Estimating Efficient Spacing for Arterials and Expressways

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●THE PROCESS of preparing a transportation plan—that is, the actual sketching of lines representing street systems—is probably the least well thought-out area in transportation and city planning. By contrast, a great deal of thought has gone into methods for gathering origin-destination travel data, and into techniques for predicting future land use and travel volumes. Much skill and invention has also gone into the development of methods for testing plans, once they have been prepared (1) but the layout of street systems itself has largely remained an intuitive affair.

THE PROBLEM

It is the purpose of this paper to present some thoughts on this subject, focused mainly on the problem of finding the most efficient spacings of arterials and expressways. It is hoped that this will lead toward a more disciplined process of planning which will be based on an understanding of the principles which affect the location of transportation networks.

Taking as given the necessity of systems of streets to move conventional rubber-tired vehicles from one part of an urban region to another, the highway transportation planning problem (2) can be defined as the process of locating street systems (here temporarily restricted to local and arterial streets, and expressways) in accordance with some previously established criteria.

Speaking broadly, these criteria fall into two groups: criteria related to land development, and criteria related to transportation. Once the criteria have been established, the development of a transportation plan can be thought of as a series of steps, as follows:

1. Finding an abstract pattern of facilities which satisfies the criteria in some optimum fashion.
2. Placing the abstract pattern on maps and adjusting it to fit the real situation.
3. Predicting future traffic volumes on the facilities.
4. Evaluating the net economic return on the investment.

This paper deals mainly with finding an abstract pattern of facilities which satisfies the criteria in an "optimum" fashion. In so doing, some examples will be given using data of the Chicago Area Transportation Study (CATS). The application of these methods to the preparation of a transportation plan for the Chicago area is now under study.

The methods used to determine a pattern of transportation facilities are mathematical, in which the transportation criteria are dealt with explicitly. The results so obtained are reviewed from the viewpoint of land planning, but in a subjective manner. A single "optimum" solution cannot be claimed, therefore. The results, however, seem to be very good, particularly in view of the adjustments which must necessarily be made when fitting them to a real situation.

In reaching the desired pattern of street facilities, approximations of items 3 and 4 are reached. That is, the mathematics used to find a "best" pattern also yield estimates of future traffic volumes. The methods also are an important part of benefit-cost analysis because they find the spacing which minimizes community transportation costs.

In the present state of the art, traffic assignment and additional benefit-cost work should follow the development of the kind of transportation plan described here. The assignment and benefit-cost work can be looked at as both a check and a refinement of these results. The optimum spacing formula described here is based on a number of simplifying assumptions and is concerned with a general homogeneous area; full-fledged assignment and benefit-cost work will account for specific, particular and local conditions. It is hoped, of course, that in the future these steps can be combined into a general theory.

CRITERIA

The following are criteria which influence the spacing of arterials and expressways in urban regions. Not all of these criteria have been used in the methods described in this paper. The explicit inclusion of more criteria into the planning processes is something which awaits the completion of further research and the development of faster and more precise methodology.

Land Planning Criteria

Sufficient area must be provided in the spaces between the streets in a network for the efficient and pleasant conducting of the semistatic activities called land uses. The required land area is, of course, related to density of land development. This is a review criterion, considered after the spacing of arterials and expressways has first been determined.

Desirable Land Use-to-Road Relationships. This criterion is concerned with the relationships between street facilities and abutting land uses. It is a detailed criterion which can be applied only when an abstract pattern is fitted to a real stiuation.

Desirable Land Development Densities, from the Viewpoint of the Cost of Construction of Buildings and Related Facilities (Excepting Roads). Not considered in this report, this criterion needs to be the subject of additional research.

Desirable Land Development Densities, from the Viewpoint of Living and Operating Costs. This is not considered here and needs to be the subject of much additional research.

Transportation Planning

Travel Costs. These costs (primarily the value of personal time) are considered explicitly in the described method.

Construction Costs. These costs (including land acquisition and construction costs) are considered explicitly.

The Balance, on Each Facility, Between Traffic Volumes and Capacities. This criterion is considered explicitly, but as a review criterion after the spacing has been determined.

The Balance, by Area, Between Vehicle-Miles of Capacity and the Vehicle-Miles of Travel Demand. This is a review criterion.

Economic Criteria

The Minimization of the Sum of Construction and Travel Costs.

Developing a Plan Most Conducive to the Economic Growth of an Urban Region. This is a most difficult topic and could not be considered at this time.

AN OUTLINE OF THE METHOD

The following is a brief description of the method used to estimate efficient spacings for arterials and expressways. For the sake of brevity, not all terms are defined or qualified, nor are all assumptions made explicit. Complete detalis are given in succeeding parts and in the Appendices.

A key notion in this approach is the minimization of a community's highway transportation costs within framework of driver behavior. Highway transportation costs are taken as the sum of (a) construction costs and (b) travel costs for vehicle occupants.

Three street types are assumed: local streets, arterials and expressways. Speeds and construction costs on each type are given.

The number of trips generated per square mile per day is given, as is the distribution of trip lengths. The distribution of trip lengths is taken as stable over time, and in particular is taken as unaffected by changes in the street network. Costs to vehicle occupants are treated as a function of travel time only.

Total transportation cost is then expressed as equal to (1) the number of miles of each street type, times its unit construction cost, plus (2) the amount of time the average vehicle occupant spends on each facility times the number of occupants, with time converted to yearly costs and capitalized for the expected life period of the street type.

The number of miles of each street type, and hence construction costs, can be related to the spacing between streets of that type for a given area. Travel costs also are a function of this spacing. The sum of these costs can then be minimized, and hence the minimum-cost spacing can be determined, using the differential calculus, or by graphical means. Minimization can be carried out with respect to the spacing of each street type, or for any subset of street types. Thus, if local and arterial spacings have been determined historically (that is, if an area has become so completely developed that the construction of new arterials cannot be contemplated) a minimum-cost solution can be obtained in terms of expressway spacing alone.

Once the minimum-cost spacing has been determined, it can be reviewed with respect to other criteria, including design criteria, capacity criteria and land-planning criteria. The application of these criteria, either mathematically or subjectively, may suggest changes in spacing.

Examples are given of minimum-cost spacing for various parts of the Chicago area and the results are reviewed.

A STATEMENT OF HIGHWAY TRANSPORTATION COSTS

This section of the paper contains an explicit mathematical statement of highway transportation costs. Terms are defined and assumptions and simplifications are noted.

A major simplification that holds throughout is that streets exist in a grid form only, and construction and travel costs are both based on a grid network. Further research is needed to apply the techniques to non-gridded street systems, but it is not anticipated that the results will be greatly different.

Total Costs

Highway transportation costs can be written:

$$C = C_1 + C_2 \tag{1}$$

where

C = Total transportation cost
C_1 = Construction cost
C_2 = Travel cost

Construction Costs

Construction costs for a square area can be expressed as:

$$C_1 = 2S^2 \left(\frac{C_X}{x} + \frac{C_Y}{y} + \frac{C_Z}{z} \right) \tag{2}$$

where X, Y and Z are labels referring to the local, arterial and expressway street system, respectively, and x, y and z are the respective distances between streets in each street system. Thus C_X is construction cost per mile of X, C_Y is construction cost per mile of Y, and C_Z is construction cost per mile of Z. Total construction cost consists of cost per mile times miles of each street system. For a square with side S, the number of X streets on a side is equal to S/x (Fig. 1). Each X street has length S, so that the

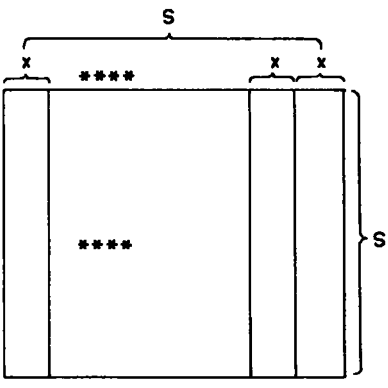

Figure 1.

number of miles of X in one direction is number of streets times length per street; this equals S (S/x) or (S^2/x). But there are the same number of X streets in a direction perpendicular to the original direction, so the total miles of X street is $2S^2/x$. Similarly, the total miles of Y and Z are $2S^2/y$ and $2S^2/z$, respectively. The multiplication of miles of street by construction cost per mile yields Eq. 2.

Travel Costs

General Form. Travel costs of vehicles is written:

$$C_2 = NK\left[\sum_{i=1}^{r-1}\frac{L_i}{v_X}F_i + \sum_{i=r}^{s-1}\left(\frac{2A}{v_X} + \frac{L_i - 2A}{v_Y}\right)F_i + \sum_{i=s}^{t}\left(\frac{2A}{v_X} + \frac{2B}{v_Y} + \frac{L_i - 2A - 2B}{v_Z}\right)F_i\right] \quad (3)$$

This equation can be expressed in words in a fairly straightforward manner.

Travel costs of vehicle occupants (C_2) consist of the following:

1. Number of daily trips (N)
2. Multiplied by hours of travel of the average trip (the expression within square brackets)
3. Multiplied by the value of an hour. The multiplications to this point yield costs per day for all vehicle occupants. This value in turn is multiplied by—
4. Number of weekday equivalents per year, which yields costs per year
5. Costs per year are assumed to occur for a given number of years and are discounted to the present at an interest rate of 5 percent. In this study, the given number of years was taken at 30 yr.

Items 3, 4 and 5 when multiplied together yield the value of K appearing in Eq. 3.

Hours of Travel of the Average Trip. The expression in brackets (Eq. 3) consists of the hours of travel of the average trip. The distribution of trip lengths, F_i, is one of the givens in this expression. Trip length is L_i, where i refers to a given class of trip lengths; $\overline{L}_i$ is average trip length of the class, and F_i is the frequency of occurrence of that class. These items are given in Table 1 for the entire Chicago Study Area; however, Table 1 gives F_i in terms of airline distance l_i rather than over-the-road distance L_i. It can be shown that L_i approximately equals 1.3 l_i. (Much of the notation of this paper is developed to handle the translation from airline to over-the-road distance. This is because there are some contexts where one form is more convenient, others where the other form is preferable.)

The expression in brackets (Eq. 3) consists of three parts; these are the average amounts of time spent in travel on the X, Y and Z system, respectively. Trips for the classes i = 1 to r - 1 are short trips which use the X network exclusively, trips for the classes i = r to s - 1 are longer trips which use both the X and Y network, and trips for the classes i = s to t are long trips which use all three networks.

It is argued that all trips begin their journeys on the X network (local streets) and if long enough, move to the Y network (arterials) and then to the Z network (expressways). The values v_X, v_Y and v_Z are the speeds that hold on the respective facilities, while A is the average distance traveled in moving from an X street to a Y street, and B is the average distance traveled in moving from a Y street to a Z street.

Thus, the first part of the bracketed expression (Eq. 3) consists of those trips with length less than 2A which presumably can use local streets only. The average trip length, $\overline{L}_i$, is divided by speed in miles per hour to yield hours traveled on the facility. This is multiplied by the frequency of occurrence of this trip type, F_i, to obtain the average travel time of this trip. The value 2A is the "over-the-road" trip length cutoff point because it is argued that a trip that uses arterials will travel A distance from origin to arterials, and then will travel A distance from arterials to destination. Hence, a trip using both locals and arterials will, on the average, travel 2A on local streets and the remainder of its trip length on arterials. This is stated formally as part of the bracketed expression (Eq. 3): $2A/v_X$ is the travel time on locals, $(\overline{L}_i - 2A)/v_Y$ is the travel time on arterials.

Finally, a "long" trip that can use expressways will travel 2A on locals, 2B on arterials, and the remainder of its trip length on expressways. The trip will go A miles on locals from origin to arterial, B miles on the arterial to an expressway, and then, leaving the expressway, will travel B miles on arterials, and A on locals to its destination. This is indicated in the third term in the bracketed expression (Eq. 3).

To recapitulate:

The component of average travel time is computed for each trip class, and the times are summed to yield total travel time for the average vehicle.

This in turn is multiplied by number of trips per day to obtain average daily travel time, which is then converted to a capitalized value, using K.

The determination of the values of A and B, and the assumptions and simplifications involved, are discussed in the section on Estimating Distance Traveled by Street Type.

TABLE 1

FREQUENCY DISTRIBUTION OF TRIP LENGTHS, CHICAGO STUDY AREA, 1956

Class, i	Range of l_i[1] (mi)	l_i[1] (mi)	F_i
1	0 to 0.99	0.5	0.202
2	1 to 1.99	1.5	0.227
3	2 to 2.99	2.5	0.121
4	3 to 3.99	3.5	0.088
5	4 to 4.99	4.5	0.070
6	5 to 5.99	5.5	0.051
7	6 to 6.99	6.5	0.043
8	7 to 7.99	7.5	0.037
9	8 to 8.99	8.5	0.027
10	9 to 9.99	9.5	0.020
11	10 to 10.99	10.5	0.018
12	11 to 11.99	11.5	0.015
13	12 to 12.99	12.5	0.011
14	13 to 13.99	13.5	0.009
15	14 to 14.99	14.5	0.007
16	15 to 15.99	15.5	0.005
17	16 to 16.99	16.5	0.005
18	17 to 17.99	17.5	0.004
19	18 to 18.99	18.5	0.003
20	19 to 19.99	19.5	0.003
21	20 +	25.0	0.034
Total			1.000

[1] Airline distance.

<u>The Value of K.</u> The value of K used in the applications of this formula was $7,500. K contains three components: time value per hour, set at $1.43; weekday equivalents in a year, set at 340; and an appropriate interest plus depreciation charge, set at 0.065 to square with a market interest rate of 5 percent and an assumed asset life of 30 yr.

Time value was based on the following considerations. Of total vehicle trips, 14.5 percent were truck trips, 85.5 percent were auto trips. The value of truck driver time was set at $3.00 per hour, which is the going wage rate. For automobile occupants, the value of time was set at $1.00 per hour for wage earners, because $1.00 is the minimum wage; and it was assumed that three-fourths of auto occupants were wage earners (to account for trips by non-wage earners), yielding $0.75 as the average value of occupant time. There were 1.56 occupants per auto, so total time value per auto was $1.17 per hour. Then weighting truck and auto hourly value by their respective percentages yielded $1.43.

The number of weekday equivalents was taken as 340, since weekends and holidays have only about 77 percent of the traffic of weekdays.

Finally, a yearly income stream can be converted to a present capitalized value by dividing by an appropriate gross interest rate. The gross interest rate consists of the market interest rate plus a depreciation component for assets of limited life. Arbitrarily setting the life of a highway at 30 yr, and taking the market rate at 5 percent, implies a gross interest rate of $6\frac{1}{2}$ percent. Now:

$$\frac{\$1.43 \times 340}{0.065} = \$7,480$$

Hence, K was taken as $7,500.

ESTIMATING DISTANCE TRAVELED BY STREET TYPE

A question posed in the preceding section was: what are the values of A and B, that is, what is the average distance in travel from a random point on the X network to the Y network, and what is the average distance from a random point on Y to the Z network? Some approximations to A and B are developed in this section and used in succeeding work.

Estimates Using an Analytic Approach

In developing estimates of A and B, a mathematical model was employed to obtain an initial set of estimates. In this model it was assumed drivers would move to a higher speed network as soon as possible and, in doing so, would take the shortest possible route in terms of distance. Estimates of A and B obtained here are termed a and b; these estimates were obtained by mathematical induction.

Three steps were involved; these were: (1) specifying prevailing conditions and assumptions; (2) expressing the first step in the form of a summation; and (3) applying standard summation formulas to obtain a general equation.

The results obtained are as follows:

$$a = \text{average trip length on X in miles} = \frac{y}{6}\,\frac{(y + x)}{(y - x)} \tag{4}$$

$$b = \text{average trip length on Y in miles} = \frac{1}{6}\,(z + 3M - \frac{y^2}{z}) \tag{5}$$

where

$$M = \frac{y}{6}\,\frac{(2y - x)}{(y - x)} \tag{6}$$

These values can be approximated as follows:

$$a = y/6 \tag{7}$$

$$b = y/6 + z/6 \tag{8}$$

An examination of the ratio of approximation to actual value indicated the approximation would generally contain an error of less than 10 percent.

Some points worthy of note are as follows:

1. a and b depend on network spacing.
2. In the application of a and b, it should be remembered that the "average trip" is being considered; thus, it is argued that expressway usage, for an average trip, occurs only for trip length greater than 2a plus 2b.
3. Because simplifying assumptions were necessary, a and b will probably vary somewhat from actual behavior.

In an attempt to take this into account, some experimental work was carried out and this led to some modification of a and b. That work is described in the section on Experimental Method. The remainder of this section describes how a and b were derived using a mathematical model. Readers uninterested in the mathematical detail may turn directly to the section on Experimental Method.

Trip Length From the X Network to the Y Network. In examining average distance from the X network to the Y network, it was assumed that all trip origins were located at intersections of X streets. (In this formulation, no trips arise on the Y network. The latter case could be developed as a variation.)

An example of the situation specified is shown in Figure 2. Here, a square is formed by four arterials, and it is assumed that the ratio of y to x is 8 to 1; for example, arterials are 1 mi apart, local streets are ⅛ mi apart. As a consequence, there are 7 local streets between two parallel arterials, and 49 points of trip origin in a square formed by four arterials.

Figure 2 shows the distance of a trip origin point from the closest street on the Y network (Y_1 through Y_4). Units of dis-

Y_4

	1	1	1	1	1	1	1	
	1	2	2	2	2	2	1	
	1	2	3	3	3	2	1	
Y_1	1	2	3	4	3	2	1	Y_3
	1	2	3	3	3	2	1	
	1	2	2	2	2	2	2	
	1	1	1	1	1	1	1	

Y_2

Figure 2. Distance of trip origin points from Y network.

tance are in terms of x units, so that a point one unit away from a Y street is x miles from Y. The circled point is 2 units from Y_2, 3 from Y_1, 5 from Y_3 and 6 from Y_4. Its distance from the Y network for trip making purposes is listed as 2, which is its shortest distance from the Y network.

Of the 49 points of trip origin, 24 are 1 unit away from Y; 16 are 2 units away; 8 are 3 units away; and 1 is 4 units away. The average distance of a trip origin from Y is thus:

$$\frac{24\ (1) + 16\ (2) + 8\ (3) + 1\ (4)}{49} = \frac{84}{49} = 1.714 \text{ units.}$$

By drawing squares with varying points in a given line of the square, an over-all formula can be derived. The distribution of distance from Y for squares of varying size is given in Table 2 and a general formula can be derived. In general, a square formed by legs of the Y network will contain

$$\frac{y}{x} - 1 \text{ points along a given X street}$$

The total number of points within a square will therefore be

$$\left(\frac{y}{x} - 1\right)^2$$

The following formula expresses the average distance traveled within the square to get to the Y network:

$$a = \frac{x}{W_2}\left[\sum_{k=1}^{H-1} 4\ (w + 1 - 2k)\ k + dH\right]$$

where

$$w = y/x - 1$$

$$H = \frac{w+1}{2}, \text{ with } \frac{1}{2} \text{ values rounded to the next highest number}$$

$$d = 0 \text{ if w is even}$$

$$\quad 1 \text{ if w is odd}$$

Expanding the right-hand side of the a relation and applying standard summation, formulas yielded:

$$a = \frac{y}{6}\left[\frac{y/x + 1}{y/x - 1}\right]$$

Trip Length From the Y Network to the Z Network. The work involved in finding b was essentially an extension of the technique used in finding a. A key aspect of the approach was to exhibit the source of trips to each "leg" in the Y network. This is shown in Figure 3 where a diamond drawn around each leg shows the source of trips to the leg. (This is for a network with z/y = 8.) Trips arriving at the leg are then sent to the Z network. In the shaded diamonds, all trips travel directly to the Z road using only the Y leg of origin. Average trip length in these diamonds equals the value of the midpoint along the leg. In the unshaded diamonds, trips travel to the nearest Y leg perpendicular to the Z network and use that to get to the Z road. Average distance traveled within the unshaded diamond equals a value labeled M rather than the midpoint. Figure 3 exhibits two diamonds

TABLE 2

DISTRIBUTION OF DISTANCE FROM Y NETWORK

Number of Points Per Line	Distance From Y Network (in x units) 1	2	3	4	5	6
1	1	0	0	0	0	0
2	4	0	0	0	0	0
3	8	1	0	0	0	0
4	12	4	0	0	0	0
5	16	8	1	0	0	0
6	20	12	4	0	0	0
7	24	16	8	1	0	0
8	28	20	12	4	0	0
etc.						

which, for their pattern, are closest to the Z network; in each is shown distance of origin point from Z and the relative frequency of trips arriving at the origin point from X streets.

The generalized version of this case was developed. The summation expressing it is:

$$b = \text{numerator}/\text{denominator}$$

$$\text{numerator} = \text{sum of Pattern I diamond values plus sum of Pattern II diamond values}$$

$$= y/2\ (z/y-1) + \sum_{k=1}^{(z/2y)-1} (ky + y/2)\ (z/y - 1 - 2k)$$

$$+ \frac{1}{2} M^{(z/y)} + \sum_{k=1}^{(z/2y)-1} (ky + M)\ (z/y - 2k)$$

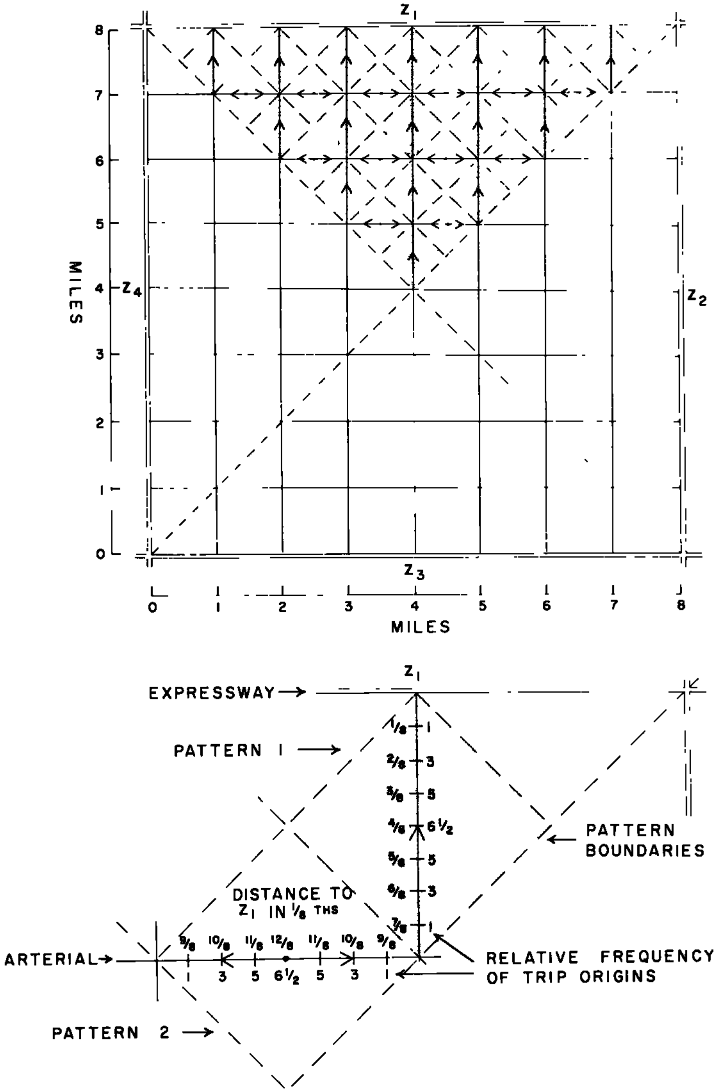

Figure 3. Source of trips for each Y leg in the Y network.

$$\text{denominator} = \text{number of cases} = \frac{1}{2} z/y + (z/y-1) + \sum_k (z/y-2k) + \sum_k (z/y-1-2k)$$

Application of summation formulas and simplifying, yields:

$$b = 1/6\left[z + 3M - y^2/z\right]$$

where

$$M = y/6 \; \frac{2y-x}{y-x}$$

Experimental Method

An experiment was conducted which provided an alternative method of estimating average trip length on the X, Y, and Z street systems and provided a number of other clues as to the usage made of these systems. Using data from the experiment, values of a and b (determined analytically) could be modified to give A and B, the average distances traveled on local and arterial streets. The experiment further permitted a simplified statement to be developed giving break points in airline trip lengths at which vehicles start to use higher speed systems. This is an approximation to reality but the evidence developed by the experiment indicates it is a reasonable approximation.

Conduct of Experiment: Terms, Definitions and Assumptions. A large sheet of paper (30 in. x 30 in.) was ruled precisely with grid lines representing the X, Y and Z systems. The X system was scaled to represent 0.125-mi intervals, the Y system, 1.0-mi intervals, and the Z system, 4.0-mi intervals.

Sticks were cut to represent airline journeys of 2, 4, 6, 8 and 10 mi. These sticks were then thrown at random to land on the paper and their positions were carefully marked. Thirty throws were made with each stick.

The airline journeys were then assigned to the X, Y and Z systems on the assumption that the trip would take the shortest time path through the gridded network. Speeds were taken as in the ratio X: Y: Z = 1:2:4. This is not unrealistic, considering local streets at 12 mph, arterials at 25 mph, and expressways at 50 mph.

The over-the-road distance traveled on each street type was recorded, and the 30 records for each airline trip length were averaged.

Over-the-road distances were computed on three different assumptions as to ramp spacing. First, ramps or connections were assumed so that a trip could enter the Z (expressway) system at each intersection of that system with the Y (arterial) system; that is, at 1-mi intervals. Second and third assumptions permitted access only at 2Y and 4Y (2- and 4-mi) intervals.

Results of Experiment. The results of the experiment are given in Table 3 and shown in Figures 4, 5, and 6. Generally these results are about what one would expect.

TABLE 3

AVERAGE OVER-THE-ROAD DISTANCE IN MILES TRAVELED ON LOCAL, ARTERIAL, AND EXPRESSWAY SYSTEMS, AS A FUNCTION OF AIRLINE TRIP LENGTH AND RAMP SPACING

Airline Trip Length (mi)	Ramp Spacing (mi) 1: Average Over-the-Road Trip Length (mi)			2: Average Over-the-Road Trip Length (mi)			4: Average Over-the-Road Trip Length (mi)		
	X	Y	Z	X	Y	Z	X	Y	Z
2	0.38	1.82	0.63	0.40	2.10	0.27	0.41	2.27	-
4	0.46	2.35	2.86	0.49	2.94	2.04	0.48	3.33	1.86
6	0.38	2.07	6.07	0.40	2.47	5.73	0.39	3.51	4.70
8	0.39	2.07	8.37	0.41	2.60	7.87	0.41	3.89	6.80
10	0.37	1.77	11.17	0.38	2.37	10.80	0.42	3.71	9.67

The results are averages for trips of different airline length. Some trips of a particular length may have a high proportion of their length on a given facility and others very little. The averages for all "throws" are as shown.

The proportions driven on local streets for trips of airline length 1 mi is understated by the graph, which simply joins plots of observations. The same amount of travel (roughly 0.4 mi) is probably driven on local streets by trips of airline length 1 mi as for longer trips.

Interpretation of Results. As airline trip length increases, it is more probable that a higher proportion will be on higher speed facilities.

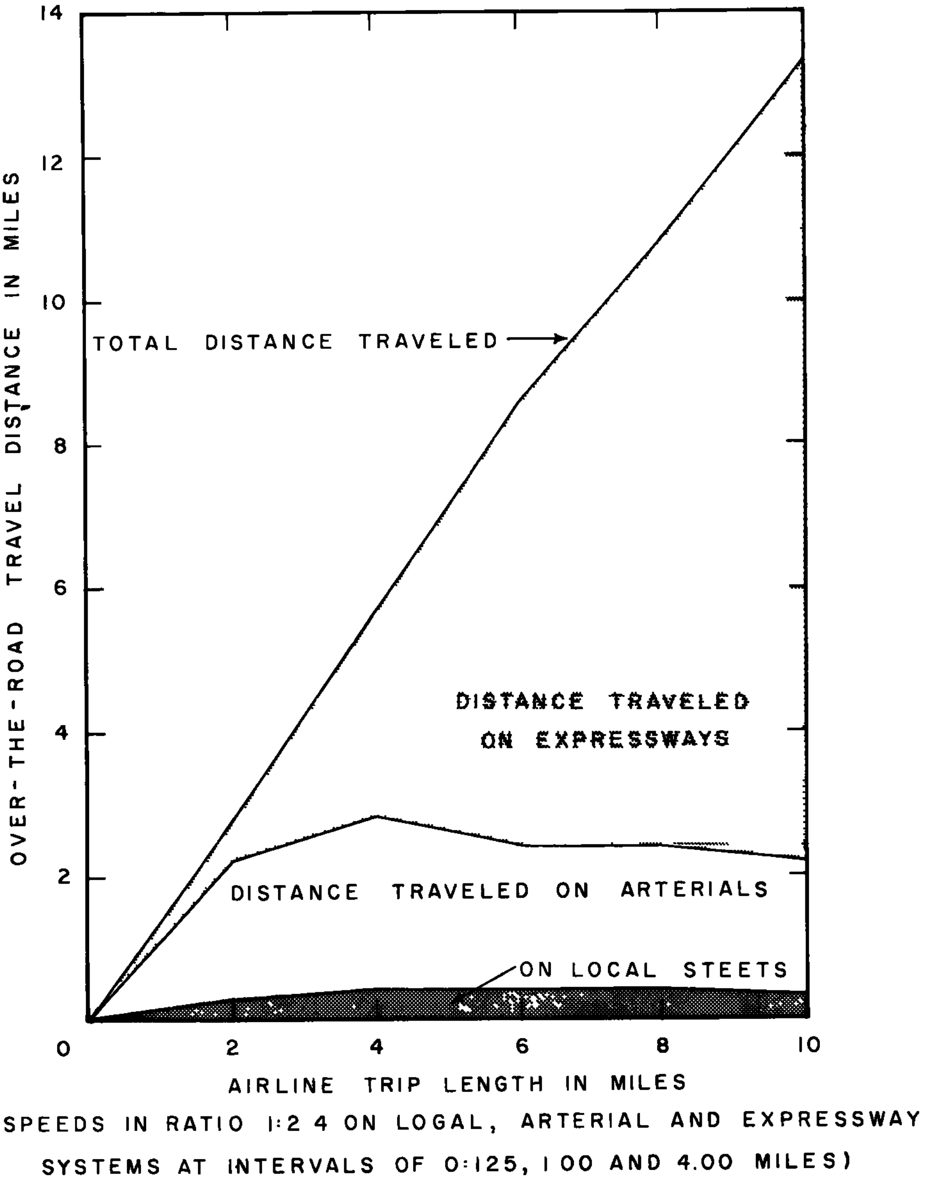

Figure 4. Use of local and arterial streets and expressways by trips of different airline length, with ramp spacing at 1-mi intervals.

When airline trip length reaches certain points, use of local and arterial streets ceases to rise and stabilizes at a certain level.

As airline trip length increases beyond a certain point, adverse travel (travel in the wrong direction in order to reach a higher speed facility) probably becomes more profitable. At this point, over-the-road travel on the lower speed facility appears to decline slightly (see, for example, the drop in arterial usage between 4 and 6 mi of airline trip length in Figure 4).

Effects of Different Ramp Spacings. As ramp spacing increases, use of expressways declines and use of arterials rises. It was estimated for the results given in Table 3 that the vehicle-miles on expressways decline about 11 percent while the vehicle-miles on arterials rise more than 20 percent as a result of the change in ramp spacing from 1 to 2 mi.

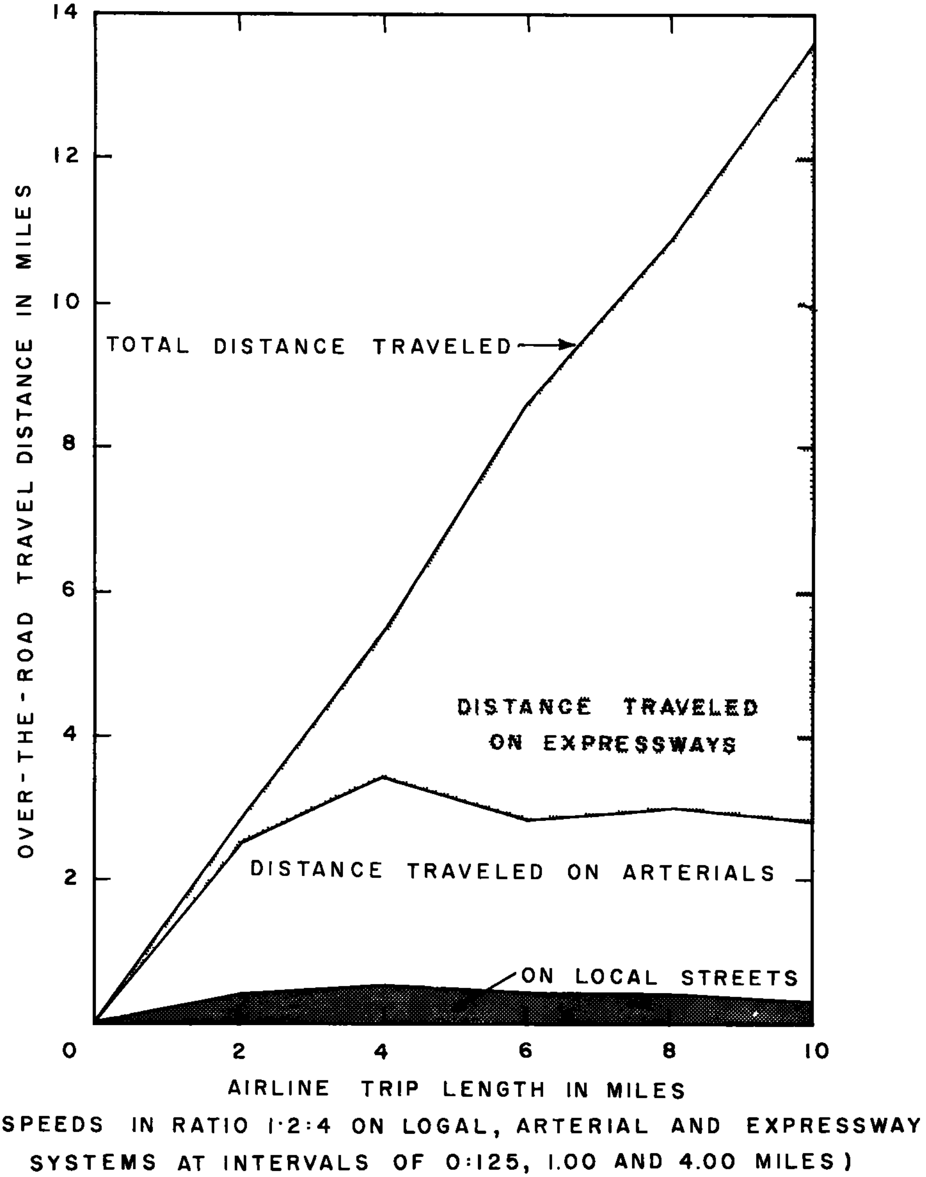

Figure 5. Use of local and arterial streets and expressways with ramp spacing at 2-mi intervals.

Combining Analytic and Experimental Methods

The analytic method is an attempt to answer the question: what is the average distance from a random point on one network to the nearest point on the next higher speed network? The experimental method furnishes approximate answers to the questions: (1) How far do vehicles actually travel in moving from one network to another? (2) Is the simplified model of driver behavior used in minimizing costs really a good representation of reality?

In answering the first question, the evidence developed by the experimental method indicated a and b were understatements of A and B, respectively. This probably occurred because the analytic method cannot account for adverse travel which is occasioned when a trip goes in the wrong direction in order to minimize total journey time. The

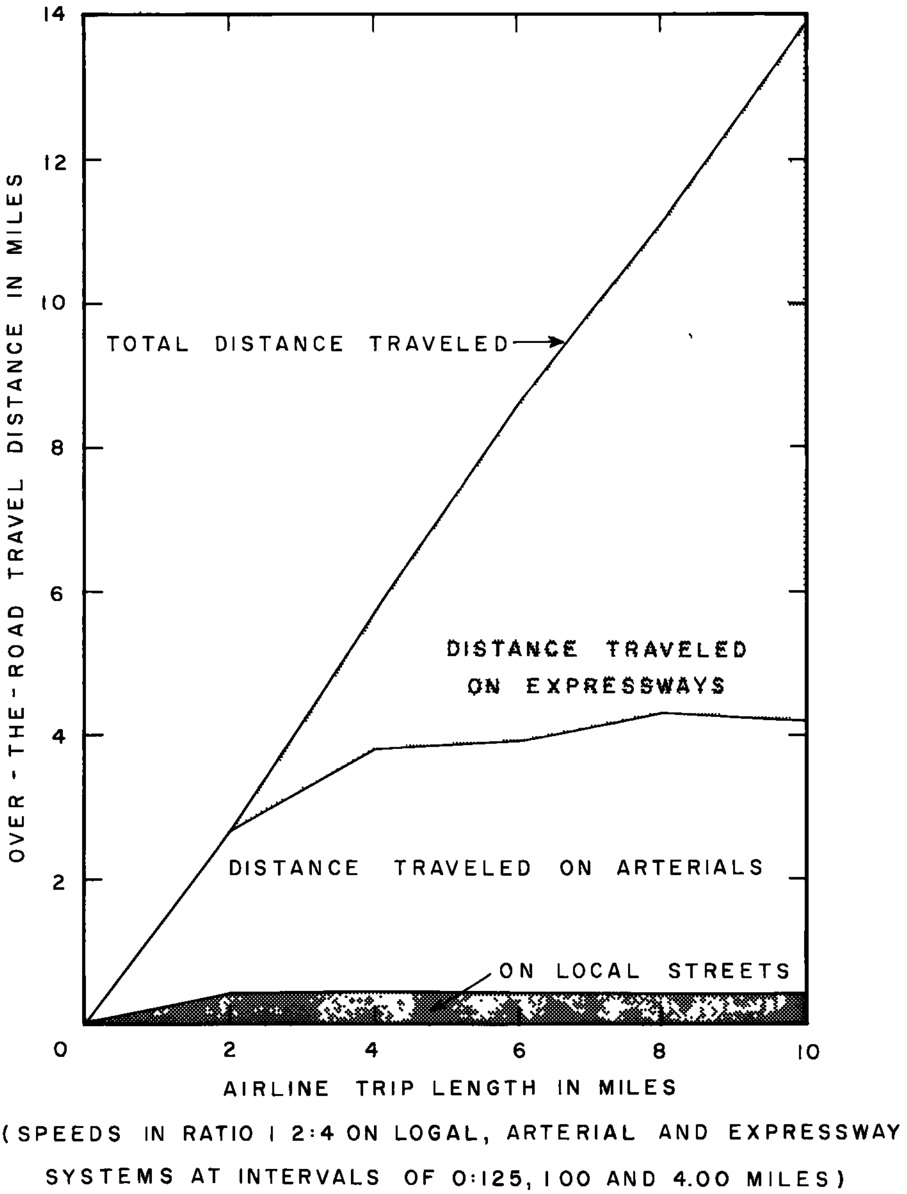

Figure 6. Use of local and arterial streets and expressways with ramp spacing at 4-mi intervals.

values of A and B obtained in the particular case examined by the experimental approach were approximately 20 percent greater than a and b. It was assumed that this effect would prevail for all cases: therefore, a and b were factored by 1.20. These factored values were taken as the values of A and B to be used in Eq. 3; that is, $1.2a = A$, $1.2b = B$, with a taken as $y/6$, b taken as $\frac{y + z}{6}$.

With respect to the second question, the model of driver behavior used in minimization, states that for trips with over-the-road trip length between 0 and 2A, only local streets are used; trips with lengths between 2A and 2B use both locals and arterials, with local use for the first 2A of length, and arterials thereafter; finally, trips with lengths greater than 2A + 2B use expressways for that part of the trip beyond 2A + 2B. In actuality, however, the distribution of travel among street types for a given trip length presents a more complicated picture. Thus, short trips with length less than 2A will not be found exclusively on local streets; some of these trips may use arterials or expressways. Similarly, trips below 2A + 2B in length may use expressways. On the other hand, it is possible that some trips longer than 2A + 2B do not use expressways. Figure 4 shows actual use of the street systems as approximated by the experimental results. Use is presented as a function of airline trip length (rather than over-the-road trip length). This shows that some use of each street type occurs for every trip length (This is within the limits of the experiment as it was conducted, which did not include estimates of the use made of different systems by trips of airline lengths less than 2 mi.)

In order to make this complicated expression of use agree with the previously posited statement of driver behavior (as stated in the basic cost equations leading to the minimum cost statements of Eqs. 2 and 3), the patterns of use shown in Figure 4 were deliberately simplified. The results appear in Figure 7, which is not a bad approximation of the results shown in Figure 4.

Had the experiment revealed that the pattern of driver behavior was markedly different from the original statement of driver use, then that statement would have had to be revised. Actually, the original statement intuitively approximated reality very closely; that is, the original statement, if graphed, would yield Figure 7. In the future the experimental approach may be conducted with greater precision by use of a computer, and the minimum-cost formula may be altered to include these more precise patterns of driver use.

In Figure 7 it should be noted that the break points between street use are α and β; this is because the trip length axis is in airline units; α and β correspond to 2A and 2A + 2B in over-the-road units. Thus:

$$\alpha = \frac{2A}{1.3} = \frac{2.4a}{1.3} \tag{9}$$

$$\beta = \frac{2(A + B)}{1.3} = \frac{2.4\,(a + b)}{1.3} \tag{10}$$

To recapitulate, it was originally stated that trips of airline lengths between o and α would only use local streets, that trips with airline lengths between α and β would also use arterial streets, and that trips with airline lengths greater than β would use all three systems. Estimates of the use made of each type were obtained analytically. These estimates were revised by experimental means, which also substantiated the original statement of use patterns.

MINIMIZATION OF TOTAL TRANSPORTATION COSTS

In the section on A Statement of Highway Transportation Costs, construction costs and travel costs were stated in terms of the spacing of y and z. The section on Estimating Distance Traveled by Street Type developed values of A and B in terms of y and z.

The development has now proceeded to a point where transportation costs can be minimized with respect to the spacing of the various street networks. It is assumed at

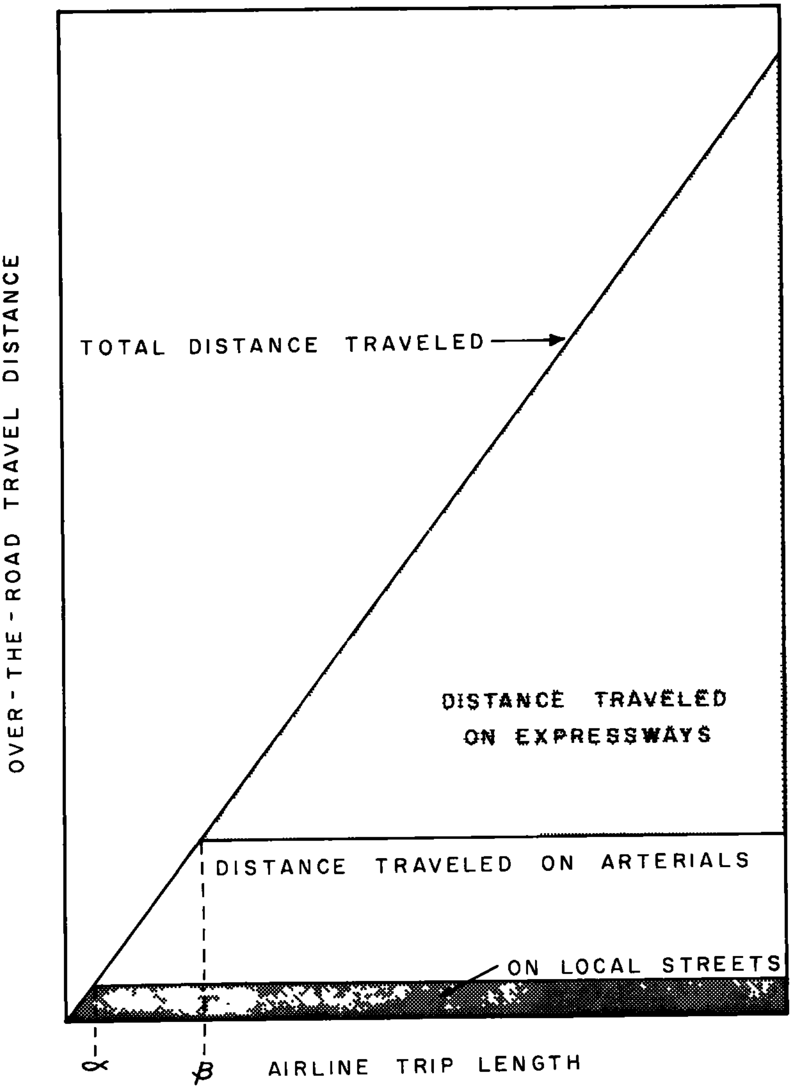

Figure 7. Simplified use of local and arterial streets and expressways by trips of different airline length.

this point that the spacing of x streets is ⅛ mi, so that minimization is carried out only with respect to y and z. Further, for some parts of urban regions, it seems realistic to argue that y spacing is fixed, having been determined historically, so that minimization for those parts is carried out only for z.

Minimization is obtained using the differential calculus; the partial derivatives of cost with respect to z and with respect to y are set equal to zero. Generally, solving the resultant equations (for y and z) would conclude the work. However, in this problem, differentiation is carried out for an expression involving a series of summations; the limits of these summations depend on A and B, and the latter depend on y and z. As a consequence, an iterative process had to be developed, so that all relationships posited did, in fact, hold. The final values of y and z obtained through the iterative technique do insure a minimization of cost; these values of z and y are defined as the optimum spacings of Z and Y streets.

The Differentiation of Cost

Using the work in the sections on A Statement of Highway Transportation Costs and Estimating Distance Traveled by Street Type, transportation costs can be written

$$C = 2S^2\left(\frac{C_X}{x} + \frac{C_Y}{y} + \frac{C_Z}{z}\right)$$

$$+ NK\left[\sum_{i=1}^{r-1} \frac{\overline{L}_i}{v_X} F_i + \sum_{i=r}^{s-1}\left[\frac{2.4\,(y/6)}{v_X} + \frac{\overline{L}_i - 2.4\,(y/6)}{v_Y}\right] F_i + \sum_{i=s}^{t}\left(\frac{2.4\,y/6}{v_X} + \frac{2.4\,(y/6+z/6)}{v_Y} + \frac{\overline{L}_i - 2.4\,(y/6+y/6+z/6)}{v_Z}\right)F_i\right] \quad (11)$$

This is a restatement of Eq. 3 with final values of A and B inserted. Minimization of cost occurs when

$$\frac{\partial C}{\partial z} = 0, \text{ and } \frac{\partial C}{\partial y} = 0$$

Thus, the derivative of cost with respect to z is:

$$\frac{\partial C}{\partial z} = \left[\sum_{i=s}^{t}\left(\frac{2.4}{6v_Y} - \frac{2.4}{6v_Z}\right) F_i\right] NK - \frac{2S^2 C_Z}{z^2} = 0 \quad (12)$$

Simplifying, and defining

$$P_s = \sum_{i=s}^{t} F_i \qquad D = \frac{N}{S^2} = \text{density} \qquad V_{YZ} = \frac{1}{v_Y} - \frac{1}{v_Z}$$

Yields this final equation for z:

$$z = 2.24\sqrt{\frac{C_Z}{K\,D\,V_{YZ}\,P_s}} \quad (13)$$

A similar process yields this equation for y:

$$y = 2.24\sqrt{\frac{C_Y}{K\,D\,(P_r\,V_{XY} + P_s\,V_{XYZ})}} \quad (14)$$

where

$$P_r = \sum_{i=r}^{s-1} F_i$$

$$V_{XYZ} = \frac{1}{v_X} + \frac{1}{v_Y} - \frac{2}{v_Z}$$

$$V_{XY} = \frac{1}{v_X} - \frac{1}{v_Y}$$

Eqs. 12-14 give the minimum cost spacings for the Y and Z networks. For a numerical solution of these equations, an iterative technique must be employed. This is because the values P_r and P_s depend on the cutoff points separating the parts of a trip in terms of the network used. This is shown in Figure 8. The value P_s is that part of the trip frequency distribution between 2B and ∞, measuring trip length in over-the-road distance.

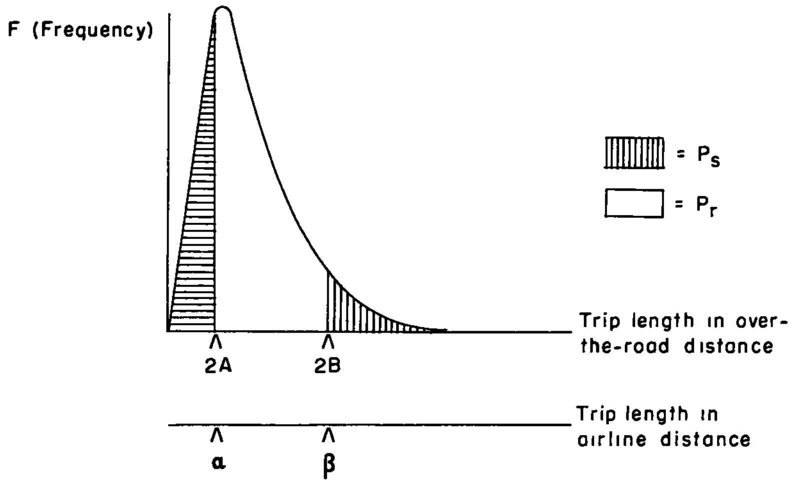

Figure 8. Trip length frequency distribution.

Thus, P_s depends on B, which in turn depends on y and z. The same sort of remarks apply to P_r. Because of this dependance on y and z, an iterative technique had to be developed to find values of y and z from Eqs. 13 and 14, which would be consistent with all the relations involved in this procedure.

The Iterative Technique

In iterating to final values of z and y, two cases were considered. These were: (1) y is given, only z is to be determined, (2) both y and z are to be determined.

For Case 1, there are in effect three equations in three unknowns. These are

$$z_1 = 2.24\sqrt{\frac{C_Z}{K\,D\,V_{YZ}\,P_s}} \tag{13a}$$

$$z_2 = 3.25\,\beta - 2y \tag{15}$$

Eq. 15 stems from

$$\beta = \frac{2.4a + 2.4b}{1.3}$$

$$P_s = P_s(\beta) \tag{16}$$

is a particular value of Case 1, and Eq. 16 is involved in the frequency distribution function.

In the iterative process, an arbitrary value of β is selected. This implies a corresponding value of P_s, from Eq. 16. The insertion of P_s in Eq. 13 yields an initial value for z_1, labeled z_1 (1). Similarly, the insertion of the original β and the given y in Eq. 15 yields an original value of z_2, labeled z_2 (1). In this initial set of computations, z_1 will probably differ significantly from z_2. Hence, a second value of β is picked, and corresponding values of z_1 and z_2 are computed, labeled z_1 (2) and z_2 (2). The second value of β is selected using this rule; if $z_2(1) > z_1(1)$, try a lower β if $z_2(1) < z_1(1)$, try a higher β. Given the values of z_1, z_2 and β for the two series of computations, a final value of β and z can be obtained by linear interpolation. The final value of z occurs where $z_1 = z_2$. The interpolation is shown in Figure 9.

The z_1 points are connected by a straight line, and the z_2 points are similarly con-

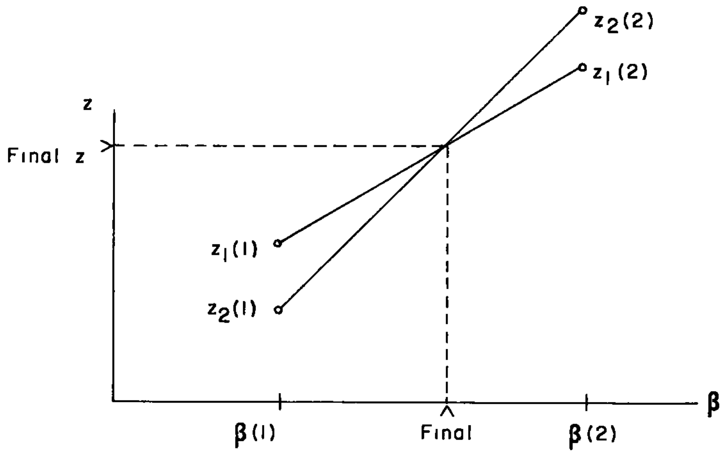

Figure 9. Iteration of z.

nected. The intersection of these lines yields values of β and z which should approximately equal the final iterative values desired.

The same sort of procedure—in expanded form—is applied in Case 2, where both y and z are to be determined. Three more equations in three more unknowns are added. These are:

$$y_1 = 3.25\,\alpha$$

from
$$\alpha = \frac{(1.2)2a}{1.3}, \quad a = y/6 \tag{17}$$

$$P_r = P_r(\alpha, P_s) \tag{18}$$

$$y_2 = 2.24\sqrt{\frac{C_Y}{K\,D\,(P_r V_{XY} + P_s V_{XYZ})}} \tag{19}$$

Iteration proceeds in this way: an arbitrary value of y is picked, termed y_1. For y_1, a final optimal z is obtained by iteration, as indicated for Case 1. This implies a corresponding P_s and P_r, which in turn implies a corresponding y_2, which probably differs from y_1. This y_2 is used to pick a new value of y_1, and the procedure is repeated, yielding a second set of values, $y_1(2)$ and $y_2(2)$. The values obtained can now be used to find a final z and y. The y_1 values are connected by a line, and the y_2 values are connected by a line. The intersection yields final values for y and z.

It may be noted that y converges quite quickly; that is, the initial y_2 is not very far from the final value of y.

A detailed description of the iterative process and a running numerical example appear in Appendix B.

Interpretations and Planning Principles

An examination of Eqs. 11-14 leads to certain conclusions or interpretations. These have some real value as providing principles which affect the layout of systems of arterials and/or expressways, as follows:

1. The minimum-cost spacing of arterials and expressways becomes greater as construction cost per miles increases.
2. The minimum-cost spacing of arterials and expressways becomes less as the value of personal time increases. Incidentally (and most undemocratically) this implies a need for closer spacing in higher income areas.

3. The minimum-cost spacing of arterials and expressways decreases as traffic density increases.
4. If the speed of expressways increases relative to that of arterials and local streets, then the minimum-cost spacing of expressways becomes less.
5. If trips become longer, then expressway minimum-cost spacing becomes less.
6. If arterial speeds increase relative to those of expressways (as through improved signalization and traffic controls) then expressway minimum-cost spacing becomes greater.

It may appear that these are common sense principles, and indeed they are. One of the values of having worked through the mathematics, however, is that these principles can be stated clearly and unequivocably, and that the quantitative effect of changes in the different variables can be estimated.

A Graphical Method of Estimating Minimum-Cost Spacings

A method of checking the minimum-cost spacing formulas given in Eqs. 11-14 was developed. This method permits a graphical presentation of the minimum-cost spacing, and is therefore of value as indicating in simple terms why the minimum-cost spacing formulas work.

This alternative method is a simplified technique. Only tow speeds are assumed—expressway speeds and non-expressway speeds. Travel is similarly assigned to the expressway network and to the non-expressway networks. Finally, either arterial or expressway spacing must be taken as a fixed item, allowing the other to vary.

Construction Cost. Construction cost of expressways (assuming arterials have a constant spacing) is a function of expressway spacing, as indicated in the following equations:

$$\text{Construction cost} = (\text{length}) \times (\text{cost per unit length}) \tag{20}$$

$$\text{Spacing} = \frac{2\ \text{area}}{\text{length}} \tag{21}$$

$$\text{Construction cost} = \frac{2(\text{area})\ (\text{cost per unit length})}{\text{spacing}} \tag{22}$$

This is a hyperbolic function, and can be graphed (Fig. 10).

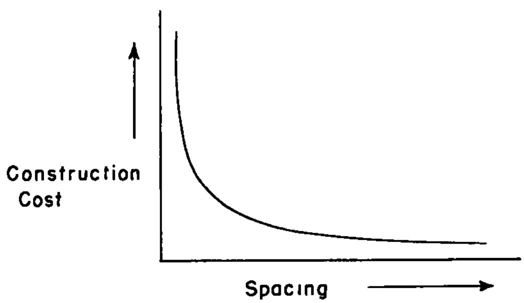

Figure 10.

Travel Cost. Travel cost is a function of the number of trips, the trip cost per hour of travel, the time period, the proportions of trip length driven on expressways and the proportion not driven on expressways. This relationship can be graphed (Fig. 11) and stated as follows:

$$\text{Travel cost} = (\text{number of trips}) \times (\text{cost per hour}) \times \left(\frac{\text{mean trip length on expressways}}{\text{expressway speed}} + \frac{\text{mean trip length not on expressways}}{\text{non-expressway speed}}\right) \times (\text{factor expanding hourly costs to long term costs}) \tag{23}$$

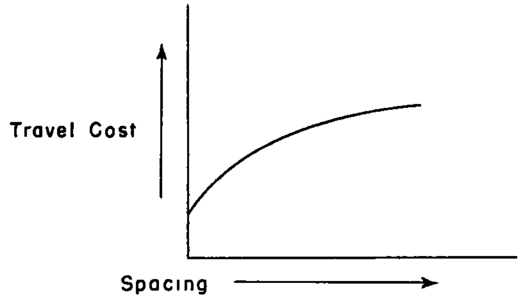

Figure 11.

Total Costs and Examples. By adding the travel cost to the construction cost as a function of spacing, one can determine the minimum total cost solution. This is shown in Figure 12.

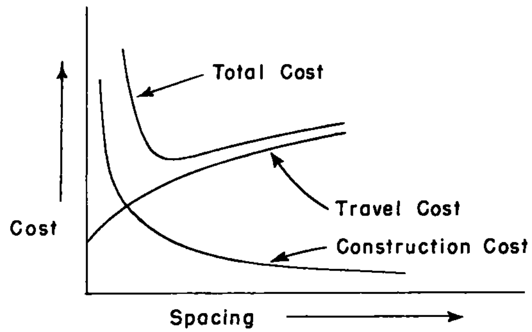

Figure 12.

Two examples are provided. The data taken for the example are the 1980 data given for Rings 4 and 7 in the section on Minimization of Total Transportation Costs. The only exception is that of arterial speed, which has been reduced in order to compensate for the amount of travel driven on local streets, which is significantly slower. (Note again that the graphic solution as stated here deals only with two speeds, instead of the three used in the more refined formula.)

TABLE 4

VALUES USED IN ESTIMATING MINIMUM-COST SPACING BY GRAPHICAL MEANS

Given	Example 1[1]	Example 2[1]
Expressway cost per mile	$8, 000, 000	$4, 000, 000
Trip density in trip destinations per square mile	20, 000	6, 200
Expressway speed	50 mph	50 mph
Non-expressway speed	12 mph	20 mph

[1] Trip length frequency distribution in both examples is that for the entire Chicago Study Area.

The results of the calculations are given in Tables 5 and 6 and are graphed in Figures 13 and 14. For comparison, the minimum-cost spacing as calculated by formula is 2.9 mi (Example 1) and 6.9 mi (Example 2). The graphical solution is "on the nose" for Example 1, and is very close in the case of Example 2, although the graphical technique gives a fuzzy answer in this latter case.

TABLE 5

EXPRESSWAY CONSTRUCTION AND TOTAL TRAVEL COSTS[1]
AS A FUNCTION OF EXPRESSWAY SPACING: EXAMPLE 1

Expressway Spacing (mi)	Expressway Construction Cost (per sq mi)	Total Travel Cost (per sq mi)	Total Costs (per sq mi)
0.0	∞	16.8	∞
0.5	32.0	20.0	52.0
1.0	16.0	24.3	40.3
2.0	8.0	27.3	35.3
3.0	5.3	29.1	34.4 (min)
4.0	4.0	32.0	36.0
6.0	2.7	37.6	40.3
8.0	2.0	42.0	44.0
10.0	1.6	46.3	47.9
16.0	1.0	52.0	53.0
20.0	0.8	54.0	54.8
∞	0	70.0	70.0

[1] In millions of dollars.

TABLE 6

EXPRESSWAY CONSTRUCTION AND TOTAL TRAVEL COSTS[1]
AS A FUNCTION OF EXPRESSWAY SPACING: EXAMPLE 2

Expressway Spacing (mi)	Expressway Construction Cost (per sq mi)	Total Travel Cost (per sq mi)	Total Costs (per sq mi)
0.0	∞	5.2	∞
0.5	16.0	6.0	22.0
1.0	8.0	6.6	14.6
2.0	4.0	7.0	11.0
4.0	2.0	7.7	9.7
6.0	1.3	8.3	9.6 (min)
7.0	1.1	8.5	9.6 (min)
8.0	1.0	8.8	9.8
10.0	0.8	9.2	10.0
15.0	0.5	10.0	10.5
20.0	0.4	10.7	11.1
∞	0	13.0	13.0

[1] In millions of dollars.

Interpretations. The graphical method of estimating minimum-cost spacing illustrates why the minimum-cost spacing formulas work. The two components of transportation cost; namely, construction cost (C_1) and travel cost (C_2), when added together, vary with the spacing of transportation facilities. In the mathematical solution the differentiation of the sum of these two components, with the derivatives set equal to zero, automatically locates the minimum point. The graphical solution does the same thing by exhibiting costs for all the spacings and the minimum point is ascertained by eye. Of course, the graphical solution is more approximate than the mathematical solution.

Construction costs are an exact function of spacing. In this example they are a dominating influence on the point of minimum total cost, because they rise so steeply when the spacing becomes tight; that is, on the order of 2 or 3 mi apart for expressways.

The position of the line of travel costs is a complicated function of relative speeds, trip density, the trip length frequency distribution, and value of time. Of these variables, trip density is very important, greatly affecting the slope of the line of travel costs. As can be seen, the travel cost line in Example 1 is very steep, reflecting the relatively high trip density of 20,000 trips per square mile. In Example 2 the flatter slope reflects the lower density of 6,200 trips per square mile.

In high density areas, the minimum cost point is sharply defined. This suggests

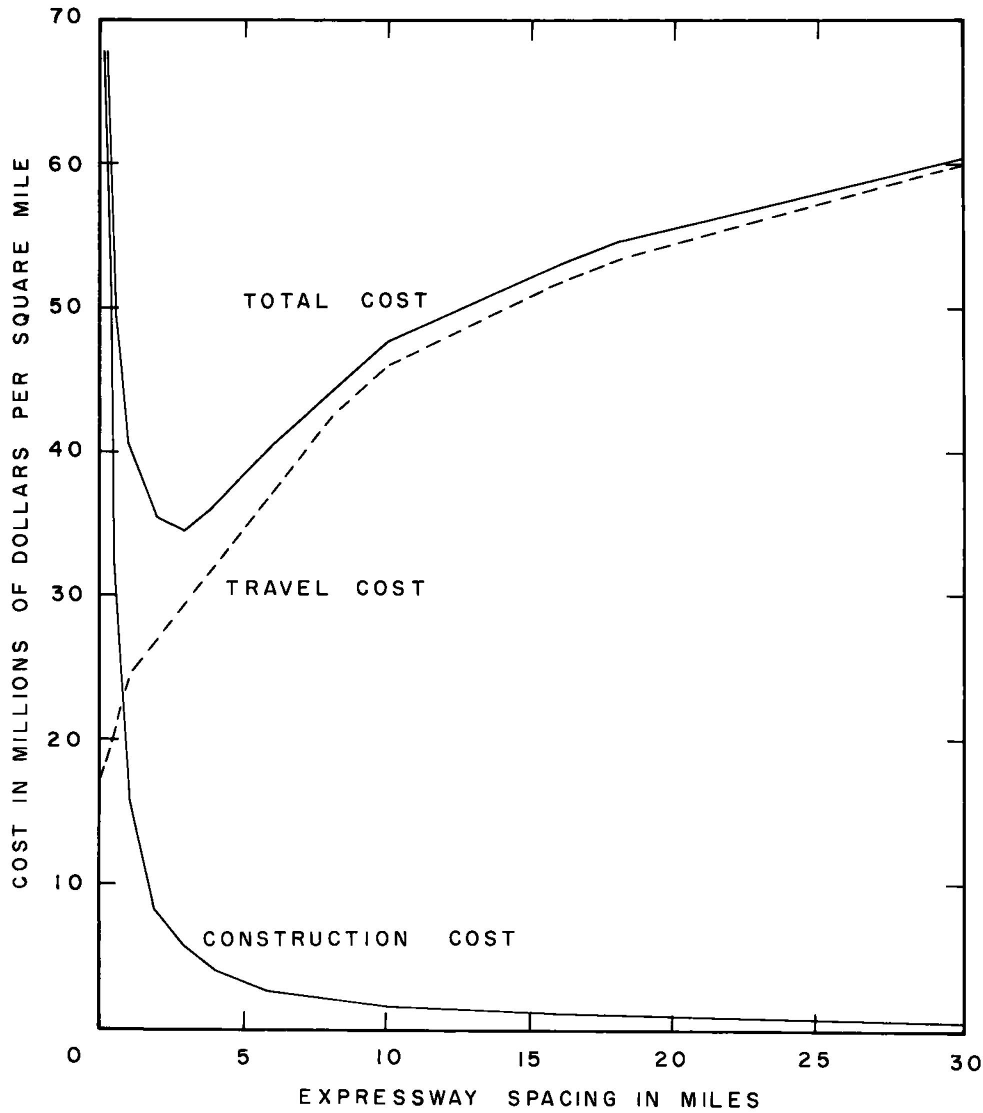

Figure 13. Minimum-cost spacing for Example 1 (source: Table 5).

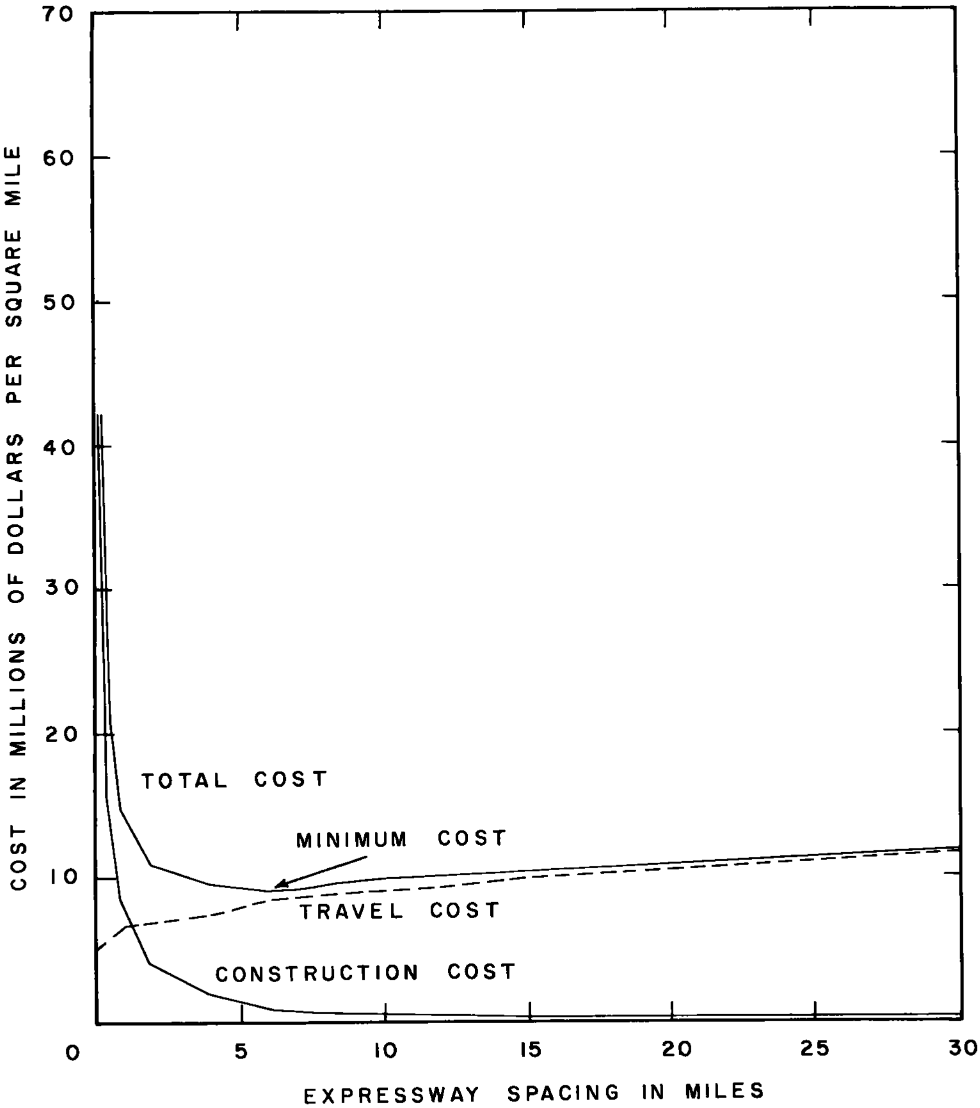

Figure 14. Minimum-cost spacing for Example 2 (source: Table 6).

that great savings can be accrued by planning an expressway network at spacings close to the minimum-cost point. In low density regions the minimum point is less well defined, which suggests that considerations other than costs may become more important in these regions.

APPLICATION OF OTHER CRITERIA

The foregoing techniques have shown the spacings of arterials and expressways which result from the minimization of the costs of travel and construction. The minimization of total costs, however, is not the sole criterion determining a "best" spacing of arterials and expressways. Other criteria were cited. Among these were the use of arterials and expressways, an over-all capacity criterion, and land planning criteria. In this section, these are taken up successively as they affect the spacing of arterials and expressways.

Use Criterion

If it should be found that the use of arterials and expressways is greater or less than their design capacity, then the minimum-cost solution may not be an optimum solution. Use is expressed here in terms of average daily volumes on the streets of each type.

If there is an imbalance between volume and design capacity (as implied by the construction cost) then modifications must be made in the minimum-cost spacing. For example, if volumes on expressways are greater than the capacity implied by a given cost level (say, $10, 000, 000 per mile for a 6-lane expressway) then a new and higher unit cost expressway (say, $13, 000, 000 per mile for an 8-lane expressway) may be used to estimate a new minimum cost. The new spacing, being wider, actually increases the volume on expressways, but the increase in volume is less than the increase in capacity implied by the higher unit cost. Hence, by a number of iterations, a minimum-cost solution can be found where capacity is in balance with expected volumes.

In order to apply the use criterion, it is necessary to estimate the volumes and vehicle-miles of travel on the various street systems. These volumes and vehicle-miles of travel vary on each street system (local, arterial, and expressway) as a function of its spacing.

The formulas for estimating volumes and vehicle-miles can be expressed in words:

$$\text{Volume on a street of given type} = \text{total trips} \times \frac{\text{(average trip length on that type)}}{\text{(miles of streets of that type)}}$$

$$\text{Vehicle-miles on a street system of given type} = \text{(total trips)} \times \text{(average trip length on that type)}$$

Crucial to solution of these equations is the part of average length driven on each type. Average (mean) trip length on each system is a function of (a) number of trips in each trip length interval (that is, the trip length frequency distribution), and (b) the proportion of each trip driven on each street type, which is a function of the spacing of arterials and the spacing of expressways. The spacing of local streets is assumed constant.

Average Trip Length in Each Type. The proportion of each trip's length on each system has been estimated experimentally and analytically, as previously described in the section on A Statement of Highway Transportation Costs (Total Costs and Construction Costs). Figure 15 is a restatement of Figure 7.

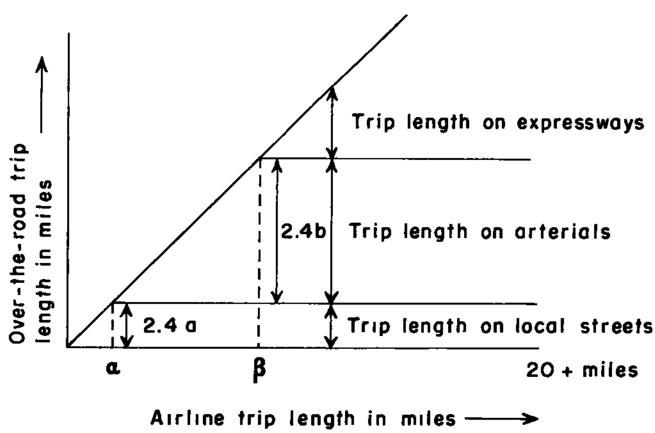

Figure 15. Distance traveled on local and arterial streets and expressways.

The point where some travel begins to be made on arterials is α and the point where some travel begins to be made on expressways is β. α is a function of the spacing of arterials (y) and β is a function of the spacing of both arterials (y) and expressways (z). Combining Eqs. 9 and 10 with Eqs. 7 and 8, it can be shown that:

$$\alpha \cong 0.31y \qquad (24)$$

$$\beta \cong 0.31z + 0.62y \qquad (25)$$

Average trip length then can be expressed as a summation of the frequencies of trips having airline length of class i times the average airline trip length of the i^{th} interval. The way the summations were prepared is shown in Figure 15.

The part of average trip length on local streets (no summation necessary)

$$\cong 2.4a \cong 0.4y \qquad (26)$$

The part of average trip length on arterials

$$= \sum_{\alpha}^{\beta} (\overline{L}_i - 2.4a)F_i + \sum_{\beta}^{20+} (2.4b)F_i \qquad (27)$$

Tables have been prepared which permit rapid estimation of average trip length. These tables were calculated using airline values for $\overline{L_i}$. To correct this to over-the-road distance, multiply the values which include the term $\overline{L_i}$ by 1.30 to approximate over-the-road mileage. The tables give values of

$$\alpha,\ \beta,\ \sum_{\alpha}^{\beta} F_i \overline{L}_i,\ \sum_{\beta}^{20+} F_i \overline{L}_i,\ \sum_{\alpha}^{\beta} F_i \text{ and } \sum_{\beta}^{20+} F_i$$

for various spacings of y and z.

The part of average trip length on expressways

$$= \sum_{\beta}^{20+} (\overline{L}_i - 2.4a - 2.4b)\, F_i \qquad (28)$$

It should be noted, that while these formulas present distance traveled on each street system by the average trip, the average distance traveled on each system by vehicles using the system can be obtained from these formulas by dividing by the corresponding frequency. Thus, the summation in Eq. 28 when divided by $\sum_{\beta}^{20+} F_i$, yields the average mileage driven on expressways by vehicles using expressways.

Note that $\overline{L_i}$ is the average over-the-road trip length of trips having airline trip length of i. The values of a and b are expressed also in over-the-road, or "L" trip lengths.

Vehicle-Miles of Travel by Type. To estimate the vehicle-miles of travel on each system, simply multiply Eqs. 26, 27, and 28 by N, which is the number of trip origins. This has been done in the following equations, and also the formulas have been put directly in terms of y and z:

$$\text{Travel}_{\text{local}} = N(0.4y) \qquad (29)$$

$$\text{Travel}_{\text{arterial}} = N\left[\sum_{\alpha}^{\beta} F_i \overline{L}_i - \sum_{\alpha}^{\beta} F_i\,(0.4y) + \sum_{\beta}^{20+} F_i\,(0.4)\,(y + z)\right] \qquad (30)$$

$$\text{Travel}_{\text{expressways}} = N \left[\sum_{\beta}^{20+} F_i \overline{L}_i - \sum_{\beta}^{20+} F_i \,(0.4z + 0.8y) \right] \quad (31)$$

Daily Volumes by Type. To obtain volumes on each street type, simply divide through Eqs. 29, 30, and 31 by the miles of streets in each type.

This can be readily obtained by the formula:

$$\text{Miles of streets} = \frac{2\ (\text{area})}{\text{spacing}}$$

$$\text{Contrariwise, spacing} = \frac{2\ (\text{area})}{\text{miles of streets}}$$

For example, the average spacing of arterials in the CATS area is:

$$\text{Spacing} = \frac{2\ (1,236\ \text{sq mi})}{2,800\ \text{mi arterials}} = 0.88\ \text{mi}$$

As a rough check on Eqs. 29, 30, and 31, it is possible to use these formulas to estimate the distribution of vehicle-miles of travel by street type for the Chicago area and to compare the results with survey data.

The average spacing of arterials and expressways can be determined as indicated previously. Average arterial spacing equals 2(1, 236)/2, 800 or 0.88 mi. Average expressway spacing (1956) equals 2(1, 236)/67 or 37 mi.

These spacings produce values of α and β of 0.27 and 12.0 mi, respectively. It should be noted that the β values do not mean that no trips of less than 12 mi in length used expressways in 1956. The value of β is an approximation which, with the very few miles of expressways which existed in the Chicago area in 1956, could not come too close to reality. Nevertheless, the results are not bad. Using these values, vehicle-miles of travel given in Table 7 were estimated by type and are compared with the vehicle-miles obtained by survey.

TABLE 7

VEHICLE-MILES OF TRAVEL—ESTIMATED AND ACTUAL, BY STREET TYPE

Street Type	Average Weekday Weighted Estimated Vehicle-Miles Using Formulas	Average Weekday Weighted Vehicle-Miles Estimated From Survey Data[1]
Local	2, 140, 000	6, 000, 000
Arterial	29, 439, 000	29, 800, 000
Expressway	4, 880, 000	3, 361, 000
Total	36, 459, 000	39, 161, 000

[1] Source: Chicago Area Transportation Study Final Report, Volume I Survey Findings, pp. 80, 81 (September 1959).

The differences between estimated and actual average weekday travel can be accounted for, in part. Actual mileage driven on local streets has always proved troublesomely high, but can be reasonably explained as caused by the high amount of circuitous travel driven on this type of street. A personal review of the distance traveled by the reader on his last trip to the neighborhood hardware store as compared with the airline distance or even the right-angle distance will demonstrate this point.

The difference between the estimated travel on expressways and the actual travel on expressways in the Chicago region can also be explained. The calculation assumes an even density of urban development and an even location of express facilities. In Chicago in 1956 the expressways were quite scattered, with a great deal of the mileage located in Rings 6 and 7, in very low-density areas. The Kingery Expressway and its extensions south, and the Edens Expressway fall into this category. As a result, use of these facilities was generally below capacity. Data from the Chicago Area Transportation Study's report indicates that expressways had in 1956 a capacity of 5, 457, 000 weighted vehicle-miles of travel, in contrast with 3, 361, 000 weighted vehicle-miles of use.

The estimate of arterial use is close to that obtained by survey, and of course constitutes the bulk of the use.

In general, therefore, the formulas for estimating vehicle-miles of travel seem to square with observed results. This is a small piece of evidence confirming the formulas. Actually, the construction of the formulas (for average travel and average volume) themselves is sufficiently precise so that the results can be used with confidence. (This statement must be qualified when there are very few miles of one type of facility, such as expressways. In such cases the theory of driver behavior upon which this work is based loses precision as a descriptive device; use of expressways in such cases becomes more an accident of the facility's location.)

Over-All Capacity Criterion

Moving gradually from the abstract to the concrete, it is desirable to provide capacity for each sub-region within the urban area sufficient to take care of the travel demand imposed on that region as of some future year. The needed capacity can be determined as suggested in the following. Needs can then be compared with the capacities to be provided by the minimum-cost systems, as a check on those systems.

The travel demand imposed on any sub-region within an urban area appears to be closely tied to the number of trip origins in that sub-region. Data on trip origins per square mile in the Chicago area were correlated with vehicle-miles of travel per square mile, as obtained by survey on a district basis. There are 44 districts in the Chicago study area. The correlation coefficient was + 0. 91. (Excluding District 01, which is the Loop area. This close correlation appears to justify the use of trip destination densities in computing minimum-cost spacings.) The plots show a fairly close fit around the regression line.

With such evidence, it appears that future vehicle-miles of travel can be estimated reasonably accurately, provided that future trip origins (or destinations) are given. These latter can be estimated from projections or plans of land use. (Actually, current assignment methods can record the vehicle-miles of travel in each route section, which can be summed up to any desired urban sub-region, thus providing what is probably a better estimate of future travel. The difficulty is that this information can only be obtained after the plan has been prepared. It is for this reason that a regression projection is used.)

Knowing future travel demand, the future deficiencies of street capacity can be estimated by subtracting present capacity from future demand for each sub-region. These are the requirements for additional capacity which must be constructed according to a plan.

It is possible to provide this new capacity by constructing expressways or arterials, or by improving arterials through various devices known to traffic engineers; that is, removing parking, one-way streets, improved signalization, or constructing median strips with "shadowed" turning lanes. Or, any combination of new construction and improvement of older streets can be undertaken. (It is difficult to ascertain what the policy should be as to the proportion of the needed new capacity which should be provided by new expressway construction or by the improvement of existing arterials. Here is where the minimum-cost spacing formulas are helpful, because they include measures of both construction problems and of service to the driving public.)

Supposing that new capacity is only to be provided by the construction of new expressways at the minimum-cost spacing for 1980 (Table 14), how much capacity will be provided? Table 8, gives these capacities by ring, and compares them with the estimated needs for 1980.

It can be seen that the 1980 minimum-cost spacing solution provides more capacity than is estimated to be needed in 1980. The average is about 6. 6 million vehicle-miles,

TABLE 8

DESIGN CAPACITY PROVIDED BY EXPRESSWAY SYSTEMS AT MINIMUM-COST SPACING COMPARED WITH CAPACITY DEFICIENCIES FOR 1980
(All capacity and travel figures in thousands of weighted vehicle-miles)

Ring	Average Distance From Loop (mi)	1956 Design Capacity[1]	Estimated 1980 Travel	Additional Capacity Needed to Provide for 1980 Travel	Capacity Provided By 1980 Optimum Spacing[2]
0	0.0	307	340	33	243
1	1.5	2,643	2,790	147	1,510
2	3.5	3,197	4,220	1,023	2,605
3	5.5	3,626	5,550	1,924	3,175
4	8.5	4,300	8,300	4,000	6,340
5	12.5	5,378	8,900	3,522	5,250
6	16.0	7,559	14,200	6,641	7,530
7	24.0	9,352	23,000	13,648	9,940
Total	-	36,362	67,300	30,938	36,593

[1] A full explanation and definition of design capacity appears on pp. 77-79, Volume I of the Final Report of the Chicago Area Transportation Study.
[2] Assuming 135,000 vehicle-miles of design capacity per mile of expressway in Rings 0-2, 108,000 in 3 and 4, 81,000 in Ring 5, and 54,000 in Rings 6 and 7.

or about one-fifth of the deficit and one-tenth of the total demand. This is not a great over-supply and would not be sufficient to suggest modification in the minimum-cost spacing, particularly in view of the expected growth beyond 1980.

By ring, however, there are some discrepancies. In Rings 0 and 1 very little additional capacity is needed, but the minimum-cost spacing formula suggests that a lot should be built. This is a peculiarity of the formula, which calculates needs on a density basis as if the area in question were large and uniformly built up at a given density. Actually, these two areas are so small (1.2 and 12.4 sq mi, respectively) that the results are not particularly applicable, because these areas contain a lesser portion of the total trip length than the formula suggests, by reason of their small extent.

In Ring 7, less capacity is provided than needed. This suggests either (a) that average capacity per mile should be increased, or (b) that spacing should be decreased. In actuality, designs for Ring 7 have been posited on a junior expressway spacing, with lower costs, greater frequency, and fairly high capacity. This has provided a much greater level of capacity in 1980, amply sufficient to meet future demands.

Land Planning Criteria

At this point, the least-cost spacing must be reviewed in terms of its effects upon land uses. This can only be a partial review, because from the land planning viewpoint the chief examination comes when a network is adjusted to the facts of topography and existing land uses. Whether a road passes between a residential neighborhood and an industrial district, or next to an airport, or through a large park is of real importance, but it cannot be taken up at this stage.

The most important principle of land planning that can be applied at the time when spacing of streets is in the abstract pattern stage is the principle of sufficient area. Roads are divisive in their influence, especially as they become wider, with heavier and faster traffic. The expressway with its 300- to 400-ft widths is a real barrier which seriously impairs communications across its right-of-way.

Therefore, the area lying between expressways must be of sufficient size for the efficient and pleasant conduct of the urban activities located there. The same is true of the areas between arterials; although, being of lesser width, they affect a different array of land uses.

Residential areas are a major worry in this connection. They are far larger than commercial or industrial districts, and hence have a greater likelihood of being damaged by intruding roads. The residential area is taken here as a unit composed of houses together with streets, small parks, schools, public buildings, and minor commercial areas.

The neighborhood is a residential area whose principal unifying function is that it serves as an elementary school district. The neighborhood is generally thought of in terms of an area containing 5,000 population. As an elementary school district, it should not be cut by arterials because these pose a major threat to the safety of children walking to school.

Hence the network of arterial streets should not be so closely spaced as to encompass areas of less than, say, 4,000 to 6,000 population.

A community is defined here as a group of neighborhoods. It should be of sufficient population size so that it can maintain certain functions internally. Suggested internal functions are: (a) schools: elementary, junior and one senior high school; (b) a local government of efficient size; (c) convenience goods stores and services; (d) cultural institutions, such as churches; (e) human needs for recognition associated with a small geographic area; and (f) human resources for adequate leadership.

Although there are no adequate standards specifying ideal community size, it is also true that there are minimums and maximums which are generally recognized. A community of less than 10,000 persons is too small, particularly from the viewpoint of adequate governmental services. A community of more than 100,000 is too large from the viewpoint of personal participation and will almost automatically fraction itself into one or more recognized sub-areas. Perhaps 30,000 to 60,000 is the ideal size range.

It may be stated, then, that expressways should not enclose areas which can house less than 20,000 to 30,000 persons. Area, of course, is a function of density and the existence of any large nonresidential uses in the same area. Community area can be calculated using the formula

$$\text{Area in square miles} = \frac{\text{Population}}{640\ D\ F\ R} \tag{32}$$

in which

D = density in dwelling units per net residential acre;

F = persons per dwelling unit; and

R = residential land as a percent of all land in that area.

Using this formula, a community of 60,000 at a density of 25 dwelling units per net acre and having 50 percent of its land in residential use, with 3.1 persons per dwelling unit, would require 2.4 sq mi of land. For such an area, expressways should be spaced not less than 1.54 mi apart. At a density of four dwelling units per net acre, this community would require 15 sq mi of land, and for this community, expressways should be not less than 3.9 mi apart in spacing.

This area criterion has been treated here in its most abstract sense, but forms the basis for that kind of review which includes the interests of the land uses.

It need not be emphasized that much further work needs to be done in the area of land planning criteria, particularly in the field of land controls and access standards abutting arterial streets, and on the problem of the collection of traffic to and its dispersion from expressway ramps, especially as this affects land uses in the vicinity of ramps.

EXAMPLES OF METHODS, USING CHICAGO AREA DATA

An initial application of the techniques developed was made using Chicago area data. Results should be treated as preliminary. So far, however, the results are rather encouraging because items that can be compared with previously available information check out quite well.

The spacing technique was applied to each of the analysis rings in the Chicago Study

Area. The rings are bounded by arcs which radiate from the Central Business District (CBD). Ring 0 is approximately coterminous with the Loop, and there are seven additional rings. Arterial spacing in Ring 0 through 5 was taken as given (Table 9) with only expressway spacing to be determined for these rings.

Both arterial and expressway spacing were determined for Rings 6 and 7. (Arterial spacing was obtained for Rings 6 and 7 because it is felt arterial changes could be planned for these rings.) In addition, for Rings 5, 6 and 7, an alternative to expressways was considered. This was the construction of "junior expressways," which would cost less and provide a lower level of service than expressways.

In applying the technique to each ring, values of all pertinent variables were estimated for each ring. This included a trip length distribution for each ring, based on CATS survey data.

The application of the technique to each ring in effect expands the ring so it becomes equivalent to the Study Area or to any very large region of the stated uniform density. In other words, the question asked is: if the entire Study Area had the properties of the ring, what would the best spacing of expressways be?

TABLE 9

SPACING DETERMINANTS FOR 1956

Ring	Trip Destinations Per Square Mile (In thousands)	Expressway Speed (mph)	Arterial Speed (mph)	Expressway Construction Cost Per Mile (In millions of dollars)	Given Arterial Spacing Rings 0 Through 5 (mi)	Rings 6 and 7 Arterial Construction Cost Per Mile (In millions of dollars)	Junior Expressway Speed (mph)	Junior Expressway Cost (In millions of dollars)
0	134.0	35	10	20	0.20	-	-	-
1	40.7	45	15	10	0.40	-	-	-
2	24.8	45	15	10	0.40	-	-	-
3	22.0	50	15	10	0.40	-	-	-
4	17.0	50	20	7	0.55	-	-	-
5	8.6	50	25	5	0.66	-	35	1
6	3.5	60	25	3	-	0.3[1]	35	1
7	1.1	60	30	1	-	0.2[1]	40	0.5

[1] In determining arterial spacing, speed on local streets is a factor; it was taken as 10 mph for Rings 6 and 7.

TABLE 10

DISTRIBUTION OF 1956 AIRLINE TRIP LENGTHS STATED AS PERCENTAGES[1]

Class i	Range Of I_1	$\overline{I_1}$	Percentage: Study Area	Ring 0	Ring 1	Ring 2	Ring 3	Ring 4	Ring 5	Ring 6	Ring 7
1	0 1	0.5	20.2	6.5	16.3	17.9	19.1	20.6	22.5	27.8	30.0
2	1 < 2	1.5	22.7	6.8	11.9	20.2	22.4	20.9	23.9	24.6	26.9
3	2 < 3	2.5	12.1	6.2	11.2	13.4	12.2	13.2	11.5	10.6	11.7
4	3 < 4	3.5	8.8	4.3	7.6	13.0	9.8	9.2	7.8	6.2	5.4
5	4 < 5	4.5	7.0	10.2	7.5	9.6	8.4	8.5	7.0	5.5	3.5
6	5 < 6	5.5	5.1	4.5	6.4	6.9	6.7	6.3	4.8	5.1	2.4
7	6 < 7	6.5	4.3	10.1	7.3	5.2	6.7	5.0	5.1	3.0	2.6
8	7 < 8	7.5	3.7	6.2	8.3	3.1	3.1	3.8	3.7	3.0	1.2
9	8 < 9	8.5	2.7	9.3	5.7	2.7	3.1	3.7	2.8	3.1	1.7
10	9 <10	9.5	2.0	9.0	4.1	1.9	1.5	2.4	2.8	2.1	1.5
11	10 <11	10.5	1.8	4.1	3.8	2.6	1.6	1.1	2.3	1.9	1.2
12	11 <12	11.5	1.5	4.6	2.5	1.2	1.4	1.0	1.7	1.8	0.7
13	12 <13	12.5	1.1	4.1	1.6	0.3	0.9	0.7	1.1	1.4	0.8
14	13 <14	13.5	0.9	2.7	1.4	0.4	0.7	0.9	0.7	1.0	1.2
15	14 <15	14.5	0.7	2.3	0.9	0.4	0.7	0.7	0.6	0.9	1.1
16	15 <16	15.5	0.5	1.0	0.2	0.3	0.3	0.5	0.1	0.3	0.8
17	16 <17	16.5	0.5	0.3	0.2	0.1	0.2	0.7	0.3	0.6	0.9
18	17 <18	17.5	0.4	1.3	0.1	-	0.4	0.2	0.3	0.2	0.5
19	18 <19	18.5	0.3	0.7	0.4	0.2	0.2	0.1	0.2	0.3	0.6
20	19 <20	19.5	0.3	0.7	0.3	0.1	0.1	0.1	0.1	0.2	0.8
21	20 +	25	3.4	5.1	2.3	0.5	0.5	0.4	0.7	0.4	4.5
Total	-	-	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

[1] For entire study area and individual rings.

Having obtained expressway spacing by ring, some inferences about the layout of the entire expressway network could be made.

Results were obtained for both 1956 and 1980 trip densities and are discussed in succeeding sections.

Spacing Results for 1956

The variables used for the determination of y and z are given in Tables 9 and 10. Table 9 gives relevant density, speeds, construction cost, and existing average arterial spacing for Rings 0 through 5. (It should be noted that some of these values are rough estimates.) Table 10 gives the trip length frequency distribution that held for each ring in 1956.

Spacing results are presented in Table 11. Results for arterial spacing in Rings 6 and 7 are of a proper order of magnitude; they agree well with actual spacing for Rings 6 and 7, which were 0.86 and 1.18 mi in 1956, respectively.

Turning to expressway spacing, it is noteworthy that the expressway spacing diverges from Rings 0 to 7, increasing with distance from the CBD. The primary explanation is that the decline in construction cost is more than offset by the decline in density of trip origins; the changes in trip length distribution and speeds are additional factors affecting results. This divergence of expressway spacing suggests a system of radial and circular expressways; this sort of pattern would fit the Z spacing results obtained.

In drawing inferences on network layout, the circumference of the central arc of each ring was divided by expressway spacing to obtain the number of radial expressways per ring. The results are given in Table 12. (The central arc of the ring is the locus of mid-points between inner and outer boundaries of the ring.) For Rings 2 through 6 an average of 11 radials per ring is obtained. There is little variation in the number of radials implied for each of these rings, which lends some support to a radial and circular system; the lack of extreme variation is another example of the "reasonable" kind of results obtained. Using 11 radials implies that expressways should be placed 33 deg apart for a circular urban region.

The pattern of circumferential and radial expressways suggested has been applied, without any adjustment, to a map of the Chicago Study Area. The results are shown in Figure 16. (In counting radials per ring, it should be noted the number listed refers to a circular area, whereas the Chicago Study Area is approximately a semicircle with a diameter on Lake Michigan.)

When the optimum system is superimposed on the Chicago area with North and South Lake Shore Drive and Congress Street Expressway as radials to the center of the city, the system resembles quite closely the existing and committed X-way system. Reading clockwise from South Lake Shore Drive, the radials correspond to the South Expressway, the Southwest Expressway, the Congress Street Expressway, the Northwest Expressway and North Lake Shore Drive. The circular expressways would correspond to the Halsted Street connector between the Northwest and South routes, a route near Western Avenue, a route near Laramie Avenue, and the tollroads. A junior expressway could be placed between the tollroad and Laramie Avenue.

The possibility of using a system of "junior expressways," or "super-arterials" has often been suggested as an alternative to the construction of full-blown expressways. There are sound reasons for this which can be supported by the methods described in this paper. These reasons can only apply effectively in low-density areas.

True expressways are very expensive. As a result, their minimum-cost spacing in low-density areas becomes quite wide, often of the order of 8 to 10 mi. With such spacing, the use of expressways becomes almost accidental; actually it is more a function of the location of the trip origin or destination than it is a function of the trip length frequency distribution. (In other words, many long trips would not be served by these widely spaced facilities.)

As a result, the use of such express facilities is less than would normally warrant the construction of such an expensive facility. Furthermore, the wide spacing of expressways causes additional travel to be undertaken on arterials, which is not desirable.

So the use of lower-cost junior expressways becomes a real prospect. The lower

TABLE 11

MINIMUM-COST SPACING RESULTS—1956

Ring	Spacing (mi) Expressways,[1] z	Arterials, y	Junior Expressways,[3] j
0	1.2	0.20[2]	-
1	2.1	0.40[2]	-
2	2.8	0.40[2]	-
3	3.0	0.40[2]	-
4	3.7	0.55[2]	-
5	6.5	0.66[2]	3.3
6	8.3	0.89[1]	6.3
7	12.1	1.34[1]	12.1

[1] Obtained from spacing formula.
[2] Set equal to actual average spacing.
[3] Spacing for the Junior Expressway System, an alternative to the expressway system. Obtained from spacing formula.

TABLE 12

NUMBER OF RADIAL EXPRESSWAYS PER RING

Ring	Circumference of Center of Ring (mi)	1956 Optimum Spacing (mi)	Number of Radials [(2) + (3)]
0	1.96	1.2	1.6
1	10.21	2.1	4.4
2	22.78	2.8	8.1
3	35.34	3.0	11.8
4	51.44	3.7	13.9
5	64.79	6.5	10.0
6	97.77	8.3	11.8

cost of these facilities, despite the lower speeds which they offer, makes their minimum-cost spacing reasonably close together—on the order of 3 to 5 mi. (The much wider spacings in Rings 6 and 7 in the 1956 results are due to the low densities obtained in those regions in 1956, when much of the area was rural.) As a result, worthwhile volumes use these facilities (40,000 to 50,000 vehicles per day) and most important, considerable reductions in arterial volumes are produced. These reductions are of the order of 25 percent.

With such information, a system of junior expressways is being considered for the Chicago area in the outer areas, with only the interstate routes constructed as fully-controlled access routes.

Spacing Results for 1980

A number of variables change between 1956 and 1980, causing some changes in the spacings obtained.

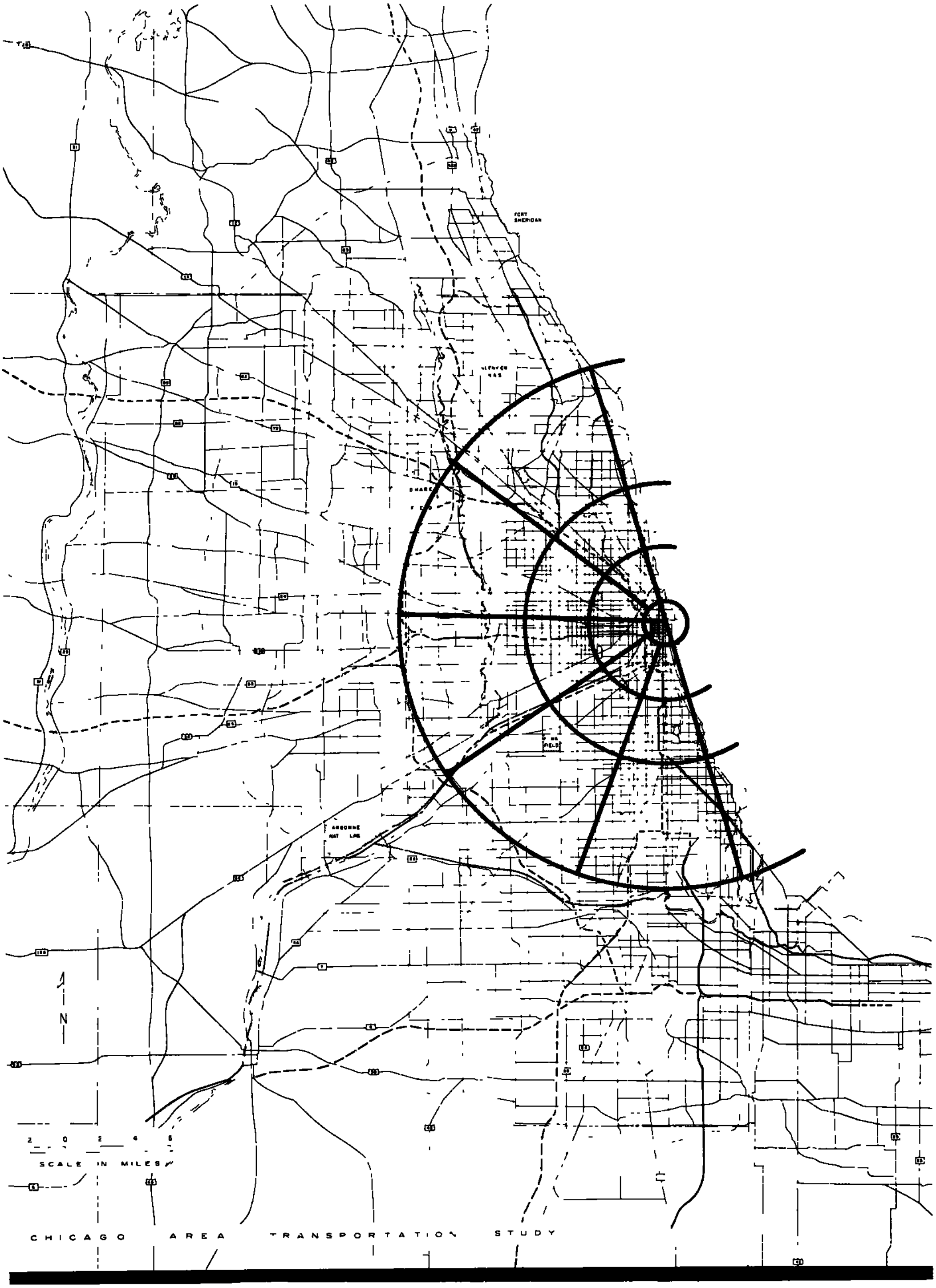

Figure 16. Minimum-cost expressway system—1956 volumes.

Table 13 gives the 1980 values of relevant variables affecting the spacing of streets, while Table 14 gives the results obtained using these values. There is not much change in expressway spacing between 1956 and 1980 for Rings 0, 1, 2 and 3. However, Rings 4 through 7 exhibit reductions in expressway spacing, indicating the need for additional expressway construction beyond the system inferred from the 1956 results; this is particularly true for Rings 4 through 7.

The fact that there is some change in minimum-cost spacing over time suggests that the planning process must be related to time, and must be extended to account for these changes.

Vehicle-Miles of Travel

Vehicle-miles of travel by facility could be estimated for both 1956 and 1980 spacings, using the techniques described in the section on Application of Other Criteria. Total Study Area vehicle-miles of travel by type of facility is given in Table 15.

TABLE 13

SPACING DETERMINANTS FOR 1980

Ring	Thousands Of Trip Destinations Per Square Mile	Expressway Speed (mph)	Arterial Speed (mph)	Expressway Cost (In millions of dollars)	Given Arterial Spacing Rings 0 Through 5 (mi)	Arterial Construction Cost Per Mile (In millions of dollars)	Junior Expressway Speed (mph)	Junior Expressway Cost (In millions of dollars)	Trip Frequency Distribution (F_1) Taken as Equal to 1956 Ring
0	152.0	35	10	25	0.20	-	-	-	0
1	47.2	45	15	12	0.40	-	-	-	1
2	28.7	45	15	11	0.40	-	-	-	2
3	25.3	45	15	10	0.40	-	-	-	2
4	19.6	50	15	8	0.55	-	-	-	4
5	13.4	50	20	6	0.66	-	30	1.5	4
6	9.0	50	25	5	-	0.5[1]	35	1.0	5
7	6.2	50	25	4	-	0.5[1]	35	1.0	5

[1] In determining arterial spacings, speeds on local streets is a factor; it was taken as 10 mph for Rings 6 and 7.

CONCLUSION: SUMMARY AND EVALUATION

Summary

A technique of estimating efficient spacings for arterials and expressways has been developed. This method was developed in order to provide a rational basis for preparing initial sketch plans, which might then be tested by computer assignment.

This method involved posing a number of criteria related to land planning, transportation planning and considerations of economy. Not all of these criteria could be considered explicitly in the first stages of estimating the most efficient spacings. Some criteria could be used as review or evaluative criteria. Other criteria could not even be included in review; their use depends upon the development of faster methods and the prosecution of other research work.

The principal technique employed was to minimize the sum of two costs related

TABLE 14

MINIMUM-COST SPACING RESULTS—1980

Ring	Spacing (mi) Expressways z	Arterials y	Junior Expressways j
0	1.3	0.20	-
1	2.2	0.40	-
2	2.7	0.40	-
3	2.8	0.40	-
4	2.9	0.55	-
5	4.0	0.66	2.5
6	6.3	0.90	3.2
7	7.0	1.10	4.0

TABLE 15

ESTIMATED VEHICLE-MILES OF TRAVEL BY STREET SYSTEM (Study Area)

	Vehicle-Miles of Travel in 000's		Percents	
	1956	1980	1956	1980
Local Streets	1,701	4,000	5.0	6.2
Arterials	10,529	20,880	31.2	32.2
Expressways	21,572	40,020	63.8	61.6
Total	33,802	64,900	100.0	100.0

to the spacing of arterials and expressways. These costs were travel costs and construction costs, both falling under the heading of transportation planning criteria.

Using a simplified network (a grid) of local and arterial streets and expressways, the construction and travel costs could be stated mathematically as functions of (a) spacing, (b) trip destination densities, (c) trip length frequency distribution, (d) unit construction costs, (e) relative speeds, and (f) value of personal time. This mathematical statement could then be minimized by means of the differential calculus. Minimum spacings can also be estimated using graphical means, for a slightly more simplified statement of total costs.

Results obtained appear to be quite reasonable. When computed for the various rings of the Chicago Study Area, the results varied, as might be expected, with closer spacings in the densely settled inner rings and wider spacings in the suburban rings.

The results are subject to review in terms of other criteria. If computation of volumes on each mile of expressway facilities indicates that in some areas the estimated volumes exceeded the capacities implied by the unit costs, then higher unit costs may have to be inserted for revised solutions. In low-density areas the wide spacing of expressways in the minimum-cost solution suggests the use of junior expressways, whose lower unit costs (but lower level of service) provide more frequent spacing and lower the volumes on arterial streets.

The results also can be reviewed in terms of estimated long-range capacity needs and in terms of land use. The required areas for the development of neighborhoods and communities seem to be provided within the minimum-cost spacings.

Evaluation

Given a technique such as that described, it is natural to ask "What good is it?" and "How real is it?" Is it a useful device? Or is it so abstract in conception that it will not be of value to the highway or city planner? Are the values used so arbitrary that the results lose their meaning?

From a conceptual viewpoint, the idea of minimizing transportation costs seems sound enough. It may be argued that the desire to expand economic growth may require more than the minimum of transportation expenditures, implied by the cost minimization technique used here. This is a moot point, and needs a good deal of development before it can be tested. Aside from this qualification, the minimization of total highway costs seems a sound first step to an optimum solution.

Are all the items of cost included? All costs have not been included yet, it is true. Operating costs and accident costs are omitted, for the understandable reason that differential operating costs and accident costs, as functions of the varying splits in use between three systems resulting from varying spacings, would be extremely hard to state. Yet on the face of it, these differentials would probably not be great enough to affect the solution markedly. These costs could be included if the problem were solved by computer, which will surely be the next step in this process. So it can be asserted that the largest items of cost are included and that therefore the solution is complete enough from this viewpoint.

Are the costs, speeds, trip densities, trip lengths and time values good estimates? This will always be subject to debate, which is desirable. The values used in the examples in this paper seem quite reasonable. Expressway and arterial construction costs will always vary from city to city, and from place to place within cities. Arterial costs are hard to evaluate, because in most cases (except in open areas) they are already in place. Arterials can be taken as fixed and minimum total costs achieved without reference to changes in arterial spacings, which obviates that difficulty. Speeds, of course, are real enough, but could be measured more precisely. Trip densities and trip lengths are obtained from survey data. Time values are highly debatable, yet study after study, even in foreign countries, indicates that they operate effectively in dictating travel paths within ranges of $0.75 to $1.35 per hour. So the data seem accurate enough.

The simple technique of graphing values of construction cost and travel costs as affected by spacing lends weight to the more precise results obtained by the formulas.

It may be concluded, therefore, that the methods employed provide "realistic" results. The accuracy of the results, however, cannot be demonstrated so simply.

In the first place, the mathematics are not precise. Approximations have been used to make hand calculations possible, and so errors may be introduced from this source alone. Further, the mathematics were calculated for a simplified grid network of streets. This may not be bad approximation, however. Finally, the values of time costs, construction costs, and all the other variables are subject to inaccuracies.

Thus, the results must be labeled as approximate. But this does not rule out the technique, because the application of the resulting minimum-cost spacing pattern to a city will probably result in distortions far greater than those produced within the formula.

There is always some danger that the production of a formula will result in its blind application by persons who believe in it without discrimination, or without understanding the processes involved. This is always a hazard, but is no excuse for not developing new techniques and methods which are based on the application of reason in systematic ways.

Having argued that the results are real, the benefits coming from the use of these techniques may be discussed. Among the foremost of these benefits is the formal statement of criteria and the development of systematic methods for reaching a solution within the framework of the criteria. This process indicates the relationship of the variables one to another.

Second, a process has been suggested for preparing plans.

Finally, and most elusive, is a way of regarding travel and a system of roads needed to move travel. One's view of how trips move about in an urban area has a profound influence on the way one designs a system. A conventional view is of trips as lines connecting origins with destinations, which are gathered together in bundles and carried from one point to another of a city—principally to the Central business district. This view is a sort of "point-to-point" view, and has been fostered by the conventional, hand drawn, desire line display. A better view, it is maintained, is to regard travel as a continuum, a layer perhaps, spread over the urban region with varying depths. This has a certain degree of truth, because at each point in the urban terrain there is a whole array of trips of different lengths, pointed in different directions, and mixed up like jackstraws.

To drain this uneven layer of travel (which can be likened to water-saturated soil of varying depth) systems of pipes must be built with diameters conforming to the trip densities of the regions through which they pass. Each pipe must carry a load suitable to its diameter, so that no part of the system is under pressure. Access to these pipes—principally the larger ones, is not at its ends, but along its length, so that the pipes drain the region effectively like a drainage field, specializing in different lengths of trips, but without regard to direction.

With such a view it becomes more important to plan a proper system than it is to plan a single expressway. And this should result in far fewer mistakes and greater economies.

Further Research Needed

The development of this method has—as always—shown that additional work needs to be done. Some of the areas where further developmental and research work are needed are listed as follows:

1. A more sophisticated mathematical description of driver route choice might be attempted. This might involve estimating the portions of trips using local, arterial and expressway facilities for regions of varying density with non-gridded street systems.
2. Accident and operating cost differences between street types might be brought into the cost equation. A relationship between these costs and travel time could be incorporated in the value of K.

3. In working with parts of an area (for example, the ring analysis carried on here), the problem of interaction between areas needs additional work. Thus a given sub-area will have through trips which use its streets (probably expressways) but have origins and destinations outside the given sub-area. The effect of through trips on minimum-cost spacing needs more study; it is a phase of the problem of generalizing the formulas so that they will deal with all sorts of situations.

4. More work is needed on the value of K, which includes the value of time. The value of time may increase over time, with increases in real income, and subsequent effects on the spacing solutions.

5. Because spacing solutions are obtained for a particular year, the problem of the planning process over time must be dealt with.

6. It may be possible to include rapid transit in the minimum-cost solution. Rapid transit is a special case generally requiring high population densities (greater than 35,000 persons per net residential square mile) to operate economically.

7. If more money is spent on the construction of a transportation facility of a given type (here the junior expressway is an obvious example) the speeds of travel on that type of facility may be made to rise. This would result in a reduction of travel costs. What is the point at which further expenditure will yield no more return in travel savings? Both increased construction costs and faster speeds would have to be fed back into the minimum-cost spacing formulas, because these affect spacings. An additional point here is that network planning may be able to specify the boundaries of costs and service requirements which need to be met by intermediate types of facilities, such as the junior expressway. This then becomes a fruitful area for research and for ingenious design, involving the traffic engineer, the design engineer, and the city planner. The latter's contribution would consist of land plotting and neighborhood design so that driveways and minor streets would not interfere with flows of traffic on intermediate facilities.

Computers may be useful in solving some of these problems. Computers can deal with much more complex cases quite rapidly and could eliminate the need for approximations while testing many more situations. The experimental determination of limits and the effect of ramp spacing could be studied quite easily; variable density situations could be inserted; and non-gridded networks could be studied.

REFERENCE

1. Carroll, J. Douglas, Jr., "A Method of Traffic Assignment to an Urban Network." HRB Bul. 224, pp. 64-71 (1959).

Appendix A--Notation Used

This appendix lists the main symbols used in this paper and the corresponding definitions.

X = Name of local street system; an X street is a local;
Y = Name of arterial system; a Y street is an arterial;
Z = Name of expressway system; a Z street is an expressway;
J = Name of junior expressway system;

x = Spacing of X system; distance between X streets;
y = Spacing of Y system; distance between Y streets;
z = Spacing of Z system; distance between Z streets;

C = Total transportation costs;
C_1 = Construction cost;
C_2 = Travel cost;

S = Side of a square. A square area is generally assumed; area = S^2;

C_X = Construction cost per mile of X;

C_Y = Construction cost per mile of Y;

C_Z = Construction cost per mile of Z;

N = Total number of trip origins or destinations in an urban region or portion thereof;
K = A constant including (1) value of an hour for occupants of an average vehicle, (2) weekday equivalents per year, and (3) the reciprocal of a gross rate of return, which includes market interest and a depreciation charge;

v_X = Speed on X system;

v_Y = Speed on Y system;

v_Z = Speed on Z system;

$V_{XY} = \frac{1}{v_X} - \frac{1}{v_Y}$;

$V_{YZ} = \frac{1}{v_Y} - \frac{1}{v_Z}$;

$V_{XYZ} = \frac{1}{v_X} + \frac{1}{v_Y} - \frac{2}{v_Z}$;

D = density = N/S^2;
L = Over-the-road distance;
l = Airline distance;
F = Trip length frequency;
i = Particular trip length class;
$\overline{L}$ = Average over-the-road trip length for a given trip length class;
$\overline{l}_i$ = Average airline trip length for a given trip length class;

$\overline{L} = 1.3\overline{l}_i$;

F_i = Frequency of given trip length class i;

a = Distance from a random point on the X network to the Y network, computed mathematically;
A = Estimated empirical over-the-road distance traveled on locals by the average vehicle in going from origin to arterial, A = 1.2a;
2A = Over-the-road distance traveled on local streets by the average trip. Maximum over-trip;
b = Distance from a random point on the Y network to the Z network, computed mathematically;
B = Estimated over-the-road distance traveled on arterials by the average vehicle in moving from arterial to expressway, B = 1.2b;
α = Maximum airline distance traveled on local streets for an individual trip;

$$\alpha = \frac{2A}{1.3} = \frac{2.4a}{1.3}$$

β = Maximum airline distance traveled on local streets plus arterials for an individual trip.

$$\beta = \frac{2A + 2B}{1.3} = \frac{2.4\ (a + b)}{1.3}$$

r, s, t are values of i;
r is that trip length for which the initial value of (l) is α, (or for which the initial value of L is 2A);
s is that trip length class for which the initial value of (l) is β (or for which the initial value of L is 2A + 2B);
t is the last trip length class;

$P = \Sigma_i F_i$ = cumulative distribution of F;

$Pr = \sum_{i\ (\alpha)}^{i\ (\beta)} F_i$ = that part of cumulative frequency of F occurring between α and β; and

$Ps = \sum_{i\ (\beta)}^{\infty} F_i$ = that part of cumulative frequency of F occurring for values of $(l) > \beta$.

Appendix B--Numerical Methods for an Iterative Solution for z and y

This appendix deals with numerical methods developed in solving the optimal spacing problem.

CASES DEALT WITH AND OUTLINED SOLUTIONS

In solving for z and y, two basic cases are discussed here:

1. Find z with y given, that is arterials are already in and no new construction is planned.
2. Find both z and y. This involves a simultaneous solution for z and y.

In the general case of finding z and y, start with an arbitrary value of y taken as given, and iterate to z. This should take only three passes. Given the selected y and z, a value of y can now be calculated by formula. This will differ from the original y started with. Use this result to select a new y and start all over, again ending in a comparison y. The values of y and z obtained to this point can be graphed and a final value of y and z can then be interpolated from the graph. This can then be checked by formula.

Case 1 is a special case of Case 2—given y, one finds only z.

The section on Methodology presents the detailed mathematics of this. On the left the general rule or operation is given. On the right, the arithmetic for a particular case is presented. Journey time through these operations is about 1 hr, once facility in them is acquired.

Setting this material down in this fashion may serve as the first step for a computer program, if such a program is wanted.

METHODOLOGY

General Rule or Operation | **Arithmetic Example**

A. The Givens: General equations

(1) $z_1 = 2.24\sqrt{\dfrac{(C_Z)}{K\,(D)\,(V_{YZ})\,(P_s)}}$

(2) $z_2 = 3.25\,\beta - 2y\ \left(\text{from } \beta = \dfrac{2.4a + 2.4b}{1.3}\right)$

(3) $P_s = P_s\,(\beta)$ β is a particular value of (1): This relation is embodied in the Table of F_i.

(4) $y_1 = 3.25\,\alpha$ From $\alpha = \dfrac{(1.2)\,2a}{1.3}$, $a = y/6$

(5) $P_r = P_r\,(\alpha,\ P_s)$ This is embodied in the Table of F_i.

(6) $y_2 = 2.24\sqrt{\dfrac{(C_Y)}{K\,(D)\,(P_r V_{XY} + P_s\, V_{XYZ})}}$

We have 6 equations in 6 unknowns: z, y, P_r, P_s, α, β. This would imply a quick solution save that two of the equations are in implicit form, and are embodied in the Table of F_i.

The notation z_1 and z_2, and y_1 and y_2 is used to indicate the divergent values of z and y which will be obtained at first from the two equations containing z, and the two containing y. When $z_1 = z_2$, and $y_1 = y_2$, iteration is complete.

The Givens

Ring 7, 1980 is used as the example.

The F^* distribution was taken as equal to that which held for Ring 5 in 1956. F^* is 1 minus the cumulative distribution of F.

F^* distribution

(1) ≥	F^*
0	1.000
1	0.775
2	0.536
3	0.421
4	0.343
5	0.273
6	0.225
7	0.174
8	0.137
9	0.109
10	0.081
11	0.058
12	0.041
13	0.030
14	0.023
15	0.017
16	0.016
17	0.013
18	0.010
19	0.008
20+	0.007

$K = 7,500$
$D = 6,200$
$C_Z = 4,000,000$
$v_X = 15$ $V_{XY} = 2/75$
$v_Y = 25$ $V_{YZ} = 1/50$
$v_Z = 50$ $V_{XYZ} = 5/75$
$C_Y = 500,000$

In the F^* distribution, (1) = airline distance. Thus, 34.3% of all trips are ≥ 4 mi in length. α and β are particular values of (1).

B. General Procedure

Step 1. For a given y, find the optimal z:

1.1 Insert specific values of given parameters, that is, K, D, V_{YZ}, C_Z, C_Y and V_{XYZ} are given initially. Rewrite Eq. 1

$$z_1 = 2.24\sqrt{\frac{(C_{Z0})}{(K_0)\,(D_0)\,(V_{YZ0})}}\ \sqrt{\frac{1}{P_s}}$$

where 0 indicates specific value

$$1.1\ z_1 = 2.24\sqrt{\frac{4,000,000}{(6,200)\,(7,500)\left(\frac{1}{50}\right)P_s}}$$

$$= 2.24\sqrt{4.30\ \frac{1}{P_s}}$$

$$= (2.24)\,(2.07)\sqrt{\frac{1}{P_s}} = 4.64\sqrt{\frac{1}{P_s}}$$

From this point on, z_1 in Eq. 1 depends only on P_S.

1.2 Obtain y_1,

Either y is given (Case 1) or y is to be determined (Case 2). In Case 2, arbitrarily pick an initial value for y, call it y_1.

1.2 $y_1 = 1.00$ (arbitrarily)

1.3 Arbitrarily select an initial value for β.

1.3 $\beta = 4$ (arbitrarily)

1.4 The value of β selected implies a corresponding value of P_s. This is obtained from the F^* Table, relating (1) and P_s. β is a specific value of (1). P_s is a value of F^*.

1.4 For $\beta = 4$, $P_s = 0.343$

1.5 Insert P_s in Eq. 1 to obtain z_1.

1.5 $z_1 = 4.64\frac{1}{0.343} = 4.64\sqrt{2.92} = 4.64 (1.71) = 7.93$

1.6 Insert β and y_1 in Eq. 2 to obtain z_2.

1.6 $z_2 = 3.25(4) - 2(1) = 13.00 - 2.00 = 11.00$

1.7 If $z_2 > z_1$ try a lower initial β.

If $z_2 < z_1$ try a higher initial β.

For this β, repeat steps 1.4, 1.5, 1.6.

1.7 $z_2 = 11.00 > z_1 = 7.93$

Try $\beta = 3$

For $\beta = 3$, $P_S = 0.421$

$z_1 = 4.64\sqrt{2.38} = 7.15$

$z_2 = 3.25(3) - 2.0 = 7.75$

1.8 We have now run through two trials. In trial 1, call the values obtained β (1), z_1 (1), and z_2 (1). In trial 2, call the values (2), z_1 (2), and z_2 (2). Now, plot the z's as a function of β. Connect z_1 (1) and z_1 (2) by a straight line, and do the same for the z_2's. Take the point of intersection of these lines as approximately the final limiting values of β and z.

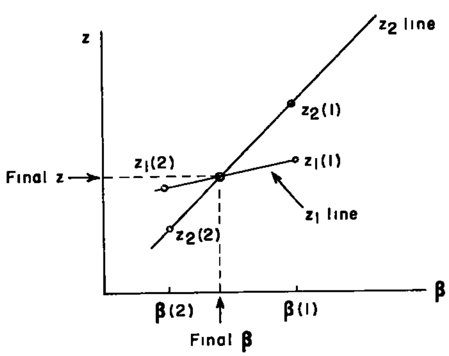

Figure 17.

1.8

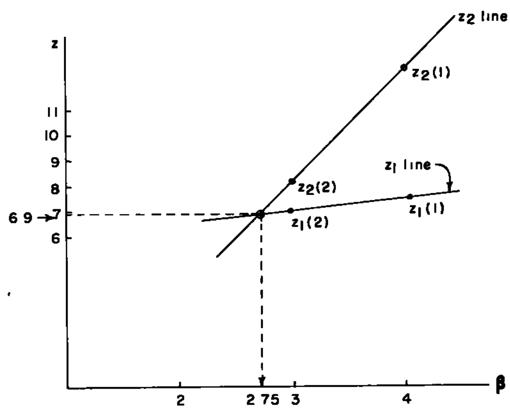

Figure 18.

1.9 Check the graphic interpolation or extrapolation by running through steps 1.4 to 1.7 and seeing if $z_1 = z_2$. If y is given (Case 1), this is the end of of the procedure

1.9 Trying $\beta = 2.75$ yields $P_s = 0.450$ (by linear interpolation of F^* between $\beta = 2$ and $\beta = 3$)

$z_1 = 4.64\sqrt{2.22} = (4.64)(1.49) = 6.91$

$z_2 = (3.25)(2.75) - 2 = 6.93$

Then z can be taken as 6.9.

2.0 If y is to be determined, use the initial value of $y = y_1$ and the z obtained to compute y_2.

2.0 $y_1 = 1.00$, $z = 6.9$

2.1 Using y_1, obtain α from Eq. 4.

2.1 $3.25\ \alpha = y$ from Eq. 4.
$\alpha = 1/3.25 = 0.308$

2.2 Given α, this implies $P_r + P_s$.

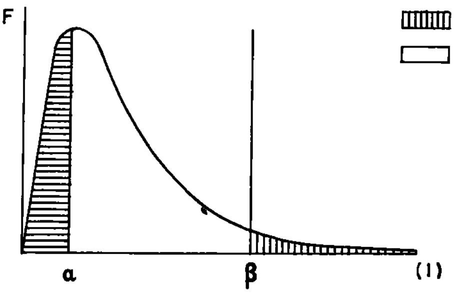

Figure 19.

2.2 For $\alpha = 0.308$
$F^* = 0.932$

i.e. For (1) = 0, $F^* = 1.000$
(1) = 1, $F^* = 0.775$

By interpolation (1) = 0.308, $F^* = 0.932$

where is a particular (1).

This value is $P_r + P_s$.

2.3 $(P_r + P_s) - P_s = P_r$ (as indicated in Eq. 5).

2.3 $P_s = 0.450$ from step 1.9
Therefore $P_r = 0.932 - 0.450 = 0.482$.

2.4 Given P_r and P_s, obtain y_2 from Eq. 6, having expressed Eq. 6 in terms of P_s and P_r only by inserting constants into Eq. 6.

2.4 $$y_2 = 2.24\sqrt{\frac{500,000}{(6,200)(7,500)\left(\frac{2}{75}P_r + \frac{5}{75}P_s\right)}}$$
from Eq. 4

simplifies to

$$y_2 = 2.02\sqrt{\frac{1}{2P_r + 5P_s}}$$

$$y_2 = 2.02\sqrt{\frac{1}{2(0.482) + 5(4.50)}}$$

$$= 2.02\sqrt{\frac{1}{3.21}} = 1.127$$

2.5 Compare y_1 and y_2. Call these $y_1(1)$, $y_2(1)$. If $y_2(1) > y_1(1)$, select a new y_1 somewhat above $y_2(1)$. Call this $y_1(2)$. Similarly if $y_2(1) < y_1(1)$, select $y_1(2)$ somewhat below $y_2(1)$.

2.5 $y_1(1) = 1.000$, $y_2(1) = 1.127$
Let $y_1(2) = 1.250$.

3.0 A final value for y is now obtained.

3.1 For $y_1(2)$ compute a new z following procedures outlined in step 1.

3.1 $y_1(2) = 1.250$
Applying step 1 yields
$z = 7.1$, $\beta = 2.95$, $P_s = 0.427$

3.2 Compute a new y_2 (2) following procedures outlined in step 2.

3.2 $\alpha = 0.385$, $P_r = 0.486$

$$y_2 (2) = 2.02 \sqrt{\frac{1}{3.107}} = 1.145$$

3.3 Set up tables of y_1, and y_2 and z for runs (1) and (2). Plot y_1 and y_2 against z. Draw a line through y_1 (1) and y_1 (2) and similarly draw a line through y_2 (1) and y_2 (2). The point of intersection of these lines yields a graphic solution for y and z.

It may be noted that y appears to converge very quickly, so that $y_2(1)$ is very close to the true y.

3.3

run	y_1	z	y_2
(1)	1.00	6.9	1.127
(2)	1.25	7.1	1.145

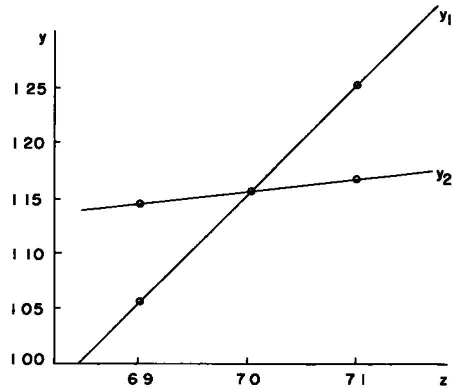

Figure 20.

4.0 Check the graphic solution.

Graphic solution: z = 7.0, y = 1.136

4.1 Insert the graphic z into Eq. 2.

4.1 $7.00 = 3.25\,\beta - 2.27$

4.2 Compute β and corresponding P_s.

4.2 So $\beta = 2.85$ and corresponding $P_s = 0.438$.

4.3 Obtain z_1 from Eq. 1 using P_s. Check against graphic z.

4.3 z_1 for this $P_s = 7.006$. Check.

4.4 Insert graphic y into Eq. 4 and obtain α.

4.4 $\alpha = 1.136/3.25 = 0.350$

4.5 Given α, one can obtain P_r and P_s by interpolation from F^* table.

4.5 For $\alpha = 0.350$, $P_r + P_s = 0.921$

$$P_s = 0.438$$

$$\therefore P_r = 0.483$$

4.6 Obtain y_2 from Eq. 6 and check against graphic y.

4.6 For this P_r and P_s, $y_2 = 1.137$. Check.

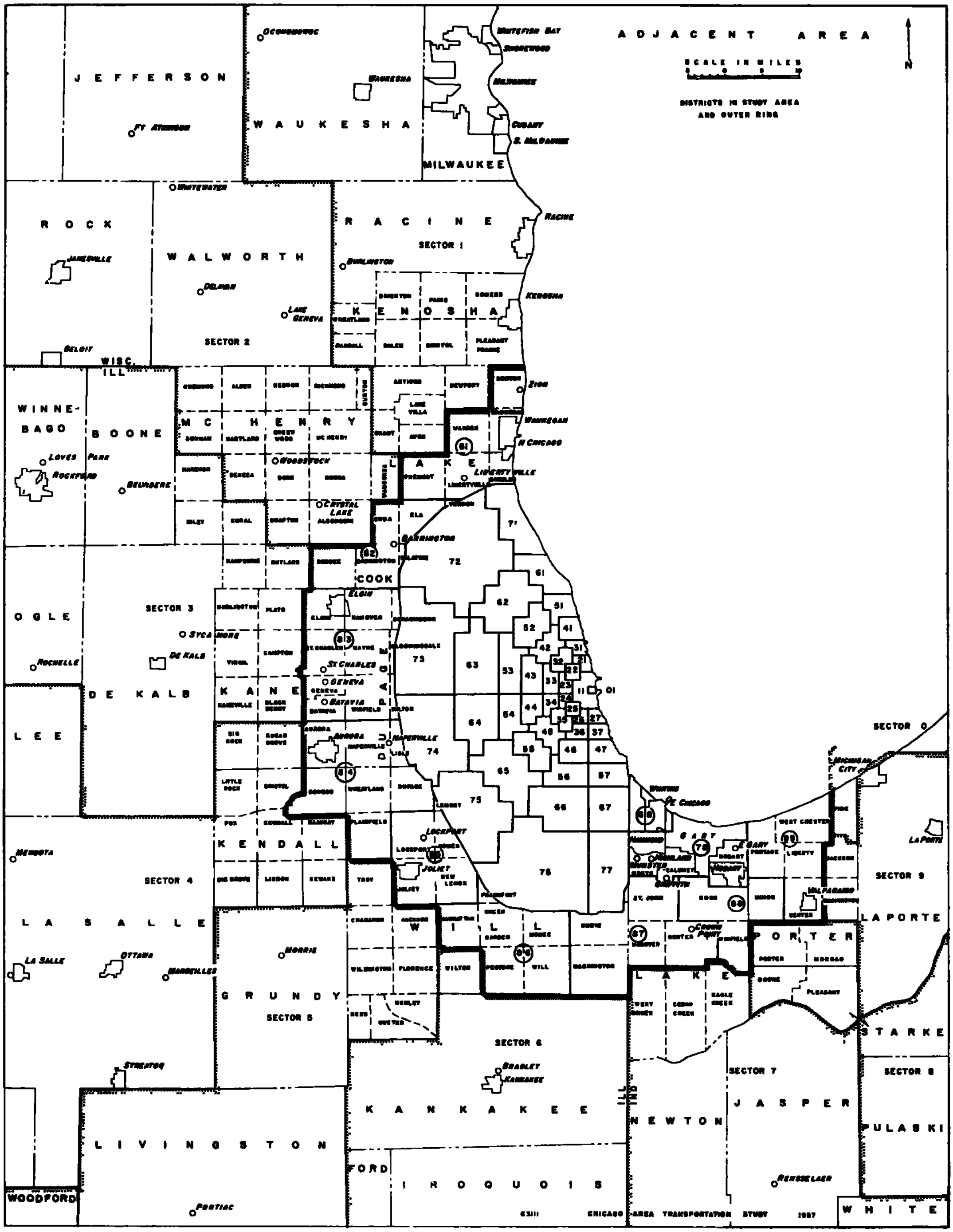

Figure 21. Study area—rings and sectors.