A Theoretical Prediction of Work-Trip Patterns

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● ORIGIN AND DESTINATION surveys have been developed into effective tools for determining travel patterns. Unfortunately, such surveys have two major shortcomings: they are quite expensive, and they are tied closely to the localities and points in time in which they are made. During the past 5 or 6 yr, considerable effort has been expended in attempts to find relationships between land-use patterns and travel patterns in cities where such surveys have been made. It is hoped that the relationships which are found may be used to synthesize patterns of movement in those communities at other points in time, or may be translatable to other cities. This paper may contribute to the synthesis of an origin and destination (O-D) survey.

Three and one-half years ago, research was begun on the possibility of creating a purely theoretical model of the movement of people in response to varying land uses based upon the principles of electrostatics. Although the ability of this theory to predict all types of movement has not been checked as yet, it does seem to be capable of predicting work-trips patterns in a metropolitan area within usable limits of accuracy. This paper presents the techniques which have evolved thus far from the research, but does not delve into the theory behind these techniques.

BASIC ASSUMPTIONS

It is an interesting paradox that nothing in nature occurs with true mathematical precision, although so many natural phenomena are so nearly predictable from mathematical relationships. When human decisions and vagaries are involved in the events, larger errors must usually be expected than may be necessary when dealing with inanimate objects.

In particular, it must be recognized that it is probably impossible to predict with any degree of accuracy the movements of any one person. In addition, the theory, herein demonstrated, requires that the following simplifying assumptions be made in order to permit mathematical formulation.

1. The region to be analyzed must be precisely bounded. An alternate form of this premise would be that every worker living within the region of interest has a job within that region, and every job within the region is filled by a worker living within the region.
2. Every worker goes to work every day, and there is no "turnover" in employment. The technique described herein depends on an initial unbalance in this respect, but the derived pattern of movement is assumed to be stable.
3. Movement between places of varying land use is independent of the mode of travel. Predictions resulting from the theory concern the movement of people, not of vehicles. If walking trips are not of interest, they must be factored out of the theoretical pattern of movement.
4. Every center of employment includes a hierarchy of employees, ranging from common laborers to company presidents. It is recognized that any one organization may have a preponderance of either higher or lower echelon workers, but it is assumed that these irregularities balance out within a comparatively small area.
5. Workers of all income levels live in all parts of the region of interest. This does not necessarily mean that laborers and managers live side by side, but there are very few tracts, of even 1 sq mi in area, within metropolitan regions which are completely homogeneous with respect to incomes of residents.
6. The separations between centers of residence and centers of employment are best measured by the straightline distances between such centers. Even though the actual movement between two centers of activity is quite limited because of physical obstructions, if the potential for movement is great enough, someone will build a Mackinac Bridge, a Lincoln Tunnel, a tunnel through the Alps, or even a tunnel under the English Channel. Any other measure of separation would seem to merely reflect

how people react to existing conditions, and not how they would behave if given a better alternative.

7. Real land-use patterns, made up of centers of residence, centers of employment, centers of shopping, centers of recreation, etc., may be represented as patterns of noncompeting centers of positive charge; that is, at no point in time does a center of recreation compete with a center of employment in the attraction of workers.

8. Human beings may be considered to be electrons. Given the initial distribution of these unit negative charges, corresponding to centers of residence, and the distribution of centers of positive charge, representing places of employment, with magnitudes equaling the numbers of persons employed, the probability of movement between places of residence and places of employment can be predicted on the basis of electrostatic field theory.

SOURCES OF INFORMATION

To apply the theory to a real community, the places of residence and of employment must be abstracted and made centers of negative and positive charge, respectively. Many governmental agencies collect data on population and on business, and many private organizations compile business statistics, so there are many sources of information on where people work and where they live.

The Federal Government publishes census data on the population, housing, retail trade, wholesale trade, service trade, and agriculture. Some useful information based on Social Security records is also published.

State governments may make available sales-tax records or unemployment-compensation records. Ohio publishes annually a "Directory of Ohio Manufacturers," which lists every manufacturing establishment in the state by name and address, with the numbers of male and female employees of each one.

City planning commissions have land-use maps, and other sources of information.

Private organizations publish a number of marketing and investment guides which contain much information on population and production characteristics of the areas covered. City directories are invaluable sources of information, as are Sanborn Fire Maps. In some cities, electric power companies and telephone companies keep records of the places where they have installations which may be helpful.

Despite the many sources of information on where people live and where they work, it must be expected that some desirable information will not be readily available whereas some of that which is available will be conflicting. Neither is it likely that all of the information can be obtained for one point in time. With due care, however, it should be possible to create a reasonably accurate pattern of distribution of places of residence, of employment, of shopping, of recreation, etc., from existing sources of information.

After the city has been translated into such an abstract field of centers of positive and negative charges, the basic equations of electrostatics may be used to predict the patterns of movement which will develop.

EQUATIONS FOR PREDICTING MOVEMENT

It is believed that the equations presented herein are capable of predicting all types of movement, but thus far they have been tested only for work trips, and this discussion will be confined to this type of trip. It will be noted that these equations are quite similar to those proposed by Casey (1) and Voorhees (2). So far as is known, these men arrived at their models empirically from the analysis of O-D surveys. As stated previously, the model presented herein was arrived at theoretically by assuming that human movement can be represented by that of electrons in a field of positive charges.

By its nature and derivation, Eq. 1 will insure that the correct number of workers will be drawn from each zone of residence.

$$V_{P_iQ_j} = \frac{\frac{Q_j}{R_{ij}} \quad P_i}{\sum_{j=1}^{m} \frac{Q_j}{R_{ij}}} \qquad (i = 1, 2, \text{———} n) \qquad (1)$$

in which

$V_{P_iQ_j}$ is defined as the probability of movement from i to j.

P_i is the number of workers living in zone i.

Q_j is the number of jobs available at center j.

R_{ij} is the straightline distance from i to j if the field contains no physical barriers. Where such barriers exist, R would have to be the straightline distance from i to the point of passage across the barrier plus that from the point of passage to j.

Eq. 2 is quite similar to Eq. 1, but it insures that the correct number of workers is assigned to each job site.

$$V_{Q_jP_i} = \frac{\frac{P_i}{R_{ij}} Q_j}{\sum_{i=1}^{n} \frac{P_i}{R_{ij}}} \qquad (j=1, 2, \text{——} n) \qquad (2)$$

in which

$V_{Q_jP_i}$ is defined as the probability of movement to j from i, and the other terms are as in Eq. 1.

Unfortunately, the direct use of Eqs. 1 and 2 will usually yield two different sets of movement, and not the unique pattern which is desired. Eq. 1 does not take into account the total number of workers assigned to each job site by the "m" solutions each for i=1, 2, ——n, and, therefore, will usually overassign or underassign workers to the various job centers. Eq. 2 does not keep track of the total number of workers drawn from each of the zones of residence by the "n" solutions each for j=1, 2, ——m, and, therefore, tends to overdraw or underdraw from the various zones.

Two alternate methods have been developed for arriving at completely balanced assignments. Inasmuch as either method may be applied to either of the basic equations, four patterns of movement may be calculated. Several sets of sample calculations have indicated that all four of these patterns will be essentially identical. It, therefore, seems safe to state that the theory develops a unique pattern of work-trip movements in any given field of activity. The first of these methods is theoretically justifiable, whereas the second reduces the number of calculations significantly.

CALCULATION BY SUCCESSIVE PARTIAL ASSIGNMENTS

The theoretically justifiable method for balancing the assignment of workers to job sites and the drawing of workers from zones of residence will be illustrated with reference to Eq. 1, although application to Eq. 2 is quite similar and no more difficult.

Let it be assumed that there are "m" places of employment scattered over the region of interest, each of which wishes to hire Q_j (j=1, 2, ——m) workers. Also let there be "n" zones of residence in the region, in each of which there are P_i (i=1, 2, ——n) workers who are seeking employment. The only restraint placed on these numbers is that $\sum Q_j = \sum P_i$. Figure 1 shows such a situation in general terms, with m = 3, and n = 2.

As stated, the direct application of Eq. 1 to this region "nm" times (once for every center of employment to interact with each zone of residence) will insure that the correct number of workers is drawn from each zone of residence, but is not likely to assign the correct number of workers to any job site. If, however, only $\frac{P_i}{x}$ workers are initially drawn from each place of residence, "x" can be chosen sufficiently large (perhaps even as 100 or 200) so that no job site will be overassigned by the first set of calculations. After this set of "nm" solutions of Eq. 1,

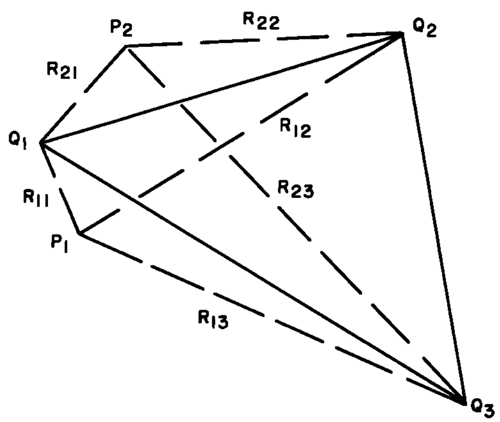

Figure 1. General hypothetical community with three centers of employment and two centers of residence.

each Q_j is reduced by the corresponding $\sum_{i=1}^{n} V_{P_iQ_j}$, the total number of workers assigned to the site from the "n" zones of residence. The next group of $\frac{P_i}{x}$ workers from each zone faces a somewhat different field than did the first group, because each Q_j is now somewhat different, and, indeed, some of them may have been reduced to zero. These steps are repeated through "xnm" solutions of Eq. 1 so that, at last every worker has been assigned to a job site, and every job site has been exactly satisfied. Table 1 gives the first few steps of this method as applied to solve the general community of Figure 1. Table 5 is a numerical solution of a particular community by this method.

TABLE 1

PORTIONS OF GENERAL SOLUTION OF WORK-TRIP PATTERN FOR COMMUNITY OF FIGURE 1 BY SUCCESSIVE PARTIAL ASSIGNMENTS

Original Assignments

From		To Q_1	To Q_2	To Q_3	Sum
P_1	$\dfrac{\frac{Q_1}{R_{11}} \; \frac{P_1}{x_1^*}}{\frac{Q_1}{R_{11}} + \frac{Q_2}{R_{12}} + \frac{Q_3}{R_{13}}} =$	$\frac{V_{P_1Q_1}}{x_1}$	$\frac{V_{P_1Q_2}}{x_1}$	$\frac{V_{P_1Q_3}}{x_1}$	$\frac{P_1}{x_1}$
P_2	$\dfrac{\frac{Q_1}{R_{21}} \; \frac{P_2}{x_1}}{\frac{Q_1}{R_{21}} + \frac{Q_2}{R_{22}} + \frac{Q_3}{R_{23}}} =$	$\frac{V_{P_2Q_1}}{x_1}$	$\frac{V_{P_2Q_2}}{x_1}$	$\frac{V_{P_2Q_3}}{x_1}$	$\frac{P_2}{x_1}$
Total Assigned to Q_j		$\frac{V_{P_1Q_1}}{x_1} + \frac{V_{P_2Q_1}}{x_1}$	$\frac{V_{P_1Q_2}}{x_1} + \frac{V_{P_2Q_2}}{x_1}$	$\frac{V_{P_1Q_3}}{x_1} + \frac{V_{P_2Q_3}}{x_1}$	

Second Assignments

From	To Q_1
P_1	see below

$$P_1 \quad \frac{\dfrac{Q_1 - \frac{V_{P_1Q_1}}{x_1} - \frac{V_{P_2Q_1}}{x_1}}{R_{11}} \; \dfrac{P_1}{x_2}}{\dfrac{Q_1 - \frac{V_{P_1Q_1}}{x_1} - \frac{V_{P_2Q_1}}{x_1}}{R_{11}} + \dfrac{Q_2 - \frac{V_{P_1Q_2}}{x_1} - \frac{V_{P_2Q_2}}{x_1}}{R_{12}} + \dfrac{Q_3 - \frac{V_{P_1Q_3}}{x_1} - \frac{V_{P_2Q_3}}{x_1}}{R_{13}}} = \frac{V_{P_1Q_1}}{x_2}$$

*Subscript on x denotes the number of partial assignment, x being a constant.

In a real community, this method of assignment will involve a large number of calculations. When the theory was used to predict work-trip patterns in Lafayette, Ind., for example, 45 zones of residence and 13 centers of employment were used. Instead of keeping x a constant, it was found possible to vary x and thereby reduce the number of calculations, but even so, 10 sets of partial assignments were needed to assign the first 99 percent of the workers. In this problem it was found possible to use x = 5 for three sets of calculations, x = 10 for one set, x = 25 for four sets, x = 50 for one set, and x = 100 for one set (P_i (3x1/5 + 1x1/10 + 4x1/25 + 1x1/50 + 1x1/100) = 0.99P_i). When the theory was applied to the metropolitan area of Cincinnati, assignments were made from 455 zones of residence to 480 centers of employment so that it did not appear feasible to use this method of partial assignments.

CALCULATION BY CORRECTION FACTORS

In an effort to reduce the number of calculations required in a complex community, an empirical method was devised for balancing the drawing of workers from zones of residence and assigning them to places of employment.

The explanation of this system, too, will be limited to use with Eq. 1, although it works equally well with Eq. 2. Eq. 1 is solved "m" times for each value of i, or "nm" times, using the full values of each P_i and Q_j. The total number of workers assigned to each job site, $\sum_{i=1}^{n} V_{P_iQ_j}$, is found, and each total is divided into its corresponding value of Q_j to get the first set of correction factors, C_{j_1}. When each value of $V_{P_iQ_j}$ is multiplied by the correction factor for the corresponding job site, C_j, and checked by adding $\sum_{i=1}^{n} V_{P_iQ_j} \cdot C_{j_1}$ and $\sum_{j=1}^{m} V_{P_iQ_j} \cdot C_{j_1}$, the former sums should exactly equal the corresponding values of Q_j from the nature of the correction factors. It has been found that the latter sums may be close enough to the corresponding values of P_i to be acceptable. If, however, these latter sums are not sufficiently close to the original values of P_i, new correction factors may be formed by dividing $\sum_{j=1}^{m} V_{P_iQ_j} \cdot C_{j_1}$ into the corresponding values of P_i. This system of calculation is given in Table 2 with reference to the community of Figure 1, with Q_3 made zero to reduce the calculations to manageable size.

It will be noted from Table 2 that the general solution by this method gives no indication that the drawing of workers from zones of residence and the assignment of workers to places of employment will ever balance out, nor what the individual assignments will be, should such a balance ever be achieved. Limited experience indicates that such a balance does occur within a few iterations of correction factors, and that the resulting assignments are essentially identical to those obtained from the method of successive partial assignments. Table 6 solves the community of Table 5 by means of correction factors to illustrate these points.

Table 7 compares the results obtained by the two alternate methods of solution.

CHECKS OF THE THEORY

To determine whether the theory, thus far demonstrated, might be capable of predicting work-trip patterns in real communities, it was applied, as noted previously, to Lafayette, Ind., and the metropolitan area of Cincinnati, Ohio.

Lafayette was used as a pilot study. Workers were assigned to zones of residence with considerable accuracy because a special census had been taken in 1953. Fairly accurate employment figures were obtained for the six largest individual employers, but jobs were assigned to another seven places of employment by estimations based on the 1947 Census of Business and 1950 Census of Population data. The work-trip pattern created by the theory was checked against the vehicular movements found by the 1953 O-D survey of the area. The 585 predicted movements resulting from the theory contained many discrepancies when compared with the corresponding O-D

TABLE 2

PORTIONS OF GENERAL SOLUTION OF WORK-TRIP PATTERN FOR COMMUNITY OF FIGURE 1 BY CORRECTION FACTORS

Original Assignments

From	To Q_1	To Q_2	
P_1	$\dfrac{\frac{Q_1}{R_{11}} P_1}{\frac{Q_1}{R_{11}} + \frac{Q_2}{R_{12}}}$	$\dfrac{\frac{Q_2}{R_{12}} P_1}{\frac{Q_1}{R_{11}} + \frac{Q_2}{R_{12}}}$	Let $Q_1 = aQ_2$ $Q_3 = O$ $P_1 = bP_2$ $R_{11} = cR_{21} = dR_{12} = eR_{22}$
P_2	$\dfrac{\frac{Q_1}{R_{21}} P_2}{\frac{Q_1}{R_{21}} + \frac{Q_2}{R_{22}}}$	$\dfrac{\frac{Q_2}{R_{22}} P_2}{\frac{Q_1}{R_{21}} + \frac{Q_2}{R_{22}}}$	

Original Assignments in Terms of Ratios

			Sums
P_1	$\dfrac{aP_1}{a+d}$	$\dfrac{dP_1}{a+d}$	P_1
P_2	$\dfrac{acP_2}{ac+e}$	$\dfrac{eP_2}{ac+e}$	P_2
Sums	$\dfrac{aP_1\,(b(ac+e)+c(a+d))}{(a+d)\,(b)\,(ac+e)} = X_1$	$\dfrac{P_1\,(bd(ac+e)+e(a+d))}{(a+d)\,(b)\,(ac+e)} = X_2$	

Correction Factors $\dfrac{Q_1}{X_1} = C_{Q_1}$ $\qquad \dfrac{Q_2}{X} = C_{Q_2}$

Original Assignments Multiplied by Correction Factors

			New Sums
P_1	$\dfrac{Q_1\,(b)\,(ac+e)}{b(ac+e) + c(a+d)}$	$\dfrac{Q_2\,(bd)\,(ac+e)}{bd(ac+e) + e\,(a+d)}$	S_1
	$\dfrac{Q_1\,(c)\,(a+d)}{b(ac+e) + c(a+d)}$	$\dfrac{Q_2\,(e)\,(a+d)}{bd(ac+e) + e(a+d)}$	S_2

$$S_1 = \frac{Q_1\,(b(ac+e))\,(bd(ac+e)+e(a+d)) + \frac{Q_1}{a}\,(bd)\,(ac+e)\,(b(ac+e)+c(a+d))}{(b(ac+e)+c(a+d))\,(bd(ac+e)+e(a+d))}$$

$$\frac{Q_1(c)\,(a+d)\,(bd(ac+e)+e(a+d)) + \frac{Q_1}{a}\,(e)\,(a+d)\,(b(ac+e)+c(a+d))}{(b(ac+e)+c(a+d))(bd(ac+e)+e(a+d))}$$

findings, but a large enough proportion of the predicted movements were sufficiently close to the observed movements that a much more detailed analysis of the Cincinnati area seemed to be justified.

A 2,000-ft grid was drawn over a map of Hamilton County, and portions of Campbell and Kenton Counties in Kentucky. Workers were assigned to 455 places of residence on this grid as accurately as available data permitted. Every known source of information on employment was studied, and 480 job sites were established in Hamilton County. No job sites were established in northern Kentucky, but the difference between the number of workers living in that area and the number of jobs which could be accounted for there, amounting to about 40,000 workers, was assigned to work in Hamilton County.

When all of the data had been assembled, it was found that 348,308 workers had been assigned to places of residence while 333,537 jobs had been assigned to places of employment. Because of the experimental nature of this project, these totals were not adjusted to bring them into balance, but it is strongly recommended that such adjustment be made if this theory is used again.

To date, only a few of the 218,400 predicted movements have been studied, but two detailed checks were made of some of the movements. The first of these compared the predicted movements between six communities of the area with the numbers of passenger car trips found by the 1954 post-card-type O-D survey of the county. The communities were chosen so that the majority of the actual trips between them might reasonably be expected

TABLE 3

COMPARISON OF PREDICTED WORK TRIPS AND 1954 O-D SURVEY TOTAL PASSENGER CAR TRIPS BETWEEN SELECTED COMMUNITIES IN THE STANDARD METROPOLITAN AREA OF CINCINNATI, OHIO

From	Cheviot 619 Jobs	N. College Hill Mt. Healthy 1,781 Jobs	Lockland 4,148 Jobs	Norwood 18,322 Jobs	Fairfax—Mariemont 3,927 Jobs	Mt. Washington 788 Jobs
(a) Movements by Theory						
Cheviot, Ohio	262.0	70.5	87.3	277.7	61.4	16.2
N. College Hill—Mt. Healthy, Ohio	16.3	688.2	144.9	302.2	60.8	14.0
Lockland, Ohio	2.3	16.0	513.5	107.7	21.5	3.5
Norwood, Ohio	12.2	56.1	188.8	3,839.9	211.8	34.3
Fairfax—Mariemont, Ohio	5.2	20.9	65.8	334.8	992.7	31.5
Mt. Washington	2.6	9.3	28.2	132.9	67.5	163.7
(b) Comparison of Movements						
Cheviot, Ohio 7,740 workers	262.0[1]					
N. College Hill— Mt. Healthy, Ohio 6,675 workers	86.8 313 (0.55)	688.2[1]	Total predicted one-way trips. Passenger car trips by O & D. Persons per passenger car.[2]			
Lockland, Ohio 2,827 workers	89.6 55 (3.20)	160.9 285 (1.12)	513.5[1]			
Norwood, Ohio 15,691 workers	289.9 191 (3.02)	358.3 527 (1.36)	296.5 491 (1.16)	3,839.9[1]		
Fairfax—Mariemont, Ohio 5,354 workers	66.6 46 (2.90)	81.7 61 (2.55)	87.3 47 (3.64)	546.6 653 (1.67)	992.7[1]	
Mt. Washington 2,347 workers	18.8 23 (1.57)	23.3 89 (0.51)	31.7 43 (1.44)	167.2 469 (0.71)	99.0 364 (0.54)	163.7[2]

[1] No check available for these predictions.

[2] Number of passenger car trips, found by O-D survey, divided by twice the predicted number of one-way trips.

to be auto-driver and auto-passenger work trips. Table 3 gives a comparison of the predicted work trips and the total passenger car trips, as found by the O-D survey, between these communities. The second check consisted of comparing the predicted places of residence of the approximately 1,400 full-time employees of the University of Cincinnati with the actual places of residence, as determined from the 1957 payroll. Table 4 gives the distribution of discrepancies between the predicted and actual places of residence, as found in this comparison.

TABLE 4

DISTRIBUTION OF ERRORS: PREDICTED NUMBER OF RESIDENTS MINUS ACTUAL NUMBER OF RESIDENTS, FULL-TIME EMPLOYEES OF UNIVERSITY OF CINCINNATI

Range of Error		Median	Frequency	Remarks
From	To	Values	of Errors	
-77.9	-76.0	-77.0	1*	Adjacent to Univ.
-75.9	-74.0	-75.0	1*	Adjacent to Univ.
-	-	-		
-21.9	-20.0	-21.0	1*	
-19.9	-18.0	-19.0	0	
-17.9	-16.0	-17.0	0	
-15.9	-14.0	-15.0	2 (6)	(6) includes avg of 4
-13.9	-12.0	-13.0	2	poor values at U.C.
-11.9	-10.0	-11.0	5	
- 9.9	- 8.0	- 9.0	2	
- 7.9	- 6.0	- 7.0	3	
- 5.9	- 4.0	- 5.0	8	
- 3.9	- 2.0	- 3.0	13	
- 1.9	- 0.0	- 1.0	16	
+ 0.1	+ 2.0	+ 1.0	24	
+ 2.1	+ 4.0	+ 3.0	9	
+ 4.1	+ 6.0	+ 5.0	9	
+ 6.1	+ 8.0	+ 7.0	6	
+ 8.1	+10.0	+ 9.0	3	
+10.1	+12.0	+11.0	1	
+12.1	+14.0	+13.0	0	
+14.1	+16.0	+15.0	2	
-	-	-		
+24.1	+26.0	+25.0	1*	
-	-	-		
+30.1	+32.0	+31.0	1*	Northern Kentucky
+32.1	+34.0	+33.0	1*	Basin area of city.
-	-	-		
+38.1	+40.0	+39.0	1	Adjacent to Univ.
-	-	-		
+48.1	+50.0	+49.0	2*	Northern Kentucky
-	-	-		
+54.1	+56.0	+55.0	1*	Adjacent to Univ.
			115	

Case 1. Including entire table.
Mean error = +0.57 Standard Deviation = 15.5
Case 2. Excluding values marked*.
Mean error = -0.37 Standard Deviation = 5.75
Case 3. Averaging errors at University and dropping values marked Basin and Kentucky. Mean error = 0.15 Standard Deviation = 8.88

TABLE 5

SOLUTION OF HYPOTHETICAL COMMUNITY OF FIGURE 2 BY EQUATION .1. AND SUCCESSIVE ASSIGNMENTS OF P_i/x WORKERS CALCULATIONS[1]

x	P_i	P_i/x	Q_1	Q_2	Q_3	$\frac{Q_1}{R_{i1}}$	$\frac{Q_2}{R_{i2}}$	$\frac{Q_3}{R_{i3}}$	k_{P_i} [2]	$\frac{V_{P_iQ_1}}{x}$	$\frac{V_{P_iQ_2}}{x}$	$\frac{V_{P_iQ_3}}{x}$	Sum
4	600	150	500	200	100	0.0250	0.0167	0.0250	14.99	56.2	37.6	56.2	150.0
4	200	50	500	200	100	0.1000	0.0182	0.0048	8.13	40.7	7.4	2.0	50.1
4	600	150	403	155	42	0.0202	0.0128	0.0105	22.99	69.7	44.1	36.2	150.0
4	200	50	403	155	42	0.0806	0.0141	0.0020	10.34	41.7	7.2	1.0	49.9
4	600	150	291	104	5	0.0147	0.0087	0.0012	40.81	90.6	52.2	7.2	150.0
Overassigns Q_3 by 7.2 - 5 = 2.2, therefore use x = 10.													
10	600	60	291	104	5	0.0147	0.0087	0.0012	40.81	35.7	21.3	2.9	60.1
10	200	20	291	104	5	0.0580	0.0095	0.0002	14.77	17.1	2.8	0.1	20.0
10	600	60	238	80	2	0.0119	0.0067	0.0005	52.36	37.4	21.0	1.6	60.0
10	200	20	238	80	2	0.0495	0.0075	0.0001	17.57	17.4	2.6	0.0	20.0
10	600	60	183	57	0	0.0092	0.0048	-	71.43	39.4	20.6	0.0	60.0
10	200	20	183	57	0	0.0365	0.0052	-	23.98	17.5	2.5	0.0	20.0
10	600	60	126	34	0	0.0063	0.0028	-	109.9	41.5	18.5	0.0	60.0
10	200	20	126	34	0	0.0232	0.0031	-	38.02	17.6	2.4	0.0	20.0
10	600	60	60	13	0	0.0034	0.0011	-	222.2	45.3	14.7	0.0	60.0
Overassigns Q_2 by 14.7 - 13 = 1.7, therefore use x = 20.													
20	600	30	60	13	0	0.0034	0.0011	-	222.2	22.7	7.3	0.0	30.0
20	200	10	60	13	0	0.0134	0.0012	-	68.5	9.2	0.8	0.0	10.0
20	600	30	35	5	0	0.0018	0.0004	-	454.5	24.5	5.4	0.0	30.0
20	200	10	35	5	0	0.0070	0.0005	-	133.3	9.3	0.7	0.0	10.0
Overassigns Q_2 by 1.1, and underassigns Q_1 by 1.2, but use as is.													

[1] Summary of assignments in Table 7.

[2] k_{P_i} is reciprocal of $\frac{Q_1}{R_{ij}}$

TABLE 6

SOLUTION OF HYPOTHETICAL COMMUNITY OF FIGURE 2 BY APPLICATION OF CORRECTION FACTOR TO INITIAL SOLUTION OF EQUATION .1.

P_i	Q_1	Q_2	Q_3	$\frac{Q_1}{R_{i1}}$	$\frac{Q_2}{R_{i2}}$	$\frac{Q_3}{R_{i3}}$	k_{P_i}	$V_{P_iQ_1}$	$V_{P_iQ_2}$	$V_{P_iQ_3}$
600	500	200	100	0.0250	0.0167	0.0250	14.99	224.9	150.2	224.9
200	500	200	100	0.1000	0.0182	0.0048	8.13	162.6	29.6	7.8
							Initial sums	387.5	179.8	232.7

Calculation of correction factors:

First set: $\frac{500}{387.5} = 1.290$; $\frac{200}{179.8} = 1.112$; $\frac{100}{232.7} = 0.430$

Second set: $\frac{600}{553.9} = 1.083$; $\frac{200}{246.0} = 0.813$

Third set: $\frac{500}{484.9} = 1.031$; $\frac{200}{207.7} = 0.963$; $\frac{100}{107.3} = 0.932$

Fourth set: $\frac{600}{595.8} = 1.007$; $\frac{200}{204.1} = 0.980$

Summary of assignments:

Movement	First Assign.	Second Assign.	Third Assign.	Fourth Assign.	Fifth Assign.
$V_{P_1Q_1}$	224.9	290.2	314.3	324.0	326.3
$V_{P_2Q_1}$	162.6	209.8	170.6	175.9	172.4
$V_{P_1Q_2}$	150.2	167.1	181.0	174.3	175.5
$V_{P_2Q_2}$	29.6	32.9	26.7	25.7	25.2
$V_{P_1Q_3}$	224.9	96.9	104.6	97.5	98.2
$V_{P_2Q_3}$	7.8	3.3	2.7	2.5	2.5
Totals assigned to:					
Q_1	337.5	500	484.9	500	498.7
Q_2	179.8	200	207.7	200	200.7
Q_3	232.7	100	107.3	100	100.7
Totals draw from:					
P_1	600	553.9	600	595.8	600
P_2	200	246.0	200	204.1	200

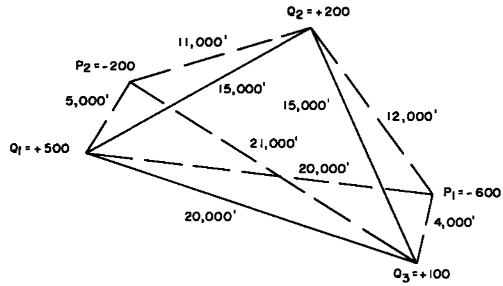

Figure 2. Hypothetical community with three centers of employment and two centers of residence.

CONCLUSIONS

This paper has attempted to present the possibilities of a new theory for predicting travel patterns in urban areas. No attempt has been made to explain the basis of the theory, but its use has been illustrated by the prediction of work-trip patterns in a simple, hypothetical community and in the metropolitan area of Cincinnati, Ohio.

Given the distribution of residences of workers and the locations of all jobs in a bounded region, it appears that the proposed method can be used to develop theoretical work-trip patterns which approach the actual patterns closely enough to be usable. Given anticipated future distributions of workers and jobs, future work-trip patterns should be predictable.

The method presented herein is much less costly and time-consuming than a comprehensive O-D survey. It is believed that further research will permit the theory to develop shopping-trip, personal-business-trip, and recreation-trip patterns, as well as work-trip patterns in urban areas.

TABLE 7

SUMMARY OF ASSIGNMENTS FROM TABLE 5 AND COMPARISON WITH ASSIGNMENTS FROM TABLE 6

x	$V_{P_1Q_1}$	$V_{P_2Q_1}$	$V_{P_1Q_2}$	$V_{P_2Q_2}$	$V_{P_1Q_3}$	$V_{P_2Q_3}$	Check Total Assignments	
4	56.2	40.7	37.6	7.4	56.2	2.0	P_1	P_2
4	69.7	41.7	44.1	7.2	36.2	1.0		
10	35.7	17.1	21.3	2.8	2.9	0.1	327.1	170.5
10	37.4	17.4	21.0	2.6	1.6	0.0	175.8	26.4
10	39.4	17.5	20.6	2.5	0.0	0.0	96.9	3.1
10	41.5	17.6	18.5	2.4	0.0	0.0		
20	22.7	9.2	7.3	0.8	0.0	0.0		
20	24.5	9.3	5.4	0.7	0.0	0.0	599.8	200.0
Sum	327.1	170.5	175.8	26.4	96.9	3.1	- by partial assignments.	
	326.3	172.4	175.5	25.2	98.2	2.5	- by correction factors.	

Check Assignments to Q_j:

	Q_1	Q_2	Q_3
	327.1	176.8	96.9
	170.5	26.4	3.1
	497.6	202.2	100.0

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