

Integrating Land Use and Traffic Forecasting

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● A TWO-YEAR STUDY of the transportation needs of the Greater Hartford Area was recently completed as a result of the combined efforts of the Connecticut Highway Department, the City of Hartford and the Bureau of Public Roads under the guidance of the Automotive Safety Foundation. As a consequence of this study there is a much clearer picture of the travel needs and desires of the area, and a better understanding of the changing urban travel patterns. Also, the study represented a wedding of the separate disciplines of the city planner and the traffic engineer, which in turn resulted in a new method of land-use analysis which may be the beginning of a better understanding of the factors affecting the growth of metropolitan areas, particularly with respect to the relationship between land use and highway development. This, in fact, may be the most important long-term outcome of the study, rather than the development of design traffic volumes which was the immediate study objective.

Actually, the study to date is only the first phase of a larger program which has as its aim the testing of alternate land-use patterns in an attempt to arrive at the most desirable arrangement from the standpoint of both highway and land-use development. This latter project is made possible only because of the basic information which has been derived from the land-use study.

The land-use analysis developed during the course of this study is actually a mathematical procedure (1) which can be used to forecast the future zone-by-zone distribution of population and employment areas. This information is a necessary prerequisite to and is used as input data for the traffic analysis.

The principal purpose of this paper is a discussion of the procedures used and the basic information learned during the development of the land-use analysis for the Hartford Area Traffic Study. However, because the purpose of the land-use study was to supply data for the traffic analysis, many of the procedures used are aimed strictly toward that end. Therefore, before discussing the details of the land-use study, it is necessary to explain briefly the traffic analysis to show just what land-use information is required.

TRAFFIC STUDY

The traffic analysis used for this study employed the Gravity Model technique developed by Voorhees (2). The gravity model derives its name from the fact that vehicle trips are distributed by a formula which closely resembles Newton's formula for the law of gravity. Newton's law states that objects attract each other in proportion to the mass of the objects and inversely as the square of the distance between them. Similarly, the gravity model distributes vehicle trips in proportion to the drawing power of an area which represents the mass, and inversely as some power of the distance between the areas. The distance is usually expressed in terms of travel time rather than in miles.

The actual formula can be expressed as:

$$\text{Trips}_{AB} = \text{Trip Production}_A \times \frac{\text{Trip Attraction}_B \times \text{Travel Time Factor}_{AB}}{\sum_A^n (\text{Trip Attraction} \times \text{Travel Time Factor})}$$

The traffic model being used by Connecticut is a four-purpose model; that is, sepa-

rate distributions are made for each of the four trip purposes, these being: work, social, commercial, and non-home-based trips. Separate land-use information is required for each trip purpose, and for both trip production and trip attraction for each zone as described in the following.

Work Trips

Work trip production is based on the labor force which, in turn, is related to the population of the zone. For the distribution of work trips a measure of attraction for each zone must be developed. This measure can be any one of a number of factors: employment, floor area, gross sales, etc. Total employment was used, inasmuch as employment figures were readily available. The factors used for the distribution of work trips then, are: labor force (which is derived from population) and total employment, each on a zone-by-zone basis.

Social Trips

Social trips are defined as those trips made for the purpose of visiting friends. They are trips between residential zones and are therefore attracted by people. The trip production for social trips is based on car ownership which is directly related to population. Hence, the information required from the land-use analysis for the distribution of social trips is population and car ownership.

Commercial Trips

Commercial trips are those trips which are made between the home and an area of commercial land use. The attraction factor for commercial trips must be some measure of the intensity of commercial activity of the zone. Retail trade employment was used for this measure. As with social trips, the trip production factor for commercial trips is car ownership, so the only new information required of the land-use analysis for the distribution of commercial trips is retail trade employment.

Non-Home-Based Trips

Non-home-based trips are defined as trips which have neither origin nor destination at the home. They are composed of housewives shopping from store to store and salesmen or doctors traveling between calls, as examples. It is seen that some of these trips are related to commercial areas as with the housewife and, in part, the salesman. However, some salesmen's trips are to industrial areas and some are to residential areas. Doctors and home delivery trucks on non-home-based trips are related to residential areas. To account for these diverse trip types an index including population, total employment and retail employment was developed for their distribution. Note that no new information is required for this distribution.

In summary, the information to be derived from the land-use analysis is the location of: (a) labor force, (b) total employment, (c) car ownership, (d) retail trade employment, and (e) population. These factors and their function in the gravity model are given in Table 1.

TABLE 1
TRIP PRODUCTION AND ATTRACTION FACTORS

Trip Type	Factors Related to:	
	Trip Production	Trip Attraction
Work	Labor force	Total employment
Social	Car ownership	Population
Commercial	Car ownership	Retail employment
	Retail employment	Retail employment
Non-home-based	Total employment	Total employment
	Population	Population

This information will supply the trip production and trip attraction information; in fact, all the data necessary for the gravity model formula except the travel time factors. A brief discussion of the use and derivation of the travel time factors follows.

Travel Time Factors

Travel time factors are used to measure the resistance of motorists to travel time. For example, if travel time were of no significance, there would be as much likelihood of a resident of Hartford working in Boston as in Hartford, and the travel time factor would then be "one" for all trips. Realistically, it is known that people attempt to minimize travel time by living near their jobs and shopping near home. What needs to be known is the distribution of travel times between people, in general, and their places of employment and shopping.

It can be recalled from the formula that the travel time factor, as used in the gravity model, is the value comparable to the inverse square relationship in Newton's formula. If these two formulae were exactly comparable, there would be no need for travel time factors and the square of the travel time could simply be placed in the denominator of the equation. This, for many years, was the approach used but with little success.

Actually, the exponent of the travel time is not two; further, it is not the same for all trip purposes, and in fact, may not even be a constant exponent. For this reason it has been necessary to develop a factor for each interval of time for each trip purpose.

Travel time factors can be developed in several ways. The traditional method, and perhaps the most straightforward way of developing travel time factors, is to conduct a small home-interview survey designed for this purpose. This was one method used for checking the Hartford Area Traffic Study. Actually, however, it has been found that travel time factors are so nearly similar nationwide that very little error would be introduced by the use of standard curves. This suggests another way of checking—using standard curves to make the gravity model computations and then checking the trip length distribution thus obtained against known values from, for example, a roadside interview origin-destination survey. By grouping inter-zonal movements by travel times and comparing the volumes produced by the model with those shown by the roadside survey, a ratio of actual to theoretical values can be developed. A plot of this ratio versus the travel time will readily show any significant bias.

Figure 1 shows a plot of the travel time factors which were used; one curve for each of the four trip purposes. From these curves it can be seen that the average trip length varies by trip purpose, and even that the trip length distribution is different. Although these curves do not necessarily represent trip length distribution, they do give an indication. For example, the shortest trip type is the non-home-based trips, with commercial trips a close second. The longest trips are work and social trips, but even these are rarely as long as 50 min, with the vast majority less than 20 min. Note also the steepness of the curves for all trip types, accentuating the low values of travel time. This indicates the tendency for very short trips, and the attempt on the part of the motorist to minimize travel time. Although these curves do not consider terminal time at either end of the trip, there is some indication that they could be made more flexible by adding terminal time.

The gravity model method has been well covered in recent publications; the AIP Journal (3), the Civil Engineering Journal (4), as well as the various reports published by the municipalities which have used this method. Of course each time the method is used, the researchers add to and/or refine the method. It is this latest refinement, the land-use analysis, which is the primary purpose of this paper.

LAND-USE ANALYSIS

The basic determinant for the generation and distribution of traffic is the land use and intensity of land use of the area. Therefore any studies of traffic must, of necessity, begin with a study of land use. Furthermore, it is the projection of future land

use which will determine the future traffic characteristics. In the Hartford Area Traffic Study a method of land-use analysis was developed which attempts to recognize the many diverse factors which influence metropolitan growth. Once these factors are known, objective analysis of them will produce much more reliable predictions of future travel.

The study of land use was based on analyses of past growth patterns for the period between 1947 and 1958. On the basis of these growth patterns, hypotheses were made of the factors influencing the trends and then these hypotheses were tested to check their validity and reliability, and to determine weights to reflect the relative magnitude of each variable. In this way the relative importance of the many factors involved was checked, and for the projection into the future, these weights were varied as necessary to adjust for changing trends.

The first task in studying land use was to determine which land-use categories would be included in the analysis; that is, which land uses should be studied separately. In order that a workable procedure be developed, it was necessary to keep the categories to a minimum, and to group the various factors by similar characteristics. At the outset, therefore, it was determined that population would be handled as a single factor. Employment, on the other hand, can be divided into any number of types depending on the particular characteristics involved. To determine just how many categories were required, a pilot study was made by the City of Hartford within the three major employment towns: Hartford, East Hartford and West Hartford. From this study it was recognized that at least three types of employment must be studied separately: manufacturing, retail, and service; the last category actually to include all types not covered by the first two. This seems to be a very realistic breakdown in that each of the three groups does include factors distinct from the others.

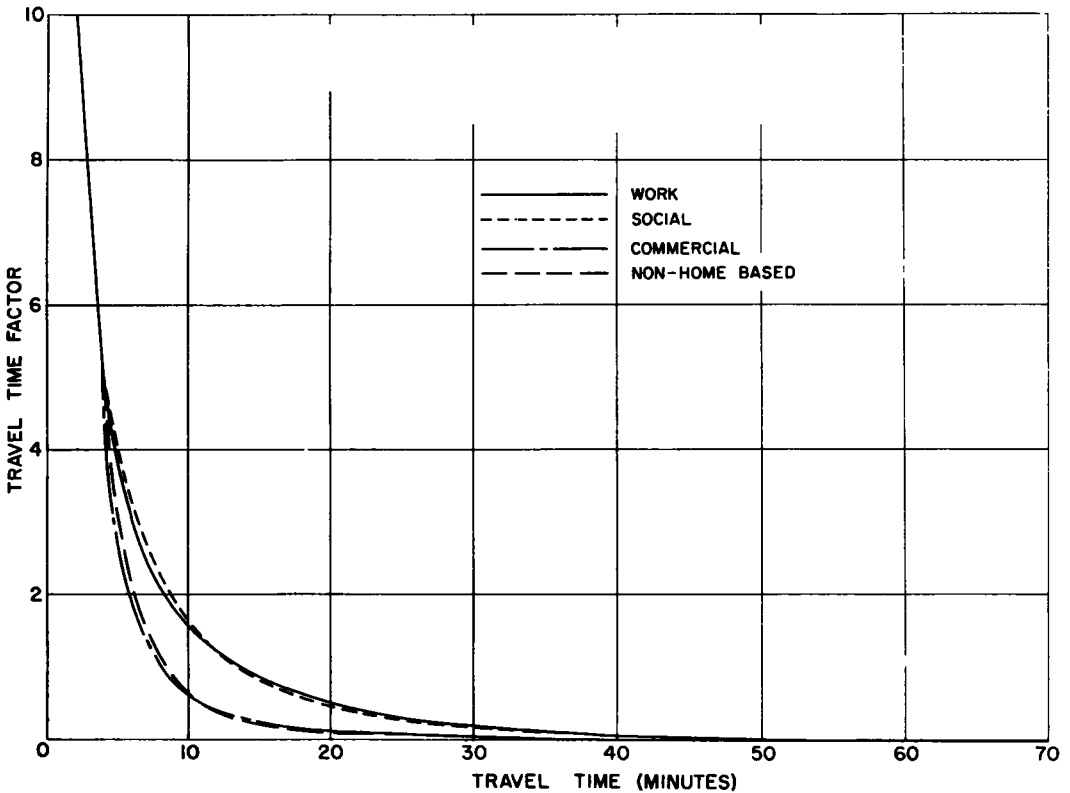


Figure 1. Travel time factors, Hartford Area Traffic Study.

For example, manufacturing employment (industry) may be dependent on rail service, whereas service employment (office buildings, principally) is relatively independent of it.

Generally, manufacturing employment will include blue-collar workers whereas service employment will include the white-collar workers, employees of insurance companies, utility companies, government agencies, etc. Retail employment must, of course, be studied separately because this is one of the categories used in the distribution of trips.

The final breakdown of land-use factors for the study was then: (a) manufacturing employment, (b) service employment, (c) retail trade employment, (d) population, and, as a by-product (e) car ownership. This breakdown seemed to work well in the Hartford area; however, some other city may require a more (or less) detailed breakdown, or perhaps different category divisions. It is expected that a larger city would require more categories.

Each of these five separate categories, which indicate land-use activity, was studied separately. The following is a discussion of the procedures used and the information derived from this study as well as the methods developed for predicting the future distribution of land use activities.

Manufacturing Employment

Study of the factors related to the distribution of new manufacturing employment began with a series of multiple correlation analyses in an attempt to determine precisely what variables affect this growth. For these analyses a number of variables were correlated with the known past growth in new manufacturing employment. Ultimately nine variables were used, these being:

Highway Accessibility to the Labor Force (1). — This is measured by the equation:

$$A_1 = \frac{P_1}{T_{1-1}^x} + \frac{P_2}{T_{1-2}^x} + \dots + \frac{P_n}{T_{1-n}^x}$$

in which

A = accessibility index of a zone to population;
 P = zone population at beginning of projection period; and
 T = travel time between employment zone and residential zone as measured by the assumed highway network at end of projection period.

Availability of Industrial Land or "Holding Capacity". — This is the additional acreage in each zone which is available for industrial development.

Tax Rate. — Reduced to a common base.

Sewer Facilities. — Primarily a consideration of the capacity of the sewer system in the zone.

Rail Service. — A subjective rating based on the adequacy of service.

Water Facilities. — Related primarily to the capacity of the system in the zone.

Travel Time to Airports. — There is only one major airport in the area and this rating varied inversely with the travel time from the airport.

Promotion. — Primarily a measure of the town's activity in promoting industrial development.

Industrial Land Bordering Expressway. — This rating was obtained by giving a numerical weight of "one" to each acre of industrial land within $\frac{1}{4}$ mile of a freeway, a weight of $\frac{1}{2}$ for acreage between $\frac{1}{4}$ and $\frac{1}{2}$ mile, and a weight of $\frac{1}{4}$ for industrial land between $\frac{1}{2}$ and 1 mile of a freeway. The expressway system at the end of the study period was used. This factor is not a measure of highway access but rather a measure of the importance of the advertising potential and prestige of being located near an expressway.

Some of the variables used in this analysis, such as sewer facilities, water facilities and promotion, must be based primarily on subjective judgment. However, most can be rated objectively by numerical analysis. All ratings were reduced to a scale varying from 1 to 50 before they were entered into the multiple correlation equation.

The coefficients developed from the multiple correlation for the three most successful analyses are shown in Figure 2. The shaded band represents the coefficient of each variable, this being the relative importance of the various factors. Shown next to each shaded band is a black band which represents the level of significance of each variable. A value of one or more (that is, above the horizontal black line) indicates a highly significant correlation; below the value of one indicates questionable significance.

The first analysis, shown at the top, considered only six variables; that is, highway access, availability of land, tax rate, sewer facilities, water facilities, and rail service. For the second analysis the factor for water facilities was deleted and airport accessibility and promotion were added. The third analysis considered all nine factors. The absolute magnitude of the weights for the various analyses differ; however, the relative size of the weights is nearly identical.

Perhaps the most significant finding from this analysis is that transportation is not the dominant or controlling factor in shaping cities. With the mobility provided by the automobile, industries have been freed of distance limitations in choice of location and are now able to give more attention to other factors.

From these data it is evident that of prime importance to the location of new industry are availability of land, and sewer facilities. Highway access, rail service and airport access are second in order of importance. Of relatively minor importance are tax rate, water facilities, industrial land bordering freeways, and promotion.

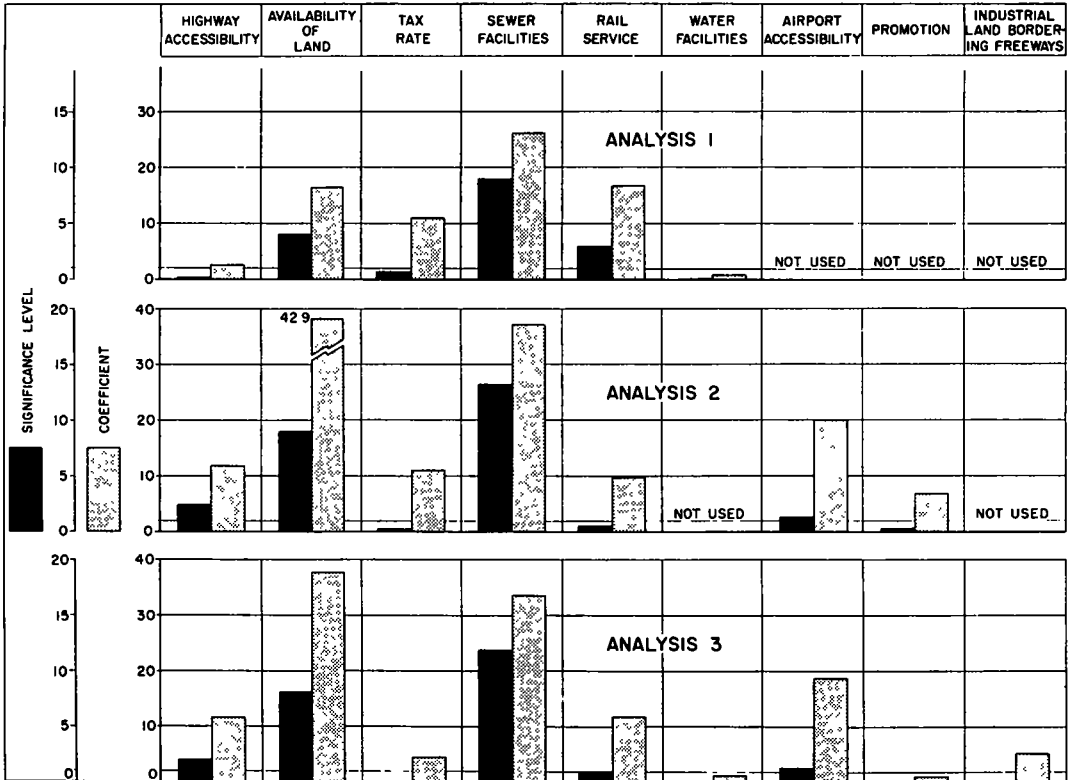


Figure 2. Multiple correlation analysis, Hartford Area Traffic Study.

From the formula developed through this analysis it is possible to calculate a growth index for each zone which, when compared to the sum of the growth indices for all zones, represents the percentage of total growth which can be expected in each zone. This formula is:

$$\text{Growth Index} = 12x_1 + 37x_2 + 5x_3 + 34x_4 + 12x_5 + 2x_6 + 19x_7 + 1x_8 + 5x_9 + 120.$$

in which the "x" values represent the magnitude of each of the 9 variables previously described and the coefficient of the "x" values is the relative importance of that variable, the height of the shaded bars in Figure 2.

The true test of an analysis such as this is how closely the formula thus derived can predict the actual growth of the area. To test this the actual known growth on a town-by-town basis was compared with the theoretical growth predicted by the formula. From this comparison it was found that in one more or less continuous corridor the actual growth was considerably higher than the theoretical growth predicted. Inasmuch as this particular corridor was the area where most of the industrial development was occurring, this higher than anticipated growth was attributed to a prestige factor for industrial development. To account for these differences a ratio of the actual to theoretical values was taken, these values were grouped by areas, and the resulting weights were used as adjustments in the formula for future predictions.

Actually, with further analysis, it is now believed that there may be a factor missing in the equation. This factor might be called a "self-generation factor" for manufacturing employment and could be related as a weight to the existing employment of the zone. The next step in the analysis of manufacturing employment must therefore be to study this in greater detail in an attempt to enter this factor in the multiple correlation analysis. This, it is believed, will be a significant addition to the equation.

Service Employment

Actually, relatively little is known about service employment except that it must be accessible to population for its labor supply and that, to a certain extent, it is related to commercial areas and tends to locate near them. Therefore, an index was developed which included both highway accessibility to population and a factor for holding this increase close to the retail areas. The highway accessibility index is the same as was used for the distribution of manufacturing employment in the preceding step. This index was multiplied by the retail trade employment for the preceding period and the product was used as a distribution factor for the distribution of the new service employment.

Retail Employment

For the study of retail trade employment the hypothesis was made that the future distribution was dependent solely on the distribution of the new increase in population. This was checked by projecting the increase from 1947 to 1955 on the basis of the hypothesis, and comparing the theoretical distribution thus obtained with the actual growth. The resulting comparison is plotted on a map of the study area (Fig. 3). It is seen that generally this is a good check; however, some of the towns in the core area have a disproportionate share of the increase, indicating that these may be developing into regional shopping areas.

Although this may be the case, as shown by this comparison, there is no past trend data to support this assumption—and in one case at least, all indications point the other way. Whereas it is recognized that there are some discrepancies in the assumption that retail trade employment follows population exactly, it is believed that this is as far as present knowledge can logically be expected to extend, and for the future prediction no adjustment was made.

An attempt was made to carry through this study the basic theme of using as few arbitrary correction factors as possible. It would be possible to account for all discrepancies noted in the past trend analysis simply by applying correction factors to the theoretical values. However, although such a procedure would result in good checks for past trends, each factor would have to be evaluated to determine if it would remain constant for the period of future projection, or whether, in reality the dis-

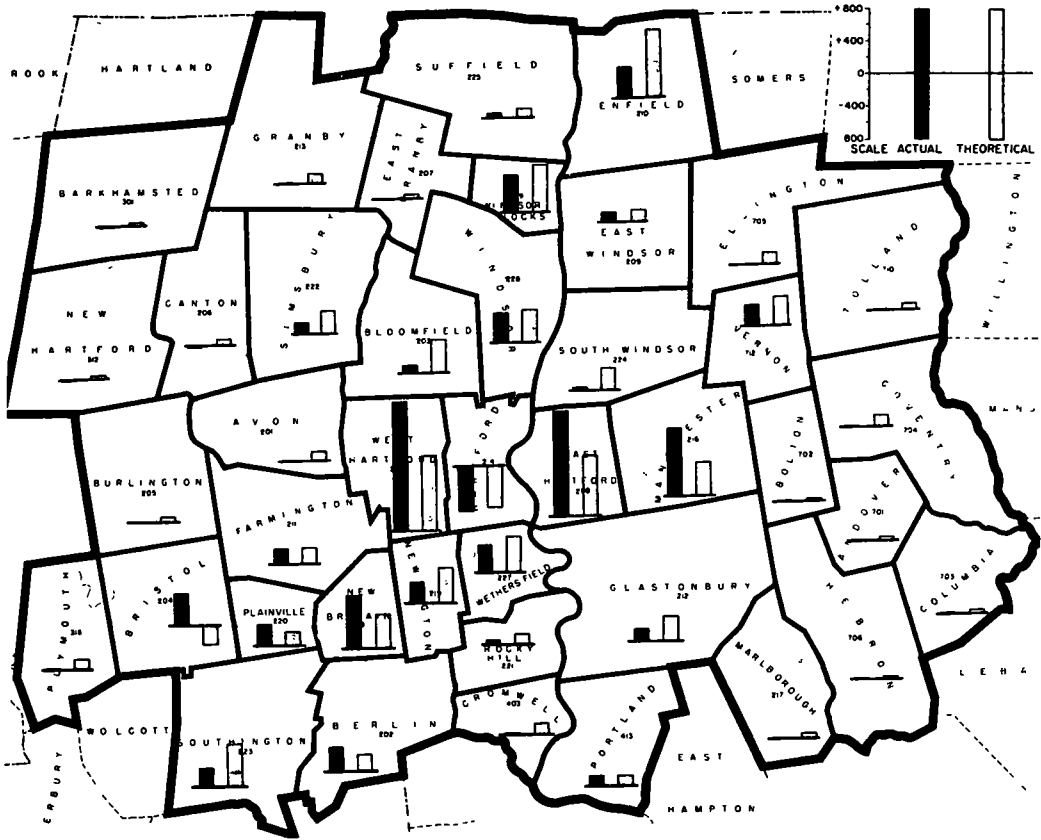


Figure 3. Retail employment change, 1947-1955, by towns—Hartford Area Traffic Study.

crepancy is not simply the result of an isolated change such as the opening of a new shopping center. For this reason no factors were used to account for small localized differences unless there was a very definite trend, and one which appeared to be stable and long term.

Similarly, no attempt was made to predict the location of new shopping areas as it was felt that realistically this was little more than a guess, and that the procedure used would result in a smaller error of estimate.

Population

As a first approximation in developing a rational method of distributing population growth it was hypothesized that the new population would distribute itself in accordance with highway accessibility to employment at the end of the study period and the "holding capacity" of the zone. In a manner similar to that used for manufacturing employment, predictions of the population growth were made for each town on the basis of accessibility and holding capacity only, and this theoretical value was then compared with the actual known growth. These values, for each town, are plotted on a map of the study area (Fig. 4). It is seen that there is considerable variation between these actual and theoretical values.

In an attempt to determine the unknown factors causing these differences, the aid of the residential developers in the area was sought. Questionnaires were mailed to all developers in the area with a request to rate any or all of the towns with which they were familiar. An over-all rating from one to five was requested for each town and in addition, the questionnaires requested that unfavorable conditions be noted. These returns, together with the researchers' knowledge of the area, formed a basis for explaining the variations noted on the map.

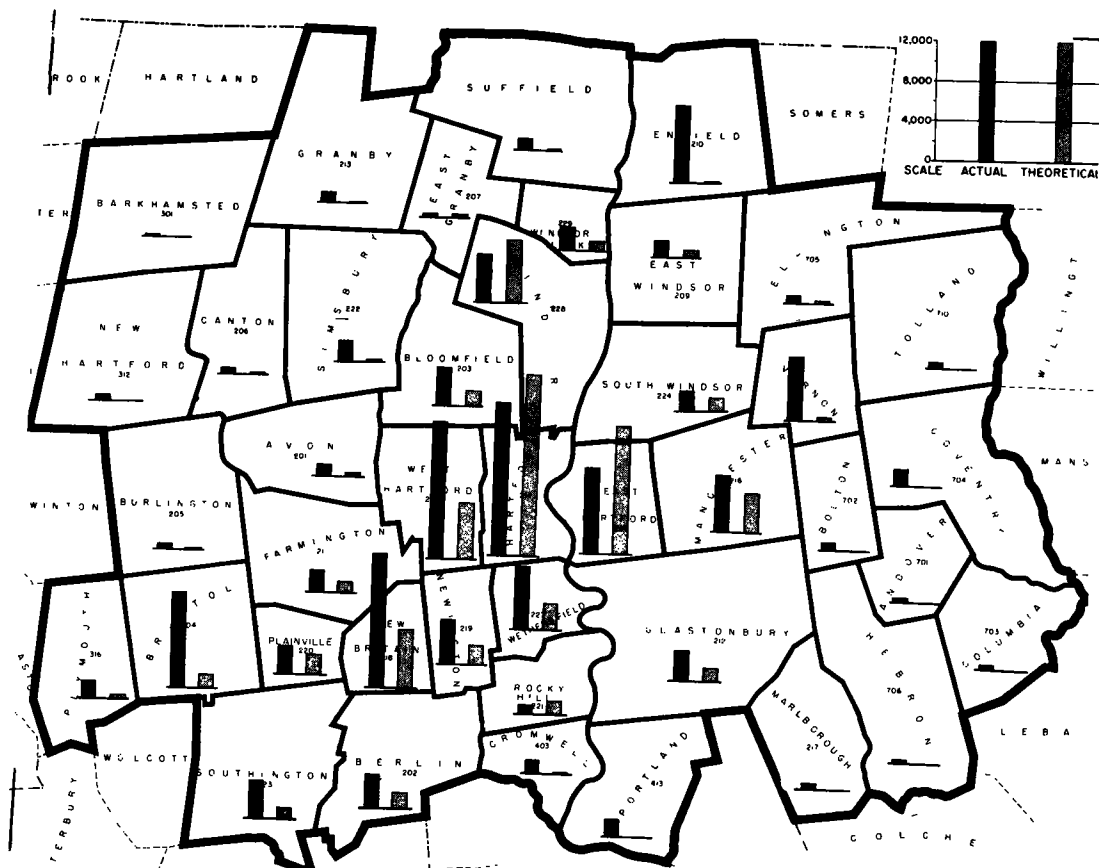


Figure 4. Population change, 1950-1958, by towns—Hartford Area Traffic Study.

For example, it was found that poor sewer and/or water facilities, or prohibitively costly installation, reduced the potential growth of a town by 50 percent. Unreasonable lot size, or house size controls, each resulted in a growth of only 75 percent of that which would have occurred under more reasonable conditions. Land shortage, or high land cost, reduced growth to 75 percent of that which might have occurred. Likewise, lack of large development tracts—that is, divided ownership—reduced growth by a factor of 0.75. In two towns it was noted that large land areas were being held for speculative purposes in the hope that prices would increase. This seemed to reduce growth to only 50 percent of that which would have normally occurred. On the plus side, the factors increasing growth appeared to be (a) lax building codes which triple growth, (b) picturesque home sites which double growth, and (c) prestige which may double or triple the rate of growth depending on the strength of this factor.

By studying the area on a town-by-town basis and applying these weights it was possible to account for most of the disparity between the actual and theoretical values to produce the comparison shown in Figure 5. This was the extent of the investigation as this provided a very close check. For projection into the future, these weights were investigated in an attempt to predict which could change, and more importantly, which were likely to change, with time. Perhaps the most important of the changing weights is prestige. Over a period of time it was observed that the prestige areas for residential development do change, generally moving toward the west as the suburban sprawl continues.

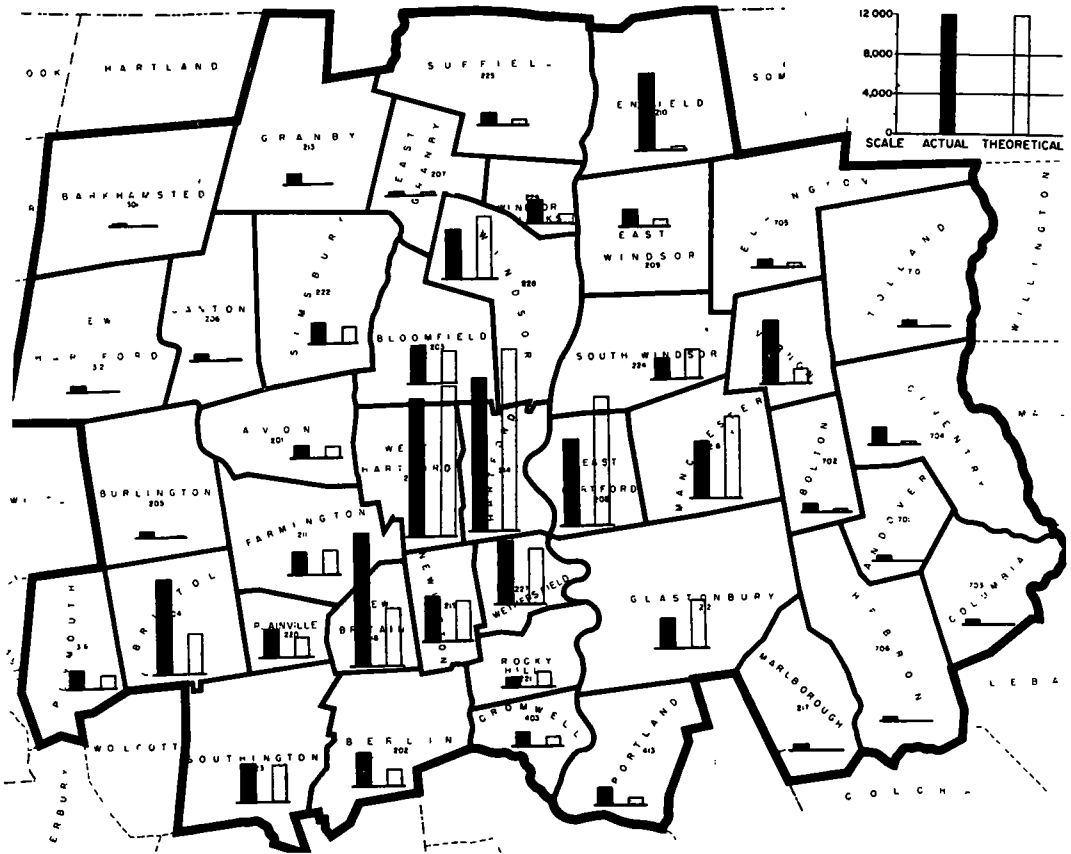


Figure 5. Population change, 1950-1958, by towns—Hartford Area Traffic Study.

Car Ownership

Although car ownership is not a land-use category, it is nevertheless a measure of land-use activity, and inasmuch as it is such an important part of the analysis, the study of these changing patterns is included here. Car ownership increases as a result of two factors: (a) more cars in the future due to a greater population, and (b) a higher rate of car ownership.

For the last 10 years or so, real income has been increasing at a rather constant rate of about 2 percent per year, nationally. This increase in buying power has enabled more families to own cars, and has also resulted in more two-car families. Of course, the car ownership rate will not continue to rise indefinitely; in fact, investigations in Beverly Hills and Washington, D. C., have shown that there appears to be a very definite ceiling on car ownership, above which it will not rise. This ceiling is different for various residential density classes. Figure 6 indicates the interrelationship between these factors for four densities of residential development. This chart shows that the rate of car ownership increases up to an income level of \$8,000 to \$10,000 per year, at which point it levels off at 0.6 cars per person for estate areas, 0.5 for suburban areas, 0.4 for two family residential development, and as low as 0.3 cars per person for apartment house developments. These ceilings check very well with the actual rates now noted for the high income level areas around Hartford for the corresponding density classes. In accordance with these curves, it is assumed that the car ownership rate for the present population will increase at a rate of three percent per year up to the ceiling car ownership rate for the particular density class.

The increase in the number of cars due to increases in population was handled simply by adding the new population to the zone at the maximum car ownership rate for that particular zone.

These assumptions on car ownership were checked by making a projection from 1950 to 1958 and comparing the projected estimates with the known growth. The theoretical and actual increases are plotted side by side on a map of the study area (Fig. 7). This is without doubt the best correlation made for the entire study, and it was felt that, at this stage, the technique could not be improved.

Land-Use Forecasting Technique

The way in which the various pieces of the land-use puzzle fit together into a whole for purposes of projection is an important part of the analysis. During

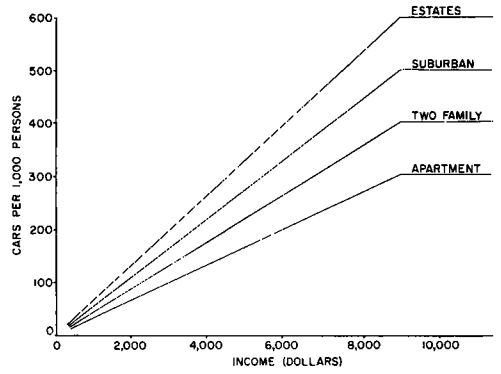


Figure 6. Car ownership ceiling for different residential density patterns. (Source Alan M. Voorhees, paper presented at Nice, France, September 1960.)



Figure 7. Car ownership change, 1950-1958, by towns—Hartford Area Traffic Study.

the course of the study it was found that some of the categories of land use were more "alert" to the influences of future change than others. By treating each category in the order of its "alertness", a very logical chain reaction of development resulted.

Increases in new employment—that is, the location of plants and offices—are the most alert to future changes which will affect their business interests. For this reason the increases in manufacturing and service employment were distributed first, and the factors determining these distributions were related to the future year. For example, the travel times reflected the assumed highway system for the target years.

After the new employment was "located" the population was distributed in relationship to it. The highway accessibility index for this distribution reflected the relationship of highways and employment in the target year.

Finally, the increase in retail employment was distributed in accordance with the increase in population just determined. Car ownership, of course, is also related to the new population distribution.

Briefly, then, the increases in employment were distributed first, it being assumed that this category was most alert to change. Second, because people choose their places of residence in relationship to their places of employment, the increase in population was distributed in this manner. Finally, the increase in retail employment, which follows population, was distributed last.

PROJECTION PROCEDURE

It is becoming apparent to the researchers close to the field of traffic forecasting that to obtain realistic estimates of future traffic, the feedback effect of changes in the highway network on the future land-use distribution must be considered. To accomplish this it is necessary to build the city up in steps over time. For example, instead of projecting from 1960 as a base year directly to the design year, 25 years or so hence, it is necessary to consider intermediate points and make projections in perhaps 5-yr intervals. The Hartford Area Traffic Study considered the years 1965, 1975, 1990 and the "horizon year", which in the Hartford area is about the year 2010. In this way the procedure was able to recognize the effect of the changes occurring throughout the period, such as the radically expanding highway system, which influences the future distribution of land use.

Such a procedure requires working through the entire projection process from beginning to end for each projection period and would result in a time-consuming, expensive operation unless a systematic, efficient flow of data from one process to the next were obtained. One advantage of the system developed for the Hartford Area Traffic Study is the complete interrelationship between the land-use and the traffic analyses. Figure 8 shows the flow of data through the process.

This diagram shows the continuous, orderly flow of land use and travel-time information toward the gravity model where the inter-zonal trips are calculated and thence into the traffic assignment. It is seen that in virtually every phase of the analysis, including the land use, travel-time information is required. The key to the process then is the tree building program, used to develop the inter-zonal travel times and trip traces, which is used over and over again, first in the land-use analysis for calculating the accessibility indices, again in the gravity model for calculating the distribution of trips, and finally in the assignment program for cumulating the link volumes. The feedback, previously

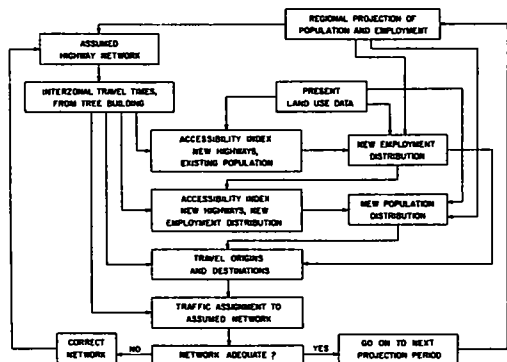


Figure 8. Flow diagram for Hartford Area Traffic Study.

mentioned, is readily apparent from Figure 8 also, for it is obvious that a major change in the highway network, such as the addition of a new expressway route, will have a profound effect on the entire process.

CONCLUSION

These investigations have shown that the traffic portion of the analysis—that is, the gravity model—is very reliable and given accurate land-use information will accurately predict the future distribution of trips. The land-use analysis, on the other hand, is probably the weakest link in the entire projection process. For this reason the major portion of the investigation for the Hartford Area Traffic Study was centered about the study of land use in an attempt to supply the most reliable information possible to the gravity model.

Although the work to date is a first step toward complete understanding, nevertheless, through this analysis many of the complex and diverse variables which affect the growth of metropolitan areas have been quantified. It is believed that the method is a vast improvement over previous methods, a step in the right direction which, with further work and study, can ultimately predict the future distribution of people and jobs with the same certainty with which population growth is now predicted.

In conclusion, the important part of the Hartford Area Traffic Study is the method of land-use analysis and the basic information learned from this analysis. Through this analysis the growth of a metropolitan area has been simulated mathematically, and it has been shown that cities grow as a result of rational decisions made by individuals which, when grouped together as a whole, are predictable. This study has afforded a great deal of insight into the future growth patterns and has, it is believed, pointed the way for future analysis. It is hoped that the work to date will form a base for additional research in this field which will be even more productive.

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