.00065	0.00041	0.00029	0.00021	0.00016	0.00013	0.00010	0.00003	0.00001	0.00001	0	0	0	0	0	0
1.2	0.01064	0.00472	0.00266	0.00170	0.00118	0.00087	0.00066	0.00053	0.00043	0.00011	0.00005	0.00003	0.00002	0.00001	0.00001	0.00001	0.00001	0
1.3	0.02460	0.01093	0.00615	0.00394	0.00273	0.00201	0.00154	0.00121	0.00098	0.00025	0.00011	0.00006	0.00004	0.00003	0.00002	0.00002	0.00001	0.00001
1.4	0.04487	0.01994	0.01122	0.00718	0.00499	0.00366	0.00280	0.00222	0.00179	0.00045	0.00020	0.00011	0.00007	0.00005	0.00004	0.00003	0.00002	0.00002
1.5	0.07182	0.03192	0.01796	0.01149	0.00798	0.00586	0.00449	0.00355	0.00287	0.00072	0.00032	0.00018	0.00011	0.00008	0.00006	0.00004	0.00004	0.00003
1.6	0.10580	0.04702	0.02645	0.01693	0.01176	0.00864	0.00661	0.00522	0.00423	0.00106	0.00047	0.00026	0.00017	0.00012	0.00009	0.00007	0.00005	0.00004
1.7	0.14713	0.06539	0.03678	0.02354	0.01635	0.01201	0.00920	0.00727	0.00589	0.00147	0.00065	0.00037	0.00024	0.00016	0.00012	0.00009	0.00007	0.00006
1.8	0.19611	0.08716	0.04903	0.03138	0.02179	0.01601	0.01226	0.00968	0.00784	0.00196	0.00087	0.00049	0.00031	0.00022	0.00016	0.00012	0.00010	0.00008
1.9	0.25302	0.11246	0.06326	0.04048	0.02811	0 02065	0.01581	0.01249	0.01012	0.00253	0.00112	0.00063	0.00040	0.00028	0.00021	0.00016	0.00012	0.00010
2.0	-	0.14140	0.07954	0.05090	0 03535	0.02597	0.01988	0.01571	0.01273	0.00318	0.00141	0.00080	0.00051	0.00035	0.00026	0.00020	0.00016	0.00013
2.5	-	0.34465	0.19386	0.12407	0.08616	0.06330	0.04847	0.03829	0.03102	0.00775	0.00345	0.00194	0.00124	0.00086	0.00063	0.00048	0.00038	0.00031
3.0	-	-	0.36797	0.23550	0.16354	0.12015	0.09199	0.07269	0.05888	0.01472	0.00654	0.00368	0.00236	0.00164	0.00120	0.00092	0.00073	0.00059
3.5	-	-	0.60758	0.38885	0.27004	0.19839	0.15190	0.12002	0.09721	0.02430	0.01080	0.00608	0.00389	0.00270	0.00198	0.00152	0.00120	0.00097
4.0	-	-	-	0.58723	0.40780	0.29961	0.22939	0.18124	0.14681	0.03670	0.01631	0.00918	0.00587	0.00408	0.00300	0.00229	0.00181	0.00147
4.5	-	-	-	0.83330	0.57868	0.42515	0.32551	0.25719	0.20833	0.05208	0.02315	0.01302	0.00833	0.00579	0.00425	0.00326	0.00257	0.00208
5.0	-	-	-	-	0.78433	0.57624	0.44119	0.34859	0.28236	0.07059	0.03137	0.01765	0.01129	0.00784	0.00576	0.00441	0.00349	0.00282
5.5	-	-	-	-	1.02621	0.75395	0.57724	0.45609	0.36944	0.09236	0.04105	0.02309	0.01478	0.01026	0.00754	0.00577	0.00456	0.00369
6.0	-	-	-	-	-	0.95925	0.73443	0.58029	0.47003	0.11751	0.05223	0.02938	0.01880	0.01306	0.00959	0.00734	0.00580	0.00470
6.5	-	-	-	-	-	1.19303	0.91342	0.72171	0.58459	0.14615	0.06495	0.03654	0.02338	0.01624	0.01193	0.00913	0.00722	0.00585
7.0	-	-	-	-	-	-	1.11484	0.88086	0.71350	0.17837	0.07928	0.04459	0.02854	0.01982	0.01456	0.01115	0.00881	0.00713
7.5	-	-	-	-	-	-	1.33927	1.05819	0.85713	0.21428	0.09524	0.05357	0.03429	0.02381	0.01749	0.01339	0.01058	0.00857
8.0	-	-	-	-	-	-	-	1.25413	1.01584	0.25396	0.11287	0.06349	0.04063	0.02822	0.02073	0.01587	0.01254	0.01016
8.5	-	-	-	-	-	-	-	1.46907	1.18995	0.29749	0.13222	0.07437	0.04760	0.03305	0.02428	0.01859	0.01469	0.01190
9.0	-	-	-	-	-	-	-	-	1.37975	0.34494	0.15331	0.08623	0.05519	0.03833	0.02816	0.02156	0.01703	0.01380
9.5	-	-	-	-	-	-	-	-	1.58554	0.39639	0.17617	0.09910	0.06342	0.04404	0.03236	0.02477	0.01957	0.01586

These properties would be determined from the geometric properties of the drain and smear pattern, the consolidation properties of the soil as determined by laboratory test, and the construction schedule. By entering either the proper curves or Table 5, the following geometric parameters are determined:

$$F_1(10,\ 2) = 0.93650,\ F_2(10,\ 2) = 0.66542,$$
$$G(10,\ 2) = 0.01273.$$

By substitution in Eqs. 27a and 27b, the numerical expressions for the excess pore pressure are:

$$\bar{u}/\bar{u}_0 = 2.66873\left[1 - e^{-1.87633T_h}\right]$$
$$0 \leq T_h \leq 0.2 \qquad (28a)$$

$$\bar{u}/\bar{u}_0 = 0.57373e^{-1.87633T_h}$$
$$0.2 \leq T_h \leq \infty \qquad (28b)$$

Direct substitution in Eqs. 28a and 28b will result in the pore pressure-time factor curve shown in Figure 22.

Variable Permeability, Construction Loading

The solution for the average excess pore pressure when the permeability of the soil varies linearly with the pore pressure and the loading is variable, has been developed using an approximate theory (1). This theory was developed for the case of drains in which no smear zone exists.

$$\frac{\bar{u}}{\bar{u}_0}(V_h) = \frac{F(n)}{8V_0}\left[1 - e^{-8V_h/F(n)}\right]$$
$$0 \leq V_h \leq V_0 \qquad (29a)$$

$$\frac{\bar{u}}{\bar{u}_0}(V_h) = \frac{F(n)}{8V_0}\left[1 - e^{-8V_0/F(n)}\right]$$
$$e^{-\left[8/F(n)\right](V_h - V_0)}$$
$$V_0 \leq V_h \leq \infty \qquad (29b)$$

$$V_h = B_h t/4d^2 \qquad (29c)$$

$$V_0 = B_h t_0/4d^2 \qquad (29d)$$

$$B_h = (\bar{k}_0 - k_f)/m\gamma_w \ln(\bar{k}_0/k_f) \qquad (29e)$$

The factor F(n) in Eqs. 29a and 29b is defined by Eq. 25d. The coefficient of consolidation permeability, B_h, must be evaluated by a laboratory test. It is best determined by using the average modulus of volume change, m, along with experimentally determined permeability values, $\bar{k}_0$ and k_f. These values of the coefficients

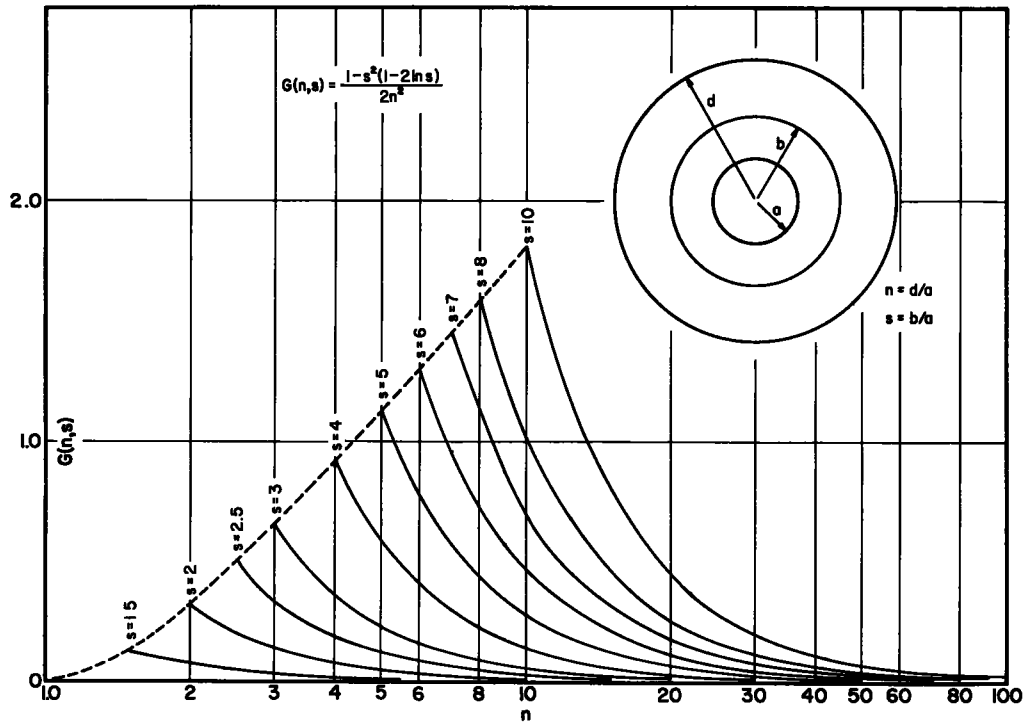

Figure 21. Geometric smear factor G (n, s) for equal-strain sand-drains (smear ratio parameter).

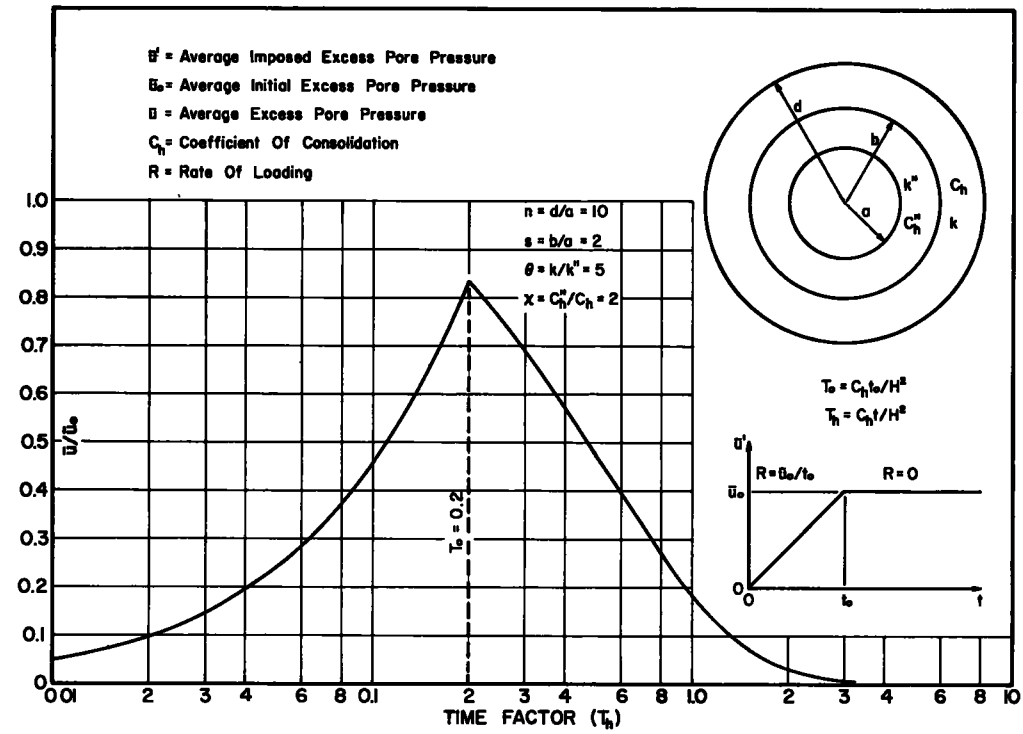

Figure 22. Average pore pressure example for construction loading of equal-strain sand-drains with constant permeability.

of permeability can be determined at beginning and end of consolidation by a radial drainage test.

The actual computation of Eqs. 29a and 29b follows similarly to the previously evaluted equations.

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The HIGHWAY RESEARCH BOARD was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the NATIONAL RESEARCH COUNCIL. The BOARD is a cooperative organization of the highway technologists of America operating under the auspices of the ACADEMY–COUNCIL and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the BOARD are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.