Time-Dependent Deflections of Prestressed Concrete Beams

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Two procedures, the "rate-of-creep" and the "superpositon" methods, are used for computing creep of concrete under variable stress from known relations for creep under constant stress and routine numerical procedures are presented for these methods.

These numerical methods are applied to two series of sustained load tests on prestressed concrete beams carried out under laboratory conditions. Measured deflections are compared with computed deflections and found to be in good agreement. The rate-of-creep method is then applied to a full size bridge beam and measured and computed deflections are again compared.

In the final section, the methods presented in the first part of the paper are used to investigate the possible ranges of time deflections in five representative composite highway bridges. A procedure is outlined for investigating the time-dependent deflections of prestressed concrete beams.

●ALL MATERIALS deform with time, although at different rates. The time-dependent deformation of concrete, which is ascribed classically to two different phenomena, creep and shrinkage, is such that the resulting dimensional changes within the life span of a structural member are usually appreciable. Consequently, prestressed concrete members undergo time-dependent deformations primarily as a result of the dimensional instability of the concrete. In addition, the relaxation of the prestressing reinforcement, the time-dependent loss of stress at constant strain which occurs when steel is subjected to a high sustained stress, results as an "elastic" change in the deformation.

Both practical experience and laboratory experiments have shown the need for being able to predict the time-dependent deformations of prestressed concrete members. Creep and shrinkage of the concrete combined with relaxation of the reinforcement cause loss of prestress, deflection of flexural members, and secondary stresses in continuous structures; and none of these can be ignored completely. In the early phases of the development of prestressed concrete as a building material, much emphasis was placed on the time-dependent loss of prestress which was considered to be the Achilles' Heel of prestressed concrete. The availability of high strength steel and the development of a better understanding of the behavior of prestressed reinforced concrete has put this problem in its proper perspective; partial loss of prestress is not necessarily accompanied by a loss in flexural strength, although the cracking moment may be reduced. More recently, interest in time-dependent effects has shifted towards the analysis of the deflections of flexural members, a consideration which has little to do with strength but which is frequently critical for serviceability.

A loaded prestressed concrete beam deflects with time under the influence of two effects: the nearly constant stresses caused by the load which tend to make it deflect in one direction, and the changing stresses caused by the prestress which tend to make it deflect in the other direction. By judicious design, these two opposing effects may be used by the engineer in such a manner that they offset each other. The primary objective of this paper is to present methods for the computation of time-dependent deflections so that the designer may use this concept to control such deflections, or, at least, to obtain a reasonable estimate of the maximum deflection.

No pretensions are made in the paper regarding the precise prediction of the qualitative or quantitative characteristics of creep, shrinkage, or relaxation as a function of time. In the analyses presented, these are assumed to be known. Because the sensitivity of these relationships, especially for creep and shrinkage, to various factors which are practically uncontrollable in engineering works is well-known, the adoption or admission of any expression giving the magnitude of creep and shrinkage would curtail unnecessarily the applicability of the analyses. Therefore, it is strongly recommended that, whenever the basic creep shrinkage and relaxation versus time relationships are not established under conditions similar to those imposed on the structure in question, deflections be computed for the possible upper and lower bounds of these relationships.

The stresses in prestressed concrete beams vary with time inasmuch as the prestress varies. Consequently, a knowledge of the relationship between creep strain and time under constant stress is not sufficient for the analysis of time-dependent deflections of prestressed concrete beams. It is necessary to know this relationship for varying stress. Because it is not practical to obtain creep strain relationships under every possible rate of varying stress, a hurdle in the analysis is the construction of a creep strain curve for a given "program" of stress variation from a creep strain curve obtained under constant stress. Two methods can be used to accomplish this, the "rate-of-creep" and the "superposition" methods. The first part of this paper is devoted essentially to a discussion of the relative merits of these two methods in view of the results of laboratory tests on beams under sustained load in a controlled environment. In addition, the applicability of the commonly used "reduced modulus" method is also considered, not because the writers feel it has possibilities but because of its time-honored place among methods of analysis for time-dependent deflections of reinforced concrete members. (These methods are described in this paper primarily from the point of view of their application in the analysis of deflections of prestressed beams. Ross (1) gives an excellent general discussion of these methods.)

A study of time-dependent deflections measured in eight laboratory tests on beams and in one highway bridge under actual working conditions constitutes the second part of the paper.

To give the reader some "feel" for the magitude of time-dependent deflections to be expected in highway bridges, and also to outline a routine method of analysis, deflections for a group of representative composite highway bridge girders are studied in the final part of the paper.

ANALYSIS

General Remarks

Time-dependent deformations in prestressed concrete are attributed to creep and shrinkage of the concrete and relaxation of the steel. Each of these is a function of numerous other effects. Creep and shrinkage are affected by almost every variable involved in the fabrication and loading of a member. In addition, exposure to different atmospheric conditions results in different rates and magnitudes of time-dependent deformations in the concrete. Relaxation of the steel may be affected by the composition of the steel and the manufacturing process as well as by the loading conditions.

In general, creep of the concrete is the predominant variable insofar as time-dependent deformations of prestressed concrete beams are concerned. It has been observed generally that creep strain is approximately proportional to stress for stresses up to about 50 percent of the concrete strength. Assuming that this relationship holds, time deformations of a member under a given constant stress can be computed by proportions if the time deformation relationship is known for a member under a different constant stress. In prestressed concrete members, the concrete stresses are varying continuously with time; consequently, the problem becomes more complicated. If the relation for creep strain versus time has not been determined for similarly varying conditions of stress, time deformations of a prestressed concrete member cannot be found by simple proportion but must be modified to take into account the variation of stress.

In the following paragraphs, three methods of computing time deformations are discussed. Two of the procedures, the "rate-of-creep" and the "superposition" methods, use a known relationship between creep strain and time at constant stress to predict the relationship under varying stress. The third method, the "reduced modulus" method, involves simply a reduction of the value of modulus of elasticity for the concrete.

Each of the three methods of analysis has advantages over the others under certain conditions. The reduced modulus method is often applied to reinforced concrete. If the stresses remain nearly constant, this method is a convenient one-step procedure for computing deformations. Because there are often large stress changes in prestressed concrete members, however, a one-step method is not reliable for computing time-dependent deformations, and either the "rate-of-creep" method or the "superposition" method should be used.

Reduced Modulus Method

This method uses a reduced modulus of elasticity for predicting creep strains in members. In order for it to be applicable, it is necessary that the creep strain versus time relationship for the concrete be obtained for the same variation of stresses as that which occurs in the member. If this is done, the reduced modulus method provides a convenient one-step method of predicting deformations. The reduced modulus is defined as:

$$E' = \frac{1}{\epsilon + c} \qquad (1)$$

in which

E' = reduced modulus of elasticity of the concrete;
ϵ = instantaneous strain of concrete under 1.0 psi stress; and
c = unit creep (creep strain per unit stress) at the time when strain is computed.

Once the reduced modulus has been determined, deformations are computed by ordinary methods.

The major disadvantage of this method lies in determining the proper relationship between strain and time. Even if the stress variation in a given member could be predetermined, the evaluation of the correct reduced modulus would involve a special test. Consequently the reduced modulus method is not convenient for predicting time-dependent deformations in prestressed concrete.

The Rate-of-Creep Method

The rate-of-creep method takes into account the fact that concrete stresses in a prestressed concrete member change with time. The relationship between creep strain and time is expressed as follows:

$$\frac{dC}{dt} = f_c \frac{dc}{dt} \qquad (2)$$

The total creep is then given by the expression:

$$C = \int_{t_1}^{t_2} f_c \frac{dc}{dt}\, dt \qquad (3)$$

in which

C = total creep strain between times t_1 and t_2;
c = unit creep strain (creep strain per unit stress), a function of time; and
f_c = concrete stress, a function of time.

Although this method does take into account changing stress, it does not consider the entire stress history of the member. Eq. 2 states that the concrete will creep at the same rate dc/dt regardless of whether stresses are increasing or decreasing. McHenry (2) found that changes in applied stress cause changes in the rate of creep which are not proportional to the change in stress. His findings indicated that if stresses are increased the unit rate of creep is increased, and if stresses are decreased the unit rate of creep decreases. For this reason the rate-of-creep method results in an overestimation of creep strains where concrete stresses are decreasing and an underestimation of creep strain where concrete stresses are increasing.

Although the rate-of-creep method can be expressed by Eq. 2, the solution of this equation is accomplished with ease only in the simplest problems. For this reason, it is convenient to use a numerical integration for the solution of most problems encountered in engineering. A simple numerical integration method for the analysis of beam deflections can be developed with the following assumptions:

1. The initial stress conditions are known for the member considered.
2. The unit creep (strain per unit stress) versus time relationship for constant stress is known and may be considered as a step function. (A step function is defined as a function which is represented graphically by a set of horizontal line segments with abrupt rise or fall from one segment to the next.)
3. The shrinkage strain versus time relationship is known for each fiber of the beams and may be considered as a step function.
4. The relaxation versus time relationship is known and may be considered as a step function.
5. Creep strain is proportional to stress up to a stress of 50 percent of the concrete strength.
6. Strains are linearly distributed over the depth of the cross-section.
7. Concrete has a linear stress-strain relationship up to 50 percent of the concrete strength under short-time loading.
8. Steel has a linear stress-strain relationship under short-time loading.

With these assumptions, the time-dependent deflection of a simply-supported prestressed beam can be computed in the following steps:

1. The gross increase in creep strain in the extreme fibers is computed by multiplying the stress in each fiber at the beginning of the time interval considered by the increment of unit creep strain for that interval.
2. The change in creep strain at the level of the steel is found on the basis of the assumption that strains are linearly distributed over the depth of the cross-section.
3. The total change in strain at the level of the steel is found by adding the creep strain (step 2) to the shrinkage strain increment for the corresponding time interval.
4. The loss in steel stress caused by creep and shrinkage is determined by multiplying the change in strain at the level of the steel (step 3) by the modulus of elasticity of the steel.
5. The total loss in steel stress for the interval considered is found by adding the loss due to creep and shrinkage (step 4) to the relaxation loss for the corresponding time interval.
6. The change in stress in the extreme fibers is found by considering the loss of steel stress as a load applied at the center of gravity of the steel and computing the corresponding "elastic" change in stress in the extreme fibers.
7. The "elastic" change in strain in the extreme fibers is computed by dividing the change in stress (step 6) by the "instantaneous modulus" of the concrete.
8. The net change in strain in the extreme fibers is found by finding the algebraic difference between the gross change in strain found in step 1 and the "elastic" change in strain found in step 7.
9. The stress in the extreme fibers at the end of the time interval is determined by finding the algebraic difference between the initial stress and the change in stress found in step 6.

Once the strain versus time relationship has been determined for a given member, curvatures can be computed and deflections found from the distribution of curvature along the span. Because most structures experience deformations from causes which can be separated conveniently, deformations can be computed for fictitious beams subjected to the separate stresses and the resulting deflections added algebraically. For example, in a highway bridge beam, separate calculations of deflections due to prestress, beam dead load, and slab dead load can be made and their results added algebraically at the appropriate times.

Because it is convenient to compute time-dependent deformations caused by "external" effects such as load and prestress separately, it is a simple matter to use different time versus unit creep relationships for loads applied at different times. In this way it is possible to eliminate a portion of the error inherent in this method. If the unit creep curve corresponding to the age at loading is used, a portion of the change in rate of creep caused by the external load is taken into account.

Superposition Method

The superposition method was first applied to time deformations of concrete by McHenry (2). He found that if stresses in concrete are changed, the increase or decrease in stress has the same effect as applying a stress of the same magnitude to an unloaded cylinder. Although it is not possible to express this method with a simple relationship as in the case of the rate-of-creep method, a numerical procedure can be used to superpose the effects of increasing or decreasing stresses.

The superposition method does take into account the entire stress history of the member. Because each incremental change in stress follows a new creep versus time curve, changes in the rate of creep are accounted for. To be able to use the method, it is necessary to know the creep strain versus time relationship measured from the time of application of any large change in stress. For small changes in stress, such as internally caused changes in prestressed concrete members, good results can be obtained by using the creep strain versus time relationship measured from the date of prestressing.

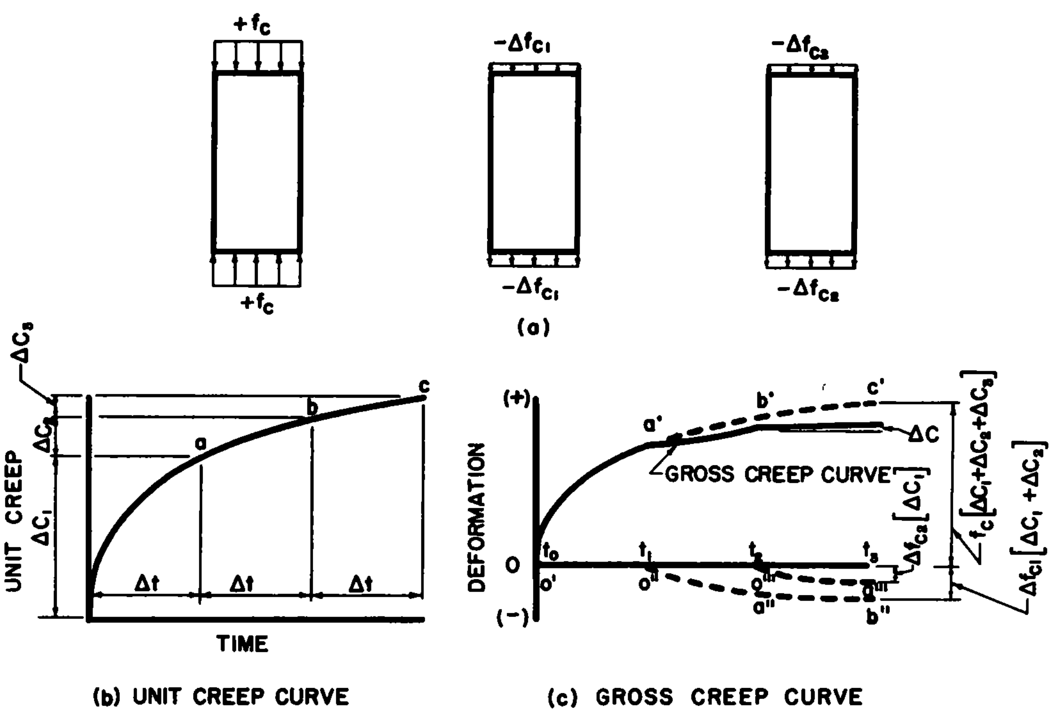


Figure 1. Illustration of gross creep computation by superposition method.

To illustrate the superposition method for computing gross creep, consider a concrete cylinder which has three different stress increments applied as shown in Figure 1(a). Stress $+f_c$ is applied at time t_0 and remains on the cylinder, an additional stress, $-\Delta f_{c_1}$ is applied at time t_1, and still another stress $-\Delta f_{c_1}$ is applied at time t_2. The final stress is $[f_c - \Delta f_{c_1} - \Delta f_{c_2}]$. The time intervals t_0 to t_1, t_1 to t_2, and t_2 to t_3 are each equal to the interval Δt shown in Figure 1(b). The unit creep curve for the cylinder is shown by line oabc in Figure 1(b).

The gross creep for each stress increment is found by multiplying the ordinates of the unit creep curve by the magnitude of the stress. The gross creep curve for the stress $+f_c$ is represented by line o'a'b'c' in Figure 1(c). At time t_3 the ordinate to this curve is $+f_c [\Delta C_1 + \Delta C_2 + \Delta C_3]$. In a similar manner, the gross creep curves for the stress increments $-\Delta f_{c_1}$ and $-\Delta f_{c_2}$ are shown by lines o'' a'' b'' and o''' a''', respectively, in Figure 1(c). The ordinate to curve o'' a'' b'' at time t_3 is $-\Delta f_{c_1} [\Delta c_1 + \Delta C_2]$ and the ordinate to curve o''' a''' at time t_3 is $-Af_{c_2} [\Delta C_1]$.

The gross creep curve for the total stress on the cylinder at any time is shown by the solid line in Figure 1(c). The ordinates to the gross creep curve are the algebraic sum of the ordinates to curves o' a' b' c', o'' a'' b'', and o''' a'''. From this it can be seen that the gross creep for the cylinder from time t_2 to time t_3 is the difference between the ordinates to the gross creep curve at times t_2 and t_3 and may be expressed by the quantity:

$$\Delta C = f_c (\Delta C_3) - \Delta f_{c_1} (\Delta C_2) - \Delta f_{c_2} (\Delta C_1)$$

The gross creep for other time increments can be computed in a similar manner.

To develop a numerical procedure for the superposition method, it is again necessary to make some assumptions regarding the behavior of the materials involved. The first eight assumptions are identical to those necessary for a solution by the rate-of-creep method. In addition, it is necessary to assume that deformations due to the separate stresses can be superposed and that the shape of the curve for creep strain versus time is known for loads applied at any time and may be considered a step function.

With these assumptions, the numerical procedure for the superposition method involves the following steps:

1. The gross increase in creep strain in the extreme fibers is computed by the method illustrated in Figure 1.
2. The change in creep strain at the level of the steel is found from the assumption that strains are linearly distributed over the depth of the cross-section.
3. The total change in strain at the level of the steel is found by adding the creep strain (step 2) to the shrinkage for the corresponding time interval.
4. The loss in steel stress due to creep and shrinkage is determined by multiplying the change in strain at the level of the steel (step 3) by the modulus of elasticity of the steel.
5. The total loss in steel stress for the interval considered is found by adding the loss due to creep and shrinkage (step 4) to the relaxation loss for the corresponding time interval.
6. The change in stress in the extreme fibers is found by considering the loss of steel stress as a load applied at the center of gravity of the steel and computing the corresponding "elastic" change in stress in the extreme fibers. This change in stress is considered as a newly applied stress in the next cycle of computations.
7. The "elastic" change in strains in the extreme fibers is computed by dividing the changes in stress (step 6) by the "instantaneous modulus" of the the concrete.
8. The net change in strain in the extreme fibers is found by finding the algebraic difference between the gross change in strain found in step 1 and the "elastic" change in strain found in step 7.

Although the assumption that the creep strain versus time curve has the same shape regardless of the time of loading is not correct, the assumption gives good results when the rate of change in stress is small. The method can be improved by determining separate curves for the time of loading corresponding to each change in stress. Because this involves either questionable assumptions or a great deal of laboratory work, it is practical only on extremely large projects.

Computations of deflections are made in the same manner as described for the rate-of-creep method. Separate computations are made for deflections caused by prestress and by individual externally applied loads. These separate effects are then superposed at the pertinent times and the net deflections obtained.

RESULTS OF TESTS AT THE UNIVERSITY OF ILLINOIS

Description of Specimens

Four prestressed concrete beams were subjected to sustained loads for a period of 2 yr. The beams were 4 by 6 in. in cross-section and had a clear span of 6 ft. Six 0.196-in. diameter prestressing wires were placed in each beam so that the center of gravity of the steel coincided with the kern limit of the cross-section (Fig. 2).

Type III portland cement was used in all specimens. The coarse aggregate had a maximum size of 3/8 in. and the sand had a fineness modulus of 3.30. Table 1 contains proportions of the mixes, slumps, 7-day compressive strengths, and moduli of elasticity for the concrete in each beam. Compressive strengths are based on 6- by 12-in. control cylinders.

The beams and cylinders were cured in the forms in laboratory air until the concrete had gained sufficient strength to allow release of the prestress wires. The wires were released when the computed bottom fiber stress in the beams was from 50 to 55 percent of the cylinder strength. The variation in concrete strength with time, as obtained

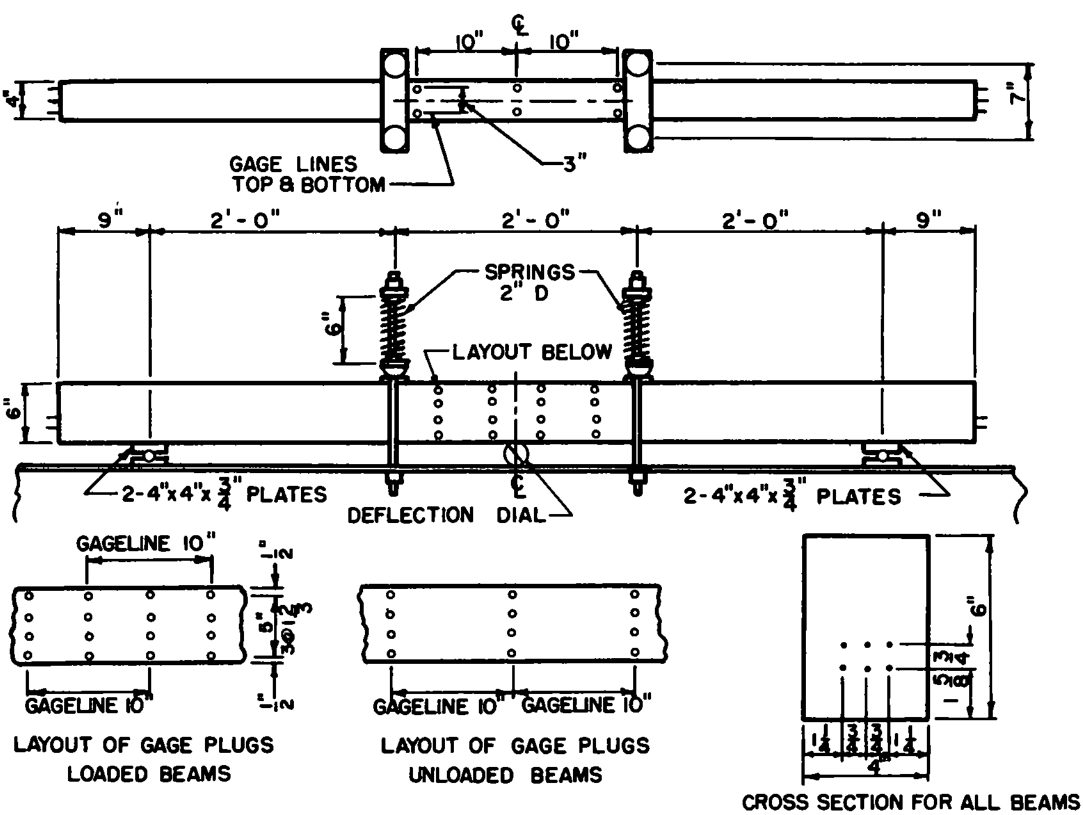


Figure 2. Dimensions and loading arrangement for beams, University of Illinois tests.

TABLE 1

PROPERTIES OF CONCRETE MIXES FOR UNIVERSITY OF ILLINOIS TESTS

Beam	Cement:Sand:Gravel (by wt)	Water:Cement (by wt)	Slump (in.)	7-Day Compressive Strength, f'_c (psi)	Modulus of Elasticity, E_c (psi x 10^6)	Cement Type
MU-1	1:2.98:3.35	0.76	3	4,070	3.08	III
MU-2	1:2.97:3.32	0.74	3	4,300	3.18	III
ML-1	1:2.97:3.36	0.75	2½	4,170	3.08	III
ML-2	1:2.96:3.33	0.80	5	3,500	2.70	III

from compressive tests of ten concrete cylinders over a period of 28 days, is shown in Figure 3. Table 2 gives the chronology of the various operations involved in the preparation of the test specimens.

Two samples of the prestressing wire used in the beams were used to determine the stress-strain curve shown in Figure 4. Two other lengths of the same wire were placed in steel frames at stress levels of approximately 51 to 55 percent of the ultimate strength to determine their relaxation properties (3). Plots of relaxation versus time for the two specimens are shown in Figure 5.

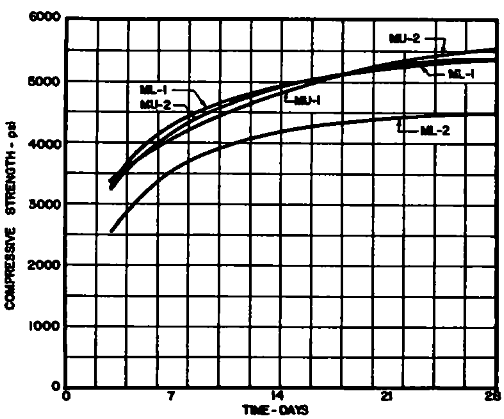


Figure 3. Concrete strength vs time based on 6- by 12-in. cylinders.

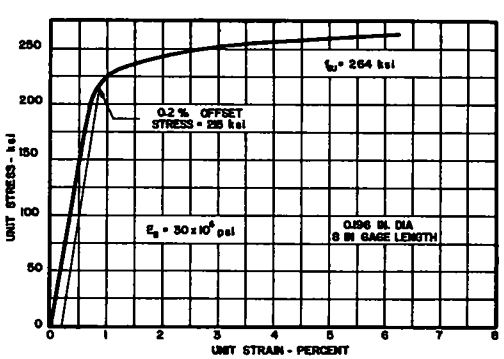


Figure 4. Stress-strain relationship for prestressing wire.

Four 4- by 16-in. control cylinders were also cast for each beam. One of the 4- by 16-in. cylinders from each batch was loaded to a stress of 2,000 psi in the sustained-loading rig shown in Figure 6. This stress was maintained throughout the duration of the beam tests to determine time-dependent strains of the concrete under constant stress. Because 30 to 40 min were required to load the cylinders in the special rigs, companion specimens were loaded rapidly to 2,000 psi in a screw-type testing machine and kept under constant load for one week so that early strain readings of the cylinders in the special testing rigs could be compared with those for the cylinders loaded "instantaneously". No significant difference was found for the two methods of loading. Figure 7 shows the relationship between time and total measured strain for each of the cylinders loaded in the special frames.

The remaining two 4- by 16-in. cylinders cast from each batch were left unloaded in order to measure time-dependent strains under zero applied stress. The time-strain relationship for the unloaded cylinders for each beam is shown in Figure 7.

The four test beams were placed in the special loading frame shown in Figure 8. Two of the beams, MU-1 and MU-2, were placed in the lower berths and had no externally applied loads. These beams were designed for a nominal prestress of 2,000 psi in the bottom fiber and zero in the top fiber. Computed stress distributions based on the measured effective prestress are shown in Figure 9.

TABLE 2

TEST CHRONOLOGY FOR UNIVERSITY OF ILLINOIS TESTS

Beam	Date of Tensioning of Wires	Date of Casting Beams and Cylinders	Date of Release of Prestress in Beam	Beam Installed[a] on Storage Frame	Beam Loaded[a] at Third Points	4 by 16-in.[a] Cylinder Loaded in Testing Machine	Initial[a] Reading on Shrinkage Cylinder	4- by 16-in.[a] Cylinder Loaded in Frame
MU-1	4-11-57	4-12-57	4-17-57	+3.0	-	+44.5	- 4.0	+19.0
MU-2	4-18-57	4-19-57	4-24-57	+3.5	-	- 3.5	+ 6.5	+28.5
ML-1	5-1-57	5-2-57	5-7-57	+3.0	+4.5	- 4.5	+ 7.0	+22.0
ML-2	5-9-57	5-10-57	5-17-57	+5.5	+6.0	- 1.5	+15.0	+14.0

[a] Hours after release of prestress (+).
Hours before release of prestress (-).

Beams ML-1 and ML-2 were placed in the two upper berths of the special frame and were loaded by means of four 2-in. diameter steel springs. Spring deflections were measured with a direct reading compressometer equipped with a 0.001-in. dial indicator. Load was applied by tightening the nuts on ½-in. diameter rods bolted to the frame angles.

Beam ML-1 was designed so that the net stress under load would be 2,000 psi in the top fiber in the middle third of the span. Beam ML-2 was designed for a constant stress under load of 1,000 psi throughout the depth of the cross-section in the middle third of the span. The computed stresses based on the measured effective prestresses are shown in Figure 9.

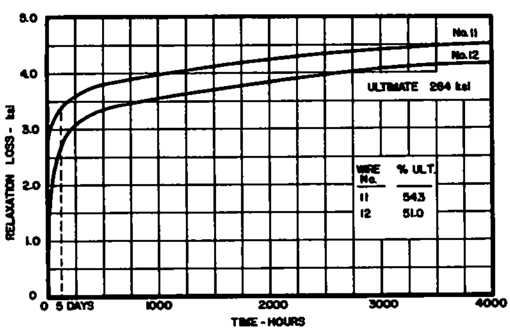


Figure 5. Plot of relaxation loss vs time for prestressing wire.

After the prestress was released, the forms were removed from the beam and the cylinders removed from their molds. The beams and cylinders, except the cylinders loaded for one week in the screw-type testing machine, were moved into a controlled temperature and humidity room within 24 hr after the release of prestress. This room is kept at a constant 50 percent relative humidity and 75 F temperature by means of automatic equipment.

The tensioning force in each prestress wire was determined by measuring electrically the compressive strain in aluminum dynamometers placed on the wire between the nut and the bearing plate at the end of the beam opposite that at which tension was applied.

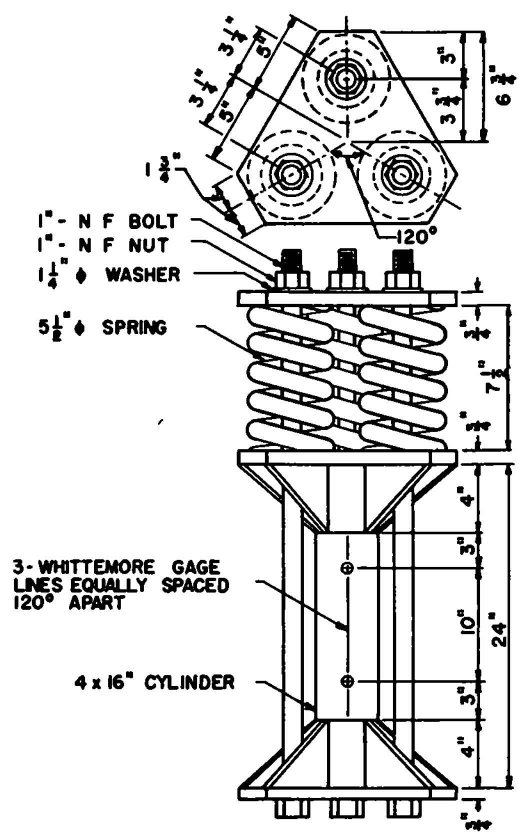


Figure 6. Dimensions and loading arrangement for the 4- by 16-in. cylinders.

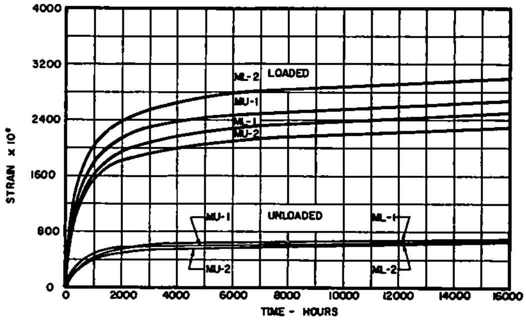


Figure 7. Measured total strains in 4- by 16-in. cylinders.

All concrete strains were measured with a 10-in. Whittemore strain gage. Each beam had four gage lines on each side, as shown in Figure 2. The unloaded beams had one set of four gage lines on each side of the centerline while the loaded beams had two sets of gage lines overlapping the centerline 5 in. The gage plugs were embedded in a high-strength gypsum plaster flush with the surface of the concrete.

Instantaneous and long-time deflections were measured at midspan of each beam with a 0.001-in. dial indicator.

For measuring strains in the 4- by 16-in. control cylinders, three sets of gage lines were arranged symmetrically about the circumference as shown in Figure 6. Holes were drilled in the cylinders and gage plugs were set in the same manner as those in the beams. For the 4- by 16-in. cylinders which were put in the special loading rigs, applied loads were measured by the deflection of the three railroad car springs (Fig. 6). Type A-7 SR-4 strain gages applied to opposite sides of the steel tie rods were used to measure loads during their initial application.

After the beams and cylinders were placed in the loading frame, deflection and strain readings were taken at time intervals corresponding to approximately equal increments of deflection. On the average, readings were taken every day for the first four days, every two days for the next four days, every week for the next two weeks, every three weeks for the next nine weeks, and every six months thereafter. Strains in the unloaded cylinders were read less frequently because of the relatively small strains involved.

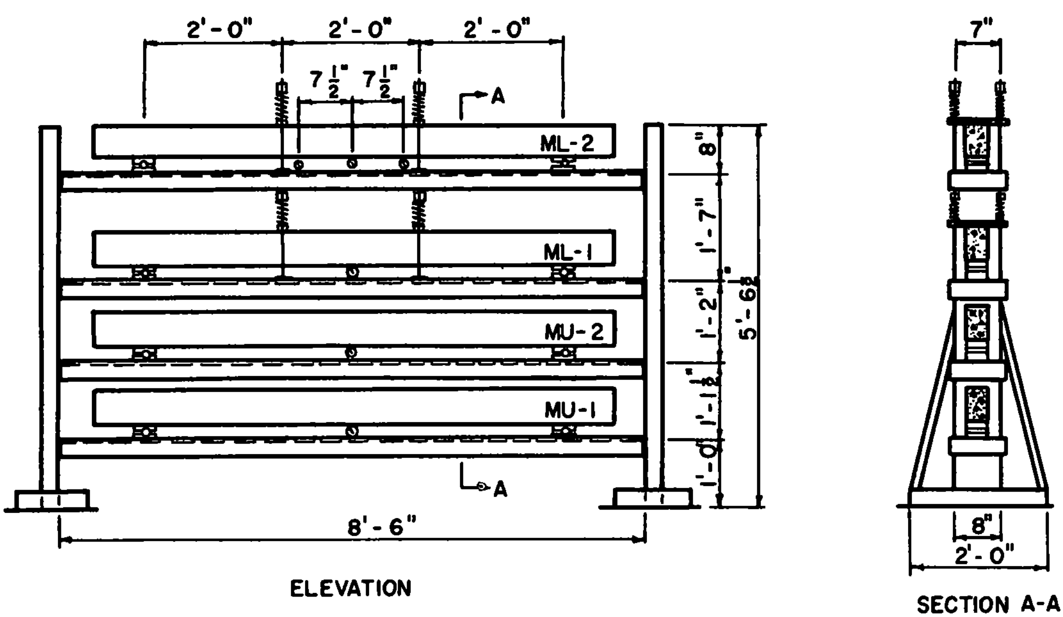


Figure 8. Beams in place on storage frame, University of Illinois tests.

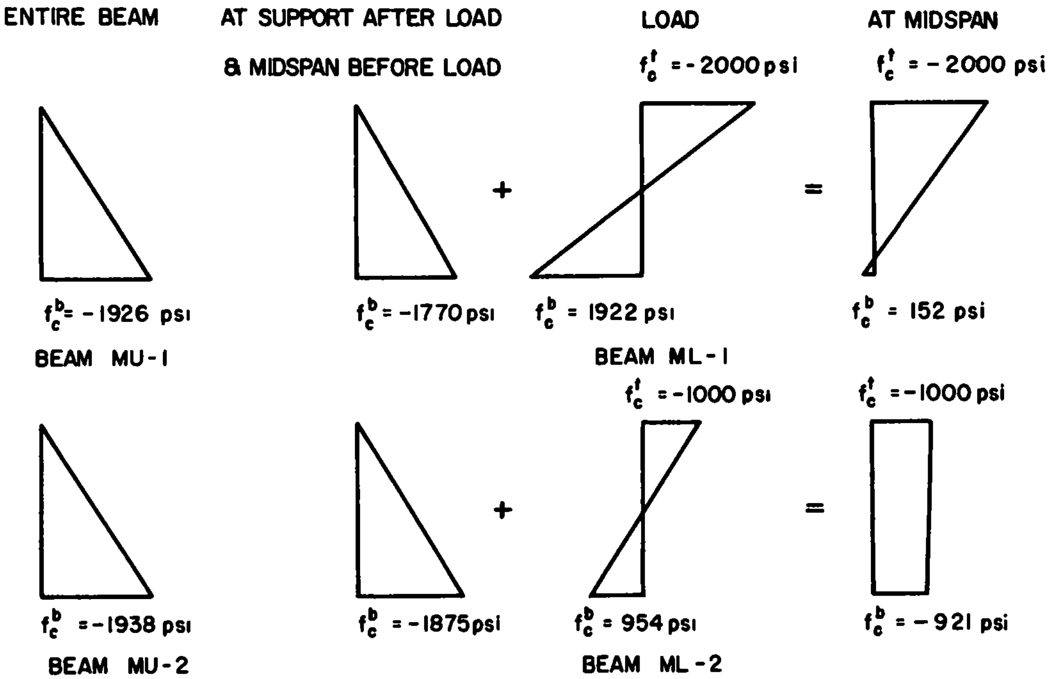


Figure 9. Computed stress distributions in beams.

Comparison of Measured and Computed Results

Figure 7 shows the measured strains in the 4- by 16-in. control cylinders. The origin of the curves for the loaded cylinders refers to the time of loading of the cylinders (Table 2). The ordinates to these curves do not include the "instantaneous" strains caused by the application of load, but represent the sum of creep and shrinkage strains. The origin of the curves for the unloaded cylinders is referred to the time immediately after the beams were placed in the loading frame.

Unit creep curves for the cylinders corresponding to each beam are shown in Figure 10. These curves were obtained by subtracting the measured strain in the unloaded cylinders at a given time from the measured strain in the loaded cylinders at the corresponding time and dividing the result by the applied stress. The origin of the unit creep curves refers to the time immediately after the cylinders were loaded (Table 2).

Distributions of beam strains at given time intervals are shown in Figure 11. The strain at each level is the average of readings from four gage lines, two on each side of the beam. The strains "before prestress" are represented by the vertical zero strain line. All other strain distribution lines represent the difference in readings after the wires were released and the readings "before prestress." For beams MU-1 and MU-2, time is measured from the release of prestress. Zero time for the loaded beams, ML-1 and ML-2, is the time at which the load was applied. Although some variation from a straight line was found in individual rows of gages, the average strain distribution was linear throughout the depth of the cross-section.

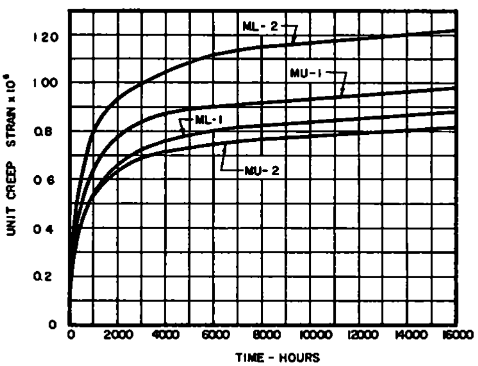


Figure 10. Unit creep relationships for University of Illinois tests.

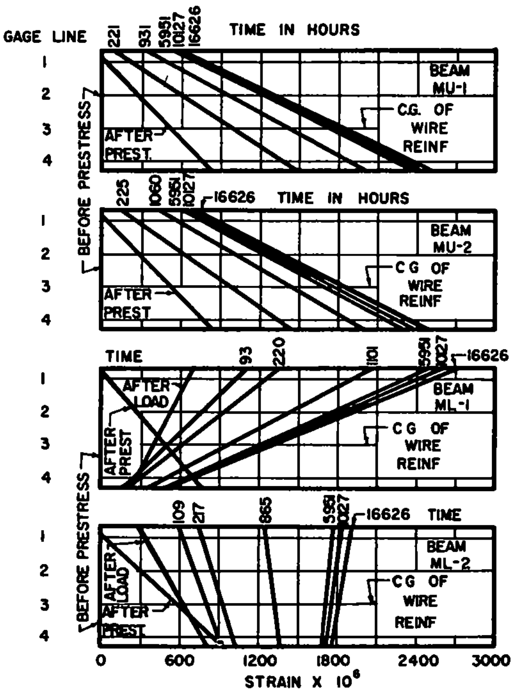


Figure 11. Successive strain distributions for University of Illinois tests.

Figures 12, 13, 14, and 15 show plots of midspan deflection versus time for beams MU-1, MU-2, ML-1, and ML-2, respectively. In all of these figures, the origin corresponds to the position of the beam before release of the prestress. Upon release, there was an instantaneous upward deflection which is indicated by the ordinate at zero time marked "after prestress." Three hours were required to transfer each beam to the loading frame. During this time, deflections were not recorded. Time-dependent deflections for the unloaded beams, MU-1 and MU-2, are referred to the reading taken immediately after positioning the beam in the loading frame. For the loaded beams, time-dependent deflections are referred to the ordinate at zero time marked "after load." This ordinate corresponds to the deflection after the load had been applied.

Time-dependent deflections of the four beams were computed by both the rate-of-creep and superposition methods. The computations were made in accordance with the methods described previously. Initial stress conditions for the beams were assumed to be as shown in Figure 9. The unit creep, shrinkage, and relaxation relationships used were those shown in Figures 10, 7, and 5, respectively. Other properties of the beams used in the computations are given in Table 3.

It was assumed that the curvature caused by the prestress was constant along the span of the beam. Curvature caused by the load was assumed to be constant in the middle third of the span and to vary linearly from the value at the load points to zero at the supports.

For both methods of computation, deflections of the loaded beams were computed for the beam under prestress alone and under load alone, and the total deflection taken as the algebraic sum of these two effects. Dead load of the beam was not considered in any of the computations. Deflections computed by the rate-of-creep and superposition methods are shown with the measured deflections in Figures 12-15.

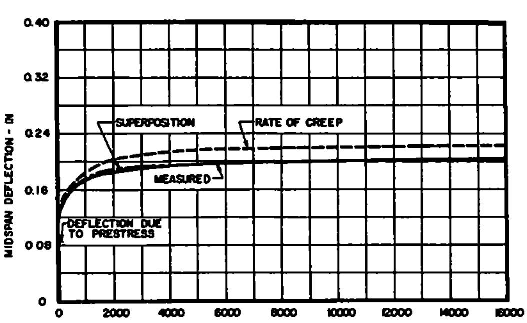


Figure 12. Measured and computed deflections for beam MU-1.

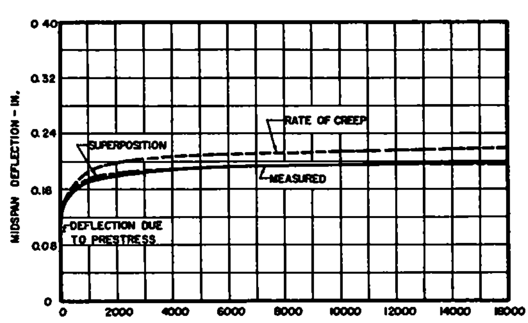


Figure 13. Measured and computed deflections for beam MU-2.

TABLE 3

PROPERTIES OF BEAMS FOR UNIVERSITY OF ILLINOIS TESTS

Beam	Width, b (in.)	Over-all Depth h (in.)	Effective Depth d (in.)	Number of Wires	Area of Steel A_s (sq in.)	Clear Span (ft)	Cylinder Strength at Release (psi)	Initial Prestress (ksi)	Total Load (lb)
MU-1	3.96	5.94	4.00	6	0.181	6	3,760	149.4	0
MU-2	3.96	5.94	4.00	6	0.181	6	3,930	149.4	0
ML-1	3.96	5.94	4.00	6	0.181	6	3,800	137.4	4,000
ML-2	3.96	5.94	4.00	6	0.181	6	3,550	149.4	2,000

Comparison of Computed and Measured Deflections

Figures 12 and 13 show that the rate-of-creep method overestimated the deflection of the unloaded prestressed concrete beams. This may be ascribed to the fact that the rate-of-creep method does not consider the entire stress history of the member. As pointed out previously, this method predicts that the concrete will creep at the rate dc/dt regardless of the stress level during the previous increment. Because decreasing stresses decrease the rate of creep, the rate-of-creep method predicts a deflection greater than that which actually occurs.

The superposition method, which considers the stress history of the concrete, gave a better prediction of the deflection of the unloaded beams.

Figures 14 and 15 show that both the rate-of-creep and superposition methods underestimated the downward deflection of the loaded beams. Because time deflections for the loaded beams were computed by first finding the upward deflections of the beam due to prestress alone and adding to these the downward deflections of a fictitious beam under external load alone, the differences between measured and computed final deflections are a combination of errors in the computations for upward and downward deflections. The rate-of-creep method overestimates the amount of deflection in a member with decreasing stress; therefore, error in computations made by this method will depend partially on the relative magnitudes of the upward and downward deflections. Because the stresses are decreasing both in the beam subjected only to prestress and in the fictitious beam subjected only to external load, the rate-of-creep method overestimates the magnitude of both the upward and downward deflection. These errors tend to cancel each other in the loaded beams; consequently, the inherent errors should have a relatively small effect on the net deflections.

Because the net deflection of the loaded beams was downward, it would be expected that the rate-of-creep method would overestimate the downward deflection. Because both methods of computation actually underestimated the deflection, it is possible that there is another source of error. One

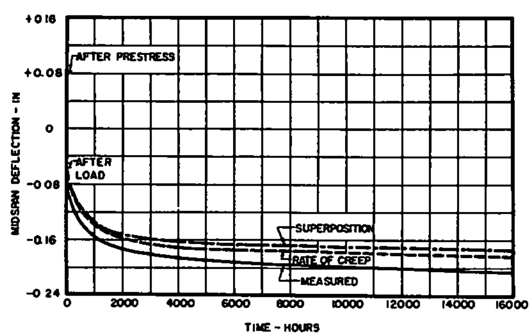


Figure 14. Measured and computed deflections for beam ML-1.

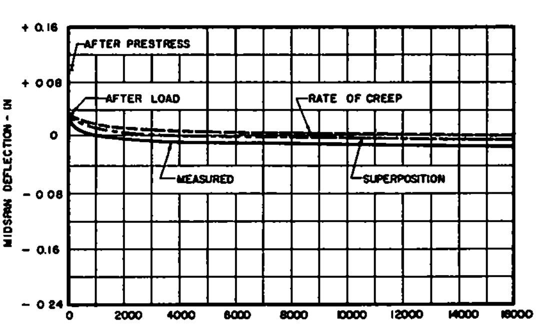


Figure 15. Measured and computed deflections for beam ML-2.

possibility is that the applied load was in error. However, the decrease in spring load caused by deflection of the beams was measured and taken into account in the computations. The unit creep and shrinkage curves are another possible source of error. However, the agreement of measured and predicted deflections for the unloaded beams indicates that the methods used to determine these curves were sufficiently accurate. If this is true, then it can be inferred that the computation of the upward deflections is probably correct, but some source of error is present in the computation of the downward deflections.

The deflections computed by the superposition method stand in the expected relation to those computed by the rate-of-creep method. Considering the many variables concerned, it is believed that either method gives as good a prediction of the time deflections of the test beams as can be expected.

The total deflection of a member may be found by the rate-of-creep method without knowing the shape of the unit creep curve, if the relations between the unit creep curve and the shrinkage and relaxation curves are known. Consequently, computations can be made using equal increments of unit creep and corresponding increments of shrinkage and relaxation. Once the computations are made, the deflection history of the beam can be obtained, if desired, by plotting the points in accordance with an assumed shape of the unit creep curve.

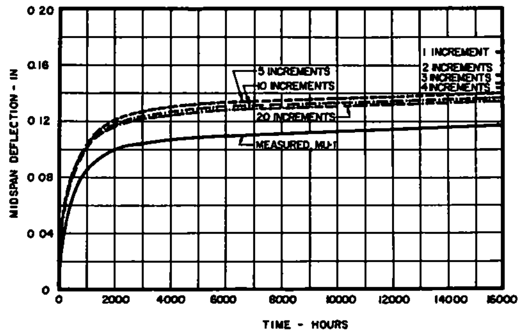


Figure 16. Effect of variation of strain increments for rate of creep method.

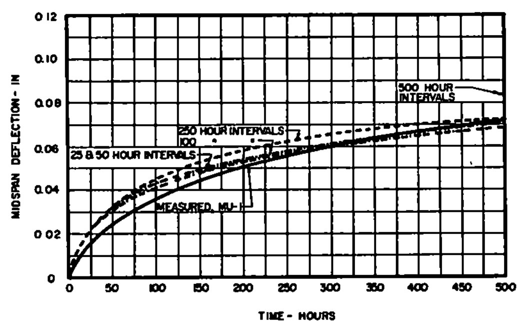


Figure 17. Effect of variation of time intervals for superposition method.

To determine the effect of the number of increments on the accuracy of the rate-of-creep method, several sets of computations were made using various numbers of increments. Figure 16 shows a comparison of measured deflections for beam MU-1 and deflections computed by the rate-of-creep method using various numbers of strain increments. From this comparison it can be seen that increments equal to about

TABLE 4

PROPERTIES OF BEAMS FOR UNIVERSITY OF FLORIDA TESTS

Beam	Width, b (in.)	Over-all Depth, h (in.)	Effective Depth, h (in.)	Area of Steel, A_s (sq in.)	Clear Span (ft)	Concrete Strength, f'_c (psi)	Modulus of Elasticity, E_c (psi x 10^6)	Initial Bottom Fiber Stress[a] (psi)
1	10	12	8	1.32	25	6,500	5.20	-2,400
2	10	12	8	1.32	25	6,500	5.20	-2,400
3	10	12	8	1.32	25	6,500	5.20	-1,500
4	10	12	8	1.32	25	6,500	5.20	-1,500

[a]Compressive stress (-).
Tensile stress (+).

one-tenth or less of the total unit strain will give answers with little error. Good results can be obtained from as few as three increments but the errors become relatively large when only one or two increments are used.

Computations for deflections are more easily made by taking time increments rather than strain increments when using the superposition method. In this method, the shape of the unit creep curve has a direct effect on the deflection computations; therefore, there is no advantage in taking equal strain increments as a basis for computation intervals. A comparison of measured deflections with those computed from the superposition method using various magnitudes of time intervals is shown in Figure 17. The curves are shown only for the first 500 hr of the test. These curves indicate that time increments up to 250 hr will give satisfactory results. This corresponds to a strain increment of about one-fifth the total. For larger time increments, the errors again become large.

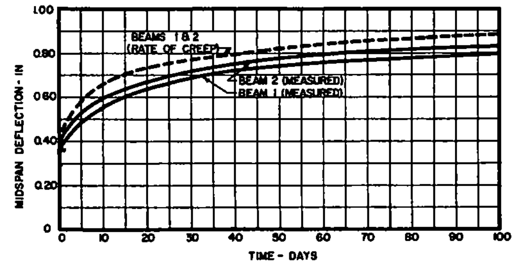


Figure 18. Measured and computed deflections for beams 1 and 2 of University of Florida tests.

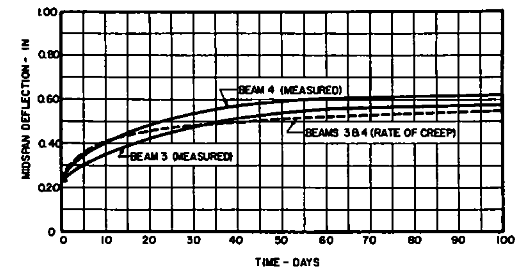


Figure 19. Measured and computed deflections for beams 3 and 4 of University of Florida tests.

RESULTS OF TESTS AT THE UNIVERSITY OF FLORIDA

Description of Tests

The results of a series of tests investigating time-dependent deformations of prestressed concrete beams have been reported by Ozell and Lofroos (4). This series consisted of four post-tensioned beams with a span of 25 ft. Each beam was 10 by 12 in. in cross-section and was post-tensioned by three straight, unbonded, "Stressteel" bars. The reinforcement was arranged so that its center of gravity coincided with the lower kern limit of the cross-section. In addition to the four beams, two shrinkage speciments, 10 by 12 in. in cross-section and 5 ft long, and thirty 6- by 12-in. cylinders were cast. Ordinary stone with a maximum size of 1 in. was used for coarse aggregate. All specimens were cured 5 days under wet burlap. After 5 days, the forms were stripped and the beams remained in the open laboratory for the rest of the test period. The duration of the test period was in excess of 100 days.

Beams 1 and 2 were post-tensioned 20 days after casting and beams 3 and 4 were post-tensioned 34 days after casting. Computed bottom fiber stresses were 2,400 psi in beams 1 and 2 and 1,500 psi in beams 3 and 4. The beams were tested under the effect of post-tensioning forces and beam dead load only. The only difference between beams 1 and 2 and beams 3 and 4 was the magnitude of the post-tensioning force.

Comparison of Measured and Computed Results

The rate-of-creep method has been used to predict the deflections of these beams. To use this method, unit creep, shrinkage, and relaxation properties of the materials must be known or assumed. Because the tests did not include the determination of all of these properties, it was necessary to estimate them.

At the University of California, tests of up to 30 yr duration have been made on a large number of concrete specimens subjected to varied conditions of load and storage and having widely different properties (5). The envelope of unit creep curves for these tests fell within a very small range. It was found that the curve for the first 720 days under load had the shape:

$$\epsilon = K \log_e (t + 1) \quad (4)$$

in which

ϵ = fraction of total creep;
K = "creep coefficient," or slope of curve on semilogarithmic plot; and
t = time in days.

These tests also indicated that it took an additional 18 yr to add 15 percent to the creep at the end of 2 yr. Because the increase in creep after 2 yr is small, it appears acceptable to assume that all of the creep takes place in the first 720 days. The shape of the unit creep curve can be then described by the following expression:

$$\epsilon = 0.152 \log_e (t + 1) \quad (5)$$

Although the shape of the shrinkage and relaxation curves does not have as large an effect on the calculated deflections as the shape of the unit creep curve, it is still necessary to use some reasonable method of approximating their shapes. In view of the many variables involved and the small effects these shapes have on computed deflections, the assumption can be made that the shrinkage and relaxation curves have the same shape as the unit creep curve.

After the shapes of the curves have been assumed, it is necessary to estimate the magnitudes of the unit creep, shrinkage, and relaxation. Troxell, Raphael, and Davis (5) point out that creep strain at 90 days may range between 40 and 70 percent of the 20-yr creep, and that the 90-day shrinkage strain may range between 40 and 80 percent of the 20-yr shrinkage. Using this as a guide to extrapolate the test results, it appears that a creep strain of two times the elastic strain and a shrinkage strain of 0.0003 would be reasonable values. A value of 4 percent was assumed for relaxation loss. The instantaneous modulus of the concrete as measured from 6- by 12-in. cylinders in the Florida tests was about 5.2×10^6 psi when the beams were loaded. Measured deflections of the beams indicated a slightly lower value, but these deflections included a small amount of creep. The higher value of "instantaneous" modulus was used in the rate-of-creep computations.

Using these assumptions, the time-dependent deflections were computed and plotted with the test results. The results for beams 1 and 2 are shown in Figure 18 and the results for beams 3 and 4 are shown in Figure 19.

Comparison of Computed and Measured Deflections

Lofroos and Ozell concluded that for beams containing the same concrete at different stress levels, large variations can occur in the "apparent modulus of elasticity" (4). Figures 18 and 19 show that the time-dependent deflections computed, by the rate-of-creep method compare closely with the measured deflections. Moreover, consistent results were obtained with only one set of assumptions for the time-dependent properties of the materials.

This comparison shows that time-dependent deflections which are computed by simply using a reduced modulus of elasticity can lead to inconsistent results for members in which stresses vary. Because the rate-of-creep method does consider the stress history of the member, more consistent results can be obtained.

MEASURED DEFLECTIONS OF A HIGHWAY BRIDGE

Description of the Bridge and Measurements

The time-dependent deformations of one beam of a highway bridge in North Africa have been reported by J. Delarue (6). This beam was a post-tensioned T-section, 28 m long and 2 m deep. The cross-sectional properties of the beam at midspan and at the supports are shown in Figure 20. The post-tensioning reinforcement consisted of seven cables each of which contained 20 wires 8 mm in diameter. The cables were draped in the shape of a parabola and had an initial tension of 140,000 psi. The concrete strength at 28 days was 8,200 psi. In addition to the test beam, several control specimens were cast from the same concrete mix. The control specimens were one meter long and were 12, 15, or 17 cm square in cross-section. One-half of these specimens were loaded axially to a compressive stress of 120 kg/cm^2 (1,710 psi) and were kept at this stress for the duration of the test. The remaining prisms were left unloaded and were used to determine the shrinkage of the concrete under zero external load. The concrete in all specimens was made with ordinary aggregate.

Post-tensioning operations were begun 60 days after the beam was cast, and required seven days to complete. After post-tensioning, the beam was supported on a clear span of 27.6 m. The beam remained on these supports for 12 months after which it was moved to the bridge site and placed in its final position. During the transportation of the beam to the bridge site, the beam was prestressed externally to prevent cracking at the top. The move was completed in one day. After the beam was put in place, 9-in. thick slab strips were cast beween the beams. Casting and curing of the slab were completed in two months. Construction equipment was then placed on the bridge to simulate a sustained live load. After three days, the construction equipment was removed and the bridge was opened to traffic approximately 500 days after the beam was cast. Observations made for the first 650 days were reported by Delarue (6). This paper reports the deflection data for 1,000 days (10). The observations are still being continued.

Computation of Deflections

The numerical procedure for the rate-of-creep method was used to analyze this beam. Because laboratory tests were made to determine the shrinkage and creep properties of the concrete, the results of these tests were used in the computations. The laboratory results obtained from control specimens of different sizes were in good agreement. Consequently, measured deformations of the control specimens which were 15 cm square in cross-section were taken as representative of the time-dependent properties of the concrete.

The curves for time versus shrinkage and time versus unit creep used in the compu-

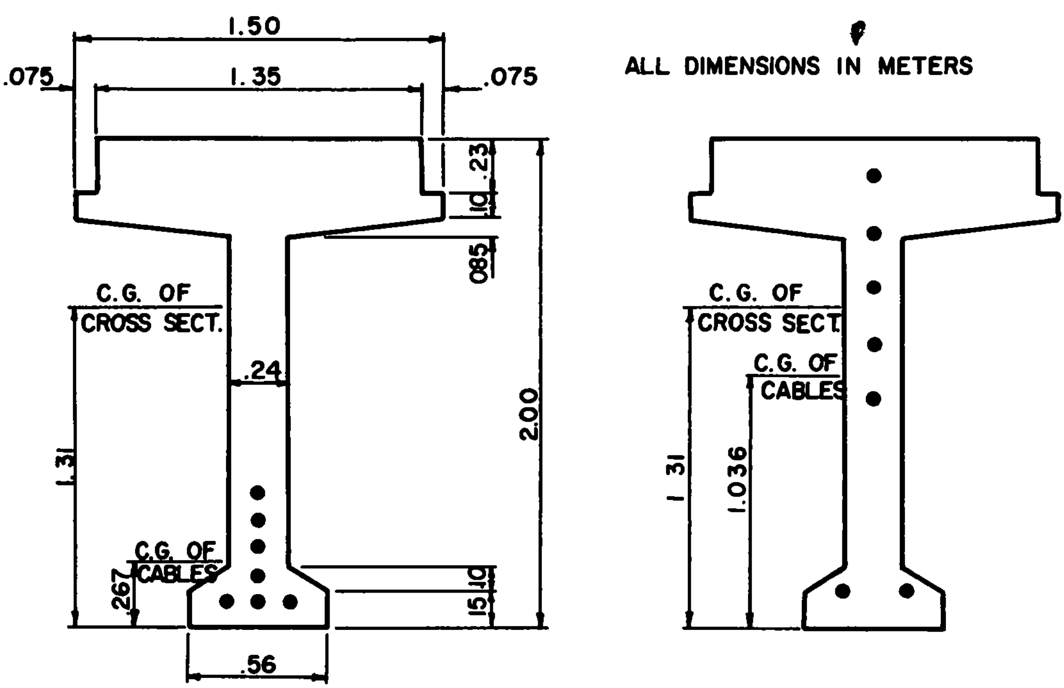


Figure 20. Cross-sectional properties of Rabat bridge test beam.

tations are shown in Figures 21 and 22, respectively. The unit creep curve was obtained by dividing the ordinates to the curve of time versus strain for the specimen under the sustained load by the magnitude of the sustained load. For simplicity, the unit creep curve was assumed to have this shape regardless of the age of the concrete at the time of loading. Relaxation of the steel was assumed to be two percent of the initial steel stress and was assumed to have the same shape as the unit creep curve. The elastic modulus of the steel was taken as 30 x 10^6 psi and the "instantaneous" modulus of the concrete was taken to be 6 x 10^6 psi as determined from the control prisms. Computed initial stresses in the concrete caused by prestress, beam dead load, and slab dead load, are given separately in Table 5. Deflections caused by temporary loads were neglected in the computations.

Time-dependent deflections due to prestress only were found by computing the curvatures at the reactions and at midspan and assuming the curvature between these

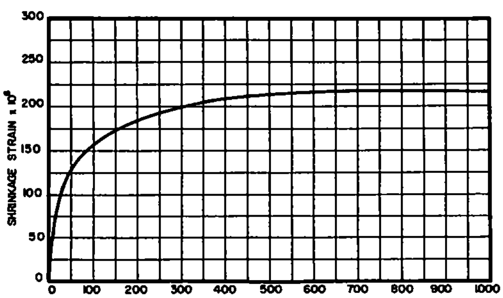


Figure 21. Shrinkage strain vs time relationship for Rabat bridge test beam.

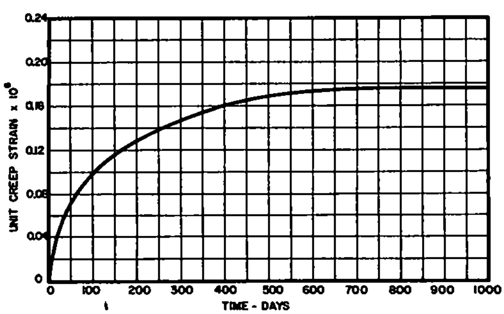


Figure 22. Unit creep relationship for Rabat bridge test beam.

points to have a parabolic distribution. For deflections due to beam dead load and slab dead load, a parabolic distribution of curvature varying from a maximum at midspan to zero at the reactions was assumed in accordance with the moment distribution. After determining the time versus deflection relationships for the separate load effects, the ordinates to the individual curves were added at the appropriate times to the "instantaneous" deflections, and the time versus total deflection relationship was obtained.

Comparisons of Computed and Measured Deflections

Figure 23 shows measured and computed deflections for the bridge beam. It can be seen that deflections computed by the rate-of-creep method are in good agreement with the measured deflections up to about 700 days. In the early stages, the rate-of-creep method tends to overestimate the upward deflection, as would be expected. Later, however, the upward deflection of the bridge beam becomes greater than that computed.

Delarue has pointed out that the deflections of the bridge appear to vary with the seasons. During the summer, deflections were found to increase, whereas during the

TABLE 5

INITIAL STRESSES IN EXTREME FIBERS OF FULL SIZE BEAM TEST

Cause of Stress	Stress in Top Fiber of Beam[a] (psi) Midspan	End	Stress in Bottom Fiber of Beam[a] (psi) Midspan	End
Post-tensioning	+720	-570	-4,230	-1,870
Beam dead load	-530	0	+960	0
Slab dead load	-290	0	+540	0

[a]Compressive stress (-)
Tensile stress (+)

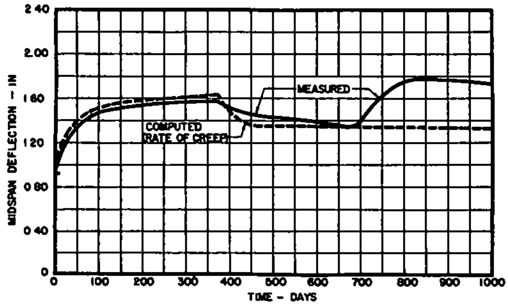


Figure 23. Measured and computed deflection for Rabat bridge test beam.

winter they decreased. The seasonal variations are due in part to the effect of moisture content on the creep and shrinkage properties of the concrete. Because the bridge was exposed to large variations in temperature and humidity, whereas the laboratory control specimens were stored under nearly constant atmospheric conditions, the shrinkage and creep characteristics of the laboratory specimens were not the same as those of the concrete in the beam. As the beam grows older, the differences in the behavior of the concrete in the laboratory and that in the beam appear to increase. At present, there is not enough information available to determine if this increase will continue. If the time versus deformation properties had been determined for the concrete under the same variations of temperature and humidity as the concrete in the beam, the rate-of-creep method would probably give a good indication of the complete deformation history of the beam including the seasonal effects which evidently can cause variations in deflection of the same magnitude as those occurring under nearly constant atmosphere conditions.

COMPUTED TIME-DEPENDENT DEFLECTIONS OF STANDARD PRESTRESSED CONCRETE BRIDGE SECTIONS

Properties of Sections

The studies of test results described in the preceding sections indicate that the basic time-dependent properties of concrete and steel can be used to compute the deflection of flexural members in which the concrete is subjected to variable stress. On the basis of these correlations of the unit creep, shrinkage, and relaxation under constant conditions with the deflection of prestressed concrete beams, it can be assumed that either the rate-of-creep or the superposition methods will provide a satisfactory basis for predicting the time-dependent deflections of prestressed concrete highway girders. However, it is necessary that the time-dependent properties be known for the materials used in the particular structure. Unfortunately, it is difficult to obtain these properties precisely. Even when laboratory tests are conducted on the materials used for a given structure, differences between laboratory and field conditions may be large enough to make the results unreliable. Laboratory tests are often conducted under constant or nearly constant conditions of temperature and humidity whereas a structure such as a highway bridge is subjected to a wide range of temperature and humidity. Quality of workmanship may also differ in the laboratory and field, thereby adding to the variation.

Any of the differences between laboratory and field conditions may cause a notable difference in the behavior of the control specimens and the structure. For this reason, it is preferable to work with reasonable approximations to the unit creep, shrinkage, and relaxation curves, unless circumstances warrant special tests. A guide to the ranges of the time-dependent parameters can be obtained from the literature. Any of the parameters which have a large effect on the results should be thoroughly investigated by using extreme values of their magnitudes.

The time-dependent deformations of several standard prestressed concrete bridge beams were analyzed by the rate-of-creep method to determine the effects of some of the many variables. For these computations it was again assumed that the deformations which occurred after about 2 yr (720 days) could be neglected and that the shape of the unit creep curve could be described by the relation:

$$\epsilon = 0.152 \log_e (t + 1) \qquad (6)$$

Although the shapes of the shrinkage and relaxation curves do not have as large an effect on the calculated deflections as the shape of the unit creep curve, it is still necessary to use some reasonable method of approximating them. In view of the many variables involved and the small effects these shapes have on the final deflections, the shrinkage and relaxation curves have been assumed to have the same shape as the unit creep curve.

Tests indicate that the magnitude of unit creep may range from 1 to 8 times the "instantaneous" deformation, depending on the properties of the material and conditions of temperature and humidity (5). For most conditions, a good estimate of creep deformation is 2 to 4 times the instantaneous deformations. More extreme values may be indicated for structures under severe atmospheric conditions or for concrete mixes of unusual proportions. For these computations, each beam was analyzed for unit creep under constant load of 2 and 4 times the instantaneous deformation.

Shrinkage strains have been found to vary in magnitude from zero, for concrete stored under very wet conditions, to more than 0.0012, for concrete stored under dry conditions (5). For highway bridges or other structures in open air, a reasonable value for shrinkage strain is 0.0003. This value was used in the analysis. Extreme values should be investigated for structures subjected to very wet or very dry conditions as well as for concrete of unusual consistency.

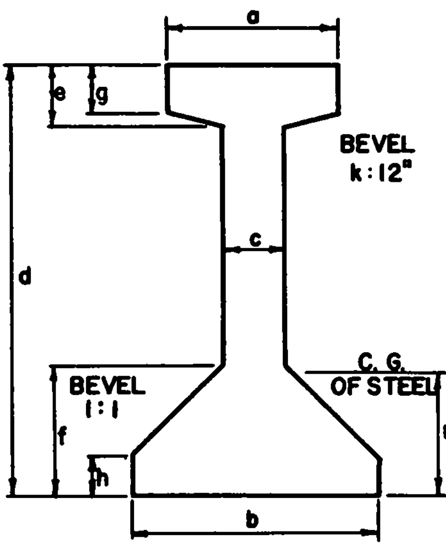
Relaxation losses after release of prestress may range from 1 to 8 percent of the initial stress (3). For these computations a rather high value of 8 percent was assumed.

Deflection calculations were made only for the dead load. Live loads were considered to be of such short duration that their effects on time deflections would be negligible.

The loads were divided into three categories: (a) the dead load of the beam, which was considered to act at all times; (b) the prestressing force, which was considered to act at all times; and (c) the dead load of the slab, which was considered to act from the time the slab was cast. The instantaneous modulus of the concrete and the magnitude of unit creep for the effects of prestress and beam dead load were based on a concrete strength of 4,000 psi, whereas those for the effect of slab dead load were based on a concrete strength or 5,000 psi.

Instantaneous deflections were computed on the basis of an elastic uncracked plain section. Time deflections were computed for the separate loading conditions and the time versus deflection relationships were found by plotting these deflections in accordance with the shape of the unit creep curve. The instantaneous deflections were then combined with the time deflections at the appropriate times to obtain the net time versus deflection relationship.

The concrete in the beams was assumed to have a strength of 4,000 psi at release



SPAN	a	b	c	d	e	f	g	h	k	m	NO. OF WIRES
40'-0"	16	18	6	36	5 1/2	10	4 1/4	4	3	9 3/4	30
45'-0"	17	18	6	40	5 1/2	10	4 1/8	4	3	10	32
50'-0"	18	18	6	44	5 1/2	10	4	4	3	11 7/8	36
60'-0"	20	24	6	48	6 1/2	14	5 1/2	5	2 1/4	9	43
70'-0"	22	26	6	56	6 1/2	14	5	4	2 1/4	10 3/4	52

DIMENSIONS IN INCHES

Figure 24. Cross-sectional properties of pretensioned standard prestressed concrete bridge sections.

of the prestress and a strength of 5,000 psi at the time the slab was cast. The instantaneous modulus for the concrete was obtained from the expression recommended by the ACI-ACE Joint Committee 332 (7):

$$E_c = 1,800,000 + 500 f'_c \qquad (7)$$

in which

E_c = instantaneous modulus, and
f'_c = concrete strength in psi.

The steel was assumed to have a prestress of 175,000 psi prior to release. The modulus of elasticity of the steel was taken as 30 x 10^6 psi.

The beams analyzed were for 40-, 45-, 50-, 60-, and 70-ft spans with 28-ft roadways. These are the pre-tensioned standard prestressed concrete bridges listed by the Bureau of Public Roads (8). They are designed for H20-S16-55 loading. Cross-sectional dimensions for these beams are shown in Figure 24.

Method of Computation

The numerical procedure for the rate-of-creep method described previously was used for the computation of deflections. The problem of time deflection in prestressed concrete highway bridges is well suited to analysis by this method because it involves nearly every possible variable. The beams for a typical bridge may be cast at a factory, and stored for 60 days or more before the slab is cast on them. In such a case, some of the time deflections will take place in the beam before the slab is cast. If a period of 60 days or more elapses between the release of prestress and casting of the slab, shrinkage strains in the slab will usually be greater than combined shrinkage and creep strains in the top fiber of the beams. This will result in forces tending to shorten the top fibers of each beam. These forces may be handled easily by the numerical

TABLE 6

CREEP PARAMETERS USED FOR PRE-TENSIONED, STANDARD PRESTRESSED CONCRETE BRIDGES

Span Length (ft)	Beam No.	Days From Prestressing to Casting of Slab	Modulus of Elasticity, E_c, at Prestressing (psi x 10^6)	Modulus of Elasticity, E_c, at Casting of Slab (psi x 10^6)	Relaxation (ksi)	ϵ_c/ϵ_i
40	1	30	3.80	4.30	12.96	2
	2	60	3.80	4.30	12.96	2
	3	30	3.80	4.30	12.96	4
	4	60	3.80	4.30	12.96	4
45	1	30	3.80	4.30	12.96	2
	2	60	3.80	4.30	12.96	2
	3	30	3.80	4.30	12.96	4
	4	60	3.80	4.30	12.96	4
50	1	30	3.80	4.30	12.96	2
	2	60	3.80	4.30	12.96	2
	3	30	3.80	4.30	12.96	4
	4	60	3.80	4.30	12.96	4
60	1	30	3.80	4.30	12.96	2
	2	60	3.80	4.30	12.96	2
	3	30	3.80	4.30	12.96	4
	4	60	3.80	4.30	12.96	4
70	1	30	3.80	4.30	12.96	2
	2	60	3.80	4.30	12.96	2
	3	30	3.80	4.30	12.96	4
	4	60	3.80	4.30	12.96	4

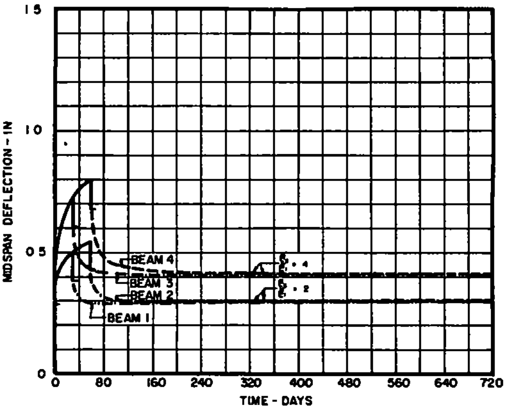


Figure 25. Computed deflection of 40-ft composite highway bridge beam.

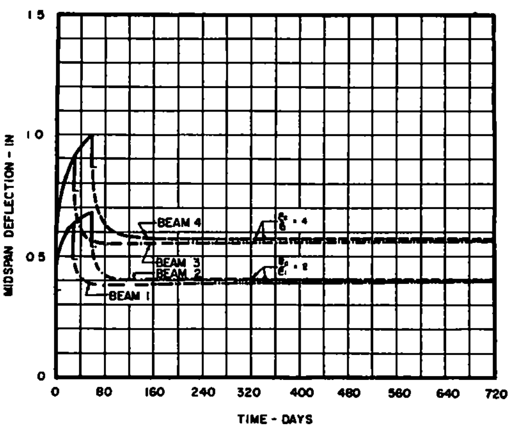


Figure 26. Computed deflection of 45-ft composite highway bridge beam.

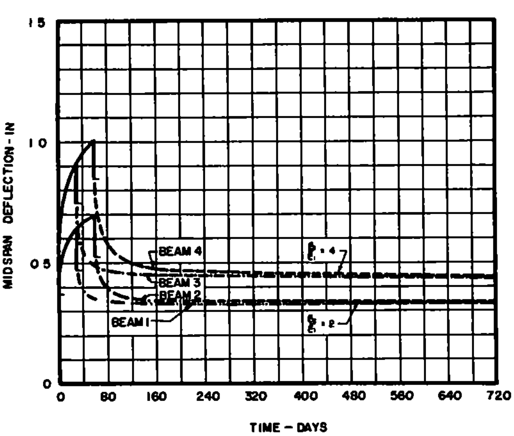


Figure 27. Computed deflection of 50-ft composite highway bridge beam.

procedure; however, they are usually small enough to be neglected. In the beams analyzed, the computed strain increments in the top fiber of the girder were almost equal to the expected shrinkage strain increments in the slab. Consequently, the forces exerted on the beams because of shrinkage of the slab were ignored.

Computed Deflections for Prestressed Bridges

Figures 25 through 29 show the time versus deflection relationships for five Pre-Tensioned Standard Prestressed Concrete Bridges with slabs cast at 30 or 60 days. Table 6 gives the parameters used in the computations for each curve. The curves include both instantaneous and time deflections.

If the girders are assumed to be straight before prestressing, it can be seen that the net camber after all deflections have taken place is quite small in every case. Final cambers range from about ⅝ in. in the 45-ft span to less than ¼ in. in the 60-ft span. It can also be seen that a variation in creep strain from two times the instantaneous strain to four times the instantaneous strain has only a small effect on the net deflection due to dead load. One reason for the small computed deflection is the fact that the dead load stress gradient over the depth of the cross-section is small in all beams considered. In addition, the l/d ratio is nearly constant for all beams. These two effects combine to keep computed net upward deflections small and also nearly equal for all of the beams considered.

In general, time deflections will be relatively small if the stress gradient corresponding to the combined effects of dead load and prestress is small. The critical stress gradient is that corresponding to the dead load and any part of the live load which may remain on the structure for long periods of time.

The curves shown in Figures 25 through 29 also indicate that loading at 30 or 60 days results in a little difference in the final deflection of the structures. This is a result of the assumption made for the time-dependent characteristics of the concrete. If the concrete exhibited a greater change in magnitude of unit creep in relation to the age at loading than that assumed, there would be a larger difference in final deflection for different dates of loading. The comparison of measured and computed deflections for the beam in Figure 23 shows that the effects of humidity and temperature may be greater than the effect of age of concrete at loading. Hajnal-Konyi has found the same effect in a number of reinforced concrete beam tests (9). In these tests, the beams were stored under shelters which were open on all four sides. A definite change in the rate and even in the sense of the deflection increase was noted during different seasons of the year. This rate was considerably lower during autumn (Great Britain). In view of these results, a more refined method of estimating the total unit creep does not appear to be justified unless the time-dependent parameters for the concrete are known for environmental conditions very similar to those for the structure.

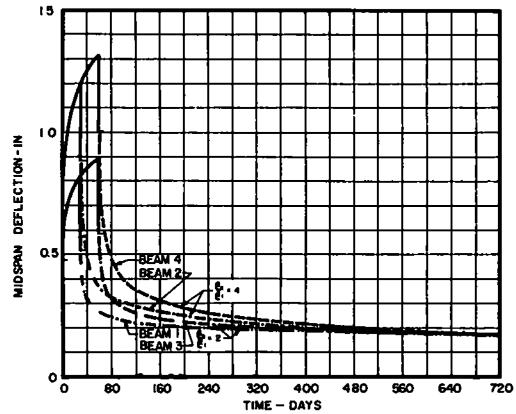


Figure 28. Computed deflection of 60-ft composite highway bridge beam.

In many cases, the important portion of the time-dependent deflection is that which occurs after the slab is cast. Figures 25 through 29 show that this deflection may range from as little as 1/10 in. for the 40-ft span to as much as 7/8 in. for the 70-ft span. Although this change in deflection can be larger than the net deflection, it is interesting to note that most of it occurs within the first 100 days after the slab is cast. This phenomenon is chiefly a function of the shape of the unit creep curve; consequently, errors in estimation of the magnitude of unit creep do not greatly affect the time at which the deflections nearly stabilize.

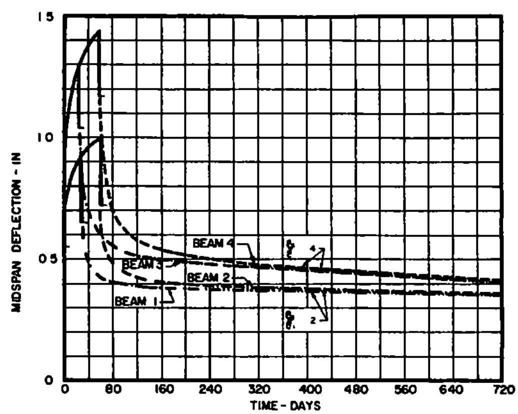


Figure 29. Computed deflection of 70-ft composite highway bridge beam.

Although this method of analysis of time-dependent deflections does not predict precisely the behavior of the beam, it is useful in determining the range in deflection. Comparison with test results indicates that either the rate-of-creep or the superposition method can be used to predict deflections if the time-dependent properties of the materials are known. Consequently, the method of analysis used for these examples can be used as a guide for determining the effects of time-dependent deformations.

SUMMARY

The objective of this paper has been to present and discuss various methods for computing the time-dependent deflections of prestressed concrete beams, to compare the deflections computed on the basis of these methods with results of tests, and to estimate the possible magnitudes of time-dependent deflections in representative prestressed concrete highway bridges.

The time-dependent deflection of prestressed concrete beams is attributed to three effects: creep and shrinkage of the concrete, and relaxation of the reinforcement. Ordinarily, creep of the concrete is the dominant factor. Because concrete is subjected to a stress which varies with time primarily as a result of the changes in the prestress level, its creep characteristics will be different from those determined under a constant stress. To analyze the time-dependent deflections of prestressed concrete beams having various proportions using only one consistent set of assumptions regarding the material properties, it is necessary to recognize this variation in stress and its possible effects on the creep characteristics. The "rate-of-creep" method recognizes the stress variation but ignores the possible changes in the creep characteristics. The "superposition" method recognizes the stress variation and also modifies the creep characteristics according to the stress variation. Both methods involve a numerical integration procedure; the second requires more numerical work.

The measured deflections from four laboratory tests on beams were compared with deflections computed using creep strains evaluated according to these two methods. The shrinkage, relaxation, and the basic creep (creep under sustained stress) relationships were determined from control tests. All specimens were kept under conditions of controlled temperature and humidity. The agreement between the measured and computed curves (Figs. 12-15) was good. Although the superposition method gave better results, the rate-of-creep method was deemed preferable for general application in view of its relative simplicity.

The measured deflections of four beams tested by Lofroos and Ozell (4) were analyzed using the rate-of-creep method, primarily to show that the deflection of beams having different levels of prestresses could be predicted on the basis of the same basic creep curve.

The time-dependent deflections of a prestressed highway bridge in Morocco have been reported by J. Delarue (6). The measured deflections were compared with values computed using the material properties determined from laboratory tests (Fig. 23). The seasonal deflections of this bridge, which may be attributed to changes in relative humidity and possibly in temperature, were of nearly the same magnitude as the computed maximum time-dependent deflection. Such a seasonal variation can be predicted only on the basis of basic properties determined at the site.

Although the precise relationships for basic creep, shrinkage, and relaxation cannot be obtained without undue expense, it is possible to obtain a fairly good estimate of the time-dependent deflections of prestressed concrete girders by computations based on reasonable maximum and minimum values of the critical variables. This was done for five representative composite highway bridges with spans ranging from 40 to 70 ft. It was found that under ordinary conditions, the deflections under dead load for these bridges should not exceed ¾ in. and could be less than ¼ in. It should be pointed out, however, that the time-dependent deflections of prestressed concrete beams can be objectionably high, especially if there is a high stress gradient in the beam under permanent load.

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