Frost Penetration Beneath Concrete Slabs Maintained Free of Snow and Ice, With and Without Insulation

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This paper presents the results of an investigation relative to the prediction of frost penetration in soils beneath concrete slabs, both insulated and uninsulated. A one-dimensional periodic heat-flow solution of the Fourier conduction equation was used to determine the effect of concrete and/or insulation in reducing frost penetration. For simplicity, the solution assumed an annual sinusiodal temperature distribution at the surface boundary. The freezing index at the concrete-soil interface (and insulation-soil interface) was found by calculating the amplitude of the sinusoidal temperature wave at this depth and then converting it to a freezing index. This freezing index was then used in the modified Berggren equation to calculate the depth of frost penetration into the underlying soil.

During the 1960-61 winter season, subsurface temperatures were recorded at three small-scale concrete-slab test sections at Waltham, Mass. The slabs were 8, 12, and 24 in. in thickness. A 2-in. layer of insulation was placed under the 8-in. slab. Slab surfaces were exposed to the natural climatic influences and were generally kept free of snow and ice.

Results indicated reasonable agreement between actual and predicted depths of frost penetration for the uninsulated slabs and suggested a solution for frost penetration problems in a multilayer profile of portland cement concrete, insulation, and soil.

•THIS STUDY was conducted to substantiate design criteria in use by the Corps of Engineers for estimating the amount of non-frost-susceptible soil required beneath uninsulated concrete pavements greater than 12 in. in thickness and to determine a suitable heat-flow computational technique for predicting the total depth of frost penetration under conditions in which an insulating layer is placed directly beneath a PCC pavement.

Corps of Engineer design criteria for estimating the amount of non-frost-susceptible soil required beneath uninsulated pavements kept free of snow or ice make use of the relationship between air freezing index, soil properties, and frost penetration shown in Figures 1 and 2. As indicated in note 2 of Figure 1, the frost penetration depths are measured from the pavement surface and are computed for 12-in. PCC pavements by means of the modified Berggren equation (1). An adjustment is made in this equation to convert the air-freezing index to a surface-freezing index in order to establish the upper boundary conditions for heat flow.

Many present-day runways are more than 12 in. thick and the effect of thicker pavements on the total frost penetration must be considered in design. This is done by assuming that 10 degree-days of air-freezing index are required for each inch of frost penetration in concrete pavement below the top 12 in. as shown in the following example.

If a PCC pavement thickness of 18 in., a design air-freezing index of 3,000 degree-days, F, and a base-course material having an average dry unit weight of 135 pcf and

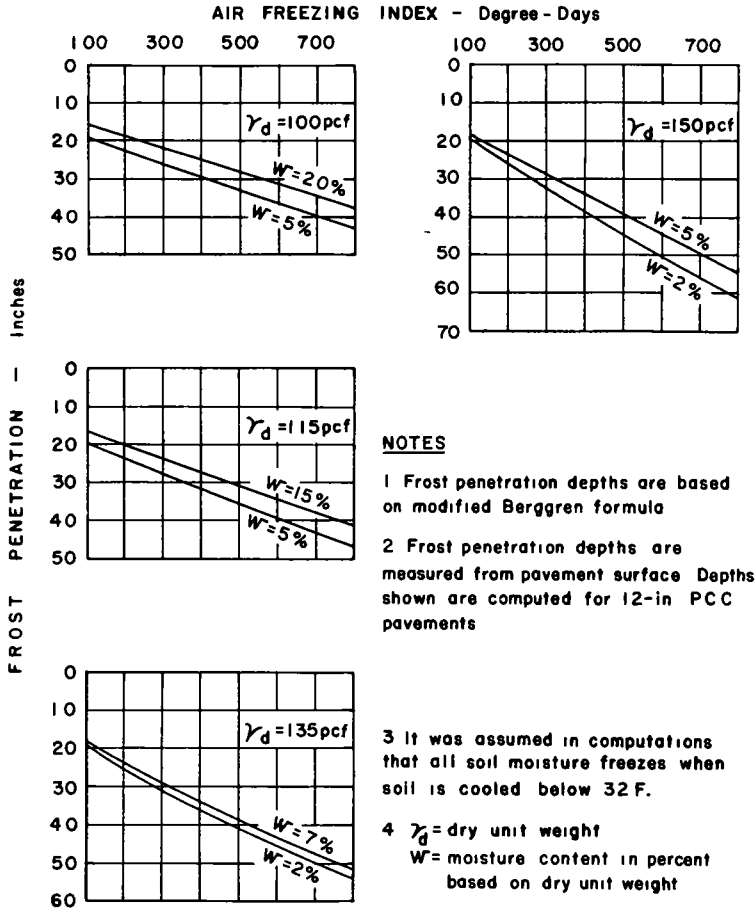

Figure 1. Relationships between air-freezing index and frost penetration into granular, non-frost-susceptible soil beneath pavements kept free of snow and ice for freezing indexes below 800.

an average water content after drainage of 5 percent is assumed, each inch of concrete pavement in excess of 12 in. is considered to reduce the air-freezing index by 10 degree-days. Thus, the modified freezing index is 3,000-10x(18-12) = 2,940 degree-days, F. From Figure 2, the combined thickness of 12-in. pavement and base required to prevent freezing of the subgrade is 138 in.; adding the originally deducted 6 in. of pavement results in a combined thickness of pavement and base of 144 in.

Inasmuch as the empirical 10 degree-day factor is based on field experience and the judgment of design engineers, it was thought advisable to check the validity of this factor by an appropriately instrumented field test in which the number of degree-days associated with the frost penetration for a 12-in. PCC slab could be compared with that for a 24-in. PCC slab.

For the case of the insulated concrete slab, the periodic heat flow method developed by A. H. Lachenbruch (2) was used to analyze frost penetration. This method was also found applicable to predict frost penetration for the case of uninsulated slabs. An insulating layer under a concrete slab might be used when site conditions or economic considerations make it an advantageous alternative to hauling non-frost-susceptible material over great distances or to establishing an aggregate washing operation. The possibility also exists that an insulating layer might be used together with a non-frost-susceptible fill to reduce the required thickness of the fill and still prevent frost penetration into an underlying frost-susceptible subsoil.

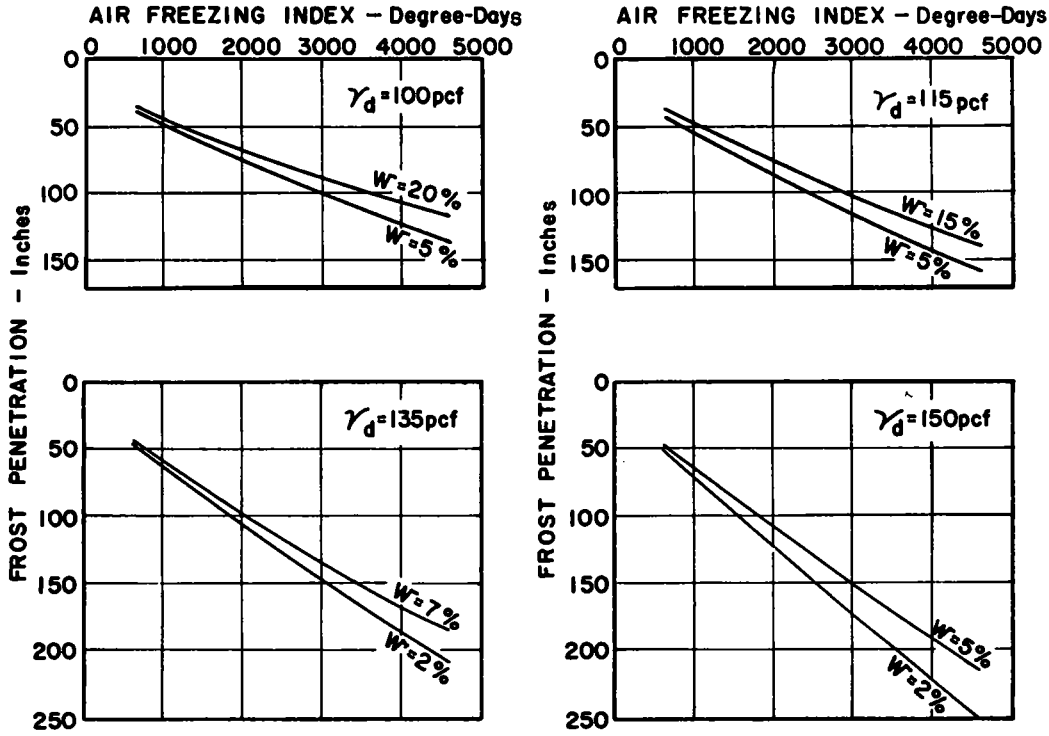

Figure 2. Relationships between air-freezing index and frost penetration into granular non-frost-susceptible pavements kept free of snow and ice (see notes on Fig. 1).

DEFINITIONS

The following specialized frost terms (3) are used in this report:

Non-Frost-Susceptible Materials.—Cohesionless materials such as crushed rock, gravel, sand, slag and cinders in which significant detrimental ice segregation does not occur under normal freezing conditions.

Average Daily Temperature.—The average of the maximum and minimum temperatures for one day or the average of several temperature readings taken at equal time intervals during one day, generally hourly.

Average Annual Temperature.—The average of the average daily temperatures for one year.

Mean Annual Temperature.—The average of the average annual temperatures for several years. In this report it is abbreviated MAT.

Degree-Days.—The degree-days for any one day equals the difference between the average daily air temperature and 32 F.

Freezing Index.—The number of degree-days between the highest and lowest points on a curve of cumulative degree-days vs time for one freezing season. The index determined for air temperatures at 4.5 ft above the ground is commonly designated as the air-freezing index, and that determined for temperatures immediately below a surface is known as the surface-freezing index.

Design-Freezing Index.—The average air-freezing index of the three coldest winters in the latest 30 yr of record. If 30 yr of record are not available, the air-freezing index for the coldest winter in the latest 10-yr period may be used.

Mean Freezing Index.—The freezing index determined on the basis of mean temper-

atures. The period of record over which temperatures are averaged is usually a minimum of 10 yr, and preferably 30, and should be the latest available.

n-Factor. —The ratio of surface freezing index to air freezing index.

FIELD INSTALLATION AND TEMPERATURE OBSERVATIONS

Three small-scale concrete test slabs were constructed on the property of the U. S. Army Engineer Division, New England, in Waltham, Mass. The slabs measured 5 by 5 ft in plan with thicknesses of 8, 12, and 24 in. A 2-in. layer of cellular-glass insulation was placed under the 8-in. slab. At the midpoint of each slab a string of 18-gage copper-constantan thermocouples was installed and connected to an automatic strip-chart recorder timed to print temperatures at approximately 75-min. intervals. The slabs were placed on the south side of the property to expose them to the warming effects of the sun. Photographs of the test area are shown in Figure 3, and plan and section in Figure 4. The slabs were inspected daily during the work week and any new-fallen or drifted snow and ice was removed from the surfaces. On two cold January weekends, however, a thin layer of snow drifted on the 24-in. slab.

The 12-in. and 24-in. slabs were built in the fall of 1959 on an 8-in. base course of silty gravelly sand (SW-SM) having a dry density of 109 pcf and a moisture content of 5.5 percent. The 8-in. slab, with a 2-in. insulating layer, was built in the fall of 1960 and was placed on top of a 4-in. base course of silty gravelly sand (SW-SM) having a dry density of 137 pcf and a moisture content of 2.6 percent. The natural soil at the site is a silty gravelly, coarse to fine sand (SM) having a dry density of about 130 pcf and a moisture content of 4 percent. No subsurface water table was found within the drilling depth of 11.5 ft for installation of the thermocouple assemblies.

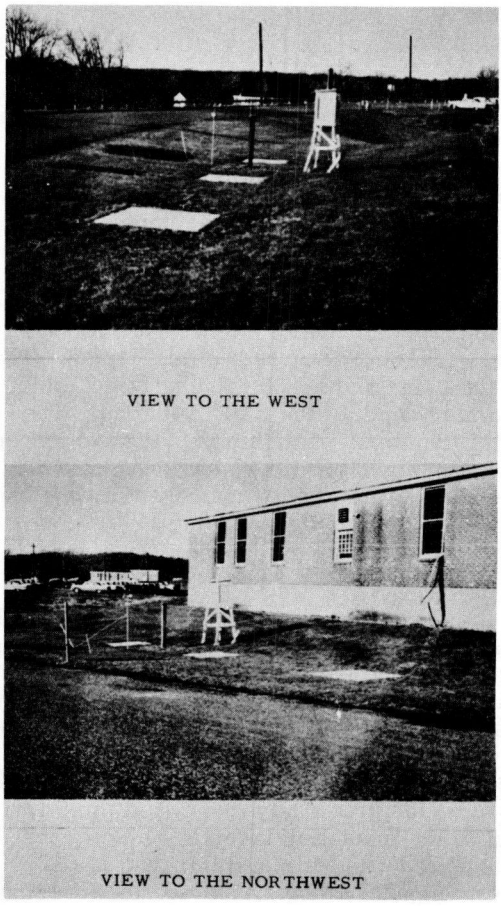

Figure 3. Test installation.

Air temperatures were continuously recorded with a thermograph placed in a standard U. S. Weather Bureau shelter at the site. The 1960-1961 air-freezing index at the site was 663 degree-days, F, and the mean annual temperature, 49 F (estimated). Because long-term weather records are not available for Waltham, Boston records were used to determine the relative intensity of the freezing season. The 1960-61 air-freezing index at the official U. S. Weather Bureau station in Boston (approximately 10 mi east of Waltham) was 428 degree-days, F. The mean freezing index for Boston is 166 degree-days, F, and the design freezing index (based on the coldest year in 10) is 620 degree-days, F. Although the 1960-61 index of 428 is not unusually high for Boston, a severe siege of cold befell the area on January 16 when temperatures remained below 32 F for 16 consecutive days. This was the longest such period in 43 years and the second longest of record.

A plot of the seasonal accumulation of degree-days for air temperature and surface temperatures for each slab is shown in Figure 5. Surface temperatures were obtained from thermocouples located $\frac{1}{8}$ in. below each slab surface. The test slabs underwent num-

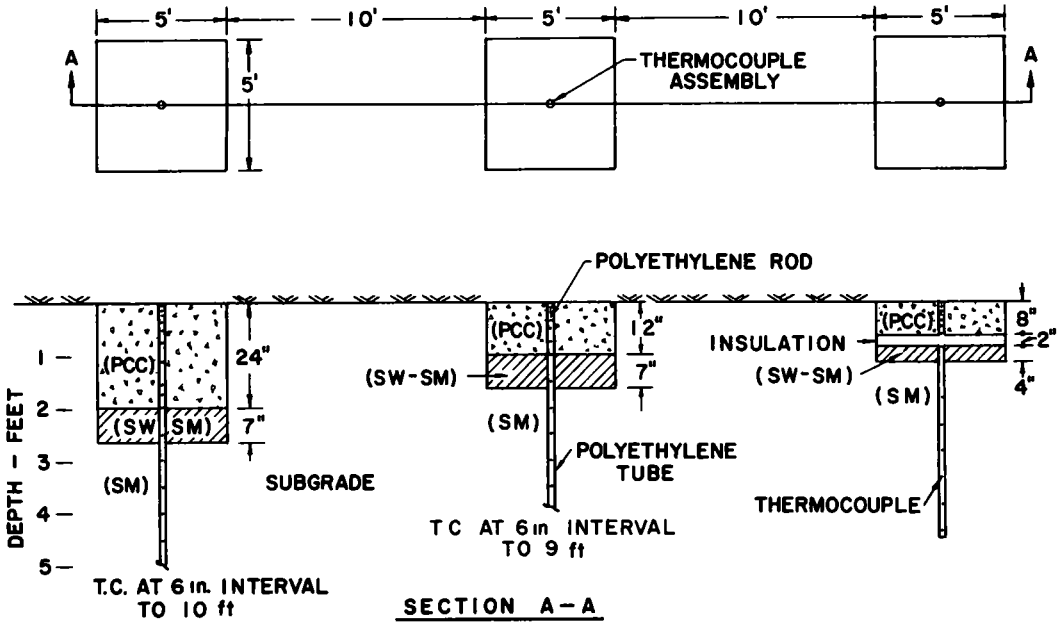

Figure 4. Plan and section of test slabs.

erous freeze-thaw cycles as shown in Figure 6. Although freezing penetrated the soil beneath the 12-in. uninsulated slab early in December and also reached the bottom of the 24-in. slab, freezing did not take place below the insulation layer of the 8-in. slab until after mid-January. The large number of freeze-thaw cycles in the uninsulated concrete slabs throughout the winter season, and the similarity between the air and the insulated slab surface cumulative degree-day measurements. The fact that the base course under the 12-in. slab froze during the early part of the season made it less susceptible to change under diurnal temperature variations (owing to latent heat effects) than the 24-in. slab, under which no base-course freezing occurred in this period. The total measured frost penetration was 24 in. at the 8-in. slab; 34 in. at the 12-in. slab; and 36 in. at the 24-in. slab, based on the maximum depth of the 32 F isotherm. The loss of heat (edge effect) around the periphery of the small-scale slabs may have tended to reduce the depth of frost penetration from what it would have been had one-dimensional (vertical) heat flow taken place. Temperature gradients for representative times during the freezing season appear in Figure 7.

VARIATION OF FREEZING INDEX WITH DEPTH

Cumulative degree-days were tabulated for the surface and for the 1-ft and 2-ft depth under the 12-in. and 24-in. slabs and for the 8-in. and 10-in. depths under the 8-in. sla (Fig. 8). Degree-days were determined by averaging the maximum and minimum daily recorded temperatures at the designated depth and accumulating the differences between the average daily temperature and 32 F. The surface-freezing indexes were 439 degree-days, F, for the 12-in. slab, 471 for the 24-in. slab, and 646 for the 8-in. insulated slab. The greater surface-freezing index of the 24-in. slab may be attributed in part to the higher reflectivity of the slab surface caused by the previously mentioned thin layer of drifted snow. Figure 8 also shows the effect of the insulating layer in reducing the accumulation of degree-days at the top of the subgrade; there is a difference of 617 degree-days, F, between the top and bottom of the insulation layer. Also, the freezing indexes at the top and bottom of the 8-in. slab were very similar, the bottom index being 23 degree-days, F, greater than at the top. (It would be logical to expect the sur-

face index to exceed the 8-in. depth index.) This observed anomaly may have been caused by minor errors in temperature measurement.

The computed n-factors for the three slabs (8-, 12-, and 24-in.) are 0.98, 0.66, and 0.71 respectively. The surface reflectivity of the insulated slab was found to be slightly higher than that of the uninsulated slabs, but this does not entirely explain the large difference in n-factors. The surface reflectivity measured with a light meter held vertically at a height of about 2 ft above the slab surface. The reading obtained with the meter facing downward was divided by the reading obtained with the meter facing upward. The discontinuity in the temperature gradient for the insulated slab (see Fig. 7) may offer some explanation; i.e., the relatively warm temperature of the soil underlying the insulation (lower boundary condition) does not have the same opportunity to affect the slab temperatures as it would if the insulation were not present. No discontinuity appears in the temperature gradients for the uninsulated slabs.

Figure 8 shows that from January 18 to February 18 the cumulative number of degree-days at the 1-ft depth in each of the uninsulated slabs was 170 degree-days, F. However, for that portion of the freezing season before January 18, the cumulative number of degree-days at the 1-ft depth in the 12-in. slab was 40 degree-days, F, greater than in the 24-in slab. Temperature records indicate that during this period the 1-ft depth temperatures in the 24-in. slab were more responsive to surface diurnal temperature fluctuations than in the case of the 12-in. slab. This effect may have been partly due to the temperature dwell at 32 F resulting from the freezing of soil pore water beneath the 12-in. slab.

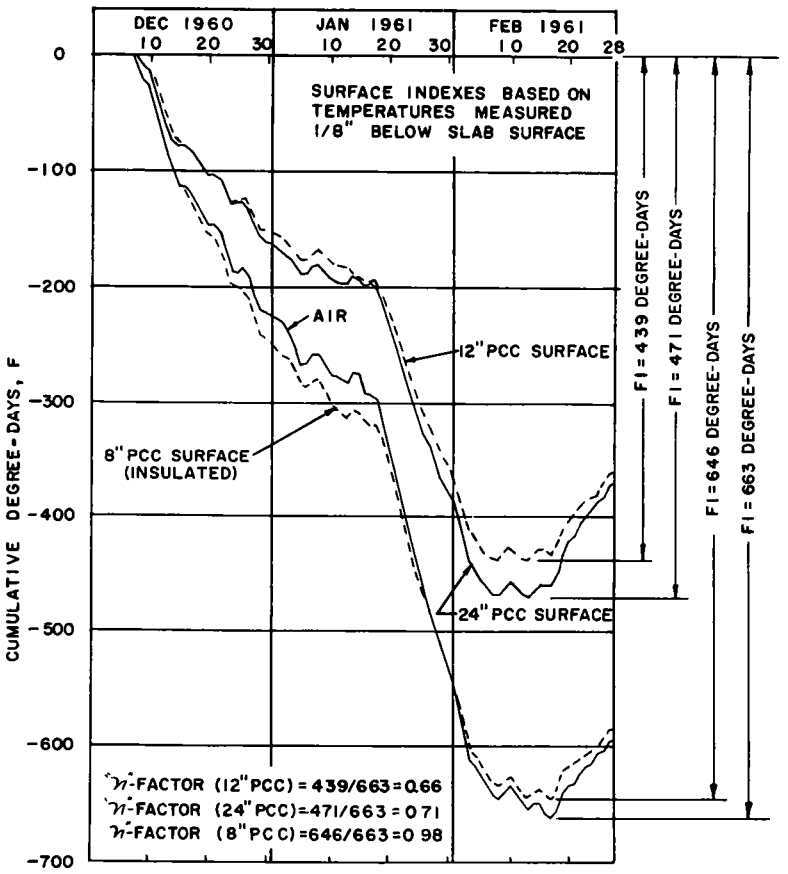

Figure 5. Air and surface cumulative degree-days.

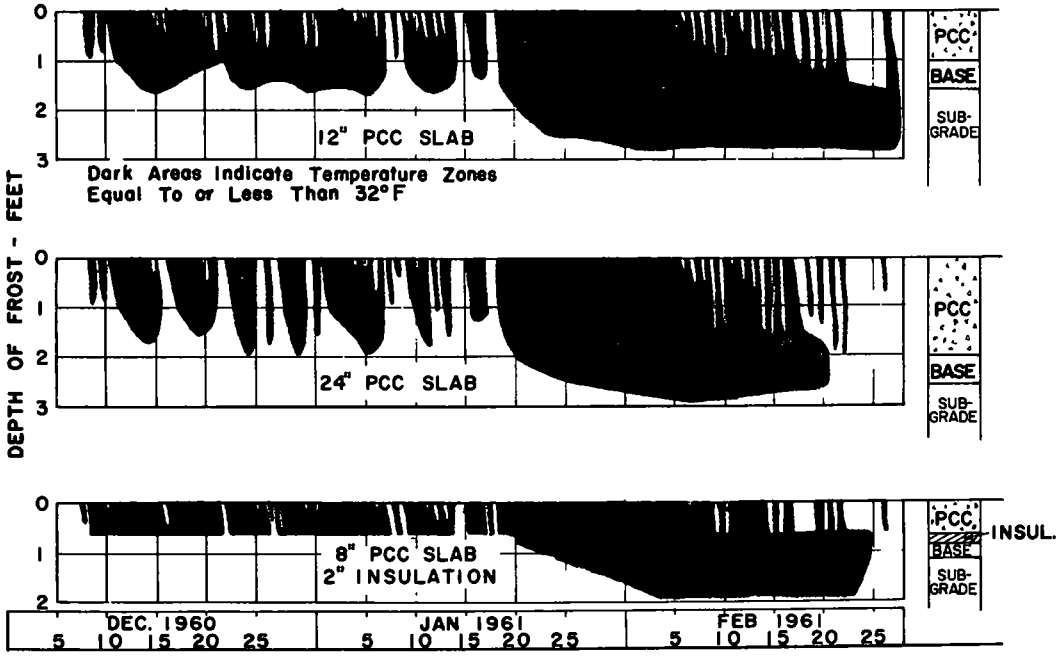

Figure 6. Frost penetration vs time.

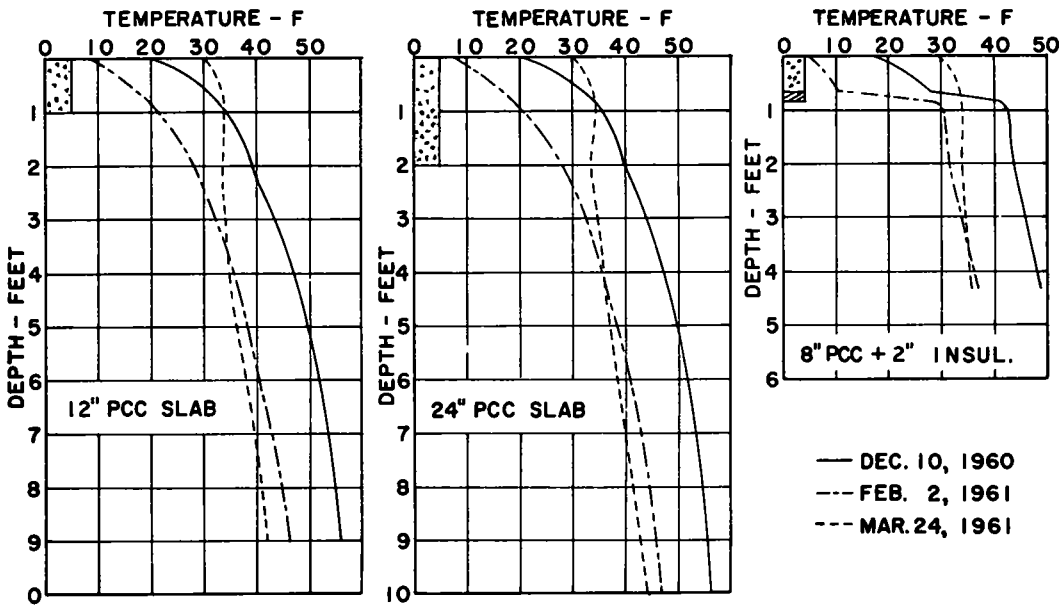

Figure 7. Temperature gradients.

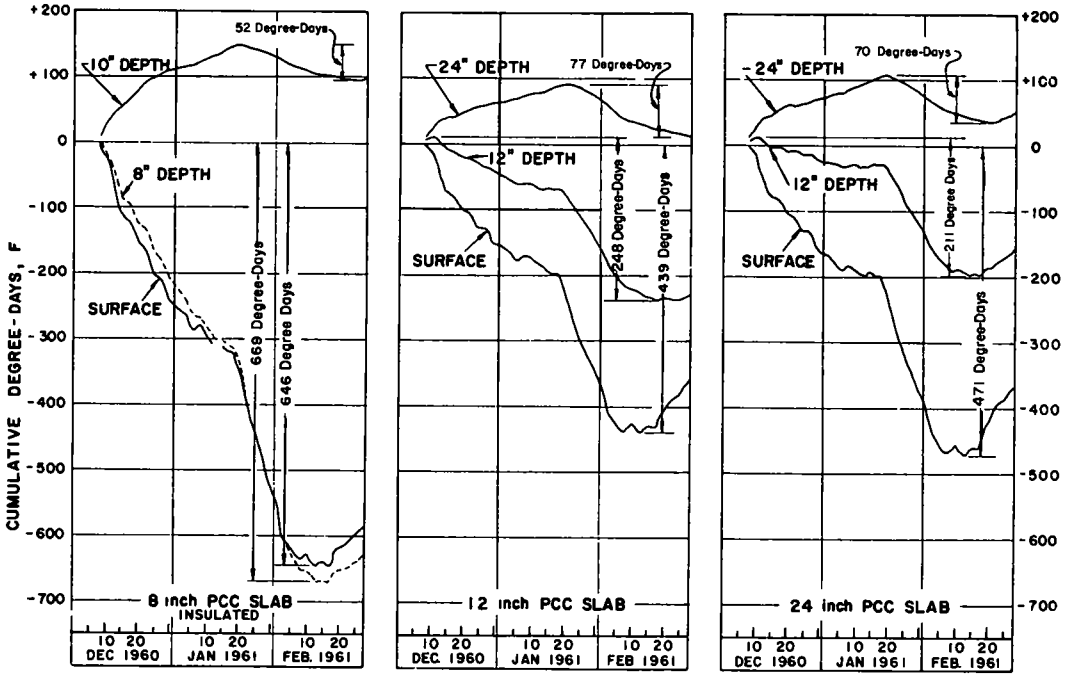

Figure 8. Variation in cumulative degree-days with depth.

The average decrease in freezing index per inch of concrete was $\frac{(439-248)}{12} = 15.9$ degree-days, F, per in. for the 12-in. slab and $\frac{(471-70)}{24} = 16.7$ degree-days, F, per in. for the 24-in. slab. In the 24-in. slab, the average decrease in freezing index was 21.7 degree-days, F, per in. in the upper foot and 11.75 degree-days, F, per in. in the lower foot. As stated previously, the present design method recommends deducting 10 degree-days, F, per in. from the air-freezing index for each inch of concrete over 12 in. To convert the 11.75 degree-days, F, per in. to an air-freezing index datum, it should be divided by the applicable n-factor, which in this case is 0.71. Thus $\frac{(11.75)}{0.71} = 16.5$ degree-days, F, per in. represents the average reduction in air freezing index per inch of concrete over 12 in. for this particular test. On the basis of this single, small-scale experiment in an area of relatively low air-freezing index, consideration might be given to raising the recommended design criteria from 10 degree-days, F, per in. to 12 degree-days, F, per in., which should more nearly predict frost penetration.

ANALYSIS OF INSULATED SLAB BY PERIODIC HEAT FLOW

The depth of frost penetration under the combined layer of portland cement concrete and insulation was analyzed using the one-dimensional periodic heat-flow solution of the Fourier conduction equation (4). This equation takes into consideration the thermal contact coefficients, β (defined later), of the heterogeneous materials comprising the cross-sectional profile of the insulated section. The mathematical development of this solution, presented by Lachenbruch (2), assumes a simple annual sinusoidal temperature distribution as the surface boundary condition and constant termal properties of each stratum. The freezing index at the insulation-soil or concrete-soil interface was determined by calculating the amplitude of the sinusoidal temperature wave at this interface and then converting this amplitude to a freezing index. This freezing index was

then applied in the modified Berggren equation to calculate the depth of frost penetration into the underlying soil. The modified Berggren equation takes into consideration the concurrent thermal effects of temperature changes and soil water phase transformations and may be applied to either homogeneous (5) or stratified (6) soil conditions.

If the effect of the thermal contact coefficient, β, is ignored, however, the amplitude of the sinusoidal temperature wave is damped logarithmically as its depth increases and the mathematical expression for this condition becomes

$$A_x = A_0 \exp(-y) \tag{1}$$

in which

$$y = x_1 \sqrt{\frac{\pi}{a_1 P}}$$

x_1 = depth from surface within upper layer, feet;
a_1 = thermal diffusivity of upper layer, ft^2/day;
P = period of sine wave, 365 days;
A_0 = amplitude of surface sine wave, F; and
A_x = amplitude of sine wave at depth x, F.

When the effect of contact coefficients are taken into consideration, Eq. 1 may be modified as follows:

$$A_x = A_0 \left[\frac{1 + M}{\sqrt{S}}\right] \exp(-y) \tag{2}$$

in which

$$(1 + M) = \frac{2\beta_1}{\beta_1 + \beta_2} \quad \text{(dimensionless)}$$

$$\text{(upper layer)} \quad \beta_1 = \sqrt{k_1 \cdot C_1}, \quad \text{Btu/ft}^2, \text{ F}\sqrt{\text{hr}}$$

$$\text{(lower layer)} \quad \beta_2 = \sqrt{k_2 \cdot C_2}, \quad \text{Btu/ft}^2, \text{ F}\sqrt{\text{hr}}$$

k_1 = thermal conductivity of upper layer, Btu/ft-hr-F;
k_2 = thermal conductivity of lower layer, Btu/ft-hr-F;
C_1 = volumetric specific heat of upper layer, Btu/ft^3-F;
C_2 = volumetric specific heat of lower layer, Btu/ft^3-F; and

$$S = 1 + 2M_1 \cos 2x_1 \sqrt{\frac{\pi}{a_1 P}} \exp\left[-2x_1 \sqrt{\frac{\pi}{a_1 P}}\right] + M_1^2 \exp\left[-4x_1 \sqrt{\frac{\pi}{a_1 P}}\right].$$

If β_1 is greater than β_2, the A_x of Eq. 2 is larger than that of Eq. 1 and conversely, if β_1 is less than β_2, the A_x of Eq. 2 is smaller. The effect of thermal contact resistance is most pronounced when analyzing heat flow through materials having considerably different β values, such as concrete, insulation, and soil.

The freezing index represented by a sinusoidal temperature wave at any depth may be determined from the relationship between the amplitude of an equivalent sinusoidal wave and the mean annual temperature, as

$$F_x = \frac{365}{\pi}\left[\sqrt{A_x^2 - \nu_0^2} - \nu_0 \cos^{-1}\frac{\nu_0}{A_x}\right] \tag{3}$$

in which

F_x = freezing index at depth x, degree-days, F;

A_x = amplitude of sine wave at depth x, F; and

ν_0 = (average annual temperature −32F), F.

Eq. 1 and 3 are shown graphically in Figure 9.

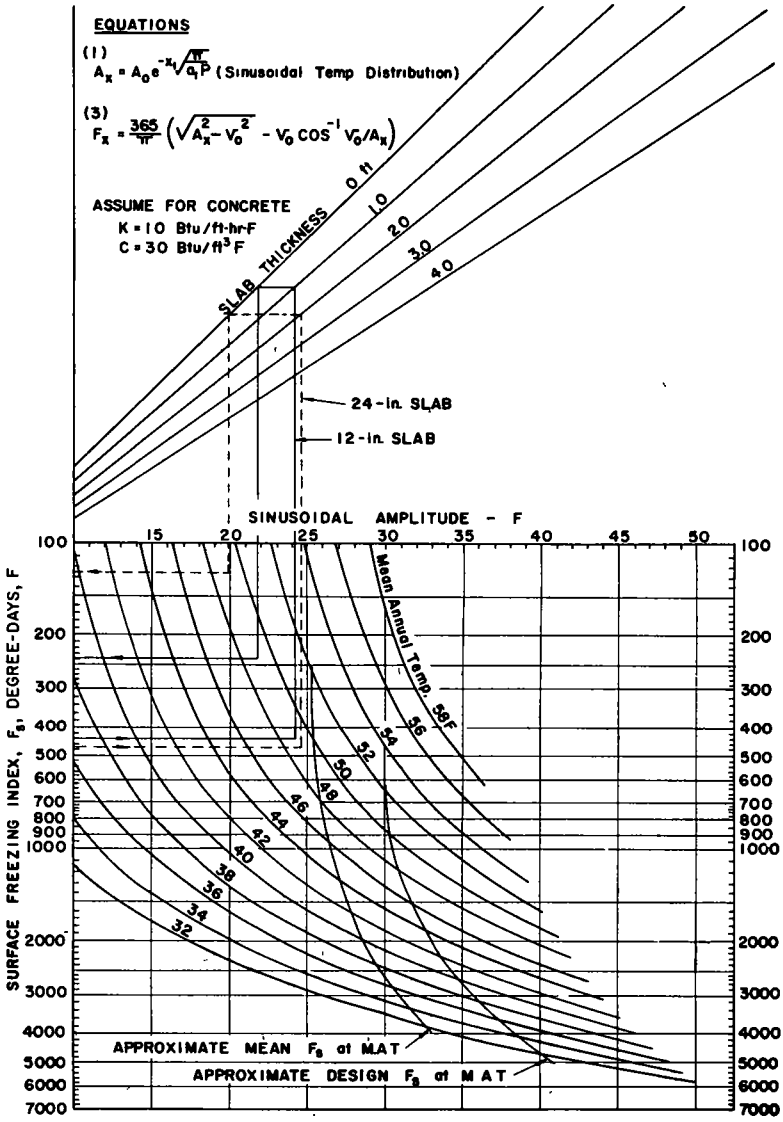

Figure 9. Freezing index at bottom of concrete slabs.

EVALUATION OF FROST PENETRATION UNDER TEST SLABS

A discussion of frost penetration, measured vs predicted, follows. The thermal properties of the materials used in the analysis are given in Table 1. The mean annual temperature used in the calculations was 49 F.

TABLE 1

THERMAL PROPERTIES

Material	Thermal Conductivity, k (Btu/ft-hr-F)	Volumetric Specific Heat, C (Btu/ft³-F)	Thermal Diffusivity, α (ft²/day)	Thermal Contact Coef., β (Btu/ft²-F-√hr)
Concrete	1.00	30.0	0.800	5.48
Insulation	0.0317	1.8	0.423	0.239
Soil:				
8-in. slab	1.20[a]	26.0	1.109	5.60
12- and 24-in. slabs	0.80[a]	23.0	0.835	4.30

[a]After Kersten (7).

Uninsulated Slabs

12-In. Slab.—Entering Figure 9 with a surface freezing index of 439 and a mean annual temperature of 49 F, the surface sinusoidal amplitude is found to be 24.25 F and the freezing index at the bottom of the slab, 240 degree-days, F, with a corresponding sinusoidal amplitude of 21.7 F. This predicted freezing index of 240 degree-days, F, is in close agreement with the measured index of 248.

The predicted frost penetration using the modified Berggren equation for a multilayer condition is 36 in. (Appendix A) in comparison to the measured depth of 34 in.

As the thermal contact coefficients of the concrete and base course are of similar magnitude, Eq. 1 was used for this solution. In general, the β-values of concrete and granular base courses will be reasonably equivalent, which permits a much simpler solution of the problem. The magnitude of the $(1 + M) / \sqrt{S}$ modifier is 1.022 for this case; this was determined (Table 1) as

$$\beta = 5.48 \text{ for concrete}$$
$$\beta = 4.30 \text{ for soil}$$
$$(1 + M) = 1.12$$
$$\sqrt{S} = 1.096$$

thus,

$$\frac{1 + M}{\sqrt{S}} = 1.022.$$

Because of the many simplifying assumptions made in applying the periodic heat flow solution to the frost penetration problem, the effect of slightly differing thermal contact coefficients may be ignored when dealing with portland cement concrete and a high-quality base-course material. However, this effect must be considered when dealing with a stratified profile involving an insulation layer.

24-In. Slab.—The surface freezing index of the 24-in. slab was 471 degree-days, F. From Figure 9, the surface sinusoidal amplitude is 24.5 F and the freezing index at the base of the 24-in. slab is 125 degree-days, F, compared to the measured 70 degree-days, F. The difference between the predicted and measured freezing indexes at the base of the slab may be due to the influence of three-dimensional heat-flow and

to the inaccuracy of representing low freezing indexes by means of a simple sinusoidal wave form.

The frost penetration calculations (Appendix B) predict a total frost penetration of 39 in. The measured frost penetration was 36 in. based on the maximum depth of the 32 F isotherm.

Insulated Slab

The surface-freezing index of the 8-in. slab was 646 degree-days, F, which in Figure 9 corresponds to a surface sinusoidal amplitude of 26.5 F for a mean annual temperature of 49 F. The calculated freezing index at the bottom of the 8-in. slab is 638 degree-days, F, compared to the measured index of 669 degree-days, F. If the effect of the different thermal contact coefficients had not been considered the predicted freezing index would have been 480 degree-days, F, at the 8-in. depth.

The calculated freezing index at the bottom of the 2-in. insulation layer was 2 degree-days, F, compared to the measured index of 55 degree-days, F; however, if the effect of contact resistances had been neglected, the predicted index would have been 580 degree-days, F. Although the predicted index was smaller than that which actually occurred, it was of the correct order of magnitude compared to what it would have been if the effect of contact resistances had been neglected. The calculation technique is presented in Appendix C.

The fact that the measured index at the bottom of insulation was greater than predicted is largely attributed to the flow of heat around, rather than through, the insulating layer, creating an other than one-dimensional problem. If one-dimensional conditions had been maintained, it is expected that the predicted index would have been more nearly correct and the depth of frost penetration in the soil would have been negligible.

CONCLUSIONS

The assumption that 10 degree-days, F, of air-freezing index are required for each inch of frost penetration in concrete pavements below the top 12 in. was found conservative in this study. The use of 12 degree-days, F, per in. is suggested as more suitable and would still permit an element of conservatism in the design. Mathematical analysis by means of the periodic heat-flow method indicates that the number of degree-days required to penetrate concrete slabs more than 12 in. thick will vary, depending on the magnitude of the surface-freezing index and mean annual temperature. In zones of relatively high freezing indexes, a greater reduction in freezing index might be allowed than in zones of low freezing index.

The periodic heat flow solution for the insulated case (Eq. 2) gave results in the proper order of magnitude, although the quantitative results were slightly untenable. This is partly attributed to the lateral heat flow around the insulation due to the small-scale slabs employed in the field experiment. However, the test did indicate that the effect of the markedly different thermal contact coefficients should be taken into consideration when dealing with periodic heat flow through a concrete, insulation, and soil profile.

The use of the periodic heat flow equation for the uninsulated case (Eq. 1) appears to be a valid solution when dealing with portland cement concrete slabs placed on sound granular base courses. Such a procedure eliminates the need for evaluating the relatively complex $(1 + M)/\sqrt{S}$ term used when termal contact coefficients are considered. The assumption of an annual sinusoidal temperature distribution at the surface boundary appears to give sufficiently accurate results even though the temperature distribution is more complicated than assumed.

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Appendix A

Problem

To predict the frost penetration in the multilayer soil profile under the uninsulated 12-in. concrete slab using the modified Berggren method.

Solution

The following physical and thermal properties are used in this solution:

Layer	Depth, x (ft)	Dry Unit Wt., γ_d (pcf)	Moist. Cont., w (%)	Latent Heat, L (Btu/ft³)	Avg Volumetric Specific Heat, C (Btu/ft³ F)	Avg. Thermal Conduc., k (Btu/ft hr F)	Unified Soil Class
A	0-0 67	109	5.5	865	23.0	0 8	SW-SM
B	0.67-2.00	130	4 0	750	26.0	1 3	SM

in which

$$L = 1.44\, w\, \gamma_d;$$

$$C_{avg} = \gamma_d \left(0.17 + 0.75 \times \frac{w}{100}\right);$$

$$k_{avg} = \tfrac{1}{2}\left(k_{frozen} + k_{unfrozen}\right) \text{ after Kersten (7); and}$$

x = depth below bottom of concrete slab.

The freezing index required to penetrate each layer is given by

$$F_n = \frac{L_n\, x_n}{24}\left(\Sigma R + \tfrac{1}{2} R_n\right)\frac{1}{\lambda_n^2}$$

in which

L_n = volumetric latent heat of layer, Btu/ft³;

x_n = thickness of frozen layer, feet;

R_n = x_n/k_n = thermal resistance of layer, ft² hr F/Btu;

ΣR = sum of resistances above layer considered:

λ_n = correction factor - function of μ and α (Fig. 10).

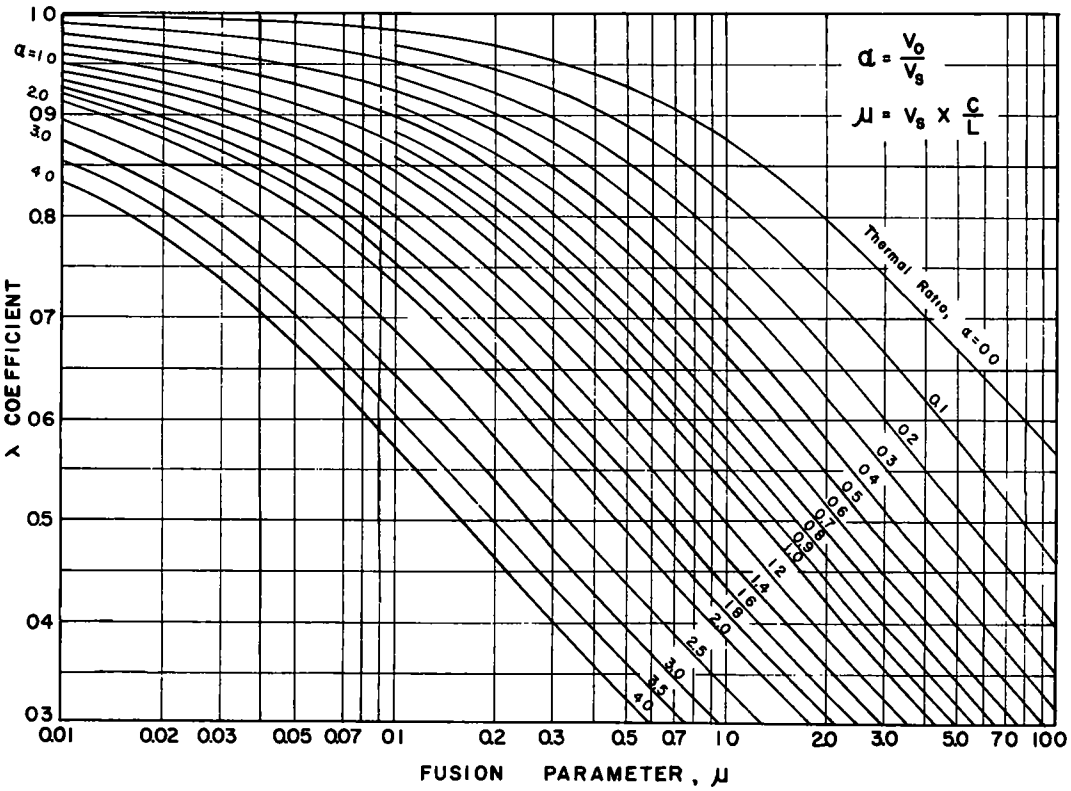

Figure 10. λ coefficient in the modified Berggren formula.

$$\mu = \frac{\overline{C}_n}{\overline{L}_n} \times \nu_s; \text{ where } \nu_s = \Sigma F/t;$$

ΣF = total freezing index at bottom of concrete, degree-days, F;
t = duration of freezing index, days (Fig. 11); and

$$\alpha = \frac{\nu_o}{\nu_s} \text{ where } \nu_o = \text{mean annual temperature - 32 F.}$$

Frost penetration in a multilayer soil profile is solved by trial and error, so that the total of the F_n values for each layer is equal to ΣF. The solution is given in Table 2 and predicts a frost penetration into the soil of 24 in. The total predicted frost penetration, which includes the concrete slab and soil, is (24 + 12) = 36 in.

The duration of the freezing season, t, at the surface of the slab was 68 days (Fig. 5). If a pure sinusoidal temperature waveform existed, the duration of the surface freezing index would have been 93 days. This is determined from the relationship between the $\frac{\nu_o}{A} = \frac{17}{24.25}$ term and the length of the freezing season, t (see Fig. 11). The length of freezing season required for a sinusoidal temperature variation at a 12-in. depth with an amplitude of 21.7 F is 77 days. Assuming proportional relationships, the predicted length of the freezing index at the 12-in. depth is 77/98 x 68 = 57 days. Thus, the duration of ΣF is taken as 57 days.

TABLE 2

FROST PENETRATION—12-IN. SLAB

Layer	x	Σx	C	k	L	Lx	ΣLx	$\overline{L}$	Cs	ΣCx	$\overline{C}$	μ	λ^2	R	ΣR	$\Sigma R + \frac{R}{2}$	F_n	ΣF_n
A	0 67	0 67	23.0	0 8	865	580	580	865	15.4	15.4	23 0	0 112	0.304	0 84	0	0.42	34	-
B	1 33	2 00	26 0	1 3	750	997	1,577	788	34.6	50 0	25 0	0 133	0 277	1 02	0.84	1 34	201	235

ΣF_n = 235 degree-days, F, compared to the total available index (ΣF) of 240 degree-days, F This is considered to be sufficiently close agreement, and the total predicted frost penetration in the soil (Σx) is 2 00 ft

ΣF = 240 degree-days, F

t = 57 days

$\Sigma F/t$ = 4.21

$\alpha = \frac{49 - 32}{4\ 21} = 4\ 04$

λ is taken from Figure 10

$\overline{L} = \frac{\Sigma Lx}{\Sigma x} = \frac{1,577}{2\ 00} = 788$

$\overline{C} = \frac{\Sigma Cx}{\Sigma x} = \frac{50.0}{2\ 00} = 25$

$\mu = \frac{\overline{C}}{\overline{L}} \times \frac{\Sigma F}{t} = \frac{25}{788} \times 4\ 21 = 0\ 133$

$R = \frac{x}{k} = \frac{1\ 33}{1\ 3} = 1\ 02$

$F_B = \frac{Lx}{24} \left(\Sigma R + \frac{R}{2}\right) \frac{1}{\lambda^2} =$

$\frac{997}{24} (1.34) \times \frac{1}{0\ 277} =$

201 degree-days, F

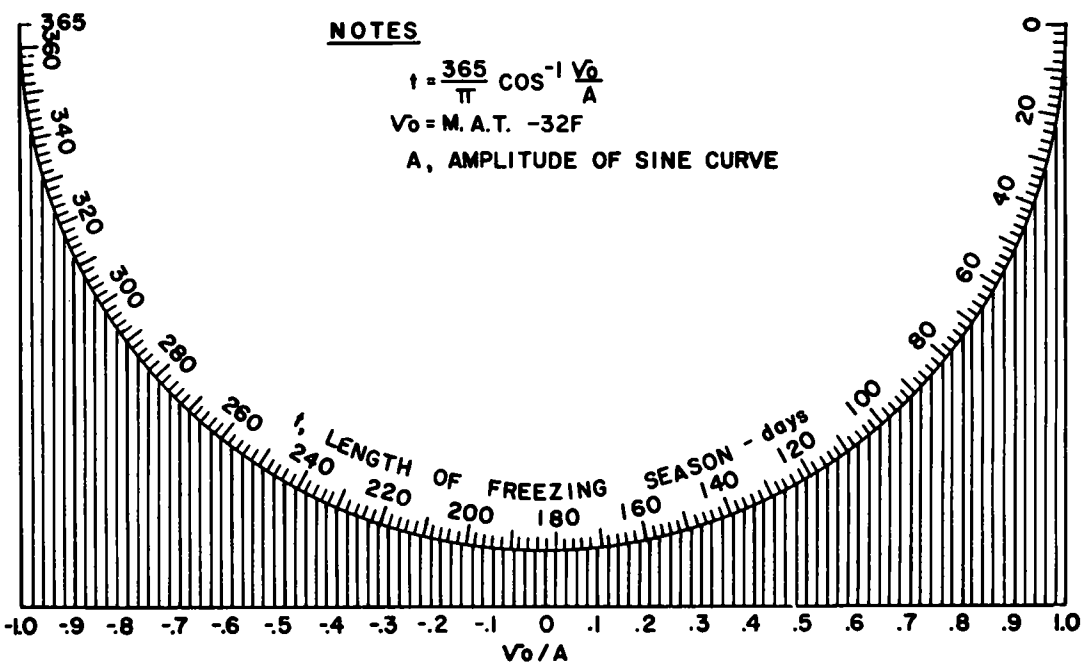

Figure 11. Length of freezing season for sinusoidal temperature variation.

Appendix B

Problem

To predict the frost penetration in the multilayer soil profile under the uninsulated 24-in. concrete slab.

Solution

The physical and thermal properties used in this solution are the same as those indicated in Appendix A. The solution is compiled in Table 3 and predicts a frost penetration into the soil of 15 in. The total frost penetration is (15 + 24) = 39 in.

The duration of the freezing season at the surface of the slab was 68 days (Fig. 5). If a sinusoidal temperature wave form had existed, the duration of the surface freezing index would have been 93 days (ν_0/A = 17/24.5). The length of freezing season required for a temperature variation at a 24-in. depth with an amplitude of 19.9 F is 63 days. The predicted length of the freezing season at the 24-in. depth is

$$\frac{63}{93} \times 68 = 46 \text{ days}$$

TABLE 3
FROST PENETRATION–24-IN SLAB

Layer	x	Σx	C	k	L	Lx	ΣLx	$\bar{L}$	Cx	ΣCx	$\bar{C}$	μ	λ²	R	ΣR	$\Sigma R + \frac{R}{2}$	F_n	ΣF_n
A	0 67	0 67	23.0	0 8	865	580	580	865	15 4	15 4	23.0	0 072	0 250	0.84	0	0 42	40	40
B	0 60	1.27	26 0	1 3	750	450	1,030	810	15 6	31 0	24.4	0 082	0.230	0.46	0 84	1 07	87	127

As shown in Table 3, ΣF_n = 127 degree-days, F, which is in agreement with the total available index of 125 degree-days, F. The total predicted frost penetration in the soil (Σx) is 15 in.

$$\Sigma F = 125 \text{ degree-days, F}$$

$$t = 46 \text{ days}$$

$$\frac{\Sigma F}{t} = 2.72$$

$$\alpha = \frac{49 - 32}{2.72} = 6.25$$

λ (is taken from Fig. 10).

$$\mu = \frac{\bar{C}}{\bar{L}} \times \nu_s$$

Appendix C

Problem

To predict the freezing index at the bottom of the insulated 8-in. slab and at the bottom of the 2-in. insulation layer. The subscript 1 refers to the concrete, 2 refers to the insulation, and 3 refers to the base course.

Solution

Heat Flow Through Concrete Slab.—Eq. 2 is used for establishing the sinusoidal temperature amplitude at the bottom of the 8-in. concrete slab. The surface freezing index was 646 degree-days, F, and the mean annual temperature is 49 F.

$$A_1 = A_0 e^{-x_1 \sqrt{\frac{\pi}{a_1 P}}} \left[\frac{1 + M_1}{\sqrt{S_1}} \right]$$

in which

$$A_0 = 26.5 \text{ F (Fig. 9)};$$

$$e^{-x_1 \sqrt{\frac{\pi}{a_1 P}}} = e^{-\frac{8}{12}\sqrt{\frac{\pi}{(0.80)\ (365)}}} = 0.933;$$

$$(1 + M_1) = \frac{2\beta_1}{\beta_1 + \beta_2}; \quad = \frac{2(5.48)}{5.48 + 0.239} = 1.915;$$

$$M_1 = 0.915;$$

$$S_1 = 1 + 2M_1 e^{-2x_1 \sqrt{\frac{\pi}{a_1 P}}} \cos 2x_1 \sqrt{\frac{\pi}{a_1 P}} + M_1^2 e^{-4x_1 \sqrt{\frac{\pi}{a_1 P}}}$$

$$= 1 + 1.580 + 0.635$$

$$= 3.215; \text{ and}$$

$$\sqrt{S_1} = 1.791.$$

Thus,

$$\frac{1 + M_1}{\sqrt{S_1}} = 1.069 \text{ and}$$

$$A_1 = (26.5)\ (0.933)\ (1.069) = 26.4 \text{ F}$$

(from Fig. 9) the freezing index, F_1 = 638 degree-days, F.

This compares favorably with the measured index of 669 degree-days, F. If the effect of thermal contact coefficients had been neglected, the sinusoidal amplitude would be (26.5 x 0.933) = 24.7 F. This would correspond to a freezing index of 480 degree-days, F, thus the damping effect is less than it would have been if Eq. 1 were used. This is attributed to the fact that β_1 (5.48) is greater than β_2 (0.239).

Heat Flow Through Insulation Layer.—The sinusoidal temperature amplitude, A_1, at the bottom of the concrete slab (top of insulation layer) is utilized in determining the temperature amplitude at the bottom of the insulation layer. The amplitude, A_2, at the bottom of the insulation is given by Eq. 2.

$$A_2 = A_1 e^{-x_2\sqrt{\frac{\pi}{a_2 P}}} \left[\frac{1 + M_2}{\sqrt{S_2}}\right]$$

in which

$$A_1 = 26.4\ F;$$

$$e^{-x_2\sqrt{\frac{\pi}{a_2 P}}} = e^{-\frac{2}{12}\sqrt{\frac{\pi}{(0.423)\ (365)}}} = 0.9766;$$

$$(1 + M_2) = \frac{2\beta_2}{\beta_2 + \beta_3} = \frac{2\ (0.239)}{0.239 + 5.60} = 0.08186;$$

$$M_2 = -0.9181;$$

$$S_2 = 1 + 2M_2 e^{-2x_2\sqrt{\frac{\pi}{a_2 P}}} \cos 2x_2\sqrt{\frac{\pi}{a_2 P}} + M_2^2 e^{-4x_2\sqrt{\frac{\pi}{a_2 P}}}$$

$$= 1 - 1.7511 + 0.7622$$

$$= 0.0151;\ \text{and}$$

$$\sqrt{S_2} = 0.1229.$$

Thus,

$$\frac{1 + M_2}{\sqrt{S_2}} = 0.666 \text{ and}$$

$$A_2 = (26.4)\ (0.9766)\ (0.666) = 17.17F$$

(from Eq. 3) the predicted freezing index, F_2 = 2 degree-days, F.

The measured freezing index was 52 degree-days, F; a probable explanation for the disparity between this value and the predicted value is given in the main text. If the effect of thermal contact coefficients had been ignored, the predicted sinusoidal amplitude would have been (26.4 x 0.9766) = 24.8 F. This represents a freezing index of 580 degree-days, F; therefore, the damping effect is greater than it would have been if Eq. 1 were used. In this case β_2 (0.239) is less than β_3 (5.60).

In analyzing the three-layer problem, a slight departure has been made from the Lachenbruch method, which for this case is more conservative and is also a simplification of the more rigorous Lachenbruch technique.