Bridge Vibrations as Influenced by Elastomeric Bearings

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Inasmuch as many highway bridges are now being built with elastomeric bearings, it was considered desirable to study the vibration effects of such bridges. A theoretical analysis is presented, followed by several example calculations of short and medium span bridges.

General relationships are presented to determine the frequency of vibration of simple span bridges in the fundamental mode and the ratio of bridge beam deflections to the flexible bearing deflection under vibration subject to vehicular loads.

Contrasting the behavior of bridges with elastomeric bearings and with conventional rigid bearings, the following general conclusions are found: (a) the dynamic bridge deflections are increased; (b) the frequency of vibration is reduced (c) the elastomeric bearings add damping to the system; and (d) the dynamic stresses in the bridge are significantly reduced.

The fact that the impact stresses are reduced is quite revealing, as elastomeric bearings may lead to further structural economy, whereby less tolerance need be allowed for impact.

● MANY modern highway bridges are now being constructed in which the conventional rigid metal end bearings are being replaced by elastomeric bearing pads, such as neoprene. The primary purpose of using such neoprene bearings for short and medium span bridges is to reduce the cost of end bearing construction while still maintaining provision for expansion, contraction, and rotation of the bridge beams.

Because such supports are much more flexible than conventional supports, these elastomeric bearings will change the vibration characteristics of such bridges. The damping properties of elastomeric pads may also exert an influence on the vibration.

A completely rigorous theoretical study of bridge vibration including all the effects of oscillating moving loads, of the irregularities of road surface, of the infinite degree of freedom of the structure, and of the many manners of damping is a well-recognized task of monumental proportions. C. E. Inglis, in his book on vibrations, goes so far as to say, perhaps not too facetiously, that these "repulsive" mathematical equations are so complex that solutions cannot be had until after much prayer and fasting. As this study represents a first attempt to evaluate, in order of magnitude, the dynamic behavior of bridges supported on elastomeric bearings, the theoretical assumptions will be simplified to the ones believed to be the most influential on the behavior.

Several examples will be presented following the theory to indicate the trends of frequency, dynamic stresses, and deflections that would possibly occur under live loads.

THEORY

Undamped Frequency with Vehicular Load

The most critical bridge frequency is the one that is in resonance with the forcing frequency, as caused by a heavily loaded truck whose spring mass is oscillating vertically (1). For a simply supported bridge this critical frequency may be analytically obtained by placing the vehicle at the midspan and determining the fundamental mode caused by the bridge and truck masses.

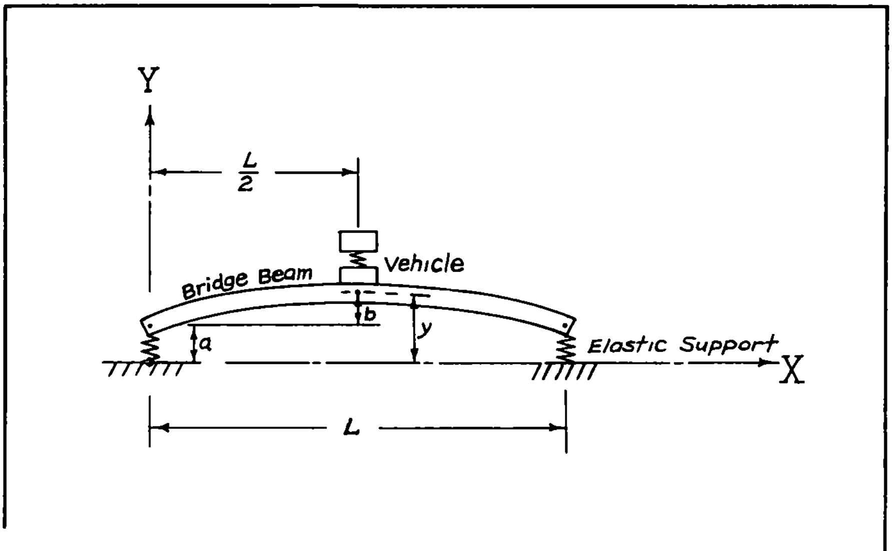

Figure 1. Vibration mode of bridge.

At resonance, the vertical oscillations of the vehicle will be in phase with the oscillations of the bridge itself. For simplicity of analysis, the oscillations of the vehicle will be assumed equal to that of the midspan of the bridge.*

Figure 1 shows the fundamental mode of vibration for a bridge beam supported on elastic bearings. Using Rayleigh's energy method of analysis (2), the total bridge deflection will be considered as

$$y = (a + b \sin \frac{\pi x}{L}) \cos pt = X \cos pt \tag{1}$$

where p is the circular frequency, t is time, a is the maximum support deflection, and b is the maximum beam deflection.

In Rayleigh's method, the maximum potential energy, V, is equated to the maximum kinetic energy, T.

From Article 4 in Timoshenko (2)

$$V = \frac{EI}{2} \int_0^L \left(\frac{d^2 X}{dx^2}\right)^2 dx + 2 \frac{k}{2} a^2 \tag{2}$$

where EI is the flexural rigidity of the beam, and k is the spring constant of the flexible support.

$$T = \frac{mp^2}{2} \int_0^L X^2 \, dx + \frac{1}{2} M v_1{}^2 \tag{3}$$

where m is the mass of the bridge beam per unit length, M is the mass of the vehicle applied to the beam, and v_1 is the maximum vertical velocity of the vehicle, taken as p(a + b) for harmonic motion.

After equating V to T and regrouping

*The approximation here lies in the possibility that the motion of the sprung mass of the vehicle may differ from that of the bridge, necessitating more involved analysis.

$$p^2 = \frac{EI\int_0^L \left(\frac{d^2 X}{dx^2}\right)^2 dx + 2ka^2}{m\int_0^L X^2\, dx + M\,(a + b)^2} \tag{4}$$

This reduces to

$$p^2 = \frac{4\pi ka^2L^3 + \pi^5 EIb^2}{2\pi ma^2 L^4 + 8abmL^4 + \pi mb^2L^4 + 2\pi Ma^2L^3 + 4\pi MabL^3 + 2\pi Mb^2L^3} \tag{5}$$

To obtain the fundamental mode, p is minimized by taking

$$\frac{\partial p^2}{\partial a} = 0 \quad \text{or} \quad \frac{\partial p^2}{\partial b} = 0.$$

Both expressions give the same results. After performing this operation and reducing, the support deflection, a, may be found in terms of the beam deflection, b, as follows:

$$a = nb \tag{6}$$

where

$$n = \pi^5 EI(Lm+M) - 2\pi KL^3(Lm+2M) + \left\{\left[2\pi KL^3(Lm+2M) - \pi^5 EI(Lm+M)\right]^2 + 16\pi^4 EIKL^3 (2Lm + \pi M)^2\right\}^{1/2} \Big/ 8KL^3\,(2Lm+\pi M)$$

To obtain the circular frequency, p, Eq. 6 is substituted into Eq. 5. Both a and b will thereby be eliminated, and p may be determined as

$$p = \left[\frac{4\pi kn^2L^3 + \pi^5 EI}{2\pi mn^2L^4 + 8nmL^4 + \pi mL^4 + 2\pi Mn^2L^3 + 4\pi MnL^3 + 2\pi ML^3}\right]^{1/2} \tag{7}$$

The fundamental frequency in cycles per unit of time is therefore

$$f = \frac{p}{2\pi}$$

For comparative purposes, the fundamental frequency for a supported bridge beam with a load at the center (but with rigid bearings) may be found as outlined in Norris et al. (1, p. 423) and Timoshenko (2, pp. 26-27). By considering one-half the beam mass acting with the central load, the beam vibration problem may be resolved into a single degree of freedom problem whose frequency solution is

$$f' = \frac{1}{2\pi}\sqrt{\frac{k'}{m\,L/2 + M}} \tag{8}$$

where

$$k' = \frac{48EI}{L^3}$$

Amplitudes and Stresses with Damping

Without specific knowledge of the magnitude of the disturbing force, such as one caused by a moving truck, the exact amplitudes and stresses in a bridge cannot be determined. However, if one assumes that the same disturbing force acts on a bridge with flexible supports as with conventional rigid supports, a ratio of amplitudes between the two may be determined.

If one degree of freedom is assumed, a dynamic model of the loaded bridge with damped elastic bearings may be shown, as in Figure 2. C is the coefficient of viscous

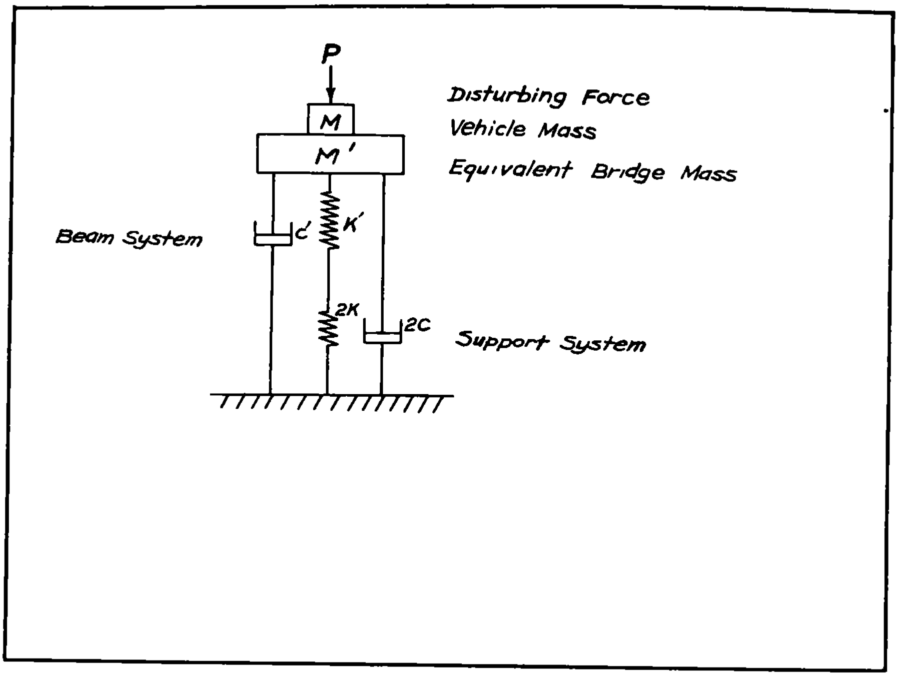

Figure 2. Dynamic model of loaded bridge.

damping for the elastic support and "C" is the coefficient of viscous damping for the beam itself.* If k is considered infinite and C considered zero, the model in Figure 2 then represents a bridge conventionally supported.

From Article 13 (2), the ratio of amplitudes for bridges with and without flexible supports for a resonance condition of forced vibration with damping may be obtained as follows:

$$\frac{\text{Amplit. elastic support}}{\text{Amplit. rigid support}} = R_A = \frac{C' f'}{C_e f} \tag{9}$$

where $C_e = C' + 2C$ = Effective damping of beam and support

$k_e = \frac{2k\,k'}{k' + 2k}$ = Effective spring constant of beam and support

Note that in Eq. 9 the bridge and vehicle masses divide out, as does the disturbing force, P. As will be shown later, this ratio, R_A, for dynamic deflections is generally greater than unity.

For comparative purposes, the ratio of live load nondynamic deflections with and without elastic supports for vehicles at crawl speed or stopped may be written as

$$R_A' = \frac{k'}{k_e} \tag{10}$$

This value of R_A' also exceeds unity.

*It is assumed that in conventionally supported bridges, the rollers or rockers are well lubricated, having negligible damping.

Since the maximum total dynamic bridge deflection consists of the support deflection, a, and the beam deflection, b, the increased bridge deflection obtained for elastically supported bridges from Eq. 9 does not necessarily imply increased dynamic flexural stresses.

If the dynamic amplitude for rigidly supported beams is taken as unity then R_A represents the dynamic amplitude for elastically supported beams. Therefore

$$a + b = R_A$$

However, using Eq. 6

$$b(1 + n) = R_A$$

Because the beam deflections are directly related to the dynamic flexural stress, S, the ratio of vibration induced stresses of bridges with and without elastic supports is given by

$$\frac{\text{Dyn. stress elastic support}}{\text{Dyn. stress rigid support}} = \frac{b}{1} = \frac{R_A}{1+n} = \frac{C'f'}{C_e f}\,\frac{1}{(1+n)} \qquad (11)$$

As will be shown later, this ratio, R_S is generally less than unity.

Again for comparative purposes, it is well to remember that no stress changes take place in the beam with vehicles at crawl speed or stopped (not considering vibration) for bridges with or without elastic supports. In simply supported beams, the beam stresses do not change under this "static" condition, because the stresses are independent of the end deflections.

EXAMPLES

Short Span Bridge

As an example of a short span highway bridge, a simply supported reinforced concrete structure, 36 ft in length is selected. H-20 loading is assumed, acting uniformly on five parallel stringers. The bearing pads are assumed made of neoprene, 1½ in. thick of 60 Durometer hardness. Based on compressive load-deflection curves by Biggs and Suer (3), the equivalent spring constant, k, for this particular pad is 32×10^4 lb per in. From Yerzley Oscillograph readings performed in accordance with test method ASTM D 945 as supplied by the E. I. DuPont de Nemours and Co. the coefficient of viscous damping, C, is computed as 3.11 lb-sec per in.*

Other pertinent data for this bridge are as follows:

Mass per unit length of beam = $m = 0.282$ lb-sec² per in.
Mass of vehicle per beam = $M = 21.7$ lb-sec² per in.
Modulus of elasticity of beam = $E = 3 \times 10^6$ lb per sq in.
Moment of inertia of beam = $I = 73 \times 10^3$ in.⁴

The coefficient of viscous damping, C, of the bridge beam itself is difficult to pinpoint, as many accumulated minor factors are involved. A number of citations to experimental damping coefficients are given by DuPont de Nemours and Co. (4), Foster and Oehler (5) and Inglis (6). A decision of judgment therefore had to be made in the selection of the coefficient, C, based on the significant structural similarities between the example bridge and the bridge in the previously cited references. For this bridge the value of C = 460 lb-sec per in. is taken as a representative mean.

By means of Eq. 6, the relationship is found that

$$a = 0.597\, b$$

*This constant was also checked by an independent method by the author.

By use of Eq. 7, the fundamental frequency is found as

$$f = 6.5 \text{ cps}$$

By means of Eq. 9, the ratio of dynamic amplitude, R_A, with and without elastic support is

$$R_A = 1.44$$

By means of Eq. 11, the ratio of dynamic stresses, R_S, with and without elastic support is

$$R_S = 0.90$$

For comparative purposes, the bridge frequency with rigid supports is computed from Eq. 8 as

$$f' = 9.5 \text{ cps}$$

Also for comparative purposes, the ratio of nondynamic live-load deflection with and without elastic supports is computed from Eq. 10 as

$$R_A' = 1.45$$

Thus to summarize this example of a short span bridge, the consequences of using elastic bearings instead of conventional bearings are the following:

1. Reduction of the frequency of vibration by a ratio $R_f = 0.685$. This reduction in frequency may have significant consequences if the frequency is reduced below 6.5 cps as tests by Foster and Oehler (5) indicate this to be the threshold of pedestrian distress. If the frequency is reduced below 3 cps, undesirable resonance between vehicles and structure may result.
2. Increase of the total dynamic amplitude by a ratio $R_A = 1.44$ of which 37.5 percent of the total amplitude lies in the support pads. However, this increased amplitude is not serious within itself.
3. Increase of the over-all damping by a ratio $R_d = 1.01$. This change is negligibly small.
4. Decrease of the dynamic beam stresses by a ratio $R_S = 0.90$. This decrease is significant as less allowance may perhaps be permissible for impact stresses.
5. Increase of the nondynamic live-load deflection by a ratio $R_A' = 1.45$ of which 31 percent of the total deflection lies in the bearings.
6. Absence of change in the nondynamic live-load or dead-load beam stresses; i.e., $R_S' = 1$.

Medium Span Bridge

A 78-ft simply supported bridge with prestressed concrete beams is taken as another example. H-20 loading is again assumed. The bearing pads are assumed to be the same as in the prior example. The data pertinent to this bridge are given as follows:

$m = 0.286$ lb-sec^2 per in.
$M = 27.7$ lb-sec^2 per in.
$E = 5 \times 10^6$ lb per sq in.
$I = 640.57 \times 10^3$ in.
$k = 32 \times 10^4$ lb per in.
$C = 3.11$ lb-sec per in.
$C' = 866$ lb-sec per in.

From Eq. 6	a	$= 0.4077\ b$
From Eq. 7	f	$= 4.49$
From Eq. 9	R_A	$= 1.20$
From Eq. 11	R_S	$= 0.85$
From Eq. 8	f'	$= 5.42$ cps
From Eq. 10	R_A'	$= 1.29$

Summarizing the results of this example, it is seen that the use of elastic bearings in place of rigid bearings achieves the following:

1. Reduction of the frequency of vibration by a ratio R_f = 0.83 putting the bridge in a more critical low frequency range.
2. Increase of the total dynamic amplitude by a ratio R_A = 1.20, in which 29 percent of the total amplitude lies in the flexible bearings.
3. Increase of the over-all damping by ratio R_d = 1.01.
4. Decrease of the dynamic beam stresses by a ratio R_S = 0.85.
5. Increase of the nondynamic live-load deflection by a ratio R_A' = 1.29, of which 22.5 percent of the total deflection lies in the bearings.
6. Absence of change in the nondynamic live-load or dead-load beam stresses, making R_S' = 1.

To illustrate the effects of thicker bearing pads, consider the same 78-ft bridge but with 3-in. thick neoprene pads of Durometer hardness 70. For this pad k = 23.3×10^4 lb per in. and C = 5.19 lb-sec per in.

Then from Eq. 6	a	= 0.542b
From Eq. 7	f	= 4.42 cps
From Eq. 9	R_A	= 1.21
From Eq. 11	R_S	= 0.79
From Eq. 8	f'	= 5.42 cps
From Eq. 10	R_A'	= 1.40

Thus, the consequences of using thick elastic bearings in place of conventional rigid bearings are the following:

1. Reduction of the frequency of vibration by a ratio R_f = 0.81.
2. Increase of the total dynamic amplitude by a ratio R_A = 1.21, in which 35 percent of the total amplitude lies in the supports.
3. Increase of the over-all damping by a ratio R_d = 1.01.
4. Decrease of the dynamic beam stresses by a ratio R_S = 0.79.
5. Increase of the nondynamic live-load deflection by a ratio R_A' = 1.40, of which 28.5 percent of the total deflection lies in the bearings.
6. Absence of change in the nondynamic live-load or dead-load beam stress.

A comparative study of the dynamic effects of the same bridge with thick and thin bearing pad shows that impact stresses are reduced an additional 6 percent for thick pads, or a total of 21 percent over rigid bearings.

CONCLUSIONS

Despite the fact that the example bridges are of different types and lengths, the general trends of amplitudes, stresses, and frequencies are similar.

Flexible supports will decrease the natural bridge frequency; however, the example problems indicate the reduction in the order of magnitude of 20 to 30 percent, being larger for shorter spans. Bridges normally stiff enough with rigid bearings may therefore become undesirably flexible with elastomeric bearings, especially if frequencies are in the range of 4 to 6 cps.

It is also obvious that elastic supports will increase both dynamic and nondynamic amplitudes and deflections. However, the examples show that the increase is slightly less for dynamic than for nondynamic live-loads.

The small amount of damping added to the system by the elastomeric supports is negligibly small, and even this increase is questionable inasmuch as the theoretical assumption is made that the conventional supports have no damping.

In cases of rusty or dirty conventional supports that have an appreciable amount of friction damping, a replacement with elastomeric bearings may actually decrease the total damping.

The most interesting and somewhat paradoxical conclusion to be drawn from this study however, is that the dynamic beam stresses actually decrease by about 15 percent, despite an increase in total deflection of about 30 percent. The explanation lies in the fact that the total deflection is due to the sum of the support and beam deflections; whereas the stresses are due only to the beam deflection portion of the total deflection. The explanation may also be stated in another manner, namely, the total impact energy is distributed to both the beam and the elastic support. Because the supports absorb a portion of this energy, there is less energy for the beam to absorb. In a general way, the elastic supports act as shock absorbers.

The examples show that the thicker bearing pads tend to reduce impact stresses materially as compared to thinner pads. Thus, the question of design arises if perhaps designing for thicker pads is not to be preferred to reduce impact stresses, providing that frequency levels are not critical. A lower impact stress would certainly be advantageous in many respects, including that of improving a bridge's fatigue life.

Unfortunately, immediate experimental verification to the theoretical conclusions is not available. Until it is, certain caution is suggested in adopting the theoretical conclusions for design use, as the theory is based on many simplifying assumptions.

Relatively easy laboratory experiments would generally be useless due to the virtual impossibility of scaling and reproducing vehicular dynamics, deck roughness, and damping. Field tests would therefore provide the only reliable confirmation for design use. In field testing, two possibilities appear feasible. The first is to design twin bridges (as on a dual highway) one with conventional bearings and the other with elastomeric bearings. Instrumenting both bridges would then provide comparative information. However, variations in deck roughness between the two bridges may conceivably be a significant factor in impact. Therefore, a second alternate for field testing would be to use but one bridge but designed for interchanging rigid bearings and elastic bearings. The second alternate is more costly, although more precise. The Bridge Division of the Virginia Department of Highways is currently considering erecting such bridges from which reliable data may be found. It is hoped that this may be done in the near future.

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