Dynamic Behavior of Simple-Span Highway Bridges

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● This paper presents the results of an analytical study on the dynamic behavior of simple-span highway bridges traversed by heavy vehicles. The factors that affect the dynamic response of such bridges are reviewed, and the influence of several of these are discussed. The factors considered include the speed of the vehicle, the spacing of the vehicle axles, the dynamic conditions of the bridge and the vehicle as the vehicle enters on the span, and the unevenness of the bridge surface. Most of these factors have been found in field tests to be responsible for major dynamic effects in highway bridges(1).

The approach used in this study consists in taking certain "representative" bridges and vehicles, and evaluating the effects of the parameters enumerated above by varying one parameter at a time. The bridges considered are of the I-beam type with span lengths in the range between 20 and 78 ft. This type of bridge consists of a series of steel girders and a reinforced concrete slab. With a few exceptions, the vehicle loading used corresponds to the heavy rear axles of a tractor-semitrailer combination with weights equal to those of the rear axles of an H20-S16 vehicle.

In the analysis, the bridge is idealized as a simply supported beam and the vehicle as a two-axle sprung load unit. Since the system is considered to have no width, the effects of the torsional oscillations of the bridge and the rolling of the vehicle about its longitudinal axis cannot be taken into account. The representation of the vehicle as a two-axle load is one of the distinguishing features of this study.

The idealized system is analyzed on the assumption that the instantaneous deflection configuration of the neutral axis of the beam is proportional to the corresponding static configuration produced by the weight of the vehicle and the weight of the bridge itself. In effect, this assumption reduces the beam to a system with a single degree of freedom and simplifies the analysis of the problem. The method has been programmed for the ILLIAC, the electronic digital computer of the University of Illinois. The results presented herein were obtained by application of this computer program.

Analytical studies of the dynamic response of highway bridges under moving vehicles have been reported in several publications. The effects of speed and of the initial vertical motion of the vehicle are discussed in Tung et al. (2) and Biggs et al. (3) by idealizing the vehicle as a single-axle load. The two-axle load used in the present study is obviously a more realistic representation of the vehicle. It enables one to take into account the effects of such variables as the spacing of the axles and the pitching motion of the vehicle. Field tests data (4) have given evidence of increased dynamic effects when the period of axle applications (i.e., the time between the passing of successive axles over a given point on the bridge) is synchronized with the fundamental natural period of vibration of the bridge.

The first analytical investigation of the dynamic effects produced by multiple-axle loads was published by Looney (5), who considered both two-axle and three-axle loads. Each axle load was represented either as a moving force of constant magnitude or as a smoothly running unsprung mass. The effect of vehicle suspension was not considered. Some exploratory studies of the influence of roadway unevenness have been reported by Scheffey (6) and Edgerton and Beecroft (7). In the present study, these parameters are studied in greater detail than in any of the previous publications.

METHOD OF ANALYSIS

System Considered

The idealized beam-load system used to represent the actual bridge-vehicle system is shown in Figure 1. It consists of a simply supported, linearly elastic beam spanning between two rigid supports on a horizontal line, and a two-axle load moving from left to right at a constant speed.

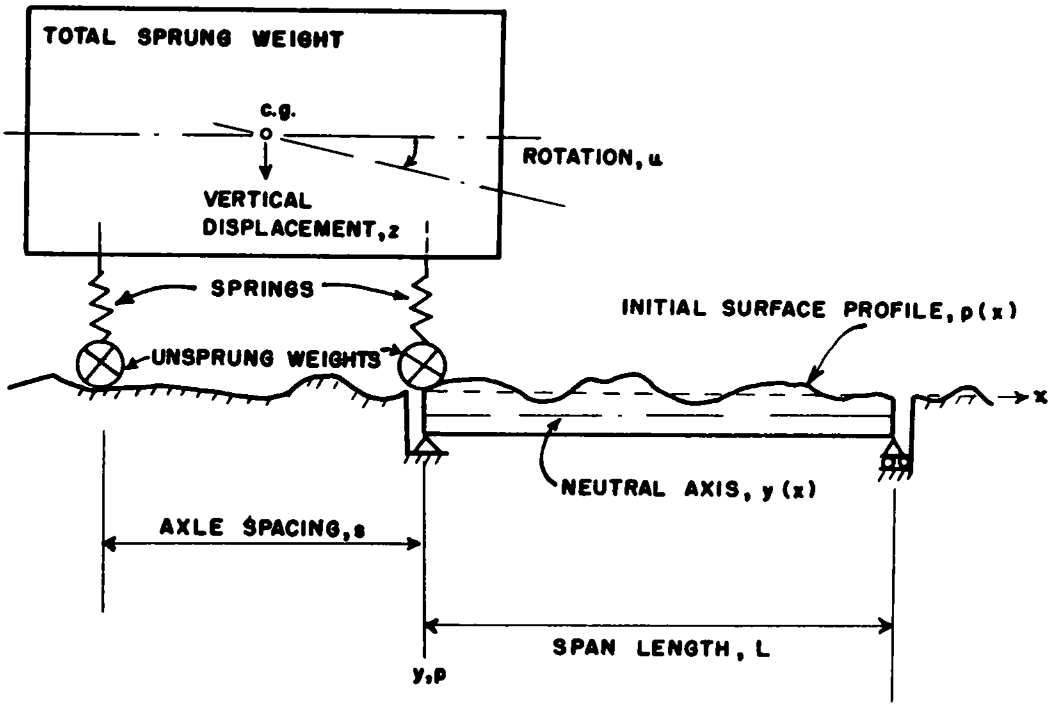

Figure 1. Idealized bridge-vehicle system.

Although the surface of the beam may be uneven, the beam itself is assumed to be of uniform mass and flexural rigidity per unit of length. In other words, the magnitude of the unevenness is assumed to be so small that its effect on the distribution of the mass and flexural rigidity of the beam along the span is negligible. This assumption is realistic because the height of the unevenness in actual bridges is usually small in comparison with the depth of the bridge. (In Figure 1 the magnitude of the unevenness is greatly exaggerated.)

The two-axle load consists of a rigid "sprung mass" connected to two "unsprung masses" through two linearly elastic springs. The unsprung mass is considered to be in direct contact with the roadway surface. The sprung mass represents the mass of the payload and chassis of the vehicle, and the unsprung masses represent the mass of the axles and tires. The springs simulate the flexibility of the suspension system and/ or tires. It should be noted that no damping is considered in either the vehicle or the bridge model.

In the following discussion, the terms "beam" and "bridge" and the terms "load" and "vehicle" are used interchangeably.

Analysis of Idealized System

The beam is analyzed as a system with a single degree of freedom. This is done by specifying the shape of its deflection curve at any instant. Specifically, it is assumed that the instantaneous dynamic deflection configuration of the beam is proportional to that produced by the weight of the moving load and the weight of the beam itself applied statically. Throughout the time that the vehicle is on the span, the axles of the vehicle are considered to be in contact with the beam or the approach pavement. The analysis is based on the ordinary beam theory which neglects the effects of shearing deformation and rotary inertia.

With the above simplifications, the behavior of the bridge-vehicle system can be described in terms of three second-order linear differential equations with variable coefficients. The three unknown functions are the vertical displacement of the center of gravity of the sprung weight of the vehicle, z (see Figure 1), the angular displacement of the sprung weight, u, and a function f(t) relating the instantaneous dynamic deflection of the beam to the corresponding static deflection. These equations were solved by a step-by-step method of numerical integration. For each time interval, the displacements z and u and the function f(t) are first determined. Next, the instantaneous reactions between the unsprung masses and the beam surface, and the inertia forces due to the mass of the beam are evaluated. Finally, the bending moments in the beam are determined from the instantaneous loading on the beam in the same manner as for a static problem. For the details of the method of analysis, the reader is referred to Wen (8).

Problem Parameters

The parameters that affect the response of the idealized system considered here may conveniently be classified as "bridge parameters" and "vehicle parameters." The bridge parameters include the span length, total weight, and fundamental natural period of vibration of the bridge; the deviation, if any, of the bridge surface from a straight line through the end supports; and the initial dynamic condition of the bridge. The term "initial condition" is used to designate the dynamic displacement and the velocity of the bridge at the instant the vehicle enters the span. It is conceivable that these quantities may be different from zero, because when the vehicle enters the span the bridge may already be in a state of oscillation due to the previous passage of another vehicle.

The vehicle parameters include the speed and total weight of the vehicle, the distribution of the weight among its unsprung and sprung components, the spacing of the axles, the effective spring constant for axle, and the initial dynamic condition of the vehicle. It is possible that at the instant the vehicle enters the span, the sprung weight of the vehicle may have a bouncing or a pitching motion, or a combination of the two. This motion may be due to the unevenness of the approach pavement, or it may result from a discontinuity at the bridge entrance.

In the analysis, these variables are combined into the following dimensionless parameters:

The Speed Parameter.—Denoted by the symbol α, this parameter is defined by the equation

$$\alpha = \frac{vT_b}{2L} \tag{1}$$

in which v is the speed of the vehicle, T_b is the fundamental natural period of vibration of the bridge, and L is the span length (for symbols see Appendix).

For the type of highway bridge considered herein, the period T_b is for all practical purposes proportional to L. Hence, the parameter α is essentially a function of the vehicle speed only.

The Weight and Weight Distribution Parameters.—The weight parameter is defined as the ratio of the total weight of the vehicle to the total weight of the bridge. The weight distribution parameters include the ratio of the static reactions on the two axles and the ratios of the unsprung weight for each axle to the total weight of the vehicle.

Frequency Parameters.—Associated with each axle j (j = 1 or 2), there is a frequency parameter defined as the ratio

$$\frac{(f_v)_j}{f_b} = \frac{\text{"Natural Frequency of } j^{th} \text{ Axle"}}{\text{Fundamental Natural Frequency of Bridge}}$$

The axle frequency, $(f_v)_j$, is defined by the equation

$$(f_v) = \frac{1}{2\pi}\sqrt{\frac{k_j}{M_j}} \tag{2}$$

in which k_j is the effective stiffness of the j^{th} axle, and M_j is the corresponding sprung mass. The sprung mass for an axle is the mass corresponding to the static reaction on the axle due to the sprung weight of the vehicle. It should be pointed out that only in special cases does the axle frequency $(f_v)_j$ represent the actual natural frequency of the vehicle in either the vertical mode or the pitching mode of vibration.

Rotary Inertia Parameter.—This parameter is a measure of the resistance of the sprung mass of the vehicle against pitching motion, and it is defined by the ratio

$$\frac{J}{\sum_{j=1}^{2} M_j a_j^2}$$

in which J is the polar moment of inertia of the sprung mass about its centroidal axis, M_j is the sprung mass for the j^{th} axle, as previously defined, and a_j is the horizontal distance between the j^{th} axle and the centroid of the sprung mass. The value of this parameter depends on the geometry of the sprung mass and the spacing of the axles. The expression in the denominator of this ratio represents the polar moment of inertia of the sprung mass when the masses M_j are concentrated at the axles. Thus, when this ratio is unity, the two-axle load can be thought of as two separate and independent single-axle loads, each consisting of a sprung and an unsprung mass.

Axle Spacing Parameter.—This parameter represents simply the ratio s/2L, where s is the spacing of the axles.

Initial Condition Parameters for Bridge.—The initial condition of the bridge may be defined by the dynamic deflection and the velocity of the structure at the instant the front axle of the vehicle enters the span. The expressions for these quantities are presented later.

Initial Condition Parameters for Vehicle.—The initial condition of the vehicle may be defined by the vertical and angular displacements and velocities of the sprung mass of the vehicle at the instant the front axle enters the span. The expressions for these quantities are also given later.

Initial Profile Parameters.—These parameters specify the shape of the bridge surface when the bridge is in a position of equilibrium under the influence of its own weight only. The number of parameters required depends on the degree of regularity of the initial profile. For example, if the profile can be represented by a mathematical expression, such as a sine function, one need specify only the amplitude and the length of the sinusoidal wave. At the other extreme, if the profile is quite irregular, one must specify its elevation at a large number of stations along the span. In this case, the number of parameters needed is equal to the number of stations used. Obviously, the latter technique may be used also for any profile representable by a simple mathematical function.

Computer Program

The method of analysis has been programmed for the ILLIAC, the high-speed digital computer of the University of Illinois. With the program developed, it is possible to consider any practical combination of the parameters enumerated in the preceding section. By an appropriate choice of the weight distribution parameters, it is also possible to consider the effects of a two-axle, totally sprung load, or of two unsprung point masses. It is also possible to consider the case of two independent one-axle loads, each consisting of a sprung and/or unsprung mass, as well as the case of a single one-axle load. These are simply "degenerated" cases of the more general two-axle load unit shown in Figure 1.

The program can handle a sinusoidal profile variation with a maximum of 33 half-sine waves along the span, and any other nonsystematic profile that can be prescribed by the values of the ordinates at 100 stations equally spaced along the span.

To use the program, one need only prepare a "parameter tape" on which are recorded the values of the parameters defining the problem to be solved. This tape is then read by the computer following a "master tape" that contains appropriate machine instructions for the analysis of the problem. The results computed include the dynamic deflection and the maximum static deflection at midspan, and the amplification factors for bending moment at midspan and at sections beneath the axles. The term "amplification factor" is defined as the ratio of the total dynamic effect at a section to the corresponding absolute maximum static effect at the same section. For example, the amplification factor for bending moment at a section beneath the rear axle of the vehicle represents the ratio of the instantaneous dynamic moment at that section to the corresponding absolute maximum static moment.

A complete solution, covering the period between the instant the front axle moves on the span and the instant the rear axle leaves the span, is obtained in about 3 min. This time can be halved if the computer is instructed to print out only the maximum values of the quantities referred to above.

SCOPE OF STUDY

Approach

The approach used in this investigation consists in considering specific bridges and vehicles and evaluating the dynamic behavior of these systems by varying each of the following parameters: (a) speed of the vehicle, (b) spacing of the axles, (c) initial condition of the bridge, (d) initial condition of the vehicle, and (e) unevenness of the bridge surface.

It is realized that the effects of these factors cannot actually be isolated because of the interrelations among their roles and influences. For example, when the effect of axle spacing is being considered, the influence of vehicle speed is inseparably involved. In this presentation, each factor will be treated separately, but mention will be made of any important interrelations that exist between the various parameters.

For each problem, the response of the bridge was evaluated for the time interval between the entry of the front axle on the span and the exit of the rear axle from the span. Except where otherwise indicated, it is considered that (a) the vehicle has no vertical or angular motion at the instant it enters the span and (b) the bridge is initially at rest, and its surface is horizontal and perfectly smooth. In other words, the bridge is assumed to be cambered for dead-load deflection.

Bridges Considered

The bridges considered in this study correspond to type SA-2-53 in the Standard Bridge Plans of the Bureau of Public Roads (9). These bridges are of the I-beam type with steel girders and a concrete deck, designed for an H20-S16 loading. Their weights and natural frequencies, calculated from the data given in the standard plans, are listed in Table 1. In determining the natural frequencies, the bridges were assumed to behave as simply supported beams. The flexural rigidity of the bridge cross-section, EI, was determined by considering noncomposite action between the slab and the beams, and the modular ratio for concrete was taken equal to 10.

TABLE 1

ESTIMATED WEIGHTS AND NATURAL FREQUENCIES OF SA-2-53 BRIDGES

Span (ft)	Total Weight (lb)	Fundamental Natural Frequency (cps)
20	98,000	12.13
45	227,000	5.41
50	257,200	4.97
60	323,500	4.08
70	385,700	3.19
78	448,200	2.81

For this class of bridges, the relationship between the speed parameter, α, defined by Eq. 1, and the speed of the vehicle, v, is given approximately by the equation

$$\alpha = 0.003\, v \tag{3}$$

in which v is in miles per hour.

Vehicles Considered

Three vehicles, A, B, and C, were used in this study. Their characteristics are shown in Table 2. The majority of the numerical results presented were obtained with Vehicle A. The characteristics of this vehicle were estimated from information obtained from 6 major truck and trailer manufacturers and from data contained in Reference 10. The front and rear axle of this vehicle are intended to represent the rear axle of a tractor and the axle of a semitrailer, respectively.

TABLE 2

CHARACTERISTICS OF VEHICLES USED

Quantity	Vehicle A		Vehicle B with Two Identical Axles	Vehicle C with Single Axle
	Front Axle	Rear Axle		
Unsprung weight (lb)	5,200	3,400	4,300	5,100
Sprung weight (lb)	26,800	28,600	27,000	26,000
Spring constant (lb/in.)	21,700	26,000	23,850	21,700
"Natural frequency" (cps)	2.81	2.98	2.90	2.86
Axle spacing (ft)	27.1		14.0	---
Polar moment of inertia of total sprung mass about centroid (kip-ft^2)	10,200		2,700	---
Gross vehicle weight (lb)	64,000		64,000	31,000

Vehicle B, with identical axles, is a simplified version of Vehicle A. The weight and spring constants for each axle of this vehicle were taken equal to the average values of the corresponding quantities for the two axles of Vehicle A. In this case, it can readily be shown that the actual natural frequency of vibration of the vehicle for both the vertical mode and the pitching mode are identical and numerically equal to the natural frequencies of the axles.

Vehicle C, a single-axle loading, simulates the front axle of Vehicle A. This vehicle was used to obtain most of the data relating to the effects of deck unevenness.

The vehicle speeds used in this study ranged approximately from 15 to 70 mph.

RESULTS OF STUDY

Representative History Curves

It is instructive to examine first the response of a particular section of a bridge to the crossing of a vehicle. The graph of this response as a function of time will be referred to as a "history curve." Such curves are given in Figures 2 and 3.

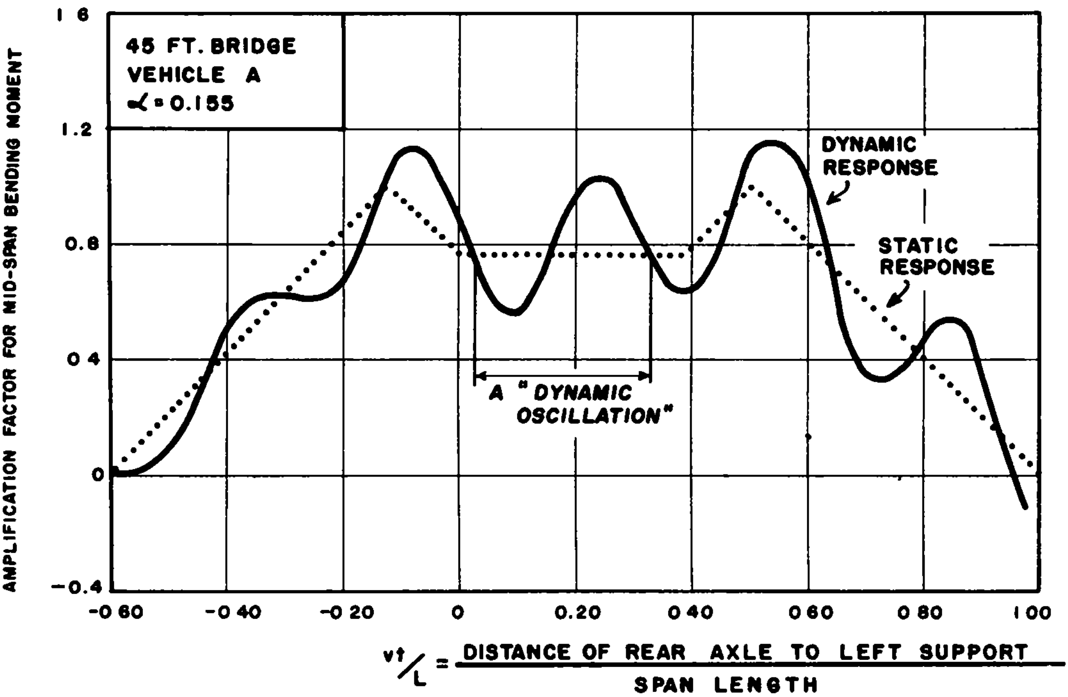

Figure 2. Representative history curves for midspan bending moment.

In Figure 2 the solid line shows the time history of the bending moment at midspan of a 45-ft bridge traversed by Vehicle A at a speed of 51 mph. The ordinate represents the amplification factor for moment at midspan, as previously defined, and the abscissa represents the quantity vt/L, in which t is time. Since both v and L are constant for a given problem, the abscissa is essentially a time coordinate. The time origin, t = 0, is taken as the instant when the rear axle enters the span. Accordingly, the time interval between the entry of the front axle on the span and the exit of the rear axle corresponds to a range of the abscissa from -s/L to unity. The symbol, s, denotes the spacing of the axles. For the case considered, s/L = 27.1/45 = 0.60.

Included in Figure 2 as a dotted line is the history curve of the corresponding static bending moment. This is essentially an influence line for moment at midspan due to the two-axle vehicle load. The difference between the ordinates of the solid curve and the dotted curve represents the history of the dynamic effect of midspan.

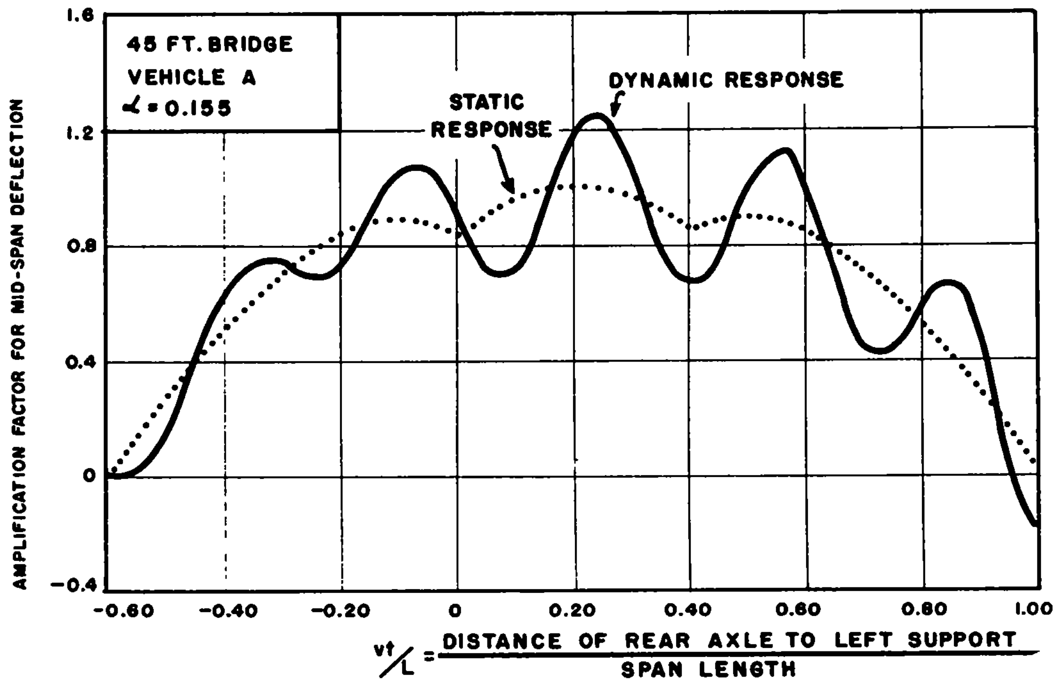

Figure 3. Representative history curves for midspan deflection.

Figure 3 shows the dynamic and static history curves for deflection at midspan for the particular bridge-vehicle system considered in Figure 2. As before, the ordinates are expressed as amplification factors.

It can be seen from Figures 2 and 3 that the dynamic history curves oscillate about the static curves with a "period" close to the fundamental natural period of vibration of the unloaded bridge, T_b. One complete wave on a history curve will be referred to as a dynamic oscillation. The reason the "period" of the dynamic oscillations is not identical to T_b is that the reactions between the axles and the bridge surface are not constant but vary with time generally in a fairly complex manner. If the "periods" of the individual waves are considered to be identical to T_b, then the speed parameter acquires an interesting physical meaning: its reciprocal

$$\frac{1}{\alpha} = 2\,\frac{L/v}{T_b} \tag{4}$$

is equal to twice the number of oscillations performed by the bridge in the time required for one axle to cross the span.

Both the ordinate and the slope of the dynamic response curves in Figures 2 and 3 are zero at the instant the front axle of the vehicle enters the span. This is as it should be, because the bridge is initially at its position of static equilibrium. The free vibration of the bridge (i.e., the motion after the vehicle has left the span) is determined by the values of the deflection and velocity of the bridge at the instant the rear axle leaves the span. From Figure 3 it can be seen that both of these quantities are different from zero in this case.

It should be noted that the maximum dynamic response may be greater or smaller than the maximum static response depending essentially on the positions of the dynamic oscillations relative to that of the maximum static response. The term "dynamic response" refers to the sum of the static effect and the dynamic effect. For a given amplitude of dynamic oscillation, the dynamic response will be greatest when the peak of a dynamic oscillation coincides with the peak static response; it will be smallest when the valley point of a dynamic oscillation coincides with the peak static response. While the position of the load producing the maximum static effect at a section is fixed, the position of the dynamic oscillations is altered when the speed of the vehicle, its initial conditions, or the initial conditions of the bridge are changed.

Effect of Vehicle Speed

The numerical results showing the effects of the various factors will be presented, in general, in the form of spectrum curves. A spectrum curve is a plot of the maximum dynamic response as a function of some parameter of the bridge-vehicle system.

Figure 4 shows spectrum curves of amplification factors for midspan bending moment and midspan deflection as functions of the speed parameter for the system considered in the preceding section; i.e., the 45-ft bridge traversed by Vehicle A. In addition to the values of α the abscissa shows the speed of the vehicle in miles per hour. The ordinates of these curves at v = 51 mph are the maximum values of the corresponding curves in Figures 2 and 3.

It is noted that these spectrum curves are undulatory and that both the length and the height of the undulations increase with increasing values of the speed parameter. The undulatory feature is a consequence of the characteristics of the dynamic history curves, as discussed in the preceding section. It has been pointed out that the magnitude of the maximum dynamic response at a section depends on the position of the dynamic oscillations in the history curve relative to that of the maximum static value. An increase in the speed parameter, α, decreases the number of oscillations that the bridge undergoes while the vehicle is on the span. In other words, as the speed parameter increases, the lengths of the individual waves in the history curve increase, and the peaks of these waves shift to the right. It is this change in the position of the dynamic oscillations relative to that of the maximum static response that produces the major change in the magnitude of the maximum dynamic effect at a section.

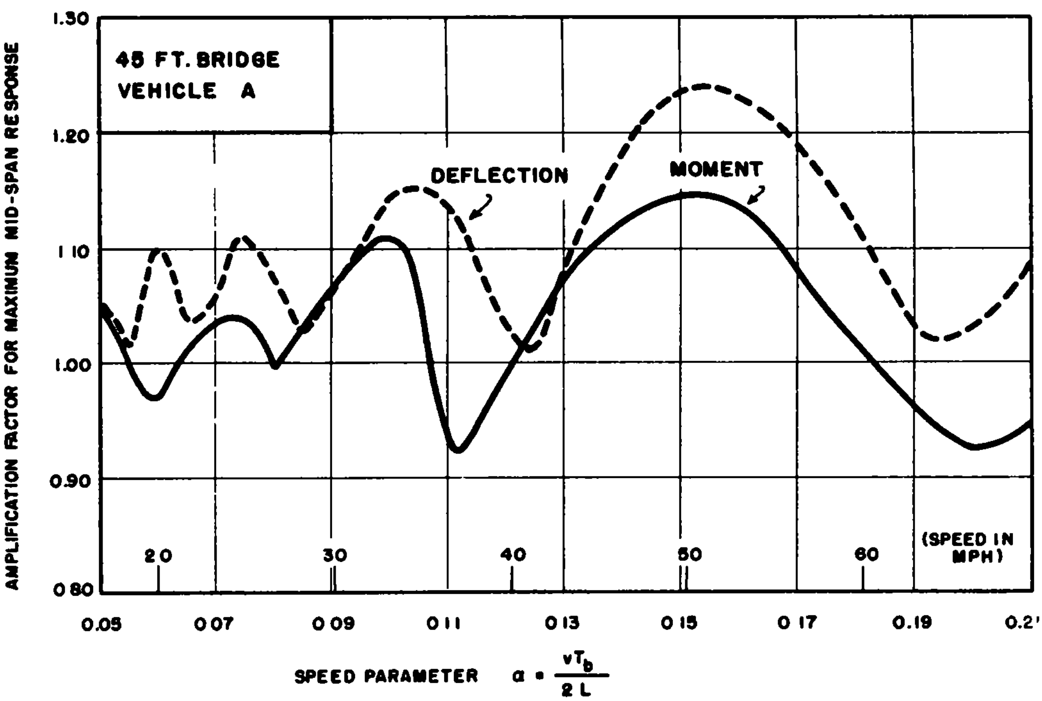

Figure 4. Effect of vehicle speed on maximum response at midspan.

The length of one undulation on the spectrum curve denotes the change in α necessary to "shift" one complete dynamic oscillation in the corresponding history curve past the position of the maximum static response. A relative maximum is obtained when the maximum static response coincides with the maximum ordinate of a dynamic oscillation, and a relative minimum is obtained when the peak static ordinate combines with the minimum ordinates of a dynamic oscillation.

Because the number of dynamic oscillations for the period that the vehicle is on the span is approximately inversely proportional to the speed parameter (see Eq. 4), a given change in speed will alter the positions of the dynamic oscillations by a smaller amount at high speeds than at low speeds. It is for this reason that the lengths of the undulations in the spectrum curves of Figure 4 increase with increasing speed.

The undulating nature of the spectrum curves, though interesting, is of a limited practical importance, because in practice the speed of a vehicle may vary within a fairly wide range. From the standpoint of application to design, it is the peak values of these curves that are significant. In this connection, it is important to note that the peak values of the response increase with increasing speed.

Figure 5 represents spectrum curves for midspan bending moment for the 20- and 70-ft spans together with the curve for the 45-ft span reproduced in Figure 4. The term "spectrum curve for bending moment" is used in lieu of the more precise term of "spectrum curve of amplification factors for bending moment" for the sake of brevity. This abbreviation will be adopted throughout the remaining part of this paper. Furthermore, unless otherwise noted, all bending moments will refer to midspan moments.

The differences in the curves in Figure 5 reflect essentially the influence of the bridge characteristics. It should be noted that when the span of the bridge is changed, both the weight ratio and the frequency ratios of the system are altered. The values of these parameters for the three spans may be determined from the data listed in Tables 1 and 2.

It is seen that the curves from the 20- and 70-ft spans exhibit the same general trends as those previously discussed for the 45-ft span. However, the magnitudes of the response for the 20-ft span are considerably larger than those for the longer spans. From available data (2) it would appear that the dynamic effects would not have been so large had a single-axle load been used to represent the vehicle.

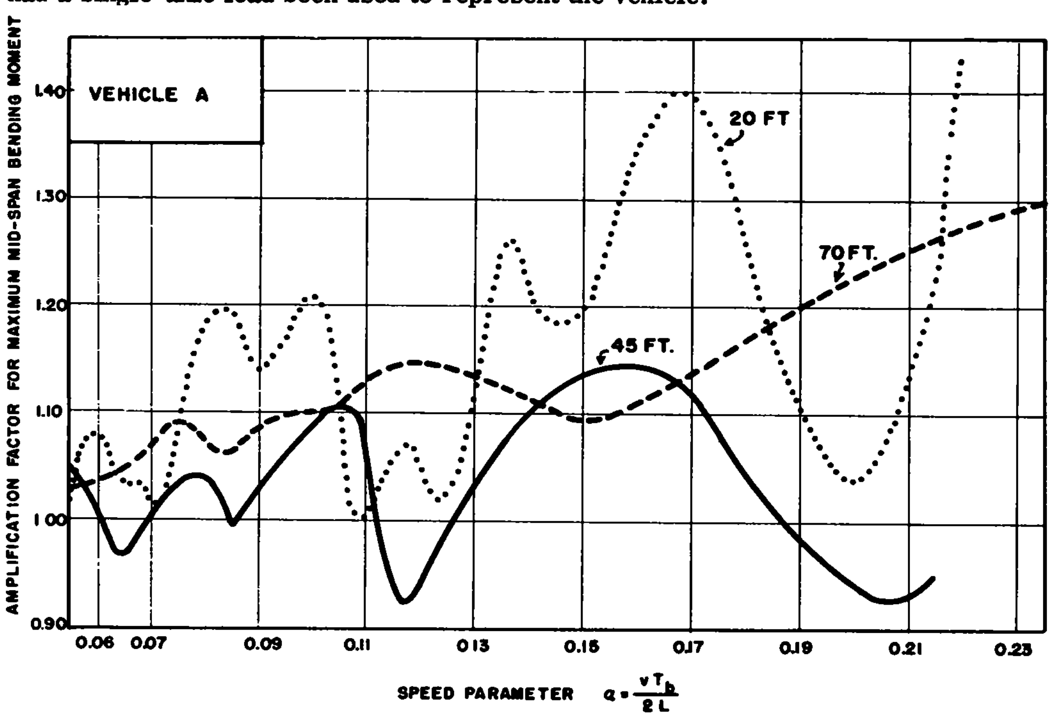

Figure 5. Effect of vehicle speed (for three span lengths).

For values of σ less than 0.21, corresponding to vehicle speeds of less than about 70 mph, the absolute maximum amplification factors for bending moment for the 20-, 45-, and 70-ft spans are 1.42, 1.15, and 1.29, respectively. The corresponding values of α are approximately 0.17, 0.16, and 0.21. The lowest amplification factor, applicable to the 45-ft span, is associated with the smallest value of the speed parameter. In this connection, it should be pointed out that the location of the peak values of the response in a spectrum curve is a function of the axle spacing. For different axle spacings, it is quite possible that the absolute maximum amplification factor for the 45-ft span may occur at a higher speed, in which case, the magnitude of the amplification factor may be considerably larger. The effect of the axle spacing is considered further in the next section.

For the same range of the speed parameter, the absolute maximum amplification factors for deflection at midspan of the 20-, 45-, and 70-ft spans are 1.67, 1.24, and 1.26, respectively. The maxima occur at approximately the same values of the speed parameter as those for the absolute maximum bending moment. The spectrum curves for deflection are not included here, but are available in Wen (11).

In Figure 5 it is of interest to note that the curve for the 20-ft span is considerably more sensitive to variations in the speed parameter than are the curves for the longer spans. For the purpose of explaining this trend, it may be assumed that the period of the bridge oscillations induced by the front axle of the vehicle is equal to the fundamental natural period of vibration of the bridge, T_b. Then the time interval between the entry of the two axles on the span is s/v, and the number of complete oscillations executed by the bridge in this time interval is

$$n_o = \frac{s/v}{T_b} \tag{5a}$$

By virtue of Eq. 1, this equation may also be written in the form

$$n_o = \frac{s}{2L\alpha} \tag{5b}$$

The quantity n_o is essentially a measure of the "phase" of the bridge oscillation at the instant that the rear axle enters onto the span. Since the response of the bridge is obviously a function of this initial "phase," any change that influences this quantity will have a corresponding influence on the magnitude of the response. From Eq. 5b it can be seen that, for a fixed value of s, the change in n_o resulting from a given change in α is greater for the shorter spans. It follows then that the response of the short spans should be more sensitive to variations in α than that of the long spans. This is in agreement with the trends shown in Figure 5.

Figure 6 shows spectrum curves for bending moments at sections beneath the axles for a 45-ft span traversed by Vehicle A. In this case the moments are normalized with respect to the absolute maximum value of the static moment beneath an axle. This reference moment, which is the same for both axles because the two axle-loads are identical, represents the maximum possible static moment in the bridge.

It can be seen in this figure that the curve for the rear axle exhibits higher peaks and is more sensitive to variations in the speed parameter than the curve for the front axle. These features are consequences of the fact that the bridge is already in a state of oscillation when the rear axle enters the span, and may be explained in a manner analogous to that used previously in this section.

Effect of Axle Spacing

The field tests reported by Foster and Oehler (4) have shown that the axle spacing of the vehicle (in combination with the vehicle speed) is one of the major factors that influence the dynamic behavior of highway bridges. In these tests, relatively large dynamic effects in the bridge were observed when the time interval between the passing of the two axles over a given point was equal to the natural period of vibration of the structure. This observation suggests that the successive applications of the axle loads may act as a periodic forcing agent on the structure and, therefore the synchronization of the period of vibration of the bridge with the period of load application may lead to a sort of "resonant" condition.

The time interval between the passing of the two axles of the vehicle over the same point is s/v. One would expect the "resonant" or "critical" condition to occur when

$$\frac{s/v}{T_b} = \frac{s/2L}{\alpha} = \text{an integer} \tag{6}$$

This theory is generally borne out by the analytical results that have been obtained, covering three span lengths of 20, 45, and 70 ft, three vehicle speeds of approximately 35, 52, and 68 mph, and axle spacings from 0 to 35 ft. Some of these results are presented and discussed in the following paragraphs.

Figure 7 shows spectrum curves for moment and deflection as a function of the axle spacing. These results are for the 45-ft span and for a value of $\alpha = 0.105$ ($v \simeq 35$ mph). The characteristics of the vehicle used are the same as those of Vehicle A, except that the axle spacing is varied over a range. The abscissa shows both the dimensionless axle spacing parameter, s/2L, and the axle spacing, in feet. It is seen that the distances between consecutive peaks in these curves are approximately constant. Furthermore, the peaks occur at values of s/2L that are essentially multiples of the speed parameter used. In other words, Eq. 6 is satisfied quite closely. However, the magnitude of the peak effects are not sufficiently large to justify the use of the term "resonance" to describe the phenomenon. In the following, the term quasi-resonance will be used.

Spectrum curves of bending moment for the 20- and 70-ft spans are given in Figure 8, in which is also reproduced the moment spectrum curve from Figure 7. All three curves are for the same loading and the same value of the speed parameter, $\alpha = 0.105$. It can be seen that the general features of these curves are essentially the same. Additional curves, obtained for values of $\alpha = 0.155$ and 0.205, show the same characteristics. Some of these data are summarized in Table 3. The complete curves are available in Wen (11).

The third column of Table 3 lists the absolute maximum amplification factors for midspan moment for the complete range of axle spacings considered. The fourth column gives the amplification factors for $s = 0$; i.e., a single axle having the same total load.

TABLE 3

SUMMARY OF MAXIMUM MOMENTS FOR A RANGE OF AXLE SPACINGS

Span Length (ft)	α	Amplification Factor for Maximum Midspan Moment: Maximum Value for $0 \leq s \leq 35$	Amplification Factor for Maximum Midspan Moment: Value for $s = 0$
20	0.105	1.17	0.96
	0.155	1.31	1.06
	0.205	1.34	0.84
45	0.105	1.18	1.00
	0.155	1.27	1.16
	0.205	1.32	0.97
70	0.105	1.11	1.02
	0.155	1.21	1.01
	0.205	1.18	1.03

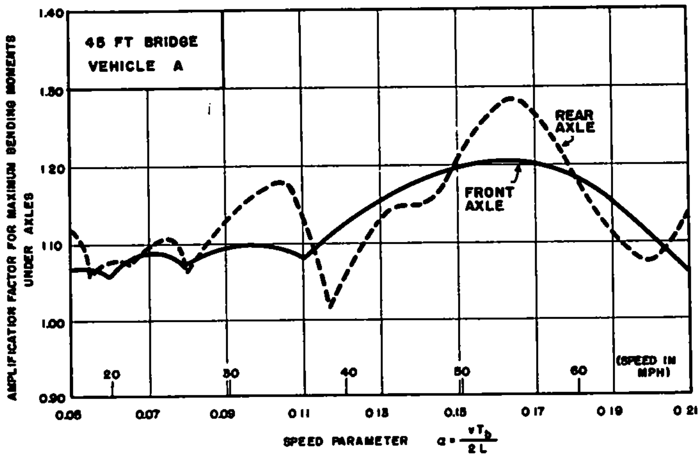

Figure 6. Effect of vehicle speed on maximum bending moments under axles.

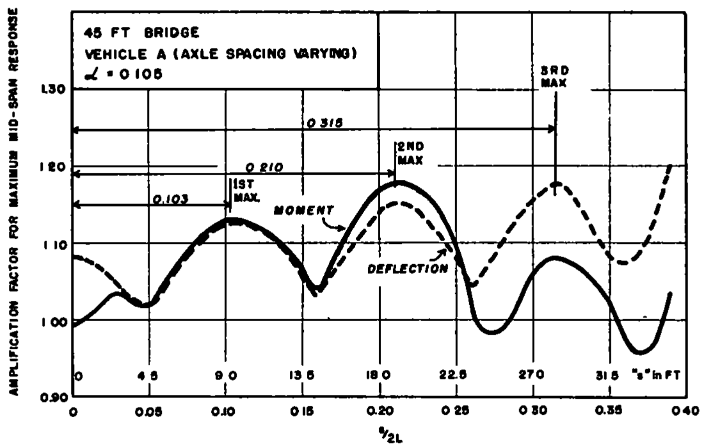

Figure 7. Effect of axle spacing (45-ft bridge).

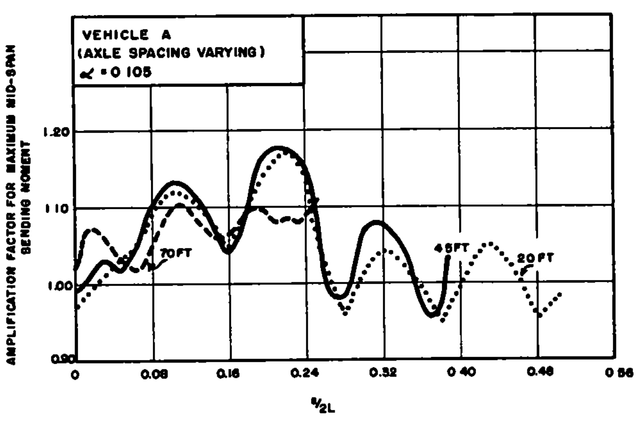

Figure 8. Effect of axle spacing (for three span lengths).

It is seen that the absolute maximum effects are, in general, significantly larger than those for the single-axle loading. However, this comparison is not a reliable measure of the effect of axle spacing, because the results for the single-axle load are quite small for the particular speeds considered. A change in the speed parameter will affect both sets of values, but the possible increase in the magnitude of the effects is expected to be more pronounced in the case of the single-axle load. This may be appreciated by noting that for the 20-ft span the amplification factor is less than one, even for a value of the speed parameter as high as $\alpha = 0.205$. It is apparent that in this case the maximum static moment combines with a negative ordinate of a dynamic oscillation. The amplitude of this oscillation is at least 0.16; i.e., 16 percent of the maximum static moment. Therefore, the amplification factor for moment will increase at least by 0.32 when the speed of the vehicle is changed so that the peak static moment combines with the maximum positive ordinate of a dynamic oscillation. For a complete evaluation of the axle-spacing parameter, one must consider also the effect of varying the speed parameter.

Table 4 shows the interrelationship of speed and axle spacing. Column 1 gives the peak values of the bending moment obtained by keeping the axle spacing at $s = 27.1$ ft and increasing the speed parameter to a maximum value of 0.21. Column 2 gives the corresponding values obtained by considering three values of the speed parameter, as previously noted, and varying the axle spacing from 0 to 35 ft. It can be seen that, whereas for the 20- and 70-ft spans, the values given in Column 1 are larger than those in Column 2, for the 45-ft span the reverse is true. In Column 3 are listed the larger of the values given in Columns 1 and 2. These results show that the peak dynamic effects increase with decreasing spans.

TABLE 4

SUMMARY OF MAXIMUM MOMENTS FOR A RANGE OF SPEEDS AND AXLE SPACINGS

Span (ft)	Maximum Amplification Factors for Midspan Moment		
	(1) Speed Varied	(2) Axle Spacing	(3) Max. Value of (1) and (2)
20	1.42	1.34	1.42
45	1.15	1.32	1.32
70	1.29	1.21	1.29

From the results presented, it may be inferred that, for a vehicle having more than two axles, the dynamic effects produced by the individual axles may be cumulative, if the spacings between consecutive axles are multiples of one another.

Concerning the relative magnitude of the dynamic effects produced by tandem axles and a single axle of the same total load, it may be noted that, no matter how small the axle spacing, Eq. 6 can always be satisfied, provided the vehicle speed is sufficiently low. However, from the discussion presented relative to the effect of vehicle speed, it follows also that quasi-resonance at low speeds is not as important as at high speeds. If one somewhat arbitrarily takes $v \cong 33$ mph as the lowest speed for which dynamic effects are of consequence, and uses $s = 4$ ft for tandem axles, then he finds that Eq. 6 can be satisfied only for spans equal to or less than 20 ft. That is, for vehicle speeds greater than about 33 mph, the quasi-resonant condition cannot be realized in bridges with spans longer than 20 ft. Hence, as one might expect intuitively, the dynamic effects produced by tandem axles may be substantially larger than those produced by a single axle only in the case of very short spans.

Effect of Initial Bridge Motion

The dynamic oscillations induced by the passage of a vehicle over a bridge may last for a considerable time after the vehicle has left the span. The influence of this motion must be considered in evaluating the response of the bridge to the passage of a second vehicle following the first closely.

It is reasonable to assume that the initial oscillation is in the fundamental mode of vibration of the bridge. The initial deflection, $y(x, t')$, may then be expressed by the equation

$$y(x, t') = y_o \sin\frac{\pi x}{L} \sin(2\pi\frac{t'}{T_b} + \beta_b) \quad (7)$$

in which y_o is the amplitude of the midspan deflection, x is the position coordinate measured from the left support, t' is time measured from the instant the front axle enters the span, and β_b is a phase angle. At $t' = 0$, the midspan deflection is $y_o \sin \beta_b$, and the corresponding velocity is $\frac{2\pi}{T_b} y_o \cos \beta_b$. The initial condition of the bridge may be specified either in terms of the initial deflection and velocity, or in terms of the amplitude y_o and the phase angle β_b. In the following, the latter alternative is used. For convenience, the amplitude y_o is specified by the dimensionless parameter

$$q_b = y_o/y_s \quad (8)$$

in which y_s is the maximum static deflection of the bridge at midspan due to the weight of the vehicle.

The influence of the phase angle, β_b, on the response of the bridge is shown in Figure 9, in which are given spectrum curves of bending moment for a 45-ft bridge traversed by Vehicle A. The value of $q_b = 0.314$. The three curves correspond to vehicle speeds of 38, 51, and 58 mph. Values of $\beta_b = 0$ and $\beta_b = 2\pi$ define the condition in which the beam has no initial deflection but has a maximum downward velocity. For $\beta_b = \pi$ there is no initial deflection, but there is a maximum upward velocity. For $\beta_b = \frac{\pi}{2}$ there is an initial maximum downward deflection but no initial velocity.

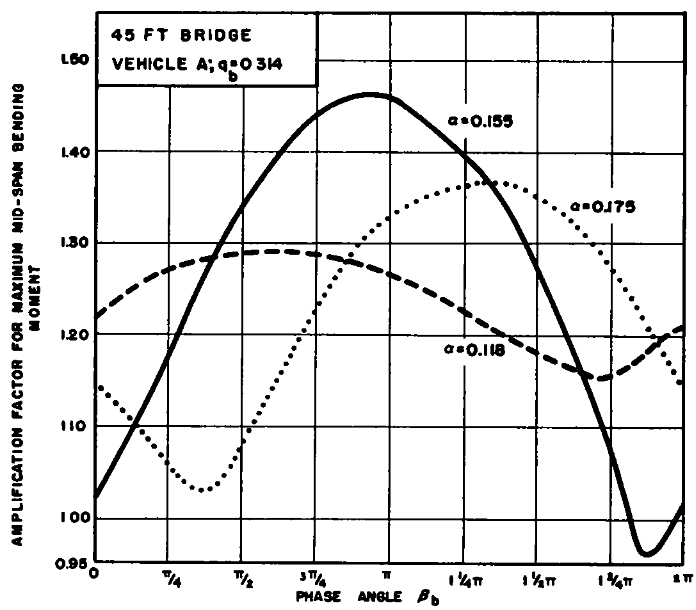

Figure 9. Effect of phase angle of bridge initial oscillation.

It can be seen from this figure that the maximum response at midspan is quite sensitive to variations in the phase angle β_b. As might be expected, the peak values of the response, which are the most significant quantities from a design standpoint, occur at different values of β_b for the different speeds.

In Figure 10 the absolute maximum values of the bending moment and deflection for three initially oscillating bridges are plotted as a function of the initial oscillation parameter q_b. These results are for Vehicle B and a speed parameter of $\alpha = 0.10$. The amplification factors plotted are the maximum possible for values of the phase angle β_b in the range between 0 and 2π In other words, the ordinates in this plot represent the peaks of curves similar to those presented in Figure 9.

It can be seen from Figure 10 that the amplification factors for both moment and deflection increase linearly with q_b and that the rate of increase is approximately one to one. Accordingly, the maximum effects produced by a vehicle in an initially oscillating bridge are approximately equal to the sum of the effects of the initial oscillation and the effects produced by the vehicle when the bridge is initially at rest. This result may be expressed by the equation

$$(AF)_{q_b} = (AF)_o + q_b \tag{9}$$

in which $(AF)_{q_b}$ is the maximum possible amplification factor for either moment or deflection at midspan for an initially oscillating bridge, and $(AF)_o$ is the corresponding quantity for the bridge when initially at rest.

In Table 5, the amplification factors for bending moment predicted by Eq. 9 are compared with the exact values as determined from the plots given in Figures 9 and 10. It can be seen that the agreement between the two sets of results is quite satisfactory. It should be pointed out, however, that additional solutions are required to establish the range of validity of Eq. 9.

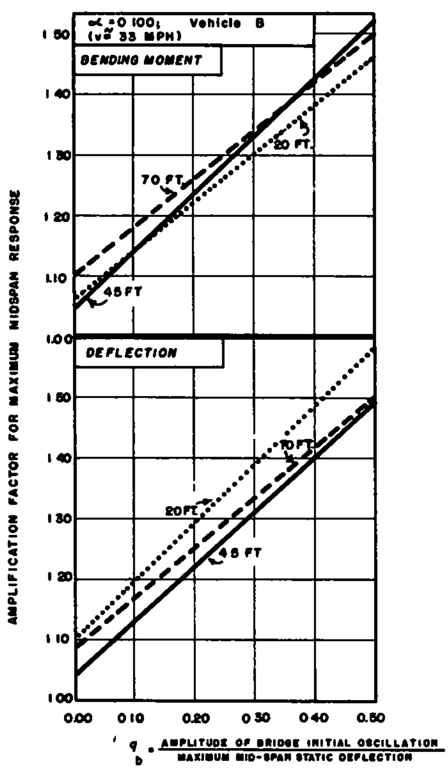

Figure 10. Effect of amplitude of bridge initial oscillation.

TABLE 5

COMPARISON OF EXACT AND APPROXIMATE VALUES OF MAXIMUM MOMENT

α	Maximum Amplification Factor for Midspan Moment		
	$q_b = 0$	$q_b = 0.314$	
		From Fig. 9	From Eq. 9
0.118	0.97	1.29	1.28
0.155	1.15	1.46	1.46
0.175	1.04	1.37	1.35

Span (ft)	Maximum Amplification Factor for Midspan Moment		
	$q_b = 0$	$q_b = 0.50$	
		From Fig. 10	From Eq. 9
20	1.06	1.46	1.56
15	1.05	1.52	1.55
70	1.10	1.50	1.60

Effects of Initial Vehicle Motion

The influence of this factor is investigated by assuming that the initial motion of the vehicle corresponds to one of its natural models of vibration. Proceeding in a manner analogous to that used in the preceding section, it can be shown that, for each motion, the initial condition of the vehicle can be specified in terms of the amplitude of the motion and a phase angle.

For the motion corresponding to the vertical or bouncing natural mode of vibration, the phase angle will be denoted by β_v, and the amplitude of the initial displacement will be expressed in terms of the dimensionless parameter, q_v, defined as

$$q_v = z_o/z_s \tag{10}$$

where z_o is the amplitude of the dynamic vertical displacement of the centroid of the sprung mass, measured from the position of static equilibrium, and z_s is the corresponding static deflection. The maximum value of q_v measured in the field (3, 12) appears to be on the order of 0.40. For the motion corresponding to the pitching or angular natural mode of vibration, the phase angle will be denoted by β_p, and the amplitude of the motion will be expressed in terms of the parameter, q_p, defined as

$$q_p = u_o/u_s \tag{11}$$

where u_o is the amplitude of the angular displacement, and u_s is the ratio of the sum of the static deformations of the springs for the two axles divided by the spacing of the axles.

The effects of the phase angles, β_v and β_p, on the response of the bridge are similar to that of the phase angle for initial bridge oscillation, β_b, considered in Figure 9, and they will not be discussed further here.

The effects of the amplitudes of initial motion are shown in Figure 11, wherein are given spectrum curves for bending moment and deflection for a 70-ft bridge traversed by Vehicle B at a speed corresponding to a value of $\alpha = 0.10$. For the solid curves the initial oscillation consists of a purely vertical motion, and for the dashed curves it consists of a purely angular motion. The ordinates in these plots represent the absolute maximum amplification factors for all possible values of the phase angle associated with the initial motion considered.

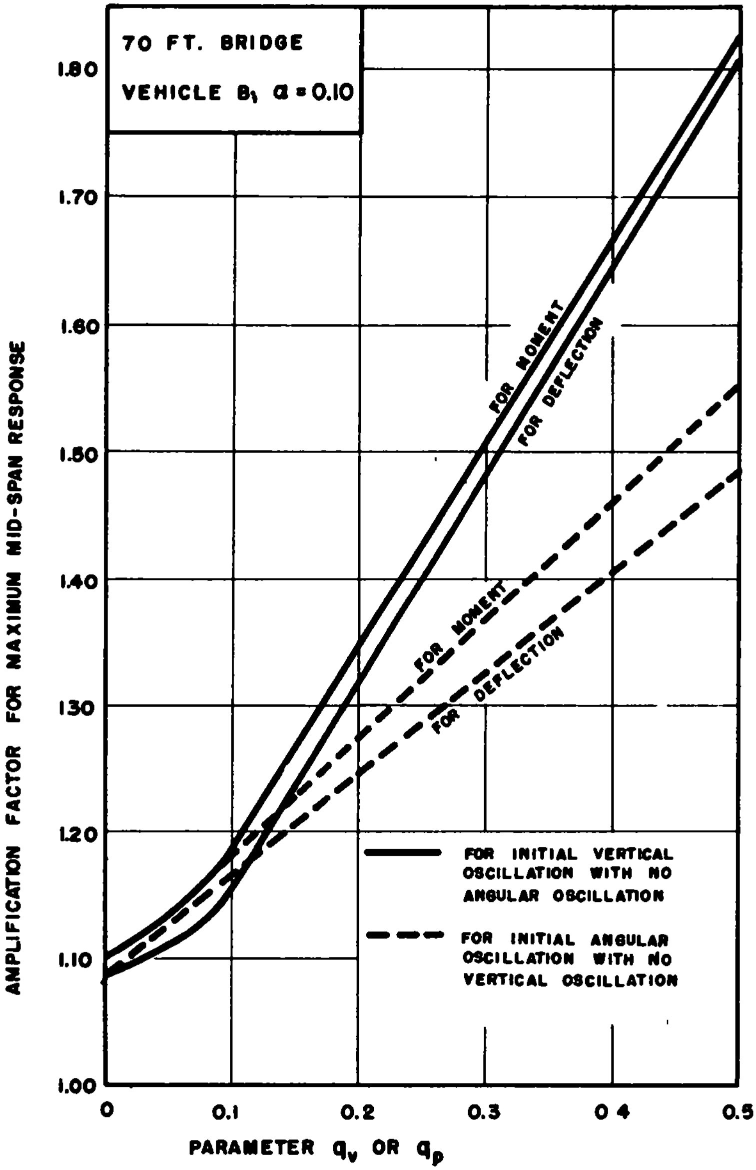

Figure 11. Effect of amplitude of initial oscillation of vehicle.

It is seen that, except for the initial portions of the solid curves that are curved, these curves increase linearly with increasing values of the abscissae. This linear relationship between the maximum response and the amplitude of the initial vehicle oscillation has been noted before (3) for the case where the vehicle was represented as a single-axle load.

For the vehicle considered, equal values of q_v and q_p define equal amplitudes of dynamic deformation for the springs of the two axles. Of course, the deformations of the individual springs are in phase for the bouncing motion and 180 deg out of phase for the pitching motion. On comparing the magnitudes of the dynamic effects produced for identical values of q_v and q_p, one finds that, when q_v and q_p are greater than about 0.10, the effects for an initial bouncing motion are appreciably larger than those for an initial pitching motion. It should be pointed out, however, that the characteristics of the vehicle used to obtain these data are such that the two natural frequencies are equal. It is conceivable that, if the two frequencies are appreciably different from one another and the pitching frequency is closer to the natural frequency of the bridge than is the bouncing frequency, the maximum effects produced by the initial pitching motion may be greater than those due to the initial bouncing motion.

For the range of amplitudes of initial vehicle oscillation considered in Figure 11, the maximum amplification factors for the bouncing and pitching motion are roughly 1.8 and 1.5, respectively. These are undoubtedly rather large dynamic effects. However, it should be remembered that in obtaining these results the vehicle has been assumed to have no damping. The effect of damping will be to reduce the magnitude of the amplification factors. This reduction is expected to be particularly significant for the longer spans for which the time required for the vehicle to cross the span at a given speed is greater than for the shorter spans.

Effect of Deck Unevenness

The fact that the condition of the bridge surface may be one of the major factors controlling the magnitude of the dynamic effects in highway bridges was demonstrated in the tests reported by Edgerton and Beecroft (7). These tests involved two bridges with identical superstructures, except that one bridge had a smoother deck than the other. For the same loading, considerably greater dynamic effects were measured in the bridge with the uneven deck.

The majority of the solutions presented in the following paragraphs are for a sinusoidal unevenness. The results of an exploratory study are also given for a deck profile representable by a series of half-sine waves of unequal lengths and amplitudes, and for a localized irregularity or a "bump" located on the otherwise smooth surface. The vehicle is represented as a single-axle load.

Sinusoidal Unevenness. — Let p(x) denote the deviation of the deck profile from the design grade, as shown in Figure 1. Then a sinusoidal unevenness may be expressed by the equation

$$p(x) = b \sin \frac{m\pi x}{L} \tag{12}$$

in which b is the amplitude of the unevenness, and m is the number of half-sine waves along the span. The length of each half-sine wave is L/m.

There are two conditions under which the dynamic effects produced by the unevenness may be expected to be of appreciable magnitude. The first corresponds to the case in which the period of the profile deviation, T_p, is equal or close to the natural period of vibration of the vehicle, T_v. The period T_p is equal to the time required for the vehicle to travel the distance covered by one complete wave, and it is given by the equation

$$T_p = \frac{2L/m}{v} = \frac{1}{m}\frac{T_b}{\alpha} \tag{13}$$

By equating the expression on the right-hand side of this equation with T_v, one finds that this critical or "resonant" condition will occur when

$$m\alpha = \frac{T_b}{T_v} \tag{14}$$

It is seen that m and α are interrelated. That is, for a given value of m there is a critical speed, and for a given speed there is a critical value of m.

The second critical condition may be expected when the period of the profile variation coincides with the natural period of vibration of the bridge, T_b. On equating these two periods one obtains the relation

$$m\alpha = 1 \tag{15}$$

Although the amplitude of variation of the force exerted by the vehicle on the bridge may be small in this case, the dynamic effects in the bridge may still be appreciable. The absolute maximum effects might be expected to occur when

$$T_p = T_v = T_b$$

In this case Eqs. 14 and 15 are identical.

In the formulation of Eqs. 14 and 15, it has implicitly been assumed that there is no coupling between the motion of the vehicle and the bridge. The extent to which these approximate equations can predict the actual critical conditions is discussed in the following.

Figure 12 gives spectrum curves for bending moment as a function of the speed parameter for values of m = 5, 9, and 15. Included in this figure as a solid curve is also the solution for a smooth surface, b = 0. The bridge involved is the 70-ft span, and the vehicle is the single-axle Vehicle C. In this case, the amplitude of the unevenness, b, is one thousandth the length of the half-sine wave. Since L = 70 ft, the amplitude b, in inches, is given by the expression

$$b = \frac{0.84}{m}$$

in which m is different from zero.

For these and all subsequent problems the vehicle is assumed to have no vertical motion as it enters the span, and the bridge is considered to be initially at rest but deflected under its own weight. In other words, the design grade is considered to coincide with the dead-load deflection configuration of the structure. The first wave of the unevenness is assumed to be located below the design grade.

The most striking feature of the results depicted in this figure is that the peak values of the response for the wavy surfaces are considerably larger than those for the smooth surface. The values of α corresponding to the maximum peak values of the response are listed in Table 6 together with the values evaluated from Eqs. 14 and 15. In general the values obtained from Figure 12 are close to the predicted value, particularly those based on Eq. 14.

According to Eqs. 14 and 15, for each of the spectrum curves in Figure 12 there should be two high peaks. The fact that there is only one peak may be attributed to the closeness of the two resonant conditions that appears to have forced the two peaks to merge into one. For the system considered, the ratio $T_b/T_v = 0.90$. Additional re-results reported elsewhere (8)(13) indicate that large dynamic effects do occur for each of the two critical conditions referred to above, but of the two conditions, the condition

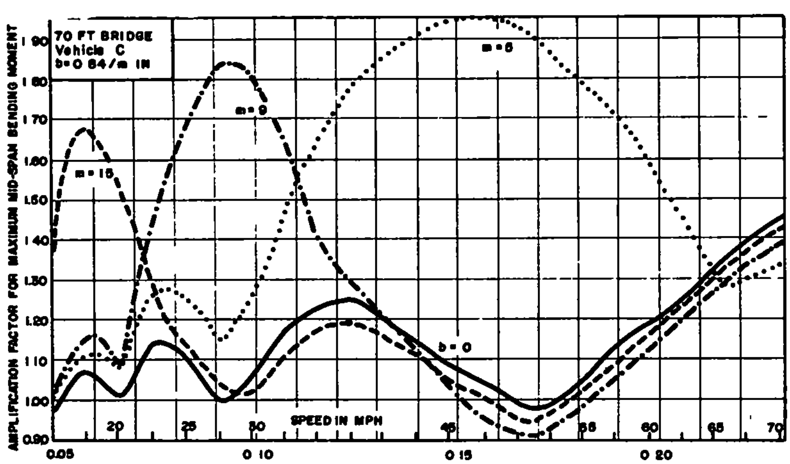

Figure 12. Effect of vehicle speed (bridge with sinusoidal deck profile).

of synchronization between the period of the profile variation and the natural period of vibration of the vehicle is, in general, the more severe. The implication of this result is that, in order for the dynamic effects in the bridge to be large, the amplitude of the variation of the interacting force must first be magnified.

In Figure 12 it is noted that the ordinates of the curves for the wavy surfaces decrease rapidly after they have passed their respective high peaks and eventually join the curve for the smooth surface. This is as it should be, inasmuch as one would normally expect the dynamic effect produced by the sinusoidal unevenness to be negligible when the period of the disturbance, T_p, is small in comparison with the period of the responding system. For a given bridge-vehicle system, T_v and T_b are constant but T_p is inversely proportional to v and m. Hence, the larger the value of m for a curve in Figure 12, the smaller is the value of v or α at which the dynamic effects of the unevenness become negligible and the curves for m = 0 to start to join the curve for b = 0.

TABLE 6

COMPARISON OF EXACT AND APPROXIMATE "CRITICAL" VALUES OF α

Value of m	Critical Value of α		
	From Fig. 12	From Eq. 14	From Eq. 15
5	0.154	0.179	0.200
9	0.093	0.100	0.111
15	0.057	0.060	0.067

In Figure 13 are given spectrum curves for bending moment for five different bridges as a function of the number of half-sine waves present along the span. In each case, the speed parameter is α = 0.1 and the amplitude of the wave is

$$b = 0.001 \frac{L}{m}$$

The vehicle used is similar to Vehicle C, except that its natural period of vibration is 0.28 sec instead of 0.35 sec. The values in parentheses in this figure represent the ratio T_b/T_v.

The values of m corresponding to the two highest peaks of the curves in Figure 13 are compared in Table 7 with the values obtained from Eqs. 14 and 15. It can be seen that the agreement between the actual and the predicted critical values is quite satisfactory.

In Figure 13 it can be seen that, except for the 78-ft span, the magnitude of the peak response increases with increasing span length. This trend is due to two factors: (a) since the parameter mb/L is constant for all span lengths, the amplitude of the waves corresponding to a fixed value of m is greater for the longer spans and (b) as the span length increases the ratio T_b/T_v approaches unity, with the result that at the critical value of m the period of the surface unevenness is in synchronism with both the natural period of the vehicle and the natural period of the bridge. Of the two factors, the latter is the more dominant. This is evidenced by the fact that the curve for the 78-ft span, which has the largest value of b but a ratio of T_b/T_v = 1.27, lies considerably below the curves for the 60- and 70-ft spans.

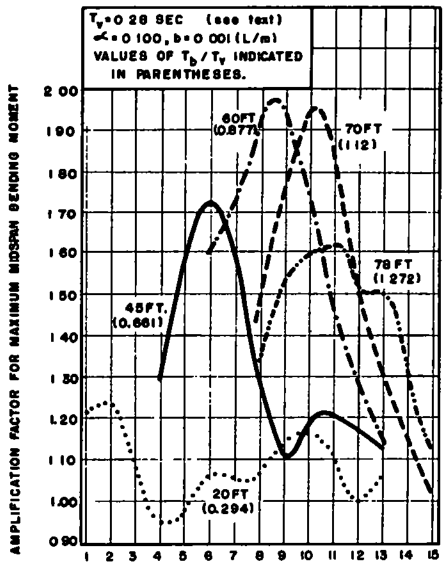

Figure 13. Effect of ratio T_b/T_v (bridges with sinusoidal deck profile).

TABLE 7

COMPARISON OF EXACT AND APPROXIMATE "CRITICAL" VALUES OF m

Span (ft)	$\frac{T_b}{T_v}$	Critical Value of m			
		From Fig. 13	From Eq. 14	From Fig. 13	From Eq. 15
20	0.294	2.0	2.9	10.0	10.0
45	9.66	6.0	6.6	10.7	10.0
60	0.875	8.8	8.7	8.8	10.0
70	1.12	10.4	11.2	10.4	10.0
78	1.27	13.0	12.7	11.0	10.0

It is important to note that although the amplitude of the surface unevenness considered in these conditions was by no means excessive, the computed effects are quite large. In this connection, it must be remembered that the effects of both bridge damping and vehicle damping were not accounted for in the analysis. Accordingly, the computed effects are generally larger than those expected in practical cases. The influence of damping is likely to be particularly important when the number of waves along the span is large or when the ratio T_b/T_v is close to unity.

Semisystematic Unevenness.—The form of the unevenness considered is shown in Figure 14. It consists of five half-sine waves, alternately of opposite signs, having different lengths and amplitudes. The length of each end wave is taken equal to 1/5 the length of the span, whereas the lengths of the remaining waves are varied over the entire possible range, keeping the waves symmetrical about the center of the span. The amplitude-to-length ratio is considered to be the same for all the waves.

The distribution of the unevenness can conveniently be specified in terms of the dimensionless parameter, λ, defined as

$$\lambda = \frac{L_2}{L_2 + L_3}$$

where L_2 and L_3 are the lengths of the second and third waves, respectively. The range of λ is from 0 to 1. As shown in Figure 14, λ = 0 corresponds to the case in which there are only three waves along the span, whereas λ = 1 corresponds to the cases in which there are four waves. For a sinusoidal unevenness λ = 0.5. The end waves are assumed to be located below the design grade.

The parameter λ is essentially a measure of the regularity of the unevenness. Its influence is shown in Figure 15, in which the absolute maximum amplification factor for midspan bending moment is plotted as a function of α for bridges of 20-, 45-, and 70-ft spans. The

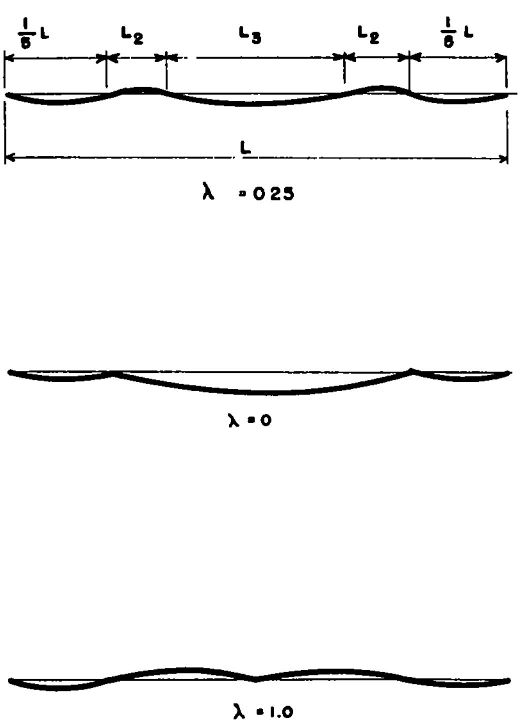

Figure 14. Sinusoidal deck profile with varying wave length.

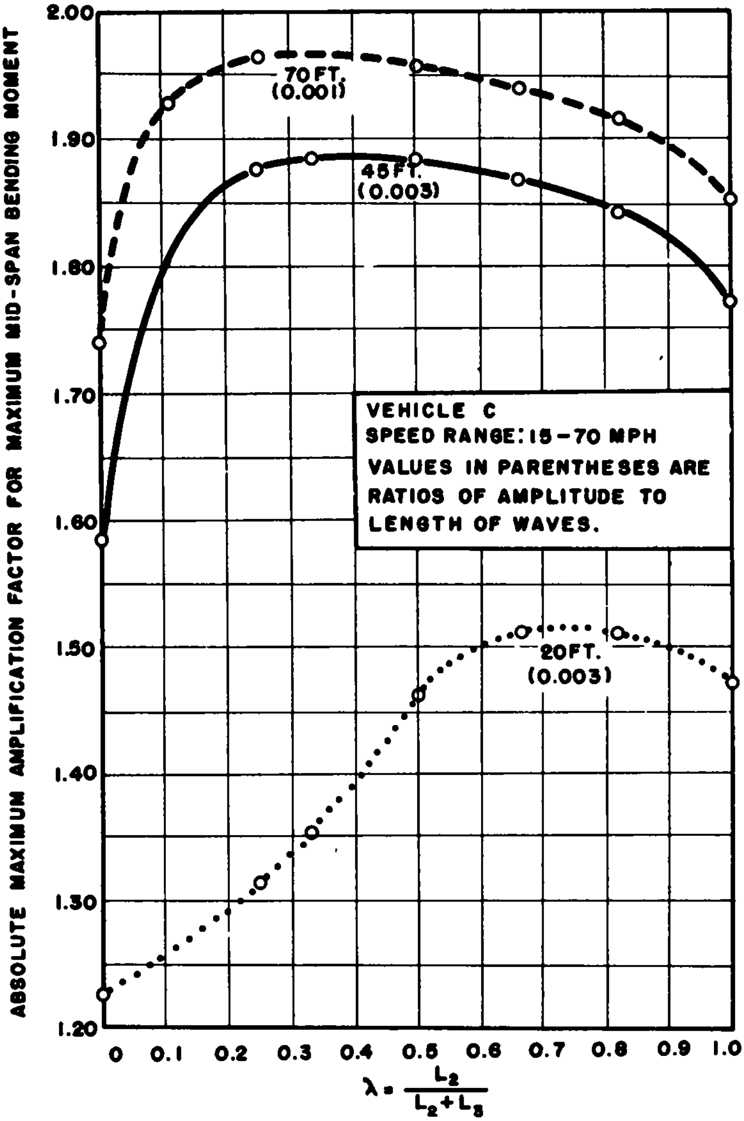

Figure 15. Effect of relative wave length.

amplification factors plotted are the maximum within the range of speeds from 15 to 70 mph. These results were obtained by use of Vehicle C.

It is noted that for a fairly wide range of values of λ the ordinates of the curves in this figure are of the same order of magnitude as the corresponding ordinate for a sinusoidal profile (λ = 0.5). This is particularly true in the case of the 70-ft span for which the natural period of the bridge and the vehicle are close to one another. For this span, the amplification factor for bending moment varies from 1.97 to 1.85 for values of the λ in the range between 0.15 and 1.0.

The practical implication of these results is that, even when the unevenness of the bridge deck is not entirely symmetric, it may be possible to estimate the absolute maximum effects by considering a sinusoidal unevenness with approximately the same number of waves and an amplitude equal to the average amplitude of the waves in the actual profile.

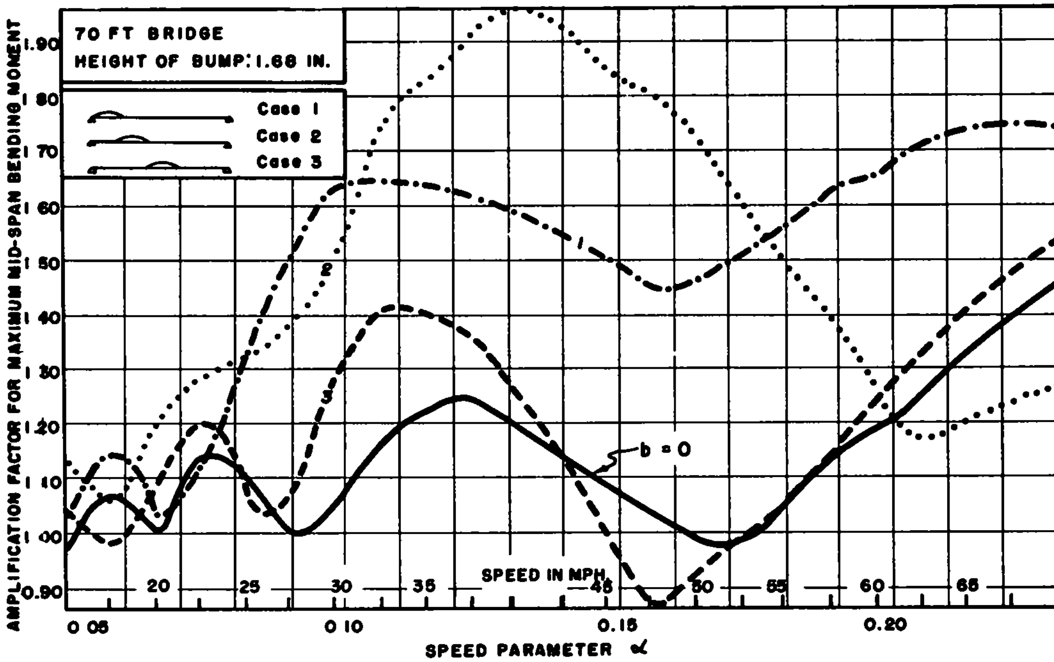

Figure 16. Effect of a "bump" on deck.

Localized Unevenness.—Figure 16 gives spectrum curves for bending moment as a function of the speed parameter for a 70-ft bridge with a localized irregularity in the form of a half-sine wave. The wave is considered to project above the design grade. Its amplitude is 1.68 in. and its length is 1/5 of the length of the bridge span. The curves labeled 1, 2, and 3 correspond to the cases where the irregularity occupies the first, second, and third fifth of the span, respectively. The curve labeled zero is for a smooth deck.

It is seen that the dynamic effects produced by the unevenness are in general of considerable magnitude, and the absolute maximum effect is produced when the irregularity is located at the second fifth of the span instead of when centered about midspan. The latter result can be explained as follows. The primary effect of the irregularity is to amplify the vertical motion of the vehicle and thus increase the magnitude of the dynamic reaction exerted by the vehicle on the bridge. But there is a time lag between the initiation of the motion of the vehicle due to the unevenness and the development of the maximum possible dynamic reaction. When the irregularity is situated at midspan, by the time the dynamic reaction attains its maximum value, the vehicle is past midspan and at a position for which a given force will stress the structure of the lesser extent than when the same force is applied at midspan.

SUMMARY

The results of an analytical study have been presented to illustrate the influence of some of the factors producing dynamic effects in simple-span highway bridges. In the majority of the solutions the vehicle was represented as a two-axle loading.

The factors considered fall into two categories. In the first belong the factors related to the behavior of bridges with a smooth surface traversed by a vehicle that is initially in its position of static equilibrium. They include the speed of the vehicle, the axle spacing, and the initial oscillation of the bridge. The common characteristics of these factors is that they prevail in all bridges to a comparable degree, and their possible ranges are reasonably certain. In the second category belong the additional factors associated with the effects of the unevenness of the bridge surface and the approaches. They include the initial oscillation of the vehicle and the characteristics of the bridge surface. The relative importance of these factors may be quite different for different bridges, as the surface irregularities depend on such factors as the type of the pavement, and the location and maintenance of the bridge. At the present time, little is known about the distribution and the magnitude of roadway unevenness for highway bridges.

In general the magnitude of the maximum dynamic effects in a bridge increase with increasing vehicle speed. For axle spacings larger than the bridge length, the dynamic effects produced by a two-axle loading were found to be larger than those produced by single-axle loading. The increase is attributed to the fact that when the rear-axle enters the span the bridge is already in a state of oscillation.

By varying the axle spacing while keeping all other variables constant, it was found that a quasi-resonance condition is developed when the time interval between the application of the two axles over a point is equal to the fundamental natural period of vibration of the bridge.

If the bridge is already in a state of oscillation when the vehicle enters the span, the dynamic effects produced by the vehicle depend on the timing of the entry of the vehicle with the oscillation of the bridge. The absolute maximum effect can be estimated approximately by superimposing on the maximum effect due to the initial oscillation the effect that would be produced by the vehicle if the structure were initially at rest.

The influence of the initial vehicle oscillation was investigated by considering both a bouncing and a pitching motion for the vehicle. For the problems studied it was found that, for the same amplitude of initial deformation in the springs, the effects of pitching are less severe than those of bouncing. The magnitude of the resulting dynamic effects were found to increase almost linearly with the amplitude of the initial oscillation considered.

The influence of the unevenness of the bridge deck was investigated by considering a sinusoidal profile variation, a semisystematic unevenness consisting of a series of half-sine waves of unequal amplitudes and lengths, and a localized unevenness. The results obtained show clearly that roadway unevenness may be a source of large dynamic effects in highway bridges.

In interpreting the practical significance of the solutions presented in this paper it must be kept in mind that the contributions of both vehicle damping and bridge damping were not accounted for in the analysis. Accordingly, the computed effects are generally larger than those to be expected in actual cases.

ACKNOWLEDGMENT

The study described in this paper was made as part of the Highway Bridge Impact Investigation, a research project conducted by the University of Illinois Engineering Experiment Station in cooperation with the Illinois Division of Highways and the Bureau of Public Roads, U. S. Department of Commerce. This paper is based in part on a doctoral dissertation by Wen (11) prepared in 1957. The data concerning the effects of roadway unevenness were drawn from Toledo-Leyva and Veletsos (13). The latter report is based on an M.S. thesis by Mr. Toledo-Leyva, formerly Research Assistant in Civil Engineering.

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Appendix

NOTATIONS

AF = amplification factor
b = amplitude of sinusoidal unevenness on bridge surface
f_b = fundamental natural frequency of vibration of bridge, in cycles per sec
L = span length of bridge
m = number of half-sine waves along the bridge span
q_b = y_o/y_s = dimensionless amplitude of initial bridge oscillation
q_p = dimensionless amplitude of initial pitching motion for vehicle, as defined by Eq. 11
q_v = dimensionless amplitude of initial vertical motion for vehicle, as defined by Eq. 10
s = spacing between axles
T_b = fundamental natural period of vibration of bridge
T_v = natural period of vibration of vehicle, idealized as a single-axle load
T_p = period of sinusoidal profile, defined by Eq. 13
t = time
v = vehicle speed
y_o = amplitude of initial bridge oscillation at midspan
y_s = maximum static deflection of bridge at midspan due to weight of vehicle
α = speed parameter, defined by Eq. 1
β_b = phase angle of initial bridge oscillation
β_p = phase angle of initial pitching motion of vehicle
β_v = phase angle of initial (vertical) bouncing motion of vehicle