Osmotic Interpretation of the Swelling of Expansive Soils

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● VOLUME CHANGES and swelling pressures represent the most important properties of soils if the latter are employed for engineering purposes, especially as supporting bases and other structural components of roads and highways. The problem of the consolidation of water-saturated soils under applied loads has been studied quite thoroughly in soil mechanics. In contrast, the external work performed by swelling soils with increase in moisture content that can seriously affect the superimposed structures has attracted only minor attention. In the particular case of subgrades, Wooltorton (1) has clearly indicated that swelling must be considered as a factor in pavement design and in Hveem's method swelling is considered of equal importance with the generally accepted criterion of minimum deformation. In the classical method of the California Bearing Ratio, its originator, Porter, indicates that no swelling should occur under a surcharge that is equal to the weight of the contemplated superstructure.

From a practical point of view, the subject of greatest interest is the external work that potentially expansive soils can perform against the superimposed structure during swelling. This work is measured by the product of volume increase and swelling pressure and is performed against the weight of the superstructure. Hence, it is not sufficient, as is generally done, to judge a soil solely on the basis of volumetric change; one must also take into account the swelling pressure which supplies the force involved in volume changes and which together with the volume changes determines the total detrimental work.

Experience has shown that both factors are functions of the initial moisture content and this value must be established for each case. This moisture content is of particular importance with respect to the swelling pressure. For air-dry soils of low moisture contents, swelling pressures of the order of 11 kg per sq cm (11.3 tons per sq ft) have been observed (2). Relatively common expansive soils at moisture contents that correspond to working conditions, lying in the vicinity of the plastic limit, give swelling pressures from about 0.5 to 3 kg per sq cm (0.5 to 3 tons per sq ft).

The range of initial moisture contents of interest in soil engineering is quite large. One may accept as a minimum moisture content the hygroscopic moisture of air-dry soils, i.e., the moisture content that is in equilibrium with the vapor pressure of the atmosphere. The hygroscopic moisture is not a constant for a given soil but depends on the temperature and the degree of humidity of the air. The latter is usually given as relative humidity and expressed as

$$H = \frac{p}{p^o} \, 100$$

where p^o and p are the respective water vapor pressures of the saturated and the actual atmosphere. The condition of air-dryness at ambient temperature (25 C) generally refers to either of two cases:

    (a) in dry air with H = 50 and

    (b) in practically saturated air with H = 96, which corresponds to the water vapor pressure of a 10 percent aqueous solution of sulfuric acid.

The highest water content for manipulated soils corresponds to the liquid limit at which the mechanical resistance of a soil is practically zero. Within this large range, the moisture contents of greatest interest are those that correspond to actual working conditions, such as the optimum moisture for compaction and the plastic limit. Increase in moisture content above the latter results in rapid decrease of the bearing capacity of cohesive soils.

From the point of view of swelling, one is not interested in fluctuations of moisture content that do not pass above the shrinkage limit because, by definition, no volume changes take place in this moisture range and the external work is zero. With respect to the shrinkage limit, it must be stated that the value obtained by the method of the U. S. Bureau of Public Roads is a minimum value. In many expansive soils at moisture contents about and below the plastic limit, the reduction in volume no longer equals the volume of water lost. Residual contraction starts when the volume change is smaller than the volume of water lost and proceeds gradually until the shrinkage limit is reached. For this reason, some institutions such as the Road Research Laboratory of England consider as shrinkage limit the intercept of the theoretical extrapolation of the indicated relationship below which appreciable contractions are not expected to occur. It is exactly in this moisture range in which high swelling pressures are found accompanied by small volume changes. These combinations are, of course, less detrimental than smaller pressures associated with relatively large volume changes.

The swelling of soils is a consequence of their ability to take in free water and in doing so to overcome the resistance of the soil-water system to expansion, the viscous resistance to the movement of water throughout the soil system, and the confining forces that are due to existing surcharges. The capacity for water sorption is in turn a result of the tendency of the system to pass into a more stable condition under loss of free energy (second principle of thermodynamics). Hence, the forces that determine the retention of water by a soil are also responsible for its swelling characteristics. Therefore, every theory or explanation of swelling must necessarily start from a consideration of the nature and magnitude of the forces that determine the suction of water into a soil at a certain initial moisture content.

The primary purpose of this paper is to point out the various factors involved in this problem and how the organization of these factors may then serve as a dependable guide for experimentation.

MECHANISM OF WATER SUCTION BY SOILS

It is necessary to examine first the various mechanisms that determine the sorption of water by soils, particularly, starting at an initial moisture content of the order of the shrinkage limit.

The following statements recapitulate the available knowledge on the fixation of water by soils (3,4,5):

1. Dry soil fixes water vapor from the atmosphere or liquid water by polar absorption. This is particularly important for the clay fraction because of its high degree of dispersion or large specific surface (surface per unit volume or mass) and its large energy per unit of surface. This is a consequence of the small size produced by physical weathering and the alterations due to chemical weathering which produce electric surface charges not balanced within the mineral structure. These ionic charges are predominantly of negative sign. They give rise to a field of potential energy that acts on the hydrated ions in the aqueous medium surrounding the particles and on the water molecules themselves because of their polar nature. The latter are oriented toward the solid surface and are immobilized to a greater or lesser degree. Opposed to this action is the kinetic energy of the ions and molecules.

Water held by this electric field is quite strongly bound and does not possess the same physical properties as free water, and in its mechanical properties, resembles more or less a rigid solid. The binding energy is greatest at the solid interface and falls off with increasing distance as an exponential function; this means that the binding energy decreases with increasing thickness of the films around the particles, which thickness is determined by the water content of the system and its total internal surface. The kinetic energy is a maximum at an infinite distance from the particle surfaces (free water) and decreases with decreasing distance from the solid surfaces.

The strongly retained (immobilized) water corresponds to a film thickness of up to 10 molecules, or approximately 2 to 3 millimicrons ($m\mu$); practically, it represents the hygroscopic moisture in an atmosphere of $H = 50$ ($pF = 6.0$) and the loss in free energy manifests itself as heat of wetting. For higher water contents as for hygroscopic moisture at $H = 96$ ($pF = 4.7$), film thicknesses of the order of hundreds of molecules (20 - 40 $m\mu$) are reached involving only partial immobilization of the water molecules. The differential heat of wetting (i.e., the heat developed by adding an additional gram of water to a moist soil system) is very small or zero at a hygroscopic moisture content corresponding to $H = 50$. These films act as semirigid or plastic bodies and flow or deform under relatively small stresses. This explains why polar adsorption reaches a minimum at hygroscopicities corresponding to $H = 100$ in the vicinity of which capillary condensation is added to polar adsorption. Water held in excess of that corresponding to hygroscopicity at $H = 100$ cannot be explained by polar adsorption.

2. Clay soils at moisture contents at or above their hygroscopicities at $H = 100$ are able to sorb more liquid water with less energy but in larger quantity. This gives rise to film thicknesses in the order of hundreds of millimicrons called lyosorptive layers by Ostwald and it is these that represent the swelling water. Commonly, this water suction is attributed to the capillary action of concave menisci in the soil system or to the resulting capillary potential which gives an adequate explanation for water sorption in coarse, granular, and silty soils. Undoubtedly, capillary suction also plays a role in water sorption by clay soils, but it does not explain water sorption into saturated clay systems in which the absence of an airphase prevents the formation of menisci. In expan-

sive, and hence, compressible soils, saturation occurs at moisture contents of the order of magnitude of the optimum for compaction; suction, however, continues up to moisture contents as high as the liquid limit and even if dispersed in an excess of free water the particles retain envelopes of more or less strongly bound water.

During the period 1931-33, Mattson (6) proposed the osmotic forces as causative agents of water retention by clays. According to him, this osmotic imbibition occurred after saturation of the polar adsorption capacity. The ideas of Mattson represent an application to soils of the swelling mechanism proposed by Wilson in 1916 generally for proteins and particularly for collagen, which is of such importance in the tanning process. Except for the work of Winterkorn (5,7,8), this interpretation has not found wide application in the field of soil engineering in which the capillary interpretation prevails despite its definite shortcomings.

From the physicochemical point of view, osmotic suction is based on two general phenomena, namely osmosis without membrane and Donnan equilibrium. These fundamental phenomena will be discussed and applied to the swelling of soils and related problems. This shall be done in a quantitative or at least semiquantitative manner.

Mattson's concept of osmotic imbibition is fundamentally based on the concept of the Donnan equilibrium while the theory treated here considers as the fundamental phenomenon osmosis without membrane combined with the concept of osmotic activity. The Donnan equilibrium modifies but does not govern osmotic suction and its influence is important only in saline soils. The osmotic swelling itself is a consequence of the cation exchange characteristic of clays. Table 1 gives data by Mattson showing that swelling increases with an increasing value of T, the base exchange capacity expressed in milliequivalents per 100 gm of soil.

TABLE 1

Clay Soil	T value	Volume (cu cm)		
		Dry	Swollen	
			H+	Na+
Nipe	4	0.90	0.92	0.90
Norfolk	21	1.04	1.42	2.30
Sharkey	80	1.12	1.75	7.20

Osmotic suction is possible only in polar solvents that are able to disperse the exchange cations. It does not take place in nonpolar solvents such as hydrocarbons and carbon tetrachloride in which, of course, capillary suction can occur.

OSMOSIS WITHOUT MEMBRANE

The molecules or ions of a substance in solution possess kinetic energy that permits them to diffuse spontaneously when the solution is in contact

with the pure solvent or with a more dilute solution, until uniform distribution is reached.

When a solution A containing a certain number of particles per unit volume (concentration) is placed in contact with a pure solvent or more dilute solution B by means of a membrane or semipervious wall a b that permits only the passage of the pure solvent, then two cases may be differentiated (Fig. 1).
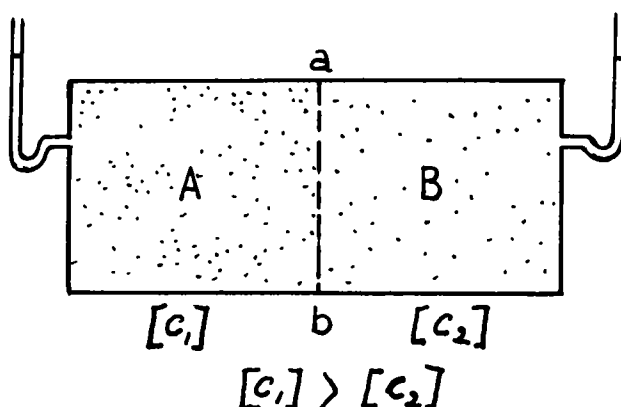

Figure 1.

Case A. — If the semipermeable wall a b is freely moveable, then it will displace itself in the direction of B while solvent from compartment B moves into compartment A until the concentrations in both compartments are equal.

Case B. — If the semipermeable wall a b is fixed, then the tendency to equalize the concentrations in both compartments will result in the movement of solvent from B to A. This causes a hydrostatic or osmotic pressure to develop in compartment A and a suction in compartment B. This process continues until the hydrostatic pressure in compartment A pushes the solvent into the compartment B with the same force as the osmotic phenomenon sucks the solvent from compartment B into compartment A. At this point, a permanent equilibrium is reached. One sees that it is the osmotic phenomenon that determines the hydrostatic or osmotic pressure and not the reverse. The osmotic pressure can be measured by the magnitude of the external pressure that must be applied at A in order to prevent osmotic liquid movement through the semipermeable wall.

In 1885 Van t'Hoff applied the gas laws to the case of dilute solutions and conceived the osmotic pressure as equivalent to the pressure that the dissolved substance would exert if it were in gas form in the same volume as occupied by the solution. In accordance with this concept, the osmotic pressure can be calculated in the following way from the molecular concentration (c)[1]/

$$P = R T (c) \text{ where}$$

[1]/Number of particles in one liter of water expressed as multiples of the Avogadro number (6.02×10^{23}).

P = osmotic pressure
R = universal gas constant = 0.082 liter atmospheres
T = absolute temperature
(c) = molar concentration.

The pressure P_0 developed when one mol of solvent (18 g of H_2O is osmotically incorporated into a liter of molar concentration (c) has been derived thermodynamically by Van t'Hoff on the basis of the osmotic work performed which itself is related to the vapor pressures at the respective concentrations. If M denotes one additional mol of solvent, p and p_0 the vapor pressures of the solution and the pure solvent, respectively, then:

$$P = \frac{RT}{M} \ln \frac{p_0}{p}$$

The law of Van t'Hoff states that the osmotic pressure depends only on the number of dispersed particles irrespective of their nature, hence they may be atoms, ions, molecules, or colloidal or even larger particles. The respective systems are called ideal solutions if they obey this law closely, and distinction is made between ideal and real solutions in the same manner as between ideal and real gases. In the case of dilute solutions, the dissolved and dispersed particles can move independently of each other and consequently follow the law quite closely. In other cases such as in concentrated electrolyte solutions, ionic attraction and repulsive forces act between the particles and result in a deviation from the ideal osmotic law. Each ion tends to surround itself with ions of opposite charge with resultant change in ionic activity. The interionic forces increase in strength with increasing concentration and increasing ionic charge. As a consequence, the real osmotic activity is smaller than that calculated for ideal solutions. For this reason, Lewis developed the concept of activity (1901-07) and introduced the coefficient of activity f, which is defined as follows:

$$f = \frac{\text{real activity}}{\text{concentration}} = \frac{a}{(c)}$$

For very dilute solutions, a approaches (c) and $f \rightarrow 1$. With increase in concentration and valency f changes as illustrated below:

Activity coefficients f

(c)	0.01	0.001	0.0001
K^+ in KCl	0.89	0.96	0.99
Cu^{++} in $CuSO_4$	0.40	0.75	0.91
Al^{+++} in $AlCl_3$	0.08	0.45	0.78

From a thermodynamic point of view, the presence of a semipervious wall is not absolutely necessary for restraint of diffusion and development of osmotic pressure. It is sufficient if, due to the influence of other forces such as adsorption and electrostatic attraction, one component is free to diffuse while the movement of the other is restrained. From this arises the concept of osmosis without membrane, which is of special importance for the swelling of clays.

One must keep in mind that osmosis is a consequence of the kinetic energy of the dispersed particles and that the forces that play the role of a semipermeable membrane must necessarily decrease the osmotic activity of the particles by restraining their freedom of movement and hence their kinetic energy. This is the essential basis for an osmotic interpretation of water sorption by clay soils above a certain initial moisture content.

Consequently, application of the law of Van t'Hoff to real systems involving osmosis without membranes must make use of the activity f(c) rather than of the concentration alone. This is important because several authors have used concentrations rather than activities in the application of Van t'Hoff's law to the osmotic swelling of clay-water systems (7,9,10). Although this has been done more for the purpose of illustration and as a guide to experimentation, it could be misleading to the uninitiated who may believe in the reality of the very high osmotic pressures that are obtained by such calculations.

OSMOTIC PRESSURE OF THE SOIL-WATER SYSTEM

As a starting point for application of the concept of osmosis without membrane to soil-water systems a peptized, salt-free, clay soil is taken at a moisture content between the liquid and the plastic limit.

The water occupying the free spaces between the solid particles acts as a dispersion medium for the exchange cations. The latter are subjected to two counteracting influences: (c) the electrostatic attraction by the electric charges on the surfaces of the solid particles and (b) the dispersive tendency which is due to the kinetic energy of the particles and because of which the latter attempt to distribute themselves uniformly throughout the liquid phase. The cations are in a hydrated condition, which means that the radius of the kinetic particulate unit is that of the particle plus its bound water molecules.

The distribution of exchange ions around particles dispersed in an aqueous medium has been studied by Duclaux (11) under the assumption that the particles are spherical and that the electrostatic and kinetic forces are in equilibrium. He arrived at the conclusion that the concentration of the cations decreases exponentially with increasing distance from the particle surfaces. Later, Winterkorn (7) applied the same calculation to plane particle surfaces that are more in accordance with the shape of real swelling clays and arrived at the conclusion that, practically, there is no change in concentration of the exchange cations with increasing distance up to a film thickness of the order of hundreds of molecules. Schmid (12) arrived at a similar result for microporous systems. Consequently, it should be permitted to assume a uniform distribution of cations in the aqueous medium for the moisture range in which we are interested. For higher moisture contents, the cations will tend to become more concentrated toward the particle surfaces in accordance with the classical theory of the Helmholtz double layer, which is applicable for dilute clay suspensions.

Accepting a uniform concentration of the exchange ions in the aqueous dispersion medium, one can calculate the molar concentration from the exchange capacity of the clay (Hissink "T"-Value); the valency Va of the cations in the case of homoionic clays or the median valency in the case of heteroionic clays and the moisture content. The exchange capacity "T" is usually expressed in milliequivalents per 100 g of soil. Hence, the number of equivalents per 100 g of soil is "T" per 1,000 and of moles "T" per 1,000 x Va when Va represents the valency. If this number of moles

exists in 100 g of soil that contains i g of water, then the molar concentration [c] or the number of moles of exchange ions per liter of aqueous dispersion medium becomes

$$[c] = \frac{\frac{"T"}{1000\ Va}}{i} 1000 = \frac{"T"}{i\ Va} \tag{1}$$

An "ideal" dispersion of cations of a certain concentration, placed in contact with pure water by means of a membrane that allows free passage of the water molecules but not of the cations, develops an osmotic suction that can be translated into a hydrostatic pressure whose magnitude according to Van t'Hoff is

$$(P_o)_{ideal} = R\ T\ \frac{"T"}{i\ Va} \tag{2}$$

where $(P_o)_{ideal}$ is the hydrostatic pressure in atmospheres that equilibrates the developed osmotic pressure, R is the gas constant (0.082 l atm), and T the absolute temperature.

The theoretical osmotic pressures, calculated under assumption of "ideal" behavior, are inversely proportional to the initial moisture content and are of the order of magnitude of tens of atmospheres. Thus, for "T" = 30, i = 28.6, T = 273 + 25 = 298 C and Va = 1, one obtains

$$(P_o)_{ideal} = 0.082 \times 298\ \frac{30}{28.6 \times 1} = 25.6\ atm\ (pF = 4.41)$$

The osmotic pressures actually developed in real soil-water systems are necessarily smaller than those just calculated for ideal systems. In a real soil-water system, the role of the assumed semipermeable membrane is filled by the electrostatic attraction between the cations and the negatively charged particle surfaces. This attraction decreases the effectiveness of the kinetic energy of the cations, which is the cause of their diffusion tendency and of the resultant osmotic pressure. Hence, one may write

$$(P_o)_{real} = (P_o)_{ideal}\ ^f$$

where f is a coefficient smaller than 1 that corresponds to the activity coefficient of Lewis and expresses the real osmotic activity of the dispersed ions under the prevailing environmental conditions.

The coefficient of activity f must decrease with decreasing moisture content, since the restraining electric attraction of the cations by the charged particle surfaces increases with increasing proximity of the cations, i.e. with decreasing moisture content. Therefore, f should be expressed as a function of the moisture content f(i), the maximum value of this function being unity. The general expression is

$$(P_o)_{real} = (P_o)_{ideal} \times f(i) = RT\ \frac{"T"}{i\ Va}\ f(i) \tag{3}$$

In equation (3), R, T, "T", and Va are constants for a particular soil at constant temperature and $(P_o)_{real}$ is a function of the moisture content. The decrease of $(P_o)_{real}$ with increasing initial moisture content is due to two factors:

(a) dilution: $(P_o)_{real}$ changes inversely with change in moisture content;

(b) activity: higher initial moisture content implies less electro-static restraining influence, i.e. greater diffusibility and less osmotic pressure. This is expressed by the factor $f(i)$.

The factor $f(i)$ can be calculated if one knows $(P_o)_{real}$. Equation (3) can then be transformed as follows:

$$f(i) = \frac{(P_o)_{real} \times i \times Va}{R\ T\ "T"}\ ; \tag{4}$$

its solution implies the experimental determination of $(P_o)_{real}$.

The logical method for the determination of $(P_o)_{real}$ is by means of the consolidation test as used in soil mechanics. If one applies to a soil-water system that is in communication with free water a constant pressure of sufficient magnitude, then free water is squeezed out of the soil with corresponding decrease in volume until the system has reached a state of equilibrium. Let P be the externally applied pressure that causes equilibrium to be reached at a water content of i. If we consider the cell of the consolidation apparatus as an osmometer, then the hydrostatic or osmotic pressure developed by a puddled soil-water system at a water con-tent of i must be compensated by the externally applied pressure in order that a permanent equilibrium may be reached. This signifies that the os-motic pressure fills a role similar to that of the pore pressure for satu-rated systems in soil mechanics and that the applied P measures directly the $(P_o)_{real}$. Therefore,

$$(P_o)_{real} = P$$

$$P = \frac{R\ T\ "T"}{Va} \times \frac{1}{1} \times f(i) \tag{5}$$

where all terms except $f(i)$ can be measured experimentally and $f(i)$ can be calculated from

$$f(i) = \frac{P\ i\ Va}{R\ T\ "T"} \tag{6}$$

for each moisture content as long as the condition $(P_o)_{real} = \underline{p}$ holds.

From theory, it may be expected that $f(i)$ is a decreasing exponential function of i. This is confirmed by plotting the logarithms of experi-mentally obtained $f(i)$ values against the respective i values. This has been done in Figure 2 which shows the straight line relationships obtained for several ionic modifications of Putnam soil. This relationship is also shown in Figure 3.

With soil A, one obtains straight lines that can be extrapolated to $f(i) = 1$ or $\log f(i) = 0$; they intercept the abscissa at a moisture con-tent i_o which can be expressed by the equation

$$\log f(i) = K(i - i_o). \tag{7}$$

Accordingly, $f(i)$ falls exponentially with increase in moisture content above a certain initial moisture content i_o for which the maximum theoreti-cal value of the osmotic pressure holds in accordance with Eq. 3. As could be expected, the value i_o is of the same order of magnitude as the shrinkage limit of the soil. Consequently, in this type of soil, the os-motic water suction gives rise to pore pressures and associated expansion,
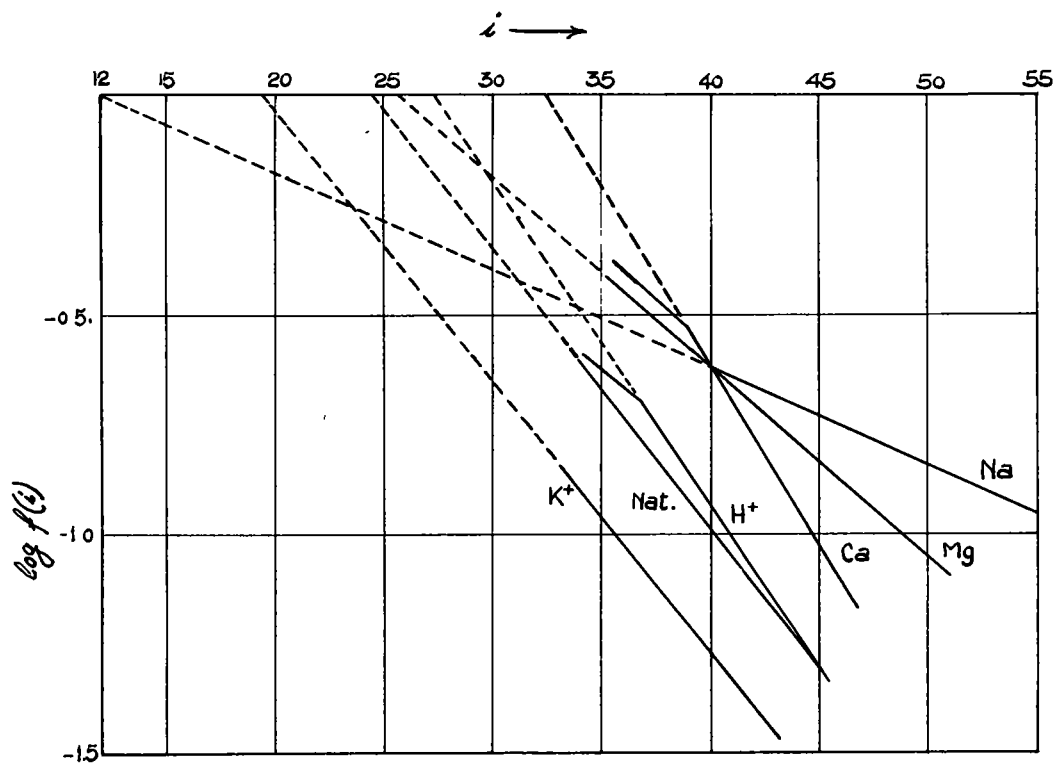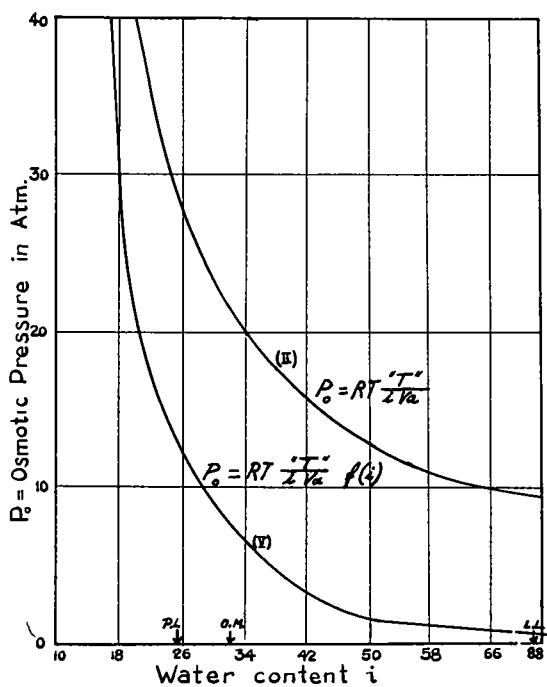
Figure 2. Relation between i and log f(i)

Figure 3. Osmotic pressure of Na-clay.

starting at moisture contents that are of the same order of magnitude as the shrinkage limit. The negative angular coefficient K can be calculated by the equation

$$K = \frac{\log f(i)}{i - i_o} \tag{8}$$

Its value determines the osmotic behavior of a soil. For the same soil, the K value depends on the nature of its exchangeable cations. Highly hydrated cations such as Na^+ give negative values of K that are smaller than those for cations that have a higher position in the Hofmeister lyotropic series such as K^+ or the bivalent cations. This results from the force that attracts the cations to the particle surfaces being a function of their charge (valency), radius, and degree of hydration.

In the case B, one obtains a straight line that is broken at point O. The straight line B follows Eq. 7, but the value i_o is larger than the shrinkage limit. Evidently, at the moisture content that corresponds to point O and is around 36 to 40 percent, the solid volume is of the same order of magnitude as the pore volume and contact is established between the more energetically bound semirigid water envelopes around the solid particles. In the case of flocculating cations, such as Ca^{++} and H^+, that cause greater attraction forces between the particles, a certain structure is developed and the consolidation pressure is partially consumed by the deformation of this structure. This results in a change of the angular coefficient. With the new angular coefficient, the straight line must tend toward i_o = shrinkage limit because the structure formed by these soils cannot resist the shrinkage forces. By deformation of this structure or by a new structural arrangement, the shrinkage limit is finally reached.
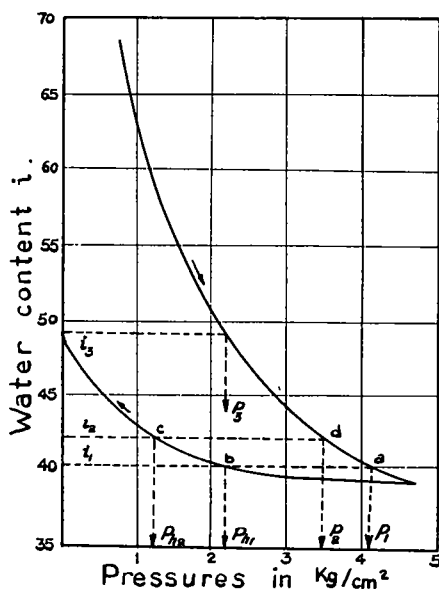

Figure 4. Consolidation-swelling curves of Na-Putnam soil.

EXPERIMENTAL DATA

The work of Winterkorn and Moorman (13) on Putnam clay soil of podsolic origin contains the data necessary for the application of the theoretical concepts that have been developed. It contains the engineering soil characteristics, the "T" value, the nature of the exchange cations from which the median valency can be calculated, and the consolidation curves for the natural soil and for the homoionic Na^+, H^+, K^+, Ca^{++}, and Mg^{++} soils. In Figure 4 the moisture contents have been calculated from the voids ratios given. The temperature has been assumed to be 25 C or 298 A.

In Table 2 are given the data taken from the work of Winterkorn and Moorman (13) and in Table 3 are found the values for f(i), K, and i_o that have been calculated by means of the Eqs. 7 and 8 from the consolidation pressures P and the equilibrium water contents i taken from the consolidation graphs of Winterkorn and Moorman (13). Figure 2 shows plots for log f(i) versus i with the straight lines extended to obtain the i_o values.

TABLE 2

CHARACTERISTICS OF THE PUTNAM CLAY SOIL[1]
AND ITS HOMOIONIC MODIFICATIONS

Characteristics	Natural	H^+	Na^+	K^+	Mg^{++}	Ca^{++}
Liquid limit	64.5	56.4	88	52.8	56.3	61.9
Plastic limit	23.5	24.8	25.4	27.7	25.4	27.0
Plasticity index	41.0	31.6	62.6	25.1	30.9	34.9
Shrinkage limit	17.9	16.3	11.8	19.4	12.4	12.4
Hygroscopicity	5.8	5.3	4.2	4.6	4.4	5.7
Vacuum moist equiv.	57.1	54.3	water-logged	53.7	58.7	58.9
Optimum moisture (Proctor)	28.6	30.9	31.3	28.4	31.2	32.2
% passing No. 200 sieve	98.1	98.7	97.5	98.5	98.6	98.5
% clay 0.005 mm	54.0	48.0	61.0	44.0	49.0	45.0
% colloids 0.001 mm	33.0	26.0	48.0	21.0	25.0	22.0

[1] Base exchange capacity, "T" = 30 me per 100 g; cations in natural soil in me per 100 g—H^+ = 12.0, Ca^{++} = 11.6, Mg^{++} = 4.5, Na^+ = 1.4, K^+ = 0;

Average valency (calc.) = $\dfrac{30}{\dfrac{12}{1} + \dfrac{11.6}{2} + \dfrac{4.5}{2} + \dfrac{1.4}{1}}$ = 1.4.

TABLE 3

OSMOTIC ACTIVITY $f(i)$ OF PUTNAM CLAY AND ITS HOMOIONIC MODIFICATIONS FROM
THE CONSOLIDATION GRAPHS OF REFERENCE

Soil Modification	i (%)	P (atm)	$f(i)$ Eq. 6	K Eq. 8	i_o	$i_o/S.L.$
Natural	33.3	4	0.255	-0.064		
	34.0	3.5	0.228	-0.064		
	34.6	3	0.198	-0.066	24	1.34
	37.7	2	0.145	-0.061		
	41.2	1	0.079	-0.064		
	45.2	0.5	0.434	-0.064		
Sodium	40.6	4	0.222	-0.0228		
	41.7	3.5	0.200	-0.0236		
	44.0	3	0.180	-0.0233	12	1.02
	49.3	2	0.135	-0.0233		
	59.1	1	0.081	-0.0232		
	68.0	0.5	0.0465	-0.0238		
Potassium	31.8	4	0.174	-0.063		
	32.5	3.5	0.156	-0.0635	19.8	1.02
	33.3	3	0.137	-0.064		
	35.5	2	0.097	-0.0645		
	39.8	1	0.0545	-0.063		
	43.0	0.5	0.0295	-0.066		
Calcium	36	4	0.398	-0.105	-	-
	37	3.5	0.354	-0.094	-	-
	38.2	3	0.315	-0.0835		
	39.8	2	0.219	-0.0795	32.2	2.6
	43.6	1	0.1195	-0.081		
	47.2	0.5	0.0645	-0.0795		
Magnesium	34.6	4	0.378	-0.046		
	35.5	3.5	0.344	-0.046		
	37.0	3	0.304	-0.0445	25.4	2.05
	39.8	2	0.218	-0.046		
	45.7	1	0.125	-0.0445		
	50.0	0.5	0.0685	-0.0435		
Hydrogen	35.5	4	0.194	-0.080	-	-
	37.0	3.5	0.177	-0.072		
	38.0	3	0.156	-0.071		
	39.8	2	0.109	-0.073	26.6	1.63
	43.6	1	0.060	-0.071		
	46.8	0.5	0.032	-0.073		

The results shown in Table 3 and Figure 2, which have been obtained by considering water suction in clay soils as an osmotic phenomenon without membrane due to the exchange ions, lead to the following conclusions:

1. The direct application of Van t'Hoff's law implies that the hydrostatic or osmotic pressure developed (Eq. 2) is directly proportional to the base exchange capacity and inversely to the valence of the exchange ions. This is in qualitative agreement with the known facts on the influence of the quantity and nature of the clay minerals and organic matter on water sorption and with the lesser activity of polyvalent cations. On the other hand, this does not explain the enormous influence of the nature of the cation, for example, Na^+ and K^+. The same law would make the osmotic pressure, under otherwise equal conditions, increase with increasing temperature, a phenomenon that according to our knowledge has not as yet been investigated experimentally. The simple application of Van t'Hoff's law considers the osmotic pressure inversely proportional to the initial water content of the soil system, which we have called the dilution factor, but it does not explain the quantitative relationship between both factors and leads to extremely high values for the osmotic pressure developed.

2. The basic hypothesis is the introduction of the concept of the osmotic activity of the exchange ions, which is a direct and necessary consequence of the fact that osmotic water suction in the case of clay soils is "osmosis without membrane." The activity is a function that decreases exponentially with increase in moisture content and the course of the decrease depends on a specific soil constant K, which is the basic factor that determines the osmotic pressure actually developed. Figure 3 shows the influence of the "dilution" factor and of the activity factor on the magnitude of the osmotic pressure developed by Na^+-Putnam clay.

The value for K depends for each soil on the nature of the exchange ions and decreases in the following order:

$$Na^+ \quad Mg^{++} \quad K+ \quad H^+ \quad Ca^{++}$$

$$K = -0.023; \quad -0.045; \quad -0.063; \quad -0.072; \quad -0.081$$

which corresponds to their degree of hydration or solvatation. The osmotic pressure developed is, of course, a decreasing function of the moisture content (Eq. 3).

3. It is an important experimental fact that the maximum value for the activity (i.e. $f(i) = 1$) corresponds to a moisture content i_0 that is practically identical with the shrinkage limit for the Na^+- and K^+-Putnam clay soils. The ratio $i_0/S.L.$ increases for the other ions in the order

$$Na^+ = K^+ \quad nat \quad H^+ \quad Mg^{++} \quad Ca^{++}$$

$$\frac{i_0}{S.L.} = 1.02 \quad 1.34 \quad 1.63 \quad 2.05 \quad 2.6$$

This signifies that the bivalent cations, and likewise H^+ which also has a flocculating effect, exhibit aggregation tendencies and form a structure as the moisture content decreases and the particles come closer together. One must keep in mind that at a water content of i_0, the pore volume and the solid volume of the system are approximately equal, giving a chance for contact between the particles, and that i_0 is that water content at which the relationship, "decrease of volume of the system =

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SLOPE STAKE BOOK MADE ON IBM 407 FROM EARTHWORK OUTPUT CARDS

PAGE 47 — BUREAU OF PUBLIC ROADS — FORM PDP 108 — COMPUTER STAKING DETAIL OR EARTHWORK COMPUTATIONS

STATION	ACTUAL PROFILE	GRADE AND PROFILE	TEMPLATE	AREA EXC SLOPE	AREA EMB ELEV OF STAKE	VOLUME EXC DIST FROM CL	VOLUME EMB CUT OR FILL
WARRENPEAK		76.4 GN 060 SL 060		2094 SF	SF	51.9 LT	U 29.9
129+75.00		5450.38GR 16.0 13.0		1.00 C	5477.8	48.4 RT	U 29.4
				1.00 C	5479.0	1875 CY	CY
				1957 SF	SF		
130+00.00		75.2 GN 055 SL 055		1771 SF	SF	47.8 LT	U 26.0
		5450.80GR 15.8 13.0		1.00 C	5474.5	48.3 RT	U 29.3
				1.00 C	5479.4	1726 CY	CY
130+50.00		71.0 GN 040 SL 040		1232 SF	SF	39.6 LT	U 18.6
		5451.46GR 15.0 13.0		1.00 C	5468.2	42.6 RT	U 23.6
				1.00 C	5474.3	2780 CY	CY
				PAGE TOTAL		6381 CY	CY
				BOOK TOTAL		40151 CY	28484 CY
				JOB TOTAL		40151 CY	28484 CY

SECTION LEFT — CENTER — SECTION RIGHT — SLOPE LEFT — SLOPE RIGHT — FORM PDP 108

GPO 941800

COORDINATES OF RIGHT-OF-WAY OWNERSHIPS FROM ORIGINAL PLAT DESCRIPTIONS

OWNERSHIP   CO ORDINATES

JOB	PARCEL	CRNER		NS CO ORD	EW CO ORD
HOLMANTAOS	HES 317	1		53,890.72	75,302.70
HOLMANTAOS	HES 317	2		53,721.40	75,251.53
HOLMANTAOS	HES 317	3		53,642.24	75,218.44
HOLMANTAOS	HES 317	4		53,541.72	75,175.36
HOLMANTAOS	HES 317	5		53,641.44	74,935.20
HOLMANTAOS	HES 317	6		53,765.13	74,423.93
HOLMANTAOS	HES 317	7		54,068.73	73,842.37
HOLMANTAOS	HES 317	8		54,177.70	73,936.32
HOLMANTAOS	HES 317	9		53,955.91	75,053.01

ERRORS

HOLMANTAOS	HES 317		1.36	0.31	53,640.88	75,218.75

FORM CDH 230
REV FEB 1960

Dayton - Kane
ROW Parcel No 2

COLORADO DEPARTMENT OF HIGHWAYS
TRAVERSE COMPUTATIONS

1. ALL SIDES AND BEARINGS KNOWN
2. TWO SIDES UNKNOWN
3. ONE SIDE & ONE BEARING UNKNOWN
4. TYPE 1 WITH AREA

PROBLEM TYPE:
2. TYPE 2 WITH AREA
3. ONE SIDE & ITS BEARING
3. TYPE 3 WITH AREA
4. TYPE 4 WITH AREA

5. TYPE 5 WITH AREA
6. TWO BEARINGS UNKNOWN

PROBLEM TYPE	GROUP	BASIC OR COURSE	TRAV NO.	AREA IN SQUARE FEET OR DISTANCE		BEARING				COSINE	SINE	LATITUDE OR AREA IN ACRES / ERROR OF CLOSURE	DEPARTURE OR SINE OF DELTA	NORTH PROGRESSIVE TOTALS	EAST	UNKNOWN
4	E	1	2											100000 000	100000 000	
			1	133 990	S	89	10	00	E	0 014543896	0 999894230	S 1 949	E 133 976	99998 051	100133 976	
			2	450 420	S	43	14	30	W	0 728470613	0 685077041	S 328 118	W 308 572	99669 933	99825 404	
			3	132 260	N	00	24	00	E	0 999975626	0 006981255	N 132 257	E 923	99802 190	99826 327	
			4	263 250	N	41	17	00	E	0 751456085	0 659783102	N 197 821	E 173 688	100000 011	100000 015	
			99									011	015			
			Area	33 674	Sq Ft							Area 773 Acres				
				33 674								773				
												Plus Arc Area 467				
												Parcel Area 1,240 Acres				

PROBLEM TYPE	GROUP	BASIC OR COURSE	TRAV NO	AREA IN SQUARE FEET OR DISTANCE		BEARING				COSINE	SINE	LATITUDE OR AREA IN ACRES / ERROR OF CLOSURE	DEPARTURE OR SINE OF DELTA	NORTH PROGRESSIVE TOTALS	EAST	UNKNOWN
2			4	263 248	S	41	16	60	W	CHORD	CALC	Acres= 0 282	− 12292 192 Sq Ft 306 258			
2			2	450 416	S	43	14	30	W	CHORD	CALC	Acres = 0 749	+ 32643 616 Sq Ft 512 393			
												Net Area 0 467 +				

RIGHT OF WAY PARCEL COMPUTATION

+area
area

Section Line

TABLE 5
RELATIONSHIP BETWEEN OSMOTIC PRESSURE AND SOIL-WATER CONSTANTS

Cation	L L	Osmotic Pressure for i = L L		P L	Osmotic Pressure for i = P L		Vacuum Moist Equiv	Osmotic Pressure for i = V M E		Opt Moist	Osmotic Pressure for i = Opt M	
		atm	pF		atm	pF		atm	pF		atm	pF
Natural	64.5	0.021	1.32	-	-	-	57.1	0.070	1.84	-	-	-
Na$^+$	88.0	0.142	2.15	25.4	14.1	4.1	water-logged	-	-	31.3	8.2	3.9
K$^+$	52.8	0 107	2.03	27.7	8.4	3.9	53.7	0.090	1.95	28.4	7.3	3.8
H$^+$	56.4	0.093	1.97	-	-	-	54.3	0.138	2.14	-	-	-
Ca^{++}	61.9	0.025	1.40	-	-	-	58.9	0.426	2.63	-	-	-
Mg	56.3	0.269	2.42	-	-	-	58.7	0.199	2.29	-	-	-

the more closely, the more permeable the soil is. Table 5 contains vacuum moisture equivalents and the corresponding osmotic suction values calculated by means of Eq. 5. The osmotic suction values approach the applied suction of 0.55 atmospheres in the following order:

$$Na^+ < nat < K^+ < H^+ < Mg^{++} < Ca^{++}$$

This order coincides with that of the permeabilities given by Winterkorn and Moorman (13). The calculated osmotic pressure for the most pervious soil (Ca^{++}) is 0.426 atm, which is closest to that for the applied suction. Waterlogging of the Na$^+$ soil, the least permeable, prevented the determination of its vacuum moisture equivalent.

It has been shown that the swelling of soils is due to the action of the osmotic pressure that is developed in the aqueous phase of a compacted soil-water system that is in contact with free water. For constant compactive effort (Standard Proctor) at optimum moisture content, it is possible to calculate by means of Eq. 5 the osmotic pressure of the respective moisture contents by using data given in the work by Winterkorn and Moorman (13). Results of such calculations are found in Table 5 for the Na$^+$- and K$^+$-Putnam clays for which the extrapolation to i_o, from which f(i) is determined, can be considered as permissible in the range of the i value at the optimum moisture content because of the coincidence of i_o and the shrinkage limit. For the other soils, it is necessary to have consolidation test data at lower moisture contents, because the optimum moisture content falls in the range where K changes or is likely to change because the shrinkage limit is smaller than i_o.

The same problem for the calculation of f(i) appears in the case of the plastic limit and for this reason only the osmotic pressures that pertain to the Na$^+$ and K$^+$ modifications are given in Table 5. The osmotic pressures calculated for these two cases give an idea of the magnitude of the suction force that clay soils are capable of developing at low initial humidities. They also reveal the P.I. as an index of osmotic activity since the larger P.I. of the Na$^+$ modification corresponds to a larger K value. This is a consequence of the relatively small influence of the nature of the exchange cations on the plastic limit and their marked influence on the liquid limit. It is the latter constant that is of major importance from the point of view considered here (see Figure 4).

The properties characteristic for clays (such as plasticity and compressibility) are intimately connected with their cation exchange capacity which in turn governs the osmotic suction. It is well known that rocks that have not suffered chemical weathering (e.g. quartz flower ground to the size of clay particles) show very low base exchange capacity and also

no plasticity or water-retaining power exceeding capillary saturation. All this leads to the conclusion that osmotic water suction plays a fundamental role with respect to these properties.

The ability of clays to retain water, even when under external pressure, above that required for capillary saturation goes parallel with their plasticity properties and several indices for their measurement have been proposed. Skempton (7) proposed the index of compressibility, which is the numerical value of the tangent of the approximately straight line obtained by plotting the logarithms of the pressures applied in the consolidation test against the corresponding water contents.

$$I_c = \frac{i_1 - i_2}{\log P_1/P_2}$$

The limiting value of this function is the inverse of $\frac{d \log P}{di}$ which implies that P varies exponentially with i. The importance of the index of compressibility is that it is inversely correlated with the liquid and plastic limits but has a general proportionality with the content in <0.002 mm clay. This is to be expected, since not only the quantity of the clay fraction but also the character of the component minerals and the nature of the exchange cations are determinant factors. The relationship between Terzaghi's coefficient of compressibility and Skempton's index of compressibility is apparent.

Operating at high moisture contents where the logarithmic relationship of Skempton represents a closer approximation, Eq. 5 should hold true if the osmotic interpretation is correct. This equation implies that P is not a simple exponential function of i, but a function of a function of i. It seemed indicated to calculate I_c in accordance with the osmotic concept and to compare it with the findings of Skempton. Using the logarithms in Eq. 5, one obtains:

$$\log P = \log (RT''T'') + \log \frac{f(i)}{i}$$

where RT"T" is a constant for each individual soil at a fixed temperature and the index of compressibility the inverse of $\frac{d \log P}{di}$:

Setting $\frac{f(i)}{i} = s$, one obtains

$$\frac{ds}{di} = \frac{i \frac{df(i)}{di} - f(i)}{i^2}$$

$$\frac{d \log P}{ds} = \frac{0.434 \, i}{f(i)}$$

$$\frac{d \log P}{di} = \frac{ds}{di} \times \frac{d \log P}{dS} = \frac{0.434 \, i \frac{d \, f(i)}{di} - 0.434 \, f(i)}{i \, f(i)}$$

Since f(i) is an exponential function of i :

$$\frac{d\ f(i)}{di} = K\ f(i) = \frac{Kf(i)}{0.434}$$

$$\frac{d\ \log P}{di} = \frac{\frac{0.434\ iK}{0.434\ f(i)} - 0.434\ f(i)}{i\ f(i)} = K - \frac{0.434}{i}$$

Consequently: $I_c = \dfrac{i}{K - \dfrac{0.434}{i}}$ ⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤⁤(9)

TABLE 6

INDEX OF COMPRESSIBILITY, NATURAL PUTNAM CLAY SOIL

i (%)	P (atm)	$\dfrac{d\ \log P}{di}$	$\dfrac{1}{\dfrac{d\ \log P}{di}}$	I_c, after Skempton
33.3	4	-0.077	12.9	-
34.0	3.5	-0.077	12.9	12.5
34.6	3	-0.078	12.8	8.8
37.7	2	-0.0725	13.8	17.6
41.2	1	-0.0745	13.2	11.6
45.2	0.5	-0.074	13.5	13.3
Mean	-	-	13.2	12.9

Table 6 gives values of I_c according to Skempton (Col. 5) and values calculated by means of Eq. 9 for the natural Putnam soil using the experimental data of Winterkorn and Moorman (13). The values calculated by means of Eq. 9 are in better agreement with each other than the I_c values after Skempton in which the term $0.434/i$ is neglected. On the other hand, these calculations show why the straight-line relationship between log P and i is only an approximation and that the true index of compressibility is $K - 0.434/i$. The approximative character of the linear relationship between log P and i has also been indicated by Macey (18) and by Terzaghi with respect to the relationship between the voids ratio and log P.

There have been other empirical formulae proposed to express the relationship between P and i. Freundlich and Posniak (15) found with other materials:

$$P = P_o \times C^k$$

where P_o and k are constants and C is the concentration expressed in grams

of colloids per 1000 gm of colloid plus water; this relationship was applied by Terzaghi to soils. Mattson (6) proposed:

$P = \dfrac{B}{i^3}$ where B is a constant and i the cubic centimeters of

water retained per gm of bentonite clay.

These relationships represent only gross approximations and consequently do not fit the experimental data very well. This, of course, is to be expected, because in accordance with osmotic considerations P is a function of a function of i.

It would be desirable to apply Eq. 5 to other soils and engineering characteristics that involve a sufficiently high water content for osmotic suction to play a determinant role. For this purpose, data on consolidation behavior, base exchange capacity, and nature of the exchange ions are required. Unfortunately, these latter properties have not been given sufficient attention by most experimenters. It is hoped that this state of affairs will be rectified and that the osmotic theory in its present form may be tested and verified on the basis of a larger body of experimental evidence. If additional experimental evidence confirms the conclusions drawn from the data on Putnam soil, then a unified physical interpretation can be given to apparently unrelated empirical soil characteristics and be connected by these characteristics with the fundamental properties of the soil at hand ("T" value and character of exchange ions) at moisture contents at which osmotic suction is a determinant factor.

On the other hand, recent investigations have demonstrated the relationship existing between the physical properties of soils and clays and their mineral components. Each of the major known groups of clay minerals has a more or less well defined range of exchange capacity and consequently of osmotic capacity:

Clay Mineral Group	"T" value in m e per 100 g
Montmorillonite	60-100
Illite	20-40
Kaolinite	3-15

Their osmotic activity, however, will depend on the nature of the exchange cations among which the Na^+ ion plays an outstanding part.

OSMOTIC INTERPRETATION OF SWELLING

In the osmotic interpretation of consolidation data obtained on remolded saturated clay soils, equilibrium was reached when the applied pressure P equals the hydrostatic pressure of osmotic origin developed by the dissolved exchange ions.

$P = (P_0)_{real}$ in equilibrium with a water content i. Before the equilibrium condition is reached, the applied constant pressure P performs consolidation work that involves plastic deformation and expelling of water through the pores of the system. For the latter, viscous flow resistance must be overcome. This resistance follows the law of Poiseuille which relates the flow rate to the pressure gradient and the pore radius. Therefore, a certain period of time is necessary for equilibrium to be

attained at each constant applied pressure and this period will be the greater the smaller is the effective pore radius. The elimination of water implies an increase in the concentration of the exchange ions and of their osmotic activity "T"/i and f(i) in Eq. 3, i.e. in the osmotic pressure $(P_o)_{real}$ which at a certain water content i reaches the value of P. The osmotic interpretation differs from the mechanical model of Terzaghi in the explanation of the equilibrium condition, but agrees with it on the influence of the time factor.

If the consolidation pressure is removed at equilibrium and contact with free water is maintained, osmotic suction starts because of the dilution tendency of the exchange ions. The system begins to swell, but the swelling is opposed by the deformation resistance of the system and the viscous resistance against the inflow of water. If the previously applied consolidation pressure has been sufficiently high to produce a high $(P_o)_{real}$ at equilibrium condition, this pressure will overcome the resistance of the system and result in swelling which may even perform external work by lifting a surcharge; in other words, a swelling pressure P_S is developed that may be measured by a surcharge that just prevents swelling. Swelling involves an increase in moisture content i; therefore, the initial $(P_o)_{real}$ existing at the removal of the surcharge decreases while the system swells until it becomes equilibrated by the combined effect of the internal resistance and the remaining surcharge. This decrease of the swelling pressure P_o with increasing i makes the swelling process essentially different from the consolidation process which proceeds at a constant pressure.

The possibility of realizing external work from the swelling process and of having a swelling pressure P_S that can be determined experimentaly is a consequence of a low initial moisture content and of an advanced state of consolidation. Such a state can be obtained by consolidation of a system from a high initial moisture content to a point where the moisture just fills the pore space.

By this reasoning, it is possible to explain the shape of both the consolidation and swelling curves as corresponding to existing equilibrium conditions. In consolidation, the applied load P balances the osmotic or hydrostatic pressure existing in the aqueous phase of the system at the equilibrium water content, whereas in swelling, the osmotic pressure is balanced by the surcharge and the internal resistance of the system to expansion and water intake.

Figure 4 presents swelling and consolidation curves derived for Na^+-Putnam clay from the data by Winterkorn and Moorman (13). We consider an initial state a with a water content i that is in equilibrium with pressure P_1 equal to $(P_o)_{real_1}$. If the pressure P_1 is reduced to P_{S2} while the system is in contact with free water, then $(P_o)_{real_1} - P_{S1}$ determines the swelling and the associated decrease in osmotic pressure and increase in i. At the moisture content i_2, the value of the osmotic pressure reaches that of the supercharge P_{S2} and swelling stops. From P_{S1} to P_{S2}, the swelling produces external work that is measured by the area i_1 i_2 c b (pressure x volume, if $i_2 - i_1$ is used to express the volume increase). If one wants to complete the cycle, one must increase the supercharge to P_2 (point d); then, following the consolidation curve, water in the amount of $i_2 - i_1$ is expelled and the initial state is reached by performing an amount of work given by the area $i_2 - i_1$ d a. The area a b c d or P_1 P_2

P_{s1} P_{s2} measures the difference between the work supplied to the system during the consolidation process by the action of the supercharge P_1 and the external work realized from the osmotic swelling. This difference is a measure of the internal resistance of the system to swelling and is called swelling hysteresis.

For small amounts of swelling ($i_2 - i_1 \longrightarrow 0$), the area P_1 P_2 P_{s1} P_{s2} is approximately equal to $P - P_s$ ($i_2 - i_1$). Hence, for the same increment ($i_2 - i_1$), $P - P_s$ is a measure of the internal resistance of the system to swelling. This can be read directly from consolidation-expansion equilibrium graphs made from plotted experimental data.

For the case in which the moisture content is i_3 and $P_s = 0$, the consolidation pressure equals P_3. This is the minimum value for an effective consolidation and corresponds to a $(P_o)_{real}$ that is equal to the internal resistance of the system. Higher pressures lead to further consolidation and to $(P - P_s) > 0$; i.e., to swelling pressures that after removal of the external load permit the system to perform external work.

A relationship between the applied consolidation pressure P and the maximum swelling pressure P_s, which the system can produce and which corresponds to a small increase in moisture content (i.e. at practically constant volume), can now be established. A coefficient a that permits the dividing of P into P_s and the internal resistance is introduced in the following manner:

$$aP = P_s; \quad (1 - a) \, P = P - P_s$$

$$aP + (1 - a) \, P = P \tag{10}$$

When $a \longrightarrow 1$, $P \longrightarrow P_s$ and $P - P_s = 0$, then one is dealing with strongly consolidated soils having a low moisture content, a high $(P_o)_{real}$, and consequently high swelling pressures if the internal resistance does not compensate for the $(P_o)_{real}$ that has been developed.

When $a \longrightarrow 0$, $P_s \longrightarrow 0$ and $P - P_s \longrightarrow P$, the system is one of low consolidation, high moisture content, low $(P_o)_{real}$, and small swelling pressure. When $(P_o)_{real}$ is equal to or smaller than the internal resistance, no expansion or production of external work is possible. For each soil there exists a maximum value for P at which $a = 0$ and $P_s = 0$.

From the consolidation-swelling graphs given by Winterkorn and Moorman (13) for natural and homoionic Putnam soils, the P and P_s values for moisture contents have been taken that were practically equal for the extreme cases, i.e. for $P_s = 0$ and, in the case of maximum values, for equal P values. These are shown in Table 7.

It can be seen that the internal resistance compensating $(P_o)_{real}$ for $P_s = 0$ corresponds to a moisture content and degree of consolidation at which contact between the solid particles or the strongly bound water films around these particles is already possible. This moisture content is distinctly higher for the Na^+ soils.

It can be expected that a is a function of the degree of consolidation which determines the free pore space, and of the moisture content, which determines $(P_o)_{real}$. The swelling curves of Winterkorn and Moorman (13) have a flattened shape and it was possible to extract only a few points that indicate in a preliminary manner that a decreases linearly with i and increases exponentially with P_1 starting at a P_o value that corresponds

TABLE 7

Soil Modifications	i (%)	P (atm)	P_s (atm)	$P-P_s$ (atm)	$a = \frac{P_s}{P}$
Natural	36.2	2.8	0	2.8	0
	34.0	4	0.9	3.1	0.25
Potassium	34.7	2.4	0	2.4	0
	31.8	4	0.5	3.4	0.125
Magnesium	37.7	2.8	0	2.8	0
	34.8	4	1.2	2.8	0.30
Sodium	49.0	2.2	0	2.2	0
	41.3	4	1.6	2.4	0.40
Hydrogen	38.4	2.8	0	2.8	0
	36.2	4	0.7	3.3	0.175
Calcium	37.0	2.6	0	2.6	0
	33.4	4	2.2	1.8	0.55

with $a = 0$. It is desirable to ratify these relationships on the basis of a larger body of experimental data.

The osmotic interpretation of the consolidation and swelling phenomena occurring in saturated clay soil systems permits the recognition and organization of the causative factors of these phenomena in the following manner:

1. The active swelling force is the osmotic pressure of the aqueous phase of the soil-water system. This force decreases with increasing water content. The soil characteristics that govern this force have been treated earlier.

2. Opposed to this active force is an internal resistance of the soil-water system expressed by the coefficient a. The internal resistance is the sum of (a) the mechanical resistance of the system to expansive deformation and (b) the viscous resistance to the incorporation of swelling water into the pores of the system. This sum is a function of the viscosity of the water and the permeability of the system.

3. The capacity for swelling and for developing a swelling pressure that is able to perform external work depends on the relative magnitude of the swelling pressure and the internal resistance.

4. Remolded, noncemented, expansive soils possess a low resistance to deformation and a high one to viscous flow. The coefficient approaches unity when a high pressure of consolidation results in a low initial moisture content and a high $(P_o)_{real}$; $a \longrightarrow 0$ under opposite conditions.

5. Undisturbed or cemented, structure-endowed, expansive soils possess a high resistance to deformation and may absorb a large part of $(P_o)_{real}$; consequently $a \longrightarrow 0$.

6. When the clay fraction of a soil occupies the spaces between the coarse granular components of a soil, then the swelling of the clay results in expansion of the system if the combined volume of clay particles and osmotic water exceeds that of the intergranular pore space. If the combined volume of clay and osmotic water is smaller than the intergranular porosity, then $a \longrightarrow 0$; if larger, then $a \longrightarrow 1$, as shown by Winterkorn and Choudhury (20) and Winterkorn (21).

7. When the initial soil-water system is not watersaturated, then water intake can take place by dislocating air without increase in total volume. It is impossible, however, to expell all air. Occluded air will be compressed by the swelling pressure in accordance with Boyle's law and dissolved in part in accordance with Henry's law. The result of this is a decrease in active swelling pressure. Altogether, the swelling press that a given soil in a given condition can develop depends fundamentally on the osmotic activity of the cations in the aqueous phase and on the value of a.

THE SWELLING OF SUBGRADES

The swelling pressure and the volume changes that may take place in a subgrade underneath a highway pavement are now examined. A subgrade soil compacted at Proctor optimum moisture content to maximum dry density, which expands against a supercharge that equals the weight of the contemplated superimposed structure is assumed. Of primary interest is the swelling pressure that may be developed with volume changes small enough to leave the structure unaffected. This means that the soil volume should remain practically constant or $\frac{\Delta V}{V} < 1$ percent.

For the determination of the maximum swelling pressure in the laboratory under conditions that duplicate as closely as possible those in the field, one can use the apparatus of Hveem (15) or its adaptation for determining the C.B.R. as used by Duarte Marchetti, and Ruiz (22) in the laboratories of the Highway Department of the Province of Buenos Aires. Test specimens are made with compaction requirements similar to those specified for the road project — at optimum moisture content as well as on the dry and wet side of it. The mold with the specimen is placed within a rigid frame. The surface of the specimen is covered with a porous plate and a compressometer is fastened between this plate and the upper arm of the frame. The mold with specimen is submerged in water and the compressometer dial registers simultaneously the swelling pressure and the expansion. One uses compressometers that, at maximum swelling force reached as a function of time, register a volume increment less than 0.5 percent of the original volume. This means that the volume remains practically constant and that a condition of permanent equilibrium exists between swelling pressure and supercharge.

Table 8 gives swelling pressures measured by this method for typical cases. These cases show as a generally observed characteristic the rapid fall of the swelling pressure with increase in initial moisture content.

The basic purpose of this work is to develop a theory that permits the interpretation of the preliminary results that have been obtained so far with different soils of the Province of Buenos Aires and that can serve as a guide to the correction of swelling, a problem of great technical and economic interest.

TABLE 8

MAXIMUM SWELLING PRESSURES AT PRACTICALLY CONSTANT VOLUME OF
TYPICAL SOILS[1]/ OF THE PROVINCE OF BUENOS AIRES

Soil	Characteristics	Compaction Moisture (%)	Max. Swelling Pressure (kg/cm^2)
A	SL = 18 LL = 34 PL = 23 Passing 200 = 98% HRB Class. A-6(9)	20 23[2]/ 25	0.172 0.080 0.011
B	SL = 10 LL = 44 PL = 25 Passing 200 = 84% HRB Class. A-7-6(12)	18 23[2]/ 28.5	1.490 0.529 0.230

[1]/Compacted by the Standard Proctor Method at, below, and above optimum moisture content.

[2]/Optimum.

The first point to be kept in mind is the initial moisture content. From the osmotic point of view this governs the active force of swelling, which is the osmotic pressure developed. Considering soils compacted by the standard Proctor method at optimum moisture content, the compaction work employed amounts to 5.5 lb x 25 blows x 18 in. = 12.4 ft-lb per cu ft = 60.4 kg per cu dm. These amounts are used up by the frictional and cohesive forces that oppose themselves to the displacement of the particles. The actual work performed in this compaction process is necessarily smaller than that required for consolidation to the same density starting with an excess of water. This is because of the work involved in overcoming the viscous resistance of the latter. Consequently the amounts of work involved in compaction and consolidation are not strictly comparable. Also, a soil-water system compacted at optimum moisture content with the Proctor standard method is not saturated, but possesses an air-pore volume of the order of 5 percent. Thus, in order for such systems to be saturated, an increase in moisture content of the order of 3 to 5 percent of the dry weight of the soil is necessary.

The fact of unsaturation implies that osmotic suction can superimpose itself on capillary suction. The capillary suction later stops at saturation while swelling continues as long as the osmotic pressure exceeds the resistance to volume increase.

The problem of establishing a moisture content for subgrade soils that is representative of that in equilibrium under field conditions is well known and need not be treated here. It need only be stated that there exist different criteria for judging the danger of water saturation and swelling under service conditions. These criteria are reflected in the different test methods proposed for obtaining data to be used in design.

As examples, the following are mentioned:

1. The criterion of Porter: saturation in the presence of free water, the specimen being confined by a supercharge equal to the pressure exerted by the pavement.

2. The criterion of Hveem: no water exudation when the specimen is submitted to a pressure of 28 kg cm^{-2}.

3. The criterion of McDowell: saturation under conditions of normal and lateral confinement that duplicate service condition.

All this shows the great need for a theory to explain the suction of water and the swelling of a soil system after the existing capillary porosity has been saturated. Such a theory is particularly important for soils of marked swelling characteristics.

The osmotic theory explains, at least qualitatively, the experimental and observational facts. For a certain degree of densification, low initial moisture contents give rise to large osmotic pressures particularly in the case of Na^+ exchange ions possessing high osmotic activity. Osmotic suction does not stop at pore saturation of a soil and may still be present to a marked degree at the moisture content of the liquid limit. The osmotic suction produces a hydrostatic pressure (analogous to the pore pressure) that acts against the solid phase and tends to expand the system against the resistance of the solid phase. The osmotic suction forces must also overcome the resistance of the porous system to the entrance and distribution of water. Simultaneously with water sorption, the osmotic pressure decreases tending to a new equilibrium value. Low initial moisture contents and associated high osmotic pressures may also exist in granular-cohesive soil systems that are possessed of a granular skeleton, the intergranular spaces of which contain the swelling clay material. Under such conditions, internal swelling may occur without external increase in the volume of the soil system (20,21).

There are two basic reasons for the importance of the character of the exchange ions in the swelling process. Ca^{++} and H^+ ions possess less osmotic activity than the Na^+, K^+, and Mg^{++} ions as expressed by the value of the constant K and a greater tendency to aggregation, which tendancy favors formation of a flocculated structure and results in increased internal resistance to swelling. This explains the reduction of volumetric swelling by the incorporation into a soil of portland cement or hydrated lime often mentioned in foreign and domestic literature (23,24,25,26). No study seems as yet to have been undertaken on the effect of these admixtures on the swelling pressure though this research with soils of the Province of Buenos Aires is being planned. Theoretically, small admixtures of hydrated lime should be sufficient for this purpose as long as they are uniformly distributed and given enough time for effective cation exchange. It is important to keep in mind that the influence of Na^+ exchange ions on the physical properties of soils are already pronounced when these ions represent only about 15 percent of the total exchange capacity (27). Hence, marked changes in volumetric expansion and swelling pressure which codetermine the swelling work may be expected already with additive concentrations well below those calculated on the basis of the "T" value.

The action of soluble salts, such as $CaCl_2$, employed in the control of soil swelling is of a more complex nature. Its theoretical treatment employs the Donnan equilibrium, to be shown later.

As previously indicated, experiments show a rapid fall of the maximum swelling pressure, at practically constant volume of the system, with a small increase in moisture content when the moisture content is in the range of the optimum for compaction. The osmotic theory permits the derivation of the function that connects P_S and i.

When Eqs. 5 and 10 are combined,

$$P_S i = \frac{R\,T\,{''}T{''}}{Va} \times a \times f(i) \qquad (11)$$

which expresses the condition of equilibrium between the excess osmotic pressure developed with respect to the internal resistance of the system and the supercharge P_S. Eq. 11 shows the rapid fall of P_S with increase in i; the term $R\,T\,{''}T{''}/Va$ is a constant for each specific soil and temperature and is directly proportional to the base exchange capacity and inversely to the valency of the exchange ions. The term $a\,f(i)$ is a combined expression of the internal resistance a and the osmotic activity $f(i)$; both functions decrease with increasing moisture content i and are influenced by the nature of the exchange cations.

The term $a\,f(i)$ can be evaluated provided that the remaining terms of Eq. 11 can be determined experimentally. Preliminary experiments show that $a\,f(i)$ falls exponentially with increasing i, because log $[a\,f(i)]$ is a decreasing straight line function of i. With some soils, this function deviates from linearity at low moisture contents. This may be interpreted as a consequence of internal expansion in systems possessing a granular skeleton with fixed intergranular porosity (20,21).

The importance of these preliminary results and of the application of the osmotic theory lies in the fact that they explain why one cannot expect simple relationships to exist between P_S and i inasmuch as the first is of necessity a function of a function of the moisture content. The swelling of soils and particularly of hydrophilic colloids in general has been considered for many years as a complex phenomenon in which purely mechanical factors are of importance though the fundamental explanation must be sought in the electric charges of the micelles and in osmotic phenomena (Ostwald and Mündler, 1919). The osmotic theory gives a first rational explanation of the influence of these factors when the moisture content of the soil is such that osmosis without membrane is the predominant phenomenon. This occurs in soils when the moisture content exceeds that of the shrinkage limit and becomes the exclusive factor in the saturated state where the capillary forces cease to exist. For the future development of this theory, it is necessary that methods be found that permit separate determination of a and $f(i)$ and the establishment of correlations that connect these functions with moisture content, degree of densification, and soil characteristics.

THE INFLUENCE OF FREE ELECTROLYTES—DONNAN EQUILIBRIUM

So far, only the influence of water on soil-water systems in the absence of free electrolytes or salts has been considered. Soils containing more than 0.2 percent of soluble salts are called saline and one differentiates between weakly salinized (0.2 to 0.5 percent) and strongly salinized (> 0.5 percent) soils (Rosov). At a soil moisture content of 20 percent the salts present in the liquid phase cause by themselves alone osmotic pressures of the order of tens of atmospheres and even

larger if they are separated from free water by means of a semipermeable membrane. In the absence of such a membrane, free diffusion takes place and no osmotic pressure is developed.

The free electrolytes interfere, however, with the osmotic pressure developed by the exchange ions which are held in the electrostatic adsorption field of the solid soil particles. This interference gives rise to a counterpressure called "Donnan effect." Consider the simplest case can be taken from Winterkorn (8), namely a homoionic Na^+ clay in a NaCl solution of molar concentration a. The contact surface AB in Figure 5 is pervious to water and free ions but not to the electrostatically restrained exchange ions. The molar concentration of the Na^+ exchange ions shall be called z.

If there were no Na^+ exchange ions, then Na^+ and Cl^- ions of the external solution would diffuse into the internal water phase until the concentration in both solutions were the same. In the presence of the Na^+ exchange ions, the external Na^+ and Cl^- ions diffuse only until thermodynamical equilibrium is established. The latter requires that the concentration product in both solutions be equal.

If q is the loss in concentration of the external solution, then the equilibrium condition is

$$q \, (z + q) \; = \; (a - q)^2$$

FIGURE 5

DONNAN EQUILIBRIUM

Internal Solution			A	External Solution		
Clay$^-$	Na^+	-		Na^+	Cl^-	Initial State
	z		B	a	a	
Clay$^-$	Na^+	Cl^-	A	Na^+	Cl^-	State of Donan Equili-
	z + q	q	B	a - q	a - q	brium

which permits the calculation of q as a function of the known z and a values:

$$q = \frac{a^2}{z + 2a}$$

The number of external ions that cross the boundary AB depends, therefore, on the relationship between a and z.

First case: a = 0. This applies to nonsaline soils, i.e., soils not containing free electrolytes. The molar concentration of the internal solution corresponds to that calculated from the base exchange capacity as employed in the preceding paragraphs.

Second case: a < z. This applies to clays of high base exchange capacity and low salt content. The equation for the Donnan equilibrium shows that the osmotic pressure developed under equilibrium conditions is smaller than that calculated from Van t'Hoff's law because it depends now

on the difference in concentration of the internal and external ions. This is the basic principle of Mattson's theory of "osmotic imbibition" (6). In this case, the difference or excess e will be equal to a, and $e = (2q + z) - 2(a - q) = 4q - 2a + z$. As the value for q is unknown, one substitutes its value as a function of z and a and obtains

$$e = \frac{4a^2}{z + 2a} - 2a + z = \frac{z^2}{z + 2a}$$

Then, in accordance with Mattson, the osmotic pressure becomes

$$P = RT \frac{z^2}{z + 2a}$$

and water will be sucked from the external to the internal solution. According to the author's concept, the right hand part of this equation should be multiplied by $f(i)$ to account for the actual osmotic activity of the exchange ions.

Third case: a > z. This applies to clays with low base exchange capacity and concentrated electrolyte solutions. In this case, $q \longrightarrow 1/2a$, which means that at the limiting case the external Na^+ and Cl^- ions tend to distribute themselves uniformly throughout the entire liquid medium, thus annulling the osmotic effect of the exchange cations. One sees clearly that the increase in free electrolyte concentration counteracts the osmotic suction and hence swelling. This is a well-known experimental fact shown, e.g., in the classical experiments by Winterkorn and Eckert and Schappler (8) who determined the free swelling of a bentonite Na^+ clay in solution of NaCl of increasing concentration and of Ca^{++}-Putnam soil in solutions of $CaCl_2$. However, experiments concerned with the influence of free electrolytes on the consolidation phenomenon and the swelling pressure are not known to the author.

Besides the use of Donnan equilibrium in a case where the clay was homoionic and had a cation in common with the electrolyte, the Donnan equilibrium can also be applied to more complicated systems where there are no common ions or where the ions are polyvalent. The resulting equations are more complicated, but the direction of the electrolyte influence is the same.

In recapitulation, the basic concept is that the osmotic suction of water is due to the retention of the exchange ions by the electrostatic charges on the surfaces of the clay particles which counteracts their kinetic energy and restrains their ability to diffuse. Osmotic suction occurs in the absence of free electrolytes in accordance with the phenomenon of osmosis without membrane to which the equation of Van t'Hoff can be applied using, however, the "osmotic activity" instead of the molar concentration. The Donnan equilibrium that is produced in the presence of free electrolytes appears to act as a reducing and not as a determinant factor with respect to osmotic suction. Mattson's ideas are useful in the explanation of the depressing action of salts on osmotic suction in the case of saline soils.

In order to clarify the different concepts, the osmotic pressure that Na^+-Putnam clay can develop at optimum moisture content from the different points of view will be calculated. It is assumed that the dissociation of NaCl in Na^+ and Cl^- ions is complete, that their activity in dilute solutions is unity, and that the "osmotic activity" $f(i)$ of the exchange Na^+ is the same as in the absence of the electrolyte. These simplifications

are acceptable in first approximation when the free electrolyte concentration is small.

Taking: Optimum moisture content = 31.3; "T" = 30; Va = 1; T = 273 + 25.

$$\log (f(i))_{i \, = \, 31.3} = 0.0235 \, (31.3 - 12) = -0.454 = 1.546;$$

$$f(i) = 0.352$$

Applying the equation of Van t'Hoff directly in the absence of free electrolyte, i.e. for a = 0, one obtains

$$z \approx [c] = \frac{"T"}{I \, Va} = \frac{30}{31.3 \times 1} = 0.96$$

$P_O = 0.082 \times 298 \times 0.96 = 23.4$ atm. This P_O holds for the case that the aqueous phase behaves as an ideal solution. In the absence of electrolytes, there exists no Donnan equilibrium and consequently the cation excess is $z = [c]$. Introduction of the osmotic activity f(i) reduces this value to $P_O = 0.082 \times 298 \times 0.96 \times 0.352 = 8.2$ atm.

We consider now the presence of 0.19 g of NaCl per 100 g of dry soil, which when dissolved in the aqueous phase of the soil-water system gives a = 0.1. This free electrolyte may diffuse freely and does not modify P_O according to Van t'Hoff's law. Application of the Donnan equilibrium reduces the active concentration to the excess of internal over external concentration and one obtains

$$e = \frac{(0.96)^2}{0.92 + (2 \times 0.1)} = 0.80$$

which gives an osmotic pressure of

$$P_O = 0.082 \times 298 \times 0.80 = 19.6 \text{ atm.}$$

This is 16 percent less than that calculated according to Van t'Hoff. Considering now the real osmotic activity of the exchange ions as affected by the reduction produced by the Donnan equilibrium, one obtains

$$P_O = 0.082 \times 298 \times 0.80 \times 0.352 = 6.9 \text{ atm.}$$

Thus, the presence of the NaCl reduces the $(P_O)_{real}$ from 8.2 to 6.9 atm.

These calculations show that the effect of an assumed Donnan equilibrium in the case of nonsaline soils is much smaller than that of the osmotic activity f(i). This is a logical consequence of the concept of osmotic suction as due to osmosis without membrane.

In turn, the Donnan equilibrium gives a first if semiquantitative explanation of the influence of free electrolytes in the case of saline soils. The calculation made before involving high concentrations of NaCl do not correspond with actual behavior, inasmuch as the simplified assumptions, by means of which calculation was made possible, are incompatible with reality.

With respect to practical applications, such as adding soluble salts for the correction of soil swelling, one must keep in mind that such salts produce ionic interchanges and modify the soil properties. Also, one can use only such salts as do not expand by crystallization in their hydrated forms. For economical and technical reasons, the most promising salt for the indicated use is calcium chloride, which has been employed in practice for a long time mainly because of its hygroscopic properties.

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