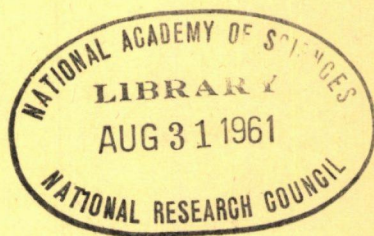


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Bulletin 283

***Photogrammetry:
Developments and Applications
1960***



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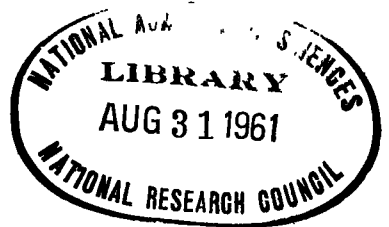
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Lens Characteristics as Related to Model Flatness

P. KATIBAH, Supervising Photogrammetrist, California Division of Highways

Stereomodel deformations which are directly attributable to optical characteristics of any part of a lens system are notably of a systematic nature. Although these errors may be within the usual specifications for contours (90 percent within $\frac{1}{2}$ contour interval), their systematic trend can conceivably have a significant influence in the calculation of earthwork quantities from highway design maps.

An analytical study has been conducted to ascertain the effect of individual optical elements of the lens system, including camera lenses, projection lenses, and diapositive glass. Because of its extensive use in highway design mapping, the study hinges on Kelsh-type plotting instruments. In conjunction with these instruments, such camera lenses as the Metrogon, Aviogon, Pleogon, and Planigon are discussed. Procedures for testing plotting equipment and isolating causes of deformation are outlined. Data are derived for compensation of combined distortion, within minimum tolerances, for the restitution of stereomodels relatively free from deformations causing systematic errors.

USE of photogrammetric data and maps for highway location and design is generally accepted by most highway departments. Whether or not photogrammetric surveys are sufficiently reliable for payment of earthwork quantities is a topic of current interest, there are great potential savings in manpower which can be realized by obtaining earthwork quantities from photogrammetric data. Research work by the California Division of Highways has shown that the most important factor in the accuracy of earthwork quantities is the vertical accuracy of the survey measurements (5). Investigation of photogrammetric map accuracies (4) had demonstrated that photogrammetric measurements can be of an accuracy which justifies statistical analysis, especially if systematic errors and blunders are eliminated.

The causes of errors in the photogrammetric system are extremely difficult to eliminate. Perfect restitution of the stereomodel, point for point, is undoubtedly an ideal situation which is not attained in everyday practice. Every stereomodel is deformed to some extent, and it is the degree of deformation which determines whether or not it will be detected. In this country, where film-base photography is the accepted medium instead of glass-plate photography, there is a tendency to dispose of all deformation as a function of instability of the film-base. Although the film-base perhaps remains the most important single cause of model deformation, other causes should not be overlooked or quickly branded as playing a relatively minor roll in the over-all problem. By analyzing the cause of individual errors, the necessary data can be compiled as a basis for distortion compensation. After compensation is accomplished, the residual errors would tend to be of a random nature rather than systematic. The results, then, would lend themselves to valid statistical analysis, which in turn would make the science of photogrammetry more useful for detailed engineering studies. There are two broad sources of error which can be accounted for to some extent by the photogrammetrist; namely, instrument calibration and the characteristics of

the lens components in the total system. The scope of this study encompasses only the latter considerations, especially those associated with Kelsh-type plotters because the instruments are undoubtedly far more important in the production of highway design mapping than any other type. There is adequate literature dealing with instrument calibration (1, 9), and no particular need exists at this time to elaborate on the subject. There is also considerable literature covering cameras and camera lenses, as well as plotting equipment, but there is a lack of coverage, available to the practicing photogrammetrist, which includes all the lenses in a photogrammetric system, relating them to the final product; that is, the stereomodel.

The significant feature of the Kelsh-type plotter is the formation of the stereomodel by direct projection of the 9- by 9-in. diapositive. This solution of the photogrammetric problem was first made by Harry T. Kelsh in the years immediately following World War II. The best known instrument of this group is the Kelsh plotter as manufactured by the Kelsh Instrument Company. A variation of it is the Nistri-Photomapper, manufactured by the O. M. I. Corporation of Rome, Italy. Other commercial makes have appeared on the market, but regardless of manufacturer they all share the identical feature of direct projection of the original negative size. This solution permitted simplification of instrument design, thereby making it possible to produce a relatively inexpensive instrument capable of forming a large-size model. In common with all direct projection instruments, the model scale is a function of the magnification factor of the projection lens.

The perfect restitution of the model depends entirely on whether or not the cone of rays emerging from the projection lens is angularly identical with the cone of rays received by the camera lens. Any deviation whatsoever of the projected rays from their original entrance paths will contribute to model deformation. Causes for deviations may be divided into three broad independent groups, as follows:

1. Mechanical—imperfections in instrument fabrication and/or unsatisfactory calibration;
2. Photographic—any shift of the image position on the aerial film or on the diapositive after exposure; and
3. Optical—lack of data pertaining to lenses, or failure to compensate for radial distortion.

Because this paper is concerned with only optical causes, mechanical and photographic causes will not be dwelled on in further discussion.

ANALYSIS OF MODEL ERRORS

A mathematical analysis of a stereomodel provides a method of predicting model deformation in terms of vertical error. The usual assumptions are that the photographs are truly vertical, and that any two exposures comprising a stereo pair are identical in scale. In addition, a base-height ratio and a width-height ratio must be assigned to determine the size of the neat model. The data for analysis are the distortion values of any lens component in the system. These are customarily given in the calibration report.

Of the various methods of computation, the one devised by J. G. Lewis (6) has been used in this report. Lewis' method analyzes vertical errors of 16 points that are well distributed in 52 locations in the total model area, with 32 of them being in the neat model area. The distribution of the points in relation to a 25-mm grid model is shown in Figure 1 and similar figures referred to in the text. The neat model in Figure 1 is the rectangular area with the corners marked by triangles. The base-height ratio is 0.62 and the width-height ratio is 1.12, which corresponds to a neat model size of 3.72 by 6.72 in. at photo scale. This is a realistic size for large-scale design mapping. Another method of computation is given by Friedman (3) which also is satisfactory providing base-height and width-height ratios are modified to fit conditions usually associated with larger scales.

The distortions in a system accumulate algebraically. One may begin an analysis with the algebraic sum of all the known distortions to determine the resulting vertical

error in the model, or the known distortions can be separated according to lens component, analyzed individually, and the separate results at each point added to arrive at the final vertical error. Either way will yield the same answers. However, if calibration data are available for the camera lens only, the components will have to be separated for analysis, and the final results in the model determined by adding the separate errors. This makes it necessary to resort to methods of analysis other than mathematical to determine errors by the projection lens of the plotting instrument. A calibration report is very seldom available for a projection lens.

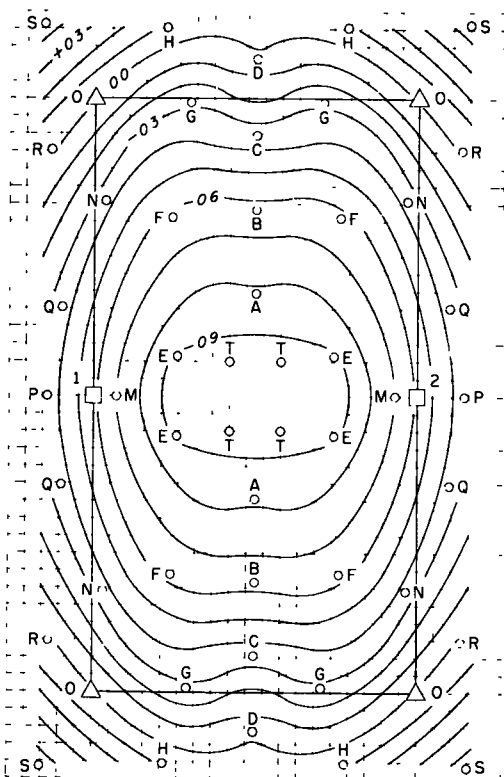
Probably the most effective method for testing performance of a projection lens is the "grid model" method. Precise grids on glass are used as diapositives in the formation of a grid model with a base-height ratio equivalent to the value used in mathematical analysis. Vertical errors are determined by reading the model at the optimum projection distance. A perfectly restituted model would read as a truly plane surface, whereas any deformation would show as a vertical departure from this criterion. Grid model deformations are a result of all errors associated with the plotter, and serve as a final test of the over-all performance of the instrument. The instrument, therefore, must be carefully calibrated and tested to minimize the influence of mechanical sources of error. Assuming that all other sources of errors have been accounted for, the resulting vertical errors in the grid model are attributable to the distortion in the projection lens.

CAMERA CALIBRATION

Most mapping projects require the use of nominal 6-in. photography, with several designs of lenses available in cameras of different manufacture. The calibration report which accompanies a cartographic camera assures the user that the camera was designed and manufactured to certain standards, and further provides detailed data pertaining to focal length, distortion pattern, and resolving power of the lens as mounted in the camera. Cameras which have not been certified by a qualified testing agency should not be used for cartographic photography.

Although calibration reports contain basic data important to the user, there is a concerning lack of uniformity among reporting agencies in the manner of presentation, which may occasion doubt as to the consistency of results. To illustrate, the following four agencies present data in varying ways and to different tolerances:

U. S. Bureau of Standards: Lists both equivalent and calibrated focal lengths to a stated tolerance of ± 0.10 mm. Six distortion values are given to a stated tolerance ± 0.02 mm, based on both E. F. L. and C. F. L.



Assumptions: Camera and projection lenses distortion-free

Instrument: 5X projection plotter
 Model Scale: 1 in. = 50 ft
 Contour Interval: 0.1 ft
 B/H = 0.62 W/H = 1.12
 25 mm grid at model scale

Figure 1. Model deformation caused by 0.06-in. thick glass.

Fairchild Camera and Instrument Corporation: Lists both equivalent and calibrated focal lengths to a stated tolerance of ± 0.10 mm. Eleven distortion values are given to a stated tolerance of ± 0.01 mm, based on both E. F. L. and C. F. L.

Zeiss-Aerotopograph: Lists calibrated focal length to a stated tolerance of ± 0.02 mm. Fourteen distortion values are given to a stated tolerance of ± 0.002 mm, presumably based on C. F. L.

Wild-Heerbrugg Instruments, Inc.: Lists calibrated focal length with no stated tolerance. Eight distortion values are given with no stated tolerance, presumably based on C. F. L.

The apparent confusion in data presentation is noted here because it is a situation with which the practicing photogrammetrist must cope. It does not necessarily mean the data are not usable: it does mean, however, that data are not transferable from the terms of one agency into the terms of another agency, so that two cameras reported individually by two agencies cannot be compared on a uniform basis. This is particularly annoying because the photogrammetrist is forced to regard any camera report as absolute, unless of course evidence exists to the contrary.

Procedures for camera calibration are explicitly explained by Sewell (7), and Washer and Case (14). The latter reference applies to U. S. Bureau of Standards methods. Washer has further enumerated and described the various sources of errors associated with camera calibration and their effect on the accuracy of the calibration (10, 11, 12, 13).

The calibration data with which the photogrammetrist is mainly concerned are the resolution, distortion pattern, and the focal length. A "high performance lens" is one which exhibits high resolving power and low distortion. These terms imply that a high performance lens is capable of producing photographs of exceptional clarity and detail, and at the same time can be used for photogrammetric measurements without correcting for displacement of images caused by deviations in the light rays passing through the lens. Because all lenses exhibit distortions, however small, no lens can be labeled "distortion-free," and the effect of the residual distortions must be analyzed to verify the ultimate effect on the stereomodel. The focal length, of course, is necessary to correctly adjust the principal distance of the projectors in order to recover the proper perspective geometry of the exposures.

RESULTS OF INVESTIGATIONS

Diapositive Glass

Kelsh plotter procedure normally requires only two lens components in completing the optical path from the exposure of a ground area to the projection of it onto the platen. The lenses involved are the camera lens and the projection lens. It is currently normal procedure to make the diapositives by contact printing through the film base, using a point-source light, in order to register a reverse image. This permits the diapositives to be placed emulsion-surface down in the projectors. This produces the same results as a one to one ratio projection printer, but eliminates an optical step.

If the diapositive is made emulsion to emulsion in a contact printer the photo-image will have to be projected through the glass. This procedure introduces an optical step because the glass is actually a lens, each surface being of infinite radius. The light rays transmitting the image through the glass will be refracted, causing a distortion which will result in model deformation. Distortion values can be readily determined for glass of any particular thickness considering the angular distance from the axis of the lens system according to the tabular values on page 47 of the "Manual of Photogrammetry" (2).

Printing diapositives emulsion-side up on 0.06-in. thick glass is still in practice, mostly because of the lower costs of materials and the facility of conventional printing methods. The 0.06-in. thick glass does not measure up to the quality of the thicker glass currently available for diapositive materials, as indicated in the brochures of the commercial outlets. Experience shows that it requires support in the middle to

prevent sag, and there is the possibility of wedge effect caused by the lack of parallelism between the two planar surfaces.

The distortion values for 0.06-in. thick glass are given in Table 1. Using these distortion values, 16 points distributed in 52 locations in the stereomodel can be computed, yielding results in terms of vertical errors, or deviations from a truly plane surface. Computational results are given in Table 2 at model scale in millimeter units, and in equivalent feet at a scale of 1 in. = 50 ft, the usual design mapping scale required by the California Division of Highways.

The location of the points in relation to the 25-mm grid model (5-mm diapositive grids enlarged 5 diameters) is shown in Figure 1. The contours have been interpolated between computed values to depict the expected model deformation, which is a "dished" effect approaching 1 ft in equivalent value at the model center.

The close agreement between computed and actual values demonstrates the validity of the computational approach, and of course is a tribute to the skillfulness of the instrument operator inasmuch as he had no prior knowledge of the computed values. The biggest spread between computed and actual values

occurred at points H and S, both far outside the neat model area within about $\frac{1}{2}$

in. from the margin of a corresponding

photograph. Point P was so far outside the neat model area that it was beyond the physical limitations of the instrument, and therefore could not be read. Inasmuch as

TABLE 1
DIAPOSITIVE GLASS DISTORTION^a

Angle Off Axis (deg)	Distortion (mm)
5	0.000
10	0.002
15	0.005
20	0.013
25	0.026
30	0.048
35	0.081
40	0.130
45	0.202

^aEmulsion surface up on 0.06-in. thick glass.

TABLE 2
VERTICAL ERRORS IN MODEL^a

Point	Comp. Vert. Error		Actual Average Reading (ft) ^d
	(mm) ^b	(ft) ^c	
A	-0.415	-0.82	-0.85
B	-0.320	-0.63	-0.75
C	-0.180	-0.36	-0.55
D	-0.015	-0.03	0.00
E	-0.465	-0.92	-0.90
F	-0.330	-0.65	-0.60
G	-0.145	-0.29	-0.10
H	-0.040	+0.08	+0.55
M	-0.360	-0.71	-0.70
N	-0.195	-0.38	-0.45
O	0	0.00	0.00
P	-0.170	-0.34	-
Q	-0.175	-0.35	-0.45
R	+0.015	+0.03	+0.05
S	+0.230	+0.45	+0.75
T	-0.485	-0.95	-0.95

^aCaused by 0.06-in. thick glass, emulsion surface up.

^bModel scale = 5 times scale of diapositive.

^cModel scale = 1 in. = 50 ft.

^dAt 1 in. = 50 ft in grid model as set up in Nistri-Photomapper.

parallax is cleared at point O, the datum plane for the measurements is established as passing through these four points, which are the corners of the neat model. Within this area the biggest spread in readings is 0.2 ft, which is actually 0.1 mm.

Although it may be physically possible to compensate for the large distortion values exhibited by 0.06-in. thick glass, the use of thicker plates with emulsion down is generally considered to produce noticeably improved results. The latter plates eliminate an optical step in the projection system as well as remove for all practical purposes the possibility of variations caused by sag, wedge, and glass quality.

Projection Lenses

Hypergon.—The conventional Kelsh plotter uses a Hypergon lens for projection. These lenses have a nominal focal length of 127 mm, or 5 in. When set with a principal distance of 6 in. in the projector, the optimum focal plane is formed at a projection distance of 30 in. The magnification factor of the lens in this specific situation is 5 diameters, and will vary directly with a change in projection distance. Thus, if the relief in a model, such as a high hill, causes a projection distance of 27 in., the resulting magnification will be 4.5 diameters.

There is considerable variation in Hypergon lenses with respect to focal length and distortion patterns. A variation in focal length does not alter the geometry of projection, therefore has no influence on model deformation. It is desirable, however, to incorporate matched lenses in a plotter, in order to assure satisfactory model definition. For instance, a variation of 0.05 in. in focal length produces a change of almost 2 in. in optimum projection distance. The important consideration is that each of the two lenses have essentially the same focal length.

The variation in distortion patterns of Hypergon lenses directly influences expected model deformation. It is customary to separate Hypergon lenses into two groups, relative to results in the model. The first group includes lenses which produce models with little or no measurable deformation, whereas the second group includes lenses which do produce models of measurable deformation. A calibration report similar to those associated with aerial cameras would be inconclusive because of the construction and assembly of the projector unit.

The distortion pattern of a Hypergon lens is typically negative; that is, light rays are distorted inward radially toward the principal point. As an example, Table 3 gives the values of a lens which would produce a model of measurable deformation.

TABLE 3

POSSIBLE HYPERGON DISTORTION

Angle Off Axis (deg)	Distortion (mm)
5	0.00
10	0.00
15	0.00
20	0.00
25	-0.01
30	-0.01
35	-0.01
40	-0.03
45	-0.03

A combination of two Hypergon lenses characterized by the distortion values given in Table 3 would produce a stereo-model as shown in Figure 2. The undulating configuration of this deformed surface generally rises above the ideal datum plane.

Not all Hypergon lenses produce a model surface as illustrated in Figure 2. As previously mentioned, it is impractic to attempt calibration of a Kelsh projector in the same manner that a camera is calibrated. The distortion of a Hypergon lens must therefore be determined by optical bench procedures for the unmounted lens. In the final analysis, the only important consideration is model deformation and not lens distortion: rather than rely solely on lens calibration, a more practical

solution is the test of over-all performance by the grid model method.

Omigon.—The Omigon lens is a wide-angle type of nominal 6-in. principal distance used in the Nistri-Photomapper. The entire projector cone is an integral unit, with the lens permanently mounted and the fiducial marks designed to establish the focal

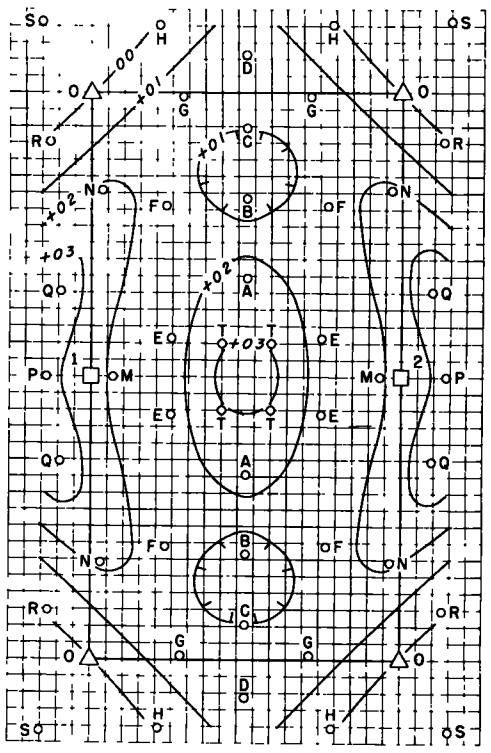
plane. The projector unit is virtually a camera, and therefore can be calibrated like a camera. Principal distance setting is accomplished at the focal plane by adjusting the micrometers at each fiducial mark.

The Omigon lens must be essentially distortion-free for use with nominally distortion-free photography, because there is no provision for any distortion compensation. The Division of Highways has one Nistri-Photomapper, and it is capable of producing an unusually flat model by the grid model test. In fact, this was the instrument used in the grid model test associated with Table 3. The report accompanying the instrument stated simply that the "cameras were practically distortion-free."

Figure 3 shows the distortion curve of a typical Omigon lens. This curve was included in a calibration report for an instrument belonging to a private concern. The curve data made it possible to compute the projected model, as shown in Figure 4. Grid model test data are not available for comparison with the computed data, but a close agreement could be expected. The computed model is very flat by comparative standards with other models, and also compares very favorably with grid model test results of the instrument belonging to the California Division of Highways.

Camera Lenses

The four camera lenses commonly used in this country for mapping photography are all of nominal 6-in. focal length, and are considered to be wide-angle lenses for use with the 9- by 9-in. format size. Each of the four designs have individual



Assumptions: Camera lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 2. Model deformation caused by possible Hypergon distortion.

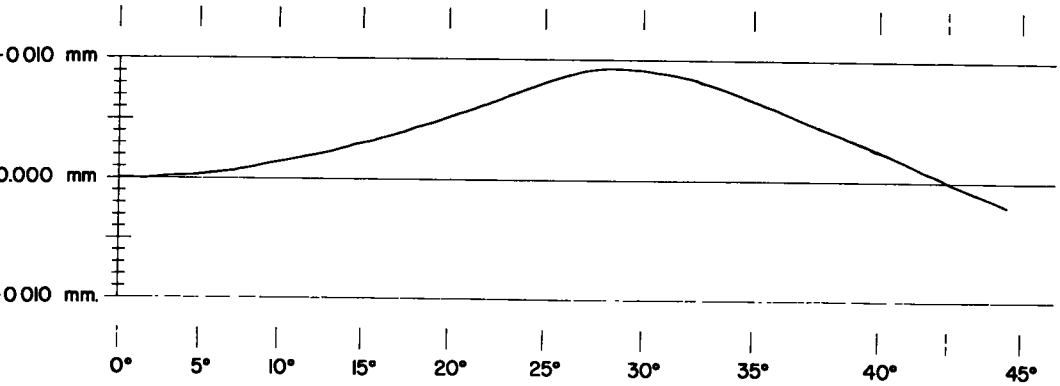
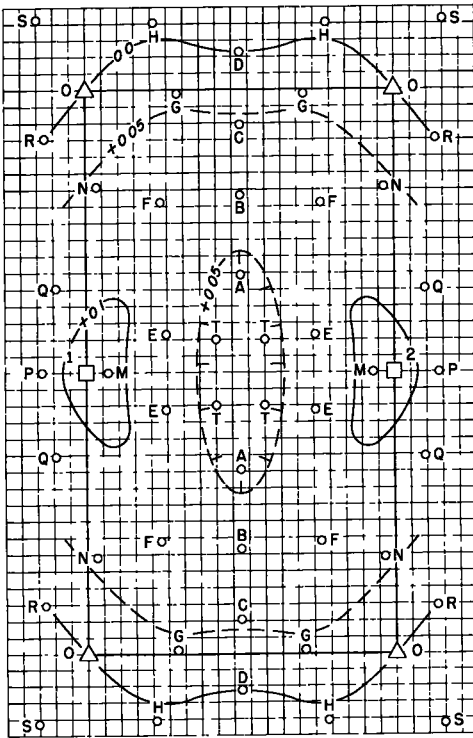


Figure 3. Omigon distortion curve.



Assumptions: Camera lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 4. Model deformation caused by typical Omigon lens.

similarity of the three curves makes it virtually impossible to arrive at an average curve for the purpose of distortion compensation. Compared with Metrogon distortion

characteristics with respect to radial distortion. The four designs are discussed separately.

Metrogon. —The Metrogon is probably the oldest design of mapping lens currently in use. The distortion characteristics follow a general curve as shown in Figure 5. Considering that this is a mapping lens the distortion pattern is quite extreme and must be compensated for in the restitution of a reliable stereomodel. Figure 6 shows the expected model configuration if Metrogon distortion is not compensated. Not all lenses will duplicate this nominal or average curve. As an actual curve departs from the average curve, model deformations will result from the lack of compensation of the residual distortion. Aspheric cams in the Kelsh Plotter for compensation of Metrogon distortion are usually ground for the average curve but may be ground specifically for an individual lens or a group of lenses if each lens in the group exhibits a similar distortion pattern.

Planigon. —The Planigon lens is a post-World War II design of American manufacture considered to be nominally distortion-free. The alternate name of Cartogon is used for the commercial version of the Planigon, and the two terms are used interchangeably.

Photogrammetrists have been aware for many years that distortion characteristics of Planigon lenses vary considerably (8). Examples of the variations are shown in Figure 7, where each of the curves was derived from calibration data for three different cameras. The dis-

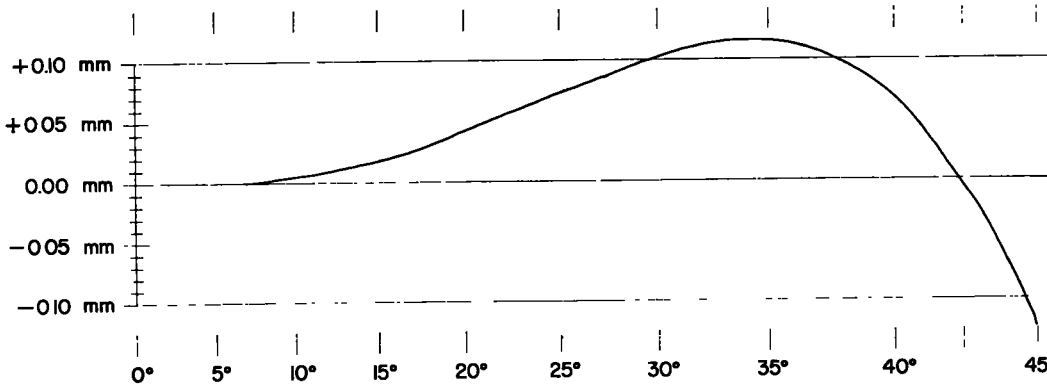


Figure 5. Nominal Metrogon distortion curve.

Planigon distortion does not cover a wide range and individual lenses may not even require compensation. The nominal Metrogon distortion curve shows a maximum value about 10 times greater than the maximum value of Planigon curve C.

Model surfaces resulting from curves A, B, and C are shown in Figures 8, 9, and 10, respectively.

Aviogon. — The Aviogon lens is an example of a nominally distortion-free design of Swiss manufacture, and is associated with the Wild RC5A and the Wild RC8 cameras.

The pattern of Aviogon distortion is particularly consistent, as indicated in Figure 11-a showing curves for three different lenses. Aviogon distortion characteristically produces "humped" models (Figs. 12, 13, 14) derived from curves D, E, and F in Figure 11-a. This characteristic "hump" suggests the possibility of compensation using an average Aviogon distortion curve.

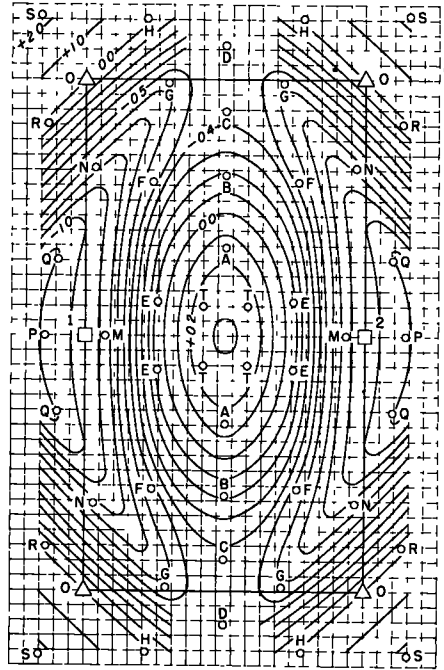
Pleogon. — The Pleogon lens is a German contribution to the list of nominally distortion-free lenses, and is associated with the Zeiss RMK 15/23 cameras.

Pleogon distortion is shown in Figure 15-a. It is noted that Pleogon distortion tends to be consistent, at least as indicated by curves G, H, J, and that the general shape of the curves is opposite the general shape of the Aviogon curves in Figure 11.

Pleogon models exhibit a slightly "dished" effect as illustrated in Figures 16, 17, and 18 which have been computed from the distortion values taken from curves G, H, J, respectively, of Figure 15-a.

Camera Calibrator Tests

The foregoing computational results pertaining to Metrogon, Planigon, Aviogon, and Pleogon lenses are based entirely on calibration data derived from



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 6. Model deformation caused by nominal Metrogon distortion.

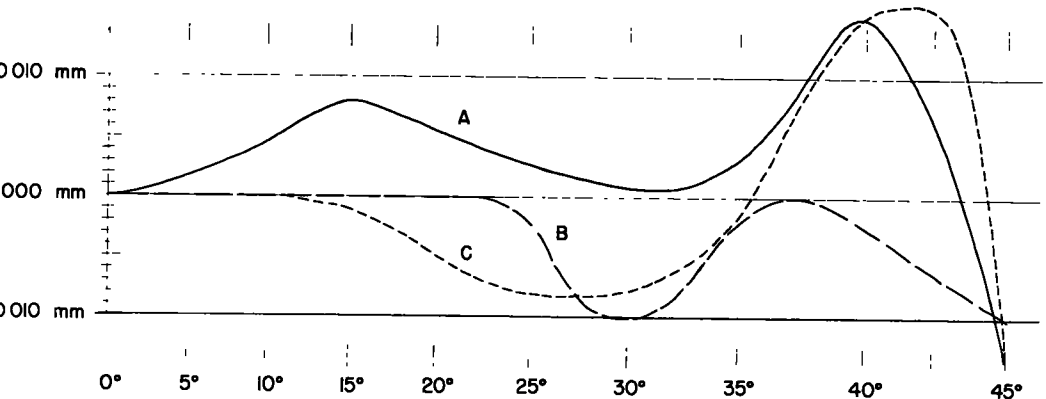
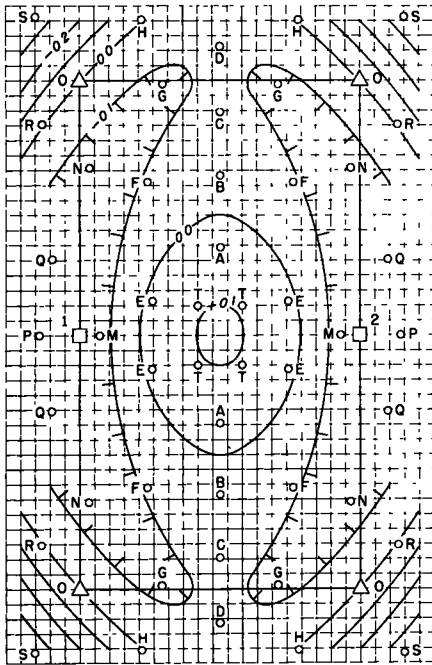


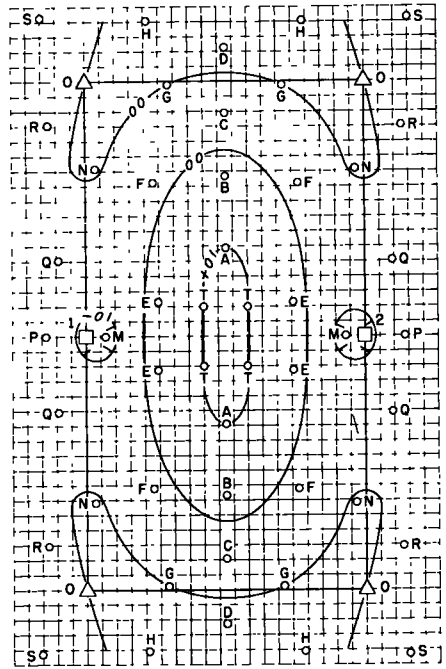
Figure 7. Variation in Planigon distortion.



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 8. Model deformation caused by Planigon curve A.



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 9. Model deformation caused by Planigon curve B.

camera reports. The distortion values tabulated in a camera report are merely averages of values taken at specified radial points along four radii within the format area, using the point of symmetry for center, as obtained from camera calibrator tests (14). The resultant distortion curve is presumably representative of the distortion curve along any radii. This is a basic assumption and produces symmetric deformations in the computed model. Thus, the foregoing figures depicting model deformations are somewhat idealized. Deviations of computed values from actual value at any point in the model are directly related to the magnitude of residual distortion resulting from curve averaging. For the purpose of computing aspheric surfaces as a means of distortion compensation, the average curve representing the lens must be the starting data, just as it is the starting data for computing model deformations.

A camera calibrator test procedure developed by the U. S. Geological Survey eliminates the derivation of distortion data by analysis of comparator measurements. In this calibrator the collimators are arranged so that nine of them will be combined to produce conjugate images for stereomodel testing. The arrangement of the nine discrete points in the model is shown in Figure 19. The real advantage of the USGS method is that the optical performance of a camera can be translated directly in terms of model deformation. Any camera can be rapidly tested for acceptance or rejection for use with a particular plotting instrument.

The values shown in Figure 19 apply to the same camera analyzed in Figure 14. Note the lack of symmetry in the deformation pattern in the stereo-performance test data, undoubtedly due to asymmetric distortion distribution of the camera lens. As

An average curve is used in mathematical analysis, computational results will not indicate asymmetric characteristics.

Combinations of Optical Components

The investigation of individual optical components can be extended to include combinations of components in the overall photogrammetric system. If the performance of the plotting instrument by the grid model method has been determined, and assuming other things being equal, vertical errors may be attributable entirely to the projection lenses. The compatibility of a particular camera with the plotting instrument can be ascertained by combining grid model results with computed results of the camera lens at each discrete point in the neat model area. The total errors represent the expected magnitudes of model deformation. Interpolated contours in any desired unit may be drawn in order to visualize the model surface.

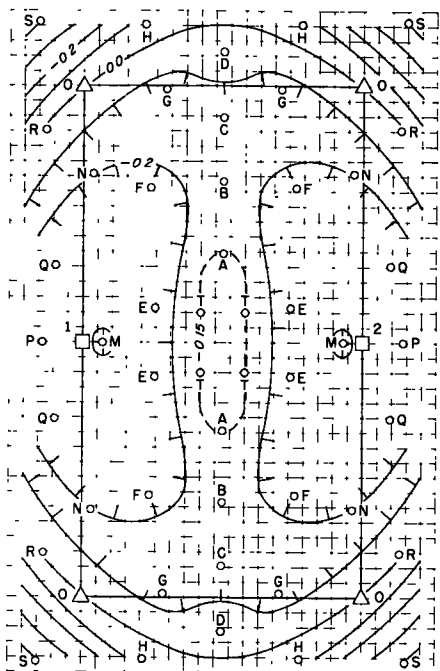
The discussion on diapositive glass provides an example. When the deformation (Fig. 1) caused by 0.06-in. thick glass, emulsion up, is combined with typical Aviogon deformation (Fig. 12) the total expected deformation is shown in Figure 20. Note that vertical errors are relatively equalized along the line between principal points, but the total deformation describes a dished model. In a similar manner, a combination of Hypergon (Fig. 2) with Aviogon (Fig. 14) will aggravate the hump in the model center, approximating +0.6 ft at model scale of 1 in. = 50 ft.

Analysis of Errors Under Operational Conditions

Photography obtained under operational conditions is not a particularly reliable medium for camera testing. The analysis of errors in routine mapping projects often turns out to be an illusive job of detective work with inconclusive results. The photogrammetrist seldom has time to make a systematic analysis, and even if he did, many blind alleys would be encountered. He may decide that if a test area were located on the ground, all the necessary data could be gathered with two overlapping exposures, a simple remedy for innumerable frustrations.

With this thought in mind, an area within a routine mapping project near Sacramento was premarked with a general distribution of lined points just prior to the taking of the photography. The resulting model area showing the point distribution is illustrated in Figure 21. The premarked points shown as crosses had been established by the mapping contractor in conformance with the contract specifications. The premarked points shown as dots had been established by Division of Highways personnel, with the elevation of each point determined by spirit levels.

The four corner points, A, B, C, and D, were used to level the model, and the elevations of all other points read accordingly. The model was actually set in four different instruments, with each instrument operated by a different individual. The resulting errors were averaged and compared with the known field elevations. These



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 10. Model deformation caused by Planigon curve C.

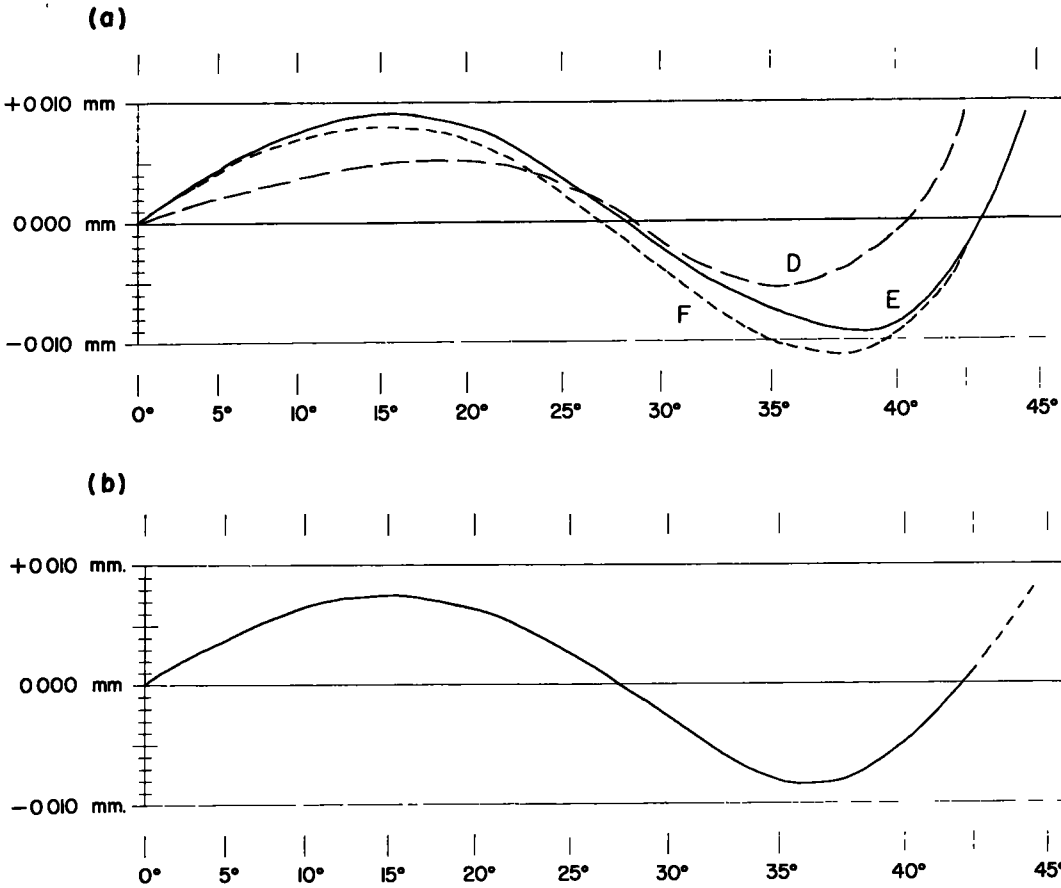
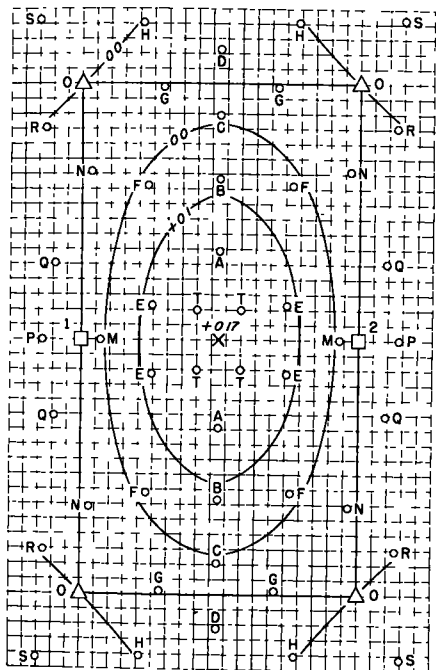


Figure 11. (a) Variation in Aviogon distortion. (b) Average of Aviogon curves D, E, F

average errors are noted along side the individual points in Figure 21.

The camera used in this test was a Wild RC8 with an Aviogon lens, in fact the same camera analyzed in Figure 14 and in Figure 20. The observed errors in the operational test model do not duplicate either of the other two test results point for point, but do show the trend of deformation and the asymmetric distribution. Some unexplainable variations exist in the operational model. For instance, a test point happened to fall adjacent to the lower left-hand corner point D, but an error of 0.2 ft was observed in the photogrammetric elevation. Other examples of this anomaly are evident. Photogrammetric elevations of premarked points are frequently difficult to determine, probably due to variations in image quality. Among the possible reasons for variation are: premarked images tend to halate; the premarking may be on sloping ground; surrounding ground cover may obscure part of the premarking.

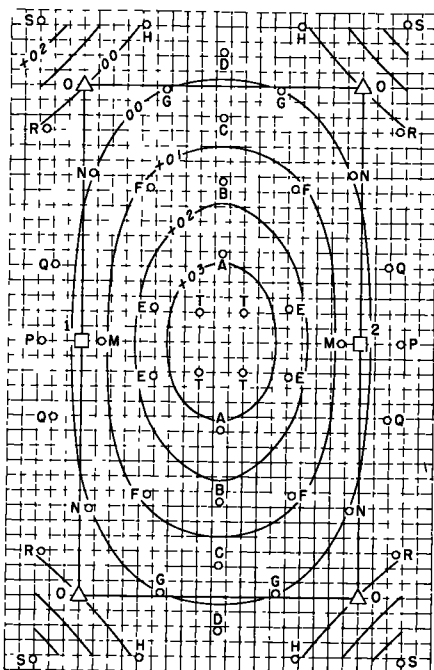
Observed errors under operational conditions may not agree with computed errors based on distortion data for reasons other than image quality. It was previously pointed out that certain assumptions were made relative to the geometry of the overlapping photographs, as follows: the base-height ratio and width-height ratio was assigned to determine the size of the neat model; both photographs comprising the stereo pair had identical scale values; both photographs were tilt-free. It is obvious that these specifications cannot be applied to operational photography, and it follows, therefore, that the geometry of actual exposure probably will differ from the assumed geometry used in mathematical analysis. Because varying geometric conditions are bound to occur,



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 12. Model deformation caused by Avigon curve D.



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 13. Model deformation caused by Avigon curve E.

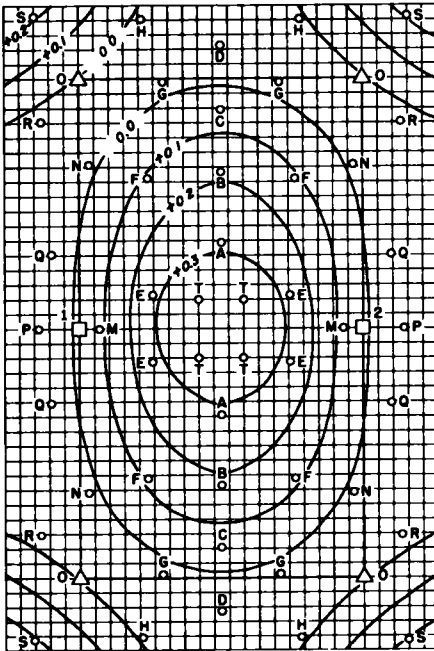
Comparison of observed results with computed results will only verify the trend of model deformation. One cannot hope to definitely repeat point for point the identical errors.

The operational test introduces other variables not related to the distortion characteristics of the various lens components. It must be remembered that each variable contributed to some extent, however small, to model deformation. These considerations are beyond the scope of this paper, but are mentioned here as a reminder that operational testing will not always produce the concrete evidence the photogrammetrist desires.

DISTORTION COMPENSATION

Standards for Compensation

The standards for compensation of distortion in the photogrammetric system depend to a large extent on photogrammetric measurement requirements. The tolerances for phototriangulation throughout many models demand a different standard than for compilation of individual models. Tolerances for compilation of highway design maps demand a different standard than compilation of military maps. Standards for compensation are not well defined and are usually resolved according to the best that can be done, because it is virtually impossible to compensate all the distortion in the system. The "best that can be done" is dependent not only on the ability of the optical designer and the machinist, but also on the type of plotting instrument and the ability of the individual operating it.



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 14. Model deformation caused by Aviogon curve F.

hibit some distortion, a decision must be made as to whether or not the camera is compatible with the plotting instrument. For this purpose the computed model will quickly reveal the expected vertical errors. In the foregoing section on camera lenses a total of nine Planigon, Aviogon, and Pleogon lenses were analyzed. The greatest deformations are shown in Figures 10, 13, and 14, amounting to 0.3 ft at 1 in. = 50 ft, or $\frac{H}{5,000}$. Assuming that an operator can level on the control within 0.1 ft, residual model errors attributable to camera lens distortion will occur, but will not be detected unless control is arranged to locate the maximum range of errors. The Nistri-Photomapper has no provision for compensation and, therefore, the residual errors represent "the best that can be done." (The Nistri-Photomapper is also available with projectors designed to accommodate nominal Metrogon photography.) At this time it is not definitely known whether or not these residuals can be compensated by the customary procedures associated with Kelsh plotters. This is discussed later in this paper.

Compensation Methods

There are, in general, three possible ways to compensate distortion: (a) compensation in diapositive printing by optical means, (b) compensation in projection by optical means, and (c) compensation in projection by mechanical means.

The first method would be applicable if diapositives were made with a 1 to 1 ratio printer. It would be possible to locate an aspheric glass corrector plate in the optical path within the printer, designed to introduce distortion values equal to the camera

A logical starting point is the plotting instrument, specifically the direct projection 5X Kelsh-type plotter. As indicated in Table 2 by the close correlation between computed values and values determined with a Nistri-Photomapper, the direct projection instrument is capable of very satisfactory results. Within the neat model area the largest discrepancy is 0.2 ft at 1 in. = 50 ft, or 0.1 mm, which when compared to the optimum projection distance of 760 mm represents an accuracy of $\frac{H}{7,600}$ in

terms of flying height. Grid model tests with this instrument using 0.250-in. thick diapositives, emulsion-surface down, indicate that the neat model area is flat within 0.05 mm, or $\frac{H}{15,000}$. Readings smaller than this cannot be made with any degree of certainty even by the sharpest sighted operators.

The value of 0.05 mm, defining the plane of grid model flatness, has been used by the California Division of Highways as a specification for a Kelsh plotter ordered from the Kelsh Instrument Company. The delivered instrument was carefully tested by two operators, with the joint conclusion that the projected model did not exceed the specified tolerance. This value is suggested as a standard for 5X projection instruments.

The second consideration is the aerial camera. Because all camera lenses ex-

lens but opposite in direction. The resulting diapositive would be free of distortion. This method is incorporated in the reduction printer for making of ER-55 diapositives from Metrogon photography. Ratio printers for contact size diapositives are not commercially available.

The second method suggests the use of aspheric glass corrector plates located in the projectors, similar to the arrangement in the Wild Autographs. As cost of the corrector plates undoubtedly would be out of proportion to the initial cost of the projection plotter, this does not appear to be a practical solution. It is also possible to design the projector lens to compensate for the camera lens distortion, in the manner associated with the Nistri-Photomapper designed to accommodate nominal Metrogon distortion. Cost of the projector units is approximately the same as for aspheric glass plates, which is about \$1,000 for one pair.

The third method is employed in the Kelsh plotter. It offers a practical, low-cost solution, and is described in the following section.

Theory of Distortion Compensation by Aspheric Cams

Figure 22 represents the geometry of distortion compensation for 5X Kelsh-type plotters. The ray 1, which emanates from some object *A* on the ground, is directed toward the perspective center of the camera lens and incident to it at the angle α measured from the axis of the lens system. In the ideal lens, the refracted ray 2 would emerge likewise at the angle α , recording the image in the undistorted position *a'*. However, in the event of distortion the refracted ray 3 emerges at the slightly different

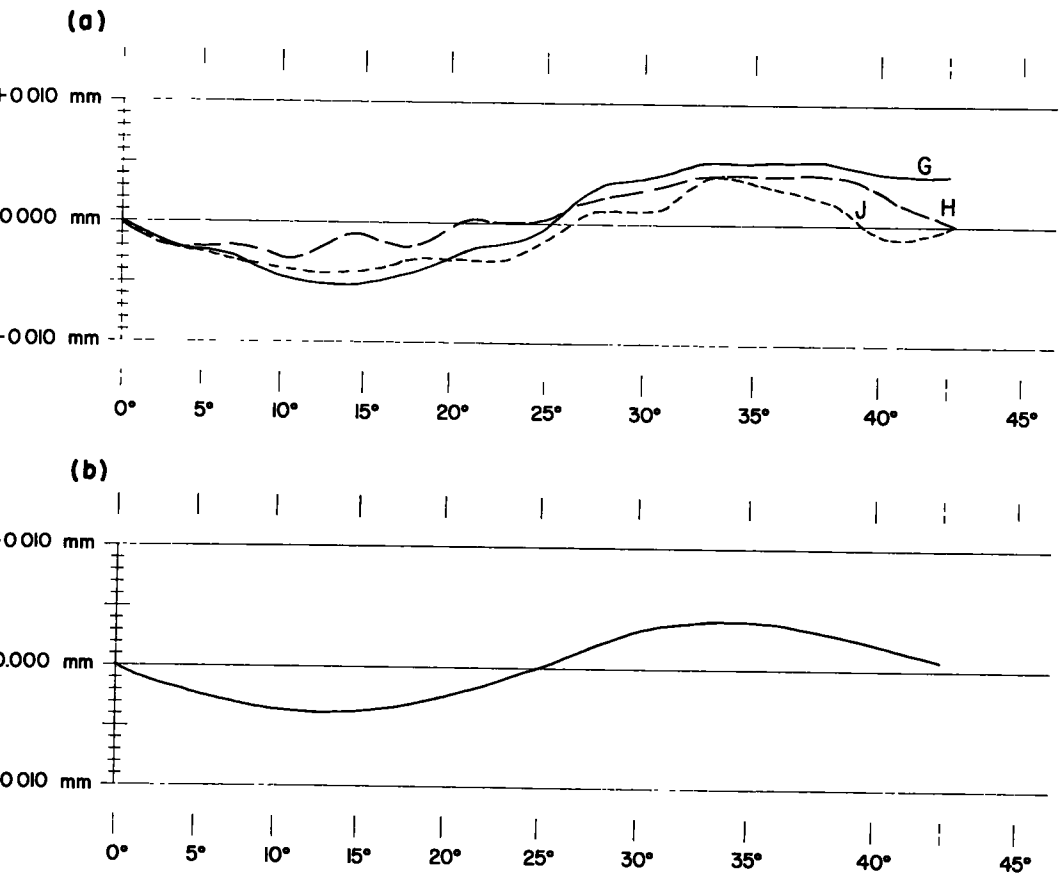


Figure 15. (a) Variation in Pleogon distortion. (b) Average of Pleogon curves G, H, J.

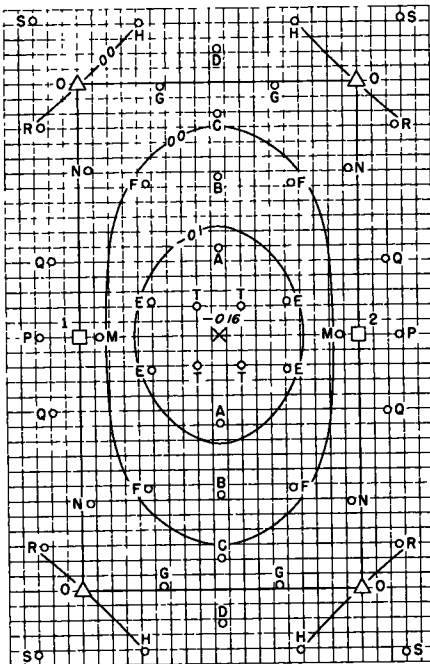
angle β , recording the image in the distorted position \underline{a} . The difference between \underline{a} and \underline{a} is a measure of radial distortion D on the negative. The distorted point \underline{a} will be recorded in turn on the diapositive and ultimately projected through the projection lens. Assuming the projection lens is distortion free, point \underline{a} will emerge along ray 4, coming to focus at $\underline{A'}$. Because the lens has a magnification factor of 5 diameters, point $\underline{A'}$ will be displaced $5D$ from the correct position \underline{A} . The next step would be to attempt to compensate for the distortion by changing the principal distance of the projector; that is, by moving the lens vertically the amount $D (\cot \alpha)$. Point \underline{a} would be projected along ray 5 parallel with the original ray 1, coming to focus at $\underline{A''}$. In this position it is displaced from position \underline{A} by the amount D . In order to make point \underline{a} focus at position \underline{A} , the principal distance is varied by $5/6 D (\cot \theta)$. This can be verified by inspection of the geometry illustrated in Figure 22.

Because the magnitude of distortion in modern camera lenses is very small, the difference between the angles α and θ is also very small. Sufficient accuracy will be attained by substituting α for θ , modifying the expression for change in principal distance, accordingly:

$$\Delta P.D. = 5/6 D (\cot \alpha)$$

This equation serves as a basis for computing the aspheric surface of the distortion correction cam, which actuates the mechanical linkage imparting vertical movement $\Delta P.D.$ to the projection lens.

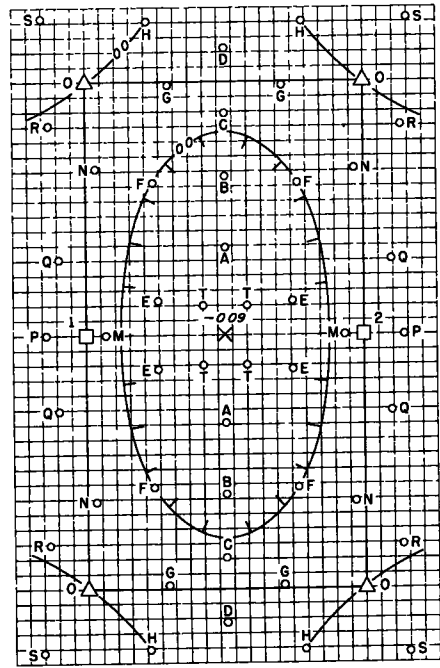
Figure 23 is a diagram of the lens assembly showing the linkage between the cam



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 16. Model deformation caused by Pleogon curve G.

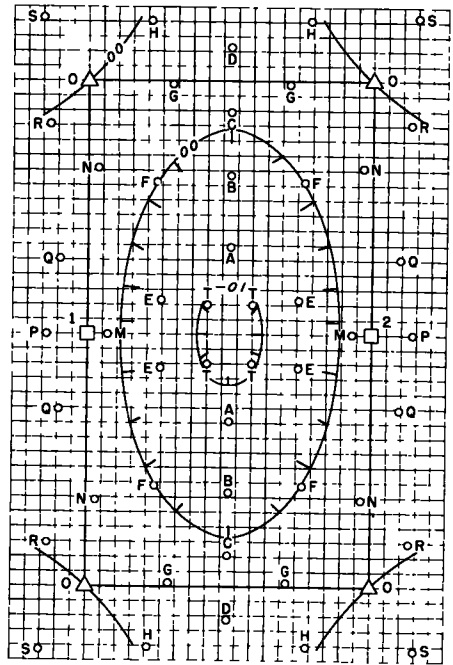


Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 17. Model deformation caused by Pleogon curve H.

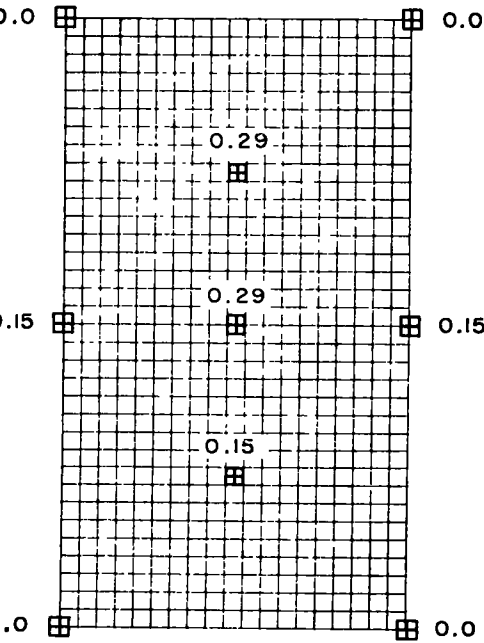
and the lens barrel. The design of the plotter permits the orientation of the cam stem to be made parallel with any ray projected from the perspective center of the lens. For example, if the projected ray defines an angle of 22 deg, the cam stem also defines an angle of 22 deg. In this way the cam is rotated in the bracket so that the cam follower moves up or down as it rides the aspheric surface. The force applied to the lever at the cam follower transmits a reaction to the pin fixed to the lens barrel. In most commercial models of the Kelsh plotter the lever ratio is 3.5:1 (Fig. 23), thereby making the vertical motion of the cam follower 3.5 times greater than the vertical motion of the lens barrel. The lens barrel is actually encased in a sleeve rigidly fixed to the bracket. The spring applies a constant downward force to the pin in the lens barrel, which in turn is applied to the lever to assure positive contact between the cam follower and the cam surface.



Assumptions: Projection lens distortion-free
Diapositives emulsion down

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 18. Model deformation caused by Pleogon curve J.



Approximate locations of test points in relation to 25 mm grid. Readings in ft at model scale of 1 in. = 50 ft, converted from mm readings by ER-55

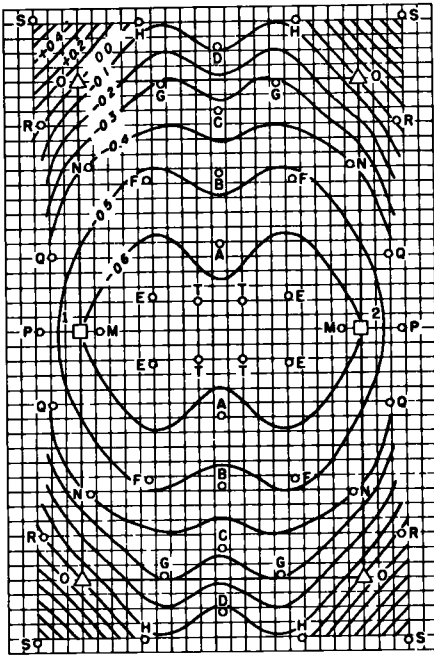
Figure 19. USGS stereo-performance test model.

The surface of the cam is expressed in terms of the vertical movement of the cam follower:

$$\text{Cam follower drop} = (3.5) \frac{5}{6} D(\cot \alpha)$$

This is simply 3.5 times greater than $\Delta P.D.$ It also represents a variation in length of selected radii to describe an aspheric surface of revolution (Fig. 24).

The value of the distortion D in the formula for "cam follower drop" is the algebraic sum of all the known distortions in the system. As an example, assume photography taken with a Metrogon lens, diapositives printed emulsion up on 0.06-in. thick glass, and projected with a Hypergon lens. A ray passing in turn through each optical component receives some degree of deviation from its original path. The distortion introduced by each component will contribute to model deformation. Rather than treat each independently,



Assumptions: Projection lens distortion-free
Diapositives emulsion up

Instrument: 5X projection plotter
Model Scale: 1 in. = 50 ft
Contour Interval: 0.1 ft
B/H = 0.62 W/H = 1.12
25 mm grid at model scale

Figure 20. Model deformation, Aviogon curve F plus 0.06-in. thick glass.

(760 mm) from the lens, a departure from this plane will introduce a vertical error in the model. This error may be evaluated, for any point, from the approximate equation:

$$\Delta Z = \frac{(Z) (\Delta P. D.)}{\text{Optimum Projection Distance}}$$

in which ΔZ is the vertical error, and Z is the departure from the optimum plane of focus. It is possible, therefore, to predict a systematic error under certain circumstances. As an illustration, suppose flat valley land was photographed with a Metrogon lens, and that the flying height was too great, causing the projection distance to be about 36 in. The optimum projection plane will be 6 in. above the terrain in the model, or 152 mm. From Table 4, the maximum $\Delta P. D.$ is 0.222 mm at 35 deg. The

$$\Delta Z = \frac{152 (0.222)}{760} = +0.044 \text{ mm.}$$

At the model scale of 1 in. = 50 ft, this error amounts to +0.09 ft. In ordinary situations this can be safely ignored, but it is present and could conceivably be added to other deformations existing at that point in the model. With the diapositives printed emulsion down, maximum $\Delta P. D.$ would be 0.126 mm, ultimately making ΔZ equal to 0.05 ft at 1 in. = 50 ft for the same point.

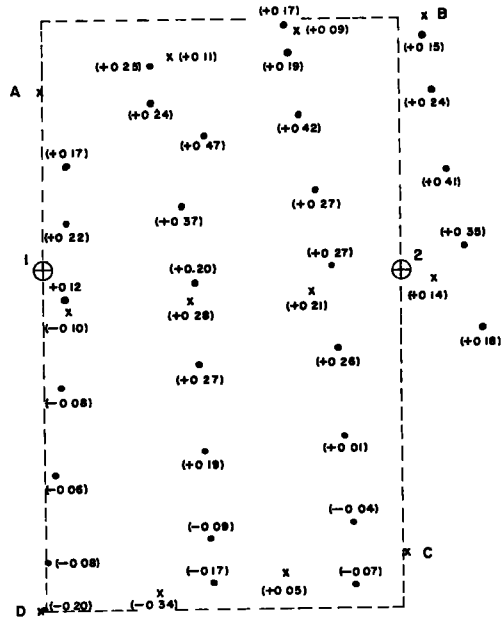


Figure 21. Stereo-performance test results under operational conditions (readings in ft at model scale of 1 in. = 50 ft).

their total distortion is used in cam computations. A typical computation is given in Table 4 showing how the data is handled.

Relief in a model also has an effect on cam performance (9). Because the cam is designed and computed to compensate at the optimum plane of focus, or 30 in.

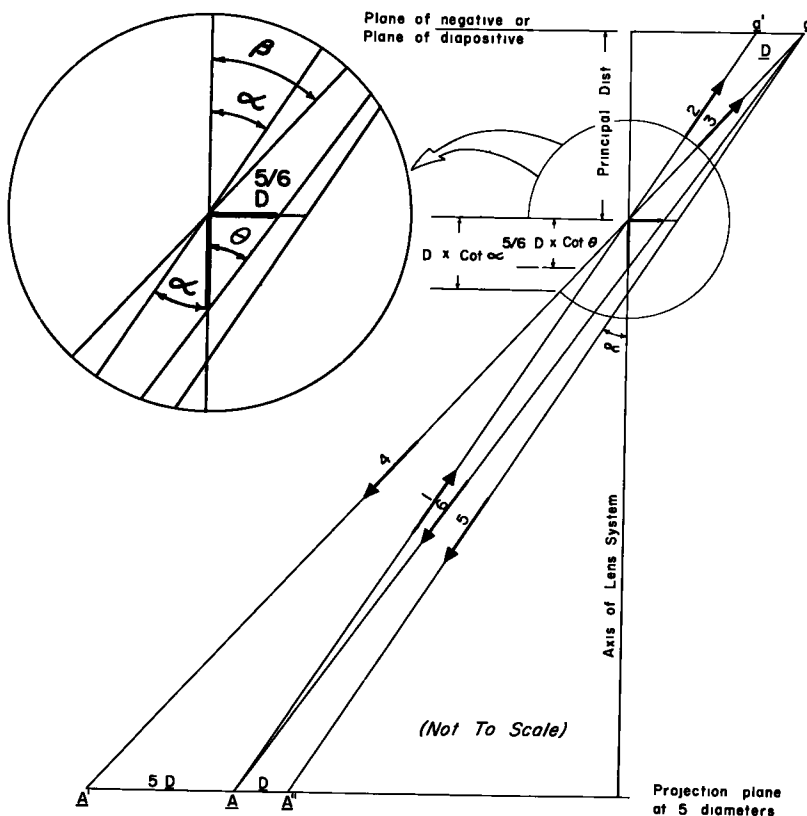


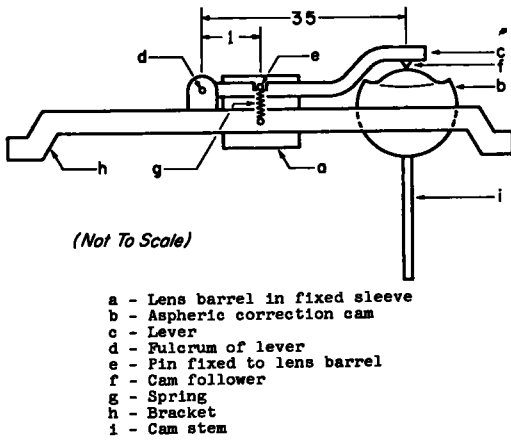
Figure 22. Geometry of distortion compensation, 5X Kelsh Plotter.

Application of Cams

Kelsh plotters come equipped with cams for compensation of Metrogon distortion. Without the cams Metrogon photography would not be usable in the Kelsh, just as it is not usable in the distortion-free Nistri-Photomapper. A cam surface may be ground to compensate distortion of a particular Metrogon, or it may be ground to compensate a group of Metrogons exhibiting distortion patterns within a specified tolerance of an average curve. As demonstrated by the sample computation (Table 4), known distortion other than Metrogon may be compensated by the cam.

The application of cams to Planigon, Aviogon, or Pleogon photography is not standard practice. A provision is made in the recent models of the Kelsh plotter to disengage the cam follower whenever low distortion photography is to be used, with the presumption that the residual distortions will not adversely affect the model datum. For most mapping requirements this procedure is justified and the resulting accuracies will be well within usual specifications. The new demand for photogrammetric data as a basis for deriving final pay quantities for earthwork makes it necessary to inquire into the possibility of using cams to correct the existing distortions in the nominally distortion-free lenses.

Aviogon lenses exhibit fairly uniform and consistent distortion patterns causing measurable deformations in Kelsh models. A cam computation correcting for the average Aviogon distortion (Fig. 11-b) is given in Table 5. The average curve is selected in this example with the idea that any of the three cameras may be used to



(Not To Scale)

- a - Lens barrel in fixed sleeve
- b - Aspheric correction cam
- c - Lever
- d - Fulcrum of lever
- e - Pin fixed to lens barrel
- f - Cam follower
- g - Spring
- h - Bracket
- i - Cam stem

Figure 23. Diagram of lens assembly.

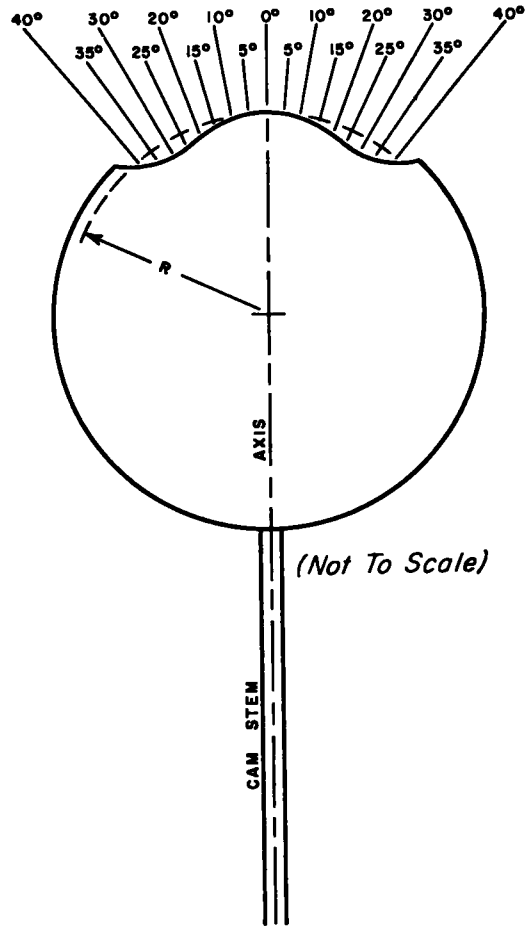
photograph a mapping project. Departure of an actual curve from the 3-curve average will leave residual distortions. Table 6 gives the computed vertical errors, due to residual distortion, for each of the three Aviogon cameras. In no instance does any value exceed 0.1 ft, which demonstrates that from a purely academic standpoint cam compensation based on the average curve should be adequate. The column listing cam follower drops (Table 5) reveals that the cam surface requires a very high degree of machining quality.

Pleogon distortion curves follow a fairly uniform and consistent pattern (Fig. 15-a). The average curve (Fig. 15-b) provides the necessary data for cam computations. At this point it must be decided whether or not compensation is actually justified because of the small model errors caused by the Pleogon lens. The average curve indicates that Pleogon distortion may be expected to be less than 0.004 mm. Models from photography with distortion values not exceeding this should be flat within 0.05 mm, or 0.1 ft at 1 in. = 50 ft. It does not seem feasible to attempt compensation of errors which cannot be definitely read in the Kelsh model.

Planigon distortion does not follow a uniform and consistent pattern (Fig. 7) and may or may not require compensation. With the cameras currently available for furnishing mapping photography, it may be difficult to group them according to similarity of distortion curves. If distortion compensation is desired for any particular camera, a separate set of cams would have to be made specifically for it.

Limitations of Cams

Photogrammetrists are not in agreement concerning the virtues of cams as a means of distortion compensation. In recent years there has been a definite trend towards their elimination from Kelsh plotters, especially since low distortion photography has been readily available. It would appear, that the suggestion of returning to cams at this late date is an anachronism.



(Not To Scale)

Figure 24. Vertical section of aspheric cam (cam follower drop is the difference along selected radii between dashed surface (radius R) and aspheric surface).

TABLE 4
SAMPLE CAM COMPUTATIONS

Angle off axis, α (deg)	5	10	15	20	25	30	35	40	45
Metrogon ^a	0.001	0.003	0.018	0.042	0.071	0.103	0.116	0.073	-0.116
Hypergon ^a	0.000	0.000	0.000	0.000	-0.010	-0.010	-0.010	-0.030	-0.030
Total lens	0.001	0.003	0.018	0.042	0.061	0.093	0.106	0.043	-0.146
0.06-in. glass	0.004	0.008	0.013	0.020	0.028	0.048	0.061	0.130	0.202
Distortion ^a	0.005	0.012	0.031	0.062	0.089	0.141	0.187	0.173	0.056
Cot α	11.43	5.67	3.73	2.75	2.15	1.73	1.43	1.19	1.00
Total distortion D	0.005	0.012	0.031	0.062	0.089	0.141	0.187	0.173	0.056
Dx cot α	0.057	0.068	0.116	0.171	0.191	0.244	0.267	0.206	0.056
Lens drop 5/6 D x cot α									
mm	0.047	0.057	0.097	0.142	0.159	0.203	0.222	0.172	0.047
in.	0.0019	0.0022	0.0038	0.0056	0.0063	0.0080	0.0088	0.0068	0.0018
Cam follower drop 5/6									
Dx cot α									
mm	0.166	0.198	0.338	0.499	0.557	0.712	0.779	0.601	0.0163
in.	0.0065	0.0078	0.0133	0.0196	0.0219	0.0280	0.0307	0.0237	0.0064

^aAll distortion values in millimeters.

TABLE 5
CAM COMPUTATIONS^a

Angle Off Axis, α (deg)	Cot α	Average Distortion D (mm)	DxCot α	Lens Drop 5/6DxCot α		Cam Follower (3.5/6DxCot α	
				(mm)	(in.)	(mm)	(in.)
5	11.43	0.0035	0.040	0.033	0.0013	0.117	0.0046
10	5.67	0.006	0.034	0.028	0.0011	0.099	0.0039
15	3.73	0.007	0.028	0.022	0.0009	0.076	0.0030
20	2.75	0.008	0.017	0.044	0.0006	0.050	0.0020
25	2.15	0.025	0.005	0.004	0.0002	0.015	0.0006
30	1.73	-0.003	-0.005	-0.004	-0.0002	-0.015	-0.0006
35	1.43	-0.008	-0.011	-0.009	-0.0004	-0.032	-0.0013
40	1.19	-0.005	-0.006	-0.005	-0.0002	-0.017	-0.0007
45	1.00	-	-	-	-	-	-

^aFor average Avigon distortion curve, Figure 11-b.

The vertical motion of the lens (lens drop) in Table 5 covers a very small range of travel from -0.0004 in. to +0.0013 in., a total distance of 0.0017 in. Within this distance the lens barrel must travel freely without binding and without lateral play. Any resistance to free travel will tend to cause stress in the mechanical linkage (Fig. 23), resulting in wear. It is very easy to overstress the bearings between the lens barrel and the sleeve, ultimately making grooves in the lens barrel. This could cause the lens to "hang-up," or at least "chatter" in the sleeve. The adjustment of the bearings at the fulcrum of the lever is also critical.

The machining tolerance for grinding the surface of the cam is particularly demanding. The maximum difference in cam radii, according to cam follower drops, is from +0.0046 in. to -0.0013 in., or a total of 0.0059 in. This difference can be increased by using a lever ratio greater than 3.5:1. A lever ratio of 4:1 will increase the difference to 0.0068 in. Adoption of a new lever ratio would require modification of the lens assembly to accommodate the new relative positions of the fulcrum, lens, and cam.

There are undoubtedly other reasons which tend to offset the effectiveness of cams, but the main point is that mechanical compensation is not always reliable enough to be depended on. The Kelsh operator must be constantly alert to the possibility of malfunction of the mechanism, and he should also be aware that projector calibration is directly affected by the adjustment of the entire lens assembly.

CONCLUSIONS

From this study some conclusions can be drawn and some opinions offered. First of all, one should realize that the question of lens distortion is not by any means the "weakest link" in the photogrammetric system. The principal concern is that it does contribute to systematic vertical errors which may or may not be significant depending

TABLE 6
RESIDUAL MODEL ERRORS^a

Point	Aviogon D		Aviogon E		Aviogon F	
	(mm)	(ft) at 1 in. = 50 ft	(mm)	(ft) at 1 in. = 50 ft	(mm)	(ft) at 1 in. = 50 ft
A	-0.032	-0.06	0.036	0.07	0.033	0.06
B	0.002	0.00	0.027	0.05	0.014	0.03
C	-0.008	-0.02	0.012	0.02	0.010	0.03
D	0.000	0.00	0.015	0.03	0.005	0.01
E	-0.020	-0.04	0.039	0.08	0.027	0.05
F	-0.025	-0.05	0.024	0.05	0.022	0.04
G	-0.008	-0.02	0.021	0.04	0.011	0.02
H	0.002	0.00	0.001	0.00	-0.002	0.00
M	-0.015	-0.03	0.024	0.05	0.022	0.04
N	-0.010	-0.02	0.028	0.06	0.016	0.03
O	0.000	0.00	0.000	0.00	0.000	0.00
P	0.000	0.00	0.031	0.06	0.013	0.03
Q	0.000	0.00	0.026	0.05	0.013	0.03
R	0.000	0.00	-0.002	0.00	0.000	0.00
S	-	-	-	-	-	-
T	-0.045	-0.09	0.042	0.08	0.037	0.07

^aAfter compensation of average distortion of Aviogon D, E, F.

on the purpose of the measurements. The magnitude of errors under consideration would not be important in a reconnaissance map or in a general purpose map of any type. These errors may possibly influence earthwork quantity calculations, especially in flat terrain, and for this reason they are significant in large-scale highway design maps.

At this point the photogrammetrist is at a crossroads. If he is working under the customary specification for vertical accuracy; namely, that 90 percent of contour elevations shall be correct within one-half contour interval, he is relatively certain that by following established procedures the maps will meet the specifications. On the other hand, if he is working under specifications also requiring that 90 percent of all spot elevations be within one-quarter contour interval, and that the mean error shall not exceed a certain value, he is not at all certain that the routine established procedures will produce the additional accuracy requirements. He is forced to re-evaluate all the procedures step by step, and to determine the possible effect that variations on procedure will have on map accuracy, substantiated by a program of accuracy testing.

The following steps are recommended as a starting point in an over-all inspection program:

1. Carefully check calibration of the plotting equipment in order to account for and eliminate mechanical sources of error.
2. Arrange for the printing of diapositives on glass which is at least 0.130 in. thick with the emulsion surface down.
3. With the use of precise grids as diapositives, analyze the projected grid model for vertical errors, with cam action disengaged. A suggested standard is 0.05-mm maximum error. If errors larger than this are observed, the cause may be found in the projection lenses.
4. If cams are used, the grid model should also be analyzed for the effectiveness of cam compensation with cam action engaged.

The foregoing steps provide an adequate check on the geometry of projection and in no way involve other sources of error which occur prior to projection. These steps are entirely within the control of the photogrammetrist.

The calibration data furnished with the various camera lenses provide information relative to operational planning procedures. Distortion values for Aviogon and Pleogon lenses are seldom given beyond 140 mm radially from the indicated principal point, which describes a cone of coverage of about 85 deg at the perspective center of the lens. The model deformation diagrams for the various lenses show that beyond the assigned limits of the neat model the variations in model datum tend to change rather suddenly, especially on the extreme edges away from the flight line. The reading of grid models also indicates that plotting instruments tend to produce unreliable elevations at the extreme edges of projection. These facts confirm that the compilation unit should be the neat model area. For a 60 percent overlap the dimensions of this area on the photograph are 3.6 by 7 in. This area presumes a cone of coverage of 60 deg, or 40 deg off axis, which allows a margin of safety because overlaps are bound to vary, thereby affecting the dimensions of the neat model. If the 7-in. dimension is held constant, then the cone of acceptable coverage must be allowed to vary with change in overlap. For instance, with an overlap of approximately 54 percent the dimensions of the neat model would be about 4.1 by 7 in., and would be equivalent to a cone of 85 deg, or 42.5 deg off axis corresponding to the calibration limits of the camera lens. Inasmuch as highway design mapping ordinarily covers a strip of terrain of uniform width, it is reasonable to limit this width to a fixed dimension on the photograph. This will insure that the optical limitations of the photogrammetric system are not exceeded.

The model deformation diagrams also serve as a guide to the planning of vertical control. The datum of the models illustrating Planigon, Aviogon, and Pleogon deformation remain fairly constant within the neat model in the immediate vicinity of the corners, with the greatest deviation tending to occur at the model center. The characteristics of model datum lead to the conclusion that:

1. Vertical control should be planned so that at least one point is located in the approximate center of the neat model and the wing point control located within the bounds of the neat model, one near each of the four corners.
2. Vertical control should not be planned beyond the limits of the neat model, especially on the extreme edges away from the flight line. It is a safe rule to restrict the outer limits of the control so that no point is nearer than 1 in. from the edge of the photograph.

The planning and selection of vertical control points as a function of operational planning within the limitations of the photogrammetric system are very important factors influencing map accuracies. The effectiveness of cams designed to compensate for distortion values needs to be investigated, because it is not a foregone conclusion that cams will be successful in eliminating systematic errors. Furnishing the instrument operator with a well-planned project, and with definite control data, is the positive approach to the solution of the map accuracy problem, for the success of the mapping is ultimately a product of his training and ability.

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Use of Aerial Surveys and Topographic Maps For Projecting Highway Alignment

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By introducing modern methods of highway projection and the efficient use of topographic maps developed from photogrammetry, the location engineer can establish alignment and grades which complement the topography and eliminate nearly all of the undesirable aspects of the older accepted methods of ground location.

In order to project accurate highway alignment, a coordinate grid system must be established over the area through which the location is to pass. The final results of any line projected through this area will be no more accurate than the original control used to establish the grid system.

The present paper is based on the assumption that the topographic map through which the highway alignment is to be projected is accurate both horizontally and vertically, that pre-established horizontal and vertical control points are in usable positions, and planimetric features are well defined.

Simple and accurate methods of calculating curve positions by coordinates, simple procedures for projecting centerline where grade or land use problems exist, the effective use of the spline, simplified use of spiral transitions and a simplified, accurately accurate but approximate, method of introducing transitions between compound curves are discussed.

Through intelligent use of the coordinate method of centerline projection, employing an accurate topographic manuscript, compiled by photogrammetric methods, better highway locations can be achieved along with a substantial savings in man-hours.

ONE of the greatest factors contributing to the building of America was the original railroad and highway location surveys through the wilderness and mountainous areas of the western United States. This stupendous achievement was performed under conditions that would now discourage the most energetic of modern-day engineers. During a few short years, the work was extended over half a continent, uncharted except for sketches drawn by optimistic (and oftentime imaginative) explorers, mountain men, trappers, scouts, and superstitious Indians.

Thousands of miles of railroads and primitive highways were located by sheer determination, either on foot or on horseback, exploring the box canyons, river drainages, and game trails. From some high knoll or other vantage point, the pioneer locator would determine a favorable position for his course to the next plateau and/or vantage point. These positions became the point of intersection between two straight lines extending in the general direction of the setting sun or a distant peak. With simple

instruments he would scribe a curve of some known radius to ease the abrupt change in direction as he wandered through the natural valleys and mountain passes. Civilization that followed with wagons and wood-burning locomotives became impatient to reach eternity and began to develop high-speed vehicles for transportation. The old roadways which had been developed through the sweat of the pioneer locator became obsolete. Enterprising emigrants began to construct heavy earth-moving equipment to level the hills and fill the ravines to accommodate their desire for high speeds and heavy loads. Taking advantage of this new earth-moving equipment, the locator lengthened the radii of his curves to give a smoother transition between his tangent sections, then relaxed, well satisfied with the results of his accomplishments. Today, as this modern earth-moving equipment is biting deeper into the hills and filling higher the valleys, some of the modern location engineers are still relaxed, standing on top of the knoll, admiring their newly established point of intersection between two tangents. Generally, because this point is on the crest of a hill, they will return to the design office and establish a grade intersection at the summit, usually from a standpoint of economics or habit. The present day driver, traveling the highways of the west, can most assuredly expect, as he reaches the crest of nearly every hill, that the road will change in direction both horizontally and vertically. Those with the clearest crystal ball and the best brakes will live the longest.

By introducing modern methods for highway projection and the efficient use of topographic maps (developed from photogrammetry) the location engineer can establish alignment and grades which complement the topography and eliminate nearly all of the undesirable aspects of the older accepted methods of making highway locations by surveys on the ground. The locator no longer has an excuse for being unprepared at every change in topography.

Photogrammetry is the tool which will provide the topographic map. The map in turn must be superimposed with planimetric control system. To effect this, a plane coordinate grid system over the area through which the location is to pass must be established. The results of any highway centerline projected through this area will be no more accurate than the original control used to establish the grid system.

Through the intelligent use of the plane coordinate method of centerline projection, employing an accurate topographic map manuscript, as compiled by photogrammetric methods, better highway locations can be achieved with substantial savings in man-hours in preliminary engineering. Lasting dividends accrue through higher benefit-cost ratios, etc., as compared to achievements by other methods.

Without accurate ground control to insure proper orientation of a stereoscopic model in a plotter, it is impossible to secure maps to the accuracy required for location and design of highways. Both horizontal and vertical controls are required in large-scale engineering mapping by photogrammetric methods. Control points should be well defined photographic images that are easily and accurately identified on the ground. Points of precise horizontal control should be appropriately preserved by station markers positioned near the center of the proposed location corridor. Centered by each station marker there must be a photographic target of proper size and shapes before the mapping photography is taken. The interval between the photographic targeted station markers used for ground control should be such as to insure at least two targets per stereoscopic model, but the interval should not exceed 1,000 ft depending of course, on map scale.

A traverse or triangulation net run through all station markers centering photographic targets provides the horizontal control required for the mapping, and bench levels run through the same targets provide a very necessary portion of the vertical control through the center of the stereoscopic models. In addition to the vertical control thus obtained through the center of each stereoscopic model, elevation of photographic image points easily identified on the ground and lying near the corners of each stereoscopic model are also required. Use of a 12-ft subtense bar in conjunction with a precise theodolite has proven to be an accurate and efficient method for obtaining the horizontal distances in the traverse as well as the elevations of the model corner points.

Compilation of topographic maps by photogrammetric methods should be based on a system of plane rectangular coordinates. Wherever possible, this system should

be tied to the state system established by the U. S. Coast and Geodetic Survey. The use of plane coordinates increases precision and eliminates cumulative plotting errors. Targeted control points, for which plane coordinates are determined, may be accurately plotted on the map manuscript on which a plane coordinate grid has been scribed. Positioned under the stereoscopic plotter, the model may then be oriented to these plotted positions to obtain proper horizontal scale.

The photogrammetrically compiled maps along with the photographs offer the design engineer the topographic information he needs to accomplish the design. With the terminal controls for the proposed location already established, intermediate topographic controls may be easily determined, and the design completed with the usual consideration given to earthwork quantities, minimum radius, maximum grade, exposure, and use, sight distances, rise and fall, etc.

Where grade is the controlling factor, a grade contour may be sketched on the map as a guide to position in projecting the highway centerline on the maps. A grade contour may be developed by setting dividers to "walk down" the contours on a uniform grade. Where grade is not the controlling factor, the best alignment consistent with cost, exposure, and use is selected. A flexible spline is an excellent tool for establishing the preliminary alignment on the map. It will naturally assume the shape of circular curves, spirals, and joining tangents, and is easily position shifted as desired.

After a preliminary centerline is established by the preceding methods, it may be penciled on the map. Further comparison of the horizontal alignment with a plotted profile may reveal a desirable revision. Curve templates or a projectoscope are useful in determining the degree and approximate length of curves required. For an exact fit of the spline or for passing through definite control positions, other methods are used. These are discussed later. For ease in staking the designed centerline in the field, it is usually advantageous to change the originally measured total delta angle sufficiently so that the length of curve is a multiple of 5 ft. Generally, the change required in the total delta angle to accomplish this is not a significant amount. Circular curves of full degree are also desirable when choice permits; otherwise circular curves of full degree plus nearest usable two-tenths of degree are used. As the circular curves and spiral curves are selected, the stationing and plane coordinates of the TS, PI, and PT are progressively computed for each curve system and these points are plotted on the map. For long curve systems, coordinate positions for about each 300 ft of arc should be calculated and accurately plotted. Curve templates or beam compasses are not reliable to establish long segments of the curves in exact position for later profile and cross-sectioning by photogrammetric methods.

Alternate methods of using a protractor or tangent offset method to lay off the angles and scale the tangent distances in plotting highway alignment are deemed obsolete. With improvements in construction methods and the availability of more accurate instruments for field work, the need for increased accuracy as well as speed in projecting highway alignment must be recognized. In the author's opinion, the best method of highway projection that will enable the location engineer to keep pace with these increasing demands is to employ plane coordinates in plotting and computing the highway alignment. Plotting by plane coordinates is no cure-all or absolute solution to all the problems encountered by the location engineer. Their proper use, however, reduces the discrepancies between the projected alignment and field staking generally encountered with other methods of projection. The plane coordinate method is adaptable to any topographic map regardless of whether it is compiled from a ground survey or by photogrammetric methods.

Details for implementing the procedure warrant a step-by-step review. The topographic map should be compiled using a plane coordinate grid, preferably tied to the state plane coordinate system. The size of the grid square depends on the scale of the map. On a map whose scale is 100 ft to 1 in., it has been found in practice that a 500-ft interval is best but a 1,000-ft interval for each grid square may be used. The plane coordinate grid is constructed so that its line intersections will be at an even 500- or 1,000-ft interval designation.

Once the approximate alignment has been adjusted to meet predetermined standards of degree of curvature and rate of grade, it will be necessary to obtain two additional

elements of information. First, the plane coordinates at the point of beginning, secondly, the bearing or azimuth of the tangent at the point of beginning. If the proposed project is in the vicinity of triangulation stations established by the USC and GS, then the plane coordinates of the control points should properly be tied to the state system by traverse from the nearest triangulation station or by triangulation from two or more of such stations which are visible from the desired beginning point of location survey. Should the beginning point and one of the horizontal control points coincide, fortunate situation directly establishes the coordinates for that point; if not, then these coordinates can be determined by carefully scaling from the nearest grid intersection, because the control points when plotted were oriented to the coordinate grid. This is the only point to be occupied on the ground for which the position need ever be scaled on the projection for the purpose of entering the coordinate system in the alignment computations. Should it be necessary to assume an origin for project plane coordinates, it would be well to assume an ordinate and abscissa sufficiently large to cover the entire length of the project and avoid negative figures. Plane coordinate values increase north and east, decrease south and west. In recording the plane coordinates and in using bearings on maps and in computations, the north-south are written first and east-west second.

To determine the bearing or azimuth of the beginning tangent, after the point of beginning has been selected, extend the tangent a sufficient distance until it crosses a plane coordinate grid line, preferably 1,000 to 2,000 ft or more beyond its beginning. Where the tangent crosses the grid line, determine the coordinates by scaling. Because the coordinates of the point of beginning are known and where the line crosses a grid line has been determined, the bearing can be determined by the difference in coordinates. For example, assume plane coordinates for the point of beginning are N 10,000 and E 10,000, and the coordinates at the grid line are determined to be N 11,000 and E 10,277.30. By subtraction, the difference in N and E coordinates is determined. In this case the north coordinates have increased 1,000 ft and the east coordinates have increased 277.30 ft. The increase in east coordinates divided by the increase in north coordinates provides an arc tangent of 0.2773000. By inspection in seven place tables of natural trigonometric functions, it is determined that the bearing angle is 15 deg 30 min. Inasmuch as the coordinates increased by north and east, the bearing of the beginning tangent will be N. 15 deg 30 min E. If azimuths from the south are to be used, add 180 deg to the bearing and drop the directional signs. Thus, this example, the beginning azimuth becomes 195 deg 30 min. When using such azimuths, south is 00 deg or 360 deg. If the coordinates and azimuths are on the appropriate state grid, the location engineer has established a definite tie in the proposed alignment which can be used for future references (Fig. 1).

The computations of plane coordinates for various curve points involves the trigonometric cosine and sine functions of the bearing or azimuth and the distance between points. Cosine times the distance will be the north or south displacement from the previous point. This displacement is generally referred to as latitude. Sine times the distance will be the east or west displacement. This displacement is generally referred to as departure. The algebraic sign to be applied to the separate latitudes and departures is positive for north and east, negative for south and west. Summation of plane coordinates is determined from the last point and is a cumulative total. To illustrate the simplicity of this method, the plane coordinates for the various curves given in Table 1 have been computed and tabulated in Figure 2. Seven place natural trigonometric functions are considered accurate enough because the sine and cosine do not change rapidly between minutes.

Although the plane coordinates for any point on each curve system can be computed this was done for only the TS, PI, and ST points. To determine the SC and CS points it is sufficiently accurate to scale in these points by X and Y offsets from the plotted TS and ST. The X and Y offset distances can be computed from unit length values listed in appropriate columns of published spiral curve tables. Once these points have been located, the curves are drawn. Because the control points of each curve system of spiral-circular curve-spiral are plotted accurately, any one part of each curve system could be drawn in error and not affect the alignment beyond the segment for which the points are in error.

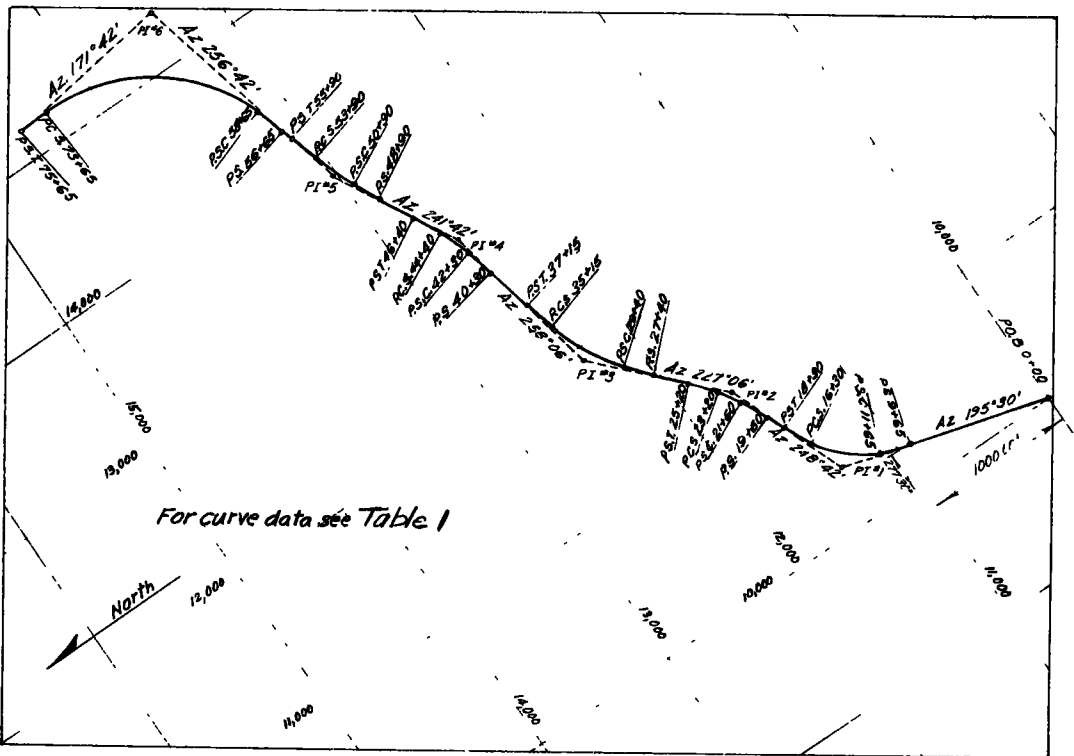


Figure 1.

TABLE 1

Curve	Sta. at TS	Total Δ		T_s (ft)	Δ Simple		D_c o	L_c (ft)	L_s (ft)	Δ Spiral	
		o	'		o	'				o	'
1	9+65	53	12	459.75	37	12	8 Rt.	465.00	200	8	00
2	19+65	21	36	282.46	9	36	6 Lt.	160.00	200	6	00
3	27+40	31	00	497.54	23	00	4 Rt.	575.00	200	4	00
4	40+30	16	24	306.58	8	24	4 Lt.	210.00	200	4	00
5	48+90	15	00	351.54	9	00	4 Rt.	300.00	200	3	00
6	56+65	85	00	1151.35	75	00	5 Lt.	1500.00	200	5	00

It is the practice of a few engineers to vary the tangent distances between curves and the total delta angle to adjust the tangent distance and curve lengths to the nearest 5-ft interval. This practice enables the location engineer to control, to a certain degree of accuracy, the field work by eliminating any odd parts of a foot in the chaining and parts of minutes in measuring angles. Even in mountainous terrain where distances are frequently critical, this method of shifting the line has proved extremely satisfactory. Figure 2 has been computed in this manner. Figure 3 has been computed to show the comparative location of the respective curve points by coordinates, had not the tangent distances between curves and the total delta angles been varied to obtain the nearest 5-ft interval in distance. This example is introduced to lend credence to the author's theory that inasmuch as the coordinates are dependent on the functions of the bearings and distance, it does not affect the accuracy of the plane coordinate method in plotting to vary the distance and angles.

LINE COORDINATES

FROM STATION _____ TO STATION _____

Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude		Departure		Coordinates	
							N	S	E	W	Y or N	X or E
POB 0+00											10,000.00	10,000.00
ST 2+65.00		195°30'		765.00	.7636305	.267233h	+929.97		+257.89		10,929.91	10,257.89
PI #1 1h+2h	73 53°12' R	195°30'		h.59.75	.3636305	.267233h	+h.3.03		+122.87		11,372.9h	10,370.76
ST 13+30		2h8°h2'		h5.75	.3632512	.931612	+167.01		+h23.35		11,539.95	10,609.11
ST 19+60		2h8°h2'		1.40.00	.3632512	.9316912	+h.7.23		+121.13		11,587.18	10,930.2h
PI #2 22h12.4h6	21°36' L	2h8°h2'		282.4h6	.3632512	.9316912	+102.61		+261.17		11,689.79	11,173.4h
ST 25+20		227°06'		282.4h6	.6807209	.732429	+172.28		+206.92		11,892.07	11,400.33
ST 27+h0		227°06'		220.00	.6807209	.732429	+h0.76		+161.16		12,031.83	11,561.49
PI #3 32+37.5h	118°00' L	227°06'		h97.5h	.6307209	.732427	+333.69		+36h.47		12,370.52	11,92.96
ST 37+15		258°06'		258.00	.20620h2	.9785090	+ 6h.96		+303.2h		12,473.12	12,12.82
ST 40+30		258°06'		306.58	.20620h2	.9745090	+ 61.22		+239.99		12,538.08	1,721.06
PI #4 43+36.58	162°h' L	2h3' h2'		306.53	.h7h082	.890477h	+h5.35		+267.95		12,601.30	13,021.05
ST 46+h0		2h3' h2'		250.00	.h7h082	.890477h	+1h3.53		+220.13		12,7h6.65	13,291.00
ST 48+90		2h3' h2'		351.5h	.h7h0882	.890477h	+166.66		+30.52		13,031.8h	13,820.65
PI #5 52+h3.5h	150°00' R	256°h2'		351.5h	.2300h7	.9731789	+ 80.87		+3h2.12		13,112.71	h.162.77
ST 52+90		256°h2'		75.00	.2300h7	.9731789	+ 17.26		+73.00		13,169.97	h.235.77
ST 56+65		256°h2'		1151.35	.2300h7	.9731789	+26h.87		+120.47		13,39h.8h	1,356.2h
PI #5 68+16.35	85°00' L	173°h2'		1151.35	.989258	.1h4.582	+1139.30		166.21		h.5h.1h	15,190.03
ST 75+65												

Figure 2.

Another instance where the plane coordinate method is particularly useful is when the total delta angle exceeds 180°. In these computations it is necessary to compute the coordinates for the points through the curve; that is, TS, SC, radius point, CS and the ST. Inasmuch as the total delta exceeds 180°, the T_S distance will not intersect. First the coordinates for the TS are computed; then on the same azimuth the coordinates for the SPI (Spiral PI) are computed using the long tangent of the spiral. From this point the azimuth is changed in the appropriate direction by the amount of the spiral delta. The distance used will be the short tangent of the spiral. The coordinates at this point define the SC. From the SC the azimuth of the radius is determined to be 90° to the short tangent. The radius of the circular curve times the cosine and sine functions of the azimuth establish the plane coordinates for the center of the curve. From the center point of the circular curve, the azimuth to the CS is changed by the amount the delta angle of that curve differs from 180°. If the angle exceeds 180°, then the difference is added; if less than 180°, the difference is subtracted. After the azimuth of the radial line from the center point of the circular curve has been determined, then the plane coordinates for the CS are computed. From this point the short tangent of the spiral will be at 90° to the radius. These coordinates will establish the SPI. The plane coordinates of the ST will be determined by computing the change in the azimuth from the delta angle of the spiral and multiplying the long tangent of the spiral by the cosine and sine of that azimuth, and then algebraically adding the results to the N and E coordinates of the SPI. This plane coordinate method of computing and plotting is useful for accurate positioning on the map of centerline in mountainous terrain.

U S DEPARTMENT OF COMMERCE
BUREAU OF PUBLIC ROADS
REGION NO 9 DENVER, COLO

LINE COORDINATES

PROJECT _____
STATE _____ Zone _____
Computation No _____ Page _____ of _____
Traverse No _____ Field Book No _____ Pgs _____ to _____
Computed by _____ Date _____
Checked by _____ Date _____

FROM STATION _____ TO STATION _____

Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude		Departure		Coordinates	
							N	S	E	W	Y or N	X or E
PCB 0+00											10,000.00	10,000.00
		195°30'		963.37	.9636305	.2672384	+328.34		+257.45			
PT 9+63.37		195°30'		160.54	.9636305	.2672384	+141.80		+124.08		10,928.34	10,257.45
PI 14+21.9	57°18' Rt.	248°48'		160.54	.3616212	.9323238	+166.55		+121.38		11,372.14	10,380.53
ST 18+29.62		248°48'		131.69	.3616212	.9323238	+147.63		+122.79		11,538.69	10,809.91
TA 19+61.3		248°48'		281.88	.3616212	.9323233	+101.93		+262.80		11,864.32	10,732.70
PI 22+43.19	21°32' Lt.	227°16'		281.88	.6794871	.7341399	+171.27		+27.05		11,688.25	11,195.50
ST 25+20.80		227°16'		221.23	.6785871	.7341399	+140.13		+162.57		11,479.51	11,402.55
TA 27+11.44		227°16'		196.64	.6785871	.7341399	+337.01		+361.79		12,029.67	11,565.06
PI 32+33.07	30°56' Rt.	258°12'		196.64	.2044961	.7386674	+101.56		+137.15		12,466.68	11,929.85
ST 37+14.76		258°12'		313.72	.2044961	.9733674	+64.16		+307.10		12,468.24	12,416.00
TA 40+24.44		258°12'		307.43	.2044961	.9733674	+62.86		+300.91		12,523.60	12,723.10
PI 43+35.89	16°28' Lt.	243°44'		307.44	.4735759	.8807530	+145.59		+270.76		12,525.26	13,024.01
ST 46+10.5		243°44'		202.00	.4735759	.3307530	+119.35		+221.95		12,710.35	13,241.77
TA 48+92.15		243°44'		352.66	.4735759	.6307130	+187.01		+110.61		12,860.20	13,116.72
PI 52+44.8	15°04' Rt.	256°48'		352.66	.2243509	.9735789	+80.53		+343.35		13,027.21	13,827.33
ST 50+24.87		256°48'		73.50	.2283509	.9735789	+16.79		+71.57		13,107.74	14,170.64
TA 56+67.87		256°48'		1153.21	.2283509	.9735789	+263.34		+1122.74		13,124.53	14,242.25
PI 62+21.88	85°06' Lt.	171°42'		1113.21	.9895258	.1443562	+1441.14			166.47	13,317.87	15,361.99
ST 75+61.87											14,529.01	15,191.52

Figure 3.

tainous country where large delta angles are sometimes necessary. To illustrate the computations of plane coordinates to be used, assume total delta = 212°24', D_C = 18° Rt, L_S = 200', Spiral delta = 18°, L_C = 980.00'. From published tables, obtain the radius = 318.31', Long tangent = 134.03', Short tangent = 67.30'. Again the work sheet is tabulated as in the preceding examples. For computations, see work sheet (Fig. 4).

Should it be necessary to change a section of the centerline after it has been plotted, it is not necessary to run the entire line again, only that section that will be affected in the change. Select a curve point that will not be affected in the change and use its plane coordinates as the coordinates for the point of beginning. For plane coordinates at the point where the line change ends, select coordinates of the center of the next succeeding circular curve. To determine the delta angle for the final curve system, it is only necessary to obtain the differences in the azimuth. Because the plane coordinates of the center of the last circular curve on the line change have been computed and the plane coordinates of the center of the circular curve ahead are known, the difference in these coordinates is ascertained—and this information is sufficient to set off a triangle which can be solved by the Law of Sines. By subtracting the T_S distance from the respective sides of the triangle, the length of the tangent between curve points is determined. This is one occasion where the delta angle and the tangent distance cannot be adjusted to the nearest 5-ft interval without affecting the entire line ahead. To account for the difference in stationing, an equation is used at the point where

U S DEPARTMENT OF COMMERCE
BUREAU OF PUBLIC ROADS
REGION NO 9 DENVER, COLO
LINE COORDINATES

PROJECT _____
STATE _____ Zone _____
Computation No _____ Page _____ of _____
Traverse No _____ Field Book No _____ Pgs _____ to _____
Computed by _____ Date _____
Checked by _____ Date _____

FROM STATION _____ TO STATION _____

Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude		Departure		Coordinates	
							N	S	E	W	Y or N	X or E
ST 100+00											10,000.00	10,000.00
SPY 101+31.03	180°00' Rt.	100°00'		131.03	.1736182	.9818078	+ 23.28		-131.00		10,023.28	9,869.00
SC 102+00	90°00' Rt.	118°00'		67.30	.16291716	.8829176	+ 31.60		- 59.43		10,023.88	9,809.57
SC 102+00	90°00' Rt.	208°00'		318.31	.882476	.16291716	+261.06		+149.44		10,335.94	9,959.01
SC 111+80	90°00' Rt.	294°02'		67.30	.1131044	.9106837	-27.80		+ 61.30		10,598.03	10,151.80
SPY 112+17.30	180°00' Rt.	312°02'		131.03	.6743024	.7384553	-90.38		+ 28.28		10,507.65	10,230.78
ST 113+80												

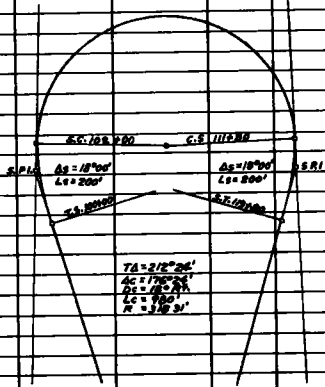


Figure 4.

the line change ends. To illustrate this solution, assume curve systems 2, 3, and 4 in Figure 2 are replaced with one curve system. Here the coordinates for the point of beginning on the line change will be those at the ST at 18+30 and the coordinates to be used on the existing line ahead will be at the ST at 46+40. The total delta will be the difference between the back azimuth, $248^\circ 42'$, and the forward azimuth $241^\circ 42'$, or $7^\circ 00'$. Because the forward azimuth is less than the back azimuth, the curve will be to the left. The following curve data will be used: total delta= $7^\circ 00'$, $T_s=275.25'$, Delta circular curve= $3^\circ 00'$, $D_c=2^\circ 00'$ Lt., $L_c=150'$, $L_s=200'$, Spiral delta= $2^\circ 00'$. This curve revision is given in Figure 5 in the same manner as the previous examples.

Preparing the centerline for plotting by plane coordinates affords definite ties to be computed in the office for use by the field survey crew. Because both the basic horizontal control stations and the centerline alignment are based on the same plane coordinate system, it is an easy computation to determine the distance and azimuth from various points along the line to the nearest horizontal control point.

Figure 1 was plotted to show the alignment illustrated by Figure 2. It will be noted that the topography has been deleted due to the scale of the drawing. The plane coordinate grid has been laid off on a 1,000-ft interval in both directions.

From these previous examples, it can readily be seen that, by using the plane coordinate method of plotting the desired alignment, the location engineer has eliminated most of the element of chance and has changed it to an element of choice.

For elevations and distances determined during the location survey staking of the highway on the ground to agree with elevations and distances obtained from the map projection, the map plotted and designed centerline must be staked on the ground in the same position in which it is projected on the map manuscript. This required duplication during the line staking operations of the accuracy accomplished in establishing

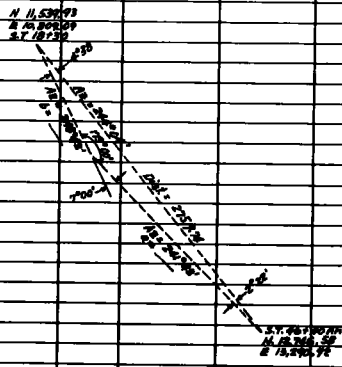
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REGION NO 9 DENVER, COLO

LINE COORDINATES

PROJECT _____
STATE _____ Zone _____
Computation No _____ Page _____ of _____
Traverse No _____ Field Book No _____ Pgs _____ to _____
Computed by _____ Date _____
Checked by _____ Date _____

FROM STATION _____ TO STATION _____

Station	Angle	Grid Azimuth	Grid Bearing	Distance	Cosine	Sine	Latitude		Departure		Coordinates	
							N	S	E	W	Yor N	X or E
ST 13+30		248°42'		659.76	.3632512	.9316912	+239.66		+614.70		11,511.23	10,309.02
PI 21+32.65		248°42'		275.25	.3632512	-.9316912	+91.18		+256.57		11,779.59	11,423.79
PI 27+65.01 7°00' Lt.		211°42'		275.25	-.4740882	.8904774	+140.50		+242.36		11,879.57	11,690.24
ST 30+39.76		211°42'		1553.99	-.4740882	.8904774	+736.73		+1368.25		12,010.07	11,921.60
(Sta. 45+93.76 Bk.) (Sta. 46+40 Ah.)											12,716.80	13,290.85



$$\sin 4^{\circ}20' = \frac{\sin 193^{\circ}00' \cdot \sin 2^{\circ}25'}{2739.74} = \frac{a}{b}$$

$$\frac{68076}{a} = \frac{12187}{2739.74} = \frac{24129}{b}$$

$$a = 1829.24$$

$$b = 935.01$$

$$1829.24 - 275.25 + 1553.99 = \text{New Y.S.}$$

$$935.01 - 275.25 + 659.76 = \text{New S.T.}$$

As S.T. to Y.S. =
180° + 2.4° + 140° 0' =
180° + 142.4° =
180° + 142.4° = 242.4°

Figure 5.

the original basic control. With precise instruments and trained personnel, such accuracy in the line staking can be satisfactorily obtained.

Even when plane coordinates are used for the alignment projection on the map, results will still be disappointing if the entire centerline is not accurately plotted point by point for each contiguous segment. Small plotting errors will not be noticeable in easy topography, but on steep side hills and other mountainous topography, plotting errors will cause considerable difference in elevation between map and ground positions. There are several factors contributing to plotting errors. Circular curve templates are not always accurately cut on uniform curvature or to the specified scale. But, changing radius curves used to sketch spiral transitions do not reflect the true position of such line segments. Manuscripts on unstable material that is affected by moisture and temperature, use of inaccurate scales, and the projection of alignment on map prints which are definitely out of scale also contribute to inaccuracies in position of the plotted line. Moreover, circular curves drawn with a beam or other compass will be displaced laterally in some segments because of a warp of the map manuscript (or the printed copy if used instead) during design. In overlooking, in not realizing, or in ignoring such causes, photogrammetry has often been blamed unjustifiably for errors which were introduced elsewhere.

Although many field tables have standardized the symbols for terms used in spiral curve transitions and circular curves, engineers still find that when referring to certain curve components, they are speaking in terms not familiar to persons using other tables. Without a thorough understanding of the actual curve problems, one may find himself solving a problem by equation substitution to determine a value that is obvious at first glance. An engineer with a good working knowledge of highway alignment

projection will refer to tables of functions to reduce the time of calculations and not for equations which will provide an answer by correct substitution and solution through the indicated steps. Some examples which are accurate within reasonable limits are, as follows:

1. The deflection angle for a circular curve in minutes is 0.3' per foot per degree of curve. The Delta or central angle is twice the deflection angle.
2. The spiral delta angle is equivalent to the deflection angle for the particular curve computed for the length of the spiral.
3. Transition spiral curves change in curvature at uniform rate throughout.

This is sometimes referred to as a rate of change or a deflection factor. This rate of change is determined by the degree of curve in degrees divided by the length in 100-ft stations of the spiral curve. Because the spiral curve deflection varies approximately as the square of the distance, the deflection from the beginning of the transition may be determined in minutes by multiplying 10 by the rate of change in curvature per 100-ft station by the distance squared; each station or 100 ft is used as a unit distance of one in computing the square.

There are times when it is desirable to compound curves to better fit the ground, drainage, and land use. Such compounds should always be connected by a spiral curve transition. To avoid an unsightly "broken back" curve, it may be necessary to combine a 2-deg curve with a 4-deg curve. For simplicity, assume the transition will be made in 200 ft. Some texts will provide satisfactory equations for solution of this problem. By substituting appropriate values in the equations, the problem may be solved. Actually the spiral delta of the transition curve, in this instance, is the sum of the degrees of the two combining curves or 6 deg. Should a 400-ft transition curve be selected, the spiral delta is 12 deg. The difference in curvature between the circular curves being joined by the spiral curve is 2 deg. This means that the curvature is changed proportionately throughout the length of the spiral curve from 2 to 4 deg or vice versa, depending on the direction in which the change is being made to sharp or less sharp curvature. Returning to the original selection of a 200-ft spiral curve for transition between a 2-deg curve and a 4-deg curve, it is observed that the degree of curvature must increase 1 deg for each 100-ft station of transition. For the 400-ft transition, the degree of curvature would only be increased 30 min for each 100-ft station. The rate of change would be equivalent to the difference in degree of the circular curves divided by the length in 100-ft stations of the spiral curve. Thus, for the 200-ft transition, the rate of change would be 4 deg minus the 2 deg divided by 2 stations which is a rate of change of 1 deg per 100-ft station. For the 400-ft transition, the rate of change would be 0.5 deg per 100-ft station. The angle of deflection to be turned from the CS of the 2-deg curve to establish the SC of the 4-deg curve would be the normal deflection for a 2-deg curve plus the rate of change for the distance of the transition. In the instance of the 200-ft transition, the deflection angle would be 1 deg plus 10(1) (2)² or 40 min. The deflection angle would be 2°40'. From the SC of the 4-deg curve, the deflection angle would be 4 deg minus 40 min or 3°20'. The sum of these angles will equal the spiral delta angle of 6 deg.

This explanation allows the use of an approximate calculation for plotting the short and long tangent of the spiral transition between two curves. There are some well-worn text books on the market which provide an accurate solution. This method used for plotting and not field staking, however, is usually accurate to 0.10 ft unless the length and rate of change of the transition curve between circular curves is quite pronounced. The spiral delta and the two deflection angles are known. In 200 ft, a transition has been made between a 2- and 4-deg curve. The average degree of curvature is 3. By the law of signs, there is an oblique triangle with three known angles. If the long chord is known, the two tangent lengths can be calculated. For an approximate solution, solve the long chord for a 3-deg curve with an arc length of 200 ft using the equation $C + 2R \text{Sine } \frac{A}{2}$. This provides the base of the triangle. Using the known angle the solution of the other two sides is elementary (Fig. 6).

The distances of T_1 and T_2 are calculated at approximately 111.20 ft and 88.98 ft, respectively (Fig. 6).

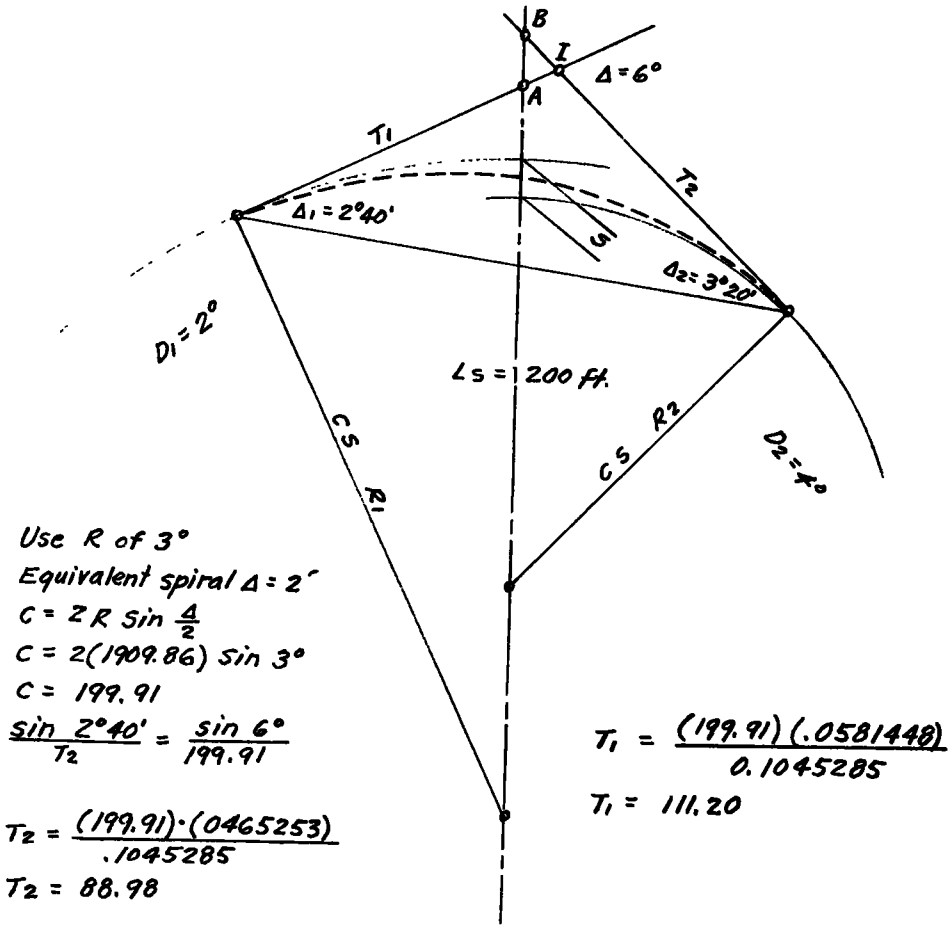


Figure 6.

Compare this approximation by solving the same problem using a method outlined in a text regarding a particular set of field tables:

Given: $D_1 = 2^\circ$ $R_1 = 2864.789$

$D_2 = 4^\circ$ $R = 1432.39$

Equivalent Spiral Angle = $\frac{200}{200} L_s (4^\circ - 2^\circ) = 2^\circ$

S = offset of the circular curves which is essential for insertion of the spiral curve. S in feet is the function for offset from tables of a unit length spiral multiplied by the spiral length. In this case $S = 0.00291 \times 200 = 0.5820$

$\Delta_1 = \frac{200}{200} \times 2^\circ = 2^\circ$, $\Delta_2 = \frac{200}{200} \times 4^\circ = 4^\circ$

$AB = R_2 \text{ Exsec } \Delta_2 - R \text{ Exsec } \Delta_1 - (\text{offset.})$
 $= (1432.39)(0.0024419) - (2864.79)(0.0006095) - 0.5820$

$AB = 1.169663$ or 1.1697

$AI = 1.1697 \times \frac{\cos 4^\circ}{\sin (2^\circ + 4^\circ)} = 11.163$

$$T_1 = 2864.79 \times \tan 2^\circ + AI \\ 2864.79 (0.0349208) + 11.163 = \underline{\underline{111.203}} \text{ or } 111.20$$

$$BI = 1.1697 \frac{\cos 2^\circ}{\sin 6^\circ} = 11.183$$

$$T_2 = 1432.39 (\tan 4^\circ) = BI \\ 1432.39 (0.0699268) - 11.183 = \underline{\underline{88.979}} \text{ or } 88.98$$

It appears from the preceding solution that a considerable amount of time was consumed in solving a problem that is more easily computed by the first approximate method. Approximate methods should never be used except when the difference in degree of circular curves between which the spiral curve is to serve as transition is small.

For an exact computation of a description of a centerline projected on a map with the spline line or to pass a circular curve through given points, a more accurate method of solution is offered. Where the spline line passes through points to which the centerline must be held, mark off stations along the arc through which the arc maintains uniform curvature. Mark the beginning station point No. 1 and the ending point No. 2. Move ahead to the next point along the spline to be held and mark stations along this arc in the same manner. These will be noted as points No. 3 and 4. Scale the coordinate positions at points No. 1 and No. 2. Scale the chord distance between these points and the mid-ordinate. Use the same procedure between points No. 3 and No. 4.

For symbols use:

R = radius

M = mid-ordinate

C = chord

S = shift in feet

Use D_1 and D_2 in degrees

L_s = length of spiral in stations

$$R = \frac{M}{2} + \frac{C^2}{8M} \quad D_c = \frac{11459.156}{M = \frac{C^2}{4M}}$$

As an example, let the chord between points No. 1 and No. 2 of the first circular curve be 495 ft and M 18.5 ft.

$$R = \frac{18.5}{2} + \frac{(495)^2}{148} = 1665 \pm \text{for trial only}$$

$$D_c = \frac{11459.156}{18.5 + \frac{(495)^2}{74}} = 3.44^\circ \pm \text{ Use } 3.6 \text{ or } R = 1591.549$$

Moreover, the scaled difference in the north coordinates is 169.5 and the east coordinates difference is 465.5, the actual length of the long chord will calculate to be 495.40 and the bearing of the chord will be N 69°59'32" E; then

$$\sin \frac{A}{2} = \frac{C}{2R} = \frac{495.40}{3183.10} = 0.15563444$$

$$\sin \frac{A}{2} = 8^\circ 57' 13''$$

Using the bearing of the chord

N69°59'32"E

Less $\frac{A}{2}$ -

$\frac{8^\circ 57' 13''}{\text{N}61^\circ 02' 19'' \text{E}}$

Plus 90°

$\frac{89^\circ 59' 60''}{\text{S}28^\circ 57' 41'' \text{E}}$

Bearing to curve center from point No. 1

Adding in the same manner from Point No. 2, the bearing to center is S 11°03'15" E.

This same procedure is used to determine the degree of curve, mid-ordinate, chord, and bearing of radial lines to the center of the next curve from points No. 3 and 4. With this information, calculate the plane coordinate of the center of circular curve No. 1 and circular curve No. 2.

(1) Use the difference in plane coordinates of the separate positions of the respective centers of curves 1 and 2.

(2) Compute from their plane coordinates, the length of line and bearing between the respective centers.

(3) Subtract the length of line between centers, as computed in step 2, from the difference in radii of the circular curves to be joined by the spiral curve. The result equals the shift "S" provided by the spline.

(4) Compute the length of spiral for trial only.

$$L_s = \sqrt{\frac{S}{0.0727(D_1 - D_2)}} \quad \begin{array}{l} S = \text{feet shift, in ft} \\ D_1 \text{ and } D_2 = \text{degrees} \\ L_s = 100\text{-ft stations} \end{array}$$

(5) Suppose $S = 1.0$, then

$$L_s = \sqrt{\frac{1.0}{0.0727(3.6 - 2.8)}} = \sqrt{17.19} = 4.15 \text{ station}$$

(6) Select length of spiral to attain even deflection factor $= \frac{D}{L_s} = \text{minutes per foot}^2$

$$\text{Example: deflection factor} = \frac{3.6 - 2.8}{10 L_s} = \frac{0.8}{L_s} \text{ when } D_1 = 3.6^\circ \text{ and } D_2 = 2.8^\circ$$

Use a length (in stations) that is divisible into $(D_1 - D_2)$ the quotient of which is divisible into D_1 and D_2 separately if possible. In this instance, use 4.0 stations.

(7) Compute spiral joining circular curves 1 and 2.

K = rate of change in degrees per 100-ft station

$$K = \frac{3.6 - 2.8}{4} = \frac{0.8}{4} = 0.2^\circ \text{ per 100-ft station}$$

$$L_{s1} = \frac{D_1}{K} = \frac{3.6}{0.2} = 18.0 \text{ stations} = 1800 \text{ ft}$$

$$L_{s2} = \frac{D_2}{K} = \frac{2.8}{0.2} = 14 \text{ stations} = 1400 \text{ ft}$$

$$L_s = L_{s1} - L_{s2} = 4 \text{ stations or 400-ft spiral}$$

(8) Compute functions of spiral between circular curve.

$$\Delta_{s1} = \frac{L_{s1} D_1}{200} = \frac{1800 \times 3.6}{200} = 32.4^\circ$$

$$\Delta_{s2} = \frac{L_{s2} D_2}{200} = \frac{1400 \times 2.8}{200} = 19.6^\circ$$

$$\Delta_s = (\Delta_{s1} - \Delta_{s2}) = 12.8^\circ$$

$$*X_1 = 1800 \times 0.9684924$$

$$Y_1 = 1800 \times 0.1842337$$

$$Q_1 = 1800 \times 0.4947175$$

$$P_1 = 1800 \times 0.0465893$$

$$U_1 = 1800 \times 0.6781866$$

$$V_1 = 1800 \times 0.3438307$$

$$X_2 = 1400 \times 0.9883610$$

$$Y_2 = 1400 \times 0.1130786$$

$$Q_2 = 1400 \times 0.4980560$$

$$P_2 = 1400 \times 0.0283882$$

$$U_2 = 1400 \times 0.6707995$$

$$V_2 = 1400 \times 0.3370936$$

T_{S1} = short tangent of spiral between two circular curves.

T_{S2} = long tangent of spiral between two circular curves.

$$T_{S1} = (Y_1 - Y_2) \cos \Delta_{S2} - (X_1 - X_2) \sin \Delta_{S2} = V_1 - \frac{(U_1 - U_2) \sin \Delta_{S2}}{\sin \Delta_S}$$

$$T_{S2} = \frac{(X_1 - X_2) \sin \Delta_{S1} - (Y_1 - Y_2) \cos \Delta_{S1}}{\sin \Delta_S} = \frac{(U_1 - U_2) \sin \Delta_{S1}}{\sin \Delta_S} - V_2$$

By taking advantage of the use of coordinates in projecting highway alignment, solutions to problems are easily solved and provide the engineer with complete control of the alignment position.

Development of Photogrammetric Methods for Right-of-Way Operations in Texas

HUBERT A. HENRY, Supervising Designing Engineer, Texas Highway Department

THE Texas Highway Department made a number of studies prior to 1958 to ascertain the feasibility of using data photogrammetrically obtained for the development of right-of-way maps and field notes for deeds. Each time the possibility was discussed with one of the photogrammetric engineering firms they agreed it probably could be done but it would be very expensive. In some instances, it was reported that special equipment for flight altitudes for photography which were not acceptable would be required to guarantee the accuracies needed.

In the specific project studies, primarily cost estimates and comparisons were made, taking into consideration the project status with regard to the amount of survey work completed by field methods, the amount of field survey work remaining to be done, time limitations due to vegetation and weather, and the relatively high cost of the photogrammetry to be used. Determinations for the pilot project were made in the early spring of 1958 and the Dallas District decided it would be economically feasible, based on preliminary cost estimates, to use photogrammetric methods to develop a project for a section on Interstate Highway 20 in the City of Dallas, provided the accuracies required could be met.

Basically, the requirements were for maps to be compiled by photogrammetric methods at a scale of 20 ft = 1 in., with the horizontal errors not to exceed 0.5 ft and the vertical errors not to exceed 0.3 ft. The specifications were prepared by Texas Highway Department personnel and proposals were given to photogrammetric engineering firms which had been prequalified by the Texas Highway Department and had at least one of several accepted "first order" plotting instruments. It was further stipulated that any firm wishing to do the work must have a responsible representative attend a conference to review the specifications and ascertain the exact intent of each item prior to letting a contract.

At the conference, those in attendance were given the opportunity to object to any provision in the specifications considered unreasonable, and the group decided as a whole if it was necessary to make a revision. There were surprisingly few things the photogrammetric firms found impossible or unreasonable in this conference with their associates.

The proposal for the photogrammetric project consisted of the usual Texas Highway Department "Standard Specifications for Aerial Surveys and Photogrammetric Maps" containing nine items covering the method of handling the contract and prosecution of the work, and "Special Specifications" consisting of fifteen sections which detailed requirements for the specific work and materials to be furnished.

The sections of the "Special Specifications" were numbered and titled. Section 1, "Areas to be Mapped," designated five areas labeled A through E. Two of these areas were 2,000 ft wide, one area 2,400 ft wide, and two areas 1,200 ft wide. The areas 2,000 ft and 2,400 ft wide were interchange areas and were almost square. The normal width of the project was 1,200 ft which required two flight widths.

Determinations of the sizes and locations of the areas were made from a small-scale aerial photographic mosaic on which a layout of the roadway had been drawn. It was decided in the early development of the method of using data photogrammetrically obtained as field notes for property ownership deeds that it would be necessary to show all four corners of a block in a subdivision from which any property was to be obtained in order to properly complete the survey into which the individual tracts would be tied. As a result, an area 1,200 ft wide was required. Experience with the method

has since proved that this is not necessary and maps purchased for the same purpose are now being used which cover an area 760 ft wide. The reduction of the map size to provide coverage with one flight width allows substantial savings in the cost.

Section 2 of the specifications, "Maps to be Furnished," detailed which areas were to be developed as contour maps, in full or in part, and which areas were to be developed as planimetric maps only. Contours were specified for the interchange areas and spot elevations for cross-sections were specified for the remaining areas.

Section 3, "Contents of the Planimetric Maps," set out details which must be shown on the maps. An exception was allowed in this section which is not now permitted—"the planimetric features must be visible on or interpretable from the photography." It has been found from experience there should be a field edit of a project of this type and it is not an extravagant expense. Details of how the drainage, wooded areas, and coordinates were to be shown on the map were given in this section; also, the indexes for the map sheets were specified.

Section 4, "Contents of Topographic Map," required the elevation data to be shown either by contour lines on a base map which is the same as the planimetric map, or by spot elevations to be shown at designated points with errors not to exceed 0.25 ft. The grid was designated for this project to be on the State Plane Coordinate System. An index of the topographic maps was also required.

Section 5, "Drafting," Section 6, "Planimetric Maps," and Section 7, "Topographic Maps," specified requirements for drafting, sheet size, reproductions and similar details for both the planimetric and topographic maps.

Section 8, "Ground Control," and Sections 9 and 10, "Accuracy of Planimetric Maps" and "Accuracy of Topographic Maps," are probably the three most important sections of the "Special Specifications." The horizontal control was required to meet second order accuracy and the vertical control required to meet third order accuracy, tied to surveys of first or second order accuracy of the U. S. Coast and Geodetic or U. S. Geological Survey. Station markers and bench marks were monumented by concrete markers with a bronze disc properly labeled.

Field notes of the horizontal and vertical control were furnished in the original field books and on IBM cards for quick checking. Sketch maps showing the location of all monuments were also required. The specified accuracies of the maps were based on the requirements that all defined cultural features were to be mapped, in their correct horizontal grid position, with errors not to exceed 0.5 ft. Ninety percent of all elevations interpolated were to be correct with errors not to exceed 0.3 ft and the remaining 10 percent with errors not to exceed 0.5 ft.

Section 11, "Cross Sections," required 90 percent of the points shown to have a vertical accuracy of 0.2 ft and the remaining 10 percent not to exceed an error of 0.2 ft. Horizontal accuracy of the points was to conform with requirements for accuracy of planimetric maps.

Section 12, "Methods of Testing," and Section 13, "Negatives and Photographs," were of the standard type specification for these items. Section 14, "Delivery Schedule," and Section 15, "Payment," established the schedules for delivery of the photographs and maps and payment for the work and materials.

The specifications included a map of the areas designated and a "Table of Symbols".

The project was contracted for on April 16, 1958. Total price for the project was \$120,000 or \$11,760 per linear mile 1,000 ft to 1,200 ft wide, with a completion time of 200 calendar days.

The work on the project was carefully checked by the Texas Highway Department District Office personnel as rapidly as possible after it was delivered by the photogrammetric engineering firm. No major errors were found and most of the minor discrepancies were the result of the lack of a field edit, which was not required by the specifications.

As a part of the method developed to use this information in preparing right-of-way maps and deeds, the City of Dallas made available to the Department previously established survey points at or near the center of each street intersection. These points are part of a survey system established and maintained by the City of Dallas. Each point was paneled prior to taking the aerial photographs for the photogrammetric project and were shown on the completed maps.

The plats of the subdivisions through which the project was to be developed were obtained for use in checking the maps. It was found that the panel points plotted by this method and those plotted from the data furnished by the City of Dallas were in remarkably close agreement. All were well within the tolerances of a second order survey. It was also found that the plats of the subdivisions and the planimetrics of the photogrammetric maps conformed throughout the project with few or no discrepancies.

The corners of each block having been established from the panel points and the subdivision plats, the property lines of each parcel involved in the right-of-way considerations were checked according to the deeds recorded and on file. The latest records of the deeds were photostated or the deed description copied on card forms and furnished to the Department to be used for plotting the property lines in the District Office.

In order to determine the properties involved in the right-of-way considerations, the generally proposed geometric layout for the highway was plotted on the subdivision plats. A copy of this map was made available to the abstract company scheduled to handle the abstract work and title insurance for the project. The abstract company furnished, as a part of the contingencies, the deed information.

A field edit was then made by engineers of the Texas Highway Department to study the improvements included in the proposed right-of-way and to make brief notes on a work copy of the map. This information was very valuable in the final determinations of the geometrics and design with regard to the estimated cost of the real estate involved. Such things as retaining walls versus slopes had to be determined. In some cases, this meant working up cost estimates on the retaining walls and having appraisals made on the additional right-of-way required for slopes. It also meant adjusting a ramp or changing the design of a fill to avoid a major right-of-way expense.

An important advantage to this method was the accurate property map developed with a speed which allowed the design engineer early consideration of all these factors, and many delays experienced in the past while waiting for information from which to make a decision were avoided.

After the geometrics had been definitely established and design features which affected right-of-way had been decided on, the exact right-of-way lines were drawn on the planimetric map.

Appraisers were then given work sheets which were prints of the area of the parcel they were to appraise. The parcel was designated by pencil shading. Copies of these sheets were also made for the property owner.

A new deed was then prepared using the deed description from the record and property survey notes scaled from the map. The new deed description was written to define clearly the property to remain and should the property owner wish to have a survey of his own made on the ground, it could affect the right-of-way line a maximum of 6 in.

The area in which this method was first used involved over 300 parcels. To date there remain less than 20 parcels to be obtained and no major problems have developed. Most of the remaining parcels are in condemnation proceedings.

The Texas Highway Department has since contracted for photogrammetric data to be developed for a project 4.3 mi long with about the same accuracy requirements, but with the width limited to 760 ft and with elevation data shown as contours only. This project was contracted for in February 1959. The per mile cost of the project was \$3,194 as compared to the original \$11,760 per mile paid for the area 1,200 ft wide. This contract is now complete, and although all phases of the project have not been developed, no major changes in the method are anticipated. The survey points established by the City of Dallas for this area were not recovered and paneled; as previously stated, these points had proved to be unnecessary.

The specifications for this second project are considered adequate as the project completed under these requirements has received warm approval from the engineers closely associated with it in the Dallas District Office. The contract price paid for the project is also considered very acceptable, and as it is the intention of the Department to use these specifications for a guide in future projects of this type, they are given in the Appendix for reference.

At the same time this method for using data photogrammetrically obtained to prepare right-of-way maps and deeds was being developed, maps were contracted for in other parts of the state to be prepared photogrammetrically with the ownership of the property in the entire map area to be plotted by the photogrammetric engineering firm. The requirements were usually for the ownership data to be prepared as an overlay to a planimetric map. The property lines were plotted in accordance with the deeds on record as of a certain date. The required accuracy of the base map was specified according to the general purpose for which the map was to be used and the ownership overlay was never required to be more accurate than the base map.

The information obtained on these overlays proved to be little more than a tabulation of the property owners which could be obtained directly from the records or from a local abstract company more economically.

The Texas Highway Department will continue working on methods and requirements of specifications for photogrammetric projects to try to reach the ultimate economic point in using data photogrammetrically obtained for the development of right-of-way maps and field notes for deeds. It was realized that one of the projects in the City of Dallas was somewhat unique in the amount of control provided by the surveys maintained by the city; however, the method is feasible for areas which require more field checking of plats and tying in previous surveys. Many of the errors of the original field surveys will become apparent on the maps and can be quickly proved in the field if necessary.

There are additional benefits from a map of the accuracy required for preparing deeds which are beginning to develop with the design studies and plan preparation of the projects. These benefits are considered by some of the design engineers using these maps as essential to the type of information they want on any future maps prepared by photogrammetric methods for their use, regardless of whether the intent is to develop right-of-way maps and field notes for deeds from the data or not. In view of the favorable price paid for the second project of this type in the City of Dallas, the additional benefits of large map scales and high orders of accuracy will be a factor to consider in determining requirements for any maps to be made of heavily developed urban areas by photogrammetric methods.

Appendix

TEXAS HIGHWAY DEPARTMENT SPECIAL SPECIFICATIONS FOR AERIAL PHOTOGRAPHS, PHOTOGRAMMETRIC MAPS, AND MOSAICS

Section 1. Area to Be Mapped

The maps and photographs to be prepared and furnished the Texas Highway Department in accordance with these specifications shall cover an area from near Loop 12 to Clarendon Drive in Dallas, Texas, as approximately outlined on the attached sketch for bidding purposes and further designated by the Texas Highway Department as Control 442, Section 2.

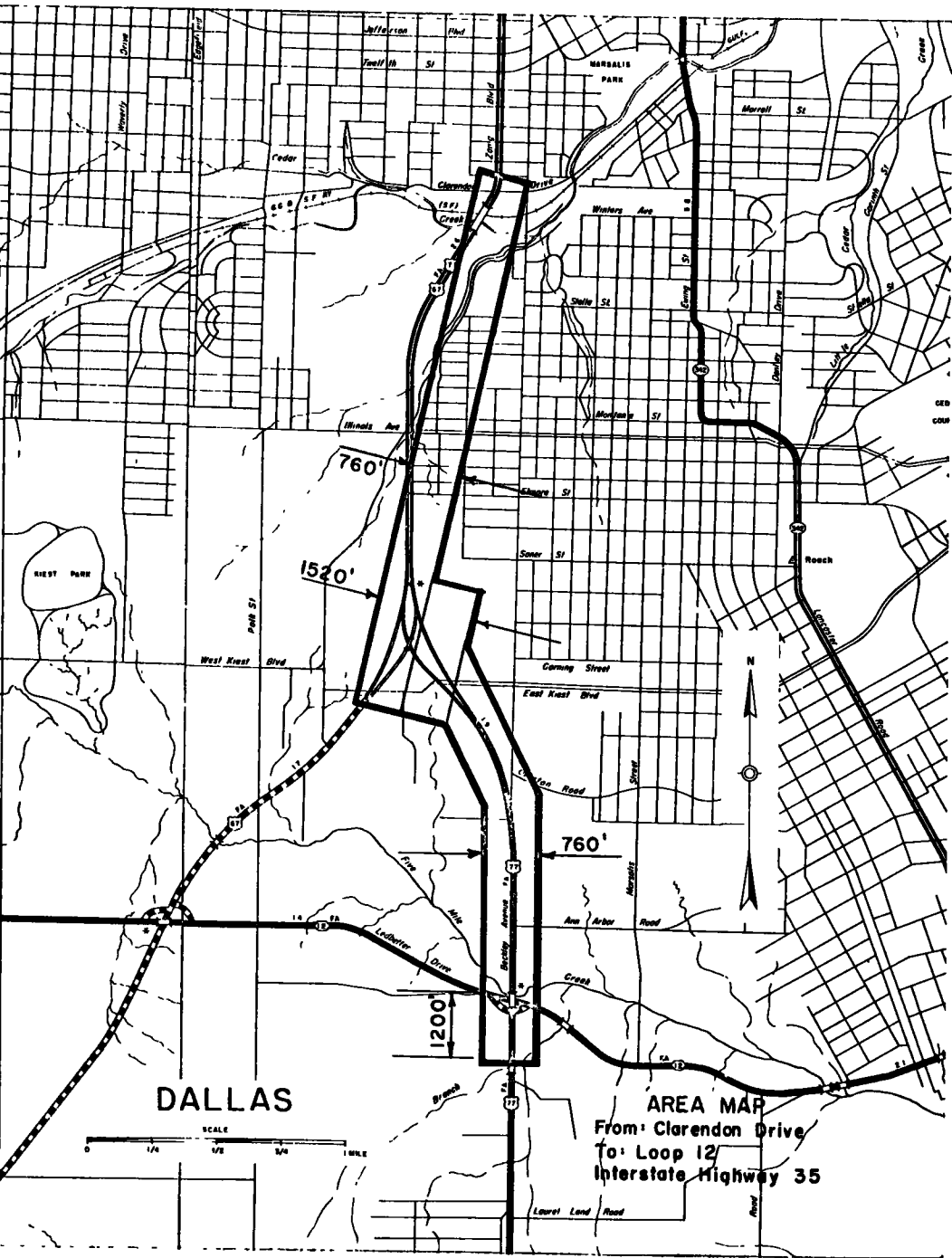
Section 2. Maps, Mosaics and Photographs to Be Furnished

(a) A topographic map manuscript and a positive copy thereof on Du Pont Matte Cronar 0.004 in. thick at a scale of 1 in. = 20 ft with a 1-ft contour interval and a planimetric map positive on Du Pont Matte Cronar (0.004 in. thick) before elevations are plotted, at a scale of 1 in. = 20 ft, are to be furnished of the area generally outlined on the attached sketch.

The positive reproductions will under no conditions be enlargements of the map manuscript.

(b) One semi-controlled mosaic shall be furnished of the area covered by the photogrammetric maps at a scale of 1 in. = 50 ft.

(c) One set of contact prints providing full stereoscopic coverage and index maps shall be furnished of the entire designated area.



Section 3. Contents of Planimetric Maps

(a) General. The maps shall contain all planimetric features which are visible or identifiable on or are interpretable from the aerial photography, including land use features such as buildings, canals, ditches, reservoirs, trails, roads (highways), rail roads, quarries, borrow pits, cemeteries, orchards, boundaries of logged-off areas and wooded areas, and individual, lone large trees that can be recognized as such; and all telephone, telegraph, and electric power transmission line poles and/or tower underground cables, pipe lines and sewers, fence lines, billboards, rock and other walls, and similar details of land use. Structures such as bridges, trestles, tunnels, piers, retaining walls, dams, power plants, transformer and other substations, transportation terminals and airfields, oil, water and other storage tanks, and the like shall be shown. These shall be shown where they occur in addition to all other land use features; sidewalks, parking strips, driveways, fire hydrants, manholes, lamp posts and similar features. The backs of all curb lines shall be plotted.

Buildings and similar dimensionable objects shall be correctly outlined and oriented and shall be drawn to proper scale.

The contractor shall place on the map the names of such streams, streets, roads, towns, etc., as can be secured.

The contractor shall show by proper symbol the principal points of each model in their exact position on the map manuscript and on the positive copies thereof.

(b) Drainage. Drainage lines shall be shown by dash and three dot symbols for all well-defined drainage features indicated when the drainage feature is $\frac{1}{10}$ mi or more length.

All drainage lines shall be stopped at a distance of at least 40 ft from the ridge line. Streams shall be shown double line; each shore being indicated by the dash and three dot symbol. The shore line of small ponds shall also be shown by the drainage line symbol and the interior lightly hatched in ink. Where drainage is known to exist, and where depressions are noted, such culvert and bridge end walls as can be seen in the original photography, or are known to exist, shall be shown on the map.

(c) Wooded Areas. Woodland outlines shall be carefully and accurately delineated. The width of clearing bands along all power transmission lines shall be accurately shown. Woodland lines must be in exact position, especially where the boundary is a road, railroad or transmission line right-of-way. Free standing trees with crown diameters greater than 10 ft shall be shown by an acceptable conventional symbol.

(d) Coordinates. Grid lines shall be shown as grid ticks from inside border to inside border at 100-ft intervals conforming to the State Plane Coordinate System.

(e) Indexes. The contractor shall furnish one complete index of the area defined on the attached sketch map, of the planimetric maps on Du Pont Matte Cronaflex (0.001 in. thick) showing proper orientation of each sheet. There shall be fixed on the index map sheets a north arrow, and the approximate position of each tenth numbered coordinate grid line. The scale of this map shall enable the contractor to represent diagrammatically all map sheets on one sheet of the size stipulated for finished maps in the proposal schedule.

Section 4. Contents of Topographic Map

(a) General. The base map showing all cultural features and the grid coordinates will be the same as the planimetric map manuscript. Relief features will be superimposed on the base map to form the topographic map manuscript.

(b) Relief. Elevations shall be based on United States Coast and Geodetic Survey datum and relief shall be shown by 1-ft contour lines. All contours shall be drawn clear and sharp with a continuous solid line except through buildings or drainage structures. Each 5-ft contour shall be accentuated and numbered. Contour numbers shall be shown at intervals not to exceed 10 in. measured along the contour line. Elevation of all road intersections, roads and railroad intersections, saddles, summits, depressions, the water level on the shore lines of all streams, drainage canals, lakes, reservoirs, ponds, etc., and the centerline of each bridge end and like structures of importance in highway engineering, shall be shown accurately within 0.25 ft of the correct elevations.

Section 5. Drafting

All drafting on the map manuscript will be of the highest standard of workmanship. The manuscript shall be scribed. All lettering shall be mechanical, neat and legible with the style in accordance with standard map practice and with weight and size relative to the importance of the physical features. The general appearance of the maps and the quality of the drafting shall be consistent from sheet to sheet. Standard symbols and line weights will be in accordance with Table "A" of these Special Specifications, or as directed by the engineer.

Section 6. Positive Reproductions

The positive reproductions of the manuscript furnished for the topographic and planimetric maps shall be on sheets with a maximum size of 40 in. wide and 96 in. long. Detail sheet sizes will be as directed by the engineer. Where sheets join there shall be a minimum lap of 2 in. and there shall be match marks within the areas of the lap on each positive reproduction. There shall be in the lower right-hand corner of each sheet a title block, the size and information contained as directed by the engineer. The Du Pont Matte Cronar positives shall show all the details of the manuscript and shall be clear of blemishes, discoloration, scratches or marks that might in any way reduce usefulness during reproduction processes.

Section 7. Ground Control Surveys

It will be the contractor's responsibility to establish second order horizontal ground control and third order vertical control as required. The survey shall be tied to the State Plane Coordinate System and monumented at not greater than $\frac{1}{2}$ -mi intervals, such as monuments as may be required shall be so located that each consecutive monument shall be intervisible.

(a) Description. Ground control shall consist of determining the ground position of pertinent data on existing triangulation and traverse station monuments and bench marks, known as the basic control, the making of surveys to determine the position of all station markers and the elevation of all bench marks set on the project and the making of essential ground control surveys for the establishment of horizontal and vertical control for the stereophotogrammetric mapping which the contractor shall do by the use of aerial photography.

(b) Requirements.

1. **Origin of Basic Control:** Primary ground control surveys shall be based on and adjusted to the basic control specified. Such basic control will consist of first or second order work of the U. S. Coast and Geodetic Survey, the U. S. Geological Survey or other competent agencies or parties who engage in making basic control surveys. The Texas Highway Department makes no guarantee of either the accuracy of the position, elevations or the physical existence of the basic control monuments and bench marks. All information available about the existing basic control shall be made known to the contractor, but the responsibility shall be his to obtain and use this information.

2. **Horizontal and Vertical Control:** Standard ground survey methods necessary to obtain the accuracies specified shall be used; these may include triangulation and traverse for horizontal control, the spirit levels and precise vertical triangulation for vertical control.

All control surveys shall originate and end on the basic control for which closures are known and available or shall be run to make a closed circuit.

All station markers and bench marks set for this project shall be included in and tied to the control survey. Horizontal and vertical control stations and all points needed by the Department prior to flying shall be accurately shown and identified on the photogrammetric maps. Station markers and bench marks shall be concrete having dimensions of not less than 6 in. in width and 3 ft in the ground with bronze markers. Bronze markers are to be supplied by the Texas Highway Department. Monuments shall be placed at points as directed by the engineer.

The contractor may use whatever methods are feasible and practicable to establish the supplemental horizontal and vertical control needed for scale adjustment and orientation of the aerial photographs in the stereophotogrammetric mapping operations; such methods, however, shall be precise enough to establish control commensurate with map accuracies required.

3. Accuracy: For the purpose of these Special Specifications the order of accuracy shall be as listed below. Maximum error of closure for this project, before adjustments are made, shall be wherein N is the number of angles between tangents of the traverse and M is the length of the level circuit in miles.

HORIZONTAL ANGLES TRIANGLE CLOSURE

Horizontal Distance	Traverse Lines	Average	Maximum	Vertical Distance
1:10,000	$(10 \text{ sec})\sqrt{N}$	3 sec	5 sec	$(0.050 \text{ ft.})\sqrt{M}$

4. Field Notes: The field notes of the horizontal and vertical control surveys shall be kept in a clear legible manner in bound engineer's field notebooks of standard manufacture. The contractor will also furnish horizontal and vertical control surveys on data processing cards (IBM) which have been punched and verified for the herein specified control survey. The data processing cards (IBM) are to be punched and verified for the herein specified surveys in accordance to instructions as outlined by D-21 of the Texas Highway Department. It will be the contractor's responsibility to contact the Texas Highway Department, D-21 in Austin directly to obtain these instructions and IBM cards which will be furnished by the state. Horizontal and vertical control information, and the data processing cards (IBM) shall be delivered at the same time the contact photographic prints, photo indexes and mosaics are delivered.

5. Records: The contractor shall provide the engineer with a written description and sketches of all azimuth marks, station markers and their references, and all bench marks. He shall also prepare a line diagram sketch map, at an appropriate scale not smaller than 1,000 ft = 1 in., of the network of horizontal control surveys completed for the project. On this map he shall designate by appropriate symbols (See Table "A") representing each kind and their respective orders of accuracy, the existing survey monuments and bench marks, the triangulation network and traverses and station markers and bench marks set on the project. This sketch map shall be appropriately titled and shall contain a graphic scale bar, directional north arrow, and applicable bearing and plane coordinate notations.

6. Materials: All materials and equipment essential for the satisfactory completion of this item shall be selected and furnished by the contractor.

7. Prior Requirements: The state highway department requires that the contractor contact the district engineer's office, Dallas, Texas, before doing any work on this contract. The amount of control, work progress outline of work to be performed and acceptability of weather conditions for the photography of this project will be subject to the approval of the district engineer. If it is determined necessary to delay the photography, such delays will be made and proper credit will be given the allowed contract time.

Section 8. Accuracy of the Planimetric Maps

All defined features checked must fall in their correct horizontal grid position within $\frac{1}{2}$ of 1 ft (0.5 ft).

Section 9. Accuracy of the Topographic Maps and Cross Section Data

(a) Cultural Features. All defined features checked must fall in their correct horizontal grid position within $\frac{1}{2}$ of 1 ft (0.5 ft).

(b) Contours. Ninety percent of all elevations interpolated shall be correct within three-tenths of the contour interval (0.3 ft) and the remaining 10 percent shall not be

an error by more than one-half contour interval (0.5 ft). Where ground is obscured by heavy cover of trees or high grass contours shall be shown as broken or dashed lines.

Section 10. Method of Testing and Inspection

The Texas Highway Department will field inspect and test the sheets as rapidly as possible after receipt from the contractor. The completeness of the cultural and topographic detail will be determined by a thorough inspection in the field. Accuracy tests will be made by the "test profile" method—that is, by running a profile over any section of the map and then comparing the elevations of test points and the horizontal positions of the cultural features indicated on the profiles with those shown on the map sheet when converted to grid position. If the section is rejected, the contractor will be required to bring it to proper accuracy at his own expense within 20 days after notification of rejection. In all cases, the highway department reserves the right to select the areas to be tested.

The highway department shall be allowed 30 days after receipt of each map for acceptance or rejection. The contractor shall be notified of acceptance or rejection within this period. This period is not included in the contract time.

In addition to field inspection and testing, the highway department reserves the right to inspect any and all phases of the work at any time.

Section 11. Photographic Mosaic

The contractor shall furnish one copy of a semi-controlled mosaic for the full coverage of the contact prints at a scale of 1 in. = 50 ft, using all of the contact prints.

The mosaic shall be an assembly of scale ratioed aerial photographs matched and mounted to form a photographic mosaic, a copy photographic print thereof and negatives of the photographic prints. The photographic prints shall be on single weight, semi-matte photographic paper and linen backed. The negatives will have a 1:1 scale with the prints. Linen backing shall be of durable grade sufficient to withstand considerable handling of the mosaics. The linen backing shall be Commodore Blue Print Cloth, E-D White, as manufactured by Special Fabrics, Inc., Saylesville, Rhode Island, or equivalent. The edges of the mosaics shall be bound with edge binding plastic tape.

All photographs used in the assembly of the mosaics shall be of such quality that the finished mosaic shall have fine grain quality, normal uniform density, and such color tone and degree of contrast that all photographic details show clearly. Color shall be uniform from one photograph to another. Abrupt changes in color will not be permissible and lines shall not show where the photographic images of the separate photographs join each other. Variation from scale shall be equivalent to, or better than, one part in five hundred (1:500). Prints shall not be more than three diameters larger than the negatives of the original photographs made for the project. Any two adjacent prints of the mosaic shall not be mismatched by more than $\frac{1}{20}$ of an inch.

All prints shall be clear and free from chemicals, stains, blemishes, uneven spots, ripples, light fog or streaks, creases, scratches and other defects which would interfere with their use or in any way decrease their usefulness.

The contractor shall place on the mosaic north arrows, title blocks and the names of the major streams, streets, roads, highways, towns, etc. The length of the mosaic sheets shall be a maximum 5 ft in length with 12-in. overlap on each sheet as designated by the engineer.

Section 12. Negatives and Photographs to Be Furnished

All photographs furnished the Texas Highway Department in accordance with this proposal shall be made from a new flight subsequent to the date of contract.

The contractor shall identify all negatives with the date of photography, code letters identify project, film roll and negative number, and shall place on the film strip the time of day exposed. The negatives will be labeled in either a northerly or easterly direction.

The contractor shall furnish one complete set of photographic contact prints on semi-matte double weight paper. The prints shall be sufficiently overlapped to permit stereoscopic study of the entire area.

The contractor shall furnish two copies of a photographic index map at a scale of 1 in. = 40 ft to readily permit the selection of prints covering any part of the project on semi-matte double weight paper. Photographic index maps should show street names at maximum 9-in. spacing to aid in orientation.

All photographs shall be clear, sharp, free of blemishes, discoloration, chemicals fog or uneven spots, light streaks, creases, scratches and other defects which would in any way reduce their usefulness.

On the back of each photographic print delivered and in the same corner as the photographic number appears on the image side, there shall be stamped: Property of the Texas Highway Department, the name and address of the contractor, the focal length in millimeters of the aerial camera and the shutter opening used. Prints of vertical photography shall also be stamped with their approximate scale.

Section 13. Delivery Schedule

Contact photographs, photo indexes and mosaics shall be delivered to the Texas Highway Department, at the address specified, within 70 days after completion of flying.

The cronar positives of the map manuscript shall be delivered as rapidly as they are completed.

The contractor will deliver all materials under this contract to Mr. Frank W. Cawthon, District Engineer, P.O. Box 3067, Dallas, Texas, within 180 days after the work order is issued.

Section 14. Payment

Partial payments not to exceed one payment per calendar month, may be made on the following basis as the work progresses:

(a) Twenty percent of the contract bid price will be paid on delivery and acceptance of the contact prints, photo indexes, mosaics and control data with punched IBM cards.

(b) Seventy percent of the contract bid price will be paid on delivery and acceptance of the planimetric and topographic maps. In the case of progressive shipments the payment for each shipment will be made on the ratio of the number delivered and accepted as to the number required. It is not required that the manuscript be delivered at this time.

(c) The remaining 10 percent will be paid on delivery and acceptance of the map manuscripts and acceptance of all work; the delivery and approval of all material required under these specifications and bid proposal.

From Map to Computer

J. L. FUNK, Photogrammetric Engineer, California Division of Highways, Sacramento

A new method of obtaining digital terrain data for earthwork quantity computation is presented. The procedure is being used in the design of a 200-mi section of Interstate highway on new location. Cross-section data are referred to a calculated centerline, which is developed from a study of aerial photographs prior to map compilation. The method permits key punching for electronic computers directly from manuscripts prepared by the photogrammetric mapping contractors.

Although the method was developed primarily for comparatively flat terrain it has also been adapted for use in taking digital data from contours of topographic maps. Time studies were made and comparative costs developed for this technique and for other methods of taking data from contours. These costs will serve as a guide in estimating the probable value of automation as a connecting link between contours of the maps and the computer. Complete automation from plotter to computer by resetting the stereomodels is also discussed briefly.

PHOTOGRAMMETRIC MAPS are a basic source of information for practically all highway design in California. They are generally at a scale of 1 in. = 50 ft and cover a band from 1,000 ft to 1,400 ft in width. In rolling terrain a contour interval of 20 ft is used. In comparatively flat terrain a grid of spot elevations at 100-ft intervals is shown either in place of, or in addition to, the contours. During design the centerline is laid out on the maps and terrain data for earthwork quantities are taken off the maps manually and processed by electronic computers. This paper describes a method recently developed to minimize the labor involved in preparing terrain data for computation.

DESCRIPTION OF PROJECT

The method was developed and is being used on a project involving 240 mi of Interstate highway on new location extending from US 99, south of Bakersfield, to US 50 west of Tracy. Most of the route is on the west side of the San Joaquin Valley and immediately adjacent to the foothills. Approximately 170 mi of the location are in flat terrain with the remainder ranging from gently rolling to rolling. There is very little development in this area with the land being used primarily for agriculture and grazing. Ground cover is negligible.

Available information used for reconnaissance studies consisted of a 1 in. = 2 mi assembly of County Road Inventory maps, 1 in. = 4,000 ft photo-indexes, and 1 in. = 4,000 ft U. S. Geological Survey quadrangle maps. In some areas these were supplemented by 1 in. = 1,000 ft uncontrolled mosaics.

Following adoption of the recommended route by the California Highway Commission, the work of preparing plans and specifications for photogrammetric mapping was started. The 240-mi route was divided into seven mapping contracts ranging from 25 to

52 mi in length. It was during this stage that plans were made for an improvement in methods of preparing terrain data for electronic computation.

In conformity with current California practice, basic horizontal control monuments were established by use of electronic distance measuring equipment at intervals of from 2 to 4 mi. Mapping contractors were required to set semi-permanent monuments in the immediate vicinity of the proposed centerline at intervals of from 1,000 to 2,000 ft. The positions of these monuments were to be established by second order traverses closing on the basic control monuments. In addition to the semi-permanent monument the mapping contractors were required to leave chaining points along the random traverse at intervals of approximately 300 ft. The specifications provided that these chaining points were to be pre-marked and used for both horizontal and vertical photo control.

The purpose of these pre-marked chaining points was twofold. First, they would strengthen the mapping in the central portion of the mapping strip where accuracy is essential for development of earthwork quantities. Second, they would provide controls for laying out and computing the proposed centerline through areas where the terrain was flat and topography would not affect the precise location.

STRIP METHOD IN FLAT TERRAIN

The specifications also provided that in areas of flat terrain, where spot elevations were to be used in place of contours, the contractor was to lay out the coordinate grid, plot the position of the pre-marked control points, and deliver the manuscripts together with a set of contact prints prior to the start of map compilation.

By studying the contact prints of the 1 in. = 250 ft photography together with available utility maps and other data, it was possible in a very short time to lay out the centerline on the 1 in. = 50 ft map manuscripts. The position of the centerline was computed and 100-ft stations were plotted on the manuscripts which were then returned to the mapping contractor for compilation.

In compiling the maps the contractor was required to show elevations at centerline and at distance of 42, 76, 103, 150, and 200 ft left and right. The first three of these distances corresponded, respectively, to the inside edge of pavement, outside edge of shoulder, and catch point of shallow cuts and fills. Beyond 200 ft left and right, elevations were required at 100-ft intervals. In addition to these specified distances, elevations were also required at all breaks.

In place of copying the terrain data on the usual forms for earthwork computation it was decided to keypunch directly from prints of the photogrammetric maps. Accordingly, strips covering a 300-ft width were cut from prints of the maps. Distances to break points, other than the specified distances previously mentioned, were then scaled and written opposite the elevations. A small portion of one of the strips is shown in Figure 1.

A holder with two spools (Fig. 2) was then constructed. A transparent template with marks at the specified distances of 42, 76, 103 and 150 ft was fastened to the holder so that zero on the template would correspond to centerline on the strip. The template is shown at the left in Figure 1.

After inserting a strip, generally about 5 ft in length, in the holder the keypunch operator turns the spool until the template is adjacent to the first station. After keypunching this station, a turn of the spool moves the strip into position for the next station. The keypunch operation is shown in Figure 3. After a minor amount of practice it was found that keypunching from the strips could be done at the same rate as from the conventional terrain data forms.

Comparative Costs

A comparison of costs between the strip method with spot elevations along cross-section lines and the conventional method of scaling distances and recording on forms is given in the first two lines of Table 1. The cost of 2.1 cents and 4.1 cents per point, respectively, are for all charges involved in getting the data to the computer and include 1.6 cents per point for keypunching.

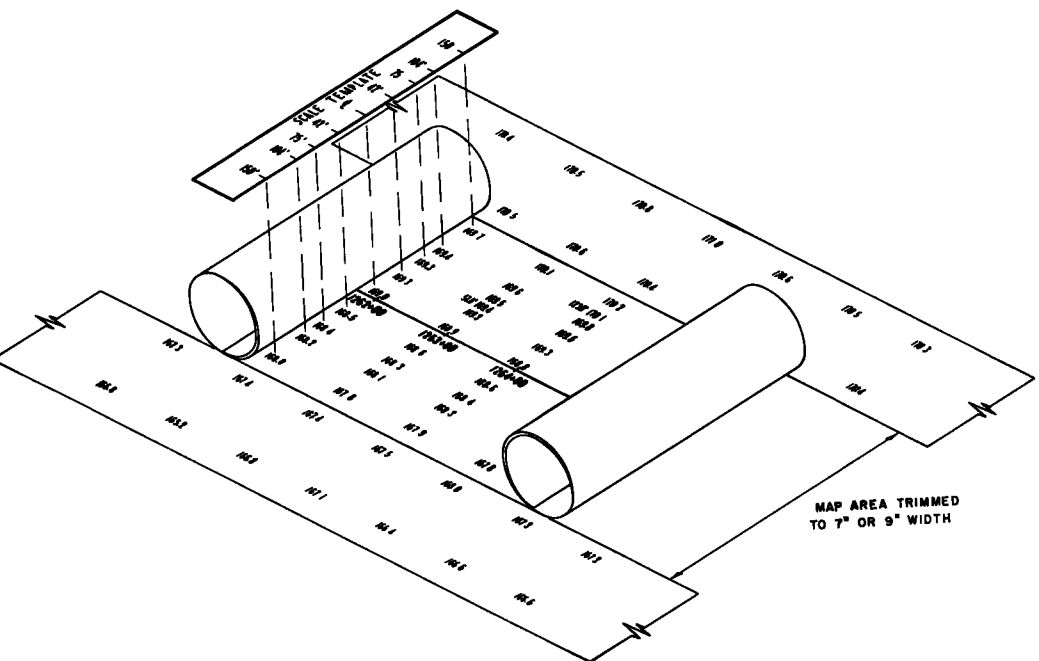


Figure 1.

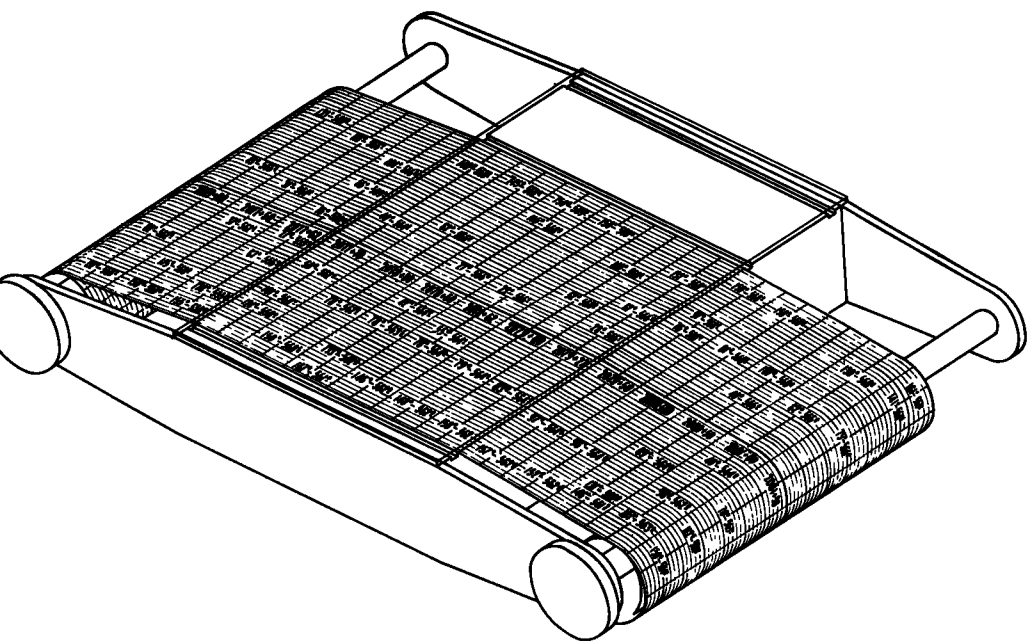


Figure 2.



Figure 3.

It should be noted that the saving of 2 cents per point is by comparison with scaling and recording points which are written along regular cross-section lines. With an arbitrary grid of spot elevations, as usually obtained in California, the mental interpolation required for determining elevations on cross-section lines would require more time. This in turn would result in greater savings for the strip method by comparison.

An alternate solution to this problem would have been to require the mapping contractor to use equipment such as the terrain data translator (1) and furnish the data on punch cards in place of writing the elevations on the maps. However, maps showing elevations are an essential tool of the designer for laying out drainage systems, interchange plans, and many other basic components of the total design. It was therefore considered that the minor savings which could be obtained by full automation of terrain data were more than offset by the value of the maps to the designer.

ROLLING TERRAIN

The success of the strip method in comparatively flat terrain led to an adaptation of its use in rolling terrain where elevations were shown by contours rather than by spot elevations along cross-section lines. In this type of terrain no attempt was made to position the centerline prior to map compilation, as the exact location could best be determined by the designer using the large-scale contour maps.

The conventional method of obtaining terrain data from contour maps is to scale distances along cross-section lines to each contour and/or breaks in the terrain.

TABLE 1
COMPARATIVE COSTS OF DIGITAL TERRAIN DATA

Method	Time (man-hr)		No. of Points	Points per Man-Hour	Cost per Point
	Record	Check			
Spot elevations, cross-section lines:					
Conventional	3.17	1.17	525	121	4.1 cents ^a
Strip	0.58	0.25	525	632	2.1 cents ^a
Contour maps:					
Conventional	10.03	6.07	1,050	65	6.2 cents ^a
Strip	6.45	1.58	1,011	126	4.0 cents ^a
Port-A-Punch	4.97	0.90	452	77	4.1 cents ^b
Dictaphone	4.83	2.25	982	138	4.0 cents ^{a, b}

^aBased on \$3.00 per hour for take-off and check plus 1.6 cents per point for keypunching and verifying.

^bIncludes 0.2 cents per point for printing tabulation prior to checking.

Elevations and distances are recorded in the form of cross-section notes which are then keypunched for electronic computation.

Using the strip method, elevations and distances are recorded on a 7-in. or 9-in. strip of transparent mylar with a printed grid as shown in Figure 4. The strip is positioned over the contour map at the station for which a cross-section is desired, a tick mark is placed at each desired contour crossing and at summits and depressions along the cross-section line. Elevations of these points are written on the strip opposite the tick marks. This operation is then repeated at the next desired station.

It was found best to record elevations at all stations on a strip before determining distances left and right of centerline. Distances are then established by visual interpolation of the tick marks on the grid and recorded on the left of the tick marks. It is obvious that the transparent grid takes the place of the usual scale for measuring distances out from centerline. Scale parallel to centerline is of no importance and can be varied as desired to provide adequate spacing for the cross-section lines. Curves are represented as straight lines on the grid with lines normal to centerline representing radial lines.

Keypunching was expedited at first by using different colored pencils for stations, elevations and distances. After the operators became accustomed to keypunching from the strips this was found unnecessary. The same holder illustrated in Figure 2 is used for keypunching.

Other Methods Tested

After adapting the strip method to taking data from contour maps, tests were made over a 1-mi section to determine comparative time and costs for: (a) conventional method, (b) strip method, (c) use of the Port-A-Punch, and (d) use of a dictaphone. The Port-A-Punch is a small device for holding punch cards which are punched manually the same time distances out are scaled and elevations determined.

The costs per point for each of these methods are given in the last column of Table 1.

The costs for the conventional, strip, and dictaphone methods include 1.6 cents per point for keypunching. This cost was determined from records compiled for 15,000 points in a two-month period. The 1.6 cent cost includes: supervision, equipment rental, office space, and such miscellaneous charges as vacation, sick leave, and retirement. The keypunching costs also include verifying by another operator.

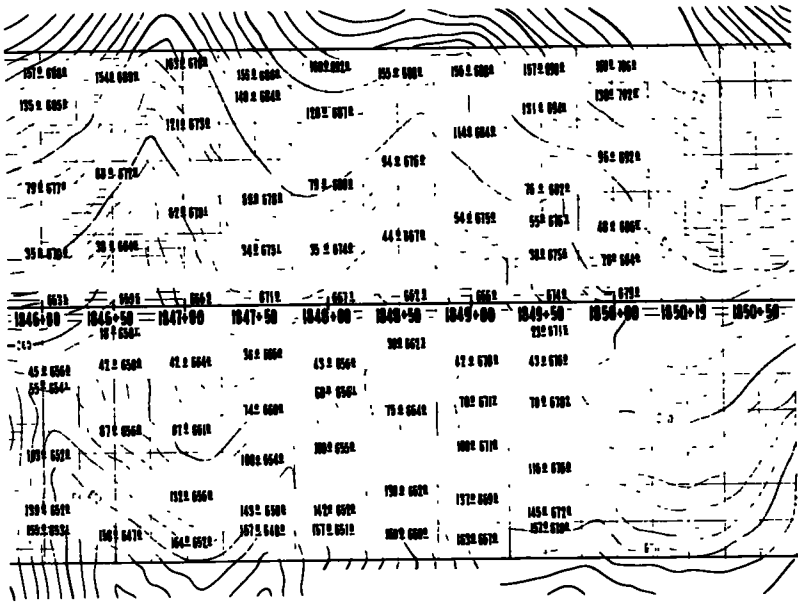


Figure 4.

Although the cost for the strip method was approximately the same as for either the Port-A-Punch or dictaphone it has several advantages over both of these methods. The most important of these is that the strip method can be checked before keypunching, whereas it was found that a satisfactory check of the Port-A-Punch and dictaphone take-off could only be made from the printed tabulation. This required returning the material to the Tabulation Section for correction. The strip method is also much simpler than either of the others, requires less equipment, and needs less practice to become proficient in its use.

Resetting Models

Another method of obtaining digital terrain data which is used in some states is to reset the models in a stereo-plotter after the centerline has been laid out on the large scale maps. Digital data are then recorded directly on punch cards with the use of special equipment. This method was tried on a previously reported (2) experimental project. The costs involved were approximately 10 cents per point as compared to 4 cents by the strip method.

It has been contended that more accurate earthwork quantities would result from resetting the models, as the elevations are measured directly in the stereo-plotter rather than from contours. However, as C.L. Miller has pointed out (3), statistical accuracy rather than point accuracy is the essential requirement for accurate earthwork quantities. Therefore unless systematic errors are completely eliminated there is no assurance that any major increase in the accuracy of earthwork quantities can be achieved by direct stereo-plotter measurements over those taken from a contour map (2).

Automation from Map to Computer

A method of taking digital terrain data from contour maps with semi-automation is possible through use of the digital scale, developed by Benson-Lehner (4), or the digital terrain data recorder, which is being developed by MIT (5). On the basis of information available at this time it appears that take-off of as many as 800 points per man-hour may be possible with such equipment, which is estimated to cost approximately \$9,000.

Take-off of 800 points per hour would result in a saving of 2 cents per point in take-off and checking, plus 1.6 cents per point in keypunching or a total of 3.6 cents per point as compared to the strip method. However, to amortize a \$9,000 cost in three years plus rental of \$50 per month for keypunch equipment it is apparent that an annual volume of 100,000 points in one office would be necessary for equal costs between automation of this type and the strip method. It is therefore obvious that the value of this type of automation will depend on: (a) the volume of points to be processed in one office, (b) the cost of the equipment and the rate of amortization to be used, and (c) the production rate possible with such equipment.

CONCLUSIONS

The following conclusions are drawn from this study:

1. The strip method of terrain data take-off has proved practical for both flat and rolling terrain. It has the advantages of extreme simplicity and low cost.
2. The savings over conventional methods are in the order of 2 cents per point. Although at first glance such savings may appear minor, it must be considered that the California Division of Highways processes earthwork quantities involving approximately 1.5 million terrain points annually. Of these it is estimated that more than a million points are taken from contour maps.
3. There is a possibility of further savings by automation of the step between map and computer. The principal obstacle to realizing such savings at present is the high initial cost of available equipment.

ACKNOWLEDGMENTS

The idea of keypunching terrain data from map strips and its adaptation to take-off from contour maps was conceived and developed by Wm. J. Ellis, a squad leader on the project where the method was first used.

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Getting from Map to Ground

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Transferring the designed centerline of a highway location from photogrammetrically compiled topographic maps to the ground can be accurately and efficiently accomplished when a system of plane coordinates is used for control of the mapping and for preparation of a mathematical description of the centerline. Accurate surveys on the ground, and/or accurate photogrammetric triangulation, through man made or natural objects or features which appear as small and easily identifiable images on the mapping photography, provide the horizontal control and checks on vertical control for compilation of the topographic maps. The ground position and plane coordinates of each of the surveyed images, and the computed plane coordinates of points of changes in curvature on the designed centerline, are the basic data from which distances and bearings are computed for measurement on the ground. Such data are the essentials for beginning centerline staking and serve as intermediate ties for checking position during the process of transfer of the centerline from maps to ground when the designed location is staked in readiness for construction of the highway.

This paper describes the techniques involved in the foregoing procedures and gives illustrations of their applications in specific cases.

●SUCCESS, or lack of it, in getting from maps to the ground is the yardstick by which the quality of maps, especially those compiled by photogrammetric methods, and their accuracy are judged. The manner in which the maps are used and the way in which transfer of position from maps to the ground is accomplished are seldom, and in some cases never, considered to be the cause of engineers' difficulties.

This paper gives some facts regarding common difficulties which are encountered in getting from map to ground, and methods of eliminating these difficulties. Unless these difficulties are removed, their multiplicity arising from improper use of maps will continue.

The procedures and techniques were developed through the employment of variations in and adaptations from work accomplished on numerous highway surveys. They may be new to many users of photogrammetry and aerial surveys for highways. It is hoped that they will give stimulus for further research and development to devise new and improve old methods and procedures of using photogrammetry and aerial surveys.

OLD METHODS

In the past, essential topographic data for highway location and design were obtained by surveys on the ground and plotted in map form at a scale of 40, 50, or 100

= 1 in. These maps were generally accompanied with a profile of the P-line (preliminary traverse) which constituted the base line of the survey. Using such data (often limited in detail, scope, and accuracy) the location engineer established the centerline of the proposed highway. Because the topographic data were usually restricted to a relatively narrow band centered approximately by the P-line, the position-projected centerline for the highway could not be very far from it. Large displacements from the P-line could be made only when the field survey crew anticipated the probable desirability of a substantial line shift and secured more than the usual band-width of field-surveyed data, or returned to the field to obtain such data.

While making a field reconnaissance, the location for the P-line was flagged on the ground. Then stakes were set at each 100-ft station, at the intermediate major breaks of ground slope, and at the instrument points where angles in this traverse were measured with a transit. A profile of the traverse was measured by spirit levels, and cross-sections were measured for a distance of approximately 100 ft from and as nearly normal to it as could be readily determined by the cross-sectioning party. After the ground profile of the P-line had been plotted, a tentative grade line (including vertical curves) was established for subsequent use in designing the highway. Following all desirable adjustments in tentative locations for the centerline of the highway—as projected on the map—so that the entire highway was fitted to the topography horizontally, vertically, and cross-sectionally, the location centerline (called the L-line) was staked on the ground.

Because the L-line had been designed in relation to the P-line, which existed on the ground, and because some segments of the L-line and P-line were coincident, and where not coincident their points of intersection and points of maximum separation were known, staking of the L-line in its designed position was never considered to be a particularly difficult problem. Without the P-line to "hold on to," however, the same engineers, who successfully staked many miles of L-line from P-line data, seemingly became frustrated and "lost" in transferring to and staking on the ground an L-line designed and delineated on photogrammetrically compiled topographic maps. They missed the "holding, guiding hand" of the P-line. Actually though, with a fair understanding of the procedures used to compile maps by photogrammetric methods, it is easy to recognize, in essence, that a P-line-like base line does exist, both on the ground and on the maps, and that the designed L-line may be mathematically tied to and staked from this "base line" to accurately position it on the ground.

METHODS WHEN AERIAL SURVEYS ARE USED

With the advent of using aerial surveys and photogrammetry in the engineering of highways, a significant change in location and design procedures evolved. Through the use of stereoscopic pairs of aerial photographs, the location engineer accomplishes a major portion of his reconnaissance surveys and route determinations in the office. His theater of operations need no longer be confined to what he can see on the ground as he trudges from one visibility vantage point to another, but includes all of the broad area for which he has aerial photographic coverage. A number of route alternatives may be determined and compared, and the best one selected for preliminary survey. No longer is such reconnaissance a one-man operation. Now the talents of specialists may be quickly used and the accomplishments of each one of them easily presented for review and correlation with the work of all others concerned.

In using aerial survey methods, the route is located on the small-scale photographs as they are examined stereoscopically while making the reconnaissance survey. Then the route is photographed and topographically mapped by photogrammetric methods. This mapping is accomplished as a preliminary survey in ample detail and at large enough scale (100 ft = 1 in. or larger) as necessary to design on the topographic maps the use of a spline line and/or appropriate curve templates the best location for the highway on the ground. Thus, by use of the aerial photographs and maps, the L-line location is fully established so that the highway fits the topography and land use, both physically and aesthetically. Consequently the location engineer may move directly from the photographs and maps to the ground without the intermediate step of P-line

staking. Only after mutual agreement on the design of the highway need the location survey staking of the L-line on the ground, in readiness for construction, be undertaken.

Survey Control

Horizontal and vertical control based on accurate surveys made on the ground and an accurate system of plane coordinates mathematically related to that control are of utmost importance in compiling maps photogrammetrically. Without accurate ground control to insure proper orientation of each stereoscopic model in the photogrammetric instruments, it is impossible to secure maps which meet the necessary accuracy requirements for location and design of highways. For plotting the surveyed control points and establishing the points of change in curvature on the designed centerline, use of the system of plane coordinates assures positioning precision and eliminates cumulative errors in using the maps and in doing all subsequent centerline and other staking. Having the system of plane coordinates based on a statewide network of basic horizontal control that is part of the national network, as established by the U. S. Coast and Geodetic Survey, is highly advantageous.

Both horizontal and vertical control points that will be used to control the mapping by photogrammetric methods are called supplemental control, and must be within the route band of topography to be photographed and mapped. Before the mapping photography is taken, a suitable photographic target must have been placed on the ground centered over each point to be used for supplemental horizontal control or such points must be natural objects or features appearing as small and easily identifiable images on the mapping photography which can be easily and accurately tied to basic control points in the project ground control surveys. Each basic control point is preserved in its position as a permanent station marker, which is usually a metal pin or concrete monument centered by a metal tablet appropriately marked to identify it.

The spacing in feet of points at which photographic targets are placed for constituting the principal supplemental horizontal control along the lengthwise direction of the route zone of photography should be not more than approximately two times the mapping photography scale expressed in feet to one inch. They should also be alternately on one side and then the other of the approximate center of the route, laterally separated in feet not less or more, respectively, than one or five times the photography scale in feet to one inch. Occasional additional targeted points to serve as basic project control points should be on or near the center of the route at a $\frac{1}{2}$ - or 1-mi interval, centered by a permanent station marker. All other targeted points should be centered by a semi-permanent station marker, which may be a tack in a wooden stake driven into the ground to where its top is flush with the ground surface.

Bench mark levels measured through the horizontal control points may provide a necessary portion of the vertical control near the center of each stereoscopic model. In addition to vertical control thus obtained, the elevation of pass points—photographic image points easily identified on smooth, preferably level, ground near the corners of each stereoscopic model—are also required.

Use of the Surveyed Control

With the plane coordinate grid system constructed on each map manuscript and each ground survey control point plotted thereon in its exact coordinate position, the stereoscopic model from which each segment of the maps is to be compiled is oriented to the control. If pre-set targets are not available and survey station markers do not appear as photographic images on the mapping photography, it is necessary to tie, on the ground, the objects or features which appear as small and easily identifiable images on the photography to the basic, permanent, station markers set and surveyed in the project control surveys. These images—for which coordinates have been computed from the control surveys data—are the control points plotted on the map manuscripts to control the mapping. In addition, the position of each permanent station marker is plotted accurately and symbolized on the applicable map, and its plane coordinates are also noted thereon for subsequent use.

With a plane coordinate grid system existing on the maps, the position of each control point accurately designated thereby, and each point on the designed centerline mathematically computed thereto, cumulative error is eliminated. Also, scaling ability is improved. This is achieved by interpolation between the plane coordinate grid lines, and, when done, the effects of changes in map scale are eliminated. The mathematically ascertained position of each surveyed control point is known. An erroneously scaled position of the highway alignment, as an L-line, is not used. Generally, the spacing interval of plane coordinate grid lines is 5 in. By converting this spacing to its ground distance equivalent, the effects of any change in map scale are applied immediately to points in the designed alignment, and to any other points, as required, during engineering use of the maps. Consequently, the designed and computed position of the L-line is exact.

The coordinate positions determined for horizontal control points on the ground, which appear as photographic images on the mapping photography, constitute a base line—a traverse similar to that used in P-line surveys on the ground. This base line exists on the ground and on the maps. The designed alignment is easily tied to the control points on this "traverse" for transfer to the ground. The distance and bearing from a control point to the nearest point on the centerline are computed by use of their plane coordinates. Then each centerline point is staked on the ground—points of change in curvature and other points on the centerline, as the nearest P. O. T. and P. O. C. on rests and other essential visibility points (Figs. 1 and 2). This procedure is applied successively for all centerline points to be staked, which separately lie in reasonable proximity to a control point, particularly one of the basic control points. In this way, each basic centerline point for instrument occupancy is staked in its proper position with an accuracy possibility as good as the ground survey control on which the plane coordinates, mapping, alignment design and computations, and centerline staking are based. From such centerline instrument points, each 100-ft station and the essential intermediate plus points are line in and measured in their proper position. It is not necessary to actually stake and occupy each P. I. of the highway tangents. Moreover, this method of using computed data for numerous centerline points and staking them from nearby project survey control points, rather than only from preceding points on the L-line, eliminates the possibilities of cumulative error in taped distances or measured angles.

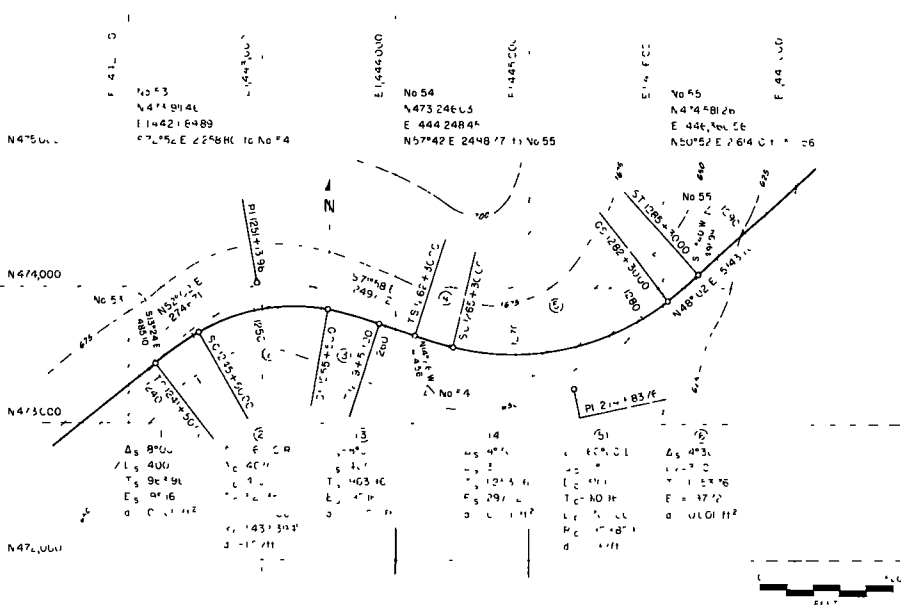


Figure 1. Section of designed and computed highway alignment showing survey ties computed from the permanent station markers for its staking on ground.

STATION	DIST.	BEARING	COSINE	SINE	COURSE		COORDINATES	
					NORTH	EAST	NORTH	EAST
No. 53							473,911.46	1,442,089.89
	485.10	S 13° 24' E	97277588	23174790	-471.89	+112.42		
1241+50.00 T.S.							473,439.57	1,442,202.31
	963.96	N 52° 02' E	61520293	78836880	+593.03	+759.96		
1251+13.96 PI							474,032.60	1,442,962.27
	963.96	S 71° 58' E	30957024	95087658	-298.41	+916.61		
1259+50.00 S.T.							473,734.19	1,443,878.88
	280.00	S 71° 58' E	30957024	95087658	-86.68	+266.24		
1262+30.00 T.S.							473,647.51	1,444,145.12
	414.56	S 14° 26' E	96843832	24925334	-401.48	+103.33		
No. 54							473,246.03	1,444,238.45
	414.56	N 14° 26' W	96843832	24925334	+401.48	-103.33		
1262+30.00 T.S.							473,647.51	1,444,145.12
	1,253.76	S 71° 58' E	30957024	95087658	-388.13	+1,192.17		
1274+83.76 PI							473,259.38	1,445,337.29
	1,253.76	N 48° 02' E	66864915	74353398	+838.39	+932.21		
1285+30.00 S.T.							474,097.77	1,446,269.50
	491.99	N 10° 40' E	98272065	18509492	+483.49	+91.06		
No. 55							474,581.26	1,446,360.56
	2,614.10	N 50° 52' E	63112719	77567936	+1,649.83	+2,027.70		
No. 56							476,231.09	1,448,388.26

Figure 2. Computation of plane coordinates.

Subsequently, accuracy in transfer of the designed and computed highway alignment from the maps to its proper position on the ground depends on the initial accuracy of the project ground control surveys, the proximity and accessibility of the targeted station markers of the control surveys to the intended ground position of the L-line, the accuracy with which the L-line, during design, was projected on the maps, and the working accuracy of the field survey party in staking it on the ground. Thus, the positioning and staking of the L-line on the ground are governed by, and related in accuracy to, the basic control established for and used to control the mapping. Accordingly, all field work is readily accomplished in a methodical, efficient, and effective manner. Should errors in closure occur, their position, cause, and magnitude are easily determined. Therefore, they can be corrected with certainty. By such use of a system of plane coordinates, accuracy requirements are met in each step of the work. The L-line is actually staked on the ground where position-designed on the maps.

Use of Planimetric Features as Position Control

Attempts have been made by some engineers to transfer a designed L-line from the maps to the ground by relating the position of the line to planimetric features which were plotted on the maps photogrammetrically. Most specifications indicate that 90 percent of all planimetric features, which were well defined on the aerial photograph

shall be plotted so that their position on the finished maps will be accurate to within at least $\frac{1}{40}$ in. of their true coordinate position, and that none of the features tested shall be misplaced on the finished maps by more than $\frac{1}{20}$ in. from their true coordinate position. It is evident from these specifications that the probability is remote indeed that a planimetric feature will be plotted precisely in its correct coordinate position. Moreover, when the L-line is transferred from the maps to the ground by relating it to such features, even if they were precisely in their true coordinate position, there is the possibility that the offset measurement from the feature to the nearest line point may be in error as much as 2 ft when the map scale is 100 ft = 1 in. This is because the ability to measure with greater accuracy on a map at this scale is limited.

Worse still is the fact that any misplacement of features used as origin for offset measurement are likely to be additive to the inaccuracy in measurements made on the maps. At the scale of 100 ft = 1 in., the on-the-ground magnitude of errors resulting from the allowed misplacement of $\frac{1}{40}$ in. on the map represents $2\frac{1}{2}$ ft and $\frac{1}{20}$ in. represents 5 ft. Such displacements enter directly into the displacement of any designed line that is staked on the ground by use of offsets measured from planimetric features equally displaced (Fig. 3). The solid line in Figure 3 is the centerline in its designed and computed position. The distances of 80, 104, 84, and 58 ft, respectively, were measured on the map to the designed line from planimetric features. The respective distances were then measured on the ground from the objects, for which the true ground position of each of the features is as shown by their dotted representations, to establish staking points for staking the map-projected alignment on the ground. Inasmuch as the map position of the features used were in error, the distances therefrom placed the alignment, as shown in the dashed position, where it is displaced horizontally from its designed position, and the central angle at the intersection of the tangents and length of the semitangents are larger than computed.

Thus, wherever the position of an L-line point is established on the ground from an object symbolized as a planimetric feature, which contains error in position on the map, it is evident that, regardless of the accuracy with which a mathematical description of the designed L-line has been computed, it will be impossible by this method to stake it on the ground in its designed position. By relating numerous points on the

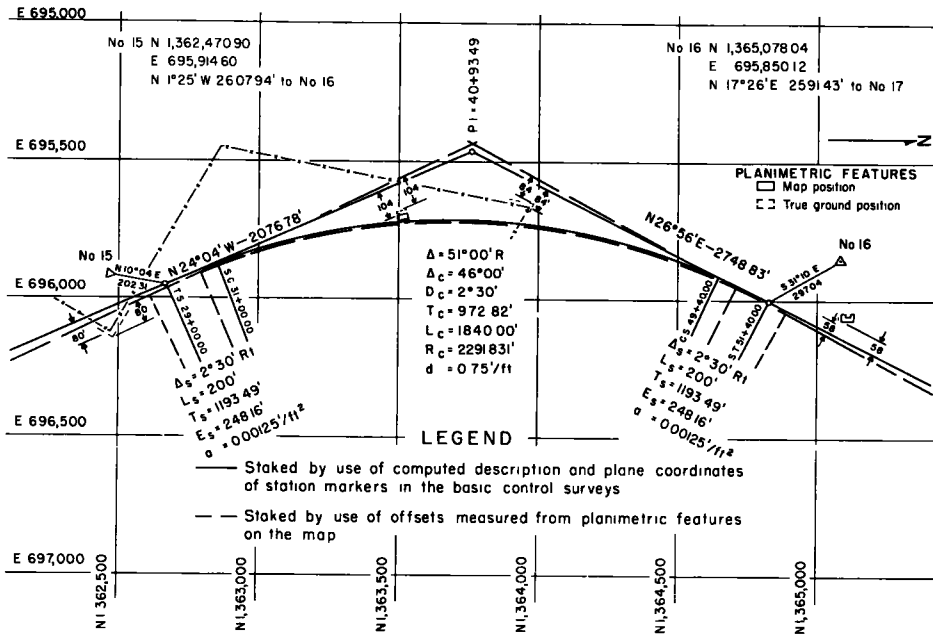


Figure 3. Comparison of staked positions of designed centerline.

designed L-line to nearby planimetric features, it will be possible to achieve only an average positioning related statistically to the average error in position of features used as points of position origin, but the pre-computed curve data, bearings, and distances could not be used. Consequently, the actual positioning of the L-line on the ground would be in variance with the topographic data used, as compiled on the maps.

On the other hand, with the designed L-line tied to control points of known plane coordinates and ground positions, the pre-computed curve data, bearings, and distances are used without alteration, and any error in field survey work while staking the line is quickly determined and corrected by checking into succeeding control point. At the same time, frustrations and delays in survey party work, and engineering deficiencies which would otherwise occur, are prevented by eliminating cumulative error. Distances on the ground from an L-line staked in that manner to objects represented by planimetric features on the maps will seldom be the same as distances determined from the map. Differences between such separately determined distances represent the actual error in plotting these features on the maps. If the maps fulfill their accuracy requirements, however, 90 percent of such differences will not exceed the ground distance represented by $\frac{1}{40}$ in. at map scale, and the remaining 10 percent no more than the distance represented by $\frac{1}{20}$ in.

Use of Maps Without Plane Coordinates

Attempts to project alignment designed on a map not containing plane coordinates have proven hazardous. Generally, in such attempts a protractor or tangent offset method is used to determine the angles and an engineer's scale to measure the tangent distances. On any map at a scale of 100 ft = 1 in., accuracy in measuring and plotting to the equivalent of 1 ft on the ground is difficult to achieve. And it is virtually impossible to determine an angle to an accuracy better than several minutes of arc by such methods, and bearings cannot be established for checking by polaris or solar observation. In addition, it is apparent that regardless of the accuracy achieved in the initial plotting, large cumulative errors will result from changes in map scale, due to shrinkage or expansion of the material on which the maps are compiled, the finished maps drafted, and, finally, map copies produced for engineering use.

Without plane coordinate grid lines having been placed on the map manuscripts and accurately transferred to the finished maps at an interval equivalent to a pre-determined ground distance, it is impossible, from map data only, to become aware specifically of, and to determine the direction and magnitude of errors resulting from any changes in the map scale caused by the shrinkage and/or expansion of the material on which map details exist. Moreover, it is unlikely that any errors may be isolated, or even discovered, until an attempt is made to stake the L-line on the ground. Then frustrations and blaming of the maps begin. Seldom is the real cause—improper use of the maps—identified.

CONCLUSION

The procedures and techniques advocated in this paper for transferring a designed L-line from a photogrammetrically compiled map to the ground have been used successfully for many years on thousands of miles of highway location. Nevertheless, achieved results, as for any similar survey, are dependent on the accuracy with which the designed alignment is projected and its plane coordinate positions are computed on the maps, and the accuracy of the field survey party in staking the L-line on the ground. In no way are inaccuracies in basic control eliminated, but they are easily localized and corrected where they occurred because the effects of cumulative error have been eliminated.

Comparison of Accuracy in Cross-Sections Determined Photogrammetrically and by Ground Measurements

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This paper presents results obtained from (a) precise field cross-sections, (b) unadjusted photogrammetric cross-sections as taken with a Benson-Lehner terrain data translator on a Kelsh plotter, and (c) adjusted photogrammetric cross-sections.

THIS COMPARISON of results for different types of surveys is from earthwork computations based on field measurements and photogrammetric measurements on Wyoming Interstate Project I-90-2(5). The test was made within a 2.5-mi section of very rough terrain where in some instances right-of-way widths were in excess of 100 ft due to maximum vertical variations up to 250 ft across the sections.

Photography was taken, by a consultant, with a Fairchild cartographic camera with B and L Metrogon lens, and a 6-in. focal length, at a flight height of 6,000 ft for a negative scale of 1,000 ft = 1 in. Ground control was furnished by the highway department engineers. The area was then mapped, by the consultant, with a 5-ft contour interval at a horizontal scale of 200 ft = 1 in. The projected alignment was staked in the field by personnel of both the consultant and the highway department, and a precise field profile obtained by highway department engineers on this alignment and furnished to the consultant. The consultant obtained the photogrammetric cross-sections with a Benson-Lehner terrain data translator on a Kelsh plotter.

For checking the accuracy of the photogrammetric cross-sections, precise field cross-sections were obtained with a Zeiss self-leveling level at an interval of approximately 1,000 ft. Right angles were turned off each station with a transit, and the distances to all cross-section breaks were measured with a steel chain.

For the earthwork computations of embankment and excavation, as given in Table 1, it was assumed that the distance between each section was 100 ft in horizontal distance. End areas for embankment were computed for a four-lane section of interstate highway with a 40-ft median width and approximately a 15-ft average fill at centerline of the eastbound lanes. An approximate 10-ft cut at centerline of the eastbound lanes was used for the excavation computations. This method gives a thorough check on each one-half of the section that would be affected by cut and fill.

Earthwork quantities were computed for (a) the precise field sections, (b) for the unadjusted photogrammetric sections, and (c) after each entire photogrammetric cross-section was "adjusted" either up or down to reduce the centerline elevation difference between photogrammetric and field to zero (0) (1). Table 1 shows that the larger percentages of "difference" between photogrammetric and field quantities and unadjusted photogrammetric quantities were in nearly all instances reduced by "adjusting" the photogrammetric cross-section. This adjustment procedure is easily handled in computer programs, and may be a standard procedure if desired.

CONCLUSIONS

The results of this study, as given in Table 1, support the conclusion of Funk (1) that adjustment of photogrammetric surveys by means of an accurate field profile will materially reduce large localized errors in earthwork quantities.

TABLE I
COMPARISON OF EXCAVATION AND EMBANKMENT QUANTITIES
IN CUBIC YARDS AND PERCENT DIFFERENCE

Station	CUBIC YARDS EXCAVATION			Percent Difference From Precise Field		CUBIC YARDS EMBANKMENT			Percent Difference From Precise Field	
	Precise Field	Photogrammetric		Photogrammetric		Precise Field	Photogrammetric		Photogrammetric	
		Unadjusted	Adjusted	Unadjusted	Adjusted		Unadjusted	Adjusted	Unadjusted	Adjusted
1695	6661	7261	6743	+ 9.00	+1.23	7603	7050	7471	- 7.27	- 1.74
1703	6986	7495	7193	+ 7.28	+2.96	8144	7726	7967	- 5.13	- 2.17
1709	6264	6677	6470	+ 6.59	+3.29	7296	6970	7139	- 4.52	- 2.15
1720	7486	7609	7512	+ 1.64	+0.35	7273	7211	7282	- 0.85	+0.12
1730	7768	7789	7637	+ 0.27	- 1.69	7481	7539	7573	+ 0.76	+ 1.23
1740	7355	7135	7229	- 2.99	- 1.71	7039	7255	7138	+ 3.07	+ 1.41
1749	9388	8688	9221	- 7.46	- 1.71	7841	8516	8104	+ 8.61	+ 3.35
1760	8218	7697	8088	- 6.34	- 1.58	6244	6783	6437	+ 8.63	+ 3.09
1770	5953	5659	5804	- 4.94	- 2.50	4707	4917	4796	+ 4.46	+ 1.89
1780	7327	7303	7379	- 0.33	+0.71	6816	6966	6906	+ 2.20	+ 1.32
1790	7138	7285	7292	+ 2.06	+ 2.16	6835	6895	6871	+ 0.87	- 0.53
1800	6357	6580	6525	+ 3.50	+ 2.64	6581	6431	6426	- 2.28	- 2.36
1810	6690	6983	6807	+ 4.38	+ 1.75	7526	7273	7420	- 3.36	- 1.41
1820	7575	7669	7186	+ 1.24	- 5.14	7449	7258	7663	- 2.56	+ 2.87
1830										
TOTAL	101,166	101,830	101,086	+ 0.66	- 0.08	98,835	98,790	99,193	- 0.05	+ 0.36

NOTE: For earthwork computations the distance between each cross-section taken was assumed to be 100'

The fact that medium-scale photography has provided such relatively small "differences" between field and photogrammetric quantities is of particular interest, because it is not general practice to cross-section from this scale of photography. It is planned at this time to pay staked quantities on this project as based on the photogrammetric cross-sections. This study supports the theory that precise control and good workmanship will produce dependable aerial surveys.

REFERENCE

1. Funk, L. L., "Adjustment of Photogrammetric Surveys." HRB Bull. 228:21-27 (1959).

HRB: OR 41

THE NATIONAL ACADEMY OF SCIENCES—NATIONAL RESEARCH COUNCIL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The ACADEMY itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the ACADEMY and the government, although the ACADEMY is not a governmental agency.

The NATIONAL RESEARCH COUNCIL was established by the ACADEMY in 1916, at the request of President Wilson, to enable scientists generally to associate their efforts with those of the limited membership of the ACADEMY in service to the nation, to society, and to science at home and abroad. Members of the NATIONAL RESEARCH COUNCIL receive their appointments from the president of the ACADEMY. They include representatives nominated by the major scientific and technical societies, representatives of the federal government, and a number of members at large. In addition, several thousand scientists and engineers take part in the activities of the research council through membership on its various boards and committees.

Receiving funds from both public and private sources, by contribution, grant, or contract, the ACADEMY and its RESEARCH COUNCIL thus work to stimulate research and its applications, to survey the broad possibilities of science, to promote effective utilization of the scientific and technical resources of the country, to serve the government, and to further the general interests of science.

The HIGHWAY RESEARCH BOARD was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the NATIONAL RESEARCH COUNCIL. The BOARD is a cooperative organization of the highway technologists of America operating under the auspices of the ACADEMY-COUNCIL and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the BOARD are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.
