Panel Discussion on Inter-Area Travel Formulas

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Mr. Lynch

Comprehensive origin-and-destination (O-D) surveys have been conducted in more than 130 urban areas during the past 16 years, and repeat surveys have been made in many of these. The means for developing and analyzing the current travel pattern is, therefore, well established. But the real problem is to predict the travel pattern at some time in the future, for it is on such a prediction that the highway network layout and design must be based.

In the field of forecasting future travel, many theories have been propounded and numerous procedures have been developed. They have one thing in common—they all recognize the fact that future travel must depend on the kind, intensity, and direction of urban development. For it is the size and distribution of the population and the commercial and industrial centers that will determine the travel pattern. Some of the methods being used or proposed are as follows:

1. The Growth Factor method, including the Fratar formula among others, projects the present travel pattern forward on the basis of anticipated growth in different areas.
2. The Gravity Model and modifications of it, used in many cities, assume that the travel between two areas depends on their attractive power and the distance between them, similar to the law of gravitation.
3. At the O-D survey committee meeting (January 11, 1960) Howe of the University of Cincinnati propounded an "electrostatic field" theory of trip attraction.
4. The model being used in Chicago introduces the concept of "intervening opportunities" on the theory that absolute distance is of lesser importance than the availability of opportunities to fulfill the travel desire at nearer locations. It assumes that travel will take place in such a manner as to minimize time.
5. The late Sam Osofsky of the California Division of Highways developed a multiple regression method of forecasting traffic volumes that he claimed to be better than either the Fratar or the Gravity Model method.
6. J.G. Waldrop, of the British Road Research Laboratory has developed a model for forecasting the distribution of traffic on a road system based primarily on the cost of travel.

Here are 6 methods based on different theories, probably producing significantly different results. They have not been fully evaluated statistically and compared, though research projects are under way with this objective. But some of them have been widely used because the road program if it is to fulfill the pressing needs of traffic cannot await the standardization of procedures, which may not be accomplished for several years.

At the present time there is considerable controversy in this field and on this panel there are three men who can be expected to bring out some of the controversial points.

Glenn E. Brokke, of the Bureau of Public Roads, is one of the principal advocates of the Growth Factor method. Alan M. Voorhees of the Automotive Safety Foundation has done much in developing and applying the Gravity Model. Morton Schneider of the Chicago Area Transportation Study was principally responsible for developing the Chicago Model.

Mr. Brokke

Two problems are involved. One is concerned with the method that one would use today to forecast trips to 1980. Here, the question is not so much how perfect is the method but rather is it better than any other fully developed procedure and does it provide an acceptable degree of accuracy?

The other problem is concerned with the research aspects. Is there any room for improvement of our present procedures? Most everyone would agree that there is. The difficulty is in evaluating improvement. Hindsight is always 20/20 but foresight is often less acute. Because all methods now being considered are of relatively recent origin, none, so far as is known, have received the acid test of time. Therefore evaluations must necessarily be somewhat synthetic and subject to the vagueness of statistical inference.

With these limitations in mind a return to the problem of which method is best for use today is in order. A test of various growth factor models two years ago determined the errors at various volume ranges (1). A re-examination of these errors in the light of the Phoenix data presented by Sosslau (2) indicates that if the results can be considered applicable to the two surveys in Washington, D.C., the growth factor method was about as accurate in projecting 1948 data to 1955 as a 1 in 35 sample O-D survey in 1955 would have been. This degree of accuracy is really quite good and should it continue to hold for more comprehensive tests over longer time intervals, it does represent an acceptable standard to be used in evaluating alternate procedures.

Within the last year or so an interarea travel formula was developed and used in a large western city. The volumes as determined from the formula and also as measured in an O-D survey were each assigned to a highway network by identical criteria. The differences in the assigned volumes indicate that this particular equation had an accuracy in duplicating present, not future, trips about equivalent to a 1 in 175 sample O-D survey.

The State of California has developed a multiple regression approach to forecasting urban area traffic volumes as reported by Sam Osofsky at the eighth WASHO Planning Conference in April 1959. This study indicates that a very considerable increase in accuracy is obtained by using an individual formula for each zone rather than area wide equations. It is to be noted that this method involves a stratification by zone rather than by trip purpose although land-use factors for the individual zones may produce somewhat similar effects.

This brings up the point of having a standard method of measuring the accuracy of various travel distribution equations. The method described by Sosslau (2) appears to be unbiased, sensitive, and relatively easy to accomplish with modern computers. Its use in forthcoming tests is anticipated.

Before leaving the urban problem, two rather new features of the growth factor method should be reported. One feature has the purpose of alleviating the difficulties caused by interzone volumes of zero. It is accomplished by combining low density zones with neighboring zones of similar character that have a more stable travel pattern. The forecast is made using the larger zones and the total volume is then prorated back to the smaller individual zones on a probability ratio.

The second feature is even more basic and may lead to a fundamental change in traffic assignment. From studies available in the Bureau of Public Roads, it can be shown that speeds during peak hours are significantly different from those during non-peak hours. In addition, traffic counts at 282 urban locations throughout the nation indicate that morning peak hour volumes on expressways may vary from 3.9 percent to 15 percent of the daily traffic with from just more than 50 percent to as much as 94.6 percent of the traffic flowing in the heavier direction. In the afternoon, peak hour volumes vary from 4.2 percent to 17.8 percent of the daily traffic with as much as 93.4 percent of the traffic moving in the heavier direction. This variation of several hundred percent in design hour volumes can seriously affect the cost and utility of proposed highway improvements. The more intimate one's knowledge is of present methods of forecasting and assignment, the more clearly one will recognize that present procedures for estimating peak hour flow and directional split involve gross approximations from area wide averages.

The growth factor method can be applied to trips as represented by individual tabulating cards just as easily as it can be applied to total zone-to-zone movements, except that the computer will run a few minutes longer. If, then, it is assumed that future trips will be made during the same time as their present counterpart, these trips can be sorted into those made during the morning peak, the afternoon peak or any time period desired. These trips can then be assigned to a highway network by direction using travel times that are appropriate for the time period involved. The result is the assigned traffic volume by direction during the morning peak, the afternoon peak and the total for the day.

The Minnesota Highway Department has used these procedures in assignments for Minneapolis and St. Paul. The data developed indicate a pattern very similar to that found on existing expressways throughout the United States. Further research is needed, however, to demonstrate the effectiveness of this procedure.

Outside of the urban field, a formula of the gravity model type appears to have much merit in predicting travel between cities. Using data obtained from the external cordon survey at Detroit, Michigan, the following equation was developed:

$$\text{Trips}_{AB} = \frac{K\ \text{Population}_A \times \text{Population}_B}{\text{Distance}_{AB}^{\ n}}$$

If the populations of the two areas are measured in thousands and the distance between them in miles, the proportionality constant "K" and the exponent "n" for distance have the following average values for various vehicle types:

Vehicle Type	K	n
Passenger cars	157	2.49
Panels and pickups	27.5	2.81
Single unit trucks	13.3	2.66
Combinations	0.32	1.58
Total trips	156	2.44

The Bureau is in the process of enlarging this study by including data from other cities and at the same time expects to investigate other factors.

The principal problem is one of evaluating the various formulas. Until this is done, discussion usually involves more heat than light. Spot discrepancies to one observer will appear accidental or trivial; while to another, they will appear basic and conclusive. Confirmation of the over-all effects of the various equations must necessarily await increased knowledge and experience in the use of computers in the traffic field.

Progress, however, is inevitable and it is believed that soon the means will be found for recognizing and alleviating areas of traffic congestion before they occur.

REFERENCES

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Mr. Voorhees

There are now two general types of techniques that are being used for traffic projections. The first type attempts to extrapolate existing patterns. In other words, on the basis of existing O-D patterns and anticipated growth within, various sections of the region traffic are projected into the future. The "growth factor" procedures fall in this category. The main advantage of these procedures is that unique travel patterns are taken into consideration. But the disadvantage is that they cannot be used effectively if there are substantial changes anticipated in land-use patterns or in transportation services. For example, the procedure cannot be used effectively to estimate traffic patterns that would result if an area were to change from an industrial to a residential area, or if a rapid transit system were developed.

The other approach is that of developing formulas or mathematical models which can be used in estimating traffic. Naturally, such techniques can be "designed" to evaluate the impact that changes in land use or transportation service would have on traffic patterns. In general, the models that are now in use can do this fairly effectively, but they do not consider the influence that social and economic factors have on travel patterns.

For example, a certain residential area might be closely tied to the downtown area because of some social and economic patterns that have developed in the community. The number of trips between these areas do not follow the "averages" that are estimated by the model. However, with proper application of mathematical models, adjustments can be made to account for these deviations.

An O-D study, for example, was conducted in Cedar Rapids, Iowa, and the state highway department analyzed this information and developed a mathematical model which reflected the "averages" derived from the observed data. They then followed this procedure in developing a complete O-D pattern for the area. Comparison was then made between the patterns obtained from the actual O-D and that derived from the model. It was found that in most cases it checked very well (within plus or minus 10 percent), though in a few instances modifications had to be made. This was achieved by using weights similar to those applied for trips to the downtown area in the recent Baltimore transportation study.

TYPES OF MODELS

Although there are many types of models that are now being considered and applied, most of them follow two general steps. First is that of determining trip production or the number of trips that start from an area, and the second step is that of determining the destination of these trips.

The procedure by which trip frequency information is calculated varies. Some base the estimates on the acres of residential, commercial or industrial land in a zone, whereas others consider car ownership, population and employment data. The use of the latter type of parameters seems to give better results from tests that have been made, as, after all, the number of people employed in an area dictates the number of work trips, not the number of acres in industrial use. If the parameters deal with car ownership, population, etc., frequency patterns are usually expressed in terms of trip purpose—work, commercial and social. If the parameters deal with acres of residential, commercial and industrial land, then the categories of trips are usually divided into land-use groups: such as, trips between residential and commercial areas, commercial and industrial areas, etc. This division of trips into categories is aimed at modifying any variations in trip behavior related to various activities. Generally, it has been found that if trips are divided into three or four categories sufficient breakdown is obtained to make the synthesis of existing traffic patterns fairly accurate. Further breakdowns would improve results somewhat, but the improvement does not warrant the extra cost.

Generally, there are about 4½ trips produced per car in the largest cities, 5 trips per car in cities between 250,000 and 500,000 and around 6 trips per car in cities of less than 100,000. It appears that this difference in trip frequency is related to the fact that in smaller communities the average trip is shorter, so more trips are made by the average individual. But, within limitations of city size, it does appear that trip production figures are very comparable throughout the country.

As indicated in Figure 1, the number of auto-travel shopping trips made from any residential area largely depends on the number of cars per dwelling unit. Generally, for every 1,000 cars in a residential area there are about 1,600 commercial trips made each day. Figure 1 shows that this number will vary depending on the type of trip. In all cases, except the work trip, the number of trips seems to increase

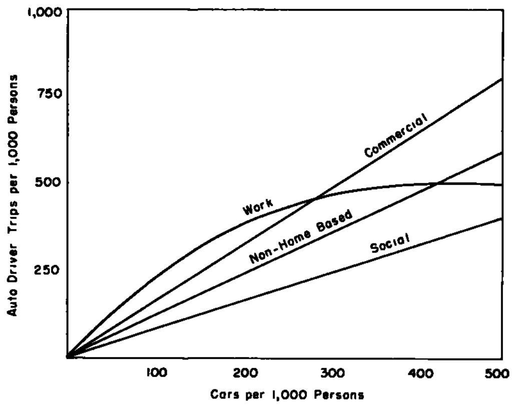

Figure 1. Auto driver trip frequency vs car ownership.

directly with car ownership. However, because there is a ceiling in the total number of work trips, due to the limitation in the size of the labor force, this pattern levels off (Fig. 1).

TRIP DISTRIBUTION

The next step is probably more difficult—that of determining the distribution of these trips. There are several mathematical procedures that have been used to estimate this. One is the multiple regression approach developed by Sam Osofsky, which has been used in California. Another is a linear programming method which has been proposed by Howard Bevis. Still another is a so-called opportunity model which is discussed further by Morton Schneider.

However, probably the most widely used model is the gravity model. In adjusting Newton's Law of Gravity to urban traffic it has been necessary to make several modifications. The adaption that seems to make most sense takes into consideration the relative travel time between various zones and the intensity of activity within these zones.

This type of model can be expressed by

$$V_{1-2} = V_1 \frac{\frac{M_2}{T^x_{1-2}}}{\frac{M_2}{T^x_{1-2}} + \frac{M_3}{T^x_{1-3}} + \cdots + \frac{M_n}{T^x_{1-n}}} \tag{1}$$

in which M designates the intensity of the activity in the zone in some appropriate unit by employment or population, T represents the travel time for the trip, and x is the empirically determined exponent.

The main advantage of this model is that it not only is sensitive to changes in travel time between the zones, but also takes into consideration competition between land uses. It is similar in concept to the opportunity model. The main difference between the two models is that they use different mathematical procedures to relate the two variables—travel time and intensity of activity within a zone (which is referred to as opportunities in Schneider's model).

In applying this type of model, generally trip patterns are divided into three or four types of trips. In the Baltimore and Hartford studies four trip purposes were used:

1. Home-based work trips.
2. Home-based commercial trips.

3. Home-based social trips.
4. Non-home based trips.

In both of these studies, in applying the model M was expressed in terms of employment in dealing with work trips; population for social trips; retail employment for commercial trips; and for non-home based trips a factor that equals the population plus 25 times the retail employment for each zone.

In the seven cities in Iowa, where the gravity model was used recently, the trip purposes were divided as follows:

1. Home-based work trips.
2. Other home based trips.
3. Non-home based trips.

In this case the M for work trips was expressed in terms of employment, other home-based trips by a factor that equaled the population plus 25 times retail employment plus employment for each zone. Non-home based trips were done in a similar manner.

In applying this model for work trips usually an iterative process is used to make the trip patterns conform to the number of workers that live or are employed in a zone. In studies that have been made in Iowa, it is quite clear that this process is not necessary for other types of trips. In fact, better results are obtained by not iterating to some predetermined number of trips based on land-use characteristics.

As already indicated, the gravity model only takes into consideration two variables. But there are other factors that influence travel habits, particularly those related to social and economic conditions. For example, Sears Roebuck, primarily because of its merchandising policies, is able to attract people from much greater distances than most stores of similar size. Social patterns in a community also influence social and recreational travel habits. Comparative tests between actual patterns and those developed from the model should be made. Significant variations should be corrected by simply adding weights to the model (3). However, this means that in forecasting traffic one has to estimate how these weights will perform through time.

To develop these weights, a systematic procedure should be developed. To reduce sample errors to a minimum it would be best to compare the observed trip patterns with the patterns estimated by the model on a district-by-district basis. This should clearly reveal the traffic patterns that are significantly influenced by social and economic forces. In addition, it would be advisable to correct for calibration errors made in determining average travel time within a zone and between adjacent zones. This should be done at the same time as the examination of district-to-district travel to detect social and economic influences. This weighting process has the additional advantage in that it will eliminate the need for an iterative procedure to bring the work trips into balance.

So, by this procedure the gravity model can be made sensitive to many factors. This flexibility is one of the main advantages of the gravity model.

Perhaps the most salient feature of the gravity model is that the parameters that are used, appear to be fairly constant and some have apparently held over a considerable period of time. For instance, the work trips in Hartford, Baltimore, San Francisco, Cedar Rapids, Iowa, or Wichita, Kansas, seem to follow the same basic patterns and these patterns can be calculated by using the gravity model. It has been noted in all these cities that travel time, if raised to the 0.8 power, will give good results. The fact that this is consistent throughout the country, and has also held over time (4), would indicate that the gravity model is approaching a universal law. In other words, the great advantage of Newton's Law of Gravity is that the distance factor in his calculation, when raised to the second power, has given good results when and wherever measured, and certainly an attempt should be made to achieve in traffic models the development of some technique that would be universally applicable.

Another advantage of the gravity model is that it is easy to understand and, therefore, easy to apply in any particular community. Numerous state highway departments and city officials have found it very easy to comprehend this procedure and, therefore have been able to follow through on their own in applying the technique.

The gravity model is also adaptable to computer programming, and has now been programmed for the IBM 704 and Univac. This permits one to use the gravity model quickly and cheaply in any particular area. Recently, one person in Frankfort, Kentucky, in the period of one month, was able to develop the existing and future traffic patterns by the use of a gravity model. During this period he also made numerous checks to compare the existing travel habits with the gravity model results.

However, there is one general weakness with the gravity model and that is that the concept of applied physics to human behavior is being used. It seems that one should be able to develop some procedure that would really be more fundamentally related to human behavior. Surely in the near future this will be done, and it will be possible to improve existing techniques. However, the more experimental results developed with the gravity model and these other techniques, the more can be learned about human behavior. Thus, a better understanding can be developed as to what should be included in a more sophisticated model that would interrelate all the factors that seem to be important in influencing urban travel.

Therefore, the most important thing is not personal liking for the gravity model or any of the other models, but that the value of applying models in urban transportation planning work is appreciated. Whatever model is selected and checked with the existing information, it will give more light as to what factors influence travel behavior. If this is done enough throughout the country, it will soon be possible to develop a sound procedure. But, until that time, in light of the experience with the gravity model, one can apply the gravity model with considerable confidence.

REFERENCES

3. Voorhees, A.M., and Morris, R., "Estimating and Forecasting Travel for Baltimore by Use of a Mathematical Model." HRB Bull. 224 (1959).
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Mr. Schneider

Perhaps the most meaningful interpretation of the Fratar method regards it as making this statement: if the trip populations of a set of zones are altered, the resulting interchanges are such that the ratios of the new interchanges to the corresponding original ones have a minimum dispersion, according to some measure, around 1, subject to the prescribed zone populations being satisfied. In its applied aspects the Fratar method is expensive and inflexible; it requires a complete O-D survey, fixed zonal and regional boundaries, and troublesome data handling and computations. In its support, the argument is commonly advanced that one cannot go too far wrong with the Fratar method, that it preserves the strongly established characteristics of the present. Undoubtedly this is true over a short term in which negligible change occurs, but considering that the avowed purpose of the technique is to deal with change, that point seems somewhat blunted. Besides, among the strongly established characteristics that it preserves is the instability of small number events.

A crucial flaw in the Fratar method is its handling of small interchanges, notably the limiting case of zero. If an interchange volume is surveyed as zero, the Fratar method cannot threaten, torture, or cajole it into becoming anything else—unless a zone volume at one end of the interchange grows from zero to something finite, and that would be an amusing situation inside a computer. To catalogue a few other conspicuous faults: the results are not independent of arbitrary procedures—a forecast performed through intermediate stages will not be the same as one performed directly in one jump; the method grants no effect whatever to changes in access, such as a whole new expressway system. An interesting extreme example is that of two cities, close to each other but separated by an impassable ravine. If a bridge is built across that ravine, the Fratar method becomes inapplicable.

On the other hand, no claim has been voiced that the Fratar method is strictly tenable, but merely that it is a good rough working tool. When it comes to this ground, these comments appear a bit captious. Working tools are not as easily come by as they may seem, and they can be more usable within their limits than elegant theory. The Fratar method certainly is neither foolish nor indefensible, but it does not contribute to an understanding of trip behavior.

The "gravity" formula declares, without preamble, that the interchange between two zones is proportional to the trip volumes at each of the zones and inversely proportional to some power of the distance (or travel time, or cost) between them. It has nothing at all to do with physical gravity, of course. Although this formulation avoids some of the shortcomings of the Fratar method, its behavior exhibits a number of peccadillos that disqualify it as a serious hypothesis (Appendix A). It can be shown that the formulation is not generally valid over an unlimited or undefined range of the distance variable, but can only be entertained within a region between some stated minimum and maximum distances. When these limits are given, however, the formula becomes a function of them, and they are quite arbitrary. Moreover, no tampering with the formula, not even a change of exponent, can yield the same results if the arbitrary boundary of the region is moved. And, to be usable, the exponent must be supposed stable from place to place and time to time. The author's experience leads him to doubt that.

Like the Fratar method, the gravity formula has a certain utility, but, in spite of a few arguments that have been voiced, no conceptual content is apparent.

The method being applied in the Chicago study rests on a premise that certainly sounds good: total travel time from a point is minimized subject to the condition that every destination point has a stated probability of being accepted if it is considered. If that probability is constant, the problems of choosing the minimizing order of consideration and deriving the consequences of probabilistic behavior are rather simple (Appendix B). It is less simple to say whether or not the working method, with its assumptions and approximations, extorted from this proposition is worthwhile.

It is not easy to assay the quality of a set of interchange predictions. Graphs and charts give an impression, of course, but cumulative curves are meretricious at best, and less seductive displays are hard on the eyes. The statistical measures suggested are not unreasonable, but they have a makeshift air to them and a slightly upsetting tendency towards comparing aggregate data and predictions; upsetting because it is obvious that any large-scale aggregation is specious. That the variance in individual interchanges need never be considered because interchanges are such small quantities and there are so many of them is a meaningless and perplexing point of view, like dismissing atomic weights because atoms are tiny and numerous. If these small quantities do not matter, why fool with them at all? If they do matter, what mystery causes deviations in them to cancel out in, say, the assignment process? Actually, deviations in interchange values may cancel each other in assignment, but the extent to which they do is a measure of the system's insensitivity to zone centroid locations (which is related to the complexity of the road network); if the assignment is insensitive to zone positions, then zones may as well be grouped into super-zones and super-interchanges developed. But the only point in assignment is to deal with the locational properties of trip ends, so as these properties are lost or dispersed the assignment itself becomes inane. If zonal definitions are properly geared to the assignment network, individual interchanges cannot be sanguinely, or even glumly, neglected.

Getting back to the merit of the Chicago method, the formula that evolves from the basic concepts turns out to be computationally convenient, and most well-behaved mathematically. Aside from precision effects, it is independent of zonal or regional boundaries; and it entails no special cases. It compares with data better than had been expected: computed interchanges assigned to a network by minimum paths does not give, on the planning scale, a noticeably different pattern from data interchanges assigned to the same network; the virulent attacks of members of the CATS staff on this method have not indicated that some other method would be just as good; using interchanges from seven scattered zones to all others in the Chicago study area as a base, it was found that seven different gravity models predicted the same data from which they were obtained less well than the over-all Chicago formula.

Not that the Chicago formula is all good. The dispersions from data are unquestionably larger than theoretical variance. The Chicago method shares two flaws with the gravity model: the number of trips received at a zone do not necessarily agree with the number provided, and there is distinct difficulty in obtaining parameters for future or unknown situations. On the first score, some comfort is derived from the received and provided totals being generally within 10 percent of each other; the discrepancies have been important clues to the functioning of the system and to defects in the formulation. On the second point, the staff is confident that fairly good estimates of unknown parameters can be obtained, and tests of that confidence are being conducted. But more than that, the formulation holds out direction and hope for defining them exactly.

Although the formulation has proved agreeably serviceable on the applied level, it seems that its most telling contributions are heuristic. It has introduced subtended volume, the volume of trip opportunities lying closer to the origin than the point of interest, as an explicit parameter and has attributed to it an explicit mechanical role. It has explicitly treated interchanges as probability numbers. It has focused fuzzy areas of study, and pointed to new ones. It has elicited sharp probings into such matters as the distributional properties of trips with respect to various parameters, and the meaning of it all. It is fair to say that discussions of trip behavior among the interested Chicago staff have taken on vitality and new color.

It is not intended to convey that the Chicago approach, even with present working simplifications removed, is a sovereign remedy. In fact, some work has been done in a different direction that may be more profitable ultimately. But the approach discussed here has given a practical method that seems better than any other available, while it has engendered much intellection and germinated many ideas. And with all respect to working tools, ideas are not so easy to come by, either.

Appendix A

The gravity formula may be written

$$V_{ij} = K_i V_i V_j / r^a_{ij} \tag{1-1}$$

in which

V_{ij} = interchange from zone i to zone j;

V_k = total trip volume at zone k;

r_{ij} = distance (or travel time, or travel cost) between zones i and j;

a = a constant exponent expressing the resistance, or something, of distance; and

K_i = a normalizing constant—that is, the constant required by the condition $\Sigma_j V_{ij} = V_i$.

Because the formula is independent of zone size, it should hold for small zones, so that Eq. 1-1 can be thought of as

$$dV_i = K_i V_i p \, dA / r^a = 2\pi K_i V_i p \, r^{(1-a)} dr \tag{1-2}$$

in which p is the average trip density in the annular region dA, at any r distance from the origin zone. From this it follows that the normalizing constant must be

$$K_i = 1/2\pi\bar{p} \int_B^C r^{(1-a)} dr \tag{1-3}$$

Here, $\bar{p}$ is an average trip density for the region between r=C and r=B, in the sense that

$$\bar{p} \int_B^C r^{(1-a)} dr = \int_B^C p r^{(1-a)} dr$$

Integration of Eq. 1-3 gives

$$K_i = \frac{2-a}{2\pi\bar{p}\,(C^{(2-a)} - B^{(2-a)})} \quad \text{if } a \neq 2 \tag{1-4a}$$

$$K_i = \frac{1}{2\pi\bar{p}\,\log (C/B)} \quad \text{if } a = 2 \tag{1-4b}$$

Now $\bar{p}$, because it is an average, is only weakly affected by changes in the limits C and B—provided the interval of integration is reasonably large and encloses all singular regions, such as the CBD—and may be considered more or less constant in this argument. Thus K_i, and through it any interchange calculation, varies with the minimum and maximum distances used. Inspection of Eqs. 1-4 shows that the sensitivity of K_i to these limits increases as the exponent, a, moves away from the value 2; as a decreases, the sensitivity is more and more to the upper limit, C, while as a increases, the sensitivity shifts to the lower limit, B.

To show that a change of exponent cannot correct for a change in K_i (due to moving the boundaries of the region), it is only necessary to compute the new exponent, a', that would make an interchange computation the same under a new constant, K_i'. This can be done by simple algebra from Eq. 1-1, and the result is

$$a' = a + \frac{\log K_i'/K_i}{\log r_{ij}} \tag{1-5}$$

This is not a constant, as required by the formulation, but a function of distance: the formulation cannot be rectified, made to yield the same calculations, from one boundary situation to another. (K_i' is, of course, a function of a' as well as of the boundaries; but the final, purified solution for a' need not be obtained, inasmuch as the argument depends only on K_i' being different from K_i. That they are different follows from the hypothesis that they are equal: then a' would equal a, which implies the contradiction that K_i' does not equal K.)

Appendix B

If the probability of a destination point being acceptable is independent of the order in which destinations are considered, the order that will minimize travel time is clearly time proximity, from near to far. And the premise may be re-stated: a trip prefers to remain as short as possible, but its behavior is governed by a probability of stopping at any destination it encounters—it cannot always just go to the nearest destination and stop; it must consider the nearest destination, and if that is unacceptable consider the next nearest, and so on. To cast this into mathematical language: the probability that a trip will terminate within some volume of destination points is equal to the probability that this volume contains an acceptable destination, times the probability that an acceptable destination closer to the origin of the trip has not been found. But the latter two probabilities may vary from point to point, so the problem must be stated in terms of limitingly small quantities. This leads to

$$dP = (1-P)LdV \tag{2-1}$$

P is the probability the trip has terminated within the destination volume, V, lying earlier in the order of consideration (or subtended volume); L is the probability density (probability per destination) of destination acceptability at the point of consideration.

If L is constant, the only case discussed here, the solution of Eq. 2-1 is

$$P = 1-ke^{-LV} \tag{2-2}$$

But K (the constant of integration) = 1, because P must be zero when V is zero, so

$$P = 1-e^{-LV} \tag{2-3}$$

The expected interchange from zone i to zone j is simply the volume of trip origins at zone i multiplied by the probability of a trip terminating in j; that is,

$$V_{ij} = V_i \left[P(V+V_j) - P(V)\right] = V_i (e^{-LV} - e^{-L(V+V_j)}) \qquad (2\text{-}4)$$

An obvious extension of this is the supposition that, although L is constant for each trip, different trips have different L's. The more general equation then is

$$V_{ij} = \int_{L_{min}}^{L_{max}} (e^{-LV} - e^{-L(V+V_j)}) Z_i dL \qquad (2\text{-}5)$$

in which Z_i is the distribution of V_i with respect to L; that is, $Z_i = \frac{dV_i}{dL}$. Further, it can be argued that the destinations are also distributed in their affinities. This may be allowed (without going into detailed reasoning) by construing V and V_j in Eq. 2-5 as effective volumes, and functions of L. The computation of Eq. 2-5 cannot be realized in practice without far more understanding. But an attempt to adjust Eq. 2-4 in the direction of Eq. 2-5 can be made by clustering trips into "kindred" sub-populations with all members of a given sub-population being governed by the same L. The approximation to Eq. 2-5 is then

$$V_{ij} = \sum_s V_{i(s)} (e^{-L_{(s)} V_{(s)}} - e^{-L_{(s)} (V_{(s)} + V_{j(s)})}) \qquad (2\text{-}6)$$

The subscript (s) is the sub-population index. This is quite analogous to the stratification commonly used with gravity, iteration, and other models, but a little different in concept.

Readers with a taste for rigor may feel there is some mathematical license in treating discrete, unitary trip ends as a continuous "volume," and that a more proper form of Eq. 2-1 would be the difference equation

$$\Delta P = (1-P)L\Delta V = (1-P)L \quad (\text{since } \Delta V = 1). \qquad (2\text{-}7)$$

But Eq. 2-7 represents a well-behaved step function—piecewise continuous and everywhere finite—so the difference between it and Eq. 2-1 is one of precision rather than of kind, and the discrepancy introduced by integrating a continuous approximation to a step function is small if the number of steps is large. Differentials are preferable to differences, in this instance, simply for reasons of tractability—continuous expressions are amenable to generalization and adjustment, and they are usually more lucid. Eq. 2-7 can be solved easily enough, if L is constant, by stating it as $P_{n+1} - P_n = (1-P_n)L$ and then writing out the recursions (Eq. 2-1 can be solved by inspection), but any departure from that special case requires considerable manipulation.

Mr. Lynch

It is fairly clear that there is no agreement among experts as to the best method for projecting future urban travel. However, they all agree on one thing—that considerably more research is needed in this field. It is hoped that research now under way, or to be undertaken in the not too distant future, will result in general acceptance of one of the methods already developed or of a new method yet to be devised.