Analyzing Vehicular Delay at Intersections Through Simulation

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The first section of this paper describes the development of a simulation model for the intersection of two 2-lane two-directional streets, with one street being controlled by stop signs. The lack of adequate mathematical distributions describing traffic behavior and the field studies performed to obtain these distributions are discussed. The elaborate techniques used to test the logic of the model before beginning the analysis are also reviewed.

The second portion of the paper presents the simulation results. The variability of vehicular delay under constant traffic conditions is described and the relationships between vehicular delay and individual approach volumes and turning movements are formulated.

A brief discussion of the value of this research and the general applicability of simulation techniques to solving traffic problems is also presented.

•A GREAT MANY traffic engineering decisions are made on the basis of experience and engineering judgment at the present time. In this growing profession, there is an increasing need for factual information concerning the effect of these decisions. The simulation of traffic situations on high-speed digital computers can be used to provide a large amount of data under controlled laboratory conditions which would be difficult, if not impossible, to obtain through field studies. The techniques and methods of vehicular simulation on computers are relatively new, and the lack of basic mathematical distributions describing traffic behavior has slowed progress. As these techniques are established, simulation can become one of the more valuable research tools available to the profession.

Almost any traffic situation is capable of simulation and, as techniques improve, can be rapidly programed. Variables or controls can be changed and their effects analyzed. Before-and-after studies can be performed in hours or days without disturbing traffic in the field. Situations can be simulated and observed which could not be risked in field installations. Peak traffic flows can be simulated for hundreds of hours under the precise conditions desired instead of obtaining one or two hours of data per day in the field under uncontrolled conditions.

One of the most important problems facing traffic engineers is congestion on city streets. There has been much discussion in the last few years concerning network analysis of street systems to provide maximum efficiency in moving traffic. This certainly is an ultimate objective and will use computers to solve the problem. But, congestion begins primarily at intersections. A network analysis combines a great number of intersections and studies the over-all operation of the system. To do this, even on large computers, the individual intersection must be analyzed in a macroscopic manner. In other words, only the significant features of intersection operation can be included in the analysis if it is to be accomplished in a reasonable time. To determine which operational features are significant, it is necessary to study intersection operation in fine detail or microscopically. The microscopic simulation of vehicular traffic at intersections is the subject of this paper.

The current research in simulation at the Institute of Transportation and Traffic Engineering is divided into two phases—intersections controlled by stop signs and

intersections controlled by traffic signals. The first phase is nearly complete and the second phase has begun. Numerous objectives are to be obtained from this research project, including the following:

1. Development of "models" by which vehicular traffic at intersections can be simulated on high-speed computers.
2. Determination of the total vehicular delay experienced at intersections with respect to approach volumes and turning movements. In the case of signalized intersections, signal timing is the third major variable.
3. Evaluation of the effect of installing a signal at an intersection on vehicular delay which will provide a basis for examining and refining existing traffic signal warrants.
4. Determination of the effect of turning movement restrictions (and signal timing in the second phase of the research) on intersection operation.

STOP INTERSECTION

The programing for this phase of the research is essentially complete. The model described was coded for an IBM 701 computer available at the University of California. Unfortunately, this computer has been out of operation since July 1, 1961. Programs coded for the 701 computer are not readily translated for use on later generation computers and, due to the complexity of this simulation program, no attempt has been made to recode the model for another computer. The 701 is expected to be in operation by January 1962. This phase of the research has, therefore, been tabled temporarily pending the availability of the computer.

Model Description

The programed model for the first phase of this project consists of the time simulation of an orthogonal intersection of two two-lane, two-way streets with the minor street being controlled by stop signs. The model (shown in Fig. 1) includes the approaches to the intersection for sufficient length that vehicles enter the system before they are influenced by any condition existing at the intersection.

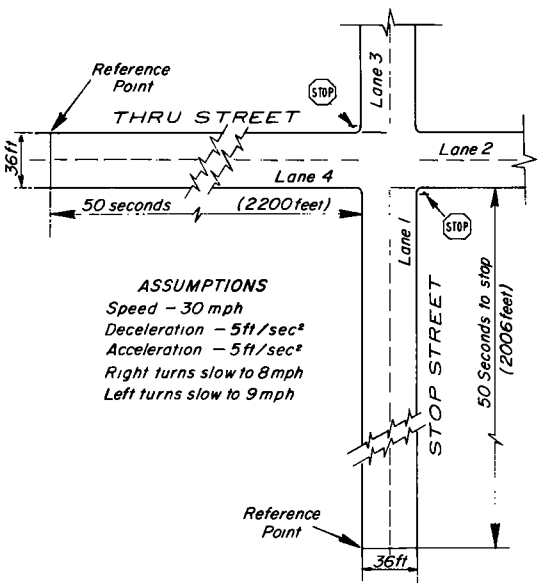

Figure 1. Simulation model for stop sign intersection.

Vehicles entering an approach are generated randomly by Monte Carlo techniques from a given distribution at a preselected hourly volume. Each of the four approaches has a preselected volume and is generated independently. As a vehicle is generated, it is randomly assigned a turning movement (right, left, or thru) based on a requested distribution of these movements. Vehicles travel from the point of generation to the intersection (or to a point where they must decelerate) at a predetermined velocity.

Minor street vehicles decelerate to a stop either at the intersection or in queue (in which case they move forward as the queue is released). A vehicle at the intersection accepts or rejects available gaps in conflicting traffic streams from given acceptance distributions based on their respective turning movements. On accepting a gap, the minor street vehicle accelerates to recovery driving velocity.

Major street vehicles are given the right-of-way over minor street vehicles when conflicts in turning movements and/or time exist. Through major street vehicles

are also given the right-of-way over opposing left-turning major street vehicles when conflicts in time occur. Major street queues, due to stopped left-turning vehicles, and delays caused by vehicles slowing to make turns, are also included in the model.

Briefly, the simulation is accomplished as follows. Each time a major street vehicle enters the intersection, the model is analyzed. If the major street vehicle is not delayed, the minor street traffic is brought up to this time (vehicles delayed or released as appropriate) and the major street vehicle is released. If the major street vehicle is delayed, the minor street traffic is also delayed and the system is checked to see when the major street vehicle might be released. This process is repetitive, generating new traffic as necessary.

Acceleration, deceleration, slowing, stopping, and queueing delays are computed for both major and minor street vehicles. Stopped time delays for minor street vehicles while waiting for an acceptable gap are also computed. All delays are accumulated and stored for each simulated hour of real time.

At the termination of each simulation run, the results of each simulated hour are printed and include the following items for each approach, for the minor street, for the major street, and for the entire intersection: elapsed real time, vehicular volume entering the system, vehicular volume released from the system, turning movement counts, turning movement percentages, number of vehicles delayed, percent of vehicles delayed, total vehicular delay, average delay per vehicle, average delay per delayed vehicle, and maximum queues experienced.

Vehicle Generation

As mentioned earlier, vehicles are generated randomly from a given distribution for each approach independently. One of the first problems encountered in the development of the simulation model was the absence of a satisfactory mathematical distribution describing gaps or headways in a traffic stream. The Poisson distribution which has been used in the analysis of other traffic problems does not adequately describe the headway distribution. A number of theoretical distributions have been proposed by various authors but no information could be found to indicate that these distributions had been tested over an extensive volume range or that the parameters had been solved in terms of volume. It was necessary, therefore, to select a distribution, test it, and solve for its parameters.

A composite distribution proposed by Andre Schuhl (1) was selected. He theorized that a traffic stream is divided into two groups. A certain proportion of the vehicles in the stream travel as they wish and are not influenced by the vehicle in front of them. For convenience, this group shall be referred to as the free-flowing vehicles. The remaining vehicles have been influenced by the vehicle in front of them and shall be called the restrained vehicles. Each of these groups has a distinct mean and obeys some Poisson-type law. The theoretical distribution for the total stream is a composite or summation of these two subdistributions. Figure 2 shows this composite distribution along with the two individual curves which have been summed. Figure 3 shows the same curves replotted so that the ordinate p is the probability of a headway or gap that is less than or equal to the time t indicated.

The equation for this composite distribution is

$$p(h \geq t) = (1 - \alpha)\, e^{-\frac{t - \lambda}{T_1 - \lambda}} + \alpha e^{-\frac{t - \tau}{T_2 - \tau}} \qquad (1)$$

in which

$p(h \geq t)$ = probability of a headway (h) greater than or equal to the time (t);
α = proportion of the traffic stream in restrained group;
$(1 - \alpha)$ = proportion of traffic stream in free-moving group;
T_1 = average headway of free-moving vehicles;
T_2 = average headway of restrained vehicles;
λ = minimum headway of free-moving vehicles;

τ = minimum headway of restrained vehicles; and
e = natural or Naperian base of logarithms.

In this equation, there are five parameters—α, T_1, T_2, λ, and τ—which are functions of the traffic volume. These can be reduced to four unknowns by transforming Eq. 1 to give

$$p(h \geqq t) = e^{a - \frac{t}{K_1}} + e^{c - \frac{t}{K_2}} \tag{2}$$

in which

$$a = \frac{\lambda}{T_1 - \lambda} + \ln(1 - \alpha) \tag{3}$$

$$K_1 = T_1 - \lambda \tag{4}$$

$$c = \frac{\tau}{T_2 - \tau} + \ln \alpha \tag{5}$$

$$K_2 = T_2 - \tau \tag{6}$$

The problem, therefore, is to find these four unknowns in terms of volume. Fortunately, Eq. 6 still describes the two subdistributions separately. Returning to the plot of the composite curve (Fig. 2), the restrained subdistribution does not affect the composite curve for larger headways. Therefore, the unknowns corresponding to the free-moving subdistribution can be determined by fitting an exponential curve to the equivalent portion of field data. Once these two unknowns are determined, the contribution of the free-moving vehicles in the lower portion of the curve can be calculated and subtracted from the original data. The residuals form the distribution of the

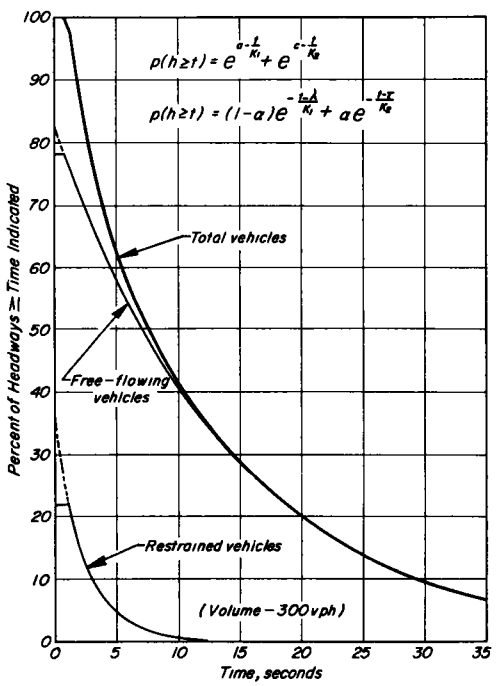

Figure 2. Composite exponential distribution.

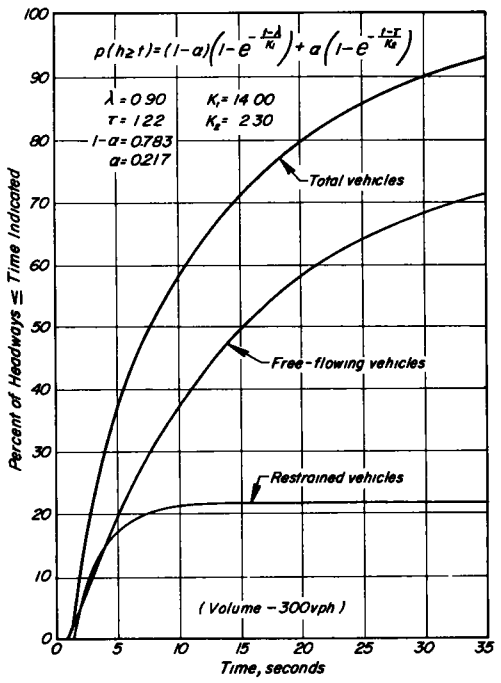

Figure 3. Composite exponential distribution.

restrained vehicles and, therefore, the remaining unknowns can be determined by fitting a second exponential curve to these points.

To evaluate these unknowns, extensive field data were collected. These data, obtained on two-lane urban streets, resulted in 585 samples with volume rates ranging from a little over 100 vph to almost 1,200 vph. Eighteen different fits were computed and tested by means of χ^2 tests for each sample.

To indicate the magnitude of the computations involved, a man with a desk calculator and a book of log tables requires from 12 to 14 hr to compute the values for one data sample. Needless to say, these computations were done on a computer which performed the same operations in approximately 18 min.

Equations for the unknowns in Eq. 2 have been made:

$$K_1 = \frac{4827.9}{V^{1.024}} = e^{8.48 - 0.024 \ln V} \tag{7}$$

$$a = -0.046 - 0.0448 \frac{V}{100} \tag{8}$$

$$K_2 = 2.659 - 0.120 \frac{V}{100} \tag{9}$$

$$c = \left[e^{-10.503 + 2.829 \ln V - 0.173 (\ln V)^2} \right] - 2 \tag{10}$$

Whereas the parameters of Eq. 2 have been solved in terms of volume, the simulation model requires the use of Eq. 1. It is impossible to transform Eq. 2 back directly to the form of Eq. 1 because α, λ, and τ are dependent on one another. The assignment of a value to one of these three parameters determines the other two. The equations of these parameters are

$$\lambda = K_1 [a - \ln(1 - \alpha)] \tag{11}$$

$$\tau = K_2 (c - \ln \alpha) \tag{12}$$

and

$$1 - \alpha = e^{a - \frac{\lambda}{K_1}} \tag{13}$$

or

$$\alpha = e^{c - \frac{\tau}{K_2}} \tag{14}$$

λ and τ are the minimum headways of the subdistributions and, because negative headways are impossible, as these minimum headways approach zero, they define the two boundary conditions. In fact, these parameters cannot even approach zero because vehicles have a finite length. A more realistic minimum headway is 0.5 sec which, at 30 mph, is a distance of 22 ft between the front bumper of the lead car and the bumper of the following car.

Figure 4 is a plot of the two conditions where λ and τ equal 0.5 sec. When t is greater than the largest λ or τ, the summation curve is identical. The only problem is the shaded area on Figure 4 between λ and τ or τ and λ as the case may be. To solve this problem, the original field data were analyzed by first grouping the data into volume ranges and then examining the leading portion of the cumulative curve for each volume range. It became apparent that λ and τ were relatively constant throughout the volume range. Best agreement between the theoretical curve and the observed data occurred where $0.9 < \lambda < 1.0$ and $1.20 < \tau < 1.36$.

Once the parameters of this distribution are determined for a given volume, the next step is to generate headways that fit this distribution randomly. This is accomplished by a technique similar to that described in an earlier paper by Gerlough (2). A flow diagram of the random generator used in this simulation model is shown in Figure 5. A separate random generator is used for each approach.

Using these random generators to generate many consecutive hours of traffic at a particular volume yields a distribution of volumes that has some spread on either side of the requested volume. This spread becomes larger as the volume increases. Because

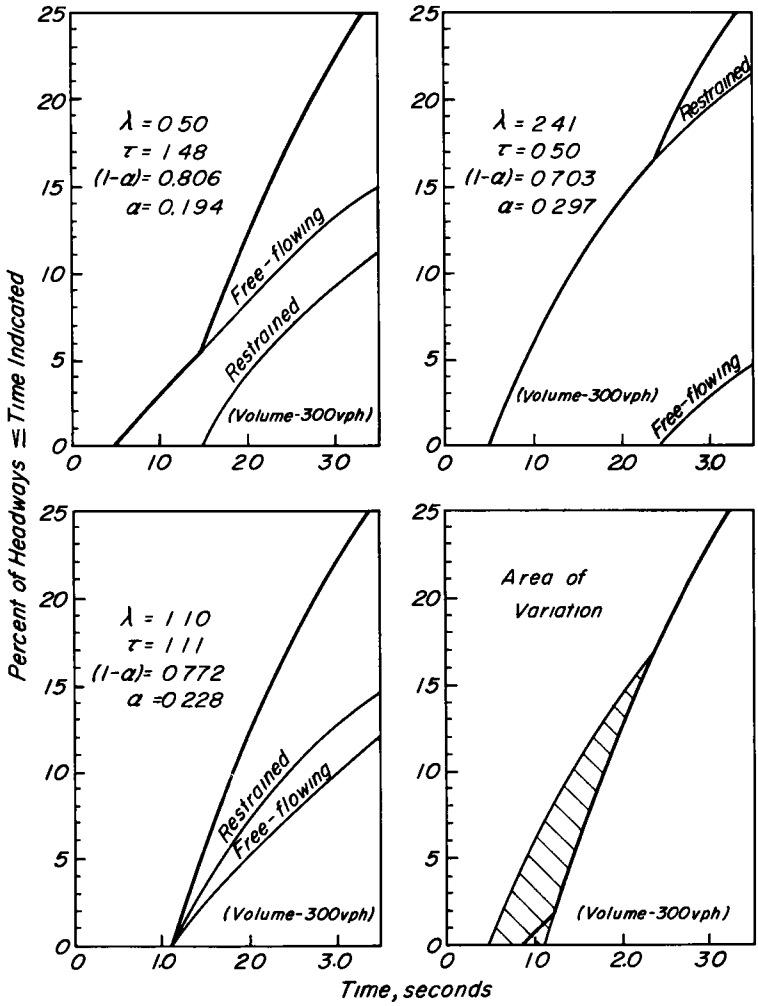

Figure 4. Effect of λ, τ, and α on composite distribution.

of this variation in generated volumes, the analysis of the simulation output becomes more complex. This problem can be overcome if the random number used to begin each hour of simulation is known to produce a volume within a small tolerance of the desired volume. To obtain random numbers that would produce the desired volumes, a separate computer program was developed, incorporating the same random generator as used in the simulation, to pregenerate vehicular volumes. As each hour is generated, the generated volume is compared to the desired volume. If the generated volume is within the tolerance (presently +2 percent and -1 percent), the generated distribution is tested against the theoretical distribution by means of the Komolgorov-Smirnov test of goodness of fit (3). When this test is successful, the computer prints the random number used to start the hour of generation, the time into the next hour of the last vehicle, and the time preceding the end of the hour of the last twelve vehicles. These vehicle times are used in ordering the random numbers for the simulation to ensure continuity and volume accuracy. Essentially, the time over the hour of the last vehicle in the preceding hour is an offset for the new hour of generation. This may result in the loss of one or more vehicles in the new hour of generation (i.e., the simulation hour may end before all vehicles in the expected volume are generated). This is the reason

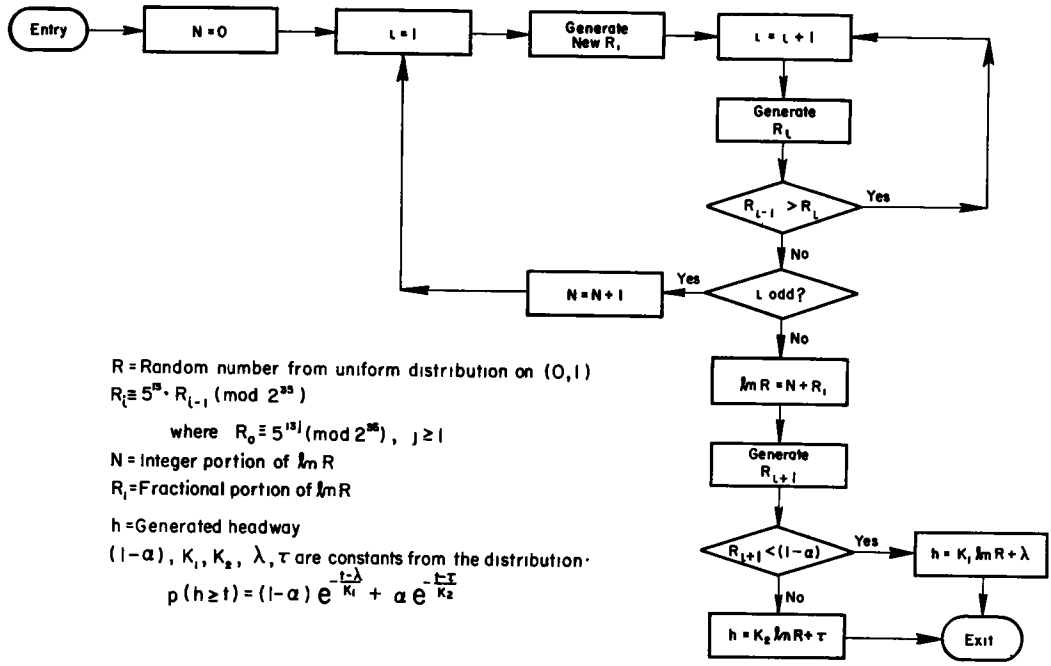

Figure 5. Random headway generator using Monte Carlo technique.

for using a larger plus than minus tolerance in the generation. Sufficient random numbers for a particular volume are ordered and used as simulation input to provide the desired number of hours of simulation.

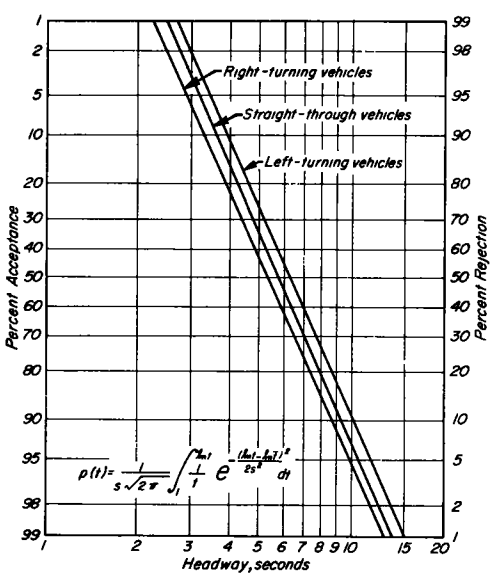

Figure 6. Gap acceptance distributions for vehicles passing a stop sign.

Gap Acceptance

Vehicles waiting at the stop sign in the simulation model accept or reject available gaps in conflicting traffic streams based on a gap acceptance distribution for their respective turning movement. These gap acceptance distributions had to be determined from field data. Howard Bissell, a graduate student at the Institute, undertook this study as a graduate research project (4). He collected and analyzed data for over 10,000 gaps and found that the gap acceptance distribution was of a lognormal form. The distributions found are shown in Figure 6.

The validity of using a lognormal distribution to describe gap acceptance was confirmed when this distribution was fitted to some data obtained in an independent study in Australia. The fit was exceptionally good. The only difference between the United States' data and the Australian data was the mean gap accepted. Apparently, Australian drivers accept shorter gaps on the average than drivers in this country.

The time required by the computer to

generate individual values of a lognormal distribution was considered excessive. Therefore, the distributions are inserted in the simulation program in tabular form and a table-look-up procedure is used to determine gap acceptance. This is accomplished in the following manner: A gap of size x is available. The probability of accepting a gap of size x is obtained from the table. A random number (between 0 and 1) is generated and compared with the probability value. If the random number is larger, the gap is rejected. Otherwise, it is accepted. With this method, there is a possibility of a vehicle rejecting a gap of a certain size and then accepting a shorter gap later. Bissell (4) found that this occurred for approximately 5 percent of the vehicles observed.

Simulation Procedure

To utilize the simulation model to provide data (in this case, vehicular delay), a number of simulation"runs" were performed. A simulation run consists of simulating a predetermined number of hours of traffic under constant conditions. After the model has been programed and tested on the computer, the first step in any simulation project is to determine the number of simulated hours under constant conditions that are required to obtain valid estimates of the output factors. In this project, vehicular delay is being measured. Sufficient hours of traffic are simulated with constant approach volumes and turning movements to analyze the delay pattern and determine the variance that occurs. On the basis of this analysis, the length of a simulation run is determined.

A series of runs are then established to begin the analysis of vehicular delay. The approach volumes used in the first 15 runs of the first series are given in Table 1. Turning movements from all four approaches are held constant with left and right turns being 10 percent each. The first five runs of this series have approach volumes which correspond to the minimum vehicular volume warrant for traffic signal installation (5).

An evaluation of the results of this first series of runs provides an insight into the relationship between delay and approach volumes and indicates the additional runs necessary to obtain sufficient data to define this relationship.

TABLE 1

SIMULATION RUNS, SERIES 1

Run Number	Approach Volume[a]						Total Volume
	Major Street			Minor Street			
	Lane 2	Lane 4	Total	Lane 1	Lane 3	Total	
1	250	250	500	150	100	250	750
2	300	200	500	150	100	250	750
3	200	300	500	150	100	250	750
4	400	100	500	150	100	250	750
5	100	400	500	150	100	250	750
6	200	200	400	150	100	250	650
7	300	100	400	150	100	250	650
8	300	300	600	150	100	250	850
9	400	200	600	150	100	250	850
10	500	100	600	150	100	250	850
11	400	200	600	200	100	300	900
12	400	200	600	100	200	300	900
13	400	200	600	200	200	400	1,000
14	400	200	600	300	100	400	1,000
15	400	200	600	400	100	500	1,000

[a]Turning movements in this series are constant with 10 percent right turns and 10 percent left turns.

After the volume relationship is determined, a further series of runs is made to evaluate the effect of turning movements. Approach volumes are held constant at various volume levels for both the major and minor streets while the percentage of turns is varied. The results of the initial runs, when analyzed, determine areas where further data are needed. This defines the runs to be included in the next series.

This procedure is continued until the various relationships under investigation are determined for the range of variables desired.

SIGNAL INTERSECTION

Programing of the second phase of this research is underway. The completed program will be coded in computer language for use on any of the later generation computers to avoid any repetition of delays experienced in the first phase of this project. A grant of $3,250 has been received from the Technical Development and Research Fund of the Institute of Traffic Engineers to support this project. These monies are to defray the cost of computer time for simulating the signalized intersection.

Model Description

The signalized model is quite similar to the first model. Physical conditions are the same—an orthogonal intersection of two two-lane, two-way streets. Vehicles are generated in the same manner as previously described. In programing the second phase, care is being taken to insure that the exact same traffic generated in the stop sign model can be reproduced in the signalized model.

Intersection control in this model is programed as a separate subroutine. There will be at least three different types of signal controllers programed as subroutines—fixed-time, semi-actuated, and full-actuated. During any simulation run, the model is preset to select the proper subroutine for the type of signal being simulated.

The signal sequence is main street (lanes 2 and 4) green, main street amber, all red, cross-street green, cross-street amber, all red. The all-red intervals will normally be set equal to zero. They have been included in the model to permit the analysis of all-red clearance periods and also to allow the inclusion and evaluation of separate pedestrian phases (scramble system). It may also be possible to include additional phases for left-turning vehicles during these intervals.

An optional feature to be incorporated into the program is the ability of vehicles waiting on a red phase to make right turns. This maneuver is legal in several western States and its inclusion in the model will permit an evaluation of its effect on vehicular delay. Vehicles permitted to make this turn will accept gaps in the cross-traffic in a similar manner as right-turning vehicles at a stop sign.

Operation of the model is quite similar to the through street in the stop sign model. Vehicles are generated at the reference point and approach the intersection. If no barrier (red signal, queue, or turning vehicle) impedes its progress, the vehicle continues through the intersection. Otherwise, it decelerates and stops, if necessary, until such time as it can proceed. Left-turning vehicles accept gaps in the opposing stream based on a gap-acceptance distribution. If left-turning vehicles are waiting as the signal changes, a maximum of two will be permitted to turn after the signal change.

Each street will be analyzed in detail during its green and amber phase. Vehicles are generated and brought to the intersection until the arrival time of the next vehicle is later than the end of the amber phase. In the case of an actuated signal, this last vehicle is used to "call the green" back to that phase. The cross-street is then brought "up-to-date" through the preceding red phase and the current green and amber phases. At the conclusion of each simulated hour, results are tabulated and stored until the completion of the run, when they are printed. The same items are obtained in this output as were obtained in the stop sign simulation to facilitate analysis through direct comparisons.

Vehicular Approach

As previously stated, vehicles are generated at the reference point and proceed

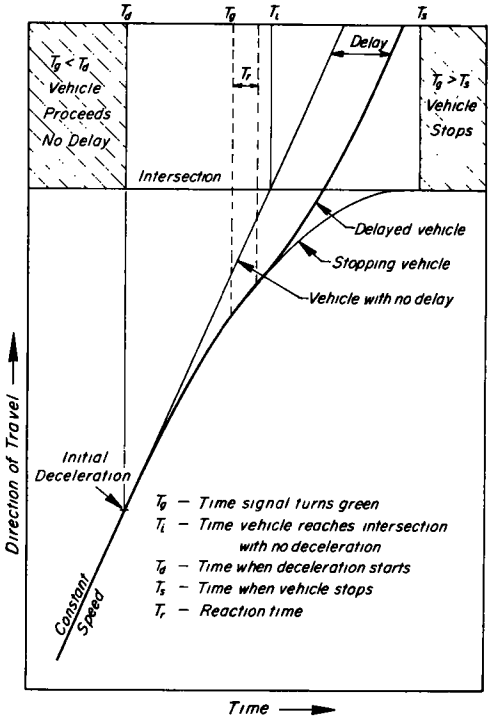

Figure 7. Vehicle approacning signal.

through the intersection unless a barrier impedes their progress. When a prior vehicle is already stopped in the approach, there is no question—the approaching vehicle stops in queue behind the stopped vehicle. Neither is there any question when the signal is red as the vehicle approaches and remains red until the vehicle stops, nor when the signal is green and remains green until the vehicle enters the intersection. But, there are two situations where an approaching vehicle's behavior is varied: (a) where the signal changes from red to green as an approaching vehicle is decelerating, and (b) where the signal changes from green to amber before an approaching vehicle reaches the intersection.

The first case is fairly simple to resolve. As a vehicle approaches an intersection where a red signal is displayed, there is some point (in distance and time) where the vehicle begins to decelerate to a stop. If the signal turns green before this point (in time), then no deceleration occurs. Similarly, if the signal does not turn green before the vehicle arrives at the intersection, the vehicle stops. Therefore, two boundary conditions have been defined. If a vehicle is between these two boundaries when the light turns green, he

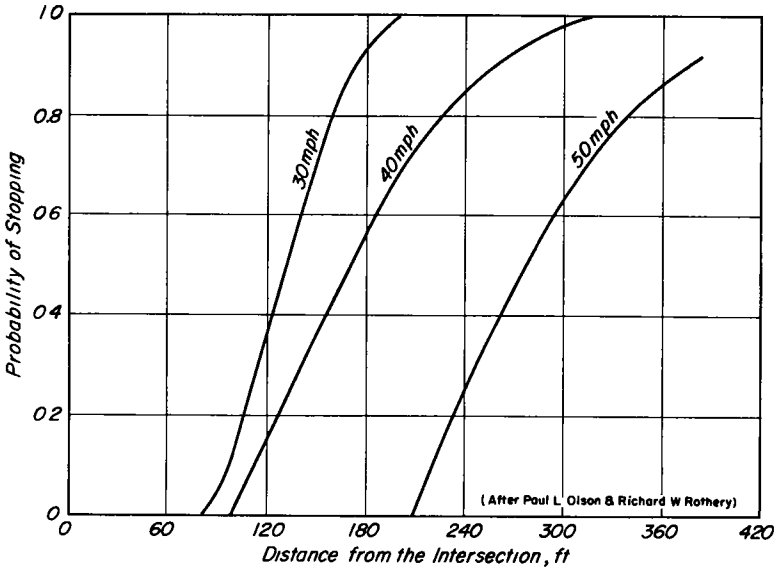

Figure 8. Stopping distributions for vehicles approaching amber signal.

is decelerating but has not stopped. After reacting to the light change, he accelerates and returns to normal driving speed. This critical interval of a vehicle's approach is shown in Figure 7. If the signal turns green during this interval and there is no other delaying factor, the vehicle proceeds without stopping and has a slowing delay proportional to the time spent in deceleration.

The signal change from green to amber (or yellow) is more complex. In this case, only one boundary condition is fixed—when the amber light appears after the vehicle arrives at the intersection (in which case, there is no delay). But, if the light changes to amber as a vehicle approaches, then the driver must decide whether he is going to stop or not. To simulate this in the model, a probability distribution must be used. Initially, a distribution curve presented by Olson and Rothery (6) will be used (see Figure 8). These curves are based on data from five intersections in Michigan. Additional data will be gathered in California to refine the distribution. Also, an attempt will be made to evaluate the difference in the legal definition of the yellow light. In California, the yellow is a "warning" period, whereas in most other States it is a "clearance" period.* It may be necessary to use two different distributions.

The procedure is similar to that using gap acceptance distributions except that instead of using a gap to enter the probability table, the distance the vehicle is from the intersection is used. A random number (between 0 and 1) is generated and compared with the probability value from the table. If the random number is greater, the vehicle proceeds; if less, it stops.

CONCLUDING REMARKS

Originally, this paper was to include results from the first phase of this research. The unfortunate shutdown and continued lack of operation of the particular computer for which the program was coded has prevented this. However, the techniques presented should be of value to other researchers concerned with intersection simulation.

A generation routine has been provided which will result in a realistic and random distribution of traffic that can be reproduced as often as required. A technique has been described by which it is possible to control the volume per hour to values within a small tolerance of the requested volume in order to simplify analysis of the simulation output. Gap acceptance distributions based on field data have been presented along with a technique for utilizing the distributions.

Two intersection models and their general operation have been described to illustrate some of the possibilities that are available through the use of simulation. As simulation techniques improve and mathematical distributions become available, almost any situation will be capable of simulation. The potential of this tool in evaluating the efficiency of any system of control is practically unlimited.

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*Sec 11-202(b) of the Uniform Vehicle Code defines the yellow light ("steady yellow alone") and reads in part:

> "1. Vehicular traffic facing the signal is thereby warned that the red or 'Stop' signal will be exhibited immediately thereafter and such vehicular traffic shall not enter or be crossing the intersection when the red or 'Stop' signal is exhibited." (emphasis added)

Sec. 21452(a) of the California Vehicle Code has the same provision except that the emphasized portion is replaced by "vehicular traffic will be required to stop."

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