Stress and Strain Factors for Three-Layer Elastic Systems

K. R. PEATTIE, Research Engineer, Thornton Research Centre, on temporary assignment at the Research Laboratory, Shell Oil Company, Wood River, Illinois.

Tables of stress factors for systems of three elastic layers under load have been published by A. Jones of the Thornton Research Centre of "Shell" Research Ltd. in connection with the development of a fundamental method of road design. For convenience in the analysis and design of road structures, it is desirable to present these factors graphically. Some stress and strain factors not directly tabulated in Jones' report have been derived from the data therein. A suitable graphical method for the presentation of the factors is described. A series of graphs covering four factors has been prepared.

•MOST METHODS of road design are empirical. They cannot be extended to cover new types of loading or materials of construction. Neither can they be used for the analysis of the behavior of roads.

A fundamental method for the design of flexible roads is being developed by Thornton. The basis of this method is to determine the thicknesses of the various layers so that the stresses and strains developed by moving traffic are within the permissible limits for the materials. It is therefore necessary to be able to calculate the values of these stresses and strains.

A real road structure may be represented by a system of elastic layers lying on a semi-infinite elastic mass. From a review of methods available for calculating stresses in such systems, it was concluded that the stresses should be obtained from rigorous solutions of the elastic equations for layered systems. Suitable solutions for a wide range of the parameters involved have been published by A. Jones of the Thornton Research Centre.

The stress factors are tabulated at wide intervals of the four parameters involved. In the analysis and design of road structures it is necessary to interpolate between the tabulated values. A convenient graphical method of doing this is described.

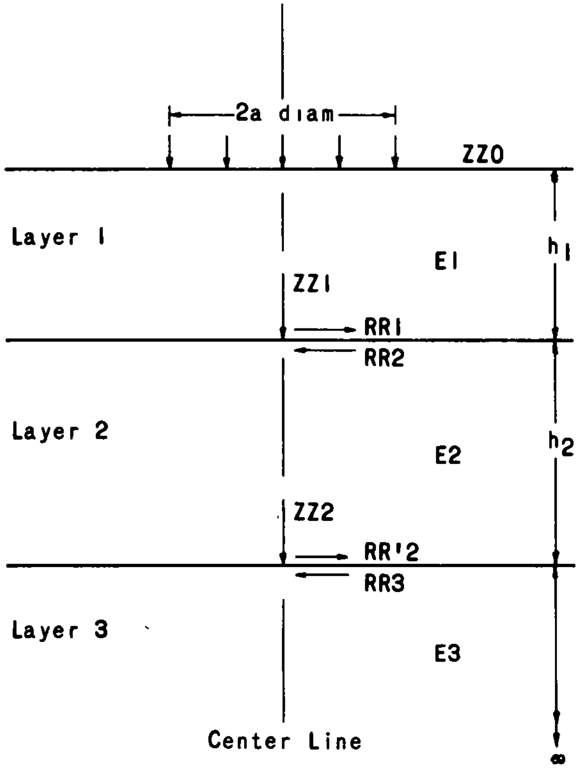
STRESS AND STRAIN FACTORS

Figure 1 shows a three-layer road structure and the stresses for which factors have been calculated. These have been confined to points at the interfaces on the vertical centerline through the loaded area because they have their maximum values under these conditions if the load is uniformly distributed.

The tables prepared by Jones (1) list the stress factors given in Table 1. The stresses are obtained by multiplying the contact stress by the stress factor.

These were the six stress factors given by Acum and Fox (2) whose tables were considerably extended by Jones (1). The difference between the vertical and horizontal stresses was tabulated for convenience in obtaining shear stresses. Because in the design and analysis of flexible pavements the stresses and strains existing at the bases of the upper two layers can be important, it would be convenient to have the stress factors, RR1 and RR'2, and the strain factors, ½ (RR1-ZZ1) and ½ (RR'2 - ZZ2), tabulated directly. They have been obtained from data in the Jones tables by computations carried out on the IBM 650 system computer at Wood River.

The horizontal strain is obtained from the strain factor by multiplying the factor by the contact stress and dividing by the elastic modulus of the layer.



Legend:
ZZO = Contact stress
ZZ1 = Vertical stress at first interface
ZZ2 = Vertical stress at second interface
RR1 = Horizontal stress at base of top layer
RR2 = Horizontal stress at top of middle layer
RR'2= Horizontal stress at base of middle layer
RR3 = Horizontal stress at top of bottom layer

Figure 1. Three-layer road structure.

TABLE 1
STRESS FACTORS TABULATED IN THORNTON REPORT

Stress	First Interface	Second Interface
Vertical	ZZ1	ZZ2
(Vertical horizontal)	(ZZ1 - RR1)	(ZZ2 - RR'2)
	(ZZ1 - RR2)	(ZZ2 - RR3)

GRAPHICAL PRESENTATION

The stress and strain factors in the tables by Jones (and in subsequent tables at Wood River) are listed in terms of the following parameters: $A = a/h_2$; $H = h_1/h_2$; $K_1 = E_1/E_2$; and $K_2 = E_2/E_3$; in which a is the radius of circular contact area; h_1 and h_2 are thicknesses of top and middle layers, respectively; and E_1, E_2, and E_3 are elastic moduli of top, middle, and bottom layers, respectively.

The stress and strain factors are tabulated for the following values of these parameters:

$A = 0.1,\ 0.2,\ 0.4,\ 0.8,\ 1.6,\ 3.2$
$H = 0.125,\ 0.25,\ 0.50,\ 1.0,\ 2.0,\ 4.0,\ 8.0$
$K_1 = 0.2,\ 2.0,\ 20.0,\ 200.0$
$K_2 = 0.2,\ 2.0,\ 20.0,\ 200.0$

These ranges were chosen to cover the conditions most likely to occur in flexible pavements. The individual values were selected to be convenient for interpolation. The data have now to be presented in a graphical form suitable for use in the analysis and design of flexible pavements.

There are four independent variables and one dependent variable involved. The dependent variable and one pair of independent variables can be represented on one "grid" figure. The parameters A and H are related to the geometry of the pavement and the load system. The parameters K_1 and K_2 are related to the elastic properties of the pavement. The same geometric arrangement may have to be analyzed for combinations of materials of different properties. It would therefore be convenient to construct

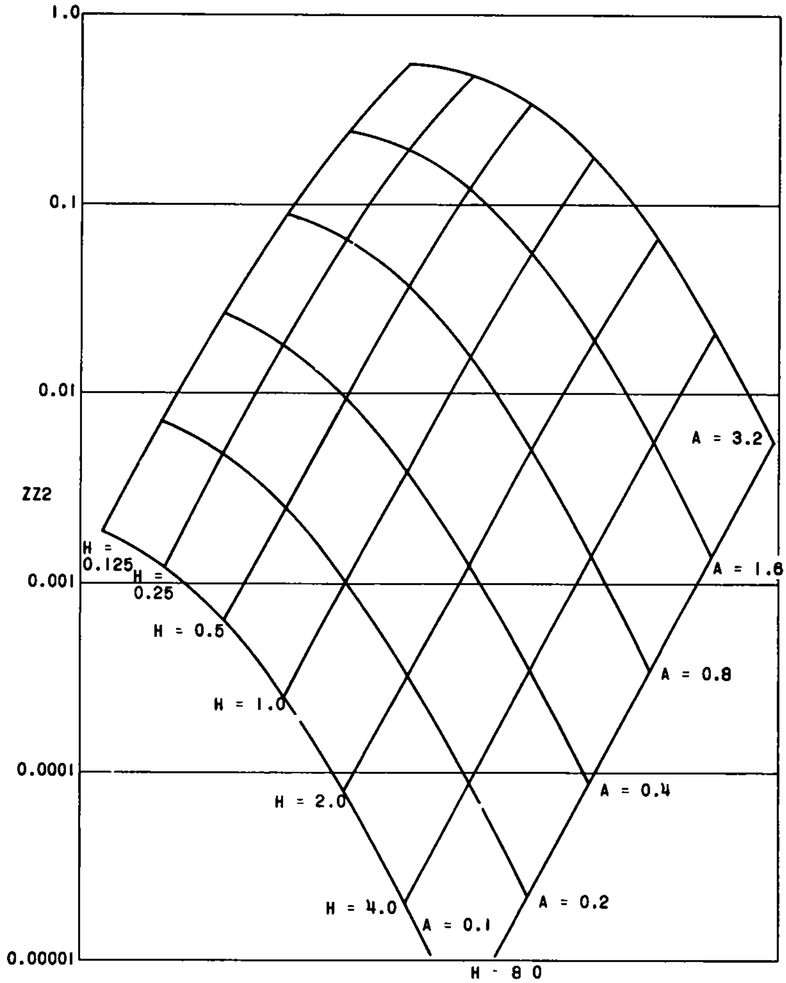


Figure 2. Vertical stress factor ZZ2 for $K_1 = K_2 = 20$.

a series of grids, each one of which covered the full range of A- and H-values for one pair of values of K1 and K2. The full range of any one stress or strain function can be covered on 16 grids.

The graphical representation of three or more variables is discussed by McIntosh (3) and the method is described in detail in Appendix B of this report. A specimen grid giving the vertical stress factor ZZ2, for $K_1 = K_2 = 20$, is shown in Figure 2.

The production of the grids for ZZ1 and ZZ2 is straightforward. At high values of A and low values of H the grid for ZZ1 becomes compressed but that is unimportant as it is in a range of very thin pavements where ZZ1 is approximately equal to unity.

The production of grids for the horizontal stress and strain factors is complicated by two features. First, the factors change sign over the range of the tables. Because only the regions of tensile stress and strain are of interest in the analysis of flexible pavements, the compressive values are disregarded in plotting. Second, when plotting

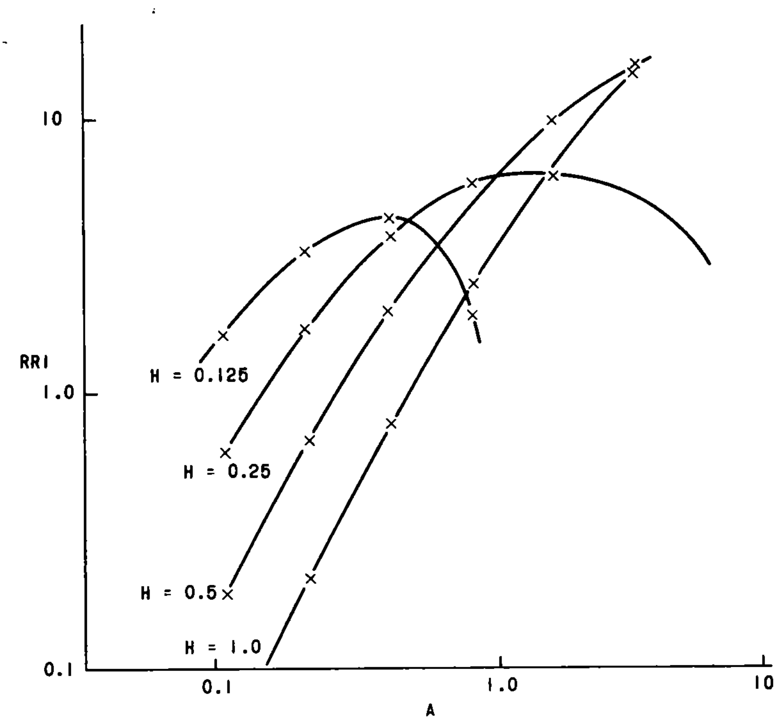


Figure 3. Horizontal stress factor RRI vs A for $K_1 = K_2 = 20$. Pavements corresponding to pairs of values of A at one value of H are compared in Table 2.

TABLE 2
VALUES TAKEN FROM FIGURE 3 FOR RR1 = 3 (a = 6 IN.)

H	A	h_1 (in.)	h_2 (in.)	Total Thickness (in.)
0.125	0.17	4.4	35.3	39.7
	0.65	1.2	9.2	10.4
0.25	0.32	4.7	18.8	23.5
	5.8	0.26	1.03	1.29
0.5	0.54	5.5	11.1	16.6

the factors against A, for a given value of H, they are sometimes seen to pass through maxima as A increases. This occurs at high values of A and low values of H.

Graphs of the radial stress factor, RR1, as a function of A are shown in Figure 3. For H = 0.125 and 0.25 two values of A correspond to each value of RR1. A similar situation has been shown to exist in a two-layer structure by van der Poel (4). In practice, the pavements denoted by the larger of each pair of values of A are inadmissible because they are so thin that the soil would be overstressed. In plotting grids of the horizontal stress and strain factors the lines are stopped when peak values are reached. This simplifies interpolation, but there must be no extrapolation.

The pavements corresponding to the values of A of 0.65 and 5.8 are very thin and are unlikely to be able to protect the soil.

A typical grid showing the values of the horizontal stress factor, RR1, for $K_1 = K_2 = 20$ is given in Figure 4. A series of grids has been plotted for the vertical stress factors ZZ1 and ZZ2 and the horizontal stress and strain factors RR1 and ½ (RR1 - ZZ1). The former were chosen because of their importance in granular bases and subgrades; the latter because of their importance in bituminous carpets.

The full series of grids, which is given in Appendix A, is composed of the following figures: Figures 8 to 23, vertical compressive stress factor ZZ1; Figures 24 to 39, vertical compressive stress factor ZZ2; Figures 40 to 51, horizontal tensile stress factor RR1; and Figures 52 to 67, horizontal tensile strain factor ½ (RR1 - ZZ1).

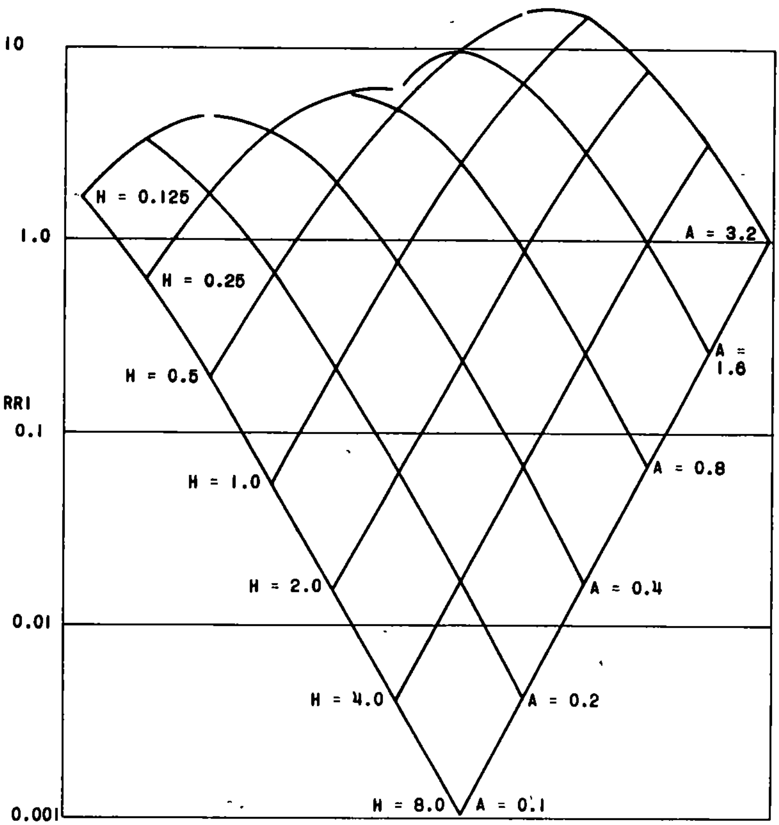


Figure 4. Horizontal stress factor RR1 for $K_1 = K_2 = 20.0$.

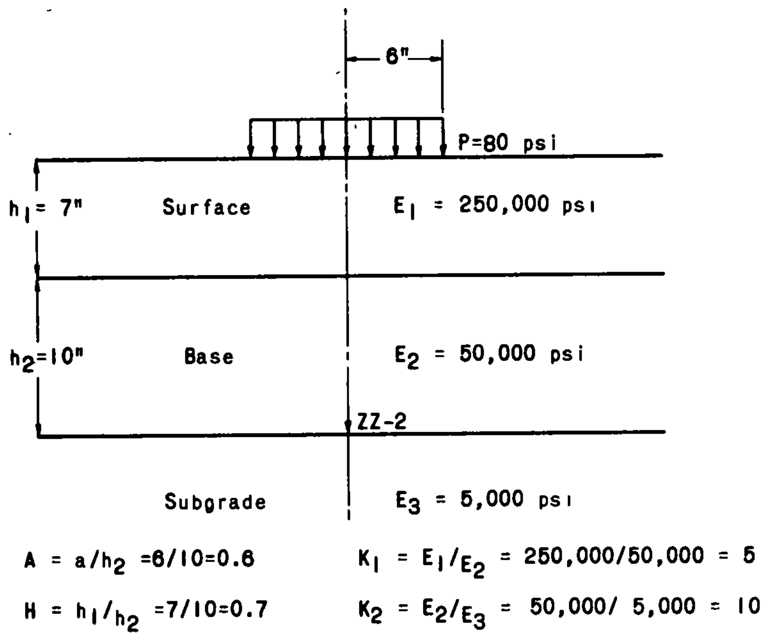


Figure 5. Hypothetical pavement.

TABLE 3
VERTICAL STRESS ON SUBGRADE FOR 9,000-LB WHEEL LOAD
(A = 0.6, H = 0.7)

	K_1 = 0.2		K_1 = 2.0		K_1 = 20.0		K_1 = 200.0	
K_2	Stress Factor ZZ-2	Stress[a] (psi)	Stress Factor ZZ-2	Stress[a] (psi)	Stress Factor ZZ-2	Stress[a] (psi)	Stress Factor ZZ-2	Stress[a] (psi)
0.2	0.19	15.2	0.21	16.8	0.13	10.4	0.05	4.0
2.0	0.13	10.4	0.11	8.8	0.05	4.0	0.017	1.36
20.0	0.045	3.6	0.026	2.08	0.013	1.04	0.0045	0.36
200.0	0.012	0.96	0.006	0.48	0.0028	0.22	0.0010	0.08

[a] For applied unit load of 80 psi.

EXAMPLE OF USE OF GRAPHS

The following numerical example demonstrates the use of the graphs. Considering a hypothetical pavement with properties and dimensions as shown in Figure 5, it is required to determine the vertical compressive stress produced on the subgrade by a uniform load of 80 psi at the surface. This load acting on an area of 6-in. radius is equal to a total load of 9,000 lb. It is similar to the load imposed by a truck wheel.

First, it is necessary to evaluate the parameters A, H, K_1, and K_2. These values shown at the bottom of Figure E are used to enter the graphs.

Next, a table like Table 3 is prepared, and stress factors (ZZ-2) are listed for different combinations of K_1 and K_2. These are the stress factors for a pavement of the given dimensions, but they represent different modular ratios between the layers. Each factor is obtained by interpolation on a separate graph. Thus, the factor of 0.19 for K_1 = 0.2 and K_2 = 0.2 is read from Figure 24 for values of the dimensional parameters A and H equal to 0.6 and 0.7. Factors for other combinations of K_1 and K_2 are read from Figures 25 through 39. Numerical stresses in separate columns of Table 3 are obtained by multiplying stress factors by the applied unit load of 80 psi.

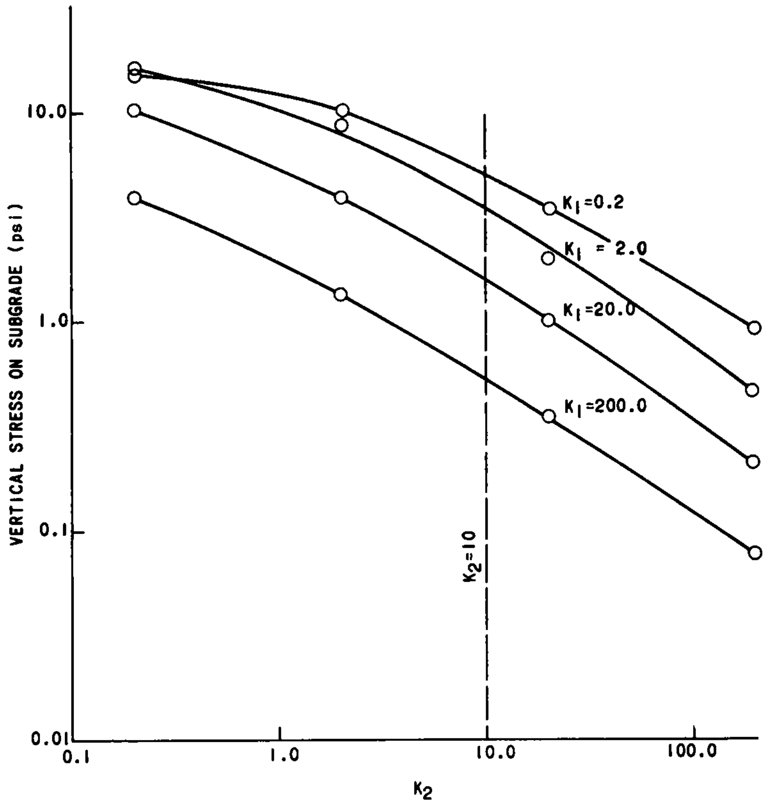


Figure 6. Relation of subgrade stress to modular ratio K_2.

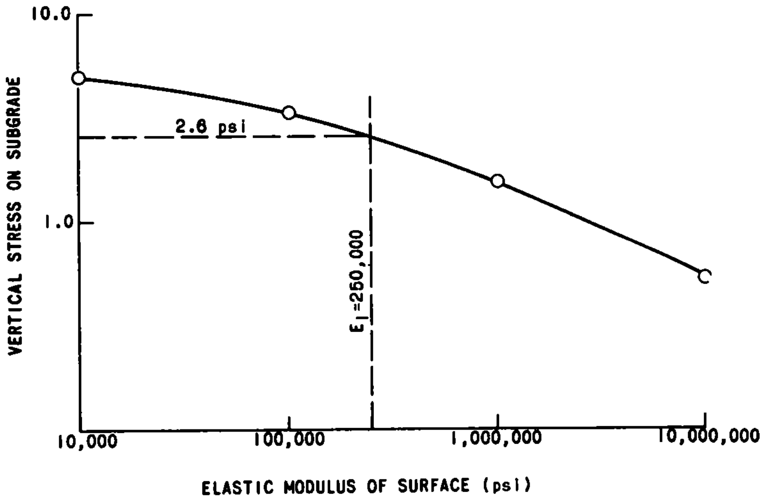


Figure 7. Relation of subgrade stress to elastic modulus of surface.

In Figure 6, subgrade stresses are plotted against corresponding values of K_1 and K_2. The figure demonstrates the influence of modular ratios on subgrade stress. For a constant subgrade modulus, an increase in base course modulus (increase in K_2) reduces subgrade stress. An increase in surface modulus (increase in K_1) also reduces stress.

Stresses for the pavement in the example are taken from Figure 6 at a value of K_2 equal to 10. These, in turn, are replotted in Figure 7. Here, subgrade stresses are shown as a function of the surface modulus $E_1(E_1 = K_1 \times E_2)$. For the designated surface modulus of 250, 000 psi, the vertical stress on the subgrade under an 80-psi load is 2.6 psi.

A graph like Figure 7 is frequently useful because it demonstrates how subgrade stresses are influenced by changes in the surface modulus.

The foregoing example illustrates the use of the graphs to calculate the theoretical stress on the subgrade. The graphs can be used in the same manner to calculate other values. These include the tensile stress or strain, and the vertical compressive stress at the bottom of the surface layer. Thus, stresses can be investigated at several critical points in a pavement structure.

CONCLUSIONS

In the design and analysis of flexible pavements it would be convenient to have the horizontal stress and strain factors directly tabulated. These factors have been calculated from the original data published by A. Jones of the Thornton Research Centre.

A suitable graphical method for presenting the stress and strain factors has been selected. A series of these graphs covering the factors commonly used in the analysis and design of flexible pavements has been produced.

REFERENCES

1. Jones. A., "Tables of Stresses in Three-Layer Elastic Systems." presented to Annual Meeting of the Highway Research Board 1962.
2. Acum, W. E. A., and Fox, L., Geotechnique, 2: 293-300 (1951).
3. McIntosh, J. D., Mag. Concrete Research, C.A.C.A. No. 3, pp. 145-148 (Dec. 1949).
4. van der Poel, C., "Building Materials." Ed. by M. Reiner, N. Holland Publishing Company, Amsterdam, Ch. 9, Fig. 2 (1956).

Appendix A

GRAPHICAL REPRESENTATION OF STRESS AND STRAIN FACTORS

The following factors are presented graphically:

ZZ1	=	Vertical compressive stress at first interface—Figures 8-23
ZZ2	=	Vertical compressive stress at second interface—Figures 24-39
RR1	=	Horizontal tensile stress at base of top layer—Figures 40-51
½ (RR1-ZZ1)	=	Horizontal tensile strain at base of top layer—Figures 52-67

The factors for all combinations of A and H appear on one grid. The grids are arranged in groups of four in ascending order of K_1. Within a group each grid corresponds to one value of K_2.

There must be no extrapolation on any of the grids.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ1

K1 = 0 2
K2 = 0 2

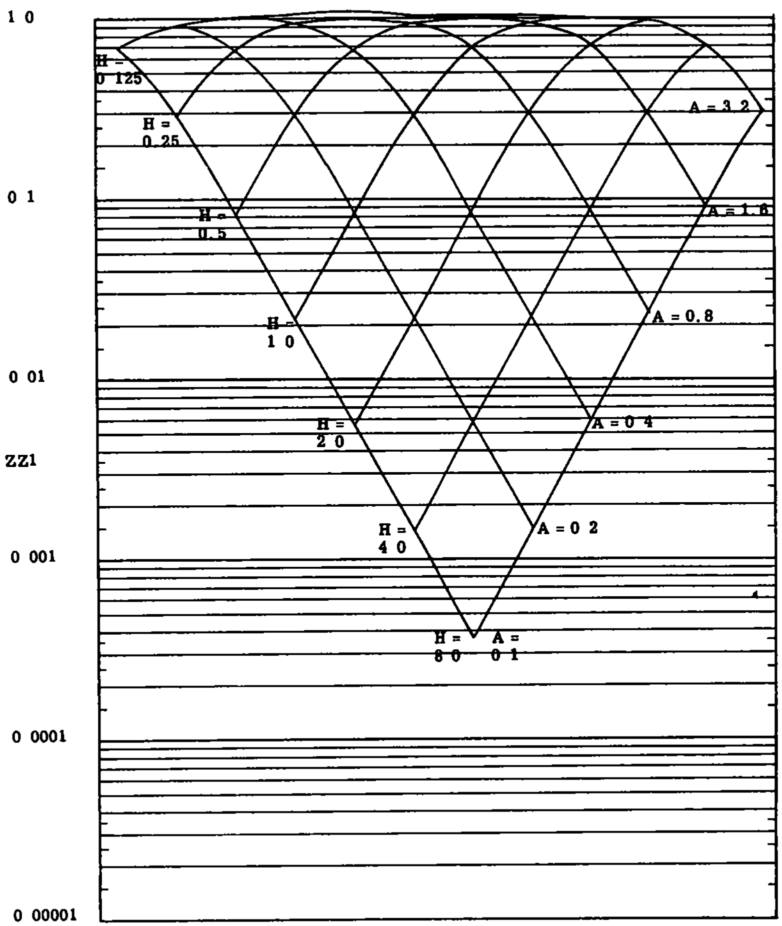


Figure 8.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ1

K1 = 0.2
K2 = 2.0

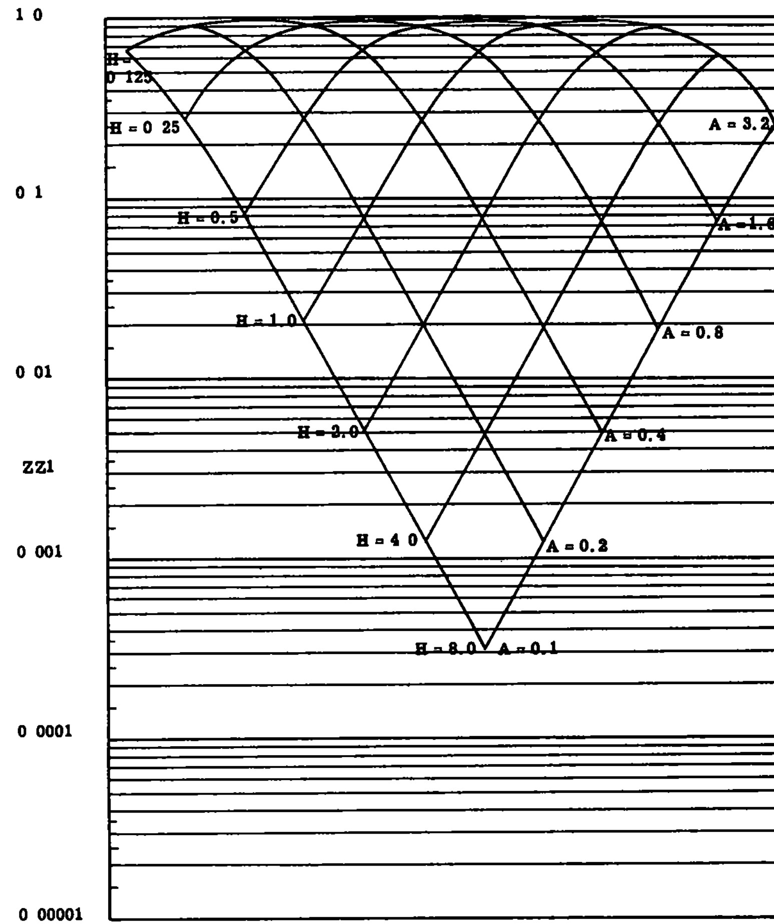


Figure 9.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ1

K1 = 0.2
K2 = 20 0

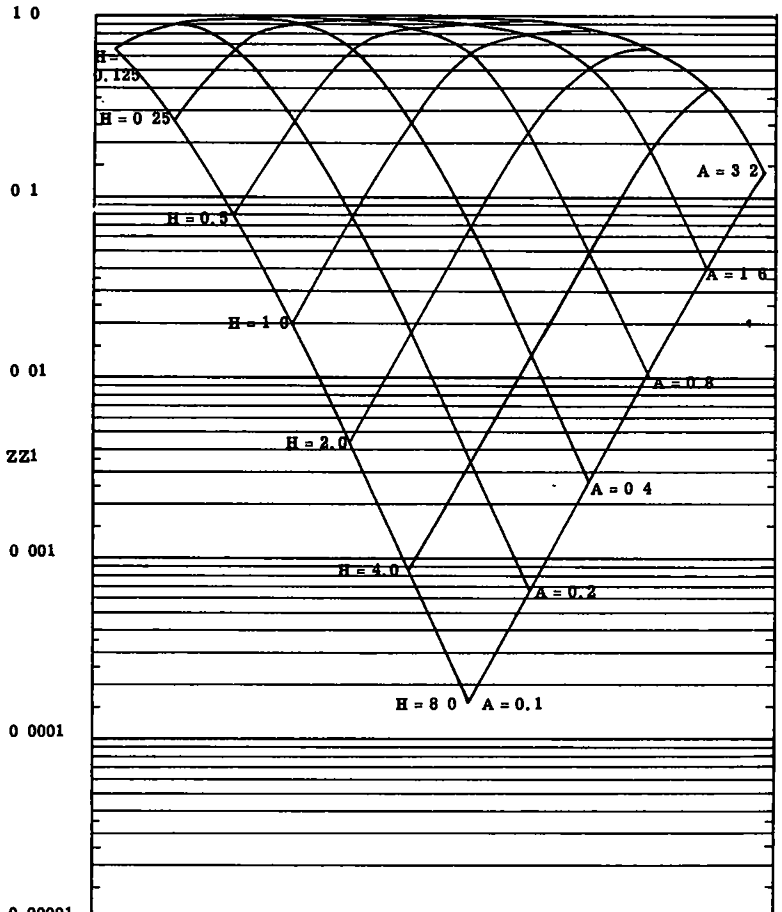


Figure 10.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ1

K1 = 0.2
K2 = 200 0

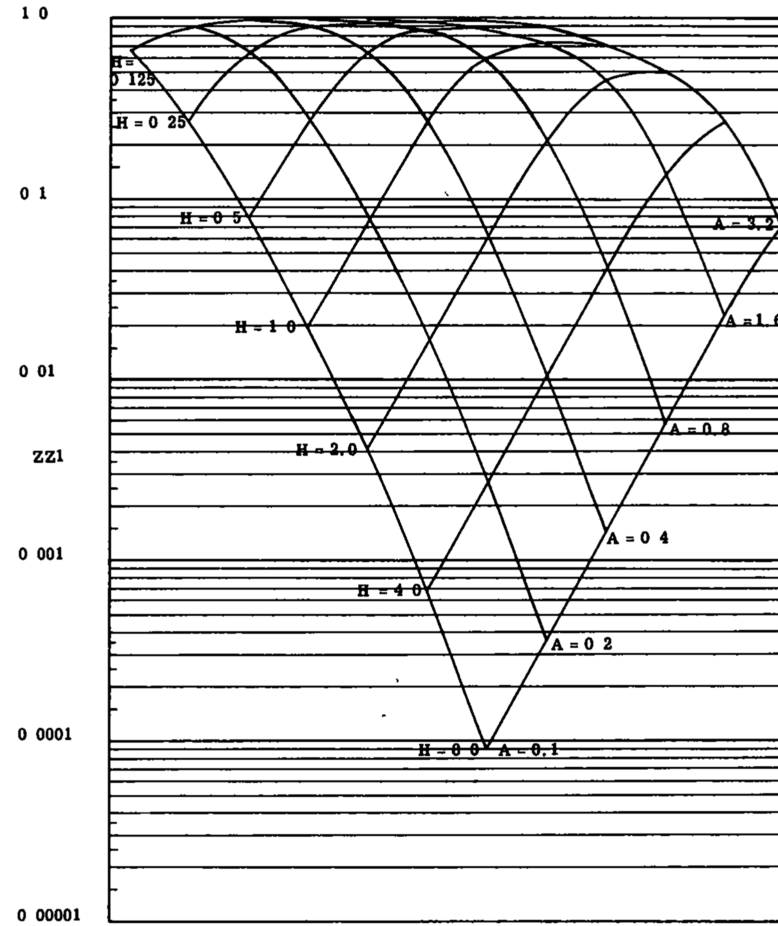


Figure 11.

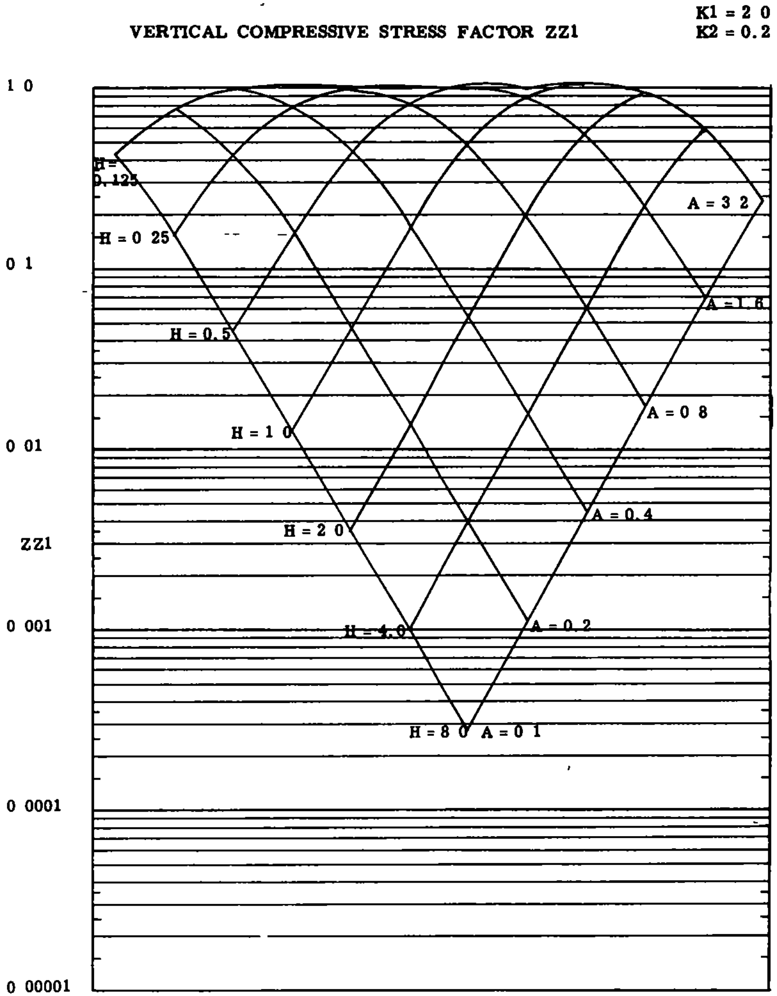


Figure 12.

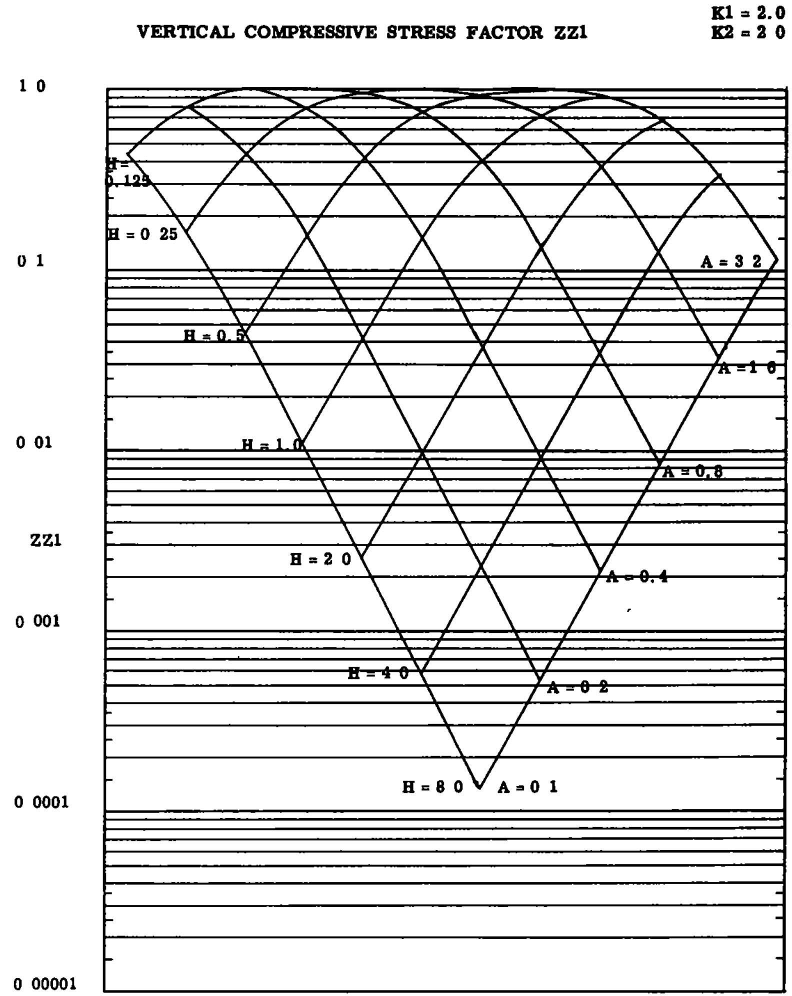


Figure 13.

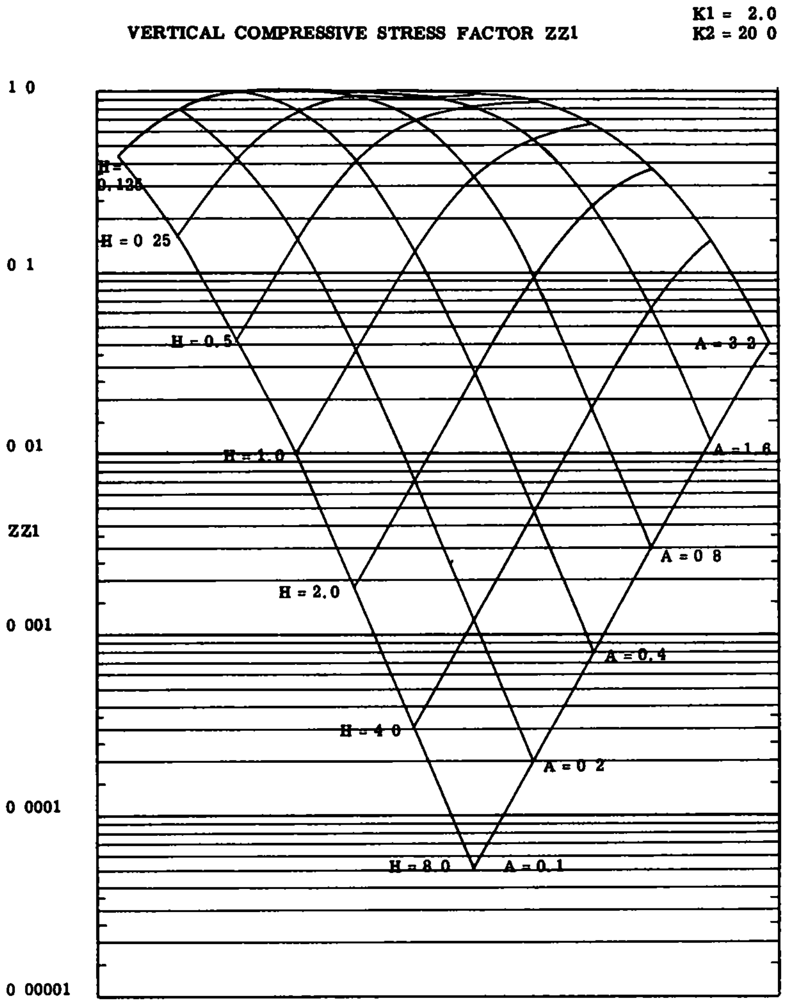


Figure 14.

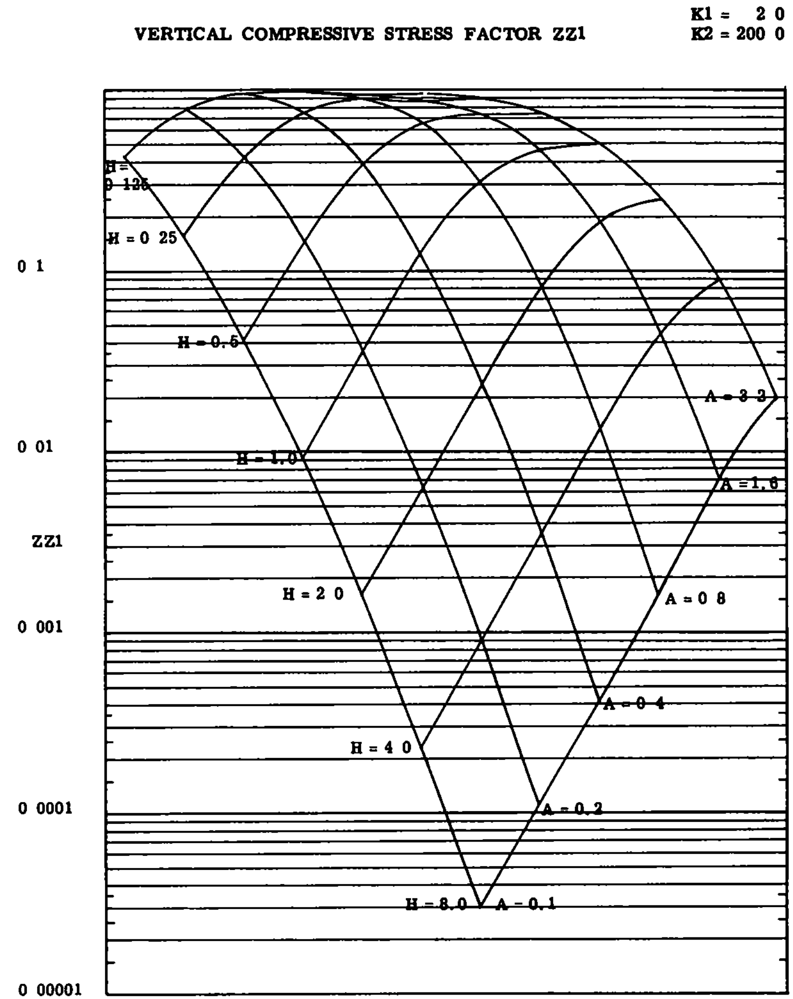


Figure 15.

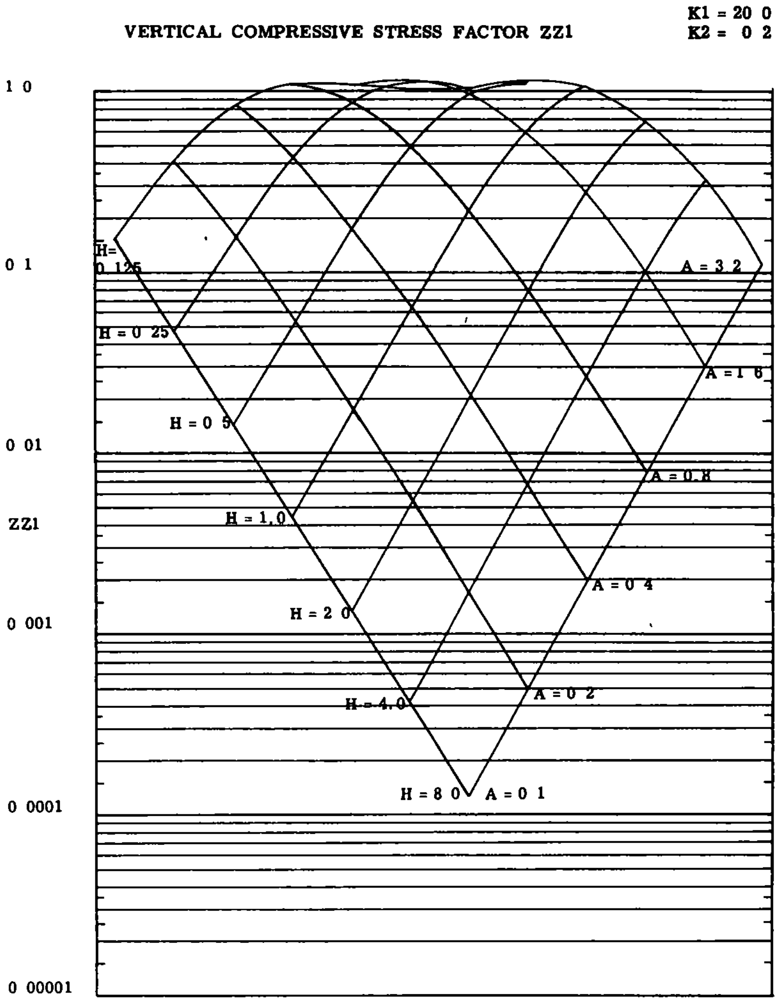


Figure 16.

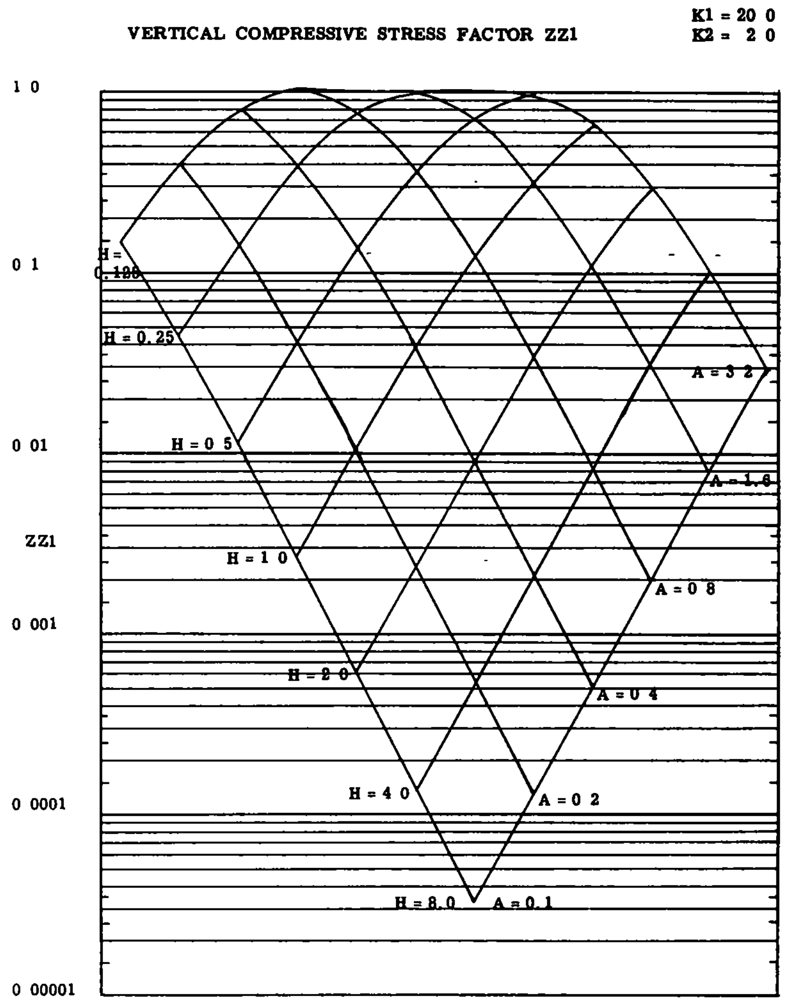


Figure 17.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ1

K1 = 200.0
K2 = 20 0

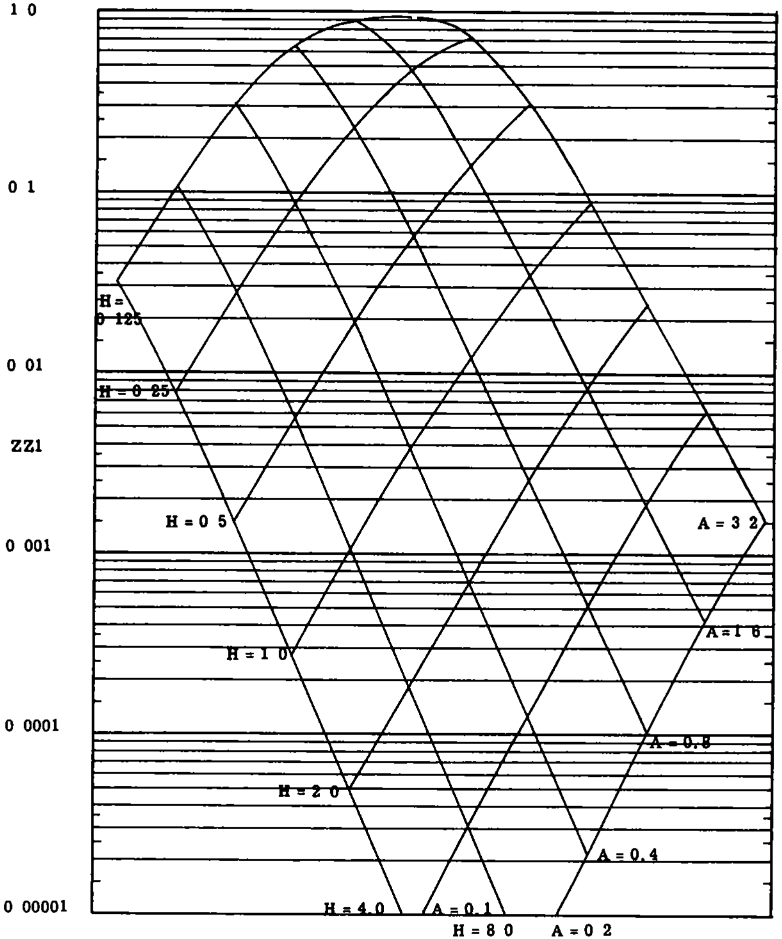


Figure 22.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ1

K1 = 200.0
K2 = 200.0

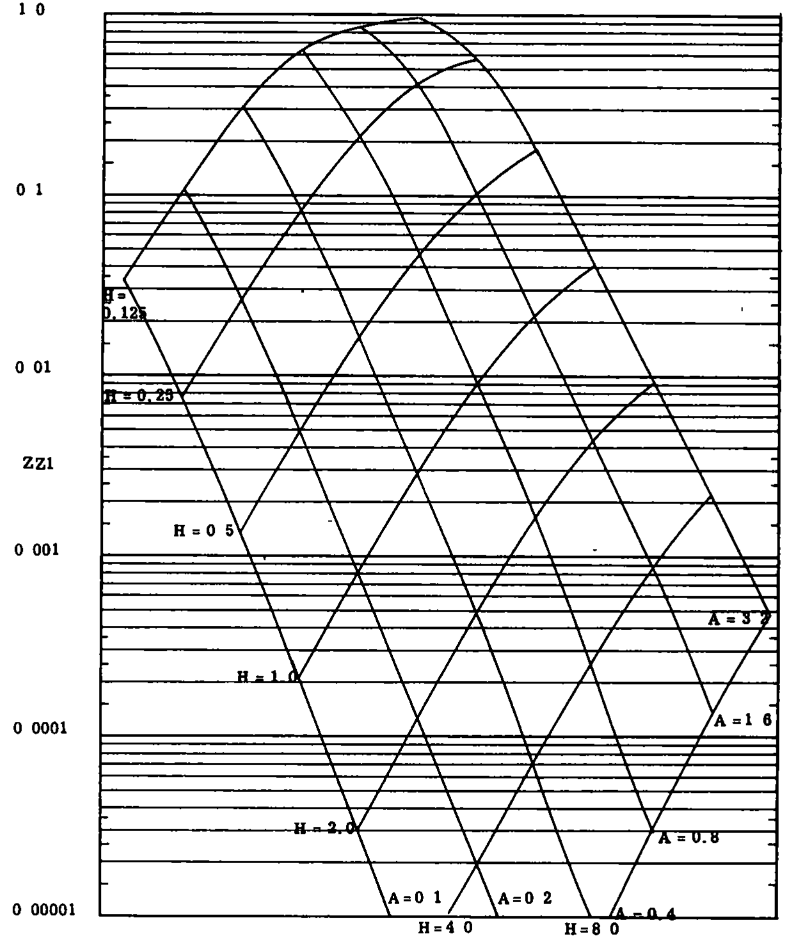


Figure 23.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 0.2
K2 = 0 2

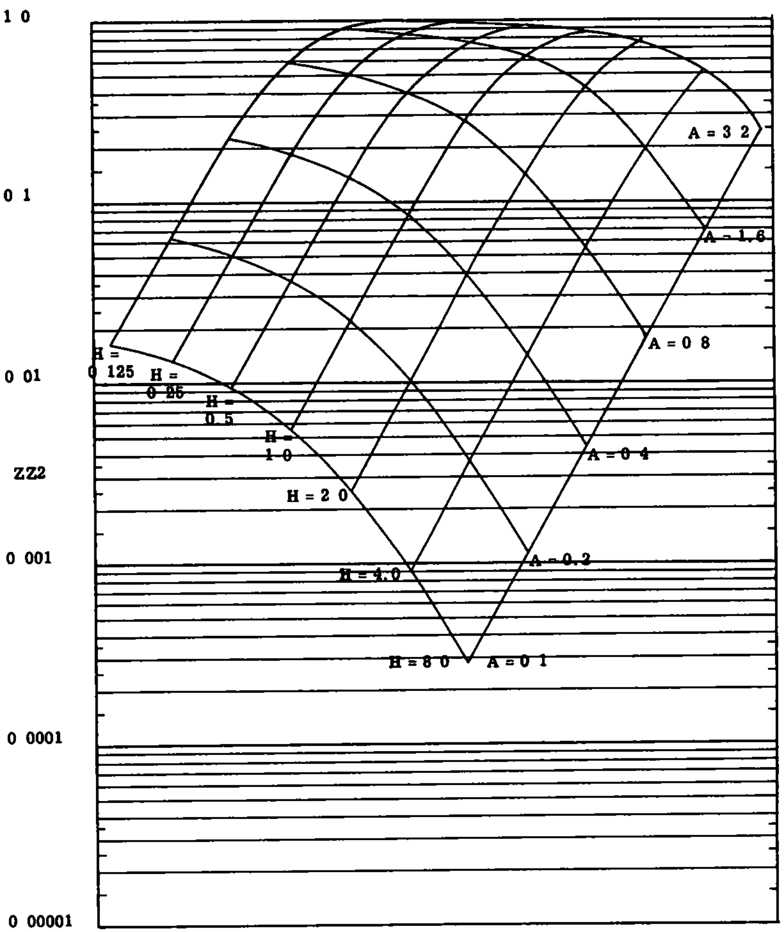


Figure 24.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 0 2
K2 = 2 0

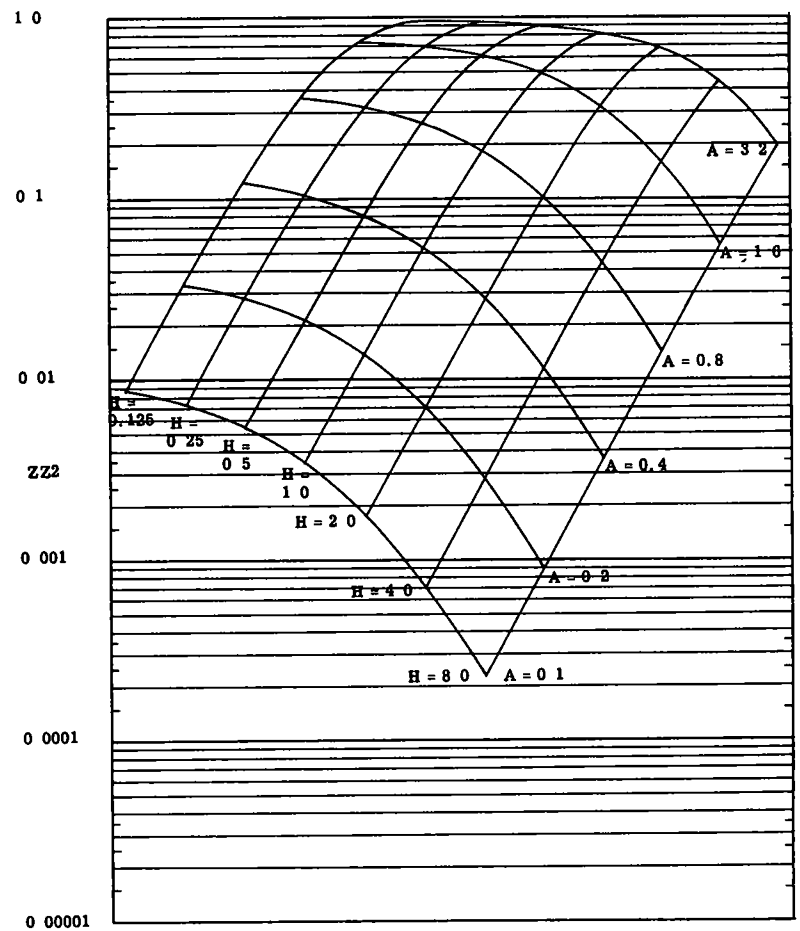


Figure 25.

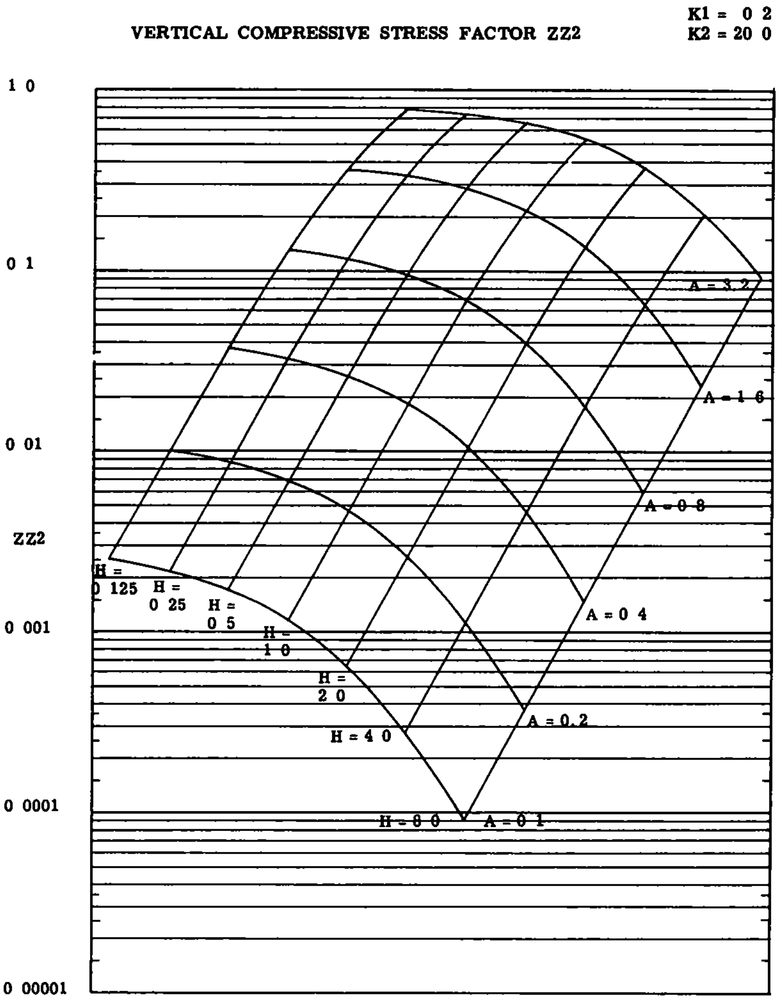


Figure 26.

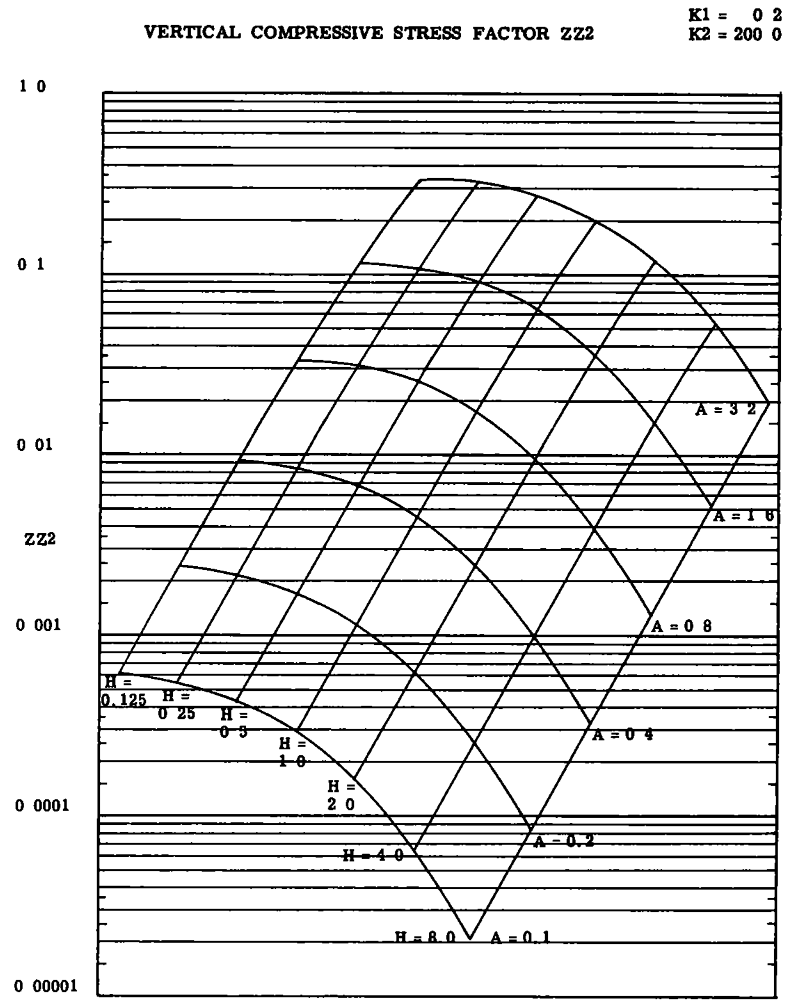


Figure 27.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 2.0
K2 = 0.2

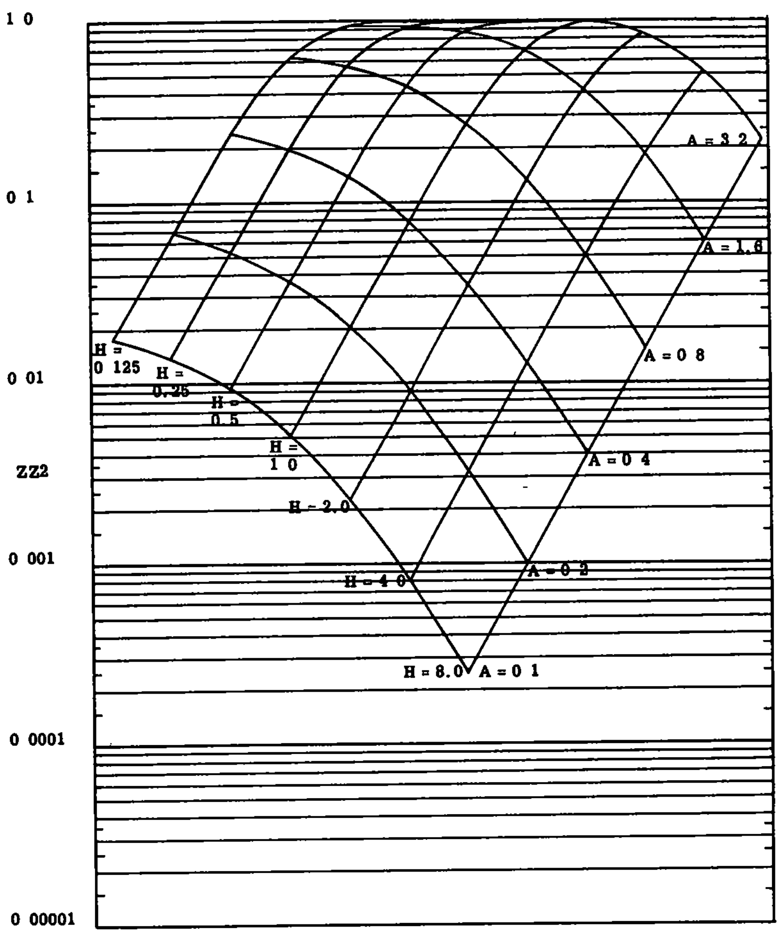


Figure 28.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 2.0
K2 = 2.0

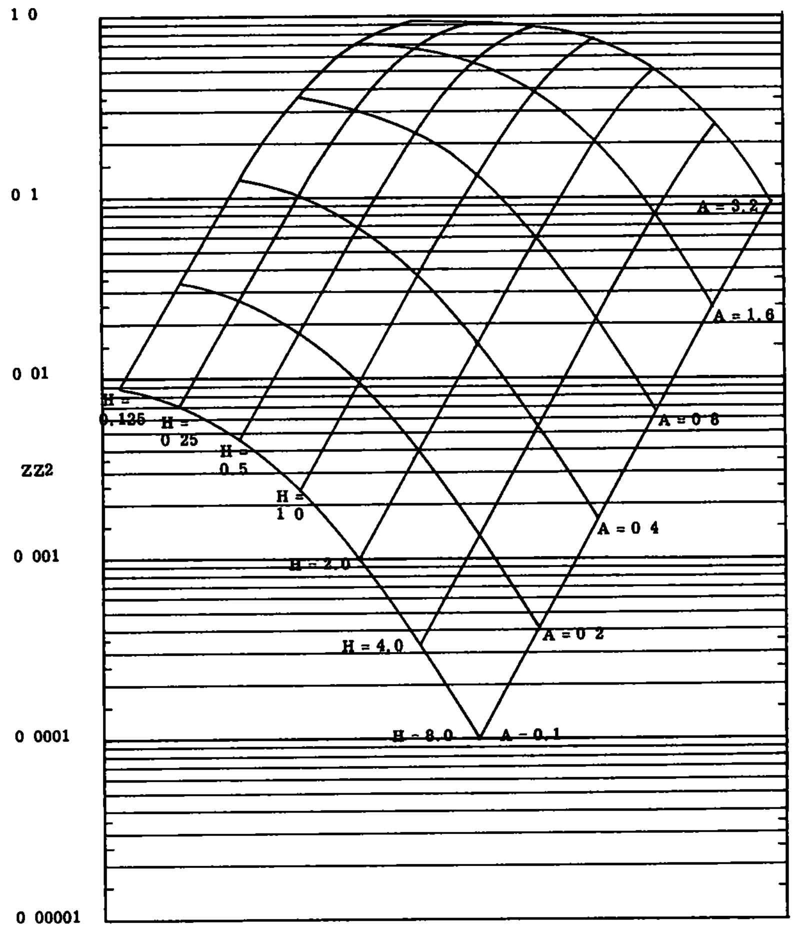


Figure 29.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 2 0
K2 = 20.0

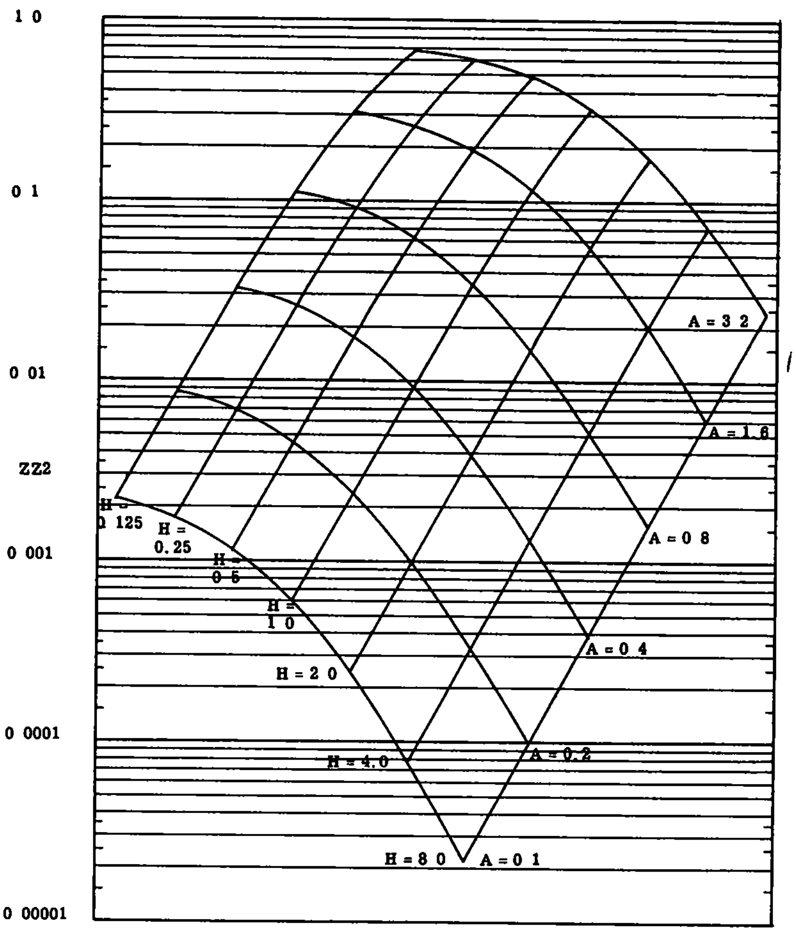


Figure 30.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 2.0
K2 = 200.0

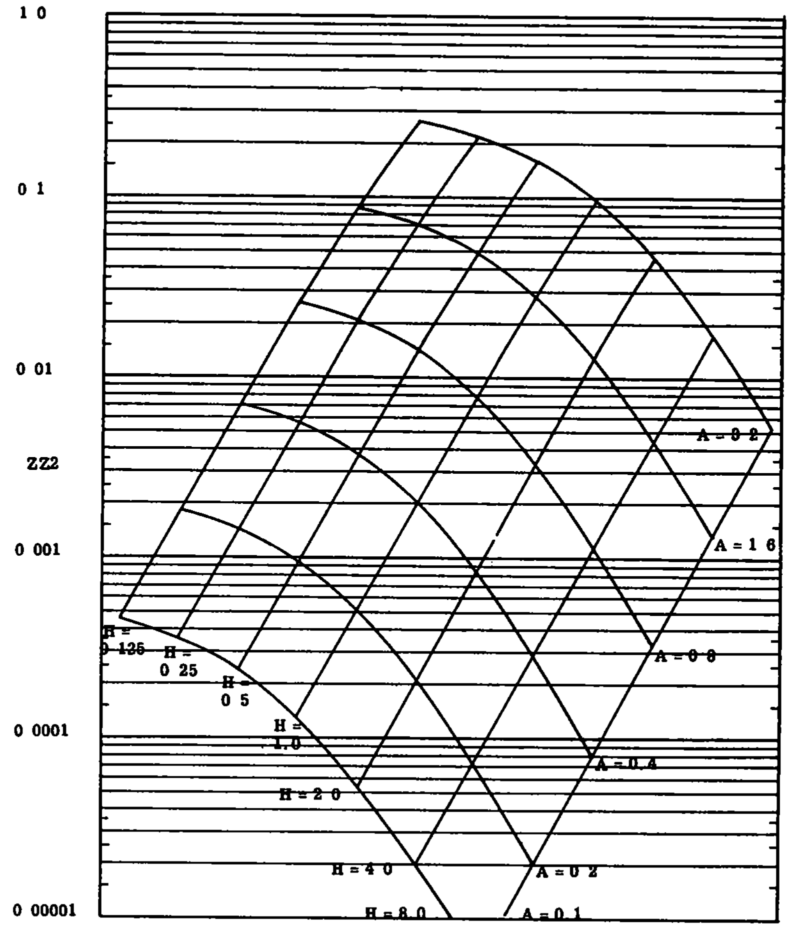


Figure 31.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 20.0
K2 = 0 2

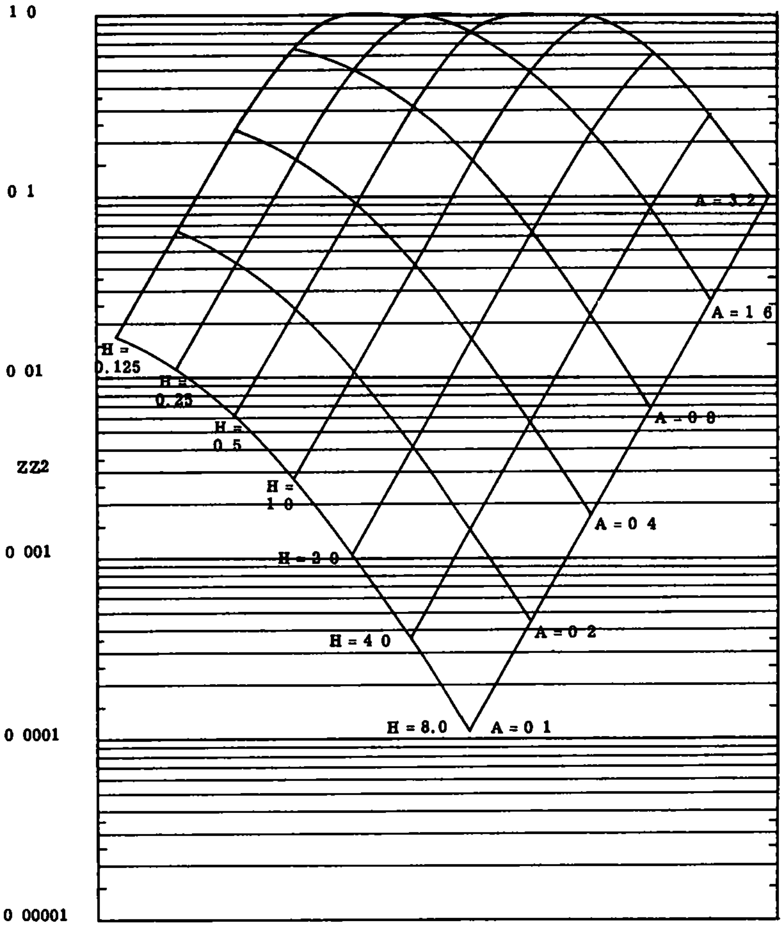


Figure 32.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 20.0
K2 = 2.0

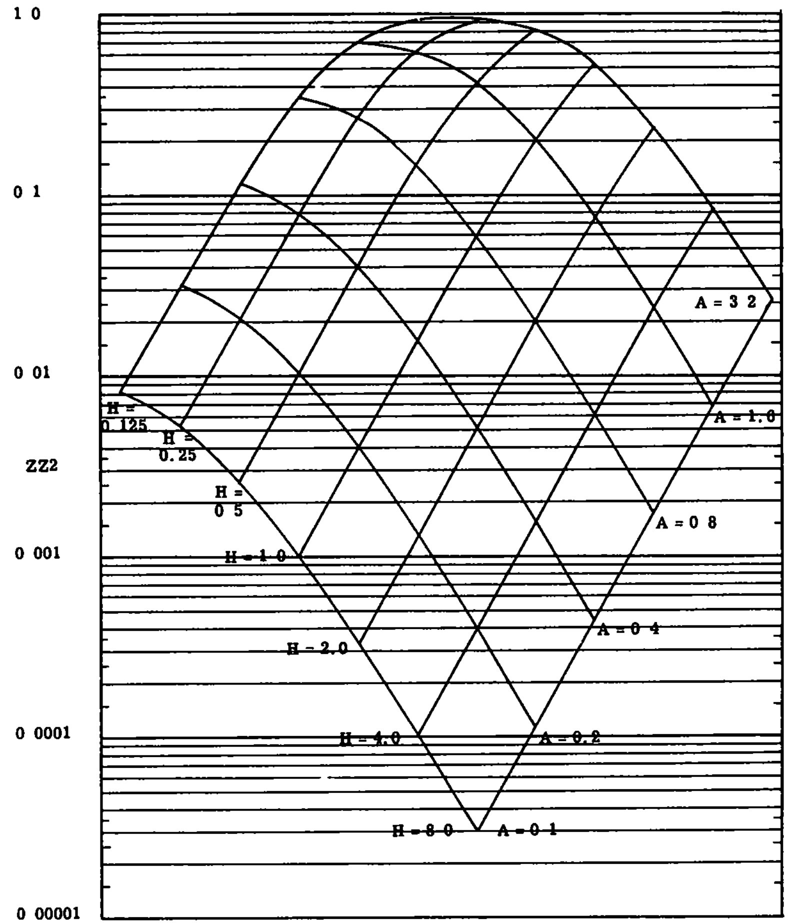


Figure 33.

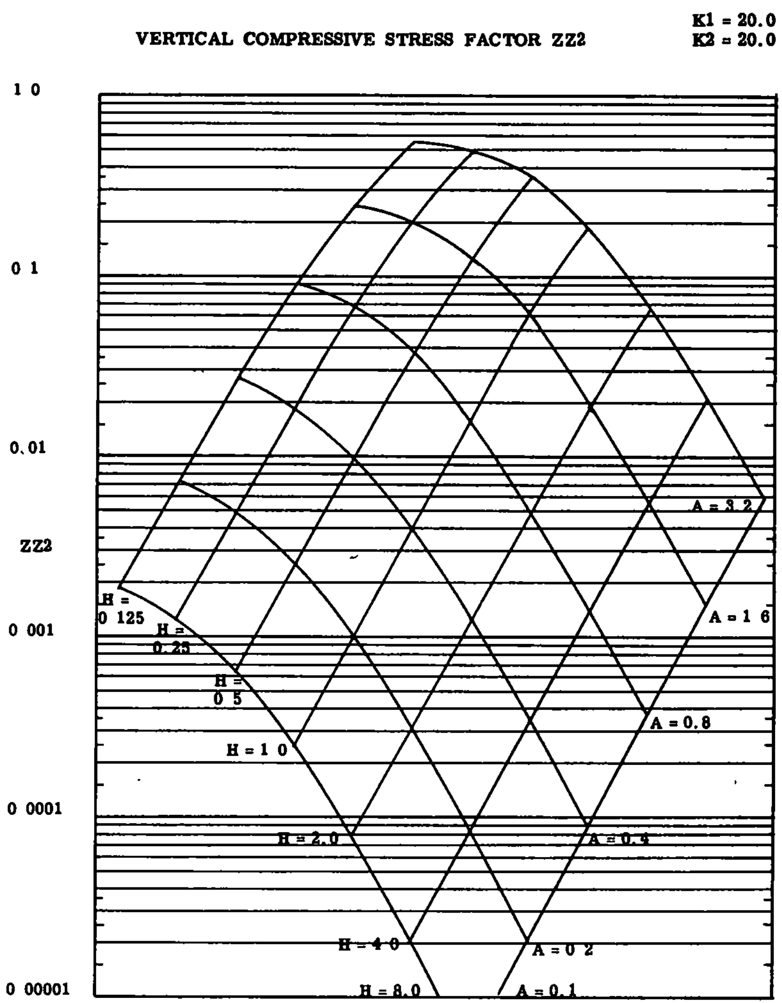


Figure 34.

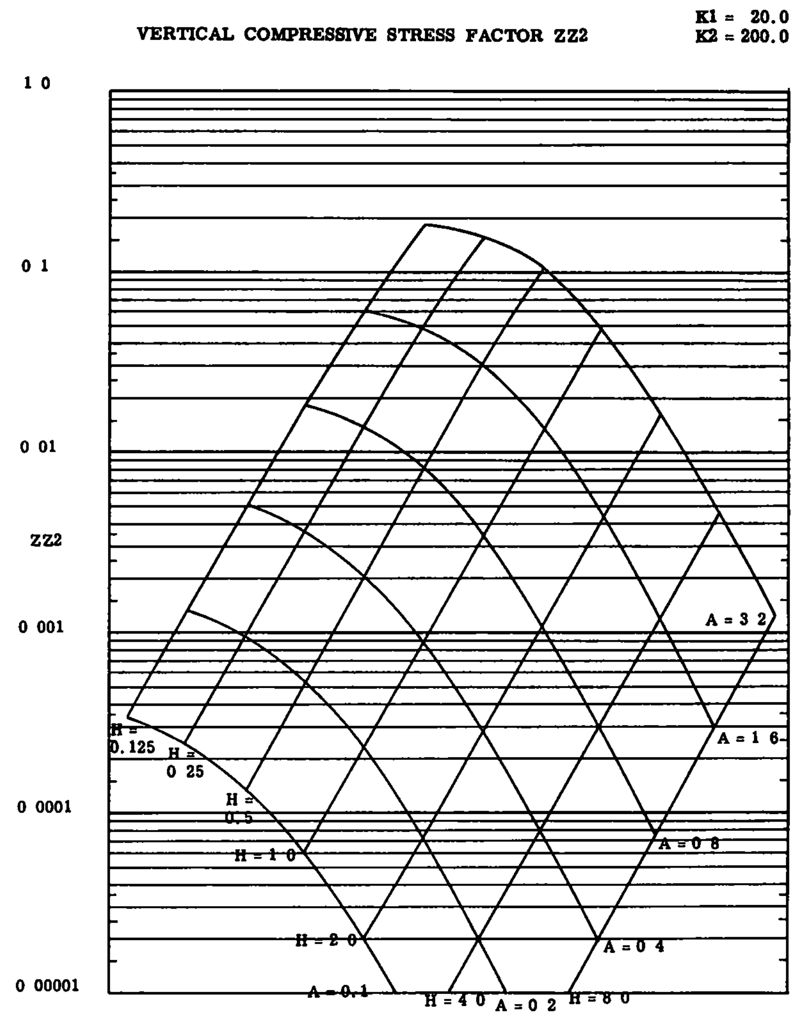


Figure 35.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 200.0
K2 = 0.2

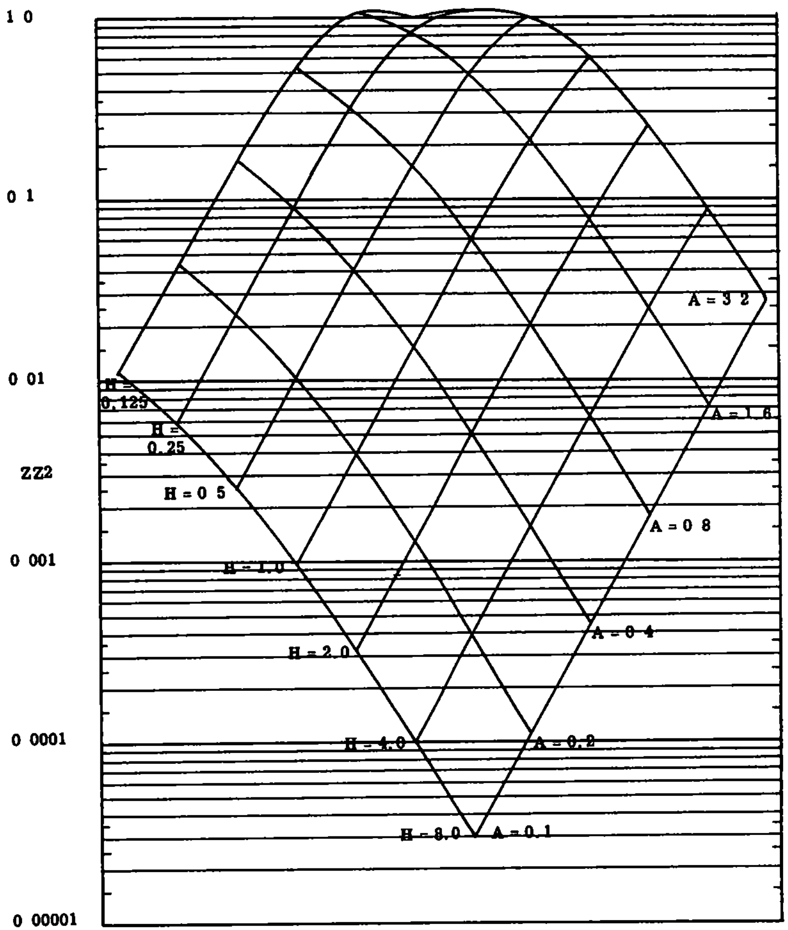


Figure 36.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 200.0
K2 = 2 0

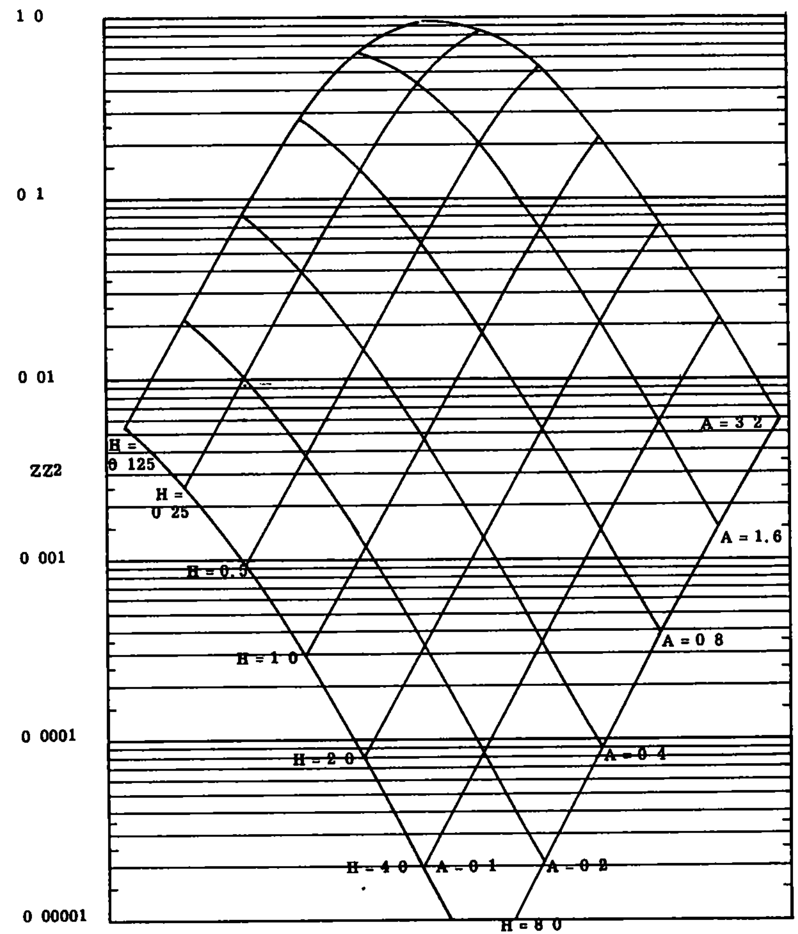


Figure 37.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 200.0
K2 = 20.0

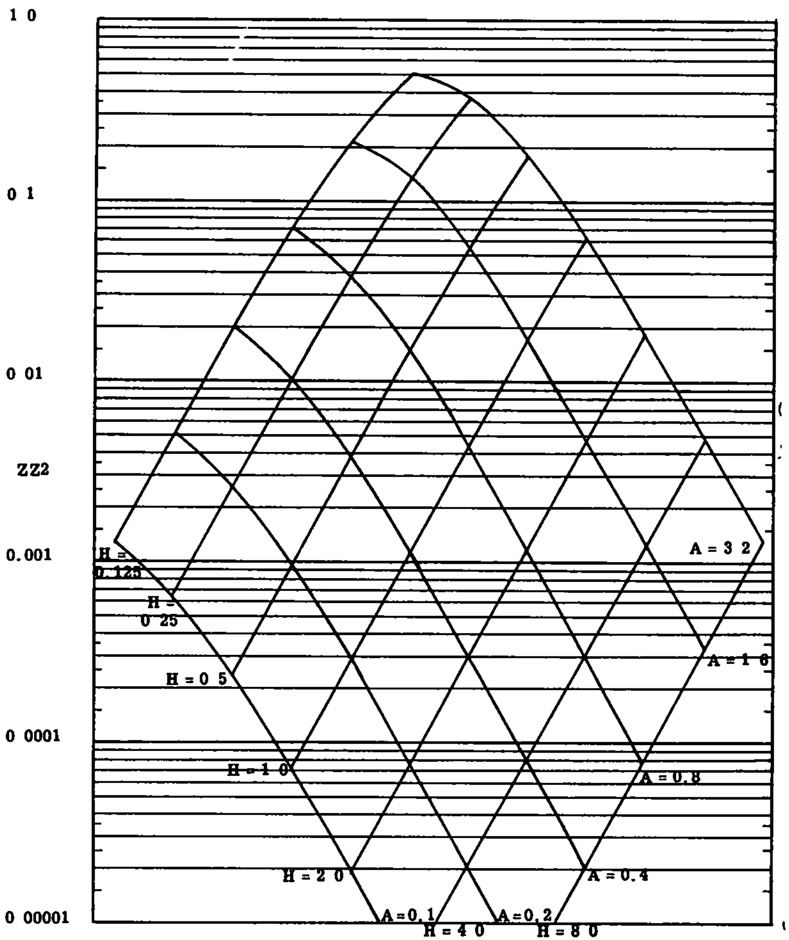


Figure 38.

VERTICAL COMPRESSIVE STRESS FACTOR ZZ2

K1 = 200.0
K2 = 200.0

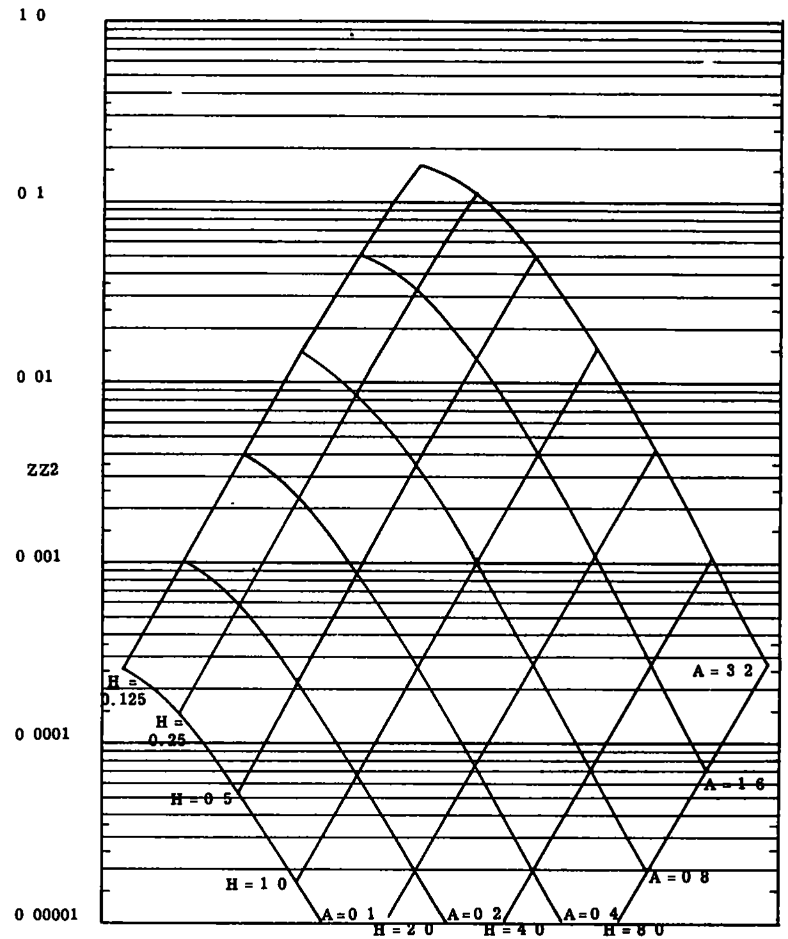


Figure 39.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 2 0
K2 = 0 2

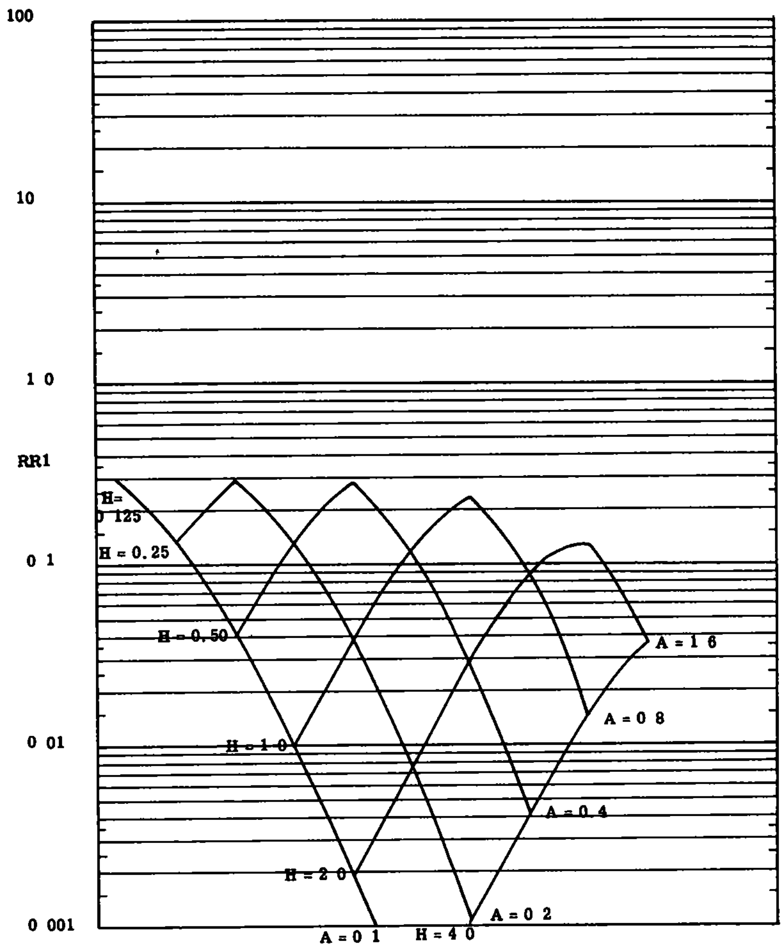


Figure 40.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 2 0
K2 = 2 0

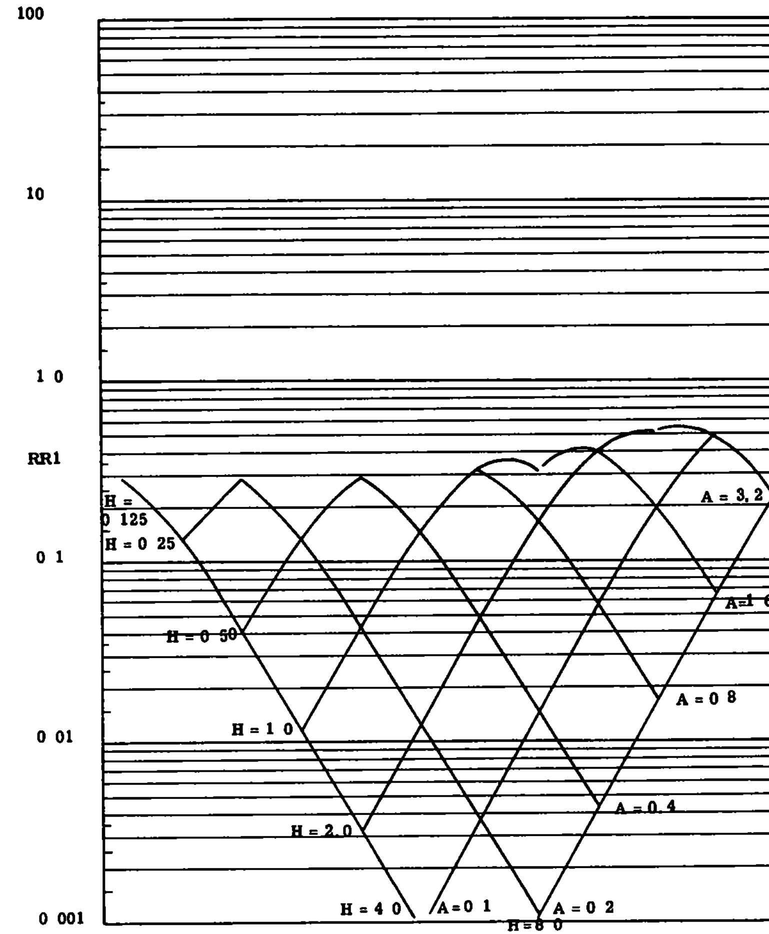


Figure 41.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 2.0
K2 = 20 0

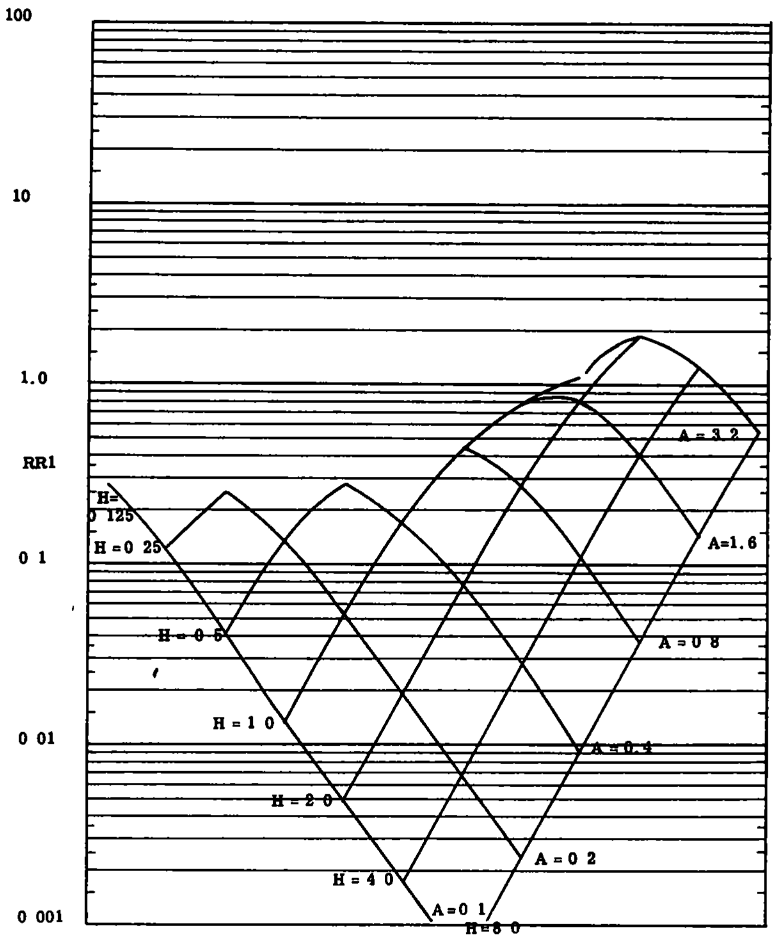


Figure 42.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 2.0
K2 = 200 0

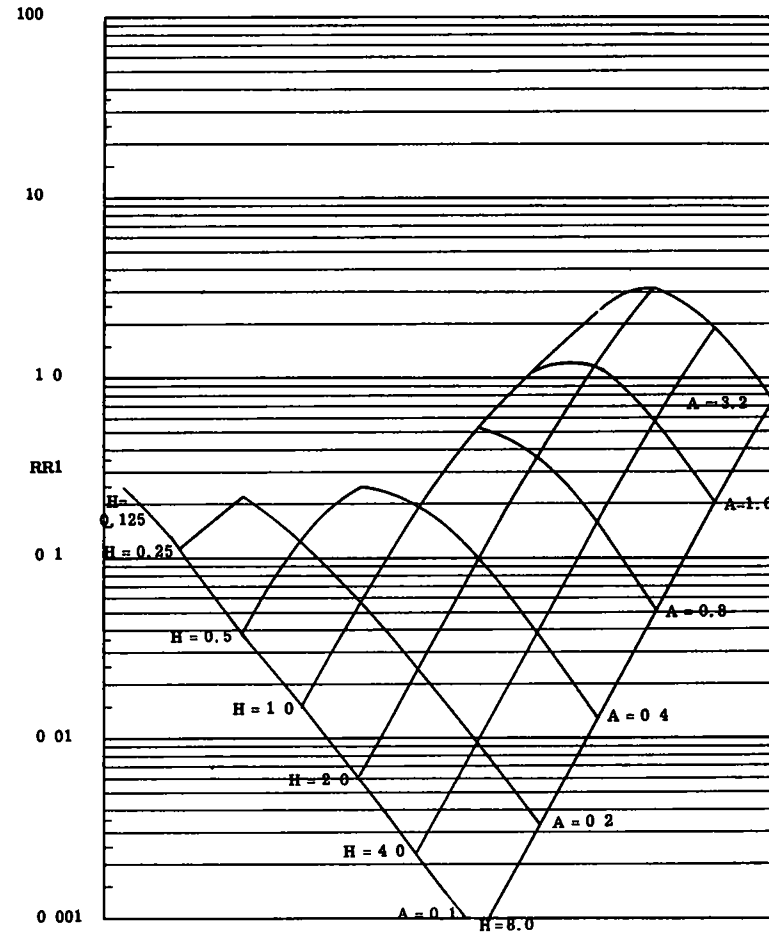


Figure 43.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 20 0
K2 = 0 2

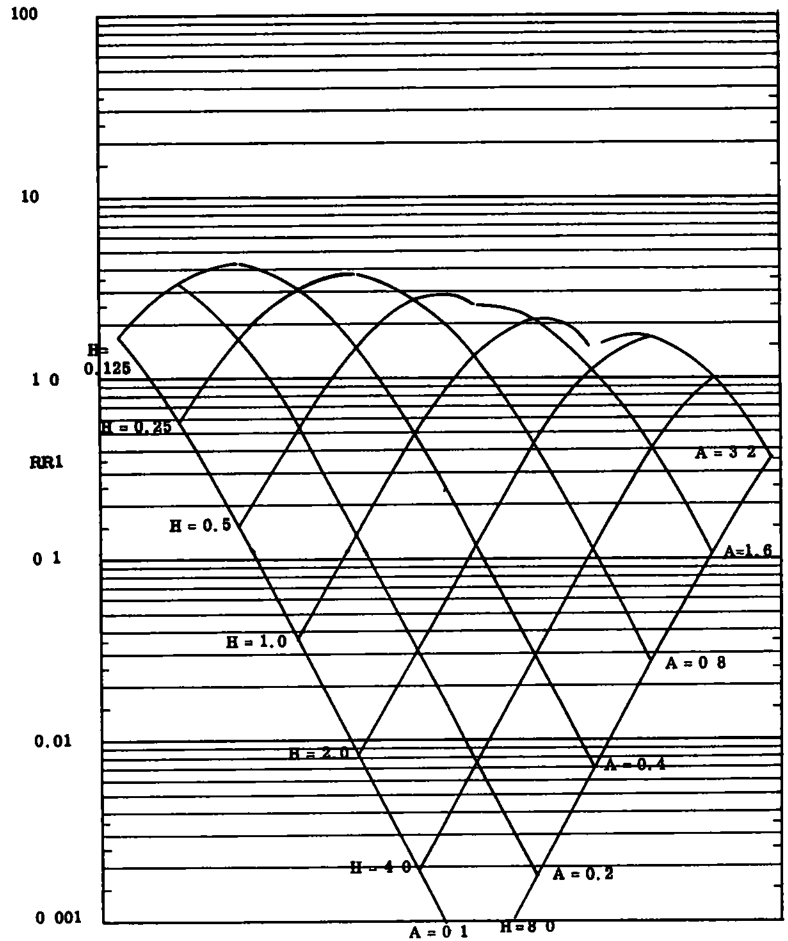


Figure 44.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 20 0
K2 = 2 0

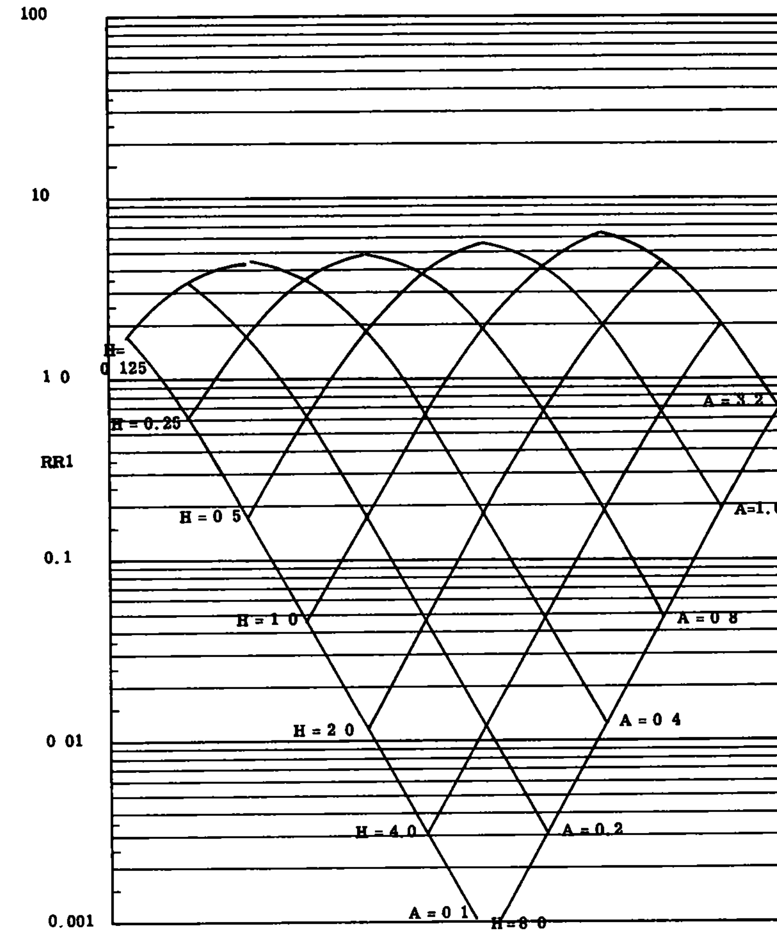


Figure 45.

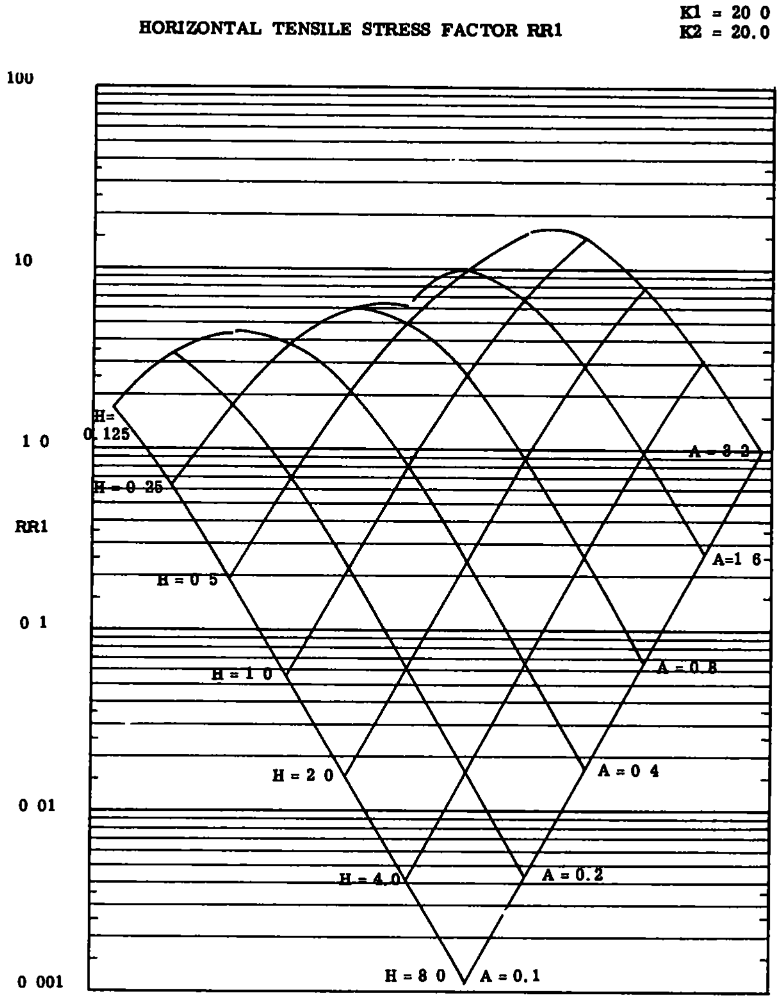


Figure 46.

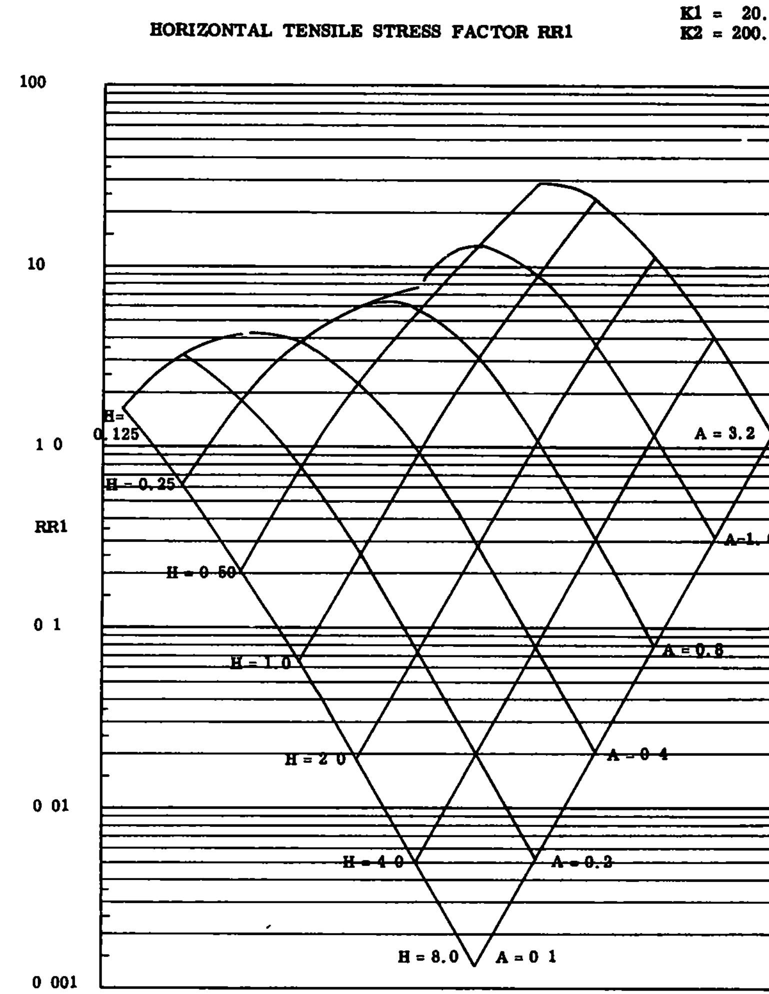


Figure 47.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 200 0
K2 = 0 2

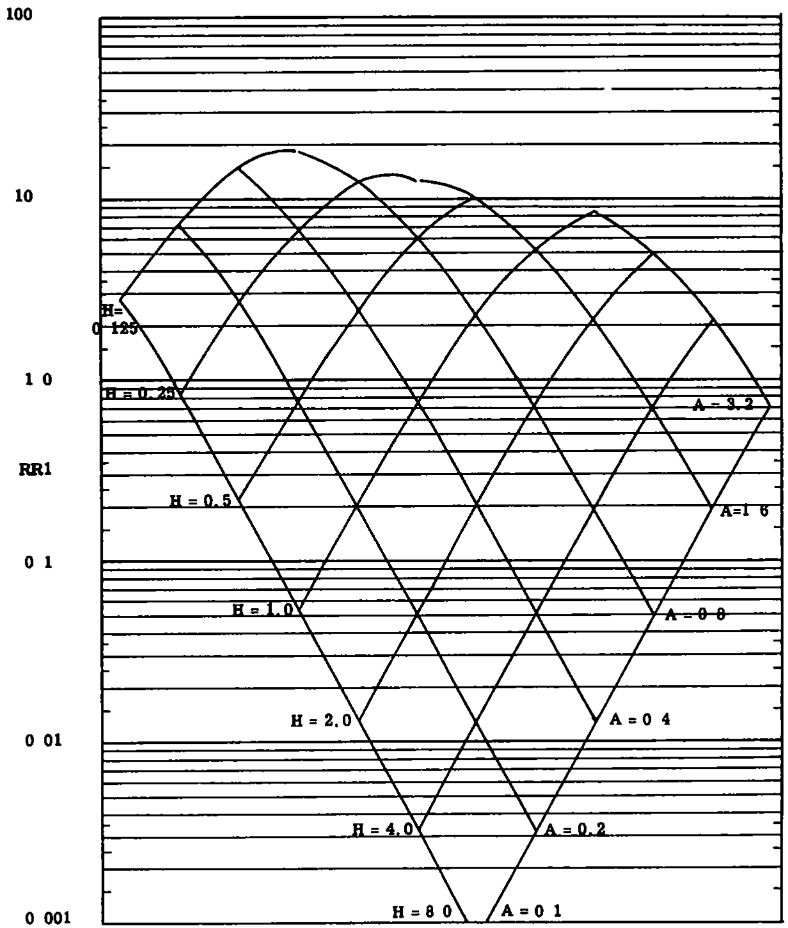


Figure 48.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 200 0
K2 = 2 0

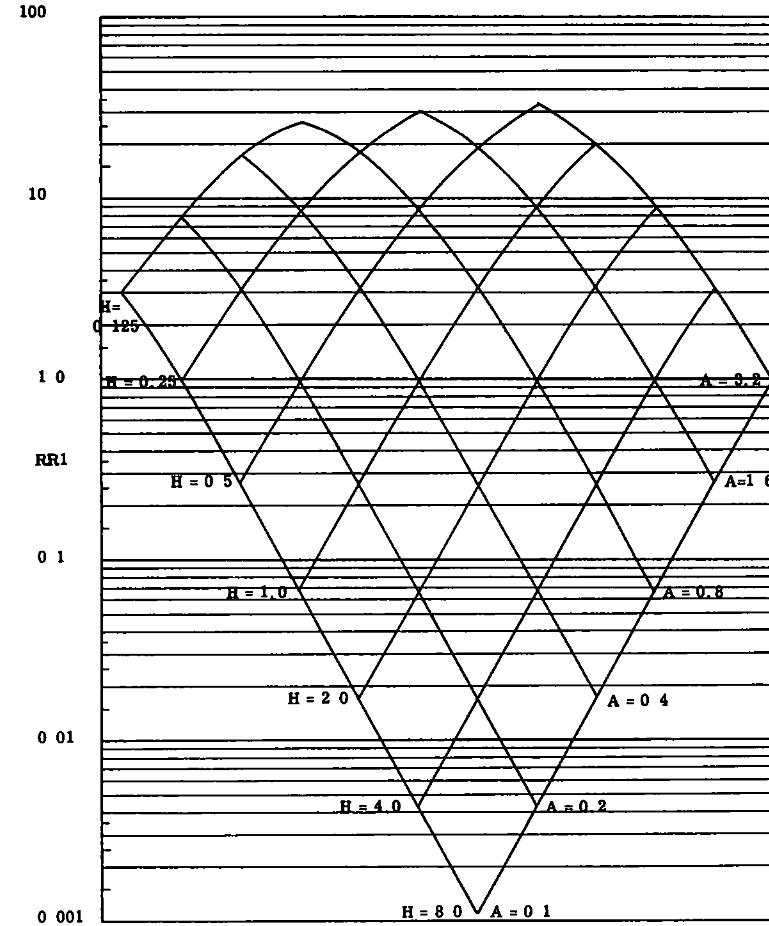


Figure 49.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 200 0
K2 = 20.0

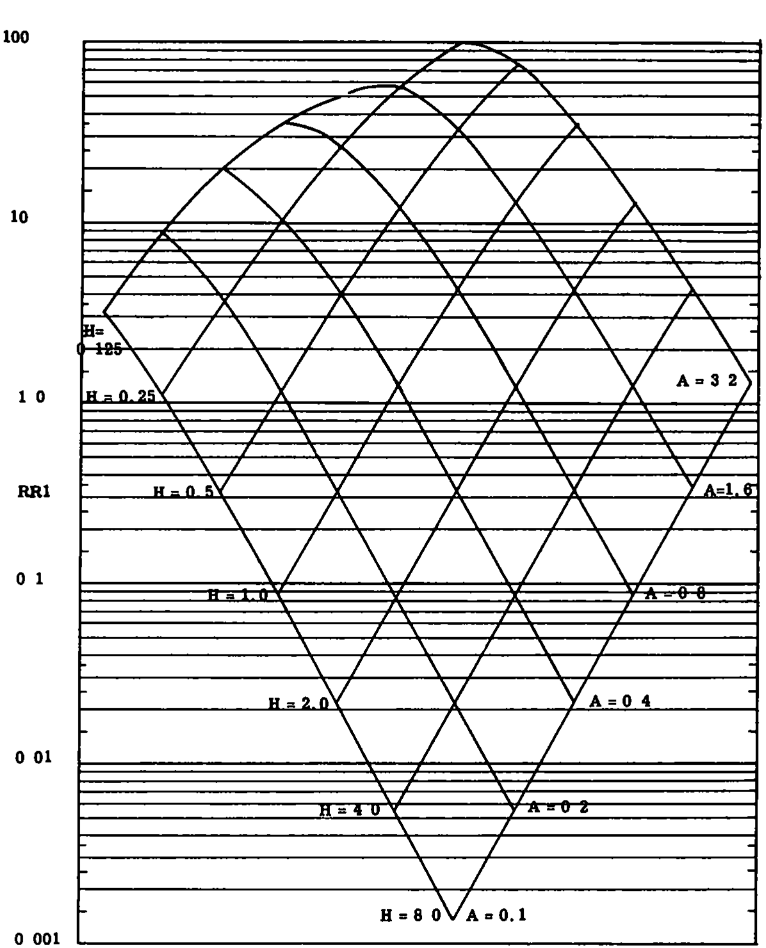


Figure 50.

HORIZONTAL TENSILE STRESS FACTOR RR1

K1 = 200.0
K2 = 200 0

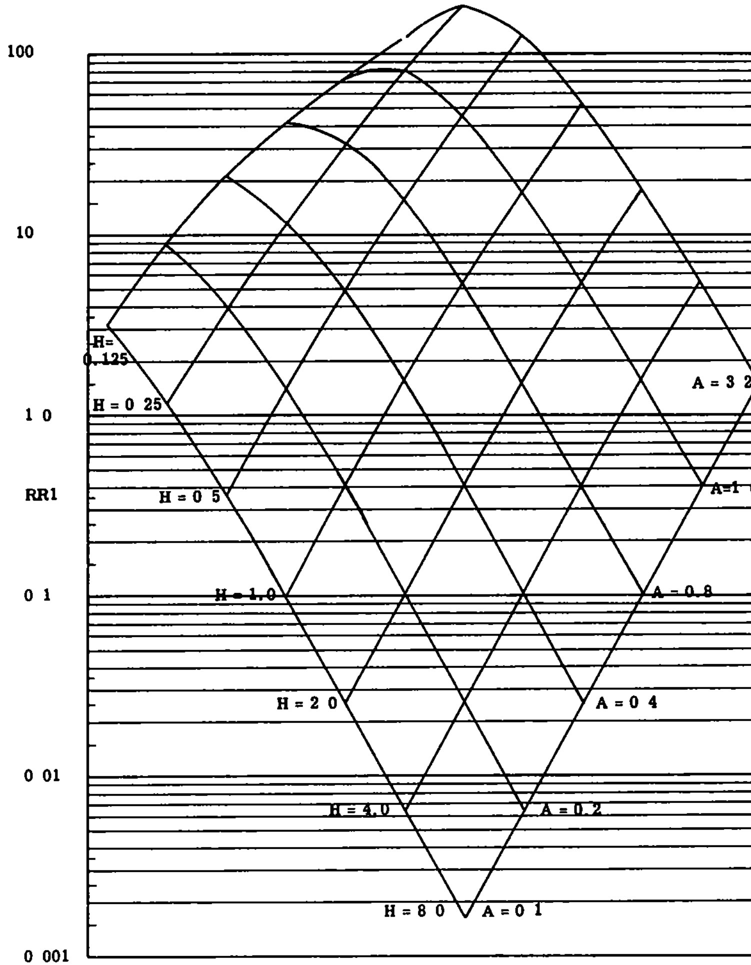


Figure 51.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 0.2
K2 = 0 2

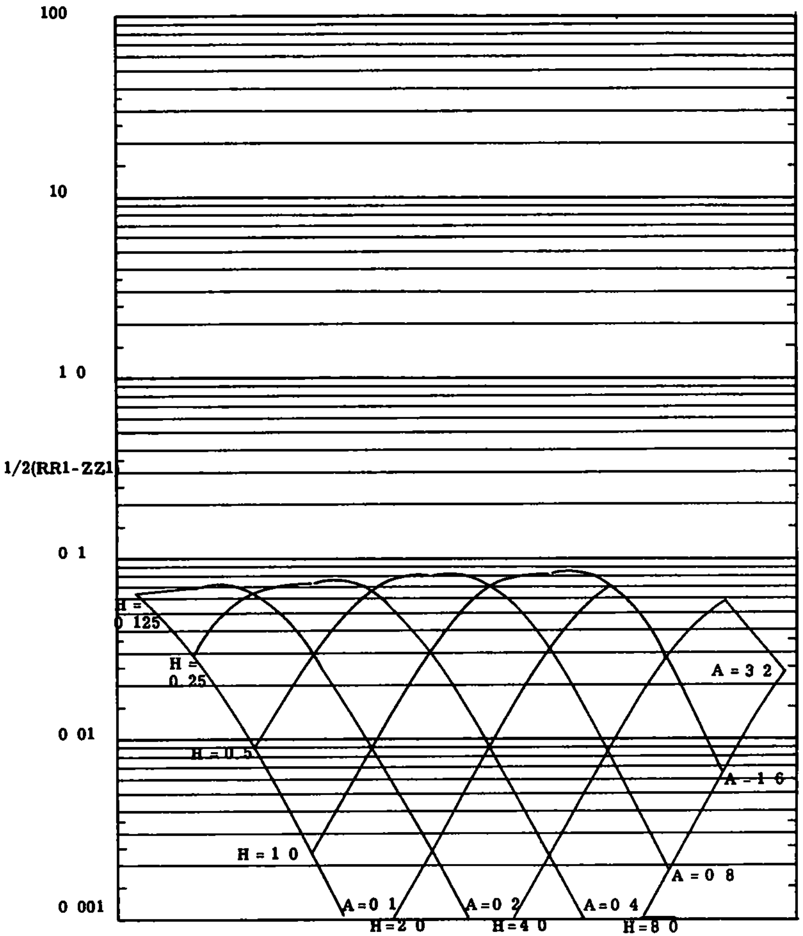


Figure 52.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 0.2
K2 = 2 0

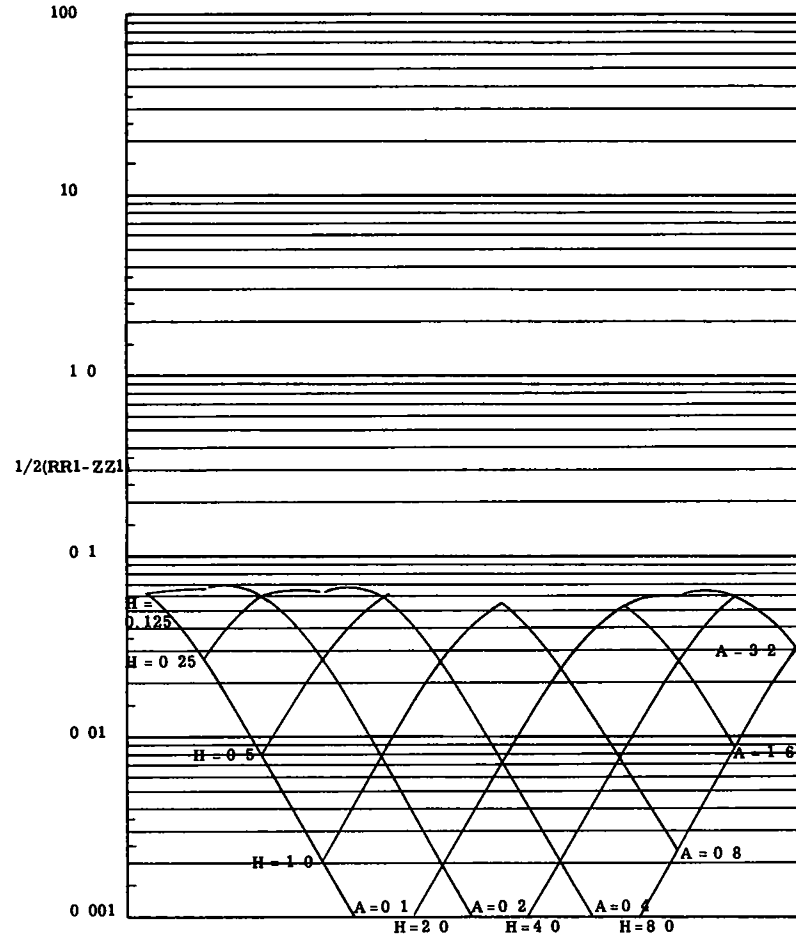


Figure 53.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 0.2
K2 = 20.0

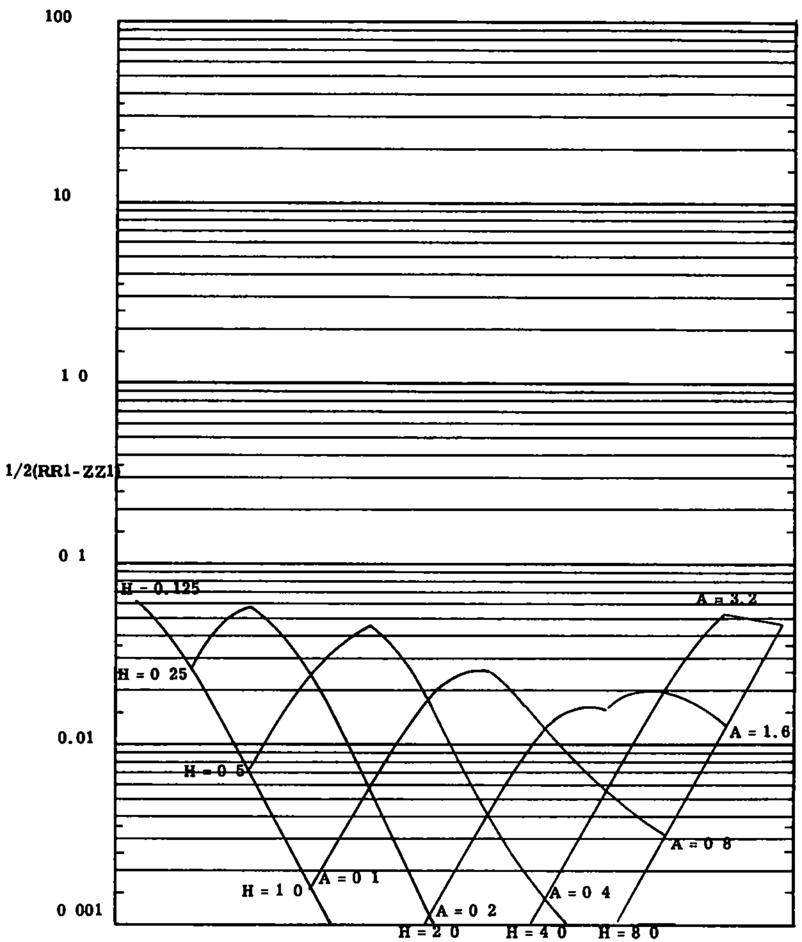


Figure 54.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 0 2
K2 = 200.0

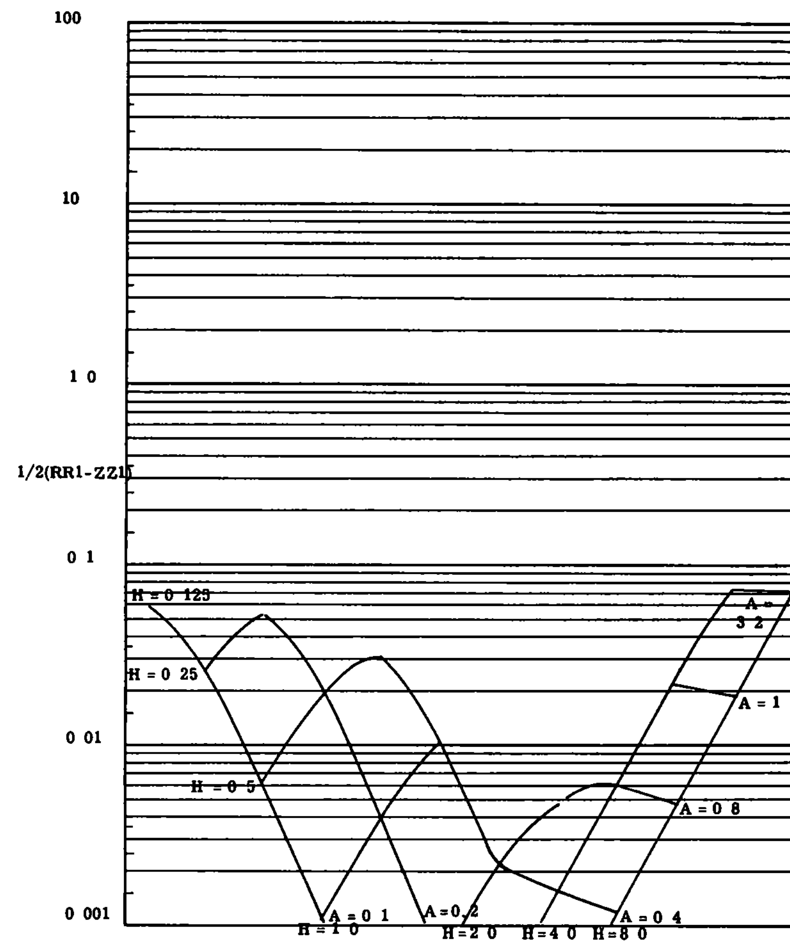


Figure 55.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 2 0
K2 = 0 2

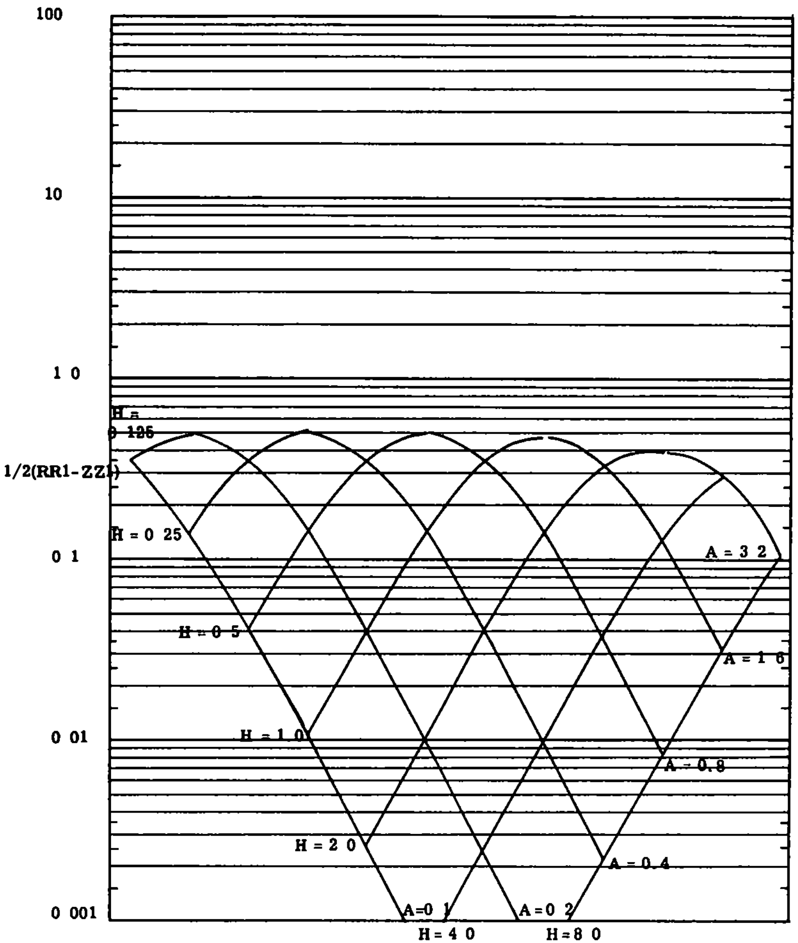


Figure 56.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 2.0
K2 = 2.0

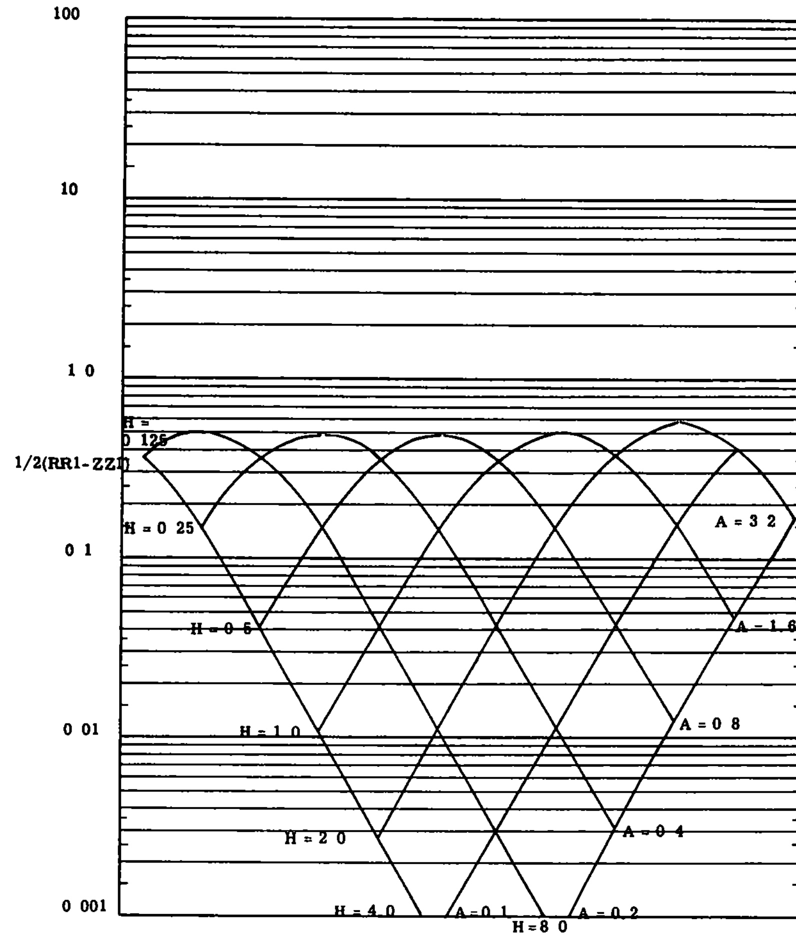


Figure 57.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 2.0
K2 = 20.0

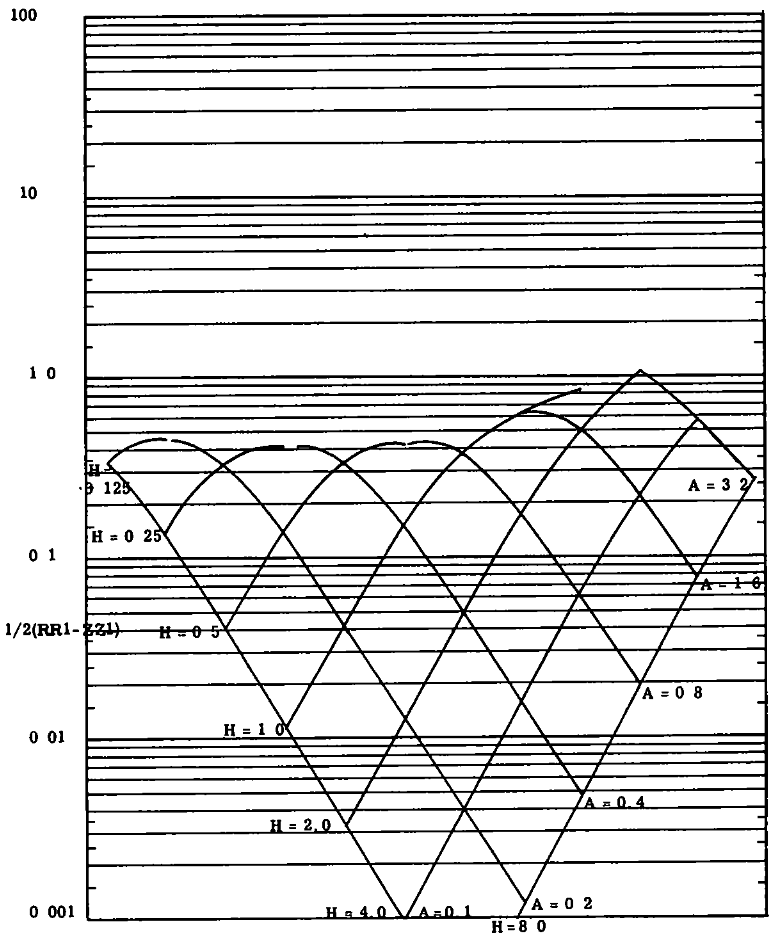


Figure 58.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 2.0
K2 = 200 0

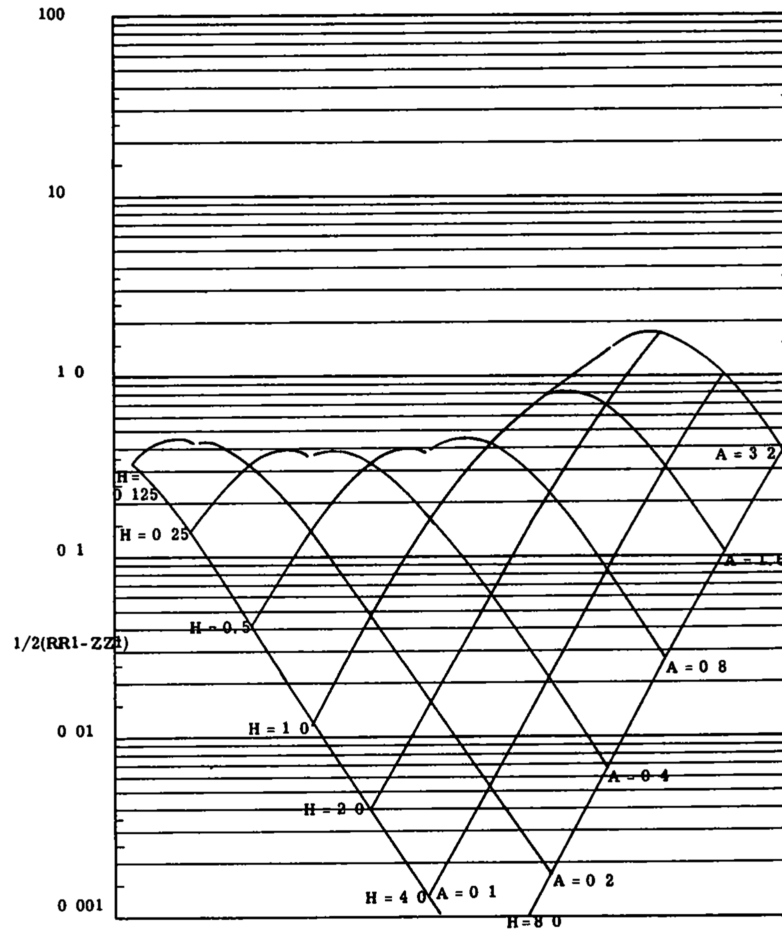


Figure 59.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 20.0
K2 = 0.2

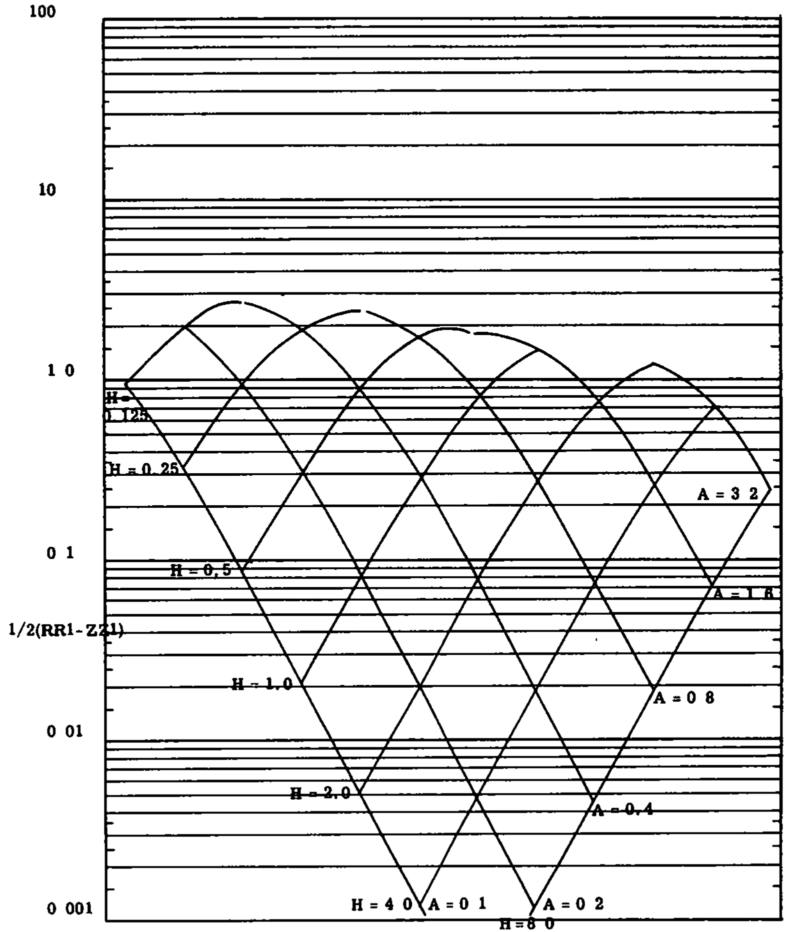


Figure 60.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 20.0
K2 = 2.0

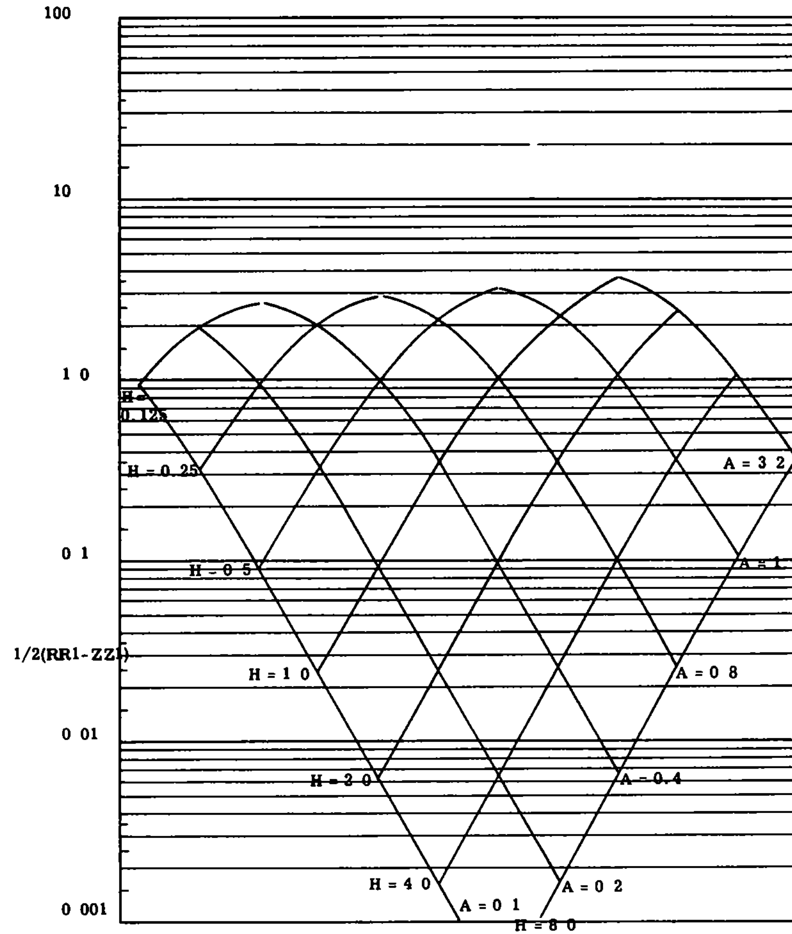


Figure 61.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 20 0
K2 = 20 0

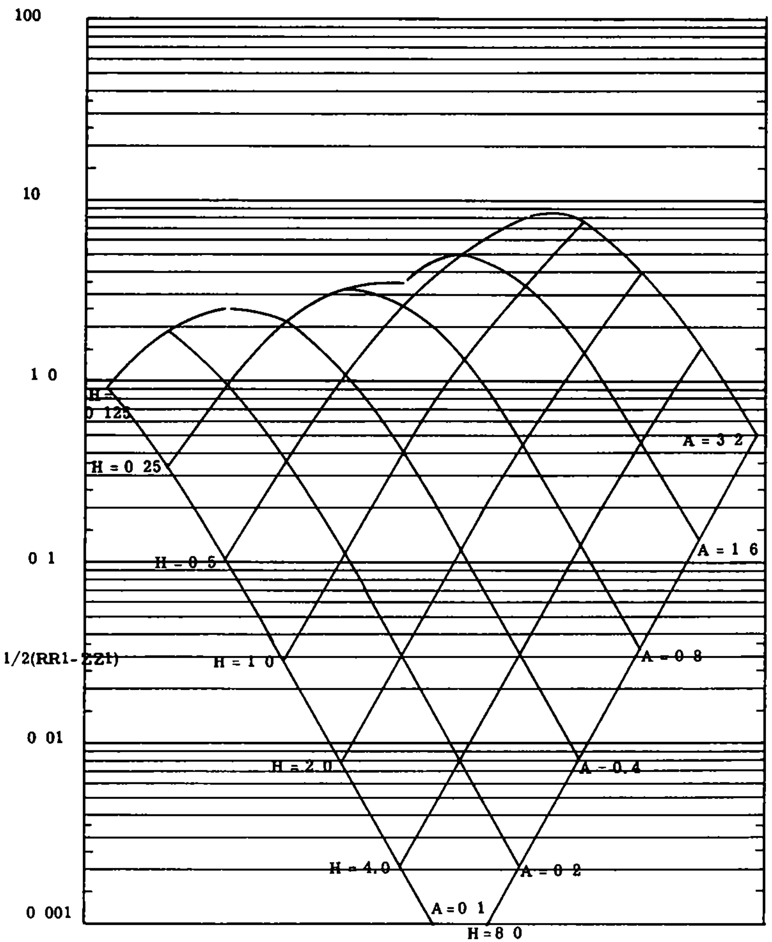


Figure 62.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 20 0
K2 = 200 0

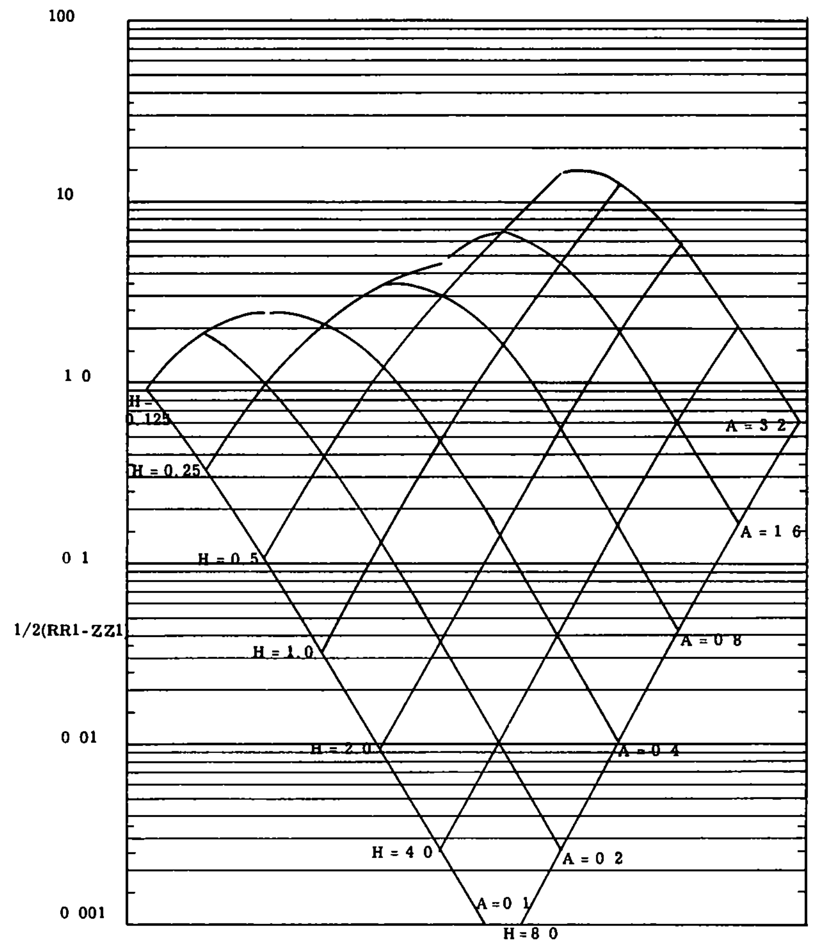


Figure 63.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 200.0
K2 = 0 2

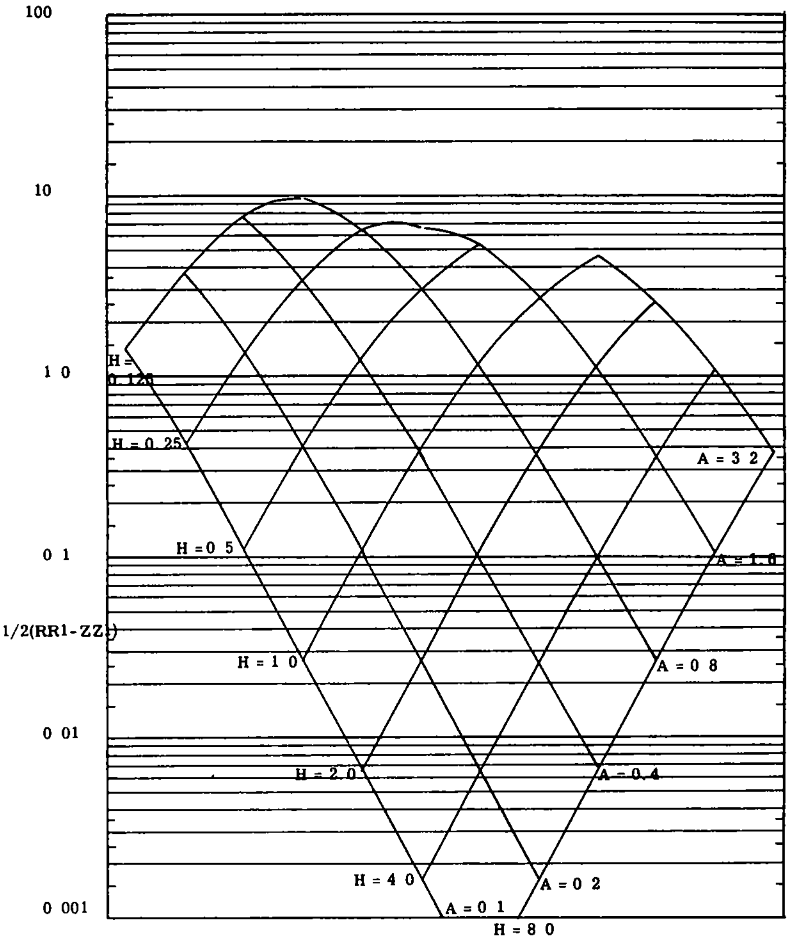


Figure 64.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1)

K1 = 200.0
K2 = 2.0

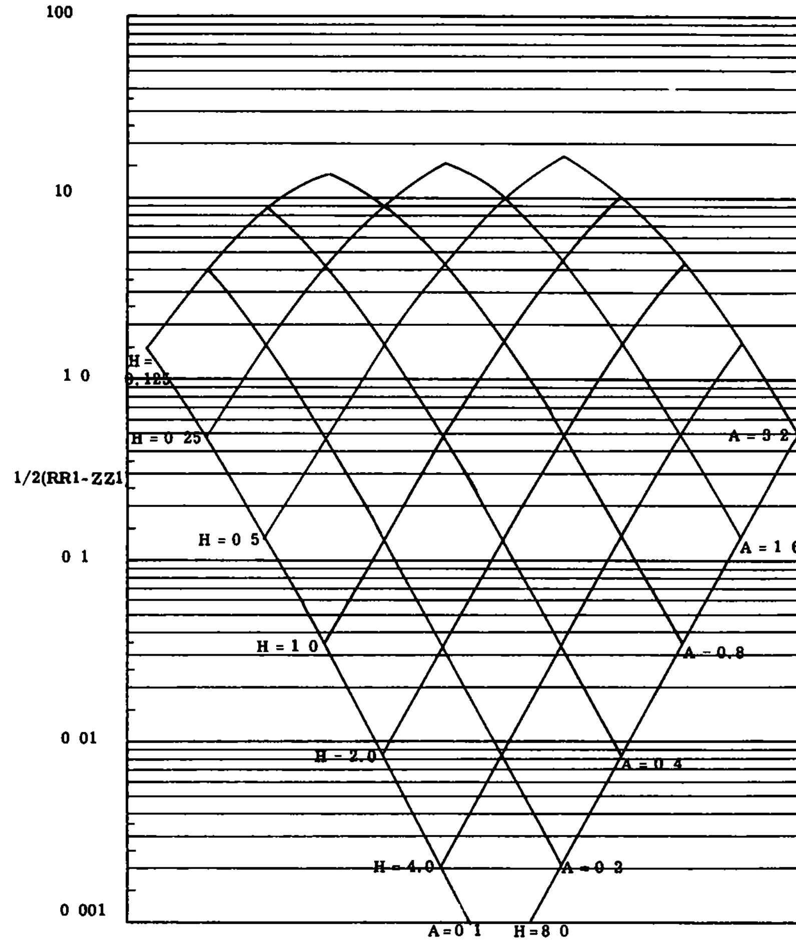


Figure 65.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1) K1 = 200.0 K2 = 20.0

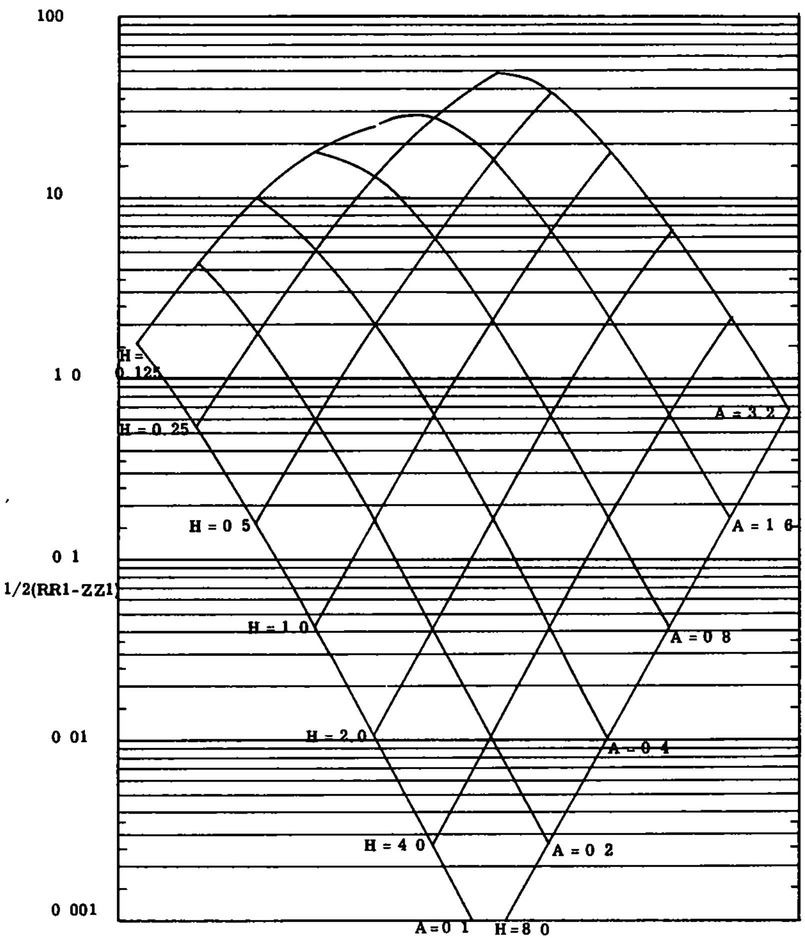


Figure 66.

HORIZONTAL TENSILE STRAIN FACTOR 1/2(RR1 - ZZ1) K1 = 200.0 K2 = 200.0

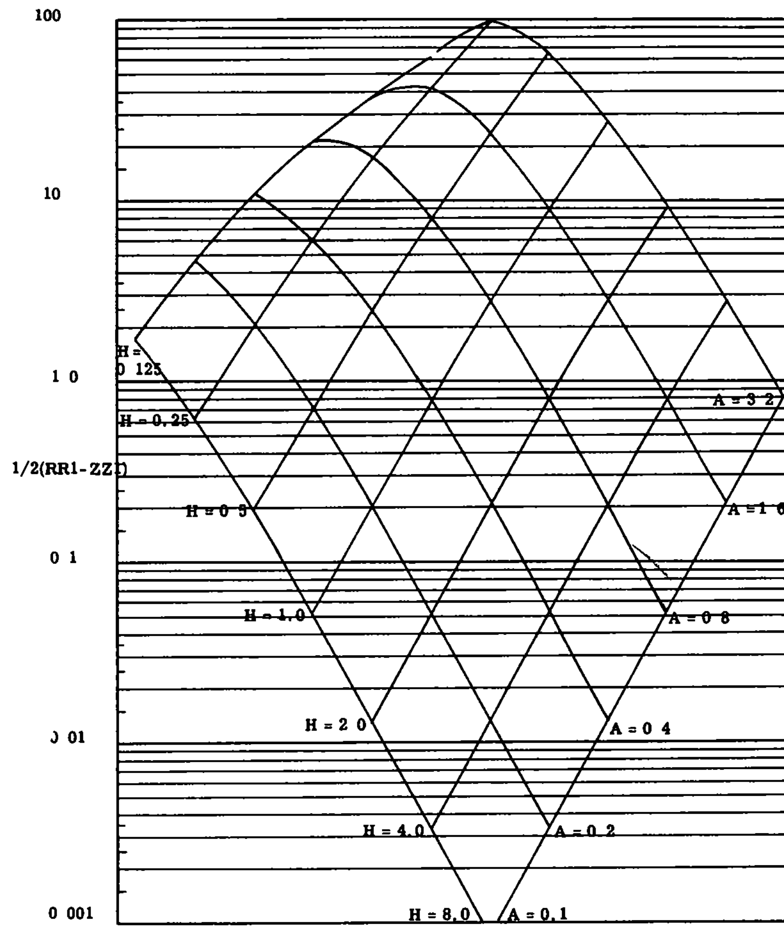


Figure 67.

Appendix B

PLOTTING OF THREE VARIABLES BY THE GRID METHOD

In the grid method of plotting (3), the dependent variable is plotted vertically in the conventional manner. The independent variables are plotted horizontally on a composite scale.

The dependent variable is plotted against one independent variable for one value of the other independent variable as shown by the solid line in Figure 68. The whole scale is then displaced horizontally and the second (dotted) line is plotted. A third displacement of the horizontal scale for A enables the (chain dotted) line for the next value of H to be plotted. Once all the plots of the stress factor against A have been drawn, the points of equal A-values are connected together to complete the grid.

The amount of displacement should be selected to give a good intersection of the grid lines; i.e., so that the lines intersect nearly at right angles. In the present series of graphs it has been found convenient to place the "0. 1" point of each A-scale at a distance from the left-hand axis proportional to the logarithm of the relevant value of H.

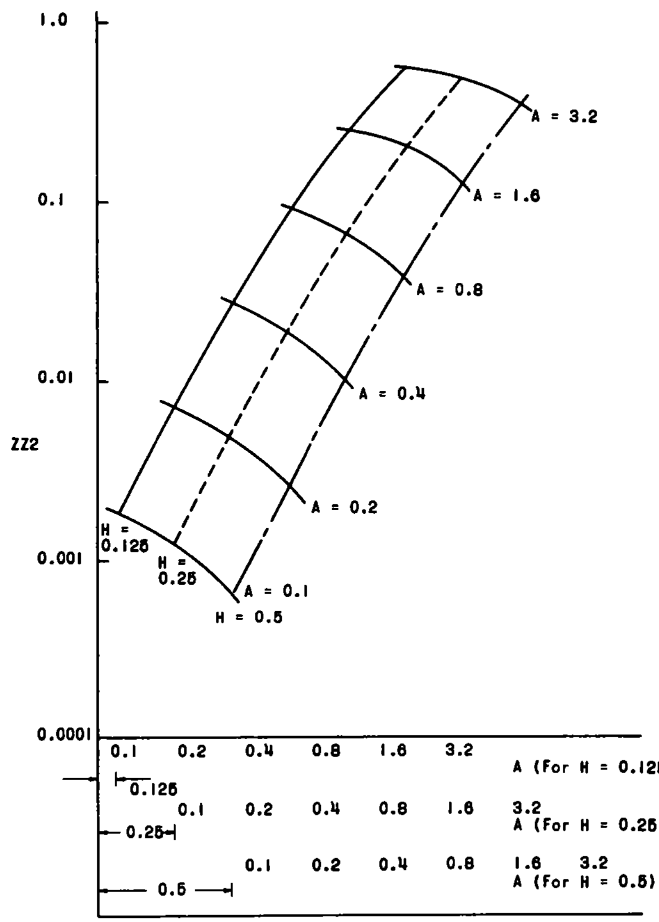


Figure 68. Construction of grid for vertical stress factor ZZ2.