Some Fundamental Relationships of Traffic Flow on a Freeway

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• FUNDAMENTAL CHARACTERISTICS of traffic flow — especially the relationships among speed, volume, and density — have been given increasing attention in recent years. One reason for the current interest in these characteristics is the hope that by studying their effects on the capacity of a highway, a way may be found to alleviate or eliminate congestion on highways, especially under maximum volume conditions.

The fundamental characteristics are dependent on the geometric design of a roadway and on the traffic flow. This dependence has been expressed in the form of a "Four Friction Concept" (7, 15), in which three frictions (intersectional, medial, and marginal) can be minimized through geometric design, while the fourth (internal-friction) is dependent on the traffic flow. If it can be assumed that the geometric design of a modern expressway controls intersectional, medial, and marginal frictions, the study of the traffic flow characteristics on such a freeway will point up the effects of internal friction only.

PURPOSE

It was the purpose of this work to study the relationships among speed, volume, and density on an expressway to obtain known or new characteristics of traffic flow at high volumes. The reason for such research is the need for specific indications of impending traffic congestion which can then be used in control processes intended to maintain maximum traffic flow.

Because traffic flow in the area of maximum volume changes from stable to unstable, and because the characteristics of this change are not well known, the primary emphasis of this work had to be extended to defining this area of change and to separating data into the ranges of critical and noncritical flow.

CRITICAL VS NONCRITICAL FLOW

It has been known for a long time that traffic flow is unstable in the region beyond maximum volume. Any volume increase beyond a certain value causes complete congestion. The precise characteristics of the point where unstable flow begins are not known, although this topic has been the subject of considerable research. One might compare the situation in traffic flow with the transition between laminar and turbulent flow in hydraulics, although in the case of fluid flow the transition point does not represent the point of maximum flow capacity. Another possible comparison is that with the yield point of steel. But the maximum strength of steel, unlike traffic flow capacity, is maintained over a wide range of strain before complete failure of the material occurs. In traffic flow, failure occurs suddenly and no definite warning has as yet been found.

Critical Flow

The unstable region of traffic flow is referred to here as "critical flow." Characteristics of this type of flow are the appearance of congestion, the drastic reduction in traffic volume and speed, and the unpredictability of data variations, exemplified by the scatter of measurements obtained in this type of flow. The motorist traveling in this flow is much more restricted by the action of vehicles around him than by his own desires. This can be concluded from the greater fluctuations in data in short successive time intervals. It is also easily verified qualitatively by each driver when caught in such flow. Specific quantitative comparisons will be given in later sections.

The study of critical flow is not of primary interest, except in determining how to avoid it. Once critical flow has been established, noncritical flow can only return when the traffic volume input is reduced sufficiently so that the speed can increase to free flow speed and stable flow.

An important fact about critical flow is that its effect spreads so rapidly and becomes so dominant. If only one point on a long, heavily traveled facility converts to critical flow, the lower capacity at this point will quickly create an overload that will spread backward fast and far. The point of conversion from noncritical to critical flow travels opposite to the direction of vehicular travel quickly, and may even spread into heavily loaded feeder or cross-routes. Thus critical flow can be quite "contagious."

On the other hand, the conversion from critical to noncritical flow requires that all points along the facility can and do convert. If only one location retains critical flow, no points behind this one can change to noncritical flow.

Noncritical Flow

Noncritical traffic flow is flow in the stable region. This flow is the primary subject of this discussion because it represents the desirable traffic flow characteristics from the point of capacity and economy.

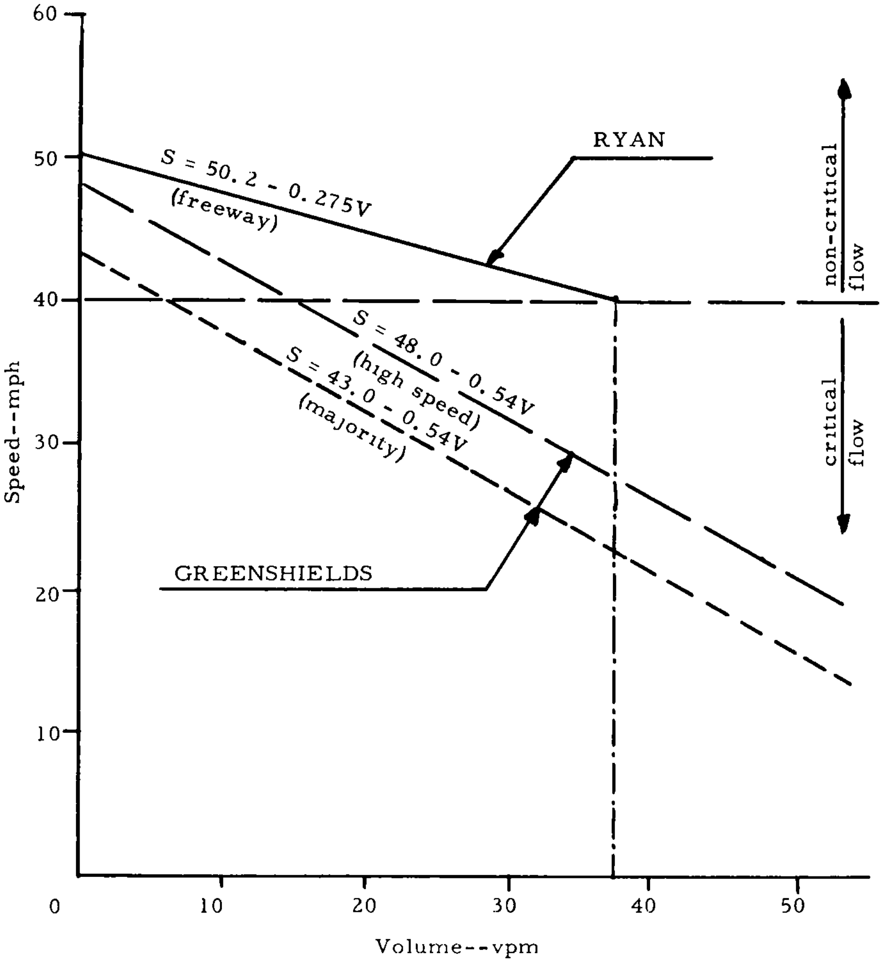

Figure 1. A comparison of speed-volume relations.

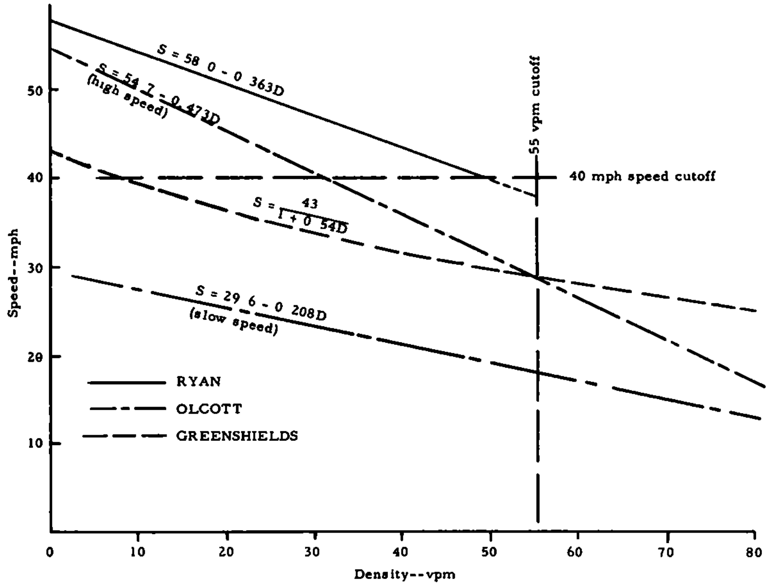

Figure 2. A comparison of speed-density relations.

Linearity of relations of the fundamental characteristics has been suggested in various and previous studies, but not necessarily for all relations. Greenshields' early study in 1934 (6) proposed a linear relation for the speed-volume relation, but slight, negligible, curvature for the speed-density relation. Linearity of volume-density relation is not generally suggested, but May and Wagner (15) present graphs in which good linearity seems to exist in the range of noncritical flow, although curvilinear regressions are proposed for the entire flow range.

Linearity of the basic relations need not extend to very low volumes, because the flow in that area is of no significance in the study of maximum flow. Furthermore, the few observations possible would not be statistically significant.

Linearity of the relations in the region of noncritical flow was assumed in this study. Regression equations were computed for the data and were tested for linearity by the F-linearity test. If linearity was rejected, as it was in all but one case for the entire peak period, a cutoff was chosen and the reduced sample again tested for linearity. In this way the limit of linearity of relationships was determined and found to agree closely with the selected boundary between noncritical and critical flow.

Another characteristic of noncritical flow is the relatively small change in measurements of the fundamental characteristics from one observation period to the next. In critical flow these changes are considerably larger. It appears that these average changes might be a measure of internal friction and be very helpful in defining the boundary between noncritical and critical flow.

The significance of change of average measurements is not evident, although a comparison to the effects of turbulence in fluid flow would seem reasonable. More study of this subject is indicated.

METHOD OF STUDY

Description of Test Area

The study site was located on the three in-bound lanes of the Edsel Ford Expressway at the Lonyo Street overcrossing in Detroit. The expressway is a six-lane, divided, grade-separated, and depressed freeway, located in an urban area approximately 2 mi west of the CBD. It is about 15 mi long and is an integral part of Interstate 94 — Detroit to Chicago.

There are two on-ramps in the vicinity of the study area. One is on the west side of the bridge about 1/4 mi away. The other ramp is on the east side of the bridge some 600 ft away. In general, this section of freeway is typical of modern freeway design.

Instrumentation

The data-gathering equipment consisted of electronic radar vehicle and speed detectors, some mounted over and some alongside the roadway. The detection equipment was not discernible to the passing motorist.

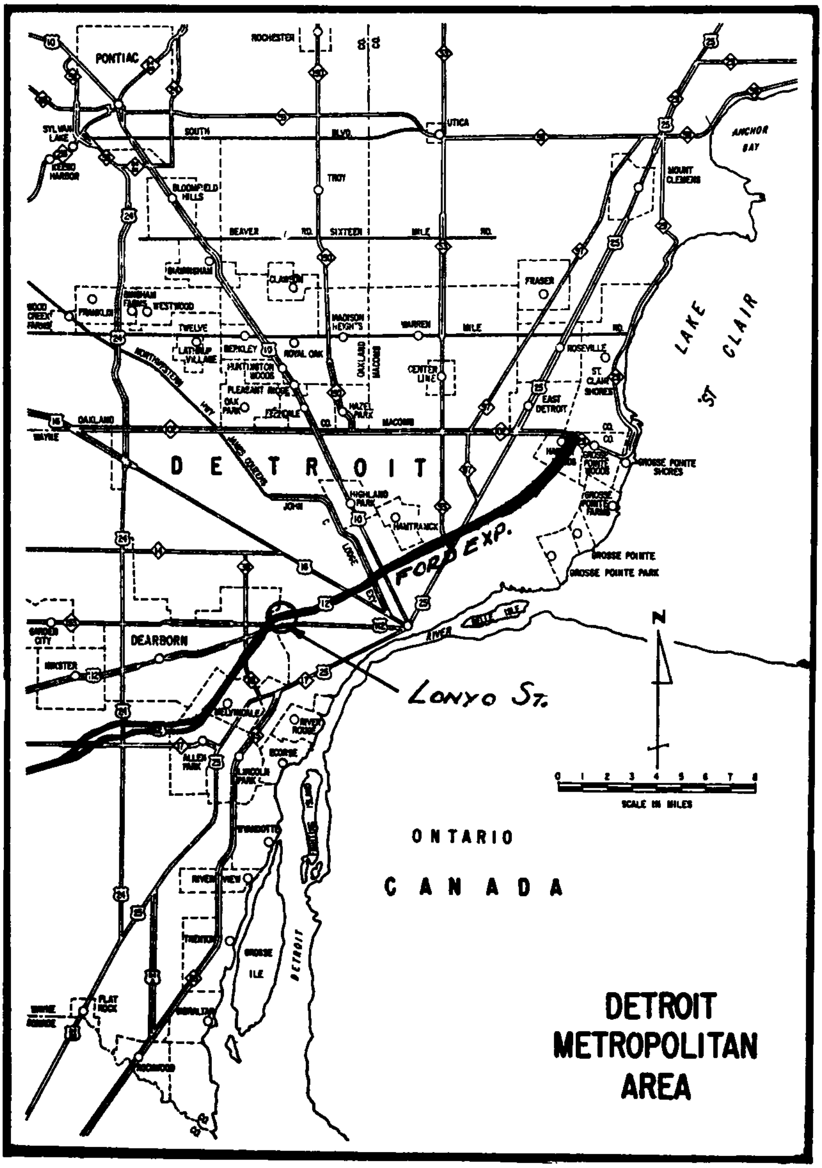

Figure 3. Map of Detroit area showing study site location.

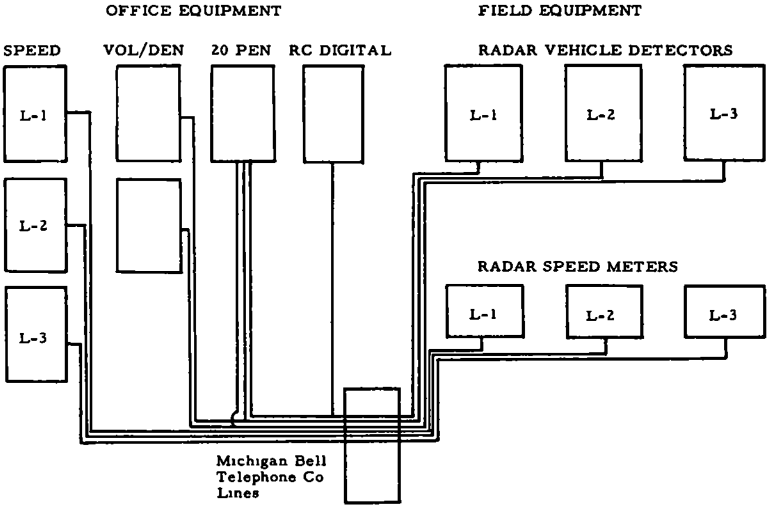

Figure 4. Instrumentation of test site.

Each vehicle that passed the field study point was detected by model RD-1A Electro-Matic overhead radar vehicle detectors and by modified models of the S-2A Electro-Matic radar speed meter. These units were connected remotely by telephone lines to an Esterline-Angus 20-pen recorder, a 1-min model of the RG Streeter-Amet printing counter, and to Esterline-Angus graphical speed and volume-density recorders.

The 20-pen recorder was used to plot volumes and time headways of vehicles by lane. The digital recorder was used as a check on the 20-pen volume record of the various lanes. The graphical speed recorders kept a plot of speeds of all vehicles according to time of occurrence, allowing for correlation of speed and volume data. The volume-density recorders produced a plot of the daily fluctuations in traffic. These could be compared for different days to see if daily characteristics were similar. All equipment was kept under constant surveillance and was checked periodically to make sure that all units were functioning properly.

Data Collected

Data for this study were taken during periods of peak traffic flow in order to have volumes sufficiently large to cause a decrease in speed and an increase in density so that a back-up of traffic would occur. Because the test siste was on the in-bound traffic lanes, data were collected in the morning from 6:45 a.m. to 8:00 a.m. This time period included peak flow and the time immediately preceding and following the peak.

All the data for this study were gathered in 1-min intervals. In the past, samples of 5-min or more intervals have generally been used and it is believed that much valuable information has therefore been leveled out and lost through averaging of data extremes over a 5-min or longer interval.

The majority of vehicles on this road were assumed to be driven by people working in or around Detroit, who were familiar with the highway, the peak flow conditions, and with the posted speed limits of 55-mph maximum and 40-mph minimum when conditions permit.

METHOD OF ANALYSIS

The analysis in this study was made on the basis of finding linear relationships in the data in the region of noncritical flow and of determining the dividing line between critical and noncritical flow. The main steps in the analysis consist of calculating several statistics, making chronological plots, and correlating the results into a set of logical deductions.

To facilitate the calculation of statistics a correlation table was used which shows numerically as well as graphically the two-way distributions of speed-volume, speed-density, or volume-density. The values in the table were used in the more refined statistical calculations. Of these calculations, the F-linearity test was used to determine whether acceptable linearity existed in a region, and in then defining noncritical flow as that region in which acceptable linearity existed.

All relationships determined were hypothesized to be linear and were tested for linearity by the F-linearity test. Calculations of the F-value had to be less than the 95 percent level values to be accepted; if larger than the 95 percent level, they were rejected. A rejection, however, is just as useful, for it helps to explain the relation as much as acceptable F-values. If the F-linearity test is rejected, a new region of noncritical flow is defined and a recalculation made. The defining of a new region is essentially the selecting of a new cutoff line between critical and noncritical flow. This process is repeated until an acceptable F-test is made.

Another statistic determined was the correlation coefficient. This statistic is used to determine whether the regression coefficient is significantly non-zero. The null hypothesis is made and tested, that is, it is believed that the regression coefficient is zero. Values of r larger than the 95 percent level values reject the hypothesis. In general, the closer the value of r to 1 the greater the significance of non-zero and of correlation of Y and X.

The standard error of estimate ($S_{y/x}$) was used to determine the relative scatter of the observed points about the regression line in the ordinate Y direction. A large standard error of estimate means that data are more widely scattered, whereas a small $S_{y/x}$ indicates the data to be relatively close to the regression line. Relating this to traffic flow then, a small $S_{y/x}$ would be more desirable as it would indicate relatively smooth-flowing traffic. A large $S_{y/x}$ would therefore indicate a wide scattering of data, which would likely be the result of congestion or interrupted flow. If the standard error of estimate is squared, the resulting value is the variance of Y left unexplained by the regression of Y on X.

To help visualize and support the statistical analysis a chronological plot of data was made. This is a successive plot of data chronologically from the beginning of the study period to the end. A plot is made of X, Y points and these points are connective successively with an arrow indicating the progression of time. Cutoff lines are also shown which help point out the differences between data in the range of critical and noncritical flow.

Analysis of Speed-Volume Relationship

In the speed-volume analysis the cutoff line was determined at approximately 40 mph, dividing flow into critical flow below and noncritical flow above the line. Figure 5 shows from the chronological plot of lane-1 data, that the character of data differs considerably for critical and noncritical flow. This difference is further demonstrated by comparing the range of speeds and the change of average speed between 1-min intervals as shown in Figure 6. Noncritical flow has a speed range of 6 mph, an average speed increase of 1.94 mph, and an average speed decrease of 1.77 mph. Critical flow has a speed range of 31 mph, an average speed increase of 3.56 mph, and an average speed decrease of 4.43 mph. These differences in flows reflect the increase in internal friction in critical flow.

When the fluctuations in speed are less pronounced, as shown in Figure 7, the apparent effect of congestion is not present and the flow soon returns into the region of noncritical flow.

Each of the lane regressions (see Table 1) were tested for linearity of relationship. The results indicated that at the 95 percent level there was no basis for rejecting the hypothesis that the speed-volume relationship was linear. The results further indicate that there is some justification for saying that speed and volume are linearly related in the area of noncritical flow but not in the area of critical flow. A calculation of the regression slope limits indicate generally that the slopes of all lanes lie within these limits, with the exception of lane 2, on Friday, which had a positive slope.

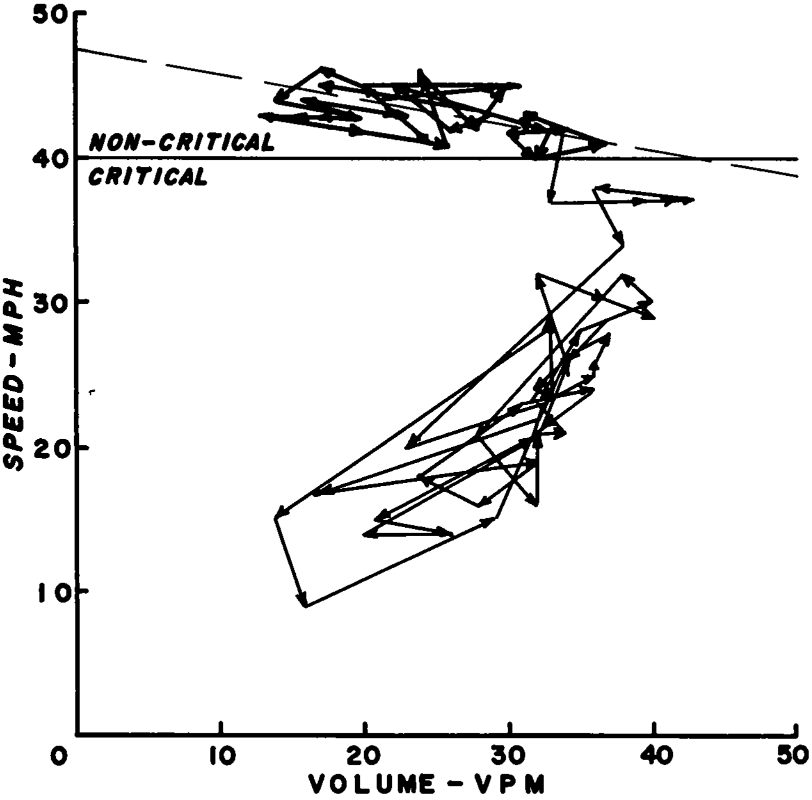

Figure 5, Chronological plot of data for lane 1, February 13, 1958.

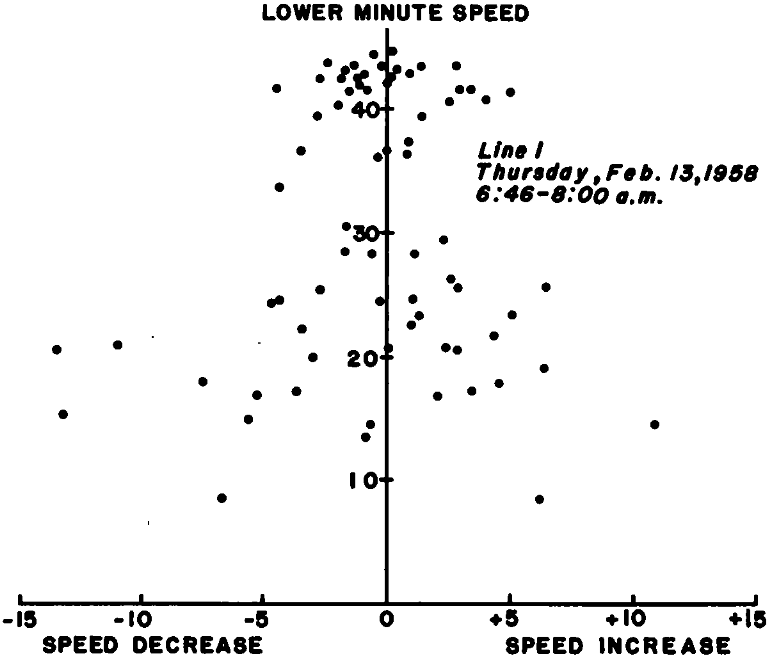

Figure 6. Change of minute average speed vs lower minute speed.

TABLE 1

REGRESSION EQUATIONS FOR FUNDAMENTAL RELATIONS

Lane		Speed Volume	Speed Density	Volume Density
1	Thur.	$S = -0.179V + 47.6$	$S = -0.142D + 48.1$	$V = 0.612D + 3.7$
	Fri.	$S = -0.166V + 52.7$	$S = -0.15\ D + 53.6$	$V = 0.695D + 3.1$
2	Thur.	$S = -0.231V + 54.6$	$S = -0.271D + 57.2$	$V = -0.584D + 7.6$
	Fri.	$S = 0.010V + 41.1$	$S = -0.310D + 57.8$	$V = 0.574D + 7.1$
3	Thur.	$S = -0.273V + 48.6$	$S = -0.323D + 54.2$	$V = 0.497D + 3.3$
	Fri.	$S = -0.231V + 49.2$	$S = -0.427D + 53.8$	$V = 0.559D + 3.5$
Avg.	Thur.	$S = -0.275V + 50.2$	$S = -0.363D + 58.0$	$V = 0.472D + 5.7$
	Fri.	$S = -0.227V + 51.1$	$S = -0.361D + 57.4$	$V = 0.459D + 7.0$

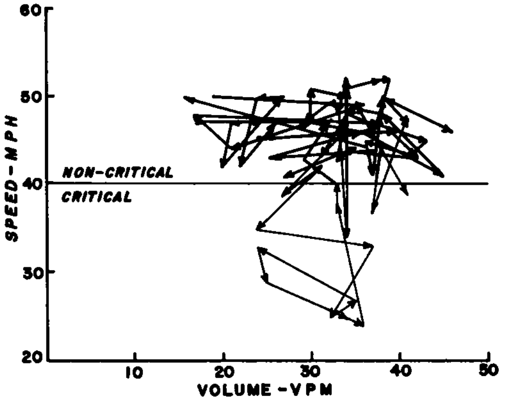

Figure 7. Chronological plot of data for lane 2, March 28, 1958.

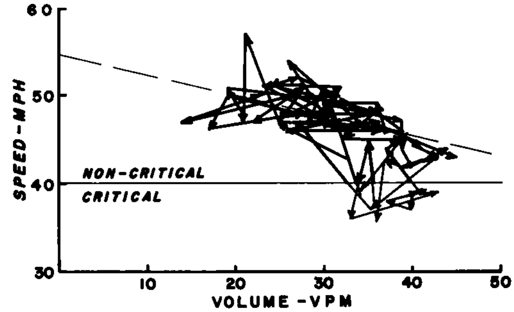

Figure 8. Chronological plot of data for lane 2, March 27, 1958.

The chronological plot of speed-volume data shows that critical flow (congestion) does not always occur immediately following peak flow. There is some delay after peak noncritical flow before congestion occurs, shown in Figures 5 through 9. Furthermore, peak-minute volumes do not necessarily occur in noncritical flow, but can also occur during critical flow when speeds are between 35 and 40 mph for lanes 1 and 2, and 30 to 35 mph for lane 3. Further, Figures 5 through 9 show that the transition to critical flow occurred on lanes 1 and 2 when speeds were less than 45 mph and when volumes were from 30 to 40 vehicles per min. On lane 3 the transition occurred when the volume range was 15 to 25 vehicles per min.

Analysis of Speed-Density Relationships

The speed cutoff line in the speed-density analysis was also 40 mph. However, in this analysis a second cutoff line for density data was determined, indicating critical flow above 55 vehicles per mile (vpm). Figure 10 shows a considerable difference in the character of data between critical and noncritical flow. In addition to the difference in speed range pointed out in the speed-volume analysis, the difference in density range, 36 vs 57 vpm, further substantiates the difference in critical and noncritical flow. Furthermore, the average density changes are 55 percent greater in critical flow than in noncritical flow. These greater changes in density reflect larger changes in headway, which points to increased internal friction and a more likely chance of congestion.

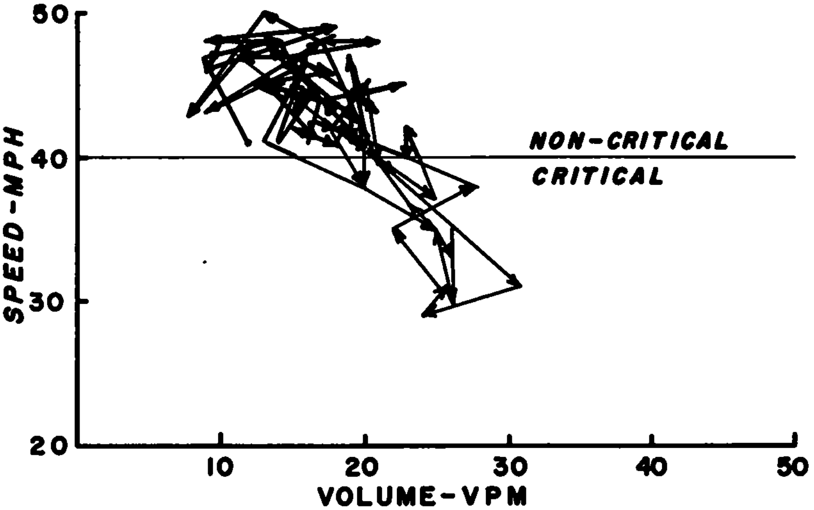

Figure 9. Chronological plot of data for lane 3, March 28, 1958.

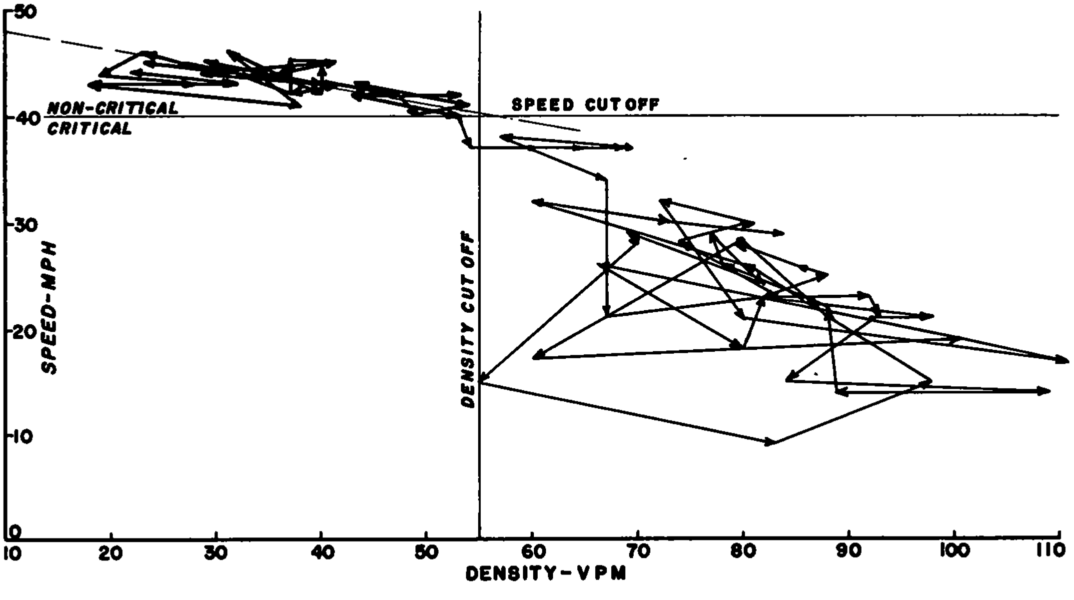

Figure 10. Chronological plot of data for lane 1, February 13, 1958.

Densities ranged from as few as 18 vpm to as many as 111 vpm. This is equivalent to a drop in headway from 293 ft to less than 48 ft. The greatest change though was only from 60 to 100.5 vpm or an increase of 40.5 vpm. This change occurred in 60 sec and decreased headways from 88 to 53 ft. Such density changes are not possible in noncritical flow, where high speeds prevail.

Lane 3 speed-density characteristics are a little different in that the plot of data (Fig. 11) indicates that the cutoff line corresponding to a speed drop below 40 mph would have to be about 30 to 35 vpm. Some of the calculations, such as the regression analysis of volume-density relation, bear this out, although the regression analysis of the speed-density relation does not.

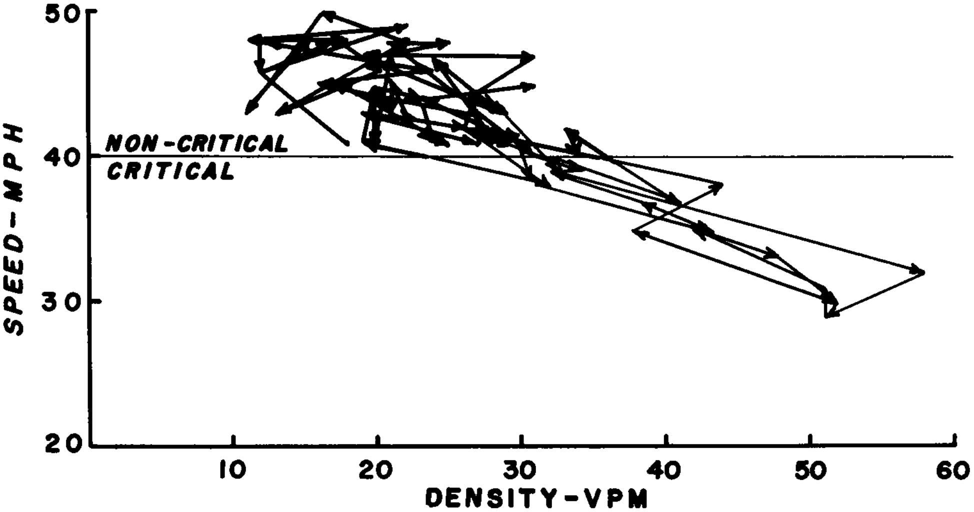

Figure 11. Chronological plot of data for lane 3, March 28, 1958.

Each of the lane regressions were tested for linearity of relationship. The results indicated that at the 95 percent level the speed-density relationship was linear with a cutoff of 55 vpm. Acceptable, although statistically less definite, linearity was determined for some higher cutoffs. A calculation of the regression slope limits indicated generally that the slopes of the speed-density relation for each lane are different from one another.

The chronological plot of speed-density data shows that maximum density does not occur with minimum speed.

Analysis of Volume-Density Relationships

The cutoff line for this analysis was a density of 55 vpm for lanes 1 and 2, and 45 vpm for lane 3. The chronological plot of data for each lane (Fig. 12) shows that the character of data differs by varying amounts in critical flow as compared to noncritical flow. Lane 1, which portrays congestion, supports the premise of difference in flows both graphically and numerically. This figure also shows the relative linearity of volume vs density up to 55 vpm.

The range in volume is not too different in critical flow as compared to noncritical flow, nor are the average volume increases and decreases widely different. The only thing pointed out here is that average decreases are larger than average increases in critical flow, whereas average increases are larger than average decreases in noncritical flow. This may be caused by the greater internal friction in congestion. Noncritical flow may have the ability to absorb these changes, whereas in critical flow there may not be enough headway to absorb an increase in density.

There are generally close similarities of operating characteristics of lanes 1 and 2, but not with lane 3. Volumes, speeds, and densities are less for lane 3, which is reflected in the lower density cutoff of 45 vpm. Inspection of the plotted data indicates a gentle curve primarily above 30 vpm. It is felt that a higher degree of linearity could be attained if the density were cut off between 30 and 35 vpm on lane 3.

The regression analysis indicated in that five of the six analyses the use of all data would not give an acceptable F-linearity test. Lanes 1 and 2 had cutoffs of 65 to 75 vpm on preliminary calculations. It was believed, however, that a higher degree of linearity could be attained with lower cutoff densities, and recalculations of data proved this in all respects. First, F-values were 0.874 for the 55 vpm cutoff and 1.21 for the 65 vpm cutoff. Second, the coefficient of correlations were considerably different, 0.956 as compared to 0.641. Third, the standard error of estimate is smaller for 55

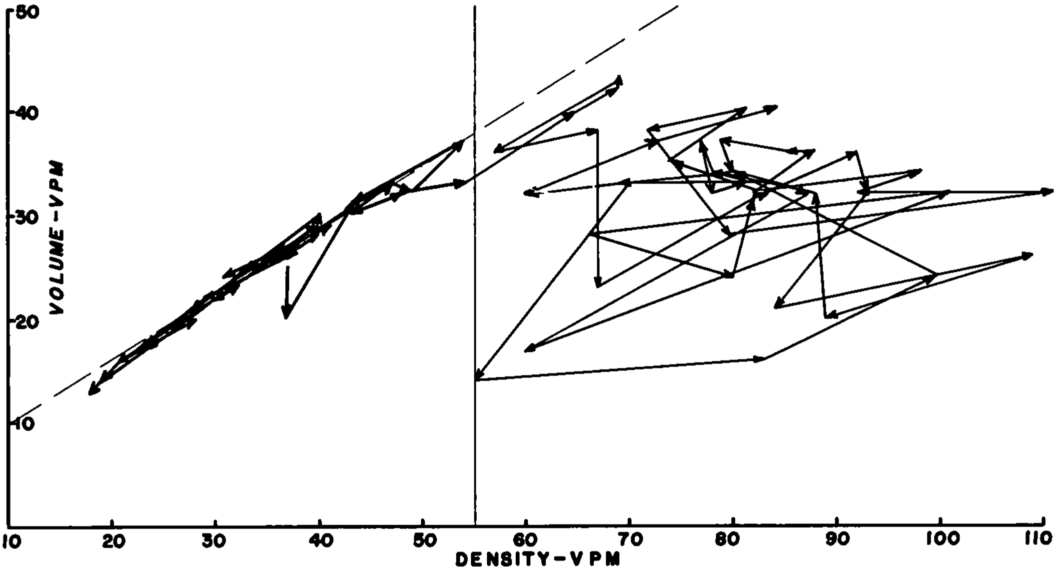

Figure 12. Chronological plot of data for lane 1, February 13, 1958.

vpm cutoff than for the higher cutoffs. These results indicate that the 55 vpm cutoff is a better estimate of the line between noncritical and critical flows.

SUMMARY AND CONCLUSIONS

A detailed analysis of traffic flow on an expressway during peak flow periods has been presented. Traffic flow was separated into noncritical and critical flow, and the analysis limited on the determination of the boundary between the two types of flows and on the characteristics of noncritical flow.

Noncritical flow is studied alone to obtain reliable data on that type of flow. 40 mph was found to be the boundary between critical and noncritical flow for the speed-volume and the speed-density relationships. A density cutoff was also chosen for the volume-density relationship, although the cutoff point is not so clearly evident for density as it seemed to be for speed.

It was found that all three relations among speed, volume, and density are linear within noncritical flow in range of observations bounded on one side by low traffic flow and on the other by the boundary with critical flow. This linearity is lost at the boundary where flow becomes unstable. Therefore, it is proposed that the end of linearity of flow marks the boundary between noncritical and critical flow.

Another significant set of data is the average change of characteristic measurements between successive 1-min observation intervals. It was found that in noncritical flow the average change in general is much smaller than in critical flow. This result, for which rational explanations may be postulated, might lead to another definition of the boundary between noncritical and critical flow. This study did not find whether this change occurs sufficiently before actual congestion to serve as an advance warning tool.

The three relationships studied showed considerable difference in correlation coefficients. The coefficient of correlation for speed is the lowest, that for density the highest. This might point to a comparative independence of speed from the other two measurements. But because density was determined from accumulations of volume and speed measurements over time, and, therefore, is not independent, this conclusion must be taken with reservation until proved by direct density measurements.

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