Use of Aerial Surveys and Topographic Maps For Projecting Highway Alignment

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By introducing modern methods of highway projection and the efficient use of topographic maps developed from photogrammetry, the location engineer can establish alignment and grades which complement the topography and eliminate nearly all of the undesirable aspects of the older accepted methods of ground location.

In order to project accurage highway alignment, a coordinate grid system must be established over the area through which the location is to pass. The final results of any line projected through this area will be no more accurate than the original control used to establish the grid system.

The present paper is based on the assumption that the topographic map through which the highway alignment is to be projected is accurate both horizontally and vertically, that pre-established horizontal and vertical control points are in usable positions, and planimetric features are well defined.

Simple and accurate methods of calculating curve positions by coordinates, simple procedures for projecting centerline where grade or land use problems exist, the effective use of the spline, simplified use of spiral transitions and a simplified, accaptably accurate but approximate, method of introducing transitions between compound curves are discussed.

Through intelligent use of the coordinate method of centerline projection, employing an accurate topographic manuscript, compiled by photogrammetric methods, better highway locations can be achieved along with a substantial savings in man-hours.

ONE of the greatest factors contributing to the building of America was the original railroad and highway location surveys through the wilderness and mountainous areas western United States. This stupendous achievement was performed under conditions that would now discourage the most energetic of modern-day engineers. During few short years, the work was extended over half a continent, uncharted except for etches drawn by optimistic (and oftentime imaginative) explorers, mountain men, my scouts, and superstitious Indians.

Thousands of miles of railroads and primitive highways were located by sheer determination, either on foot or on horseback, exploring the box canyons, river drainages, d game trails. From some high knoll or other vantage point, the pioneer locator uld determine a favorable position for his course to the next plateau and/or vantage int. These positions became the point of intersection between two straight lines ding in the general direction of the setting sun or a distant peak. With simple

instruments he would scribe a curve of some known radius to ease the abrupt change in direction as he wandered through the natural valleys and mountain passes. Civilization that followed with wagons and wood-burning locomotives became impatient to reach eternity and began to develop high-speed vehicles for transportation. The old roadways which had been developed through the sweat of the pioneer locator became obsolete. Enterprising emigrants began to construct heavy earth-moving equipment to level the hills and fill the ravines to accommodate their desire for high speeds and heavy loads. Taking advantage of this new earth-moving equipment, the locator lengthened the radii of his curves to give a smoother transition between his tangent sections, then relaxed, well satisfied with the results of his accomplishments. Today, as this modern earth-moving equipment is biting deeper into the hills and filling higher the valleys, some of the modern location engineers are still relaxed, standing on top of the knoll, admiring their newly established point of intersection between two tangents. Generally, because this point is on the crest of a hill, they will return to the design office and establish a grade intersection at the summit, usually from a standpoint of economics or habit. The present day driver, traveling the highways of the west, can most assuredly expect, as he reaches the crest of nearly every hill, that the road will change in direction both horizontally and vertically. Those with the clearest crystal ball and the best brakes will live the longest.

By introducing modern methods for highway projection and the efficient use of topographic maps (developed from photogrammetry) the location engineer can establish alignment and grades which complement the topography and eliminate nearly all of the undesirable aspects of the older accepted methods of making highway locations by surveys on the ground. The locator no longer has an excuse for being unprepared at every change in topography.

Photogrammetry is the tool which will provide the topographic map. The map in turn must be superimposed with planimetric control system. To effect this, a plane coordinate grid system over the area through which the location is to pass must be established. The results of any highway centerline projected through this area will be no more accurate than the original control used to establish the grid system.

Through the intelligent use of the plane coordinate method of centerline projection, employing an accurate topographic map manuscript, as compiled by photogrammetric methods, better highway locations can be achieved with substantial savings in man-hours in preliminary engineering. Lasting dividends accrue through higher benefit-cost ratios, etc., as compared to achievements by other methods.

Without accurate ground control to insure proper orientation of a stereoscopic model in a plotter, it is impossible to secure maps to the accuracy required for location and design of highways. Both horizontal and vertical controls are required in large-scale engineering mapping by photogrammetric methods. Control points should be well defined photographic images that are easily and accurately identified on the ground. Points of precise horizontal control should be appropriately preserved by station markers positioned near the center of the proposed location corridor. Center by each station marker there must be a photographic target of proper size and shapes before the mapping photography is taken. The interval between the photographic targeted station markers used for ground control should be such as to insure at least two targets per stereoscopic model, but the interval should not exceed 1,000 ft depending of course, on map scale.

A traverse or triangulation net run through all station markers centering photographic targets provides the horizontal control required for the mapping, and bench levels run through the same targets provide a very necessary portion of the vertical control through the center of the stereoscopic models. In addition to the vertical control thus obtained through the center of each stereoscopic model, elevation of photographic image points easily identified on the ground and lying near the corners of each stereoscopic model are also required. Use of a 12-ft subtense bar in conjunction with a precise theodolite has proven to be an accurate and efficient method for obtaining the horizontal distances in the traverse as well as the elevations of the model corner points.

Compilation of topographic maps by photogrammetric methods should be based on a system of plane rectangular coordinates. Wherever possible, this system should

be tied to the state system established by the U.S. Coast and Geodetic Survey. The use of plane coordinates increases precision and eliminates cumulative plotting errors. Targeted control points, for which plane coordinates are determined, may be accurately plotted on the map manuscript on which a plane coordinate grid has been scribed. Positioned under the stereoscopic plotter, the model may then be oriented to these plotted positions to obtain proper horizontal scale.

The photogrammetrically compiled maps along with the photographs offer the design engineer the topographic information he needs to accomplish the design. With the terminal controls for the proposed location already established, intermediate topographic controls may be easily determined, and the design completed with the usual consideration given to earthwork quantities, minimum radius, maximum grade, exposure, and use, sight distances, rise and fall, etc.

Where grade is the controlling factor, a grade contour may be sketched on the map as a guide to position in projecting the highway centerline on the maps. A grade contour may be developed by setting dividers to "walk down" the contours on a uniform grade. Where grade is not the controlling factor, the best alignment consistent with cost, exposure, and use is selected. A flexible spline is an excellent tool for establishing the preliminary alignment on the map. It will naturally assume the shape of circular curves, spirals, and joining tangents, and is easily position shifted as desired.

After a preliminary centerline is established by the preceding methods, it may be penciled on the map. Further comparison of the horizontal alignment with a plotted profile may reveal a desirable revision. Curve templates or a projectoscope are useful in determining the degree and approximate length of curves required. For an exact fit of the spline or for passing through definite control positions, other methods are used. These are discussed later. For ease in staking the designed centerline in the field, it is usually advantageous to change the originally measured total delta angle sufficiently so that the length of curve is a multiple of 5 ft. Generally, the change required in the total delta angle to accomplish this is not a significant amount. Circular curves of full degree are also desirable when choice permits; otherwise circular curves of full degree plus nearest usable two-tenths of degree are used. As the circular curves and spiral curves are selected, the stationing and plane coordinates of the TS, PI, and CT are progressively computed for each curve system and these points are plotted on the map. For long curve systems, coordinate positions for about each 300 ft of arc should be calculated and accurately plotted. Curve templates or beam compasses are not reliable to establish long segments of the curves in exact position for later profile and cross-sectioning by photogrammetric methods.

Alternate methods of using a protractor or tangent offset method to lay off the angles and scale the tangent distances in plotting highway alignment are deemed obsolete. With improvements in construction methods and the availability of more accurate instruments for field work, the need for increased accuracy as well as speed in projecting highway alignment must be recognized. In the author's opinion, the best method of highway projection that will enable the location engineer to keep pace with these increasing demands is to employ plane coordinates in plotting and computing the highway alignment. Plotting by plane coordinates is no cure-all or absolute solution to all the problems encountered by the location engineer. Their proper use, however, reduces the discrepancies between the projected alignment and field staking generally encountered with other methods of projection. The plane coordinate method is adaptable to any topographic map regardless of whether it is compiled from a ground survey or by photogrammetric methods.

Details for implementing the procedure warrant a step-by-step review. The topographic map should be compiled using a plane coordinate grid, preferably tied to the state plane coordinate system. The size of the grid square depends on the scale of the map. On a map whose scale is 100 ft to 1 in., it has been found in practice that a 500-ft interval is best but a 1,000-ft interval for each grid square may be used. The plane coordinate grid is constructed so that its line intersections will be at an even 500- or 1,000-ft interval designation.

Once the approximate alignment has been adjusted to meet predetermined standards of degree of curvature and rate of grade, it will be necessary to obtain two additional

elements of information. First, the plane coordinates at the point of beginning, second-
ly, the bearing or azimuth of the tangent at the point of beginning. If the proposed pro-
ject is in the vicinity of triangulation stations established by the USC and GS, then the
plane coordinates of the control points should properly be tied to the state system by
traverse from the nearest triangulation station or by triangulation from two or more
of such stations which are visible from the desired beginning point of location survey.
Should the beginning point and one of the horizontal control points coincide, fortunate
situation directly establishes the coordinates for that point; if not, then these coordi-
nates can be determined by carefully scaling from the nearest grid intersection, be-
cause the control points when plotted were oriented to the coordinate grid. This is
the only point to be occupied on the ground for which the position need ever be scaled
on the projection for the purpose of entering the coordinate system in the alignment
computations. Should it be necessary to assume an origin for project plane coordi-
nates, it would be well to assume an ordinate and abscissa sufficiently large to cover
the entire length of the project and avoid negative figures. Plane coordinate values
increase north and east, decrease south and west. In recording the plane coordinates
and in using bearings on maps and in computations, the north-south are written first
and east-west second.

To determine the bearing or azimuth of the beginning tangent, after the point of
beginning has been selected, extend the tangent a sufficient distance until it crosses a
plane coordinate grid line, preferably 1,000 to 2,000 ft or more beyond its beginning.
Where the tangent crosses the grid line, determine the coordinates by scaling. Be-
cause the coordinates of the point of beginning are known and where the line crosses
a grid line has been determined, the bearing can be determined by the difference in
coordinates. For example, assume plane coordinates for the point of beginning are N
10,000 and E 10,000, and the coordinates at the grid line are determined to be N 11,0
and E 10,277.30. By subtraction, the difference in N and E coordinates is determined
In this case the north coordinates have increased 1,000 ft and the east coordinates
have increased 277.30 ft. The increase in east coordinates divided by the increase in
north coordinates provides an arc tangent of 0.2773000. By inspection in seven place
tables of natural trigonometric functions, it is determined that the bearing angle is
15 deg 30 min. Inasmuch as the coordinates increased by north and east, the bearing
of the beginning tangent will be N. 15 deg 30 min E. If azimuths from the south are
to be used, add 180 deg to the bearing and drop the directional signs. Thus, this ex-
ample, the beginning azimuth becomes 195 deg 30 min. When using such azimuths,
south is 00 deg or 360 deg. If the coordinates and azimuths are on the appropriate
state grid, the location engineer has established a definite tie in the proposed alignme
which can be used for future references (Fig. 1).

The computations of plane coordinates for various curve points involves the trigon
metric cosine and sine functions of the bearing or azimuth and the distance between
points. Cosine times the distance will be the north or south displacement from the
previous point. This displacement is generally referred to as latitude. Sine times th
distance will be the east or west displacement. This displacement is generally referr
to as departure. The algebraic sign to be applied to the separate latitudes and depar-
tures is positive for north and east, negative for south and west. Summation of plane
coordinates is determined from the last point and is a cumulative total. To illustrate
the simplicity of this method, the plane coordinates for the various curves given in
Table 1 have been computed and tabulated in Figure 2. Seven place natural trigonome
tric functions are considered accurate enough because the sine and cosine do not chan
rapidly between minutes.

Although the plane coordinates for any point on each curve system can be compute
this was done for only the TS, PI, and ST points. To determine the SC and CS points
it is sufficiently accurate to scale in these points by X and Y offsets from the plotted
TS and ST. The X and Y offset distances can be computed from unit length values
listed in appropriate columns of published spiral curve tables. Once these points hav
been located, the curves are drawn. Because the control points of each curve system
of spiral-circular curve-spiral are plotted accurately, any one part of each curve sy
tem could be drawn in error and not affect the alignment beyond the segment for whic
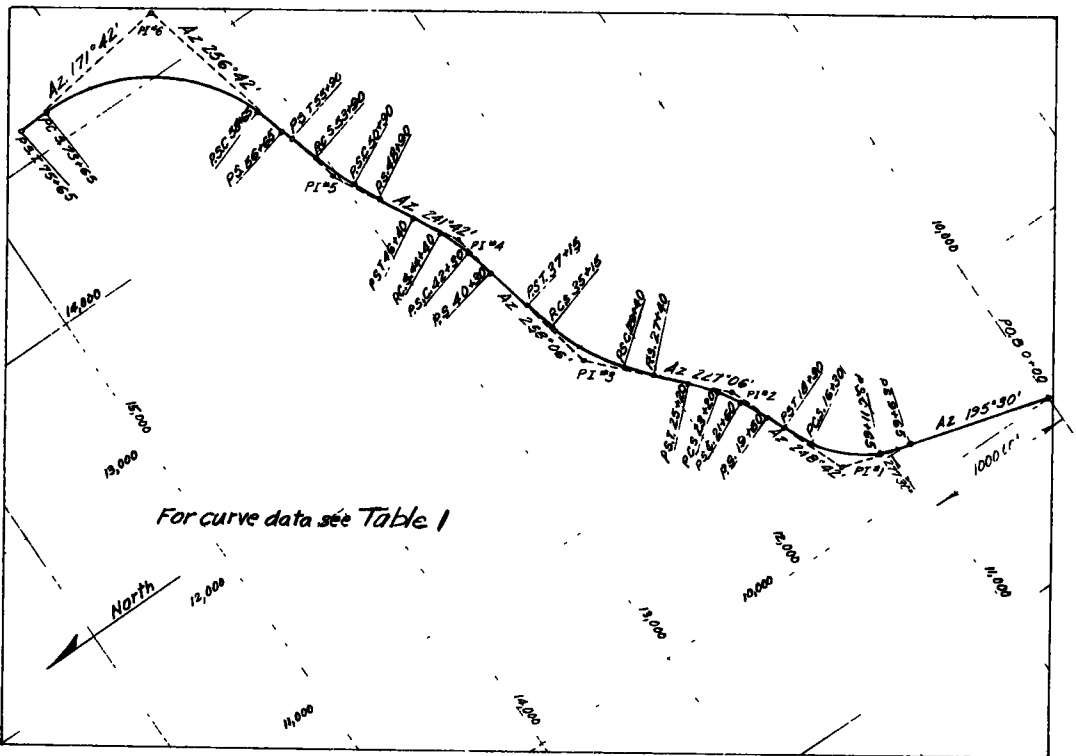the points are in error.

Figure 1.

TABLE 1

Curve	Sta. at TS	Total Δ (o)	(')	T_s (ft)	Δ Simple (o)	(')	D_c (o)	L_c (ft)	L_s (ft)	Δ Spiral (o)	(')
1	9+65	53	12	459.75	37	12	8 Rt.	465.00	200	8	00
2	19+65	21	36	282.46	9	36	6 Lt.	160.00	200	6	00
3	27+40	31	00	497.54	23	00	4 Rt.	575.00	200	4	00
4	40+30	16	24	306.58	8	24	4 Lt.	210.00	200	4	00
5	48+90	15	00	351.54	9	00	4 Rt.	300.00	200	3	00
6	56+65	85	00	1151.35	75	00	5 Lt.	1500.00	200	5	00

It is the practice of a few engineers to vary the tangent distances between curves and the total delta angle to adjust the tangent distance and curve lengths to the nearest -ft interval. This practice enables the location engineer to control, to a certain de- ree of accuracy, the field work by eliminating any odd parts of a foot in the chaining nd parts of minutes in measuring angles. Even in mountainous terrain where distances re frequently critical, this method of shifting the line has proved extremely satis- ctory. Figure 2 has been computed in this manner. Figure 3 has been computed to ow the comparative location of the respective curve points by coordinates, had not e tangent distances between curves and the total delta angles been varied to obtain e nearest 5-ft interval in distance. This example is introduced to lend credence to e author's theory that inasmuch as the coordinates are dependent on the functions of e bearings and distance, it does not affect the accuracy of the plane coordinate meth- l in plotting to vary the distance and angles.

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Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude N	Latitude S	Departure E	Departure W	Y or N	X or E
POB 0+00											10,000.00	10,000.00
		195°30'		265.00	.9636305	.2672334	+929.91		+257.89			
Ta 3+65.00											10,929.91	10,257.89
		195°30'		449.75	.9636305	.267238L	+443.03		+122.87			
PI#1 14+24.79 53°12' Rt											11,372.94	10,330.76
		248°42'		451.75	.3632512	.9316912	+167.01		+423.35			
ST 13+30											11,539.95	10,809.11
		248°42'		130.00	.3632512	.9316912	+ 47.23		+121.11			
TS 19+60											11,587.18	10,930.24
		248°42'		282.46	.3632512	.9316912	+102.61		+263.17			
PI #2 22+42.46	21°36' Lt.										11,689.79	11,173.41
		227°06'		282.46	.6807209	.732429	+172.28		+206.92			
ST 25+20											11,862.07	11,400.33
		227°06'		220.00	.6807209	.7325429	+149.76		+161.16			
TS 27+40											12,031.83	11,561.49
		227°06'		497.54	.6807209	.7325422	+333.69		+364.47			
PI #3 32+37.54	31°00' Rt.										12,370.52	11,922.96
		258°06'		497.54	.206204	.9735090	+102.60		486.86			
ST 37+15											12,473.12	12,012.82
		258°06'		315.00	.2062042	.9785090	+ 64.96		+303.24			
TS 40+30											12,538.08	12,721.06
		258°06'		306.58	2062042	.9745090	+ 63.22		+239.99			
PI #4 43+36.58	16°24' Lt.										12,601.30	13,021.05
		241°42'		306.53	.4740832	.8304774	+145.35		+262.95			
ST 46+40											12,746.65	13,291.00
		241°42'		250.00	.4740832	.8304774	+118.53		+220.13			
TS 48+90											12,865.18	13,511.13
		241°42'		351.54	.4740882	.8304774	+166.66		+304.52			
PI #5 52+41.54	15°00' Rt.										13,031.84	13,820.65
		256°42'		351.54	.2300497	.9731789	+ 80.87		+342.12			
ST 55+90											13,112.71	14,162.77
		256°42'		75.00	.2300497	.9731789	+ 17.26		+73.00			
TS 56+65											13,129.97	14,235.77
		256°42'		1151.35	.2300497	.9731789	+264.87		+1120.47			
PI #6 68+16.35	85°00' Lt.										13,394.84	15,356.24
		171°42'		1151.35	.9895258	.1443542	+1139.30			166.21		
ST 75+65											14,534.14	15,190.03

Figure 2.

Another instance where the plane coordinate method is particularly useful is when the total delta angle exceeds 180°. In these computations it is necessary to compute the coordinates for the points through the curve; that is, TS, SC, radius point, CS and the ST. Inasmuch as the total delta exceeds 180°, the T$_S$ distance will not intersect. First the coordinates for the TS are computed; then on the same azimuth the coordinates for the SPI (Spiral PI) are computed using the long tangent of the spiral. From this point the azimuth is changed in the appropriate direction by the amount of the spiral delta. The distance used will be the short tangent of the spiral. The coordinates at this point define the SC. From the SC the azimuth of the radius is determined to be 90° to the short tangent. The radius of the circular curve times the cosi and sine functions of the azimuth establish the plane coordinates for the center of the curve. From the center point of the circular curve, the azimuth to the CS is changed by the amount the delta angle of that curve differs from 180°. If the angle exceeds 180°, then the difference is added; if less than 180°, the difference is subtracted. After the azimuth of the radial line from the center point of the circular curve has been determined, then the plane coordinates for the CS are computed. From this poi the short tangent of the spiral will be at 90° to the radius. These coordinates will es tablish the SPI. The plane coordinates of the ST will be determined by computing the change in the azimuth from the delta angle of the spiral and multiplying the long tange of the spiral by the cosine and sine of that azimuth, and then algebraically adding the results to the N and E coordinates of the SPI. This plane coordinate method of com puting and plotting is useful for accurate positioning on the map of centerline in moun

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Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude N	Latitude S	Departure E	Departure W	Y or N	X or E
POB 0+00											10,000.00	10,000.00
		195°30'		963.37	.9636305	.2672384	+928.34		+257.45			
Ts 9+63.37											10,928.34	10,257.45
		195°30'		460.54	.9636305	.2672384	+443.80		+123.08			
PI 14+23.91	53°18' Rt.										11,372.14	10,380.53
		248°48'		460.54	.3616242	.9323238	+166.55		+423.38			
ST 18+29.62											11,538.69	10,809.91
		248°48'		131.69	.3616242	.9323238	+ 47.63		+122.79			
Ts 19+61.31											11,586.32	10,932.70
		248°48'		281.88	.3616242	.9323238	+101.93		+262.80			
PI 22+43.19	21°32' Lt.										11,688.25	11,195.50
		227°16'		281.88	.678.871	.734.199	+191.27		+2.7.05			
ST 25+20.20											11,379.54	11,402.55
		227°16'		221.23	.6785871	.734.199	+150.13		+162.51			
Ts 27+41.48											12,029.67	11,565.06
		227°16'		496.64	.6785871	.794.199	+337.01		+364.79			
PI 32+33.07	30°56' Rt.										12,366.68	11,929.85
		258°12'		496.64	.2044961	.7388674	+101.56		+437.15			
ST 37+14.76											12,468.24	12,416.00
		258°12'		313.72	.2044961	.9733674	+ 64.16		+307.10			
Ts 40+2'.48											12,523.40	12,723.10
		258°12'		307.41	.2044761	.9733674	+ 62.86		+300.91			
PI 43+35.89	16°28' Lt.										12,575.26	13,024.01
		241°44'		307.41	.4735759	.8807530	+145.59		+270.76			
ST 46+40.15											12,740.35	13,294.77
		241°44'		252.00	.4735759	.8807530	+119.35		+221.95			
Ts 48+92.15											12,860.20	13,516.72
		241°44'		352.66	.4735759	.8807530	+167.01		+310.61			
PI 52+44.81	15°04' Rt.										13,027.21	13,827.33
		256°48'		352.66	.2243509	.9735783	+ 80.53		+343.35			
ST 5.+24.47											13,107.74	14,170.68
		256°48'		73.50	.2283509	.9735783	+ 16.79		+ 71.57			
Ts 56+67.87											13,124.53	14,242.25
		256°48'		1153.21	.2283509	.9735789	+263.34		+1122.74			
PI 63+21.08	85°06' Lt.										13,317.87	15,364.99
		171°42'		1143.21	.9895258	.1443562	+1141.14			166.47		
ST 75+67.37											14,529.01	15,193.52

Figure 3.

tainous country where large delta angles are sometimes necessary. To illustrate the computations of plane coordinates to be used, assume total delta = 212°24', D_C = 18° Rt, L_S = 200', Spiral delta = 18°, L_C = 980.00'. From published tables, obtain the radius = 318.31', Long tangent = 134.03', Short tangent = 67.30'. Again the work sheet is tabulated as in the preceding examples. For computations, see work sheet (Fig. 4).

Should it be necessary to change a section of the centerline after it has been plotted, it is not necessary to run the entire line again, only that section that will be affected in the change. Select a curve point that will not be affected in the change and use its plane coordinates as the coordinates for the point of beginning. For plane coordinates at the point where the line change ends, select coordinates of the center of the next succeeding circular curve. To determine the delta angle for the final curve system, it is only necessary to obtain the differences in the azimuth. Because the plane coordinates of the center of the last circular curve on the line change have been computed and the plane coordinates of the center of the circular curve ahead are known, the difference in these coordinates is ascertained—and this information is sufficient to set off a triangle which can be solved by the Law of Sines. By subtracting the T_S distance from the respective sides of the triangle, the length of the tangent between curve points is determined. This is one occasion where the delta angle and the tangent distance cannot be adjusted to the nearest 5-ft interval without affecting the entire line ahead. To account for the difference in stationing, an equation is used at the point where

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FROM STATION_____ TO STATION_____

Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude		Departure		Coordinates	
							N	S	E	W	Y or N	X or E
2= 100+00											10,000.00	10,000.00
		100°00'		134.03	.1736482	.9848078	+ 23.28			-131.00	10,023.28	9,869.00
SPI 101+34.03	18°00' Rt.											
		118°00'		67.30	.4694716	.8829476	+ 31.60			- 59.43	10,054.88	9,809.57
SC 102+00	90°00' Rt.											
		208°00'		318.31	.882,476	.4694716	+281.06		+149.44		10,335.94	9,959.01
Radius Point	03°36' Lt.											
		204°24'		318.31	.9106837	.4131044	+289.89		+131.49		10,625.83	10,090.50
CS 111+80	90°00' Rt.											
		294°24'		67.30	.4131044	.9106837		-27.80	+ 61.30		10,598.03	10,151.80
SPI 112+47.30	18°00' Rt.											
		312°24'		134.03	.6743024	.7384553		-90.38	+ 98.98		10,507.65	10,250.78
ST 113+80												
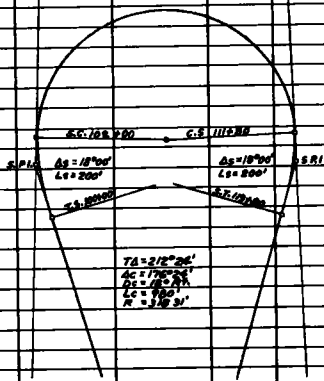
Figure 4.

the line change ends.   To illustrate this solution, assume curve systems 2, 3, and 4 in Figure 2 are replaced with one curve system.   Here the coordinates for the point of beginning on the line change will be those at the ST at 18+30 and the coordinates to be used on the existing line ahead will be at the ST at 46+40.   The total delta will be the difference between the back azimuth, 248°42', and the forward azimuth 241°42', or 7°00'.   Because the forward azimuth is less than the back azimuth, the curve will be to the left.   The following curve data will be used: total delta=7°00', T_s=275.25', Delta circular curve=3°00', D_c=2°00' Lt., L_c=150', L_s=200', Spiral delta=2°00'.   This curve revision is given in Figure 5 in the same manner as the previous examples.

Preparing the centerline for plotting by plane coordinates affords definite ties to be computed in the office for use by the field survey crew.   Because both the basic horizontal control stations and the centerline alignment are based on the same plane coordinate system, it is an easy computation to determine the distance and azimuth from various points along the line to the nearest horizontal control point.

Figure 1 was plotted to show the alignment illustrated by Figure 2.   It will be noted that the topography has been deleted due to the scale of the drawing.   The plane coordinate grid has been laid off on a 1,000-ft interval in both directions.

From these previous examples, it can readily be seen that, by using the plane coordinate method of plotting the desired alignment, the location engineer has eliminated most of the element of chance and has changed it to an element of choice.

For elevations and distances determined during the location survey staking of the highway on the ground to agree with elevations and distances obtained from the map projection, the map plotted and designed centerline must be staked on the ground in the same position in which it is projected on the map manuscript.   This required duplication during the line staking operations of the accuracy accomplished in establishing
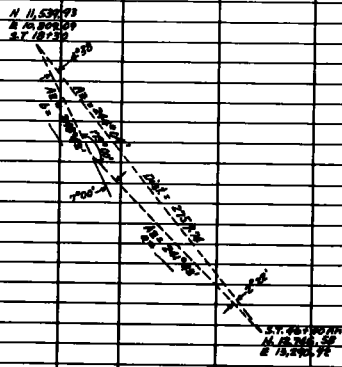
LINE COORDINATES

FROM STATION_____TO STATION _____

PROJECT_____
STATE _____ Zone _____
Computation No _____ Page _____ of _____
Traverse No _____ Field Book No ____ Pgs ____ to _____
Computed by _____ Date_____
Checked by _____ Date_____

Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude		Departure		Coordinates	
							N	S	E	W	Y or N	X or E
ST 13+30											11,531.23	10,309.09
		248°42'		659.76	.3632512	.9316912	+239.66		+614.70			
TS 24+37.76											11,779.59	11,423.79
		248°42'		275.25	.3632512	.9316912	+99.78		+256.57			
PI 27+65.01 7°00' Lt.											11,879.57	11,680.24
		241°42'		275.25	.4740882	.8804774	+130.50		+242.36			
ST 30+19.76											12,010.07	11,927.60
		241°42'		1553.99	.4740882	.8804774	+736.73		+1368.25			
TS 45+93.75 Bk) (ST 46+40 Ah)											12,746.80	13,290.85

Figure 5.

...he original basic control. With precise instruments and trained personnel, such ac-curacy in the line staking can be satisfactorily obtained.

Even when plane coordinates are used for the alignment projection on the map, results will still be disappointing if the entire centerline is not accurately plotted point by point for each contiguous segment. Small plotting errors will not be noticeable in easy topography, but on steep side hills and other mountainous topography, plotting errors will cause considerable difference in elevation between map and ground positions. There are several factors contributing to plotting errors. Circular curve templates are not always accurately cut on uniform curvature or to the specified scale. But, changing radius curves used to sketch spiral transitions do not reflect the true position of such line segments. Manuscripts on unstable material that is affected by moisture and temperature, use of inaccurate scales, and the projection of alignment on map prints which are definitely out of scale also contribute to inaccuracies in position of the plotted line. Moreover, circular curves drawn with a beam or other compass will be displaced laterally in some segments because of a warp of the map manuscript (or the printed copy if used instead) during design. In overlooking, in not realizing, or in ignoring such causes, photogrammetry has often been blamed unjustifiably for errors which were introduced elsewhere.

Although many field tables have standardized the symbols for terms used in spiral curve transitions and circular curves, engineers still find that when referring to certain curve components, they are speaking in terms not familiar to persons using other tables. Without a thorough understanding of the actual curve problems, one may find himself solving a problem by equation substitution to determine a value that is obvious at first glance. An engineer with a good working knowledge of highway alignment

projection will refer to tables of functions to reduce the time of calculations and not for equations which will provide an answer by correct substitution and solution through the indicated steps. Some examples which are accurate within reasonable limits are, as follows:

1. The deflection angle for a circular curve in minutes is 0.3' per foot per degree of curve. The Delta or central angle is twice the deflection angle.

2. The spiral delta angle is equivalent to the deflection angle for the particular curve computed for the length of the spiral.

3. Transition spiral curves change in curvature at uniform rate throughout.

This is sometimes referred to as a rate of change or a deflection factor. This rate of change is determined by the degree of curve in degrees divided by the length in 100-ft stations of the spiral curve. Because the spiral curve deflection varies approximately as the square of the distance, the deflection from the beginning of the transition may be determined in minutes by multiplying 10 by the rate of change in curvature per 100-ft station by the distance squared; each station or 100 ft is used as a unit distance of one in computing the square.

There are times when it is desirable to compound curves to better fit the ground, drainage, and land use. Such compounds should always be connected by a spiral curve transition. To avoid an unsightly "broken back" curve, it may be necessary to combine a 2-deg curve with a 4-deg curve. For simplicity, assume the transition will be made in 200 ft. Some texts will provide satisfactory equations for solution of this problem. By substituting appropriate values in the equations, the problem may be solved. Actually the spiral delta of the transition curve, in this instance, is the sum of the degrees of the two combining curves or 6 deg. Should a 400-ft transition curve be selected, the spiral delta is 12 deg. The difference in curvature between the circular curves being joined by the spiral curve is 2 deg. This means that the curvature is changed proportionately throughout the length of the spiral curve from 2 to 4 deg or vice versa, depending on the direction in which the change is being made to sharp or less sharp curvature. Returning to the original selection of a 200-ft spiral curve for transition between a 2-deg curve and a 4-deg curve, it is observed that the degree of curvature must increase 1 deg for each 100-ft station of transition. For the 400-ft transition, the degree of curvature would only be increased 30 min for each 100-ft station. The rate of change would be equivalent to the difference in degree of the circular curves divided by the length in 100-ft stations of the spiral curve. Thus, for the 200-ft transition, the rate of change would be 4 deg minus the 2 deg divided by 2 stations which is a rate of change of 1 deg per 100-ft station. For the 400-ft transition, the rate of change would be 0.5 deg per 100-ft station. The angle of deflection to be turned from the CS of the 2-deg curve to establish the SC of the 4-deg curve would be the normal deflection for a 2-deg curve plus the rate of change for the distance of the transition. In the instance of the 200-ft transition, the deflection angle would be 1 deg plus $10(1)$ $(2)^2$ or 40 min. The deflection angle would be $2°40'$. From the SC of the 4-deg curve, the deflection angle would be 4 deg minus 40 min or $3°20'$. The sum of these angles will equal the spiral delta angle of 6 deg.

This explanation allows the use of an approximate calculation for plotting the short and long tangent of the spiral transition between two curves. There are some well-written text books on the market which provide an accurate solution. This method used for plotting and not field staking, however, is usually accurate to 0.10 ft unless the length and rate of change of the transition curve between circular curves is quite pronounced. The spiral delta and the two deflection angles are known. In 200 ft, a transition has been made between a 2- and 4-deg curve. The average degree of curvature is 3. By the law of signs, there is an oblique triangle with three known angles. If the long chord is known, the two tangent lengths can be calculated. For an approximate solution, solve the long chord for a 3-deg curve with an arc length of 200 ft using the equation $C + 2R \, Sine \frac{A}{2}$. This provides the base of the triangle. Using the known angle the solution of the other two sides is elementary (Fig. 6).

The distances of T_1 and T_2 are calculated at approximately 111.20 ft and 88.98 ft, respectively (Fig. 6).
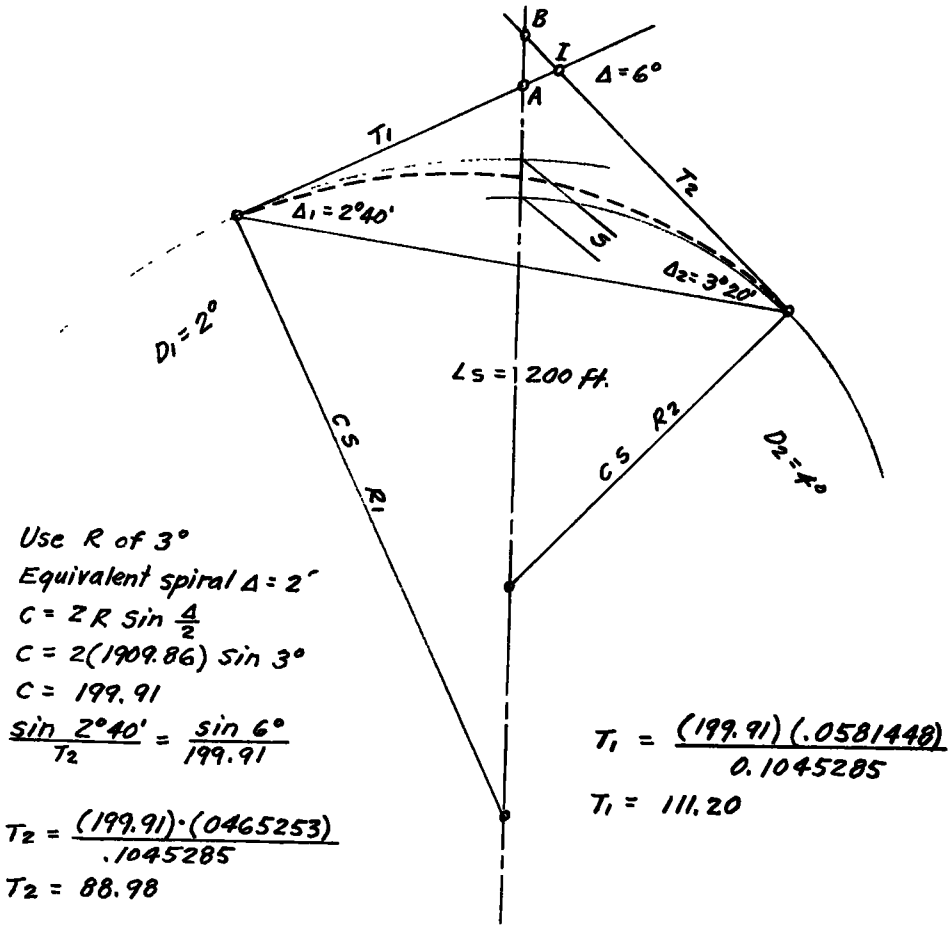
Figure 6.

Compare this approximation by solving the same problem using a method outlined in a text regarding a particular set of field tables:

Given: $D_1 = 2^O$ $R_1 = 2864.789$
$D_2 = 4^O$ $R = 1432.39$

Equivalent Spiral Angle $= \dfrac{200}{200}$ L_s $(4^O-2^O) = 2^O$

S = offset of the circular curves which is essential for insertion of the spiral curve.
S in feet is the function for offset from tables of a unit length spiral multiplied by the spiral length. In this case S = 0.00291 x 200 = 0.5820

$\Delta_1 = \dfrac{200}{200}$ x $2^O = 2^O$, $\Delta_2 = \dfrac{200}{200}$ x $4^O = 4^O$

AB = R_2 Exsec Δ_2 - R Exsec Δ_1 - (offset.)
= (1432.39)(0.0024419) - (2864.79)(0.0006095) - 0.5820
AB = 1.169663 or 1.1697

AI = 1.1697 x $\dfrac{\text{Cos } 4^O}{\text{Sin } (2^O + 4^O)}$ = 11.163

T_1 = 2864.79 x Tan 2^O + AI

2864.79 (0.0349208) + 11.163 = <u>111.203</u> or 111.20

BI = 1.1697 $\dfrac{Cos\ 2^O}{Sin\ 6^O}$ = 11.183

T_2 = 1432.39 (Tan 4°) = BI

1432.39 (0.0699268) - 11.183 = <u>88.979</u> or 88.98

It appears from the preceding solution that a considerable amount of time was consumed in solving a problem that is more easily computed by the first approximate method. Approximate methods should never be used except when the difference in degree of circular curves between which the spiral curve is to serve as transition is small.

For an exact computation of a description of a centerline projected on a map with the spline line or to pass a circular curve through given points, a more accurate method of solution is offered. Where the spline line passes through points to which the centerline must be held, mark off stations along the arc through which the arc maintains uniform curvature. Mark the beginning station point No. 1 and the ending point No. 2. Move ahead to the next point along the spline to be held and mark stations along this arc in the same manner. These will be noted as points No. 3 and 4. Scale the coordinate positions at points No. 1 and No. 2. Scale the chord distance between these points and the mid-ordinate. Use the same procedure between points No. 3 and No. 4.

For symbols use:

R = radius
M = mid-ordinate
C = chord
S = shift in feet
Use D_1 and D_2 in degrees
L_S = length of spiral in stations

$$R = \frac{M}{2} + \frac{C^2}{8M} \qquad\qquad D_c = \frac{11459.156}{M = \dfrac{C^2}{4M}}$$

As an example, let the chord between points No. 1 and No. 2 of the first circular curve be 495 ft and M 18.5 ft.

$$R = \frac{18.5}{2} + \frac{(495)^2}{148} = 1665 \pm \text{ for trial only}$$

$$D_c = \frac{11459.156}{18.5 + \dfrac{(495)^2}{74}} = 3.44^O \pm \text{ Use 3.6 or R = 1591.549}$$

Moreover, the scaled difference in the north coordinates is 169.5 and the east coordinates difference is 465.5, the actual length of the long chord will calculate to be 495.40 and the bearing of the chord will be N $69^O59'32''$ E; then

$$\text{Sin } \frac{\Delta}{2} = \frac{C}{2R} = \frac{495.40}{3183.10} = 0.15563444$$

$$\text{Sin } \frac{\Delta}{2} = 8^O57'13''$$

Using the bearing of the chord $\qquad\qquad$ N$69^O59'32''$E

$\qquad$ Less $\frac{\Delta}{2}$ - $\qquad\qquad\qquad$ $\dfrac{8^O57'13''}{\text{N}61^O02'19''\text{E}}$

$\qquad$ Plus 90° $\qquad\qquad\qquad\qquad$ $\dfrac{89^O59'60'}{\text{S}28^O57'41''\text{E}}$

Bearing to curve center from point No. 1

Adding in the same manner from Point No. 2, the bearing to center is S 11°03'15" E.

This same procedure is used to determine the degree of curve, mid-ordinate, chord, and bearing of radial lines to the center of the next curve from points No. 3 and 4. With this information, calculate the plane coordinate of the center of circular curve No. 1 and circular curve No. 2.

(1) Use the difference in plane coordinates of the separate positions of the respective centers of curves 1 and 2.

(2) Compute from their plane coordinates, the length of line and bearing between the respective centers.

(3) Subtract the length of line between centers, as computed in step 2, from the difference in radii of the circular curves to be joined by the spiral curve. The result equals the shift "S" provided by the spline.

(4) Compute the length of spiral for trial only.

$$L_S = \sqrt{\frac{S}{0.0727(D_1 - D_2)}}$$
S = feet shift, in ft
D_1 and D_2 = degrees
L_S = 100-ft stations

(5) Suppose S = 1.0, then

$$L_S = \sqrt{\frac{1.0}{0.0727 (3.6 - 2.8)}} = \sqrt{17.19} = 4.15 \text{ station}$$

(6) Select length of spiral to attain even deflection factor $= \dfrac{D\ 10}{L_S}$ = minutes per foot^2

Example: deflection factor $= \dfrac{3.6 - 2.8}{10\ L_S} = \dfrac{0.8}{L_S}$ when $D_1 = 3.6°$ and $D_2 = 2.8°$

Use a length (in stations) that is divisible into $(D_1 - D_2)$ the quotient of which is divisible into D_1 and D_2 separately if possible. In this instance, use 4.0 stations.

(7) Compute spiral joining circular curves 1 and 2.
   K = rate of change in degrees per 100-ft station

$$K = \frac{3.6 - 2.8}{4} = \frac{0.8}{4} = 0.2° \text{ per 100-ft station}$$

$$L_{S_1} = \frac{D_1}{K} = \frac{3.6}{0.2} = 18.0 \text{ stations} = 1800 \text{ ft}$$

$$L_{S_2} = \frac{D_2}{K} = \frac{2.8}{0.2} = 14 \text{ stations} = 1400 \text{ ft}$$

$$L_S = L_{S_1} - L_{S_2} = 4 \text{ stations or 400-ft spiral}$$

(8) Compute functions of spiral between circular curve.

$$\Delta_{S_1} = \frac{L_{S_1} D_1}{200} = \frac{1800 \times 3.6}{200} = 32.4°$$

$$\Delta_{S_2} = \frac{L_{S_2} D_2}{200} = \frac{1400 \times 2.8}{200} = 19.6°$$

$$\Delta_S = (\Delta_{S_1} - \Delta_{S_2}) = 12.8°$$

*X_1 = 1800 x 0.9684924 X_2 = 1400 x 0.9883610
Y_1 = 1800 x 0.1842337 Y_2 = 1400 x 0.1130786
Q_1 = 1800 x 0.4947175 Q_2 = 1400 x 0.4980560
P_1 = 1800 x 0.0465893 P_2 = 1400 x 0.0283882
U_1 = 1800 x 0.6781866 U_2 = 1400 x 0.6707995
V_1 = 1800 x 0.3438307 V_2 = 1400 x 0.3370936

*Functions taken from "Circular Curve and Unit Length Spiral Tables," by William T. Pryor, Bureau of Public Roads.

T_{S1} = short tangent of spiral between two circular curves.

T_{S2} = long tangent of spiral between two circular curves.

T_{S1} = $(Y_1 - Y_2) \cos \Delta s_2 - (X_1 - X_2) \sin \Delta s_2 = V_1 - \dfrac{(U_1 - U_2) \sin \Delta s_2}{\sin \Delta s}$

T_{S2} = $\dfrac{(X_1 - X_2) \sin \Delta s_1 - (Y_1 - Y_2) \cos \Delta s_1}{\sin \Delta s} = \dfrac{(U_1 - U_2) \sin \Delta s_1}{\sin \Delta s} - V_2$

By taking advantage of the use of coordinates in projecting highway alignment, solutions to problems are easily solved and provide the engineer with complete control of the alignment position.