

## Stresses Under Circular Flexible Foundations

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This paper outlines a new simple, and convenient method of computing stresses underneath uniformly loaded flexible circular foundation of a definite given radius: (a) along the vertical axis underneath the circular surface flexible foundation; and (b) along horizontal plane at any outward distance from the vertical axis at right angles to it, and parallel to the horizontal plane surface boundary.

- EMPLOYING the geometrical dimensions outlined in Figure 1, and the original preliminary assumptions of Boussinesq (1) for point loads, Love (2) developed the basic differential equations for the deflections and stresses beneath uniformly loaded flexible circular surface foundations. To use these differential equations conveniently in practi-

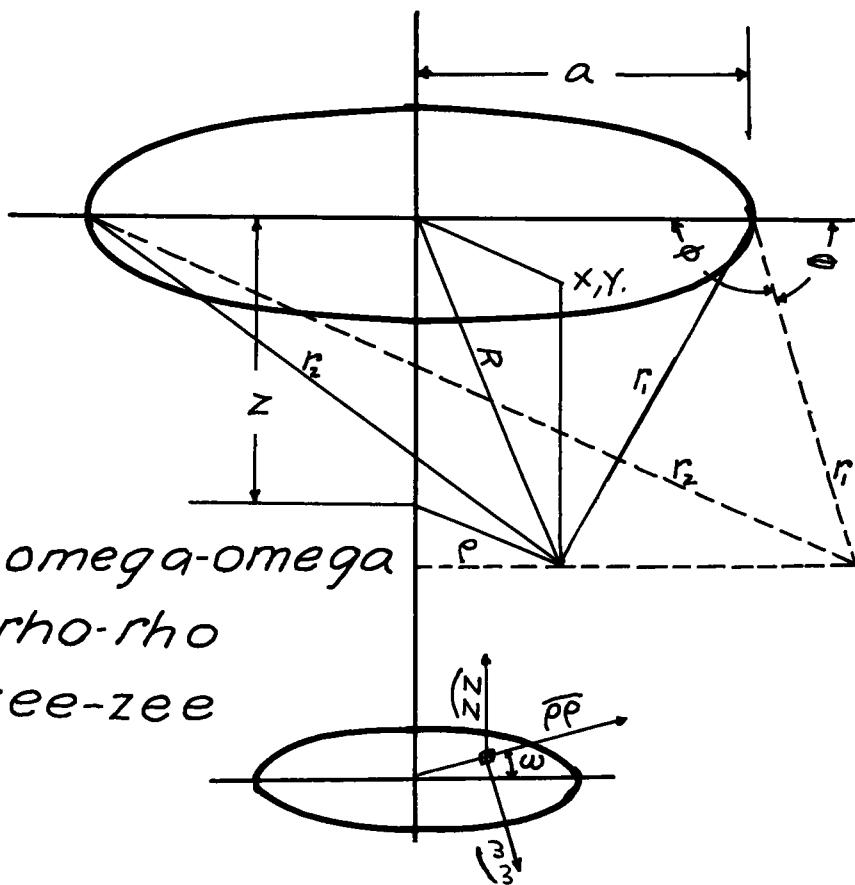


Figure 1. Geometrical dimensions used to develop theoretical stresses underneath a circular area.

cal foundation design problems Tufts (3), employing elliptic integral tables, developed "influence values" (that is, expressing the differential equations in terms of dimensionless ratios the quantities being dimensions that can be measured from a load plan). In this form the righthand member of the equation can be solved for various values of the dimensionless ratios. These influence value tables are set forth in the Appendix.

With the aid of these influence tables in the Appendix the author drew up quickly and easily the stress contour curves outlined in Figures 2 to 16 for a flexible surface foundation of radius  $A$  and a definite uniform surface loading in pounds per square inch or square foot. These stress contour curves are self explanatory.

For purposes of this paper, the following nomenclature or symbols have been adopted:

- $a$  = radius of circular loaded area;
- $p$  = unit vertical pressure on loaded area;
- $z, \omega, \rho$  = cylindrical coordinates (Fig. 1);
- $\hat{z}$  = vertical normal stress;
- $\hat{\rho}\hat{\rho}$  = radial normal stress;
- $\hat{\omega}\hat{\omega}$  = tangential normal stress;
- $\hat{p}\hat{z}$  = shear stress in the  $\hat{\rho}\hat{z}$  plane;
- $T_1$  = major principal stress;
- $T_\pi$  = minor principal stress;
- $S$  = stress difference =  $T_1 - T_\pi$ ;
- $s$  = maximum shearing stress =  $\frac{1}{2} S$ ; and
- $\mu$  or  $\sigma$  = Poisson's ratio.

Attention is directed to the fact that the value of Poisson's ratio ( $\mu$ ) does not affect the stresses acting vertically but does cause stress variations in all stresses acting at an angle with the vertical.

The vertical stress contours have been found to be of considerable value in the design of flexible base courses for bituminous surfaces.

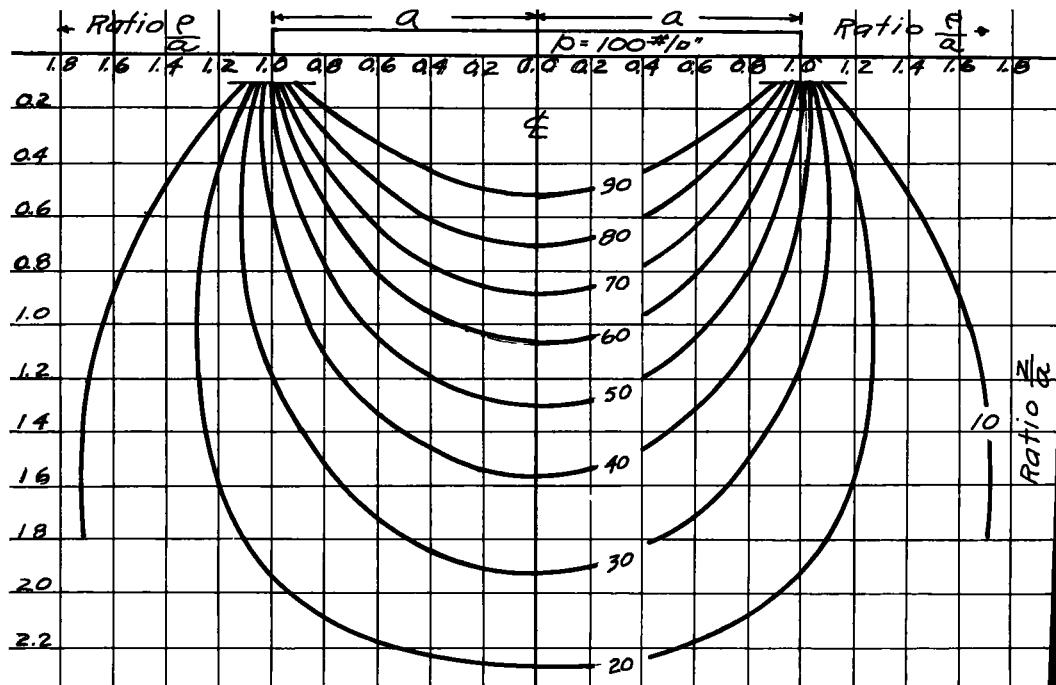
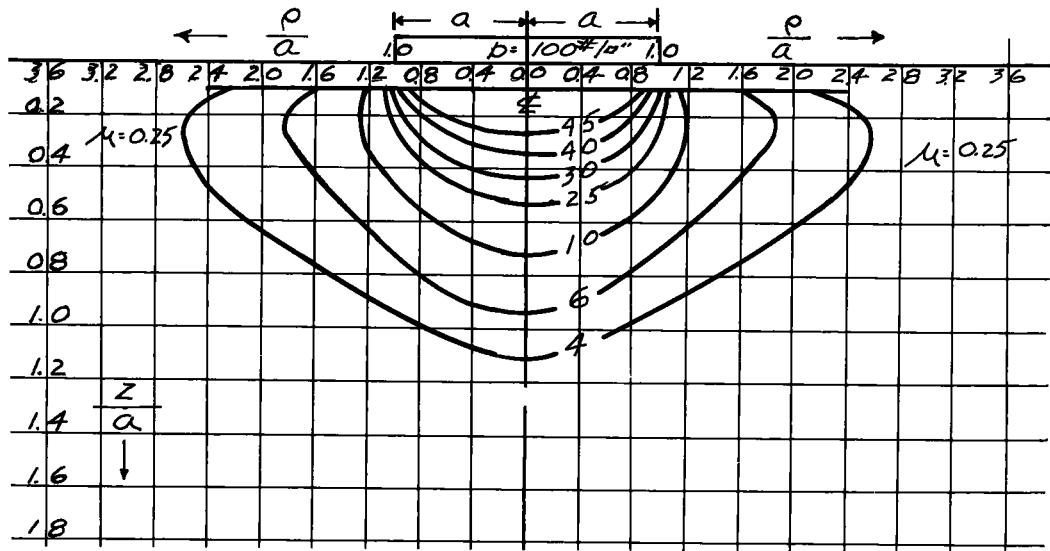
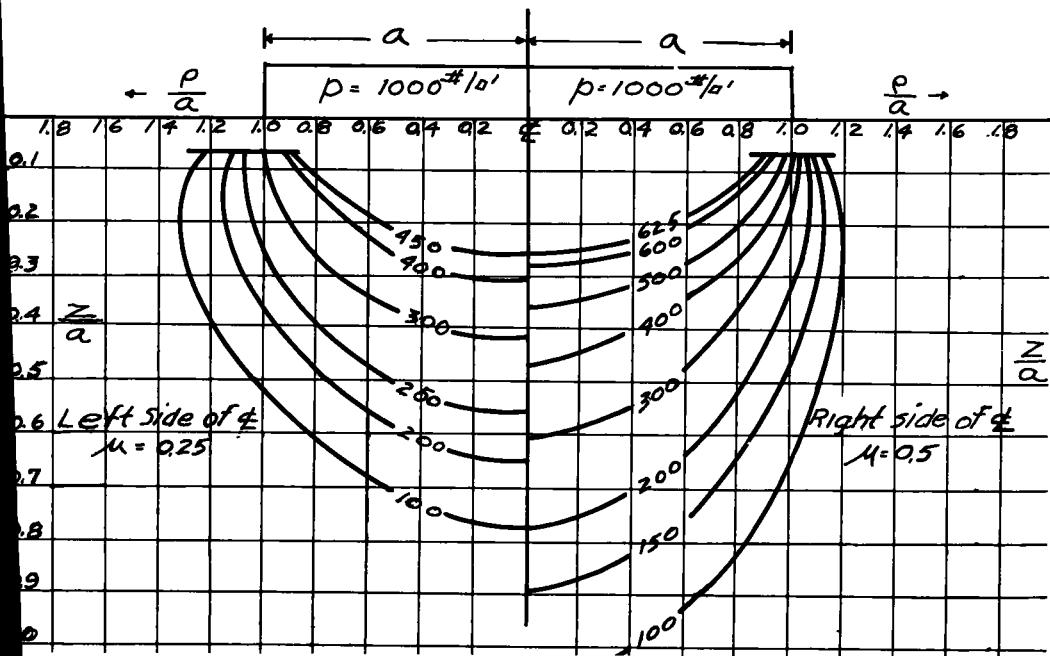


Figure 2.  $\hat{z}\hat{z} = p_z$  contours of stress.

Figure 3. Tangential  $\bar{\sigma}_\theta$  stress contours.Figure 4. Tangential  $\bar{\sigma}_\theta$  stress contours.

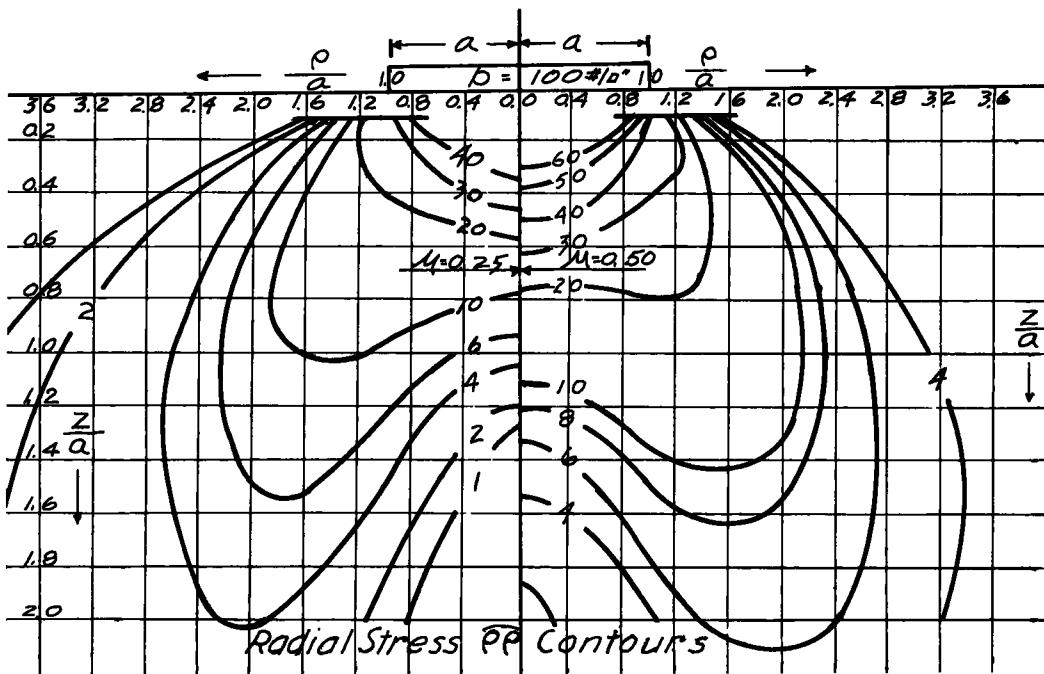
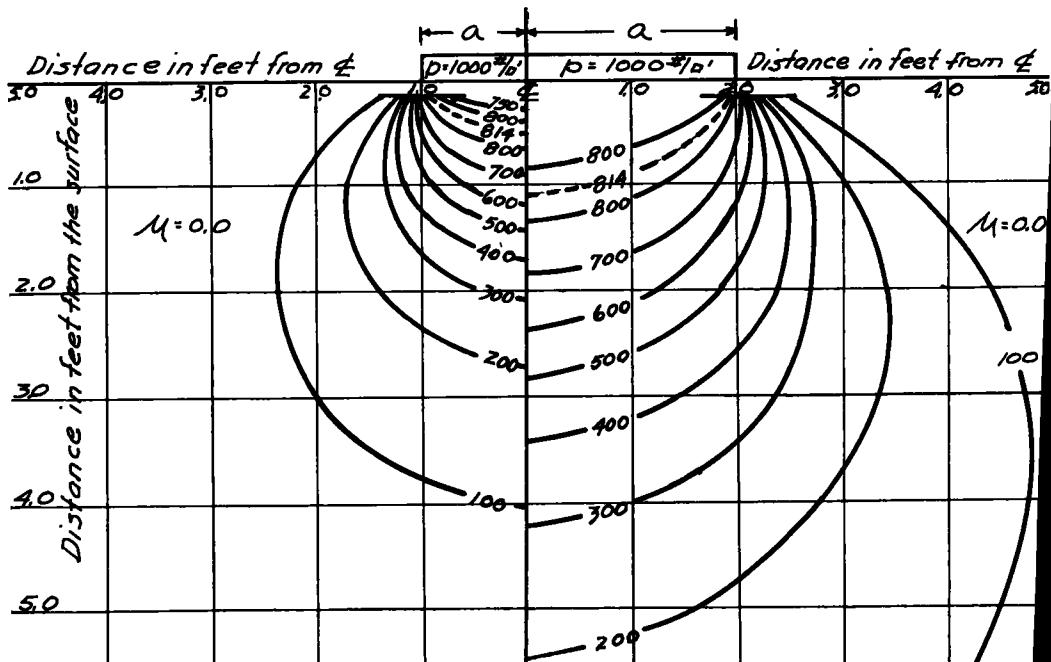
Figure 5. Radial stress  $\bar{p}$  contours.

Figure 6. Stress-difference (S) contours.

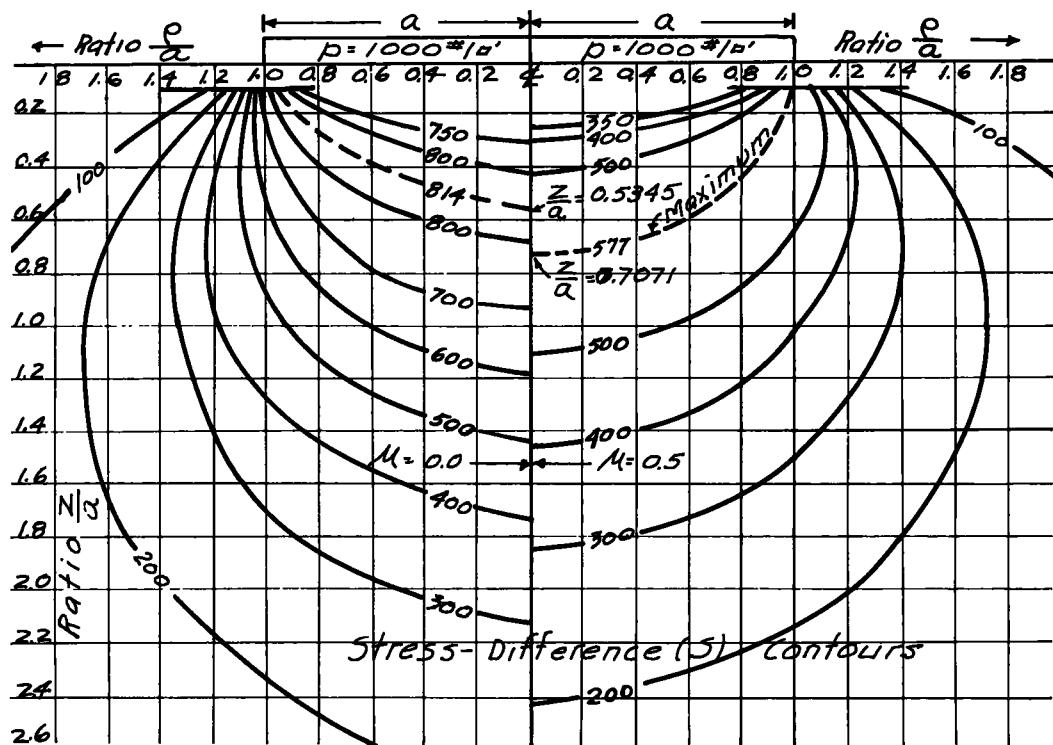


Figure 7. Stress-difference (S) contours.

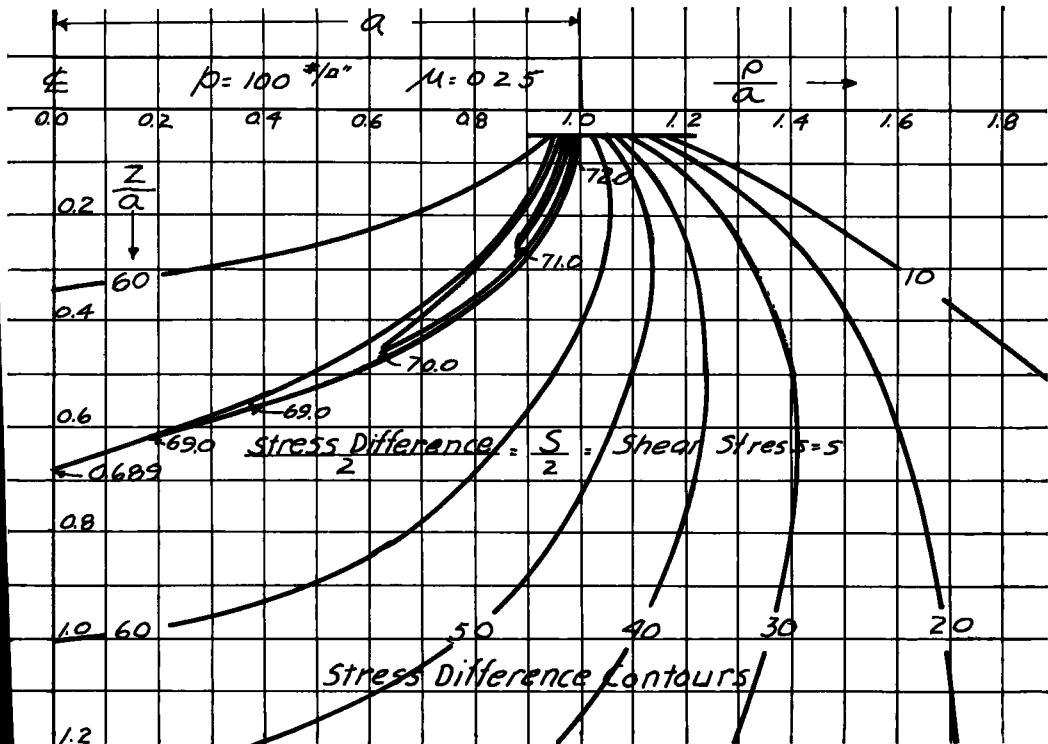


Figure 8. Stress-difference contours.

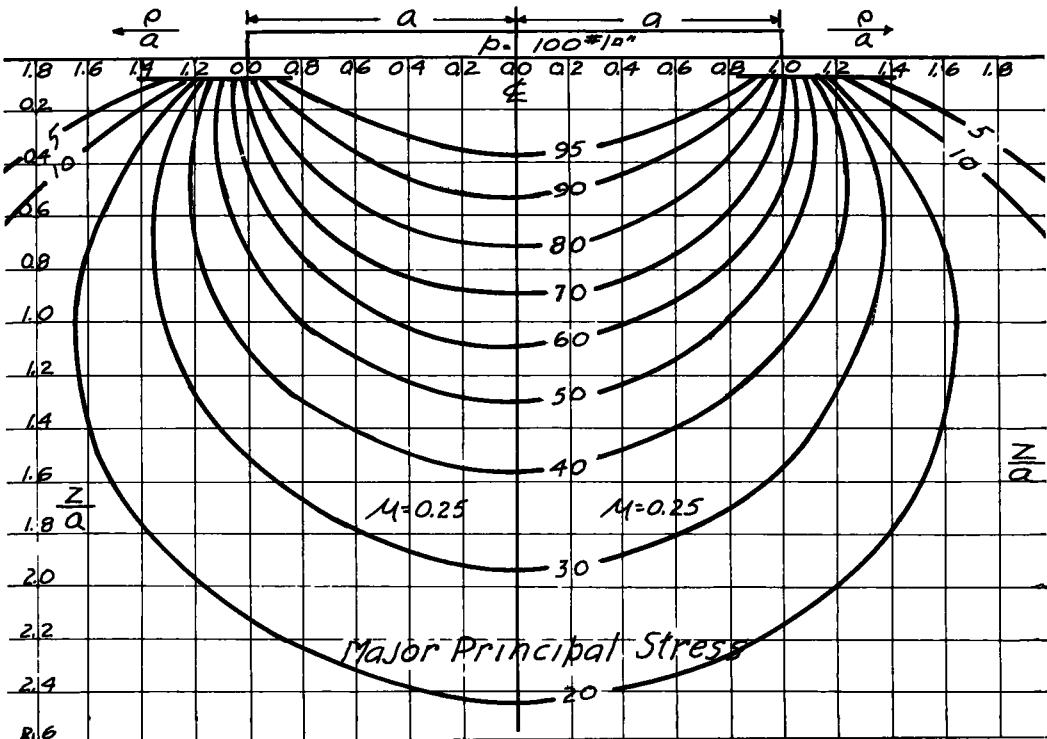


Figure 9. Major principal stress.

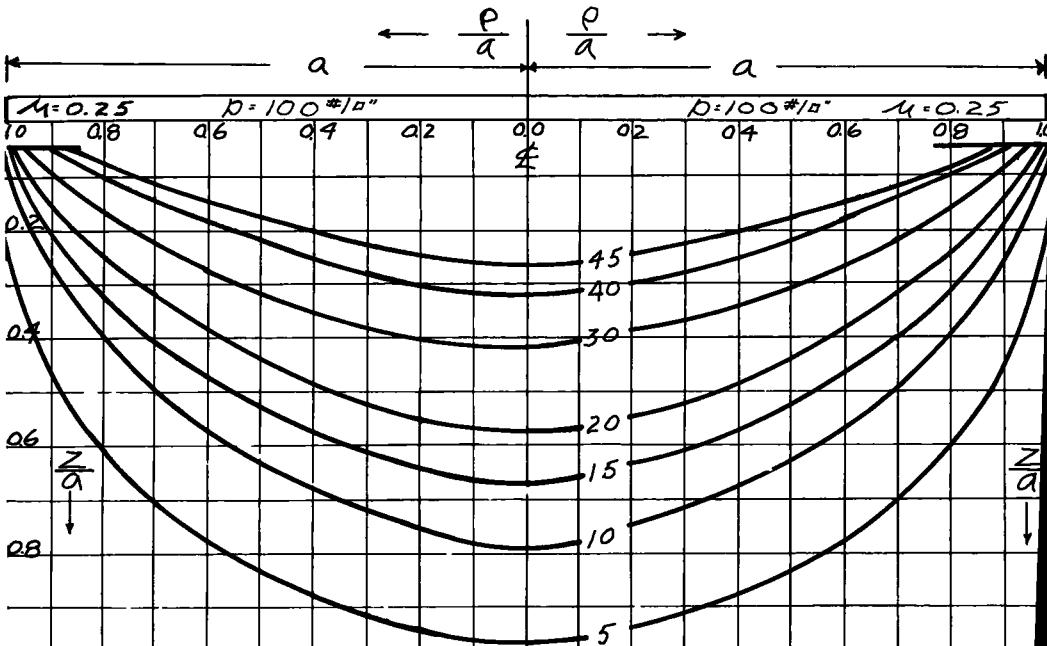
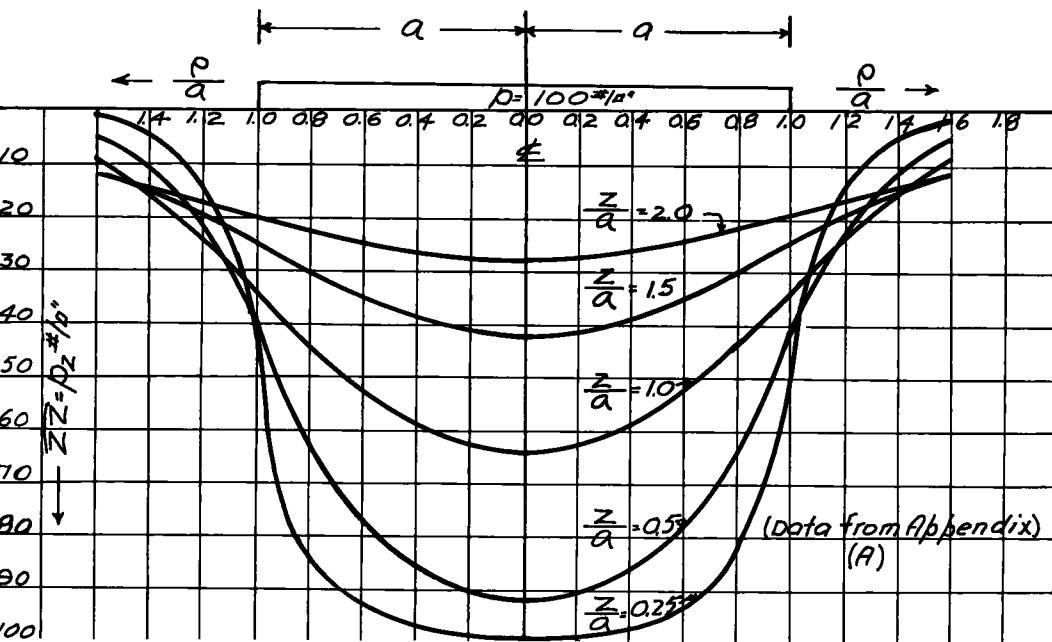
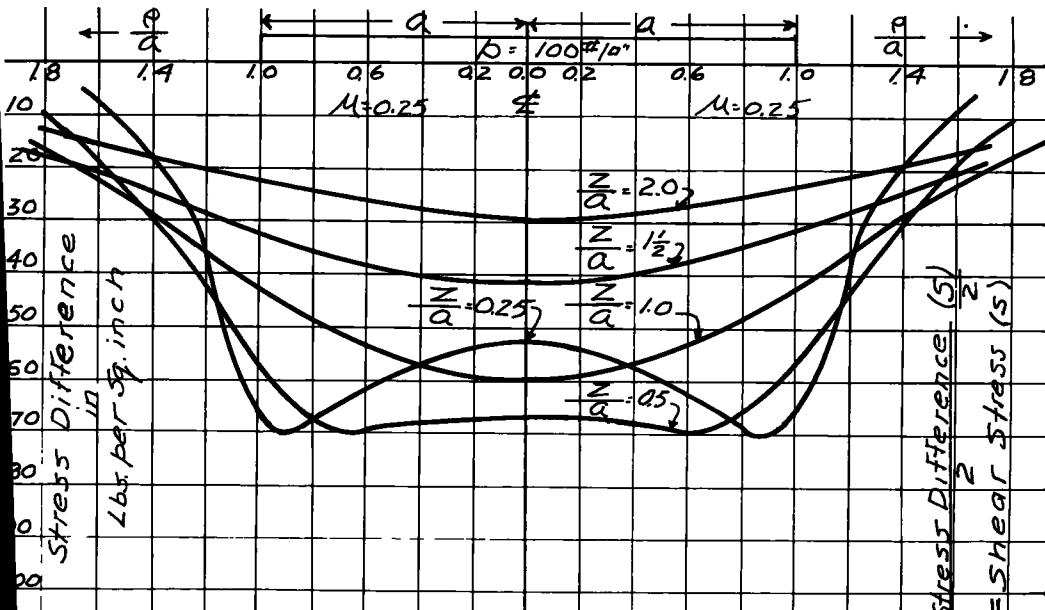


Figure 10. Minor principal stress contours.

Figure 11. Vertical stress  $\bar{z} = p_z$ .Figure 12. Stress difference (S) and shear curves  $= \frac{S}{2} = s$ .

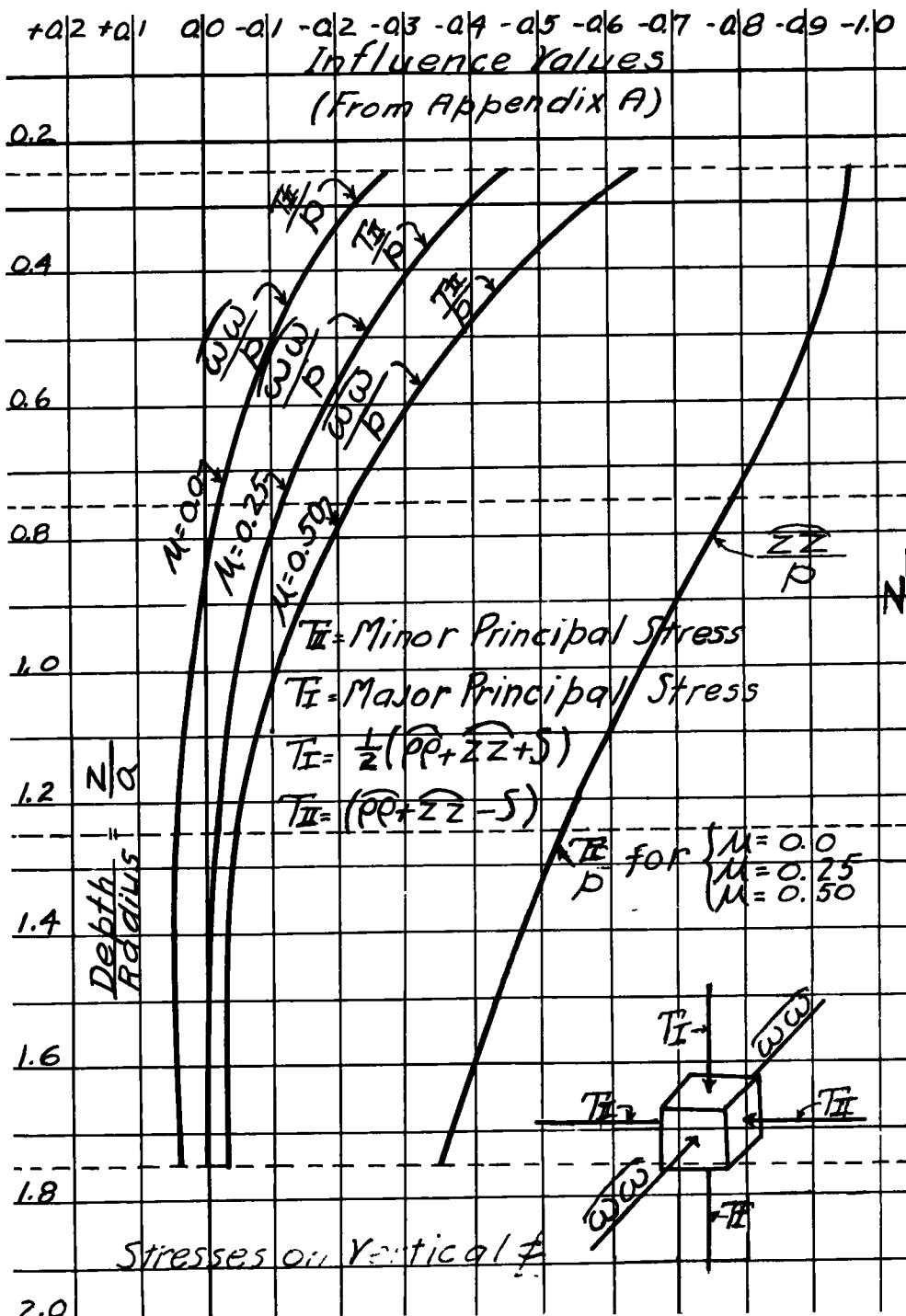


Figure 13. Stresses on vertical centerline.

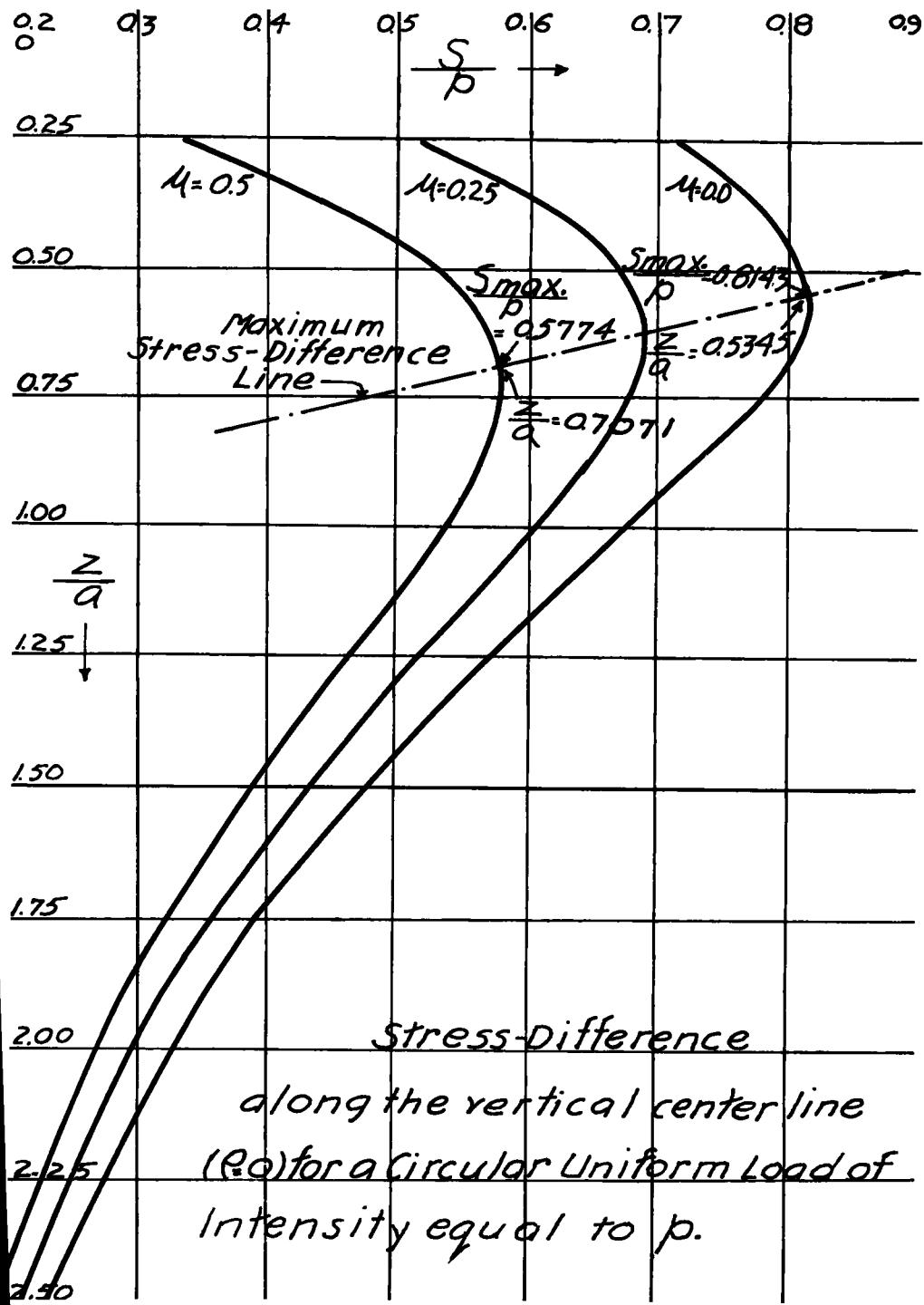


Figure 14. Stress-difference along the vertical centerline ( $\rho = 0$ ) for a circular uniform load of intensity equal to  $p$ .

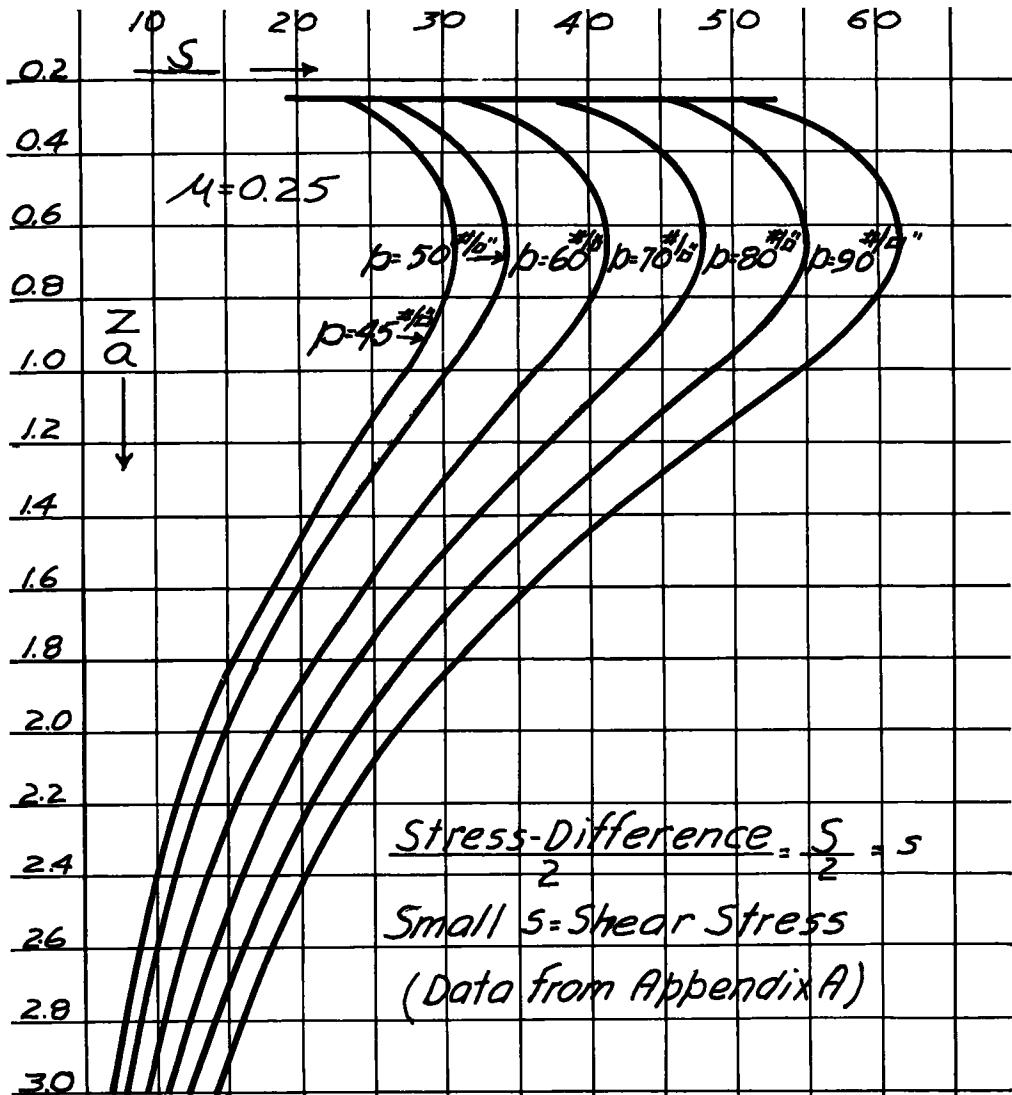


Figure 15. Stress-difference along vertical centerline beneath the circular foundation.

#### REFERENCES

1. Boussinesq, J. V., "Application des Potentiels à l'Etude de l'Equilibre et du Mouvement des Solides Elastiques." Gauthier-Villars, Paris (1885).
2. Love, A. E. H., "The Stress Produced in a Semi-Infinite Solid by Pressure on Part of the Boundary." Philosophical Trans., Royal Society (London, England), Series A, Vol. 228 (1929).
3. Tufts, W. M., "Public Aids to Transportation." Vol. 14, pp. 248-250, U. S. Govt. Print. Off. (1940).

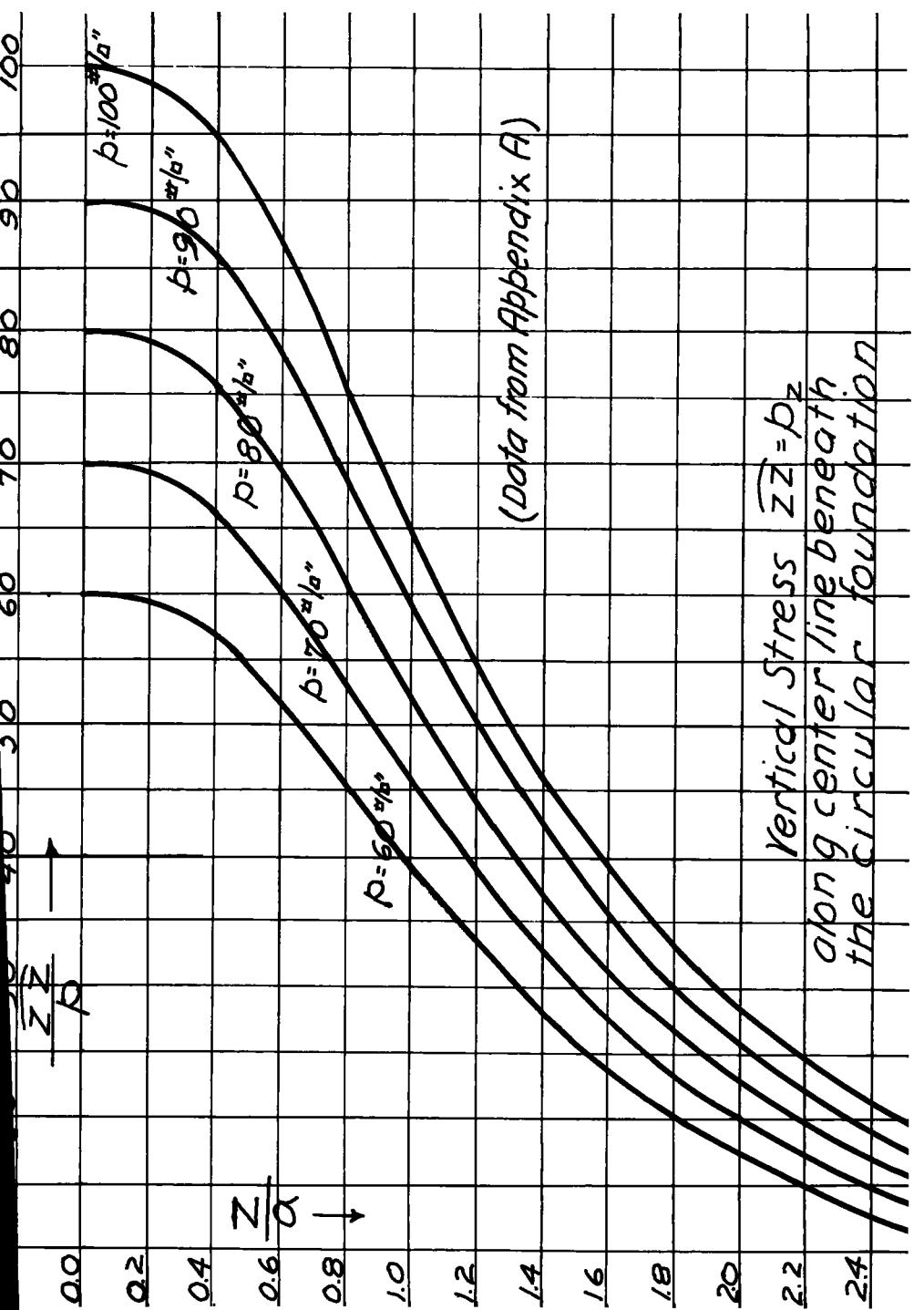


Figure 16. Vertical stress  $\frac{Z}{P_z} = p_z$  along centerline beneath the circular foundation.

## Appendix

$\frac{P}{\alpha}$	$\frac{P\varrho}{\alpha}$			$\frac{\bar{\omega}\bar{\omega}}{\alpha}$			$\frac{\bar{z}z}{\alpha}$	$\frac{Pz}{\alpha}$	$\frac{S}{\alpha}$		
	$M=0$	$M=0.25$	$M=0.5$	$M=0$	$M=0.25$	$M=0.5$			$M=0$	$M=0.25$	$M=0.5$
$\frac{Z}{\alpha} = 0.25$											
0	-0.2613	0.4590	0.6433	0.2646	-0.4540	0.6433	-0.9857	0	0.7211	0.5318	0.3424
0.0670	-0.2647	0.4538	0.6428	-0.2633	0.4516	-0.6119	-0.9854	-0.0024	0.7209	0.5318	0.3429
0.3131	-0.2431	0.4304	0.6177	-0.2576	0.4405	-0.6234	-0.9801	0.0294	0.7394	0.5529	0.3672
0.5670	-0.1828	0.3649	0.5471	-0.2401	0.4027	-0.5612	-0.9542	0.0781	0.7871	0.6097	0.4361
0.7500	-0.1111	0.3846	0.4580	-0.2173	0.3469	-0.1765	-0.8810	0.1977	0.8361	0.6702	0.5322
0.8834	-0.0696	0.2273	0.3970	-0.1978	0.2820	-0.3662	-0.7243	-0.2562	0.8314	0.7097	0.6090
1.0000	-0.0322	0.2235	0.3849	-0.1832	0.2131	-0.2430	-0.4596	-0.2923	0.7069	0.6390	0.6023
1.1166	-0.0892	0.2252	0.3613	-0.1735	0.1553	-0.1371	-0.2086	-0.2312	0.4776	0.4628	0.4870
1.2500	-0.0771	0.1635	0.2812	-0.1620	0.1149	-0.0677	-0.0797	-0.1297	0.2609	0.2745	0.3320
1.4330	-0.0116	0.0842	0.1900	0.1435	-0.0859	-0.0204	-0.0211	-0.0525	0.1105	0.1311	0.1961
1.6869	0.0494	0.0286	0.1015	0.1172	-0.0638	-0.0055	-0.0054	0.0227	0.0675	0.0909	0.1061
2.9178	0.0460	0.0079	-0.0701	-0.0669	0.0391	-0.0014	-0.0003	0.0057	0.0471	0.0113	0.0206
$\frac{Z}{\alpha} = 0.50$											
0	-0.0975	-0.2357	-0.3739	-0.0975	-0.2357	-0.3739	-0.9106	0	0.8130	0.6149	0.5367
0.1340	-0.0951	-0.2327	-0.3703	-0.0961	-0.2329	-0.3698	-0.9066	-0.0292	0.8136	0.6164	0.5395
0.2859	-0.0840	-0.2208	-0.3567	-0.0932	-0.2246	-0.3559	-0.8913	-0.0654	0.8170	0.6832	0.5504
0.5000	-0.0678	-0.1938	-0.3249	-0.0851	-0.2009	-0.3168	-0.8396	-0.1287	0.8183	0.6951	0.5754
0.7688	-0.0474	-0.1615	-0.2875	-0.0736	-0.1532	-0.2328	-0.6775	-0.2259	0.7753	0.6913	0.5968
1.0000	-0.0352	-0.1809	-0.2861	-0.0701	-0.1054	-0.1406	-0.4175	-0.2621	0.6258	0.5721	0.5403
1.1820	-0.0914	-0.1836	-0.2758	-0.0726	-0.0769	-0.0817	-0.2224	-0.2143	0.4481	0.4303	0.4318
1.5000	-0.0559	-0.1261	-0.1962	-0.0736	-0.0510	-0.0263	-0.0604	-0.1019	0.2038	0.2140	0.2448
1.7141	-0.0267	-0.0748	-0.1430	-0.0635	-0.0240	-0.0446	-0.0268	-0.0590	0.1181	0.1316	0.1657
2.0722	-0.0139	-0.0333	-0.0899	-0.0793	-0.0324	-0.0038	-0.0087	-0.0260	0.0565	0.0586	0.0924
2.8660	-0.0175	-0.0075	-0.0324	-0.0394	-0.0202	-0.0000	-0.0013	-0.0069	0.0227	0.0142	0.0337
3.8356	-0.0175	-0.0075	-0.0106	-0.0237	-0.0103	-0.0026	-0.0026	-0.0019	0.0203	0.0039	0.0137
$\frac{Z}{\alpha} = 0.75$											
0	-0.0080	-0.1080	-0.2080	-0.0080	-0.1080	-0.2080	-0.7840	0	0.7760	0.6760	0.5760
0.1062	-0.0091	-0.1085	-0.2079	-0.0065	-0.1060	-0.2078	-0.7803	-0.0244	0.7734	0.6743	0.5753
0.3707	-0.0063	-0.1031	-0.1999	-0.0071	-0.0975	-0.1879	-0.7554	-0.1045	0.7585	0.6660	0.5749
0.5670	-0.0100	-0.1025	-0.1951	-0.0073	-0.0846	-0.1620	-0.6621	-0.1601	0.7264	0.6447	0.5662
0.7990	-0.0287	-0.1138	-0.1889	-0.0104	-0.0661	-0.1212	-0.5241	-0.2101	0.6506	0.5984	0.5326
1.0000	-0.0546	-0.1316	-0.2087	-0.0166	-0.0510	-0.0854	-0.3745	-0.2211	0.5457	0.5054	0.4722
1.2010	-0.0730	-0.1412	-0.2094	-0.0247	-0.0398	-0.0548	-0.2773	-0.1940	0.4206	0.3993	0.3889
1.4330	-0.0722	-0.1304	-0.1886	-0.0328	-0.0318	-0.0209	-0.1241	-0.1412	0.2872	0.2825	0.2897
1.7500	-0.0492	-0.0956	-0.1419	-0.0980	-0.0260	-0.0139	-0.0872	-0.0804	0.1652	0.1610	0.1698
2.0711	-0.0292	-0.0662	-0.1031	-0.0378	-0.0222	-0.0066	-0.0639	-0.0446	0.0960	0.0895	0.0977
2.6084	-0.0039	-0.0300	-0.0561	-0.0328	-0.0175	-0.0022	-0.0367	-0.0186	0.0495	0.0377	0.0418
3.7990	-0.0082	-0.0057	-0.0196	-0.0212	-0.0108	-0.0003	-0.0190	-0.0041	0.0227	0.0110	0.0105
$\frac{Z}{\alpha} = 1.00$											
0	-0.0309	-0.0479	-0.1161	-0.0303	-0.0479	-0.1161	-0.6464	0	0.6768	0.6036	0.5303
0.1609	-0.0276	-0.0449	-0.1174	-0.0313	-0.0410	-0.1132	-0.6758	-0.0424	0.6708	0.5990	0.5273
0.4228	-0.0200	-0.0503	-0.1205	-0.0272	-0.0373	-0.1019	-0.5388	-0.1081	0.6498	0.5781	0.5136
0.6360	-0.0047	-0.0620	-0.1287	-0.0224	-0.0317	-0.0889	-0.5103	-0.1521	0.5981	0.5417	0.4880
0.9125	-0.0247	-0.0850	-0.1453	-0.0127	-0.0242	-0.0662	-0.3779	-0.1813	0.5059	0.4658	0.4305
1.1763	-0.0491	-0.1031	-0.1563	-0.0013	-0.0191	-0.0796	-0.2461	-0.1713	0.3948	0.3712	0.3306
1.4663	-0.0594	-0.1046	-0.1499	-0.0101	-0.0163	-0.0225	-0.1363	-0.1325	0.2760	0.2670	0.2654
1.8391	-0.0183	-0.0544	-0.1205	-0.0193	-0.0149	-0.0105	-0.0598	-0.0812	0.1627	0.1642	0.1734
2.4281	-0.0223	-0.0476	-0.0732	-0.0230	-0.0132	-0.0074	-0.0713	-0.0349	0.0698	0.0760	0.0893
3.1445	-0.0019	-0.0223	-0.0397	-0.0206	-0.0108	-0.0011	-0.0050	-0.0158	0.0217	0.0327	0.0444
4.7321	-0.0008	-0.0049	-0.0131	-0.0132	-0.0067	-0.0009	-0.0060	-0.0029	0.0077	0.0068	0.0137

$\frac{\rho}{Q}$	$\frac{\rho\rho}{P}$	$\frac{\omega\omega}{2}$	$\frac{ZZ}{P}$	$\frac{PZ}{P}$	$\frac{S}{P}$	
	$M=0.0$	$M=0.25$	$M=0.50$	$M=0.0$	$M=0.25$	$M=0.50$

$$\frac{L}{d} = 1.25$$

$$\frac{Z}{C} = 1.50$$

<i>O</i>	-0.0440	+0.0020	-0.0399	0.0440	0.0020	-0.0399	0.0240	0	0.4680	0.4260	0.3840
0.1340	-0.0416	-0.0000	-0.0417	0.0499	0.0031	-0.0387	0.0203	-0.0235	0.4642	0.4229	0.3815
0.4540	-0.0310	-0.0094	-0.0499	0.0398	0.0022	-0.0355	0.0193	-0.0746	0.4404	0.4025	0.3653
0.7355	0.0127	-0.0255	-0.0637	0.0372	0.0019	-0.0294	0.0240	-0.1060	0.3979	0.3661	0.3357
1.0000	-0.0069	0.0422	-0.0777	0.0251	0.0011	-0.0229	-0.2560	-0.1191	0.3449	0.3202	0.2976
1.2645	-0.0233	-0.0376	-0.0879	0.0166	-0.0000	-0.0167	-0.1892	-0.1165	0.2860	0.2685	0.2540
1.5460	-0.0338	-0.0626	-0.0914	0.0083	-0.0016	-0.0114	-0.1292	-0.1019	0.2250	0.2194	0.2073
1.8660	-0.0366	-0.0616	-0.0866	0.0007	-0.0032	-0.0072	-0.0800	-0.0798	0.1624	0.1576	0.1567
2.2597	-0.0310	-0.0087	-0.0736	-0.0054	-0.0047	-0.0046	-0.0155	-0.0551	0.1108	0.1106	0.1143
2.7076	-0.0215	-0.0180	-0.0545	-0.0085	-0.0057	-0.0019	-0.0197	-0.0722	0.0649	0.0669	0.0732
3.5981	-0.0093	-0.0212	-0.0329	-0.0107	-0.0057	-0.0007	-0.0066	-0.0146	0.0294	0.0327	0.0393
4.2186	-0.0042	-0.0137	-0.0227	-0.0101	-0.0052	-0.0003	-0.0072	-0.0085	0.0170	0.0198	0.0195

$$\frac{z}{a} = 1.75$$

<b>O</b>	<b>0.0410</b>	<b>0.0080</b>	<b>-0.0249</b>	<b>0.0410</b>	<b>0.0080</b>	<b>-0.0249</b>	<b>0.3155</b>	<b>O</b>	<b>0.3865</b>	<b>0.3535</b>	<b>0.3206</b>
<b>0.1840</b>	<b>0.0387</b>	<b>0.0061</b>	<b>-0.0266</b>	<b>0.0409</b>	<b>0.0084</b>	<b>-0.0241</b>	<b>0.3406</b>	<b>-0.0251</b>	<b>0.3826</b>	<b>0.3502</b>	<b>0.3179</b>
<b>0.5311</b>	<b>0.0270</b>	<b>-0.0045</b>	<b>-0.0361</b>	<b>0.0364</b>	<b>0.0074</b>	<b>-0.0216</b>	<b>-0.3058</b>	<b>-0.0663</b>	<b>0.3583</b>	<b>0.3292</b>	<b>0.3006</b>
<b>0.8469</b>	<b>0.0091</b>	<b>-0.0200</b>	<b>-0.0496</b>	<b>0.0298</b>	<b>0.0061</b>	<b>-0.0177</b>	<b>-0.2532</b>	<b>-0.0902</b>	<b>0.3188</b>	<b>0.2948</b>	<b>0.2719</b>
<b>1.1531</b>	<b>-0.1077</b>	<b>-0.0350</b>	<b>-0.0623</b>	<b>0.0221</b>	<b>0.0044</b>	<b>-0.0134</b>	<b>-0.1947</b>	<b>-0.0978</b>	<b>0.2706</b>	<b>0.2525</b>	<b>0.2362</b>
<b>1.4689</b>	<b>-0.0217</b>	<b>-0.0458</b>	<b>-0.0704</b>	<b>0.0141</b>	<b>0.0023</b>	<b>-0.0095</b>	<b>-0.1386</b>	<b>-0.0921</b>	<b>0.2184</b>	<b>0.2062</b>	<b>0.1963</b>
<b>18160</b>	<b>-0.0281</b>	<b>-0.0500</b>	<b>-0.0717</b>	<b>0.0066</b>	<b>0.0002</b>	<b>-0.0062</b>	<b>-0.0911</b>	<b>0.0769</b>	<b>0.1657</b>	<b>0.1589</b>	<b>0.1549</b>
<b>2.2254</b>	<b>-0.0283</b>	<b>-0.0469</b>	<b>-0.0652</b>	<b>0.0000</b>	<b>-0.0018</b>	<b>-0.0036</b>	<b>-0.0523</b>	<b>-0.0568</b>	<b>0.1160</b>	<b>0.1137</b>	<b>0.1143</b>
<b>2.7500</b>	<b>-0.0275</b>	<b>-0.0373</b>	<b>-0.0522</b>	<b>-0.0049</b>	<b>-0.0034</b>	<b>-0.0018</b>	<b>-0.0259</b>	<b>-0.0361</b>	<b>0.0723</b>	<b>0.0732</b>	<b>0.0769</b>
<b>3.49993</b>	<b>-0.0129</b>	<b>-0.0240</b>	<b>-0.0351</b>	<b>-0.0076</b>	<b>-0.0042</b>	<b>-0.0007</b>	<b>-0.0102</b>	<b>-0.0187</b>	<b>0.0375</b>	<b>0.0399</b>	<b>0.0449</b>
<b>4.7529</b>	<b>-0.0036</b>	<b>-0.0108</b>	<b>-0.0192</b>	<b>-0.0076</b>	<b>-0.0039</b>	<b>-0.0002</b>	<b>-0.0077</b>	<b>-0.0062</b>	<b>0.0139</b>	<b>0.0162</b>	<b>0.0206</b>

$$\frac{Z}{a} = 2.00$$

<i>O</i>	<i>0.0367</i>	<i>0.0103</i>	<i>-0.0161</i>	<i>0.0367</i>	<i>0.0103</i>	<i>-0.0161</i>	<i>-0.2885</i>	<i>O</i>	<i>0.3211</i>	<i>0.2947</i>	<i>0.2683</i>
<i>0.2721</i>	<i>0.0731</i>	<i>0.0071</i>	<i>-0.0189</i>	<i>0.0763</i>	<i>0.0105</i>	<i>-0.0153</i>	<i>-0.2767</i>	<i>-0.0285</i>	<i>0.3150</i>	<i>0.2894</i>	<i>0.2640</i>
<i>0.6473</i>	<i>0.0213</i>	<i>-0.0017</i>	<i>-0.0287</i>	<i>0.0316</i>	<i>0.0091</i>	<i>-0.0135</i>	<i>-0.2430</i>	<i>0.0606</i>	<i>0.2907</i>	<i>0.2682</i>	<i>0.2461</i>
<i>1.3527</i>	<i>-0.0093</i>	<i>-0.0306</i>	<i>-0.0519</i>	<i>0.0180</i>	<i>0.0050</i>	<i>-0.0080</i>	<i>-0.1459</i>	<i>-0.0002</i>	<i>0.2107</i>	<i>0.1975</i>	<i>0.1859</i>
<i>1.7279</i>	<i>0.0196</i>	<i>-0.0386</i>	<i>-0.0575</i>	<i>0.0101</i>	<i>-0.0027</i>	<i>-0.0054</i>	<i>-0.0998</i>	<i>-0.0721</i>	<i>0.1649</i>	<i>0.1566</i>	<i>0.1502</i>
<i>2.1547</i>	<i>-0.0274</i>	<i>0.0401</i>	<i>-0.0565</i>	<i>0.0243</i>	<i>0.0005</i>	<i>-0.0037</i>	<i>-0.0612</i>	<i>-0.0571</i>	<i>0.1201</i>	<i>0.1161</i>	<i>0.1143</i>
<i>2.6782</i>	<i>-0.0217</i>	<i>-0.0352</i>	<i>-0.0400</i>	<i>-0.0011</i>	<i>0.0014</i>	<i>-0.0018</i>	<i>-0.0729</i>	<i>0.0393</i>	<i>0.0794</i>	<i>0.0786</i>	<i>0.0802</i>
<i>3.3835</i>	<i>-0.0151</i>	<i>-0.0256</i>	<i>-0.0361</i>	<i>-0.0071</i>	<i>-0.0029</i>	<i>-0.0008</i>	<i>-0.0145</i>	<i>-0.0226</i>	<i>0.0453</i>	<i>0.0466</i>	<i>0.0502</i>
<i>4.4681</i>	<i>0.0068</i>	<i>0.0142</i>	<i>-0.0215</i>	<i>-0.0061</i>	<i>-0.0032</i>	<i>-0.0003</i>	<i>-0.0047</i>	<i>-0.0100</i>	<i>0.0201</i>	<i>0.0222</i>	<i>0.0261</i>

$\frac{\rho}{\alpha}$	$\frac{\rho\rho}{P}$				$\frac{\omega\omega}{P}$				$\frac{zz}{P}$	$\frac{\rho z}{P}$	$\frac{s}{P}$			
	1=0.0	1=0.25	1=0.50	1=-0.0	1=0.25	1=-0.50					1=0.0	1=0.25	1=0.50	
$\frac{z}{a} = 2.50$														
0	0.0283	0.0104	-0.0075	0.0283	0.0104	-0.0075	-0.1996	-0.1996	0	0.2278	0.2100	0.1921		
0.3301	0.0274	0.0078	-0.0098	0.0273	0.0104	-0.0070	-0.1933	-0.1933	-0.0212	0.2230	0.2057	0.1885		
0.7813	0.0159	0.0010	0.0179	0.0241	0.0090	-0.0067	-0.1681	-0.1681	-0.0442	0.2041	0.1890	0.1742		
1.2187	0.0034	-0.0123	-0.0290	0.0189	0.0071	-0.0048	-0.1327	-0.1327	-0.0553	0.1734	0.1635	0.1524		
1.6699	-0.0076	-0.0218	-0.0360	0.0131	0.0048	-0.0035	-0.0955	-0.0955	-0.0557	0.1420	0.1337	0.1264		
2.1658	-0.0146	-0.0270	-0.0394	0.0078	0.0226	-0.0023	-0.0621	-0.0621	-0.0481	0.1073	0.1024	0.0988		
2.7505	-0.0164	-0.0269	-0.0374	0.0025	0.0006	-0.0013	-0.0357	-0.0357	-0.0359	0.0743	0.0723	0.0718		
3.500	-0.0136	-0.0220	-0.0303	0.0013	0.0010	-0.0006	-0.0174	-0.0174	-0.0227	0.0456	0.0457	0.0472		
4.5704	-0.0081	-0.0142	-0.0204	0.0035	0.0019	-0.0002	-0.0064	-0.0064	-0.0117	0.0234	0.0246	0.0271		
$\frac{z}{a} = 3.00$														
0	0.0218	0.0089	-0.0039	0.0218	0.0089	-0.0039	-0.1462	-0.1462	0	0.1680	0.1551	0.1423		
0.7375	0.0148	0.0025	-0.0098	0.0199	0.0083	-0.0032	-0.1300	-0.1300	-0.0281	0.1554	0.1440	0.1327		
1.2625	-0.0056	-0.0060	-0.0175	0.0159	0.0066	-0.0027	-0.1046	-0.1046	-0.0392	0.1353	0.1261	0.1172		
1.8038	-0.0036	-0.0141	-0.0245	0.0113	0.0047	-0.0020	-0.0761	-0.0761	-0.0414	0.1101	0.1035	0.0976		
2.3989	-0.0098	-0.0190	-0.0292	0.0067	0.0027	-0.0013	-0.0497	-0.0497	-0.0366	0.0833	0.0793	0.0762		
3.5173	-0.0115	-0.0184	-0.0254	0.0010	0.0002	-0.0005	-0.0203	-0.0203	-0.0229	0.0458	0.0450	0.0452		
$\frac{z}{a} = 4.00$														
0	0.0136	0.0061	-0.0013	0.0136	0.0061	-0.0013	-0.0819	-0.0819	0	0.1005	0.0931	0.0856		
0.2947	0.0121	0.0049	-0.0023	0.0191	0.0066	-0.0009	-0.0816	-0.0816	-0.0058	0.0984	0.0912	0.0840		
1.0000	0.0086	-0.0115	-0.0056	0.0124	0.0058	-0.0009	-0.0761	-0.0761	-0.0176	0.0918	0.0825	0.0789		
1.7053	0.0028	-0.0038	-0.0105	0.0096	0.0044	-0.0008	-0.0764	-0.0764	-0.0239	0.0760	0.0710	0.0663		
2.4559	-0.0029	-0.0031	-0.0148	0.0066	0.0030	-0.0006	-0.0463	-0.0463	-0.0243	0.0653	0.0651	0.0580		
3.3094	-0.0061	-0.0112	-0.0164	0.0036	0.0016	-0.0004	-0.0351	-0.0351	-0.0205	0.0504	0.0477	0.0452		
4.3564	-0.0071	-0.0113	-0.0156	0.0010	0.0003	-0.0005	-0.0358	-0.0358	-0.0144	0.0443	0.0422	0.0393		