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## Highway Aerial Surveys-Controls and <br> Rights-of-Way <br> 1962

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# Horizontal Control Staking by Triangulation With Computations by Computer 

ROBERT L. LEWIS, Supervising Desıgning Engineer, Texas Highway Department


#### Abstract

This paper describes the development and use of a method for establishing a horizontal control system for staking complicated geometrics on the ground. Major control points were established overlooking the project. Construction stakes could then be "cut in" from at least two control points by using directional theodolites. The data for each control point, including clockwise angles from an established reference point to desired construction stakes, were predetermined. Calculations based on coordinates were made by using various geometric programs designed for the IBM 650 computer. The output information for each control point was printed and bound into a convenient field book size volume, thus completely eliminating the need for field computations.

The Austin project is nearing completion and this method of staking has proved effective in obtaıning information necessary for plan preparation, adjustments of utility lines, and the most important phase-that of translating the geometric design of the plans to construction stakes in the field.


- HIGHWAY ENGINEERS are constantly faced with the problem of setting construction stakes for complicated intersections, structures, and expressway geometrics. Translating the design and layout information of the plans to construction stakes in the field is frequently a difficult task. This is especially true where physical features, obstructions, and heavy traffic congestion prevent laying out reference lines or centerlines in the field. In most cases, the modern urban highway is constructed along an existing alignment or amid a network of streets where it is virtually impossible to interrupt the flow of traffic and to provide the necessary safety to field surveying personnel. This danger and difficulty interferes with all surveying operations for gathering design data and setting stakes for the construction of the project.

This paper describes how the writer, in conjunction with the Texas Highway Department's Computer Section, solved many of these problems and established a new set of staking procedures for a project in Austin, Texas. These procedures constituted a method whereby construction stakes for horizontal control were set by being "cut in" from control points outside the limits of the construction area.

Interstate 35, from 19th Street to the Colorado River, is a new six-lane freeway under construction along old East Avenue, a main north-south artery through Austin. Approximately 30,000 vehicles each day used the avenue and heavy traffic congestion existed at all the cross-streets. This heavy traffic, combined with rough terrain, poor sight distance, and a lack of alternate routes available to detour traffic made conventional staking procedures a real hazard to field personnel. Chaining across the flow of heavy traffic was almost impossible.

The IBM 650 computer was used to calculate and tabulate all the necessary coordinates, angles, and distances in a form that could be readily used in the field by construction personnel. These predetermined data, geared to a field survey procedure, were used to set desired construction stakes quickly and accurately. The purpose of this paper is to acquaint the reader with the basic computer programs used. Then
there is a discussion to show how these programs were combined to develop the triangulation method of horizontal control staking as used on Interstate 35 in the City of Austin.

## COMPUTER PROGRAMS

The programs described are primarily basic geometric programs designed to solve any combinations of straight lines and curves. The rectangular coordinate system forms the basis for these programs, and the system may be either a recognized coordinate system such as the Texas coordinate system (Lambert) or a coordinate system selected arbitrarily for the problem. The Lambert scale factor can be incorporated where the Texas coordinate system is chosen. For the computer to calculate correct bearings, an arbitrary coordinate system should be so selected that the entire problem, including distant reference points, will always remain in the first quadrant (positive), and that the Y -axis of the system be due north and the X -axis be due east.

These programs are so designed that they may be used separately or, with the exception of the "traverse," may all be combined to extend calculations from one program into that of another.

This discussion is concerned only with a brief description of each program as it is used in the construction staking, without discussing the mathematics or the details involved in programing these problems. Additional information can be obtained from the Operations Division of the Texas Highway Department in Austin, Texas. The sample problem used on the Austin project, described later, will best illustrate the use of these programs and allow the reader to observe the input and output data for each program.

## Interdependent Traverse Program

The interdependent traverse program, developed by the California Highway Department and modified by the Texas Highway Department (1), calculates unknown sides and unknown bearings, determines the area, and provides as output a systematic listing of courses with their computed or known factors. The traverse program, with coordinates furnished for the beginning point, will calculate coordinates for each subsequent points traversed. The program also provides for some interdependency of traverses. The interdependency feature allows data from specified courses to be stored for later use and permits a subsequent traverse to call for stored data.

One traverse problem will handle up to 90 courses. A maximum of $\mathbf{2 5}$ unknowns may be kept in storage at any given time to be used in subsequent problems.

## B-10 Program

The B-10 program ( $\underline{2}, \mathrm{Pt} . \mathrm{I}$ ) computes two separate types of calculations as follows:

1. The coordinates of a point when the centerline station and offset are known. The centerline may be either a straight or curved line.
2. The centerline station and offset when the coordinates of the point are known.

Coordinates for all points computed in the B-10 program may be stored for future use in other programs. This is a very important feature in that this eliminates manual listing of coordinates on the input data sheet of subsequent programs. Coordinates of a desired point on a particular centerline can be computed and coded for re-use as reference coordinates for the line. This feature makes it possible to start with only one point of known coordinates and traverse through a series of straight or curved lines.

One B-10 program has a capacity of 10 reference or station lines and will compute the answers for 99 points. If more points are needed, another problem with a different part number must be run.

## B-11 Program

If coordinates of any two points are known, the B-11 program (2, Pt. III) will compute the distance and bearing between them (the distance and bearing defines a straight
line). Rather than having to manually write out the coordinates for each point on the B-11 input form, coordinates previously determined by the B-10 program, for one or both points may be called for by inserting the point number from the appropriate problem. Here again, this stored problem feature plays an important part in that writing out all the coordinates necessary for the input would be time consuming, costly, and a source of many human errors.

## B-11A Program

The B-11A program (2, Pt. IV) is used in conjunction with the B-11 program. The purpose of this program is to define a straight line and when used with the B-11 program it computes the clockwise angle between the line defined by the $\mathrm{B}-11 \mathrm{~A}$ and the line or lines defined by the B-11 program. This program furnishes the terminal data for the triangulation staking; i.e., the clockwise angles the instrumentmen must turn in the field to locate and establish the desired point for a construction stake.

## TRIANGULATION STAKING FOR INTERSTATE 35 IN AUSTIN

As mentioned earlier, Interstate 35, from 19th Street to the Colorado River, followed East Avenue, an old established north-south artery through Austin. The traffic on East Avenue was very heavy and there was no way to detour or relieve the congestion of traffic within the project area. The rough terrain further complicated the traffic problem, making it almost impossible to establish conventional centerline control in the field for the purpose of gathering design information and for construction purposes.

A study of the various programs available from the Computer Section of the Operations Division, Texas Highway Department, indicated that coordinates could easily be obtained for any predetermined point within the project by using the B-10 program and that, with the addition of the B-11A program to the available B-11 program, additional information could be secured to simplify staking these points in the field.

A topographic map, prepared from recorded subdivision plats, was field checked against the old monument line of East Avenue. This map was then used for design


Figure 1. Typical traverse-mput form.
studies and as a base to prepare the project schematic layout. From the approved schematic layout, a project reference line or centerline was developed on the map for the entire length of the project. Reference lines were established for each frontage street and for each throughway lane in those sections of the project where they maintained separate alignments. Had photogrammetric methods been used to provide controlled topographic maps, this phase of development would have been much easier.

The centerline and one point on the north end of the project was oriented to the Texas coordinate system. This was a simple matter since this point was in sight of four USC \& GS intersection stations and one USC \& GS triangulation station. With the project centerline oriented with the Texas coordinate system, the interdependent traverse program was used to calculate distances, as well as tie together and check the various reference lines.

Traverses were run either to calculate or to check the geometric layouts for all ramps, connections, turnouts, and all transitions and channelized street connections. The traverse program, from the input (Fig. 1), calculated the coordinates (Fig. 2) for every point included in each traverse.

With the various reference lines set and the geometric layouts of the project established, a series of $\mathrm{B}-10$ problems were prepared to calculate coordinates when either the station and offset were given or to calculate the station and offset with respect to the reference line when coordinates were furnished.

First, an attempt was made to predetermine all the points needed to control the project reference line or centerline and to establish sufficient offset points to this reference line for taking cross-section data and setting points to be used during construction as reference hubs. It was desirable to have several offset points on each side of the project reference line in order to eliminate chaining across the lanes of traffic within the project. A print of the project schematic showing the reference line and cross-section lines was prepared and all the desired points for control of these lines were plotted on this map. Points were chosen for each station and half-station along the project centerline and the P.C., P.I., and P.T. of each curve. Adequate offset points were also included. A second layout (Fig. 3), similar to the one used to control the project reference line, was prepared to show points required to stake frontage roads, ramps, connections, turnouts, transitions, inlets, and other miscellaneous points


Figure 2. Typical traverse-output form.
needed during construction. These points were assigned numbers beginning with point No. 1 at the beginning of the project and numbered consecutively until all the desired points were identified.

Separate B-10 program input sheets were than prepared for points on each layout with the point number on the $B-10$ corresponding to the same point number on the layout. The B-10 input sheet (Fig. 4) is self-explanatory regarding the form. It might be pointed out that because only 99 points can be written for each B-10 problem, new B-10 problems with a change in part number were prepared and a notation made in the remarks column indıcating the hundred series belonging to the point number.

Where coordinates for a particular point were found in a traverse program, these were entered in the X - and Y -columns of the $\mathrm{B}-10$; thus, in addition to calculating the station and offset, it stored the coordinates of that point for future use.

The output information for all these predetermmed points submitted on the B-10 problems was listed (Fig. 5) and the data stored for future use in subsequent problems. To establish terminology, all points listed on the B-10, either as a centerline point or as an offset point, were designated as station points.

Because the B-10 program had calculated the coordinates for all station points desired for construction, a method was needed to set quickly and safely these points in the field. It was thought that these station points could conceivably be set in the field if they could be "cut in" from two control points outside the area of construction. In other words, the station point became the intersection point of two lines whose bearing could be calculated.

A study of the B-11 program (Figs. 6 and 7) showed that from a control point of known coordinates, the distance and bearing to all desired station points could be accurately determined. The preparation of the input data sheet (Fig. 6) did not require coordinates for each station point to be manually written, but only the point number


Figure 3. Station points for frontage roads, ramps, connections, etc.


Figure 4. B-10 input form.


Figure 5. B-10 output data sheet.


Figure 6. B-ll input form.


Figure 7. B-11 output data sheet.
from the appropriate $\mathrm{B}-10$ program entered in the station point column was required. By using the B-11A program (Fig. 8) in conjunction with the B-11 program, the clockwise angle (Fig. 9) from some known reference point to each station point was determined. Then, from any two such control points, a station point within view could be set by intersection (Fig. 10). A station point could also be set by turning the prescribed angle and measuring the computed distance from a nearby control point.

On the Austin project, permanent markers designated as control points were set approximately 500 ft apart, along and just outside of the construction area. These control points, identified by number, were arbitrarily set to command a view over approximately 800 ft of the project length and with the idea that each station point to be set must also be within view of at least two and preferably three control points, except for those station points within convenient measuring distance of one contrnl point. In addition, each control point was set in view of a known reference point. Because coordinates for the star on the top of the State Capitol dome were available and this point was visible from almost all parts of the project, this provided an excellent reference point for most of the control points. A control point located on high ground or on top of some adjacent building provided the best vantage point for observing the project area.

After control points were set, conventional triangulation and traverse methods of surveying were used to determine accurately the coordinates for each control point. These control points were then plotted on the strip maps with their identifying numbers so that their relation to the station points could be observed.

The B-11 and B-11A programs (Figs. 6 and 8) were then submitted for each control point. The coordinates for the control point and the reference point were entered as input data together with the appropriate $\mathrm{B}-10$ problem for the stored station points. At the end of the B-11 input listing, it was found desirable to enter the coordinates of several adjacent control points or of any other secondary reference point in the station point column. This not only served as a check for the control points themselves, but it was thought that certain construction features or weather conditions may subsequently prevent the observer from seeing the primary designated reference point.


Figure 8. B-11A input form.

The output listing from the computer provided, for each control point, the distance and bearing to each station point (Fig. 7) and the clockwise angle from the reference point to each station point (Fig. 9). Two copies of these output data were bound in book

## CONTROL POINT NO. L-I7 REF. PT: CAPITOL STAR

B11 A

| DATE | PR REF DEG MIN SEC CODE FROM |  |
| :--- | :--- | :--- |
| 111461 | NO LN | B11 |
| CODE |  |  |

$11800151301 \quad 1 \quad 870706.86 \quad 13$


| 8 | 1 | 62.40 | 50.80 | 20 |
| :--- | :--- | :--- | :--- | :--- |
| 9 | 1 | 78 | 37 | 42.23 |


| 1 | 1 | 730301.14 | 22 |
| :---: | :---: | :---: | :---: |

1117723
$121.611018 .73 \quad 24$
$131 \quad 635602.60 \quad 25$
$141 \quad 53 \quad 18$ 56.82 26
$151 \quad 415419.89 \quad 27$
$161484543.95 \quad 28$
$171580524.28 \quad 29$
$181905409.49 \quad 30$
$191285228.43 \quad 31$
$201 \quad 3634$ 54.97 $\quad 32$
$211 \quad 31 \quad 5046.56 \quad 33$

Figure 9. B-llA output data sheet.
form. With this information available, it was then possible to set any one of the preselected station points on the ground without having to make field calculations. The identifying number shown on the strip map (Fig. 3) for each station point corresponds to the same point number on the B-10 (Fig. 5), B-11 (Fig. 7), and B-11A (Fig. 9) programs. A typical station point (Point 19, Fig. 10) has been marked on the examples to indicate the steps through the various programs and to show that data for any desired point could be found quickly.

The following procedure was used in the field to set construction stakes. The survey crew consisted of a party chief, two instrumentmen, and a rodman. The party chief, using the strip map showing the numbered station points and the location of control points (Fig. 3), directed the instrumentmen to two chosen control points in command of the area to be staked. The party chief informed each instrumentman of the station point number to be set. Communications between members of the party were accomplished by hand signals. At times during construction, three two-way hand radio units were successfully used to improve communications. Each instrumentman using the data for his respective control point turned the computed clockwise angle to that station point. A rodman, obtaining line from each instrumentman, located and set the station point. The rodman, after a little experience, was able to find this point of intersection rapidly.

Two directional theodolites were used in the field to turn the required angles. These instruments read directly the clockwise angle and only that angle when the zero angle was observed on the reference point. This established a simple procedure for turning and checking angles because the clockwise angle calculated and printed by the computer was read and set directly in the theodolite. The instrumentmen, observing zero angles on the reference point, proceeded to "cut in" as many points as necessary to establish construction control. The instrumentmen, to check instrument orientation, turned


Figure 10. Typical station point set from control points.
calculated angles to other established points in the area. These points were adjacent control points, secondary reference points, or a station point previously set and checked.

## COMMENTS AND OBSERVATIONS

The Austin project is now nearing completion and this system of staking has proved very effective. Other than a few field tests to prove the practicability of setting up such a control for the Austin project, the first real test came in the design stage when it was necessary to obtain original cross-sections for earth work computations. Crosssections every 50 ft along the $1.7-\mathrm{mi}$ long project centerline were taken and the horizontal control for approximately 70,000 linear ft of these sections was staked in one week under adverse traffic conditions. The cross-section lines and points were obtained without having to chain across moving traffic lanes or without having to stake the project centerline. Also, this method was used extensively to set location stakes for the relocation of utility lines on this project.

In the beginning, the idea was to use this method to stake only roadway geometrics; however, during construction when proved an accurate, simple, and effective way to set points, the system was enlarged to include bridge construction points. This Austin project was constructed in segments or sections conforming to a planned sequence of work to allow traffic to pass through it uninterrupted. Without this method of staking, it would have been difficult to furnish the contractor with sufficient construction stakes.

How accurate were points set in the field using this method of triangulation staking? It must be realized that errors such as careless handling of the instrument, wind vibrations, heat from the sun, mistakes in reading the proper angle, and other errors common to angular measurements had to be considered in the field staking. The control points were set and coordinates determined to an accuracy greater than second order horizontal control as defined by the U.S. Coast and Geodetic Survey. However, because a number of station points were set from one control point, the position of each station point, relative to each other, depended primarily on the accuracy of the clockwise angle turned at the control point. The calculations and listing provided by the computer gave angles to the nearest 0.01 sec . With the instruments used on this project, angles were consistently measured within 5 sec of the desired angle. By using a long backsight on the reference point and observing a short foresight ( 500 ft or less), the error caused by the inability to turn the exact angle was minimized. Many points were set and checked in the field and the usual maximum variation amounted to approximately 0.01 ft . In staking an overpass structure, column points in a bent were staked by triangulation and then checked by chaining the prescribed distances between them. Only one column was out of position by about 0.02 ft and this error was corrected when the angles turned at the control points were rechecked. Accurate setting of station points

TABLE 1
THE ERROR RESULTING FROM VARIOUS ANGULAR ERRORS, (FT)

| Angular <br> Error <br> (sec) | 100 | 200 | 300 | 400 | 500 | 1,000 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.0005 | 0.0010 | 0.0015 | 0.0019 | 0.0024 | 0.0049 |
| 1 | 0.0010 | 0.0019 | 0.0029 | 0.0039 | 0.0049 | 0.0097 |
| 2 | 0.0015 | 0.0029 | 0.0044 | 0.0058 | 0.0073 | 0.0145 |
| 3 | 0.0019 | 0.0039 | 0.0058 | 0.0078 | 0.0097 | 0.0194 |
| 4 | 0.0024 | 0.0048 | 0.0073 | 0.0097 | 0.0121 | 0.0242 |
| 5 | 0.0029 | 0.0058 | 0.0087 | 0.0116 | 0.0145 | 0.0291 |
| 6 | 0.0034 | 0.0068 | 0.0102 | 0.0136 | 0.0170 | 0.0339 |
| 7 | 0.0039 | 0.0078 | 0.0116 | 0.0155 | 0.0194 | 0.0388 |
| 8 | 0.0044 | 0.0087 | 0.0131 | 0.0175 | 0.0218 | 0.0436 |
| 9 | 0.0048 | 0.0097 | 0.0145 | 0.0194 | 0.0242 | 0.0485 |
| 10 |  |  |  |  |  |  |

for this type of construction depended on having strong angles of intersections in addition to repeating the angle measurement several times. Table 1 gives the reader an idea of the angle accuracy necessary to establish transit line within desired limits. This table indicates the errors resulting from various angular errors at given distances from a control point.

Although not directly concerned with horizontal control staking, other uses were made of the data furnished by the computer. The listed data from the B-10 was used to plot the various layouts required in plan preparation. These data were not only an aid to plotting, but furnished an excellent check on the geometric calculations. The B-10A program (2, Pt. II), a program used in conjunction with the B-10 program, was later submitted to obtain the profile grade elevations for points along the desired reference lines. The stored data from these B-10 problems were used later in conjunction with the B-12 program (2, Pt. V), program designed to find the coordinates of the intersection point of two straight lines, points of a straight line and circle, or points of two circles. This program was used to develop design sections for the multi-lane earthwork program by finding at each cross-section station, the perpendicular distance from the project centerline to other adjacent roadways.

The development and use of this staking procedure during the last two years on the Austin project has proved the practicability of using this method of staking on this and other future complicated projects. Intersections containing numerous compound curves, small radius curves, curved bridges, and other construction features that are normally difficult to stake could very well be adapted to this method of horizontal control. Because reference lines are normally developed for the design layouts, it is not necessary to replace completely the reference line method of staking with triangulation staking. They can complement each other and there may be times during construction or on certain portions of the project where it would be more desirable and convenient to set stakes from reference lines.

## ACKNOWLEDGMENTS

The writer is indebted to many people who have contributed and exemplified splendid co-operation towards the development of this staking method and the preparation of this report. Consideration must be given to contributions made by members of District Fourteen of the Texas Highway Department, under the able direction of Ed Bluestein, District Engineer, with special attention focused on members in the Austin Residency under the supervision of T.A. Long, Supervising Resident Engineer. Without the aid and assistance given by personnel and equipment in the Computer Section of the Operations Division, Texas Highway Department, it would have been unthinkable to attempt such a staking method as described in this report.

The efforts of all who have contributed and encouraged the development of this report are greatly appreciated.

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# Use of the Zeiss Stereotope for Highway Engineering Purposes 

O.W. MINTZER, Associate Professor, Department of Civil Engineering, Ohio State University, R.K. BASTIAN, Capt., Corps of Engineers, and O.S. SAHGAL, Associate Professor, Punjab (India) Engineering College

The need to evaluate a third order photogrammetric instrument for general use to procure data for highway engineering promoted the idea of testing the Zeiss stereotope. To determine whether the stereotope would meet better than third-order accuracy requirements, a comparative study of first and third order photogrammetric instruments was made at the Ohio State University. This paper reports an evaluation of the stereotope in terms of the accuracy requirements set for procurement of cross-sectiondata used in computing earthwork quantities.

The comparative study consisted of two parts. The first part was a calibration test of the stereotope to determine the feasibility of using it for large-scale mapping and cross-section data procurement. The second part consisted of evaluating it in terms of spot-height accuracy and earthwork quantity measurements of final-pay quantities. To evaluate the accuracy of its measuring of spot heights, the latter were compared with those read by means of the Wild A-7 autograph.

The photography of a $1-\mathrm{mi}$ section of interstate route had been taken for determining final-pay quantities on an interstate highway improvement project constructed in 1960 by the Ohio Department of Highways.

It was found that with normal ground-control data and aerial photographic operations the stereotope would be an instrument as satisfactory as the Kelsh plotter for procuring cross-section data for final-pay quantity measurements, provided that care is taken in performing the parallax measurements, using either dimensionally stable plastic-base print film or glass diapositives.

- ACCURACY of the large-scale maps required in the highway applications of photogrammetry in the past has made necessary the use of second and higher order plotting instruments. In general, the higher order, the more expensive instruments, are not within the financial grasp of highway departments of the county level of government or of most underdeveloped countries. Therefore, there is the need to determine if a third order instrument would be satisfactory. This need promoted a comparative study at the Ohio State University. First and third order photogrammetric instruments were used to determine cross-section data. The spot-height measurements of discrete points along the profile and cross-sections of a roadway were used in the evaluation of instrument accuracy.

Information was not available on the Zeiss stereotope's capability in large-scale mapping or in spot-height measurement of elevations for determining cross-sections. However, Quinnell (30), in 1959, tried the instrument for reconnaissance survey data and found that it afforded a simple but economical method of obtaining engineering data by means of photogrammetry.

This paper reports the results of the study to establish what use can be made of the instrument in highway engineering. The study was performed in two parts. The first
part was to determine the feasibility of using the stereotope for measurements of crosssection data for use in earthwork volume calculations. An instrument calibration experiment was set up. Figures 1 and 2 show the stereotope as used for spot-heighting measurements. Bastian (19), in 1960, determined that it was feasible to use it to determine cross-section data for earthwork volumes.

The second part of the study determined its accuracy, again using the performance of the Wild A-7 autograph as the standard. Sahgal (29), in 1961, found that because the third order instrument performed well to produce accurate cross-section data, the stereotope could be used in many phases of highway engineering applications of photogrammetry.

## CALIBRATION TEST

## Procedure

The camera was set up and leveled 23 ft from the mock-up of a cut-fill roadway section (Fig. 3). The camera stations were designated from left to right as north base and south base, respectively (Fig. 4). The point of intersection of the line of sight of the telescope axis and the plane of the surface of the mock-up was marked with a small cross consisting of masking tape, the exact point of intersection being marked on the tape with a horizontal and a vertical pencil line. The camera was turned toward the south base for the tilt angles. Points on the mock-up were designated 0, 1, 2, 3, 4, and 5 to represent the principal points of photographs for the $0^{\circ}$ to $5^{\circ}$ of tilt (Fig. 5). Four reference points were marked on the floor to locate each camera station exactly, inasmuch as no plum system was available. The optical axis of the instrument was lined in first direct, then reversed, and the angles to the principal points were re-read to eliminate orientation errors. The maximum difference between orientation angles


Figure 1. Zeiss stereotope used at Ohio State University (1960-61).


Figure 2. Schematic diagram of Zelss stereotope showing arrangement of photographs for viewing and parallax measurements (after Bastian, 19).


Figure 3. Illustration of cut-fill roadway mock-up section photographed and measured using Zeass-stereotope calıbration test (after Bastian, 19).
was less than 30 sec , representing a linear distance of only 0.0037 ft , which was considered accurate within the experimental limits. Direct and reverse readings did not differ by more than 0.001 ft . A level rod was read to determine the height of the instrument. The north base line of sight was 0.018 ft below the south base, and this was taken into account.

Six glass-plate negatives were made with the camera pointing at the marked principal points from each base. Before dismantling the camera equipment, measurements were taken with a steel tape between machined surfaces on the camera mount in position as a check on the camera base test. The distance ( 15.672 ft ) between north and south bases compared exactly with the measurement determined later between the camera stations, and the distance between the $0^{\circ}$ principal-point marks on the mock-up, corrected for the 5 percent slope, was within 0.001 ft of the base distance (Fig. 6).

A Wild T-2 theodolite was set up and accurately sighted on the stations defined by the four control points. Three or four sets of angles were taken at each station, reading


Figure 4. Camera station arrangement when negatives were exposed for calibration test (after Bastian, 19).
both the horizontal and vertical angles of the control points, W, X, Y, and Z. The mean errors in the horizontal and vertical angles were 3.19 and 2.63 sec , respectively, representing an elevation difference of only 0.0006 ft which was disregarded.

The glass-plate negatives, on which the photographs had been exposed using (an f:32 aperture) distortion-free lens, were oriented in the Wild A-7 autograph, and the elevations of each point determined. These elevations, determined to 0.001 ft , were used as the true elevation of each point and the results obtained with the stereotope were compared with them. Figure 7 shows one set of stereophotographs used in the experiment.

One of the sources of error in the measurements obtained with an instrument like the stereotope is inherent in the photographic positive print material. The errors inherent in paper print material are significant. DuPont "Cronopaque" print film on Cronar


Figure 5. A, B, C, D, and E identify cross-sections, with 1 through 9 indicating spotheight points; crosses in left center indicate principal points for tilted photographs (after Bastian, 19).


Figure 6. Camera in position at north base.


Figure 7. One set of stereophotographs used in experiment.


Figure 8. Error plotted against cumulative frequency for spot-height elevations read on paper prints.
polyester photographic film base holds size, is flexible, and is easy to handle in the instrument. For a $30-\mathrm{in}$. length of Cronopaque film a change in length from 0.008 to 0.1 in . occurs for a $20^{\circ}$ increase in temperature, and a 20 percent increase in relative humidity causes a change of 0.007 to 0.01 in . in a $30-\mathrm{in}$. length of film. Thus, under the calibration test conditions and later the accuracy test conditions, the Cronopaque film was assumed to be dimensionally stable. A comparative test of the accuracy of spot-height values was obtained first on ordınary single-weight print paper, then on Cronopaque print film. The mean error and standard deviation gave a means of evaluating the magnitude of the error for ordinary print paper. Figures 8 and 9 show the error plotted against cumulative frequency in percent. For this special case, the standard deviation for the measurements made on the Cronopaque film was about one-fourth that for single-weight print paper. The mean and standard deviation of error can be reduced appreciably using a stable print film such as DuPont Cronopaque.

After the model was oriented in the stereotope, three micrometer readings were taken at each of the pre-selected spots in cross-sections A, B, C, and E. The readings were averaged and the value of the parallax constant, $C$, added to obtain the parallax of each point. Elevations of the points were computed from parallax values in the conventional manner. The elevation of each point was compared to the elevation determined with the Wild A-7. The differences in the elevations given by the Wild A-7 and the stereotope were considered errors.

After the control data were numerically set on the stereotope computers, the floating dot was set at the proper position of each control point, the right photo was shifted, and the model was oriented. Then the dot was set on each point and micrometer readings recorded for use in calculating the parallax. The corresponding spot elevations were


Figure 9. Error plotted against cumulative frequency in percent for spot-height elevations read on Cronopaque film prints.
calculated in the conventional manner: Given a micrometer reading of 17.538 mm an average of three readings on point A, adding to this the parallax correction, $C=59.446$, there is then parallax $P=76.984 \mathrm{~mm}$. Using the parallax formula there is

$$
h=H-\frac{b f}{P}=25.000-\frac{15.683 \times 114.81}{76.984}=1.607 \mathrm{ft},
$$

the value of the height of point $A$ from datum. The development of the parallax formula is given elsewhere (15).

TABLE 1
SUMMARY OF RESULTS OF CALIBRATION TEST ${ }^{\text {a }}$

| Cross-Section Point | Wild A-7 <br> Elevation | Stereotope Elevation Differences* (degree of tilt)** |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Zero | 1 Left | 2 Left | 1 Left, 3 Right | 2 Left, 3 Right | 3 in Both |
| A1 | 0. 779 | -0.003 | +0.014 | *** | -0.003 | -0.009 | +0.004 |
| 2 | 0.764 | +0.003 | +0.012 | +0.009 | +0.009 | +0.001 | +0.003 |
| 3 | 0.760 | -0.003 | +0.013 | +0.005 | +0.009 | +0.008 | +0.001 |
| 4 | 1. 546 | -0.001 | +0.008 | +0.010 | +0.067 | +0.051 | +0.070 |
| 5 | 1.545 | 0.000 | +0.009 | +0.014 | +0.058 | +0.051 | +0.071 |
| 6 | 1. 540 | +0.009 | +0.009 | +0.009 | +0.057 | +0.052 | +0.071 |
| 7 | 0.734 | +0.003 | +0.013 | -0.003 | +0.015 | -0.006 | +0.008 |
| 8 | 0.733 | +0.004 | +0.008 | -0.001 | +0.016 | -0.007 | +0.001 |
| 9 | 0.737 | 0.000 | +0.009 | *** | +0.012 | -0.017 | +0.003 |
| B1 | 1.362 | +0.007 | +0.075 | +0.141 | -0.098 | -0.098 | +0.033 |
| 2 | 1.355 | +0.006 | +0.071 | +0.147 | -0.087 | -0.040 | +0.035 |
| 3 | 1.337 | +0.007 | +0.074 | +0.148 | -0.084 | -0.037 | +0.036 |
| 4 | 1.733 | +0.008 | +0.072 | +0.147 | -0.053 | -0.017 | +0.060 |
| 5 | 1.728 | +0.006 | +0.072 | +0.144 | -0.051 | -0.026 | +0.063 |
| 6 | 1.726 | +0.010 | +0.074 | +0.144 | -0.035 | -0.029 | +0.063 |
| 7 | 1.322 | +0.007 | +0.062 | +0.148 | -0.083 | -0.044 | +0.041 |
| 8 | 1.306 | +0.009 | +0.067 | +0.145 | -0.072 | -0.060 | +0.041 |
| 9 | 1.303 | +0.013 | +0.059 | +0.139 | -0.070 | -0.043 | +0.041 |
| C1 | 1.930 | +0.008 | +0.073 | +0.192 | -0.117 | -0.046 | +0.047 |
| 2 | 1.934 | +0.003 | +0.090 | +0.183 | -0.117 | -0.046 | +0.042 |
| 3 | 1.920 | +0.004 | +0.096 | +0.184 | -0.112 | -0.044 | +0.047 |
| 4 | 1.916 | +0.010 | +0.097 | +0.189 | -0.110 | -0.044 | +0.051 |
| 5 | 1.911 | +0.006 | +0.089 | +0.179 | -0.107 | -0.047 | +0.052 |
| 6 | 1.905 | +0.004 | +0.072 | +0.177 | -0.105 | -0.048 | +0.046 |
| 7 | 1.911 | +0.004 | $+0.080$ | +0.177 | -0.109 | -0.046 | +0.044 |
| D1 | 2.533 | +0.001 | +0.064 | +0.139 | -0.060 | -0.044 | +0.030 |
| 2 | 2. 555 | -0.002 | +0.061 | +0.144 | -0.074 | -0.044 | +0.036 |
| 3 | 2. 524 | +0.001 | +0.065 | +0.140 | -0.071 | -0.030 | +0.037 |
| 4 | 2.123 | -0.008 | -0.075 | -0.155 | -0.078 | -0.036 | +0.023 |
| 5 | 2.113 | +0.008 | +0.077 | +0.151 | -0.089 | -0.031 | +0.029 |
| 6 | 2.118 | +0.007 | +0.073 | +0.148 | -0.079 | -0.048 | +0.024 |
| 7 | 2.519 | +0.005 | +0.078 | +0.148 | -0.067 | -0.034 | +0.036 |
| 8 | 2. 529 | +0.005 | +0.077 | +0.140 | -0.066 | -0.042 | +0.029 |
| 9 | 2. 503 | +0.012 | +0.073 | +0.144 | -0.063 | -0.035 | +0.035 |
| E1 | 3.123 | +0.007 | +0.009 | *** | +0.007 | +0.006 | +0.003 |
| 2 | 3.114 | +0.007 | +0.017 | +0.024 | +0.009 | +0.012 | +0.009 |
| 3 | 3.117 | +0.007 | +0.014 | +0.024 | +0.009 | +0.009 | +0.011 |
| 4 | 2.314 | +0.007 | +0.029 | +0.057 | +0.009 | +0.013 | +0.003 |
| 5 | 2.306 | +0.010 | +0.039 | +0.053 | +0.011 | +0.013 | -0.002 |
| 6 | 2.313 | +0.003 | +0.036 | +0.046 | +0.007 | +0.003 | +0.003 |
| 7 | 3.119 | +0.006 | +0.024 | +0.018 | +0.001 | +0.011 | +0.007 |
| 8 | 3.121 | +0.007 | +0.013 | +0.016 | -0.000 | +0.009 | +0.008 |
| 9 | 3.106 | +0.007 | +0.009 | *** | +0.005 | +0.017 | +0.009 |

[^0]Results of Calıbration Test
A summary of the results obtained with stereotope compared to the elevations determined with the Wild A-7 is given in Table 1. A plus difference indicated that the elevation determined with the stereotope is higher than the elevation determined with the Wild A-7. The various stereoscopic models are designated by the amount of tilt present in the left and right photograph. The tulted model $1^{\circ}$ in the left and $0^{\circ}$ in the right photo refers to the north base photograph having the camera optical axis tilted $1^{\circ}$ toward the south base. The tilt angle is to the right from north base toward south base in the positive X direction.

The elevations of the four control points obtained by both plotting instruments compared closely. For these points, the maximum errors of the Wild A-7 and the steretope were, respectively, 0.002 and 0.003 ft . This comparison indicates that the accuracy with which the models were oriented in each instrument was comparable.

For the photographs with no tilt, the errors appear to be random, but with a predominance of positive values. There is an indication that the operator had a tendency to read the elevations high. It was observed during the calibration test that the instrumentation reported herein lends itself to testing operator acuity and efficiency.

The C-factor was found for the non-tilted model. The error was no greater than 0.0102 ft for 90 percent of the points and the "flying height" was 23.00 ft . These values give a C-factor equivalent to $2,230$.


Figure 10. Error plotted against position of cross-section as related to principal center of photograph.


Figure 11. Photographs (a), (b), and (c) taken of US 62 for final pay quantaty measurements controlled by pavement station marks; these stations, the other control stations, and cross-sections to be measured were plotted on manuscript as shown in these diagrams (after Sahgal, 29).

The mean error of the spot-height elevations was 0.0059 ft and the probable error was 0.0039 ft . Expressing these errors in terms of the "flying height,"

$$
\text { mean error }=\frac{0.0059}{23.00}=\frac{1}{3,900} \text { of "flying heıght" }
$$

and

$$
\text { probable error }=\frac{0.0039}{23.00}=\frac{1}{5,900} \text { of "flying height." }
$$

There were sizeable errors in the elevations measured in the tilted models. The largest portion of any error is probably due to the tilt that cannot be entirely eliminated with the stereotope computers. The maxımum error of the tilted ( $2^{\circ}$ to $0^{\circ}$ ) model was about $1 / 125$ of the flying height.

Funk (7) found that in practice the profile of a constructed section of roadway is usually accurately determined and the known profile elevations can be used to adjust the remaining cross-section elevations. The suggested adjustment was applied to the cross-sections of the $0^{\circ}-0^{\circ}$ model, reducing the mean and probable errors respectively from 0.0059 to 0.0037 and from 0.0039 to 0.0025 ft . But similar adjustments to the centerline profile data in tilted models did not give good results.

Figure 10 shows the error plotted against distance from the principal point across the mock-up from section to section. The error of greater magnitude was always near the center of the neat model. This confirms what was noted by Funk (9) in 1958.

## ACCURACY TEST

## Description of Photogrammetric Procedure

There were seven models of the photography which had been flown by the Ohio Department of Highways at an altitude of $1,650 \mathrm{ft}$ above the average terrain providing a plotting scale of $1 / 2,400$ for the stereotope and a manuscript plotting scale of $1 / 625$ for the Wild A-7 autograph. The photographs were printed on $0.13-\mathrm{in}$. glass plates and on dimensionally stable Cronopaque film. As the work with the stereotope was carried out in an air-conditioned room, for the $9-$ by $9-\mathrm{in}$. format the effect of dimensional changes due to temperature and humidity variation was neglected for the Cronopaque film. There were 45 vertical control points spread over the seven stereomodels, including 20 points fixed on the pavement edge.

To identify the cross-section lines to be measured in each instrument, the cross-

TABLE 2

## SUMMARY OF STANDARD DEVIATIONS AND ARITHMETIC MEANS FOR STEREOTOPE ${ }^{\text {a }}$

| Model <br> No. | No. of <br> Observations | Results Before and After Adjustment <br> Standard Deviation (ft) | Arithmetic Mean <br> (ft) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

[^1]section lines were marked on each Cronopaque print. These prints were placed emul-sion-side down on top of a light table and the corresponding diapositives, emulsion-side down, were then placed on top of the respective prints, using the fiducial marks as guides. The position of cross-section lines were marked on the clear side of the diapositives with a China marking pencıl while being viewed orthogonally from the top. These cross-section lines were used as the guide lines for the plotting of the manuscript in the Wild A-7 autograph operation.

The interior, exterior, and absolute orientations were executed in the Wild A-7 autograph in the conventional manner. The scaling of the models was carried out numerically by reading the machine coordinates of the stations, computing the distance between them and comparing the model distances with corresponding true ground distances. The leveling of the model was then carried out by reading the elevations on vertical control points which had been measured to 0.01 ft in the field survey. Then, after the absolute orientation and the plotting of the planimetric position of each point, the spot-height readings of 2,042 points were taken and recorded. Along with these values the horizontal distances to the left or the right of centerline on the cross-sections were scaled off from the manuscript and recorded. Also, height readings were taken at the principal points of each photograph and recorded for later use in the stereotope orientation.

The planimetric positions of the cross-section stations at $50-\mathrm{ft}$ intervals and all control points were plotted on Mylar film manuscript by means of the Wild A-7 autograph. Figure 11 shows the manuscript identification of the stations and cross-section points used in spot-heighting operation. The numbers 96 to 101 are the photo print numbers in sequence. The stations, $467+00$ to $485+00$, identify the route alignment.

In the case of the stereotope, there was only an "interior orientation" which constituted the correct positioning of the left-hand and the right-hand photographs of every model in sequence. The photographs were positioned on their respective photo carriers, in the manner described next. The principal and conjugate points were located and transferred in the conventional manner to establish the line of flight or the $\mathbf{X}$-axis of the measurement system. The photocarriers, mirrors, and ocular viewers were adjusted for comfortable stereovision. Displacements in the $Z$ and $Y$ directions were affected by the large mirrors and the small mirrors, respectively. The two measuring marks held in reticules were brought on top of the two engraved crosses on two photo carriers, with the computers being set at a value of 10.0 (which is the zero setting). Then the parallax value recorded at the X -parallax measuring thimble was checked. This value


Figure 12. Normal curve showing distribution of errors for unadjusted stereotope data (after Sahgal, 29).
should be 15.00 mm in this position; otherwise, there is some zero error in the micrometer setting. The "zeroing in" must be taken care of before operating the stereotope.

The photographs, with X - and Y -axes denoted, were then fixed on their respective carriers with magnets and tape. The coordinate axes were then matched with the engraved lines on the photo carriers.

After the preceding procedure, the "leveling" of the stereomodel was done using the stereotope computers. Aerial photographs are seldom vertical and, due to distortion characteristics of the taking camera lens, the height value read in the instrument's operation has a small systematic error; therefore, it was necessary to level the model carefully. The true parallax values at the vertical control points were set by the computers in each case. Then the photo carriers were adjusted to correspond to the respective values read at the X-parallax micrometer. Further details on the orientation procedure for the instrument are given elsewhere (3,16, and 17). Each model was observed to determine the planimetric and parallax position of the 2,042 points to establish the cross-section data for the earthwork volumes.

## Results Using Interstate Project Photos

For the spot-height elevation data obtained, this study derived the standard deviation, the arithmetic mean, developed a comparison of the frequency distribution with the theoretical error or probability curve, and calculated the C -factor as derived from the theoretical contour interval that would comply with the specification requirement "that 90 percent of the points tested must be within one-half contour interval." The 90 percent value is derived from the error frequency distribution. In addition the accuracy of earthwork volumes calculated from the cross-section data were evaluated.

The measurements taken by means of the Wild A-7 autograph were used as the standard of comparison. Spot-height elevations for the cross-sections were taken using each of the instruments. There were more than 100 sections measured. The number of discrete points $(2,042)$ was considered satisfactory as a statistical sample. To determine a measure of the vertical accuracy of the stereotope, spot-height elevations were obtained conventionally by means of the parallax readings. These elevations were compared with the corresponding values computed from data given by the Wild A-7 autograph operation. The differences from the standard were classified as


Figure 13. Normal curve showing distribution of errors for adjusted stereotope data (after Sahgal, 29).
errors. The residual or difference between the standard value and that given by the stereotope was considered either a random or systematic error, or it was a blunder.

The standard deviation of errors in the spot-height measurements of all 2,042 points was found to be $\pm 0.76 \mathrm{ft}$ with an arithmetic mean of +0.07 ft . The standard deviation varied from a low of $\pm 0.54 \mathrm{ft}$ to a high of $\pm 0.94 \mathrm{ft}$, with a corresponding arithmetic mean varying from -0.20 to +0.08 ft , respectively, and from model to model. Table 2 gives the summary of standard deviations and arithmetic means for all models.

In 1959 Funk (9) indicated that in Kelsh plotter operations to obtain cross-section data for earthwork-volume computations there was an improvement after adjusting the height readings to the data from the field-measured centerline profile. A similar adjustment was made for the cross-section data obtained using the stereotope, but the improvement was only slight; in fact, for some models the adjusted-to-centerline values produced poorer results. The over-all standard deviation reduced from $\pm 0.76$ to $\pm 0.74 \mathrm{ft}$, although the arithmetic mean degraded from $\pm 0.07$ to -0.13 ft . Table 2 gives the results in all models before and after adjustment.

Figures 12 and 13 show the distribution of the errors in the form of a "normal curve." The "normal" curve found in this study was not bell-shaped but had similar characteristics as in the normal curve. Figures 14 and 15 show the cumulative frequency distribution plotted with the abscissa scale graduated according to the area under the normal curve. Any line of this graph passing through a point defined by an elevation error of zero at a cumulative percentage error of 50 would correspond to the normal curve. Dotted lines on these figures represent the normal distribution of errors. The cumulative percentage distribution agreeing with the position of dotted lines would meet with the National Map Accuracy Standards for a contour interval of 2.0 ft . The continuous lines shown in Figure 15 indicates the degree of agreement or disagreement of the results of the stereotope operation with the Accuracy Standards requirement.


Figure 14. Errors in unadjusted stereotope data plotted against cumulative frequency distribution (after Sahgal, 29).

The C-factors computed for the cross-section data, with the contour intervals corresponding to the National Map Accuracy Standards, were read from figures of cumulative percentage distribution of errors. For the stereotope 90 percent of the discrete points were found to be within $\pm 2.4 \mathrm{ft}$ and 3.0 ft for the unadjusted and adjusted stereotope cross-section values. With a flying height of $1,650 \mathrm{ft}$, the following $\mathbf{C}$-factors were computed: stereotope unadjusted, 688; stereotope adjusted, 550.

A further accuracy test was devised in determination of the precision of the parallax readings withın a given model. This test was carried out using 32 points in one model selected at random; i.e., along the median, slopes, top of pavement, top of buildings, and on ground areas. The floating dot was placed on the image always from above. The "true reading" for any point was taken as the arithmetic mean of the ten readings on each point. The errors were computed for the ten readings in terms of the standard deviation for the 320 readings taken with each instrument. For the stereotope the standard deviation was found to be equal to $\pm 0.02 \mathrm{~mm}$ of X -parallax, which is equivalent to $\pm 0.32 \mathrm{ft}$ of ground height. One may speculate that the error is explained as a combination of the following: (a) inherent error in the stereotope, (b) the photographic quality (poor image contrast), and (c) erratic visual acuity of the operator. Comparing this error of 0.32 ft with the over-all standard deviation of 0.76 ft , one may speculate that the balance of the error is primarily random. For the Wild A-7 autograph the corresponding error in reading precision was found to be equivalent to $\pm 0.21 \mathrm{ft}$.

## Earthwork Computations

Another measure of accuracy of the stereotope was based on earthwork quantities. Both the adjusted and the unadjusted values observed in the instrument's operation were again compared with the Wild A-7 autograph measurements. The computations were made on the IBM 650.


Figure 15. Errors in adjusted stereotope data plotted against cumulative frequency distribution (after Sahgal, 29).

To facilitate the card-punching for volume computations, the earthwork cross-section data were tabulated for presentation on profile-grade data forms. The cross-section data were recorded in the conventional method used in field surveys.

There were several sets of cross-section data prepared as input for earthwork computations: (a) unadjusted sections from the stereotope, (b) adjusted sections from the stereotope, and (c) the original ground line sections.

The earthwork program instructed the electronic computer to use the original and final cross-section data and by means of the conventional end-area method to determine the earthwork volumes for each set of instrument cross-section readings.

Table 3 shows that the excavation volumes contained the largest errors. When adjustment to centerline profile was made, the accuracy of the earthwork volume data did not improve; i.e., it degraded from +1.4 to -3.8 percent, thus verifying the callbration test.

## Results Using Kelsh Plotter

Comparing the spot-elevation accuracy of the stereotope with that of the Wild A-7 autograph gives one picture of instrumental performance that can be expected. However, how well a third order instrument performs in comparison with a second order instrument in general use in the United States is of interest. Therefore, this study also included evaluating the spot-herght accuracy of the Kelsh plotter. The same photography was used and all the accuracy tests enumerated above were applied to the data procured by the plotter. For all models and 2,042 readings, the values for the standard deviation for a given arithmetıc mean were $\pm 0.94$ and $\pm 0.05 \mathrm{ft}$, respectively, but when adjusted to the true elevation of the centerline profile, the standard deviation 1 m proved to $\pm 0.69 \mathrm{ft}$ and the arithmetic mean was -0.06 ft . The C -factor was computed to be 550 initially; however, with adjustment, the C-factor improved to 750. The earthwork volume error was +1.4 percent before and -1.1 percent after data adjustment. Furthermore, the precision of readings or spot-height pointing error was equivalent to $\pm 0.36 \mathrm{ft}$. It can now be observed that the stereotope performed with similar results to those of the plotter to procure cross-sections data for earthwork volume calculations.

## EVALUATION OF ZEISS STEREOTOPE

The comparison of the stereotope, a third order instrument, with a first order instrument may hardly seem worthwhile. Most highway engineers would be reluctant to make such a study. And the results of this study will be seriously questioned by photogrammetrists. Yet there is value in learning that under certain conditions the stereotope compares favorably with, at least, second order instruments, such as the Kelsh plotter. The primary comparison is one of accuracy of spot-height elevations. Using

TABLE 3
EARTHWORK QUANTITIES IN TERMS OF ERRORS BASED ON COMPUTATIONS OF EXCAVATION AND EMBANKMENT VOLUMES ${ }^{a}$

| Unadjusted Stereotope |  |  |  | Adjusted Stereotope |  |  |  | $\begin{gathered} \text { Over-all } \\ \text { Error } \\ \text { (\%) } \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Volume(cu yd) |  | Error (\%) |  | Volume(cu yd) |  | Error (\%) |  |  |  |
| Fill | Cut | Fill | Cut | Fill | Cut | Fill | Cut | Unadj. | Adj. |
| 14,285 | 3,134 | +19.6 | -25. 7 | 10,238 | 5,439 | -14.3 | +29.0 | +44.3 | -39.1 |
| 10,929 | 2,467 | +14.3 | -37.5 | 6,951 | 5,225 | -27.3 | +32.4 | +50.7 | -69.3 |
| 22,980 | 3,019 | +6.7 | + 0.6 | 19,936 | 4,057 | -7.4 | +35.1 | $+7.7$ | -14.3 |
| 99, 097 | 2,338 | + 1.9 | $+9.7$ | 98,385 | 2,036 | + 1.2 | -4.7 | + 1.7 | +1.3 |
| 116,056 | 2,902 | - 1.3 | - 6.3 | 118, 274 | 2,819 | + 0.6 | -9.0 | -0.4 | + 0.9 |
| 2,767 | 13,236 | - 3.15 | +11.0 | 3,146 | 13,399 | -22.2 | +12.3 | +32.8 | +30.0 |
| 16,916 | 5,270 | - 8.4 | + 9.6 | 18,442 | 4,791 | -0.1 | -0.4 | -14.7 | 0.00 |
| 283, 033 | 32,366 | $+0.96$ | +2.3 | 275, 382 | 37,766 | - 1.8 | +14.0 | +1.4 | - 3.8 |

[^2]the standard of comparison (the Wild A-7 autograph), it was learned in the first part of the study that the cross-section elevations read with the stereotope on dimensionally stable print film made from non-tilted negatives exposed by a distortionless lens in close-up type photography had a mean and probable error equivalent to $1 / 3,900$ and $1 / 5,900$ of the "flıght heıght," respectively, the "flight height" being 23 ft . Thesefindings justified the second part of the study; i.e., to determine the use of the stereotope in highway engineering, large-scale mapping-type photogrammetric practice.

The results compared with those from the autograph, used as the standard of accuracy, indicated that errors in the results from the stereotope operation have a standard deviation of $\pm 0.76 \mathrm{ft}$ (for 2,042 points in elght models) with an arithmetic means of +0.07 . As a measure of accuracy, the earthwork volumes calculated from crosssections measured by the stereotope were in error by +1.4 percent. The C-factor calculated for the stereotope was 688. Because its accuracy was of a similar order to the Kelsh plotter, it can be stated that use of the former for data processing for highway engineering is warranted; i.e., for earthwork quantity and other large-scale mapping measurements having similar accuracy requirements.

When the idea of testing a third order photogrammetric instrument for highway engineering large-scale mapping purposes first was initiated in 1958, it was thought that the stereotope was a competitively priced instrument at about $\$ 2,500$. Currently it is available for about $\$ 4,500$. The increase in price is partly due to the higher value of the West German mark relative to the dollar that has occurred in the past year. With the Kelsh plotter at $\$ 6,500$ and the Balplex at $\$ 5,000$, one wonders if the stereotope is price-competitive, even though it has met the accuracy tests with favor.

Three highway departments have this instrument currently in use. There are eight government agencies, eight educational institutions, and nine private firms that have one in operation in the United States. It is believed that this instrument is used, in the latter cases, primarily for small-scale mapping.

## CONCLUSIONS

The results of this two-part study have brought forward some interesting, although not absolutely conclusive, ideas:

1. Under controlled conditions, such as distortionless, non-tilted, close-up photography printed on stable film, vertical accuracies with mean and probable errors of $1 / 3,900$ of flight height, $1 / 5,900$ of flight height and C-factor of 2,230 respectively can be expected.
2. Errors are greater in tilted photographs and affect the spot-heights of crosssections falling within the central portion of the neat model. The maximum error could be as high as $1 / 125$ of the flight height.
3. In practice, the spot-height measurements of cross-sections by means of the stereotope can be expected to have similar accuracies to those made with the Kelsh plotter.
4. The adjustment of stereotope-measured cross-sections to known centerline profile data does not increase the vertical or earthwork-quantity accuracies.
5. The volumes of earthwork quantities computed by means of stereotope-measured cross-sections have reasonable errors of less than 5 percent.
6. The C-factors for data procured by means of the stereotope and the Kelsh plotter proved to be much lower than expected, an observation that leads to the conclusion that further study of both instruments is in order.

## RECOMMENDATIONS FOR FURTHER STUDY

The calibration-test photography, data, and instrumentation have interesting possibilities; i.e., the vertical photography would be useful in testing operator accuracy, acuity, and efficiency. The effects of various lighting on the subsequent surface texture and to effects on photogrammetric measurements would be a useful study.

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# Remote Base Line Method of Measuring Horizontal and Vertical Control 

WILLIAM T. PRYOR, Chief, Aerial Surveys Branch, Interstate Division, Office of Engineering, Bureau of Public Roads
$\bullet$ IN CURRENT highway engineering practice much of the essential surveying required in the reconnaissance stages and in the preliminary survey stage of highway location and design is accomplished by aerial photogrammetric methods. Wherever a high degree of accuracy is required from such methods, as it is in the preliminary survey and design stage, accurate horizontal and vertical control is essential for scaling and leveling the aerial photographs in precision photogrammetric instruments before any measurements are made or mapping is done by use of the photographs. This is necessary whether the photographs are vertical, or are obliquely convergent in or obliquely transverse to the line of aircraft flight.

By use of electronc measuring instruments and precision theodolites, basic horizontal and vertical control can now be established quite easily to third and second orders of accuracy, as required. From specific points in the system of basic control on each survey project, appropriately placed and spaced at a convenient interval along the lengthwise direction of the highway route, supplemental horizontal and vertical control must be measured for each stereoscopic model that will be required from the aerial photography for mapping and making other measurements needed to design the highway location and prepare highway construction plans. A large portion of the expenditures in all control surveying for highway surveys done photogrammetrically is for supplemental control.

Historically, the subtense bar and trig-traverse method have been employed, as well as the usual traverse and triangulation methods, in establishing supplemental control. Recently, trilateration methods of measuring the length of all three sides of each triangle in a network of basic control has become possible by use of electronic measuring instruments, separately known as the geodimeter, tellurometer, and electrotape model DM 20; also these instruments are used to measure the slope distances in precise traverses for which precision theodolites are used to measure the horizontal and vertical angles. All of these methods have particular applications in which they are advantageous and others in which they are disadvantageous.

With the subtense bar, accuracy decreases in direct proportion to the distance the bar is away from the angle-measuring instrument. Also, there is need for an exact method of automatically making corrections should the subtense bar not be truly perpendicular to line of sight when the angle measurements are made. A variety of random errors, ranging from small to large magnitudes, usually occur when only traverse methods are employed. Sufficient and consistent precision can be attamed in all triangulation provided the topography and land use permit the attainment of appropriately shaped traingles and quadrilaterals. Usually in the preliminary stage of highway surveys, however, such shaping is difficult and often impossible to achieve. This is because of the small size and numerous triangles and quadrilaterals that must be measured if triangulation is relied on for establishing all the supplemental control required along the photographed route for which a narrow band of topography is to be mapped. The trig-traverse method has its physical limitations, and unless the base line is varied proportionately in length to the distance between it and the angle-measuring instrument and to the accuracy needed, accuracy will decrease, as with the subtense bar, the farther the base line is from the instrument. Electronic measuring requires adequate visibility along each leg of each triangle, and is more efficiently and effectively accomplished on long rather than short distances.

To overcome some of the weaknesses and physical operational problems inherent in the preceding methods, the author developed the remote base line method. This method admits of easy variation in the length of the base line according to accuracy requirements, sighting conditions, character of topography, and type and intensity of land use along the route to be surveyed. Effectiveness of the method is greatest where traffic and other land uses, vegetation, and topography greatly interfere with, if not prohibit, use of the usual traverse and electronic measuring methods, and where triangulation methods cannot be used efficiently.

The remote base line method is based on the principle of mathematically passing a curcular curve through three points-the setup point of the angle-measuring instrument and tack points in two stakes, each constituting an end of the remote base line. The fourth measurement point is a tack in a stake on the remote base line somewhere between its ends. The fourth point is necessary for accomplishing two angle measurements required for computing the distance from the point where the angle-measuring instrument is set up to the remote base line.

Electronic methods of computation make the remote base line method more effective and efficient than it would have been a number of years ago when computations were made by use of logarithms and/or desk-type calculators. Now, all the field engineer has to do is accurately measure short distances and small angles, keep appropriate notes, and let the electronic computer test the accuracy of his measurements and determine all horizontal distances, bearings and/or azımuths, plane coordinates, and elevations. Then the essential supplemental control data (horizontal and vertical) are immediately available in usable form for photogrammetric accomplishment of the mapping and for making all other measurements required in performance of the highway engineering, as cadastral surveys, and profile and cross-sections.

## PRINCIPLES OF METHOD

The basic traverse measured by the remote base line method is shown in the center of Figure 1, and supplemental control points appear on either side of this traverse. Although the area encompassed by such a control survey may be extensive, the only distance measurements required are the horizontal length of two segments of the base of each triangular figure.

In Figure 2, points B, F, and D are visible from the angle-measuring instrument at point A. Point A may be a triangulation station or a point on a traverse, the plane coordinates of which are known or can be readily determined. Angles $\mathbf{W}, V_{1}$, and $V_{2}$ are measured with a precision theodolite with which angles can be read to the nearest second of arc. An instrument measuring angles to a fraction of a second of arc will assure the attainment of better results in less time. Angle $W$ establishes the direction of the line of sight $A B$, and in combination with angles $V_{1}$ and $V_{2}$ also establishes the direction of lines of sight AF and AD. The line of sight AF and the remote base line BFD need not be perpendicular to each other. A skew of $30^{\circ}$ may be used without decreasing accuracy of the distances determined from the measured angles and the measured base line segments $b_{1}$ and $b_{2}$, which are represented by lines BF and FD, respectively, provided $b_{1}$ and $b_{2}$ are of sufficient length to compensate for the effect of a skew in the remote base line.

The greater the skew, however, the more exactly points B, F, and D must be on a straight line.

It is intended, in the usual case, that tacks be placed in wooden stakes driven in the ground to mark the position of and to identify the end points B and D, and the intermediate point $F$ on the remote base line. Aligning these points by instrument is not necessary. The line on which they are placed may be established by a taut tape. Exception to the taut tape method of aligning the three points to form the remote base line need be made only when a large angle of skew is necessary between the line of sight and the base line. The position of each stake must be visible from the instrument. A line rod consecutively centered on the tack in each of the three stakes may provide the sight for angle measuring of long distances. A plumb bob centered and carefully stabilized over each tack would be better for short distances. Better results will be
achieved more consistently under all circumstances, however, if sighting targets mounted on optical plumbing tripods are used and accurately centered over each of the three base line points. Length of base line segments $b_{1}$ and $b_{2}$ need not be equal. In the skew position, the segment with its end farther from the instrument should be longer (Fig. 3).

The combined length of $b_{1}$ plus $b_{2}$ should be increased in direct proportion to the distance the base line is from the angle-measuring instrument to attain uniformity in accuracy of distances determined from the length of each segment of the remote base line and the measured angles. Accuracy will also be improved by using a calibrated tape and taking into consideration the pounds of tension on the tape, using the tape in a


Figure 1. Remote base line and angle method of basic traverse and supplemental control surveying.
supported position wherever possible throughout the length of each segment measured, and correcting each taped distance for the effects of temperature on its length, because each base line segment will be used in computing distances from instrument to the base line. For example, an error of 0.01 ft in measurement of base line segments $\mathrm{b}_{1}$ and $\mathrm{b}_{2}$ when they are 50 ft long (unless correction is made for such error) will automatically reduce the accuracy of distances computed from the measured angles and base line segments to $1: 5,000$, and an error of 0.01 ft in a base line segment that is 100 ft long reduces the accuracy of distances determined therefrom to $1: 10,000$. Other errors in length of base line segments, such as those resulting from not reading the tape to the smallest possible increment in dimension, will affect the basic accuracy in a proportional manner. All errors in base line measurements are algebralcally additive to the consequences of error in angle measurements. For desirable precision, therefore, it is essential that the length of each base line segment be measured as accurately as possible and adjusted for variations in length due to temperature. This will require use of a tape with graduations of 0.01 ft and careful estimation of smaller dimensions.

By measurement of vertical angles as well as the horizontal angles, the elevation of each essential point can be determined to sufficient accuracy for supplemental control in mapping by photogrammetric methods.

If the survey party stakes the three points on the remote base line where they can be


BASE LINE B-D IS MEASURED TO COMPUTE DISTANCES A-B AND A-D
FROM INSTRUMENT AT POINT A, A TRIANGULATION STATION OR A TRAVERSE
POINT. USING ANGLES $V_{1}$ AND $V_{2}$ MEASURED AT THAT POINT
Figure 2. Remote base line.method for determining horizontal distances.
seen from the instrument without need for any clearing of lines of sight, and measures the length of each base line segment with a calibrated tape supported throughout its length, only a short time would be needed by the instrumentman to turn, read, and record the angles measured between these points. If several taping crews are used so


Illustrated are the required length of Separate Segments $b_{1}$ and $b_{2}$ of the Remote Base Line and relative size of Angles $V_{1}$ and $V_{2}$ for achieving same accuracy as attanable if the base line were in position $B^{\prime} F D^{\prime}$ nearly perpendicular to the central line of sight $A F$

Figure 3. Effects of remote base line being skewed.
that one crew is always in position to give the instrumentman a reading while the other crews are getting into position on another base line, the control surveying work by this method would proceed rapidly.

Figure 1 shows that a remote base line need be measured only at every other basic control point. If time is taken to measure a remote base line at each basic control point, however, a common side for the basic control traverse can be computed from both the forward and backsight measurements. This procedure provides a check on both the measurements and the computations.

## Equations

The radius of the circle passing through points A, B, and D (Fig. 2) canbe computed when desired by use of either of these equations:

$$
R=\frac{\left(b_{1}+b_{2}\right)}{2 \sin \left(V_{1}+V_{2}\right.}
$$

or

$$
R=\frac{\left(b_{1}+b_{2}\right) \operatorname{cosec}\left(V_{1}+V_{2}\right)}{2}
$$

Point E is at the intersection of the circle and an extension of the line of sight AF. In these equations, $b_{1}$ and $b_{2}$ are the separate segments of the remote base line from points $B$ to $F$ and $F$ to $D$, respectively, and $V_{1}$ and $V_{2}$ are the angles measured by the instrument at point $A$ between sightings on points $B, F$, and $D$.

The distances to be computed from the measured angles and measured base line segments are $X=A B, Y=A D$, and $Z=A F$. Through the similarity of triangle $B F E$ to triangle AFD and triangle DFE to triangle AFB, and application of the law of sines and cosines, equations were derived for computation of distances $\mathbf{X}, \mathrm{Y}$, and Z . For each distance, an alternate equation is offered. The numerator remains the same in the alternate equation, but the denominator changes from principal terms of $b_{1}$ and $\sin V_{1}$ to $b_{2}$ and $\sin V_{2}$.

$$
X=\sqrt{\frac{\left(b_{1}\right)^{2}\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{2}}{\left(b_{1}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)+\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{1}-2 b_{1}\left(b_{1}+b_{2}\right) \cos V_{2} \sin V_{1} \sin \left(V_{1}+V_{2}\right)}}
$$

and,

$$
X=\sqrt{\frac{\left(b_{1}\right)^{2}\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{2}}{\left(b_{2}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)+\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{2}-2 b_{2}\left(b_{1}+b_{2}\right) \cos V_{1} \sin V_{2} \sin \left(V_{1}+V_{2}\right)}}
$$

Also;

$$
Y=\sqrt{\frac{\left(b_{2}\right)^{2}\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{1}}{\left(b_{1}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)+\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{1}-2 b_{1}\left(b_{1}+b_{2}\right) \cos V_{2} \sin V_{1} \sin \left(V_{1}+V_{2}\right)}}
$$

and,

$$
Y=\sqrt{\frac{\left(b_{2}\right)^{2}\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{1}}{\left(b_{2}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)+\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{2}-2 b_{2}\left(b_{1}+b_{2}\right) \cos V_{1} \sin V_{2} \sin \left(V_{1}+V_{2}\right)}}
$$

Also;

$$
Z=\sqrt{\frac{\left(b_{1}\right)^{2}\left(b_{2}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)}{\left(b_{1}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)+\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{1}-2 b_{1}\left(b_{1}+b_{2}\right) \sin V_{1} \sin \left(V_{1}+V_{2}\right) \cos V_{2}}}
$$

and,

$$
Z=\sqrt{\frac{\left(b_{1}\right)^{2}\left(b_{2}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)}{\left(b_{2}\right)^{2} \sin ^{2}\left(V_{1}+V_{2}\right)+\left(b_{1}+b_{2}\right)^{2} \sin ^{2} V_{2}-2 b_{2}\left(b_{1}+b_{2}\right) \sin V_{2} \sin \left(V_{1}+V_{2}\right) \cos V_{1}}}
$$

Separate use of each of the two equations in an electronic computer program for $\mathbf{X}, \mathbf{Y}$, or Z , whichever distance is required, provides a reasonable test of the consistency attained in measurement of the angles and the separate segments of the remote base line. If the distance obtained from one equation is equal or nearly so to the distance obtained from the alternate equation, it can be assumed that the angle measurements and base line segment measurements are correct, or that they are equally smaller or larger than their true values. Either of the latter two situations is more unlikely than the first. Consequently, attainment in computations from the alternate equations of near equality within accuracy limits, as perhaps $1: 5,000$ or $1: 10,000$, of the distance from instrument setup point to base line point is good assurance that angles $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ were measured correctly and that distances $\mathrm{b}_{1}$ and $\mathrm{b}_{2}$ are likewise measured correctly.

## Strength of Triangles

Efficiency in making the measurements is improved when the remote base line is as nearly perpendicular to the line of sight from point $A$ to point $F$ as possible (Fig. 2). Also, distances $b_{1}$ and $b_{2}$ should be as nearly equal as possible. Under such circumstances, whenever $b_{1}$ and $b_{2}$ are measured with the same accuracy and are not equal in length, the accuracy of distance determinations from $A$ to $B(X)$ and from $A$ to $D(Y)$ will not be equal, provided the error in measurement of angles $V_{1}$ and $V_{2}$ is equal. The accuracy attainable in distances $X$ and $Y$ is directly proportional to the magnitude of angles $V_{1}$ and $V_{2}$, and to the straightness of the base line especially when it is greatly skewed with respect to the line of sight AF (Z).

## Angular Error

Whenever the angular measurements between the separate end points and the intermediate point on the remote base line are larger by an amount (e), than the true values of $V_{1}$ and $V_{2}$, the computed distances from points $B$ and $D$ on the base line to the instrument point $A$ would be distances $d_{5}$ and $d_{6}$, as shown in Figure 4. Whenever the measurement is smaller by the same amount, the computed distances from the same points of the base line to the instrument would be distances $d_{3}$ and $d_{4}$. The distances determined from correctly measured angles would be $d_{1}$ and $d_{2}$, respectively. The effect of errors, $e$, in measurement of angles is the recording and using of values that are larger or smaller than true values for angles $V_{1}$ and $V_{2}$. Thus e may be plus or minus in quantity, resulting from such factors as errors in instrument, in sighting, in reading, and in atmospheric conditions.

Distances $X, Y$, and $Z$ are inversely proportional to the error $e_{1}$ and $e_{2}$ of angles $V_{1}$ and $V_{2}$, respectively. A negative $e$ will cause the computed distances to be toolarge. A positive e will cause the computed distances to be too small.

The denominator of the fraction expressing accuracy is designated "a." For example, if the accuracy achieved was $1: 10,000$, then a is 10,000 . This denominator can be computed by use of equations separately containing the sine of the applicably measured angle and the sine of the same angle plus or minus the angular error e (see Fig. 4 for visualization of the angles for which the respective sine is used in these equations).

Based on angle $V_{1}$,

$$
+a_{1}=\frac{\sin \left(V_{1}-e\right)}{\frac{\sin V_{1} \sin (m-e / 2)}{\sin m}}-\sin \left(V_{1}-e\right)
$$

$$
-a_{1}=\frac{\sin \left(V_{1}+e\right)}{\sin \left(V_{1}+e\right)-\frac{\sin V_{1} \sin (m+e / 2)}{\sin m}}
$$

Based on angle $\mathrm{V}_{2}$,

$$
+a_{2}=\frac{\sin \left(V_{2}-e\right)}{\frac{\sin V_{2} \sin (m+e / 2)}{\sin m}-\sin \left(V_{2}-e\right)} \quad-a_{2}=\frac{\sin \left(V_{2}+e\right)}{\sin \left(V_{2}+e\right)-\frac{\sin V_{2} \sin (m-e / 2)}{\sin m}}
$$

It is readily apparent from Figure 4 that the larger incremental error ( $d_{3}-d_{1}$ ) occurs when $-e$ is used rather than $+e$. In applying these equations, with the $V$-angles the same and the e's equal but opposite in sign, the computed +a values will be smaller numerically than the -a's. Consequently, conservatively speaking, the +a values should be used in determining accuracy.

As the angles ( $V \pm e$ ) decrease, when the base line segments $b_{1}$ and $b_{2}$ remain constant in length, the $X, Y$, and $Z$ distances increase. The beneficial effect of achieving a decrease in error $e$ is to admit the use of a greater distance between the angle-measuring instrument and base line without decreasing accuracy of the distance measurements. When e has been reduced as much as possible, the only other way in which accuracy can be increased is to increase the size of angles $V_{1}$ and $V_{2}$ by lengthening the base line segments $b_{1}$ and $b_{2}$.

## Use of Chart on Accuracy in Distance from Instrument to Remote Base Line

The accuracy chart (Fig. 5) indicates the size of angles required for attainment of various accuracies, when the error e in seconds, in measurement of the V-angles, can be anticipated or is known. The ordinate of the chart represents the accuracy. The abscissa of this chart represents the separately measured V-angles in degrees. If only a taut tape is used to align the three points, use of the remote base line method for determining horizontal distances becomes somewhat physically impractical for V -angles greater than $30^{\circ}$; therefore, larger angles are not included. Ascertaining e is easy. Simply, the error of closure is computed as an accuracy fraction, and this accuracy is used as subsequently outlined, step by step, with respect to the applications of Figure 5.

The line representing each angular error splits into two diverging lines. For each angular error the upper line represents the accuracy attainable from each $V$-angle when the m -angle is $60^{\circ}$ (Fig. 4). This condition occurs when the base line has been rotated $30^{\circ}$ from a normal position around point $F$, and the end point of the segment of the base line to which the m -angle is measured is moved farther away from the point of instrument setup. For each possible V-angle, a skew in this direction increases the accuracy from that which is obtainable for the same angle when the base line is in the normal position. The lower line on the chart for each angular error represents the accuracy attainable from each angle when $m$ is $120^{\circ}$, and the end point of the applicable segment of the base line is moved toward the instrument setup point. This skew of the base line causes a decrease in accuracy as compared to the same angular measurement to a base line in the normal position. The normal position of the base line will produce an accuracy for each respective error in measurement of the V -angles which will lie halfway between the accuracies attainable in the separate skewed positions of the base line.

From Figure 3, as the $60^{\circ}$ position of the base line produces greater accuracy and the $120^{\circ}$ position less accuracy for the same $V$-angles, it canbe stated that any skewing of the base line results in increased accuracy on the side where m is less than $90^{\circ}\left(\mathrm{m}_{2}\right)$ and decreased accuracy on the side where $m$ is greater than $90^{\circ}\left(m_{1}\right)$. To compensate for this variation in accuracy, caused by skew of the base line, $V_{1}$ must be larger than $\mathrm{V}_{1}$ ' for the base line segment nearer the instrument, and $\mathrm{V}_{\mathbf{2}}$ should be smaller than $\mathrm{V}_{2}$ ' for the base line farther from the instrument.

To assure attainment of the required size in each angle for achievement of accuracy required in distance determinations, taking the expected error in angle measurements into consideration, the length required in each segment of the remote base line can be determined by use of the empirical equations in Table 1. In these equations, V is the number of degrees required, as ascertained from Figure 5 according to the accuracy required and the error expected in measurement of the angles; and Z is the number of $100-\mathrm{ft}$ stations from the angle-measuring instrument to the intermediate point on the base line.


Figure 4. Effects of errors in measurement at instrument point $A$ of angles $V_{1}$ and $V_{2}$.

The exact equations for computing the required length of the separate segments $b_{1}$ and $b_{2}$ of the remote base line, in terms of the sine of $V_{1}$ and $V_{2}$, respectively, and $Z$ ( $100-\mathrm{ft}$ stations) are
if $b_{1}$ is the near segment, $b_{1}(100-f t$ stations $)=\frac{Z \sin V_{1}}{\sin \left(m_{1}-\overline{V_{1}}\right)}$
if $b_{2}$ is the farther segment, $b_{2}(100-f t$ stations $)=\frac{Z \sin V_{2}}{\sin \left(m_{2}-V_{2}\right)}$


Figure 5. Accuracy in distance from instrument to remote base line according to seconds of error in the angles measured in degrees.

TABLE 1
EQUATIONS FOR COMPUTING LENGTH OF REMOTE BASE LINE SEGMENTS

| Position of Base Line with Respect to Line of Sight on Intermediate Point |  |  |
| :--- | :--- | :--- |
| Perpendicular | Skewed |  |
| Both Segments | Nearer Segment | Farther Segment |
| No. of $\mathrm{ft}=1.75 \mathrm{~V} \mathrm{Z}$ | No. of $\mathrm{ft}=2.0 \mathrm{~V} \mathrm{Z}$ | No. of $\mathrm{ft}=2.5 \mathrm{~V} \mathrm{Z}$ |

To ascertain from the chart (Fig. 5) the angles required to achieve a desired accuracy, when an angular error is anticipated, the procedure is as follows:

1. On the left side of the chart the line at the ordinate representing the accuracy required is selected.
2. That line is horizontally followed to the right to where it intersects the line representing the seconds of angular error expected.
3. From this intersection, one projects vertically to the abscissa of the chart and the minimum V -angle (in degrees) that must be measured is read.
4. From Table 1 the length of each segment of the base line that will provide that angle is determined.

As the chart of Figure 5 has a logarithmic scale in each direction, it is necessary to use a scale of the proper logarithmic cycle length when interpolating. By placing such a scale vertically (parallel with the ordinate) on the chart, lines representing other angular errors can be interpolated as desired. V-angles, other than those for which lines are drawn on the chart, can be interpolated by using the same logarithmic scale in a horizontal (parallel to the abscissa) position.

Knowing any two values, the third value may be ascertained from this chart. Thus the error in measurement of the $V$-angles which must not be exceeded can be found when the accuracy to be attained and the V-angle measured or to be measured are known.

An equation has been derived for approximation of the curves on Figure 5. The equation ignores the deviation from a straight line for the larger values of measured angles, and results for $V$-angles greater than $15^{\circ}$ are conservative. The equation provides the minimum $\mathbf{V}$-angle that should be measured from the instrument setup position between the intermediate point $F$ on the remote base line, and the ends, $B$ and D, of that line to assure an accuracy of one part in so many thousand feet in the distance from the instrument to the base line, depending on the seconds of error expected in measurement of the angles.

$$
\mathrm{V}=\mathrm{aek}^{2}
$$

in which
$\mathrm{V}=$ measured angle, in degrees;
a = denominator of fraction expressing accuracy;
$\mathrm{e}=$ seconds of error in measured angle; and
$k=$ the constant $1 / 80$.
With denominators represented by the usual orders of accuracy substituted for a in the equation, the equation for each particular accuracy may be reduced,
for an accuracy of $1: 2,500$, to $\quad V=\frac{25 e}{36}$
for an accuracy of $1: 5,000$, to $\quad V=\frac{25 e}{18}$
for an accuracy of $1: 10,000$, to $V=\frac{25 e}{9}$
for an accuracy of $1: 25,000$, to $V=\frac{125 e}{18}$

Table 2 was prepared by using the preceding equations to compute the allowable $e$ in measured V-angles for first, second, third, and fourth order accuracies. These values are independent of the distance between the instrument and the base line.

Tables 3, 4, 5, and 6, each for a different order of accuracy, indicate the distance admissible from the instrument to the ends of the remote base line according to error

TABLE 2
ALLOWABLE e IN MEASURED ANGLES FOR FIRST, SECOND, THIRD, AND FOURTH ORDER ACCURACIES

| v <br> (deg) | $\mathrm{e}(\mathrm{sec})$ |  |  |  |
| :--- | :---: | :---: | :---: | ---: |
|  | $1: 25,000$ | $1: 10,000$ | $1.5,000$ | $1: 2,500$ |
| 0.25 | 0.036 | 0.09 | 0.18 | 0.36 |
| 0.50 | 0.072 | 0.18 | 0.36 | 0.72 |
| 0.75 | 0.108 | 0.27 | 0.54 | 1.08 |
| 1.00 | 0.144 | 0.36 | 0.72 | 1.44 |
| 1.25 | 0.180 | 0.45 | 0.90 | 1.80 |
| 1.50 | 0.216 | 0.54 | 1.08 | 2.16 |
| 1.75 | 0.252 | 0.63 | 1.26 | 2.52 |
| 2 | 0.288 | 0.72 | 1.44 | 2.88 |
| 3 | 0.432 | 1.08 | 2.16 | 4.32 |
| 4 | 0.576 | 1.44 | 2.88 | 5.76 |
| 5 | 0.720 | 1.80 | 3.60 | 7.20 |
| 6 | 0.864 | 2.16 | 4.32 | 8.64 |
| 7 | 1.008 | 2.52 | 5.04 | 10.08 |
| 8 | 1.152 | 2.88 | 5.76 | 11.52 |
| 9 | 1.296 | 3.24 | 6.48 | 12.96 |
| 10 | 1.44 | 3.6 | 7.2 | 14.4 |
| 15 | 2.16 | 5.4 | 10.8 | 21.6 |
| 20 | 2.88 | 9.0 | 14.4 | 28.8 |
| 25 | 3.60 | 10.8 | 21.6 | 46.0 |
| 30 | 4.32 |  |  | 43.2 |

TABLE 3
MAXIMUM ALLOWABLE DISTANCE FROM INSTRUMENT TO ENDS OF REMOTE BASE LINE FOR VARIOUS VALUES OF e AND b FOR ACCURACY OF 1:25, 000

| e | Maximum Allowable Distance (ft) for Segment $\mathrm{b}_{1}$ or $\mathrm{b}_{2}$ of Remote Base Line of |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3.28 Ft | 5 Ft | 6 Ft | 25 Ft | 50 Ft | 75 Ft | 100 Ft | 125 Ft | 150 Ft | 200 Ft |
| 0.1 | 270 | 412 | 495 | 2,062 | 4,125 | 6,187 | 8,250 | 10,313 | 12,375 | 16,501 |
| 0.2 | 135 | 206 | 247 | 1,031 | 2,062 | 3,093 | 4,125 | 5,156 | 6,187 | 8,250 |
| 0.3 | 90 | 137 | 165 | 687 | 1,375 | 2,062 | 2,750 | 3,437 | 4,125 | 5, 500 |
| 0.4 | 67 | 103 | 123 | 515 | 1, 031 | 1,546 | 2,062 | 2,578 | 3, 093 | 4,125 |
| 0.5 | 54 | 82 | 99 | 412 | 825 | 1,237 | 1,650 | 2, 062 | 2,475 | 3,300 |
| 0.6 | 45 | 68 | 82 | 343 | 687 | 1,031 | 1,375 | 1,718 | 2,062 | 2,750 |
| 0.8 | 33 | 51 | 61 | 257 | 515 | 773 | 1,031 | 1,289 | 1,546 | 2,062 |
| 1 | 27 | 41 | 49 | 206 | 412 | 618 | 825 | 1, 031 | 1,237 | 1,650 |
| 2 | 13 | 20 | 24 | 103 | 206 | 309 | 412 | 515 | 1,618 | 825 |
| 3 | 9 | 13 | 16 | 68 | 137 | 206 | 275 | 343 | 412 | 550 |
| 4 | 7 | 10 | 12 | 51 | 103 | 154 | 206 | 257 | 309 | 412 |
| 5 | 5 | 8 | 9 | 41 | 82 | 123 | 165 | 206 | 247 | 330 |

in angular measurement, from 0.1 to 30 sec , and length of base line segments, from 3.28 to 200 ft . These distances are maximum for governing conditions.

TABLE 4
MAXIMUM ALLOWABLE DISTANCE FROM INSTRUMENT TO ENDS OF REMOTE BASE LINE FOR VARIOUS VALUES OF e AND b FOR ACCURACY OF 1:10, 000

| e | Maximum Allowable Distance (ft) for Segment $\mathrm{b}_{1}$ or $\mathrm{b}_{2}$ of Remote Base Line of |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3.28 Ft | 5 Ft | 6 Ft | 25 Ft | 50 Ft | 75 Ft | 100 Ft | 125 Ft | 150 Ft | 200 Ft |
| 0.1 | 676 | 1,031 | 1,237 | 5,156 | 10,313 | 15,469 | 20,626 | 25,783 | 30, 939 | 41, 253 |
| 0.2 | 338 | 515 | 618 | 2,578 | 5,156 | 7, 734 | 10,313 | 12,891 | 15,469 | 20,626 |
| 0.3 | 225 | 343 | 412 | 1,718 | 3,437 | 5,156 | 6,875 | 8, 594 | 10,313 | 13,751 |
| 0.4 | 169 | 257 | 309 | 1,289 | 2,578 | 3,867 | 5,156 | 6, 445 | 7,734 | 10,313 |
| 0.5 | 135 | 206 | 247 | 1, 031 | 2,062 | 3,093 | 4,125 | 5,156 | 6,187 | 8,250 |
| 0.6 | 112 | 171 | 206 | 859 | 1,718 | 2,578 | 3,437 | 4,297 | 5,156 | 6, 875 |
| 0.8 | 84 | 128 | 154 | 644 | 1, 289 | 1,933 | 2,578 | 3,222 | 3,867 | 5,156 |
| 1 | 67 | 103 | 123 | 515 | 1,031 | 1,546 | 2, 062 | 2, 578 | 3,093 | 4,125 |
| 2 | 33 | 51 | 61 | 257 | 515 | 773 | 1,031 | 1,289 | 1,546 | 2, 062 |
| 3 | 22 | 34 | 41 | 171 | 343 | 515 | 687 | 859 | 1,031 | 1,375 |
| 4 | 16 | 25 | 30 | 128 | 257 | 386 | 515 | 644 | 773 | 1, 031 |
| 5 | 13 | 20 | 24 | 103 | 206 | 309 | 412 | 515 | 618 | 825 |
| 6 | 11 | 17 | 20 | 85 | 171 | 257 | 343 | 429 | 515 | 687 |
| 7 | 9 | 14 | 18 | 73 | 147 | 220 | 294 | 368 | 441 | 589 |
| 8 | 8 | 12 | 15 | 64 | 128 | 193 | 257 | 322 | 386 | 515 |
| 9 | 7 | 11 | 13 | 57 | 114 | 171 | 229 | 286 | 343 | 458 |
| 10 | 6 | 10 | 12 | 51 | 103 | 154 | 206 | 257 | 309 | 412 |

TABLE 5
MAXIMUM ALLOWABLE DISTANCE FROM INSTRUMENT TO ENDS OF REMOTE
BASE LINE FOR VARIOUS VALUES OF e AND b FOR ACCURACY OF $1: 5,000$ BASE LINE FOR VARIOUS VALUES OF e AND b FOR ACCURACY OF 1:5,000

| c | Maximum Allowable Distance (ft) for Segment $\mathrm{b}_{1}$ or $\mathrm{b}_{2}$ of Remote Base Line of |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3.28 Ft | 5 Ft | 6 Ft | 25 Ft | 50 Ft | 75 Ft | 100 Ft | 125 Ft | 150 Ft | 200 Ft |
| 0.1 | 1,353 | 2,062 | 2,475 | 10,313 | 20,626 | 30, 939 | 41, 253 | 51, 566 | 61,879 | 82, 506 |
| 0.2 | 676 | 1,031 | 1,237 | 5,156 | 10,313 | 15,469 | 20, 626 | 25, 783 | 30,939 | 41, 253 |
| 0.3 | 451 | 687 | 825 | 3,437 | 6,875 | 10,313 | 13, 751 | 17, 188 | 20,626 | 27, 501 |
| 0.4 | 338 | 515 | 618 | 2, 578 | 5, 156 | 7, 734 | 10,313 | 12,891 | 15,469 | 20,626 |
| 0.5 | 270 | 412 | 495 | 2,062 | 4,125 | 6,187 | 8,250 | 10,313 | 12,375 | 16, 501 |
| 0.6 | 225 | 343 | 412 | 1,718 | 3,437 | 5,156 | 6,875 | 8, 594 | 10,313 | 13,751 |
| 0.8 | 169 | 257 | 309 | 1,289 | 2, 578 | 3,867 | 5,156 | 6,445 | 7,734 | 10,313 |
| 1 | 135 | 206 | 247 | 1,031 | 2,062 | 3,093 | 4,125 | 5,156 | 6,187 | 8, 250 |
| 2 | 67 | 103 | 123 | 515 | 1, 031 | 1,546 | 2, 062 | 2, 578 | 3,093 | 4,125 |
| 3 | 45 | 68 | 82 | 343 | 687 | 1,031 | 1,375 | 1,718 | 2,062 | 2,750 |
| 4 | 33 | 51 | 61 | 257 | 515 | 773 | 1,031 | 1,289 | 1,546 | 2,062 |
| 5 | 27 | 41 | 49 | 206 | 412 | 618 | 825 | 1,031 | 1,237 | 1,650 |
| 6 | 22 | 34 | 41 | 171 | 343 | 515 | 687 | 859 | 1,031 | 1,375 |
| 7 | 19 | 29 | 35 | 147 | 294 | 441 | 589 | 736 | 883 | 1,178 |
| 8 | 16 | 25 | 30 | 128 | 257 | 386 | 515 | 644 | 773 | 1,031 |
| 9 | 15 | 22 | 27 | 114 | 229 | 343 | 458 | 572 | 687 | 916 |
| 10 | 13 | 20 | 24 | 103 | 206 | 309 | 412 | 515 | 618 | 825 |
| 15 | 9 | 13 | 16 | 68 | 137 | 206 | 275 | 343 | 412 | 550 |
| 20 | 6 | 10 | 12 | 51 | 103 | 154 | 206 | 257 | 309 | 412 |

TABLE 6
MAXIMUM ALLOWABLE DISTANCE FROM INSTRUMENT TO ENDS OF REMOTE BASE LINE FOR VARIOUS VALUES OF e AND b FOR ACCURACY OF 1:2,500

| e | Maximum Allowable Distance (ft) for Segment $\mathrm{b}_{1}$ or $\mathrm{b}_{2}$ of Remote Base Line of |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3.28 Ft | 5 Ft | 6 Ft | 25 Ft | 50 Ft | 75 Ft | 100 Ft | 125 Ft | 150 Ft | 200 Ft |
| 0.1 | 2, 706 | 4,125 | 4,950 | 20,626 | 41, 253 | 61,879 | 82,506 | 103,132 | 123, 759 | 165, 012 |
| 0.2 | 1,353 | 2,062 | 2,475 | 10,313 | 20,626 | 30, 939 | 41, 253 | 51, 566 | 61,879 | 82,506 |
| 0.3 | 902 | 1,375 | 1,650 | 6,875 | 13, 750 | 20,626 | 27, 501 | 34,376 | 41, 252 | 55, 002 |
| 0.4 | 676 | 1,031 | 1,237 | 5,156 | 10,313 | 15, 469 | 20,626 | 25, 783 | 30,939 | 41, 253 |
| 0.5 | 541 | 825 | 990 | 4,125 | 8, 250 | 12,375 | 16, 501 | 20,626 | 24, 751 | 33, 002 |
| 0.6 | 451 | 687 | 825 | 3,437 | 6,875 | 10,313 | 13, 751 | 17,188 | 20,626 | 27, 501 |
| 0.8 | 338 | 515 | 618 | 2,578 | 5,156 | 7, 734 | 10,313 | 12, 891 | 15,469 | 20,626 |
| 1 | 270 | 412 | 495 | 2,062 | 4,125 | 6,187 | 8, 250 | 10, 313 | 12,375 | 16,501 |
| 2 | 135 | 206 | 247 | 1,031 | 2, 062 | 3, 093 | 4,125 | 5,156 | 6,187 | 8,250 |
| 3 | 90 | 137 | 165 | 687 | 1,375 | 2,062 | 2, 750 | 3,437 | 4,125 | 5, 500 |
| 4 | 67 | 103 | 123 | 515 | 1,031 | 1,546 | 2,062 | 2,578 | 3, 093 | 4,125 |
| 5 | 54 | 82 | 99 | 412 | 1, 825 | 1,237 | 1,650 | 2,062 | 2,475 | 3,300 |
| 6 | 45 | 68 | 82 | 343 | 687 | 1, 031 | 1,375 | 1, 718 | 2, 062 | 2,750 |
| 7 | 38 | 58 | 70 | 294 | 589 | 883 | 1, 178 | 1,473 | 1, 767 | 2,357 |
| 8 | 33 | 51 | 61 | 257 | 515 | 773 | 1, 031 | 1,289 | 1, 546 | 2, 062 |
| 9 | 30 | 45 | 55 | 229 | 458 | 687 | 916 | 1,145 | 1,375 | 1,833 |
| 10 | 27 | 41 | 49 | 206 | 412 | 618 | 825 | 1, 031 | 1,237 | 1,650 |
| 15 | 18 | 27 | 33 | 138 | 276 | 414 | 550 | 691 | 829 | 1,106 |
| 20 | 13 | 20 | 24 | 103 | 206 | 309 | 412 | 515 | 618 | 825 |
| 30 | 9 | 13 | 16 | 68 | 137 | 206 | 275 | 343 | 412 | 550 |

## TEMPERATURE EFFECT ON LENGTH OF BASE LINE

The effects of changes in temperature on the length of the remote base line have been investigated and are subsequently presented. The assumptions in this investigation are that a steel tape calibrated at a temperature of 68 F is used and that the coefficient of expansion of that tape is 0.00000645 ft per foot for each degree of change in temperature. Therefore, for a $10^{\circ}$ change in temperature a $100-\mathrm{ft}$ steel tape would change 0.00645 ft in length.

In Figure 6, point A represents the correct position of the angle-measuring instrument, and points $A_{1}, A_{2}, A_{3}$, and $A_{4}$ represent imaginary points at which the true angles $V_{1}$ and $V_{2}$ are measurable between points sighted on the remote base line.

Points B, F, and D are points on the base line that are the actual horizontal distances $b_{1}$ and $b_{2}$ from each other. Distances $b_{1}$ and $b_{2}$ are not necessarily equal, but they are the exact distance between points $B$ and $F$, and $F$ and $D$, as if measured with a tape of exact length when the temperature is 68 F .

Points $B_{1}$ and $F_{1}$ represent the situation when $b_{1}$ is numerically short because correction is not made for measurement with a tape that has expanded in length; likewise, points $F_{2}$ and $D_{3}$ when $b_{2}$ is numerically short. Points $B_{2}$ and $F_{2}$ represent the situation when $b_{1}$ is numerically long because correction is not made for measurement with a tape that has contracted in length; similarly, points $F_{1}$ and $D_{4}$ when $b_{2}$ is numerically long.

One-half the error of each segment of the remote base line when too long numerically is represented by $+t / 2$ and when too short numerically by $-t / 2$. The $+t / 2$ on one end of each segment of the base line added to the other end represents the excess in length, +t , caused by making the measurements when temperatures are colder than 68 F . The two separate deficiencies in length of $-t / 2$ amount to the total deficiency, $-t$, caused by making the measurements when temperatures are warmer than 68 F .

After the measurements are made, if distances from point $A$ to the base line are


Figure 6. Effects of using longer or shorter segments for remote base line than their actual length.
computed by use of angles $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ without applying the appropriate correction, $\pm \mathrm{t}$, for temperature to each of the taped distances, $b_{1}$ and $b_{2}$, the computed distances would be too long or too short. The positive error in computed length is represented by $\mathrm{T}_{2}$ for the distance from $\mathrm{A}_{2}$ to $\mathrm{B}_{2}$, and by $\mathrm{T}_{4}$ for the distance from $\mathrm{A}_{4}$ to $D_{4}$, when $b_{1}$ and $b_{2}$, respectively, are numerically too long. The negative error in computed length is represented by $\mathrm{T}_{1}$ for the distance from $A_{1}$ to $B_{1}$, and by $T_{3}$ for the distance from $A_{3}$ to $D_{3}$, when $b_{1}$ and $\mathrm{b}_{2}$, respectively, are numerically too short.

The denominator, a, of the fraction expressing accuracy that will result from the error, $t$, when the appropriate correction is not made to attain true distance from instrument to base line by use of measurement with the tape, of each segment of the base line, is equal to $b_{1} / t_{1}$ and $b_{2} / t_{2}$ for the segments of the base line, in which $t_{1}$ is the error in length $b_{1}$, and $t_{2}$ the error in length $b_{2}$. Wherever both segments of the base line are measured simultaneously with the same tape, a, the resultant denominator of the fraction expressing accuracy, will be the same for both segments. This is because $t_{1}$ will be larger or smaller than $t_{2}$ in the same proportion $b_{1}$ is larger or smaller than $b_{2}$.

Mathematical proof of the foregoing statement is as follows:

$$
\frac{b_{1}}{\overline{A B}}=\frac{b_{1}+t_{1}}{\overline{A_{2} B_{2}}}
$$

also

$$
\frac{\mathrm{b}_{2}}{\overline{\mathrm{AD}}}=\frac{\mathrm{b}_{2}+\mathrm{t}_{2}}{\overline{\mathrm{~A}_{4} \mathrm{D}_{4}}}
$$

Then the equations for a, the denominator of the fraction expressing accuracy, is

$$
+\mathrm{a}_{1}=\frac{\overline{\mathrm{AB}}}{\overline{\mathrm{~A}_{2} \mathrm{~B}_{2}}-\overline{\mathrm{AB}}}
$$

and

$$
+a_{1}=\frac{\overline{\mathrm{AD}}}{\overline{\mathrm{~A}_{4} \mathrm{D}_{4}}-\overline{\mathrm{AD}}}
$$

by substitution for $\overline{\mathbf{A}_{2} \mathbf{B}_{2}}$ in preceding,

$$
+a=\frac{\overline{A B}}{\frac{\overline{A B}\left(b_{1}+t\right)}{b_{1}}-\overline{A B}}=\frac{b_{1}}{b_{1}+t_{1}-b_{1}}=\frac{b_{1}}{t_{1}}
$$

and

$$
-\mathbf{a}_{1}=\frac{\mathbf{b}_{1}}{\mathbf{t}_{2}}
$$

or in the general case,

$$
a_{1}=\frac{b_{1}}{ \pm t_{1}}
$$

Likewise, it can be similarly proven that

$$
+a_{2}=\frac{b_{2}}{t_{2}} \quad-a_{2}=\frac{b_{2}}{-t_{2}} \quad a_{2}=\frac{b_{2}}{ \pm t_{2}}
$$

The numerical consequences of disregarding changes in tape length caused by temperatures being colder or warmer than 68 F are given in Table 7.

## SAMPLE FIELD NOTES

The sample field notes (Table 8) suggest procedure for recording the horizontally measured angles and the length of each segment of the remote base line, and a place in the notes for recording the vertical angle measured at instrument station to a sighting on the remote base line station for which the particular measurements are applicable, the height of the instrument's axis above the station it occupies, and the reading on the level rod when it is on the particular point of the remote base line for which the vertical angle is measured.

The headings in the sample notes are considered self-explanatory. In the note-keeping system suggested by this sample, each station is numbered consecutively in the order in which it is observed in the process of measuring the required angles. Each point on the remote baseline which is an instrument point in the controls survey traverse, as points 3,5 , and 6 in the sample, are indicated by $\Delta$; likewise, point 4 , which is a supplemental control point measured by use of a remote base line to one side of the traverse. Each of the supplemental points would be targeted on the ground or would be

TABLE 7
EFFECT OF TEMPERATURE CHANGE ON STEEL TAPE

| Temperature <br> $\left({ }^{\circ} F\right)$ | True Length of <br> $100-$ Ft Tape <br> $(\mathrm{ft})$ | Error $^{\mathrm{a}}$ <br> $(\mathrm{ft})$ | Resultant Accuracy |
| :---: | :---: | :---: | :---: |
| -22 | 99.94195 | +0.05805 |  |
| -12 | 99.94840 | +0.05160 | $1: 1,722$ |
| -02 | 99.95485 | +0.04515 | $1: 1,937$ |
| +08 | 99.96130 | +0.03870 | $1: 2,214$ |
| +18 | 99.96775 | +0.03225 | $1: 2,583$ |
| +28 | 99.97420 | +0.02580 | $1: 3,100$ |
| +38 | 99.98065 | +0.01935 | $1: 3,875$ |
| +48 | 99.98710 | +0.01290 | $1: 5,167$ |
| +58 | 100.99355 | +0.00645 | $1: 7,750$ |
| +68 | 100.00000 | 0 | $1: 15,503$ |
| +78 | 100.01290 | -0.00645 |  |
| +88 | 100.01935 | -0.01290 | $1: 15,505$ |
| +98 | 100.02580 | -0.01935 | $1: 7,753$ |
| +108 | 100.03225 | -0.02580 | $1: 5,169$ |
| +118 | 100.03870 | -0.03225 | $1: 3,877$ |
| +128 | 100.04515 | -0.03870 | $1: 3,102$ |
| +138 |  | -0.04515 | $1: 2,585$ |

[^3]TABLE 8
SAMPLE FIELD NOTES, LEFT PAGE

| Instrument Station | Index Station | Remote Base Line Station |  |  | Vertical Data |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | End B | Interm F | End D |  |
| Inverted Direct | $\begin{gathered} 162^{0} 04^{\prime} 56^{\prime \prime} \\ 50^{\circ} 01^{\prime} 46^{\prime \prime} \end{gathered}$ | $\|$$350^{\circ} 09^{\prime} 58^{\prime \prime}$ $352^{\circ} 41^{\prime} 08^{\prime \prime}$ <br> $193^{\circ} 06^{\prime} 48^{\prime \prime}$ $195^{\circ} 377^{\prime} 58^{\prime \prime}$ <br> 51.831 47.212 |  | $\begin{aligned} & 354^{0} 588^{\prime} 14^{\prime \prime} \\ & 197^{\circ} 555^{\prime \prime} \end{aligned}$ | $\begin{gathered} 271^{\circ} 20^{\prime} 14^{\prime \prime} \\ 91^{\circ} 20^{\prime} 1^{\prime \prime \prime} \\ \text { HI } 5.06 \\ \text { R } \end{gathered} 4.37$ |
| 5 | 3 | 6B | $6 \Delta$ | 6D |  |
| Inverted Direct | $\begin{gathered} 177^{0} 35^{\prime} 24^{\prime \prime} \\ 11^{\circ} 05^{\prime} 26^{\prime \prime} \end{gathered}$ | $\begin{array}{\|l\|} \hline 345^{\circ} 577^{\prime} 35^{\prime \prime} \\ 179^{\circ} 27^{\prime} 37^{\prime \prime} \end{array}$ | $\begin{aligned} & 347^{\circ} 42^{\prime} 47^{\prime \prime} \\ & 181^{\circ} 12^{\prime} 49^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 348^{0} 54^{\prime} 34^{\prime \prime} \\ & 182^{\circ} 24^{\prime} 36^{\prime \prime} \end{aligned}$ | $\begin{gathered} 267^{0} 29^{\prime} 30^{\prime \prime \prime} \\ 87^{0} 29^{\prime} 31^{\prime \prime} \end{gathered}$ |
|  | 2 | 5 B 57.100 | $\mathrm{FF}^{38.605}$ | $5 \Delta$ | R ${ }^{\text {R }} 4.97$ |
| Inverted Direct | $\begin{gathered} 33^{0} 111^{\prime} 17^{\prime \prime} \\ 238^{\circ} 151^{\prime \prime} \end{gathered}$ | $\begin{array}{\|l\|} \hline 116^{\circ} 01^{\prime} 27^{\prime \prime} \\ 321^{\circ} 05^{\prime} 21^{\prime \prime} \\ \hline \end{array}$ | $\begin{aligned} & 119^{\circ} 06^{\prime} 39^{\prime \prime \prime} \\ & 324^{\circ} 10^{\prime} 33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 121^{\circ} 44^{\prime} 49^{\prime \prime \prime} \\ & 326^{\circ} 48^{\prime} 43^{\prime \prime} \end{aligned}$ | $267^{0} 13^{\prime} 13^{\prime \prime}$ $87^{\circ} 13^{\prime \prime} 14^{\prime \prime}$ |
|  |  | 30.141 | $1 \quad 25.914$ |  | HI 5.03 |
| 3 | 2 | 4B | 4F | 4 | R 6.21 |
| Inverted Direct | $\begin{array}{r} 170^{\circ} 11^{\prime} 50^{\prime \prime \prime} \\ 1^{\circ} 18^{\prime} 27^{\prime \prime} \end{array}$ | $\begin{array}{\|l\|} \hline 353^{\circ} 10^{\prime} 54^{\prime \prime} \\ 184^{\circ} 172^{\prime \prime} 32^{\prime \prime} \end{array}$ | $\begin{aligned} & 355^{\circ} 38^{\prime} 48^{\prime \prime} \\ & 186^{\circ} 45^{\prime} 26^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 358^{\circ} 41^{\prime} 32^{\prime \prime} \\ & 189^{\circ} 48^{\prime} 10^{\prime \prime} \end{aligned}$ | $\begin{array}{r} 271^{\circ} 13^{\prime} 16^{\prime \prime} \\ 91^{\circ} 13^{\prime} 18^{\prime \prime} \end{array}$ |
|  |  |  |  |  |  |
|  |  | 41.18 | 180.470 |  | HI 4.81 |
| 2 | 1 | $3 \Delta$ | 3F | 3D | R 5.98 |

a well-defined, pinpointing type of image of a natural object on the ground. Many of the instrument points on the controls survey traverse would also be targeted so as to appear on the aerial photographs when taken. The other two points on each remote base line which are not used as an instrument point are given the same sequential number as the instrument station point, but are distinguished from that point by letter postscript in the same manner the remote base line points are designated in the sample field notes, as $4 \mathrm{~B}, 4 \mathrm{~F}, 6 \mathrm{D}$, and so forth.

The length of each segment of the remote base line is recorded between the point designations on the left page of the notes (Table 8). Anticipating that direct and indirect readings in a clockwise direction would be made of the horizontal angles in order to compensate for likely errors of adjustment in the instrument, the sample notes contain index readings of the instrument, as sighted on the indexing station (backsight station), and angle readings for both direct and inverted sightings on each of the three points on the remote base line.

The vertical angle is also measured and recorded from a direct and an inverted position of the telescope of the instrument. For European-made theodolites a vertical angle reading of $90^{\circ}$ means the telescope is level in the direct sighting position and a reading of $270^{\circ}$ that it is level in the inverted position. For American-made transits the vertical angle of zero means the telescope is level. In the vertical data column the height of instrument (HI) and rod reading (R) at time the vertical angles were measured are recorded on the notation line of each instrument station.

On the right-hand page of the field notes, (not shown) the survey project traverse and the supplemental points measured are dıagramed. This diagraming is advisable to aid users of the notes in ascertaining their general and physical relationships. Whenever there is a change in temperature, the actual temperature should be noted for appropriate adjustment to actual length of the recorded length of each segment of each remote base line. The type of angle-measuring instrument and calibration length of the tape used should also be recorded in the notes.

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## Discussion

R.T. HOWE. - Have any field tests been made? Should a 1-min or a 1 -sec instrument be used?
W. T. Pryor. - The Aerial Surveys Branch has used the method for surveying supplemental control, and it has been similarly used in Region 9 of Public Roads. An instrument reading to 1 sec is used and the angle measurement estimated to a fraction of a second. It is difficult to get sufficient precision with a $1-$ min instrument and achieve an angle measurement to an accuracy of 1 sec or a few seconds of arc unless many measurements are made cumulatively and then averaged. Consequently, if such an instrument were used, the base line would have to be quite long to maintain an accuracy of $1: 5,000$ and exceptionally long to get an accuracy of $1: 10,000$. For efficiency and attaining consistent effectiveness a T-2 or T-3 type theodolite is needed.
D.E. Winsor. -With increased length of base, the accuracy is increased proportionally. We had chained a highway survey traverse 20 miles in length and with Mr. Pryor's method we took only one day to find errors causing the initially measured traverse to not close properly on the basic control.
E.S. Preston. - Methods are clearly explained in this paper. In it, errors are more completely dealt with than in any publication that comes to mind. The method is made possible by the measurement of angles directly to 1 sec of arc or smaller increment of angle measurement, and is made practicable by electronic computation of the resection.

Before the advent of electronic computers and of electronic devices for measuring distances the Ohio Department of Highways was partial to a method wherein the bases were established from subtense measurements. The ratio between distance measured and the angle measurement base was kept to about $20: 1$, except in the use of the subtense bar as base. By this practice the angle measurement base, using it like a subtense bar base, was increased from the usual 2 to as near 140 ft as possible. Pointing errors were thereby decreased. The distance between measurement stations was thus around $1 / 2 \mathrm{ml}$, which is about the average distance between hilltops in Ohio. The angle measurement base of about 140 ft used in this method of control surveying was measured by use of the subtense bar rather than the usual tape. This procedure permitted use of a smaller survey party than would otherwise be required; also, temperature, slope, and tape tension corrections were not necessary and computations were reduced to a minimum.

Such methods as those suggested in the paper should be widely practiced where electronic distance-measuring equipment is not available or applicable.

WILLIAM T. PRYOR, Closure-Mr. Preston has strengthened the meaning of the concepts and application principles stated in the paper. In so doing, he presented facts regarding special use of the subtense bar in Ohio. The subtense bar and the remote base line methods are somewhat similar. Their main difference is that one of the lines of sight in the subtense bar procedure must be perpendicular to the base. This is not a requirement in the remote base line method.

In effect, Ohio lengthens the measurement base from a bar of only 2 m to about 140 ft to increase accuracy and the distance that can be measured. The subtended angle between ends of the subtense-bar-measured base is measured from a distance of approximately $1 / 2 \mathrm{mi}$. This procedure requires instrumentation at the base to insure its perpendicularity to one of the lines of sight, as well as to measure its length. The need for perpendicularity and for unobstructed lines of sight can cause difficulties in positioning the $140-\mathrm{ft}$ base, wherever interferences occur due to vegetation, man-made structures, and/or irregularities in the topography.

As seen in Table 5, the $140-\mathrm{ft}$ base line used in Ohio, at a distance of about $1 / 2 \mathrm{mi}$ from the angle measuring instrument, admits the achievement of a measurement accuracy of $1: 5,000$ when the measured angle does not have an error of more than 2 sec , assuming the base line itself, as measured by use of the subtense bar, is exact. If the $140-\mathrm{ft}$ base line measurement is in error 0.028 ft , however, and the measured angle contains no error, the accuracy of measurement will still be only $1: 5,000$. These facts indicate strongly the importance of the remote base line being measured accurately, whether utilized by the procedures outlined in the paper or by subtense bar base in the manner explained by Mr. Preston. When precision is achieved in measurement of the remote base line and in measurement of subtended angles results will be satisfactory.

Advantages of the remote base line method over the subtense bar procedure are as follows:

1. Freedom for establishing the remote base where it can be measured with ease, with little or no interference from topography, man-made objects, or vegetation.
2. No need for instrumentation on the base.
3. Flexibility in positioning points on the remote base where they can be seen from the angle measuring instrument.

# Aerial Photography in Right-of-Way Acquisition: A Symposium 

## Semi-Controlled Aerial Photographs as a <br> Right-of-Way Surveying Tool

ERWIN D. HOVDE, Supervising Highway Engineer, California Division of Highways


#### Abstract

With ever-increasing survey costs, it becomes necessary to develop techniques and methods of acquiring information that reduce these costs and still retain accuracies that are acceptable. To use aerial photographs as a satisfactory surveying tool and map substitute, it is necessary that the photographs be enlarged precisely. To obtain precise photographic enlargements, adequate targets must be set on the ground before photography, within the area to be photographed, to obtain both an enlargement factor and scaling factors.

It has been experienced in working with aerial photographs enlarged to a scale of 20 ft to 1 in . that the determination of the dimensions of areas, such as frontage roads, that are to be conveyed to other political subdivisions saves a great deal of office and survey time. The results obtained have been found as accurate as those normally obtained by conventional survey methods on the ground.

Planimetry can be located with sufficient precision for most of the work to be acceptable. This can be done at a lower cost, covering a greater area, and with better detailed coverage, than by conventional survey methods.

The merit of using semi-controlled aerial photographs lies in the fact that the method is fast, inexpensive, and has acceptable accuracy as compared with conventional survey methods. In addition, the number of mistakes or errors is reduced and these errors are readily isolated.


- THE USE of aerial photographs for obtaining quantitative information is not new, as this procedure has been used for many years. Over the years the use of photographs has been expanded and techniques refined, so that they are now used for many more things than originally contemplated. Aerial survey is a form or refinement of the use of the aerial photographs, and without the vision of its potentials the techniques that are now used in aerial work probably would never have been developed.

Experience by the San Francisco Office of the Division of Highways has been primarily in the use of enlarged aerial photographs for the determination of fence lines for relinquishment purposes, and for obtaining horizontal positions of planimetric features associated with other right-of-way problems. There are other uses for which this tool can be used, but this paper is primarily based on the experiences in these two fields.

In California, areas in frontage roads, relocated streets, etc., which are not part
of the freeway proper but which have been constructed or reconstructed as part of a freeway project, are turned over to the local political subdivision by relinquishment; that is, the State gives all of its rights to the county or the city concerned. Between the freeway proper and the frontage roads, there is normally a fence which becomes in most cases the line of demarcation for the relinquishment. Records indicated that costs of conventional surveys were very high for determining the fence line location with the accuracy desired.

As a means of reducing costs and speeding up the work, it was felt necessary to develop techniques and methods that would reduce the time and effort being spent in determining and preparing relinquishments. The use of aerial photographs suggested itself as a possible solution to the over-all problem.

After considerable investigation and experimentation, techniques for using largescale enlargements were developed which permit the location of these fences with greater over-all accuracy than was normally obtained by conventional survey methods. This is particularly true for fence lines that have large radius curves, and fences with many angle points.

## GENERAL REMARKS

To obtain photographic enlargements that will produce the information wanted with the accuracy desired, it is necessary to develop field control for the aerial photographs. The process is to set targets on the ground before photography so that a minimum of three targets will appear on each photograph. If the photography negative scale is to be approximately 120 ft to 1 in ., the targeted points should be set at an interval of about 350 ft within the area or along highway route zone to be photographed. These are located by any method that will permit their plane coordinates to be determined and the distance between any two points calculated. In areas where there is a known base line but not in the position that is wanted for the aerial photographs, it is necessary to establish additional targeted points within the area concerned. Thought should begiven in positioning the separate target points on the ground so that the resulting control lines between target points are within close proximity of the wanted planimetry. The closer the wanted planimetry is to the control line, the greater the accuracy of the information obtained.

It is important that the aerial photographer be given a sketch map with suggested flight lines shown thereon. This procedure becomes particularly important when the photography flight is to be made so as to attain a vertical photography scale of about 120 ft to 1 in . The lateral displacement of the flight line can be such that the information wanted is not centered in the photographs. As with all aerial photographs, the information in the center portions of each vertical photograph contains less radial displacement of images due to ground relief, and also it is the area on the photographs where lens distortions are smallest. Therefore, care should be used to plan each photography flight line to center approximately on the information desired.

After the contact prints have been obtained, the targeted points are identified and marked on each print (Fig. 1). The points are indicated on the back of the contact prints with a sketch of each target. The exact survey point with respect to the target must be indicated as well as the ground distance between points. For instance, with a triangular-shaped target, the point of the triangle used as the survey point on the ground is shown for each target, and each measured or computed distance between these points is indicated on the back of the contact print for use in the photographic laboratory.

Figure 2 is the reverse side of the contact print shown in Figure 1 and shows the information furnished to the photographic laboratory. Target points are shown with the ground distance between targets, along with the dimension in inches that should be obtained on the photographic enlargement. This information provides the dark-room photographer with the necessary data to produce photographic enlargements of the vertical photographs which will be reasonably close to the desired scale. In all cases of further reference, scales as stated are approximate scales because the accuracy of any photographic enlargement is affected by such causes as relief displacements,


Figure 1. Target points marked on photograph.
inexact rectification, and differential paper shrinkage. In practice, the causes affecting accuracy are accounted for by resorting to scaling factors

## SCALING FACTORS

Scaling factors are numbers used to adjust scaled distances. The factors are derived for each control distance to show the variation between the desired scale and the actual scale of the photographic enlargement for the distance concerned. Because of the different causes of scale variation (relief displacements, inexact rectification, differential paper shrinkage, etc.) the factor applies only to that portion of the photographic enlargement which lies in the immediate vicinity between the two target points defining a control line. Thus, there are as many scaling factors as there are separate control lines on any one enlargement.

To illustrate the derivation of a scaling factor, the following example is given, using the control line W29-W30 (Fig. 2): desired scale of enlargement, 20 ft to 1 in .; actual scale between W29-W30, $15.03 \mathrm{in} . / 297.7 \mathrm{ft}=1 / 19.8 \mathrm{ft}$ which is 19.8 ft to $1 \mathrm{in} . ;$ scaling factor, $1 \mathrm{in} . / 20 \mathrm{ft} \div 1 \mathrm{in} . / 19.8 \mathrm{ft}=19.8 / 20 \mathrm{ft}=0.99$. A more direct approach is as follows: field-measured distance, W29-W30, is 297.7 ft ; distance measured on the photographic enlargement with engineer's scale is 300.8 ft ; scaling factor $=297.7 \mathrm{ft}$ / $300.8 \mathrm{ft}=0.99$.

Consequently, all other distances measured on the enlargement in the immediate vicinity of control line W29-W30 are adjusted by multiplying such distances by the scaling factor, 0.99.

In rolling topography or in sidehill topography in which the difference in elevation between the target line and the area being determined is appreciable, it may be necessary to establish additional points laterally in order to refine the scaling factors. For example, Figure 1 has frontage roads on both sides of a freeway which are at different
elevations. It was necessary to establish points on each frontage road in order to determine a control line on each frontage road at approximately the elevation of the dimensional data wanted. Points E26 to E30 are in effect lateral points with respect to the control line established through points W29 to W32. In this case and in similar cases, it is normally desirable to order two separate photographic enlargements, one centered over control line W29 through W32, and the second centered over control line E26 through E30. In this way, reliable scaling factors may be determined for each frontage road, thus effectively eliminating large-scale differences between the roads.

Usually, it is not possible to use a single factor for the whole enlargement. Determination of factors between each targeted point results in better scaled information and, in addition, draws immediate attention to any error or deviation that might be encountered. This permits the isolation of error in the mathematical or survey location of any of the targeted points as variations from the expected factors normally inducate that there has been an error. This is not true however, on the extreme edges of the photographs where a certain amount of lens distortion will contribute to variations in


Figure 2. Reverse side of contact print shown in Figure 1.
excess of the expected factors for the main portion of each vertical photograph. It has been found that 90 percent of each photographic enlargement is sufficiently free from distortion for the purposes of scaling when the aerial photographs have been taken with a $12-\mathrm{m}$. focal length lens.

It is not feasible for the photographer to enlarge to an exact scale as there are variables over which he has little or no control. Rectification along a control line is determinable and can be taken care of, as well as lateral rectification, whichever is needed. However, there is shrinkage in paper and sometimes this shrinkage is not uniform throughout any one enlargement. As a consequence, some latitude must be given in acceptance of the enlargement. The necessary dark room equipment consists of a rectifying enlarger, such as a Saltzman projector with a tilting easel to permit adjustments for uniform slope about both the X and Y axes.

It has been demonstrated that the scaling factors obtained on the enlargements range from approximately 0.99 to 1.01 . (The California Division of Highways obtains the enlargements by contract under specifications a portion of which is, "When more than 2 premarked control points are provided, the variation in relative scales between any 2 points in a single enlargement shall not exceed one half of one percent except for variation in relative scale caused by relief displacement due to non-uniformity of slope. The variation in scale on any one enlargement shall not exceed 2 percent of the average scale specified.") These factors are easy to work with, for it is apparent that a deviation of 1 percent from unity in the factor can be applied easily to any scaled dimension. In most cases, it is no greater than the error of scaling with an engineer's scale.

Deviations caused by non-uniform slopes cannot be eliminated by rectification, and enlargements of segments of the aerial photograph are therefore required. Two or more photographic enlargements from each film negative result in scale factors closer to unity. Two or more photographic enlargements should always be obtained and used where there is non-uniformity of ground slope. This is contingent on having sufficient target points set or identified with their plane coordinates within each image area on the photograph to have adequate control lines to determine factors within the specific photographic enlargement area of concern. If the slope between the east and west frontage roads shown in Figure 1 were uniform, rectification about both axes would result in factors close to unity.

Continuing with this thought, it is easily seen that in those cases where rolling topography is encountered, sufficient points set laterally from the target line may be required to facilitate the determination of additional factors at different distances from this control line. This would be partıcularly applicable along highway locations where there is a large area for which planımetric positions are needed. By setting additional targets laterally from the main target line, additional factors can be determined so that scaling of planimetry is more accurate.

For level ground, the factor determined for the control or base line can be applied to short lateral distances with the same accuracy as distances along the control line. Over rolling ground, however, or where there is a difference in elevation of considerable amount between the target line and the planimetry desired, the scaling factor will be affected, and it is necessary in this situation to establish control points in the area where dimensional data are wanted. It is not necessary to run lines but only to determine point location. The easiest field method available is the one which should be used.

Errors of point location are easily detected on a photographic enlargement if the error is of any magnitude. The location of the point in error can be re-surveyed if necessary and another factor determined for use on the enlargement.

## OBTAINING FENCE LINE LOCATIONS

The limitation on the amount of photographic enlargement, before appreciable fuzziness occurs, seems to be about 6 diameters. That is, an aerial film negative exposed at a scale of 120 ft to 1 in . is limited to an enlargement of 6 diameters to a scale of 20 ft per in. In general, on a $50-\mathrm{ft}-\mathrm{per}-\mathrm{in}$. scale photographic enlargement, scaling should be within 1 ft of true position; thus, on a $20-\mathrm{ft}$-per-in. scale enlargement scaling is usually correct to within 0.3 ft and nearly a certainty not to exceed 0.5 ft of true
position. All points will not necessarily fall within this range but nearly all will be within these limits unless there are relatively large changes in elevations. Relative positions of the scaled points will be better and more certain if additional targets are set and additional control lines with factors are determined.

On receipt of the photographic enlargements, each target point is identified with its survey designation, and the control lines between the exact survey points defined by the targets are carefully drafted on the enlargements as shown in Figure 3. The survey distances and bearings for these control lines are either available from completed field work, or must be computed by inversing between established plane coordinate values of the survey points. The corresponding distances are also scaled on the photographic enlargements for determination of the scaling factors as previously discussed.

Figure 3 is a portion of the enlargement of Figure 1, being part of the west frontage road, and is used as an illustration. For purposes of this illustration, the fence post shadows in the figure have been deliberately darkened. In actual practice, the density of shadow in the enlargements is adequate and sufficient to define the fence line from the post shadows.

On this photograph only points W30, W31, and W32 are shown. The ground distance between points W30 and W31 is 200 ft , whereas the scale distance is 199.6 ft , which results in a scale factor of 1.002 . Between points W31 and W32 the ground distance, 253.4 ft , scales 252.0 ft , resulting in the scale factor of 1.006 . The portion between W30 and W31 is used as an example.

The corner of the fence, point 4, which is the end of the curve labeled 2, is determined by scaling the distances A and B. These scale distances are then multiplied by 1.002 and a coordinate determined for the location of this point on the fence. For determining the tangent (the line labeled 1) distances C and D , and also E , are scaled as shown in Figure 3. The scaled distances are adjusted by multiplying by the scale factor, and plane coordinates for the points on the tangent are computed. The curve is determined by trial on the photographic enlargement by use of transparent curves to obtain the best fit. The tangent line is prolonged to assist in determining the tangential point


Figure 3. Portion of Figure 1.
of the curve. The location of the beginning of curve, point 3 , is reasonably close. After the radius has been determined, the curve is computed to obtain the true position of beginning of curve and rechecked on the photographic enlargement for fit. With the plane coordinates for points 3 and 4 , and the plane coordinates of points on the tangent line 1 , having been determined, the bearing and distance for the tangent and the curve data can then be computed.

The same process is used throughout the project for locating the fence both on the portion shown in Figure 3, and also for the remainder on both the east and west side frontage roads on their respective enlargements. The photographic enlargement readily shows a multitude of points available for the location of relinquishment fence lines and as much scaling as needed can be done to average the errors of scaling and the errors in the photograph. All the required work for determining curves and tangents is done in the office by scaling, using transparent curves and making such calculations as are needed.

The aerial photographs are taken soon after the fence is built and before development of brush cover, thus precluding trouble from heavy shadow in the vicinity of the fence lines. If the image areas of concern have heavy shadows in critical areas, a certain amount of additional but minor field work would be required.

The development of agreements with the political subdivisions involved with relinquishments sometimes create cut-off lines that are not normal. There are times when a traffic island in a channelization is used as a control for the cut-off hne. Precise location of all points of the islands is not usually determined, and there are occasions when the cut-off point in the channelization is changed from the point originally proposed. It is a simple matter to make this change on the photographic enlargement by rescaling and recomputing any portion of the changed relinquishment line. If conventional survey methods for makıng ties to planimetry are used, addıtional ties are usually necessary to obtain the information wanted.

The advantage of having a complete vertical photograph of the channelization has additional merit in the use by others for corrections of improper traffic functioning of the intersection or for the study of drainage problems that might develop.

In the field it is difficult at best to determine minor variations in the fence particularly in those cases where the angular deflections are small. These are not always visible to the eye without actually sighting down a fence, and even then the exact point of deviation is not always determinable. Additional ties are then required if the survey is done by conventional methods to be certain that the point of intersection is bracketed. This is necessary to locate definitely the two tangents, so that their intersection can be determined with reasonable accuracy.

Curves with exceedingly long radiuses result in lines that appear as tangents on the ground, thus requiring a great many field ties to determine a curve that will fit accurately and be closely allied with the actual curvature of the fence. Fence lines may vary from the anticipated pattern shown on the design plans due to obstructions or the necessity of modifying the location of the fence during the construction stages. These resultant variations which do not appear to be correct from the plot of survey notes require a field check to ascertain that the deviation is as shown in the field notes. The photographic enlargement, however, clearly shows the small angular deflections, tangents, long curves, and a complete picture of the fence. The graphical determination of the tangent and curves is easily and accurately obtained from the photographic enlargement. There is also the advantage of being certain that there have been no errors in survey and that the dimensional data obtained are correct.

## OBTAINING OTHER PLANIMETRIC DATA

The techniques used with photographic enlargements for determining the location of planimetry, other than fence lines, is not a great deal different from that used for relinquishments, as it is necessary to determme the position of target points. It usually differs only in the width of the band for which planımetric information is needed.

Figure 4 shows a typical example of a large area in which planımetry is required. The control base line is shown by points A, B, and C. After the vertical photographs
had been taken, it was found that more planimetric information was needed in this area; therefore, points D, E, and F were located by ground survey methods subsequent to the receipt of the enlarged photographs.

To use this method, it was necessary to determine points within the photograph that can be physically located and defined on the ground and on the enlargement. Point D is the intersection of the driveway and the walk, and the location of this point is easily found and its plane coordinate position obtained by survey on the ground. Point E is the outside intersection of the painted lines of the school ground for the basket ball court, and point $F$ is the intersection of the road gutter and the driveway return. Additional points could be used as there are many points within the area which can be identified and defined with reasonable accuracy.

With the addition of points $\mathrm{D}, \mathrm{E}$, and F , which have been indicated with squares in Figure 4 to differentiate them from the original points A, B, and C, indicated with circles, additional control or base lines are computed, such as lines AD, DB, DF , CF, EC, and AE. These new control lines permit the determination of additional scaling factors. The required planimetry can then be scaled using the same techniques described for the fence line in Figure 3.

Photographic enlargements can be made to any scale desired. Most right-of-way work is done at the scale of, 20 ft to 1 in . for relinquishments and the scale of 50 ft to 1 in . for planimetry. By enlarging the vertical photographs to a scale of 50 ft to 1 in., the planimetry can be traced from the enlargement to the hard copy tracings for design control.

A plane coordinate grid can be superimposed on the photographic enlargement if desired, as plane coordinates for the various targeted control points are known and this plane coordinate grid with the scale factors can be used in the same manner any other similar grid is used. Slight shiftings of the tracing would of course be required to accommodate for the scale factor, and adjust for non-uniformity of ground slope. (The plane coordinate grid, as constructed on the enlarged photograph, will be imperfect with respect to square shape and uniformity of distance between the grid lines and will not coincide with map grids because of the scale factors.) The resultant tracing would be satisfactory for the location of the planimetry for appraisal purposes.


Figure 4. Example of area in which planimetry is required.

If it is necessary to know the location of a corner of a bullding with an exactitude that is greater than could be obtained from the photographic enlargement, a field survey tie to the controlling building is usually required.

The photographically enlarged aerial vertical photograph serves well in obtaining planimetric information needed for any partıcular purpose. In addition, the location of the possessory lines of ownerships of various properties are easily seen and the probable lines of ownership are reasonably well delineated along with the culture of the area.

A study of the school grounds in Figure 4 shows some of the information that can be obtained from photographic enlargements. The school ground is suggested for study as it is relatively open and has painted courts, fences, buildings, and trees.

If it is necessary in the determination of the planimetry to know the overhang of eaves and porches, a minimal amount of field survey work will be required. There will also be field survey work required when the buildings are located under heavy, dense shadows that overhang the lines of the buildings. This supplemental survey does not normally entail a complete or new survey but only field checking with notations on the photographic enlargement in order to clarify and furnish the additional information when this information is of importance.

## CONCLUSIONS

Precise location of any single point is not obtained from a photographic enlargement, but for the location of planimetry and the determination of fence lines for relinquishment and allied purposes the use of such enlargements is an inexpensive, time and labor saving device. The use of this method should be considered wherever this media will be advantageous.

District IV of the Division of Highways, which is the San Francisco Bay area, has been using 20 ft -per-in. scale photographic enlargements for determining the relinquishment areas for about four years. During this time, relinquishment lines for about 100 mi of road have been determined. This work has all been done by the use of aerial enlargements in lieu of conventional ground survey methods.

By working with the construction forces, control points are preserved for targeting, thereby reducing the amount of additional control survey required for making the photographic enlargements. In one instance, on a $5-\mathrm{mi}$ stretch of road in the Santa Cruz mountains, the additional survey required for the additional targets was completed with one survey party in one day. It was estimated that a conventional survey would have taken six weeks for a survey party to make the necessary field ties to the fence and that the office work would have been quadrupled.

In determining planımetry, the savings are primarily dependent on the density of cultural development. On a $3-\mathrm{mi}$ portion of road with planimetry of medium density, it was estimated that there were savings of three weeks of a six-man survey party and savings of office time in the plotting of the notes on a hard copy of an additional three man-weeks.

The potentials in savings in dollars and in manpower is to a large extent dependent on the complexity of the area in question, whether it be for relinquishment or for the obtaining of planimetric information. In all cases, it has been found that the office work is reduced by one-half to three-quarters by the use of the enlargements over conventional survey methods.

Aerial photographs enlarged to scale are used in the office in conjunction with property determination and are of great value to the right-of-way engmeer. A plot of the properties, as determined by the right-of-way engineer on the photographic enlargements, gives reassurance that the location of the properties is ascertained within reasonable limits. This permits the work to be done more quickly and eliminates a certain amount of mistakes in judgment. The use of photographic enlargements defines and locates errors that may occur through misinterpretation of available information or possible errors in mathematics.

The author does not believe that this method will do everything that is done by ground survey methods. It does not give accuracies that are adequate for the determination of
property lines. Proper field ties to the property corners and monuments that might be in existence within the area must be made. The potential deviation in scaling a distance of as much as 1 ft on a $50-\mathrm{ft}-\mathrm{to}-1$-in. scale photographic enlargement results in the location of property lines from aerial photographs with error differentials that are too great to be tolerable.

For those areas of low land value, perhaps the accuracies obtained by this method would be acceptable for property line locations. In these areas, however, the properties are usually large and the number of survey ties required is relatively small. In areas where land values are high, the tolerances that would be acceptable in the location of properties would not be met by the use of photographic enlargements. As a consequence, it is necessary to make field survey ties to property corners and lines, in order to determine their most probable location and to elimınate the possibility of paying for land more than once.

The use of large-scale photographic enlargements facilitates the work of the rightof -way engineer and produces the information desired at low cost. The author does not consider that the enlarged aerial photographs will satisfy all survey requirements. It is not a cure-all but a tool to be used in conjunction with conventional survey methods of acquiring essential data. Considerable savings are possible with the use of photographic enlargements with adequate control, and the results are comparable with those normally obtained by usual ground survey methods where the precise location of points is not a necessity.

## Preparation of Right-of-Way Plans from Aerial Mosaics

## KEN MOREDOCK, Pennsylvania Department of Highways, Harrisburg

- WITH THE expansion of the National System of Interstate and Defense Highways and the acceleration of completing plans and construction, the acquisition of the right-ofway is a very important facet of the over-all program.

The Commonwealth of Pennsylvania, in conjunction with the Bureau of Public Roads, has adopted the policy that construction cannot start before condemnation and clearing of the right-of-way has been completed.

The former procedure was to do the property investigation and then negotiate after the plans were signed by the Governor. This is in compliance with the condemnation law in the Commonwealth of Pennsylvania. The same plans were prepared for condemnation and construction. This naturally hindered the operation of the Right-of-Way Department with the result that valuable lead time was lost to complete the necessary work involved in property acquisition.

At the inception of the Interstate Program in Pennsylvania the Right-of-Way Department could foresee that additional measures would have to be taken to facilitate this standard method of right-of-way acquisition.

Because a great deal of preliminary work had to be done and it was necessary to obtain values for programing, it was decided that certain right-of-way information should be acquired in the prelıminary design stage.

The original contracts provided that "Transparent right-of-way maps and aerial mosaics, both 24 by 48 in ., should show the highway right-of-way lines, property lines and property owners between and for a distance of at least 100 feet beyond the right-of-way lines, and the approximate acreage to be acquired from each owner." A copy of the pertinent deed for each property owner indıcated was to be furnished.

During the early part of the preliminary design for Interstate 80, known as the Keystone Shortway, the Right-of-Way Department, in close cooperation with one of the consulting engineers, developed a more refined and workable method for producing the requirements of the Department.

The Keystone Shortway is a completely new route, approximately 300 mi of new


Figure 1. Major highway systems in Pennsylvania.
location, running in an east-west direction through the center of the Commonwealth, from the Ohio-Pennsylvania Boundary Line at Sharon to the Delaware River at Stroudsburg (Fig. 1). The estimated preliminary right-of-way cost for the project was over $\$ 9,000,000$. This paper describes the method now used by the Department.

## METHOD OF PREPARATION

The preliminary design is usually made on photogrammetric maps scaled to $1 \mathrm{in} .=$ 200 ft . Using this method affords the opportunity to produce semi-controlled negatives for right-of-way use.

The first step for the right-of-way work is to obtain the county tax assessor's maps for the area contained on the photogrammetric maps. Generally the tax assessor's maps contain the name of the property owner and the total acreage for the parcel. If the names are not on the map they can be obtained and documented with the aid of the tax assessor's office.

When the study lines are located by the designer they are replotted on the tax assessor's map to determine which parcels are affected. The deed search is tabulated on a form as shown in Figure 2. The deeds for these parcels are then copied using a photocopier. In the event that this is not permitted, copies are requested from the county clerk. This work is also performed for contiguous properties so that if any shift in the proposed alignment becomes necessary and discrepancies in deeds need correction, the information is available.

On receipt of the deeds, each parcel is plotted and closed using the photogrammetric maps to help identify physical boundaries and verify bearings and distances. The parcels are plotted on a contınuous sheet of vellum and to a scale of $1 \mathrm{in} .=200 \mathrm{ft}$, the same as the original photogrammetry.

Vellum is used because it is more stable and can therefore be used to establish the amount of taking and residue.

On the final determination of the highway, the alignment is transferred to the vellum by tracing the location from the final linen. The "taking" limits are established from either the cross-sections or computer listings, depending on the method used to determine the earthwork quantities. The required right-of-way limits thus established allow the following items to be obtained for tabulation:

1. Parcel number, given as a means of identification.
2. Acreage taken for right-of-way.
3. Residue on left.
4. Residue on rıght.
5. Deed book number.
6. Page number.

## BUCHART ENGINEERING

YORK, PENNSYLVANIA
BY
DATE $\qquad$

DEED SEARCH RECORD
PROPERTY OW NER ___ PARCEL NO. $\qquad$



The acreage for items 2, 3, and 4 is obtained by the use of a planimeter and is identified in the tabulation to a cut-off date which is noted on the sheet.

The tax assessor's map and the continuous vellum sheet containing the deed plotting

is also helpful to the designer. Cases arise where property lines cannot be shown on the photogrammetric maps used for design. By reviewing the previously mentioned maps, it can be determined which properties will be severed and access denied. A study can then be made to determine whether an access road or damages would be more economical. This detail can be very helpful at the public hearings, if questioned.

After the location of the recommended alignment is approved, semi-controlled film negatives are made to a scale of approximately $1 \mathrm{in} .=200 \mathrm{ft}$. These negatives are produced from the negatives of the original flight made for the photogrammetric mapping. The orientation is not necessarily the same as the planimetric maps. It is possible to "center" the alignment more closely on the right-of-way negatives. Generally these negatives are 22 by 30 in . A 6 -in. Cronaflex piece is attached on the right-hand edge. This $6-\mathrm{in}$. sheet has the table heading printed on and is unattached so the information can be either printed on or typed using a "vari-type" machine. The sheet number is also inserted. In Figure 3 the sheet number C/43 indicated that the area was in Clarion County and the 43 is cross-referenced to the topographic plan sheet number 43. In cases where the right-of-way sheet is partially on two plan sheets both numbers were included.

The purpose of preparing these data on a film negative (Cronaflex negative can also be used) is that blueprints can be produced in any quantity desired. Furthermore, a regular photographic print can be developed if needed.

On receipt of this material, appraisal can be made by the Department to determine the value of the property to be acquired. This allows the programing of funds for the project.

When the project is let for final design a somewhat similar procedure has not been established. As previously mentioned, construction cannot start until all buildings are removed. The policy now dictates that separate condemnation plans be prepared and submitted before the final construction plans are completed. This work is done on a scale of $1 \mathrm{in} .=50 \mathrm{ft}$., the same as the construction plans. However, this scale only indicates, in the majority of cases, a portion of the property involved. It is therefore required that a drawing, preferably $8 \frac{1}{2}$ by 11 in ., be prepared to show the effect of the roadway on the entire parcel. This is drawn at a suitable scale, which may be varied, to indicate the desired results. Figure 4 is a typical property plat.

On receipt of the condemnation plans scaled to 1 in . $=50 \mathrm{ft}$, and the individual property plats, the Department can have appraisals made by local appraisers so that a fair settlement may be negotiated. Because the condemnation plans are now separated from the construction plans, the necessary signature by the Governor may be obtained before the final construction drawings. This results in the provision of the necessary lead time required by the Right-Of-Way Department and produces a much smoother and expeditious operation.

Perhaps the greatest advantage of the $24-$ by $36-\mathrm{in}$. photo-type of plan to all phases of highway right-of-way work is inherent. The construction-type drawing, as the name implies, is just that-a drawing. The designer takes a plain sheet of vellum and starts to add lines, or construction details, plus topographic features provided by his Survey Corps.

The emphasis lies in survey lines, then center or base line, then edge of pavement, curb, medial and shoulder lines, then drainage lines plus the topographic features adjacent to these lines which in his opinion might influence construction. The topography in this instance is secondary. If this is compared with the photo map showing information essential to the right-of-way function, plus all building outlines, vegetative cover, and soil uses, the distinctive advantage that all of this information is provided automatically by a camera becomes clear. The photo-type of plan was shown to the designer who had previously useder modified type of construction plan with all of its imposed lines, and his first remark was, "Why, it's so simple." Is this not, then, a true test of the purpose of any plan?

The first person, other than a highway employee, to whom the right-of-way plans are assigned, is an appraiser, who in Pennsylvania is an independent real estate broker trained to recognize property values and judge the impact of damage to these property values as a result of the condemnation. Often the most qualified appraiser's first step in making an appraisal is to make inquiry as to what the plan means in relation to a
property; this step has been clairfied to a great extent by the photogrammetric-type right-of-way plan. The appraiser, from this plan, can readily determine the type of land taken, such as wooded areas, cultivated areas, and scrub land. The plan also indicates quite clearly the existing water courses (such as streams, washes, and gulles) and also indicates the utilization of the land, such as contour farming and how it will be affected by the condemnation. By these additional topographic features he has first-hand information first. Tree lines used in conjunction with property plots outline the property lines in most instances and provide the appraiser with severance information. The appraiser is a busy man so any tool provided of this nature for his use makes his job that much easier. On an aerial photograph, trees are clearly indicated, as well as field roads used by the owner to travel his property. Such things are essential to the appraiser and, with his field information on the buildings, he may complete his appraisal in his office with the and of this plan. He can also, in quite a few cases, evaluate this property with the next without undue influence from the owners. In built-up areas he can work from simple base lines and right-of-way lines.

There was one instance in which the appraiser working with construction-type drawings allowed damage due to a change in the traffic pattern, assuming the necessity of the owner to build a new access road to part of his farm land. The center line and survey line, in this instance, appeared about 4 ft apart and he misinterpreted them for a raised divisor. The use of the photo type of plan with the explanation that there would be a flat divisor would have saved the appraiser an additional trip into the field. Incidentally, had he referred to the typical sections of the roadway accompanying both types of plans, he would not have started on the wrong assumption, but he is human too, so much can be said of the advantages of right-of-way plans from aerial mosaics from the appraiser's point of view.

A person with the toughest assignment is the negotiator trying to explain any drawings (with which he is already familiar) to a property owner who has never seen highway drawings of any kind. It becomes more a problem of education than negotiation. As soon as the owner sees a photo-type set of plans he recognizes landmarks that are familiar to him. It is not curiosity on the part of the owner, but he will be found using the photogrammetric plans to familiarize the negotiator with his lands. The negotiator indulges him and the groundwork is laid for successful negotiation, which is the original purpose. Many of the negotiators have said that the first thing a farmer usually does when he is shown this type of plan is to excuse himself and spend the next hour or so digging up an old, cracked, and faded photograph of his property before he fixed it up. Here, too, it should be pointed out that when one is dealing with large land areas, such as farms, the most practical way to survey such an area is simply by photogrammetric methods.

The photo-type plan does not pin down the negotiator to needless and confusing information to the owner on the more detailed drawings. This permits the negotiator to assume the role of a negotiator, rather than a construction expert. Should the negotiator fail in his efforts, the next group of laymen to use the photo plans would be the Board of View or jury. Here, too, the advantage of simple, clear plans, with only necessary detail is apparent. The photo plans reflect the conditions impartially. The Courts are primarily interested in the accuracy of property lines and the extent of the taking. This type of plan immediately supplements this evidence, becoming so to speak, a visual aid, and accomplishes considerable savings in time. Expert witnesses, viewing the plans for the first time, may temper their judgment by realizing the plans leave no vagaries, especially to severance damage; appraisal breakdown by type of land becomes apparent and conclusions again are more coherent.

In Interstate work, as well as the 100 percent State projects, condemnation letters and a complete set of cutouts colored to show the taking of owner's interests, are left with him for study at his leisure shortly after condemnation. Here, in addition to the advantages listed under negotiations, the owner himself is given lead time and an opportunity to bring up design features that may be of special advantage to himself and to the State. Such items as drives and crop planting can be taken care of long before construction starts. Here again, he is provided with information that is understood and helpful.

In addition to these many advantages, there are certain minor disadvantages that are now being eliminated as well as possible. As stated earlier, it has been the practice to color the condemnation area of a property to indicate property lines, as the plan itself obviously overlaps several properties. This method in some instances tends to obscure some of the detail on a photo-type print that could allow something to be overlooked. The suggestion that the lines be shaded or merely outlined is in the process of being adopted. As far as toning, the photo line process gives more weight control to precise lines and this is supplemented presently by outlining the buildings in the drafting rooms, but photogrammetric engineers are doing research on this problem and should eventually eliminate the need of any human doctoring, thereby reducing the possibility of error, which is always true with conventional plans.

Therefore, it is felt that the advantages of the photogrammetric-type of plan for use in right-of-way functions in Pennsylvania are inherent, and their continued use in the Right-of-Way Department is favorably anticipated.

## Use of Aerial Enlargement Transparencies

## In Right-of-Way Acquisition

CLARENCE E. HART, Office Engineer, Right-of-Way Divisıon, Maine State Highway Commission

> This paper discusses the problems arising from the colonial background, method of land division, and customs of conveyancing in relation to the requirements of Bureau of Public Roads Memorandums and those of the State's new Land Damage Board Law. Use and characteristics of a relatively new photographic plastic from which prints can be made in any reproduction machine, at any process stage, for use in mapping, appraisal, timber cruising, negotiation, court presentation, location and economic studies, and for public hearing plans, all with a reasonable expenditure of time and expense, are also discussed. Samples of the material and prints are included.

- IN MAINE, the Engineering Section of the Right-of-Way Division prepares all maps and plans required for the right-of-way function. These consist of the regular taking maps at scales of 1 in . equals 30,50 , or 100 ft ; the $50-\mathrm{ft}$ scale predominating; and such other plans necessary for land-use studies, apprassal and negotiation use, and case presentation before the Land Damage Board and in Superior Court.

The requirements of Policy and Procedure Memorandum 40-3. 1 and Maine's recent Land Damage Board Law make it necessary to determine accurately the lines of ownership of the entire perimeter of every property, even if the property contains hundreds of acres and the take is only a narrow strip for widening purposes. Policy and Procedure Memorandum 40-3. 1 requires the ties to intersection of new right-of-way with property lines, and the specification of total, remainder, and severance areas. The new Land Damage Board Law requires that the owner be furnished with a statement of determined damages including before-and-after land use and value. Formerly entire ownership boundaries were determined only for urban projects, entire takings, the Interstate, and in some cases on cutoffs involving severance.

This is a serious problem in Maine because of its history and character. The State dates back to early colonial times, the first semi-permanent settlement being Fort St. George in 1607. It is predominantly rural with relatively low land values. There are only six urban areas of more than 10,000 population and only fourteen more over 5,000. The customs of land division and conveyancing are very naccurate and indefinte. The
percentage of properties for which accurate plans can be found is only about 1 to 2 percent. At least 95 percent of the conveyances contain vague, general descriptions; reference to an old lot plan (at a scale of 40 to 100 rods to the inch); bounds in rods which are found to be generally unreliable; or are bounded by owners A, B, C, and the road. In these cases, which involve such a great majority of all those encountered, the only solution is determination of physical occupancy on the ground with the assistance of the owners.

One solution wouldbe to make an engineering survey of every property. As damage payments run mostly to only three figures, some to four figures, a few to five figures, and occasionally to six figures, the expense of a survey, especially of properties involving hundreds of acreas, cannot be justified as it would most often be many times the appraised damages.

Some means of solving this problem by a reasonably simple, quickly performed, and economical routine is an absolute necessity. During the last few years several methods have been tried, none of which was satisfactory from the aspect of results, cost, and time.

About a year ago the Highway Commission found that new mediums had been developed, one by DuPont under the trade name of Cronaflex, a second by Kodak under the name Kodalith Ortho Matte Film. The Commission is using Cronaflex. These are essentially a matte-surface mylar plastic with a photographic emulsion on the underside. They are extremely stable, tough, nearly fireproof, waterproof, and with proper workmanship will produce transparencies that can be run through black and white, blueprint, or


Figure 1. Cronaflex enlargement (22- by 30-in.) at scale of 1 in . equals 300 ft on Route 4 in the City of Auburn-Cronaflex on left and black and white print made from the Cronaflex on right. Shows material as received from aerial photo contractor; original contacts at 1 in . equals l,000 ft.
ozalid machines to reproduce as many prints as may be desired. Good black and white prints are being obtained on a Copyflex machine at speeds of 12 to 17 ft per min using Bruning No. 10 standard paper at a cost of about $\$ 0.30$ per sheet, speed depends of course on photographic density. The top surface takes ink or pencil lines and lettering perfectly without any special preparation, also self-adhesive tapes have been found to be very satisfactory. They stick well, are stable through the reproduction machines, but can be easily removed for purposes of making changes or corrections. Pencil erasability is perfect without any effect on the matte surface and ink is easily removed with a water-dampened swab.

A present contract with James W. Sewall Company, Old Town, Maine, provides for three parallel flights, the center flight being on the specified location line. Contact prints of all three flights are furnished at a scale of 1 in . equals 800 ft with a 60 percent overlap for stereo study of location and soils interpretation work. The contractor also furnishes to the Right-of-Way Division one set of transparency enlargements at a scale of 1 in . equals 200 ft of all photos in the central strip on sheets 22 by 30 in . This gives about 4 to 6 in . of overlap along the flight line and a lateral coverage of $3,000 \mathrm{ft}$ on either side. Whenever found necessary, the enlargements for either or both side flights can be quickly obtained.

Older photography is being supplied on transparencies at scales of 1 in . equals 300 ft and 1 in . equals 400 ft depending on the scale and quality of original photography. Detail on these is not as good as the photography under the present contract but is still considered to be the best medium available for the uses desired. On the enlargements


Figure 2. Enlargement (200 ft to 1 in. ) on US 1 in Town of Cyr. Cronaflex on left; black and white print on right; match lines for adjoining sheets shown, original contacts at scale of 1 in . equals 800 ft .
of 1 in . equals 200 ft , detail is so good that individual telephone or power poles, plowed furrows, and potato barrels placed in the fields for the picking job, can be identified.

Cost as compared with former aerial photography is as follows: The former contracts called for contacts of 1 in . equals $1,000 \mathrm{ft}$ and enlargements of 1 in . equals 500 ft for every other photo. Prices for special single flights were $\$ 60$ per mi within 50 mi of Bangor, $\$ 70$ per mi from 50 to 100 mi from Bangor, and $\$ 75$ per mi over 100 mi from Bangor, with a minimum charge of $\$ 250$ for any project. Prices under the present contract calling for contacts of 1 in . equals 800 ft and enlargements of 22 by 30 in . for every photo for special single flights are $\$ 80$ per mi within 50 mi of Bangor, $\$ 90$ per mi from 50 to 100 mi from Bangor, and $\$ 95$ per mi over 100 mi from Bangor, with a minimum charge of $\$ 340$ for any project. Enlargements of 4 x on $22-$ by $30-\mathrm{in}$. Cronaflex are furnished from any desired portions of the side flights at $\$ 15$ each or about $\$ 25$ per mi. Enlargements from the older photography at 300 - or $400-\mathrm{ft}$ scale averages about $\$ 25$ per mi per flight required.

In October 1960 an agreement was made under present contract specifications for photography on 147.2 mi on fourteen projects for a total cost of $\$ 10,659.76$ or $\$ 72.42$ per mi. Six projects were more than 100 mi from Bangor, four projects over 50 mi from Bangor, and four within 50 mi . This is approximately the same cost as the older contracts.


Figure 3. Completed Cronaflex for secondary Federal-aid project in Limestone. Match lines and Leroy lettering in ink; remaining lettering in pencil; property and right-ofway lines done with plastic tape; identification of westerly boundaries of properties simplified; land use readily identifiable; overlap showing very slight mismatch as evidenced by screen dots showing.

The procedure is as follows:

1. The transparencies are delivered labeled with flight and photo number (Fig. 1). They are indexed, matched on a lighttable, match lines drawn and titled in ink, survey centerline put on with plastic tape (Chartpak) and stationing indicated with pencil, every fifth station being identified (Fig. 2). Prints are then made and delivered to personnel of the preliminary information section. They locate and show on the prints the property lines as claimed by the owners, make out property owner report, also show location of property lines (within limits of the sheet) on prints of plan and profile sheets for exact station ties.
2. Prints of plan and profile and aerial enlargements are then given to the legal section for title work for use as a master working plan.
3. On completion of titles they are reviewed, any conflicts between field information and record documentation brought to light and means of settling worked out. This may involve further field and/or registry work.
4. Plans and title summary are returned to the Engineering Section and property lines for the project checked and plotted. Area of take is calculated from a right-ofway map. Areas of remainder, severance if any, and land use areas, divided into wooded and other use, calculated from transparency prints and put onto transparencies. Old and new right-of-way lines, and property lines are done with plastic tape. Name, parcel numbers, areas, etc., in HB pencil (Fig. 3).
5. Complete sets of prints of plans including construction plan and cross-sections, right-of-way maps, and aerial enlargements are delivered to the appraisal section.
6. Following appraisal and condemnation the same sets of plans are furnished the negotiator. Procedure from this point is regular routine (Fig. 4).


Figure 4. Portion of strip map as furnished appraiser and negotiator for major relocation of Route 100 in Auburn in vicinity of interchange with Maine Turnpike. Four previous minor relocations shown; map solved problem on property near center; deed described original tract, excepting and reserving lands previously conveyed; outsales readily identified; relocation is controlled access.

At present all enlargements are made by screen process without correction for tip or tilt. Even without correction the accuracy of scale is remarkably good. Scaling along the flight line is regularly within 5 ft per mi with an occasional sheet that will approach an error of 10 ft per mi . The lateral error increases to 10 or 15 ft with occasional excessive tilt showing greater error. The 22-by $30-\mathrm{in}$. enlargements cover only 60 percent of the contact along the flight line and about 85 percent laterally. As there is an overlap on all sheets of 3 to 4 in ., the used portion along survey line is under 50 percent. If side flights are required for extended lateral coverage, the overlap reduces the lateral use to 50 percent of the center flight and to about 65 percent of the side flight. This reduces both matching and scaling errors. Relief errors of course cannot be eliminated. From the checking that has been done to date, it is believed the areas calculated from the prints of these enlargements average in the vicinity of 1 percent error.

These transparencies and the prints made from them are the best solution found so far to solve quickly, easily, and economically the following right-of-way acquisition problems:

1. Field location of the lines of ownership is simplified because the lines or boundaries as shown by the owners as their use and occupancy are in most cases readily identifiable, less than 10 percent having to be measured on the ground.


Figure 5. Portion of Public Hearing Plan_relocation of Cold Brook Road in Hampden. Cold Brook Interchange on Interstate 95 is about 0.9 mi westerly of photo. US IA is vertical band at right of photo. US 202 from Augusta intersecting at Hampden Upper Corner in lower right-hand corner; present Cold Brook Road (lower black band) runs below high knoll at intersection with US IA which at that point has 7 percent grade. Proposed relocation shown by upper black band. On original, dark bands are different colored plastic tapes, channelized intersection with islands and passing lane on US IA being indicated; building and other identifications are typed stickers; ground survey made from plan line.
2. As a master plan for title search, the scale is large enough to permit plotting the one or more deed descriptions covering the ownership. Determination of deed and occupancy conflicts, spotting the probability of unrecorded deeds, and other title problems are readily apparent and can be worked out without difficulty by standard procedures.
3. Determination of total area, areas of remainders, and approximate areas of land usage become routine calculation.
4. They are very useful as a typing map for timber cruises.
5. For use by the appraiser for land use, comparative market, and neighborhood influence studies, and as a means of studying relation of take to improvements and remainders.
6. For use by the negotiator as a "selling" tool. A firm offer is now made to the owner with the notification of taking and the negotiator must "sell" this offer to the owner. The owner can be shown the over-all picture of his property and the manner in which the highway will affect it. He also gets a clearer idea of the detail and amount of work, study, and thought that has gone into the appraisal on which the offer is based. Reaction from appraisers, negotiators, and owners to date has been very favorable.
7. As an exhibit before the Land Damage Board and in Superior Court as a means of illustrating of the State's case. This has been done in the past by drawing up a special plan for each property at scales varying from 1 in . equals 10 ft to 1 in . equals 400 ft to obtain a plan of the entire property of a size easily readable by a board or jury, with the various features color outlined or shaded to depict the various factors to be discussed. In complicated cases one or more mylar overlays have been used to show certain specific features. This has been time consuming as it required mechanical enlargement; also only the more significant features could be shown. The prints themselves are now being used, where the scale is satisfactory, and color outlined and shaded. Where enlargement is necessary photostating produces a faded and blurred base that can be traced to show all of the more important features of the photo with reasonable expenditure of time.

The Construction Divisions are making two uses of the prints:

1. For location studies and surveys. If a 2 - or $3-\mathrm{ml}$ section of a route of 20 mi or more between important terminuses is to be reconstructed, the location to be surveyed can be planned to fit the most desirable over-all location. Economic effect is more easily determined; cutoffs can be more intelligently planned; and survey control points can be chosen. The Commission has been told by the location engineers that the prints are proving of excellent value for these purposes; also that surveys run on the ground from lines laid out on the prints have only required minor adjustment to make them fit the chosen terminuses and control points. It is expected this use will become regular procedure even though on the more important projects the 1620 computer and planimetric maps will be used for location studies.
2. They are also an excellent exhibit for use at public hearings on proposed projects (Fig. 5). Prints are mounted on plywood panels, the alignment shown with colored plastic tape, connections and interchanges shown (alternate designs being taped on mylar for superimposition). All roads shown on the prints, all important topographic features, buildings that will be affected, and other items of interest are labeled. The explanation of the purposes and effects of the proposed project is more easily made, and most important, is more clearly understood by the citizens at the hearing. They can identify buildings, road intersections, streams, ponds, and other things with which they are familiar and arrive at a reasonably clear understanding of the relationship of the proposed location with these knowns.

The public reaction has been very favorable to this use, and there appears to be less opposition now to proposals, which is believed due to the better understanding of the proposal.

A public hearing was recently held on about 25 mi of Interstate location. The prints covered five 4 - by 8 -ft panels of plywood. The cost for the prints was about $\$ 30.00$. Two men matched, mounted, taped, and labeled them in less than three days. The strip
covered varied in width between 2 and $31 / 2 \mathrm{mi}$ over the $25-\mathrm{mi}$ length. These were enlargements of 1 in . equals 300 ft made from photography of 1 in . equals $1,000 \mathrm{ft}$.

There are now enlargements for immediate and future use, for nearly 300 mi of highway locations for all presently planned Interstate, Primary and Secondary FederalAid, and State Projects (except spot hazard removal work).

Both time and money are being saved by using this medium. Also, better information is being obtained, providing the appraisers with means of making better and more realistic appraisals, and have an additional and better means of presenting the case to the owner and when necessary before the Board and in Court.

The Legal Section has stated they believe about $\$ 10$ per title or $\$ 150$ to $\$ 200$ per mi is being saved. Drafting time saved per project is running $\$ 50$ to $\$ 100$ per mi. Appraisal cost has been reduced about $\$ 10$ per appraisal or about $\$ 150$ to $\$ 200$ per mi. The other possible savings have not been estimated. Time-saving and better results are the most important things that use of this procedure has accomplished. The Commission is now current with the construction divisions and is expected to remain so even though it is furnishing appraisal and negotiation sections with material that was previously not possible.

## Use of Aerial Mosaics and Photogrammetry

## In Right-of-Way Acquisitions

L. E. McMAHON, Data Processing Engineer, Michigan State Highway Commission

- THE Michigan State Highway Department executed its first contract for an aerial survey on September 22, 1925. An excerpt from the contract states "we will desire to do enough work in the neighborhood to find out the value of this 'sort' of mapping to us in hilly country." As a result of this contract, made with Talbert Abrams, the Department was furnished with an uncontrolled mosaic of photographs taken at a scale of approximately 800 ft per in. The aerial mosaci was subsequently used to establish the alignment for a particularly scenic section of US 31 near Beulah. The photographs were taken with a World War I Eastman camera from a standard J-1 90-hp World War I training plane.

Apparently the highway officials at that time recognized the value of this "sort" of mapping immediately, because the Department has had contracts with Abrams Aerial Survey Corporation for aerial mapping every year since.

Immediately after World War I, the people of Michigan demanded a system of weatherproof roads to get them out of the mud. By the middle of the 1950's their children and grandchildren were demanding a road system that would free them from the strangling python of traffic that had grown up to choke the very life of the State.

The national government being aware of highway needs of the entire nation, provided a great boost with legislation authorizing a national system of interstate highways to be completed in 1975. The Michigan legislature, heedful of the demands of the populace, authorized the Highway Department to borrow money through the sale of bonds to speed the construction of the Interstate System, and a system of arterial freeways and farm-to-market State roads. Michigan expects to complete this system by 1967, and at a cost of $\$ 2$ billion.

Confronted by a program of this scope, the State administration is using every advanced engineering tool available in order to meet the program deadlines. One of these valuable tools is photogrammetry. Photogrammetric methods are used in all phases of work prior to contract letting. One of these phases includes the use of aerial mosaics and photogrammetry in the acquisition of rights-of-way for the projects. It has been found that aerial mosaics can be used to a great advantage in land evaluation work during the study of alternate project alignments, and that the same mosaic can be used to
start the actual right-of-way acquisition processes, and actively to purchase parcels for the road project as soon as the alternate lines are weighed and the final alignment chosen.

Figure 1 shows a portion of a photographic mosaic used in the location studies for


Figure 1. Portion of photographic mosaic used in location studies for Interstate 75 northwest of Detroit.

Interstate 75 northwest of Detroit. Three locations were studied within the given area. After weighing the advantages of each, the line shown in detail was selected. Land ownership boundaries are indicated for each land parcel affected by each line for the use of appraisers evaluating right-of-way costs.

It is from such mosaics that right-of-way acquisition processes are started on projects that are expected to require considerable amounts of time to secure the land parcels. In cases, this plan of action has saved a considerable amount of time. Of course, a few slips have occurred in getting too far ahead of the design process. However, it is believed that the few errors made are insignifıcant when the over-all accomplishment is considered.

Figure 2 shows a portion of Interstate 94 just north of the City of Detroit and in the City of Saint Clair Shores. This project is an extension of the now-completed Edsel Ford Freeway which traverses Detroit from west to east. The orıginal right-of-way for this project was ordered from a $200-\mathrm{ft}-\mathrm{per}-1 \mathrm{n}$. mosaic used for the route location studies. All parcels wholly enclosed within the necessary right-of-way, as determined on the route location studies, were purchased or negotiations for purchase were started before the beginning of the detailed plans. The view is the complete coverage of a finished right-of-way plan sheet showing the following:

1. The right-of-way requirements for that portion of the project shown.
2. The platted subdivisions, streets and service alleys.
3. Excess properties resulting from the purchase of entire lots or parcels.
4. The construction limits for the project (slope lines).

This plan is used by right-of-way personnel for the the following purposes:

1. Appraisals of properties to be purchased and retaned for highway use.
2. Appraisals of excess or residual parcels to be disposed of by public sale.
3. Negotiations with property owners.
4. For visual record of lands and improvements prior to construction.

Right-of-way plan sheets such as that shown in the figure are reproductions scaled


Figure 2. Portion of Interstate 94 north of Detroit in city of Salnt Clair Shores.
to $1 \mathrm{in} .=40 \mathrm{ft}$. Scaled dimensions are shown for residual areas. These plans are considered as acceptable by the Bureau for reimbursement for rights-of-way purchased for Interstate projects. Other reproductions of these drawings are used in the detailed construction plans to carry items of site clearing. For this use, it is found that aerial plans such as these are more accurate than are manually drawn plans, and are much less costly.

Figure 3 shows a matching of an aerial photograph with a planimetric map of the same area of coverage at the site of a rural road crossing over Interstate 96 near Lansing. Plan base sheets of this nature are produced in the Department's photographic laboratories for use by designers in preparing the details for ultimate highway improvements.

The lower half shows the plan view of the road improvement as designed. Except for the road profiles and other special detailed drawings which are furnished on supplementary sheets, all details including the necessary rights-of-way are shown on this portion of the plan. Permanent right-of-way limits are shown by the heavier lines. Right-of-way lines enclosing land parcels required on a temporary basis (for example, grading easements, land for temporary roads, and borrow areas) are shown by the lighter lines. Dimensions for right-of-way takings, as referenced to an established road centerline, are shown on this plan.

The upper part of this plan is a photographic coverage of the same area. The photographic portion shows the proposed right-of-way lines for the project as an aid to right-of-way personnel.


Figure 3. Matching of aerial photograph of rural road crossing over Interstate 96 near Lansing with planimetric map of same area.


Figure 4. Segment of Interstate 96 southeast of Lansing.

It should be apparent that plan base sheets such as these save a considerable amount of design time for road projects. It has been found that the plan sheet combination, along with a minimum of field survey information and the stereo-pairs covering the area, gives the design personnel about all of the information needed to produce a complete set of construction plans. The photographic section is most helpful, because it provides details not otherwise available at such small cost.

Right-of-way buyers find the plan combination to be most helpful in their negotiations with property owners. The photographic portion, because of its complete showing of recognizable features, is of great aid to land owners in their understanding of the road problems.

Figure 4 shows a segment of Interstate 96 southeast of Lansing. The figure is an oblique photograph of a parcel of land acquired for highway use through negotiation. This particular view was used by the Right-of-Way Appraisal Section in illustrating the appraisal procedures approved by the Bureau for right-of-way acquisition on the Interstate system. Oblique views of properties such as this are often used by right-of-way personnel in court cases as are mosaics and larger scale plan views.

Although the value, in dollars and cents, of this type of presentation in courts is not known, it is believed that the pictorial representation of the site conditions are better understood by all participants in the action. Planimetric maps have been used on occasions to the advantage of the State. In one case, a saving in the neighborhood of $\$ 30,000$ was affected through the use of planimetric map on a drainage dispute. In another, the ability to make a reasonably close estimate from a planimetric map of a
disputed volume of core sand available on a land tract being severed by a highway project resulted in a saving to the State.

Photogrammetric methods are of proven value in the right-of-way acquisition processes. The development of newer and better techniques are eagerly awaited.

# Use of Photographic Enlargements in Right-of-Way Problems in Kansas 

GLENN ANSCHUTZ, Engineer of Aerial Surveys and Photogrammetry Section, Kansas Highway Commission

> The use of photographic enlargements has been made by various departments of the Kansas Highway Commission for the solutions to several and varied problems. One of the greatest uses of photographic enlargements has been in the area of right-of-way problems, especially in the trials and court cases arising in the settlements of area acquisition. Although Kansas can report some use in the preconstruction stages of the right-of-way processes, this report concerns itself primarily with the use of photographic enlargements in the post-construction phases of right-of-way problems.
> Use of enlargements in various court cases arising in the settlement of various actions is discussed, as is a second use of enlargements being made by resident engineers after projects have been completed.
> The report also reviews the promising feature of enlargements in their use in these areas and the difficulties that have been and could be encountered with their use.

- AERIAL photographs have been found useful in practically all of the stages of highway location and design in Kansas. Among the uses of contact-size aerial prints, enlargements, and photographic mosaics, one could list these in the areas of preliminary location surveys, project reports, public meetings, drainage area determinations, materials investigations, right-of-way appraisals, land use studies, highway inventories, condition surveys, traffic counts and turning movements, and right-of-way negotiations and condemnations. This report is concerned with photographic enlargements of aerial prints or mosaics as they are used in post-construction right-of-way and allied problems.

During the past two years, Kansas has attempted to photograph all major construction projects before the first signs of construction to obtain a record of the route before construction changes have occurred. This photography is taken even when normal on-the-ground surveying methods have been used to obtain design data. Aerial photographs have also been taken of many highway projects after construction has been completed or nearly completed.

For various reasons, the Kansas Highway Commission may be involved in damage claims, land separation cases, or easement questions after construction has been initiated or completed. When such cases arise, enlargements of aerial photography taken of the area in question before construction are made to some convenient scale so that the enlargements may be used as a public display. The same area is photographed at a time when construction changes are being made or have been completed. Enlargements of these photographs are also made at the same scale as the pre-construction enlargements. To these enlargements are affixed taped lines showing such items as property lines, section lines, location of the new route and easement lines, directional arrows, scales, date of photography, and similar data, depending on the scope and type of case in question. The enlargements, when placed in evidence, are displayed in
positions adaptable to visual comparisons. With such data, an attorney of the Commission or proper authority representing the Commission can present a display showing all points in question before and after construction, (Figures 1 and 2).

The advantages of the use of enlargements for such post-construction questions are in many cases difficult to evaluate in cost figures. The feeling of the majority of the individuals using such photography can be summarized by the following quotations from a letter received in March 1961 from an attorney representing the Kansas Highway Commission on many such cases:

During the past several years I have had the opportunity to rely on the Photogrammetry Section and...[the Design Department, Drafting Section] for the photographing and preparation of aerial plats of the "units" through which the State Highway Commission has taken land for highway purposes. The use of such controlled aerial photography taken at an established height at an established angle on a date "on or about the legal date of taking" so that such photography


Figure 1.
would be admissible over objections of opposing counsel has provided the jury with visual evidence of the topography, soil type and use of the area under consideration; and in my experience greatly facilitated the ease of the presentation of highway condemnation cases and without question has assured desired results in defensible cases. There are of course cases in which an aerial [photograph] will indicate a level piece of fertile bottomland. Such will not hurt the condemnor's position, for it is assumed that the original appraisal was based on that condition of fertility. If not then such photography would indicate the necessity of an adjustment in said appraisal before embarking on a defense against the landowner's appeal. Whereas considerable time may be expended in the explanation of the appearance of a piece of land, words are at best an inadequate tool for such a task and the fact remains that the jury may choose to believe the opposing side's description without visual substantiation of one's case. In the main, the areas under consideration do not measure up to the quality that the testimony of appraisers hired by the landowners would indicate and the damages are, if I may be permitted this opinion, generally somewhat exaggerated. With


Figure 2.
aerial evidence of the land used together with expert explanation (generally by the resident engineers) as to how to examine the aerials for the contour of the land, the waste areas and sometimes the soil type and the effect of construction, a jury has a better opportunity of weighing the evidence to determine which side's evaluation of the property is worthy of the greater credence.

In one case on the interstate recently for example, we were faced with a 320 acre unit-the interstate took 20 acres therefrom separating approximately 30 acres to the south. The landowners entire testimony was directed to the separation caused by the nonaccess feature of the improvement effecting an increased distance of 5 to 6 miles from 1 side of this unit to the other. Our aerial photography indicated that nearly all of the area to the south of the new highway had been waste land and that, therefore, its original value was debatable. The exorbitant damages claimed by the landowners due to separation were thereby visually destroyed and the jury verdict was within the preappraisal and, therefore, within the amount offered to the landowners in settlement before trial.

Recently also in the Geary County condemation project in the appeal involving the channel change of the Smoky Hill River the use of aerial photography of the river before the new construction proved visual evidence of the river's meanderings and cutting over the years of which the landowners were advised and no doubt affected their decision to reduce their claim by $\$ 5,000$ to a justifiable figure at which we were able to settle.

No attempt has been made on each of the specific instances in which aerials photographed by the S.H.C. [State Highway Commission of Kansas] have been used by this attorney to estimate the specific savings in time and money; for I am not trained in making such comparisons between different fact situations having numerous variables. However, I have an opinion based on participation in the trial of approximately 100 cases over the past six years and settlement of numerous others including participation in the trial and/or the settlement of each appeal on the interstate system through Geary, Dickinson and Saline counties, from which I can only conclude that the use of such photography regulated so that it can be readily admitted and studied so that it can be thoroughly explained has a tremendous impact on the outcame of a case with a jury which is eager to get the facts.

The photogrammetry section is new. Some of its facilities are even newer. In the past we have relied heavily on the hardworking section of the design department which willingly provided drawings of the property involved. It is not without due appreciation of what was available and used, but the advent of the availability of the controlled photography has assured admissibility of an aerial photograph designed to portray all of the facts pertinent to the land involved. In the transition period I have used A.S.C. [Agriculture Stabilization and Conservation Service Photography] to show the "before taking" (and also "before photogrammetry") photos and photogrammetry's [Photogrammetry Section, Kansas Highway Commission] pictures to show after construction shots which were advantageous. The first full project upon which I was able to get complete coverage of the alignment "on or about the date of taking" before construction was Interstate 70 in Saline County; and such photography was to evolve another use. From the negatives the complete alignment was provided the resident engineer and used to aid in survey work during construction as well as to provide the appraisers with the full picture.

Such evidence stresses the many advantages of such enlargements in the areas of direct savings and data presentation.

A second use of post-construction photography and photographic enlargements has recently been initiated by the engineers in the construction administration activity of the Kansas Highway Commission. Enlargements of certain routes located in areas of
high build-up or possible build-up have been produced and bound into a book form. The scale of such enlargements has usually been the scale of the original construction plans for the highway projects in that area. These books of enlargements have been delivered to the resident or division engineer of the area who has in turn placed the right-of-way lines on the enlargements in ink or colored pencil and has added information concerning the access permitted under law to each section or route covered in the book. The book is then used as a reference for the engineer as highway entrance requests, drainage changes, and maintenance problems arise. It makes a ready reference for such engineers and saves many trips to the site on many problems. When inspection of the routes are made, the book of enlargements is carried with the engineer and many notes added to the photography. Such changes and notes keep the photography current. When the routes have been changed considerably, a new set of photographic enlargements are produced from more recent photography of the area and the process is repeated.

After routes or portions of a system of routes have been completed, photography of broad areas showing the traffic pattern, especially urban areas, is flown and enlargements of this post-construction is made available to the resident engineer. The photography may be vertical or oblique, depending on the size of the area, the complexities of the traffic control, and the use to be made of the photography.

As the public is instructed in the use of the routes, the resident engineer finds the enlargements to be excellent tools in carrying the information to the news media and any irregularites in the control of traffic can be usually pinpointed. The newspaperman


Figure 3.
and the newsman of the local television stations seem to appreciate such service. It is evidently an excellent public service and serves the Highway Commission in the area of public relations.

The birth of another use of post construction photographic enlargements has been proposed by the Kansas Highway Patrol. Enlargements of interchanges, especially those enterchanges which seem to generate high traffic volumes and or traffic accidents, could be filed in the offices of the Highway Patrol Unit covering these particular interchanges. Accident reports made on the interchange can then be reviewed in the patrol office, especially in times of inclement weather, and the report can be written giving a detailed description of the physical characteristics of the interchange as well as the particular conditions concerning the accident, the weather, and other data. When cases arising in civil courts concerning such accidents that require the testimony of the investigating patrolman, he can take the enlargement to court to explain the accident situation and give the court a visual explanation of the area. It is felt that contact-size prints will first be distributed to the patrol offices so that a knowledge of the existence of such photography is first available (Fig. 3). As accidents or other incidents occur that may justify the enlargement of the print, enlargements will be made and sent to the requesting individual. In this manner, all offices will in time have enlargements of such areas.

This use has not been tried to a point where an evaluation of the results can be made; however, the few instances where it has been used have been gratifying and thus far considered successful.

The post-construction use of photographic enlargements of prints and mosaics is a valuable asset to a highway department and makes a fine additional service which can be rendered by a photogrammetric unit possessing the facilities to produce such enlargements. It is felt that this is a growing field and more uses of such photography will be found.

## Discussion

EDMUND SWASEY, U.S. Geological Survey-Highway right-of-way acquisition people use aerial photographs for interpretation, display, and measurement. They also compile planimetric maps, but in many cases prefer to use the aerial photograph because of the wealth of information recorded on it. The orthophotoscope is an instrument that should be considered for its application in illustrating and solving highway right-of-way problems. Using standard vertical photography as source material, this instrument produces the equivalent of an orthographic photograph in which the imagery is preserved and orthographically positioned. Corrections for tilt and relief displacements are selfincorporated.

THE National Academy of Sciences-National Research CounCLL is a private, nonprofit organization of scientists, dedicated to the furtherance of science and to its use for the general welfare. The Academy itself was established in 1863 under a congressional charter signed by President Lincoln. Empowered to provide for all activities appropriate to academies of science, it was also required by its charter to act as an adviser to the federal government in scientific matters. This provision accounts for the close ties that have always existed between the Academy and the government, although the Academy is not a governmental agency.

The National Research Council was established by the Academy in 1916, at the request of President Wilson, to enable scientists generally to associate their efforts with those of the limited membership of the Academy in service to the nation, to society, and to science at home and abroad. Members of the National Research Council receive their appointments from the president of the Academy. They include representatives nominated by the major scientific and technical societies, representatives of the federal government, and a number of members at large. In addition, several thousand scientists and engineers take part in the activities of the research council through membership on its various boards and committees.

Receiving funds from both public and private sources, by contribution, grant, or contract, the Academy and its Research Council thus work to stimulate research and its applications, to survey the broad possibilities of science, to promote effective utilization of the scientific and technical resources of the country, to serve the government, and to further the general interests of science.

The Highway Research Board was organized November 11, 1920, as an agency of the Division of Engineering and Industrial Research, one of the eight functional divisions of the National Research Council. The BOARD is a cooperative organization of the highway technologists of America operating under the auspices of the Academy-Council and with the support of the several highway departments, the Bureau of Public Roads, and many other organizations interested in the development of highway transportation. The purposes of the Board are to encourage research and to provide a national clearinghouse and correlation service for research activities and information on highway administration and technology.


[^0]:    ${ }^{\text {after Bastian (19). }}$
    *Difference equals Stereotope elevation less A-7 elevation.
    **Refers to tilt in left and right photographs, respectively.
    ***ilot legible-ink on photograph.

[^1]:    ${ }^{\mathrm{a}}$ After Sahgal (29).
    ${ }^{\mathrm{b}}$ Col. A=initial stereotope readings; Col. $\mathrm{B}=\mathrm{adjusted}$ stereotope readings.

[^2]:    $\mathbf{a}_{\text {After }}$ Sahgal (29).

[^3]:    ${ }^{\text {a }}$ Assuming measured distance is 100 ft .

