253

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Bulletin 253

Traffic Origin-and-Destination Studies

Appraisal of Methods

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LAND USE FORECASTING FOR TRANSPORTATION PLANNING

Estimating Efficient Spacing for Arterials and Expressways

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●THE PROCESS of preparing a transportation plan—that is, the actual sketching of lines representing street systems—is probably the least well thought-out area in transportation and city planning. By contrast, a great deal of thought has gone into methods for gathering origin-destination travel data, and into techniques for predicting future land use and travel volumes. Much skill and invention has also gone into the development of methods for testing plans, once they have been prepared (1) but the layout of street systems itself has largely remained an intuitive affair.

THE PROBLEM

It is the purpose of this paper to present some thoughts on this subject, focused mainly on the problem of finding the most efficient spacings of arterials and expressways. It is hoped that this will lead toward a more disciplined process of planning which will be based on an understanding of the principles which affect the location of transportation networks.

Taking as given the necessity of systems of streets to move conventional rubber-tired vehicles from one part of an urban region to another, the highway transportation planning problem (2) can be defined as the process of locating street systems (here temporarily restricted to local and arterial streets, and expressways) in accordance with some previously established criteria.

Speaking broadly, these criteria fall into two groups: criteria related to land development, and criteria related to transportation. Once the criteria have been established, the development of a transportation plan can be thought of as a series of steps, as follows:

1. Finding an abstract pattern of facilities which satisfies the criteria in some optimum fashion.
2. Placing the abstract pattern on maps and adjusting it to fit the real situation.
3. Predicting future traffic volumes on the facilities.
4. Evaluating the net economic return on the investment.

This paper deals mainly with finding an abstract pattern of facilities which satisfies the criteria in an "optimum" fashion. In so doing, some examples will be given using data of the Chicago Area Transportation Study (CATS). The application of these methods to the preparation of a transportation plan for the Chicago area is now under study.

The methods used to determine a pattern of transportation facilities are mathematical, in which the transportation criteria are dealt with explicitly. The results so obtained are reviewed from the viewpoint of land planning, but in a subjective manner. A single "optimum" solution cannot be claimed, therefore. The results, however, seem to be very good, particularly in view of the adjustments which must necessarily be made when fitting them to a real situation.

In reaching the desired pattern of street facilities, approximations of items 3 and 4 are reached. That is, the mathematics used to find a "best" pattern also yield estimates of future traffic volumes. The methods also are an important part of benefit-cost analysis because they find the spacing which minimizes community transportation costs.

In the present state of the art, traffic assignment and additional benefit-cost work should follow the development of the kind of transportation plan described here. The assignment and benefit-cost work can be looked at as both a check and a refinement of these results. The optimum spacing formula described here is based on a number of simplifying assumptions and is concerned with a general homogeneous area; full-fledged assignment and benefit-cost work will account for specific, particular and local conditions. It is hoped, of course, that in the future these steps can be combined into a general theory.

CRITERIA

The following are criteria which influence the spacing of arterials and expressways in urban regions. Not all of these criteria have been used in the methods described in this paper. The explicit inclusion of more criteria into the planning processes is something which awaits the completion of further research and the development of faster and more precise methodology.

Land Planning Criteria

Sufficient area must be provided in the spaces between the streets in a network for the efficient and pleasant conducting of the semistatic activities called land uses. The required land area is, of course, related to density of land development. This is a review criterion, considered after the spacing of arterials and expressways has first been determined.

Desirable Land Use-to-Road Relationships. This criterion is concerned with the relationships between street facilities and abutting land uses. It is a detailed criterion which can be applied only when an abstract pattern is fitted to a real stiuation.

Desirable Land Development Densities, from the Viewpoint of the Cost of Construction of Buildings and Related Facilities (Excepting Roads). Not considered in this report, this criterion needs to be the subject of additional research.

Desirable Land Development Densities, from the Viewpoint of Living and Operating Costs. This is not considered here and needs to be the subject of much additional research.

Transportation Planning

Travel Costs. These costs (primarily the value of personal time) are considered explicitly in the described method.

Construction Costs. These costs (including land acquisition and construction costs) are considered explicitly.

The Balance, on Each Facility, Between Traffic Volumes and Capacities. This criterion is considered explicitly, but as a review criterion after the spacing has been determined.

The Balance, by Area, Between Vehicle-Miles of Capacity and the Vehicle-Miles of Travel Demand. This is a review criterion.

Economic Criteria

The Minimization of the Sum of Construction and Travel Costs.

Developing a Plan Most Conducive to the Economic Growth of an Urban Region. This is a most difficult topic and could not be considered at this time.

AN OUTLINE OF THE METHOD

The following is a brief description of the method used to estimate efficient spacings for arterials and expressways. For the sake of brevity, not all terms are defined or qualified, nor are all assumptions made explicit. Complete detalis are given in succeeding parts and in the Appendices.

A key notion in this approach is the minimization of a community's highway transportation costs within framework of driver behavior. Highway transportation costs are taken as the sum of (a) construction costs and (b) travel costs for vehicle occupants.

Three street types are assumed: local streets, arterials and expressways. Speeds and construction costs on each type are given.

The number of trips generated per square mile per day is given, as is the distribution of trip lengths. The distribution of trip lengths is taken as stable over time, and in particular is taken as unaffected by changes in the street network. Costs to vehicle occupants are treated as a function of travel time only.

Total transportation cost is then expressed as equal to (1) the number of miles of each street type, times its unit construction cost, plus (2) the amount of time the average vehicle occupant spends on each facility times the number of occupants, with time converted to yearly costs and capitalized for the expected life period of the street type.

The number of miles of each street type, and hence construction costs, can be related to the spacing between streets of that type for a given area. Travel costs also are a function of this spacing. The sum of these costs can then be minimized, and hence the minimum-cost spacing can be determined, using the differential calculus, or by graphical means. Minimization can be carried out with respect to the spacing of each street type, or for any subset of street types. Thus, if local and arterial spacings have been determined historically (that is, if an area has become so completely developed that the construction of new arterials cannot be contemplated) a minimum-cost solution can be obtained in terms of expressway spacing alone.

Once the minimum-cost spacing has been determined, it can be reviewed with respect to other criteria, including design criteria, capacity criteria and land-planning criteria. The application of these criteria, either mathematically or subjectively, may suggest changes in spacing.

Examples are given of minimum-cost spacing for various parts of the Chicago area and the results are reviewed.

A STATEMENT OF HIGHWAY TRANSPORTATION COSTS

This section of the paper contains an explicit mathematical statement of highway transportation costs. Terms are defined and assumptions and simplifications are noted.

A major simplification that holds throughout is that streets exist in a grid form only, and construction and travel costs are both based on a grid network. Further research is needed to apply the techniques to non-gridded street systems, but it is not anticipated that the results will be greatly different.

Total Costs

Highway transportation costs can be written:

$$C = C_1 + C_2 \qquad (1)$$

where

C = Total transportation cost
C_1 = Construction cost
C_2 = Travel cost

Construction Costs

Construction costs for a square area can be expressed as:

$$C_1 = 2S^2 \left(\frac{C_X}{x} + \frac{C_Y}{y} + \frac{C_Z}{z} \right) \qquad (2)$$

where X, Y and Z are labels referring to the local, arterial and expressway street system, respectively, and x, y and z are the respective distances between streets in each street system. Thus C_X is construction cost per mile of X, C_Y is construction cost per mile of Y, and C_Z is construction cost per mile of Z. Total construction cost consists of cost per mile times miles of each street system. For a square with side S, the number of X streets on a side is equal to S/x (Fig. 1). Each X street has length S, so that the

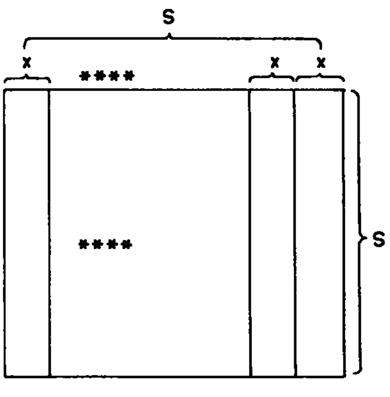


Figure 1.

number of miles of X in one direction is number of streets times length per street; this equals S (S/x) or (S^2/x). But there are the same number of X streets in a direction perpendicular to the original direction, so the total miles of X street is $2S^2/x$. Similarly, the total miles of Y and Z are $2S^2/y$ and $2S^2/z$, respectively. The multiplication of miles of street by construction cost per mile yields Eq. 2.

Travel Costs

General Form. Travel costs of vehicles is written:

$$C_2 = NK \left[\sum_{i=1}^{r-1} \frac{L_i}{v_X} F_i + \sum_{i=r}^{s-1} \left(\frac{2A}{v_X} + \frac{L_i - 2A}{v_Y} \right) F_i + \sum_{i=s}^{t} \left(\frac{2A}{v_X} + \frac{2B}{v_Y} + \frac{L_i - 2A - 2B}{v_Z} \right) F_i \right] \quad (3)$$

This equation can be expressed in words in a fairly straightforward manner.

Travel costs of vehicle occupants (C_2) consist of the following:

1. Number of daily trips (N)
2. Multiplied by hours of travel of the average trip (the expression within square brackets)
3. Multiplied by the value of an hour. The multiplications to this point yield costs per day for all vehicle occupants. This value in turn is multiplied by—
4. Number of weekday equivalents per year, which yields costs per year
5. Costs per year are assumed to occur for a given number of years and are discounted to the present at an interest rate of 5 percent. In this study, the given number of years was taken at 30 yr.

Items 3, 4 and 5 when multiplied together yield the value of K appearing in Eq. 3.

Hours of Travel of the Average Trip. The expression in brackets (Eq. 3) consists of the hours of travel of the average trip. The distribution of trip lengths, F_i, is one of the givens in this expression. Trip length is L_i, where i refers to a given class of trip lengths; $\bar{L}_i$ is average trip length of the class, and F_i is the frequency of occurrence of that class. These items are given in Table 1 for the entire Chicago Study Area; however, Table 1 gives F_i in terms of airline distance l_i rather than over-the-road distance L_i. It can be shown that L_i approximately equals 1.3 l_i. (Much of the notation of this paper is developed to handle the translation from airline to over-the-road distance. This is because there are some contexts where one form is more convenient, others where the other form is preferable.)

The expression in brackets (Eq. 3) consists of three parts; these are the average amounts of time spent in travel on the X, Y and Z system, respectively. Trips for the classes i = 1 to r - 1 are short trips which use the X network exclusively, trips for the classes i = r to s - 1 are longer trips which use both the X and Y network, and trips for the classes i = s to t are long trips which use all three networks.

It is argued that all trips begin their journeys on the X network (local streets) and if long enough, move to the Y network (arterials) and then to the Z network (expressways). The values v_X, v_Y and v_Z are the speeds that hold on the respective facilities, while A is the average distance traveled in moving from an X street to a Y street, and B is the average distance traveled in moving from a Y street to a Z street.

Thus, the first part of the bracketed expression (Eq. 3) consists of those trips with length less than 2A which presumably can use local streets only. The average trip length, $\bar{L}_i$, is divided by speed in miles per hour to yield hours traveled on the facility. This is multiplied by the frequency of occurrence of this trip type, F_i, to obtain the average travel time of this trip. The value 2A is the "over-the-road" trip length cutoff point because it is argued that a trip that uses arterials will travel A distance from origin to arterials, and then will travel A distance from arterials to destination. Hence, a trip using both locals and arterials will, on the average, travel 2A on local streets and the remainder of its trip length on arterials. This is stated formally as part of the bracketed expression (Eq. 3): $2A/v_X$ is the travel time on locals, $(\bar{L}_i - 2A)/v_Y$ is the travel time on arterials.

Finally, a "long" trip that can use expressways will travel 2A on locals, 2B on arterials, and the remainder of its trip length on expressways. The trip will go A miles on locals from origin to arterial, B miles on the arterial to an expressway, and then, leaving the expressway, will travel B miles on arterials, and A on locals to its destination. This is indicated in the third term in the bracketed expression (Eq. 3).

To recapitulate:

The component of average travel time is computed for each trip class, and the times are summed to yield total travel time for the average vehicle.

This in turn is multiplied by number of trips per day to obtain average daily travel time, which is then converted to a capitalized value, using K.

The determination of the values of A and B, and the assumptions and simplifications involved, are discussed in the section on Estimating Distance Traveled by Street Type.

TABLE 1

FREQUENCY DISTRIBUTION OF TRIP LENGTHS, CHICAGO STUDY AREA, 1956

Class, i	Range of l_i[1] (mi)	l_i[1] (mi)	F_i
1	0 to 0.99	0.5	0.202
2	1 to 1.99	1.5	0.227
3	2 to 2.99	2.5	0.121
4	3 to 3.99	3.5	0.088
5	4 to 4.99	4.5	0.070
6	5 to 5.99	5.5	0.051
7	6 to 6.99	6.5	0.043
8	7 to 7.99	7.5	0.037
9	8 to 8.99	8.5	0.027
10	9 to 9.99	9.5	0.020
11	10 to 10.99	10.5	0.018
12	11 to 11.99	11.5	0.015
13	12 to 12.99	12.5	0.011
14	13 to 13.99	13.5	0.009
15	14 to 14.99	14.5	0.007
16	15 to 15.99	15.5	0.005
17	16 to 16.99	16.5	0.005
18	17 to 17.99	17.5	0.004
19	18 to 18.99	18.5	0.003
20	19 to 19.99	19.5	0.003
21	20 +	25.0	0.034
Total			1.000

[1] Airline distance.

The Value of K. The value of K used in the applications of this formula was $7,500. K contains three components: time value per hour, set at $1.43; weekday equivalents in a year, set at 340; and an appropriate interest plus depreciation charge, set at 0.065 to square with a market interest rate of 5 percent and an assumed asset life of 30 yr.

Time value was based on the following considerations. Of total vehicle trips, 14.5 percent were truck trips, 85.5 percent were auto trips. The value of truck driver time was set at $3.00 per hour, which is the going wage rate. For automobile occupants, the value of time was set at $1.00 per hour for wage earners, because $1.00 is the minimum wage; and it was assumed that three-fourths of auto occupants were wage earners (to account for trips by non-wage earners), yielding $0.75 as the average value of occupant time. There were 1.56 occupants per auto, so total time value per auto was $1.17 per hour. Then weighting truck and auto hourly value by their respective percentages yielded $1.43.

The number of weekday equivalents was taken as 340, since weekends and holidays have only about 77 percent of the traffic of weekdays.

Finally, a yearly income stream can be converted to a present capitalized value by dividing by an appropriate gross interest rate. The gross interest rate consists of the market interest rate plus a depreciation component for assets of limited life. Arbitrarily setting the life of a highway at 30 yr, and taking the market rate at 5 percent, implies a gross interest rate of 6½ percent. Now:

$$\frac{\$1.43 \times 340}{0.065} = \$7,480$$

Hence, K was taken as $7,500.

ESTIMATING DISTANCE TRAVELED BY STREET TYPE

A question posed in the preceding section was: what are the values of A and B, that is, what is the average distance in travel from a random point on the X network to the Y network, and what is the average distance from a random point on Y to the Z network? Some approximations to A and B are developed in this section and used in succeeding work.

Estimates Using an Analytic Approach

In developing estimates of A and B, a mathematical model was employed to obtain an initial set of estimates. In this model it was assumed drivers would move to a higher speed network as soon as possible and, in doing so, would take the shortest possible route in terms of distance. Estimates of A and B obtained here are termed a and b; these estimates were obtained by mathematical induction.

Three steps were involved; these were: (1) specifying prevailing conditions and assumptions; (2) expressing the first step in the form of a summation; and (3) applying standard summation formulas to obtain a general equation.

The results obtained are as follows:

$$a = \text{average trip length on X in miles} = \frac{y}{6}\,\frac{(y + x)}{(y - x)} \qquad (4)$$

$$b = \text{average trip length on Y in miles} = \frac{1}{6}\,(z + 3M - \frac{y^2}{z}) \qquad (5)$$

where

$$M = \frac{y}{6}\,\frac{(2y - x)}{(y - x)} \qquad (6)$$

These values can be approximated as follows:

$$a = y/6 \qquad (7)$$

$$b = y/6 + z/6 \qquad (8)$$

An examination of the ratio of approximation to actual value indicated the approximation would generally contain an error of less than 10 percent.

Some points worthy of note are as follows:

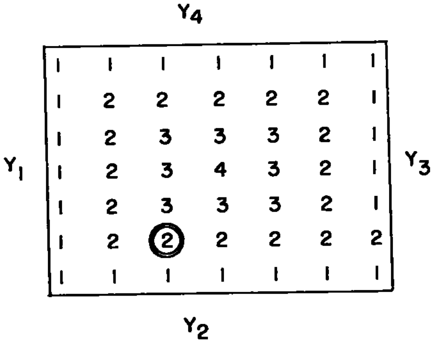
1. a and b depend on network spacing.
2. In the application of a and b, it should be remembered that the "average trip" is being considered; thus, it is argued that expressway usage, for an average trip, occurs only for trip length greater than 2a plus 2b.
3. Because simplifying assumptions were necessary, a and b will probably vary somewhat from actual behavior.

In an attempt to take this into account, some experimental work was carried out and this led to some modification of a and b. That work is described in the section on Experimental Method. The remainder of this section describes how a and b were derived using a mathematical model. Readers uninterested in the mathematical detail may turn directly to the section on Experimental Method.

Trip Length From the X Network to the Y Network. In examining average distance from the X network to the Y network, it was assumed that all trip origins were located at intersections of X streets. (In this formulation, no trips arise on the Y network. The latter case could be developed as a variation.)

An example of the situation specified is shown in Figure 2. Here, a square is formed by four arterials, and it is assumed that the ratio of y to x is 8 to 1; for example, arterials are 1 mi apart, local streets are ⅛ mi apart. As a consequence, there are 7 local streets between two parallel arterials, and 49 points of trip origin in a square formed by four arterials.

Figure 2 shows the distance of a trip origin point from the closest street on the Y network (Y_1 through Y_4). Units of dis-



1	1	1	1	1	1	1
1	2	2	2	2	2	1
1	2	3	3	3	2	1
1	2	3	4	3	2	1
1	2	3	3	3	2	1
1	2	②	2	2	2	2
1	1	1	1	1	1	1

Figure 2. Distance of trip origin points from Y network.

tance are in terms of x units, so that a point one unit away from a Y street is x miles from Y. The circled point is 2 units from Y_2, 3 from Y_1, 5 from Y_3 and 6 from Y_4. Its distance from the Y network for trip making purposes is listed as 2, which is its shortest distance from the Y network.

Of the 49 points of trip origin, 24 are 1 unit away from Y; 16 are 2 units away; 8 are 3 units away; and 1 is 4 units away. The average distance of a trip origin from Y is thus:

$$\frac{24\ (1) + 16\ (2) + 8\ (3) + 1\ (4)}{49} = \frac{84}{49} = 1.714 \text{ units.}$$

By drawing squares with varying points in a given line of the square, an over-all formula can be derived. The distribution of distance from Y for squares of varying size is given in Table 2 and a general formula can be derived. In general, a square formed by legs of the Y network will contain

$$\frac{y}{x} - 1 \text{ points along a given X street}$$

The total number of points within a square will therefore be

$$\left(\frac{y}{x} - 1\right)^2$$

The following formula expresses the average distance traveled within the square to get to the Y network:

$$a = \frac{x}{W_2}\left[\sum_{k=1}^{H-1} 4\ (w + 1 - 2k)\ k + dH\right]$$

where

$w = y/x - 1$

$H = \frac{w + 1}{2}$, with $\frac{1}{2}$ values rounded to the next highest number

$d = 0$ if w is even

$\phantom{d = {}}1$ if w is odd

Expanding the right-hand side of the a relation and applying standard summation, formulas yielded:

$$a = \frac{y}{6}\left[\frac{y/x + 1}{y/x - 1}\right]$$

Trip Length From the Y Network to the Z Network. The work involved in finding b was essentially an extension of the technique used in finding a. A key aspect of the approach was to exhibit the source of trips to each "leg" in the Y network. This is shown in Figure 3 where a diamond drawn around each leg shows the source of trips to the leg. (This is for a network with z/y = 8.) Trips arriving at the leg are then sent to the Z network. In the shaded diamonds, all trips travel directly to the Z road using only the Y leg of origin. Average trip length in these diamonds equals the value of the midpoint along the leg. In the unshaded diamonds, trips travel to the nearest Y leg perpendicular to the Z network and use that to get to the Z road. Average distance traveled within the unshaded diamond equals a value labeled M rather than the midpoint. Figure 3 exhibits two diamonds

TABLE 2

DISTRIBUTION OF DISTANCE FROM Y NETWORK

Number of Points Per Line	Distance From Y Network (in x units)					
	1	2	3	4	5	6
1	1	0	0	0	0	0
2	4	0	0	0	0	0
3	8	1	0	0	0	0
4	12	4	0	0	0	0
5	16	8	1	0	0	0
6	20	12	4	0	0	0
7	24	16	8	1	0	0
8	28	20	12	4	0	0
etc.						

which, for their pattern, are closest to the Z network; in each is shown distance of origin point from Z and the relative frequency of trips arriving at the origin point from X streets.

The generalized version of this case was developed. The summation expressing it is:

$$b = \text{numerator/denominator}$$

$$\text{numerator} = \text{sum of Pattern I diamond values plus sum of Pattern II diamond values}$$

$$= y/2\ (z/y-1) + \sum_{k=1}^{(z/2y)-1} (ky + y/2)\ (z/y - 1 - 2k)$$

$$+ \frac{1}{2} M^{(z/y)} + \sum_{k=1}^{(z/2y)-1} (ky + M)\ (z/y - 2k)$$

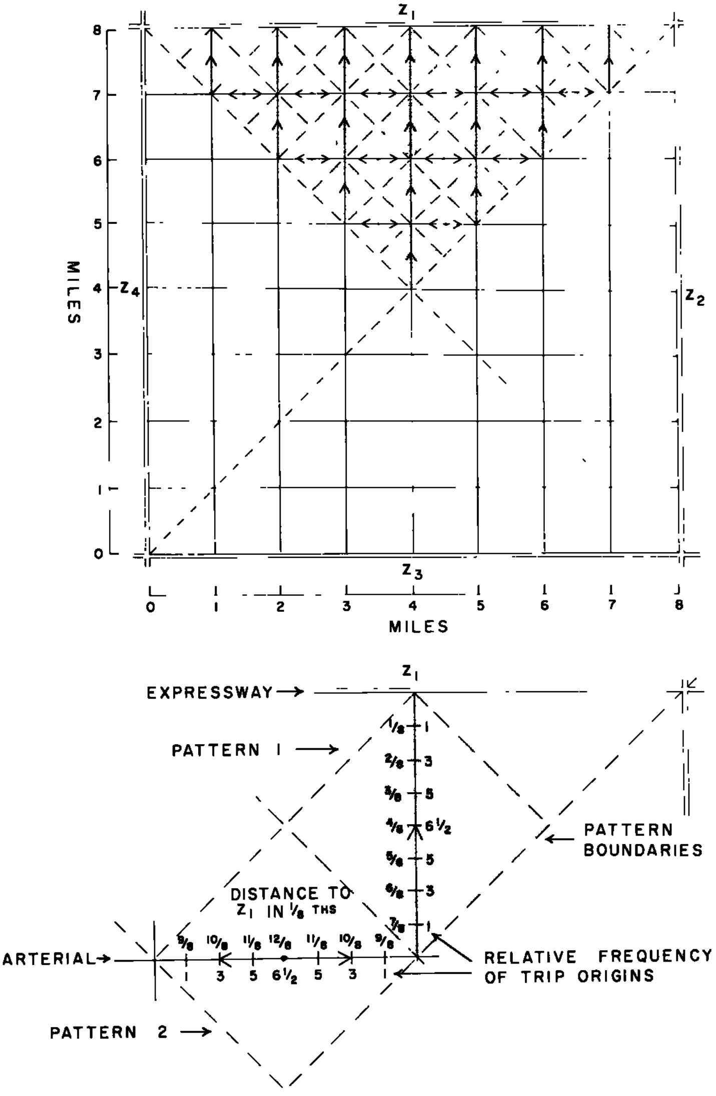


Figure 3. Source of trips for each Y leg in the Y network.

$$\text{denominator} = \text{number of cases} = \frac{1}{2} z/y + (z/y-1) + \sum_k (z/y-2k) + \sum_k (z/y-1-2k)$$

Application of summation formulas and simplifying, yields:

$$b = 1/6 \left[z + 3M - y^2/z \right]$$

where

$$M = y/6 \; \frac{2y-x}{y-x}$$

Experimental Method

An experiment was conducted which provided an alternative method of estimating average trip length on the X, Y, and Z street systems and provided a number of other clues as to the usage made of these systems. Using data from the experiment, values of a and b (determined analytically) could be modified to give A and B, the average distances traveled on local and arterial streets. The experiment further permitted a simplified statement to be developed giving break points in airline trip lengths at which vehicles start to use higher speed systems. This is an approximation to reality but the evidence developed by the experiment indicates it is a reasonable approximation.

Conduct of Experiment: Terms, Definitions and Assumptions. A large sheet of paper (30 in. x 30 in.) was ruled precisely with grid lines representing the X, Y and Z systems. The X system was scaled to represent 0.125-mi intervals, the Y system, 1.0-mi intervals, and the Z system, 4.0-mi intervals.

Sticks were cut to represent airline journeys of 2, 4, 6, 8 and 10 mi. These sticks were then thrown at random to land on the paper and their positions were carefully marked. Thirty throws were made with each stick.

The airline journeys were then assigned to the X, Y and Z systems on the assumption that the trip would take the shortest time path through the gridded network. Speeds were taken as in the ratio X: Y: Z = 1:2:4. This is not unrealistic, considering local streets at 12 mph, arterials at 25 mph, and expressways at 50 mph.

The over-the-road distance traveled on each street type was recorded, and the 30 records for each airline trip length were averaged.

Over-the-road distances were computed on three different assumptions as to ramp spacing. First, ramps or connections were assumed so that a trip could enter the Z (expressway) system at each intersection of that system with the Y (arterial) system; that is, at 1-mi intervals. Second and third assumptions permitted access only at 2Y and 4Y (2- and 4-mi) intervals.

Results of Experiment. The results of the experiment are given in Table 3 and shown in Figures 4, 5, and 6. Generally these results are about what one would expect.

TABLE 3

AVERAGE OVER-THE-ROAD DISTANCE IN MILES TRAVELED ON LOCAL, ARTERIAL, AND EXPRESSWAY SYSTEMS, AS A FUNCTION OF AIRLINE TRIP LENGTH AND RAMP SPACING

Airline Trip Length (mi)	Ramp Spacing (mi)								
	1			2			4		
	Average Over-the-Road Trip Length (mi)			Average Over-the-Road Trip Length (mi)			Average Over-the-Road Trip Length (mi)		
	X	Y	Z	X	Y	Z	X	Y	Z
2	0.38	1.82	0.63	0.40	2.10	0.27	0.41	2.27	-
4	0.46	2.35	2.86	0.49	2.94	2.04	0.48	3.33	1.86
6	0.38	2.07	6.07	0.40	2.47	5.73	0.39	3.51	4.70
8	0.39	2.07	8.37	0.41	2.60	7.87	0.41	3.89	6.80
10	0.37	1.77	11.17	0.38	2.37	10.80	0.42	3.71	9.67

The results are averages for trips of different airline length. Some trips of a particular length may have a high proportion of their length on a given facility and others very little. The averages for all "throws" are as shown.

The proportions driven on local streets for trips of airline length 1 mi is understated by the graph, which simply joins plots of observations. The same amount of travel (roughly 0.4 mi) is probably driven on local streets by trips of airline length 1 mi as for longer trips.

Interpretation of Results. As airline trip length increases, it is more probable that a higher proportion will be on higher speed facilities.

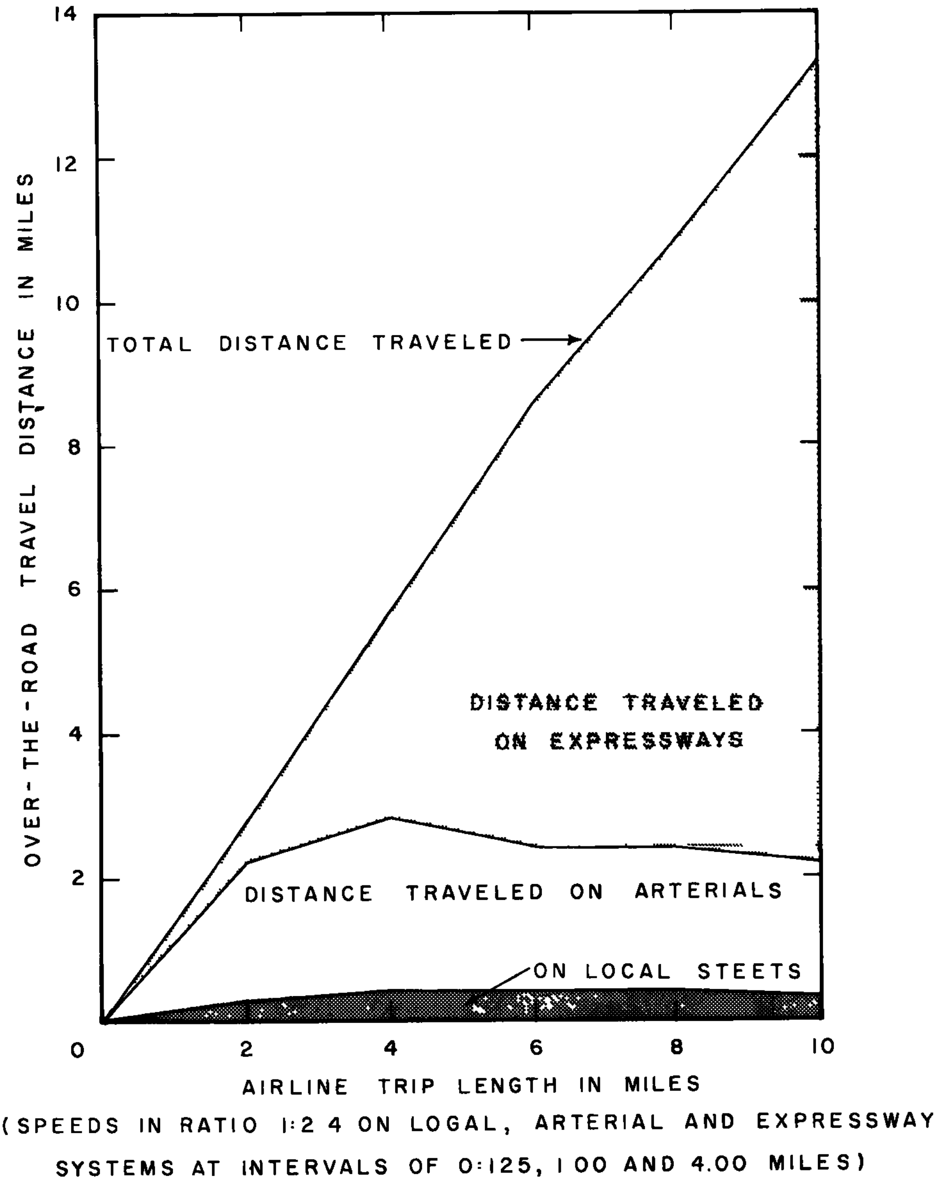


Figure 4. Use of local and arterial streets and expressways by trips of different airline length, with ramp spacing at 1-mi intervals.

When airline trip length reaches certain points, use of local and arterial streets ceases to rise and stabilizes at a certain level.

As airline trip length increases beyond a certain point, adverse travel (travel in the wrong direction in order to reach a higher speed facility) probably becomes more profitable. At this point, over-the-road travel on the lower speed facility appears to decline slightly (see, for example, the drop in arterial usage between 4 and 6 mi of airline trip length in Figure 4).

Effects of Different Ramp Spacings. As ramp spacing increases, use of expressways declines and use of arterials rises. It was estimated for the results given in Table 3 that the vehicle-miles on expressways decline about 11 percent while the vehicle-miles on arterials rise more than 20 percent as a result of the change in ramp spacing from 1 to 2 mi.

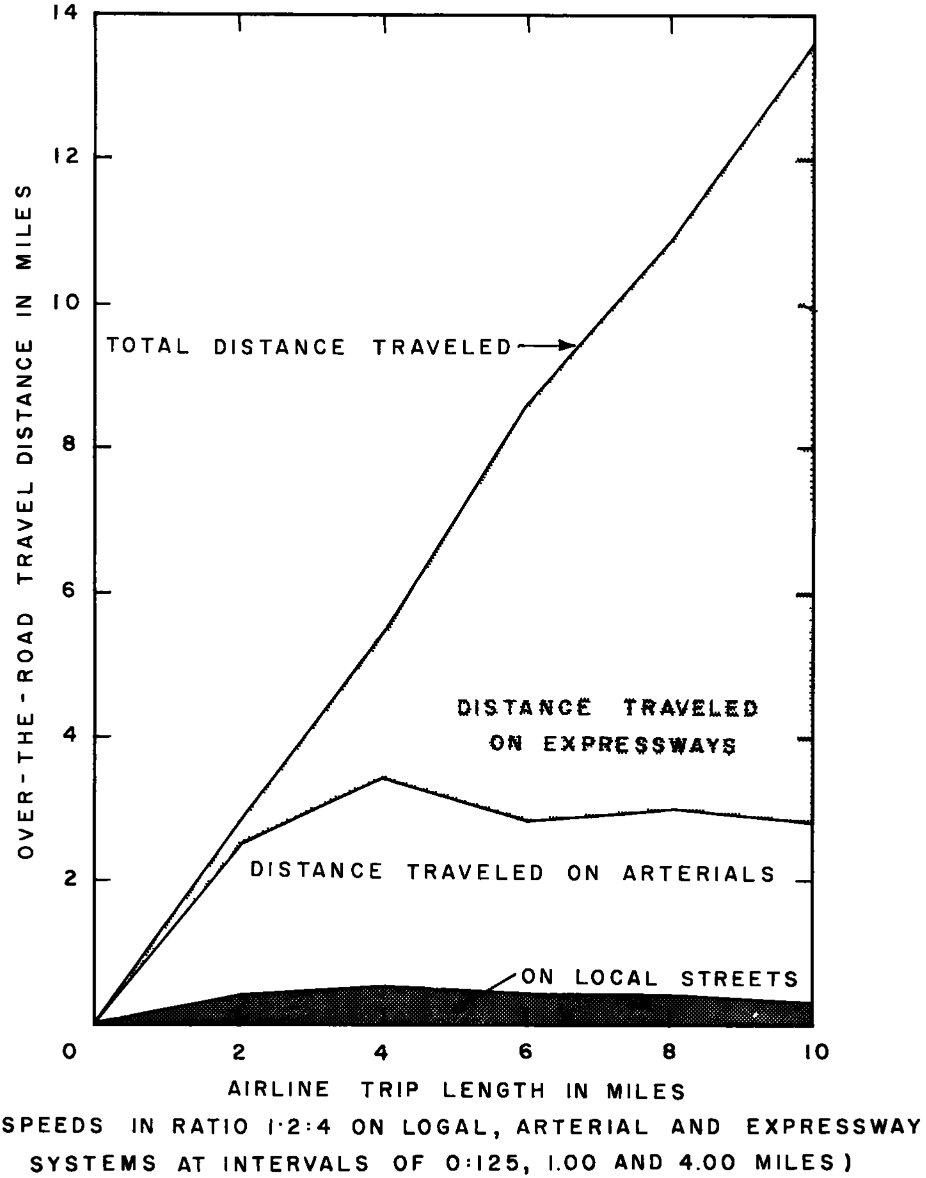
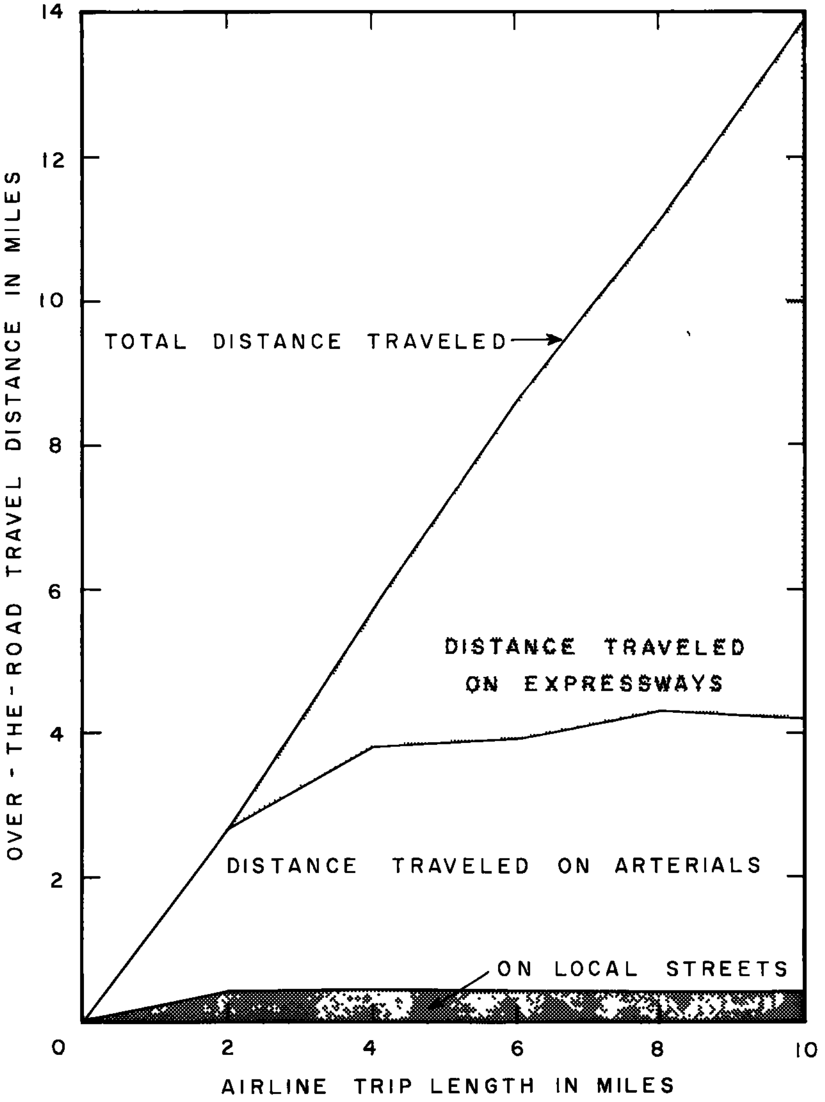


Figure 5. Use of local and arterial streets and expressways with ramp spacing at 2-mi intervals.

Combining Analytic and Experimental Methods

The analytic method is an attempt to answer the question: what is the average distance from a random point on one network to the nearest point on the next higher speed network? The experimental method furnishes approximate answers to the questions: (1) How far do vehicles actually travel in moving from one network to another? (2) Is the simplified model of driver behavior used in minimizing costs really a good representation of reality?

In answering the first question, the evidence developed by the experimental method indicated a and b were understatements of A and B, respectively. This probably occurred because the analytic method cannot account for adverse travel which is occasioned when a trip goes in the wrong direction in order to minimize total journey time. The



(SPEEDS IN RATIO 1 2:4 ON LOCAL, ARTERIAL AND EXPRESSWAY SYSTEMS AT INTERVALS OF 0:125, 1.00 AND 4.00 MILES)

Figure 6. Use of local and arterial streets and expressways with ramp spacing at 4-mi intervals.

values of A and B obtained in the particular case examined by the experimental approach were approximately 20 percent greater than a and b. It was assumed that this effect would prevail for all cases: therefore, a and b were factored by 1.20. These factored values were taken as the values of A and B to be used in Eq. 3; that is, 1.2a = A, 1.2b = B, with a taken as y/6, b taken as $\frac{y + z}{6}$.

With respect to the second question, the model of driver behavior used in minimization, states that for trips with over-the-road trip length between 0 and 2A, only local streets are used; trips with lengths between 2A and 2B use both locals and arterials, with local use for the first 2A of length, and arterials thereafter; finally, trips with lengths greater than 2A + 2B use expressways for that part of the trip beyond 2A + 2B. In actuality, however, the distribution of travel among street types for a given trip length presents a more complicated picture. Thus, short trips with length less than 2A will not be found exclusively on local streets; some of these trips may use arterials or expressways. Similarly, trips below 2A + 2B in length may use expressways. On the other hand, it is possible that some trips longer than 2A + 2B do not use expressways. Figure 4 shows actual use of the street systems as approximated by the experimental results. Use is presented as a function of airline trip length (rather than over-the-road trip length). This shows that some use of each street type occurs for every trip length (This is within the limits of the experiment as it was conducted, which did not include estimates of the use made of different systems by trips of airline lengths less than 2 mi.)

In order to make this complicated expression of use agree with the previously posited statement of driver behavior (as stated in the basic cost equations leading to the minimum cost statements of Eqs. 2 and 3), the patterns of use shown in Figure 4 were deliberately simplified. The results appear in Figure 7, which is not a bad approximation of the results shown in Figure 4.

Had the experiment revealed that the pattern of driver behavior was markedly different from the original statement of driver use, then that statement would have had to be revised. Actually, the original statement intuitively approximated reality very closely; that is, the original statement, if graphed, would yield Figure 7. In the future the experimental approach may be conducted with greater precision by use of a computer, and the minimum-cost formula may be altered to include these more precise patterns of driver use.

In Figure 7 it should be noted that the break points between street use are α and β; this is because the trip length axis is in airline units; α and β correspond to 2A and 2A + 2B in over-the-road units. Thus:

$$\alpha = \frac{2A}{1.3} = \frac{2.4a}{1.3} \tag{9}$$

$$\beta = \frac{2(A + B)}{1.3} = \frac{2.4\,(a + b)}{1.3} \tag{10}$$

To recapitulate, it was originally stated that trips of airline lengths between o and α would only use local streets, that trips with airline lengths between α and β would also use arterial streets, and that trips with airline lengths greater than β would use all three systems. Estimates of the use made of each type were obtained analytically. These estimates were revised by experimental means, which also substantiated the original statement of use patterns.

MINIMIZATION OF TOTAL TRANSPORTATION COSTS

In the section on A Statement of Highway Transportation Costs, construction costs and travel costs were stated in terms of the spacing of y and z. The section on Estimating Distance Traveled by Street Type developed values of A and B in terms of y and z.

The development has now proceeded to a point where transportation costs can be minimized with respect to the spacing of the various street networks. It is assumed at

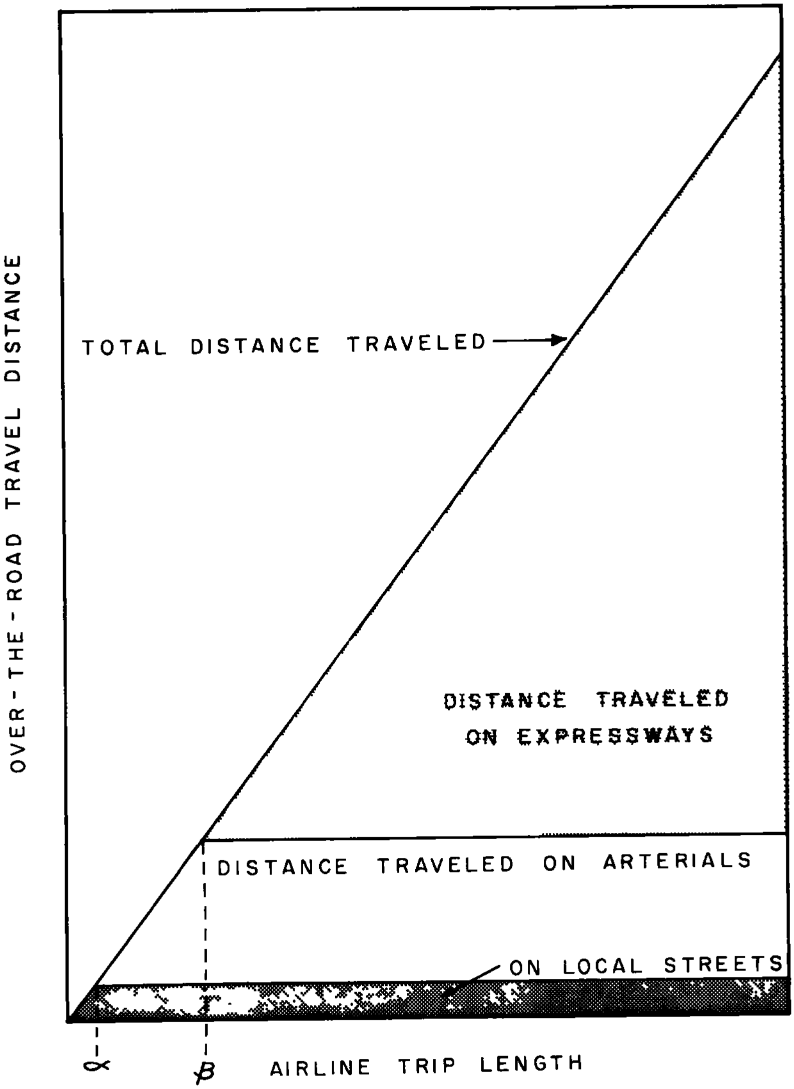


Figure 7. Simplified use of local and arterial streets and expressways by trips of different airline length.

this point that the spacing of x streets is ⅛ mi, so that minimization is carried out only with respect to y and z. Further, for some parts of urban regions, it seems realistic to argue that y spacing is fixed, having been determined historically, so that minimization for those parts is carried out only for z.

Minimization is obtained using the differential calculus; the partial derivatives of cost with respect to z and with respect to y are set equal to zero. Generally, solving the resultant equations (for y and z) would conclude the work. However, in this problem, differentiation is carried out for an expression involving a series of summations; the limits of these summations depend on A and B, and the latter depend on y and z. As a consequence, an iterative process had to be developed, so that all relationships posited did, in fact, hold. The final values of y and z obtained through the iterative technique do insure a minimization of cost; these values of z and y are defined as the optimum spacings of Z and Y streets.

The Differentiation of Cost

Using the work in the sections on A Statement of Highway Transportation Costs and Estimating Distance Traveled by Street Type, transportation costs can be written

$$C = 2S^2 \left(\frac{C_X}{x} + \frac{C_Y}{y} + \frac{C_Z}{z} \right)$$

$$+ NK \left[\sum_{i=1}^{r-1} \frac{\overline{L}_i}{v_X} F_i + \sum_{i=r}^{s-1} \left[\frac{2.4\,(y/6)}{v_X} + \frac{\overline{L}_i - 2.4\,(y/6)}{v_Y} \right] F_i + \sum_{i=s}^{t} \left(\frac{2.4\,y/6}{v_X} + \frac{2.4\,(y/6+z/6)}{v_Y} + \frac{\overline{L}_i - 2.4\,(y/6+y/6+z/6)}{v_Z} \right) F_i \right] \qquad (11)$$

This is a restatement of Eq. 3 with final values of A and B inserted. Minimization of cost occurs when

$$\frac{\partial C}{\partial z} = 0, \text{ and } \frac{\partial C}{\partial y} = 0$$

Thus, the derivative of cost with respect to z is:

$$\frac{\partial C}{\partial z} = \left[\sum_{i=s}^{t} \left(\frac{2.4}{6v_Y} - \frac{2.4}{6v_Z} \right) F_i \right] NK - \frac{2S^2 C_Z}{z^2} = 0 \qquad (12)$$

Simplifying, and defining

$$P_s = \sum_{i=s}^{t} F_i \qquad D = \frac{N}{S^2} = \text{density} \qquad V_{YZ} = \frac{1}{v_Y} - \frac{1}{v_Z}$$

Yields this final equation for z:

$$z = 2.24 \sqrt{\frac{C_Z}{K\,D\,V_{YZ}\,P_s}} \qquad (13)$$

A similar process yields this equation for y:

$$y = 2.24 \sqrt{\frac{C_Y}{K\,D\,(P_r\,V_{XY} + P_s\,V_{XYZ})}} \qquad (14)$$

where

$$P_r = \sum_{i=r}^{s-1} F_i$$

$$V_{XYZ} = \frac{1}{v_X} + \frac{1}{v_Y} - \frac{2}{v_Z}$$

$$V_{XY} = \frac{1}{v_X} - \frac{1}{v_Y}$$

Eqs. 12-14 give the minimum cost spacings for the Y and Z networks. For a numerical solution of these equations, an iterative technique must be employed. This is because the values P_r and P_s depend on the cutoff points separating the parts of a trip in terms of the network used. This is shown in Figure 8. The value P_s is that part of the trip frequency distribution between 2B and ∞, measuring trip length in over-the-road distance.

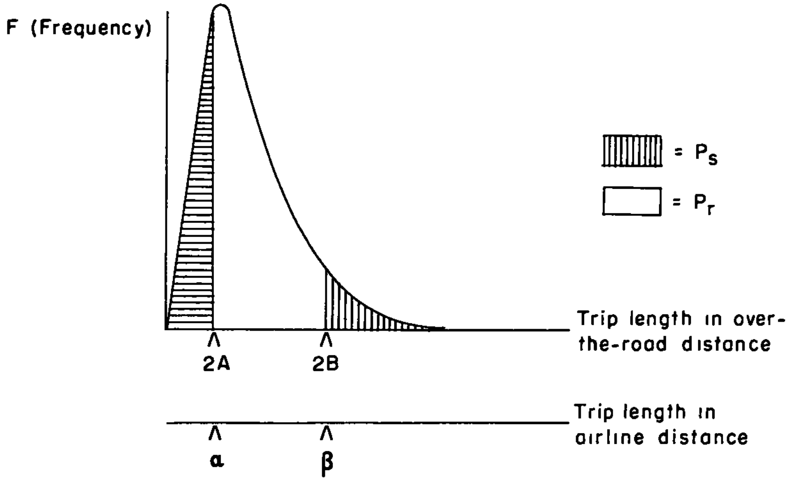


Figure 8. Trip length frequency distribution.

Thus, P_s depends on B, which in turn depends on y and z. The same sort of remarks apply to P_r. Because of this dependance on y and z, an iterative technique had to be developed to find values of y and z from Eqs. 13 and 14, which would be consistent with all the relations involved in this procedure.

The Iterative Technique

In iterating to final values of z and y, two cases were considered. These were: (1) y is given, only z is to be determined, (2) both y and z are to be determined.

For Case 1, there are in effect three equations in three unknowns. These are

$$z_1 = 2.24\sqrt{\frac{C_Z}{K\,D\,V_{YZ}\,P_s}} \qquad (13a)$$

$$z_2 = 3.25\,\beta - 2y \qquad (15)$$

Eq. 15 stems from

$$\beta = \frac{2.4a + 2.4b}{1.3}$$

$$P_s = P_s(\beta) \qquad (16)$$

is a particular value of Case 1, and Eq. 16 is involved in the frequency distribution function.

In the iterative process, an arbitrary value of β is selected. This implies a corresponding value of P_s, from Eq. 16. The insertion of P_s in Eq. 13 yields an initial value for z_1, labeled z_1 (1). Similarly, the insertion of the original β and the given y in Eq. 15 yields an original value of z_2, labeled z_2 (1). In this initial set of computations, z_1 will probably differ significantly from z_2. Hence, a second value of β is picked, and corresponding values of z_1 and z_2 are computed, labeled z_1 (2) and z_2 (2). The second value of β is selected using this rule; if $z_2(1) > z_1(1)$, try a lower β if $z_2(1) < z_1(1)$, try a higher β. Given the values of z_1, z_2 and β for the two series of computations, a final value of β and z can be obtained by linear interpolation. The final value of z occurs where $z_1 = z_2$. The interpolation is shown in Figure 9.

The z_1 points are connected by a straight line, and the z_2 points are similarly con-

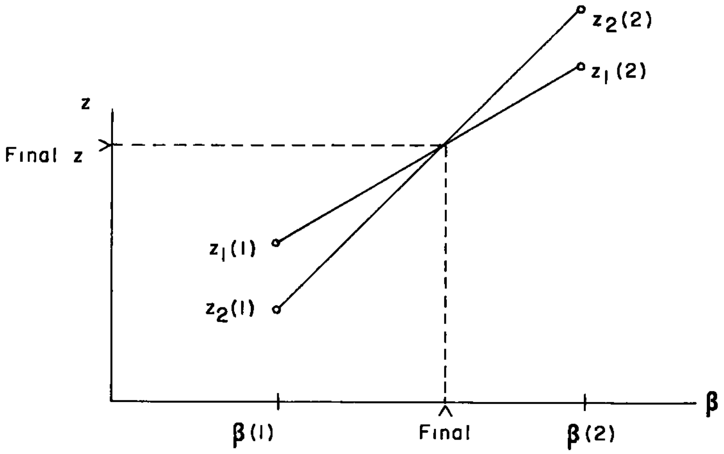


Figure 9. Iteration of z.

nected. The intersection of these lines yields values of β and z which should approximately equal the final iterative values desired.

The same sort of procedure—in expanded form—is applied in Case 2, where both y and z are to be determined. Three more equations in three more unknowns are added. These are:

$$y_1 = 3.25\,\alpha$$

from

$$\alpha = \frac{(1.2)2a}{1.3}\ , \ a = y/6 \tag{17}$$

$$P_r = P_r\,(\alpha\,,\ P_s) \tag{18}$$

$$y_2 = 2.24\sqrt{\frac{C_Y}{K\,D\,(P_r\,V_{XY} + P_s\,V_{XYZ})}} \tag{19}$$

Iteration proceeds in this way: an arbitrary value of y is picked, termed y_1. For y_1, a final optimal z is obtained by iteration, as indicated for Case 1. This implies a corresponding P_S and P_r, which in turn implies a corresponding y_2, which probably differs from y_1. This y_2 is used to pick a new value of y_1, and the procedure is repeated, yielding a second set of values, y_1 (2) and y_2 (2). The values obtained can now be used to find a final z and y. The y_1 values are connected by a line, and the y_2 values are connected by a line. The intersection yields final values for y and z.

It may be noted that y converges quite quickly; that is, the initial y_2 is not very far from the final value of y.

A detailed description of the iterative process and a running numerical example appear in Appendix B.

Interpretations and Planning Principles

An examination of Eqs. 11-14 leads to certain conclusions or interpretations. These have some real value as providing principles which affect the layout of systems of arterials and/or expressways, as follows:

1. The minimum-cost spacing of arterials and expressways becomes greater as construction cost per miles increases.
2. The minimum-cost spacing of arterials and expressways becomes less as the value of personal time increases. Incidentally (and most undemocratically) this implies a need for closer spacing in higher income areas.

3. The minimum-cost spacing of arterials and expressways decreases as traffic density increases.
4. If the speed of expressways increases relative to that of arterials and local streets, then the minimum-cost spacing of expressways becomes less.
5. If trips become longer, then expressway minimum-cost spacing becomes less.
6. If arterial speeds increase relative to those of expressways (as through improved signalization and traffic controls) then expressway minimum-cost spacing becomes greater.

It may appear that these are common sense principles, and indeed they are. One of the values of having worked through the mathematics, however, is that these principles can be stated clearly and unequivocably, and that the quantitative effect of changes in the different variables can be estimated.

A Graphical Method of Estimating Minimum-Cost Spacings

A method of checking the minimum-cost spacing formulas given in Eqs. 11-14 was developed. This method permits a graphical presentation of the minimum-cost spacing, and is therefore of value as indicating in simple terms why the minimum-cost spacing formulas work.

This alternative method is a simplified technique. Only tow speeds are assumed—expressway speeds and non-expressway speeds. Travel is similarly assigned to the expressway network and to the non-expressway networks. Finally, either arterial or expressway spacing must be taken as a fixed item, allowing the other to vary.

Construction Cost. Construction cost of expressways (assuming arterials have a constant spacing) is a function of expressway spacing, as indicated in the following equations:

$$\text{Construction cost} = (\text{length}) \times (\text{cost per unit length}) \qquad (20)$$

$$\text{Spacing} = \frac{2\ \text{area}}{\text{length}} \qquad (21)$$

$$\text{Construction cost} = \frac{2(\text{area})\ (\text{cost per unit length})}{\text{spacing}} \qquad (22)$$

This is a hyperbolic function, and can be graphed (Fig. 10).

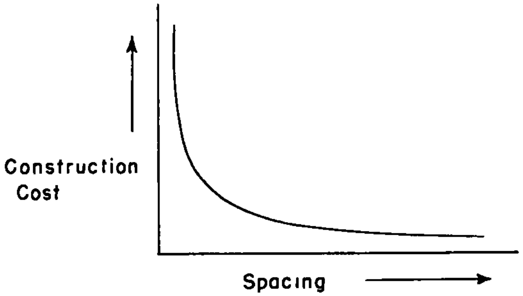


Figure 10.

Travel Cost. Travel cost is a function of the number of trips, the trip cost per hour of travel, the time period, the proportions of trip length driven on expressways and the proportion not driven on expressways. This relationship can be graphed (Fig. 11) and stated as follows:

$$\text{Travel cost} = (\text{number of trips}) \times (\text{cost per hour}) \times \left(\frac{\text{mean trip length on expressways}}{\text{expressway speed}} + \frac{\text{mean trip length not on expressways}}{\text{non-expressway speed}}\right) \times (\text{factor expanding hourly costs to long term costs}) \qquad (23)$$

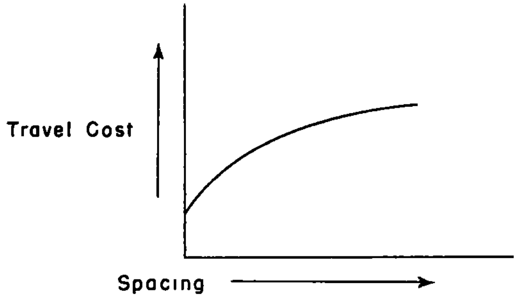


Figure 11.

Total Costs and Examples. By adding the travel cost to the construction cost as a function of spacing, one can determine the minimum total cost solution. This is shown in Figure 12.

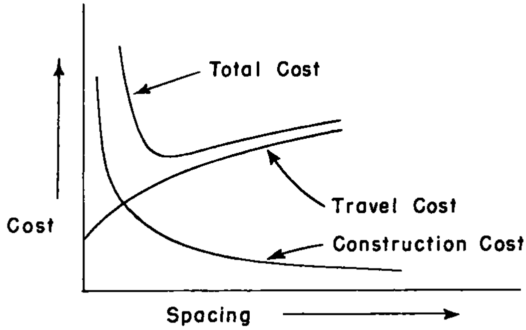


Figure 12.

Two examples are provided. The data taken for the example are the 1980 data given for Rings 4 and 7 in the section on Minimization of Total Transportation Costs. The only exception is that of arterial speed, which has been reduced in order to compensate for the amount of travel driven on local streets, which is significantly slower. (Note again that the graphic solution as stated here deals only with two speeds, instead of the three used in the more refined formula.)

TABLE 4

VALUES USED IN ESTIMATING MINIMUM-COST SPACING BY GRAPHICAL MEANS

Given	Example 1[1]	Example 2[1]
Expressway cost per mile	$8, 000, 000	$4, 000, 000
Trip density in trip destinations per square mile	20, 000	6, 200
Expressway speed	50 mph	50 mph
Non-expressway speed	12 mph	20 mph

[1] Trip length frequency distribution in both examples is that for the entire Chicago Study Area.

The results of the calculations are given in Tables 5 and 6 and are graphed in Figures 13 and 14. For comparison, the minimum-cost spacing as calculated by formula is 2.9 mi (Example 1) and 6.9 mi (Example 2). The graphical solution is "on the nose" for Example 1, and is very close in the case of Example 2, although the graphical technique gives a fuzzy answer in this latter case.

TABLE 5

EXPRESSWAY CONSTRUCTION AND TOTAL TRAVEL COSTS[1] AS A FUNCTION OF EXPRESSWAY SPACING: EXAMPLE 1

Expressway Spacing (mi)	Expressway Construction Cost (per sq mi)	Total Travel Cost (per sq mi)	Total Costs (per sq mi)
0.0	∞	16.8	∞
0.5	32.0	20.0	52.0
1.0	16.0	24.3	40.3
2.0	8.0	27.3	35.3
3.0	5.3	29.1	34.4 (min)
4.0	4.0	32.0	36.0
6.0	2.7	37.6	40.3
8.0	2.0	42.0	44.0
10.0	1.6	46.3	47.9
16.0	1.0	52.0	53.0
20.0	0.8	54.0	54.8
∞	0	70.0	70.0

[1] In millions of dollars.

TABLE 6

EXPRESSWAY CONSTRUCTION AND TOTAL TRAVEL COSTS[1] AS A FUNCTION OF EXPRESSWAY SPACING: EXAMPLE 2

Expressway Spacing (mi)	Expressway Construction Cost (per sq mi)	Total Travel Cost (per sq mi)	Total Costs (per sq mi)
0.0	∞	5.2	∞
0.5	16.0	6.0	22.0
1.0	8.0	6.6	14.6
2.0	4.0	7.0	11.0
4.0	2.0	7.7	9.7
6.0	1.3	8.3	9.6 (min)
7.0	1.1	8.5	9.6 (min)
8.0	1.0	8.8	9.8
10.0	0.8	9.2	10.0
15.0	0.5	10.0	10.5
20.0	0.4	10.7	11.1
∞	0	13.0	13.0

[1] In millions of dollars.

Interpretations. The graphical method of estimating minimum-cost spacing illustrates why the minimum-cost spacing formulas work. The two components of transportation cost; namely, construction cost (C_1) and travel cost (C_2), when added together, vary with the spacing of transportation facilities. In the mathematical solution the differentiation of the sum of these two components, with the derivatives set equal to zero, automatically locates the minimum point. The graphical solution does the same thing by exhibiting costs for all the spacings and the minimum point is ascertained by eye. Of course, the graphical solution is more approximate than the mathematical solution.

Construction costs are an exact function of spacing. In this example they are a dominating influence on the point of minimum total cost, because they rise so steeply when the spacing becomes tight; that is, on the order of 2 or 3 mi apart for expressways.

The position of the line of travel costs is a complicated function of relative speeds, trip density, the trip length frequency distribution, and value of time. Of these variables, trip density is very important, greatly affecting the slope of the line of travel costs. As can be seen, the travel cost line in Example 1 is very steep, reflecting the relatively high trip density of 20,000 trips per square mile. In Example 2 the flatter slope reflects the lower density of 6,200 trips per square mile.

In high density areas, the minimum cost point is sharply defined. This suggests

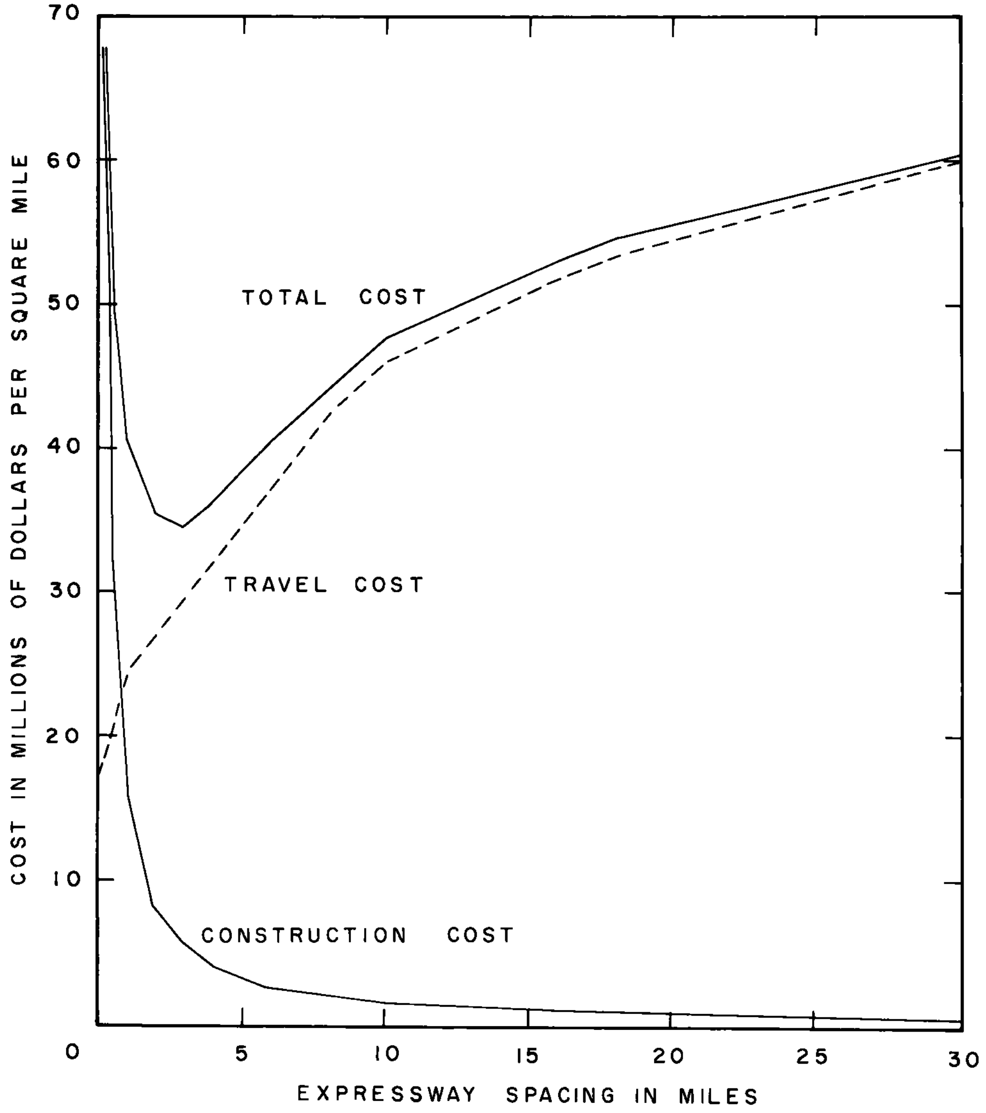


Figure 13. Minimum-cost spacing for Example 1 (source: Table 5).

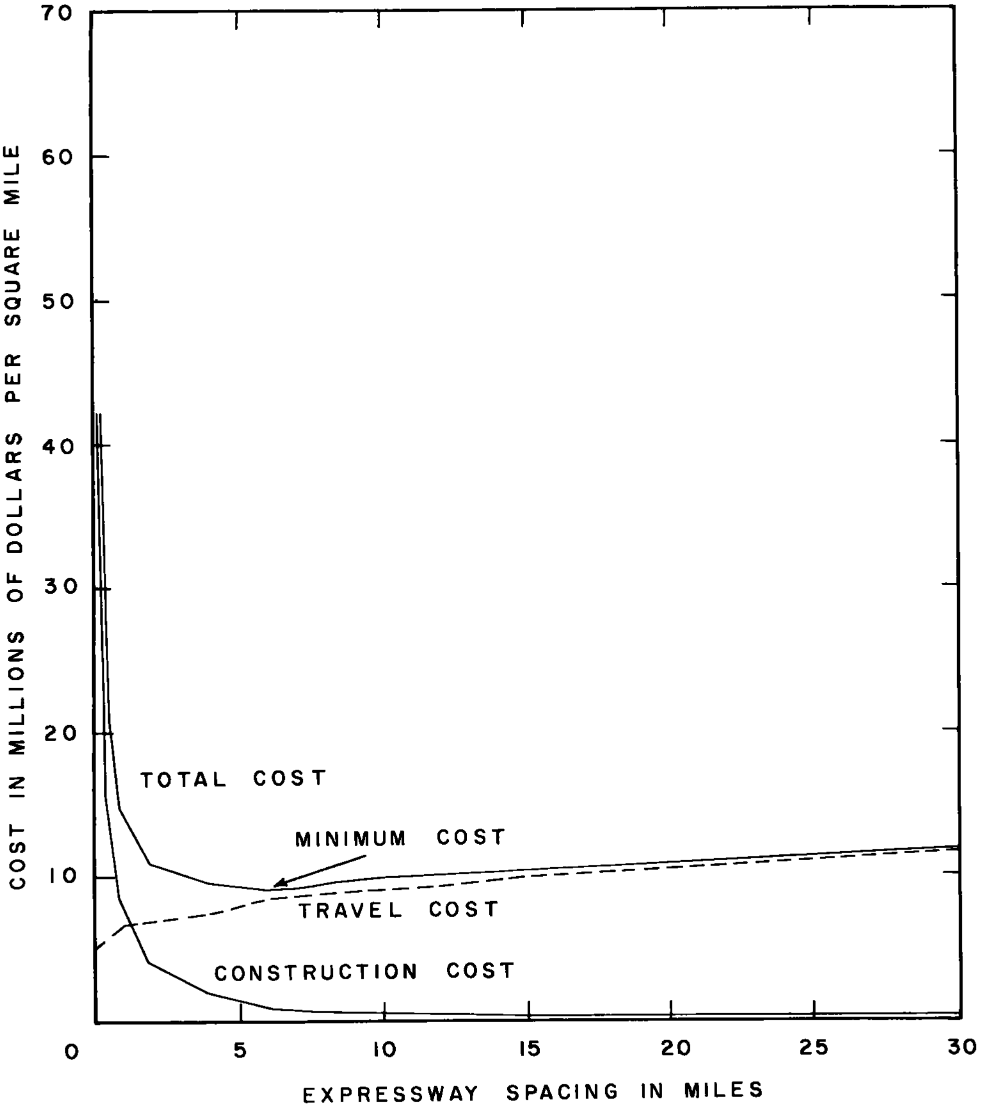


Figure 14. Minimum-cost spacing for Example 2 (source: Table 6).

that great savings can be accrued by planning an expressway network at spacings close to the minimum-cost point. In low density regions the minimum point is less well defined, which suggests that considerations other than costs may become more important in these regions.

APPLICATION OF OTHER CRITERIA

The foregoing techniques have shown the spacings of arterials and expressways which result from the minimization of the costs of travel and construction. The minimization of total costs, however, is not the sole criterion determining a "best" spacing of arterials and expressways. Other criteria were cited. Among these were the use of arterials and expressways, an over-all capacity criterion, and land planning criteria. In this section, these are taken up successively as they affect the spacing of arterials and expressways.

Use Criterion

If it should be found that the use of arterials and expressways is greater or less than their design capacity, then the minimum-cost solution may not be an optimum solution. Use is expressed here in terms of average daily volumes on the streets of each type.

If there is an imbalance between volume and design capacity (as implied by the construction cost) then modifications must be made in the minimum-cost spacing. For example, if volumes on expressways are greater than the capacity implied by a given cost level (say, $10, 000, 000 per mile for a 6-lane expressway) then a new and higher unit cost expressway (say, $13, 000, 000 per mile for an 8-lane expressway) may be used to estimate a new minimum cost. The new spacing, being wider, actually increases the volume on expressways, but the increase in volume is less than the increase in capacity implied by the higher unit cost. Hence, by a number of iterations, a minimum-cost solution can be found where capacity is in balance with expected volumes.

In order to apply the use criterion, it is necessary to estimate the volumes and vehicle-miles of travel on the various street systems. These volumes and vehicle-miles of travel vary on each street system (local, arterial, and expressway) as a function of its spacing.

The formulas for estimating volumes and vehicle-miles can be expressed in words:

$$\text{Volume on a street of given type} = \text{total trips} \times \frac{(\text{average trip length on that type})}{(\text{miles of streets of that type})}$$

$$\text{Vehicle-miles on a street system of given type} = (\text{total trips}) \times (\text{average trip length on that type})$$

Crucial to solution of these equations is the part of average length driven on each type. Average (mean) trip length on each system is a function of (a) number of trips in each trip length interval (that is, the trip length frequency distribution), and (b) the proportion of each trip driven on each street type, which is a function of the spacing of arterials and the spacing of expressways. The spacing of local streets is assumed constant.

Average Trip Length in Each Type. The proportion of each trip's length on each system has been estimated experimentally and analytically, as previously described in the section on A Statement of Highway Transportation Costs (Total Costs and Construction Costs). Figure 15 is a restatement of Figure 7.

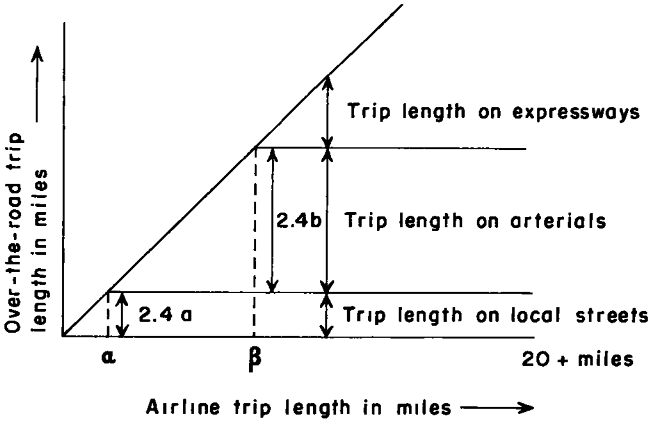


Figure 15. Distance traveled on local and arterial streets and expressways.

The point where some travel begins to be made on arterials is α and the point where some travel begins to be made on expressways is β. α is a function of the spacing of arterials (y) and β is a function of the spacing of both arterials (y) and expressways (z). Combining Eqs. 9 and 10 with Eqs. 7 and 8, it can be shown that:

$$\alpha \cong 0.31y \tag{24}$$

$$\beta \cong 0.31z + 0.62y \tag{25}$$

Average trip length then can be expressed as a summation of the frequencies of trips having airline length of class i times the average airline trip length of the i^{th} interval. The way the summations were prepared is shown in Figure 15.

The part of average trip length on local streets (no summation necessary)

$$\cong 2.4a \cong 0.4y \tag{26}$$

The part of average trip length on arterials

$$= \sum_{\alpha}^{\beta}(\overline{L}_i - 2.4a)F_i + \sum_{\beta}^{20+}(2.4b)F_i \tag{27}$$

Tables have been prepared which permit rapid estimation of average trip length. These tables were calculated using airline values for $\overline{L}_i$. To correct this to over-the-road distance, multiply the values which include the term $\overline{L}_i$ by 1.30 to approximate over-the-road mileage. The tables give values of

$$\alpha,\ \beta,\ \sum_{\alpha}^{\beta}F_i\,\overline{L}_i,\ \sum_{\beta}^{20+}F_i\overline{L}_i,\ \sum_{\alpha}^{\beta}F_i \text{ and } \sum_{\beta}^{20+}F_i$$

for various spacings of y and z.

The part of average trip length on expressways

$$= \sum_{\beta}^{20+}(\overline{L}_i - 2.4a - 2.4b)\,F_i \tag{28}$$

It should be noted, that while these formulas present distance traveled on each street system by the average trip, the average distance traveled on each system by vehicles using the system can be obtained from these formulas by dividing by the corresponding frequency. Thus, the summation in Eq. 28 when divided by $\sum_{\beta}^{20+} F_i$, yields the average mileage driven on expressways by vehicles using expressways.

Note that $\overline{L}_i$ is the average over-the-road trip length of trips having airline trip length of i. The values of a and b are expressed also in over-the-road, or "L" trip lengths.

Vehicle-Miles of Travel by Type. To estimate the vehicle-miles of travel on each system, simply multiply Eqs. 26, 27, and 28 by N, which is the number of trip origins. This has been done in the following equations, and also the formulas have been put directly in terms of y and z:

$$\text{Travel}_{\text{local}} = N(0.4y) \tag{29}$$

$$\text{Travel}_{\text{arterial}} = N\left[\sum_{\alpha}^{\beta}F_i\overline{L}_i - \sum_{\alpha}^{\beta}F_i\,(0.4y) + \sum_{\beta}^{20+}F_i\,(0.4)\,(y+z)\right] \tag{30}$$

$$\text{Travel}_{\text{expressways}} = N \left[\sum_{\beta}^{20+} F_i \bar{L}_i - \sum_{\beta}^{20+} F_i \, (0.4z + 0.8y) \right] \tag{31}$$

Daily Volumes by Type. To obtain volumes on each street type, simply divide through Eqs. 29, 30, and 31 by the miles of streets in each type.

This can be readily obtained by the formula:

$$\text{Miles of streets} = \frac{2\ (\text{area})}{\text{spacing}}$$

$$\text{Contrariwise, spacing} = \frac{2\ (\text{area})}{\text{miles of streets}}$$

For example, the average spacing of arterials in the CATS area is:

$$\text{Spacing} = \frac{2\ (1,236 \text{ sq mi})}{2,800 \text{ mi arterials}} = 0.88 \text{ mi}$$

As a rough check on Eqs. 29, 30, and 31, it is possible to use these formulas to estimate the distribution of vehicle-miles of travel by street type for the Chicago area and to compare the results with survey data.

The average spacing of arterials and expressways can be determined as indicated previously. Average arterial spacing equals 2(1,236)/2,800 or 0.88 mi. Average expressway spacing (1956) equals 2(1,236)/67 or 37 mi.

These spacings produce values of α and β of 0.27 and 12.0 mi, respectively. It should be noted that the β values do not mean that no trips of less than 12 mi in length used expressways in 1956. The value of β is an approximation which, with the very few miles of expressways which existed in the Chicago area in 1956, could not come too close to reality. Nevertheless, the results are not bad. Using these values, vehicle-miles of travel given in Table 7 were estimated by type and are compared with the vehicle-miles obtained by survey.

TABLE 7

VEHICLE-MILES OF TRAVEL—ESTIMATED AND ACTUAL, BY STREET TYPE

Street Type	Average Weekday Weighted Estimated Vehicle-Miles Using Formulas	Average Weekday Weighted Vehicle-Miles Estimated From Survey Data[1]
Local	2,140,000	6,000,000
Arterial	29,439,000	29,800,000
Expressway	4,880,000	3,361,000
Total	36,459,000	39,161,000

[1] Source: Chicago Area Transportation Study Final Report, Volume I Survey Findings, pp. 80, 81 (September 1959).

The differences between estimated and actual average weekday travel can be accounted for, in part. Actual mileage driven on local streets has always proved troublesomely high, but can be reasonably explained as caused by the high amount of circuitous travel driven on this type of street. A personal review of the distance traveled by the reader on his last trip to the neighborhood hardware store as compared with the airline distance or even the right-angle distance will demonstrate this point.

The difference between the estimated travel on expressways and the actual travel on expressways in the Chicago region can also be explained. The calculation assumes an even density of urban development and an even location of express facilities. In Chicago in 1956 the expressways were quite scattered, with a great deal of the mileage located in Rings 6 and 7, in very low-density areas. The Kingery Expressway and its extensions south, and the Edens Expressway fall into this category. As a result, use of these facilities was generally below capacity. Data from the Chicago Area Transportation Study's report indicates that expressways had in 1956 a capacity of 5,457,000 weighted vehicle-miles of travel, in contrast with 3,361,000 weighted vehicle-miles of use.

The estimate of arterial use is close to that obtained by survey, and of course constitutes the bulk of the use.

In general, therefore, the formulas for estimating vehicle-miles of travel seem to square with observed results. This is a small piece of evidence confirming the formulas. Actually, the construction of the formulas (for average travel and average volume) themselves is sufficiently precise so that the results can be used with confidence. (This statement must be qualified when there are very few miles of one type of facility, such as expressways. In such cases the theory of driver behavior upon which this work is based loses precision as a descriptive device; use of expressways in such cases becomes more an accident of the facility's location.)

Over-All Capacity Criterion

Moving gradually from the abstract to the concrete, it is desirable to provide capacity for each sub-region within the urban area sufficient to take care of the travel demand imposed on that region as of some future year. The needed capacity can be determined as suggested in the following. Needs can then be compared with the capacities to be provided by the minimum-cost systems, as a check on those systems.

The travel demand imposed on any sub-region within an urban area appears to be closely tied to the number of trip origins in that sub-region. Data on trip origins per square mile in the Chicago area were correlated with vehicle-miles of travel per square mile, as obtained by survey on a district basis. There are 44 districts in the Chicago study area. The correlation coefficient was + 0.91. (Excluding District 01, which is the Loop area. This close correlation appears to justify the use of trip destination densities in computing minimum-cost spacings.) The plots show a fairly close fit around the regression line.

With such evidence, it appears that future vehicle-miles of travel can be estimated reasonably accurately, provided that future trip origins (or destinations) are given. These latter can be estimated from projections or plans of land use. (Actually, current assignment methods can record the vehicle-miles of travel in each route section, which can be summed up to any desired urban sub-region, thus providing what is probably a better estimate of future travel. The difficulty is that this information can only be obtained after the plan has been prepared. It is for this reason that a regression projection is used.)

Knowing future travel demand, the future deficiencies of street capacity can be estimated by subtracting present capacity from future demand for each sub-region. These are the requirements for additional capacity which must be constructed according to a plan.

It is possible to provide this new capacity by constructing expressways or arterials, or by improving arterials through various devices known to traffic engineers; that is, removing parking, one-way streets, improved signalization, or constructing median strips with "shadowed" turning lanes. Or, any combination of new construction and improvement of older streets can be undertaken. (It is difficult to ascertain what the policy should be as to the proportion of the needed new capacity which should be provided by new expressway construction or by the improvement of existing arterials. Here is where the minimum-cost spacing formulas are helpful, because they include measures of both construction problems and of service to the driving public.)

Supposing that new capacity is only to be provided by the construction of new expressways at the minimum-cost spacing for 1980 (Table 14), how much capacity will be provided? Table 8, gives these capacities by ring, and compares them with the estimated needs for 1980.

It can be seen that the 1980 minimum-cost spacing solution provides more capacity than is estimated to be needed in 1980. The average is about 6.6 million vehicle-miles,

TABLE 8

DESIGN CAPACITY PROVIDED BY EXPRESSWAY SYSTEMS AT MINIMUM-COST SPACING COMPARED WITH CAPACITY DEFICIENCIES FOR 1980

(All capacity and travel figures in thousands of weighted vehicle-miles)

Ring	Average Distance From Loop (mi)	1956 Design Capacity[1]	Estimated 1980 Travel	Additional Capacity Needed to Provide for 1980 Travel	Capacity Provided By 1980 Optimum Spacing[2]
0	0.0	307	340	33	243
1	1.5	2,643	2,790	147	1,510
2	3.5	3,197	4,220	1,023	2,605
3	5.5	3,626	5,550	1,924	3,175
4	8.5	4,300	8,300	4,000	6,340
5	12.5	5,378	8,900	3,522	5,250
6	16.0	7,559	14,200	6,641	7,530
7	24.0	9,352	23,000	13,648	9,940
Total	-	36,362	67,300	30,938	36,593

[1] A full explanation and definition of design capacity appears on pp. 77-79, Volume I of the Final Report of the Chicago Area Transportation Study.

[2] Assuming 135,000 vehicle-miles of design capacity per mile of expressway in Rings 0-2, 108,000 in 3 and 4, 81,000 in Ring 5, and 54,000 in Rings 6 and 7.

or about one-fifth of the deficit and one-tenth of the total demand. This is not a great over-supply and would not be sufficient to suggest modification in the minimum-cost spacing, particularly in view of the expected growth beyond 1980.

By ring, however, there are some discrepancies. In Rings 0 and 1 very little additional capacity is needed, but the minimum-cost spacing formula suggests that a lot should be built. This is a peculiarity of the formula, which calculates needs on a density basis as if the area in question were large and uniformly built up at a given density. Actually, these two areas are so small (1.2 and 12.4 sq mi, respectively) that the results are not particularly applicable, because these areas contain a lesser portion of the total trip length than the formula suggests, by reason of their small extent.

In Ring 7, less capacity is provided than needed. This suggests either (a) that average capacity per mile should be increased, or (b) that spacing should be decreased. In actuality, designs for Ring 7 have been posited on a junior expressway spacing, with lower costs, greater frequency, and fairly high capacity. This has provided a much greater level of capacity in 1980, amply sufficient to meet future demands.

Land Planning Criteria

At this point, the least-cost spacing must be reviewed in terms of its effects upon land uses. This can only be a partial review, because from the land planning viewpoint the chief examination comes when a network is adjusted to the facts of topography and existing land uses. Whether a road passes between a residential neighborhood and an industrial district, or next to an airport, or through a large park is of real importance, but it cannot be taken up at this stage.

The most important principle of land planning that can be applied at the time when spacing of streets is in the abstract pattern stage is the principle of sufficient area. Roads are divisive in their influence, especially as they become wider, with heavier and faster traffic. The expressway with its 300- to 400-ft widths is a real barrier which seriously impairs communications across its right-of-way.

Therefore, the area lying between expressways must be of sufficient size for the efficient and pleasant conduct of the urban activities located there. The same is true of the areas between arterials; although, being of lesser width, they affect a different array of land uses.

Residential areas are a major worry in this connection. They are far larger than commercial or industrial districts, and hence have a greater likelihood of being damaged by intruding roads. The residential area is taken here as a unit composed of houses together with streets, small parks, schools, public buildings, and minor commercial areas.

The neighborhood is a residential area whose principal unifying function is that it serves as an elementary school district. The neighborhood is generally thought of in terms of an area containing 5, 000 population. As an elementary school district, it should not be cut by arterials because these pose a major threat to the safety of children walking to school.

Hence the network of arterial streets should not be so closely spaced as to encompass areas of less than, say, 4, 000 to 6, 000 population.

A community is defined here as a group of neighborhoods. It should be of sufficient population size so that it can maintain certain functions internally. Suggested internal functions are: (a) schools: elementary, junior and one senior high school; (b) a local government of efficient size; (c) convenience goods stores and services; (d) cultural institutions, such as churches; (e) human needs for recognition associated with a small geographic area; and (f) human resources for adequate leadership.

Although there are no adequate standards specifying ideal community size, it is also true that there are minimums and maximums which are generally recognized. A community of less than 10, 000 persons is too small, particularly from the viewpoint of adequate governmental services. A community of more than 100, 000 is too large from the viewpoint of personal participation and will almost automatically fraction itself into one or more recognized sub-areas. Perhaps 30, 000 to 60, 000 is the ideal size range.

It may be stated, then, that expressways should not enclose areas which can house less than 20, 000 to 30, 000 persons. Area, of course, is a function of density and the existence of any large nonresidential uses in the same area. Community area can be calculated using the formula

$$\text{Area in square miles} = \frac{\text{Population}}{640\ D\ F\ R} \qquad (32)$$

in which

D = density in dwelling units per net residential acre;

F = persons per dwelling unit; and

R = residential land as a percent of all land in that area.

Using this formula, a community of 60, 000 at a density of 25 dwelling units per net acre and having 50 percent of its land in residential use, with 3. 1 persons per dwelling unit, would require 2. 4 sq mi of land. For such an area, expressways should be spaced not less than 1. 54 mi apart. At a density of four dwelling units per net acre, this community would require 15 sq mi of land, and for this community, expressways should be not less than 3. 9 mi apart in spacing.

This area criterion has been treated here in its most abstract sense, but forms the basis for that kind of review which includes the interests of the land uses.

It need not be emphasized that much further work needs to be done in the area of land planning criteria, particularly in the field of land controls and access standards abutting arterial streets, and on the problem of the collection of traffic to and its dispersion from expressway ramps, especially as this affects land uses in the vicinity of ramps.

EXAMPLES OF METHODS, USING CHICAGO AREA DATA

An initial application of the techniques developed was made using Chicago area data. Results should be treated as preliminary. So far, however, the results are rather encouraging because items that can be compared with previously available information check out quite well.

The spacing technique was applied to each of the analysis rings in the Chicago Study

Area. The rings are bounded by arcs which radiate from the Central Business District (CBD). Ring 0 is approximately coterminous with the Loop, and there are seven additional rings. Arterial spacing in Ring 0 through 5 was taken as given (Table 9) with only expressway spacing to be determined for these rings.

Both arterial and expressway spacing were determined for Rings 6 and 7. (Arterial spacing was obtained for Rings 6 and 7 because it is felt arterial changes could be planned for these rings.) In addition, for Rings 5, 6 and 7, an alternative to expressways was considered. This was the construction of "junior expressways," which would cost less and provide a lower level of service than expressways.

In applying the technique to each ring, values of all pertinent variables were estimated for each ring. This included a trip length distribution for each ring, based on CATS survey data.

The application of the technique to each ring in effect expands the ring so it becomes equivalent to the Study Area or to any very large region of the stated uniform density. In other words, the question asked is: if the entire Study Area had the properties of the ring, what would the best spacing of expressways be?

TABLE 9

SPACING DETERMINANTS FOR 1956

Ring	Trip Destinations Per Square Mile (In thousands)	Expressway Speed (mph)	Arterial Speed (mph)	Expressway Construction Cost Per Mile (In millions of dollars)	Given Arterial Spacing Rings 0 Through 5 (mi)	Rings 6 and 7 Arterial Construction Cost Per Mile (In millions of dollars)	Junior Expressway Speed (mph)	Junior Expressway Cost (In millions of dollars)
0	134.0	35	10	20	0.20	-	-	-
1	40.7	45	15	10	0.40	-	-	-
2	24.8	45	15	10	0.40	-	-	-
3	22.0	50	15	10	0.40	-	-	-
4	17.0	50	20	7	0.55	-	-	-
5	8.6	50	25	5	0.66	-	35	1
6	3.5	60	25	3	-	0.3[1]	35	1
7	1.1	60	30	1	-	0.2[1]	40	0.5

[1] In determining arterial spacing, speed on local streets is a factor; it was taken as 10 mph for Rings 6 and 7.

TABLE 10

DISTRIBUTION OF 1956 AIRLINE TRIP LENGTHS STATED AS PERCENTAGES[1]

Class 1	Range Of l_1	$\overline{l_1}$	Percentage								
			Study Area	Ring 0	Ring 1	Ring 2	Ring 3	Ring 4	Ring 5	Ring 6	Ring 7
1	0 1	0.5	20.2	6.5	16.3	17.9	19.1	20.6	22.5	27.8	30.0
2	1 < 2	1.5	22.7	6.8	11.9	20.2	22.4	20.9	23.9	24.6	26.9
3	2 < 3	2.5	12.1	6.2	11.2	13.4	12.2	13.2	11.5	10.6	11.7
4	3 < 4	3.5	8.8	4.3	7.6	13.0	9.8	9.2	7.8	6.2	5.4
5	4 < 5	4.5	7.0	10.2	7.5	9.6	8.4	8.5	7.0	5.5	3.5
6	5 < 6	5.5	5.1	4.5	6.4	6.9	6.7	6.3	4.8	5.1	2.4
7	6 < 7	6.5	4.3	10.1	7.3	5.2	6.7	5.0	5.1	3.0	2.6
8	7 < 8	7.5	3.7	6.2	8.3	3.1	3.1	3.8	3.7	3.0	1.2
9	8 < 9	8.5	2.7	9.3	5.7	2.7	3.1	3.7	2.8	3.1	1.7
10	9 < 10	9.5	2.0	9.0	4.1	1.9	1.5	2.4	2.8	2.1	1.5
11	10 < 11	10.5	1.8	4.1	3.8	2.6	1.6	1.1	2.3	1.9	1.2
12	11 < 12	11.5	1.5	4.6	2.5	1.2	1.4	1.0	1.7	1.8	0.7
13	12 < 13	12.5	1.1	4.1	1.6	0.3	0.9	0.7	1.1	1.4	0.8
14	13 < 14	13.5	0.9	2.7	1.4	0.4	0.7	0.9	0.7	1.0	1.2
15	14 < 15	14.5	0.7	2.3	0.9	0.4	0.7	0.7	0.6	0.9	1.1
16	15 < 16	15.5	0.5	1.0	0.2	0.3	0.3	0.5	0.1	0.3	0.8
17	16 < 17	16.5	0.5	0.3	0.2	0.1	0.2	0.7	0.3	0.6	0.9
18	17 < 18	17.5	0.4	1.3	0.1	-	0.4	0.2	0.3	0.2	0.5
19	18 < 19	18.5	0.3	0.7	0.4	0.2	0.2	0.1	0.2	0.3	0.6
20	19 < 20	19.5	0.3	0.7	0.3	0.1	0.1	0.1	0.1	0.2	0.8
21	20 +	25	3.4	5.1	2.3	0.5	0.5	0.4	0.7	0.4	4.5
Total	-	-	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

[1] For entire study area and individual rings.

Having obtained expressway spacing by ring, some inferences about the layout of the entire expressway network could be made.

Results were obtained for both 1956 and 1980 trip densities and are discussed in succeeding sections.

Spacing Results for 1956

The variables used for the determination of y and z are given in Tables 9 and 10. Table 9 gives relevant density, speeds, construction cost, and existing average arterial spacing for Rings 0 through 5. (It should be noted that some of these values are rough estimates.) Table 10 gives the trip length frequency distribution that held for each ring in 1956.

Spacing results are presented in Table 11. Results for arterial spacing in Rings 6 and 7 are of a proper order of magnitude; they agree well with actual spacing for Rings 6 and 7, which were 0.86 and 1.18 mi in 1956, respectively.

Turning to expressway spacing, it is noteworthy that the expressway spacing diverges from Rings 0 to 7, increasing with distance from the CBD. The primary explanation is that the decline in construction cost is more than offset by the decline in density of trip origins; the changes in trip length distribution and speeds are additional factors affecting results. This divergence of expressway spacing suggests a system of radial and circular expressways; this sort of pattern would fit the Z spacing results obtained.

In drawing inferences on network layout, the circumference of the central arc of each ring was divided by expressway spacing to obtain the number of radial expressways per ring. The results are given in Table 12. (The central arc of the ring is the locus of mid-points between inner and outer boundaries of the ring.) For Rings 2 through 6 an average of 11 radials per ring is obtained. There is little variation in the number of radials implied for each of these rings, which lends some support to a radial and circular system; the lack of extreme variation is another example of the "reasonable" kind of results obtained. Using 11 radials implies that expressways should be placed 33 deg apart for a circular urban region.

The pattern of circumferential and radial expressways suggested has been applied, without any adjustment, to a map of the Chicago Study Area. The results are shown in Figure 16. (In counting radials per ring, it should be noted the number listed refers to a circular area, whereas the Chicago Study Area is approximately a semicircle with a diameter on Lake Michigan.)

When the optimum system is superimposed on the Chicago area with North and South Lake Shore Drive and Congress Street Expressway as radials to the center of the city, the system resembles quite closely the existing and committed X-way system. Reading clockwise from South Lake Shore Drive, the radials correspond to the South Expressway, the Southwest Expressway, the Congress Street Expressway, the Northwest Expressway and North Lake Shore Drive. The circular expressways would correspond to the Halsted Street connector between the Northwest and South routes, a route near Western Avenue, a route near Laramie Avenue, and the tollroads. A junior expressway could be placed between the tollroad and Laramie Avenue.

The possibility of using a system of "junior expressways," or "super-arterials" has often been suggested as an alternative to the construction of full-blown expressways. There are sound reasons for this which can be supported by the methods described in this paper. These reasons can only apply effectively in low-density areas.

True expressways are very expensive. As a result, their minimum-cost spacing in low-density areas becomes quite wide, often of the order of 8 to 10 mi. With such spacing, the use of expressways becomes almost accidental; actually it is more a function of the location of the trip origin or destination than it is a function of the trip length frequency distribution. (In other words, many long trips would not be served by these widely spaced facilities.)

As a result, the use of such express facilities is less than would normally warrant the construction of such an expensive facility. Furthermore, the wide spacing of expressways causes additional travel to be undertaken on arterials, which is not desirable.

So the use of lower-cost junior expressways becomes a real prospect. The lower

TABLE 11

MINIMUM-COST SPACING RESULTS—1956

Ring	Spacing (mi) Expressways,[1] z	 Arterials, y	 Junior Expressways,[3] j
0	1.2	0.20[2]	-
1	2.1	0.40[2]	-
2	2.8	0.40[2]	-
3	3.0	0.40[2]	-
4	3.7	0.55[2]	-
5	6.5	0.66[2]	3.3
6	8.3	0.89[1]	6.3
7	12.1	1.34[1]	12.1

[1] Obtained from spacing formula.

[2] Set equal to actual average spacing.

[3] Spacing for the Junior Expressway System, an alternative to the expressway system. Obtained from spacing formula.

TABLE 12

NUMBER OF RADIAL EXPRESSWAYS PER RING

Ring	Circumference of Center of Ring (mi)	1956 Optimum Spacing (mi)	Number of Radials [(2) + (3)]
0	1.96	1.2	1.6
1	10.21	2.1	4.4
2	22.78	2.8	8.1
3	35.34	3.0	11.8
4	51.44	3.7	13.9
5	64.79	6.5	10.0
6	97.77	8.3	11.8

cost of these facilities, despite the lower speeds which they offer, makes their minimum-cost spacing reasonably close together—on the order of 3 to 5 mi. (The much wider spacings in Rings 6 and 7 in the 1956 results are due to the low densities obtained in those regions in 1956, when much of the area was rural.) As a result, worthwhile volumes use these facilities (40,000 to 50,000 vehicles per day) and most important, considerable reductions in arterial volumes are produced. These reductions are of the order of 25 percent.

With such information, a system of junior expressways is being considered for the Chicago area in the outer areas, with only the interstate routes constructed as fully-controlled access routes.

Spacing Results for 1980

A number of variables change between 1956 and 1980, causing some changes in the spacings obtained.

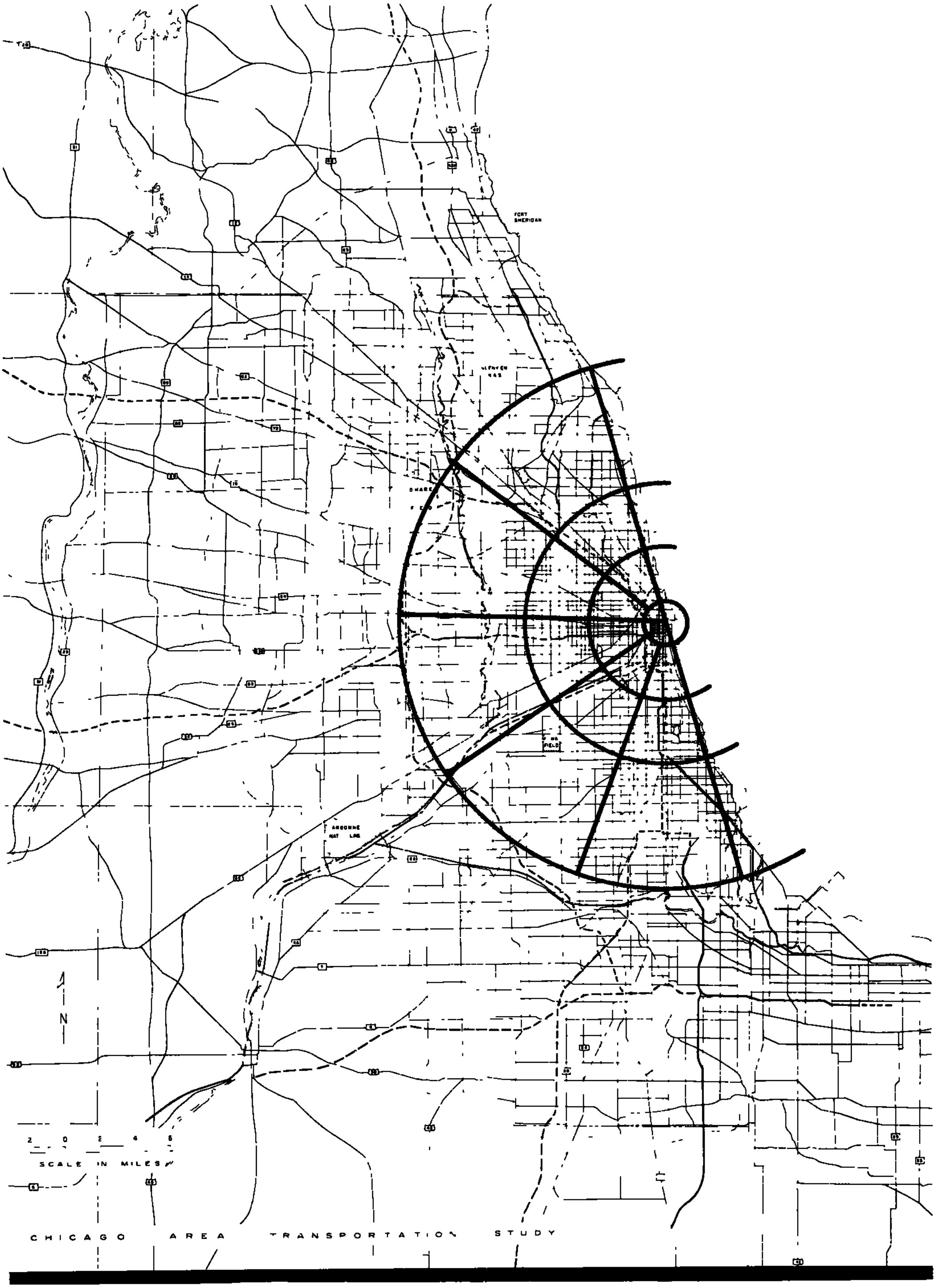


Figure 16. Minimum-cost expressway system—1956 volumes.

Table 13 gives the 1980 values of relevant variables affecting the spacing of streets, while Table 14 gives the results obtained using these values. There is not much change in expressway spacing between 1956 and 1980 for Rings 0, 1, 2 and 3. However, Rings 4 through 7 exhibit reductions in expressway spacing, indicating the need for additional expressway construction beyond the system inferred from the 1956 results; this is particularly true for Rings 4 through 7.

The fact that there is some change in minimum-cost spacing over time suggests that the planning process must be related to time, and must be extended to account for these changes.

Vehicle-Miles of Travel

Vehicle-miles of travel by facility could be estimated for both 1956 and 1980 spacings, using the techniques described in the section on Application of Other Criteria. Total Study Area vehicle-miles of travel by type of facility is given in Table 15.

TABLE 13

SPACING DETERMINANTS FOR 1980

Ring	Thousands Of Trip Destinations Per Square Mile	Expressway Speed (mph)	Arterial Speed (mph)	Expressway Cost (In millions of dollars)	Given Arterial Spacing Rings 0 Through 5 (mi)	Arterial Construction Cost Per Mile (In millions of dollars)	Junior Expressway Speed (mph)	Junior Expressway Cost (In millions of dollars)	Trip Frequency Distribution (F_1) Taken as Equal to 1956 Ring
0	152.0	35	10	25	0.20	-	-	-	0
1	47.2	45	15	12	0.40	-	-	-	1
2	28.7	45	15	11	0.40	-	-	-	2
3	25.3	45	15	10	0.40	-	-	-	2
4	19.6	50	15	8	0.55	-	-	-	4
5	13.4	50	20	6	0.66	-	30	1.5	4
6	9.0	50	25	5	-	0.5[1]	35	1.0	5
7	6.2	50	25	4	-	0.5[1]	35	1.0	5

[1] In determining arterial spacings, speeds on local streets is a factor; it was taken as 10 mph for Rings 6 and 7.

TABLE 14

MINIMUM-COST SPACING RESULTS—1980

Ring	Spacing (mi) Expressways z	Spacing (mi) Arterials y	Spacing (mi) Junior Expressways j
0	1.3	0.20	-
1	2.2	0.40	-
2	2.7	0.40	-
3	2.8	0.40	-
4	2.9	0.55	-
5	4.0	0.66	2.5
6	6.3	0.90	3.2
7	7.0	1.10	4.0

TABLE 15

ESTIMATED VEHICLE-MILES OF TRAVEL BY STREET SYSTEM (Study Area)

	Vehicle-Miles of Travel in 000's		Percents	
	1956	1980	1956	1980
Local Streets	1,701	4,000	5.0	6.2
Arterials	10,529	20,880	31.2	32.2
Expressways	21,572	40,020	63.8	61.6
Total	33,802	64,900	100.0	100.0

CONCLUSION: SUMMARY AND EVALUATION

Summary

A technique of estimating efficient spacings for arterials and expressways has been developed. This method was developed in order to provide a rational basis for preparing initial sketch plans, which might then be tested by computer assignment.

This method involved posing a number of criteria related to land planning, transportation planning and considerations of economy. Not all of these criteria could be considered explicitly in the first stages of estimating the most efficient spacings. Some criteria could be used as review or evaluative criteria. Other criteria could not even be included in review; their use depends upon the development of faster methods and the prosecution of other research work.

The principal technique employed was to minimize the sum of two costs related to the spacing of arterials and expressways. These costs were travel costs and construction costs, both falling under the heading of transportation planning criteria.

Using a simplified network (a grid) of local and arterial streets and expressways, the construction and travel costs could be stated mathematically as functions of (a) spacing, (b) trip destination densities, (c) trip length frequency distribution, (d) unit construction costs, (e) relative speeds, and (f) value of personal time. This mathematical statement could then be minimized by means of the differential calculus. Minimum spacings can also be estimated using graphical means, for a slightly more simplified statement of total costs.

Results obtained appear to be quite reasonable. When computed for the various rings of the Chicago Study Area, the results varied, as might be expected, with closer spacings in the densely settled inner rings and wider spacings in the suburban rings.

The results are subject to review in terms of other criteria. If computation of volumes on each mile of expressway facilities indicates that in some areas the estimated volumes exceeded the capacities implied by the unit costs, then higher unit costs may have to be inserted for revised solutions. In low-density areas the wide spacing of expressways in the minimum-cost solution suggests the use of junior expressways, whose lower unit costs (but lower level of service) provide more frequent spacing and lower the volumes on arterial streets.

The results also can be reviewed in terms of estimated long-range capacity needs and in terms of land use. The required areas for the development of neighborhoods and communities seem to be provided within the minimum-cost spacings.

Evaluation

Given a technique such as that described, it is natural to ask "What good is it?" and "How real is it?" Is it a useful device? Or is it so abstract in conception that it will not be of value to the highway or city planner? Are the values used so arbitrary that the results lose their meaning?

From a conceptual viewpoint, the idea of minimizing transportation costs seems sound enough. It may be argued that the desire to expand economic growth may require more than the minimum of transportation expenditures, implied by the cost minimization technique used here. This is a moot point, and needs a good deal of development before it can be tested. Aside from this qualification, the minimization of total highway costs seems a sound first step to an optimum solution.

Are all the items of cost included? All costs have not been included yet, it is true. Operating costs and accident costs are omitted, for the understandable reason that differential operating costs and accident costs, as functions of the varying splits in use between three systems resulting from varying spacings, would be extremely hard to state. Yet on the face of it, these differentials would probably not be great enough to affect the solution markedly. These costs could be included if the problem were solved by computer, which will surely be the next step in this process. So it can be asserted that the largest items of cost are included and that therefore the solution is complete enough from this viewpoint.

Are the costs, speeds, trip densities, trip lengths and time values good estimates? This will always be subject to debate, which is desirable. The values used in the examples in this paper seem quite reasonable. Expressway and arterial construction costs will always vary from city to city, and from place to place within cities. Arterial costs are hard to evaluate, because in most cases (except in open areas) they are already in place. Arterials can be taken as fixed and minimum total costs achieved without reference to changes in arterial spacings, which obviates that difficulty. Speeds, of course, are real enough, but could be measured more precisely. Trip densities and trip lengths are obtained from survey data. Time values are highly debatable, yet study after study, even in foreign countries, indicates that they operate effectively in dictating travel paths within ranges of $0.75 to $1.35 per hour. So the data seem accurate enough.

The simple technique of graphing values of construction cost and travel costs as affected by spacing lends weight to the more precise results obtained by the formulas.

It may be concluded, therefore, that the methods employed provide "realistic" results. The accuracy of the results, however, cannot be demonstrated so simply.

In the first place, the mathematics are not precise. Approximations have been used to make hand calculations possible, and so errors may be introduced from this source alone. Further, the mathematics were calculated for a simplified grid network of streets. This may not be bad approximation, however. Finally, the values of time costs, construction costs, and all the other variables are subject to inaccuracies.

Thus, the results must be labeled as approximate. But this does not rule out the technique, because the application of the resulting minimum-cost spacing pattern to a city will probably result in distortions far greater than those produced within the formula.

There is always some danger that the production of a formula will result in its blind application by persons who believe in it without discrimination, or without understanding the processes involved. This is always a hazard, but is no excuse for not developing new techniques and methods which are based on the application of reason in systematic ways.

Having argued that the results are real, the benefits coming from the use of these techniques may be discussed. Among the foremost of these benefits is the formal statement of criteria and the development of systematic methods for reaching a solution within the framework of the criteria. This process indicates the relationship of the variables one to another.

Second, a process has been suggested for preparing plans.

Finally, and most elusive, is a way of regarding travel and a system of roads needed to move travel. One's view of how trips move about in an urban area has a profound influence on the way one designs a system. A conventional view is of trips as lines connecting origins with destinations, which are gathered together in bundles and carried from one point to another of a city—principally to the Central business district. This view is a sort of "point-to-point" view, and has been fostered by the conventional, hand drawn, desire line display. A better view, it is maintained, is to regard travel as a continuum, a layer perhaps, spread over the urban region with varying depths. This has a certain degree of truth, because at each point in the urban terrain there is a whole array of trips of different lengths, pointed in different directions, and mixed up like jackstraws.

To drain this uneven layer of travel (which can be likened to water-saturated soil of varying depth) systems of pipes must be built with diameters conforming to the trip densities of the regions through which they pass. Each pipe must carry a load suitable to its diameter, so that no part of the system is under pressure. Access to these pipes—principally the larger ones, is not at its ends, but along its length, so that the pipes drain the region effectively like a drainage field, specializing in different lengths of trips, but without regard to direction.

With such a view it becomes more important to plan a proper system than it is to plan a single expressway. And this should result in far fewer mistakes and greater economies.

Further Research Needed

The development of this method has—as always—shown that additional work needs to be done. Some of the areas where further developmental and research work are needed are listed as follows:

1. A more sophisticated mathematical description of driver route choice might be attempted. This might involve estimating the portions of trips using local, arterial and expressway facilities for regions of varying density with non-gridded street systems.
2. Accident and operating cost differences between street types might be brought into the cost equation. A relationship between these costs and travel time could be incorporated in the value of K.

3. In working with parts of an area (for example, the ring analysis carried on here), the problem of interaction between areas needs additional work. Thus a given sub-area will have through trips which use its streets (probably expressways) but have origins and destinations outside the given sub-area. The effect of through trips on minimum-cost spacing needs more study; it is a phase of the problem of generalizing the formulas so that they will deal with all sorts of situations.
4. More work is needed on the value of K, which includes the value of time. The value of time may increase over time, with increases in real income, and subsequent effects on the spacing solutions.
5. Because spacing solutions are obtained for a particular year, the problem of the planning process over time must be dealt with.
6. It may be possible to include rapid transit in the minimum-cost solution. Rapid transit is a special case generally requiring high population densities (greater than 35,000 persons per net residential square mile) to operate economically.
7. If more money is spent on the construction of a transportation facility of a given type (here the junior expressway is an obvious example) the speeds of travel on that type of facility may be made to rise. This would result in a reduction of travel costs. What is the point at which further expenditure will yield no more return in travel savings? Both increased construction costs and faster speeds would have to be fed back into the minimum-cost spacing formulas, because these affect spacings. An additional point here is that network planning may be able to specify the boundaries of costs and service requirements which need to be met by intermediate types of facilities, such as the junior expressway. This then becomes a fruitful area for research and for ingenious design, involving the traffic engineer, the design engineer, and the city planner. The latter's contribution would consist of land plotting and neighborhood design so that driveways and minor streets would not interfere with flows of traffic on intermediate facilities.

Computers may be useful in solving some of these problems. Computers can deal with much more complex cases quite rapidly and could eliminate the need for approximations while testing many more situations. The experimental determination of limits and the effect of ramp spacing could be studied quite easily; variable density situations could be inserted; and non-gridded networks could be studied.

REFERENCE

1. Carroll, J. Douglas, Jr., "A Method of Traffic Assignment to an Urban Network." HRB Bul. 224, pp. 64-71 (1959).

Appendix A--Notation Used

This appendix lists the main symbols used in this paper and the corresponding definitions.

X = Name of local street system; an X street is a local;
Y = Name of arterial system; a Y street is an arterial;
Z = Name of expressway system; a Z street is an expressway;
J = Name of junior expressway system;

x = Spacing of X system; distance between X streets;
y = Spacing of Y system; distance between Y streets;
z = Spacing of Z system; distance between Z streets;

C = Total transportation costs;
C_1 = Construction cost;
C_2 = Travel cost;

S = Side of a square. A square area is generally assumed; area = S^2;

C_X = Construction cost per mile of X;

C_Y = Construction cost per mile of Y;

C_Z = Construction cost per mile of Z;

N = Total number of trip origins or destinations in an urban region or portion thereof;
K = A constant including (1) value of an hour for occupants of an average vehicle, (2) weekday equivalents per year, and (3) the reciprocal of a gross rate of return, which includes market interest and a depreciation charge;

v_X = Speed on X system;

v_Y = Speed on Y system;

v_Z = Speed on Z system;

$V_{XY} = \frac{1}{v_X} - \frac{1}{v_Y}$;

$V_{YZ} = \frac{1}{v_Y} - \frac{1}{v_Z}$;

$V_{XYZ} = \frac{1}{v_X} + \frac{1}{v_Y} - \frac{2}{v_Z}$;

D = density = N/S^2;
L = Over-the-road distance;
l = Airline distance;
F = Trip length frequency;
i = Particular trip length class;
$\overline{L}$ = Average over-the-road trip length for a given trip length class;
$\overline{l}_i$ = Average airline trip length for a given trip length class;
$\overline{L} = 1.3\overline{l}_i$;

F_i = Frequency of given trip length class i;

a = Distance from a random point on the X network to the Y network, computed mathematically;

A = Estimated empirical over-the-road distance traveled on locals by the average vehicle in going from origin to arterial, $A = 1.2a$;

$2A$ = Over-the-road distance traveled on local streets by the average trip. Maximum over-trip;

b = Distance from a random point on the Y network to the Z network, computed mathematically;

B = Estimated over-the-road distance traveled on arterials by the average vehicle in moving from arterial to expressway, $B = 1.2b$;

α = Maximum airline distance traveled on local streets for an individual trip;

$$\alpha = \frac{2A}{1.3} = \frac{2.4a}{1.3}$$

β = Maximum airline distance traveled on local streets plus arterials for an individual trip.

$$\beta = \frac{2A + 2B}{1.3} = \frac{2.4\,(a + b)}{1.3}$$

r, s, t are values of i;
r is that trip length for which the initial value of (1) is α, (or for which the initial value of L is 2A);
s is that trip length class for which the initial value of (1) is β (or for which the initial value of L is 2A + 2B);
t is the last trip length class;

$P = \Sigma_i F_i$ = cumulative distribution of F;

$Pr = \sum_{i\,(\alpha)}^{i(\beta)} F_i$ = that part of cumulative frequency of F occurring between α and β; and

$Ps = \sum_{i\,(\beta)}^{\infty} F_i$ = that part of cumulative frequency of F occurring for values of $(1) > \beta$.

Appendix B--Numerical Methods for an Iterative Solution for z and y

This appendix deals with numerical methods developed in solving the optimal spacing problem.

CASES DEALT WITH AND OUTLINED SOLUTIONS

In solving for z and y, two basic cases are discussed here:

1. Find z with y given, that is arterials are already in and no new construction is planned.
2. Find both z and y. This involves a simultaneous solution for z and y.

In the general case of finding z and y, start with an arbitrary value of y taken as given, and iterate to z. This should take only three passes. Given the selected y and z, a value of y can now be calculated by formula. This will differ from the original y started with. Use this result to select a new y and start all over, again ending in a comparison y. The values of y and z obtained to this point can be graphed and a final value of y and z can then be interpolated from the graph. This can then be checked by formula.

Case 1 is a special case of Case 2—given y, one finds only z.

The section on Methodology presents the detailed mathematics of this. On the left the general rule or operation is given. On the right, the arithmetic for a particular case is presented. Journey time through these operations is about 1 hr, once facility in them is acquired.

Setting this material down in this fashion may serve as the first step for a computer program, if such a program is wanted.

METHODOLOGY

General Rule or Operation

A. The Givens: General equations

(1) $z_1 = 2.24\sqrt{\dfrac{(C_Z)}{K\,(D)\,(V_{YZ})\,(P_s)}}$

(2) $z_2 = 3.25\,\beta - 2y \left(\text{from } \beta = \dfrac{2.4a + 2.4b}{1.3}\right)$

(3) $P_s = P_s\,(\beta)$ β is a particular value of (1): This relation is embodied in the Table of F_i.

(4) $y_1 = 3.25\,\alpha$ From $\alpha = \dfrac{(1.2)\,2a}{1.3}$, $a = y/6$

(5) $P_r = P_r\,(\alpha\,,\ P_s)$ This is embodied in the Table of F_i.

(6) $y_2 = 2.24\sqrt{\dfrac{(C_Y)}{K\,(D)\,(P_r V_{XY} + P_s\,V_{XYZ})}}$

We have 6 equations in 6 unknowns: z, y, P_r, P_s, α, β. This would imply a quick solution save that two of the equations are in implicit form, and are embodied in the Table of F_i.

The notation z_1 and z_2, and y_1 and y_2 is used to indicate the divergent values of z and y which will be obtained at first from the two equations containing z, and the two containing y. When $z_1 = z_2$, and $y_1 = y_2$, iteration is complete.

Arithmetic Example

The Givens

Ring 7, 1980 is used as the example.

The F^* distribution was taken as equal to that which held for Ring 5 in 1956. F^* is 1 minus the cumulative distribution of F.

F^* distribution

(1) ≥	F^*
0	1.000
1	0.775
2	0.536
3	0.421
4	0.343
5	0.273
6	0.225
7	0.174
8	0.137
9	0.109
10	0.081
11	0.058
12	0.041
13	0.030
14	0.023
15	0.017
16	0.016
17	0.013
18	0.010
19	0.008
20+	0.007

K = 7,500
D = 6,200
C_Z = 4,000,000
$v_X = 15$ $V_{XY} = 2/75$
$v_Y = 25$ $V_{YZ} = 1/50$
$v_Z = 50$ $V_{XYZ} = 5/75$
C_Y = 500,000

In the F^* distribution, (1) = airline distance. Thus, 34.3% of all trips are ≥ 4 mi in length. α and β are particular values of (1).

B. General Procedure

Step 1. For a given y, find the optimal z:

1.1 Insert specific values of given parameters, that is, K, D, V_{YZ}, C_Z, C_Y and V_{XYZ} are given initially. Rewrite Eq. 1

$$z_1 = 2.24\sqrt{\frac{(C_{Z0})}{(K_0)\,(D_0)\,(V_{YZ0})}}\ \sqrt{\frac{1}{P_s}}$$

where 0 indicates specific value

$$1.1\ z_1 = 2.24\sqrt{\frac{4{,}000{,}000}{(6{,}200)\,(7{,}500)\left(\frac{1}{50}\right)P_s}}$$

$$= 2.24\sqrt{4.30\,\frac{1}{P_s}}$$

$$= (2.24)\,(2.07)\sqrt{\frac{1}{P_s}} = 4.64\sqrt{\frac{1}{P_s}}$$

From this point on, z_1 in Eq. 1 depends only on P_S.

1.2 Obtain y_1, Either y is given (Case 1) or y is to be determined (Case 2). In Case 2, arbitrarily pick an initial value for y, call it y_1.	1.2 $y_1 = 1.00$ (arbitrarily)
1.3 Arbitrarily select an initial value for β.	1.3 $\beta = 4$ (arbitrarily)
1.4 The value of β selected implies a corresponding value of P_s. This is obtained from the F^* Table, relating (1) and P_s. β is a specific value of (1). P_s is a value of F^*.	1.4 For $\beta = 4$, $P_s = 0.343$
1.5 Insert P_s in Eq. 1 to obtain z_1.	1.5 $z_1 = 4.64\frac{1}{0.343} = 4.64\sqrt{2.92} = 4.64$ $(1.71) = 7.93$
1.6 Insert β and y_1 in Eq. 2 to obtain z_2.	1.6 $z_2 = 3.25(4) - 2(1) = 13.00 - 2.00$ $= 11.00$
1.7 If $z_2 > z_1$ try a lower initial β. If $z_2 < z_1$ try a higher initial β. For this β, repeat steps 1.4, 1.5, 1.6.	1.7 $z_2 = 11.00 > z_1 = 7.93$ Try $\beta = 3$ For $\beta = 3$, $P_s = 0.421$ $z_1 = 4.64\sqrt{2.38} = 7.15$ $z_2 = 3.25(3) - 2.0 = 7.75$
1.8 We have now run through two trials. In trial 1, call the values obtained β (1), z_1 (1), and z_2 (1). In trial 2, call the values (2), z_1 (2), and z_2 (2). Now, plot the z's as a function of β. Connect z_1 (1) and z_1 (2) by a straight line, and do the same for the z_2's. Take the point of intersection of these lines as approximately the final limiting values of β and z.	1.8

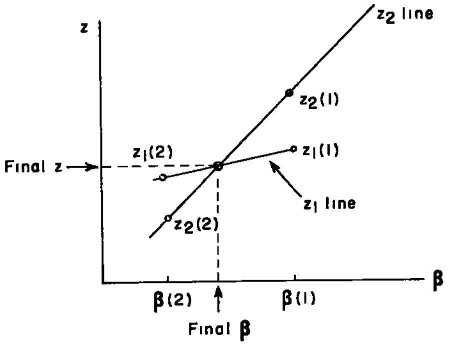


Figure 17.

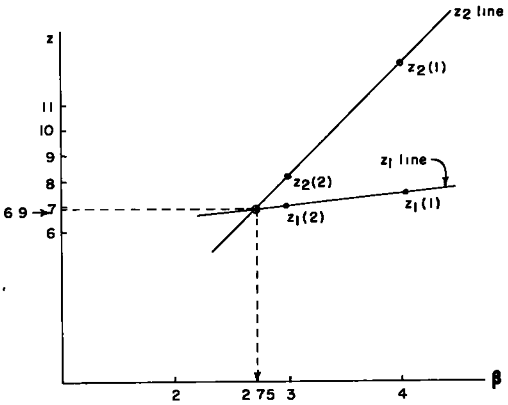


Figure 18.

1.9	Check the graphic interpolation or extrapolation by running through steps 1.4 to 1.7 and seeing if $z_1 = z_2$. If y is given (Case 1), this is the end of of the procedure	1.9	Trying $\beta = 2.75$ yields $P_s = 0.450$ (by linear interpolation of F^* between $\beta = 2$ and $\beta = 3$) $z_1 = 4.64\sqrt{2.22} = (4.64)(1.49) = 6.91$ $z_2 = (3.25)(2.75) - 2 = 6.93$ Then z can be taken as 6.9.
2.0	If y is to be determined, use the initial value of y = y_1 and the z obtained to compute y_2.	2.0	$y_1 = 1.00$, $z = 6.9$
2.1	Using y_1, obtain α from Eq. 4.	2.1	$3.25\,\alpha = y$ from Eq. 4. $\alpha = 1/3.25 = 0.308$
2.2	Given α, this implies $P_r + P_s$.	2.2	For $\alpha = 0.308$ $F^* = 0.932$ i.e. For (1) = 0, $F^* = 1.000$ (1) = 1, $F^* = 0.775$ By interpolation (1) = 0.308, $F^* = 0.932$ where is a particular (1). This value is $P_r + P_s$.

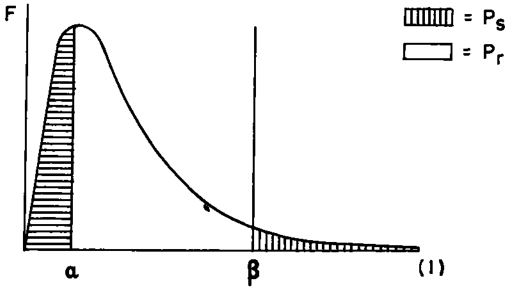


Figure 19.

2.3	$(P_r + P_s) - P_s = P_r$ (as indicated in Eq. 5).	2.3	$P_s = 0.450$ from step 1.9 Therefore $P_r = 0.932 - 0.450 = 0.482$.
2.4	Given P_r and P_s, obtain y_2 from Eq. 6, having expressed Eq. 6 in terms of P_s and P_r only by inserting constants into Eq. 6.	2.4	$y_2 = 2.24\sqrt{\frac{500,000}{(6,200)(7,500)\left(\frac{2}{75}P_r + \frac{5}{75}P_s\right)}}$ from Eq. 4 simplifies to $y_2 = 2.02\sqrt{\frac{1}{2P_r + 5P_s}}$ $y_2 = 2.02\sqrt{\frac{1}{2(0.482) + 5(4.50)}}$ $= 2.02\sqrt{\frac{1}{3.21}} = 1.127$
2.5	Compare y_1 and y_2. Call these $y_1(1)$, $y_2(1)$. If $y_2(1) > y_1(1)$, select a new y_1 somewhat above $y_2(1)$. Call this $y_1(2)$. Similarly if $y_2(1) < y_1(1)$, select $y_1(2)$ somewhat below $y_2(1)$.	2.5	$y_1(1) = 1.000$, $y_2(1) = 1.127$ Let $y_1(2) = 1.250$.
3.0	A final value for y is now obtained.		
3.1	For $y_1(2)$ compute a new z following procedures outlined in step 1.	3.1	$y_1(2) = 1.250$ Applying step 1 yields $z = 7.1$, $\beta = 2.95$, $P_s = 0.427$

3.2 Compute a new y_2 (2) following procedures outlined in step 2.

3.2 $\alpha = 0.385$, $P_r = 0.486$

$$y_2(2) = 2.02\sqrt{\frac{1}{3.107}} = 1.145$$

3.3 Set up tables of y_1, and y_2 and z for runs (1) and (2). Plot y_1 and y_2 against z. Draw a line through y_1 (1) and y_1 (2) and similarly draw a line through y_2 (1) and y_2 (2). The point of intersection of these lines yields a graphic solution for y and z.

It may be noted that y appears to converge very quickly, so that y_2(1) is very close to the true y.

3.3

run	y_1	z	y_2
(1)	1.00	6.9	1.127
(2)	1.25	7.1	1.145

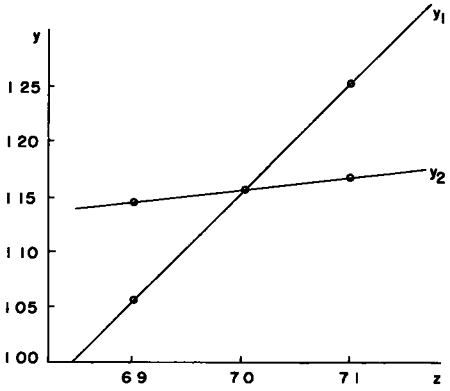


Figure 20.

4.0 Check the graphic solution.

Graphic solution: z = 7.0, y = 1.136

4.1 Insert the graphic z into Eq. 2.

4.1 $7.00 = 3.25\,\beta - 2.27$

4.2 Compute β and corresponding P_s.

4.2 So $\beta = 2.85$ and corresponding $P_s = 0.438$.

4.3 Obtain z_1 from Eq. 1 using P_s. Check against graphic z.

4.3 z_1 for this $P_s = 7.006$. Check.

4.4 Insert graphic y into Eq. 4 and obtain α.

4.4 $\alpha = 1.136/3.25 = 0.350$

4.5 Given α, one can obtain P_r and P_s by interpolation from F^* table.

4.5 For $\alpha = 0.350$, $P_r + P_s = 0.921$

$P_s = 0.438$

$\therefore P_r = 0.483$

4.6 Obtain y_2 from Eq. 6 and check against graphic y.

4.6 For this P_r and P_s, $y_2 = 1.137$. Check.

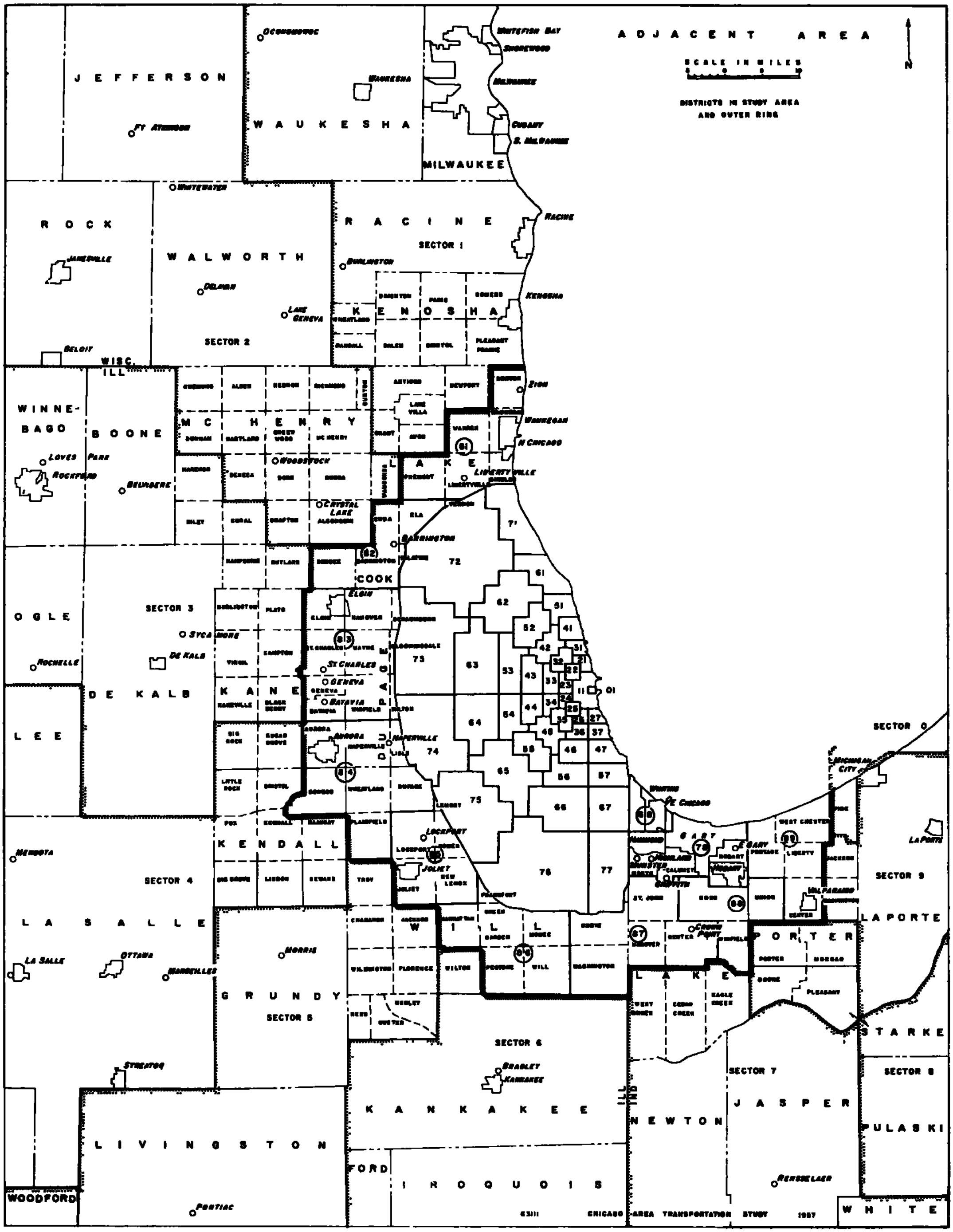


Figure 21. Study area—rings and sectors.

Generation of Person Trips by Areas Within the Central Business District

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City planners have for some time been saying that the traffic which flows in and out of a city each day is generated by the buildings in the center. So far as could be ascertained, such statements had not been checked, yet if some relationship did exist it would probably provide new ways of predicting future travel to the CBD by using floor-space forecasts. An investigation was undertaken to see if a relationship between floor space in use and travel to the CBD could be demonstrated, and secondly, if such a relationship existed in one city, to develop similar ones for other cities and endeavor by means of a comparison to estimate the form of a similar relationship in cities of different sizes and different predominant economic activities.

The method developed used the results of O-D studies and planning commissions' CBD floor-space surveys. Floor space in use in each O-D zone was classified into the three broad groups of retail, service (office and manufacturing), and warehousing. These figures were then considered as variables causing the difference in the number of persons shown by the O-D survey to have destination in the various CBD zones. A statistical regression technique was then used to determine a relationship between the variables.

The relationship between floor-space usage and the number of persons with destination in an O-D zone was found to be particularly well-developed. The mathematical relationship determined by the regression analysis was able to predict the person destinations with generally less than 20 percent error and in the zones of high attraction within the order of 2 percent. With this repeated in all of the seven cities studied, the truth of the relationship suspected by some city planners was demonstrated conclusively. It is believed that the extent of the error could be further reduced by considering a larger number of floor-space variables or by modifying the mathematical law slightly.

The investigation, which considered only metropolitan areas with populations of 250,000 persons or more, included the cities of Philadelphia, Pa., Detroit, Mich., Baltimore, Md., Seattle, Wash., Vancouver, B.C., Tacoma, Wash., and Dallas, Tex. A comparison of the equations for Philadelphia, Detroit, Baltimore and Seattle showed great similarities. Vancouver and Dallas, which were based on somewhat less complete floor-space information, also showed sufficient similarities to suggest that floor space in all these cities attracts people to the center at approximately the same rate. Tacoma, on the other hand, appears to represent another group, although the analysis in that city was seriously limited by the data available, and the result must be regarded as inconclusive.

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There is reason to believe that the investigation has shown that the number of people attracted to an area in a city's center is closely related to the floor area being used for various purposes in the section of the CBD considered. This relationship appears sufficiently well-developed to form a suitable basis for estimating future travel to the center. An equation of the type developed, used in conjunction with good economic forecasts of future floor-space use, should provide reasonably accurate forecasts of future travel to the center. The limited results show that there is a good deal of similarity among cities where the attraction of people to the CBD is considered. If more equations could be developed for cities in which O-D and floor-space studies have been made, it may be possible to formulate equations to suit other cities in which O-D studies have not been performed, which would be of great value in transportation planning. The similarities among the equations for Philadelphia, Detroit, Baltimore and Seattle are so great that it seems reasonable to suggest that if an equation is developed for a city in this population range, it will change very little as the city grows. In other words, although the central floor area may increase and the CBD grow in size, the trip attraction rate would not be expected to change appreciably.

● IT IS GENERALLY recognized that the success of highway planning in a city depends upon the ability of the planner to predict the future volumes of traffic for which the planned street system should prove adequate. The present prediction techniques, based on the projection of recent rates of increase, cannot be regarded as entirely satisfactory, and the development of some sounder methods appears to be warranted. Ideally, a prediction should take into account the changing habits and customs of people and possible technological changes affecting their mode of travel, as well as considering possible land-use changes affecting the travel patterns.

Some workers in the field of city planning have been saying that the traffic which flows in and out of a city each day is generated by the buildings, or rather the businesses that occupy and use the buildings in the center. So far as could be ascertained, such statements have not been checked, yet if some relationship did exist it could provide a new way of predicting future travel to the central business district. To investigate this, the Ontario Joint Highway Research Program sponsored research at Queen's University to see if a relationship between amount of floor area in use in various classifications and travel to the central business district (CBD) could be demonstrated. As a second part to the study it was hoped to estimate the form of similar relationships in cities of different sizes and with different predominant economic activities by means of a comparison of floor area attraction relationships from a number of cities.

All highway planning is concerned with the provision of facilities for the movement of people and goods at some future time. As such it seems that a prediction in terms of the volume and requirements of movements of people and goods rather than cars and trucks will form the most satisfactory base for planning. Such a prediction, if it takes into account foreseeable land-use changes as well as possible changes in peoples' habits and customs, will provide a picture of the required movements which will remain constant and relatively unaffected by technological change. A transportation system can then be designed to suit these volumes, utilizing the expected mode of travel and foreseeable technological change in the means of travel to decide on the characteristics of the individual branches in the network.

Based on the idea of obtaining a knowledge of future person movement, the relationship investigated was between floor space in use and the number of person destinations in the area rather than the number of automobiles attracted. To relate floor area to person destinations may appear a formidable task, requiring a large amount of detailed material giving the specific destinations and purposes which brought people to the center. However, it was believed that sufficient information was presently contained in the results of comprehensive home-interview origin-and-destination (O-D) traffic surveys and CBD floor-area studies to enable an investigation to be made based on these two surveys.

The floor-area studies give the floor area in use in the O-D survey zones, so that for each zone the floor area in use in a number of main classifications could be determined. If floor area is a measure of attraction of people to the center then there should be some relationship between these figures and the O-D survey totals for the number of people with destination in these zones in an average 24-hr period. The division of the CBD into zones in the traffic survey provides a series of floor-area versus person destination observations necessary for the statistical derivation of a regression equation. The coefficients found for the equations then provide estimates of the attraction effect of the different floor-area use types considered in the analysis.

For an investigation of this kind, in which an attempt was made to formulate a new approach to traffic prediction, it was thought justifiable to adopt an over-simplified approach to the problem. Complex relationships could probably be developed on the basis of fine differences in floor-area use classification. However, it was felt that the grouping of floor areas into three broad-use categories would prove simple but adequate. Although it may not be extremely accurate it could show whether the traffic pattern was related to floor area in use and also indicate the necessity of making any refinements in the floor-area classification.

The three broad categories chosen—retail, service-office and manufacturing-warehousing—have fairly distinct characteristics and tend, by their grouping, to give character to different parts of the city center. It was reasoned that different types of floor-area use, classified into any one of these groups, would most probably give rise to different rates of attraction. This could be a serious drawback to using such a small number of classifications. Nevertheless, the spread within a category may not be too large and if it is remembered that the success of any traffic prediction by this method will depend on the ability to forecast floor-area use, which cannot be done accurately in small groupings, it can be seen that the selection of broad grouping is justified. The use classifications listed by the various planning authorities are given in Appendix A grouped into the categories used in this study.

MATHEMATICAL MODEL

A multiple regression analysis enables the investigation and determination of a relationship from observed phenomena which may be exceedingly difficult to determine from theoretical considerations. However, its use is limited for it must depend on theoretical or intuitive understanding of the relationship in order to determine the form of the mathematical law to be investigated. A consideration of the conditions in the core of a CBD seemed to indicate that the correct form of a mathematical relationship between floor-area and person destinations would be linear. Therefore it was decided to see how well the observations fitted an equation of the form:

$$Y = b_1X_1 + b_2X_2 + b_3X_3 + k \tag{1}$$

in which

Y = number of person destinations in a zone in the CBD in an average 24-hr period from within the metropolitan area;

X_1 = area of retail floor space in use in the zone expressed in thousands of sq ft;

X_2 = area of service-office floor space in use in the zone expressed in thousands of sq ft;

X_3 = area of manufacturing-warehousing floor space in use in the zone expressed in thousands of sq ft;

b_1 = coefficient of retail floor space generation when considered in conjunction with service-office and manufacturing-warehousing space;

b_3 = coefficient of service-office floor space generation when considered in conjunction with retail and manufacturing-warehousing space;

b_3 = coefficient of manufacturing-warehousing floor space generation when considered in conjunction with retail and service-office space; and

k = constant.

A linear law of this type implies that each additional square foot of space in use, within each type, in a zone, will cause an identical increase in the number of people attracted. Obviously there are certain parts of what might be classed as a CBD to which a law such as this would not be expected to apply at all. For example, if the area is slowly growing in a certain direction, a zone in the growing fringe may be lacking somewhat in attraction. With additional development such a zone can supply a greater diversity of goods and services and as a result becomes a more popular area. In such a case, the addition of extra floor space has a cumulative effect, and instead of each additional square foot in use attracting equal numbers of people, increasing numbers are attracted and the correct mathematical model would probably be curvilinear with a power law at least for the retail variable. Manufacturing-warehousing and service-office are unlikely to be greatly influenced by the cumulative effect and the equation variables most probably always have a power index of close to unity. However, when a zone becomes well-developed and has large amounts of different types of floor space in use, any cumulative effect will undoubtedly dissipate and the attraction law will probably be linear. Even with greatly increased development in a zone the linear relationship should continue to represent the conditions of attraction, for a law of diminishing returns would restrain any retail business from using more and more floor space if each additional square foot used attracted fewer and fewer people.

If this reasoning is correct, then the linear law will only apply to the core of a CBD and will probably not represent the conditions of attraction to a growing and dying fringe. Because of this, it is important to fix the core area. This was not possible in the investigation because the analysis was made using the area classed as CBD by the traffic and floor-space studies, and as a result it was expected that some of the fringe zones would not be well described by the selected relationship.

Because the larger cities were more likely to have made floor-space studies, the investigation was confined to cities classified by the 1950 U.S. Bureau of Census as having standard metropolitan areas with a resident population of 250, 000 persons or more. Some of the existing O-D studies were not suitable for a regression analysis because the CBD was divided into only one or two zones, while in the case of other cities, floor-space surveys were not available. Nevertheless, sufficient information was available to undertake analyses of the cities given in Table 1.

An investigation of this table will show the difficulty of obtaining close time agreement between land use and travel information. It was believed that an analysis relating these two surveys would be valid in the case of Philadelphia, Detroit and Vancouver because of the small time difference involved, but the other cities presented a problem.

Inasmuch as it was believed that floor area in use was a reliable measure of the business activity which attracted people to the central area, it was necessary to estimate how well the later floor-space information represented the conditions at the time of the O-D study. This would be an extremely difficult, if not impossible, condition to demonstrate. Some change is always occurring in any city and this must have affected the use of floor space within the various zones of the O-D studies. However, a statistical regression technique actually balances out variations in the observations to determine the equation that best describes the information available. Because of this the redistribution of floor-space use that has occurred in the time between the two surveys may not be too critical as long as the general use has been relatively stable. The regression equations determined for cities with considerable time difference between surveys will probably not fit the data as well as those for the cities with negligible time difference, but the resulting equations will most probably provide a reasonable estimate of the attraction laws which would have been obtained had coincidental data been available.

It was possible to estimate the general state of floor-space use in Baltimore and Seattle using the work of Norwood and Boyce of the University of Washington (1). Their

TABLE 1

CITIES, POPULATION, AND DATES OF FIELD SURVEYS COMPARED IN REGRESSION ANALYSIS

Met. Area	State or Province	1950 Population in Standard Met. Area	O-D Field Survey Date	CBD Floor-Space Survey Date
Philadelphia	Pennsylvania	3,671,048	1947	1949
Detroit	Michigan	3,016,197	1953	1954
Baltimore	Maryland	1,337,373	1946	1957
Seattle	Washington	732,992	1946	1956
Vancouver	British Columbia	600,000	1955[1]	1954
Tacoma	Washington	275,876	1948	1958
Dallas	Texas	614,799	1951	[2]

[1] Vancouver's travel information comes from a survey of vehicle owners and public transit users, and as a result may not be as reliable as that provided by the home-interview survey.

[2] See notes for Table 10, Appendix B.

work, in which census returns and questionnaires were used to estimate CBD change, showed that retail sales had decreased with a corresponding decrease in retail workers. It was impossible to determine retail floor-space change from the census returns but the data did show that there had been no change in the over-all amount of office space in use in central Baltimore, and only a 15 percent decrease in the amount in use in Seattle during the 10-yr period, 1946-1954. These over-all pictures, although failing to produce definite proof, appear to indicate a fairly stable state of floor-space use and as a result the use of a regression analysis might be expected to provide reasonably valid results. Unfortunately no information was available for Tacoma and it was necessary to assume that a similar situation existed there.

The fitting of a regression equation to a set of observations is a statistical technique well described in a number of text books (2). In this analysis use was made of a set of statistics known as Gaussian multipliers to find the equation of "best fit." As a result of the analysis, the coefficients b_1, b_2, b_3 and the constant, k, in Eq. 1 were determined. These coefficients estimate the generative power of the three types of floor space used, in terms of the number of people attracted per 1,000 ft of floor space.

RESULTS OF THE ANALYSIS

The investigation set out to determine how useful a knowledge of floor space alone would be in estimating the attraction of people to the city center. The ability of the selected relationship to represent the actual conditions may be seen by examining Tables 4 to 10 (Appendix B). These tables show the O-D zone number, the floor area in use in each of the three classifications adopted, the number of person destinations in the zone as shown by the traffic survey and the number of person destinations estimated by the regression equation. It should be noted that estimated attraction corresponds to the observed surprisingly well. Although some zones show large errors, in nearly every city the zones which attract the greatest number of people are estimated with small percentage error, and generally the error in most zones is less than 20 percent. The closeness of the estimated and observed attraction of people to the zones, in the various cities indicates fairly conclusively that a knowledge of floor space in use provides a reasonable index of the attraction of the center. The magnitude of these errors could probably be reduced by considering a larger number of floor-space variables or by modifying the form of the mathematical law. In most cases the zones showing large errors are situated on the edge of the area classed as CBD by the traffic study, and are most likely outside the core area where the linear law could be expected to apply.

As would be expected, there is a greater range of error shown between estimated and observed attraction of zones in the cities with considerable time difference between surveys. Although the expected variations are apparent, the general trend in the floor-space attraction relationship is distinct and well-developed.

It should be mentioned that the development of a regression equation does not prove that variations in floor-space use actually cause the differences in attraction to a zone. However, the existence of these relationships and the fact that most people journeying to the center do so to transact some form of business with an occupier or user of floor space makes an inference of this kind highly probable. It seems therefore, that for highway planning it will be reasonable to use sound economic forecasts of future floor-space use in the central area as an index of the center's expected attraction, and, by means of an appropriate regression equation, estimates of the number of people who will be attracted in the future to areas within the CBD of the various cities can be made.

In discussing the form of mathematical model studied, the point was made that the linear law would probably only hold for the core area of the CBD. This was indicated in the analysis by the fact that the fringe zones generally showed a greater error of estimate than the central zones. Although the investigation of what may be classed as core would have been difficult, the use of the method depends on the fixing of some boundary within which equations of the type suggested may be used with confidence. The Murphy-Vance CBD outline (3) seemed to offer much promise as a frame within which the linear equation would probably apply.

The area available for analysis in Philadelphia was large and it was possible to compare the area delimited by the Murphy-Vance technique as the CBD with the area in which the linear law appeared to apply (Fig. 1). This analysis was complicated by the fact that in some cases the O-D study boundaries did not coincide with those used in the floor-space survey and zones had to be grouped with a resulting loss in sensitivity. Nevertheless, if the area in which person destinations could be estimated by means of the linear equation with less than 20 percent error is compared with the area delimited by the Murphy-Vance technique, reasonable agreement can be seen. If the use of a

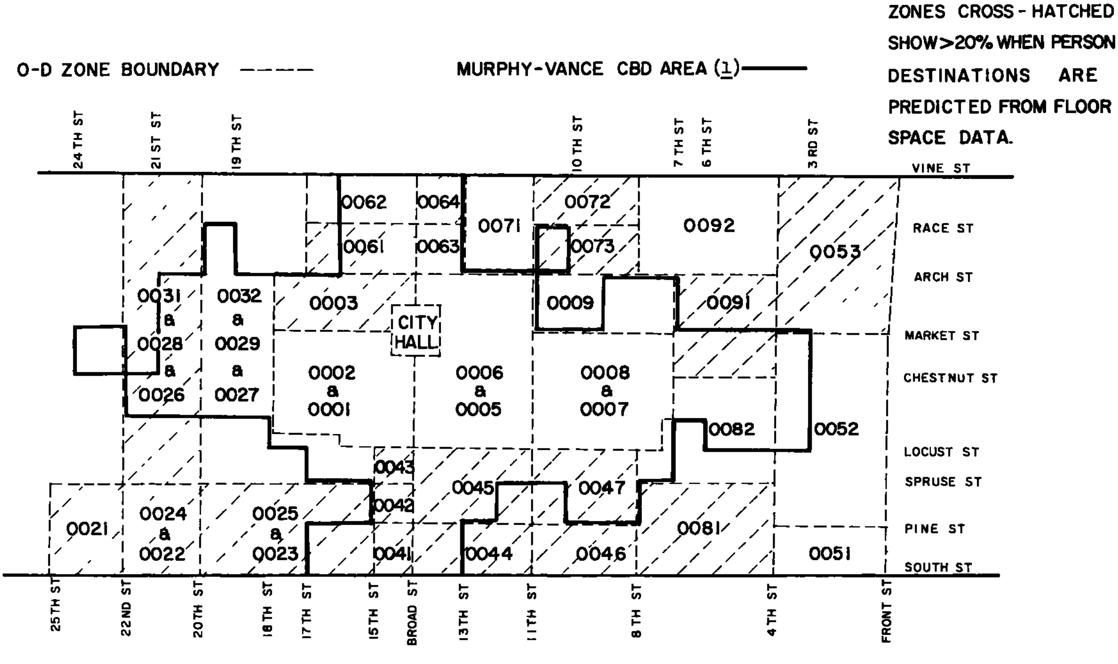

Figure 1. Philadelphia central area.

linear equation of the type suggested is restricted to the Murphy-Vance CBD area, the application appears to be valid.

COMPARISON OF CITIES

Any use of the equations developed would naturally depend on a continuation of the trip generative power of the three types of floor space at the determined value, or on reasonable estimates of the changes in the equation coefficients as the particular city grew. It was thought that similarities and differences in the attraction equations might be explainable on the basis of some characteristics of the city, such as size or predominant economic activity. Consequently, the attraction equations for the various cities were compared. The cities compared included a wide range of conditions (Table 2).

TABLE 2

LOCATION, POPULATION, AND SERVICE CLASSIFICATION OF CITIES COVERED IN ANALYSIS

Met. Area	State or Province	1950 Population in Standard Met. Area	Service[1] Classifications
Philadelphia	Pennsylvania	3,671,048	Finance Insurance Real estate
Detroit	Michigan	3,016,197	Manufacturing
Baltimore	Maryland	1,337,373	Diversified
Seattle	Washington	732,992	Strongly financial Insurance Real estate
Vancouver	British Columbia	600,000	-
Tacoma	Washington	275,876	Public Administration and finance
Dallas	Texas	614,799	Very strongly financial and wholesaling

[1] Nelson, J., "A Service Classification of American Cities." Economic Geography, XXXI No. 3, pp 189-210 (July 1955).

Table 3 sets out the various equations estimating trip attraction determined by the regression analysis. A study of this table immediately shows interesting similarities among the equations. This is particularly marked in the cases of Philadelphia, Detroit and Baltimore and to a lesser extent in the case of Seattle. The analyses of Vancouver, Tacoma and Dallas were limited by the data available as explained in the notes attached to Tables 8, 9 and 10, respectively (Appendix B). Still some similarities are apparent in all but the equation for Tacoma.

Statistical techniques such as the fitting of a regression line give results which are, in fact, estimates of an unknown set of "true" values. The equations determined cannot be regarded as the "true" equations which underly the observed differences but only estimates of these equations. The similarity of the equations for Philadelphia, Detroit and Baltimore could mean that, in actual fact, they are all estimates of the one "true" equation, and that the attraction of people to the zones in the central areas of all these cities is governed by the one law. While the X_3 coefficient in the equation for Seattle tends to made this city appear different from the first three mentioned, the extent

TABLE 3

EQUATIONS GOVERNING THE ATTRACTION OF PERSONS TO AREAS WITHIN THE CBD'S OF SEVEN CITIES ON THE BASIS OF FLOOR AREA IN USE

City	State or Province	Equation of Attraction
Philadelphia	Pennsylvania	$Y=14.602X_1 + 5.858X_2 + 1.276X_3 - 3,470$
Detroit	Michigan	$Y=13.918X_1 + 4.613X_2 + 1.717X_3 - 2,280$
Baltimore	Maryland	$Y=12.871X_1 + 4.524X_2 + 1.343X_3 - 1,080$
Seattle	Washington	$Y=13.678X_1 + 4.382X_2 + 0.152X_3 - 200$
Vancouver	British Columbia	$Y=14.322X_1 + 10.534X_2 + 3.670X_3 + 1,560$
Tacoma	Washington	$Y= 7.709X_1 + 2.493X_2 - 17.692X_3 + 3,590$
Dallas[1]	Texas	$Y=16.191X_1 + 3.546X_2 + 12.265X_3 - 8,570$

[1] The floor-space information in Dallas was estimated from a map and as a result cannot be classed as accurate. This equation is included to show the rough similarity between it and the first three.

of the difference is not sufficient to rule out the possibility that the attraction equation for Seattle may be the same as that governing trip patterns in Philadelphia, Detroit and Baltimore.

On the other hand, each of the unknowns in the equation could be influenced by some characteristic of the city or the city center. Although this was not thought to be the case, because of the great similarity of the equations, a comparison of the equation coefficients was made on the basis of various indices. The indices selected were population, retail floor space per head, retail floor space per $1,000 of retail sales, office space per head and per worker in selected service trades, manufacturing-warehousing space per head and per worker in production and wholesale activities. These comparisons failed to show any trend and seemed to give strength to the belief that the equations presented probably estimate one general equation applying to large cities irrespective of variations in size, location, or predominant economic activity.

One interesting fact emerging from this comparison was that Vancouver showed a significantly higher ratio of service workers per 1,000 sq ft of service-office floor space than any of the other cities. This ratio was, of course, influenced by differences in the material reported in the Canadian and American censuses, and by variations in the size of the area classed as the CBD. This fact could possibly account for the high service-office space generation coefficient shown in the equation for that city, but as explained in the notes attached to Table 8, the assumption made in converting acres of site area to floor area in the manufacturing-warehousing classification could seriously influence the Vancouver analysis.

No definite conclusions can be reached in the case of Tacoma, for the equation is based on only five observations and the estimate provided by the analysis may be greatly in error. However, the extent of the difference between this equation and the other equations indicates that this city center may be governed by an equation different from that for other cities studied. Because Tacoma was the only city included in the analysis with a population less than 600,000 persons, it may well represent another group of cities.

Some questions may be asked about the meaning and magnitude of the constant term of the equation. Physically, it is present because the observations indicate that the equation does not pass through the origin and represents one of the unknowns in the use of a regression analysis. The effect of the constant term on the estimates is generally small and the apparent wide difference between its value as determined in Philadelphia and Seattle is not as critical as the differences in the values of the respective coefficients.

CONCLUSIONS

This investigation has been able only to form an introduction to what may be a very profitable field of study. The results are such that it is possible to say that the number of people attracted to an area in a city's center appears to be closely related to the amount of floor space being used for various purposes in the section of the CBD considered. It seems that, for highway planning, it would be valid to use sound economic forecasts of future floor-space use in the central area as an index of the area's expected future attraction, and by means of equations similar to the ones developed, estimate the number of people likely to be attracted to the CBD zones.

Although these results cover only a limited range they show that there is a good deal of similarity among cities when the attraction of people to the CBD is considered. If more equations could be developed for cities in which O-D and floor-area studies have been made, it may be possible to formulate equations to suit other cities in which a comprehensive traffic study has not been performed which, of course, would be of great value in transportation planning.

The similarities among the equations for Philadelphia, Detroit, Baltimore and Seattle are so great that it seems reasonable to suggest at this stage that if such an equation is developed for a city in this population range it will change little as the city grows. Changes appear to occur in the amount and distribution of the floor area in use rather than in the trip generation rates.

One interesting result from the analysis is that the use of person movements, irrespective of mode of travel, has indicated great similarities between travel to the center of Philadelphia and Detroit. These similarities would have been masked by differences in automobile ownership and use in the two cities, and an apparent dominance of the central area in Philadelphia due to mass transportation facilities.

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Appendix A

STUDY FLOOR-SPACE CLASSIFICATION

Metropolitan Area	Retail	Service-Office	Manufacturing-Warehousing
Philadelphia	Retail	Wholesaling without stocks Business service Consumer service Hotels	Manufacturing Wholesaling with stocks Undetermined
Detroit	Retail business	Office building Parking lots Parking garages Institutions Utilities Hotels Terminals Open storage	Warehouses Light industry Heavy industry
Baltimore	Retail	Business service Consumer service Wholesale without stocks	Manufacturing Wholesale with stocks Unknown
Seattle	Retail	Hotels and recreation Eating places Automotive uses Banks and miscellaneous Business offices Public offices Institutional and organizational	Commercial Industrial Public Semi-public
Vancouver	Retail	Office Public Other	Industry and warehousing
Tacoma	Core retail Intensive retail Extensive retail	Office and services Consumer services Amusement and recreation Hotel Parking Public Semi-public	Warehousing Wholesale Manufacturing-industrial
Dallas	Classified from map	Parking space omitted	

Note: Residential and vacant space excluded from the analysis.

Appendix B

Philadelphia, All CBD Zones

Table 4 shows that about one-half of the areas are predicted by the regression equation with an error in excess of 30 percent. However, it can be seen that the zones with appreciable prediction error are actually the smaller generators, in all accounting for only one-quarter of the total person destinations in the center.

This analysis presented some problems because the boundaries of the blocks used in the floor-space study did not coincide in every case with the traffic zone boundaries. This meant that some O-D zones had to be grouped and as a result there was considerable variation in the land area of the zones and groups of zones used in the analysis. This variation appears to add to the error of prediction because of the greatly increased effect of the constant term in the equation on the smaller land areas. In the other cities considered, the zones were of a more uniform size from the point of view of land area. Although there is a wide variation in the amount and distribution of floor space in each zone, the predictions in the other cities are generally closer than in this case. It was thought that the great difference in the land area of the zones considered affected the regression analysis.

The coefficient of multiple determination indicates that 96 percent of the variation in person destinations can be explained on the basis of differences in the amounts of floor area in use in the zones. This estimate depends on the floor space and person destination variables being jointly normally distributed. This is difficult to assess but it appears that this coefficient may not be too misleading.

Detroit CBD

Table 5 indicates a good relationship between the variables. Use had to be made of the larger survey districts rather than zones, but only one district is considerably in error when attraction is explained by the equation. This district is situated on the edge of the area classed as CBD.

Baltimore CBD

Table 6 shows that although a relationship does exist, the accuracy of estimation is not as great as for the other cities. This is probably due to the time lag between the two surveys, although in Seattle, where a similar difference occurred, the errors are not as great. Two zones 0041 and 0051 show considerable error but again these are the smaller generators and are situated on the edge of the area classed as CBD.

Seattle CBD

Table 7 shows a well-developed relationship, the coefficient of multiple determination indicating that 96 percent of the variation is explained by the floor-space relationship. The area delimited as the CBD by the Murphy-Vance method was available for the city and could be compared with the results of the regression analysis. The only two zones falling completely outside the area were zones 006 and 008 which have the largest errors of estimation.

Vancouver CBD

Analysis for this city was complicated by the fact that manufacturing-warehousing space was given in the form of acres of site area (Table 8). The calculations made are based on the site area converted to square feet. This assumption is probably incorrect and must play some part in causing the constant in the regression equation to be positive whereas the other equations show negative constants. Because the negative constant is repeated so often and seems logical from a consideration of the curvilinear law, which it is thought would suit CBD fringe zones, this analysis seems to be in error.

TABLE 4

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATED PERSON DESTINATIONS; AND ERROR IN ESTIMATE—PHILADELPHIA ALL CBD ZONES

REGRESSION EQUATION $\hat{Y} = 14\ 602 X_1 + 5\ 858 X_2 + 1\ 276 X_3 - 3470$

O-D ZONE	FLOOR SPACE · 1000's Sq. Ft RETAIL X_1	SERVICE-OFFICE X_2	MANF'G.-WHSG X_3	O-D PERSON DESTINATIONS (24 hrs.) Y	EST'D PERSON DESTINATIONS (24 hrs.) $\hat{Y}$	$\hat{Y} - Y$	% ERROR
0001, 0002	1809	11,118	1473	88,490	89,950	+ 1460	+ 16
0003	41	2130	144	28,860	9,790	−19,020	− 66
0005, 0006	4366	6811	2290	103.690	103,100	− 690	− 06
0007, 0008	2975	3818	2633	77,200	65.700	−11,500	−15
0043	23	781	0	3700	1440	2260	−61
0042	15	165	12	1790	−2270 *		
0041	105	160	87	3700	− 980 *		
0063	86	729	612	2540	2840	+ 300	+ 12
0064	97	431	257	2180	800	−1300	−60
0009	2684	441	1706	36,590	40,480	+3890	+ 11
0061	71	658	196	8340	1670	−6670	−80
0062	174	1172	196	6260	6190	− 70	− 11
0027,0029,0032	906	3581	604	26,510	31,510	+5000	+ 19
0045	346	1484	281	4550	10,630	+ 6080	+ 134
0047	231	1067	246	3850	6470	+2620	+ 68
0082	92	2783	1642	13.660	16.270	+2610	+ 19
0091	843	1921	2351	17.220	23,090	+ 5870	+ 34
0073	300	491	1502	3270	5700	+2430	+74
0026, 0028, 0031	668	1897	628	10.950	18,200	+7250	+66
0023, 0025	558	1703	252	9540	14,980	+5440	+57
0044	348	250	370	1970	3550	+1580	+80
0046	383	226	325	1940	3860	+1920	+99
0081	629	755	860	5500	11,230	+5730	+104
0052	440	2360	3779	23,430	21,600	−1830	− 8
0053	261	1158	4234	6970	12,530	+5560	+ 80
0051	242	223	565	2190	2090	− 100	− 4·5
0092	147	859	2702	6200	7160	+960	+ 15
0072	154	277	227	2030	690	−1340	−66
0071	205	333	1390	4080	3250	− 830	−20
0022, 0024	176	344	150	2690	1310	−1380	−51
0021	117	211	76	2450	− 430 *		

Variance of Estimate $S^2(\hat{y}) = 30{,}178{,}700$

Standard Error of Estimate $S(y) = 5490$ Person Destinations

Coefficient of Multiple Determination $R^2 = 0\ 960$

	X_1	X_2	X_3	K
95 % Confidence Range of Equation Parameters	11 873 17 331	4 709 7007	− 0 693 3 245	+2120 −9080

TABLE 5

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATED PERSON DESTINATIONS; AND ERROR IN ESTIMATE—DETROIT CBD ZONES

REGRESSION EQUATION :— $\hat{Y} = 13.918\ X_1 + 4.613 X_2 + 1.717\ X_3 - 2280$

O–D DISTRICT	FLOOR SPACE 1000's Sq. Ft RETAIL X_1	SERVICE –OFFICE X_2	MANU'F. -WAREHOUSING X_3	O-D PERSON DESTINATIONS (24 hrs) Y	EST'D PERSON DESTINATIONS (24 hrs) $\hat{Y}$	$\hat{Y} - Y$	% ERROR
00	5400	2721	86	85,850	85,580	−270	− 0.3
01	2454	14,162	1573	99,670	99,900	+230	+ 0.2
11	480	3259	3348	25,260	25,180	− 80	− 0.3
12	440	1494	1154	8,210	12,710	+4500	+55
13	140	761	10	2,760	3,200	+440	+16
15	193	1968	789	11,800	10,840	−960	− 8
17	426	2680	721	19,030	17,250	−1780	− 9
19	102	1330	1578	10,000	7,980	− 2020	− 20

VARIANCE of ESTIMATE $S^2(\hat{Y}) = 7,804,000$

STANDARD ERROR of ESTIMATE $S(\hat{Y}) = 2,790$ PERSON DESTINATIONS

COEFFICIENT of MULTIPLE DETERMINATION $R^2 = 0.997$

	X_1	X_2	X_3	K
95 % CONFIDENCE RANGE	12.086	3.864	−1.392	+ 6180
of EQUATION PARAMETERS	15.750	5.362	4.862	−10740

TABLE 6

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATED PERSON DESTINATIONS; AND ERROR IN ESTIMATE—BALTIMORE CBD

REGRESSION EQUATION — $\hat{Y} = 12\,871 X_1 + 4\,524 X_2 + 1\,343 X_3 - 1080$

O-D ZONE	FLOOR SPACE· 1000's Sq Ft			O-D PERSON DESTINATIONS (24 hrs)	EST'D PERSON DESTINATIONS (24 hrs)	$\hat{Y}-Y$	% ERROR
	RETAIL X_1	SERVICE-OFFICE X_2	MANF'G-WHSG. X_3	Y	$\hat{Y}$		
010	289	888	1456	9,780	8,670	−1110	− 11
011	1356	1245	1538	19,410	23,770	+4360	+ 22
012	596	1607	1538	20,300	15,990	−4310	− 21
020	661	2739	689	21,230	20,750	− 480	− 2
021	88	1348	1384	9,910	8,040	−1870	− 19
022	258	1562	70	15,300	9,440	−5860	− 38
023	106	430	269	3,670	2,680	+ 990	+ 27
030	1323	852	177	27,830	20,180	−7650	− 27
031	1203	1504	143	18,110	21,500	+3390	+ 19
040	194	1723	53	7,570	9,310	+1740	+ 23
041	140	1445	721	2,620	8,260	+5640	+215
051	560	877	459	3,460	10,880	+7420	+214

VARIANCE of ESTIMATE $S^2(\hat{Y}) = 31{,}720{,}600.$ STANDARD ERROR of ESTIMATE $S(\hat{Y}) = 5{,}630$ PERSON DESTINATIONS

COEFFICIENT of MULTIPLE DETERMINATION $R^2 = 0.667$

	X_1	X_2	X_3	K
95 % CONFIDENCE RANGE	5.046	−2.005	−5.193	+4810
of EQUATION PARAMETERS	20.818	10.953	7.859	−30,580

TABLE 7

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATED PERSON DESTINATIONS; AND ERROR IN ESTIMATE—SEATTLE CBD

REGRESSION EQUATION :— $\hat{Y} = 13.678 X_1 + 4\,382 X_2 + 0.152 X_3 - 200$

O-D ZONE	FLOOR SPACE 1000's Sq Ft			O-D PERSON DESTINATIONS (24 hrs)	EST'D. PERSON DESTINATIONS (24 hrs)	$\hat{Y} - Y$	% ERROR
	RETAIL X_1	SERVICE -OFFICE X_2	MANU'F —WAREHOUSING X_3	Y	$\hat{Y}$		
012	138	1200	320	5,800	6,990	-1190	-20
013	1248	678	73	22,760	19,850	+2910	+13
014	1118	1374	112	19,850	21,130	-1280	-6
015	62	870	148	4,160	4,480	-320	-8
016	1380	940	25	21,110	22,800	-1690	-8
017	370	2356	23	16,160	15,190	+970	+6
002	105	629	397	3,420	4,050	-630	-18
003	191	2096	0	11,750	11,600	+150	+1
004	70	2143	0	9,170	10,150	-980	-11
005	44	1276	238	6,920	6,030	+890	+13
006	29	328	530	2,960	1,710	+1250	+42
007	25	1702	22	8,950	7,600	+1350	+15
008	22	591	191	1,340	2,720	-1380	-103

VARIANCE of ESTIMATE $S^2(\hat{Y}) = 2{,}539{,}700$ STANDARD ERROR of ESTIMATE $S(\hat{Y}) = 1590$ PERSON DESTINATIONS

COEFFICIENT of MULTIPLE DETERMINATION $R^2 = 0\,965$

	X_1	X_2	X_3	K
95% CONFIDENCE RANGE	11 077	1.781	−10.637	+5720
of EQUATION PARAMETERS	16.279	6.983	+10.941	−6110

TABLE 8

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATED PERSON DESTINATIONS; AND ERROR IN ESTIMATE—VANCOUVER CBD

REGRESSION EQUATION · $\hat{Y} = 14\ 322\ X_1 + 10{\cdot}534\ X_2 + 3\ 670\ X_3 + 1560$

O-D ZONE	FLOOR SPACE: 1000's Sq Ft.			O-D PERSON DESTINATIONS (24 hrs.)	EST'D. PERSON DESTINATIONS (24 hrs.)	$\hat{Y}-Y$	% ERROR
	RETAIL X	SERVICE-OFFICE X	MAN'F'G-WHSG. X	Y	$\hat{Y}$		
900	382	3692	44	46,900	46080	- 820	- 17
901	1674	2124	87	48 640	48,230	- 410	- 0·8
902	43	142	44	4400	3830	- 570	- 13
910	176	1273	610	18,530	19,730	+ 1200	+ 6
911	3	513	0	1860	7,000	+ 5140	+276
920	321	690	566	14,220	15,500	+ 1280	+ 9
921	4	45	392	2 630	3530	+ 900	+ 34
930	10	88	1350	12,580	6020	-6560	- 52
940	86	503	174	14,110	8730	-5380	- 38
950	1294	1348	1176	39,460	38,610	- 850	- 2
951	443	717	1525	16,450	21,050	+4600	+ 28

VARIANCE of ESTIMATE $S^2(\hat{Y}) = 15,342,400$ STANDARD ERROR of ESTIMATE $S(Y) = 3920$ PERSON DESTINATIONS

COEFFICIENT of MULTIPLE DETERMINATION $R^2 = 0{\cdot}963$

	X_1	X_2	X_3	K
95% CONFIDENCE RANGE	8·126	7 223	- 1·887	+ 10,420
of EQUATION PARAMETERS	20·518	13·845	9·227	- 7300

This is a problem encountered in any use of a multiple regression technique when the line of best fit may produce coefficients which are meaningless from a common-sense point of view. If more observations were available the resulting equation might be quite different from the one found in this analysis.

Tacoma CBD

With only five observations to work from, the results shown in Table 9 cannot be regarded as significant. An equation of this type, with four unknowns, can be calculated to fit perfectly four observations. Therefore, with only five observations, the regression equation found from the analysis may be so influenced by errors in the observations that it is completely misleading as a guide to the "true" relationship. The large negative coefficient on manufacturing-warehousing floor space is obviously incorrect. The comments made with regard to the positive constant given in the notes to Table 8 apply also to the constant and negative coefficient in this equation.

Dallas CBD

Table 10 shows a floor-space estimate based on a land-use map marked with building heights. To produce a meaningful area in the service-office classification, parking space was eliminated from the floor space. The results show only that some relationship is present between person destinations and floor space and that the general form of the equation appears satisfactory for this city.

TABLE 9

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATED PERSON DESTINATIONS; ERROR IN ESTIMATE—TACOMA CBD ZONES

REGRESSION EQUATION — $\hat{Y} = 7\,709X_1 + 2\,493X_2 - 17\,698X_3 + 3590$

O-D ZONE	FLOOR SPACE' 1000's Sq Ft			O-D PERSON DESTINATIONS (24 hrs)	EST'D PERSON DESTINATIONS (24 hrs)	$\hat{Y} - Y$	%ERROR
	RETAIL X_1	SERVICE - OFFICE X_2	MAN FG. -WAREHOUSING X_3	Y	$\hat{Y}$		
000	226	902	78	6450	6200	-250	— 3.8
001	100	719	29	4610	5640	+1030	+ 22
002	1174	1025	63	13,540	14,050	+ 510	+ 3.7
003	300	726	133	6970	5360	-1610	-23
004	218	319	194	2360	2630	+270	+ 16

VARIANCE of ESTIMATE $S^2(\hat{Y}) = 6300$

STANDARD ERROR of ESTIMATE $S(\hat{Y}) = 80$ PERSON DESTINATIONS

COEFFICIENT of MULTIPE DETERMINATION $R^2 = 0{\cdot}997$

	X_1	X_2	X_3	K
95% CONFIDENCE RANGE	5 676	−2 589	−0·672	+6470
of EQUATION PARAMETERS	9 742	7 575	−34 724	+ 700

TABLE 10

FLOOR SPACE; PERSON DESTINATIONS; ESTIMATES PERSON DESTINATIONS; AND ERROR IN ESTIMATE—DALLAS CBD

REGRESSION EQUATION:— $\hat{Y} = 16\ 191 X_1 + 3\ 546 X_2 + 12\ 652 X_3 - 8570$

O-D ZONE	FLOOR SPACE 1000's Sq Ft			O-D PERSON DESTINATIONS (24 hrs)	EST'D PERSON DESTINATIONS (24 hrs)	$\hat{Y} - Y$	% ERROR
	RETAIL X_1	SERVICE -OFFICE X_2	MANU'F -WAREHOUSING X_3	Y	$\hat{Y}$		
01	509	740	874	18,380	13,350	−5030	− 27
02	15	404	546	3,840	10	−3834	−100
03	344	289	316	3,010	2,020	− 990	− 33
04	1474	5674	183	40,130	37,730	−2400	− 6
05	243	1558	1071	11,730	14,440	+2710	+ 23
06	940	2577	356	14,870	20,290	+5420	+ 36
07	676	542	114	3,070	5,740	+2670	+ 87
08	1499	1979	0	23,730	22,720	−1010	− 4
09	181	615	857	4,940	7,380	+2440	+49

VARIANCE of ESTIMATE $S^2(\hat{Y}) = 19{,}504{,}600$

STANDARD ERROR of ESTIMATE $S(\hat{Y}) = 4{,}420$ PERSON DESTINATIONS

COEFFICIENT of MULTIPLE DETERMINATION $R^2 = 0\ 920$

	X_1	X_2	X_3	K
95 % CONFIDENCE RANGE	0 392	0 413	−3905	+14,690
of EQUATION PARAMETERS	32 774	7 505	29 209	−33,670

Tests of Interactance Formulas Derived from O-D Data

F. HOUSTON WYNN, Principal Associate, Wilbur Smith and Associates, New Haven, Conn.; and C. ERIC LINDER, Chief, Statistical Section, Wilbur Smith and Associates, Columbia, S.C.

Several methods of forecasting future travel patterns in large urban areas are currently in wide use. All of these involve many complexities and require the use of modern high-speed data-processing equipment. The problems of trip projection involve two distinct areas of analysis: (1) estimates of the number of trips which the land uses in each subdivision or zone in the study area are likely to generate, and (2) the patterns of inter-zonal origins and destinations which are likely to result when trips are distributed between logical termini.

In practice, it is possible to make reasonable and satisfactorily accurate estimates of the number of trip ends that will be generated in each zone from consideration of various land use and population factors. The rates of trip production can be determined from analyses of the current origin-destination (O-D) data, or they can be estimated with a fair degree of accuracy from relationships found in other metropolitan areas with similar characteristics.

The projection of inter-zone travel patterns is a much more complex matter. Basically, all trip distribution methods may be placed in two classes, depending on the manner in which the current travel pattern is used in estimating future desires:

1. "Analogy" Methods—projections in which growth factors are applied to current interzonal movements.
2. "Synthetic" Methods—projections in which travel characteristics are derived from current O-D data, and applied to future land-use estimates to synthesize travel patterns.

●MANY THINGS must be considered when selection of a projection method is made. When current, accurate, home-interview O-D data are available, the relative ease of application would tend to recommend the "analogy" approach to traffic forecasting, especially if projections consist of simply up-dating a survey made a few years earlier. However, in most cases, highway planners need traffic forecasts of travel patterns as they will be at least 20 yr later. In this interim, it is expected that the limits of urbanization in most metropolitan areas will spread out to include areas which today are only partially developed, are open farm land or are totally undeveloped. Future traffic in these peripheral areas, which will be devoted to housing and industrial uses 20 yr hence, will be a wholly new development which cannot be anticipated by the application of growth factors to existing patterns.

For example, travel to and from a peripheral zone which today encompasses only sparse residential development may be oriented largely toward the central portion of the urban area; in any case, the number of movements reported in the current data will be few. If considerable housing is anticipated in the zone, and commercial and industrial development is expected in neighboring peripheral areas, it would not be logical to expect that future travel patterns would continue to be centrally oriented; new travel desires will develop between these outlying areas. Application of "growth factors" to current movements will only result in enormous expansion of the few reported interzonal interchanges, with a consequent failure to develop any traffic to the nearby shopping and employment opportunities.

Comparison of urban travel patterns at different years shows that travel desires are dynamic and respond readily to changes in social and economic conditions. For example, analogy with past conditions would fail to reveal the dramatic change in the peak-hour directional split that reportedly has occurred on the George Washington Bridge in recent years. Whereas in the past, peak-hour traffic was composed largely of middle- and upper-income New Jersey residents commuting to their places of employment in New York, recent industrial developments on the New Jersey side of the river have attracted employees from lower-income groups in the public and other low-rent housing developments in New York. Consequently, peak-hour traffic today on the bridge is almost equal in both directions. Other studies have revealed significant changes in travel habits due to the advent of television and widespread evening shopping (1).

Because of this inherent weakness in the "analogy" concept of traffic projection, considerable research has been done with the aim of establishing finite relationships which could be applied to a given set of land-use characteristics, so as to produce a synthetic travel pattern for any metropolitan area. Although a comparison of the validity of all projection methods would be desirable at this time, the scope of this paper is limited to a discussion of current travel patterns for three metropolitan areas which were developed by use of "interactance formulas" derived from existing O-D data. Inasmuch as the interactance formulas may be used in developing travel patterns for any series of zones and for any given year for which planning data are available, it has been possible to synthesize current travel patterns which are directly comparable with the home-interview data.

In these studies, the interactance formulas were applied to the actual numbers of trip ends reported in O-D surveys completed in three cities since 1957 (2, 3, 4). Synthetic travel patterns were developed for each area, and compared with the results of the expanded home-interview data. Various statistical tests have been applied to evaluate the reliability of the results. The procedures used in the study and the principal findings of the tests are summarized. The comparisons of the synthetic and reported travel patterns reveal that the interactance formulas produce results comparable in reliability to those obtained directly from the expanded home-interview data.

ORIGIN-DESTINATION SURVEYS

Comprehensive O-D surveys have been completed during the past two years by this consultant in St. Louis, Mo., Kansas City, Kan.-Mo., and Charlotte, N.C. These cities present a wide range in population, area, density and travel habits, and have been selected to make the detailed analysis or test of the interactance formulas. Time and limited resources have precluded extensive tests of all projections heretofore. It is hoped that others will undertake further research in this area.

Characteristics of the survey areas are presented in Table 1. The populations range from slightly more than 202,000 in Charlotte, to 1,275,000 in St. Louis. Figure 1 shows the survey area, including the zones and analysis "rings" used in the St. Louis and Charlotte studies. Charlotte had 84 internal zones and Kansas City and St. Louis had 173 and 235 zones, respectively.

A comprehensive home-interview-type O-D study was conducted in each area. Home-interview data were obtained from 5 to 10 percent of the dwellings in each study area and expanded to represent the entire trip population. Consequently, individual zone-

TABLE 1

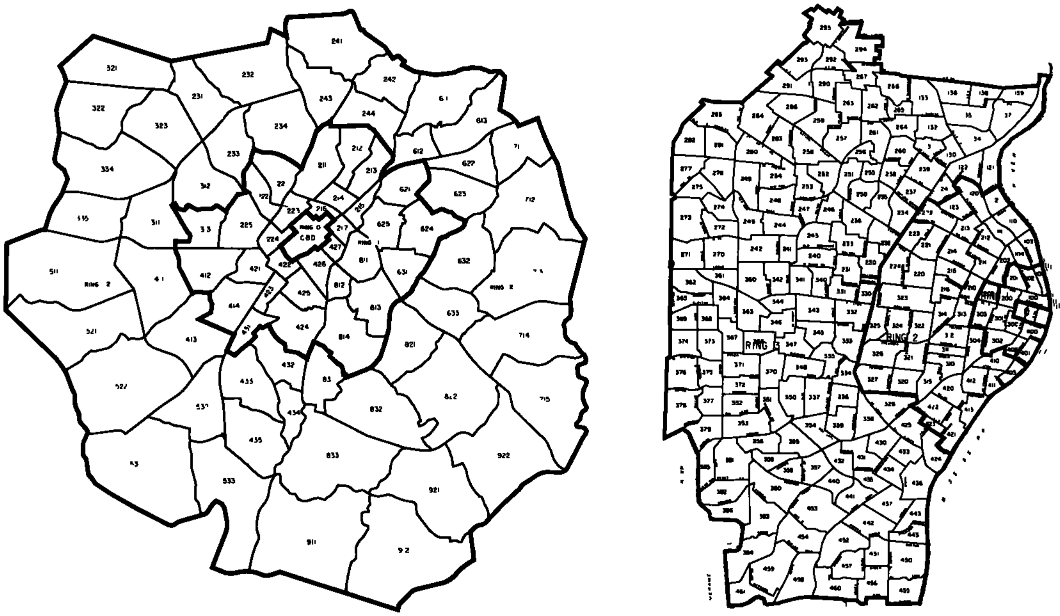
CHARACTERISTICS OF SURVEY AREAS

Characteristics	St. Louis, Mo.[1] (1957)	Kansas City, Mo. (1957)	Charlotte, N.C. (1958)
Population[2]	1,275,454	857,550	202,261
Dwelling units[2]	408,596	280,107	59,040
Sample size (% dwelling units)	5	5	10
Survey area (sq mi)	280	396	115
Persons per sq mi	4,550	2,160	1,760
Trips reported[3]	2,472,154	1,894,563	478,402
Average number of trips per person	1.94	2.22	2.36
Cars owned[2]	366,963	264,448	61,802
Average number of cars per dwelling unit	0.90	0.95	1.05
Number of zones	235	173	84

[1] The St. Louis study area consisted of the City of St. Louis and adjacent urbanized portions of St. Louis County, Missouri. The study did not include approximately 450,000 persons residing in Illinois portions of the Greater St. Louis area.

[2] Source: Home-interview data.

[3] Source: Table "B-1." Totals include all person trips, internal and external, reported in the dwelling unit interviews.



CHARLOTTE

ST. LOUIS

Figure 1. Traffic analysis rings—Charlotte, N.C. and St. Louis, Mo.

to-zone movements, most of which are small in magnitude, are subject to wide fluctuation due to sample size. However, the discrepancies caused by sample variability are minimized when a large directional movement is considered.

To determine the completeness of the data obtained in the home and external interviews, a "screenline check" was made. The results of the screenline checks for St. Louis, Kansas City and Charlotte are summarized in Table 2 with graphic comparisons given in Figure 2. The percent of reported trips, as compared to ground count, was 78.8 percent in Charlotte, which is slightly less than the usual minimum requirement. After considerable study it was ascertained that the screenline was subject to extensive "double crossing." Percentages of screenline crossings were 84.4 and 87.2 percent for St. Louis and Kansas City, respectively.

Another validity check consists of a comparison of the number of external cordon crossings made by residents of the area, as obtained in the home and roadside interviews. Because of the larger sample size of the latter, the expanded roadside interview

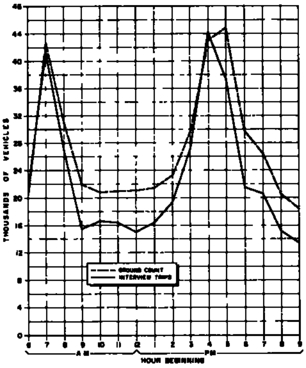
TABLE 2

TESTS OF RELIABILITY—SELECTED STUDIES

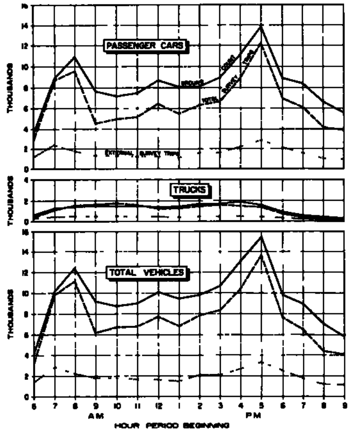
Study	Percent of Ground Count Screenline—16 Hrs (All vehicles)	Percent of External Trips Reported in Internal Survey (All vehicles)
St. Louis, Mo., 1957	84.4	76.9
Kansas City, Mo., 1957	87.2	[1]
Charlotte, N.C., 1958	78.8[2]	94.4

[1] Not reported, due to different methods of data collection in the Missouri and Kansas portions of survey area.

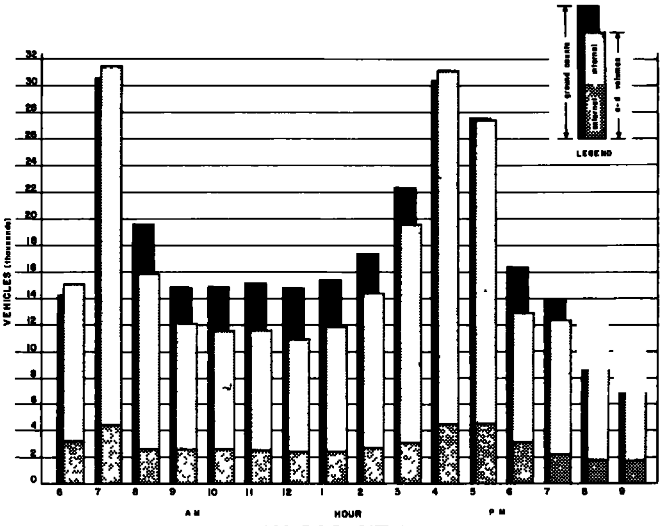
[2] It is estimated that a minimum of 11,882 cars and trucks crossed the screenline twice. Deducting these vehicles from the ground counts produces an over-all screenline comparison of about 84 percent. In addition, it is believed that a large number of vehicles registered outside the survey area made additional trips completely within the survey limits, although these were not reported in the survey data.



ST. LOUIS
PASSENGER CARS
AND TRUCKS



CHARLOTTE
PASSENGER CARS
AND TRUCKS



KANSAS CITY
PASSENGER CARS
AND TRUCKS

Figure 2. Comparison of traffic crossing screenlines—St. Louis, Mo.; Kansas City, Mo.; and Charlotte, N.C.

data are generally accepted as having greater reliability. The results of this comparison for the studies under consideration are given in Table 2. It is an accepted fact that in most home-interview O-D studies, this comparison reveals even greater variability in the data than is expected in the screenline check.

Analyses of O-D Data

It has been found in these and other O-D studies that between 80 and 90 percent of all person trips reported in the dwelling-unit interviews either begin or end at home. Trips have, therefore, been reclassified into five basic purposes, depending on purpose at the non-home terminus. Trips between home and work were classified as "work" trips. "Commercial" trips were defined as those between home and "personal business," "medical-dental," "shopping," and "eat meal" purposes. "Social" and "school" trips were defined as trips between home and each of these purposes, respectively. Trips between work and commercial generators, with neither terminus at home, were classified as "miscellaneous" trips.

Several advantages are gained by analyzing and projecting person trips separately for each of these basic purposes. It is obvious that trip production for each purpose will be related to different land-use variables, thereby greatly systematizing the procedure of estimating trip ends.

Of even greater significance, however, is the distinct difference in distribution patterns for each of the various purpose categories. The importance of this finding cannot be overemphasized in highway planning work. In Table 3, cumulative distribution patterns are compared for four of the basic purposes. Although the results vary considerably among the three metropolitan areas studied, due mainly to differences in maximum driving radius, it is clear in each case that work trips are longer than trips for the other purposes. In Kansas City, for example, although almost one-half of the total trips for all purposes were less than 9 min in length, only about one-fourth of the auto driver work trips were in this time category. The same phenomenon is observed in the data for St. Louis and Charlotte.

TABLE 3

COMPARISON OF TRIP LENGTHS BY PURPOSE—SELECTED STUDIES

				Cumulative Percent of Trips by Residents, Internal Survey											
Trip Length	Work Trips			Commercial Trips			Social Trips			School Trips			Total Trips		
(min)	St. Louis[2]	Kansas City[2]	Charlotte[2]	St. Louis	Kansas City	Charlotte	St. Louis	Kansas City	Charlotte	St. Louis	Kansas City	Charlotte	St. Louis	Kansas City	Charlotte
0	3.6	2.2	5.8	15.2	10.8	18.0	11.5	14.3	15.1	21.7	30.5	23.4	10.1	9.8	12.9
0 - 3	15.3	13.9	11.5	45.4	35.1	27.4	34.8	39.4	22.5	59.2	51.7	35.8	31.7	29.9	21.5
3 - 6	22.6	17.2	25.8	55.9	43.7	48.1	45.6	46.8	40.6	68.3	52.5	57.9	40.8	36.8	40.0
6 - 9	35.0	26.4	46.5	68.6	56.7	68.0	58.7	55.8	60.3	76.8	65.2	78.2	53.2	47.9	60.2
9 - 12	47.6	40.6	66.4	78.2	68.0	82.0	67.9	65.0	75.2	83.4	70.6	87.3	63.7	59.5	76.4
12 - 15	59.3	54.9	80.5	84.8	77.3	90.3	76.7	75.8	85.2	87.9	75.5	93.1	72.7	70.4	86.6
15 - 18	69.1	66.8	90.0	89.4	84.3	96.2	84.0	82.4	92.5	91.9	78.5	96.5	80.0	78.8	93.5
18 - 21	77.1	76.5	95.7	92.8	90.1	98.8	88.8	87.5	96.7	94.4	85.7	99.3	85.5	85.4	97.3
21 - 24	84.3	83.2	98.5	95.2	94.0	99.5	92.6	91.8	98.6	96.7	88.0	99.9	90.1	89.9	99.0
24 - 27	89.5	88.7	99.5	97.2	95.8	100.0	94.7	94.3	99.6	97.8	97.6	100.0	93.3	93.4	99.7
27 - 30	93.7	93.4	99.9	98.6	97.9	100.0	96.6	96.3	99.9	98.4	99.9	100.0	95.9	96.1	100.0
Over 30	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Avg Length (min)	14.4	14.8	11.0	7.6	9.1	6.9	9.7	9.3	8.1	6.0	8.0	5.7	10.7	10.8	8.1

[1] Intra-zone trips.

[2] St. Louis and Kansas City data are for all modes; Charlotte, auto drivers only.

A series of special tabulations was prepared for analysis of the trip data. These included tabulations showing the number of trips for each purpose by zone residents, the number of "purpose" motivated trips made to and from each zone, and the distribution of each class of trip to other zones by successive 2- or 3-min time increments.

In the studies of generation of trips by zone residents, it was found that median family income, vehicle ownership, population density, and relative decentralization were the most significant factors. Although considerable variability was observed between populations of different characteristics in certain categories of trips (such as social and school trips), consideration of the four variables in conjunction with each other tends to give a balanced picture of the factors influencing trip production by residents. Similar studies of trip production at non-home termini revealed even greater variations between zones, but it was possible to develop general patterns relating trip production to land-use characteristics for zones in each of the "rings" into which the study areas had been subdivided.

The analysis of trip production rates tends to become complex, but once the basic relationships have been established, estimation of the number of trip ends in each zone at a given future year, by application of these relationships to anticipated land-use development, is a fairly straightforward process.

A means of estimating the number of trip ends in each zone must be developed to distribute trips between the respective home and non-home termini and thus formulate a future travel pattern. Obviously, this procedure involves many complexities, because trips originating in any one zone of the study area could conceivably have destinations in all of the other zones into which the area has been subdivided. The problem resolves itself into determining exactly what proportion of the trips (for each purpose) originating in Zone "A" shall have destinations in each of the other zones. Although this distribution could be accomplished by analogy with trips reported in the current O-D study, several basic factors discount the validity of this procedure:

1. The survey data represent only 5 to 10 percent samples, and many of the smaller inter-zonal movements were completely unreported.
2. Zones greatly increasing or decreasing in relative importance as trip generators will attract different proportions of trips from other zones than they do today.

Previous studies have demonstrated a definite inverse relationship between the number of trips a pair of zones will exchange and the driving time between the zones (1, 5, 6, 7, 8). Additional research in conjunction with the St. Louis, Kansas City, and Charlotte O-D studies has confirmed these findings.

Off-peak auto driving times were computed for all possible pairs of zones. Trips reported in the home interviews were summarized separately for home and non-home termini in each zone, including a further breakdown showing the number of trips for each purpose made to all zones within successive 2- or 3-min time increments.

The driving time between a pair of zones must be coupled with the trip generation potential of each zone and of all other zones in the study area to develop reasonable estimates of the rate of travel between the zones. For example, to estimate the number of shopping trips that would be made by the residents of Zone "A" to the retail outlets in Zone "B", consideration must be given to the total number of home-based shopping trips made by the residents of Zone "A"; the total number of trips made for shopping purposes to Zone "B" by residents of all zones; and the total number of shopping trips made to all other zones, as well as the travel time between Zones "A" and "B".

Derivation of Interactance Formulas

The trip length studies were used in conjunction with the available population statistics to develop a series of curves relating the average off-peak driving time and the relative number of trip attractions to travel desires for each purpose between zones at successive increments of distance. Because trips tended to become shorter towards the periphery of each of the areas studied, separate analyses were made for each of several concentric "rings," based on relative decentralization (Fig. 1).

Although the slopes of the curves for each purpose differed considerably, they all exhibited the inverse relationship between trip production and travel time, to varying degrees, as illustrated in the curves for St. Louis (Figs. 3 and 4), Kansas City (Figs. 5 and 6), and Charlotte (Fig. 7).

Comparison of the curves for different purposes developed in each study revealed that attraction rates for work trips had the slowest rate of decrease as driving time

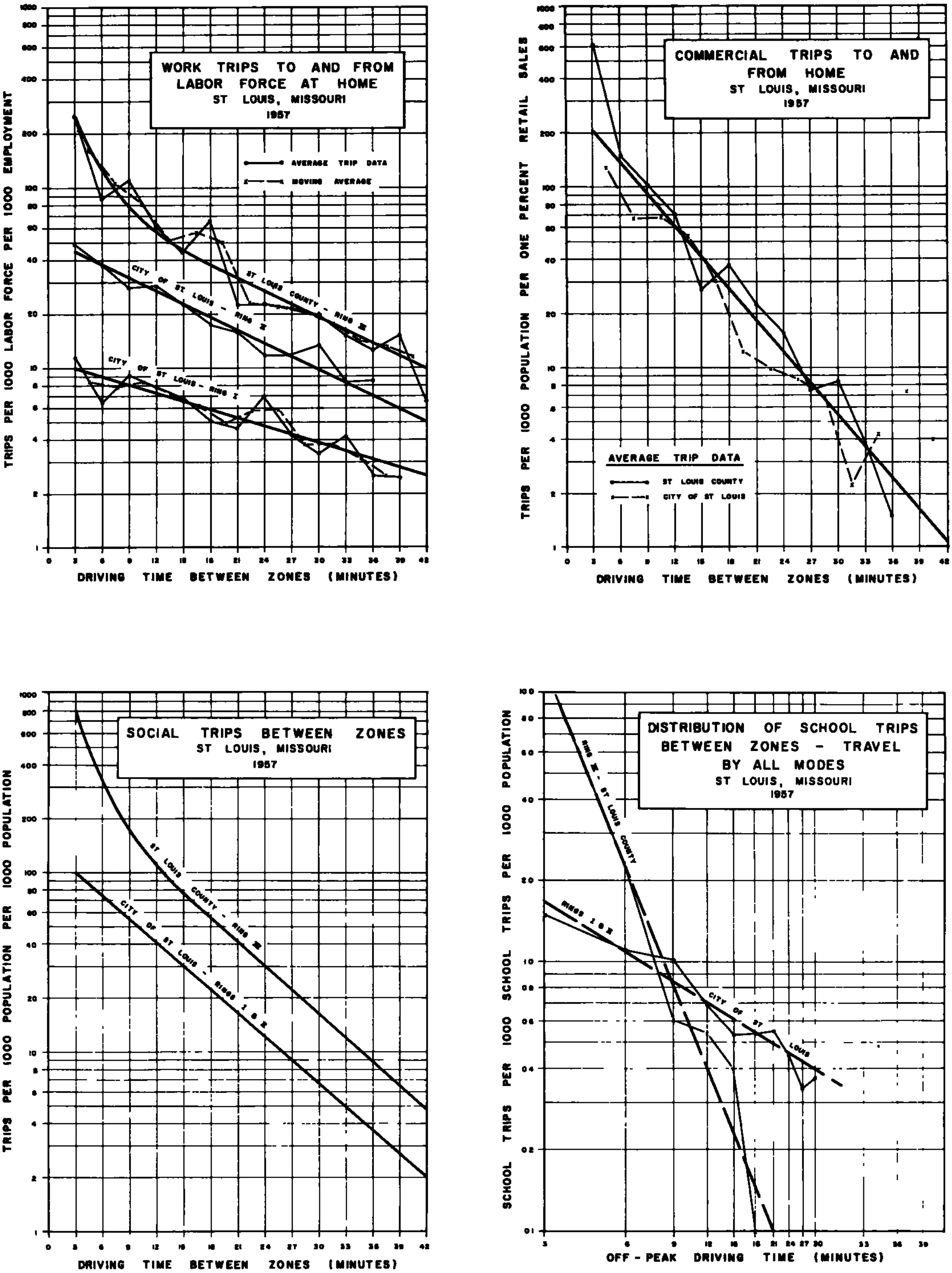


Figure 3. Inter-zonal distribution curves— St. Louis, Mo.

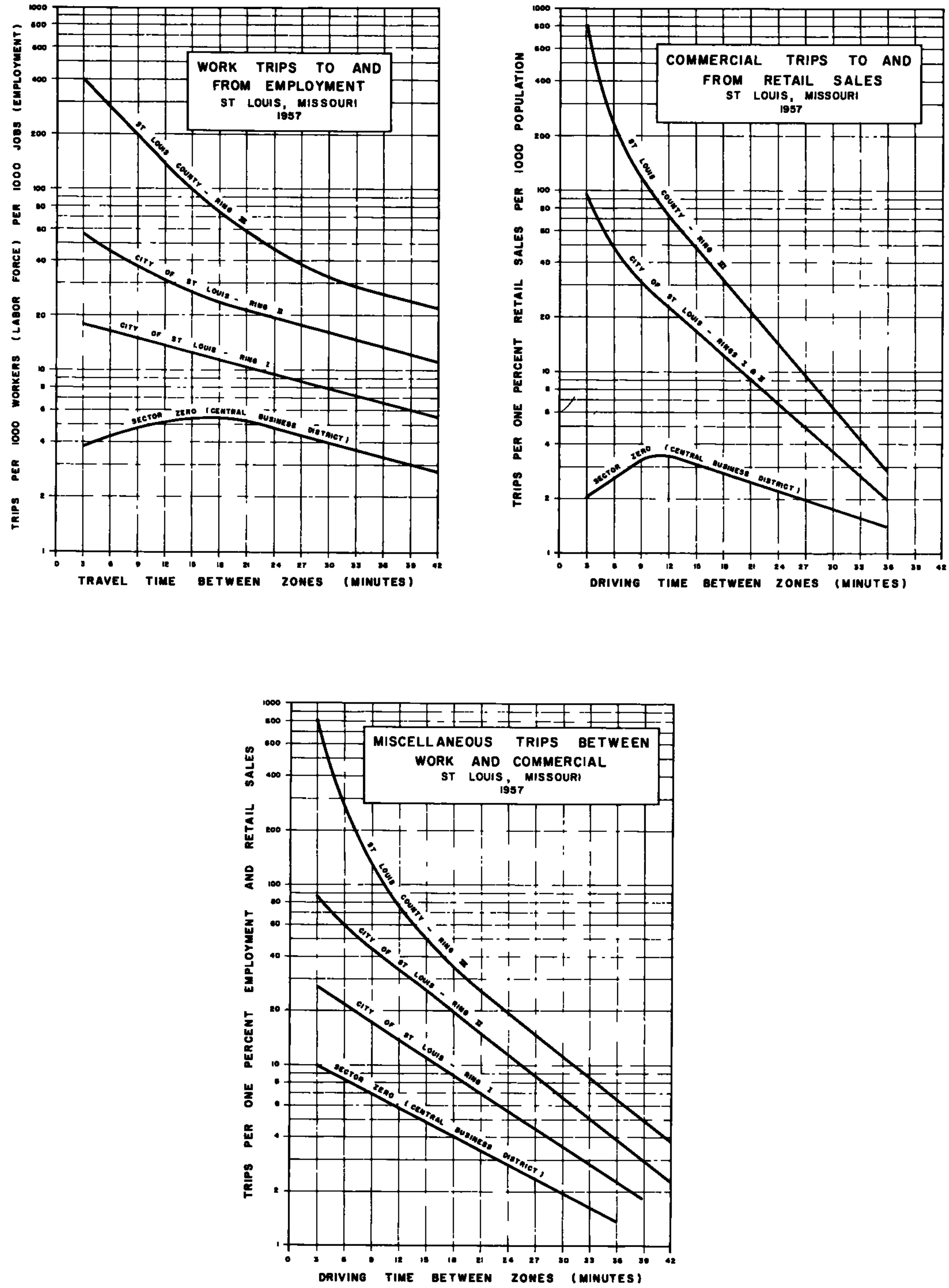
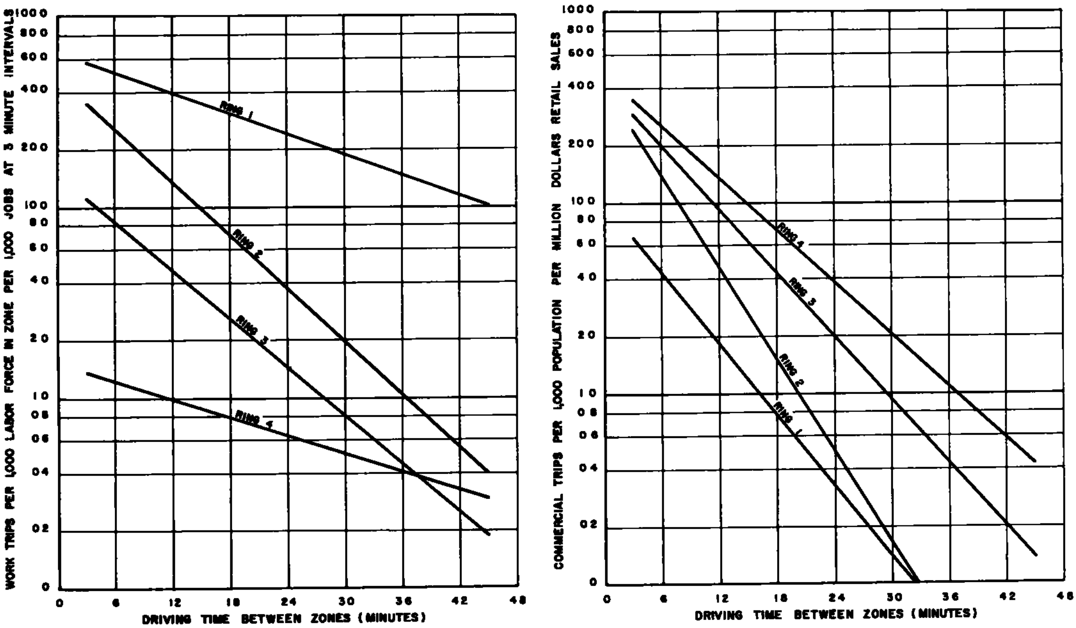
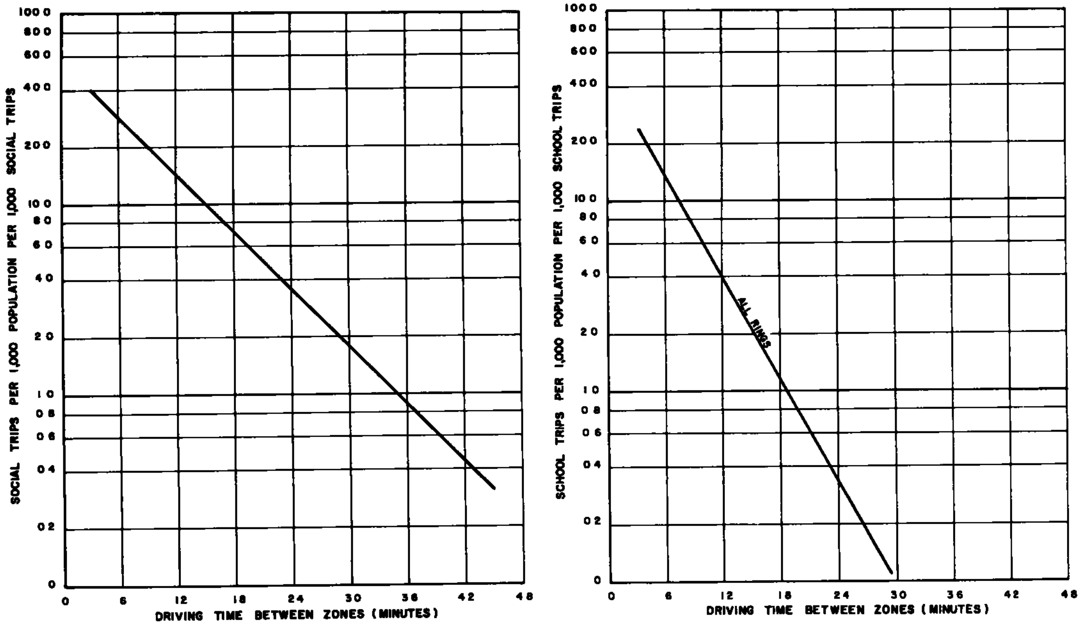


Figure 4. Inter-zonal distribution curves— St. Louis, Mo.



DISTRIBUTION OF WORK TRIPS BETWEEN ZONES
(TRIPS TO AND FROM HOME-ALL MODES)

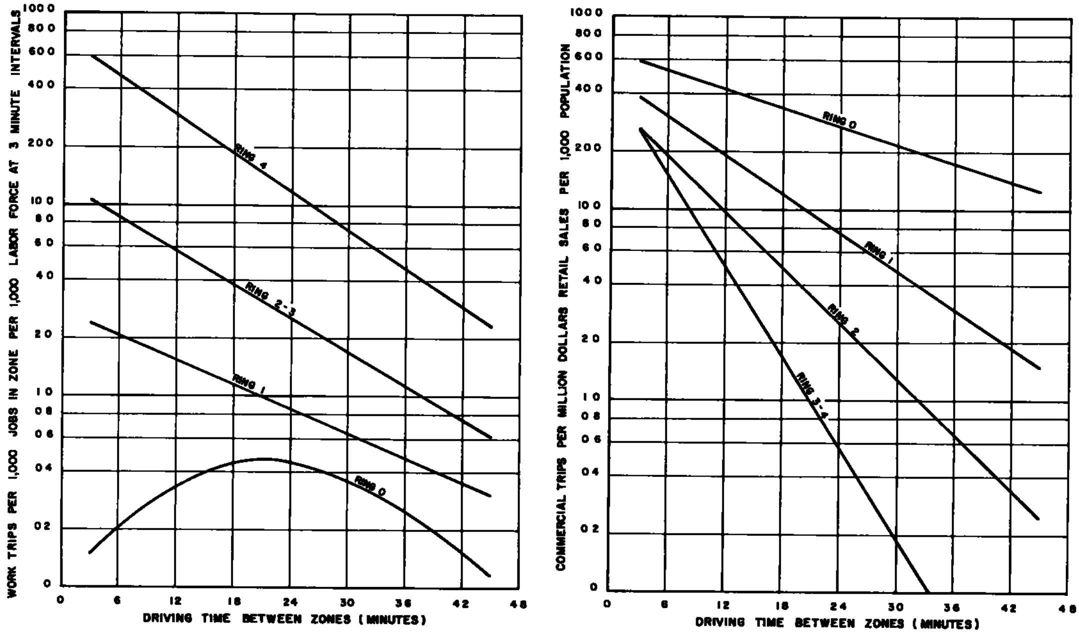
DISTRIBUTION OF COMMERCIAL TRIPS BETWEEN ZONES
(TRIPS TO AND FROM HOME-ALL MODES)



DISTRIBUTION OF SOCIAL TRIPS BETWEEN ZONES
(TRIPS TO AND FROM HOME-ALL MODES-ALL RINGS)

DISTRIBUTION OF SCHOOL TRIPS BETWEEN ZONES
(TRIPS TO AND FROM HOME-ALL MODES)

Figure 5. Inter-zonal distribution curves—Kansas City, Mo.

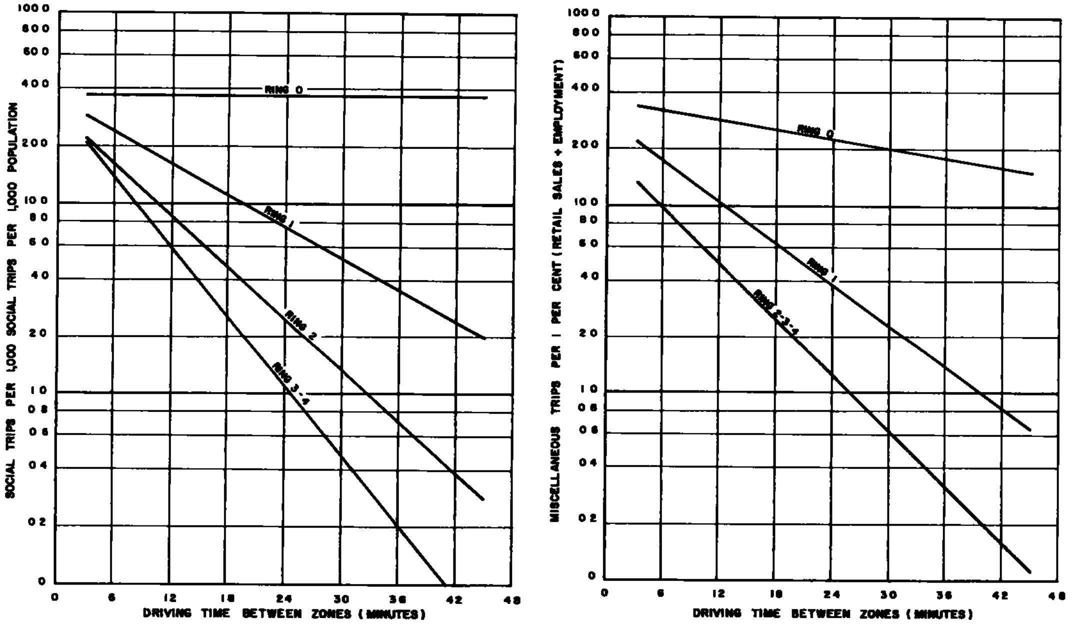


DISTRIBUTION OF WORK TRIPS BETWEEN ZONES

(TRIPS TO AND FROM EMPLOYMENT-ALL MODES)

DISTRIBUTION OF COMMERCIAL TRIPS BETWEEN ZONES

(TRIPS TO AND FROM COMMERCIAL PURPOSES-ALL MODES)



DISTRIBUTION OF SOCIAL TRIPS BETWEEN ZONES

(TRIPS TO AND FROM SOCIAL PURPOSES-ALL MODES)

MISCELLANEOUS TRIPS BY INTERNAL RESIDENTS-ALL MODES

(TRIP RATE VS AUTO DRIVING TIME)

Figure 6. Inter-zonal distribution curves—Kansas City, Mo.

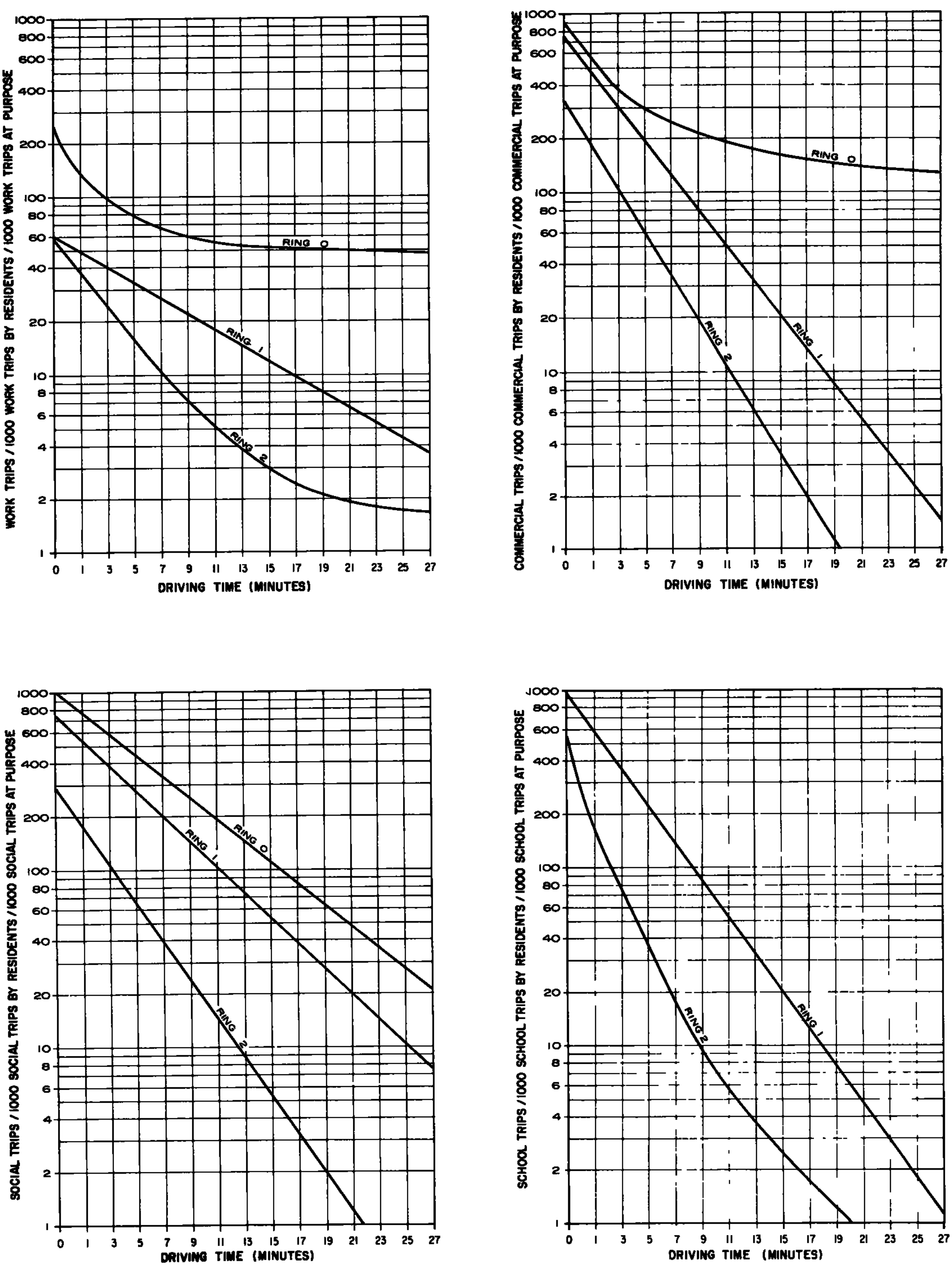


Figure 7. Inter-zonal distribution curves— Charlotte, N.C.

increases. The distributions of social trips also reflected a willingness on the part of area residents to make somewhat longer trips for this purpose. Trip attraction rates for commercial and miscellaneous trips decreased rapidly with increased travel time between zones, whereas the curves for school trips, most of which are local in nature, show the most rapid rate of decay.

Synthesis of O-D Patterns

Relative "rates of attraction" between zones, as derived from the interactance formulas, are applied to estimated trip end totals in each zone to distribute the trips and produce a synthetic travel pattern. The same procedure is used in the synthesis of either current or future travel desires, the only difference being that the number of trip ends reported in the survey data is used in the first case, whereas future trip end estimates based on anticipated land-use development are used in the latter.

The computations involved in applying interactance formulas to distribute trip end estimates have been accomplished with an IBM 650 computer. Two input cards were prepared for each possible pair of zones, with each zone considered separately as the trip's origin. Distribution rates from the interactance formulas were applied on the basis of the appropriate curve for the origin zone in each case. (Therefore, in cases of travel between zones in different "rings," slightly varying estimates of trips were produced, if each of the "rings" exhibited distinctive distribution patterns.)

In making the first approximation of trips between zones, the attraction rate derived from the interactance curve was multiplied by the corresponding "attraction" in the destination zone, and the resulting products were totaled for each origin zone and balanced to the predetermined "control total." After the initial estimates of trips for each purpose had been made for each zone, these estimates were summated and divided into the respective control totals to establish balancing factors. The initial estimates were then multiplied by the balancing factors and punched out as the first approximation of inter-zonal travel for each purpose.

This procedure is repeated with each zone in the study area considered as origin, resulting in two separate estimates of travel for each purpose between each pair of zones. Averaging the values obtained in the first approximation does not necessarily yield values which will summate to the control total for the individual zones. It may be necessary to carry the averaging process through two or more cycles (iterations) to produce a well-balanced estimate of travel in which all of the trips attributed to each zone are approximately equal to the control total (9). It has been found in the application of the interactance formulas that one or two iterations will usually suffice to produce well-balanced inter-zonal trip distribution.

Variability of Home-Interview O-D Data

Before a meaningful analysis can be made of the reliability of synthetic travel patterns, the variabilities inherent in the original expanded home-interview data must be considered. Due to limited sample size, movements reported in the survey data are subject to variations of different magnitudes from the true values that would be obtained from a 100 percent sample. An evaluation of the variations expected in the survey data is needed to give the proper perspective to the differences observed between the survey data and the projected data.

The travel patterns developed from the home-interview data and by the application of interactance formulas both consisted of very large numbers of relatively small inter-zonal movements. The smallest movement possible in the expanded home-interview data was limited by sample size, although inter-zonal movements as small as one or two trips per day undoubtedly do occur between many zone-pairs. A volume of this magnitude has no significance when considered separately, although it has been demonstrated that traffic flows consist largely of aggregations of such movements.

It is obvious that the magnitudes of individual movements, as reported in the expanded survey data, are subject to wide fluctuations due to sample variability. Inter-zonal volumes of 300 trips or less, which comprise 50 percent or more of the total traffic in the large urban areas considered in this study, are particularly susceptible to wide fluctuations.

The variability exhibited between different 5 percent samples has been demonstrated with data from a home-interview study made in Milwaukee in 1945 (10). A 50 percent sample of the dwelling units, consisting of 10 separate 5 percent samples, was taken in the census tract selected for study. The number of trip ends reported in the tract varied from about 3, 900 to 5, 300 among the various samples. Each of these extreme variations represents about 20 percent of the average value of 4, 600 trip ends, which was obtained by expanding the 50 percent sample. The percentage of deviation between the two extreme values of expanded samples was even larger (about 36 percent). Although these actually observed variations appear larger than the expected variations discussed in theoretical treatments of sampling, Lynch concludes from the Milwaukee test results that "a five percent sample is in keeping with the over-all requirements as to cost and accuracy" for use in highway planning work.

The Milwaukee study was concerned with the sample variability of a relatively large number of trips. Even more pronounced deviations are expected when smaller volumes are considered. A discussion of the degree of reliability which could be placed in data secured by interviewing different sample sizes is given by Anderson (11). The findings of this investigation are partially summarized in Table 4, which gives the standard error for different trip magnitudes obtained from expanded 5 and 10 percent samples.

The assumptions made in this study were that the sample under consideration was complete; was selected by an approved random technique; and was derived from a normally distributed population. However, none of these conditions is fully met in the home-interview sampling technique. It is known from the screenline deficiencies that every one of the home-interview studies is incompletely reported to some degree; a stratified ordinal method of sample selection has been used in preference to random sampling techniques; normal distribution is doubtful if the sample is small. Therefore, although Table 4 indicates the relative reliability of 5 and 10 percent samples, the values obtained should be considered as favorable evaluations of the data.

It is evident from this study that individual inter-zonal movements of small magnitude, which constitute the bulk of urban traffic, are subject to wide sampling variations, ranging as high as 90 percent of the true value in the case of an expanded movement of 20 trips derived from a 5 percent sample. Somewhat greater confidence may be placed in the values for small movements derived from a 10 percent sample. The degree of error in movements of 1, 000 trips or more is similar for both sample sizes given, because expected deviations decrease rapidly in the larger trip volume groups.

TABLE 4

EXPECTED ERROR FROM 5 AND 10 PERCENT RANDOM SAMPLES AND VARIOUS VOLUMES OF INTERCHANGE[1]

Volume of Interchange From Expanded Sample	Standard Error of Estimate (Percentage)	
	5% Sample	10% Sample
20	90	70
40	68	48
60	55	40
80	48	35
100	43	30
200	30	23
300	25	18
400	23	16
500	20	15
600	18	14
700	17	13
800	16	12
900	16	11
1, 000	16	10
2, 000	10	8
3, 000	8	6
4, 000	7	5
5, 000	7	5
6, 000	6	4
7, 000	6	4
8, 000	5	4
9, 000	5	3
10, 000	5	3

[1] Source: Anderson, O. K., "Statistical Evaluation of Origin-Destination Data." Unpublished Thesis, Bureau of Highway Traffic, Yale University, May 1951, pp. 33-37.

Although the variability associated with sampling has little significance to the highway planner, who is concerned with the relatively stable large volumes of trips assigned to proposed facilities, the results of the statistical tests described later are biased to the extent that the synthetic movements are being compared with survey data which in themselves are subject to varying degrees of error. For example, chances are about two out of three that the true value of a movement of 100 trips as derived from a 5 percent sample lies between 57 and 143 trips; if the latter value should happen to coincide with the true value, and the interactance curves develop an equal volume, an error of 43 trips would nevertheless be recorded between the survey and synthetic data. Because movements developed from only a few samples are not likely to show a normal distribution, it is doubtful that the distortion in the comparisons will average out even when many small movements are considered.

RELIABILITY OF INTERACTANCE FORMULAS

The application of interactance curves to predict reasonable patterns of inter-zonal travel is a relatively new technique which has been developed by empirical methods. The procedures discussed herein are the outgrowths of direct investigations.

Interactance curves have been derived and used for trip distribution in a number of studies during the past few years. (First use of the interactance formulas by this firm was for a 25-yr projection of travel in Philadelphia, Pa., made in 1955. The technique has since been used in Omaha, Neb. (1956); Washington, D.C. (1956-58); Miami, Tampa and Tallahassee, Fla., (1957-58); St. Louis, Mo. and Kansas City, Mo., (1957-58); Charlotte, N.C. (1959); Pheonix, Ariz., (1959); and Philadelphia, Pa. (revised, 1959).) Techniques of the method have been improved with each new experience.

In the course of the trip projection work undertaken in St. Louis, Kansas City, and other recent studies, an effort has been made to develop some measure of the reliability of the techniques described previously. Due to limitations in both budget and time, these tests have not been as comprehensive as possible for detailed research, but the insight they provide is most encouraging. The results of each of the tests described herein have been found to be of direct value in improving the trip distribution technique.

To test the reliability of the interactance formulas for the purpose of predicting inter-zone trip distribution, trip attraction rates determined from the curves given in Figures 3 to 7 have been applied to the 1957 inter-zone trip ends generated by residents of each study area. The numbers of inter-zone trip ends generated in each zone, as reported in the home-interview survey, have been distributed to all other zones in the study area. Two independent estimates of travel were thus determined for each purpose of trips between every pair of zones. The two estimates were averaged by the method of successive approximations. Average vehicle occupancy rates for each trip purpose were applied to develop the number of auto drivers and auto passengers in each interzonal movement. Trip patterns were developed for combined travel by each mode.

In this manner a synthetic travel pattern was developed between all internal zones in each survey area. The projected movements are directly comparable with the expanded home-interview data. However, the complexity of the comparison is emphasized by the fact that in the St. Louis study area of 235 zones, there are 27,495 possible movements to be considered, exclusive of intra-zone movements.

Comparison of Inter-Zonal Movements

The comparative analysis of expanded interview data with synthesized trip distribution patterns can be accomplished only by statistical methods. Direct comparison of the two sets of data reveals that the synthetic estimates of travel between most zone pairs differed by varying degrees from the values obtained in the home interviews. To evaluate the magnitude of these differences, the root-mean-square error has been computed for stratified volume classes of 100 trips, up to magnitudes of 1,000 trips, similar to procedures used in other recent studies of trip projection techniques (8, 12).

As a measure of variability, the root-mean-square error indicates the limits within which about two-thirds of the deviations between the observed and estimated values will fall. About 95 percent of all variations lie within twice the root-mean-square error, while almost all the differences between the theoretical and observed values are included by three times the root-mean-square error.

The root-mean-square error for each trip magnitude group was computed by summing the squares of the differences between the survey and synthesized volumes for each pair of zones, dividing the total squared differences by the number of zone pairs in the group, and finding the square root of the quotient. The procedures can be expressed by:

$$R = \sqrt{\frac{\left(T_{AB} - T^{1}_{AB}\right)}{n}} \qquad (1)$$

in which

R = root-mean-square error of volume group

T_{AB} = survey volume between Zones "A" and "B"

T^1_{AB} = synthesized volume between Zones "A" and "B"

n = number of zone-pairs in volume group (survey data)

St. Louis Results

The results of this statistical test for St. Louis, for trips by all modes and by auto drivers, are given in Table 5, with graphic presentation in Figure 8. The number of zone pairs, the root-mean-square error of estimate, and the average percentage of error are given for each volume class. Similar characteristics are observed in the deviations exhibited by auto driver trips and trips by all modes, although the auto driver trips generally showed less variability. The root-mean-square error for all volumes of auto driver trips was 70 trips or about 61 percent of the average inter-zonal auto driver movement of 114 trips; the comparable error for trips by all modes was 120 trips, representing about 65 percent of the average inter-zonal movement of 186 person trips.

The magnitude of the root-mean-square error for each volume class increases as the average size of movement increases. However, the proportional error decreases somewhat in the larger volume classes, up to magnitudes of 1, 000 trips. Although the proportional deviations may seem large at first consideration, they compare quite favorably with values obtained in other recent studies (8, 12). The effect of sample variability and other factors must also be considered to arrive at a realistic evaluation of the technique.

Although considerable variability is observed between the survey data and the theoretical movements when individual zone-to-zone interchanges were considered, a critical evaluation of the results of this test is difficult. Because the St. Louis study area was subdivided into a large number of relatively small zones, the volume of movement between zone pairs tended to be small. The average inter-zonal movement for all modes of travel between zones was only 186 trips. Zone pairs exchanging 1, 000 or fewer trips by all modes accounted for over 75 percent of all travel in the study area. The expanded trip data prepared from the home-interview reports show only 298 zone pairs, or slightly more than 1 percent of the possible movements, with more than 1, 000 trips between them. Trip movements of 2, 000 or greater were reported for only 95 zone pairs, accounting for about 10 percent of all travel within the study area.

Most of these heavy trip movements occurred between adjacent zones. Of the 298 movements which exceeded 1, 000 trips per day, 214 involved pairs of adjacent zones; only 84 movements were between non-adjacent zone pairs. Of the 95 zone pairs which generated travel exceeding 2, 000 trips per day, 87 were adjacent to one another and

TABLE 5

ROOT-MEAN-SQUARE ERRORS OF ESTIMATE, SYNTHETIC TRIP DISTRIBUTION ST. LOUIS, 1957—AUTO DRIVERS AND ALL MODES

Trip Volume	Auto Drivers			All Modes		
	Number of Zone Pairs	Root-Mean-Square Error	Percent	Number of Zone Pairs	Root-Mean-Square Error	Percent
Less than 100	6, 799	50	100	6, 287	66	132
100 - 200	1, 475	94	63	2, 073	112	75
200 - 300	498	127	51	943	147	59
300 - 400	221	167	48	473	191	55
400 - 500	115	187	42	290	215	48
500 - 600	73	239	43	145	236	43
600 - 700	50	330	51	131	277	43
700 - 800	38	342	46	102	307	41
800 - 900	25	445	52	64	328	39
900 - 1, 000	24	412	43	52	420	47
Over 1, 000	79	845		298	863	
All volumes		70			120	

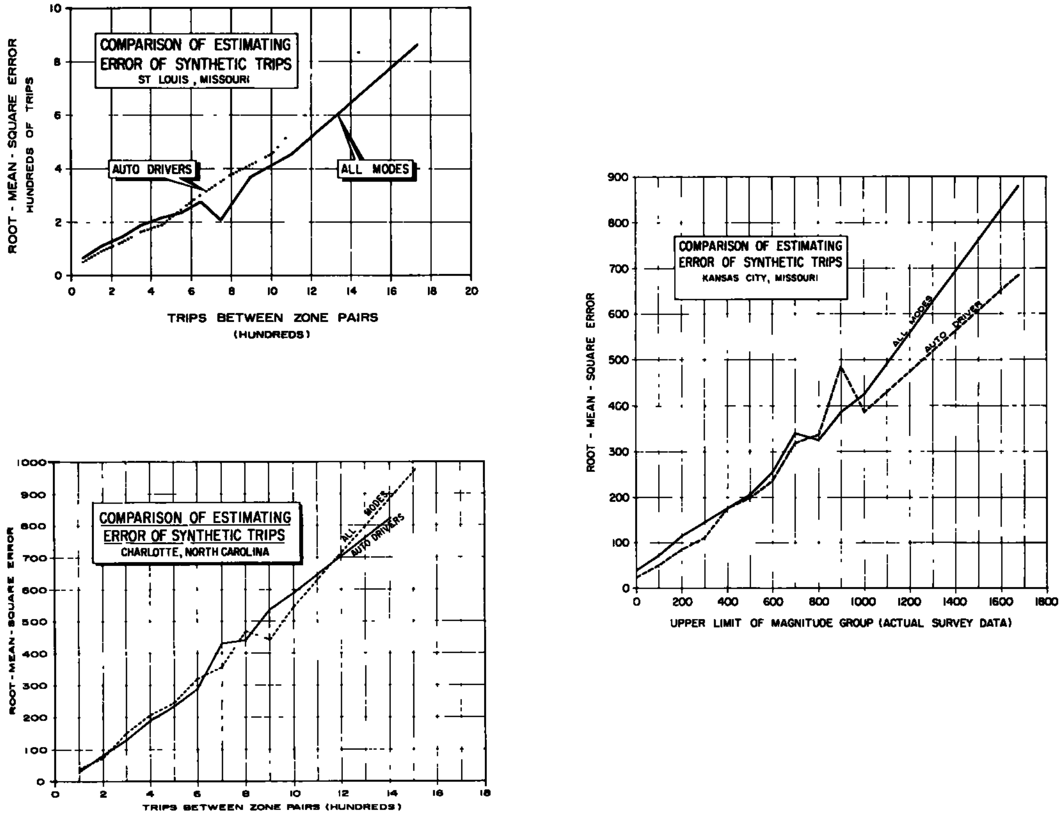


Figure 8. Comparison of estimating error of synthetic trips— St. Louis, Mo.; Kansas City, Mo.; and Charlotte, N.C.

only 8 represented travel to zones beyond the immediately adjacent tier.

There were, of course, even fewer movements of large magnitude for travel by automobile drivers. Only 114 zone pairs generated more than 1,000 auto driver trips per day in the 1957 study. Of these movements, 107 involved adjacent zones and 7 represented travel to zones beyond the adjacent tier.

Comparison of the root-mean-square errors for different volume classes (Table 5), reveals that the average proportional error developed in the synthetic estimates decreases in the larger trip volume groups, up to volumes of 1,000 trips. Although relatively few in number, the larger inter-zonal movements, composed primarily of trips between adjacent zones, showed more variability between survey data and theoretical data than observed in smaller movements.

The interactance formulas are sensitive to the measure of distance. Unless the intensity of land-use development in each zone is uniform throughout, each zone is devoted to only one use, and zones are all of approximately the same diameter, there is bound to be some distortion in the use of travel times related to zone centroids. The zoning in most cities will not meet these criteria unless a vast number of small zones is created. In the studies reported, zoning was designed to produce areas with an average diameter of about 3-min driving time. Exceptions were series of small zones in the Central Business Districts and small numbers of larger, low-density zones in peripheral areas. Large parks, cemeteries, railroad yards, etc., were separately identified in zoning so that reasonably uniform land-use intensities were encompassed by most of the important zones.

The interactance formulas produce their best estimates for trips with lengths

TABLE 10

COMPARISON OF ACTUAL AND THEORETICAL TRIP LENGTH DATA
St. Louis Central Business District—Truck Trip Lengths

Miles	Truck Driver Trips					Vehicle-Miles	
	Survey No. of Trips	Theoretical No. of Trips	Ratio	Survey Cum. % of Total	Theoretical Cum. % of Total	Survey Data	Theoretical Data
0 - 1	10,026	5,535	0.55	41.7	26.3	5,013	2,768
1 - 2	4,021	5,209	1.30	58.4	51.1	6,032	7,814
2 - 3	4,041	4,399	1.09	75.2	72.0	10,103	10,998
3 - 4	2,108	3,156	1.50	84.0	87.0	7,378	11,046
4 - 5	1,390	1,483	1.07	89.8	94.1	6,255	6,674
5 - 6	492	703	1.43	91.8	97.4	2,706	3,867
6 - 7	479	349	0.73	93.8	99.1	3,114	2,269
7 - 8	402	120	0.30	95.5	99.7	3,015	900
8 - 9	283	36	0.13	96.7	99.9	2,406	306
9 - 10	273	16	0.06	97.8	100.0	2,594	152
10 - 11	177	-	-	98.5		1,859	-
11 - 12	85	-	-	98.9		978	-
12 - 13	114	-	-	99.4		1,425	-
13 - 14	77	-	-	99.7		1,040	-
14 - 15	56	-	-	99.9		812	-
15 - 16	28	-	-	100.0		434	-
16 - 17	7	-	-			116	-
Totals	24,059	21,006	0.87			55,280	46,794
Average length						2.3 mi	2.2 mi

synthetic travel pattern of truck trips developed about 85 percent of the total number of truck vehicle-miles reported in the O-D study.

Kansas City Trip Length Comparisons

The results of the trip length study for the Kansas City CBD are shown in Figure 9. The projected results of the auto driver trip distributions were in close agreement with the survey data. Although in the category of short trips a slight deficiency was noted in the projected results (8 percent of the projected trips versus 11.5 percent in the survey data were less than 1 mi long), the two sets of data steadily converge until both reveal that slightly more than one-half of all auto driver trips were less than 3 mi in length. A slight excess of trips over 3 mi is observed in the projected results, but at no point do the cumulative distribution curves of the two sets of data vary more than 4 percent. Almost 90 percent of the auto driver trips in both data were less than 7 mi in length. A similar comparison was made of auto passenger trips, where even closer agreement between the two sets of data was observed; at no point does the variation between the two cumulative distribution graphs exceed 1.5 percent of the total passenger trips.

TABLE 11

COMPARISON OF ACTUAL AND THEORETICAL TRIP LENGTH DATA
Kansas City Central Business District—Auto Driver Trip Lengths

Miles	Auto Driver Trips					Vehicle-Miles	
	Survey No. of Trips	Theoretical No. of Trips	Ratio	Survey Cum. % of Total	Theoretical Cum. % of Total	Survey Data	Theoretical Data
0 - 1	9,674	7,049	0.73	11.54	8.04	4,837	3,525
1 - 2	7,839	7,360	0.94	20.89	16.44	11,759	11,040
2 - 3	11,729	15,221	1.30	34.88	33.81	29,323	38,053
3 - 4	12,941	15,417	1.19	50.32	51.40	45,294	53,960
4 - 5	9,642	11,326	1.17	61.82	64.32	43,389	50,967
5 - 6	4,221	4,085	0.97	66.86	68.98	23,216	22,468
6 - 7	9,450	10,892	1.15	78.13	81.41	61,425	70,798
7 - 8	8,712	6,984	0.80	88.52	89.38	65,340	52,380
8 - 9	2,985	3,474	1.16	92.08	93 34	25,373	29,529
9 - 10	3,667	3,667	1.00	96.46	97.52	34,837	34,837
10 - 11	1,897	1,559	0.82	98.72	99.30	19,919	16,370
11 - 12	985	529	0.54	99.90	99.90	11,328	6,084
12 - 13	72	78	1.08	99.99	99.99	900	975
Totals	83,814	87,641	1.05			376,940	390,986
Average length						4.5 mi	4.5 mi

The comparisons of theoretical and actual trip lengths are summarized for the Kansas City CBD in Table 11. Of even greater significance to the highway engineer than the number of trips is the number of vehicle-miles developed in the study area. It is apparent that the vehicle-miles developed in the projections for each mile increment corresponds closely with the number reported in the survey data, with equally close agreement observed in the cumulative percentage distribution. Although the total number of vehicle-miles developed by to and from the Kansas City CBD by the interactance formulas was about 7 percent more than that derived from the expanded survey data, both distributions revealed an average length of 4.5 mi for auto driver trips. As in St. Louis, comparisons of survey and theoretical trip lengths were made for a limited sample of movements throughout the remainder of the study area. Although the results were not as favorable as those in St. Louis, the study showed that the interactance formulas were producing trip lengths in proportions very similar to those revealed in the survey data.

Charlotte Trip Length Comparisons

The relatively small number of zone pairs in the Charlotte study area permitted a comparative analysis of survey and synthetic trip lengths throughout the entire area. As given in Table 12, the synthetic distribution of trips resulted in fairly close agreement with the proportions of trips in each 1-mi increment as reported in the Charlotte survey data. The cumulative distribution chart of these data (Fig. 9) shows that short trips are slightly overestimated by application of the interactance formulas. Fewer than 10 percent of the total trips exceeded 5 mi in length in both the synthetic and the survey data. The average auto driver trip length of 2.7 mi in the synthesis corresponds with an average of 3.1 mi derived from the survey data. The total number of vehicle-miles developed in the synthesis is 642,022 or about 90 percent of the total of 719,778 shown in the survey data.

Evaluation of Trip Length Studies

The trip length studies indicate that trips of various lengths are being projected in approximately correct proportions by use of interactance formulas. Because distance is one of the most significant factors in determining assignable proportions of traffic, this fact is of importance in making an evaluation of the synthetic travel patterns. The proportion of assignable trips increases constantly between zone pairs benefiting from the expressway system as the driving time between the zones increases. Consequently, a large proportion of the longer trips will be diverted to expressways where this service is provided; in the case of expressway systems as comprehensive as those proposed for St. Louis and Kansas City, the great majority of the longer inter-zonal movements will benefit from the system.

TABLE 12

COMPARISON OF ACTUAL AND THEORETICAL TRIP LENGTH DATA
Charlotte Metropolitan Area—Auto Driver Trip Lengths

Miles	Auto Driver Trips					Vehicle-Miles	
	Survey Trips	Theoretical Trips	Ratio	Survey Cum. % of Total	Theoretical Cum. % of Total	Survey Data	Theoretical Data
0 - 1	22,946	27,960	1.22	9.9	11.9	11,473	13,980
1 - 2	49,463	60,088	1.22	31.1	37.5	74,195	90,132
2 - 3	53,908	59,134	1.10	54.3	62.7	134,770	147,835
3 - 4	43,259	41,968	0.97	72.9	80.6	151,407	146,888
4 - 5	27,305	22,925	0.84	84.6	90.4	122,873	103,163
5 - 6	18,440	12,888	0.70	92.6	95.9	101,420	70,884
6 - 7	10,015	5,800	0.58	96.9	98.3	65,098	37,700
7 - 8	4,511	2,533	0.56	98.8	99.4	33,833	18,998
8 - 9	1,813	888	0.49	99.6	99.8	15,411	7,548
9 - 10	674	342	0.51	99.9	99.9	6,403	3,249
10 - 11	199	115	0.58	100.0	100.0	2,090	1,208
11 - 12	70	38	0.54	100.0	100.0	805	437
Totals	232,601	234,679				719,778	642,022
Average length						3.1 mi	2.7 mi

It has been demonstrated in each of the study areas under consideration, that the preponderance of large inter-zonal movements consists of trips between adjacent zones. In most cases, only a very small proportion of trips between adjoining zones is assignable to limited-access facilities. Therefore, the weakness of the interactance formulas in predicting large movements between particular pairs of adjacent zones to a high degree of accuracy loses significance when the projected travel data are used in the estimation of future design-hour expressway volumes. The ability of the interactance formulas to estimate inter-zonal movements of small magnitudes, which are not revealed in the expanded home-interview data because of sample size, has been demonstrated. This, coupled with the fact that underestimation of short trips (when that occurs), is compensated for by trips in the intermediate range of trip lengths rather than by long trips, confirms the applicability of theoretical travel patterns to assignment purposes.

It would have been desirable to make assignments of the synthesized data to the expressway systems proposed in St. Louis and Kansas City. A direct comparison of the expressway volumes thus estimated with the volumes developed in the 1957 assignments made in these cities would have been of great value, but the time and expense involved precluded this analysis.

Screenline Comparison

Comparisons were made of the numbers of internal truck and person trips crossing the St. Louis screenline. The synthetic trip distribution resulted in about 95 percent of the total person movements crossing the screenline, as recorded in the ground counts given in Table 13. Comparable 24-hr expanded survey movements accounted for 85 percent of ground counts. Truck movements crossing the screenline, as revealed in the synthetic and survey data, accounted for 98 and 97 percent of the actual ground counts, respectively. These results indicate that the synthetic trip distribution is capable of producing a screenline check within the degree of accuracy required of survey data by the U.S. Bureau of Public Roads and also of giving close comparison with the expanded survey data.

TABLE 13

COMPARISON OF SCREENLINE CROSSINGS
Survey and Synthesized Data—St. Louis, Missouri, 1957

Item	Total Internal Person Trips	Internal Truck Trips
Ground count (24 hr)	643,803[1]	33,468
Expanded survey data	548,635	31,585
Ratio: survey data/ground count	0.852	0.943
Synthetic data	611,574	32,482
Ratio: synthetic data/ground count	0.950	0.971

[1] Estimated by applying average occupancy factor of 1.52 persons per vehicle and adding transit ground counts. External trips have been eliminated.

SUMMARY AND CONCLUSIONS

In the last several years, interactance formulas have been developed from O-D data collected in urban areas of widely different characteristics. Although it has not been possible to test all of these relationships to synthetically duplicate existing travel patterns, a series of checks has been made on the St. Louis, Kansas City, and Charlotte studies to evaluate the validity of these formulas.

Comparison of individual movements, by computation of the root-mean-square error, showed that the interactance formulas were producing average deviations which were less, in nearly all cases, than those reported in similar research projects by other investigators. Inasmuch as these deviations were large, however, additional comparisons of actual and theoretical trip lengths were made, as a better index of the suitability of using synthetic travel patterns in assigning volumes to urban expressways. The results of these tests were highly satisfactory, revealing the ability of this synthetic approach to approximate the total number of vehicle-miles within 10 percent in most cases. In addition, a screenline check of the synthetic travel pattern developed for St. Louis showed closer agreement with ground counts than was obtained from the expanded survey data.

The insight provided by these successful tests should prove of inestimable value in devising further improvements in the development and application of interactance models. The possibility of analyzing other combinations of trip purposes, special consideration for adjacent zone pairs, and additional land-use variables, are only a few of the possible factors to be studied toward the ultimate goal of developing a universally applicable method for synthesizing travel patterns.

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Interpretation of Desire Line Charts Made on a Cartographatron

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This paper discusses the development of an electronic plotting device and a somewhat different method of summarizing trip desire lines. It evaluates the utility of the desire line. It also evaluates the particular output of this machine, using examples from the Chicago area.

●OF ALL the items recorded in origin-destination (O-D) surveys, the most crucial is, as the name suggests, the address of trip origin and destination. These surveys are designed and carried out principally as a means of insuring that new facilities be located and designed to serve traffic demand. This demand must be fixed geographically in order to achieve this purpose. The origin and destination points provide the two fixed points of demand with service required in between.

Nearly all O-D surveys plot out "desire lines." These are straight lines connecting origin points with destinations. But plotting a line for each journey would be so difficult that different forms of summarization have been developed to present these facts. Basically, this summarization has two aspects—first, summarizing the detailed addresses of origin and destination into area groups and second, grouping the lines themselves.

The first involves establishment of geographic units such as zones, districts, tracts, blocks, etc. The form of this summary imposes problems of detail and also of presentation.

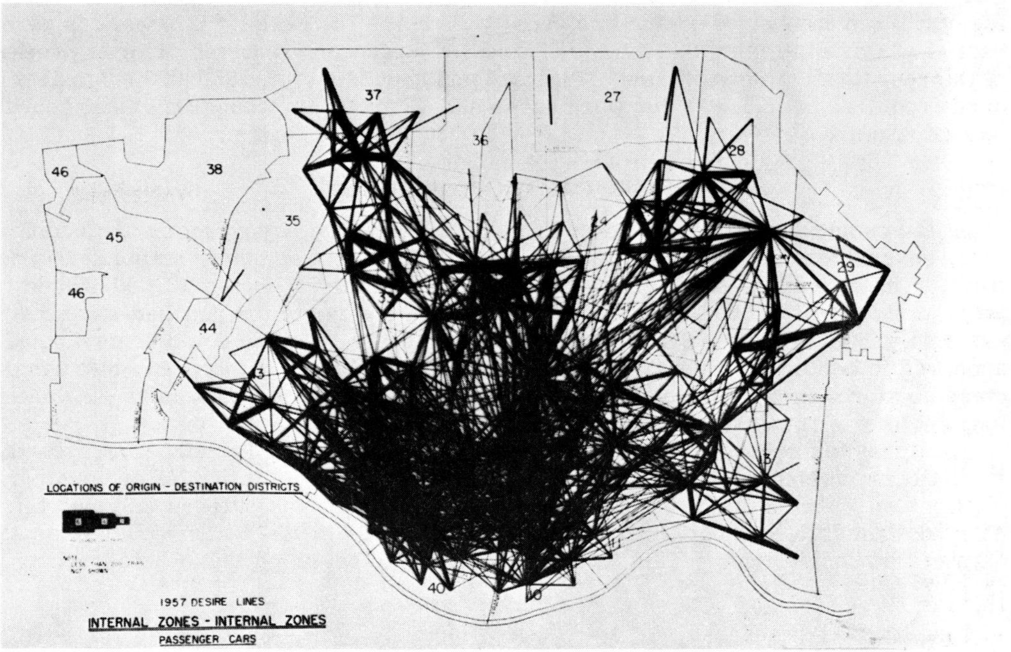
The second form of summary involves grouping desire lines in a particular way. This has generally involved collecting lines into summary bands as volumes increase and lines are closer together. Of course, there are the problems of vehicle vs person trips—separate displays for transit and auto drivers, etc., which increase the problems of more complex detail versus desired simplification.

All of these summarizations have been developed to bring the myriad of detail into some more digestible form—to simplify the picture. But with simplification, one runs the risk of distortion so that extremes are to be avoided. The method found most suitable for the Chicago study involved the use of grid coordinates and the development of desire line density charts.

GRID AND DESIRE LINE DENSITY CHARTS

The grid coordinate method of coding trip origins and destinations and the method for tracing and accumulating all desire lines across the grid was first developed by the California Highway Department (1). This method permitted the use of a large number of small zones (grid squares) and the presentation of desire lines in the form of a density map. The Detroit Metropolitan Area Traffic Study adopted this method and extended it by segregating desire lines by direction and by preparing maps (Fig. 1) on tabulating equipment (2). The machine processing and mapping of trip data was a major step forward.

However, the size of the survey area, the number of trips involved, and the degree of detail required, place limits on the use of this method. Further, the amount of data to be handled is limited by the number of columns in the punched card. The Detroit survey area enclosed some 709 sq mi which were divided into about 2,900 one-quarter square-mile grid units. Over this grid system the trips were traced and accumulated.



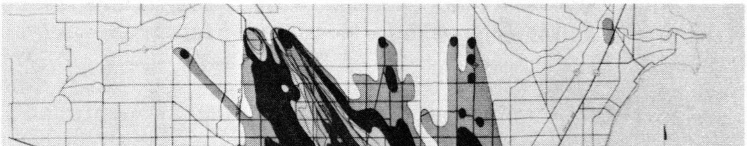


Figure 1. Trip desire line chart and trip desire line density chart (upper—desire line chart from the St. Louis Metropolitan Area Transportation Study, 1959; lower—desire line density chart from the Detroit Metropolitan Area Traffic Study, Vol. I, 1955).

The resultant summary cards, one for each grid square, permitted the selective printing of a large number of desire line density charts. The scale of this work is perhaps best shown by examining the volume of punched-card work required prior to preparation of the coordinate summary card. To trace and accumulate the 250, 000 original trip cards required over 1, 500, 000 work cards and 10 weeks to produce the final coordinate summary cards.

THE CARTOGRAPHATRON

At the outset of the Chicago Area Transportation Study, to overcome the problem of size, several attempts were made to design a computer method for mapping trip desire lines. The size of the study area, 1, 236 sq mi, and the estimated 10, 000, 000 daily trips (or nearly 370, 000 individual records) made the use of the punched-card method extremely cumbersome. Similarly, while a computer program could be developed, it appeared to be far too expensive—both in time and dollars. It was clear that use of the computer for summarizing desire lines would have tied up this expensive machine for long periods of time when it could be used for more urgent work.

As a possible solution, personnel of the Armour Research Foundation proposed that trips be traced electronically and displayed on a cathode-ray tube. After considerable exploratory work on this and other proposals, a contract was entered into with the Armour Research Foundation to design and construct a device which would automatically display trip desire lines. This came, eventually, to be called the "Cartographatron" (Fig. 2).

Operation of the Cartographatron

The Cartographatron is an electronic analog device which can display a dot or line of required density and location on the face of a cathode-ray tube. The input is on magnetic tape, prepared on the Burroughs ElectroData "Datatron 205," and the output is a 4- x 5-in. photographic negative (3). The block diagram (Fig. 3), shows the major

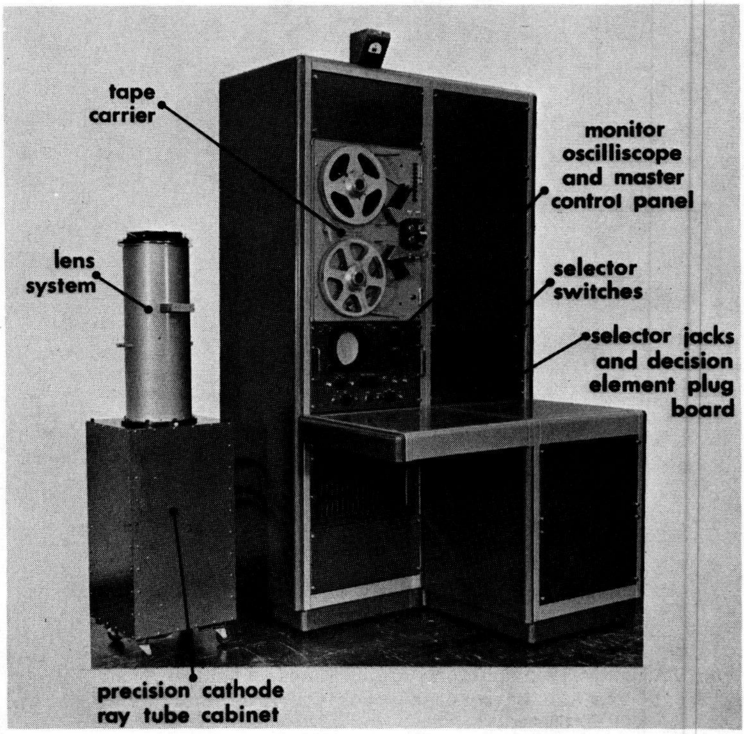


Figure 2. Front view of Cartographatron.

steps in the operation of the Cartographatron. The magnetic input tape contains the coordinate values of the origin of the trip, direction of the trip, the speed and distance of the trace, plus trip characteristic data. Speed in this sense is related to the expansion factor of the trip being displayed. If the trip has a high expansion factor, then the speed will be slow to permit a greater amount of light to be transmitted to the photographic negative. Distance is airline distance between origin and destination. As this information is being read from the tape, a dot of light appears on the face of the cathode-ray tube and moves across the tube to the destination of the trip. The light displayed is recorded on a 4- x 5-in. photographic negative. Because the shutter of the camera is fixed in an open position, all traces are added to the same negative. In this way the photographic plate works as a "memory" or summarizing device. The resulting density at any point on the negative is in proportion to the number and speed (factored trip weight) of traces which passed over that point.

In operation, the Cartographatron reads and displays trip records at approximately the rate of 48 per second. A record constitutes one trace or one unexpanded trip. Allowing time for the changing of tapes, it is possible to display the entire trip file (369, 194 records on 21 reels) for the Chicago area in approximately 3½ hr. Simply sorting this volume of cards on one column could be done at the rate of 600-1, 000 cards per minute. By comparison, the Cartographatron is 3 to 5 times as fast and displays the data besides.

One significant economy lies in the conversion of numeric output to maps. The methods used in all previous studies have required numerous hours of map preparation, posting, drafting and coloring. The Cartographatron accomplished all of this photographically thus eliminating many man hours of work.

Selective Displays of Travel Data

In addition to speed, the ability to select trip characteristics must be considered as a major feature of the Cartographatron. Actually, desire line maps are wanted by analysts in highly selective forms. The ability to select over 4, 000, 000 displays from the Chicago data must be counted as a useful feature. Table 1 gives the trip information identified for the purpose of selection by the Chicago study.

For this information on trip characteristics, a maximum of 22 digits may be coded onto tape for selection. Also, no more than three digits may be used at any one time in selecting a particular display. An example of how this selection works may be shown by assuming that only the desire lines of all internal person trips shopping at department

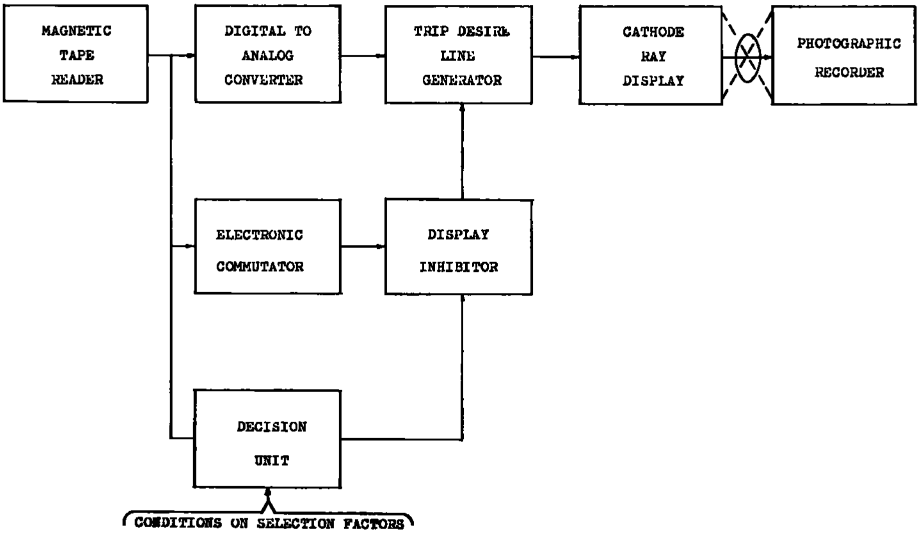


Figure 3. Cartographatron block diagram.

TABLE 1

	Internal (Home Interview) 248,063 Records		External (Roadside Interview) 72,754 Records		Suburban Railroad (External) 5,028 Records		Truck and Taxi 40,751 Truck Records 2,598 Taxi Records
1	Priority mode	1	Station number	1		1	Vehicle type
2	Age	2		2	Departure time O	2	Direction of principal route
3	Occupation	3	Hour period	3		3	Business and industry
4	Industry	4		4	Blocks to station	4	
5	Direction	5	Direction	5		5	Direction
6		6		6		6	
7	Land use O	7	Ring O	7	Total trip time	7	Land use O
8		8	Sector O	8		8	
9	Land use D	9	Ring D	9		9	Land use D
10		10	Sector D	10	Arrival time final D	10	
11	Ring D	11	Airline distance	11		11	Ring D
12	Sector D	12		12	Blocks from station	12	Sector D
13	Mode	13	Vehicle type	13		13	Day of week
14	Parking type	14	Trip purpose (trucks)	14	Miles on railroad	14	Screenline crossing (total)
15	Trip purpose - from	15	Garage code	15		15	Trip purpose - from
16	Trip purpose - to	16	Trip purpose	16	Mode - to station	16	Trip purpose - to
17	Time of arrival	17	D North or south of screenline	17	Trip purpose	17	Time of arrival
18		18	D in or out of study area	18	Auto owner	18	
19	Elapsed time	19	Thru trips	19		19	Elapsed time
20		20	Land use D	20	Airline distance	20	
21	Airline distance	21		21		21	Airline distance
22		22	Station code 1-26 or 30-56			22	

stores are wanted. To select shopping trips only, the main selector number 16 (trip purpose to) is plugged in and button number 3 (code for shopping) is depressed. At the same time selectors number 9 and 10 (land-use destination) are plugged in and buttons 5 and 2 (department stores) are depressed. The display may now be run. The Cartographatron will read all records on the tape, but will display only those meeting the selection requirements. Concurrently with the run, the machine keeps a full count of all records inspected and also a count of those actually displayed.

Other Applications

There is, in addition to the line generation feature, the possibility of point generation or dot maps. As of this time, the only dot mapping done by the Chicago Area Transportation Study has consisted of displaying the origin dot of trip records. The same characteristics as those described for line generation are used. There is, however, the possibility of transferring the land-use survey data, the population data and, perhaps, also street capacity data to magnetic tape and displaying density maps. Another possible application, yet untried, is to prepare scattergrams and correlations on magnetic tape and display them on the Cartographatron. Still a further application which has had limited use by the Chicago study is to use the equipment as a card counting device. While the count is that of unfactored trip records, the speed in which at least a preliminary count could be made of special selections is considerably greater than that of punched card work. The Cartographatron has two counters; one shows the number of records inspected and the other the number displayed.

Application of this equipment to other transportation studies and O-D surveys is limited only in that input requirements be met. The Cartographatron is designed to operate on any geographic area for which data may be coded to a grid system. The grid system used in Chicago covers a 90- by 90-mi area. As shown in Figure 4, this area has been divided into one-quarter square miles by the one-half mile grid system.

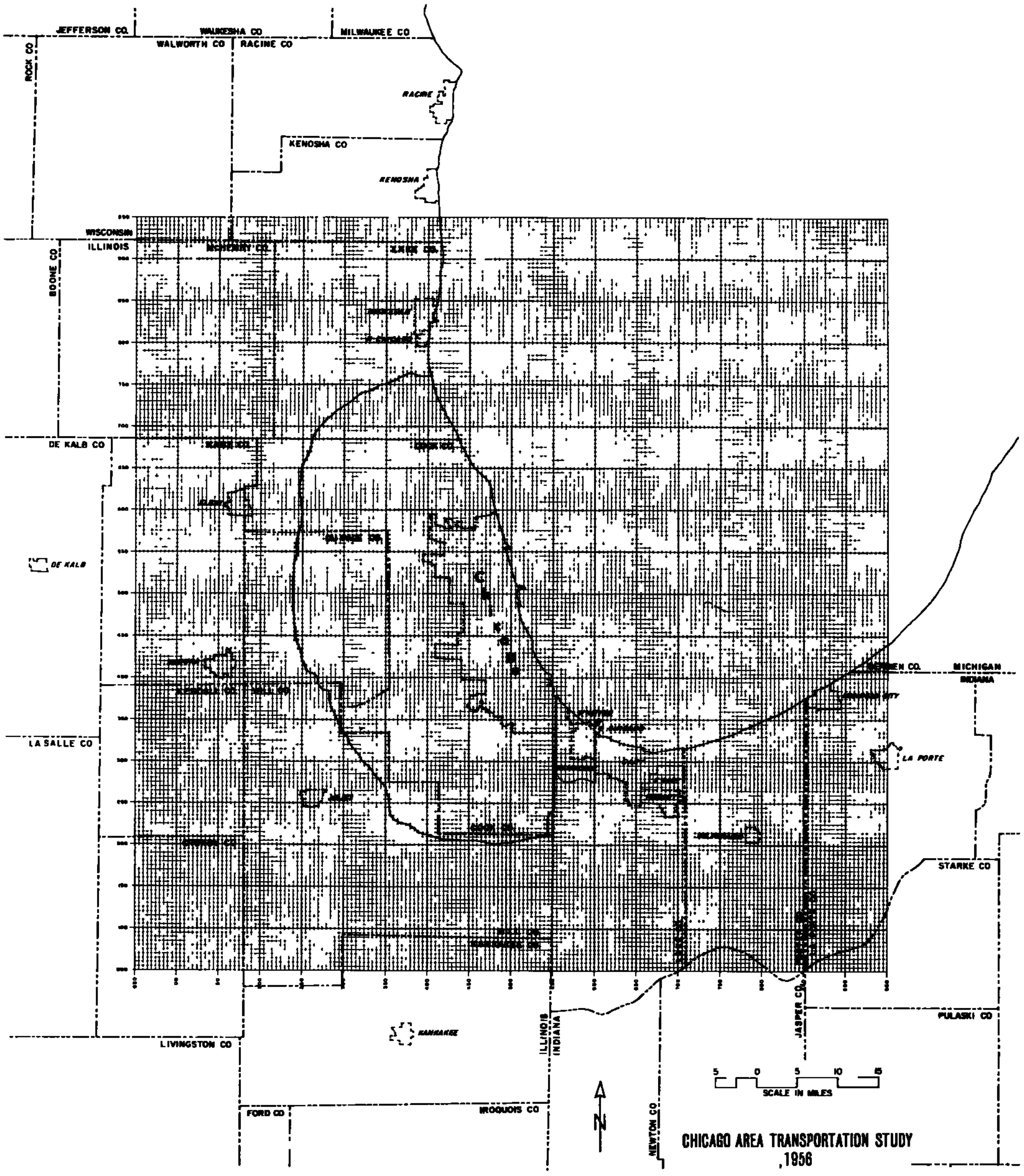


Figure 4. One-half mile grid coordinate system.

The Cartographatron will operate on detail coded to the tenth mile within this system or on a grid of 1,800 by 1,800 units. There are presently arrangements for coding and displaying Pittsburgh Area Transportation Study data on the Cartographatron. The Pittsburgh Grid System is essentially the same as used in Chicago.

Cost of Cartographatron Work

The cost of machine development and construction has been absorbed by the State of Illinois and participating agencies so that the operating cost estimates given here are merely direct out-of-pocket costs.

Cost per lane of tape (20, 000 records)	$350.00

This is based on $65 for tape, $200 for machine (computer) running time, and $85 for programming and other preparatory work.

Cost of running and processing one display from one lane of tape	$ 6.00

This cost includes the operator's time, photo-lab labor and supplies, and operating overhead (the unit cost per lane drops as the number of lanes in one display increases).

Although the cost of preparing a single map would be large because of tape preparation costs, the cost of all subsequent displays is only the running cost and represents savings as more displays are required. It should be noted, also, that these costs include photographic work which might otherwise require substantial rough drafting time and be difficult, then, to reproduce.

QUANTITATIVE MEASURES

A question frequently asked by visitors is whether it is possible to measure trip density on these displays. The answer is "Yes." The key to scaling the the Cartographatron display is the addition on each display of a calibration raster. (The Cartographatron displays reproduced in this paper do not reflect the sharpness of detail or pattern discernible in the original photo-print. Limitations of available reproduction facilities have caused this difference between the original and the reproduced copy.) As shown in Figure 5, the raster now used by the Chicago study consists of an eight-by-eight block pattern. The density function of this raster may be plotted as a normal photographic density curve. In Figure 5 the "desire line" miles are known for each of the 64 units of this scale. Each unit represents an area of 6¼ sq mi, or 25 ¼-sq mi grids. The scale is programed so as to employ two variables on both the x and the y axis. These are: (1) the number of times the base display is repeated in each row and column and (2) the expansion factor for each row and column.

The Densitometer

By comparing the density of areas within the calibration raster with densities of areas within the display, it is possible to scale the display in terms of desire line miles per unit of area. The equipment employed to make these density measurements is a Densitometer. Figure 6 shows the Densitometer used by the Chicago study.

The operation is simple: a light of constant intensity is transmitted through the 4- x 5-in. display negative via a small aperture to a photoelectric cell (the probe unit). The amount of light passing through the negative is translated to voltage and is displayed in numeric terms on the meter. This meter has a maximum reading range of 450 units. The basic densitometer equipment has been modified by the addition of controls for carrying the negative over the light source and a plotting device consisting of a 4X enlargement pantograph. Areas of 0.005 in. in diameter at negative scale (approximately 1/36th of a square mile in Chicago) may be read.

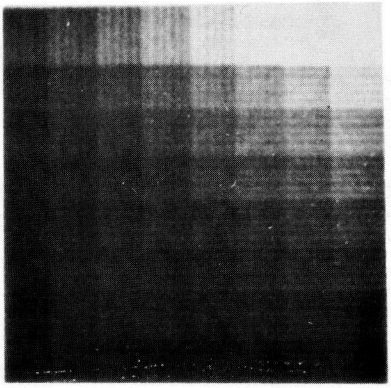


Figure 5. Calibration scale.

Whereas measurements may be made at this scale, they would ordinarily be expressed in terms of the input unit, one-quarter square mile in Chicago. The use of this equipment is limited to the development of profiles and "spot elevations." A complete description of the surface of a

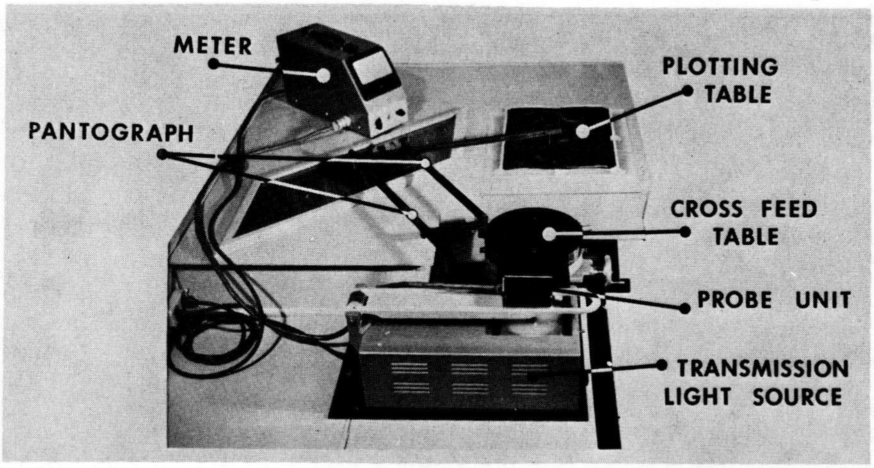


Figure 6. Densitometer unit.

display, because of the number of readings, would require the use of a recording microdensitometer. It is possible to read the value at any point within 2 to 3 percent of the precise, true value using the equipment shown.

PATTERNS

The California and Detroit mapping of trip desire line densities has employed the use of the "isoline." This line has been compiled and drafted by hand over a printed or posted density map. Each line is compiled so as to enclose areas of similar densities and has been subject to the judgment of the compiler as to exact alignment and shape. Although the resulting maps may be considered to be an accurate picture of the density patterns, they are subject to considerable human judgment. It is doubtful that any two individuals could or would isoline a desire line density map in exactly the same fashion.

In contrast, the original compilation of isolines on the Cartographatron displays is accomplished in the photo-lab. The photographer, by varying the printing exposure, varies the density pattern on the prints (Fig. 7). The scaling of isoline depicted in this fashion is done by preparing profiles of the display on the Densitometer.

The above description of "density splitting" as a basis for isolining is intended only to show that maps similar to those produced in California and in Detroit may be prepared from Cartographatron displays. However, the value of the isoline, as such on Cartographatron displays, has been questioned by the staff of the Chicago study. Because an individual with normal vision is capable of reading up to 20 shades of grey, it has been argued that the basic patterns depicted by isolines may be observed without any cartographic aids on the display. Certainly, the principal patterns are readily apparent. Depending on the data displayed and the use intended, the addition of quantitative measures may be desirable. Such measures can be accomplished on any Cartographatron display.

USEFULNESS OF DESIRE LINE DISPLAYS

The preceding portions have described the design and workings of the Cartographatron. It is a unique machine. It summarizes great quantities of information in visual form. It is fast. It is electronic. It proceeds from coded data to virtual final presentation without clerks or draftsmen. All of this is of interest but there are many who will say, "Of what real use are desire lines? What is the advantage of this stepped-up data processing device in actual application?"

Justification clearly depends on use and usefulness of O-D survey data in supplying both increased knowledge and the increased ability to make the right decision in planning

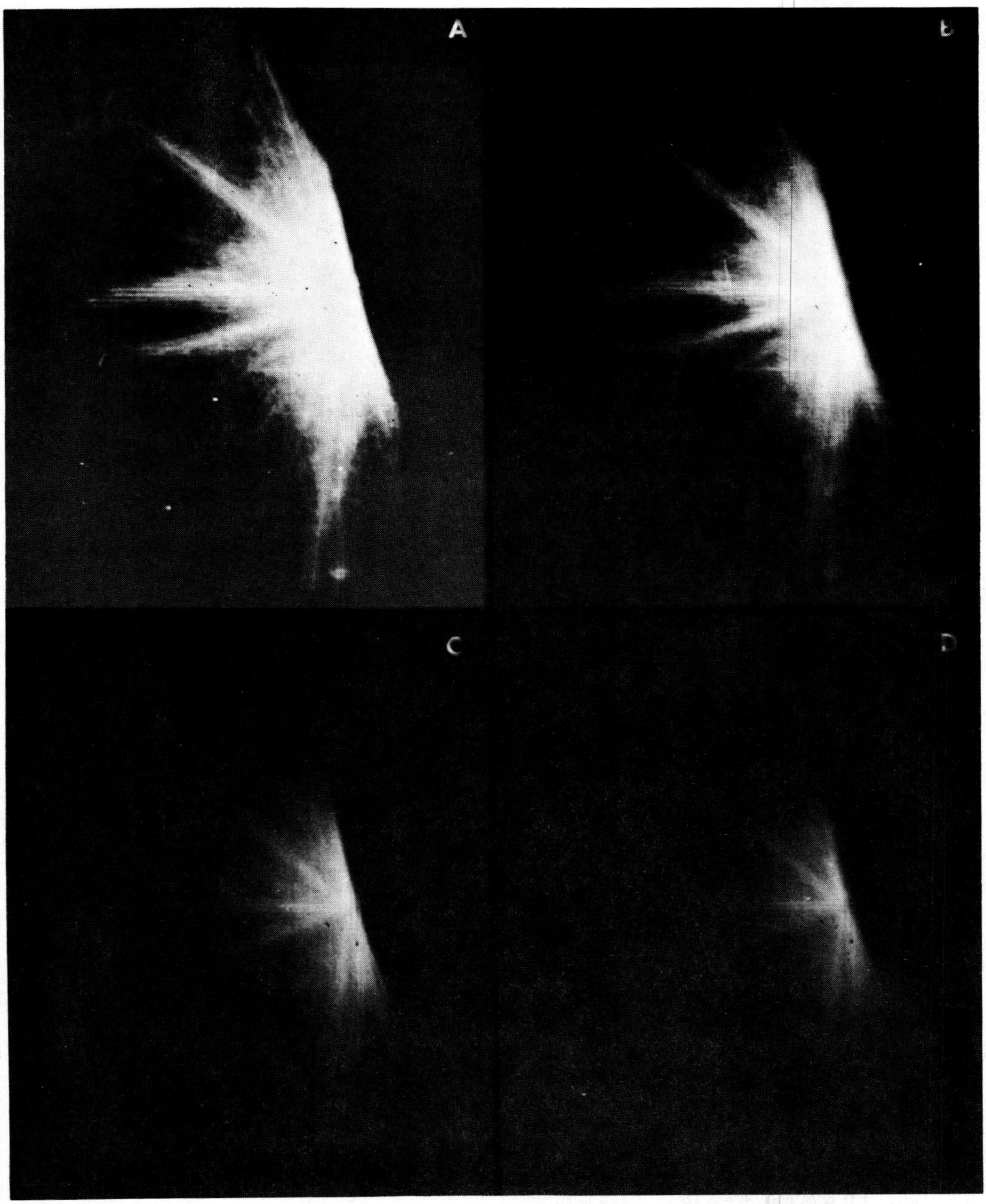


Figure 7. Density splits.

and building new transportation facilities. This matter has been puzzled over at some length in trying to measure the costs and risks of developing the equipment as against the utility of the ultimate output. It is felt that desire line summaries were critically necessary in understanding the large mass of O-D data collected.

Although one cannot exactly measure the benefits of such a machine against its development cost, two great values are apparent at this time. The first is that these desire line prints create careful and unbiased images of traffic patterns in the minds of the team of analysts and technical personnel who will have to interpret proposals. The second and less tangible benefit lies in the "sales value" of these presentations.

The first benefit of common images is by far the more significant one. Members of the study staff, as well as personnel from related agencies and decision-making officials, all have difficulty understanding large masses of data and seeing a great metropolitan region as a whole rather than a series of locations or political jurisdictions. These summary pictures create accurate, regional images of the travel data. These images being common background help to relate one person's work to another's. Both have common information. Common technical denominators produce much more effective team work and collaboration.

Obviously, these pictures, being new and carefully developed, were excellent means of communication to citizens, local groups and interested agencies. It is not simply that they were made "electronically" (which, being a "good" word, insures the respect of the layman) but rather that the impression gained of travel behavior, as the viewer has experienced it, is suddenly confirmed. He knows this is right. The data are reliable. What he has seen agrees with his observations. Once this respect for accuracy is established, communication is easier and more profitable.

To capsulize this argument, one picture is worth a thousand words. But the proof is in the result. Therefore, the remainder of the paper consists of examples.

Interpretative Examples

Figures 8-11 show four maps of the Chicago region. The first three provide data which can be obtained from secondary sources but which are easily portrayed because they are stationary. These figures also have much bearing on trip desire line patterns.

Figure 8, showing political boundaries, is simply taken from a highway road map of the region. Outlined is the cordon line marking the boundaries of the internal survey area.

Figure 9 shows the population by place of residence using isolines to identify areas of common density. This is made in the same way as a dot map but groups regions carrying similar dot densities into isolines.

Figure 10 shows the amount of floor area at each ¼-sq mi grid of the region. Floor area measurements were not made in much of the suburban area because of costs of secondary source data. However, more than 85 percent of total floor area is represented on this model. Floor area is a critical index of trip generation as can be seen by comparison with Figure 11 which shows the number of person trips beginning at each grid square on an average week day. This model will look very much like that for floor area excepting only the suburban areas where no floor area measured are shown.

All of these reflect the structure of the Chicago region and provide all the needed evidence to form a mental image of what the major desire line patterns would be. Reference to succeeding illustrations will demonstrate that the pictures are persuasive as to accuracy and that for the first time there is clarity to an otherwise vague image. The basic argument for the value of desire line displays is based on the justness of this claim.

The aggregate display of desire lines of all person trips is shown in Figure 12. This is what your mental image should have been. This is the sum of 10, 500, 000 person trips made by over 3, 000, 000 travelers on an average week day. All modes of travel are included.

In Figure 13 only the desire lines of travelers on suburban railroads or elevated and subway trains are shown. This is a unique pattern. These are travelers with very special requirements. As we would expect, they focus on the CBD. They are generally long trips. The sunburst shown here still has a sharp pattern which marks the "fingers" of land development that extend outward along commuting railroads. The desire lines are drawn only between "home" and "office" and do not necessarily have a relation to the rail line over which they traveled.

Figure 14 shows the patterns of bus riders. These are shorter journeys and are heavily concentrated in the densely built-up part of the region, primarily within the City of Chicago. There is little or no visible bus use in the outer suburban parts of

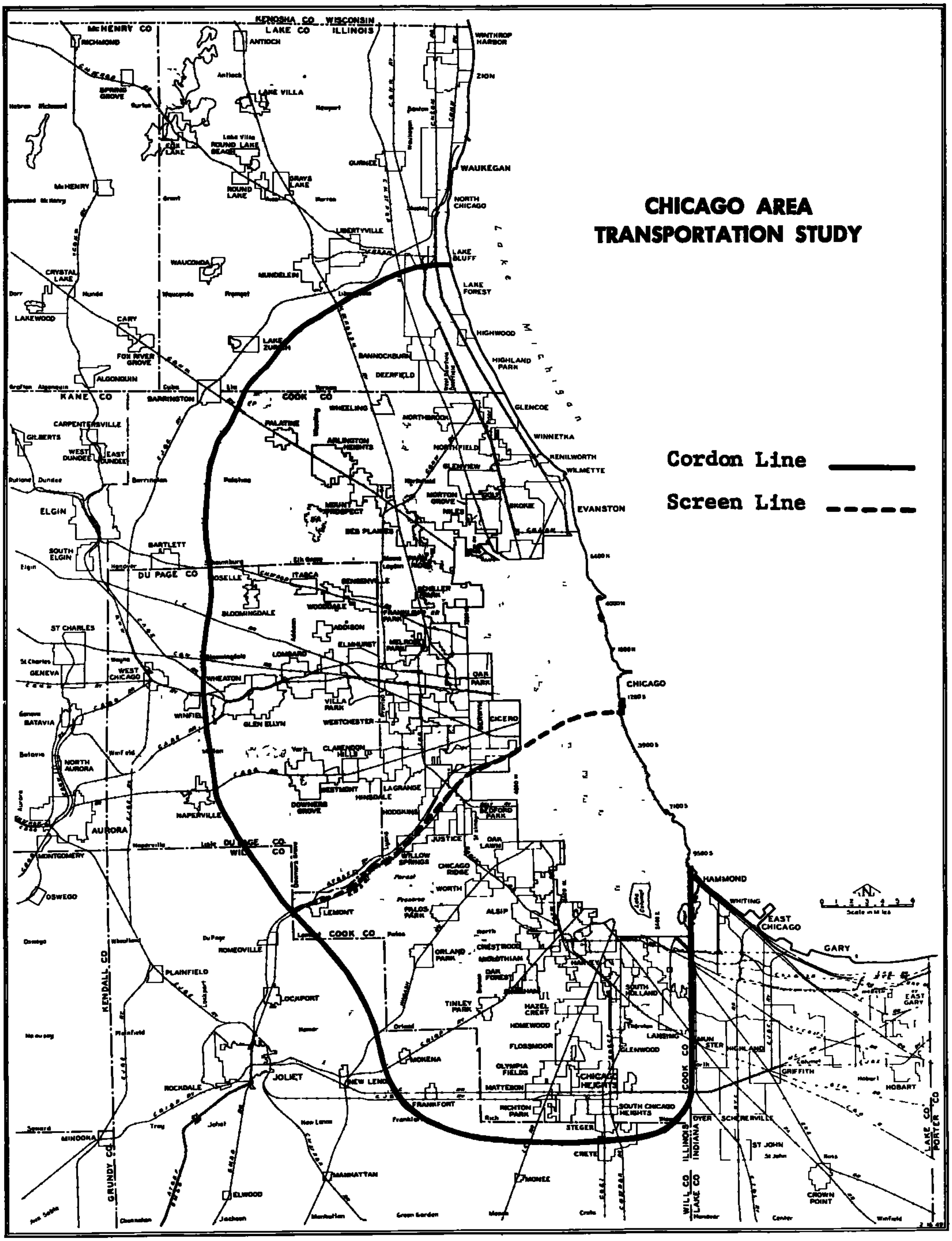


Figure 8. Study area and political boundaries.

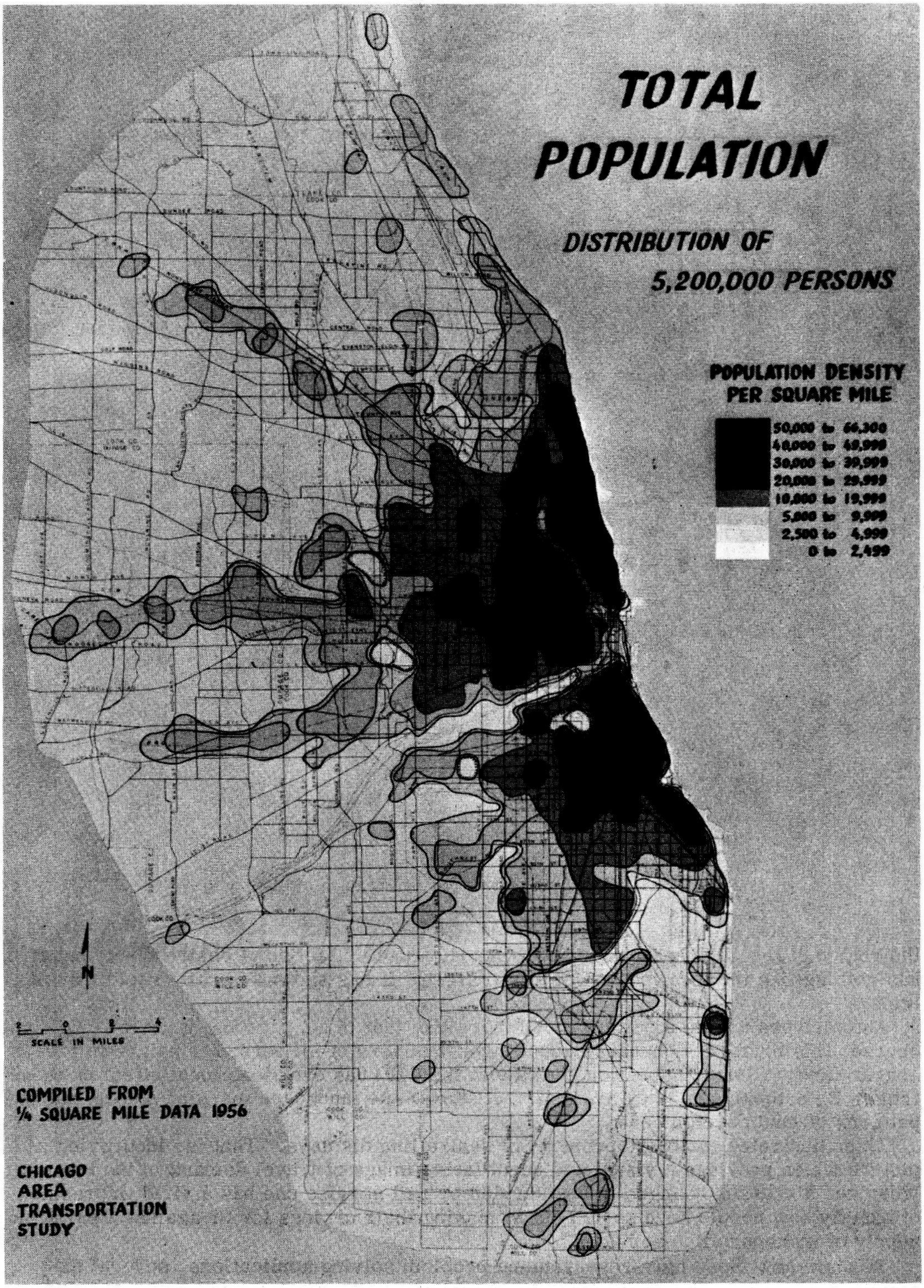

TOTAL POPULATION
DISTRIBUTION OF 5,200,000 PERSONS
POPULATION DENSITY PER SQUARE MILE
50,000 to 66,300
40,000 to 49,999
30,000 to 39,999
20,000 to 29,999
10,000 to 19,999
5,000 to 9,999
2,500 to 4,999
0 to 2,499
N
2 0 2 4
SCALE IN MILES
COMPILED FROM 1/4 SQUARE MILE DATA 1956
CHICAGO AREA TRANSPORTATION STUDY

Figure 9.

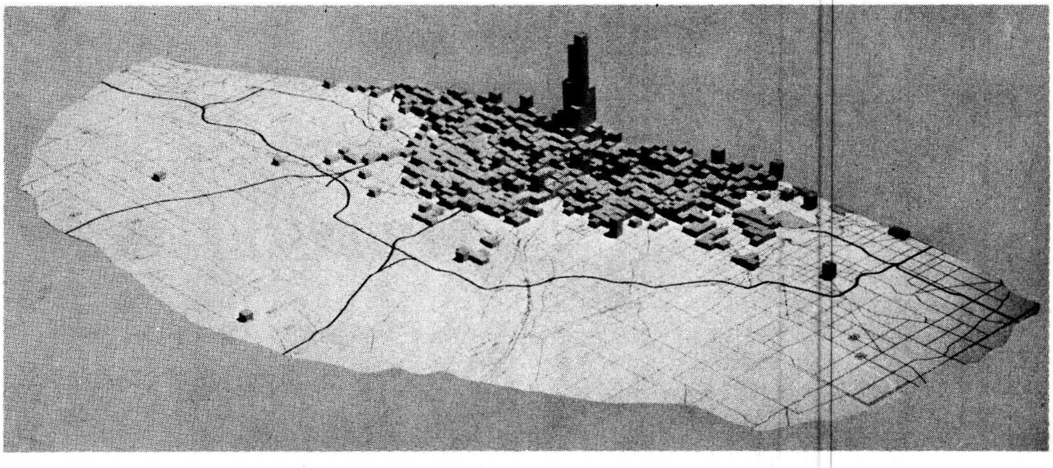


Figure 10. Model of total floor area.

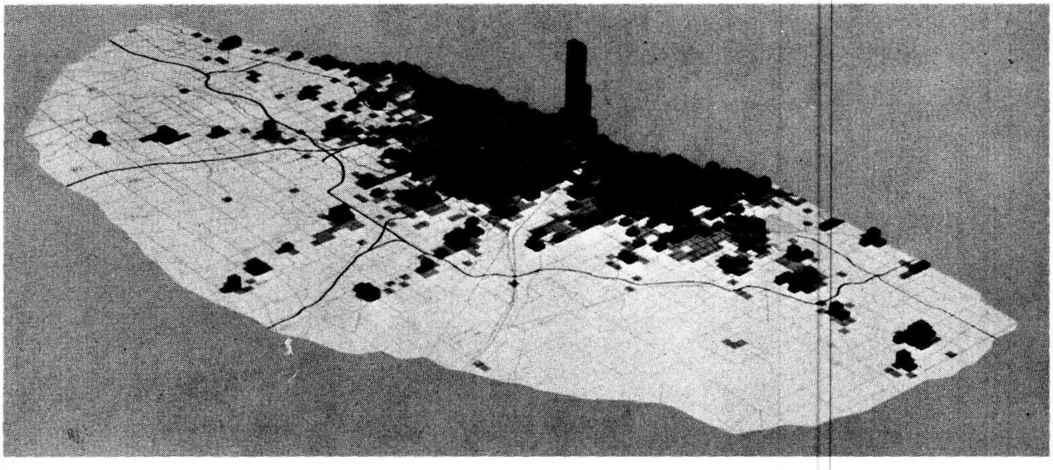


Figure 11. Model of total person trip destinations.

the region. The CBD is the focus of the most dense collection of desire lines, but it has nothing like the focal power for bus trips that it has shown for rail, rapid transit trips.

Auto drivers create a pattern much more like that of all persons (Fig. 15). Of course, this must be so because about three-quarters of all person trips are in passenger cars. Closer inspection shows that the CBD has even less focal effect on these trips. By comparison, they are distributed over the landscape marking out the major patterns of land development.

This illustrates the basic property of desire line displays. That is, identifying a picture which provides a visual and quantitative image of travel demand in the region. When one discusses changes in bus service or rail service one has a vivid impression of exactly how people have sorted out in making their choices for or against a particular supply of transportation services.

To show how these have research and problem solving applications, several examples are given. The first arises from a critical argument on applications of O-D data and predictions.

It has been argued with reason, that facilities must be planned for peak-hour requirements rather than for average daily needs. It is further argued that peak-hour travel should be forecast and that this can be accomplished by forecasting journeys to work. On the opposite side of this argument (and this side was taken) are the people who argue that peak-hour demands can be better inferred as a function of total daily travel. The Cartographatron provides a ready means to examine which is the more plausible contention.

To do this, the desire lines were displayed—first, for all auto driver trips originating in the two peak hours between 7 and 8:59 a.m. (Fig. 16). Next, for comparison, and at similar scale, are the auto driver journeys going from home to work (Fig. 17). In addition, for comparison, the auto driver journeys starting other than during the 4-hr

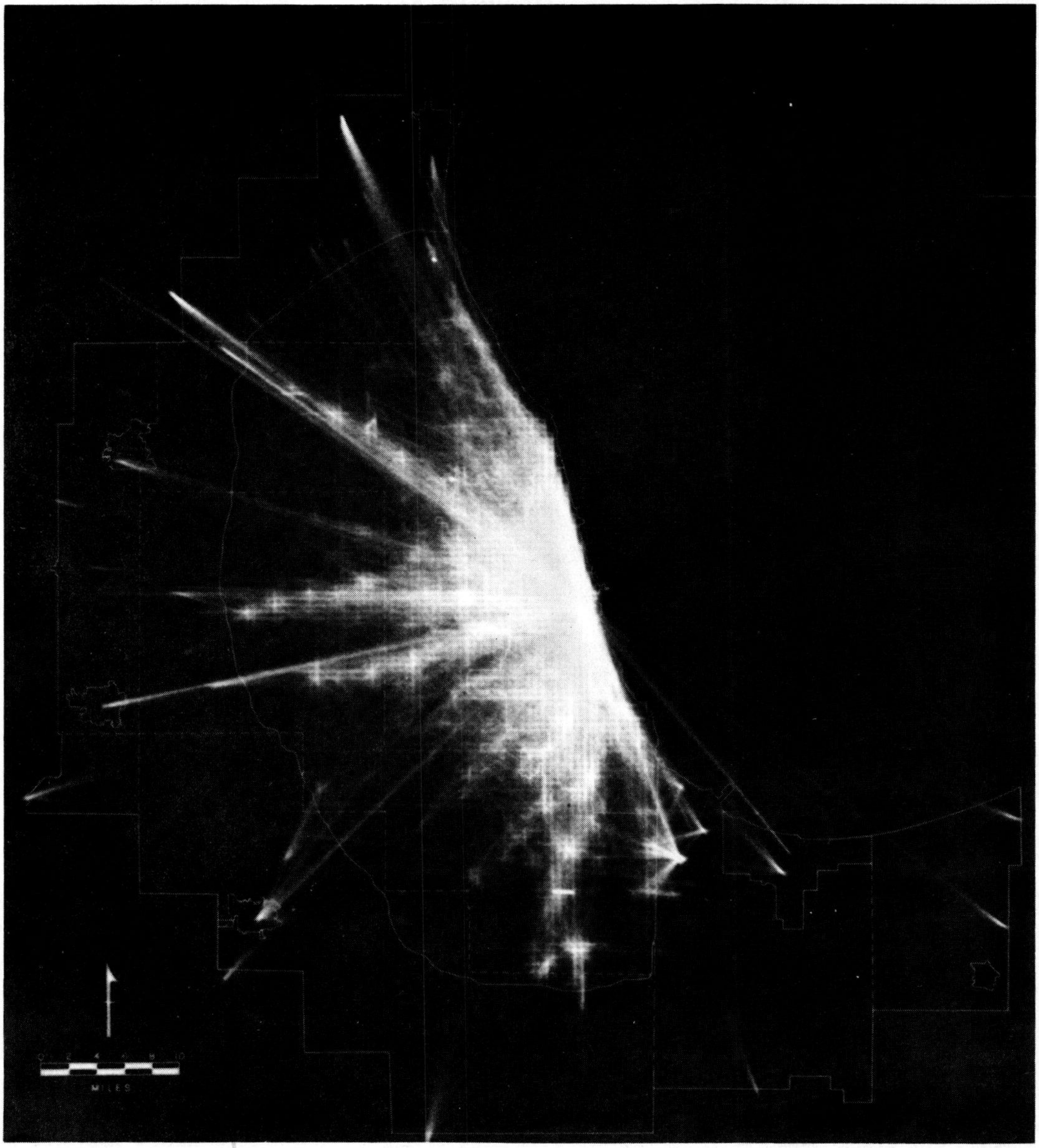


Figure 12. All person trips.

periods 7 a.m. or 8 a.m. and 5 p.m. or 6 p.m. are displayed (Fig. 18). Now the reader may judge for himself whether these are significant differences in the several patterns. Do the work trips look more like peak-hour travel than the all auto driver trips? Is the peak pattern different from the off-peak pattern? How do all of these compare with total daily travel (Fig. 19)?

For over-all patterns, the work trips are as much different from peak-hour patterns as peak-hour travel is different from total daily travel. Off-peak travel, however, seems to have quite marked differences. This answer was less pleasing than had been hoped by the author but it is possible to judge for oneself because of the properties of this machine.

Another local problem arises from the continual concern over truck needs. Some

Figure 13. Rapid transit trips.

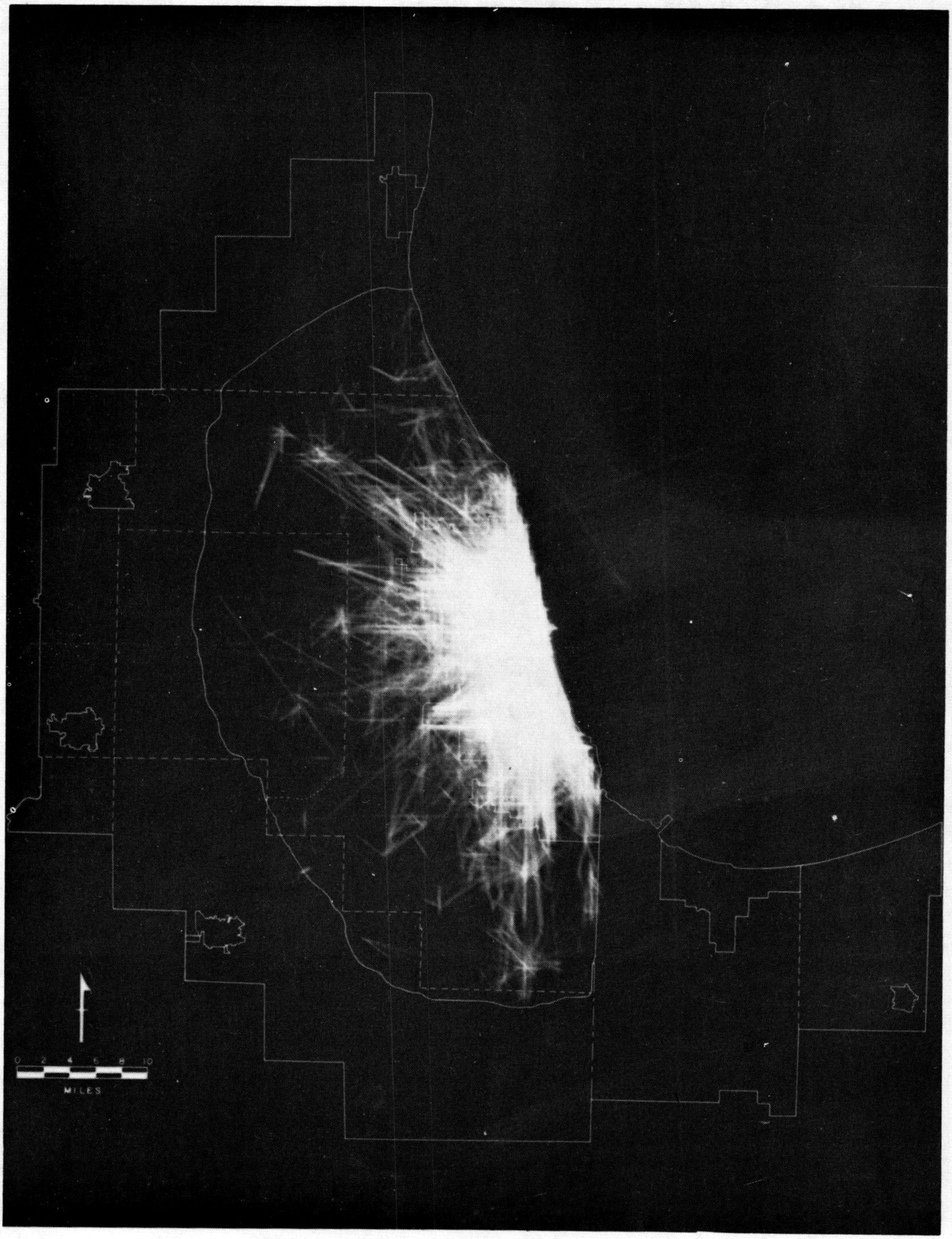


Figure 14. Bus trips.

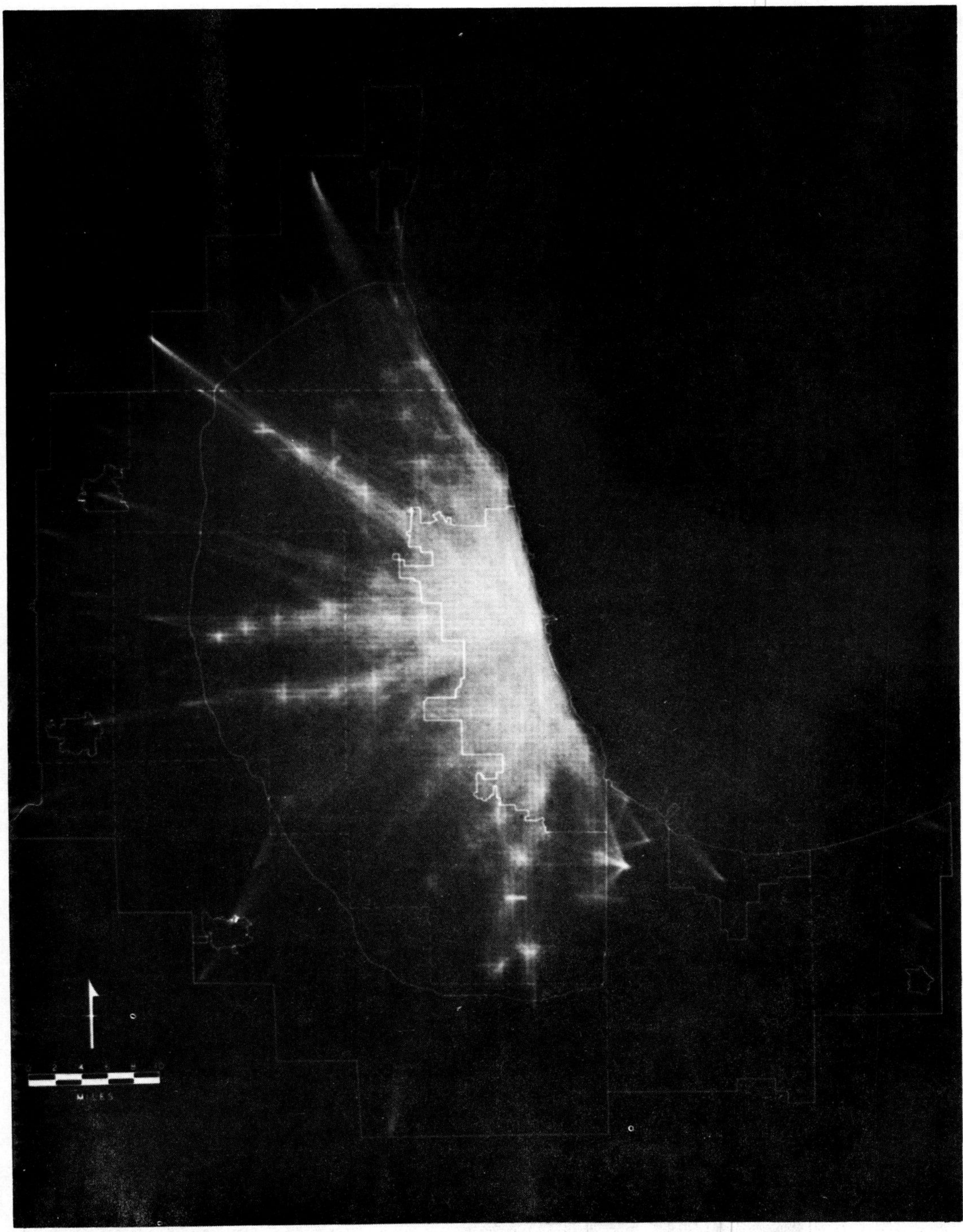


Figure 15. Automobile trips.

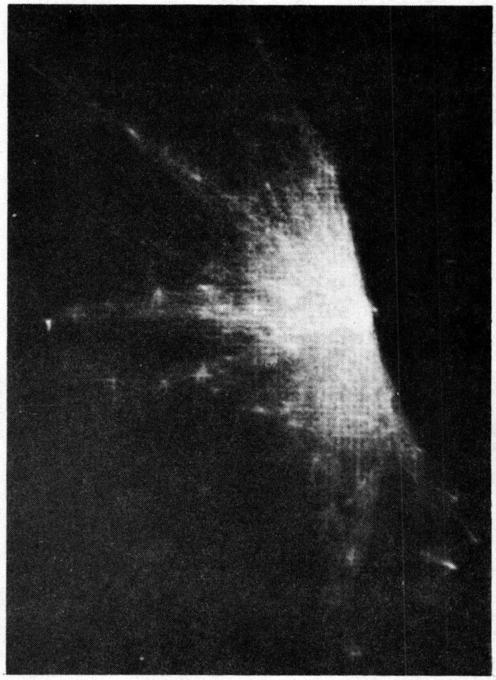


Figure 16. Peak hour automobile trips.

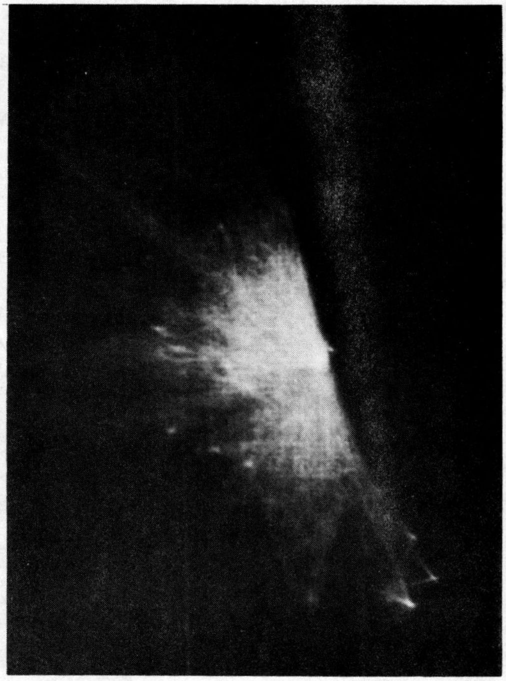


Figure 17. Automobile work trips (vehicle-miles equal to Figure 16).

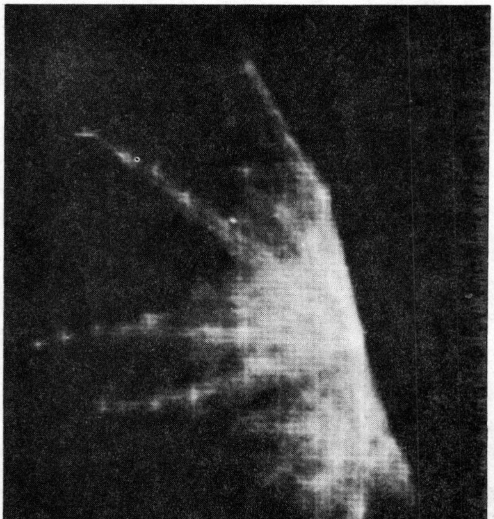


Figure 18. Off-peak automobile trips (vehicle-miles equal to Figure 16).

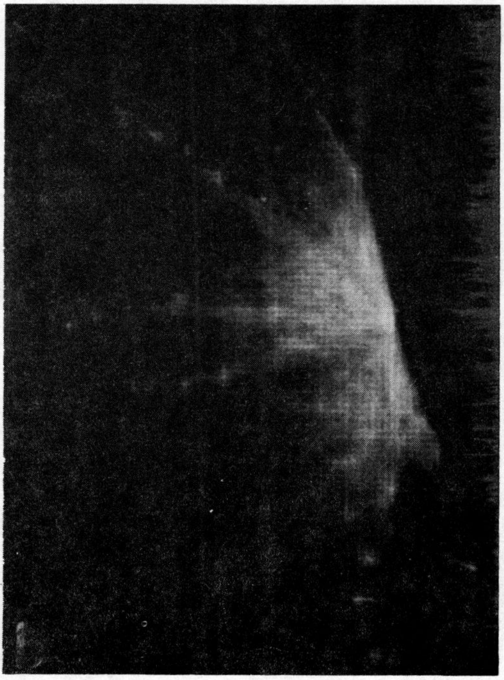


Figure 19. Automobile trips (vehicle-miles equal to Figure 16).

advise special facilities to get trucks off certain streets whereas truckers, sensitive to taxes and urban delays, are extremely concerned that planning be geared to their requirements. In Figure 20 all truck trips are displayed. Here again the picture is generally similar to that of auto drivers. This is of substantial significance because it suggests that additional studies to determine whether special facilities are needed for trucks might not be rewarding because trucks go where autos go and can therefore use the same facilities. (More than this, they do not clash during peak-hour traffic.)

One of the applications of any O-D survey is that of fixing locations for new expressways. If it is reasoned that short trips are of little significance as potential expressway users, then the vehicle trips of greater than say 10 mi of desire line length may be selected out (Fig. 21). This is one of the more difficult charts to read. Obviously, there is no great single concentration of these lines. It would be very difficult to band

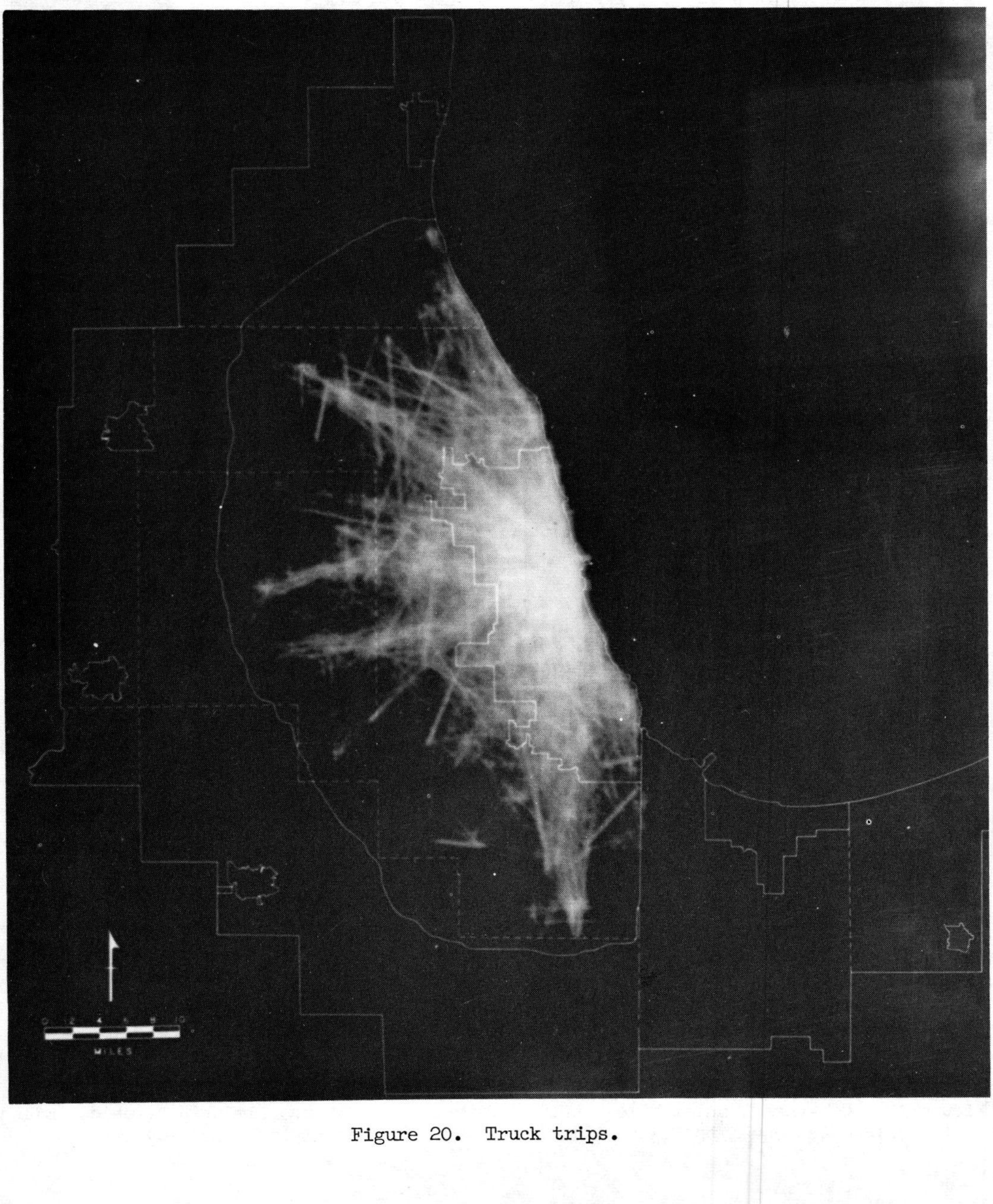


Figure 20. Truck trips.

them together into major sectoral groupings to suggest particular expressway locations.

Quite surprisingly, however, much information as to location of expressways was gleaned by plotting only the shortest trips. In Figure 22 only vehicle trip desire lines of less than 3 mi are displayed. Suddenly, little constellations of travel are isolated from one another. These journeys mark out the small community areas where travel is short. The local, internal travel of communities should move freely on collector and arterial streets. The piercing of these patterns by express highways designed to serve longer journeys may not be suitable. Thus, there is strong visual evidence of community definition as an aid in determining most suitable locations for fully controlled access routes.

One final use may round out the examples of applications which are constantly enlarging. This involves furnishing data to other public agencies—in this instance, the

Figure 21. Vehicle trips greater than 10 mi long.

Chicago Department of City Planning. They have been working for some time on proposals for modernizing older shopping centers in the city. They have been interested in possible schemes for revising traffic facilities to improve access to these older commercial centers. To study this more closely, they requested tabulations of trips to selected locations in the city. It was easy to supply this information in a visual output.

Figure 23 shows the origin pattern of all travelers going to shop in the vicinity of 63rd and Halsted Streets. Here is an example of dot maps and also a quick and usable output.

Figure 24 shows the accumulated desire lines of these shoppers. Needless to say, this was much more immediately usable data to the planners and they have reported that the materials were of great interest to the local merchants in the area. Additional requests are now on hand. Service of this kind increases the acceptance of general

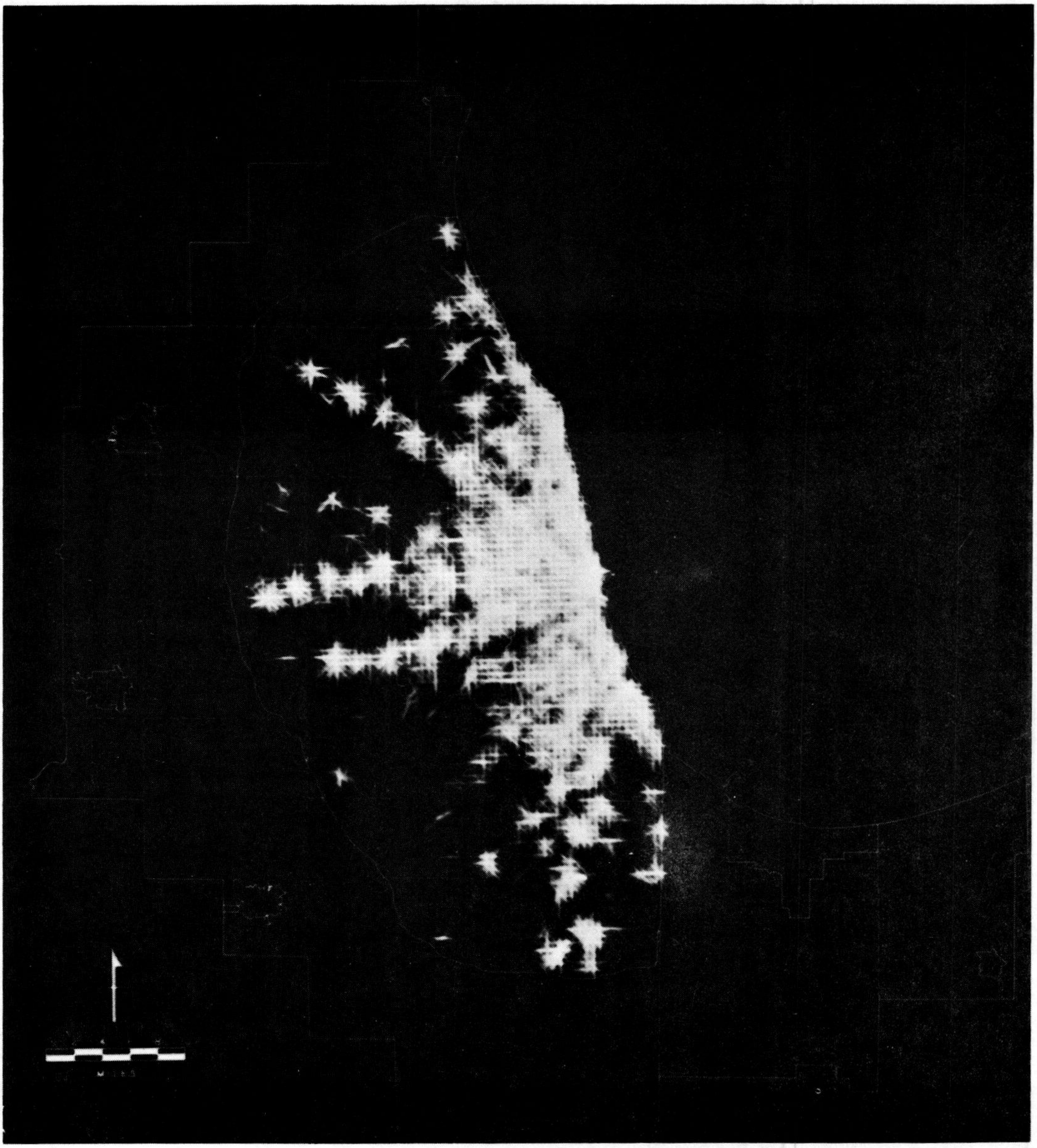


Figure 22. Vehicle trips less than 3 mi long.

survey data and will be very useful in detailed study of specific local problems.

New and varied uses of this machine are continually being found. The exact measure of utility cannot be established, nor is it possible to measure exact worth. The Pittsburgh Area Transportation Study is using it to display their data. The visual image of travel demand and pattern obtained from these displays is an essential piece of knowledge for an accurate understanding of the regional travel demand of any large urban area. It is recognized that, once known, the value of the knowledge is discounted—particularly if it conforms to preconceived notions. However, exact information, carefully presented will continue to have significant meaning in properly evaluating any problem. To work out the program of transportation facilities best suited to an urban region is of such compelling importance that carefully assembled factual data of many types must be on hand to insure that decisions are made in the public interest. Seen from this viewpoint, the machine and its output are well worth the development cost.

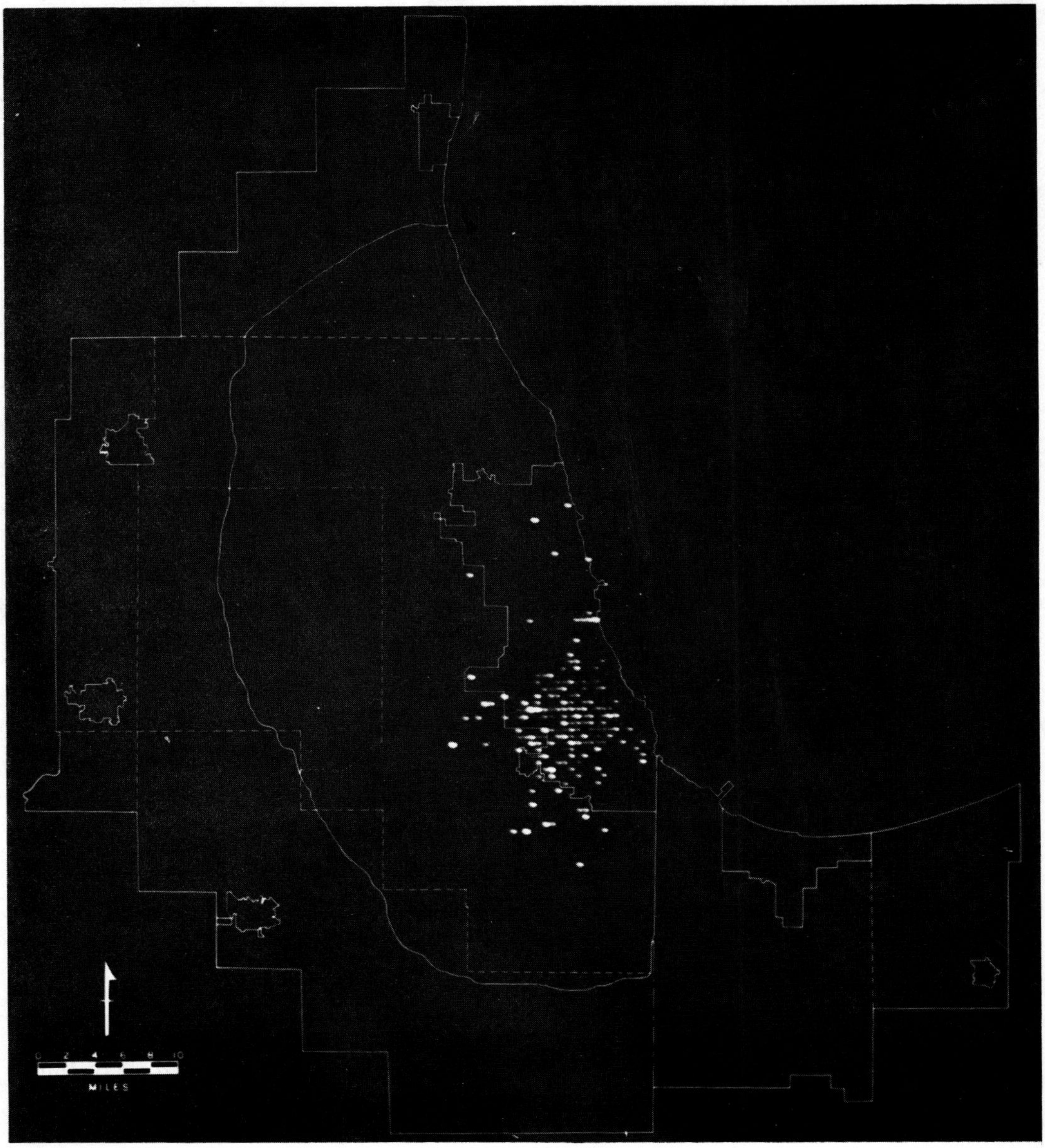


Figure 23. Origin of shopping trips to one district.

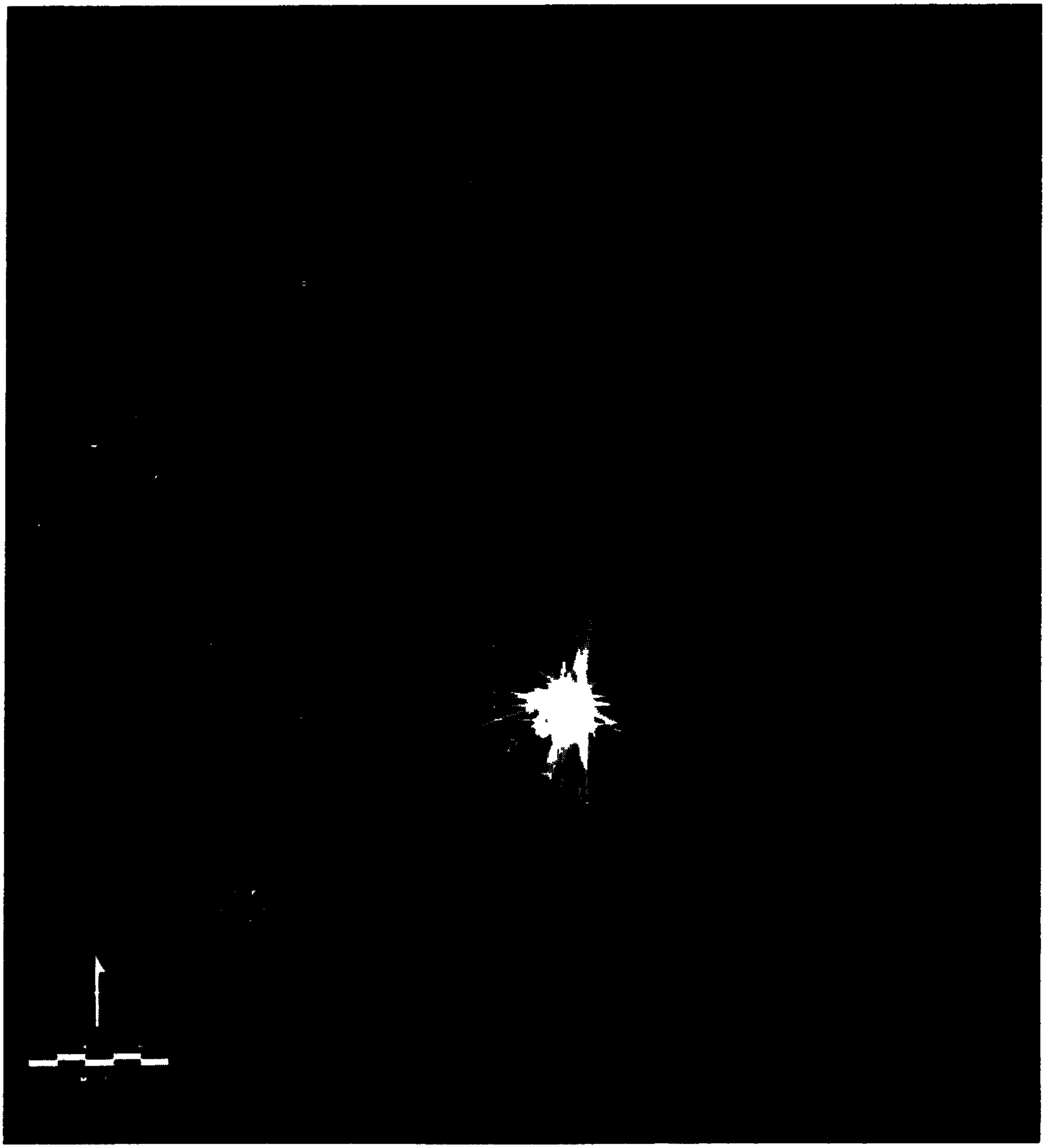


Figure 24. Desire lines of shopping trips to one district.

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The Continuing Traffic Study: Methods of Keeping O-D Data Up-to-Date

ALBERT J. MAYER and ROBERT B. SMOCK, Detroit Area Traffic Study

● IT IS NOW widely recognized that a home-interview O-D study is the only available objective basis for planning an urban transportation system. Consequently, scores of cities have undertaken such studies, a fact which is noteworthy because it symbolizes a widespread desire to base social action on something more substantial than guesswork. But the studies may easily become as misleading as guesswork if they are not constantly re-evaluated to measure the effects of ever-accelerating social and technological change. The solution is a continuing study of traffic.

It is as a representative continuing effort that the Detroit Area Traffic Study (D.A.T.S.) is discussed. The Study, now an integral part of Wayne State University, is sponsored by the Michigan State Highway Department, the City of Detroit, the Wayne County Road Commission, and the U.S. Department of Commerce, Bureau of Public Roads, as a follow-up to the Detroit Area Origin and Destination Study of 1953.

The follow-up includes the two basic types of research which are usually termed "applied" and "pure." The emphasis of applied traffic research is on supplying current data on such matters as desire-line magnitudes and inter-zonal trip volumes needed by traffic engineers for guidance in making decisions about practical problems. Pure research, as conducted by the D.A.T.S., consists of a never-ending attempt to find new sources of data and new analysis techniques. Such basic research, properly done, should serve to prevent the compounding of errors that may be in an original set of data, and should reduce prediction errors by revealing the "why" of travel behavior as a supplement to the "what" disclosed by conventional O-D studies. Pure research can, of course, be done most efficiently in a university setting, as at Wayne State University, where sociologists, economists, planners, and mathematicians, can be brought together to work in cooperation with traffic engineers.

In the day-to-day operation of the D.A.T.S., the pure and applied research approaches to a continuing study of traffic are necessarily and most profitably intertwined. Getting the answer to a traffic engineer's specific question has made the opportunity for some experiment in pure research. Or, a new procedure developed by pure research. Or, a new procedure developed by pure research is tested in such a way as to yield information of use to participating agencies interested primarily in solutions to everyday problems. There is emerging from such activities in the D.A.T.S. a reasonably well-integrated methodology for improving knowledge of and control over traffic in the metropolitan area.

INTER-ZONAL TRAFFIC VOLUMES

Three Basic Variables

Among the data necessary for planning an efficient metropolitan traffic network, those dealing with traffic volumes between various sub-areas of the metropolis are most important from the travel engineer's point of view. Such data were supplied to the D.A.T.S., as a continuing study, by the O-D survey of 1953. The data were based on counts of traffic between a large number of "traffic analysis zones," and included predictions of what inter-zonal volumes would be in 1980. It is such predictions that the continuing traffic study must constantly re-examine in the light of changing conditions.

The phrase "changing conditions" refers primarily to changes in population, land use, and tripmaker characteristics, because these are known to be the important variables in traffic generation. Among the three variables, population is the most significant. This is, in a sense, unfortunate, because reliable and complete population counts are seldom made with sufficient frequency for traffic study purposes. In the 10-yr intervals between the Federal censuses, population changes can be so extensive as to call for complete alteration in the plans for a traffic network.

One solution to the problems implied is to use dwelling units as a basis for population estimates. In Detroit, this is made possible by the fact that minor civil divisions send counts of all building and demolition permits to the Metropolitan Area Regional Planning Commission for an annual net summary. The traffic study, with a careful check on the number of occupied DUs and on the average number of occupants per DU, converts the data into current population estimates for each traffic analysis zone. Thus, the population aspects of future inter-zonal traffic are subjected to an annual review—a review which often reveals the need for greater or lesser changes in established plans.

Changes in land use are another important source of variation in traffic volumes. This means that the continuing traffic study must maintain a current inventory of land use. Fortunately, urban planning commissions share the traffic study's need to know about land use, and are therefore likely to keep records on major changes. But more than major changes must be known if traffic prediction is to be accurate. Even block statistics are insufficient because, for example, a single supermarket in an otherwise residential block can alter localized estimates considerably. Hence, in Detroit both regional and city planning agencies are cooperating with the D.A.T.S. in an effort to create and maintain a continuing, parcel-by-parcel inventory—but the difficulties are formidable.

Even when minimum necessary land-use information is available, personnel of the continuing traffic study must still try to solve the difficult problem of estimating the effect of land-use changes on rates of traffic generation. At present, crude estimates are based on number of employees per acre or on size of parking lots. Attempts have been made to refine the estimates, but the results have not been heartening. We now have research findings to substantiate such problematical points as the fact that some residential areas attract large numbers of work trips and the fact that there is no direct correlation between floor space on commercial land and the traffic-generating capacity of the land.

In short, far too little is yet known about the relationship between the characteristics of tripmakers and the nature of their trips. Perhaps some significantly new information will be provided by the exhaustive analysis now being made by the D.A.T.S. of the 1953 O-D questionnaires—an analysis which the original surveyors had neither the time nor the money to complete. But such basic research is still at the stage of raising more problems than it solves.

Iteration: Possibilities and Problems

Among the methods intended to increase the accuracy of inter-zonal traffic volume predictions, the process known as "iteration" is widely used. For those unfamiliar with iteration, a general description is given here (in three steps) as a necessary prelude to understanding the technique's possibilities and problems:

1. First, an estimate is made of total area growth which will occur as of a given year. Such large-scale estimates are known to be reasonably accurate because the effects of various types of inherent error tend to counteract one another—a "smoothing" that does not occur when the area involved is smaller.
2. The growth for the entire area is then used as a basis for predicting the total trip-ends in the area as of the chosen date.
3. Predicted trip-ends for each traffic analysis zone, relative to each other zone, are then calculated on the basis of estimated growth in the various zones. The resulting figures are initial predictions of inter-zonal traffic volumes. But experience has shown that such predictions, based as they are on the relatively small traffic analysis zones, are usually erroneous (just as predictions about individual persons, in comparison with predictions about groups of people, are likely to be unreliable). They can be corrected through the iteration technique. Although the technique appears complicated to the uninitiated, it is based on these two simple facts: (a) when the trip-ends for all the separate zones are totaled, they should equal the trip-ends predicted (in step 2) for the whole area; and, (b) if the growth for each zone, and the growth for the whole area, are stated as ratios and termed growth factors, then the growth factors for all the zones,

when averaged, should equal the growth factor for the whole area. If these equalities, termed "balance" (or "convergence") in the language of iteration, do not exist, then it is apparent that there are errors in the predictions; and, it can be presumed, for the reasons stated above, that it is the zonal or inter-zonal, and not the whole area, predictions that are in error. The iteration formula achieves balance, and thus corrects the wrong predictions, by altering the growth factor for each zone by an amount that is (when calculations are completed) proportionate to the zone's size and growth in comparison with the total area's size and growth. In turn, the corrected zonal growth factors produce corrected inter-zonal volume predictions.

Although iteration, as described, potentially increases the accuracy of inter-zonal traffic volume predictions, it has serious limitations. One obvious flaw is the assumption that O-D data are reliable; another is the exaggeration of errors inherent in data obtained through home interviews—a zone in which 100 trips are missed in the O-D study, and for which 100 percent growth is predicted, will be credited after iteration with 200 less trips than it will probably generate. Another limitation of iteration is associated with its proportional increase of growth factors for each zone—although it may be known that a particular area with no present traffic volume will grow, its "post-iteration" volume will be reported as zero since any proportion of zero is still zero. It is also true that iteration, being purely mechanical, cannot predict changes in the character of inter-zonal volumes. These difficulties, and related problems, have encouraged personnel of the D. A. T. S. to develop some new methods of studying inter-zonal traffic volumes (discussed in the section on "New Directions in Traffic Research.")

DESIRE LINES

Important as inter-zonal traffic volumes are, knowledge about them is not sufficient for planning an effective traffic network. The routes people will wish to use in getting from zone A to zone Z, from zone B to zone X, and so on, must also be known. The phrase, "routes people will wish to use" needs special emphasis to underscore the fact that a traffic plan designed primarily to improve presently congested facilities may simply perpetuate out-of-the-way routes tripmakers are forced to follow because facilities for more direct routes are non-existent or totally inadequate. To avoid such perpetuation of the undesired, the pattern of "desire lines" must be considered.

A desire line is simply a straight "as-the-crow-would-fly" path traced from the origin to the destination of a particular trip. When such lines—based on inter-zonal traffic volume data—are drawn on a map for all trips made on an average weekday, the result is the "desire line density map." A map showing desire-line densities graphically demonstrates: (a) the inadequacies of existing facilities; (b) where new expressways and improved surface arterials would be most effective; and (c) where mass transit facilities should be located. Thus, knowledge about desire-line density changes—changes which will occur in accordance with alterations in inter-zonal traffic volumes—is a basic concern of the continuing traffic study.

TRAFFIC ASSIGNMENT

In Detroit, as in other cities, desire-line densities established by the original O-D study were used as a basis for formulating a new expressway plan. The proposed network was tested by "assigning" predicted inter-zonal traffic volumes to it. Assignment was in accordance with what may be termed a best-path procedure— that is, it was accomplished by (a) calculating the time and distance of both the best expressway route and the best surface street route between each pair of zones, and (b) assigning a part of each inter-zonal volume to the expressways, the part being proportional to the time-and-distance advantage of the expressway route.

A count was kept of the number of vehicles assigned to each segment of the network. These counts served the two functions of determining whether the segments planned could handle the tripmakers desiring to use them, and demonstrating that the degree of use they would receive would be sufficient to justify the cost of constructing them.

If, as in Detroit, the ideal expressway network proves too costly to be built in the foreseeable future, personnel of the continuing traffic study are faced with still another important task. They must test, through traffic assignment where necessary and with adequate new techniques when possible, the constantly modified partial networks proposed as a compromise between the real and the ideal so far as traffic movement is concerned.

NEW DIRECTIONS IN TRAFFIC RESEARCH

Some of the standard methods of obtaining, using, and up-dating data describing inter-zonal traffic volumes, desire-line densities, and traffic assignment have been mentioned. The attendent problems described or implied justify a constant effort to refine the O-D data and techniques with which every continuing traffic study must begin. In Detroit, refinement efforts are presently proceeding in three major directions and are here termed "The Sample Survey," "Zone-Assignment," and "Computer Models."

The Sample Survey

In 1959, the D.A.T.S. began outlining the plans necessary for re-surveying its entire area on a small sample basis. Because the sample will be carefully drawn to represent its universe, the reliability of the results should closely approximate the reliability of a full-scale study. It is intended that the sample will include as many as possible of the households involved in the original O-D study, injecting a dynamic time-perspective lacking to date in most travel surveys. It should be clear that this sample re-survey will permit the following to be accomplished at the moderate cost:

1. It will test and refine the methods of the original O-D study.
2. It will reveal travel pattern changes caused by new developments (such as a rebuilt central business district), thus providing a basis for estimating the degree of error in the predictions of the purely mechanical iteration procedure.
3. It will provide an opportunity to test the contention that the only way to get full and accurate travel information is to interview every tripmaker in the household.
4. It will provide an opportunity to get some more data than is practical in the full-scale survey. Useful information can be obtained about weekend travel, the frequency of inter-city travel, exact trip purpose and route selection, reasons for use or non-use of expressways, and the like.
5. It will provide an opportunity to test travel-prediction formulas by revealing population characteristics associated most closely with amount, distance, direction, and mode of travel (a technique sometimes termed the gravity-model method).
6. It will facilitate the search for effective ways to combine original study area data with more recent data describing relatively new population concentrations.

Zone-Assignment

The important part played by desire-line densities in the effective traffic plan has been discussed previously. Not discussed, but perhaps equally important from the practical point of view, is the fact that the desire-line type of information has heretofore been dependent on a complex and time-consuming process of drawing, counting, and describing tens of thousands of separate lines. The process has, indeed, involved so much effort that it has probably been too often neglected with the ultimate result that some unrealistic traffic plans have been evolved from inter-zonal traffic volume data alone.

These considerations have prompted personnel of the D.A.T.S. to develop a new way of summarizing the desire-line-type of information. Termed "zone-assignment," the technique involves the assignment of inter-zonal traffic volumes to a network of linked zone centers on the basis of the best-path procedure. This permits desire-line-type densities to be presented in terms of just a few hundred links. Thus, desire-line densities are in effect simplified, yet they can be depicted both tabularly and graphically at a zonal level of detail.

Computer Models

Because the parts of every metropolitan traffic system are interdependent, alterations in one part of a system will have a greater or lesser effect on almost all other parts. This phenomenon is quite obvious where a network of expressways is imposed on an existing traffic pattern; less obvious, but in the aggregate perhaps just as important, are the results of seemingly minor changes. For example, a point of traffic congestion may appear or disappear when some new facility is built at a location far distant from the point.

In either case—where proposed or actual changes are major or minor—it is expedient to know what modifications may be needed in an existing street system for smooth accommodation to change. There are a number of methods whereby such knowledge may be obtained, but in terms of efficiency it seems evident that the most effective technique would be an electronic computer procedure which would assign inter-zonal volumes to a model of an entire transportation system, surface arterials as well as expressways included. In other words, what is needed is a machine simulating traffic flow and therefore showing the effects of modifications to the flow. To such a machine could be "fed" one proposed modification after another, until the flashing lights on the front of the machine indicated the "layout" having the least degree of congestion the particular community could afford. Accurate "traffic assignment" (as defined previously) would thus be achieved with speed and ease.

Although the perfect machine model of a complex traffic system remains but a distant goal, a significant step toward the goal has been achieved in Detroit. Here, a skeletal computer model has been completed—skeletal because of the limitations of available computers. The model incorporates traffic volumes between 1, 000 major intersections, termed links, and each link has been given values relative to its time-distance from every other link. When actual and predicted inter-zonal traffic volumes are assigned to the model, it produces a fairly detailed picture, suitable for tabular and for flow-map presentation, of the congestion existing on Detroit's major streets and expressways. It has already shown that some expressways, built to carry 100, 000 vehicles per day, would have to carry more than 300, 000 vehicles daily if all tripmakers were actually able to follow their best-path when moving from origin to destination; and it has shown that some arterial streets which theoretically should have near-zero traffic volume because they play a part in so few best-paths, are actually severely congested because they are carrying part of the excess load that cannot get onto the expressways. Such findings suggest that time-distance values assigned to the links in a computer model of a traffic system must include a variable that will reflect time and speed loss due to congestion; such findings also suggest that an efficient model must include other variables to reflect tripmaker choice, in proportion to amount of congestion, of second and third best-paths. These two improvements in the model will be accomplished if the basic research in traffic assignment now being conducted produces the results hoped for.

SUMMARY

It may be said that nearly all the methods available to social science research can be used by the continuing traffic study in keeping up-to-date, and in refining, the major categories (inter-zonal traffic volumes, desire-line densities, and traffic assignment) of O-D data. None of the available methods—including population estimates and forecasts, land-use inventories, iteration, prediction formulas, zonal growth formulas, small-scale re-interviewing, computer models, and the like—are cheap, quick, or easy. But their cost is relatively modest when viewed, first, as the only possible protection for the large investment of the full-scale O-D study, and second, as reliable insurance that the community involved will be faced with as few as possible of the social problems associated with an inadequate traffic plan.

Appraisal of Sample Size Based on Phoenix O-D Survey Data

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The objective of this research was to determine the relationship between home-interview type of O-D survey sampling rates and the error that can be expected in volumes accumulated from a survey. The data analyzed were taken from the home-interview phase of the 1957 Phoenix-Maricopa County traffic study.

Inasmuch as individual zone-to-zone movements have insufficient volume to be reliable indicators, a method called "trip tracing" was employed to accumulate individual zone-to-zone trip volumes for statistical study. By this method, trips are traced across the survey area in a straight line between origin and destination and accumulated in ¼-mi sections of a grid system superimposed over the survey area. An electronic computer was utilized for the trip-tracing procedure and for much of the statistical computations required for the study.

The trip data from the Phoenix-Maricopa survey were systematically sampled to provide ½, ⅓, and ⅒ subsamples of the original 1-in-15 dwelling-unit sample. The expanded results of the original survey and each of the subsamples were individually processed through the trip tracing program. This procedure provided the data necessary to determine the accuracy that could be expected in any volume due to sampling.

The results indicate that the accuracy of accumulated trip volumes is considerably less than the accuracy predicted by a purely theoretical approach. However, the results agree qualitatively with theory in that the accuracy of trip volumes varies with the square root of sample size and very nearly with the square root of the volume. The results of this research should allow the selection of a sample rate commensurate with the funds available and the degree of accuracy required.

●TRAVEL BY individuals has the characteristic of being habitual. Furthermore, the travel habits of different individuals are similar for the many types of trips generally made. For these reasons, sampling procedures, such as the home-interview type of O-D survey developed by the Bureau of Public Roads, can be used to determine travel information for a metropolitan area.

The accuracy of information developed from a home-interview survey, and indeed from any type of sampling procedure, is to an important degree dependent on the sampling rate used. Sampling rates employed in O-D surveys have usually been based on population and area. In a large, densely populated area a smaller sampling rate is used than in a small, less densely populated area. The sampling rate determined is based, predominantly, on the number of interviews needed to provide a reliable representation of the over-all travel in the city. The zone-to-zone movements developed, however, have been used to present a picture of travel desires and to aid in the location of needed transportation facilities.

The results of an O-D survey are also accumulated on, or assigned to, proposed highway facilities for design considerations. How accurate are these accumulated values that have been expanded from the survey sample to represent average daily movements? The purpose of this study was to determine the relationship between the home-interview type of O-D survey sampling rates and the accuracy of volumes that are accumulated therefrom.

Measures of the accuracy of accumulated volumes have been determined for various sampling rates and predictive equations developed for determining the sampling rate needed for a desired accuracy at any volume. Because sample rate affects the accuracy of volumes used for design purposes, an estimate can be made of the frequency that facilities so designed will be loaded beyond capacity or conversely, the frequency that such facilities will have more lanes than are actually required.

SOURCE OF DATA

The data used for this study were developed from the home-interview phase of the 1957 Phoenix-Maricopa County, Ariz., traffic survey.

The survey area covered 225 sq mi and contained a population of 397,000 persons. Of this, the City of Phoenix occupied 36.3 sq mi and included 192,500 persons. The survey area was divided into 135 zones, which were smaller in area in the downtown section of Phoenix and larger in the outlying areas.

The sample rate used in the home-interview phase of the O-D survey was 1 in every 15 dwelling units, or 6.67 percent. This rate resulted in a total sample of 8,743 dwelling units. The trips analyzed were those having both origin and destination within the survey area. All trips, regardless of mode of travel, were considered. The total expanded number of internal trips was 839,398. Separated by mode, it was as follows:

Automobile drivers	548,439
Automobile, truck and taxi passengers	233,536
Bus passengers	57,423

The survey personnel had checked the expanded household data against the 1950 decennial census and the 1953 special census with satisfactory results. Also the expanded travel data had been compared with two screenline ground counts made during the survey. The survey data accounted for 89.3 percent of the vehicles crossing the screenlines from 6 a.m. to 10 p.m. on an average weekday.

METHOD OF STUDY

Individual zone-to-zone movements obtained from a home-interview type of O-D survey cannot be used as the basis for sound conclusions regarding sample size due to the predominance of small volume movements between many zones (1). Any sample size, within the limits of economic feasibility, would be too small to produce accurate expanded values for these small movements; but the individual zone-to-zone movements do not necessarily have to be accurate if their summation reasonably represents travel accumulations throughout a city.

A method referred to as trip-tracing (1), has provided the means for determining and checking distribution of travel throughout an area. By this method zone-to-zone movements are traced across a city in a straight line between origin and destination. After the equation of the straight line between the two zones is obtained, the points of intersection of this line with previously established gridlines are determined. The volume of trips being traced is added to volumes previously accumulated, for other zone-to-zone movements, at the section of the gridline being crossed. After all zone-to-zone movements have been similarly traced across the city, the result is the accumulated number of trips per section of gridline.

The length of the sections of gridline, in which the volumes are accumulated, is chosen in accordance with the size of the accumulations desired. If large accumulated volumes are desired, large sections are used, and vice versa.

A grid system was superimposed over the Phoenix-Maricopa County traffic study area (Fig. 1). The southwest corner of the city was designed as origin of the rectangu-

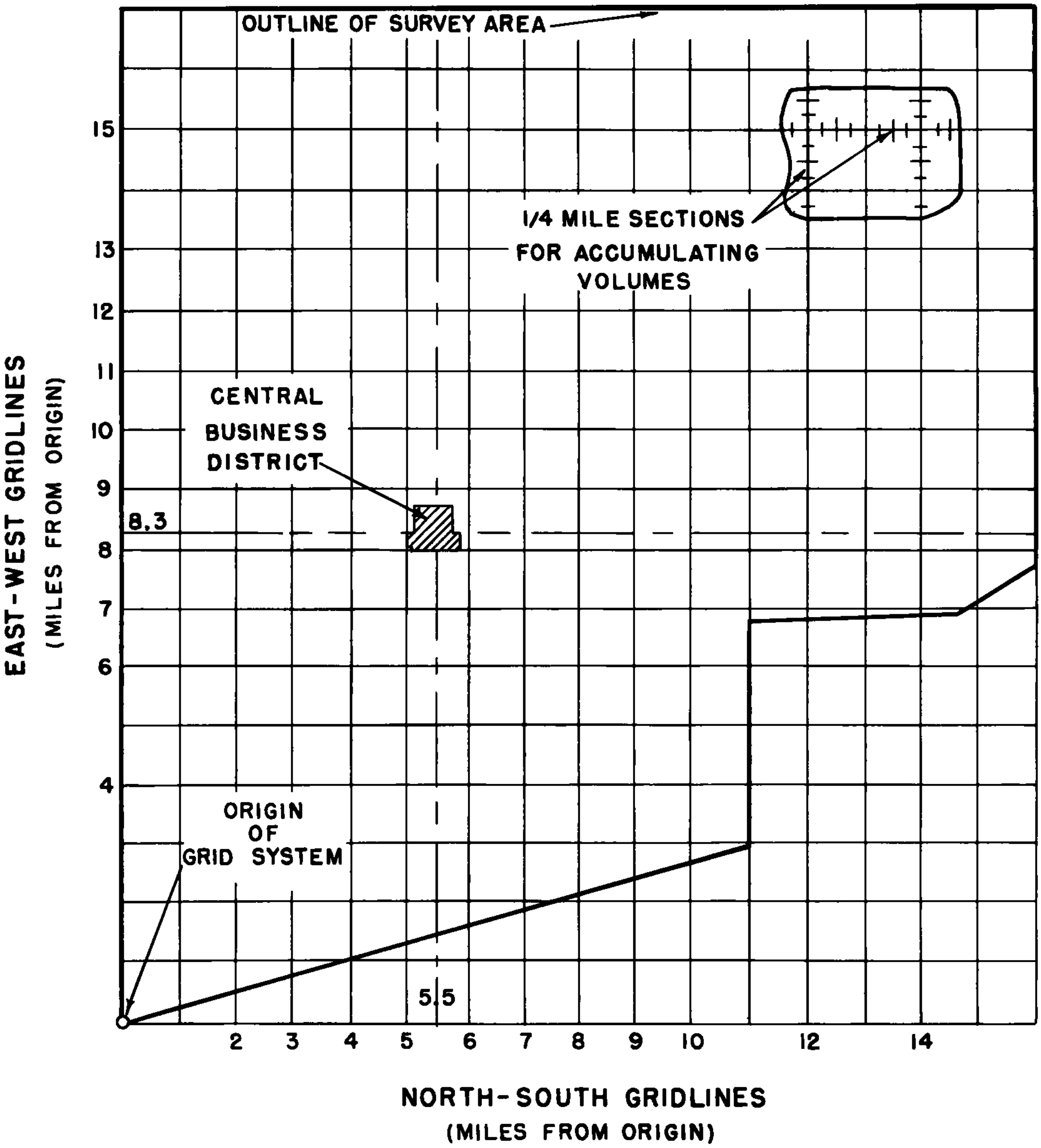

Figure 1. Design of grid system.

lar coordinate system. North-south gridlines were placed at 2, 3, 4, 5, 5.5, 6, 7, 8, 9, 10, 12, and 14 mi from the origin and east-west gridlines at 4, 6, 7, 8, 8.3, 9, 10, 11, 12, 13, and 15 mi from the origin. The closer spacing of gridlines was at the more densely populated part of the study area. Each gridline was broken into ¼-mi sections for volume accumulation purposes. The north-south gridlines were 17 mi long and the east-west lines, 16 mi long. This resulted in 1,520 ¼-mi sections (17 mi x 4 sections per mile x 12 gridlines) + (16 mi x 4 sections per mile x 11 gridlines). The volumes accumulated on the ¼-mi sections ranged from 0 to 35,000 trips.

Features of the procedure that have been described are as follows:

1. The result of the trip-trace accumulation is a spatial distribution of trips throughout the city representing the travel desires of the population. The trips are traced in

such a manner that each zone-to-zone movement is made by the most desirable path—a straight line.

2. Long trips are weighted more heavily than short trips because more gridlines are crossed. This is analogous to a long trip using more of a road network than a shorter trip.

3. The resulting accumulation of trips presents a picture of travel analogous to accumulations of trips on a street system.

The trip-tracing procedure is too lengthy for manual computation methods. It is, however, readily adaptable to computation on an electronic computer of medium size. Therefore, a computer program was developed to trace the Phoenix-Maricopa County trips across the study area. Computer programs were also developed to handle the

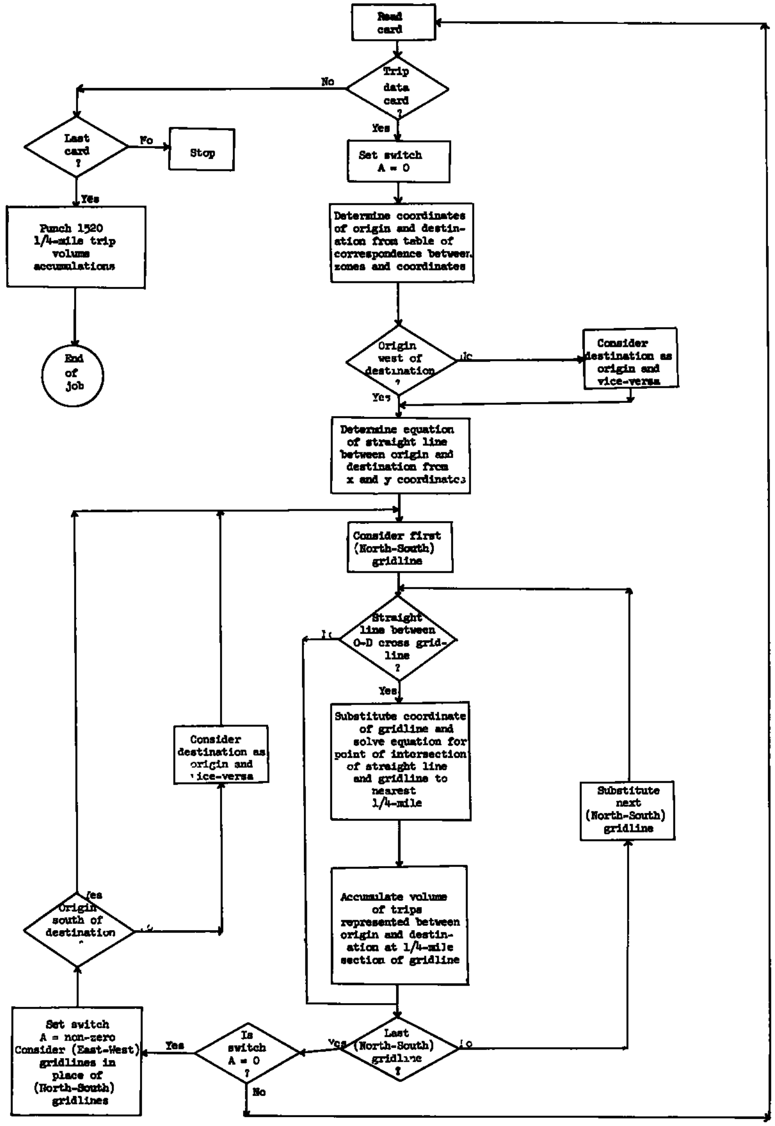

Figure 2. Generalized electronic computer flow chart for tracing trips across a grid system.

various statistical computations that were necessary for this study. A simplified flow chart of the trip-tracing program is shown in Figure 2. Generally, the following computations are carried out:

1. From the coordinates of the two zones being handled, determine the equation of the straight line passing between them.
2. Determine, to the nearest quarter of a mile, the points of intersection of the line and the north-south and east-west gridlines.
3. Add the number of trips being traced to the volumes previously accumulated in computer memory slots representing the 1, 520 ¼-mi sections.

All of the internal Phoenix-Maricopa County home-interview trip cards, representing the expanded results of the 1-in-15 dwelling-unit sample, were processed through the trip-tracing program to determine the spatial distribution of trips throughout the survey area. The original deck of cards was then sorted by sample number into 2 one-half subsamples, each representing a 1-in-30 dwelling-unit sample. Each one-half subsample was then separately run through the computer to determine the spatial distribution of the two 1-in-30 dwelling-unit samples. Similarly, the original deck was systematically stratified by sample number into 3 one-third subsamples, and 10 one-tenth subsamples, and each subsample processed through the trip-tracing program.

STATISTICAL PROCEDURE

Running of the total Phoenix-Maricopa County traffic survey data resulted in accumulated average daily volumes, in the 1, 520 ¼-mi sections, ranging from 0 to 35, 000 person-trips. Similarly, each of the subsamples processed through the trip-tracing program resulted in accumulated volumes in each section. It should be noted, that, as each subsample zone-to-zone movement was traced, the volume was expanded to represent actual movement. For example, as each one-third subsample trip card was processed, the trip factor on the card was multiplied by 3.

The data resulting from the trip-tracing program were analyzed by comparing, on a ¼-mi section basis, the expanded subsample accumulations against the total sample accumulations. It was not, however, considered practical to report the error for each section. Instead, the 1, 520 ¼-mi sections were stratified into 15 volume groups, and the individual errors were accumulated and summarized for each volume group. Such a process produced, for each of the three subsamples tested, 15 errors and the average volume at which the error occurred. The range, the number of sections, and the average volume at each volume group are given in Table 1. The sections of gridline were stratified into volume groups in accordance with the volumes accumulated in the sections from the expanded total sample.

The summarization of results, per volume group, for each subsample consisted of determining the differences in volumes accumulated per section from the original sample and the subsample, squaring the differences and accumulating the results of the squaring. The resulting summation was then divided by the number of sections in the volume group and the square root of the quotient taken. The result of this procedure is the root-mean-square error (RMS error) of the subsample as compared with the total sample. The equation for determining this error is:

$$\text{RMS error} = \sqrt{\frac{\sum_{i=1}^{i=n} (V_{ss} - V_s)^2}{n}} \qquad (1)$$

in which

RMS error = root-mean-square error
V_{ss} = volume accumulated in section i from subsample
V_s = volume accumulated in section i from total sample
n = number of sections in volume group

TABLE 1

VOLUME GROUP SUMMARY

Volume Group No.	Volume Group Range (Average Daily Person Trips)			Number of Sections Having Volumes Within Range[1]	Mean Volume of Group[2] (Average Daily Person Trips)
1	000.1	-	199.9	268	91.1
2	200.0	-	399.9	88	283.2
3	400.0	-	599.9	69	486.7
4	600.0	-	799.9	45	682.0
5	800.0	-	999.9	50	893.3
6	1,000.0	-	1,499.9	80	1,244.5
7	1,500.0	-	1,999.9	56	1,763.4
8	2,000.0	-	2,499.9	58	2,257.4
9	2,500.0	-	3,499.9	86	2,998.6
10	3,500.0	-	4,499.9	62	4,001.4
11	4,500.0	-	5,499.9	42	5,020.9
12	5,500.0	-	7,999.9	67	6,759.3
13	8,000.0	-	9,999.9	41	8,961.8
14	10,000.0	-	14,999.9	34	12,401.3
15	15,000 and over			28	21,689.9

[1] The number of sections does not equal 1,520 because some sections did not have any trips traced across them.

[2] The mean volume was determined from the total sample.

The RMS error is comparable, statistically, to the standard deviation of a group of values around their mean. For example, if the RMS error for a one-third subsample volume compared with the original sample volume is 50 person-trips, one would make little error by assuming that two-thirds of the expanded volumes obtained from the subsample would lie within 50 person-trips of the total sample volume.

RMS errors were developed for each subsample. That is, a RMS error was developed for each of the one-half subsamples, for each of the one-third subsamples, and for each of the one-tenth subsamples. Little is gained by reporting and analyzing each subsample error. Therefore, a mean error was determined from the 3 one-third subsamples, from the 2 one-half subsamples, and from the 10 one-tenth subsamples.

A RMS error was computed for each volume class, and the "percent root-mean-square error" was determined by dividing the numerical error by the average volume of the volume class being considered. The results of the one-third subsample comparison are given in Table 2.

RESULTS

The results of the trip-tracing program and the statistical procedure used are the percent root-mean-square (percent RMS) errors for the one-half, the one-third, and the one-tenth subsamples, each measured against the total Phoenix-Maricopa County sample. These results are given in Table 3 for each volume group.

As was to be expected, the percent RMS error for any particular volume group is invariably greatest for the smallest subsample rate, and vice versa. For example, for an average volume of 1,763 trips, volume group number 7, the percent RMS error for the one-half subsample is 15.3; for the one-third subsample, 23.5; and for one-tenth subsample 49.4. In addition, the percent RMS error, for each subsample, decreases as the volume increases. These decreases in percent RMS error, as volume increases, approximately follow a straight line if plotted on logarithmic paper.

It should be understood that the one-half subsample errors are in reality the percent RMS errors between a 1-in-30 and a 1-in-15 dwelling-unit sample. Likewise, the

TABLE 2

RESULTS OF ONE-THIRD SUBSAMPLE COMPARISON WITH TOTAL SAMPLE

Volume Group No.	Mean Volume of Group (Total Sample)	Mean Volume[1] 1st One-Third Subsample	Mean Volume[1] 2d One-Third Subsample	Mean Volume[1] 3d One-Third Subsample	Mean Error[2] 1st One-Third Subsample	Mean Error[2] 2d One-Third Subsample	Mean Error[2] 3d One-Third Subsample	RMS Error[3] 1st One-Third Subsample	RMS Error[3] 2d One-Third Subsample	RMS Error[3] 3d One-Third Subsample	Percent RMS Error[4] 1st One-Third Subsample	Percent RMS Error[4] 2d One-Third Subsample	Percent RMS Error[4] 3d One-Third Subsample	Average RMS Error of Three Samples: Absolute Error $\frac{\text{RMS Error 1st + 2d + 3d}}{3}$	Average RMS Error of Three Samples: Percent Error $\frac{\text{Absolute Error}}{\text{Original Sample Volume}}$
1	91.1	83.2	90.6	99.4	-7.9	-0.4	+8.3	77.5	75.1	78.9	85.07	82.44	86.61	77.2	84.74
2	283.2	231.2	309.5	308.9	-52.0	+26.3	+25.7	160.5	153.5	168.3	56.67	54.20	59.43	160.8	56.78
3	486.7	392.9	545.0	522.2	-93.8	+58.3	+35.5	205.6	237.4	225.7	42.24	48.78	46.37	222.9	45.80
4	682.0	623.2	710.9	711.9	-58.8	+28.9	+29.9	307.8	253.6	295.6	45.13	37.18	43.34	285.7	41.89
5	893.3	749.9	977.1	952.7	-143.3	+83.8	+59.5	300.5	306.4	267.8	33.64	34.30	29.98	291.6	32.64
6	1,244.5	1,153.8	1,325.3	1,254.2	-90.6	+80.9	+9.7	308.9	335.2	349.7	24.82	26.93	28.10	331.3	26.62
7	1,763.4	1,772.5	1,883.2	1,634.4	+9.1	+119.8	-128.9	390.8	427.0	424.9	22.16	24.21	24.10	414.2	23.49
8	2,257.4	2,123.7	2,631.0	2,287.5	-133.7	+373.6	+30.1	470.6	537.9	391.4	20.85	23.83	17.34	466.6	20.67
9	2,998.6	2,886.6	3,118.3	2,990.9	-112.0	+119.7	-7.7	513.6	578.4	503.6	17.13	19.29	16.79	531.9	17.74
10	4,001.4	3,938.8	3,995.6	4,069.8	-62.6	-5.8	+68.4	666.1	654.7	644.7	16.65	16.36	16.11	655.2	16.37
11	5,020.9	4,903.9	5,136.4	5,022.5	-117.0	+115.5	+1.6	727.0	667.7	729.5	14.48	13.30	14.53	708.1	14.10
12	6,759.3	6,490.2	6,628.3	7,159.4	-269.1	-131.0	+400.1	774.5	751.7	922.5	11.46	11.12	13.65	816.2	12.08
13	8,961.8	8,896.1	8,923.1	9,066.4	-65.7	-38.8	+104.5	878.5	780.4	910.2	9.80	8.71	10.16	856.4	9.56
14	12,401.3	12,285.2	12,461.4	12,457.2	-116.1	+60.1	+55.9	909.7	913.7	1,156.3	7.34	7.37	9.32	993.2	8.01
15	21,689.9	21,555.2	22,046.2	21,468.2	-134.7	+356.3	-221.7	1,741.0	1,808.9	1,655.6	8.03	8.34	7.63	1,735.2	8.00

[1] Mean volume $= \sum \frac{\left(\text{Segment volumes in volume group}\right)}{\text{Number of sections}}$

[2] Mean error $= \sum \left[\frac{\left(\text{Subsample volume} - \text{Original sample volume} \right)}{\text{Number of sections}} \right]$

[3] RMS error $= \sqrt{\frac{\sum \left[\left(\text{Subsample volume} - \text{Original sample volume} \right) \right]^2}{\text{Number of sections}}}$

[4] Percent RMS Error $= \frac{\text{RMS error}}{\text{Mean volume}}$

one-third subsample rate is actually a 1-in-45 dwelling-unit sample and the one-tenth subsample a 1-in-150 dwelling-unit sample, the error in each case being measured against the 1-in-15 sample.

The values given in Table 3 can be used in the following manner. If it is desired to estimate trips from a home-interview survey at the 4,000 average daily person-trip level, volume group number 10, for some specific design purpose, the use of a 1-in-30 dwelling-unit sample would produce a volume that is within 11.7 percent of the value that would have been obtained with a 1-in-15 sample two-thirds of the time. Likewise, the use of a 1-in-45 dwelling-unit sample would result in a volume that is within 16.4 percent of the value obtained by a 1-in-15 sample two-thirds of the time. If the probability of being within the 1-in-15 sampling rate volume 95 percent of the time is desired, two times the percent RMS error would be used. An expectancy of 99 percent would require three times the percent RMS error.

By using the values given in Table 3, the expected results of a 1-in-30, a 1-in-45, and a 1-in-150 dwelling-unit sample can be compared with that of a 1-in-15 sample. However, the prime purpose of this study was to determine the error between the volume determined from any dwelling-unit sample and the actual volume, the actual volume being the average daily person-trips measured over the study period. Evidently, only through an overwhelming expenditure of time and money could every person in a city be interviewed every day during the study period. However, through statistical procedures, using the comparisons of the 1-in-30, the 1-in-45, and the 1-in-150 dwelling-unit sampling rate with the 1-in-15 sample, an estimate of the error between any size sample and the total population can be determined.

The error between a volume determined from any of the subsamples and the true volume consists of two parts: (1) the error between the subsample volume and the total Phoenix-Maricopa County sample volume, and (2) the error between the total sample volume and the true volume. In statistical computations, for the analysis of variance, the total variance of a group of samples is equal to the "between sample" variance plus the "within sample" variance:

$$\sigma^2_{\text{total}} = \sigma^2_{\text{within}} + \sigma^2_{\text{between}} \quad (2)$$

TABLE 3

PERCENT ROOT-MEAN-SQUARE ERROR OF SUBSAMPLE VOLUME AS MEASURED AGAINST TOTAL SAMPLE VOLUME

Volume Group No.	Percent Root-Mean-Square Error of Subsample Volume Measured Against Total Sample Mean Volume of Group		
	One-Half Original Sample	One-Third Original Sample	One-Tenth Original Sample
1	67.5	84.7	193.0
2	41.4	56.8	120.7
3	28.4	45.8	88.2
4	27.6	41.9	80.2
5	23.1	32.6	67.7
6	18.0	26.6	55.7
7	15.3	23.5	49.4
8	13.4	20.7	45.0
9	14.2	17.7	36.6
10	11.7	16.4	33.8
11	10.2	14.1	29.0
12	9.3	12.1	26.4
13	6.7	9.6	21.4
14	5.4	8.0	18.5
15	3.8	8.0	16.0

in which

σ^2 = variance

The percent RMS errors computed for this study, as mentioned previously, are statistically comparable to the standard deviation of a group of values about their mean. Therefore, an equation for relating percent RMS errors, comparable to Eq. 2 is:

$$E^2_{ss-o} = E^2_{ss-s} + E^2_{s-o} \tag{3}$$

in which

E_{ss-o} = percent RMS error of subsample volume measured against true volume

E_{ss-s} = percent RMS error of subsample volume measured against total sample volume

E_{s-o} = percent RMS error of total sample volume measured against true volume

An equation for estimating the error for a sample from the error found for another independently selected sample is:

$$E_{ss-o} = E_{s-o}\sqrt{\frac{N_s}{N_{ss}}} \tag{4}$$

in which

N_s = number of interviews taken in original survey

N_{ss} = number of interviews represented in subsample

Assuming that this is an acceptable approximation to the situation being considered and substituting Eq. 4 into Eq. 3:

$$\frac{N_s}{N_{ss}} \; E^2_{s-o} = E^2_{ss-s} + E^2_{s-o}$$

or

$$E^2_{s-o} = \frac{E^2_{ss-s}}{\frac{N_s - N_{ss}}{N_{ss}}}$$

therefore,

$$E_{s-o} = \frac{E_{ss-s}}{\sqrt{\frac{N_s}{N_{ss}} - 1}} \tag{5}$$

Considering Eq. 5 for determining the percent RMS error between the total sample volume and the actual volume from the error determined from the one-tenth subsample, Eq. 5 then becomes:

$$E_{s-o} = \frac{E_{1/10-s}}{\sqrt{\frac{N_s}{1/10\,N_s} - 1}} = \frac{E_{1/10-s}}{\sqrt{10-1}}$$

or,

$$E_{s-o} = \frac{E_{1/10-s}}{3} \tag{6}$$

Likewise, the equations for determining the error of the total sample from the one-half and one-third subsample errors are:

$$E_{s-o} = \frac{E_{1/2-s}}{\sqrt{1}} = E_{1/2-s} \tag{7}$$

$$E_{s-o} = \frac{E_{1/3-s}}{\sqrt{2}} = \frac{E_{1/3-s}}{1.41} \tag{8}$$

Eqs. 6, 7, and 8 were used to determine independent estimates of the percent RMS errors in the total sample volumes, measured from the actual volume, from the one-tenth, one-half, and one-third subsample errors, respectively. If each of these independent calculations produced consistent estimates of the RMS error for the 1-in-15 sample, it appears reasonable to assume that other sampling rates would be equally consistent. These independent estimates of the error in the total sample are given in Table 4.

A comparison of the estimated percent RMS errors of the 1-in-15 dwelling-unit sample, for each volume group, shows little variation. The mean of the three estimates was, therefore, determined and plotted on logarithmic paper (Fig. 3). A least-squares fit determined from the points is also shown in Figure 3. The equation of the developed line is:

$$\text{percent RMS error of 1-in-15 dwelling-unit sample} = \frac{629.0}{\text{volume}^{0.4884}} \tag{9}$$

The coefficient of correlation developed is approximately one, indicating an almost perfect functional relationship between the two variables considered. In other words, the variation in percent RMS error is explained almost entirely by the variation in

TABLE 4

ESTIMATED PERCENT ROOT-MEAN-SQUARE ERROR OF ORIGINAL SAMPLE VOLUME AS MEASURED AGAINST POPULATION VOLUME

Volume Group No.	Estimated Percent Root-Mean-Square Error from			
	One-Half Sample	One-Third Sample	One-Tenth Sample	Mean
1	67.5	59.9	64.3	63.9
2	41.4	47.3	40.2	43.0
3	28.4	32.4	29.4	30.1
4	27.6	29.7	26.7	28.0
5	23.1	23.1	22.6	22.9
6	18.0	18.8	18.6	18.5
7	15.3	15.9	16.5	15.9
8	13.4	14.7	15.0	14.4
9	14.2	12.6	12.2	13.0
10	11.7	11.6	11.3	11.5
11	10.2	10.0	9.7	10.0
12	9.3	8.6	8.8	8.9
13	6.7	6.8	7.1	6.9
14	5.4	5.7	6.2	5.8
15	3.8	5.7	5.4	5.0

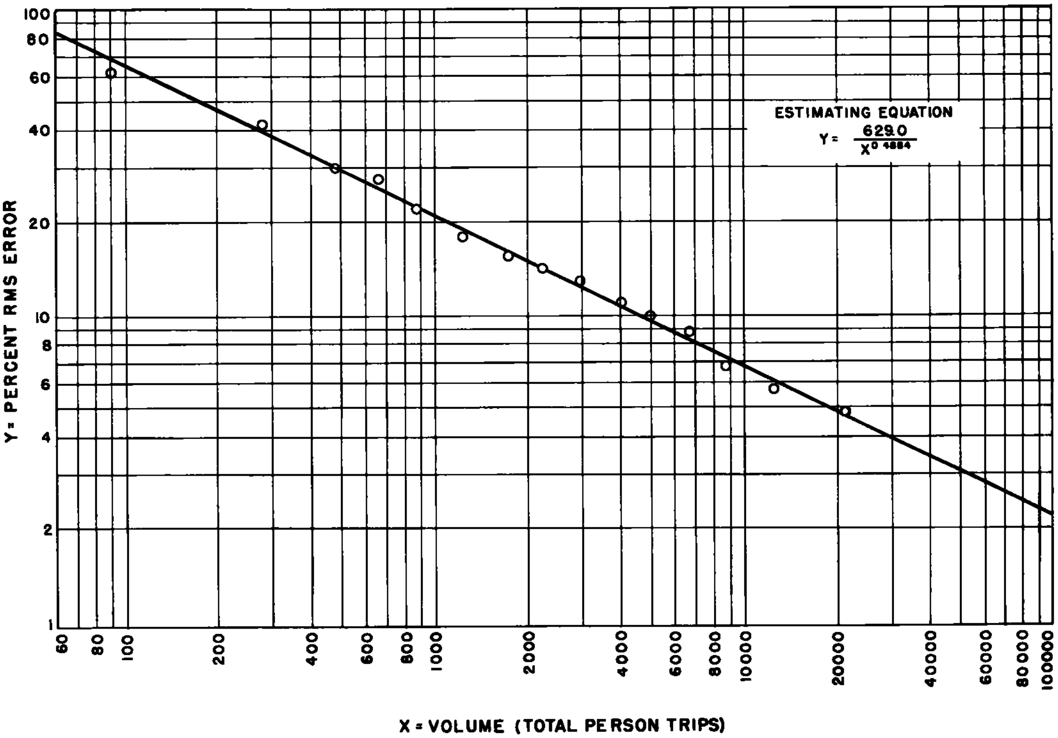

Figure 3. Relation of percent root-mean-square error and total person trips for 1-in-15 dwelling-unit sample.

volume. Because this is a logarithmic relationship the standard error of estimate for the line is a constant percent of the estimated value rather than a constant percent error. The error, which is 5.65 percent of the estimated values, means that at a volume of 1,000 trips, the standard error is about 1.2 percent (5.65 percent x 21 percent) and at 10,000 trips the standard error is about 0.4 percent (5.65 percent x 6.8 percent).

Eq. 4 can be used, in the following form, for determining the relationship between percent RMS error and volume for any sample rate:

$$E_{i\text{-}o} = E_{1/15\text{-}o} \cdot \sqrt{N_i} \tag{10}$$

in which

$E_{i\text{-}o}$ = error of any sample i

$E_{1/15\text{-}o}$ = error of 1-in-15 dwelling-unit sample

N_i = number of times the sampling rate of survey i is less than 1 in 15. For example, if 1-in-60 rate is used, N_i would be 4. If the rate was 1 in 5, N_i would be 1/3.

If a 1-in-1 sample were taken, every person in the city interviewed once, the equation for determining percent RMS error would be:

$$\text{percent RMS error} = \frac{629.0 \times \sqrt{N_i}}{\text{volume}^{0.4884}}$$

in which

$N_i = 1/15$

therefore

$$\begin{array}{l}\text{percent RMS error for 1-in-1} \\ \quad\text{dwelling-unit sample}\end{array} = \frac{162.4}{\text{volume}^{0.4884}} \tag{11}$$

It should be noted that the error for a 1-in-1 dwelling-unit sample is correctly not zero, because a 1-in-1 sampling rate is not a 100 percent sample for determining average daily traffic during the survey period. Every person in a city would have to be interviewed about his travel for every day during the survey in order to obtain the universe of travel for that period.

Eq. 11 can be used for determining the equation for percent RMS error at any sample rate. Simply multiply Eq. 11 by the square root of the denominator of the sample rate ratio used. For example, if a 1-in-30 home-interview sample rate is used, multiply Eq. 11 by the square root of 30. Figure 4 shows predictive lines for estimating percent RMS errors for volumes between 100 and 100,000 person-trips per day for various sampling rates.

COMPARISON OF RESULTS

A theoretical approach to the problem of estimating the accuracy of various sample sizes relies on the estimation of the standard deviation in volumes determined from the samples. The theory states that the expected deviation (σ) is expressed as follows:

$$\sigma = \sqrt{(\text{volume}) \times \left(\frac{\text{percent sample}}{100}\right) \times \left(1 - \frac{\text{percent sample}}{100}\right)}$$

Inasmuch as the probable volume $(\overline{V})$ is equal to the volume obtained from the survey times the sample rate:

$$\frac{\sigma}{\overline{V}} = \sqrt{\frac{(\text{volume}) \times \left(\frac{\text{percent sample}}{100}\right) \times \left(1 - \frac{\text{percent sample}}{100}\right)}{(\text{volume}) \times \left(\frac{\text{percent sample}}{100}\right)}}$$

or

$$\text{percent } \sigma = 100 \sqrt{\frac{(100 - \text{percent sample})}{(\text{volume}) \times (\text{percent sample})}}$$

A comparison of the errors predicted by the preceding theory with the relationship developed in this paper is given in Table 5 for various volumes and sample sizes. It can be seen that the observed root-mean-square errors are from 1.7 to 1.9 times as great as the error predicted by theory. This difference may be due to nonsampling errors such as response and coding errors. For example, a study made in Cincinnati, Ohio, shows that the respondent reports too few trips in some cases and too many trips in other cases.

TABLE 5

COMPARISON OF OBSERVED ERRORS AND THEORETICAL SAMPLING ERRORS

Sample Rate (%)	Volume	Observed Error (%)[1]	Theoretical Sampling Error (%)[2]	Observed Error / Theoretical Sampling Error
1	100	171.31	99.50	1.7
	1,000	55.64	31.46	1.8
	10,000	18.07	9.95	1.8
	100,000	5.87	3.15	1.9
3	100	98.91	56.87	1.7
	1,000	32.21	17.98	1.8
	10,000	10.43	5.69	1.8
	100,000	3.39	1.80	1.9
4	100	85.65	48.99	1.7
	1,000	27.82	15.49	1.8
	10,000	9.04	4.90	1.8
	100,000	2.94	1.55	1.9
5	100	76.60	43.60	1.8
	1,000	24.90	13.80	1.8
	10,000	8.10	4.36	1.8
	100,000	2.62	1.38	1.9
10	100	51.50	30.00	1.7
	1,000	16.70	9.50	1.8
	10,000	5.43	3.00	1.8
	100,000	1.76	0.95	1.9

[1] This is the percent root-mean-square error as developed in this paper.
[2] This is the theoretical percent standard deviation error.

As Table 5 shows, the observed error is almost two times as great as the theoretical error. This means that in order to maintain a desired degree of accuracy, it is necessary to increase the sampling rate to almost four times the rate indicated by the theoretical standard deviation computation.

USE OF RESULTS

The curves plotted in Figure 4 have been developed for total person-trips and can be utilized to determine the sample rate to be used when desiring an estimate of volume with a desired degree of accuracy. For example, if it is desired to estimate an average daily volume at the 10,000 person-trip level, and be within 8 percent of the true value 95 percent of the time, from Figure 4 at volume equal to 10,000 and percent RMS error equal to 4 percent (8 percent ÷ 2 for 95 percent confidence), it is found that a

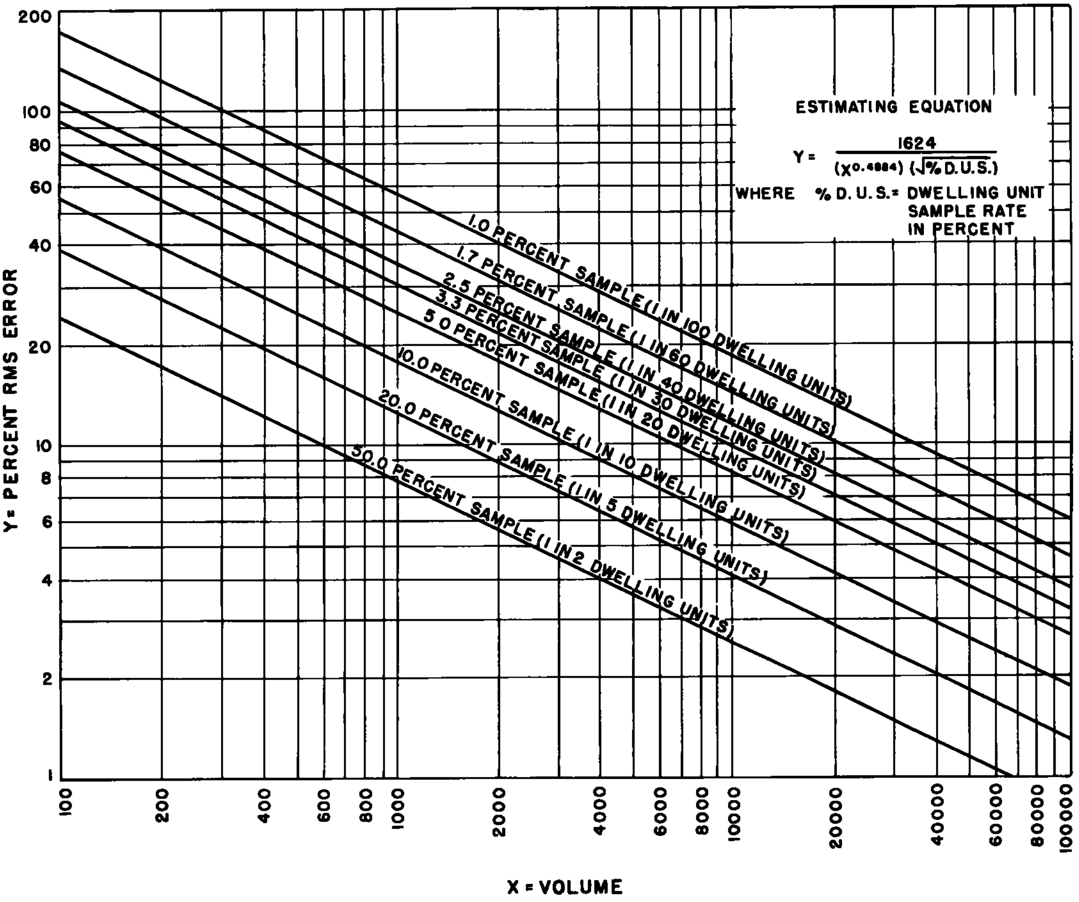

Figure 4. Relation of percent root-mean-square error and volume for various dwelling unit sample rates.

1-in-5 sample should be taken. Similarly, the curves can be used to estimate the error in accumulated volumes after the results of an O-D survey are obtained. For rates not plotted, Eq. 11 would be used in the following form:

$$\text{Dwelling-unit sample rate in percent} = \left[\frac{1,624}{(\text{percent RMS error})(\text{volume}^{0.4884})}\right]^2 \qquad (12)$$

For the problem explained, a dwelling-unit sample rate of 20.4 percent would result from the use of Eq. 12 indicating that 1 out of every 5 dwelling units should be interviewed.

More often than not, the highway engineer is interested in the number of vehicle-trips rather than the number of person-trips—vehicle-trips being the figure used for highway design purposes. Therefore, the question is: Can the curves developed for person-trips be used as an indicator of error for vehicle-trips?

The volume of automobile vehicle-trips throughout a city is less than the volume of person-trips, but is similarly distributed. The errors, if developed between the subsamples and total sample for automobile vehicle-trips, should not, therefore, be any different from the errors determined for all person-trips. That is, a percent RMS error for 10,000 automobile vehicle-trips should be no different from the error determined for 10,000 person-trips. The curves presented in Figure 4 and the equations developed can therefore be used for total person-trips, automobile vehicle-trips, bus-passenger trips, truck- and taxi-passenger trips, and automobile passenger-trips. The results of this study can also be applied to any home-interview type of O-D survey be cause accumulation of trips and not individual zone-to-zone movements are being compared.

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Panel Discussion on Inter-Area Travel Formulas

JOHN T. LYNCH, Chief, Planning Research Branch, U.S. Bureau of Public Roads, Moderator;
GLENN E. BROKKE, Highway Research Engineer, U.S. Bureau of Public Roads;
ALAN M. VOORHEES, Traffic Planning Engineer, Automotive Safety Foundation, Washington, D.C.; and
MORTON SCHNEIDER, Chief of System Research, Chicago Area Transportation Study

Mr. Lynch

Comprehensive origin-and-destination (O-D) surveys have been conducted in more than 130 urban areas during the past 16 years, and repeat surveys have been made in many of these. The means for developing and analyzing the current travel pattern is, therefore, well established. But the real problem is to predict the travel pattern at some time in the future, for it is on such a prediction that the highway network layout and design must be based.

In the field of forecasting future travel, many theories have been propounded and numerous procedures have been developed. They have one thing in common—they all recognize the fact that future travel must depend on the kind, intensity, and direction of urban development. For it is the size and distribution of the population and the commercial and industrial centers that will determine the travel pattern. Some of the methods being used or proposed are as follows:

1. The Growth Factor method, including the Fratar formula among others, projects the present travel pattern forward on the basis of anticipated growth in different areas.
2. The Gravity Model and modifications of it, used in many cities, assume that the travel between two areas depends on their attractive power and the distance between them, similar to the law of gravitation.
3. At the O-D survey committee meeting (January 11, 1960) Howe of the University of Cincinnati propounded an "electrostatic field" theory of trip attraction.
4. The model being used in Chicago introduces the concept of "intervening opportunities" on the theory that absolute distance is of lesser importance than the availability of opportunities to fulfill the travel desire at nearer locations. It assumes that travel will take place in such a manner as to minimize time.
5. The late Sam Osofsky of the California Division of Highways developed a multiple regression method of forecasting traffic volumes that he claimed to be better than either the Fratar or the Gravity Model method.
6. J.G. Waldrop, of the British Road Research Laboratory has developed a model for forecasting the distribution of traffic on a road system based primarily on the cost of travel.

Here are 6 methods based on different theories, probably producing significantly different results. They have not been fully evaluated statistically and compared, though research projects are under way with this objective. But some of them have been widely used because the road program if it is to fulfill the pressing needs of traffic cannot await the standardization of procedures, which may not be accomplished for several years.

At the present time there is considerable controversy in this field and on this panel there are three men who can be expected to bring out some of the controversial points.

Glenn E. Brokke, of the Bureau of Public Roads, is one of the principal advocates of the Growth Factor method. Alan M. Voorhees of the Automotive Safety Foundation has done much in developing and applying the Gravity Model. Morton Schneider of the Chicago Area Transportation Study was principally responsible for developing the Chicago Model.

Mr. Brokke

Two problems are involved. One is concerned with the method that one would use today to forecast trips to 1980. Here, the question is not so much how perfect is the method but rather is it better than any other fully developed procedure and does it provide an acceptable degree of accuracy?

The other problem is concerned with the research aspects. Is there any room for improvement of our present procedures? Most everyone would agree that there is. The difficulty is in evaluating improvement. Hindsight is always 20/20 but foresight is often less acute. Because all methods now being considered are of relatively recent origin, none, so far as is known, have received the acid test of time. Therefore evaluations must necessarily be somewhat synthetic and subject to the vagueness of statistical inference.

With these limitations in mind a return to the problem of which method is best for use today is in order. A test of various growth factor models two years ago determined the errors at various volume ranges (1). A re-examination of these errors in the light of the Phoenix data presented by Sosslau (2) indicates that if the results can be considered applicable to the two surveys in Washington, D.C., the growth factor method was about as accurate in projecting 1948 data to 1955 as a 1 in 35 sample O-D survey in 1955 would have been. This degree of accuracy is really quite good and should it continue to hold for more comprehensive tests over longer time intervals, it does represent an acceptable standard to be used in evaluating alternate procedures.

Within the last year or so an interarea travel formula was developed and used in a large western city. The volumes as determined from the formula and also as measured in an O-D survey were each assigned to a highway network by identical criteria. The differences in the assigned volumes indicate that this particular equation had an accuracy in duplicating present, not future, trips about equivalent to a 1 in 175 sample O-D survey.

The State of California has developed a multiple regression approach to forecasting urban area traffic volumes as reported by Sam Osofsky at the eighth WASHO Planning Conference in April 1959. This study indicates that a very considerable increase in accuracy is obtained by using an individual formula for each zone rather than area wide equations. It is to be noted that this method involves a stratification by zone rather than by trip purpose although land-use factors for the individual zones may produce somewhat similar effects.

This brings up the point of having a standard method of measuring the accuracy of various travel distribution equations. The method described by Sosslau (2) appears to be unbiased, sensitive, and relatively easy to accomplish with modern computers. Its use in forthcoming tests is anticipated.

Before leaving the urban problem, two rather new features of the growth factor method should be reported. One feature has the purpose of alleviating the difficulties caused by interzone volumes of zero. It is accomplished by combining low density zones with neighboring zones of similar character that have a more stable travel pattern. The forecast is made using the larger zones and the total volume is then prorated back to the smaller individual zones on a probability ratio.

The second feature is even more basic and may lead to a fundamental change in traffic assignment. From studies available in the Bureau of Public Roads, it can be shown that speeds during peak hours are significantly different from those during nonpeak hours. In addition, traffic counts at 282 urban locations throughout the nation indicate that morning peak hour volumes on expressways may vary from 3.9 percent to 15 percent of the daily traffic with from just more than 50 percent to as much as 94.6 percent of the traffic flowing in the heavier direction. In the afternoon, peak hour volumes vary from 4.2 percent to 17.8 percent of the daily traffic with as much as 93.4 percent of the traffic moving in the heavier direction. This variation of several hundred percent in design hour volumes can seriously affect the cost and utility of proposed highway improvements. The more intimate one's knowledge is of present methods of forecasting and assignment, the more clearly one will recognize that present procedures for estimating peak hour flow and directional split involve gross approximations from area wide averages.

The growth factor method can be applied to trips as represented by individual tabulating cards just as easily as it can be applied to total zone-to-zone movements, except that the computer will run a few minutes longer. If, then, it is assumed that future trips will be made during the same time as their present counterpart, these trips can be sorted into those made during the morning peak, the afternoon peak or any time period desired. These trips can then be assigned to a highway network by direction using travel times that are appropriate for the time period involved. The result is the assigned traffic volume by direction during the morning peak, the afternoon peak and the total for the day.

The Minnesota Highway Department has used these procedures in assignments for Minneapolis and St. Paul. The data developed indicate a pattern very similar to that found on existing expressways throughout the United States. Further research is needed, however, to demonstrate the effectiveness of this procedure.

Outside of the urban field, a formula of the gravity model type appears to have much merit in predicting travel between cities. Using data obtained from the external cordon survey at Detroit, Michigan, the following equation was developed:

$$\text{Trips}_{AB} = \frac{K\ \text{Population}_A \times \text{Population}_B}{\text{Distance}_{AB}^{\ n}}$$

If the populations of the two areas are measured in thousands and the distance between them in miles, the proportionality constant "K" and the exponent "n" for distance have the following average values for various vehicle types:

Vehicle Type	K	n
Passenger cars	157	2.49
Panels and pickups	27.5	2.81
Single unit trucks	13.3	2.66
Combinations	0.32	1.58
Total trips	156	2.44

The Bureau is in the process of enlarging this study by including data from other cities and at the same time expects to investigate other factors.

The principal problem is one of evaluating the various formulas. Until this is done, discussion usually involves more heat than light. Spot discrepancies to one observer will appear accidental or trivial; while to another, they will appear basic and conclusive. Confirmation of the over-all effects of the various equations must necessarily await increased knowledge and experience in the use of computers in the traffic field.

Progress, however, is inevitable and it is believed that soon the means will be found for recognizing and alleviating areas of traffic congestion before they occur.

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Mr. Voorhees

There are now two general types of techniques that are being used for traffic projections. The first type attempts to extrapolate existing patterns. In other words, on the basis of existing O-D patterns and anticipated growth within, various sections of the region traffic are projected into the future. The "growth factor" procedures fall in this category. The main advantage of these procedures is that unique travel patterns are taken into consideration. But the disadvantage is that they cannot be used effectively if there are substantial changes anticipated in land-use patterns or in transportation services. For example, the procedure cannot be used effectively to estimate traffic patterns that would result if an area were to change from an industrial to a residential area, or if a rapid transit system were developed.

The other approach is that of developing formulas or mathematical models which can be used in estimating traffic. Naturally, such techniques can be "designed" to evaluate the impact that changes in land use or transportation service would have on traffic patterns. In general, the models that are now in use can do this fairly effectively, but they do not consider the influence that social and economic factors have on travel patterns.

For example, a certain residential area might be closely tied to the downtown area because of some social and economic patterns that have developed in the community. The number of trips between these areas do not follow the "averages" that are estimated by the model. However, with proper application of mathematical models, adjustments can be made to account for these deviations.

An O-D study, for example, was conducted in Cedar Rapids, Iowa, and the state highway department analyzed this information and developed a mathematical model which reflected the "averages" derived from the observed data. They then followed this procedure in developing a complete O-D pattern for the area. Comparison was then made between the patterns obtained from the actual O-D and that derived from the model. It was found that in most cases it checked very well (within plus or minus 10 percent), though in a few instances modifications had to be made. This was achieved by using weights similar to those applied for trips to the downtown area in the recent Baltimore transportation study.

TYPES OF MODELS

Although there are many types of models that are now being considered and applied, most of them follow two general steps. First is that of determining trip production or the number of trips that start from an area, and the second step is that of determining the destination of these trips.

The procedure by which trip frequency information is calculated varies. Some base the estimates on the acres of residential, commercial or industrial land in a zone, whereas others consider car ownership, population and employment data. The use of the latter type of parameters seems to give better results from tests that have been made, as, after all, the number of people employed in an area dictates the number of work trips, not the number of acres in industrial use. If the parameters deal with car ownership, population, etc., frequency patterns are usually expressed in terms of trip purpose—work, commercial and social. If the parameters deal with acres of residential, commercial and industrial land, then the categories of trips are usually divided into land-use groups: such as, trips between residential and commercial areas, commercial and industrial areas, etc. This division of trips into categories is aimed at modifying any variations in trip behavior related to various activities. Generally, it has been found that if trips are divided into three or four categories sufficient breakdown is obtained to make the synthesis of existing traffic patterns fairly accurate. Further breakdowns would improve results somewhat, but the improvement does not warrant the extra cost.

Generally, there are about 4⅓ trips produced per car in the largest cities, 5 trips per car in cities between 250,000 and 500,000 and around 6 trips per car in cities of less than 100,000. It appears that this difference in trip frequency is related to the fact that in smaller communities the average trip is shorter, so more trips are made by the average individual. But, within limitations of city size, it does appear that trip production figures are very comparable throughout the country.

As indicated in Figure 1, the number of auto-travel shopping trips made from any residential area largely depends on the number of cars per dwelling unit. Generally, for every 1,000 cars in a residential area there are about 1,600 commercial trips made each day. Figure 1 shows that this number will vary depending on the type of trip. In all cases, except the work trip, the number of trips seems to increase

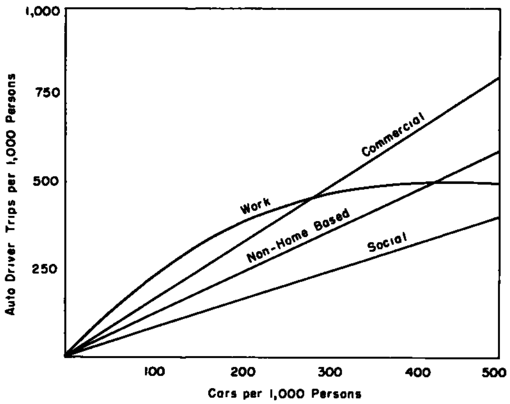

Figure 1. Auto driver trip frequency vs car ownership.

directly with car ownership. However, because there is a ceiling in the total number of work trips, due to the limitation in the size of the labor force, this pattern levels off (Fig. 1).

TRIP DISTRIBUTION

The next step is probably more difficult—that of determining the distribution of these trips. There are several mathematical procedures that have been used to estimate this. One is the multiple regression approach developed by Sam Osofsky, which has been used in California. Another is a linear programming method which has been proposed by Howard Bevis. Still another is a so-called opportunity model which is discussed further by Morton Schneider.

However, probably the most widely used model is the gravity model. In adjusting Newton's Law of Gravity to urban traffic it has been necessary to make several modifications. The adaption that seems to make most sense takes into consideration the relative travel time between various zones and the intensity of activity within these zones.

This type of model can be expressed by

$$V_{1-2} = V_1 \frac{\frac{M_2}{T^x_{1-2}}}{\frac{M_2}{T^x_{1-2}} + \frac{M_3}{T^x_{1-3}} + \cdots + \frac{M_n}{T^x_{1-n}}} \tag{1}$$

in which M designates the intensity of the activity in the zone in some appropriate unit by employment or population, T represents the travel time for the trip, and x is the empirically determined exponent.

The main advantage of this model is that it not only is sensitive to changes in travel time between the zones, but also takes into consideration competition between land uses. It is similar in concept to the opportunity model. The main difference between the two models is that they use different mathematical procedures to relate the two variables—travel time and intensity of activity within a zone (which is referred to as opportunities in Schneider's model).

In applying this type of model, generally trip patterns are divided into three or four types of trips. In the Baltimore and Hartford studies four trip purposes were used:

1. Home-based work trips.
2. Home-based commercial trips.

3. Home-based social trips.
4. Non-home based trips.

In both of these studies, in applying the model M was expressed in terms of employment in dealing with work trips; population for social trips; retail employment for commercial trips; and for non-home based trips a factor that equals the population plus 25 times the retail employment for each zone.

In the seven cities in Iowa, where the gravity model was used recently, the trip purposes were divided as follows:

1. Home-based work trips.
2. Other home based trips.
3. Non-home based trips.

In this case the M for work trips was expressed in terms of employment, other home-based trips by a factor that equaled the population plus 25 times retail employment plus employment for each zone. Non-home based trips were done in a similar manner.

In applying this model for work trips usually an iterative process is used to make the trip patterns conform to the number of workers that live or are employed in a zone. In studies that have been made in Iowa, it is quite clear that this process is not necessary for other types of trips. In fact, better results are obtained by not iterating to some predetermined number of trips based on land-use characteristics.

As already indicated, the gravity model only takes into consideration two variables. But there are other factors that influence travel habits, particularly those related to social and economic conditions. For example, Sears Roebuck, primarily because of its merchandising policies, is able to attract people from much greater distances than most stores of similar size. Social patterns in a community also influence social and recreational travel habits. Comparative tests between actual patterns and those developed from the model should be made. Significant variations should be corrected by simply adding weights to the model (3). However, this means that in forecasting traffic one has to estimate how these weights will perform through time.

To develop these weights, a systematic procedure should be developed. To reduce sample errors to a minimum it would be best to compare the observed trip patterns with the patterns estimated by the model on a district-by-district basis. This should clearly reveal the traffic patterns that are significantly influenced by social and economic forces. In addition, it would be advisable to correct for calibration errors made in determining average travel time within a zone and between adjacent zones. This should be done at the same time as the examination of district-to-district travel to detect social and economic influences. This weighting process has the additional advantage in that it will eliminate the need for an iterative procedure to bring the work trips into balance.

So, by this procedure the gravity model can be made sensitive to many factors. This flexibility is one of the main advantages of the gravity model.

Perhaps the most salient feature of the gravity model is that the parameters that are used, appear to be fairly constant and some have apparently held over a considerable period of time. For instance, the work trips in Hartford, Baltimore, San Francisco, Cedar Rapids, Iowa, or Wichita, Kansas, seem to follow the same basic patterns and these patterns can be calculated by using the gravity model. It has been noted in all these cities that travel time, if raised to the 0.8 power, will give good results. The fact that this is consistent throughout the country, and has also held over time (4), would indicate that the gravity model is approaching a universal law. In other words, the great advantage of Newton's Law of Gravity is that the distance factor in his calculation, when raised to the second power, has given good results when and wherever measured, and certainly an attempt should be made to achieve in traffic models the development of some technique that would be universally applicable.

Another advantage of the gravity model is that it is easy to understand and, therefore, easy to apply in any particular community. Numerous state highway departments and city officials have found it very easy to comprehend this procedure and, therefore have been able to follow through on their own in applying the technique.

The gravity model is also adaptable to computer programming, and has now been programmed for the IBM 704 and Univac. This permits one to use the gravity model quickly and cheaply in any particular area. Recently, one person in Frankfort, Kentucky, in the period of one month, was able to develop the existing and future traffic patterns by the use of a gravity model. During this period he also made numerous checks to compare the existing travel habits with the gravity model results.

However, there is one general weakness with the gravity model and that is that the concept of applied physics to human behavior is being used. It seems that one should be able to develop some procedure that would really be more fundamentally related to human behavior. Surely in the near future this will be done, and it will be possible to improve existing techniques. However, the more experimental results developed with the gravity model and these other techniques, the more can be learned about human behavior. Thus, a better understanding can be developed as to what should be included in a more sophisticated model that would interrelate all the factors that seem to be important in influencing urban travel.

Therefore, the most important thing is not personal liking for the gravity model or any of the other models, but that the value of applying models in urban transportation planning work is appreciated. Whatever model is selected and checked with the existing information, it will give more light as to what factors influence travel behavior. If this is done enough throughout the country, it will soon be possible to develop a sound procedure. But, until that time, in light of the experience with the gravity model, one can apply the gravity model with considerable confidence.

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Mr. Schneider

Perhaps the most meaningful interpretation of the Fratar method regards it as making this statement: if the trip populations of a set of zones are altered, the resulting interchanges are such that the ratios of the new interchanges to the corresponding original ones have a minimum dispersion, according to some measure, around 1, subject to the prescribed zone populations being satisfied. In its applied aspects the Fratar method is expensive and inflexible; it requires a complete O-D survey, fixed zonal and regional boundaries, and troublesome data handling and computations. In its support, the argument is commonly advanced that one cannot go too far wrong with the Fratar method, that it preserves the strongly established characteristics of the present. Undoubtedly this is true over a short term in which negligible change occurs, but considering that the avowed purpose of the technique is to deal with change, that point seems somewhat blunted. Besides, among the strongly established characteristics that it preserves is the instability of small number events.

A crucial flaw in the Fratar method is its handling of small interchanges, notably the limiting case of zero. If an interchange volume is surveyed as zero, the Fratar method cannot threaten, torture, or cajole it into becoming anything else—unless a zone volume at one end of the interchange grows from zero to something finite, and that would be an amusing situation inside a computer. To catalogue a few other conspicuous faults: the results are not independent of arbitrary procedures—a forecast performed through intermediate stages will not be the same as one performed directly in one jump; the method grants no effect whatever to changes in access, such as a whole new expressway system. An interesting extreme example is that of two cities, close to each other but separated by an impassable ravine. If a bridge is built across that ravine, the Fratar method becomes inapplicable.

On the other hand, no claim has been voiced that the Fratar method is strictly tenable, but merely that it is a good rough working tool. When it comes to this ground, these comments appear a bit captious. Working tools are not as easily come by as they may seem, and they can be more usable within their limits than elegant theory. The Fratar method certainly is neither foolish nor indefensible, but it does not contribute to an understanding of trip behavior.

The "gravity" formula declares, without preamble, that the interchange between two zones is proportional to the trip volumes at each of the zones and inversely proportional to some power of the distance (or travel time, or cost) between them. It has nothing at all to do with physical gravity, of course. Although this formulation avoids some of the shortcomings of the Fratar method, its behavior exhibits a number of peccadillos that disqualify it as a serious hypothesis (Appendix A). It can be shown that the formulation is not generally valid over an unlimited or undefined range of the distance variable, but can only be entertained within a region between some stated minimum and maximum distances. When these limits are given, however, the formula becomes a function of them, and they are quite arbitrary. Moreover, no tampering with the formula, not even a change of exponent, can yield the same results if the arbitrary boundary of the region is moved. And, to be usable, the exponent must be supposed stable from place to place and time to time. The author's experience leads him to doubt that.

Like the Fratar method, the gravity formula has a certain utility, but, in spite of a few arguments that have been voiced, no conceptual content is apparent.

The method being applied in the Chicago study rests on a premise that certainly sounds good: total travel time from a point is minimized subject to the condition that every destination point has a stated probability of being accepted if it is considered. If that probability is constant, the problems of choosing the minimizing order of consideration and deriving the consequences of probabilistic behavior are rather simple (Appendix B). It is less simple to say whether or not the working method, with its assumptions and approximations, extorted from this proposition is worthwhile.

It is not easy to assay the quality of a set of interchange predictions. Graphs and charts give an impression, of course, but cumulative curves are meretricious at best, and less seductive displays are hard on the eyes. The statistical measures suggested are not unreasonable, but they have a makeshift air to them and a slightly upsetting tendency towards comparing aggregate data and predictions; upsetting because it is obvious that any large-scale aggregation is specious. That the variance in individual interchanges need never be considered because interchanges are such small quantities and there are so many of them is a meaningless and perplexing point of view, like dismissing atomic weights because atoms are tiny and numerous. If these small quantities do not matter, why fool with them at all? If they do matter, what mystery causes deviations in them to cancel out in, say, the assignment process? Actually, deviations in interchange values may cancel each other in assignment, but the extent to which they do is a measure of the system's insensitivity to zone centroid locations (which is related to the complexity of the road network); if the assignment is insensitive to zone positions, then zones may as well be grouped into super-zones and super-interchanges developed. But the only point in assignment is to deal with the locational properties of trip ends, so as these properties are lost or dispersed the assignment itself becomes inane. If zonal definitions are properly geared to the assignment network, individual interchanges cannot be sanguinely, or even glumly, neglected.

Getting back to the merit of the Chicago method, the formula that evolves from the basic concepts turns out to be computationally convenient, and most well-behaved mathematically. Aside from precision effects, it is independent of zonal or regional boundaries; and it entails no special cases. It compares with data better than had been expected: computed interchanges assigned to a network by minimum paths does not give, on the planning scale, a noticeably different pattern from data interchanges assigned to the same network; the virulent attacks of members of the CATS staff on this method have not indicated that some other method would be just as good; using interchanges from seven scattered zones to all others in the Chicago study area as a base, it was found that seven different gravity models predicted the same data from which they were obtained less well than the over-all Chicago formula.

Not that the Chicago formula is all good. The dispersions from data are unquestionably larger than theoretical variance. The Chicago method shares two flaws with the gravity model: the number of trips received at a zone do not necessarily agree with the number provided, and there is distinct difficulty in obtaining parameters for future or unknown situations. On the first score, some comfort is derived from the received and provided totals being generally within 10 percent of each other; the discrepancies have been important clues to the functioning of the system and to defects in the formulation. On the second point, the staff is confident that fairly good estimates of unknown parameters can be obtained, and tests of that confidence are being conducted. But more than that, the formulation holds out direction and hope for defining them exactly.

Although the formulation has proved agreeably serviceable on the applied level, it seems that its most telling contributions are heuristic. It has introduced subtended volume, the volume of trip opportunities lying closer to the origin than the point of interest, as an explicit parameter and has attributed to it an explicit mechanical role. It has explicitly treated interchanges as probability numbers. It has focused fuzzy areas of study, and pointed to new ones. It has elicited sharp probings into such matters as the distributional properties of trips with respect to various parameters, and the meaning of it all. It is fair to say that discussions of trip behavior among the interested Chicago staff have taken on vitality and new color.

It is not intended to convey that the Chicago approach, even with present working simplifications removed, is a sovereign remedy. In fact, some work has been done in a different direction that may be more profitable ultimately. But the approach discussed here has given a practical method that seems better than any other available, while it has engendered much intellection and germinated many ideas. And with all respect to working tools, ideas are not so easy to come by, either.

Appendix A

The gravity formula may be written

$$V_{ij} = K_i V_i V_j / r^a_{ij} \qquad (1\text{-}1)$$

in which

V_{ij} = interchange from zone i to zone j;
V_k = total trip volume at zone k;
r_{ij} = distance (or travel time, or travel cost) between zones i and j;
a = a constant exponent expressing the resistance, or something, of distance; and
K_i = a normalizing constant—that is, the constant required by the condition $\Sigma_j V_{ij} = V_i$.

Because the formula is independent of zone size, it should hold for small zones, so that Eq. 1-1 can be thought of as

$$dV_i = K_i V_i p \, {}^{dA}/_{r^a} = 2\pi K_i V_i p \, r^{(1-a)} dr \qquad (1\text{-}2)$$

in which p is the average trip density in the annular region dA, at any r distance from the origin zone. From this it follows that the normalizing constant must be

$$K_i = 1/2\pi\bar{p} \int_B^C r^{(1-a)} dr \qquad (1\text{-}3)$$

Here, $\bar{p}$ is an average trip density for the region between r=C and r=B, in the sense that

$$\bar{p} \int_B^C r^{(1-a)} dr = \int_B^C p r^{(1-a)} dr$$

Integration of Eq. 1-3 gives

$$K_i = \frac{2-a}{2\pi\bar{p}\,(C^{(2-a)} - B^{(2-a)})} \quad \text{if } a \neq 2 \tag{1-4a}$$

$$K_i = \frac{1}{2\pi\bar{p}\log(C/B)} \quad \text{if } a = 2 \tag{1-4b}$$

Now $\bar{p}$, because it is an average, is only weakly affected by changes in the limits C and B—provided the interval of integration is reasonably large and encloses all singular regions, such as the CBD—and may be considered more or less constant in this argument. Thus K_i, and through it any interchange calculation, varies with the minimum and maximum distances used. Inspection of Eqs. 1-4 shows that the sensitivity of K_i to these limits increases as the exponent, a, moves away from the value 2; as a decreases, the sensitivity is more and more to the upper limit, C, while as a increases, the sensitivity shifts to the lower limit, B.

To show that a change of exponent cannot correct for a change in K_i (due to moving the boundaries of the region), it is only necessary to compute the new exponent, a', that would make an interchange computation the same under a new constant, K_i'. This can be done by simple algebra from Eq. 1-1, and the result is

$$a' = a + \frac{\log K_i'/K_i}{\log r_{ij}} \tag{1-5}$$

This is not a constant, as required by the formulation, but a function of distance: the formulation cannot be rectified, made to yield the same calculations, from one boundary situation to another. (K_i' is, of course, a function of a' as well as of the boundaries; but the final, purified solution for a' need not be obtained, inasmuch as the argument depends only on K_i' being different from K_i. That they are different follows from the hypothesis that they are equal: then a' would equal a, which implies the contradiction that K_i' does not equal K.)

Appendix B

If the probability of a destination point being acceptable is independent of the order in which destinations are considered, the order that will minimize travel time is clearly time proximity, from near to far. And the premise may be re-stated: a trip prefers to remain as short as possible, but its behavior is governed by a probability of stopping at any destination it encounters—it cannot always just go to the nearest destination and stop; it must consider the nearest destination, and if that is unacceptable consider the next nearest, and so on. To cast this into mathematical language: the probability that a trip will terminate within some volume of destination points is equal to the probability that this volume contains an acceptable destination, times the probability that an acceptable destination closer to the origin of the trip has not been found. But the latter two probabilities may vary from point to point, so the problem must be stated in terms of limitingly small quantities. This leads to

$$dP = (1-P)L\,dV \tag{2-1}$$

P is the probability the trip has terminated within the destination volume, V, lying earlier in the order of consideration (or subtended volume); L is the probability density (probability per destination) of destination acceptability at the point of consideration.

If L is constant, the only case discussed here, the solution of Eq. 2-1 is

$$P = 1 - ke^{-LV} \tag{2-2}$$

But K (the constant of integration) = 1, because P must be zero when V is zero, so

$$P = 1 - e^{-LV} \tag{2-3}$$

The expected interchange from zone i to zone j is simply the volume of trip origins at zone i multiplied by the probability of a trip terminating in j; that is,

$$V_{ij} = V_i \left[P(V+V_j) - P(V)\right] = V_i (e^{-LV} - e^{-L(V+V_j)}) \qquad (2\text{-}4)$$

An obvious extension of this is the supposition that, although L is constant for each trip, different trips have different L's. The more general equation then is

$$V_{ij} = \int_{L_{min}}^{L_{max}} (e^{-LV} - e^{-L(V+V_j)}) Z_i dL \qquad (2\text{-}5)$$

in which Z_i is the distribution of V_i with respect to L; that is, $Z_i = \frac{dV_i}{dL}$. Further, it can be argued that the destinations are also distributed in their affinities. This may be allowed (without going into detailed reasoning) by construing V and V_j in Eq. 2-5 as effective volumes, and functions of L. The computation of Eq. 2-5 cannot be realized in practice without far more understanding. But an attempt to adjust Eq. 2-4 in the direction of Eq. 2-5 can be made by clustering trips into "kindred" sub-populations with all members of a given sub-population being governed by the same L. The approximation to Eq. 2-5 is then

$$V_{ij} = \sum_s V_{i(s)} (e^{-L_{(s)}V_{(s)}} - e^{-L_{(s)}(V_{(s)} + V_{j(s)})}) \qquad (2\text{-}6)$$

The subscript (s) is the sub-population index. This is quite analogous to the stratification commonly used with gravity, iteration, and other models, but a little different in concept.

Readers with a taste for rigor may feel there is some mathematical license in treating discrete, unitary trip ends as a continuous "volume," and that a more proper form of Eq. 2-1 would be the difference equation

$$\Delta P = (1-P)L\Delta V = (1-P)L \quad (\text{since} \Delta V = 1). \qquad (2\text{-}7)$$

But Eq. 2-7 represents a well-behaved step function—piecewise continuous and everywhere finite—so the difference between it and Eq. 2-1 is one of precision rather than of kind, and the discrepancy introduced by integrating a continuous approximation to a step function is small if the number of steps is large. Differentials are preferable to differences, in this instance, simply for reasons of tractability—continuous expressions are amenable to generalization and adjustment, and they are usually more lucid. Eq. 2-7 can be solved easily enough, if L is constant, by stating it as $P_{n+1} - P_n = (1-P_n)L$ and then writing out the recursions (Eq. 2-1 can be solved by inspection), but any departure from that special case requires considerable manipulation.

Mr. Lynch

It is fairly clear that there is no agreement among experts as to the best method for projecting future urban travel. However, they all agree on one thing—that considerably more research is needed in this field. It is hoped that research now under way, or to be undertaken in the not too distant future, will result in general acceptance of one of the methods already developed or of a new method yet to be devised.

Multiple Screenline Study to Determine Statewide Traffic Patterns

MARK K. GREEN, Director of Traffic Analysis, Pennsylvania Department of Highways Planning Division

●FOR MANY YEARS past the Planning Division of the Pennsylvania Department of Highways has been conducting O-D studies throughout the state. The studies have varied in type and scope according to the population density of the area under study.

The extent of the studies ranged from the "Comprehensive Internal-External" type of survey used in large metropolitan areas, the "Parking-External" type of survey used for the smaller urban areas, and roadside interview surveys operated at a number of locations in rural areas of the state.

Although the surveys conducted varied in type, scope and costs, the motivation was the same—to study the highway needs of a relatively small area.

Over a period of years of analyzing the O-D information obtained both at spot locations in rural areas and that obtained from the external phase of the metropolitan area surveys throughout the state, a somewhat sketchy picture of the statewide traffic pattern was developed.

However, when it became apparent that an Interstate System of Highways was due to become something more than a mere dream, and that Congress was ready to implement a substantially increased over-all highway program, it became evident to the Planning Division of the Pennsylvania Department of Highways that the Division needed to know a lot more about its statewide traffic pattern than was known.

A comprehensive picture of the traffic pattern throughout the state would not only provide the information needed to determine the justification of proposed major highway locations but also enable the state to minimize the efforts of pressure groups proposing locations that cannot be justified by the facts.

The problem arose as to how to obtain this needed information quickly, economically, and in a form that would lend itself to rapid and easy processing.

After careful consideration of various approaches to this problem it was decided that a series of screenlines be established to obtain trip O-D information of vehicles using the highways at the four borders of the state and those using the state highway system at four locations within the state.

In addition, Pennsylvania Turnpike users would be interviewed at the west gateway, the east gateway, and at each interchange.

The primary purpose of this report is to present an outline and brief discussion of the Pennsylvania experiences in conducting and developing this multiple screenline study and to outline what has been accomplished.

Consistent to this general objective, no attempt is made to present all the detail as to the field operations or the techniques used in processing the survey information.

THE SURVEY

The field operations of the survey were conducted during the period from June 25, to September 13, 1957. Three interview crews were used throughout the survey.

Each crew consisted of an average of 11 men and a supervisor. Personnel used for the interviewing were students on summer leave from college and preparatory schools except for the crew supervisor and the portable machine man.

Each of the three crew supervisors were men with several years experience in conducting field operations for the Planning Division.

The Screenlines

Eight screenlines were established at which traffic was interviewed on all highways carrying significant traffic volumes. In general, highways carrying less than 500 vehicles per day were not interviewed. Exceptions were made in a few cases when considerable gap in the screenline would occur by not interviewing roadways under 500 vehicles.

The eight screenlines were established as follows:

1. An east-west screenline along the northern border of the state.
2. An east-west screenline extending from the west border to the east border approximately midway between the north border and the south border.
3. A north-south screenline extending from the north border to the south border on a line approximately midway between Lancaster and Reading.
4. A north-south screenline extending from the north border to the south border approximately midway between the east border and the west border.
5. A north-south screenline extending from the north border to the south border on a line approximately midway between Greensburg and Johnstown.
6. A north-south screenline along the western border of the state.
7. An east-west screenline along the southern border of the state.
8. A north-south screenline along the eastern border of the state.

The relative location of the eight screenlines is shown in Figure 1.

Screenlines one, six, seven and eight are in effect a cordon line bounding the state, while screenlines two, three, and four and five are internal screenlines.

Also shown in Figure 1 are 22 groupings of certain stations from the eight screenlines which may be used to form a cordon line around each of eight regional areas in the state.

Pennsylvania Turnpike Traffic

Turnpike traffic was interviewed at each interchange and at both the eastern and western gateway stations. All interviewing was conducted at toll booths, thus eliminating possible hazards from stopping high-speed traffic on the main roadway of the turnpike.

Trip O-D information was obtained only from drivers leaving the turnpike. Information as to the location where the driver entered the turnpike was obtained through the cooperation of the turnpike employees who handled the toll transaction.

Thus, with both the point of entry and point of exit recorded for each turnpike trip, mainline station data was constructed to complement other screenline data available at each of the three north-south internal screenlines.

Interview Stations

Two hundred stations were scheduled for operation. Three stations were not operated on schedule as the normal traffic pattern had been considerably disrupted by construction operations at or in the vicinity of the station. However, origin-destination at these locations was taken in the summer of 1959.

Traffic was interviewed in one direction at each station location. However, on major through highways, the direction of interviewing was reversed at the several screenlines in order to check the directional consistency or balance of flow as to trips crossing more than one screenline.

In selecting the location of the interview stations, one consideration was to avoid densely populated metropolitan areas where large volumes of commuter and other local traffic would be intercepted.

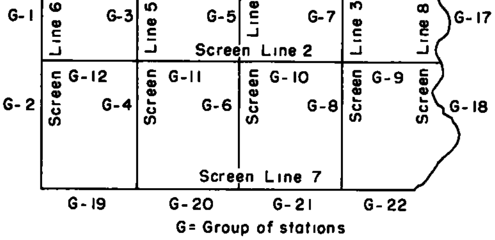


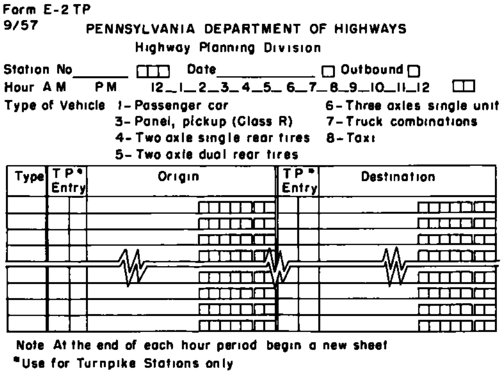
Figure 1.

The Interview Form

The interview form adopted for use in

the field is shown in Figure 2. It will be noted that only information as to the origin and destination of his trip need be obtained from the vehicle operator. Use of the short form interview blank required minimum delay to the motorist and considerably reduced the number of field personnel required for the survey.

In view of the fact that more detailed and precise information would need be obtained to analyze specific route locations, it was decided the origin-destination information to be obtained from the vehicle operator for the purpose of the statewide survey should be that of his "over-all trip," rather than from the "point of his last stop" to that of his "next intended stop."



Form E-2 TP
9/57
PENNSYLVANIA DEPARTMENT OF HIGHWAYS
Highway Planning Division

Station No______ ☐☐☐ Date__________ ☐ Outbound ☐
Hour A M PM 12_1_2_3_4_5_6_7_8_9_10_11_12 ☐☐
Type of Vehicle 1- Passenger car 3- Panel, pickup (Class R) 4- Two axle single rear tires 5- Two axle dual rear tires 6- Three axles single unit 7- Truck combinations 8- Taxi

Type	T P* Entry	Origin	T P* Entry	Destination

Note At the end of each hour period begin a new sheet
*Use for Turnpike Stations only

Figure 2. Reproduction of short form interview sheet.

A total of 279,892 usable interviews were obtained. The interviews represent a total of 764,250 vehicle trips. Figure 3 shows the number of interviews as compared to the ground count at each screenline.

Cost of Field Operations

The cost of the field operations amounted to $45,835.41.

The average cost per interview for the 279,892 interviews amounted to $0.1638.

This cost per interview considerably exceeds the average cost per external interview in metropolitan area surveys. However, a substantial portion of the cost per interview ($0.715) is represented by travel and personnel maintenance costs required due to a constantly shifting base of operations and the necessity of interviewing many routes carrying relatively small volumes of traffic.

A breakdown of the field costs is as follows:

Cost For Field Operations

Personal cars - (8 cents per mi)		$ 3,266.09
State cars & corps busses		2,486.35
Generators & trailers (for night operations)		608.00
	Sub-Total	6,360.44
Expense accounts - (food & lodging)		$13,660.77
Wages - (salary & hourly)		25,814.20
Total cost for field operations		$45,835.41

Coding of Data

The basic coding of the data is on a five digit code.

Information for trip origins-destination within the United States is available at the county level.

However, for the preparation of a general report all data is reduced to an area level as follows:

Trips to or from adjacent states were assigned to areas determined by known routes of travel into or out of Pennsylvania.

Trips to or from states other than adjacent states were grouped on a regional basis.

The basic coding for areas other than Pennsylvania and adjacent states is shown at the three digit level.

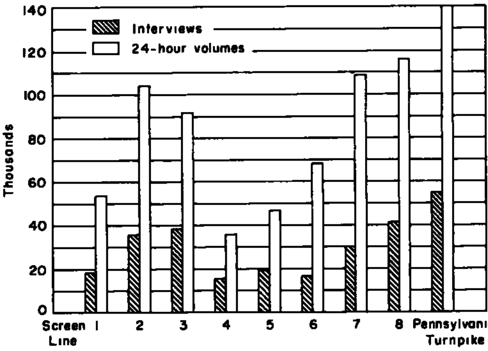


Figure 3. Number of interviews obtained compared to the number of vehicles counted in each survey operation.

Tables

The tabular information was then transferred to triangulated tables which present the total (non-directional) number of trips between any two areas and the number of intra-trips within the area in which the station was operated.

The tables are arranged to present the information in three major groupings:

1. Out of state to out of state traffic.
2. "Out of State" to "Pennsylvania" traffic.
3. "Pennsylvania to Pennsylvania" traffic.

Tables were prepared for each of the 198 interview stations. Composite tables were also prepared to show the origin-destination of all trips crossing each screenline.

Figures 4, 5, 6, and 7 are examples of the many possibilities for compiling significant trip data from the tables.

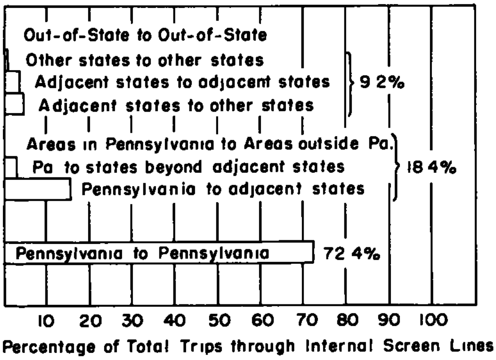


Figure 4.

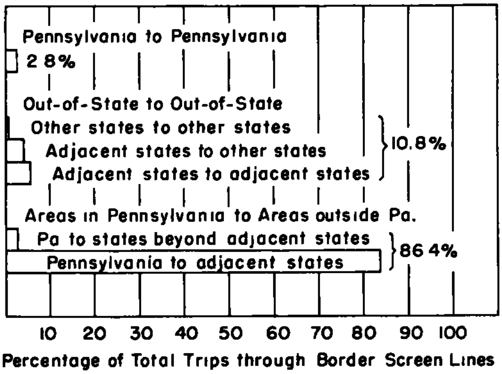


Figure 5.

Figure 7 emphasizes the high percentage of trips crossing the border screenlines that had a Pennsylvania origin or destination in areas adjacent to the screenline which they crossed.

It will be noted that a high percentage of the "out of state" termini for trips crossing the border screenlines were in "adjacent state" areas contiguous to the Pennsylvania border.

Thus it is reasonable to assume that the characteristics of the traffic crossing border screenlines is similar to the characteristics of traffic crossing the internal screenlines.

Screenline Check of Trip Data

A screenline check was made by tabulating all cards with the trip origin-destination as a major control and the number of trips through each screenline as the intermediate control.

Screenline checks of the data on the whole were excellent where trip volumes were significant. Some examples are given in Table 1.

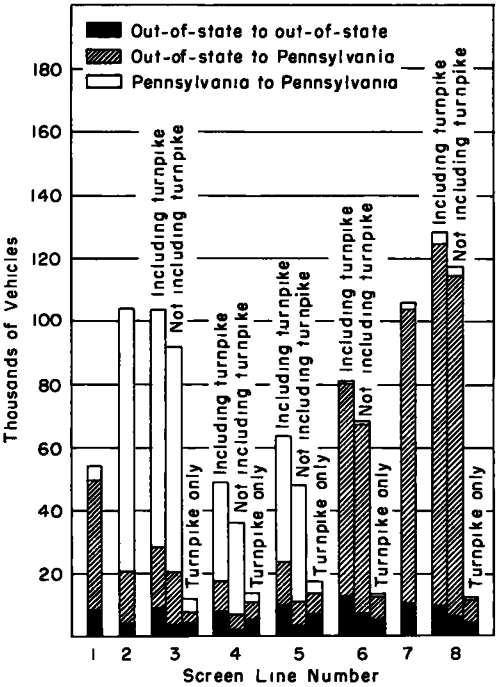


Figure 6. Showing the volume of "out of state to out of state," "out of state to Pennsylvania" and "Pennsylvania" to "Pennsylvania" traffic at each screenline.

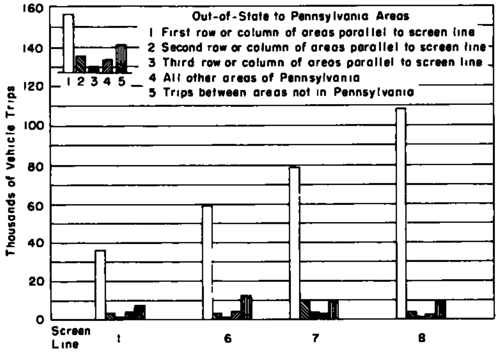


Figure 7. Number of trips through each border screenline to and from areas in Pennsylvania and between areas not in Pennsylvania.

Graphic Presentations

It was found that there are almost unlimited possibilities in preparing graphic presentations of the information, and each of these in turn could be presented for any or all of several vehicle classifications.

However, the number of graphic presentations necessarily must be limited for reasons of time and cost to those which have a practical value.

A basic concept in the preparation of the desired line exhibits is to show the relationship between the trip origins and destinations to population density and the major road network.

TABLE 1

O-D Area	Trips by Screenline (no.)							
	6	5	4	3	8[1]	1	2	7
1 - 19	82	95	86	80	49	-	5	5
1 - 18	12	17	11	14	12	-	2	-
1 - 74	6	12	14	-	-	-	-	-
1 - 99	12	20	11	8	-	-	-	-
1 - 20	5	-	-	-	-	5	-	
1 - 21	69	44	30	29	30	23	2	-
2 - 18	105	88	95	89	138	2	3	6
2 - 19	385	347	336	325	339	15	7	52
2 - 74	23	21	22	-	6	-	-	-
2 - 99	98	87	69	86	9	4	-	28
2 - 20	44	36	22	13	19	24	12	2
2 - 21	366	246	220	244	300	42	27	27
3 - 18	416	374	357	379	396	8	6	2
3 - 19	1,345	1,319	1,285	1,379	1,062	52	30	19
3 - 74	167	184	185	9	-	-	9	4
3 - 99	539	511	485	552	7	14	16	29
3 - 20	138	34	31	44	68	120	24	4
3 - 21	662	454	432	383	497	216	52	20
5 - 18	229	210	215	213	141	-	13	7
5 - 19	402	384	388	393	351	5	30	-
5 - 74	199	192	194	-	-	-	19	-
5 - 99	353	312	303	316	-	-	9	6
5 - 20	85	14	16	8	25	67	8	2
5 - 21	303	153	149	145	218	305	15	-

[1] Two key stations not available.

Turnpike Highlights

The Pennsylvania Turnpike at the western gateway was found to carry a volume of 13,960 vehicles. This volume represented approximately 17 percent of the total volume of vehicles using the highways interviewed at the western border screenline.

However, the "out of state" to "out of state" traffic using the Turnpike represented 45.4 percent of all "out of state" to "out of state" traffic found crossing the western border screenline.

The high percentage of long-haul traffic attracted to the Turnpike is readily accounted for by the extremely rugged character of the terrain in the west central section of Pennsylvania where a series of mountain ranges spread across the state in a generally southwest-northeast direction.

The Turnpike with its tunnels permits traffic to go through the mountain ranges instead of going over them and thus provides a minimum grade route with fully controlled access which affords substantial savings in both time and operation cost to users.

Land Use Forecasting for Transportation Planning

WALTER G. HANSEN, Highway Research Engineer, Bureau of Public Roads

●THIS REPORT is concerned with a recent attempt to develop a land-use forecasting procedure to be incorporated into the urban transportation planning process. The approach taken and the methods used were conditioned to a large extent by the function that such a procedure is required to perform in the larger process of developing urban transportation plans. Therefore, a brief review of this over-all planning process may be helpful in understanding and evaluating the investigations and results reported in this paper.

Figure 1 is a block diagram of the transportation planning process. The approach to transportation planning outlined in this diagram has evolved from the many transportation studies which have been made during the past decade. In particular, credit must be extended to the Detroit and Chicago studies which have contributed so heavily to the development of this process and the various techniques required.

In short, the planning process is composed of policy decisions and a series of estimating techniques, the latter being arranged in such a manner that it is possible to test and evaluate a transportation program in light of (a) anticipated future travel demands and (b) community land-use plans and objectives. Both of these checks must be made if a reasonable transportation program is to be developed (1).

Examination of Figure 1 shows that several conditions must be met by the land-use forecasting procedure if it is to fulfill its function. These conditions and characteristics are:

1. It is a distributive process. Previously developed aggregate forecasts of population and economic activity are to be distributed to transportation planning zones throughout the urban area.
2. It deals with only the net increase of population and economic activity. The existing pattern of land use is assumed to remain stable over the period of the forecast.
3. It must be sufficiently mechanical to be susceptible to machine processing.
4. The transportation network must be made explicit in the forecasting procedure. If this is not done it will be impossible to evaluate a transportation program in light of community land-use plans and objectives.

A RESIDENTIAL LAND-USE MODEL

It was decided that the first investigations of land-use forecasting would be limited to the development of a residential land-use forecasting model. This was done, first, because this type of land use constitutes the bulk of urban development, and second, it was felt that residential growth was more susceptible to systematic prediction because it is the result of numerous individual, relatively minor decisions rather than a few large decisions as is the case with commercial and industrial development.

Among the many factors which could be expected to influence the pattern of residential growth—topography, existing land-use patterns, transportation facilities, public services, and land development regulations, to mention a few—city planners, economists, and urban geographers have long emphasized the importance that accessibility has on the pattern of urban development. The general hypothesis, that the more accessible an area is to the various activities in a community the greater is its growth potential and probable intensity of development, has been expressed in one form or another for some time. This general proposition has received close to universal acceptance despite the complete lack of any attempts to quantify the relationship and test the hypothesis.

Consistent with this intuitive acceptance of a relationship between accessibility and land development, it was felt that a residential land-use model based on a realistic

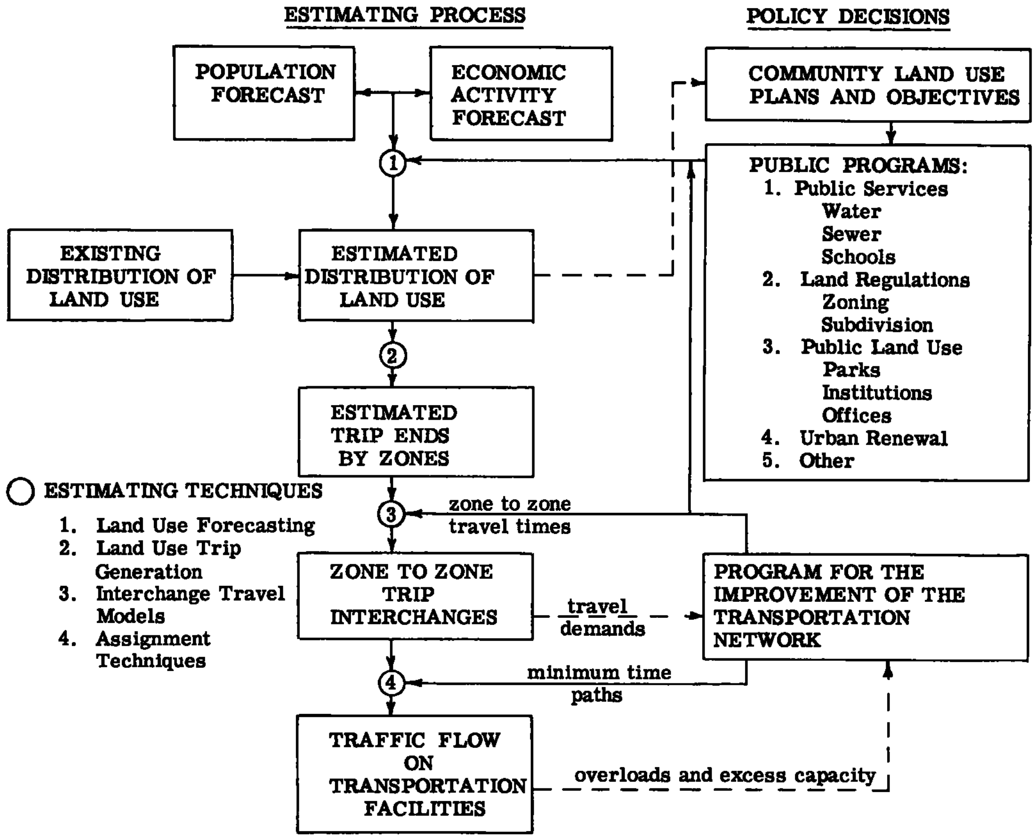


Figure 1. Transportation planning process.

measurement of accessibility along with the availability of vacant developable land could be established. Such a model would take the following form:

$$\frac{G_i}{G_T} = \frac{O_i\, f(A_i)}{O_i f(A_i) + O_j f(A_j) + \ldots\ O_n f(A_n)} \qquad (1)$$

in which

G_i is equal to the residential growth in zone i.

G_T is equal to the total residential growth in the urban area.

O_i is the vacant developable land in zone i.

A_i is a measure of the accessibility of zone i.

f is a function.

The model states that the proportion of the total urban area residential growth which can be expected to take place in any zone i is equal to that zone's attractiveness for development relative to the summation of the attractiveness of all zones, where the attractiveness of a zone is equal to the amount of developable land in a zone modified by some function of the accessibility of that zone.

As used in this investigation, accessibility is defined as the potential of opportunities for interaction. Stated in another manner it may be thought of as a measure of the effective opportunities for interaction at any zone i, created by the actual opportunities at any zone j. Stated in this manner it is obvious that the accessibility at zone i to a particular activity at zone j (say employment) varies directly with the size of the activity located at zone j (number of jobs) and varies inversely with some function of the distance separating zones i and j. More formally this concept is expressed in the following formula:

$$ {}_iA_j = \frac{S_j}{f(D_{i-j})} \tag{2} $$

in which

${}_iA_j$ is a measure of the accessibility at zone i to an activity located in zone j.

S_j is a measure of the size of the activity located in zone j; for example, number of jobs, population, retail sales.

D_{i-j} is a measure of the distance between or separation of zones i and j.

It was assumed that the function of distance in Eq. 2 would be exponential; that is, the measurement of distance would be raised to some power. This assumption was based on results of various examinations which have been made of urban travel patterns. In addition, to meet the condition that the transportation network be made explicit in the land-use forecasting model, the distance separating the various zones must be expressed in terms of travel time. Under these conditions the formula for calculating the total accessibility of a zone becomes:

$$ A_i = \frac{S_i}{T_{i-i}^{b}} + \frac{S_j}{T_{i-j}^{b}} + \cdots \frac{S_n}{T_{i-n}^{b}} = \sum_{i}^{x=n} \frac{S_x}{T_{i-x}^{b}} \tag{3} $$

in which

T_{i-x} equals the travel time between zone i and any zone x.

b is an exponent describing the magnitude of the effect that the separation T_{i-x} has on the possibility of interaction between zones i and x.

This measurement of accessibility contains elements of both the existing pattern of land use and the transportation system. It is a relative measure of the effective distribution of an activity around a zone.

Using Eq. 3 and data for the Washington, D.C., metropolitan area, an empirical examination was made to determine the relationship between this measurement of accessibility and residential growth. O-D studies, conducted in 1948 and repeated in 1955, supplied the bulk of the information required to calculate the accessibility measurements and to determine the pattern of residential growth over the 7-yr period. (The travel data used in the study were collected in 1948 and 1955 in two O-D surveys conducted by the Regional Highway Planning Committee for the Washington Metropolitan Area which was financed jointly by the highway departments of the District of Columbia, Maryland, Virginia, and by the Bureau of Public Roads. Land-use and economic data were obtained from the National Capital Planning Commission and the National Capital Regional Planning Council.)

Because a majority (80 percent) of all personal travel originating in residential areas is for work, shopping, or social purposes, this study was limited to an examination of the relationships between residential growth and accessibility to employment, shopping, and social opportunities.

Table 1 gives the types of data which were used to calculate the various measures of accessibility. The exponents of distance shown in this table were based on information made available by the Baltimore Transportation Study of 1957-58. (The exponent values are tentative. Additional research is being done by the Bureau of Public Roads to develop and statistically evaluate these exponents for the Washington, D.C., area.)

TABLE 1

DATA FOR ACCESSIBILITY CALCULATIONS

Accessibility to	Units Used to Express Activity Level of Zone S_i	Exponent of Distance[1] b
Employment opportunities	Number of jobs	2.20
Shopping opportunities	Annual retail sales	3.00
Social opportunities	Population	2.35

[1] Distance is expressed in minutes of off-peak driving time plus 5 to 8 min of terminal time.

Travel time was expressed in minutes of driving time plus 5 to 8 min of terminal time, depending on the location and density of the zone.

These examinations led to the development of the following residential land-use model:

$$\frac{G_i}{G_T} = \frac{O_i \; A\text{-}E_i^{3.04}}{O_i \; A\text{-}E_i^{3.04} + O_j \; A\text{-}E_j^{3.04} + \ldots O_n \; A\text{-}E_n^{3.04}} \qquad (4)$$

in which

G_i and G_T are increases in the number of dwelling units.

$A\text{-}E_i$ equals the accessibility to employment at zone i.

Other models could have been developed based on accessibility to retail activity or population or some combination of all three accessibility indexes; however, accessibility to employment was found to be most closely correlated to residential growth and it was not felt that the inclusion of the other measures of accessibility would result in any substantial improvement of the estimating accuracy of the model. This formula varies from one previously reported (2).

The model (Eq. 4) was used to estimate the distribution of the 1948-1955 residential growth (zonal increase in dwelling units) for the Washington, D.C., area. Comparison of these estimated growths to the actual increase of dwelling units in each zone showed that approximately three-quarters of the estimates were within 50 percent of the actual growths.

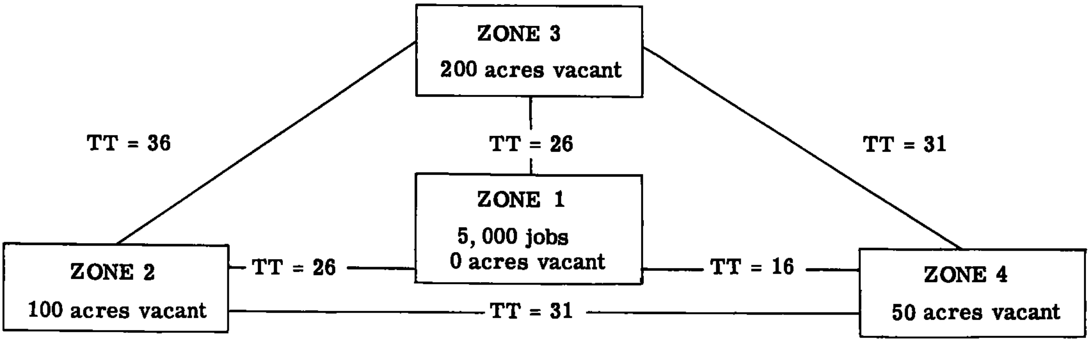
Although these results are quite promising and indicate that accessibility and the availability of land are major determinants in the pattern of residential growth, they do not in themselves produce sufficiently accurate estimates for planning purposes. Other factors must be examined and included in the model if a usable estimating process is to be developed.

The immediate value of the described model is that it makes it possible to examine empirically the effects that other factors such as zoning, taxes, and land costs have on the pattern of residential growth. The effect of such factors can be determined by studying their relationship to the differences between the actual distribution of growth and the estimated distribution of growth based on accessibility. In brief, the model can be used to establish a common reference base.

These examinations would provide both the city planner and the transportation planner with a deeper insight into the process of urban development.

An examination of this type is now being made in the Hartford, Conn., area. Several factors which seem to exert a substantial influence on residential growth have been identified: land costs, cost of providing utilities, zoning policies, land holding, and prestige. When these and other related factors with their relationship to residential growth have been quantified, it should be possible to develop a sufficiently accurate residential land-use model.

The following example is presented to help clarify the mechanics of the model and to demonstrate its potential value to transportation planning. The model used in this example is based solely on accessibility to employment and vacant land, and should not therefore be interpreted as a proposal, but rather as an illustration.



Area Num.	Accessibility to Employment $A_i = \sum_1^n \frac{\text{jobs}}{TT^{2.2}}$	Development Ratio $KD_i = A_i^{3.04}$	D_iO_i	% of Total Development $\frac{D_iO_i}{\sum_1^n DO}$	Residential Growth G_i (D. U.)
Case I					
2	$\frac{6,000}{26^{2.2}} = 4.6$	105	10,500	6.5	130
3	$\frac{6,000}{26^{2.2}} = 4.6$	105	21,000	13.0	260
4	$\frac{6,000}{16^{2.2}} = 13.3$	2,600	130,000	80.5	1,610
		$\sum_1^n DO =$	161,500	100.0	$G_T = 2,000$
Case II					
2	$\frac{6,000}{21^{2.2}} = 7.5$	460	46,000	23.4	468
3	$\frac{6,000}{26^{2.2}} = 4.6$	105	21,000	10.6	212
4	$\frac{6,000}{16^{2.2}} = 13.3$	2,600	130,000	66.0	1,320
		$\sum_1^n DO =$	197,000	100.0	$G_T = 2,000$
Case III					
2	$\frac{5,000}{26^{2.2}} + \frac{1,000}{36^{2.2}} = 4.2$	79	7,900	1.7	34
3	$\frac{5,000}{26^{2.2}} + \frac{1,000}{9^{2.2}} = 11.9$	1,850	370,000	80.0	1,600
4	$\frac{5,000}{16^{2.2}} + \frac{1,000}{31^{2.2}} = 11.6$	1,700	85,000	18.3	366
		$\sum_1^n DO =$	462,900	100.0	$G_T = 2,000$

TT = Travel time in minutes (driving plus terminal time).
Note: Intrazone travel time equals 9 min.

Figure 2. Illustration of land-use model.

Illustration of Model

Figure 2 is a diagram of a four-area hypothetical metropolitan region, showing the existing employment, vacant developable land, and the travel times between areas. Estimated growth for the entire metropolitan region, by some future point in time, is 2, 000 dwelling units and 1, 000 jobs. The distribution of residential growth will be forecast for each of the following cases:

Case I. The travel times between areas are the same in 1965 as at present and the increase in employment takes place in zone 1.

Case II. By 1965 an express highway is built between zones 1 and 2 reducing the travel time from 26 min to 21 min. The increase in employment takes place in zone 1.

Case III. The travel times between zones are the same in 1965 as at present and the increase in employment takes place in zone 3.

The calculations in Figure 2 demonstrate the potential value of this and similar land-use models to city and transportation planning. The model can assist the planner in assessing the probable effects of a given action; for example, the construction of an express highway (Case II) or a policy of decentralizing employment (Case III). Of particular importance is the fact that this determination of consequences need not be limited to some predetermined area of "influence," but can be assessed for all areas within the metropolitan region. For example, comparing Case II to Case I in the preceding illustration, the fact that zones 3 and 4 will experience less than expected growth due to the construction of the expressway may be more important than the increased growth in zone 2. Present estimating procedures would be unable to make such assessments.

By incorporating this land-use forecasting model into the transportation planning process (Fig. 1), it would be possible to determine the impact that these changes would have on the transportation system.

It should be pointed out that the reliability of this model for forecasting purposes is sensitive to the quantity of residential growth being distributed. The model distributes an increment of growth according to a constant set of factors. In reality, however, growth is a continuous process and the factors affecting its distribution, accessibility, vacant land, and other elements, are constantly changing. If a large increment of growth expected to occur over a fairly long period of time (20 yr) is distributed in a single application of the model, the resulting errors are likely to be quite large.

This does not mean that the model cannot be used for long-range forecasts; quite the contrary. When combined with estimates of the probable density of development in each zone, successive applications of the model can be used to synthesize urban growth up to any point in time. It is this aspect of the illustrated residential land-use model which offers the greatest potential value to the transportation planning process. When coupled with the traffic estimating procedures, it will be possible to examine the reasonableness of a transportation program in light of travel demands at several points in time. Such an examination would prove invaluable in determining priorities. It would also be possible to determine the timing and sequence of constructing segments of an expressway system which would best promote a desired distribution of metropolitan residential growth.

As other factors are examined and incorporated into the model—in particular these factors which are subject to change by conscious public action, such as zoning and the provision of public services—the model should become increasingly valuable for planning purposes. For example, it would be possible to coordinate a program of zoning regulations with a transportation program so that the transportation system will operate with maximum efficiency.

In conclusion, it is felt that a prototype of the land-use forecasting procedure required for transportation planning purposes has been developed. However, much hard work remains to be done before the potential value of this procedure can be realized. It is hoped that this paper will stimulate others to investigate this potentially productive area of research.

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New York Port Authority's 1958 O-D Survey Using Continuous Sampling

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●THE PORT OF NEW YORK AUTHORITY in 1958 adopted a continuous sampling method of conducting roadside O-D surveys. Previously, so-called "one shot" surveys in which one to three days of traffic were surveyed as representative of an entire year's traffic had been employed. The thinking behind this new method and the technique employed are reviewed and a resumé of the 1958 Survey results is given.

Briefly, this new technique is based on a carefully designed and controlled probability sample which builds up the interviews obtained to the required number for any desired degree of reliability by sampling over a considerable length of time rather than by sampling heavily in a short period of several days. Thus, by obtaining a few hundred interviews each day, a total of 93,329 interviews were accumulated over the course of the year 1958. These interviews were obtained under all of the varying conditions which existed in the field throughout the year.

ADVANTAGES OF CONTINUOUS SAMPLING

There are a number of advantages to be derived from the use of this new traffic survey technique. Among the more important of these, as they apply to the Port Authority's O-D survey problem, are the following:

1. By spreading the sampling over a long period of time, this system avoids the possibility inherent in "one shot" surveys that seasonal or other variations might make traffic unrepresentative on the day or days surveyed. In continuous sampling, a sufficient variety of traffic patterns over the days and seasons are covered so that the survey results are representative of normal traffic patterns. By the same token, spreading the interviews in this way makes it possible to measure seasonal variations.
2. This new technique provides up-to-date information on O-D patterns at all times because the survey data is continuously being obtained and can be analyzed periodically. This has already been useful in analyzing traffic developments in the light of weather variations, recession and construction of new facilities.
3. Building the size of the sample over a long period, rather than by intensive sampling during one day or several days, has eliminated the necessity of hiring large numbers of unskilled, temporary employees as interviewers and coders. For the continuous sampling, four regularly employed interviewers did all of the interviewing as well as the coding. These men were trained in interviewing methods, had a thorough knowledge of the geography of the Metropolitan Area, and were skilled in the coding of the data.
4. Because continuous sampling avoids intensive sampling at any facility at any one time, there is little chance of causing congestion at the facilities while obtaining the O-D information. Furthermore, skilled interviewers greatly reduce the time required for each interview and are thus able to get complete information on each interview without undue delay to the motorist.
5. By using a probability sample, it is possible to compute mathematically the degree of reliability achieved by the survey and thus keep a constant check of the accuracy of the results.

The design of the sample is as follows:

1. The sample was a probability sample requiring that every vehicle trip made during the year over the Port Authority tunnels and bridges have a known probability of selection. This was achieved by a randomized selection procedure at every stage of the sample design.
2. The level of reliability required in the survey specified that with the expected

over-all sample size of from 90 to 100 thousand interviews, a sample estimate of 1 percent, or 1,000 interviews, would have a coefficient of variation (maximum margin of error) of the order of 3 to 6 percent.

3. There was built into the sample design a procedure for developing proper measurements of the standard errors of the various sample estimates.

DEVELOPMENT OF GENERAL SAMPLE DESIGN

The general sample design had to be such that all of these statistical requirements for a probability sample were met, but it also had to be able to accommodate the practical problems encountered in the actual conduct of the survey. These two considerations necessitated development of a number of procedures, as follows:

1. The survey budget permitted the assignment of four permanent field survey interviewers to this project. These interviewers were responsible for both the field work and the office work in connection with this survey. Because this was the limit of the manpower available, it was determined that the field work had to be restricted to eleven tours of duty per week.
2. Reasonable working hours and the necessity of minimizing fluctuations in hourly traffic volumes within shifts were used as the criteria in designating the hours for shifts which became the primary sampling units. The shifts were set up as 8-hr periods to be spent at one facility on any one of the seven days of the week. The hours of the shifts decided upon were 11 p.m. to 7 a.m., 7 a.m. to 3 p.m. and 3 p.m. to 11 p.m.
3. To insure good coverage of O-D patterns, directional flows, vehicle types and vehicular volumes at each facility during each 8-hr period, the interviewer moves from one lane to another each hour in a prescribed pattern of rotation. The design includes specified relief periods each hour, but it is assumed, with good reason, that the traffic patterns during these relief periods are not different from those sampled during the interviewing periods.
4. Because of the distances involved in covering the entire toll plaza of a facility, it was necessary to subdivide each facility into two or more locations. The locations were selected in such a manner as to allow interviewers to enumerate the number of lanes open within a location in each hour. The interviewer rotates among the lanes at one specified location for 4 hr and then moves on to another location.
5. Experience with previous "one shot" surveys supplemented by field testing indicated that an average work load of 40 interviews per hour could be achieved by the interviewers. This does not mean that in busy periods more than 40 interviews per hour could not be secured. It does mean that throughout the conduct of the survey an average rate of about 40 interviews per hour was achievable.
6. A knowledge of hourly traffic variations via each facility on the different days of the week was utilized in selecting uniform sampling rates during each shift that would allow for reasonable work loads for the interviewers. The possibility of selecting any shift at any facility was proportionate to the amount of traffic volume expected to occur during the shift.
7. To assure a self-weighing sample, a procedure of balancing actual interviews against expected interviews was introduced. Thus, interviews could be duplicated or omitted depending on such factors as the ratio of actual to expected lanes open, loss of interviewing time due to bad weather and non-response.

1958 SURVEY RESULTS

Much of the data obtained in the survey is of interest only to those concerned with traffic in the New York-New Jersey Metropolitan Area. However, the following material is of general interest particularly as it shows what can be accomplished by this type of survey:

1. More than 83 percent of the trips over the Port Authority bridges and tunnels are made entirely within the 18-County Metropolitan Area. Nearly 14 percent have either origin or destination in the bi-state Port District and only 3 percent are through-trips with neither origin nor destination in the District.

2. Trends in O-D patterns since 1949 emphasize the importance of providing future peripheral facilities and routes both for autos and trucks. Traffic growth was most pronounced in the outlying counties of the Port District whereas the Manhattan Central Business District (Manhattan south of 59th Street), decreased sharply as a traffic generator from 39.3 percent of total trans-Hudson vehicular origins and destinations in 1949 to 33.9 percent in 1958.

3. As might be expected, weekday patterns are considerably different than Sunday patterns. Peripheral traffic (that is, vehicles with neither origin nor destination in Manhattan or Hudson County) constituted 53 percent of weekday trans-Hudson traffic as compared with 69 percent on Sundays.

4. Seasonal analysis shows a pronounced shift in traffic during the summer months away from the core areas toward the periphery. As might be expected, the largest seasonal variations were observed in traffic to and from the resort areas located in Union County and south. Furthermore the seasonal variations in these areas are large enough to have a definite effect on peak traffic loads.

5. The hourly and directional analyses made possible by this type of survey clearly indicate that wide differences in O-D patterns exist between averages for an entire weekday or Sunday and patterns which occur during peak periods in each direction. For instance, the Manhattan CBD, where 32 percent of trans-Hudson autos originated or terminated their trips on an average weekday in 1958, accounted for 39 percent of the eastbound traffic from 7 to 10 a.m., but only 17.5 percent of the westbound volume during those hours. In the afternoon peak hours, 4 to 7 p.m., the Manhattan CBD accounted for 39 percent of the westbound traffic and 24 percent of the eastbound volume. Here is a definite indication that the subject of reversible approach lanes is certainly well worth investigating.

6. Only 47 percent of the trans-Hudson auto trips during 1958 were made for business purposes, 51 percent for recreation and personal reasons and a surprisingly low 2 percent for shopping. During morning peak periods on weekdays, however, 86 percent of the trips were for business purposes. This proportion dropped in the afternoon peak periods to 68 percent for eastbound traffic and 72 percent for westbound traffic.

7. There has always been a heavy reverse flow volume eastbound into New York City from 4 to 7 p.m. By combining hourly purpose and hourly license plate analyses, it was shown that this heavy volume was composed of the normal flow of New Yorkers returning home from work in New Jersey plus a large number of New Jersey residents driving into New York City for pleasure purposes.

8. The average number of persons per trans-Hudson auto during the peak hours eastbound in the morning and westbound in the afternoon tends to be somewhat lower than the non-rush loads. Car pooling is not a large factor in this movement to date probably because the bulk of this commuting is still done by common carrier. Just the opposite is true, however, for the "blue-collar" workers commuting to New Jersey industrial locations, who obviously make much greater use of car pooling.

To sum up, it is felt that this new method of continuous sampling in O-D work has been successful, and it is planned to continue its use in the future. Statistically, it is found that the level of reliability in the survey results has been high and one of the reasons for this is the use of skilled rather than non-skilled interviewers. The survey has given much better seasonal, hourly and supplemental information than obtained previously, and this information has been and will be kept current in the future. The cost of the survey for the year was considerably less than the cost of conducting a "one-shot" survey. Finally, this method gives great flexibility in the types of information obtained from the survey. For instance, the Port of New York Authority has been experimenting with obtaining routing information to supplement its knowledge of origins and destination. Also, in the 1960 Survey peak periods are being concentrated on in order to obtain more detailed and accurate information concerning peak traffic flows which are so important in the analysis of demand versus capacity considerations.

A Theoretical Prediction of Work-Trip Patterns

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● ORIGIN AND DESTINATION surveys have been developed into effective tools for determining travel patterns. Unfortunately, such surveys have two major shortcomings: they are quite expensive, and they are tied closely to the localities and points in time in which they are made. During the past 5 or 6 yr, considerable effort has been expended in attempts to find relationships between land-use patterns and travel patterns in cities where such surveys have been made. It is hoped that the relationships which are found may be used to synthesize patterns of movement in those communities at other points in time, or may be translatable to other cities. This paper may contribute to the synthesis of an origin and destination (O-D) survey.

Three and one-half years ago, research was begun on the possibility of creating a purely theoretical model of the movement of people in response to varying land uses based upon the principles of electrostatics. Although the ability of this theory to predict all types of movement has not been checked as yet, it does seem to be capable of predicting work-trips patterns in a metropolitan area within usable limits of accuracy. This paper presents the techniques which have evolved thus far from the research, but does not delve into the theory behind these techniques.

BASIC ASSUMPTIONS

It is an interesting paradox that nothing in nature occurs with true mathematical precision, although so many natural phenomena are so nearly predictable from mathematical relationships. When human decisions and vagaries are involved in the events, larger errors must usually be expected than may be necessary when dealing with inanimate objects.

In particular, it must be recognized that it is probably impossible to predict with any degree of accuracy the movements of any one person. In addition, the theory, herein demonstrated, requires that the following simplifying assumptions be made in order to permit mathematical formulation.

1. The region to be analyzed must be precisely bounded. An alternate form of this premise would be that every worker living within the region of interest has a job within that region, and every job within the region is filled by a worker living within the region.
2. Every worker goes to work every day, and there is no "turnover" in employment. The technique described herein depends on an initial unbalance in this respect, but the derived pattern of movement is assumed to be stable.
3. Movement between places of varying land use is independent of the mode of travel. Predictions resulting from the theory concern the movement of people, not of vehicles. If walking trips are not of interest, they must be factored out of the theoretical pattern of movement.
4. Every center of employment includes a hierarchy of employees, ranging from common laborers to company presidents. It is recognized that any one organization may have a preponderance of either higher or lower echelon workers, but it is assumed that these irregularities balance out within a comparatively small area.
5. Workers of all income levels live in all parts of the region of interest. This does not necessarily mean that laborers and managers live side by side, but there are very few tracts, of even 1 sq mi in area, within metropolitan regions which are completely homogeneous with respect to incomes of residents.
6. The separations between centers of residence and centers of employment are best measured by the straightline distances between such centers. Even though the actual movement between two centers of activity is quite limited because of physical obstructions, if the potential for movement is great enough, someone will build a Mackinac Bridge, a Lincoln Tunnel, a tunnel through the Alps, or even a tunnel under the English Channel. Any other measure of separation would seem to merely reflect

how people react to existing conditions, and not how they would behave if given a better alternative.

7. Real land-use patterns, made up of centers of residence, centers of employment, centers of shopping, centers of recreation, etc., may be represented as patterns of noncompeting centers of positive charge; that is, at no point in time does a center of recreation compete with a center of employment in the attraction of workers.

8. Human beings may be considered to be electrons. Given the initial distribution of these unit negative charges, corresponding to centers of residence, and the distribution of centers of positive charge, representing places of employment, with magnitudes equaling the numbers of persons employed, the probability of movement between places of residence and places of employment can be predicted on the basis of electrostatic field theory.

SOURCES OF INFORMATION

To apply the theory to a real community, the places of residence and of employment must be abstracted and made centers of negative and positive charge, respectively. Many governmental agencies collect data on population and on business, and many private organizations compile business statistics, so there are many sources of information on where people work and where they live.

The Federal Government publishes census data on the population, housing, retail trade, wholesale trade, service trade, and agriculture. Some useful information based on Social Security records is also published.

State governments may make available sales-tax records or unemployment-compensation records. Ohio publishes annually a "Directory of Ohio Manufacturers," which lists every manufacturing establishment in the state by name and address, with the numbers of male and female employees of each one.

City planning commissions have land-use maps, and other sources of information.

Private organizations publish a number of marketing and investment guides which contain much information on population and production characteristics of the areas covered. City directories are invaluable sources of information, as are Sanborn Fire Maps. In some cities, electric power companies and telephone companies keep records of the places where they have installations which may be helpful.

Despite the many sources of information on where people live and where they work, it must be expected that some desirable information will not be readily available whereas some of that which is available will be conflicting. Neither is it likely that all of the information can be obtained for one point in time. With due care, however, it should be possible to create a reasonably accurate pattern of distribution of places of residence, of employment, of shopping, of recreation, etc., from existing sources of information.

After the city has been translated into such an abstract field of centers of positive and negative charges, the basic equations of electrostatics may be used to predict the patterns of movement which will develop.

EQUATIONS FOR PREDICTING MOVEMENT

It is believed that the equations presented herein are capable of predicting all types of movement, but thus far they have been tested only for work trips, and this discussion will be confined to this type of trip. It will be noted that these equations are quite similar to those proposed by Casey (1) and Voorhees (2). So far as is known, these men arrived at their models empirically from the analysis of O-D surveys. As stated previously, the model presented herein was arrived at theoretically by assuming that human movement can be represented by that of electrons in a field of positive charges.

By its nature and derivation, Eq. 1 will insure that the correct number of workers will be drawn from each zone of residence.

$$V_{P_iQ_j} = \frac{\dfrac{Q_j}{R_{ij}} \quad P_i}{\sum\limits_{j=1}^{m} \dfrac{Q_j}{R_{ij}}} \qquad (i = 1, 2, \text{———} n) \qquad (1)$$

in which

$V_{P_iQ_j}$ is defined as the probability of movement from i to j.

P_i is the number of workers living in zone i.

Q_j is the number of jobs available at center j.

R_{ij} is the straightline distance from i to j if the field contains no physical barriers. Where such barriers exist, R would have to be the straightline distance from i to the point of passage across the barrier plus that from the point of passage to j.

Eq. 2 is quite similar to Eq. 1, but it insures that the correct number of workers is assigned to each job site.

$$V_{Q_jP_i} = \frac{\frac{P_i}{R_{ij}} \quad Q_j}{\sum_{i=1}^{n} \frac{P_i}{R_{ij}}} \qquad (j=1, 2, \text{——} n) \tag{2}$$

in which

$V_{Q_jP_i}$ is defined as the probability of movement to j from i, and the other terms are as in Eq. 1.

Unfortunately, the direct use of Eqs. 1 and 2 will usually yield two different sets of movement, and not the unique pattern which is desired. Eq. 1 does not take into account the total number of workers assigned to each job site by the "m" solutions each for i=1, 2, ——n, and, therefore, will usually overassign or underassign workers to the various job centers. Eq. 2 does not keep track of the total number of workers drawn from each of the zones of residence by the "n" solutions each for j=1, 2, ——m, and, therefore, tends to overdraw or underdraw from the various zones.

Two alternate methods have been developed for arriving at completely balanced assignments. Inasmuch as either method may be applied to either of the basic equations, four patterns of movement may be calculated. Several sets of sample calculations have indicated that all four of these patterns will be essentially identical. It, therefore, seems safe to state that the theory develops a unique pattern of work-trip movements in any given field of activity. The first of these methods is theoretically justifiable, whereas the second reduces the number of calculations significantly.

CALCULATION BY SUCCESSIVE PARTIAL ASSIGNMENTS

The theoretically justifiable method for balancing the assignment of workers to job sites and the drawing of workers from zones of residence will be illustrated with reference to Eq. 1, although application to Eq. 2 is quite similar and no more difficult.

Let it be assumed that there are "m" places of employment scattered over the region of interest, each of which wishes to hire Q_j (j=1, 2, ——m) workers. Also let there be "n" zones of residence in the region, in each of which there are P_i (i=1, 2, ——n) workers who are seeking employment. The only restraint placed on these numbers is that $\sum Q_j = \sum P_i$. Figure 1 shows such a situation in general terms, with m = 3, and n = 2.

As stated, the direct application of Eq. 1 to this region "nm" times (once for every center of employment to interact with each zone of residence) will insure that the correct number of workers is drawn from each zone of residence, but is not likely to assign the correct number of workers to any job site. If, however, only $\frac{P_i}{x}$ workers are initially drawn from each place of residence, "x" can be chosen sufficiently large (perhaps even as 100 or 200) so that no job site will be overassigned by the first set of calculations. After this set of "nm" solutions of Eq. 1,

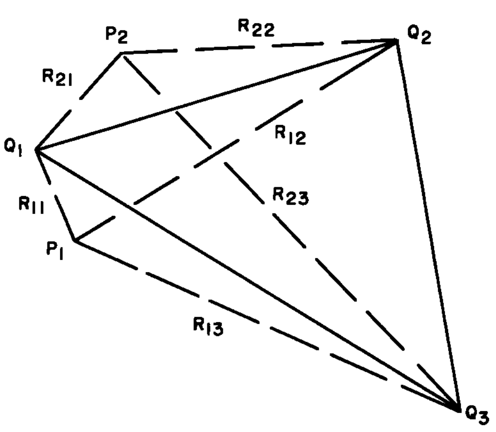

Figure 1. General hypothetical community with three centers of employment and two centers of residence.

each Q_j is reduced by the corresponding $\sum_{i=1}^{n} V_{P_iQ_j}$, the total number of workers assigned to the site from the "n" zones of residence. The next group of $\frac{P_i}{x}$ workers from each zone faces a somewhat different field than did the first group, because each Q_j is now somewhat different, and, indeed, some of them may have been reduced to zero. These steps are repeated through "xnm" solutions of Eq. 1 so that, at last every worker has been assigned to a job site, and every job site has been exactly satisfied. Table 1 gives the first few steps of this method as applied to solve the general community of Figure 1. Table 5 is a numerical solution of a particular community by this method.

TABLE 1

PORTIONS OF GENERAL SOLUTION OF WORK-TRIP PATTERN FOR COMMUNITY OF FIGURE 1 BY SUCCESSIVE PARTIAL ASSIGNMENTS

Original Assignments

From		To Q_1	To Q_2	To Q_3	Sum
P_1	$\dfrac{\frac{Q_1}{R_{11}}\ \frac{P_1}{x_1^*}}{\frac{Q_1}{R_{11}}+\frac{Q_2}{R_{12}}+\frac{Q_3}{R_{13}}} =$	$\dfrac{V_{P_1Q_1}}{x_1}$	$\dfrac{V_{P_1Q_2}}{x_1}$	$\dfrac{V_{P_1Q_3}}{x_1}$	$\dfrac{P_1}{x_1}$
P_2	$\dfrac{\frac{Q_1}{R_{21}}\ \frac{P_2}{x_1}}{\frac{Q_1}{R_{21}}+\frac{Q_2}{R_{22}}+\frac{Q_3}{R_{23}}} =$	$\dfrac{V_{P_2Q_1}}{x_1}$	$\dfrac{V_{P_2Q_2}}{x_1}$	$\dfrac{V_{P_2Q_3}}{x_1}$	$\dfrac{P_2}{x_1}$
Total Assigned to Q_j		$\dfrac{V_{P_1Q_1}}{x_1}+\dfrac{V_{P_2Q_1}}{x_1}$	$\dfrac{V_{P_1Q_2}}{x_1}+\dfrac{V_{P_2Q_2}}{x_1}$	$\dfrac{V_{P_1Q_3}}{x_1}+\dfrac{V_{P_2Q_3}}{x_1}$	

Second Assignments

From	To Q_1
P_1	$\dfrac{\dfrac{Q_1-\frac{V_{P_1Q_1}}{x_1}-\frac{V_{P_2Q_1}}{x_1}}{R_{11}}\ \dfrac{P_1}{x_2}}{\dfrac{Q_1-\frac{V_{P_1Q_1}}{x_1}-\frac{V_{P_2Q_1}}{x_1}}{R_{11}}+\dfrac{Q_2-\frac{V_{P_1Q_2}}{x_1}-\frac{V_{P_2Q_2}}{x_1}}{R_{12}}+\dfrac{Q_3-\frac{V_{P_1Q_3}}{x_1}-\frac{V_{P_2Q_3}}{x_1}}{R_{13}}} = \dfrac{V_{P_1Q_1}}{x_2}$

---

*Subscript on x denotes the number of partial assignment, x being a constant.

In a real community, this method of assignment will involve a large number of calculations. When the theory was used to predict work-trip patterns in Lafayette, Ind., for example, 45 zones of residence and 13 centers of employment were used. Instead of keeping x a constant, it was found possible to vary x and thereby reduce the number of calculations, but even so, 10 sets of partial assignments were needed to assign the first 99 percent of the workers. In this problem it was found possible to use $x = 5$ for three sets of calculations, $x = 10$ for one set, $x = 25$ for four sets, $x = 50$ for one set, and $x = 100$ for one set ($P_i\ (3x1/5 + 1x1/10 + 4x1/25 + 1x1/50 + 1x1/100) = 0.99P_i$). When the theory was applied to the metropolitan area of Cincinnati, assignments were made from 455 zones of residence to 480 centers of employment so that it did not appear feasible to use this method of partial assignments.

CALCULATION BY CORRECTION FACTORS

In an effort to reduce the number of calculations required in a complex community, an empirical method was devised for balancing the drawing of workers from zones of residence and assigning them to places of employment.

The explanation of this system, too, will be limited to use with Eq. 1, although it works equally well with Eq. 2. Eq. 1 is solved "m" times for each value of i, or "nm" times, using the full values of each P_i and Q_j. The total number of workers assigned to each job site, $\sum_{i=1}^{n} V_{P_iQ_j}$, is found, and each total is divided into its corresponding value of Q_j to get the first set of correction factors, C_{j_1}. When each value of $V_{P_iQ_j}$ is multiplied by the correction factor for the corresponding job site, C_j, and checked by adding $\sum_{i=1}^{n} V_{P_iQ_j} \cdot C_{j_1}$ and $\sum_{j=1}^{m} V_{P_iQ_j} \cdot C_{j_1}$, the former sums should exactly equal the corresponding values of Q_j from the nature of the correction factors. It has been found that the latter sums may be close enough to the corresponding values of P_i to be acceptable. If, however, these latter sums are not sufficiently close to the original values of P_i, new correction factors may be formed by dividing $\sum_{j=1}^{m} V_{P_iQ_j} \cdot C_{j_1}$ into the corresponding values of P_i. This system of calculation is given in Table 2 with reference to the community of Figure 1, with Q_3 made zero to reduce the calculations to manageable size.

It will be noted from Table 2 that the general solution by this method gives no indication that the drawing of workers from zones of residence and the assignment of workers to places of employment will ever balance out, nor what the individual assignments will be, should such a balance ever be achieved. Limited experience indicates that such a balance does occur within a few iterations of correction factors, and that the resulting assignments are essentially identical to those obtained from the method of successive partial assignments. Table 6 solves the community of Table 5 by means of correction factors to illustrate these points.

Table 7 compares the results obtained by the two alternate methods of solution.

CHECKS OF THE THEORY

To determine whether the theory, thus far demonstrated, might be capable of predicting work-trip patterns in real communities, it was applied, as noted previously, to Lafayette, Ind., and the metropolitan area of Cincinnati, Ohio.

Lafayette was used as a pilot study. Workers were assigned to zones of residence with considerable accuracy because a special census had been taken in 1953. Fairly accurate employment figures were obtained for the six largest individual employers, but jobs were assigned to another seven places of employment by estimations based on the 1947 Census of Business and 1950 Census of Population data. The work-trip pattern created by the theory was checked against the vehicular movements found by the 1953 O-D survey of the area. The 585 predicted movements resulting from the theory contained many discrepancies when compared with the corresponding O-D

TABLE 2

PORTIONS OF GENERAL SOLUTION OF WORK-TRIP PATTERN FOR COMMUNITY OF FIGURE 1 BY CORRECTION FACTORS

Original Assignments

From	To Q_1	To Q_2	
P_1	$\dfrac{\dfrac{Q_1}{R_{11}} P_1}{\dfrac{Q_1}{R_{11}} + \dfrac{Q_2}{R_{12}}}$	$\dfrac{\dfrac{Q_2}{R_{12}} P_1}{\dfrac{Q_1}{R_{11}} + \dfrac{Q_2}{R_{12}}}$	Let $Q_1 = aQ_2$ $Q_3 = O$ $P_1 = bP_2$ $R_{11} = cR_{21} = dR_{12} = eR_{22}$
P_2	$\dfrac{\dfrac{Q_1}{R_{21}} P_2}{\dfrac{Q_1}{R_{21}} + \dfrac{Q_2}{R_{22}}}$	$\dfrac{\dfrac{Q_2}{R_{22}} P_2}{\dfrac{Q_1}{R_{21}} + \dfrac{Q_2}{R_{22}}}$	

Original Assignments in Terms of Ratios

			Sums
P_1	$\dfrac{aP_1}{a+d}$	$\dfrac{dP_1}{a+d}$	P_1
P_2	$\dfrac{acP_2}{ac+e}$	$\dfrac{eP_2}{ac+e}$	P_2
Sums	$\dfrac{aP_1 (b(ac+e)+c(a+d))}{(a+d) (b) (ac+e)} = X_1$	$\dfrac{P_1 (bd(ac+e)+e(a+d)}{(a+d) (b) (ac+e)} = X_2$	

Correction Factors $\dfrac{Q_1}{X_1} = C_{Q_1}$ $\quad \dfrac{Q_2}{X} = C_{Q_2}$

Original Assignments Multiplied by Correction Factors

			New Sums
P_1	$\dfrac{Q_1 (b) (ac+e)}{b(ac+e) + c(a+d)}$	$\dfrac{Q_2 (bd) (ac+e)}{bd(ac+e) + e (a+d)}$	S_1
	$\dfrac{Q_1 (c) (a+d)}{b(ac+e) + c(a+d)}$	$\dfrac{Q_2 (e) (a+d)}{bd(ac+e) + e(a+d)}$	S_2

$$S_1 = \frac{Q_1 (b(ac+e)) (bd(ac+e)+e(a+d)) + \frac{Q_1}{a} (bd) (ac+e) (b(ac+e)+c(a+d))}{(b(ac+e)+c(a+d)) (bd(ac+e)+e(a+d))}$$

$$\frac{Q_1(c) (a+d) (bd(ac+e)+e(a+d)) + \frac{Q_1}{a} (e) (a+d) (b(ac+e)+c(a+d))}{(b(ac+e)+c(a+d))(bd(ac+e)+e(a+d))}$$

findings, but a large enough proportion of the predicted movements were sufficiently close to the observed movements that a much more detailed analysis of the Cincinnati area seemed to be justified.

A 2,000-ft grid was drawn over a map of Hamilton County, and portions of Campbell and Kenton Counties in Kentucky. Workers were assigned to 455 places of residence on this grid as accurately as available data permitted. Every known source of information on employment was studied, and 480 job sites were established in Hamilton County. No job sites were established in northern Kentucky, but the difference between the number of workers living in that area and the number of jobs which could be accounted for there, amounting to about 40,000 workers, was assigned to work in Hamilton County.

When all of the data had been assembled, it was found that 348,308 workers had been assigned to places of residence while 333,537 jobs had been assigned to places of employment. Because of the experimental nature of this project, these totals were not adjusted to bring them into balance, but it is strongly recommended that such adjustment be made if this theory is used again.

To date, only a few of the 218,400 predicted movements have been studied, but two detailed checks were made of some of the movements. The first of these compared the predicted movements between six communities of the area with the numbers of passenger car trips found by the 1954 post-card-type O-D survey of the county. The communities were chosen so that the majority of the actual trips between them might reasonably be expected

TABLE 3

COMPARISON OF PREDICTED WORK TRIPS AND 1954 O-D SURVEY TOTAL PASSENGER CAR TRIPS BETWEEN SELECTED COMMUNITIES IN THE STANDARD METROPOLITAN AREA OF CINCINNATI, OHIO

From	Cheviot 619 Jobs	N. College Hill Mt. Healthy 1,781 Jobs	Lockland 4,148 Jobs	Norwood 18,322 Jobs	Fairfax–Mariemont 3,927 Jobs	Mt. Washington 788 Jobs
			(a) Movements by Theory			
Cheviot, Ohio	262.0	70.5	87.3	277.7	61.4	16.2
N. College Hill–Mt. Healthy, Ohio	16.3	688.2	144.9	302.2	60.8	14.0
Lockland, Ohio	2.3	16.0	513.5	107.7	21.5	3.5
Norwood, Ohio	12.2	56.1	188.8	3,839.9	211.8	34.3
Fairfax–Mariemont, Ohio	5.2	20.9	65.8	334.8	992.7	31.5
Mt. Washington	2.6	9.3	28.2	132.9	67.5	163.7
			(b) Comparison of Movements			
Cheviot, Ohio 7,740 workers	262.0[1]					
N. College Hill–Mt. Healthy, Ohio 6,675 workers	86.8 313 (0.55)	688.2[1]	Total predicted one-way trips. Passenger car trips by O & D. Persons per passenger car.[2]			
Lockland, Ohio 2,827 workers	89.6 55 (3.20)	160.9 285 (1.12)	513.5[1]			
Norwood, Ohio 15,691 workers	289.9 191 (3.02)	358.3 527 (1.36)	296.5 491 (1.16)	3,839.9[1]		
Fairfax–Mariemont, Ohio 5,354 workers	66.6 46 (2.90)	81.7 61 (2.55)	87.3 47 (3.64)	546.6 653 (1.67)	992.7[1]	
Mt. Washington 2,347 workers	18.8 23 (1.57)	23.3 89 (0.51)	31.7 43 (1.44)	167.2 469 (0.71)	99.0 364 (0.54)	163.7[2]

[1] No check available for these predictions.

[2] Number of passenger car trips, found by O-D survey, divided by twice the predicted number of one-way trips.

to be auto-driver and auto-passenger work trips. Table 3 gives a comparison of the predicted work trips and the total passenger car trips, as found by the O-D survey, between these communities. The second check consisted of comparing the predicted places of residence of the approximately 1,400 full-time employees of the University of Cincinnati with the actual places of residence, as determined from the 1957 payroll. Table 4 gives the distribution of discrepancies between the predicted and actual places of residence, as found in this comparison.

TABLE 4

DISTRIBUTION OF ERRORS: PREDICTED NUMBER OF RESIDENTS MINUS ACTUAL NUMBER OF RESIDENTS, FULL-TIME EMPLOYEES OF UNIVERSITY OF CINCINNATI

Range of Error		Median Values	Frequency of Errors	Remarks
From	To			
-77.9	-76.0	-77.0	1*	Adjacent to Univ.
-75.9	-74.0	-75.0	1*	Adjacent to Univ.
-	-	-		
-21.9	-20.0	-21.0	1*	
-19.9	-18.0	-19.0	0	
-17.9	-16.0	-17.0	0	
-15.9	-14.0	-15.0	2 (6)	(6) includes avg of 4
-13.9	-12.0	-13.0	2	poor values at U.C.
-11.9	-10.0	-11.0	5	
- 9.9	- 8.0	- 9.0	2	
- 7.9	- 6.0	- 7.0	3	
- 5.9	- 4.0	- 5.0	8	
- 3.9	- 2.0	- 3.0	13	
- 1.9	- 0.0	- 1.0	16	
+ 0.1	+ 2.0	+ 1.0	24	
+ 2.1	+ 4.0	+ 3.0	9	
+ 4.1	+ 6.0	+ 5.0	9	
+ 6.1	+ 8.0	+ 7.0	6	
+ 8.1	+10.0	+ 9.0	3	
+10.1	+12.0	+11.0	1	
+12.1	+14.0	+13.0	0	
+14.1	+16.0	+15.0	2	
-	-	-		
+24.1	+26.0	+25.0	1*	
-	-	-		
+30.1	+32.0	+31.0	1*	Northern Kentucky
+32.1	+34.0	+33.0	1*	Basin area of city.
-	-	-		
+38.1	+40.0	+39.0	1	Adjacent to Univ.
-	-	-		
+48.1	+50.0	+49.0	2*	Northern Kentucky
-	-	-		
+54.1	+56.0	+55.0	1*	Adjacent to Univ.
			115	

Case 1. Including entire table.
Mean error = +0.57 Standard Deviation = 15.5

Case 2. Excluding values marked*.
Mean error = -0.37 Standard Deviation = 5.75

Case 3. Averaging errors at University and dropping values marked Basin and Kentucky. Mean error = 0.15 Standard Deviation = 8.88

TABLE 5

SOLUTION OF HYPOTHETICAL COMMUNITY OF FIGURE 2 BY EQUATION .1. AND SUCCESSIVE ASSIGNMENTS OF P_i/x WORKERS CALCULATIONS[1]

x	P_1	P_i/x	Q_1	Q_2	Q_3	$\frac{Q_1}{R_{i1}}$	$\frac{Q_2}{R_{i2}}$	$\frac{Q_3}{R_{i3}}$	k_{P_i}[2]	$\frac{V_{P_1Q_1}}{x}$	$\frac{V_{P_iQ_2}}{x}$	$\frac{V_{P_1Q_3}}{x}$	Sum
4	600	150	500	200	100	0.0250	0.0167	0.0250	14.99	56.2	37.6	56.2	150.0
4	200	50	500	200	100	0.1000	0.0182	0.0048	8.13	40.7	7.4	2.0	50.1
4	600	150	403	155	42	0.0202	0.0128	0.0105	22.99	69.7	44.1	36.2	150.0
4	200	50	403	155	42	0.0806	0.0141	0.0020	10.34	41.7	7.2	1.0	49.9
4	600	150	291	104	5	0.0147	0.0087	0.0012	40.81	90.6	52.2	7.2	150.0
Overassigns Q_3 by 7.2 - 5 = 2.2, therefore use x = 10.													
10	600	60	291	104	5	0.0147	0.0087	0.0012	40.81	35.7	21.3	2.9	60.1
10	200	20	291	104	5	0.0580	0.0095	0.0002	14.77	17.1	2.8	0.1	20.0
10	600	60	238	80	2	0.0119	0.0067	0.0005	52.36	37.4	21.0	1.6	60.0
10	200	20	238	80	2	0.0495	0.0075	0.0001	17.57	17.4	2.6	0.0	20.0
10	600	60	183	57	0	0.0092	0.0048	-	71.43	39.4	20.6	0.0	60.0
10	200	20	183	57	0	0.0365	0.0052	-	23.98	17.5	2.5	0.0	20.0
10	600	60	126	34	0	0.0063	0.0028	-	109.9	41.5	18.5	0.0	60.0
10	200	20	126	34	0	0.0232	0.0031	-	38.02	17.6	2.4	0.0	20.0
10	600	60	60	13	0	0.0034	0.0011	-	222.2	45.3	14.7	0.0	60.0
Overassigns Q_2 by 14.7 - 13 = 1.7, therefore use x = 20.													
20	600	30	60	13	0	0.0034	0.0011	-	222.2	22.7	7.3	0.0	30.0
20	200	10	60	13	0	0.0134	0.0012	-	68.5	9.2	0.8	0.0	10.0
20	600	30	35	5	0	0.0018	0.0004	-	454.5	24.5	5.4	0.0	30.0
20	200	10	35	5	0	0.0070	0.0005	-	133.3	9.3	0.7	0.0	10.0
Overassigns Q_2 by 1.1, and underassigns Q_1 by 1.2, but use as is.													

[1] Summary of assignments in Table 7.

[2] k_{P_i} is reciprocal of $\frac{Q_j}{R_{ij}}$

TABLE 6

SOLUTION OF HYPOTHETICAL COMMUNITY OF FIGURE 2 BY APPLICATION OF CORRECTION FACTOR TO INITIAL SOLUTION OF EQUATION .1.

P_1	Q_1	Q_2	Q_3	$\frac{Q_1}{R_{i1}}$	$\frac{Q_2}{R_{i2}}$	$\frac{Q_3}{R_{i3}}$	k_{P_i}	$V_{P_iQ_1}$	$V_{P_1Q_2}$	$V_{P_iQ_3}$
600	500	200	100	0.0250	0.0167	0.0250	14.99	224.9	150.2	224.9
200	500	200	100	0.1000	0.0182	0.0048	8.13	162.6	29.6	7.8
							Initial sums	387.5	179.8	232.7

Calculation of correction factors:

First set: $\frac{500}{387.5} = 1.290$; $\frac{200}{179.8} = 1.112$; $\frac{100}{232.7} = 0.430$

Second set: $\frac{600}{553.9} = 1.083$; $\frac{200}{246.0} = 0.813$

Third set: $\frac{500}{484.9} = 1.031$; $\frac{200}{207.7} = 0.963$; $\frac{100}{107.3} = 0.932$

Fourth set: $\frac{600}{595.8} = 1.007$; $\frac{200}{204.1} = 0.980$

Summary of assignments:

Movement	First Assign.	Second Assign.	Third Assign.	Fourth Assign.	Fifth Assign.
$V_{P_1Q_1}$	224.9	290.2	314.3	324.0	326.3
$V_{P_2Q_1}$	162.6	209.8	170.6	175.9	172.4
$V_{P_1Q_2}$	150.2	167.1	181.0	174.3	175.5
$V_{P_2Q_2}$	29.6	32.9	26.7	25.7	25.2
$V_{P_1Q_3}$	224.9	96.9	104.6	97.5	98.2
$V_{P_2Q_3}$	7.8	3.3	2.7	2.5	2.5
Totals assigned to:					
Q_1	337.5	500	484.9	500	498.7
Q_2	179.8	200	207.7	200	200.7
Q_3	232.7	100	107.3	100	100.7
Totals draw from:					
P_1	600	553.9	600	595.8	600
P_2	200	246.0	200	204.1	200

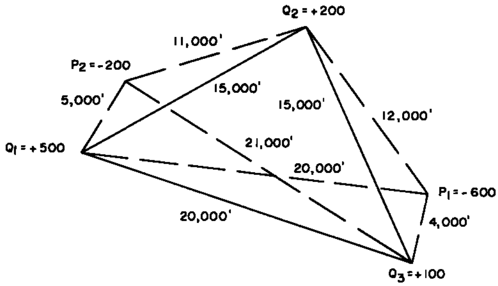

Figure 2. Hypothetical community with three centers of employment and two centers of residence.

CONCLUSIONS

This paper has attempted to present the possibilities of a new theory for predicting travel patterns in urban areas. No attempt has been made to explain the basis of the theory, but its use has been illustrated by the prediction of work-trip patterns in a simple, hypothetical community and in the metropolitan area of Cincinnati, Ohio.

Given the distribution of residences of workers and the locations of all jobs in a bounded region, it appears that the proposed method can be used to develop theoretical work-trip patterns which approach the actual patterns closely enough to be usable. Given anticipated future distributions of workers and jobs, future work-trip patterns should be predictable.

The method presented herein is much less costly and time-consuming than a comprehensive O-D survey. It is believed that further research will permit the theory to develop shopping-trip, personal-business-trip, and recreation-trip patterns, as well as work-trip patterns in urban areas.

TABLE 7

SUMMARY OF ASSIGNMENTS FROM TABLE 5 AND COMPARISON WITH ASSIGNMENTS FROM TABLE 6

x	$V_{P_1Q_1}$	$V_{P_2Q_1}$	$V_{P_1Q_2}$	$V_{P_2Q_2}$	$V_{P_1Q_3}$	$V_{P_2Q_3}$	Check Total Assignments	
4	56.2	40.7	37.6	7.4	56.2	2.0	P_1	P_2
4	69.7	41.7	44.1	7.2	36.2	1.0		
10	35.7	17.1	21.3	2.8	2.9	0.1	327.1	170.5
10	37.4	17.4	21.0	2.6	1.6	0.0	175.8	26.4
10	39.4	17.5	20.6	2.5	0.0	0.0	96.9	3.1
10	41.5	17.6	18.5	2.4	0.0	0.0		
20	22.7	9.2	7.3	0.8	0.0	0.0		
20	24.5	9.3	5.4	0.7	0.0	0.0	599.8	200.0
Sum	327.1	170.5	175.8	26.4	96.9	3.1	- by partial assignments.	
	326.3	172.4	175.5	25.2	98.2	2.5	- by correction factors.	
Check Assignments to Q_j:			Q_1	Q_2	Q_3			
			327.1	176.8	96.9			
			170.5	26.4	3.1			
			497.6	202.2	100.0			

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Critique of Home-Interview Type O-D Surveys in Urban Areas

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●IT HAS BEEN STATED (1) that "it has now been 15 years since the home-interview method of making origin-and-destination surveys in urban areas was developed. The method has now been used in 126 urban areas and repeat surveys have been made or started in 10 of these. The field methods and the information obtained have been changed very little since the earliest surveys. Present emphasis is on the improvement of analysis methods."

It has been said that where a method has been in use for 5 years, it should be reviewed; if it has been in use for 10 years, it is ready for drastic revision; if it has been used for 20 years, it should be presumed to be obsolete. On the basis of this rule of thumb, the home-interview method of collecting O-D data is rapidly approaching obsolescence, if it has not already reached it.

INHERENT DEFECTS OF HOME-INTERVIEW SURVEYS

A home-interview survey of the origins and destinations of trips, in any given study area, does reveal a fairly faithful picture of all trips made in the course of a 24-hr weekday, for all purposes, by all modes of travel. Such a survey does also have the advantages that 5 percent sample interviews, conducted at randomly selected homes, can be tied to a known universe of homes and an intercensal estimate for the year of the survey, of total population in each of the O-D zones of residence in the study area. But having said all this, the remaining characteristics of the home-interview method of assembling O-D trip data, for purpose of planning limited-access highways, and mass transportation, particularly in urban areas, are replete with inherent defects which are practically incurable with any statistical methods, or by electronic computers.

For one thing, only a fraction of the usual 5 percent sample of the primary trips (those originating at or destined for homes in any given zone in the study area), consist of journeys to and from work or business or other trips that would utilize existing arterials or expressways, or would utilize proposed expressways in the future. Substantial portions of such primary trips are usually made on local streets—picking up and delivering children at schools, visiting local shopping areas, friends and homes, and for other local area chores. Other large portions of the sampled trips are made along directions crosswise to existing arterials or proposed expressways. Consequently, of the expanded 5 percent home samplings of primary trips, only fractions can actually be utilized to the base year trip potentials for proposed expressways of the future.

Lynch further states that "throughout the past 10 years, much research has been conducted on trip production and trip attraction in relation to land use." For purposes of measuring the drawing powers of different types of land-use generators, like sites of employment in the study area, the numbers of inter-zonal trips actually recorded in an over-all 5 percent home-interview sampling, are so few and inadequate as to make them rather crude instruments for such measurements. But to make matters worse, before such meager samplings can be used to correlate with land-use data, they must first be broken down by significant trip purposes, such as journey-to-work trips, business trips, trips for shopping, recreation, amusement, etc. In some instances, such as journey-to-work trips, the fraction of the sample must be further reduced to reflect only the significant morning and evening rush hour travel. These types of breakdowns often reduce some samplings of inter-zonal trips, to such small numbers as to be wholly worthless for research purposes.

Nor does the "fattening up" of sample zonal trips, by combining trips in several zones into larger areas, to counteract the trip purpose and rush hour breakdowns, for example, serve to cure the inherent defects of the original meager zone samplings. The pairs of combined zones now become areas too large to be characterized by average travel times, distances and costs, between their centroids. Lynch states that "it is necessary, however, to smooth out the reported times because of the small number of reported trips between some pairs of zones." Correlations with average travel time, distance and cost determinants thus become blurred and blunted, evidenced by the wide scatters on correlation graphs. Imagination must often be stretched to the elastic limit to glean the types of quantitative relationships between zonal trips and some of the factors of trip generation. Then, to explain exceptions to the derived relationships, a number of additional factors, quite unpredictable in the future, have usually been introduced.

But that is not all. In any study area, zonal trip ends as usually tabulated, constitute a kind of "chemical mixture" that consists of (a) primary trips originating in or destined for homes in individual zones and produced by residents domiciled in those zones, plus (b) trips attracted to those same zones to non-residential land uses located in those same zones, for purposes of work, shopping, etc. Consequently, as a result of these "chemical mixtures" of primary and attracted trip ends, correlations between these mixtures of zonal trip ends and the autos domiciled in the corresponding zones, could not possibly yield satisfactory relationships. The trips generated by non-residences in the zones are definitely not related to either households nor autos domiciled in those zones. Lynch apparently realizes this when he says, "difficulties have been encountered because of mixed land uses within zones and lack of precise knowledge as to the character of the establishments at the end of each trip."

Before any meaningful correlations could be derived between trip determinants and trips they produced, it is therefore essential, first, to break down this "chemical mixture" of primary and attracted trip ends into "elemental" primary trip ends. To break down such mixtures, it is necessary to go back to the original home-interview schedules and distill out of them the pure "elemental" primary trips for each zone—that is, trips of one or all purposes, originating in or destined exclusively for residences in those zones. Only such "elemental" primary trip ends could yield meaningful correlations with autos domiciled in those zones. This step has now actually been taken in the analysis of the National Capital Region, by retabulating original trips. The category of trips with the ludicrous "purpose," "to home" has been mechanically eliminated; in its place, the more realistic category of "work trips to and from homes" has been substituted.

Where "elemental" primary trip ends could be made available, such trips which began or ended at homes may be expected to yield excellent correlations with autos domiciled in the corresponding zones.

Car densities in small areas (expressed as cars per acre) are intimately related to the corresponding household densities (expressed as households per acre). In suburban sparsely settled communities cars are absolute essentials. In more densely populated communities, auto densities are also higher but not proportionately so, because in such areas car ownerships become less essential by reason of convenient public transportation and of cars being more costly to own and operate.

Auto ownership densities thus increase with household densities, not in proportion but rather at declining rates. In the New York-New Jersey Metropolitan District for example, as of 1955, in Somerset County, N.J., the average household density for the county was around 2 households per acre, auto density was about 3 autos per acre; in Bergen County, N.J., where household was about 4.5 households per acre, auto density was about 5.5; in Essex County, N.J., the corresponding figures were 8 and 8.5; in Hudson County they were 15 and 13; and in Manhattan there were 52 households per acre and about 21 autos per acre.

Because autos per acre do not increase in proportion to households per acre, autos per household in densely populated areas usually constitute, currently, less than a car per household. In Somerset County, with 2 households per acre, there were 140 autos per 100 households; in Bergen County with 4.5 households per acre 120; in Hudson with 15 households per acre 80; in Manhattan with 52 households per acre only 40 autos per 100 households.

Household densities would appear to constitute far more stable indicators of auto ownership than are such other indicators as groups of income levels. Besides numbers of households and residential acreage for small areas are far more predictable, in the future, than income levels.

Also, distances from the CBD are not good fundamental determinants of either car ownerships or trips. Distance from the CBD is only a space parameter, akin to a time parameter. Mere distance from the CBD, which itself has been changing significantly in character, like mere passage of time, are both generally weak and, at times, unreliable determinants for forecasting purposes. More fundamental and considerably more predictable determinants, like household densities, are needed to measure zonal auto ownerships which in turn, are the determinants for generated zonal trip ends. The urban area is never a homogeneous continuum that spreads out in circles from a center, like the centroid of the CBD, out to the suburbs, in all directions. There are dense zones in the distant suburbs and less dense zones in close-in areas near the CBD. In any given urban study area, individual residential zones of widely varying household densities make for correspondingly widely varying auto ownership densities, and thus for widely varying volumes of auto ownerships at similar distances from the CBD. The last, in turn, generate widely varying volumes of zonal trip ends. Aggregate households in the study area and the spacial distribution of widely varying household densities, thus determine, to a large extent, the aggregate absolute number of trip ends in any given urban area.

Also in any given urban area, every O-D zone is not only a residence but also a non-residence zone. The same type of "chemical breakdown" is thus equally essential, in order to obtain trip ends in every O-D zone as non-residence zone—that is, trips originating in or destined for work places, shopping areas, recreational, amusement and cultural areas, but excluding homes as origins or destinations. If such a breakdown were actually made, by going back to the original schedules, such "elemental" trip end data to and from non-residence zones, could then be correlated with such correlative land-use data as, gainfully employed at sites of employment for journey-to-work trips, floor space in commercial buildings for business trips, floor space in retail establishments for shopping trips, and floor space in other buildings for amusement and other trips, etc. These types of land-use data have only very recently become available in connection with the home-interview O-D trip surveys, and in only a few cities. And yet these are some of the fundamental determinants of the relative number of trip ends in non-residence zones, just as households and household densities are the fundamental determinants of the absolute number of trip ends in residential zones.

The final group of essential fundamental data (in addition to land use data) that should have been, but which were not, usually assembled in the past, in conjunction with and, where possible, also simultaneously with, the collection of home-interview O-D data, are the travel impedances some of which must be obtained through test runs between residence and non-residence zones. Test runs should have been made between every pair of zones in any given study area and along various alternate routes and modes of travel, by riding autos, buses and railroads, where the last are important. In connection with such test runs, data should have been assembled, on travel distances and travel times along actual routes, tolls at bridges, tunnels and highways, parking fees in non-residence zones, and on annoying, irritating and potentially hazardous aspects of routes, like direct left turns, clover leaf left turns, parked cars, pedestrains crossing, etc.

HOW SHOULD O-D TRIP DATA BE ASSEMBLED IN THE FUTURE?

Having criticized the home-interview O-D survey, it is fair to ask whether there is a better method of assembling trip data.

There are really only three possible methods of assembling O-D trip data. One is to interview persons enroute from origins to destinations, at roadside stations. The second method is to interview persons at home about trips made in a recent period by members of the household. The third method is to interview persons (and to obtain, by other means, other correlative data) at non-residence locations, where there are large concentrations of persons, with respect to their trips for the specific purposes which brought them there.

It would seem to this researcher that, for planning future urban expressways, sample trip and correlative data on trip determinants should really be assembled, not in the homes but in areas where people are concentrated during the day. Such areas are: central business districts, large commercial and industrial sites of employment, shopping areas, amusement and recreation areas, and in general, sites of public assembly. Those are the areas on which traffic converges, where traffic is concentrated on approach highways and where consequently additional vehicular capacities are usually urgently needed on peak weekdays, in peak leisure time periods, and in peak hours.

Trip data to and from non-residences, if assembled at these sites, could at the same time, just as readily include correlative data on modes of transportation that were used to reach areas of concentration, travel distances, times and travel costs, as well as the conveniences and inconveniences of alternate routes and modes of travel between homes and such areas of concentration. Thus, for example, data on journey-to-work trips could be assembled at selected sites of employment, together with the correlative data on locations of employee residences, travel times, distances and costs to and from employees' homes. Such work trip data could then be correlated with data on employed labor forces residing in small residential zones, whence employees had been drawn. Such correlations would, at the same time, disclose the varying strengths of selected and stratified sample sites of employment in their ability to draw different classes of employees from different types of residential zones located at varying travel distances, times and costs from these sites of employment. The varying power of attraction of any given site, zone or area of concentration of persons, whether it be a large or small traffic generator, for trips from different resident zones, measured by the varying proportions of zonal employed labor forces which it draws, would be, to a large degree, inversely proportional to the travel distances, times, costs and other travel impedances between that site, zone or area and the residences scattered over the study area.

This type of inverse relationship is often referred to as the "gravity model." However, it is believed that this inverse relationship does not follow the so-called "gravity formula," which is a power function, but rather an inverse exponential type of mathematical function.

But to validate any hypothesis, which postulates inverse relationships with travel impedances, sample O-D data are, of course, essential. To establish adequate confidence in any proposed hypothesis which sets forth the fundamental determinants of trip generation, sample O-D trip data, plus the correlative supplemental data on determinants, must be quite substantial. Sample O-D trip data should therefore be assembled at locations, not where people are diffused over the study area as in their respective homes, but where people are concentrated, as at sites of employment. Trip data should therefore be assembled at work places, in retail establishments, in office buildings, in manufacturing plants, at recreational and amusement areas, and at cultural centers. In short, O-D trip data, to be useful for research and for the application of the relationships for planning purposes, whether for future expressways or for the revitalization of CBD's, or for the planning or replanning of commercial and industrial areas, must be assembled at sites where people are concentrated; also O-D trip data should reflect travel in rush hours, whether to or from work on weekdays, or to or from recreational areas in peak leisure time periods.

A SUGGESTED PROGRAM FOR THE FUTURE

Three types of research projects suggest themselves for the future: (1) re-analyze O-D trip data, which have been assembled over the years by the home-interview method, in relation to minimum fundamental trip determinants which prevailed at the time of the O-D surveys; (2) in the future assemble O-D trip data, together with correlative data on travel distances, times, modes and costs, at locations where people are concentrated; and (3) inaugurate the collection and compilation of a minimum of data on independent fundamental trip determinants, for small zones in the study area on a continuing basis.

Today there is a wealth of trip volume data between small areas in more than 100 cities, for which millions of dollars have been spent to assemble by the home-interview method and more millions for analyses of these data. Some of these original data should be "exhumed" and repunched on new cards. Some fundamental supplemental data, on trip determinants, reflective of the time of the survey period, should be punched into those cards. They should be retabulated. They should be re-analyzed, with a view of establishing quantitative relationships between trips and the minimum number of fundamental factors of trip generation that prevailed in different cities at the time of the O-D surveys.

Then after such quantitative relationships had been established, but before they are applied to estimate future changes in trip volumes, these relationships should be tested to see how close the differences in trip determinants in various zones of the study area actually account for significant differences in trip generations in those zones as of the survey period.

At this point, it would also have to be realized that a philosophical step would have to be introduced between the current trip determinants and their use for estimating future travel patterns. It is this: Currently, a unit difference in any trip determinant like distance, time or cost would be associated with a related difference in generated trips, as between two locations in the study area. It would have to be assumed that a corresponding unit change between two points in time, in any trip determinant, would be associated with an equivalent change in generated trips.

A great wealth of understanding would flow from such re-analyses of home-interview O-D trip data. Minimum real and essential trip determinants would be identified; the most effective methods of assembling the types of data required for planning purposes would be revealed.

Such re-analyses would, on the one hand, point to a serious consideration in the future for tapering off home-interview O-D surveys and on the other hand, point to the need for assembling future O-D trip data (together with correlative data on travel distances, times, and travel impedances, as well as cost via various modes of travel used to and from homes), at sites and areas of concentrated economic, social and recreational activities in urban areas.

In addition, the need to collect and compile a minimum of supplemental fundamental determinants for small zones on a continuing basis would become evident. These trip determinants would consist of the following data: population, households, autos, numbers of gainfully employed at sites or zones of employment, net residential acreages, floor space at industrial, commercial, management, office, amusement and recreational sites.

By assembling O-D trip data in areas where people are concentrated, a much richer body of data would thus become available for identifying the minimum underlying fundamental determinants of urban transportation and for the forging of more powerful tools for planning. Realistic and understandable relationships between generated trips and fundamental trip determinants that would be predictable, to a large extent, could indicate which determinants of trip generation could be controlled, through land-use planning, for example, and which changes in determinants must simply be anticipated. Such relationships could thus become effective aids and even powerful tools for the formulation of city planning and transportation policy decisions.

REFERENCE

1. Lynch, J. T., "Home-Interview Surveys and Related Research Activities." HRB Bul. 224, pp. 85-88 (1959).

Discussion

E. WILSON CAMPBELL, Assistant Director, Chicago Area Transportation Study—

The writer agrees with Mr. Cherniack that the home-interview method has inherent defects. However, little else is found in his paper with which the writer can agree. The fact that there are defects in the technique does not prevent the home-interview method from becoming a useful tool for urban transportation planning. There are defects in the way steel structures are designed and constructed, in the way cars are driven, in relations with people; in fact, there is hardly a process which is free from defect. However, the fact that there are defects does not prevent these processes or relationships from being useful when handled properly.

The author states that only a fraction of trips collected with the usual 5 percent sample consist of journeys that would utilize existing arterials and expressways, and that substantial portions of these primary trips are usually made on local streets. The Chicago Area Transportation Study home interviews are based on a sample rate of 1 in 30 (3.3 percent). The vehicle-miles of travel accounted for by these reported trips have been calculated, and through a system of sample on the ground counts, vehicle-miles of travel in the study area for the same year have been estimated. The total vehicle-miles of travel estimated by the two techniques agree within 10 percent of this total mileage. Eighty percent is on streets classified as arterial or expressway. The close correlation of these results certainly explodes the theory that trips resulting from interviews at home account for a small portion of the travel on arterials or expressways.

Mr. Cherniack's next point is that in a 5 percent sample, the number of interzonal trips reported are so few and inadequate as to make them rather crude instruments for purposes of measuring drawing powers of different types of land use. In Chicago this was not the case. As a result of the home-interview survey, a record of over 350,000 trips was obtained. Thus, it was possible to group trips by purpose and by land use at the destination and still have enough trips in a group to analyze the variation of attractiveness or drawing power of the same type of land use in 45 different geographic areas.

The author's next major point is "in any study area, zonal trip ends as usually tabulated constitute a kind of 'chemical mixture' that consists of (a) primary trips originating in or destined for homes in individual zones and produced by residents domiciled in those zones, plus (b) trips attracted to those same zones to non-residential land uses located in those same zones, for purposes of work, shopping, etc." His concerns are several. First that relationships of autos domiciled in the zone to population or to net residential density or other variables could not be determined accurately. Second that correlations of trips to various land uses which are indicators of trip generation are clouded by this chemical mixture. What he says is true if the kind of land at which the trips begin or end on cannot be identified.

This problem has been eliminated in Chicago. Four years ago when the home-interview data was collected the kind of land use at each end of the trip was determined from the respondent. This information, along with the trip purpose, tells a great deal about the trip. For example it is not only known that a trip ends in a zone, but that it is going to a residential, a commercial, an industrial use, etc. In addition it is known that a person going to a residential land use is going "to home" thus he is domiciled there. Or conversely it is known that a person is going to a residential area "to work." He is a carpenter, or painter, or domestic servant, etc. He is not domiciled there. Similar examples can be made for trips to all kinds of land uses. This precision helps to pull out information required to sharpen the relationships of auto ownership, traffic generation, etc.

Next, Mr. Cherniack says that to complete the essential fundamental data, "test runs should be made between every pair of zones in any given study area and along alternate routes and modes of travel." This is an impossible task in an area as large as the Chicago Metropolitan area. More important, however, is that this type of information is all but useless for planning purposes. The planning period is some distant target year 20 to 30 years in the future. Twenty years from now the travel speeds on arterial and expressway, train, or bus will bear no resemblance to the speeds inventoried on today's roads under today's traffic conditions.

It is the feeling in Chicago that a sampling of speeds on different types of streets in different areas during the peak hour under today's conditions is useful. These sample speeds related to net residential density or trips per square mile might be useful in estimating average speeds on facilities which have the same characteristics predicted for the future.

Next Mr. Cherniack suggests that the way to proceed in obtaining trip data is to go to the areas where people concentrate during the day, such as a CBD, industrial sites, shopping centers, etc. First, these are the areas where the predictions from the home-interview study are best. For example, in Chicago checks were made on the reported trips to work at several industrial plants of varying size. Based on the trips reported "to work in" at these plants from the home-interview study, it was possible to check within 95 percent the number of people reported at work by the plants for these same time periods.

The great disadvantage of interviewing at place of concentration is that there is no total frame of reference which can be used as a guide. That is, what is the total universe of trips per day? How many vehicle-miles of travel are driven in the area each day? This information cannot be obtained by this type of survey. The importance of knowing the total scale should not be underestimated.

Information about the household, which is useful in projecting future trips and other related trip information cannot be obtained by interviewing at the work end of the trip. The influence of "home" in organizing the travel in an area cannot be overlooked. Eighty percent of all trip ends are at home. That is to say, 80 percent of all trips begin or end at the home. This is a substantial indicator that trips inventoried at the home produce a good sample of the total trip pattern.

Mr. Cherniack states that the gravity formula which is a power function does not adequately describe the power of attraction of any given site. He feels that an inverse exponential relationship would produce better results. The writer heartily agrees with Mr. Cherniack on this point. Using a formula of this kind and 1956 trip ends and assigning to the existing arterial and expressway network we have been able to duplicate the vehicle-miles travelled in this system within 5 percent of the estimated vehicle-miles travelled in 1956.

In summary, the writer cannot agree with Cherniack that the home-interview study because of its inherent weaknesses is valueless. Nor can he agree with Mr. Cherniack's statement that the proper way to obtain this data in the future is by collecting trip information at points where people concentrate. On the contrary, the writer feels that the home-interview study is and has been a useful tool. The field of urban studies is relatively new and the tools are being developed. The home-interview technique is a stepping stone to the development of new tools. In the near future trips may be estimated based on land-use forecast, coupled with population and economic projections. This synthesis of trip data would result in great economies in data collection. This new tool would not have been possible without utilizing data collected by the home-interview method.

JOHN T. LYNCH, Chief, Planning Research Branch, U.S. Bureau of Public Roads— The writer agrees with Mr. Cherniack that the home-interview type of O-D surveys, as now conducted in urban areas, may become obsolete in the not too distant future. It is hoped that the analyses being made of the extensive data that have been collected, and particularly the study of changes in the travel pattern over periods of ten years or more, will make possible the development of a cheaper and more accurate method of forecasting future travel. This is the objective of much of the research now under way. But without the benefit of the data from these statistically controlled surveys, and especially from the repeat surveys now being conducted in a number of cities, the development of a sound and proven method would not be possible.

The home-interview method of conducting O-D surveys was adopted after a careful study of numerous shortcut methods previously used. Although some of these had provided satisfactory answers to specific problems of limited scope, they had proved to be entirely inadequate for the planning of extensive urban highway networks. In most of the cities where they had been tried, comprehensive home-interview type surveys have subsequently been undertaken.

One of the shortcut methods that was found to be inadequate was the assembling of data "at locations where people concentrate," a procedure now advocated by Mr. Cherniack. This was tried in Washington in 1939-40, in Cleveland and in Detroit about 1944, and also in other places. The difficulties encountered were numerous and the results of limited value.

For one thing, information was obtained about only a fraction of the traffic that would use a freeway or arterial network. Even for those routes that lead to the CBD, much of the traffic has neither origin nor destination in that district. In cities of about one million population, not more than one-quarter of the trips by all modes of travel have been found to have either origin or destination in the CBD. Most of the remaining trips were on arterial routes for a portion of their length. How can a freeway or arterial route be planned and designed properly if information is lacking for an unknown and relatively large portion of the travel that would use it?

Aside from the omission of important segments of the travel, such surveys were found to be unsatisfactory for other reasons. Among the most important of these were inability to obtain information from a scientifically selected sample, lack of a satisfactory universe for expansion purposes, and inability to evaluate the accuracy of the results. There were varying degrees of cooperation from different establishments, resulting in undersampling for some types of work trips and oversampling for others. It was impossible to obtain representative information for travel other than work travel, even that occurring during the peak hour. Information about shopping trips to the big department stores could sometimes be obtained, but not about those to the innumerable smaller establishments. There was no satisfactory method of accounting for the many duplications where shoppers went from store to store.

In the home-interview type of survey, the sample is selected on a systematic areawide basis, a procedure developed by the highly competent statisticians of the Bureau of the Census and advocated by them for this purpose. This is the only type of survey of which the writer has knowledge where information is obtained about all of the travel, by all modes; where the sample is selected on a sound statistical basis; and where the results can be reliably appraised. In addition to internal checks of statistical reliability, there are many checks that can be made with independent data such as population, automobile ownership, and screenline counts.

As Mr. Cherniack points out, the sample is much too small (generally about 5 percent in the larger cities) to permit an accurate determination of the zone-to-zone movements. If the true number of trips between a certain pair of zones is 50, for example, the number of such trips included in a 5 percent sample might, by chance, be 0, 1, 2, 3, 4, 5, or even more, which would be expanded to 0, 20, 40, 60, 80, 100 or more, with a high percentage error in most cases. But in estimating traffic volumes, concern is not with individual zone-to-zone movements, but rather the accumulation of a large number of such movements on an expressway, arterial, or transit line. Tests have shown that the errors for such accumulation are generally acceptable. One test that has been made is to determine from the O-D data the number of trips that would have been expected to use certain bridges or highway sections and compare the results with actual ground counts. Another more comprehensive, if somewhat more theoretical method has been to establish a grid on a map of the entire area by drawing lines, say 1 mi apart in a north-south direction and 1 mi apart in an east-west direction, and calculate from the original sample and a number of subsamples the number of zone-to-zone movements that would cross different sections of this grid if made in an airline. From the variations of the results obtained from the different subsamples, the errors in the original sample can be estimated by the use of a statistical formula. A report on a test of this kind is included elsewhere in this Bulletin (see p. 114).

Mr. Cherniack is quite right in saying that the combining of zones into larger ones to increase the number of trips in the zone-to-zone movements introduces too great an inaccuracy in such factors as distance and travel time to be an acceptable procedure for the purpose of determining traffic movements. The accumulation of the smaller number of trips between smaller zones, as discussed previously, is much better for the purpose of assigning trips to highway and transit facilities. For purposes of determining trip production in relation to land use, a different procedure is available. Information is now being obtained concerning the land use at each end of a trip, and trips to like land use can therefore be combined to obtain an adequate sample for different areas of the city. The availability of electronic computers makes the task a relatively simple one.

Another source of error, not mentioned by the author, is probably considerably more important than the error due to the smallness of the sample. This is a "response" error, due to the fact that the interview often must be conducted with someone other than the person who performed the travel—usually the housewife. An attempt is being made to correct this in the more recent surveys by a double-interview procedure. On the first call a form is left with the request that all persons making trips on the following day record the origin and destination of each trip, the time of departure and arrival, the mode of travel, and the trip purpose. The forms are picked up and the other desired information is obtained on a subsequent day.

The writer certainly agrees with Mr. Cherniack that the mass of data collected in the numerous O-D surveys should be correlated with other data and extensively analyzed in order to establish fundamental facts that will aid in forecasting future travel. This is being done, with the aid of electronic computers, to the extent that funds and personnel permit. Data from the two surveys in Washington, D.C., in 1948 and 1955, the two in Phoenix, Arizona, in 1947 and 1957, and the two in St. Paul-Minneapolis in 1949 and 1958 are being used for this purpose. In Detroit, a continuing organization is reanalyzing the data from the original survey, and is collecting and analyzing data from a few zones on a continuing basis, as Mr. Cherniack suggests. As the results of these studies become available, it is hoped that improved methods of determining the future travel pattern in relation to urban development can be devised.

C.A. STEELE, Chief, Highway Economics Branch, Highway Needs and Economy Division, U.S. Bureau of Public Roads—These comments are prompted by the impact that this paper might have on the motor-vehicle-use and other similar interview-type studies. Although the urban O-D studies and the motor-vehicle-use studies are made for widely different purposes, many of the basic sampling and analysis techniques employed are the same or nearly the same.

The home-interview method of collecting data for the motor-vehicle-use studies and their predecessors, the so-called road-use studies, has now been in use for nearly 30 years. On the basis of the "rule of thumb" cited in the paper this method should now be presumed to be obsolete. However, during the 30-yr period so many fundamental changes have been made in the selection of the sample, the design of the interview forms, the nature of the data collected, and the methods of collecting them, that motor-vehicle-use study interviews obtained today bear only a superficial resemblance to those obtained 25 or 30 years ago. For example, the statistically supportable "probability" sampling method has replaced the old "purposive" sampling method originally employed. Furthermore, in designing modern motor-vehicle-use study samples, stratification is used wherever possible to improve the coverage of certain types of areas or characteristics that it is desired to represent, and to reduce the size of the over-all sample that it is necessary to obtain. As a result, the motor-vehicle-use studies home-interview samples taken today are not obtained on a flat across-the-board percentage basis as was formerly done, but a separate sampling rate is set for each stratum that it is desired to sample which will be sufficient to reflect with reasonable accuracy those characteristics that it is desired to analyze most completely.

In the design of stratified samples due consideration is given to the matter of household densities mentioned by Mr. Cherniack. The point he makes that auto ownership densities increase with household densities, but at declining rates rather than in proportion, is a good one, and has been given at least indirect recognition in the design of motor-vehicle-use study samples in several states. It is to be hoped that the decennial census of 1960 will provide much more complete information on housing and households than has been available from previous censuses, although the information obtained in the 1950 census was extremely helpful in the design of samples for the motor-vehicle-use studies.

Mr. Cherniack's concern throughout his paper seems to be with the collection and analysis of O-D travel data to aid in the design of specific facilities. It has long been recognized that the purpose for which a travel study is to be made will have an important bearing on the type of data collection to be employed, and the design of the sample to be used in obtaining the information. The U.S. Bureau of Public Roads and the state highway departments have had recourse to all of the three methods mentioned in Mr. Cherniack's paper, or to combinations thereof, in the many studies that they have made.

In addition to the more common applications of these sampling methods there are a few which are not so well-known but in each of which the specific method employed was best adapted to obtaining the desired information. Thus, roadside interviews were used to obtain information for rural road-service studies in Oregon and Washington in an attempt to define the radius of "access" and "neighborhood" trips for a relative-use analysis of highway benefits. The interviewing of travelers at their destinations—places of work, department stores, etc.—was done extensively in connection with the war industry transportations studies made during World War II for the purpose of developing information basic to the rationing of motor fuel and motor vehicle tires.

For the purposes of the studies just described these methods of interviewing were the best available. For the purposes of the motor-vehicle-use studies, however, there is as yet no known substitute for the home-interview type study where it is desired to obtain characteristics of motor vehicle ownership and use, especially the use of passenger cars. Inasmuch as it has been found that the use of commercial trucks is often not directly related to households, a fourth method of sample selection and interviewing has been developed and applied in a number of states within recent years. Here the sample of trucks on which interviews are to be made is obtained from the registration lists and the interview itself is made either with the owner of the vehicle or, if he is not the principal driver, with both the owner or his representative and the principal operator.

Each of the interviewing methods listed above has its particular virtues but each also has its shortcomings. Although the destination type of interview mentioned by Mr. Cherniack can be used to good advantage for certain types of studies, such as determining how the workers at a given plant travel to and from work, such interviewing, especially when conducted at retail establishments, is likely to produce extensive duplication in the information reported through the interviewing of the same person at several locations, and is also likely not to give a good distribution of sources from which the travel originated.

ROBERT T. HOWE, Associate Professor of Civil Engineering, University of Cincinnati—A casual reading of this paper does not do justice to the importance of the author's basic ideas. Unfortunately he implies, but does not specify, what appears to be his fundamental objective. This discussion first considers what appears to be this basic goal, and then comments in detail on three of the author's suggestions.

It would seem that what Mr. Cherniack wants is a completely bounded study of origins and destinations. Inasmuch as he does not refer to this objective specifically, many of his same comments about the weaknesses of present home-interview O-D surveys may be turned on his suggested "destination-interview" survey. It is assumed, therefore, that the data, which he proposes to collect from samples of workers at places of employment, shoppers in business districts, and recreation-seekers at places of amusement, would be expanded on the basis of the numbers of households in the actual zones of residence of the interviewers. Actually, it would be possible to expand existing home-interview data on a comparable basis, rather than on the ratio basis usually used. Censuses of business, and various state employment publications, combined with information from a city directory, can give a comprehensive picture of the actual distribution of jobs in an area as a control for expanding work-trip patterns from a statistical sample of such trips originating in homes.

All centers of entertainment must keep attendance records for tax purposes, and these would form a control on the expansion of recreation trips. Department stores and chain drug and food stores may not have records of the number of customers served in each store each day, but at least they have some record of the number of sales made each day, and this information could be used to control the expansion of shopping trips.

Mr. Cherniack fluctuates in his use of the term "trip" from total trips (including walking), to auto plus transit trips, to auto trips alone. It is inconceivable to the writer that the number of households per acre in any way influences the total number of trips generated per household in any way. The figures given on automobile and household densities are most interesting, and are certainly significant in the generation of automobile trips.

To check the author's contentions concerning the relationships between auto density and household density, the writer analyzed ten zones of the 1954 O-D Survey of Cincinnati and calculated household densities from the 1950 Census data and auto densities from the 1954 registration figures. The particular zones were chosen because the census tract and O-D boundaries coincided. Table 1 summarizes this analysis and seems to confirm the data given in the paper.

TABLE 1

DATA ON AUTO REGISTRATION AND HOUSEHOLD DENSITY IN TEN ZONES OF CINCINNATI, OHIO

Suburb	Area (acres)	Distance from CBD (mi)	Households per Acre	Automobiles per Acre
California	1,250	7.2	0.2	0.2
Winton Place	1,860	5.1	0.9	0.3
Riverside	875	6.0	0.5	0.5
Reading and Lockland	3,100	9.5	1.4	1.7
College Hill	2,120	7.2	1.4	1.8
Clifton	1,350	3.4	2.3	2.2
Avondale	1,960	3.6	4.5	3.8
Norwood	1,950	5.1	5.8	5.1
Westend (south)	360	1.0	20.5	6.5
Westend (north)	340	1.1	26.4	8.9

In their report on the Baltimore study, Voorhees and Morris (HRB Bull. 224) state that they used the numbers of employees in business districts as a measure of attraction of such areas for shopping trips, because employment data, and no others, were readily available. The writer believes that the number of employees in places of shopping, recreation, etc., is the only valid measure of attraction of such centers. Employment can usually be varied appreciably in the short term, and therefore, reflects changes in attraction with much greater sensitivity than does floor space, which can only be altered substantially in the long term.

An excellent example of the difference between number of employees and floor space as measures of attraction is the situation in a major suburban shopping district in Cincinnati. About 1940, a large, local grocery chain, built a "supermarket," Store A, in this district. About 1946, another large grocery chain built a competing unit, Store K, almost directly across the street from Store A. Both buildings are of similar size, and have similar facilities. Both chains carry on major advertising campaigns, and both give trading stamps. Neither side of the street has obvious shopping advantages or disadvantages, although Store K has a much better parking lot. Both stores have four check-out lanes. In the autumn of 1958, Store A was employing two check-out clerks on Saturday afternoons, and these were not too busy. At the same time, Store K was employing four such clerks, and they appeared to be very busy. In October of 1959, Store A closed and completely vacated its building. The floor areas of these buildings has not been changed since they were erected, but the numbers of employees and the powers of attraction have changed greatly.

As another example, consider the fact that a gasoline service station, occupying a 100- x 200-ft lot and employing four men in a 16-hr day, is not likely to attract many more customers than a specialty shop employing four persons for a total of 10 hr a day in a 10- x 20-ft room.

From another point of view, if employment data are gathered to control the expansion of work-trip patterns, as suggested previously, the same data can be used to control the expansion of shopping and similar trip patterns.

At one point in his discussion of future trip patterns, the author says "The power of attraction of any given site—would be found to be inversely proportional to the travel time—and the residences scattered over the study area." This appears to be a slip, because it is inconceivable that the number of trips between zones of residence and zones of non-residence could vary inversely as the populations of the residence zones.

The exponential-type function suggested in place of the "gravity model" has a serious disadvantage if Eq. 1 is approximately what the author has in mind.

$$A_j = k \frac{P_i}{e^x} \tag{1}$$

in which

A_j = number of trips attracted to a non-residence center j from residence center i;
P_i = population of residence center i;
k^1 = constant of proportionality; and
x = a measure of the distance from i to j.

When there is no impedance to the movement, and x reduces to zero, Eq. 1 assigns the total population to the co-terminous non-residence center, and this is scarcely a defensible assignment.

In summary, that which seems to be the author's goal, to completely bound the conditions for a given type of movement, in order to more accurately expand the O-D trip pattern for that type of movement, is completely valid and desirable. It appears, however, that some of his specific suggestions for achieving this goal are of dubious merit.

NATHAN CHERNIACK, Closure.—In supporting his disagreements with the author, Mr. Campbell has leaned heavily on the Chicago study data and experience. The Chicago and Detroit studies have taken long strides in meeting some of the author's criticisms leveled against the home-interview type surveys which have preceded those in Detroit and Chicago. But there is still a long way to go toward improving present methods of collecting data on urban travel and its determinants, as well as improving present methods of analysis.

For example, the author has brought together (Table 1) selected data from the Appendix to the Chicago Area Transportation Study (CATS), Volume One, for 77 districts in Chicago on total dwelling places and autos owned (Table 19, page 108), for residential land use expressed in acres (Table 21, page 110) and all residential person trip destinations (Table 23, page 113). From these basic data, the author calculated for each of the 77 districts, dwelling places per acre, autos per acre, and trips per acre.

Plotting autos per acre versus dwelling places per acre, yields a "scatter diagram" (Fig. 1) which demonstrates what the author had suggested in his paper; namely, "auto ownership densities thus increase with household densities not in proportion, but rather at declining rates." In fact, a simple parabola through the origin, used as a fast first approximation, indicates that autos per acre in the Chicago Study Area increase as dwelling places per acre, raised to a power of about 0.66. Autos per acre for any one of the 77 districts can thus be estimated from this simple first approximation to within a standard deviation of 15 percent of recorded autos per acre in each of the 77 districts in Chicago. (Much closer relationships could, of course, be obtained with more careful mathematical analysis.)

Also, a "scatter diagram" of person-trips per acre versus autos per acre (Fig. 2)

TABLE 1

DWELLING PLACES PER ACRE AUTOS OWNED PER ACRE PERSON TRIPS PER ACRE IN CHICAGO 1956-57

District	Total Dwelling Places[1]	Autos Owned[1]	Person-Trips Destination Residential[2]	Residential Acreage[3]	Dwelling Places Per Acre	Autos Owned Per Acre	Person-Trips Per Acre
01	12,756	1,298	24,524	11.4			
11	137,811	48,871	240,179	1,071.5	128.6	45.6	224.2
21	59,631	29,455	130,577	866.6	68.8	34.0	150.7
22	38,723	23,175	87,552	750.8	51.6	30.9	116.6
23	53,564	29,987	122,120	960.1	55.8	31.2	127.2
24	28,832	17,732	66,747	569.3	50.6	31.1	117.2
25	19,825	11,931	41,859	396.2	50.0	30.1	105.7
26	13,842	8,673	34,353	359.2	38.5	24.1	95.6
27	42,577	15,061	82,836	545.7	78.0	27.6	151.8
31	71,830	40,976	156,397	1,135.7	63.2	36.1	137.7
32	56,559	41,955	169,230	1,825.0	31.0	23.0	92.7
33	50,347	36,803	149,316	1,574.7	32.0	23.4	94.8
34	32,631	23,070	89,319	894.9	36.5	25.8	99.8
35	22,592	15,933	62,988	884.9	25.5	18.0	71.2
36	30,670	19,474	87,182	1,001.0	30.6	19.5	87.1
37	81,787	37,332	174,061	1,052.8	77.7	35.5	165.3
41	72,092	58,668	214,166	2,282.8	31.6	25.7	93.8
42	59,889	50,078	182,776	3,174.5	18.9	15.8	57.6
43	68,558	62,669	243,255	4,505.0	15.2	13.9	54.0
44	39,692	37,995	143,746	2,805.7	14.1	13.5	51.2
45	24,386	21,769	89,145	1,895.3	12.9	11.5	47.0
46	67,524	54,141	221,899	3,031.6	22.3	17.9	73.2
47	103,975	68,297	302,674	2,771.8	37.5	24.6	109.2
51	37,371	33,509	145,845	2,511.9	14.9	13.3	58.1
52	38,676	41,952	163,705	4,312.5	9.0	9.7	38.0
53	32,152	34,952	137,777	3,748.6	8.6	9.3	36.8
54	29,306	23,548	86,985	2,632.2	7.7	8.9	33.0
55	19,595	20,040	81,071	2,045.9	9.6	9.8	39.6
56	35,778	38,510	158,752	4,092.3	8.7	9.4	38.8
57	42,232	37,106	169,175	2,615.1	16.1	14.2	64.7
61	20,145	25,148	116,513	4,630.2	4.4	5.4	25.2
62	26,552	32,303	135,244	4,560.4	5.8	7.1	29.7
63	33,023	38,434	147,178	5,902.7	5.6	6.5	24.9
64	22,499	27,815	120,288	4,810.2	4.7	5.8	25.0
65	12,988	16,298	62,550	2,757.1	4.7	5.9	22.7
66	41,101	43,416	196,282	4,559.4	9.0	9.5	43.0
67	31,356	26,883	141,321	2,293.5	13.7	11.7	61.6
71	16,709	20,649	85,163	3,581.1	4.7	5.8	23.8
72	22,174	26,443	105,950	5,917.2	3.7	4.5	17.9
73	16,838	20,025	84,285	5,314.6	3.2	3.8	15.9
74	17,086	19,987	79,084	5,042.6	3.4	4.0	15.7
75	3,801	4,239	17,020	1,359.9	2.8	3.1	12.5
76	35,176	38,126	181,946	6,117.2	5.8	6.2	29.7
77	15,015	16,920	73,492	2,403.7	6.2	7.0	30.6
Total	1,730,666	1,341,646	5,606,527	115,574.8	15.0	11.6	48.5

[1] Chicago Area Transportation Study, CATS, Vol. 1, Table 19, p. 108.
[2] Chicago Area Transportation Study, CATS, Vol. 1, Table 23, p. 113.
[3] Chicago Area Transportation Study, CATS, Vol. 1, Table 21, p. 110.

indicates that equally good approximations of person-trips per acre could be estimated from autos per acre. Another first approximation, this time a straight line through the origin, indicates that there are about 4.2 person-trips per acre for every auto per acre. Person-trips per acre can thus be computed from autos per acre from this simple first approximation to within a standard deviation of about ± 15 percent of recorded person-trips per acre, in each of the 77 districts in Chicago.

These two diagrams for the Chicago Study Area thus confirm the author's expressed judgment that "household densities would appear to constitute far more stable indicators of auto ownership than are such indicators as group income levels. Also, distances from the CBD are not good fundamental determinants of either car ownerships or trips. Distance from the CBD is only a space parameter akin to a time parameter." That is exactly what Figure 32, page 61, of Volume One (CATS) illustrates. Examining Figure 32 carefully, the author is firmly convinced of this judgment also expressed in his paper, that "mere distance from the CBD, which has been changing significantly in character, like mere passage of time, are both generally weak and at times unreliable determinants for forecasting purposes. More fundamental and considerably more predictable determinants, like household densities, are needed to measure zonal auto ownerships, which in turn are the determinants for generated zonal trip ends."

One need but compare the author's two scatter diagrams derived from the basic

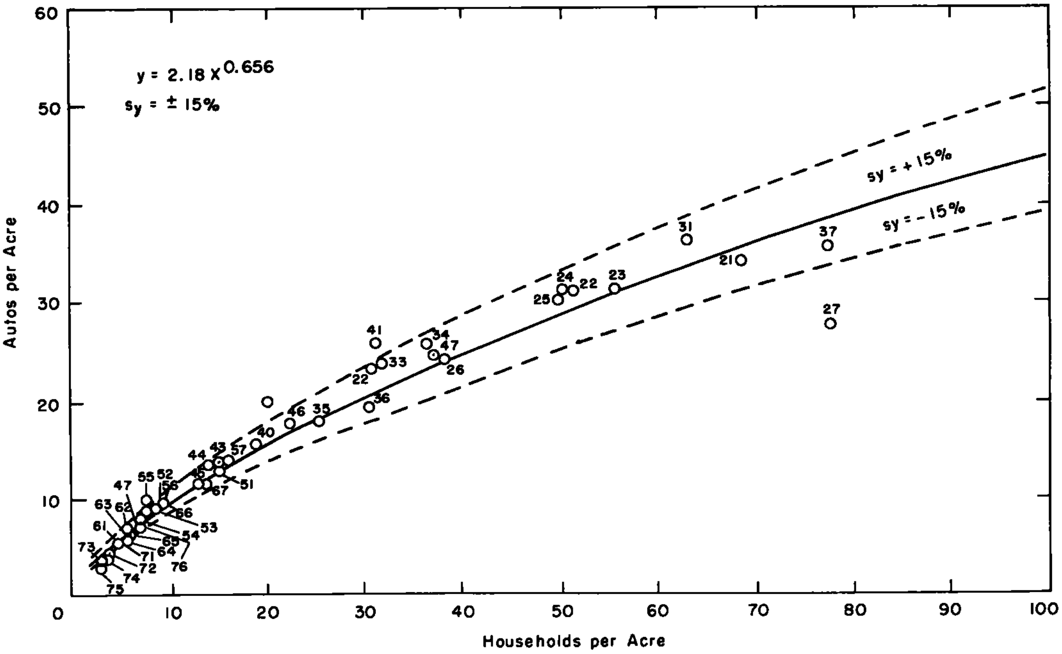

Figure 1. Relation between autos per acre and households per acre for 77 districts in Chicago, 1956-57 (from Table 1).

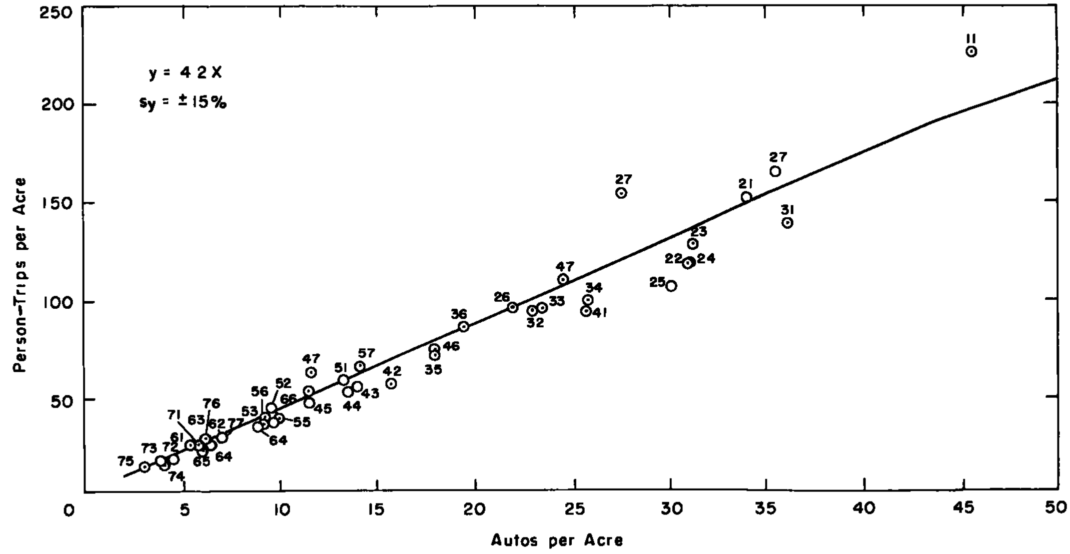

Figure 2. Relation between person-trips per acre and autos per acre for 77 districts in Chicago 1956-57 (from Table 1).

data in the Chicago Area Study—autos per acre versus dwelling places per acre, and person-trip destinations per residential acre versus autos per acre—with Figure 32, page 61 of Volume One, CATS, as well as the formulas in Appendix Table 37, to determine for one's self which are the simpler, more fundamental, more accurate determinants of person-trips in the 77 individual districts in Chicago. These simple determinants of dwelling places and residential acreages may readily be kept up-to-date in the continuing study. Also, auto ownerships in selected districts could be sampled from time to time and thus trips per residential acre estimated. A continuous check can thus be had of both the end results and the relationships established on the basis of 1956 data.

Although such forecasts will yield total person trips in the 77 districts in the study area, they do not pinpoint the journey-to-work trips, most of which occur in peak periods, built up on arterials close to concentrations of sites of employment, and absorb substantial portions of highway capacities. Accommodating journey-to-work trips close to concentrations of sites of employment is and will be the critical urban transportation problem. Consequently, the author insists that eventually O-D surveys will have to be made at locations where people are concentrated.

At the outset of his discussion, Mr. Campbell states that he "would agree with Mr. Cherniack that the home-interview method has inherent defects. However, little else is found in his paper with which the writer can agree." The author is happy to find that the "little else" with which Mr. Campbell can agree with the author is in a highly important research area, and is also supported by the results of the Chicago study. Mr. Campbell phrases his agreement with the author as follows: "Mr. Cherniack states that the gravity formula, which is a power function, does not adequately describe the power of attraction of any given site. He feels that an inverse exponential relationship would produce better results. The writer heartily agrees with Mr. Cherniack on this point."

It is kind of Mr. Lynch to express agreement with the author on several of his criticisms of the home-interview type of O-D surveys, which Mr. Lynch has pioneered and which he has been personally instrumental in improving continuously over some 15 years.

Nevertheless, there still remains a large area of disagreement. The author, therefore, feels impelled to speak out on the need for continuing to assemble essential data, and by methods which would be most effective and most adequate for a deeper understanding of the urban traffic pattern. Understanding of urban travel patterns and urban transportation needs, is still largely in the astrology stage and just beginning to show glimpses of the astronomy stage. Yet there is a tendency to speak and write on this subject as if the astronautical stage has been reached, and that all that is now needed are the proper electronic computers and all transportation problems would be solved.

The author's suggested procedure for assembling data at locations where people concentrate (which is the heart of his paper) is described by Mr. Lynch as "one of the short-cut methods that was found to be inadequate... The difficulties encountered were numerous and the results of very limited value." The author's suggested method is far from being just another "short-cut method." It requires considerable thought to design such surveys properly. Consequently, there are many difficulties to be anticipated in conducting such surveys. But does the fact of difficulty of a given type of O-D survey necessarily preclude its use if this procedure is, in fact, an effective means for acquiring a deeper understanding of the major facets of the urban traffic and transportation problem? The author is not ready to concede Mr. Lynch's opinion that the results of data collected at locations where people concentrate would have "very limited value."

Mr. Lynch reveals the weaknesses of the data actually assembled in Washington (in 1939-40), and in Cleveland and Detroit (about 1944), by pointing out that the data thus obtained revealed "only a fraction of the traffic that would use a freeway or arterial network. Even for those routes that led to the Central Business District (CBD), much of the traffic has neither origin nor destination in that district." Apparently in the Washington, Cleveland and Detroit tests the data assembled at the sites of employment and business were located only in the CBD's. The author, however, did not suggest the "short cut" of limiting the assembly of such data only to CBD locations where people were concentrated. He suggested such locations everywhere within the study area (at the Pentagon, for example, as well as the Treasury Building in Washington, D.C.).

So, when Mr. Lynch asks how a freeway or arterial route can be planned and designed properly if information is lacking for an unknown and relatively large portion of the travel that would use it, the author answers that of course it cannot, on the basis of the short-cut methods that were used and with such incomplete and inadequate data as were assembled at the CBD sites of employment in Washington. No wonder "the results were of very limited value."

If, however, one recognizes the now commonly known facts that sites of employment and concentrated shopping areas have become quite diffused over urban areas, and that CBD's are not the only such areas of concentration of people in metropolitan districts, one would proceed to assemble sample data at all locations where people are concentrated. He would thus obtain data that would reveal all actual and potential locations within urban areas where traffic concentrations occur on the arterials and which could be anticipated in the future on proposed expressways.

Mr. Lynch objects to the assembly of data at locations where people concentrate on several other technical grounds, such as "the inability to obtain information from a scientifically selected sample, the lack of a satisfactory universe for expansion purposes, and the inability to evaluate the accuracy of the results." To the author, these reasons smack of statistical idolatry. Are we so engrossed in making sacrifices to statistical idols as to forego the pursuit of a better understanding of urban traffic and transportation through the medium of pure statistical explorations without benefit of published, detailed theoretical statistical maps? What if outer space scientists took that same attitude? They do not. They send up missiles costing millions to explore outer space. We as social scientists should also do some exploring. Even where a precise value cannot be put on the entire universe (although not being altogether naive about its size), data should still be assembled at locations where people concentrate, obtaining their travel habits and correlating them with data on land uses and travel impedances, to obtain a better understanding of people's travel habits than is now possessed.

Mr. Lynch dredges up a number of other difficulties encountered in the collection of data at locations where people were concentrated. "There were varying degrees of cooperation from different establishments resulting in under-sampling for some types of work trips and over-sampling for others." To detect this under- and over-sampling statistically, one had to have some approximations of the respective universes; and if there were such approximations, the estimated under- and over-samplings could be statistically corrected, at least approximately.

Mr. Lynch goes on to say that "it was impossible to obtain representative information for travel other than work travel, even that occurring during the peak hour." But a well-known slogan says, in part, "The impossible takes a little longer." It can be done.

Mr. Lynch continues: "Information about shopping trips to the big department stores could sometimes be obtained, but not about those to the innumerable smaller establishments." The author's procedure would be to assemble the data available and analyze what is at hand. In this way, exploration will have been made further into the dark and light sufficient to outline the whole will have been shed.

Continuing, Mr. Lynch says: "There was no satisfactory method of accounting for the many duplications where shoppers went from store to store." Each store or group of stores would constitute a small, statistical universe for the study of such travel behaviors. However, when it came to adding the universes to other segments which contained duplications, there would be set up an approximate control on the aggregates so as to eliminate most of the duplications, numerically.

Mr. Lynch continues: "In the home-interview type of survey... in addition to internal checks of statistical reliability, there are many checks that can be made with independent data such as population, automobile ownership and screenline counts." Checks of statistical reliability are highly theoretical and are based on the assumption that the sample is purely random, whereas in actual practice it may be far from being random.

Data on population in intercensal years are, at best, extrapolated "guesstimates" based on the previous decennial population census data, supplemented by recorded births and deaths since the census year plus guesses as to net migrations into or out of the area. "Checking" expanded sample person trips with such population "guesstimates" is like the blind leading the blind.

Few urban areas have current auto registrations tabulated by small areas, that could be used to check the expanded 5 percent samplings of autos registered in the O-D zones. To be sure, data on individual addresses of car owners are available at the vehicle registration bureaus of the respective states. But in how many home-interview studies have the auto registrations as of the year of the survey, been tabulated by O-D zones to check the expanded 5 percent samplings of car ownership in these zones?

Screenline vehicle traffic counts are, in fact, excellent checks on expanded vehicle trip samplings obtained from home interviews, but only for a fraction of the total trips generated in the total study area. Besides, such vehicle counts have invariably revealed under-enumerations in the aggregates, particularly in off hours. Moreover, if roadside O-D interviews had been made at the screenline (as they have not been), it would have been discovered that errors in the expanded vehicle trip samplings from individual O-D zones would be quite large. Also, if O-D surveys were simultaneously made at sites of employment they would probably reveal, quite dramatically gross errors in certain expanded home-interview zone-to-zone movements.

Mr. Lynch does agree that 5 percent samples are "much too small to permit an accurate determination of the zone-to-zone movements." But, he continues, "In estimating traffic volumes, concern is not with individual zone-to-zone movements, but rather with the accumulation of a large number of such movements on an expressway, arterial or transit line. And tests have shown that the errors for such accumulations are generally acceptable." Again the author must disagree on two grounds. On the first, the author maintains that concern is with zone-to-zone movements, which are the elements of the trip aggregates that impinge upon proposed expressways at their various entrance and exit ramps. Mr. Lynch makes the implicit assumption that in the accumulation of a large number of such zone-to-zone movements, the individual errors will be compensating and the algebraic sum of the individual errors will be smaller than the error in any individual zone-to-zone movement. This is not necessarily so: the errors may be cumulative, not compensating. Besides, the theoretical tests described by Mr. Lynch are not completely satisfying.

The second ground of disagreement is that, again, concern is with zone-to-zone movements because these movements must be utilized to obtain sound relationships with trip determinants such as land uses, as well as distances, times, costs and other impedances between pairs of zones, which relationships can eventually be fed into electronic computers so that there would be some degree of confidence that the computer answers will be realistic. And the individual expanded zone-to-zone movements, which are known to contain large errors, cannot be cured statistically by merely combining trips between small zones into those between large zones and thereby reducing the size of the errors.

It is gratifying that Mr. Lynch admits that the author is correct in maintaining that "the combining of zones into larger ones, to increase the number of trips in the zone-to-zone movements, introduces too great an inaccuracy in such factors as distance and travel time to be an acceptable procedure for the purpose of determining traffic movements." He goes on to say, however, that "the accumulation of the smaller number of trips between smaller zones... is much better for the purpose of assigning trips to highway and transit facilities." They would be better if the original bases for assignments that were established from the individual zone-to-zone movements were in fact valid, but these bases are themselves weak because, for correlation purposes, the original samples of the individual zone-to-zone movements were so anemic.

Mr. Lynch goes on to say that "information is now being obtained concerning the land use at each end of a trip, and trips to like land use can therefore be combined to obtain an adequate sample for different areas of the city. The availability of electronic computers makes the task a relatively simple one." A beginning is just being made on assembling data on areas devoted to residential, commercial and industrial uses, in an effort to obtain approximations of the trips generated by significantly different types of land use. Such land-use data for small O-D zones, together with the corresponding data on the trips that focus on locations where people concentrate, in the urban areas where O-D data have been assembled, constitute a veritable mine of urgently needed researchable data. Such assembled land-use and travel-impedance data, even without benefit of electronic computers, would be far more valuable than the availability of electronic computers without such data. If a deeper understanding of urban traffic patterns is to be acquired and sound bases for an intelligent appraisel of the needs for urban transit facilities are to be established, it is the simultaneous collection of such land-use and travel-impedance data for the O-D zones in urban areas where trip data will be assembled, which is now the crying need, rather than availability of electronic computers.

Mr. Lynch volunteers a serious source of error in the home-interview type of O-D surveys which the author had not mentioned in his original paper. "This is a 'response' error due to the fact that the interview often must be conducted with someone other than the person who performed the travel—usually the housewife." In assembling trip data at locations where people concentrate, such as at sites of employment, the data for, say, 100 journey-to-work trips may be obtained not by ringing 5 home doorbells but by ringing just one doorbell—the personnel officer's. The data on 100 journey-to-work trips would thus be obtained with no "response" error of the type that would result from multiplying by 20 the responses from the wives of the 5 workers. Besides, a sample questionnaire distributed among the workers at sites of employment would also yield individual journey-to-work distances, times, costs and modes of travel between homes and work places.

The author has taken time to spell out his disagreements with Mr. Lynch in order to call attention to the fact that there is still a far way to go in developing a profound understanding of urban travel patterns sufficient to plan wisely for urban transportation needs. This results from the fact that despite all the voluminous trip data that have been collected by home-interview surveys over some 15 years, there is still a lack because of not having contemporaneously collected data on trip determinants. Short-changed trip determinants consist of such data for O-D zones as areas occupied by residences, commercial and industrial establishments, as well as travel impedances for individual zone-to-zone movements, both of the type that are directly measurable (highway distances, travel times and costs) and those that are only indirectly measurable (travel irritations and annoyances) by recording traffic lights, left turns, parked cars, and other known and suspected travel irritants.

In his discussion Mr. Steele places the home-interview type of O-D survey in its proper perspective and in the light of all the sampling techniques which have been used by the U. S. Bureau of Public Roads for various data collection purposes. He also sets forth some effective fundamental principles on sampling techniques and data collection. Consequently, the author must necessarily agree with most of Mr. Steele's discussion. In fact, the author wishes to take the liberty of using certain of Mr. Steele's statements of sampling principles and practices to underline the author's own suggestion for collecting O-D data at places where people concentrate.

Mr. Steele first points out that "so many fundamental changes have been made in the selection of the sample, the design of the interview forms, the nature of the data collected, and the methods of collecting them, that motor-vehicle-use study interviews obtained today bear only a superficial resemblance to those obtained 25 or 30 years ago." Excellent! These changes are all to the good. Obsolescence of the methods of collection is thereby postponed.

But Mr. Steele goes on to point out that in certain types of sampling, stratification has been used, and for good reasons. To the author's knowledge, however, home-interview surveys have still been based on random, rather than stratified samplings. Under random samplings, it is implicitly assumed that homes are distributed in a study area much like the molecules of a gas are in a receptacle. It is known, however, that household densities vary widely in different sections of the study area and are not randomly distributed. Sampling of households stratified on the basis of household density, for example, would improve coverage, as Mr. Steele suggests. The author, however, is not aware that home-interview samplings had been stratified except for special types of dormitories.

Assuming that a parallel system of sample interviews were also made in areas where people are concentrated, such as in CBD's as per the author's suggestion, one would not adopt the naive attitude that there is no awareness that clusters of sites of employment exist. It would have to be recognized that there were financial, theatrical, shopping and other such clusters. Stratification of sample interviews in areas where people are concentrated would thus be a sine qua non of sampling in such areas, because as Mr. Steele indicates, stratification under those conditions would improve coverage immensely.

Mr. Steele hopes that the decennial census of 1960 will provide much more complete information on housing and households than has been available from previous censuses. It probably will. But the acreages necessary to calculate household densities expressed as households per residential acre are still lacking. Residential acreages will still have to be compiled by the local planning agencies in the respective study areas.

Mr. Steele states a fundamental canon of statistics when he says: "It has long been recognized that the purpose for which a travel study is to be made will have an important bearing on the type of data collection to be employed and the design of the sample to be used in obtaining the information." Then he goes on to say: "For purposes of motor-vehicle-use studies, however, there is as yet no known substitute for the home-interview type study where it is desired to obtain characteristics of motor-vehicle ownership and use, especially the use of passenger cars." Here the author disagrees with Mr. Steele that the essential purpose of current travel studies is to obtain characteristics of passenger car ownership and use. Instead, the purpose of the travel study is to obtain quantitative measures and a thorough understanding of urban traval patterns in study areas, on the basis of which transport systems may be conceived that would adequately meet current and future needs of urban travel, particularly in peak periods when the capacities of existing and proposed transport facilities will be largely absorbed. Consequently, it is essential to interview especially those who travel in peak periods (workers on weekdays) and to interview them at their destinations which are closest to locations where the capacities of transport systems are and will usually be largely absorbed in peak periods.

Mr. Steele states: "Mr. Cherniack's concern throughout his paper seems to be with the collection and analysis of origin-destination travel data to aid in the design of specific facilities." Instead, the author's concern is with urban transportation systems, including existing rail as well as highway facilities, existing rail transit where available, and mass transit by express buses, and not just with ownership and travel by autos on the highways. That is why the author desires as accurate a quantification of the characteristics of the journey-to-work pattern as possible.

The War Industry transportation studies made during World War II for purposes of motor fuel and tire rationing, to which Mr. Steele refers, are exactly the types of studies the author has in mind in connection with his suggestions contained in the paper. Such studies, when amplified with supplemental data, would yield valuable "isochron lines" which would indicate how far away, timewise, various sites of employment drew 50 percent or 75 percent, or other percentages, of their employees. These studies would also reveal the varying percentages of the labor pools in small residential areas which different sites of employment drew at varying travel times between plants and homes. These are the types of travel data for which there is presently a great need, for the purpose of planning urban highway transport systems of the future to handle particularly the journey-to-work travel in peak periods.

It is refreshing to have Professor Howe say: "To check the author's contentions concerning the relationship between auto density and household density, the writer analyzed ten zones of the 1954 survey of Cincinnati and calculated household densities from the 1950 Census data and auto densities from the 1954 registration figures," and incidentally that "Table 1 summarizes this analysis and seems to confirm the data given in the paper."

Professor Howe did just what the author recommended in his paper; namely, to "exhume" home-interview data, to retabulate them and to re-analyze them, and then discover some significant relationships previously not revealed.

Table 2 and Figures 3 and 4 were prepared on the basis of the data dredged up by

TABLE 2

AUTOS PER ACRE AND HOUSEHOLDS PER ACRE IN 10 ZONES OF CINCINNATI, OHIO, AND 18 N.Y.-N.J. COUNTIES

New York - New Jersey Counties				Locations in Cincinnati, Ohio				
	County	Households[1] per Acre	Autos[2] per Acre		Suburb	Households[3] per Acre	Autos[4] per Acre	Distance From CBD (mi)
1	Suffolk, N.Y.	1.3	2.2	1	California	0.2	0.2	7.2
2	Morris, N.J.	1.8	2.3	2	Winton	0.9	0.3	5.1
3	Somerset, N.J.	2.2	3.0	3	Riverside	0.5	0.5	6.0
4	Middlesex, N.J.	2.7	3.4	4	Reading and Lakeland	1.4	1.7	9.5
5	Rockland, N.Y.	2.7	3.6	5	College Hill	1.4	1.8	7.2
6	Richmond, N.Y.	3.0	3.5	6	Clifton	2.3	2.2	3.4
7	Monmouth, N.J.	3.2	4.0	7	Avondale	4.5	3.8	3.6
8	Nassau, N.Y.	3.3	4.3	8	Norwood	5.8	5.1	5.1
9	Westchester, N.Y.	3.9	4.8	9	Westend (South)	20.5	6.5	1.0
10	Bergen, N.J.	4.5	5.3	10	Westend (North)	26.4	8.9	1.1
11	Union, N.J.	4.6	6.0					
12	Passaic, N.J.	5.3	5.5					
13	Essex, N.J.	7.8	8.5					
14	Queens, N.Y.	8.4	7.2					
15	Hudson, N.J.	15.4	13.0					
16	Bronx, N.Y.	22.9	9.5					
17	Brooklyn, N.Y.	38.1	16.5					
18	Manhattan, N.Y.	52.0	20.9					

[1] Estimated for 1955.
[2] Registrations as recorded in 1955.
[3] As of 1950.
[4] Registrations as of 1954.

Professor Howe. In Figure 1b the center of the CBD has been plotted on the right, and distances therefrom to the left, in order to make this plot visually comparable to Figure 1a. It will be seen that households per acre is a much superior determinant of autos per acre than is the distance from the CBD. Also, from any curve fitted to the relationship of autos per acre versus households per acre, a derivative equation may be readily obtained of autos per household by dividing both sides of the equation by households per acre.

Figure 4 shows autos per acre versus households per acre in 10 zones in Cincinnati and 18 New York and New Jersey counties in the New York-New Jersey metropolitan district. The similarity is marked for two such widely separated urban areas.

When Professor Howe says: "It is inconceivable to the writer that the number of households per acre in any way influences the total number of trips generated per household," he apparently does not realize that households per acre is only an indicator of autos per acre, and that, in the same manner, autos per acre then also becomes an indicator of not only auto trips but also of person-trips both by auto and by common carriers, where the latter are in fact available in the respective zones.

He also suggests that "the number of employees in places of shopping, recreation, etc., is the only valid measure of attraction of such centers," instead of floor-space data as proposed by the author. It would seem that, for purposes of long-term projections, it would be much more convenient to measure areas on a map and then to establish factors of floor space per 100 workers for various non-residential uses. For any future year, areas of non-residential uses would first be forecast and then translated into person-trips on the basis of the future intensity of use of floor space by workers and occupants. For example, future office space in CBD's would first be forecast. Then, in the light of the increasing use of office equipment for data processing, requiring greater areas of office space, estimates of office employees for the future would call for higher factors of floor space per 100 office workers.

In a paper by Harper and Edwards (HRB Bull. 253) of which the author was not aware when he wrote his own paper), they state: "Some workers in the field of city planning have been saying that the traffic which flows in and out of a city every day is generated by the buildings, or rather by the businesses which occupy and use the buildings in the center. So far as could be ascertained, such statements have not been checked... To investigate, the Ontario Joint Highway Research Program sponsored research at Queen's University to see if a relationship between amount of floor space in use in various classifications and travel to the CBD could be demonstrated." Of some 120 cities where O-D studies

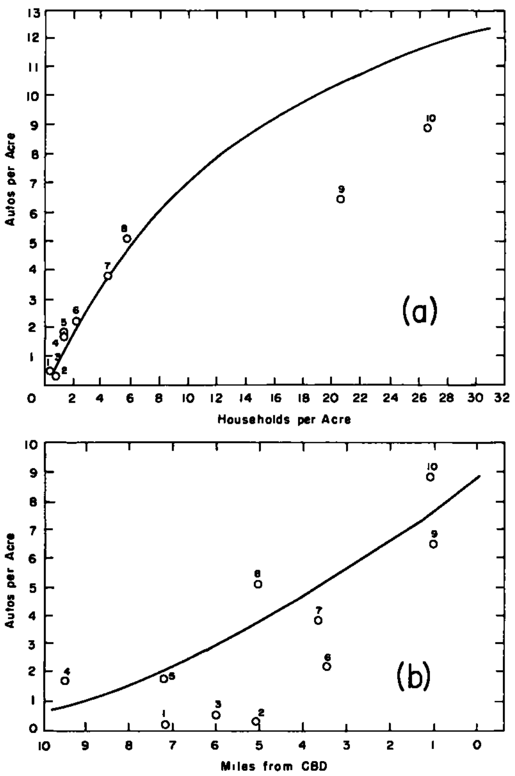

Figure 3. Auto distribution vs (a) household distribution and (b) distance from the CBD for 10 locations in Cincinnati, Ohio.

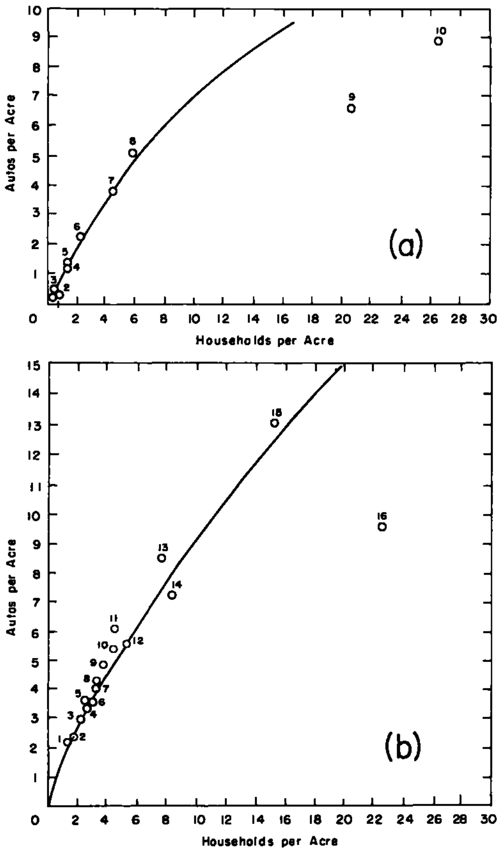

Figure 4. Comparison of auto vs household concentration for (a) 10 locations in Cincinnati, Ohio, and (b) 18 New York and New Jersey counties.

had been made, they could find only Philadelphia, Detroit, Baltimore, Seattle, Vancouver and Tacoma where they could estimate floor areas in CBD's. They conclude: "The results are such that it is possible to say that the number of people attracted to an area in a city center appears to be closely related to the amount of floor space being used for various purposes in the section of the CBD considered. It seems that, for highway planning, it would be valid to use sound, economic forecasts of future floor-space use in a central area as an index of the area's future attraction."

Not only has some support been received from Harper and Edwards in the matter of using floor space as an indicator of trips to and from non-residence areas, but there is a painful awareness of the general paucity of data on employment in small areas of employment. Consequently, the author is herein pleading for some types of data not now available, as indicators of the daytime population in non-residence areas of economic activities. Data are needed to give meaning and quantitative expression to relationships between person-trips and land use, which are assumed to be generally available but which do not now, in fact, exist.

Again, when Professor Howe says that censuses of business, and various state employment publications, combined with information from a city directory, can give a comprehensive picture of the actual distribution of jobs in an area, the author takes that statement with an oversized grain of salt. In this area, he can only comment that Professor Howe needs to get his hands dirty with statistical data on economic activities and social behavior in order to temper his optimism with humility.

Planners talk freely and qualitatively about land use and its traffic generating characteristics; so much so that many students of planning take it for granted that quantitative land-use data are generally and readily available in usable form for correlating with person-trip data; that a number of such relationships had been generally established.

ERRATA

BULLETIN 253

In Bulletin 253, pages 187 and 188, the following corrections should be made to the equations:

Eq. 3 $A_j = k\,1\,P_i e^{R(d-x)}$

Eq. 4 $e^{R(d-x)} = E$

Eq. 5 $E = e^{R(d-x)}$

Eq. 6 $\log_e E = \log_e \left(e^{R(d-x)}\right) = R(d-x)$

Eq. 8 $\log_e E = (d-x) \log_e (1+r)$

Eq. 9 $E = (1+r)^{(d-x)}$

Eq. 10 $A_j = k\,1\,P_i (1+r)^{(d-x)}$

This just is not so. Statistical data on areas devoted to residence and non-residence uses are not generally and readily available in such form that they could be used as factors of trip generation, either now or in the future.

The exponential-type mathematical function, as an expression of the inverse relationship between trips and impedances like distance, does not seem to appeal to Professor Howe and so he dismisses it quite abruptly; he prefers the "gravity model"— a power function. It is the privilege of any researcher to choose the mathematical function which, in his judgment, best expresses the law he is trying to state mathematically. Unfortunately, economic and social data never precisely define the mathematical function. The researcher must select from a family of curves the one which in his judgment is best suited to express the law he is seeking. It is the author's judgment that the exponential-type function does this best for the inverse relationship quoted previously. The soundness of the author's judgment may be demonstrated with Professor Howe's own Eq. 1 and his own nomenclature.

For use in a practical problem, Eq. 1 may at first be rewritten as

$$A_j = k\,P_i e^{-x} \qquad (2)$$

Then several constants are introduced which, however, do not change the form of the equation as an exponential-type.

$$A_j = k\,1\,P_i\,e^{-R(d-x)} \qquad (3)$$

Now it is assumed that the total number of employees that work in non-residence zone j, and that are drawn from all residence zones in the study area, may be obtained from Census data. From these data, k may then be obtained by dividing the total employees who work in non-residence zone j by the employed labor force (ELF) in the study area. The constant k would thus be expressed as "employees in zone j per 10,000 employed labor force in the study area."

P_i is the population in residence zone i. In the equations, it is merely an indicator of the ELF residing in zone i. If ELF of zone i happens to be available, wonderful! But that is a rarity. By applying the constant 1 or "ELF per 1,000 population," obtainable from the last available decennial Census data, a guesstimate may be made of the ELF in residence zone i.

The product $k1P_i$ then is equal to employees in non-residence zone j per 10,000 ELF in study area multiplied by the ELF in residence zone i. This product thus yields "probable employees in non-residence zone j drawn from residence zone i," if travel distance, time, cost, etc., were not real travel impedances. This computed number of employees drawn from zone i consequently reflects only the size of P_i in zone i, but not its travel impedance from zone j.

For the function

$$e^{-R(d-x)} = E \qquad (4)$$

which reflects the effects of travel impedances, the average "employees per 10,000 ELF for the study area," or k, lies on a circle at a mean travel distance d from the non-residence zone j to all residence zones in the study area from which employees were drawn. This distance d (or time) is obtainable as part of the employee data that the author has recommended be assembled. When x is equal to the mean distance d, in Eq. 4, e becomes equal to one. A_j for trips from centers of residence zones lying on the circle of mean distance, d, then becomes equal to $k1P_i$, or equal to the probable number of employee trips. When x is smaller than d and d-x is positive, the entire exponent is negative. Thus, where the center of a residence zone is closer than the mean distance, d, the number of trips drawn therefrom is larger than average; when x is larger than d, and d-x is negative, or where a residence zone is farther away than the mean distance, d, the number of trips drawn therefrom is fewer.

By not probing his own mathematical equation too deeply, Professor Howe has come to an apparently erroneous conclusion that the exponential-type function "assigns the total population to the co-terminus non-residence center and this is scarcely a defensible assignment."

Incidentally, R in Eq. 4 has a special significance. It expresses neatly how the differential percentage rates of trips per 10,000 ELF vary with numerical differences in travel impedances. The closer or farther away a residence zone is from the area of employment, the greater or fewer the "trips per 10,000 ELF" it furnishes this area of employment. It is these differential rates of attraction which it is desired to establish for various types of areas of employment and for other areas of attraction.

A mathematical transformation of Eq. 4 will bring this out sharply.

SEE ERRATA SLIP, PRECEDING PAGE

$$E = e^{-R(d-x)} \tag{5}$$

$$\log_e E = \log_e\left(e^{-R(d-x)}\right) = -R(d-x) \tag{6}$$

$$\text{Then let } R = \log_e\left(1-r\right) \tag{7}$$

$$\log_e E = -(d-x) \log_e (1+r) \tag{8}$$

$$E = (1+r)^{-(d-x)} \tag{9}$$

then

$$A_j = k\,1\,P_i\,(1+r)^{-(d-x)} \tag{10}$$

The constant r in Eqs. 9 and 10 represents the increased (percentage) differential rate, compounded, of trips per 10,000 ELF, with each numerical unit of reduction in travel distance x, from the average travel distance, to reach non-residence zone j or, in general, with each numerical unit of reduction in travel impedance like time, cost, and other impedances, expressed in cents, which would be substituted for d and x in Eq. 9 or Eq. 10.

In the future, given new areas of employment, the employee reservoir, and the corresponding spatial distribution of employees in the various residence zones from which these areas of employment would draw employees, can thus be computed from Eqs. 9 and 10.

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