# Structural Considerations and Development Of Aluminum Alloy Culvert 

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> This paper evaluates and compares an appraisal of the structural load capacity of aluminum alloy culvert pipe with existing methods of analysis. Not only are theoretical considerations discussed, but the work is supported by extensive structural fill tests. A fill height table based on these considerations is included for use in design.
-THE INTRODUCTION of aluminum alloy culvert pipe into the highway construction field required that an apprassal of its structural load capacity be evaluated and compared with existing methods of analysis. To do this it was necessary to re-evaluate the several methods of theoretical analysis as they would apply to the characteristics of aluminum alloy culvert. Generally, there are two recognized approaches to the design of flexible metal culvert: the moment of deflection method proposed by Spangler (2), and the compression ring theory proposed by White (8). In general, the distinction between the two approaches occurs by assuming whether the pipe will fail by collapse or buckling or by fallure of the joint seam in shear or bending. It was felt that both methods must be evaluated in aluminum culvert design to determine which method of failure might occur first under normal conditions.

In the evaluation of flexible culvert by Spangler it was noted that he was unable to substantiate performance of the culvert pipe in deflection with evaluation of bending stress. It was felt that this could be explained by a modification of the load distribution applied to the culvert pipe. If it could be demonstrated that the stresses developed conform as well as the deflection, a single solution would include axial reaction and a means of comparison with the compression ring theory.

Finally, concurrent with predicitions of load-carrying capacity, it was felt that a series of structural fill tests on aluminum culvert were necessary to demonstrate that actual test results would follow predicted results. Accordingly, a series of tests on culver in 24-, $36-$, $48-$, and $60-\mathrm{m}$. sizes were conducted by Kaiser Aluminum at Permanente, Callf., in the Spring of 1961. The prime purpose of these tests was to obtain general corroboration of structural performance of aluminum pipe over a series of sizes. Sections included examples of full circle, vertically elongated, and strutted culvert.

Using the results of the tests and analytical evaluation to confirm the results, a fill height table could be developed (Table 1). Its placement is at this point so that the results can be available for review before the supporting details are developed.

## DESCRIPTION OF TEST

The site selected for the structural fill tests was at Permanente, Calif., on the eastern side of the Santa Cruz Mountains approximately 60 ml south of San Francisco. The native soil is a sandy loam which was determined to have an optımum density of 136 pcf. Moisture content of the soll during the test period was 10 to 15 percent. The material had a liquid limit of 26.6 , a plastic limit of 10.8 , and a plasticity index of 15.8. The specific area selected is shown in Figure 1 and was centered into a small natural canyon which was shaped to contour for placing the pipe. A large quantity of

TABLE 1
FILL HEIGHT RECOMMENDATIONS FOR ALUMINUM ALLOY-CORRUGATED CULVERT ${ }^{\text {a }}$

| Culvert <br> Diameter (in.) | Type of Shape | Minımum <br> Recommended <br> Cover (in.) | Maximum Recommended Fill Ht, for Gauges and Thicknesses (ft.) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 16 | 14 | 12 | 10 |
|  |  |  | 0.060 | 0.075 | 0.105 | 0.135 |
| 8 | Full circle | 6 | 50 |  |  |  |
| 10 | Full circle | 7 | 40 |  |  |  |
| 12 | Full circle | 8 | 35 |  |  |  |
| 15 | Full circle | 8 | 32 |  |  |  |
| 18 | Full circle | 8 | 26 |  |  |  |
| 21 | Full circle | 9 | $25^{\text {b }}$ |  |  |  |
| 24 | Full circle | 9 | $15^{\text {b }}$ | $24^{\text {b }}$ |  |  |
|  | 5\% vertically elongated | 8 | $16^{\text {b }}$ | 26 |  |  |
| 30 | Full circle | 9 |  | $21^{\text {b }}$ |  |  |
|  | 5\% vertically elongated | 9 |  | 24 |  |  |
| 36 | Full circle | 10 |  |  | $21^{\text {c }}$ |  |
|  | $5 \%$ vertically elongated | 10 |  |  | $21^{\text {c }}$ |  |
| 42 | Full circle | 12 |  |  | 16 |  |
|  | $5 \%$ vertically elongated | 12 |  |  | 20 |  |
|  | 5\% field-strutted | 12 |  |  | 30 |  |
| 48 | Full circle | 15 |  |  | 15 |  |
|  | $5 \%$ vertically elongated | 15 |  |  | 18 |  |
|  | 5\% field-strutted | 15 |  |  | 30 |  |
| 54 | Full circle | 15 |  |  | 15 |  |
|  | $5 \%$ vertically elongated | 15 |  |  | 18 |  |
|  | 5\% field-strutted | 15 |  |  | 25 |  |
| 60 | Full circle | 18 |  |  |  |  |
|  | 5\% vertically elongated | 18 |  |  |  |  |
|  | 5\% field-strutted | 18 |  |  |  |  |
| 66 | Full circle | 21 |  |  |  |  |
|  | 5\% vertically elongated | 21 |  |  |  |  |
|  | 5\% field-strutted | 21 |  |  |  |  |
| 72 | Full circle | 24 |  |  |  |  |
|  | 5\% field-strutted | 24 |  |  |  |  |
| 78 | $5 \%$ freld-strutted | 24 |  |  |  |  |

${ }^{\mathrm{a}}$ Loading $=$ AASHO-H20 Highway, July 1961, shape $=22 / 3$ by $1 / 2 \mathrm{in}$. Values are for 80- to 85-percent compaction.
${ }^{\mathrm{b}}$ Based on use of $3 / 8-1 \mathrm{n}$. aluminum alloy rivets. If conventional $5 / 16-1 \mathrm{n}$. aluminum alloy rivets used, fill heights should be reduced to 21 ft for $21-0.060 \mathrm{in}$., 13 ft for 24-0.060 $\mathrm{nn} ., 21 \mathrm{ft}$ for 24-0.075 in ., and 19 ft for 30-0.075 m .
cBased on use of $1 / 2-1 n$. aluminum alloy rivets. If conventional 3/8-in. aluminum alloy rivets used, fill heights should be reduced to 18 ft .
borrowed material from an immediately adjacent hillside was available for the backfill operation and is located on the left side of Figure 1. The test was scheduled to commence in early April shortly after the winter rains had subsided. In this particular year the rains lasted somewhat longer so the actual placement of the material did not commence until April 24, 1961. The test period lasted approximately six weeks during which time no measurable rainfall was recorded. Average daily temperatures were approximately 70 F .


Figure 1. General view of test site showing pipe runs in approximate position.

The test site is not be used for traffic but can and will be left intact so that longterm readings can be taken to determine what consolidation effects develop. A drawing of the average sectional profile is shown as Figure 2.

## DEEP FILL SECTIONS

Four series of culvert pipes of the conventional $2 \frac{2}{3}$-in. pitch by $1 / 2$-in. depth corrugation were fabricated by three independent culvert manufacturers in California using standard fabrication practice for inclusion in these tests. The test runs consisted of $24-$, $36-$, $48-$, and $60-\mathrm{in}$. pipe running parallel and spaced a sufficient distance apart to theoretically satisfy the case of positive projection loading conditions and minimize effect of one pipe on another. Each run of pipe consisted of a series of $10-\mathrm{ft}$ long test sections selected and fabricated to demonstrate a series of engineering conditions from which information could be developed. A drawing has been prepared showing the arrangement of these sections (Fig. 2). In addition, Figure 3 shows the four parallel runs before completion of the installation of access tunnels and backfilling with each test section identified. The original surface below the pipe was tapered and leveled with approximately 1 ft of clean sand so that all four runs of pipe could be bedded with the tops at a common elevation.

The $24-\mathrm{in}$. run had four test sections: 16 gauge vertically elongated riveted, 16 gauge ( $0.060-\mathrm{in}$. thickness), 14 -gauge ( $0.075-\mathrm{in}$. thickness) full circle riveted, and a 16-gauge full circle piece which was spot welded with a Sciaky three-phase spot welding machine. The vertically elongated piece was separated from the full circle pieces by a 4 -ft preformed transition section and two couplings. This prevented carryover of structural effect from one shape to the other.

The $36-\mathrm{in}$. run had a section of 12 -gauge ( $0.105-\mathrm{in}$. thickness) full circle riveted, a section of 12 -gauge full circle with two spot welds per pitch, and a section of 12 -gauge vertically elongated riveted pipe. For the $36-\mathrm{in}$. and larger pipes, $3 / 4-\mathrm{in}$. plywood was used to block the ends with sufficient timber support to prevent the plywood from moving out of position during the test. At the downstream end a $24-\mathrm{in}$. hole was cut into the plywood through which a $24-\mathrm{in}$. culvert access tunnel was installed. This method of construction insured that the test sections would be under the maximum fill and that access to the pipe could still be maintained.

The 48 -in. run had a 12 -gauge full circle double-riveted section, a 12 -gauge vertically elongated double-riveted section, and a 12 -gauge timber strutted section using 4 - by 4 -in. timber struts against 4 - by 4 -in. sole and top runners with the struts placed on approximately $5-\mathrm{ft}$ centers.


Figure 2. General arrangement, aluminum culvert tests, concentrated load and deep fill.


Figure 3. General arrangement of deep fill pipe showing location of test sections.

The $60-\mathrm{in}$. run had a section of 10 -gauge ( $0.135-\mathrm{in}$. thickness) full circle doubleriveted, a section of 10 -gauge timber strutted with timber strutting following the procedure used in installation of the $48-\mathrm{in}$. section, and a section of 10 -gauge wire strutted double-riveted pipe. The wire strutting was accomplished in the factory by threading $3 / 18-\mathrm{in}$. wire loops across the centerlıne of the pipe at $2-\mathrm{ft}$ intervals and twisting and upsetting the pipe until a 5 percent pre-set elongation was obtained. The transition in this case was done by making the full circle section 14 ft long and deforming the end.

Vertically elongated pipe in the $24-, 36-$, and 48 -in. sizes was elongated to shape in the factory without stays or other prestressing means. Timber strutting and wire strutting prestressed the pipe by elongating the vertical axis. Vertical elongation in all cases was 5 percent of the diameter of the pipe.

A test section of $36-\mathrm{in}$. 12 -gauge arch pipe was installed at an elevation of approximately 8 ft above and to one side of the deep fill runs so that the structural characteristics of arching can be evaluated. It is located within the indentation in the soil over the $24-\mathrm{in}$. pipe of the deep fill in Figure 4.

## Concentration Load Sections

So that means of investigation of performance of aluminum pipe under minimum fill or concentration load conditions could be investigated, three test sections were bedded into the test site above and to the left of the deep fill sections. These sections were $24-\mathrm{in}$. flat sheet, $24-\mathrm{in}$. corrugated culvert, and $36-\mathrm{in}$. corrugated culvert. The shape of the hill was such that the concentration load pipe could be bedded on 1 ft of clean sand over native, undisturbed material. The $24-\mathrm{m}$. flat sheet was 14 gauge and riveted, the $24-\mathrm{in}$. diameter culvert was 16 gauge corrugated and riveted, and the $36-\mathrm{in}$. diameter was 14 gauge corrugated and riveted. These pipes were placed parallel so that the top centers would be at the same elevation in a manner similar to the deep fill. Figure 4 shows the position of the three test sections above the deep fill pipe when 15 ft of fill was placed.

## Instrumentation

In conducting a series of tests of this magnitude, it is highly desirable to provide as much instrumentation as possible which can be used to develop basic data. The variables affecting the performance of the culvert were recognized to be soil load and distribution, pipe deflection, and pipe stresses. Accordingly, a series of pressure cells, deflection indicators, and strain gauges were installed on the various test sections to serve as the instrumentation for this test. In addition, the Pittsburgh Testing Laboratory was retained to determine compaction curves and compaction values of the soil and to serve as supervisor of instrumentation for this test.

Pressure cells used in this test were made by using an incompressible fluid (i.e., water) to transmit pressure in a system consisting of a common hot water bottle as the pressure-sensing bladder, to which was attached $1 / 4-\mathrm{in}$. diameter aluminum tubing and an indicating pressure gauge. Pressure cells were placed at selected locations at the invert, sides, and across the tops of the deep fill and concentration load tests. Care was taken to level the cells by installing the gauges at the same elevation as the bags. The cells across the top of the pipe were used to demonstrate pressure actually applied to the pipe, and hence, loading actually applied to the pipe. Pressure cells placed against the side of the culvert were used to measure the modulus of soil reaction developed during the loading process on the culvert. Pressure cells at the invert were installed to give some indication of the pressure transmission through the structure. The specific location of these pressure cells is indicated on the master arrangment drawing (Fig. 2) for the culvert installation.

To determine accurately total and incremental deflections for zero to maximum fill conditions, deflection indicators were fabricated and placed in selected culvert sections. These were made up from $3 / 4$-in. wood dowels which telescoped into $3 / 4-\mathrm{in}$. I. D. alummum conduit and were spring loaded to hold position. A short section of conduit was screwed to a section of dowl and washers and spring were inserted inside the conduit against the dowel. Another portion of dowl was inserted into the cavity. An aluminum plate was welded to the conduit with scaled readings in $1 / 4$-in. increments with an indi-
cator attached to the floating dowel. These indicators were subsequently read by field glasses and spotlight from inside the access tunnel. In addition to the above, initial and final deflections under fill were measured for all test sections not equipped with indicators.

A series of SR-4 foil strain gauges were attached to the quadrant points for each test section. A total of six gauges was selected for each group; a pair at the top, a pair at one side, one at the invert, and one in the opposite side. The paired gauges were placed on the inside crest and inside crest and inside valley so that bending could be indicated. The gauges were installed by first sanding smooth and thoroughly cleaning the aluminum surface with acetone, after which the strain gauges were attached using Duco cement. To insure proper linearity of the gauges, a template was used which conformed to the contour of the culvert and accurately indicated the center of the valley and the ridge. Proper pressures were applied to the gauges during the curing stage by placing pieces of foam rubber over the gauges and strutting between the walls of the culvert. No. 20 wire was then soldered to the gauge leads and initial tests for grounding were made at this point. The outside surfaces of the culvert at the gauge locations were then warmed up and Petrocene wax was applied over the gauges to a thickness of approximately $3 / 16 \mathrm{in}$. in an effort to prevent water intrusion. A final ground check was then taken. All six gauges for each station were collected into a harness wired to a common six-pole selector switch which in turn was wired to a 20 -channel switching instrument. Three compensating gauges were used. These were adhered to pieces of aluminum plate, waterproofed, and taped to the inside wall of the culvert in the gauge location areas.

Where gauges were paired, it was deemed advisable to install one gauge on the inside ridge and the opposite gauge on the inside of the valley, thus preventing any contact


Figure 4. General view from below showing $24-, 36-, 48-$, and $60-\mathrm{in}$. runs across the bottom, the arch pipe above 24-in. one, and the three concentration load pipes at top; fill over deep pipe was 15 ft at this point.
with the soil. It is necessary then to extend the reading of the valley gauge to the extreme fiber by applying an algebraic proportion. Once this is done, both the average compressive stress across the section, which is an indication of the shear strength across the joint, and the maximum bending stress could be determined for each pair.

## Backfill

Each of the test sections were carefully bedded into a shaped sand base with the bedding approximately $12-$ to $15-\mathrm{in}$. wide and extending the full length. The initial backfilling of the deep fill sections was with a D4 with a loader attachment. The D4 was selected because of its maneuverability in the bottom of the canyon. A loader attachment was desirable because of the relatively long distance that material had to be moved to be placed to backfill properly the pipe to the top. To accomplish the backfill properly, placement of material commenced in one corner and radiated to the side and forward from that point as in Figure 5. Proper compaction was developed by spreading the material during placement and working with pneumatically operated tampers. This procedure was contınued in approximately $1-\mathrm{ft}$ lifts untıl all pipes were completely covered. Compaction tests in this area showed the average Proctor density to be 83 percent.

One of the purposes of this test was to establish the performance of aluminum pipe under what is accepted to be normal or standard backfill conditions. It is recognized that many specifications require compactions of 95 percent in backfilling operation, but it is also a common fact that the average installation does not secure optimum compaction but merely secures "good" compaction. The structural fill tests were designed to follow the latter pattern and from them it was possible to conclude that normal compaction around culvert pipe would be of the order of 80 to 85 percent Proctor density. This practice and theory was followed throughout the entire fill operation.

Once the material was compacted to the top of the pipe, the D4 loader was dismissed and a D7 loader brought onto the job in its place. The D7 loader served as the means of placement and compaction throughout the remainder of the test. The average compaction attained under the track of the D7 loader by compacting lifts of approximately $8-1 \mathrm{n}$. thickness was 80 percent. Backfill was increased to progressive compacted fill heights in $1-\mathrm{ft}$ increments through 6 ft , in $2-\mathrm{ft}$ increments through 12 ft , and fills of $15,20,22,26$, and 30 ft .

When the fill was 5 ft over the deep fill pipe, both the timber and the wire struts were removed so that data could be obtained on the performance of pipe that had been previously elongated under the application of increasing loads instead of the common practice of leaving the struts in place until the fill was completed. During the removal of the struts, damage was sustained on the strain gauges for the 60 -in. timber-strutted section destroying the apparent valıdity of stress results obtaned from this section.

Pressure, strain, and deflection readings were recorded for each fill height increment for each section in each run.

The concentration load tests were developed in an equal manner to the deep fill tests except that fill heights of 1,2 , and 3.5 ft were the only values considered. At each fill level a series of strain, pressure, and deflection readings were taken which would serve as the basis for analysis of data. Concentrated loads were developed by running one track of the D7 loader onto a series of $3 / 4-1$. thick steel plates with 4 - by $4-\mathrm{in}$. blocking above the plates to insure the total load was impressed on the plates. The plate sizes were 15,18 , and 24 in . square. Thus, a total of 12 complete sets of readings were taken.

## LOADING ANALYSIS

There are several methods of estimating the load actually applied to the culvert pipe in this evaluation. A series of pressure cells were placed on the top of each pipe run to attempt to measure the actual pressure transmitted at the top of the pipe. The pressure exerted by the pıpe at the center on the soil was also measured to assess the modulus of passive resistance. Attempts were made to observe and estimate the settlement



Figure 5. (a) Initial spreading of backfill with D4 loader. (b) progressive backfilling around deep pipe with pneumatic compacting; and (c) leveling backfill over deep fill with 1 - ft fill (pipes in foreground are access tunnels).
of the pipe and the soil adjacent to the pipe so that the settlement ratio of the installation could be predicted.

As indicated earlier, the pipe was installed as a positive projection, the most severe design loading condition. However, the tests demonstrated that the installation had a high negative settlement ratio, and the solution in this case would approach the ditch condition. The actual load on the pipe would be considerably below the predicted load from the dead weight of the fill if this were the case. This condition did, in fact, occur and the computations must reflect this.

The pressure values are plotted in Figure 6. The pressure transmitted to the center of the pipe was shown to be considerably below the normal average pressure across the section after 5 ft of fill was reached. The probable pressure distribution at the top of the pipe is shown in Figure 7. A theoretical uniform pressure equivalent is also shown on the curve for purposes of evaluation using existing theories. It is probable that actual pressure distribution is trapezoidal in nature following the pipe deflection


Figure 6. Pressure cell data.
values, but it is felt that a mean average uniform pressure will produce acceptable results in this case. This mean pressure will be used in computation of load factors following and is given in Table 2, Columns 3, 4, and 5.

The proper evaluation of flexible culvert by means of the Spangler (2) method depends on an accurate appraisal of the product of the settlement and projection ratio. Although the projection ratio, the ratio of distance from undisturbed surface of soil to top of pipe to pipe diameter, in this case could be accurately developed, the settlement ratio can only be estimated. Three comparisons can be developed to check the settlement ratio: pressure ratio, soil compaction deflection and pipe deflection, and visual observation of behavior of soll and pipe.

Moving now to the analysis of settlement ratio, it is necessary to evaluate first the derivation of the settlement ratio with Figure 8 for description.

$$
\begin{equation*}
r_{S}=\frac{\left(S_{M}+S_{g}\right)-\left(\Delta y+S_{f}\right)}{S_{M}} \tag{1}
\end{equation*}
$$

in which

TABLE 2
PRESSURE ACROSS PIPE AND PIPE LOAD FACTORS FROM PRESSUREa

| Culvert <br> Diameter <br> (in.) | Fill <br> Height <br> H (ft) | Pressure (psi) |  |  | $\begin{gathered} \text { Load } \\ W \\ (\mathrm{lb} / \mathrm{ft}) \end{gathered}$ | $\begin{gathered} \mathrm{C}_{\mathrm{C}} \\ \text { from } \\ \text { Pressure } \\ \text { Data } \end{gathered}$ | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | $\begin{gathered} r_{s} p \\ \text { from } \\ \text { Pressure } \\ \text { Data } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{P}_{1}$ | $\mathrm{P}_{2}$ | $\begin{gathered} P \\ \text { Mean } \end{gathered}$ |  |  |  |  |
| 24 | 10 | 7.6 | 5 | 6.3 | 1,810 | 4.15 | 5 | -0.10 |
|  | 20 | 15.2 | 5.5 | 10.3 | 2,960 | 6.78 | 10 | -0.35 |
|  | 30 | 22.8 | 6.0 | 14.4 | 4, 150 | 9.50 | 15 | -0.40 |
| 36 | 10 | 7.6 | 3.3 | 5.5 | 2,380 | 2.42 | 3.3 | -0. 50 |
|  | 20 | 15.2 | 4.6 | 9.9 | 4,280 | 4.36 | 6.7 | -0.50 |
|  | 30 | 22.8 | 6.0 | 14.4 | 6,220 | 6.34 | 10 | -0. 50 |
| 48 | 10 | 7.6 | 3.3 | 5.5 | 3,160 | 1.81 | 2.5 | -0. 50 |
|  | 20 | 15.2 | 4.6 | 9.9 | 5,700 | 3.26 | 5.0 | -0. 50 |
|  | 30 | 22.8 | 6.0 | 14.4 | 8,290 | 4.75 | 7.5 | -0.50 |
| 60 | 10 | 7.6 | 3.3 | 5.5 | 3,960 | 1.46 | 2 | -0.50 |
|  | 20 | 15.2 | 4.6 | 9.9 | 7,130 | 2.61 | 4 | -0. 50 |
|  | 30 | 22.8 | 6.0 | 14.4 | 10,360 | 3.80 | 6 | -0.50 |

${ }^{\mathrm{a}} \mathbf{P}_{1}=$ average pressure across top of pipe due to weight of $\mathrm{f}_{11 l}$ (psi), $\mathbf{P}_{2}=$ measured pressure at top center of pipe (psi). $\mathrm{P}_{\text {mean }}=\left(\mathrm{P}_{1}+\mathrm{P}_{2}\right) / \mathrm{z}, \mathrm{W} \quad 12 \mathrm{~PB}_{\mathrm{c}}, \mathrm{B}_{\mathrm{c}}=$ pipe diameter in feet, and $\mathrm{C}_{\mathrm{c}}=\mathrm{W} / \mathrm{wB}_{\mathrm{c}}{ }^{2}$.
$r_{s}=$ settlement ratio,
$\mathrm{S}_{\mathrm{M}}=$ settlement of bottom critical plane,
$S_{g}=$ settlement of bottom critical plane below soil,
$\Delta y=$ deflection of top of pipe, and
$S_{f}=$ settlement of bottom pipe.
This settlement ratio, when combined with the projection ratio, is the means by which which the load on the pipe can be determined. The computed load is expressed from

$$
\begin{equation*}
\mathrm{w}=\mathrm{C}_{\mathrm{c}} \mathrm{wB}_{\mathrm{c}}{ }^{2} \tag{2}
\end{equation*}
$$

in which
$\mathrm{W}=$ load on pipe ( lb per ft of length) (see eq. 1);
$\mathrm{C}_{\mathrm{c}}=$ load coefficient, a function of fill height, settlement ratio, and projection ratio;
$\mathrm{w}=$ density of soil (pcf); and
$\mathrm{B}_{\mathrm{C}}=$ width or diameter of pipe ( $\mathrm{f} \ddagger$ )
Product of $\mathbf{C}_{\mathbf{c}} \mathbf{B}_{\mathbf{c}}$ equals H if the settle-ment-projection ratio is zero; the full vertical wedge of the soll acts downward and $\mathrm{W}=\mathrm{wHB}_{\mathrm{c}} . \mathrm{C}_{\mathrm{c}}$ can be computed by know-


Figure 7. Pressure distribution across prpe plane under 30 ft of fill.

$$
\text { SETTLEMENT RATIO }=r_{s}=\frac{\left(S_{M}+S_{g}\right)-\left(\Delta y+S_{f}\right)}{S_{M}}
$$



Figure 8. Settlement ratio for flexible pipe.
ing the average pressure on the soil above the pipe and comparing this with the density of soll and diameter. This is done in Table 2 and forms the basis for computations of load. This method serves also as a means for determination of the actual settlement ratio.

Observation showed the values of $\mathrm{S}_{\mathrm{g}}$ and $\mathrm{S}_{\mathrm{f}}$, the settlement into the ground, to be very small and these values can be neglected in this test. $\Delta \mathrm{y}$ (which is equal to $\Delta \mathrm{x}$ ) could be readily measured and is given in Table 3 as measured $\Delta x$. This leaves the evaluation of $S_{M}$ necessary to complete the settlement ratio. Observation showed the soll to compress only a portion of that of the pıpe. Thus, in the limit if $S_{M}$ were zero, the settlement ratio would be negatıve infinite or equal in magnitude to the complete ditch condition. This confirms the trend shown in the pressure data to an approximate degree.

Marston and Spangler (3) were able to evaluate $\mathrm{C}_{\mathrm{c}}$ into a family of curves relating the settlement-projection ratio and the ratio of full height to pipe diameter. The computed values of $\mathbf{C}_{\mathrm{c}}$ and $\mathbf{r}_{\mathrm{s}} \mathrm{p}$ taken from the pressure data are given in Table 2.

As an additional method of determining the settlement ratio, a series of laboratory compaction tests were conducted on $6-1 \mathrm{n}$. diameter by $6-\mathrm{in}$. deep soll specimens. These specimens were initıally compacted to 74,83 , and 93 percent Proctor density and then
subjected to up to $40-\mathrm{psi}$ compressive stress. These results were then converted to percent consolidation (Fig. 9). Because the average compaction around the pipe was 83 percent, these data would indicate the percent settlement of the soll ( $\mathrm{S}_{\mathrm{M}} / \mathrm{D}$ ) to be 0.8 at 30 ft of fill. If the deflection of pipe only were considered, the computed settlement ratio would be approximately -5.0 . Although the value determined in this simple means is substantially over the value computed from pressure data, the deviation in the load coefficient within this test is quite small. Considering the extreme difficulty in comparing this cylinder compaction test with field performance, which is of larger scale and is three dimensional, it is felt that reasonable correlation of experimental data was obtained. Certainly the true soil settlement would be higher than the laboratory value making the check between pressure determination and settlement determination in this test quite good.

## SUPPORTING STRENGTH ANALYSIS

Before undertaking a discussion of the results of the structural fill tests, further analysis of loading and support strength should be undertaken. The method that considers the bending stresses of the culvert pipe with inclusion of soil resistance is the method of rational moments and deflection as proposed by Spangler (2). In this approach a loading pattern on the flexible pipe is developed which consists of a uniform pressure across the full top of the culvert, a parabolic-shaped soll resistance at the centerline, and a uniform but shorter width bearing pressure across the invert. This pressure distribution is shown as Figure 10. In this method of evaluation it is assumed that the pipe sustains a high deflection to ultimate fallure before the average compressive stress on the sheet is sufficient to separate the riveted joints. The form of the equation proposed by Spangler is

$$
\begin{equation*}
\Delta x=\frac{\mathrm{D}_{\mathrm{L}} \mathrm{KW}_{\mathrm{c}^{3}}}{\mathrm{EI}+0.061 \mathrm{E}^{\prime} \mathrm{r}^{3}} \tag{3}
\end{equation*}
$$

in which

$$
\begin{aligned}
\Delta \mathrm{x}= & \text { horizontal deflection (in.) ( } \Delta \mathrm{x} \text { is equal to } \Delta \mathrm{y}, \\
& \text { the vertical deflection); } \\
\mathrm{D}_{\mathrm{L}}= & \text { deflection lag factor, based on observed con- } \\
& \text { tinuing deflection as soll completes its con- } \\
& \text { solidation; } \\
\mathrm{K}= & \text { bedding constant; } \\
\mathrm{W}_{\mathrm{C}}= & \text { soll load on pipe (lb per in.); } \\
\mathrm{r}= & \text { mean pipe radius (in.); } \\
\mathrm{E}= & \text { modulus of elasticity of pipe metal (psi); } \\
\mathrm{I}= & \text { moment of inertia of pipe section (in. } 4 \\
& \text { per in.); } \\
\mathrm{E}^{\prime}= & \text { modulus of soil reaction (psi), a measure of } \\
& \text { resistance of soll to horizontal expansion } \\
& \text { under load, equal to er; } \\
\mathrm{e}= & \text { modulus of passive pressure of soıl (psi/m.). }
\end{aligned}
$$

Significantly, this equation takes into account the structural effect of both the metal comprising the pipe and the soil resisting the expansion of the pipe at the centerline. Soil analysis will develop the load applied, bedding constant, and modulus of passive resistance of the soil. The remainder of the items are geometry of the structure.

In the tests conducted, $W_{c}, K$, and $E^{\prime}$ are values determined by performance of the soll; $\mathrm{E}, \mathrm{I}$, and r are determined by the geometry of the pipe itself; and $\Delta \mathrm{x}$ is the measured or predicted deflection of the pipe. $D_{L}$, the deflection lag factor, is unity when taken on the immediate completion of the test, increasing to a suggested 1.50 for design purposes when full consolidation of a typical soll has been completed.

The initial step in evaluation of the data is to calculate deflection from actual test results using Eq. 3 and compare it with the measured deflection. After corroboration of data, the second step is to develop design fill heights using a "standard" set of soll conditions.

TABLE 3
MODULUS OF PASSIVE RESISTANCE OF SOIL ${ }^{\text {a }}$

| $\begin{aligned} & \text { Fill } \\ & (\mathrm{ft}) \end{aligned}$ | 48-In. Culvert |  |  | 60-In. Culvert |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P$ Pressure (ps1) | $\Delta \frac{\mathrm{x}}{2}$ | $\begin{gathered} \mathrm{e} \\ (\mathrm{ps} 1 / \mathrm{n} .) \end{gathered}$ | $\bar{P}$ Pressure $\left(\mathrm{ps}_{1}\right)$ | $\Delta \frac{\mathrm{x}}{2}$ | $\begin{gathered} \mathrm{e} \\ \left(\mathrm{psi} / 1 \mathrm{n}_{.}\right) \end{gathered}$ |
| 5 | 5.0 | 0.25 | 20.0 | 5.0 | 0.12 | 41.6 |
| 10 | 9.1 | 0.34 | 26.7 | 7.1 | 0.25 | 28.4 |
| 15 | 15.2 | 0.56 | 27.2 | 7.5 | 0.34 | 22.0 |
| 20 | 20.8 | 0.81 | 25.7 | 9.1 | 0.50 | 18.2 |
| 25 | 20.5 | 1.31 | 15.6 | 9.0 | 0.75 | 12.0 |
| 30 |  |  |  | 17.1 | 1.37 | 12.5 |
| Average $\mathrm{e}=$ |  |  | 23.0 |  | = | 22.4 |
| $\mathrm{E}^{\prime}=\mathrm{er}=$ |  |  | 558 |  | = | 678 |

${ }^{\text {a }}$ A typical value of $E$ ' of 600 psi is used in checking results.

TABLE 4


Figure 9. Percent settlement of soil samples vs applied pressure.


Figure 10. Pressure distribution against pipe per spangler.

Referring once again to Eq. 3, the aluminum culvert equation for test results can be reduced by substitution of known terms: $\mathrm{D}_{\mathrm{L}}=1.00 ; \mathrm{K}=0.108 ; \mathrm{W}_{\mathrm{c}}=1 / 12 \mathrm{C}_{\mathrm{c}} \mathrm{wB}_{\mathrm{c}_{2}}{ }^{2}$ (in which $\mathrm{B}_{\mathrm{C}}=2 \mathrm{r} / 12$ and $\mathrm{w}=135 \mathrm{pcf} \times 0.80=109$, and therefore, $\mathrm{W}_{\mathrm{c}}=0.252 \mathrm{C}_{\mathrm{c}} \mathrm{r}^{2}$ ); $\mathrm{E}=10 \times 10^{6} \mathrm{ps} 1$; and $\mathrm{I}=0.0332 \mathrm{t} \mathrm{in} .4 / \mathrm{in} .\left(2 \frac{2}{3}-\mathrm{by} 1 / 2-\mathrm{in}\right.$. pattern) with $\mathrm{t}=$ thickness of sheet (m.).

The modulus of passive resistance, E ', can be determined from the pressures developed in the soll and the deflection of the pipe into the soil. During the structural deep fill tests both values were recorded on the $48-$ and $60-\mathrm{m}$. sizes. These are shown in Fig. 9 from calculation of $\mathrm{e}=\frac{\mathbf{P}}{\Delta \mathbf{x} / 2}$.

The results of Eq. 3 are given in Table 4. For computation, $\mathrm{C}_{\mathbf{c}}$ is taken from the pressure data previously developed with a typical settlement-projection ratio of -0.50 . The comparative analysis deflection compares well with the measured deflection, particularly after all allowances are made for the evident varıations in soil compaction and inaccuracies in measurement and prediction of pressures. The evaluation is considered accurate for further use of the basic equation in development of design fill values.

Using the accepted standard soil conditions for design purposes, Eq. 3 can be reformed by substituting $\Delta x=5$ percent of diameter $=0.10 \mathrm{r} ; \mathrm{D}_{\mathrm{L}}=1.50$, as suggested by Spangler; $K=0.110 ; W_{c}=1 / 12 C_{C} W_{B B}{ }^{2}$ (in which $B_{c}=2 r / 12$ and $\mathrm{w}=100 \mathrm{pcf}$, and therefore, $\mathrm{W}_{\mathrm{c}}=0.231 \mathrm{C}_{\mathrm{c}} \mathrm{r}^{2}$ ); $\mathrm{E}=10 \times 10^{6} ; \mathrm{I}=0.0332$ t; and $\mathrm{E}^{\prime}=700 \mathrm{psi}$ to form:

$$
\begin{equation*}
C_{c}=\frac{112}{r}+0.870 \times 10^{6} \frac{t}{r^{4}} \tag{4}
\end{equation*}
$$

TABLE 4
CALCULATED AND MEASURED DE FLECTIONS OF ALUMINUM CULVERT PIPE

| Culvert <br> Diameter (in.) | Thickness (in.) | $\begin{aligned} & F_{1 l l} \\ & (\mathrm{ft}) \end{aligned}$ | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | $\mathrm{C}_{\mathrm{c}}$ | $\begin{gathered} \Delta \mathrm{x} \\ \text { Calculated } \end{gathered}$ | $\begin{gathered} \Delta x \\ \text { Measured } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | 0.060 | 5 | 2.5 | 2.5 | 0.217 | 0.12 |
|  |  | 10 | 5.0 | 4.3 | 0.374 | 0.25 |
|  |  | 15 | 7.5 | 5.6 | 0.486 | 0.37 |
|  |  | 20 | 10 | 6.8 | 0.590 | 0.37 |
|  |  | 25 | 12.5 | 8.2 | 0.712 | 0.37 |
|  |  | 30 | 15 | 9.5 | 0.825 | 0.50 |
| 24 | 0.075 | 5 | 2.5 | 2.5 | 0.206 | 0.06 |
|  |  | 10 | 5.0 | 4.3 | 0.354 | 0.31 |
|  |  | 15 | 7.5 | 5.6 | 0.461 | 0.44 |
|  |  | 20 | 10 | 6.8 | 0.560 | 0.81 |
|  |  | 25 | 12.5 | 8.2 | 0.675 | 0.94 |
|  |  | 30 | 15 | 9.5 | 0.782 | 1.06 |
| 36 | 0.105 | 5 | 1.7 | 1.5 | 0.321 | 0.12 |
|  |  | 10 | 3.3 | 2.4 | 0.514 | 0.18 |
|  |  | 15 | 5.0 | 3.3 | 0.707 | 0.50 |
|  |  | 20 | 6.7 | 4.4 | 0.942 | 0.75 |
|  |  | 25 | 8.3 | 5.3 | 1.133 | 1.00 |
|  |  | 30 | 10 | 6.3 | 1.350 | 1.00 |
| 48 | 0.105 | 5 | 1.3 | 1.1 | 0.450 | 0.50 |
|  |  | 10 | 2.5 | 1.9 | 0.778 | 0.68 |
|  |  | 15 | 3.8 | 2.6 | 1.064 | 1.12 |
|  |  | 20 | 5.0 | 3.3 | 1.35 | 1.62 |
|  |  | 25 | 6.2 | 4.0 | 1.64 | 2.62 |
|  |  | 30 | 7.5 | 4.8 |  |  |
| 60 | 0.135 | 5 | 1.0 | 0.9 | 0.586 | 0.25 |
|  |  | 10 | 2.0 | 1.6 | 1.043 | 0.50 |
|  |  | 15 | 3.0 | 2.2 | 1.43 | 0.68 |
|  |  | 20 | 4.0 | 2.7 | 1.76 | 1.00 |
|  |  | 25 | 5.0 | 3.3 | 2.14 | 1.50 |
|  |  | 30 | 6.0 | 3.9 | 2. 54 | 2.75 |

${ }^{a}$ Using value of $r_{s} p \quad-0.50$.

Once $\mathrm{C}_{\mathrm{c}} 1 \mathrm{~s}$ determined from this equation, a settlement-projection ratio must be assumed from which a fill height-pipe diameter ratio and finally fill height can be evolved. The most severe design requirement would be the complete projection condition of high positive settlement-projection ratio. It is probable, however, that the settlement-projection ratio in practice will not reach this value and only rarely reach +0.5 . Normal design tables for aluminum culvert herein are then based on the settlement ratio of zero in conformance with standard practice on other materials. Actually, such a decision builds into the aluminum culvert a safety margin somewhat higher than for the other materials due to the strong tendency toward ditch condition loading. This margin has been considered in later selection and evaluation of safety factors on pipe stress. To show the range possible in evaluation of culvert support strength, the range of fill heights between complete ditch condition and complete projection condition is given in Table 5. The projection values become of interest in the review of vertical elongation at a later point.

Spangler noted that though he was able to confirm the deflection theory with field experience the predicted stresses exceeded the probable stress levels by a wide margin.

TABLE 5
THEORETICAL FILL HEIGHTS AS DEVELOPED FROM SETTLEMENT RATIO THEORY OVER RANGE FROM COMPLETE DITCH TO COMPLETE PROJECTION CONDITION

| Culvert <br> Diameter (in.) | $\begin{aligned} & \text { Thickness } \\ & \text { (in.) } \end{aligned}$ | $C_{c}$ | Complete Ditch $\left(r_{s} p=-2.0\right)$ |  | Imperfect Ditch$\left(\mathrm{r}_{\mathrm{s}} \mathrm{p}=-0.5\right)$ |  | Static Weight ( $\mathrm{r}_{\mathrm{s}} \mathrm{p}=0$ ) |  | Imperfect Projection $\left(\mathrm{r}_{\mathrm{s}} \mathrm{p}=+0.5\right.$ ) |  | Complete Projection |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | H | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | H | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | H | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | H | $\frac{\mathrm{H}}{\mathrm{~B}_{\mathrm{c}}}$ | H |
| 15 | 0.060 | 28.95 |  |  |  |  | 28.95 | 36.2 |  |  |  |  |
|  | 0.075 | 32.55 |  |  |  |  | 32.55 | 40.7 |  |  |  |  |
| 18 | 0.060 | 19.22 |  |  | 31.8 | 47.7 | 19.22 | 28.8 | 12.8 | 19.2 | 6.9 | 10.3 |
|  | 0.075 | 21.00 |  |  | 35 | 52.5 | 21.00 | 31.4 | 14 | 21.0 | 7.4 | 11.1 |
| 21 | 0.060 | 14.30 |  |  | 23.6 | 41.3 | 14.30 | 25.0 | 9.5 | 16.6 | 5.5 | 9.6 |
|  | 0.075 | 15.28 |  |  | 25.3 | 44.2 | 15.28 | 26.8 | 10.1 | 17.7 | 5.7 | 10.0 |
| 24 | 0.060 | 11.47 |  |  | 19.0 | 38.0 | 11.47 | 22.9 | 7.6 | 15.2 | 4.65 | 9.3 |
|  | 0.075 | 12.05 |  |  | 20.0 | 40.0 | 12.05 | 24.1 | 7.9 | 15.8 | 4.80 | 9.6 |
| 30 | 0.075 | 8.54 |  |  | 13.8 | 34.5 | 8.54 | 21.3 | 5.5 | 13.7 | 3.85 | 9.6 |
|  | 0.105 | 9.03 |  |  | 14.8 | 37.0 | 9.03 | 22.5 | 5.9 | 14.7 | 4.00 | 10.0 |
| 36 | 0.075 | 6.73 |  |  | 10.8 | 32.4 | 6.73 | 20.2 | 4.3 | 12.9 | 3.30 | 9.9 |
|  | 0.105 | 6.96 |  |  | 11.3 | 33.9 | 6.96 | 20.9 | 4.5 | 13.5 | 3.4 | 10.2 |
|  | 0.135 | 7.20 |  |  | 11.6 | 34.8 | 7.20 | 21.6 | 4.7 | 14.1 | 3.45 | 10.3 |
| 42 | 0.105 | 5.71 | 17.0 | 59.4 | 9.1 | 31.8 | 5.71 | 20.0 | 3.7 | 12.9 | 3.0 | 10.5 |
|  | 0.135 | 5.85 | 17.5 | 61.2 | 9.4 | 32.8 | 5.85 | 20.5 | 3.8 | 13.3 | 3.05 | 10.7 |
| 48 | 0.105 | 4.88 | 13.6 | 54.4 | 7.8 | 31.2 | 4.88 | 19.5 | 3.1 | 12.4 | 2.65 | 10.6 |
|  | 0.135 | 4.96 | 14.2 | 56.8 | 8.0 | 32.0 | 4.96 | 19.8 | 3.2 | 12.8 | 2.70 | 10.8 |
| 54 | 0.105 | 4.28 | 13.3 | 59.9 | 6.7 | 30.2 | 4.28 | 19.2 | 2. 75 | 12.4 | 2.45 | 11.0 |
|  | 0.135 | 4.32 | 13.4 | 60.3 | 6.8 | 30.6 | 4.32 | 19.4 | 2.8 | 12.6 | 2.45 | 11.0 |
|  | 0.165 | 4.37 | 13.4 | 60.3 | 6.8 | 30.6 | 4.37 | 19.7 | 2.8 | 12.6 | 2.50 | 11.2 |
| 60 | 0.135 | 3.84 | 9.4 | 47.0 | 6.0 | 30.0 | 3.84 | 19.2 | 2.45 | 12.2 | 2.25 | 11.2 |
|  | 0.165 | 3.87 | 9.4 | 47.0 | 6.0 | 30.0 | 3.87 | 19.3 | 2.47 | 12.3 | 2.25 | 11.2 |
| 66 | 0.135 | 3.47 | 8.1 | 44.5 | 5.45 | 30.0 | 3.47 | 19.1 | 2.2 | 12.1 | 2.10 | 11.5 |
|  | 0.165 | 3.49 | 8.1 | 44.5 | 5.45 | 30.0 | 3.49 | 19.2 | 2.2 | 12.1 | 2.1 | 11.5 |
| 72 | 0.165 | 3.16 | 6.4 | 38.4 | 4.8 | 28.8 | 3.16 | 19.0 | 2.0 | 12.0 | 2.0 | 12.0 |

Strain and pressure data results on the test confirmed this conclusion. However, the experimental data also indicated the probable cause for the disparity in the stress data. If the load distribution on the pipe is modified to reflect a variance in applied pressure across the top and bottom planes, the solution of the deflection equation and determination of moments, reaction, and stresses produced results somewhat more consistent with the experimental results. Assuming a modified pressure distribution across the planes of the installation as shown in Figure 11, the equation for the moment at any point can be written as

$$
\begin{align*}
& M_{r} \varphi=M_{A}+R_{A} r(1-\cos \varphi)-\mathbf{P}_{A^{\prime}} \frac{\mathbf{r}^{2}}{2} \sin ^{2} \varphi- \\
& \mathbf{P}_{A^{\prime}} A^{\prime} \frac{\mathbf{r}^{2}}{6}\left(\sin ^{3} \varphi-3 \sin ^{2} \varphi\right)-\mathbf{P}_{\mathrm{B}} \frac{r^{2}}{6}(1-\cos \varphi)^{3}- \\
& \mathbf{P B}^{\mathbf{r}^{2}} \frac{\cos ^{3} \varphi}{3} \cos ^{3} \tag{5}
\end{align*}
$$

in which

$$
\begin{aligned}
& \text { Limits }=0<\varphi<\pi \text { and } \pi / 2<\varphi<\varphi<\pi \\
& \mathbf{A}=\left(\mathbf{P}_{\mathbf{A}^{\prime}}-\mathbf{P}_{\mathbf{A}}\right) / \mathbf{P}_{\mathbf{A}^{\prime}}
\end{aligned}
$$

Solution for moment and deflection produces

$$
\begin{align*}
\mathbf{M}_{A} & =\mathbf{P}_{A}{ }^{\prime} r^{2}(0.250-0.179 \mathrm{~A})-0.155 \mathrm{P}_{\mathrm{B}} \mathrm{r}^{2}  \tag{6a}\\
\mathrm{R}_{\mathrm{A}} & =0.50 \mathrm{P}_{\mathrm{B}} \mathbf{r}^{2}  \tag{6b}\\
\Delta \mathrm{x} & =\frac{\mathrm{JW}_{\mathbf{c}^{3}} \mathbf{r}^{3}}{\mathrm{EI}+0.054 \mathrm{er}^{4}}  \tag{6c}\\
\mathrm{~J} & =\frac{0.0835-0.055 \mathrm{~A}}{1-0.50 \mathrm{~A}} \tag{6d}
\end{align*}
$$

The results of this analysis confırm the equation developed by Spangler, deviating by a few percent due to the triangular pressure distribution assumption as contrasted to the parabolic distribution of Spangler. Study of the results of these equations indicates the prospective error in results by using triangular and trapezoidal distribution will introduce negligible errors.

In comparing Eq. 6 with the experımental results on $48-$ and $60-\mathrm{in}$. pipes, the deflections and pressures ( $\mathrm{P}_{\mathrm{A}}, \mathrm{P}_{\mathrm{A}^{\prime}}$, and $\mathrm{P}_{\mathrm{B}}$ ) are given in Table 6. Comparison of results for pressure is based on use of measured total bending and compressive stress as a means to evalutate pressures. The pressures $\mathbf{P}_{A}{ }^{\prime}$ and $P_{B}$ were held and $P_{A}$ varied as required to develop the solution.

The results, though preliminary, show conclusively that by modification of applied pressure distribution to the pipe the predicted and measured stresses can be brought into line and the deflection remains virtually unchanged. This would indicate the approach of Spangler to be valid for description of performance of flexible culvert pipe. Unfortunately, prediction of results is difficult when based on pressures because a small variation in pressure produces a large difference in moment and hence stress.

## STRESS EVALUATION

The culvert pipe in the structural test was installed with a large number of strain gauges placed to indicate load strain and hence stress in bending and compression at the quarter points. Before discussion of results can commence, it is necessary to establish the design stress levels that may be used for compression purposes.

The culvert material used in the test was typical culvert sheet having the mechanical properties given in Table 7.

$$
W_{C}=\left(P_{A^{\prime}}+P_{A}\right) r
$$



Figure 1l. Modified pressure distribution across planes of installation.

The selection of a suitable safety factor shall be made by application of the design values proposed for an alloy of similar mechanical properties in the ASCE Specification for Structures of Aluminum Alloy 6061-T6 (4). This specification covering an alloy with ultimate strength of $38,000 \mathrm{psi}$ and yeld strength of $35,000 \mathrm{psi}$ is as follows: safety factor on ultimate strength $=2.71$; safety factor on yield strength in tension $=$ 2.33; and safety factor on yield strength in compression and buckling =2.50. These values are further explained in Part IV, Section A, Summary of Allowable Stresses, Item A-1:

TABLE 6
MEASURED AND THEORETICAL PRESSURES ON 48- AND 60-IN. SECTIONS

| Pressure Type | 48-In. Culvert, 24-Ft Fill (ps1) | $60-\mathrm{In}$. Culvert, 30-Ft Fill (psi) |
| :---: | :---: | :---: |
| Meas. side pressure $\mathrm{P}_{\mathrm{B}}$ | 21 | 17 |
| Meas. pressure directly over pipe $\mathrm{P}_{A}$ | 5.2 | 6 |
| Pressure on adjacent soil $\mathrm{P}_{\mathrm{A}}$ at pipe top | 18.5 | 22.8 |
| Meas. avg. max. stress in pipe | 20,000 | 25,000 |
| $\mathrm{P}_{\mathrm{A}}$ computed pressure over pipe ${ }^{\text {a }}$ | 12.9 | 7.8 |

[^0]TABLE 7
MECHANICAL PROPERTIES OF TYPICAL CULVERT SHEET

| Type | Ultimate Tensile <br> Strength <br> (psi) | Yield <br> Strength <br> (psi) | Elongation <br> in 2 In. <br> $(\%)$ |
| :--- | :---: | :---: | :---: |
| Flat corrugated sheet | 35,000 | 29,000 | 9 |
| Culvert product <br> Recom. design min. properties of <br> completed culvert | 37,000 | 32,000 |  |

> A-l. Baslc Tensile Design Stress. -- The basic tensile design stress of ly kips per sq in. represents a factor of safety of 2.33 based on the specifled tensile yleld strength. This ls a larger factor of safety with respect to yleld strength than ls ordinarily encountered in specifications for structural steel. In selecting this rather large factor of safety on yleld strength, the conmittee was anfluenced to a considerable extent by the fact that there is a smaller spread between yield strength and tensile strength in this alumınum alloy than ls commonly encountered in structural steels.

Using the preceding values, a design maxımum stress for combined bending and axial compression is $S=11,200 \mathrm{psi}$, which is used to develop the fill heights from the strain data on the pipe.

A proper value of the safety factor for the resistance to rivet crushing, shear, or pull-out can be developed from the ultimate joint strength. This approach is similar to the well-known compression ring theory. The computed strength of the rivet joint is based on a shear strength of $18,500 \mathrm{psi}$ and an ultimate bearing strength on the sheet of $65,000 \mathrm{psi}$. It is suggested that for design purposes a safety factor of 3.3 be used to set the crushing design limits or a value lower if that value is consistent with basic steel pipe design practices. This allows for a normal safety factor of 3.0 with a 10 per cent reduction to account for the increase of typical properties over minimum propertie of the joint. Simularly, if spot welding is to be used a safety factor of 3.3 is suggested to apply against typical spot strengths. This value is consistent with design considerations for other components of the system, and considering that where deep fill design is encountered, excellent control of construction with attendant accuracy of prediction of load, the safety factor is considered ample.

Evaluation of safety factors in design based on soil factors is difficult. It is important to distinguish between the development of load on the pipe due to the soll settlement and internal shear and performance of the soil-pipe combination once this load has been established. Safety factors are applied to performance under load and judgement governs the load application. It is accepted practice to consider a final deflection of 5 percent of the diameter to the design limit. This is based on the assumption that 20 percent deflection is tantamount to fallure with an apparent safety factor of 4.0 to arrive at 5 percent.

## RESULTS

There are several methods of evaluation of flexible metal pipe with respect to its load-carrying ability. Three theoretical approaches may be used to evaluate the predicted load-carrying ability: elastic instability, crushing, and deflection analysis.

These may now be corroborated by comparing measured deflection and stress data obtained and from the combined strain evaluation, an accurate basis for fill height table determination for corrugated aluminum culvert may be prepared.

## Full Circle Pipe

The basic method of evaluation of pipe is in the full circle form. This lends itself to analysis of the structure using the assumption that there is little or no initial stress in the pipe. Load-carrying ability of flexible pipe, and aluminum pipe in particular, is critically dependent on the distribution of load and the total load or full height. It is necessary then to make a basic assumption of the magnitude of load before completing the analysis. The worst design load condition occurs when complete projection is assumed. To attain this condition physically, the soil must be so loosely filled about the pipe that the soll adjacent to the pipe can compress substantially more than the culvert. Practically, this condition is difficult to attain in flexible pipe, and of attaned at all, service of the pipe would be hazardous because of the very low compaction and the pipe would compress out of round readily. The probable ultimate limit design condition to be encountered in the field would be the case where the soll would compress twice as much as the pipe resulting in a settlement-projection ratio of +0.5 , but even this is unlikely. For design purposes it is best to assume the condition where the entire weight of soll and live load directly above the pipe is carried directly to the pipe. This is the case where the settlement-projection ratio is zero or $\mathrm{C}_{\mathrm{c}}=\mathrm{H} / \mathrm{B}_{\mathrm{c}}$, resulting in the load on the pipe being computed from $W=w H B_{c}$. This condition is used for the design fill analysis comparison on full circle pipe.

Having established the design load conditions, the test results and prediction equations may now be compared. Table 8 gives the composite result from which a fill table for full circle pipe can be extended.

A series of 14 stations were used to evaluate performance of culvert pipe. Strain curves were developed into bending and compressive stress curves as load progressed. Using design stresses and safety factors the recommended maximum fills for bending, joint compression, and deflection can be evaluated.

In summing up the results of the several methods of evaluation of full circle pipe and comparing this with experımental data, a normal design fill height can be developed for each diameter and gauge. In the selection of the height it is necessary to consider the relative weight or validity of each set of predictions and test results. The crushing or shear concept serves as an excellent check which the design levels should not exceed, but it is not the major loading condition experienced in full circle pipe as reflected from this test. The prime weight should be given to the deflection or moment computation and the stress values determined. The final selection of the fill height is shown in the last column of Table 8 and serves the basis for prediction of the fill height for all diameters and gauges. Extrapolation to sizes other than tested can be made by extension of the deflection equation.

Taking a specific look at the deep fill results shows some disparity from a precisely normal pattern. For example, the $36-\mathrm{m}$. pipe consistently performed better than average, whereas the $24-\mathrm{in}$. one was equally below average. The 48 - and the $60-\mathrm{in}$. ones were about as expected. These differences can readily be attributed to differences in compaction around the pipe and settlement of the soll adjacent to the pipe as a small difference in either can produce a substantial difference in results.

However, the placing of the pipe followed acceptable good practice with nominal effort being made to produce better-than-average conditions. The conditions of the test are typical of average requirements of compactable soll and reasonable backfill compaction around the pipe. The results shown wall hold for compaction around the pipe of 80 percent (by the Proctor method) or more. When the compaction drops below 75 percent the design fill height or load-carryıng ability is severely restricted. Simply stated, no flexible or rigid culvert can be expected to carry significant load without some compaction of backfill material.

Just as reduction in compaction will cause a reduction in load capacity, closely controlled good compaction will result in greater ability to carry load. When the backfill

TABLE 8
DESIGN FILL HEIGHTS BY ANALYTICAL AND MEASURED MEANS ${ }^{\text {a }}$

| Culvert Diameter (in.) | Thickness (in.) | Fill Height |  |  | Mean Compressive Stress for Shear from Strain Gauges (fig.) | Extreme Fiber <br> Stress From Strain Gauges (fig.) | Recommended <br> Design Fill <br> Height for <br> Aluminum <br> Alloy Full <br> Circle <br> Culvert |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Elastic InstabilityDetermined ${ }^{b}$ | Crushing Or ShearDetermined $C$ | Deflection SettlementDetermined |  |  |  |
|  |  |  |  | $\overline{r_{s} \mathrm{p}}=0 \quad \mathrm{r}_{s} \mathrm{p}+=0.5$ |  |  |  |
| 24 | 0.060 | 8 | 15 | 15 | 12 | 12 | 13 |
|  |  |  |  |  | 16 | 19 |  |
|  |  |  |  |  | -- | 18 |  |
| 36 | 0.075 | 16 | 24 | 2416 | 20 | 19 | 21 |
|  | 0.105 | 10 | 18 | 13 | 22 | 15 | 18 |
|  |  |  |  |  | 21 | 27 |  |
|  |  |  |  |  | 28 | 19 |  |
| 48 | 0.105 | 6 | 24 | 12 | -- | 15 | 15 |
|  |  |  |  |  | 26 | 15 |  |
|  |  |  |  |  | 36 | 18 |  |
| 60 | 0.135 | 4 | 20 | 12 | 18 | 12 | 14 |
|  |  |  |  |  | 22 | 15 |  |

[^1]TABLE 9
FILL HEIGHT COMPARISONS AND RECOMMENDATIONS VERTICALLY ELONGATED CULVERT

| Culvert Diameter (in.) | Thickness (in.) | Crushing or Shear Fill Herght Based on Load at |  | Deflection Settlement Determined Fill Height |  | Recommended Fill Height |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Vertically <br> Elongated Only | Prestress Strutting |
|  |  | $\mathrm{r}_{\mathrm{s}} \mathrm{p}=0$ | $\mathrm{r}_{\mathrm{s}} \mathrm{P}=-0.50$ |  |  | $\mathrm{r}_{\mathrm{sp}}=-0.20$ | $\mathrm{r}_{\mathrm{s}} \mathrm{p}=-0.50$ |
| 36 | 0.105 | 17 | 27 | 26 | 33 | 18 | 27 |
|  | 0.135 | 18 | 29 | 27 | 34 | 18 | 29 |
| 42 | 0.105 | 29 | 47 | 24 | 31 | 20 | 31 |
|  | 0.135 | 29 | 47 | 25 | 32 | 25 | 32 |
| 48 | 0.105 | 25 | 41 | 24 | 31 | 18 | 31 |
|  | 0.135 | 25 | 41 | 26 | 33 | 21 | 33 |
|  | 0.165 | 25 | 41 | 23 | 30 | 23 | 30 |
| 54 | 0.105 | 22 | 36 | 23 | 30 | 18 | 25 |
|  | 0.135 | 22 | 36 | 23 | 30 | 20 | 30 |
|  | 0.165 | 22 | 36 | 23 | 30 | 22 | 30 |
| 60 | 0.135 | 20 | 32 | 23 | 30 | 17 | 25 |
|  | 0.165 | 20 | 32 | 23 | 30 | 20 | 25 |
| 66 | 0.135 | 18 | 29 | 23 | 30 | 15 | 25 |
|  | 0.165 | 18 | 29 | 23 | 30 | 18 | 25 |
| 72 | 0.135 | 17 | 28 | 23 | 30 | 13 | 20 |
|  | 0.165 | 17 | 28 | 23 | 30 | 17 | 20 |
| 78 | 0.165 | 16 | 26 | 23 | 30 | 12 | 16 |
| 84 | 0.165 | 15 | 24 | 23 | 30 | 10 | 15 |

is placed in accordance with the best specifications and inspected at 90 to 95 percent Proctor density, the load-carrying capacity of aluminum pipe can be expected to increase. The design mechanism to predict this increase would be to select a high negative settlement-projection ratio or to use the ditch condition analysis. A good approximation of this would be to decrease the settlement ratio from 0 to -0.40 , the condrtions experienced in these tests with only 80 percent compaction. This approach would increase load capacity of alumınum pipe by approxımately a factor of about 1.25. It should be emphasized, however, that design in this range must be based on thorough knowledge of the soil properties and degree of compaction and continuous inspection of the installation.

## Vertically Elongated Pipe

Vertically elongated pipe (Fig. 12a) is defined as that pipe which is distorted from the full circle shape by elongating the vertical axis and, consequently, shortening the horizontal axis an equal amount. There are three accepted methods of elongation, each with a clearly defined purpose to improve the load-carrying ability. The three means shown in Figure $12 \mathrm{~b}, 12 \mathrm{c}$, and 12 d are factory elongation without struts or stays, factory elongation with wire struts to preload the pıpe, and field elongation vertical with struts to preload the pipe. Factory elongation is usually accomplished by compressing the pipe and twisting wire strutting at nominal $24-\mathrm{in}$. spacing. Field vertical strutting is usually done with timber cap and sole runners and column spacing at 5 - to 6 -ft centers placed by jacking or wedg. ing. Each method of elongation produces an improvement in load-carrying ability which can be approximated.

When pipe is elongated by permanent deformation up to 5 percent, the general stress behavior under load is approximately the same as if it were full circle. There is some initial prestress in the pipe metal of a sign opposite that of the loading due to plastic flow in forming so that the actual exteme fiber stress at any load condition is somewhat less than the values that would be indicated. However, in normal cases this prestress is low, typically less than $3,000 \mathrm{psi}$, and is not included in calculations. Having established that the load-carrying analysis of factory-elongated pipe is nearly the same as for full circle pipe, additional fill heıght capabilities must be attanable by means other than simple stress or deflection. This is done in two related ways. First, the pipe can settle up to 5 percent and still be in the round shape. This allows 5 percent of the diameter to be extended into the soil at the side to improve the soil modulus of passive resistance markedly without signs of distortion. In properly compacted soils this would also serve to make the soll adjacent to the pipe much less liable to settlement, thus producing a high negative settlement-projection ratio and distributing the fill load to the surrounding soll. In ordinary compacted soils, the initial expansion can occur with low stress due to the lower passive modulus allowing the pipe to take the new position without high stress and then be in position to absorb stress from higher loads beyond this point. In summary, vertically elongated culvert should have loador fill-carrying capacity estimated to be 25 percent more than full circle in normal ( 80 percent compaction) soils. Comparing this with the basic fill height table with a settlement ratio of zero, the increase would be equivalent to the fill capacity with a settlement ratio of -0.20 and essentially the same as full carcle pipe with 95 percent compaction. Vertical elongation would be expected to contribute little where soil compaction of 95 percent is attained except that the intial shape offers opportunity to provide continuity of high load-carrying ability of a few relatively soft pockets might be encountered. One cannot expect to attain higher than 25 percent improvement by elongation and control of soil as the stress developed in the pipe will effectively limit further gain.

When vertical elongation is accomplished by field timber strutting or by shop or field wire strutting by tensioning, the analysis of behavior changes somewhat to further improve the load-carrying ability of the pipe. In addition to the improvement noted, due to more negative settlement-projection ratio the pipe is no longer limited by developed bending stresses. In the elongation process, bending moments are $1 \mathrm{~m}-$ pressed into the structure in accordance with the equations shown on Figure 12. Max-


Figure 12. Vertically elongated pipe.
imum stress developed in aluminum alloy culvert is limited to about $\mathbf{2 6 , 0 0 0} \mathrm{psi}$, the elastic limit.

Taking a typical analysis of $60-\mathrm{in}$. pipe of $0.135-\mathrm{in}$. thickness with a maximum strutting prestress of $22,400 \mathrm{psi}$ at the point of load and $12,900 \mathrm{psi}$ at the side, the stress condition under load can be approximated by superimposing the prestresses on the load stresses developed in the test. Referring to the measured stress, the maximum pipe stress under 14 ft of load will be approximately $11,200 \mathrm{psi}$ in bending. If it is assumed that timber strutting has been used, the extreme bending stress at the side is now $+12,900-11,200 \mathrm{psi}$ or +700 psi . If the loading stress is extended by linear means to a point where the combined prestress and load stress reach the maximum design bending stress, the fill height determined by bending stress would be approximately doubled.

If the analysis by strutting is transferred to its effect on the soil around the structure, marked improvement in performance can again be predicted. The struts are usually
left in place until the fill is completed, allowing the load pressure to compact the soil around the pipe further. When the struts are finally removed, the expansion of the pipe into this compacted area coupled with equal settlement immediately over the pipe will cause a high negative settlement ratio to occur, perhaps approaching complete ditching condition. In addition, a higher modulus of passive resistance can be expected due to the high percent of compaction and lack of compressibility. In general, the decrease in settlement ratio and increase in modulus of passive resistance could be expected to produce design fill heights of nearly double the full circle values reported for prestress-strutted pipe.

Considering that the effect of bending stress under high load now becomes small for prestress-elongated pipe the shear or crushing stress analysis becomes the governing design condition. Maximum load-carrying ability of prestress-elongated culvert becomes simply an analysis relating soil load to joint strength by means of a suitable safety factor analysis (Table 9). In the installation of elongated pipe it is important that careful control of compaction is exercised so that maximum advantage be taken of the settlement ratio or the reasons for use of elongated pipe are wasted. With all this care in the installation and with provisions for deflection lag it is felt that the earlier safety factor of 3.3 can still be applied to ultimate joint strength through shear or crushing.

## Concentration Loads

As a part of the complete program on evaluating aluminum culvert, a series of concentration load tests were run on sections of $24-\mathrm{in}$. diameter $0.060-\mathrm{m}$. sheet and $36-\mathrm{in}$. diameter $0.075-\mathrm{in}$. sheet sections. Fills of 1,2 , and 3.5 ft were placed and compacted over the pipe with an average compaction of 87 percent Proctor density. Load distribution causing pressure in the soil and stress in the pipe was developed through a series of steel plates, 15,18 , and 24 in . square, used to support one side of a D7 loader with a reaction of $19,350 \mathrm{lb}$ directly over the pipe. Pressures were measured at the top, sides, and invert of the pipe and strain readings were taken in the same six locations in the circle used in the deep fill tests.

The pressures recorded were compared with the predicted pressures that could be encountered at the various points in the soll. As a basis for comparison, pressure distribution by both the method of $45^{\circ}$ divergent uniform pressure and the Boussinesq pressure bulb were prepared. These predictions compared well with the measured pressures. After review of the pressure distribution it was felt that the $45^{\circ}$ di vergent uniform pressures were sufficiently accurate to use as the standard method of analysis for concentration loading.

The uniform pressure equation is

$$
\begin{equation*}
F=P(a+2 h)(b+2 h) \tag{7}
\end{equation*}
$$

in which

$$
\begin{aligned}
& \mathrm{F} \quad=\text { force on plate or surface; } \\
& \mathbf{P} \quad=\text { pressure at any point below surface within } \\
& \text { pressure rectangle; } \\
& \mathrm{a}, \mathrm{~b}
\end{aligned}=\text { side dimensions of plate; and } \quad \begin{aligned}
& \mathrm{h} \quad=\text { depth to pressure. }
\end{aligned}
$$

To evaluate the relationship between surface load, pressure, and strain, each plate size-fill height combination could be converted to an equivalent pressure at the top of the pipe through Eq. 7 and the strain data plotted as a function of pipe top pressure. A typical curve is Figure 13. From these data a smooth curve of strain at the top of the pipe vs pressure at the top of the pipe can be plotted. Using these strain data, stress curves for average compression, crushing or shear, and bending can be shown in Figure 14. Using the safety factor analysis and design stresses of $11,200 \mathrm{psi}$ in bending and 2,100 psi in crushing, critical pipe top pressures of 26.4 and 27.2 psi for the $24-\mathrm{in}$. and $36-\mathrm{in}$. ones were developed. Once critical pressure is established (in this case, the standard was selected as 25 psi for the $36-\mathrm{in}$. diameter), the surface


Figure 13. Strain vs pressure at top of pipe.
load for each fill height can be computed to satisfy this condition. Referring to Eq. 7, the following modification produces, assuming a wheel contact area of 6 by 20 in .:

$$
\begin{equation*}
\mathrm{F}^{\prime} \mathrm{I}=100 \mathrm{~K}(3+\mathrm{h})(10+\mathrm{h}) \tag{8}
\end{equation*}
$$

in which

$$
\begin{aligned}
& F^{\prime}=\text { surface load (wheel) in pounds; } \\
& I=\text { impact factor; } \\
& h=\text { depth to top of pipe; and } \\
& K=\text { a factor allowing for change in pipe } \\
& \quad \text { diameter ( } 1.0 \text { for } 36-\mathrm{in} . \text { diameter). }
\end{aligned}
$$

Referring to the AASHO Standard Specification for Bridges (5), an impact factor of 1.30 is recommended for culvert with low fill. This value is used in development of a curve relatıng maxımum traffic wheel load to minımum fill height over pipe for compacted and well-graded or paved fill shown as Figure 15. An impact factor of 2.0 was


Figure 14. Average compressive stress and maximum bending stress at top of culvert vs pressure at top of culvert.


REFERENCE: STRUCTURAL
FILL TEST 1961 REPORT
Figure 15. Maxımum wheel loads vs fill height over culvert for 36-in. diameter and less.
used in preparation of the second curve which estımates maxımum wheel loads over minimum compacted fill during construction or on badly graded surfaces. Although the results shown are approximations, they nevertheless serve as a useful guide to the limiting of loading during construction, a persistent problem in the field. This design level is at best an approximation. Unfortunately, high speeds with large construction equipment may have much higher impact factors and because of the shock nature of loading the distributed area may be substantially smaller. It would be wise to have a minimum cover equal to the pipe diameter.

The minimum fill height must increase as the pipe diameter increases because the loading is in the moment form. The factor K would be expected to be related by the diameter ratio, using 36 in . as standard. However, in the case of 24 in ., the multiplier is 1.08 from the tests (see Table 10).

TABLE 10

| RELATION OF DIAMETER <br> TO FACTOR K |  |  |
| :--- | :---: | :---: |
| Diameter (in.) | K |  |
| 24 and less | 1.08 |  |
| 30 | 1.00 |  |
| 36 | 1.00 |  |
| 42 | 0.86 |  |
| 48 | 0.75 |  |
| 54 | 0.67 |  |
| 60 | 0.60 |  |
| 66 | 0.54 |  |
| 72 | 0.50 |  |

Referring to the curves of wheel load vs fill height, the limit conditions for the well-established AASHO H20-S16 and H15S12 are shown for conventional highway use to be 8 in. and 6.5 in ., respectively, for pipe of $36-\mathrm{in}$. diameter. The minimum fill for any other size would be determined by multiplying the $8-\mathrm{in}$. and $6.5-\mathrm{in}$. values by the reciprocal of K . This serves as the basis for minimum fill recommendations in Table 1.

Neither the stresses nor pressures measured at the sides and invert of the pipe were considered in the previous determınation. This is because measured values of both were but a small section of the top load conditions showing conclusively that the load at the top is distributed to the soll rapidly in a beam-type action and that structural considerations need not carry below the top area.

## FUTURE WORK

It can generally be concluded from these tests that aluminum alloy culvert pipe is structurally capable of supporting high soll fills and is also capable of resisting wheel or impact loads under low fills.

Future work in the area of flexible culvert should be directed toward a more thorough and detailed knowledge of the behavior of soil pressure and culvert pipe resistance arour the complete curcle. These tests would be expected to show a means of establishing better corroboration between deflection and stress evaluation. Concurrently, pressure traverses across the pipe top are necessary to establish exact patterns of pressure distribution. Finally, strain readings taken during the entire cycle of strutting and soil loading, first, with struts in place to completion of fill and, second, with struts removed under partial fill, should be evaluated.

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## Discussion

## SOME DO'S AND DONT'S IN THE DESIGN AND INSTALLATION OF FLEXIBLE PIPE CULVERTS

M. G. SPANGLER, Research Professor, Iowa State University, Ames-These remarks deal solely with the mechanics of the supporting strength of flexible pipe culverts and have no reference to the kind of material of which such conduits are made. They are equally applicable to pipes of any material, whether corrugated iron or steel, or welded smooth steel pipe, just so they are flexible in character.

Approximately 25 years ago the writer made rather extensive measurements of the magnitude and distribution of loads and pressures on flexible pipes under earth cover. The results of these measurements led to the development of the following load hypothesis for this type of structure:

1. The vertical load on a pipe may be determined by Marston's theory of loads on conduits and is distributed approximately uniformly over the breadth of the pipe.
2. The vertical reaction on the bottom of the pipe is equal to the vertical load and is distributed approximately uniformly over the width of bedding of the pipe.
3. The passive horizontal pressures on the sides of a pipe are distributed parabolically over the middle $100^{\circ}$ of the pipe, and the maxımum unit pressure is equal to the modulus of passive resistance pressure of the sidefill material multiplied by one-half the horizontal deflection of the pipe.

This hypothesis is shown graphically in Figure 16. It is emphasized that the pressures on the sides of the pipe, which hold the structure in equilibrium, are passive pressures. They do not and cannot develop unless and until the pipe deflects and moves outward against the sidefills an amount sufficient to mobilize the passive resistance characteristics of the enveloping soil. In other words, the pipe must deflect and the sides must move outward against the soil in order to develop the lateral pressures required for equilibrium.

Measurements of radial pressures on two of the experimental pipes together with pipe deflections, are shown in Figures 17 and 18. It is indicated that these pressures are essentially uniformly distributed around the periphery of the pipe. They appear to be quasi-hydrostatic in character. Other experimenters have obtained similar results. This uniform pressure situation has led some observers to conclude that the stresses in the pipe wall are compressive stresses only, as they would be in a circular vessel subjected to hydrostatic pressure. This conclusion is erroneous, in the writer's opinion. It must be realized that the pipes had to deflect for this quasi-hydrostatic pressure to develop, and when a


Figure 16. Hypothetical load system on flexible pipe culvert.


Figure 17. Radial pressures and deflections, 42 in .8 ga. pipe, experiment 1.
pipe deflects there is bending moment in the pipe wall. Therefore, a flexible pipe ring under earth load is subjected to a combination of thrust and bending moment, not just thrust alone. There are radial shears in the pipe wall also, but these are relatively unimportant.

Under the load and stress situation postulated, flexible pipes tend to progress toward failure by one or both of two types of structural action. One of these is deflection of the pipe ring. As the vertical earth load and the equal and opposite vertical reaction on the bottom of the pipe increase, the pipe deflects; that is, the vertical diameter shortens and the horizontal diameter lengthens. The amount of deflection depends on the magnitude of the load, the stiffness of the pipe wall, and the passive resistance characteristics of the enveloping soll. If the deflection is excessive, the pipe will collapse. Therefore a prediction of the amount of deflection that will develop in a given installation is necessary in the design process, and the deflection must be held within tolerable limits.

The other structural action that may lead to failure is the seam stress; that is, the combination of tangential thrust and bending moment stress at bolted or riveted longitudinal lap seams in the pipe wall. In smooth steel casing pipes, such as those used to protect gas and petroleum products transmission lines at railway and highway crossings, these stresses are of no significance because the longitudinal seam is usually a welded joint that is as strong as the pipe wall at any point. However, in the culvert pipes fabricated of corrugated metal sheets, particularly those of the larger diameters, the design of the longitudinal seams to resist the combined tangential thrust and bending moment stress is of very great importance.

Equations have been developed to facilitate investigation of both these types of structural action. In this development, the loaded pipe and the enveloping soil have been


Figure 18. Radial pressures and deflections, 42 in .10 ga. pipe, experiment 2.
treated as an indeterminate elastic body. The prediction equation for deflection is

$$
\begin{equation*}
\Delta x=\frac{D_{L} K W W_{c} r^{3}}{E I+0.061 E^{\prime} r^{3}} \tag{8}
\end{equation*}
$$

in which

$$
\begin{aligned}
& \Delta \mathrm{x}=\text { horizontal deflection, in. } ; \\
& \mathrm{D}_{\mathrm{L}}=\text { a deflection lag factor; } \\
& \mathrm{K}=\text { bedding constant, } \\
& \mathrm{W}_{\mathrm{C}}=\text { load on pipe, lb per in. of length; } \\
& \mathrm{r}=\text { mean radius of pipe, in.; } \\
& \mathrm{E}=\text { modulus of elasticity of pipe metal, psi; } \\
& \mathrm{I}=\text { moment of inertia of section thru pipe wall, } \\
& \text { in. 4/in. } ; \\
& \mathrm{E}^{\prime}=\text { er, modulus of soil reaction, psi; and } \\
& \mathrm{e}=\text { modulus of passive pressure of soil, ps1/in. }
\end{aligned}
$$

Equations for moment and thrust at any point in the pipe wall are moment, thrust and shear due to vertical load (One-half bedding angle, $\alpha=45{ }^{9}$ ):

$$
0<\varphi<45^{0} \quad \left\lvert\, \begin{align*}
& \mathrm{M}_{\mathrm{D}}=\mathrm{W}_{\mathrm{Cr}}\left(0.183-0.026 \cos \varphi \cdot 0.354 \sin ^{2} \varphi\right)  \tag{9}\\
& \mathrm{R}_{\mathrm{D}}=\mathrm{W}_{\mathrm{C}}\left(0.026 \cos \varphi+0.707 \sin ^{2} \varphi\right)  \tag{10}\\
& \mathrm{S}_{\mathrm{D}}=\mathrm{W}_{\mathrm{C}}(0.707 \sin \varphi \cos \varphi-0.026 \sin \varphi) \tag{11}
\end{align*}\right.
$$

$$
\left.\begin{array}{ll}
45^{\circ}<\varphi<90^{\circ}
\end{array} \left\lvert\, \begin{array}{l}
M_{D}=W_{c}(0.360-0.026 \cos \varphi-0.500 \sin \varphi) \\
R_{D}=W_{C}(0.026 \cos \varphi+0.500 \sin \varphi) \\
S_{D}=W_{c}(0.5 \cos \varphi-0.026 \sin \varphi)
\end{array}\right.\right\} \begin{aligned}
& \mathbf{M}_{D}=W_{c}\left(0.11-0.026 \cos \varphi-0.25 \sin ^{2} \varphi\right) \\
& R_{D}=W_{c}\left(0.026 \cos \varphi+0.5 \sin ^{2} \varphi\right) \\
& S_{D}=W_{c}(0.5 \sin \varphi \cos \varphi-0.026 \sin \varphi)
\end{aligned}
$$

Moment, thrust and shear due to horizontal load:

$$
\begin{array}{l|l}
0<\varphi<40^{\circ} & \begin{array}{l}
\mathrm{MD}_{\mathrm{D}}=\mathrm{hr}^{2}(0.345-0.511 \cos \varphi) \\
\mathrm{R}_{\mathrm{D}}=0.511 \mathrm{hr} \cos \varphi \\
\mathrm{~S}_{\mathrm{D}}=0.511 \mathrm{hr} \sin \varphi
\end{array} \\
40^{\circ}<\varphi<140^{\circ} & \begin{array}{l}
\mathrm{M}_{\mathrm{D}}=\mathrm{hr}\left(0.199-0.5 \cos ^{2} \varphi+\right. \\
\mathrm{R}_{\mathrm{D}}=\mathrm{hr}\left(\cos ^{2} \varphi-0.568 \cos ^{4} \varphi\right)
\end{array}
\end{array}
$$

in which (see Fig. 19)

$$
\begin{aligned}
& M_{D}=\text { bending moment at any point } D ; \\
& R_{D}=\text { tangential thrust at any point } D ; \\
& S_{D}=\text { radial shear at any point } D ; \text { and } \\
& h \quad=\frac{E^{\prime} \Delta X}{2 r}
\end{aligned}
$$

To obtain the moment or thrust at any point on the pipe ring, the values at the point due to vertical load and horizontal pressure should be added algebraically. Moment and thrust diagrams based on these equations are shown in Figure 19. The calculated flexural stress at all points around the periphery of the $42-\mathrm{m} .8$ ga. pipe with standard corrugations under 15 ft of fill in experiment 1 are shown in Figure 20. Calculated compressive stresses due to tangential thrust in the plpe wall are also shown in this figure.

The calculated flexural stresses are very much greater than the compressive stresses due to thrust. Also, the flexural stresses are very high and well above the probable elastic limit of the metal. This, of course, does not mean that actual stresses were as high as the calculated values, because the flexure formula is not valid when stresses exceed the elastic limit. It does mean, however, that the metal in the pipe ring was stressed in the plastic range over a substantial portion of the periphery. This conclusion is qualitatively verified by the fact that when the fill soil in this experiment was removed the pipe sections were permanently deformed, indicating that they had been stressed in the plastic range under the fill load.

In spite of the indication that stresses in a flexıble pipe culvert may readily exceed the elastic limit of the metal, the writer does not advocate that such pipes should be designed on the basis of stress level. When the pipe metal is stressed in the plastic range it simply means that the rate of deflection of the pipe under load increases, and the sides of the pipe will push harder against the soil. This mobilizes the passive resistance pressure of the soil at a more rapid rate, and equilibrium forces are built up against the sides of the pipe. Also, observations indicate that flexible metal pipes fail by excessive deflection rather than by rupture of the pipe wall. Therefore, a limiting deflection of the pipe rather than a limiting stress level is belleved to be the most import criterion for design.

The corrugated metal pipe industry has long advocated a deflection limit of 5 percent of the nominal pipe diameter as a design criterion and the writer's experience tends to indicate that this figure is satisfactory. However, in the American Society of Civil Engineers Manual of Practice 37 (Water Pollution Control Federation Manual of Practice No. 9) on "Design and Construction of Sanitary and Storm Sewers," a factor of safety of 1.25 is applied to the limiting deflection of 5 percent.

Moment Diagrams



Tangential Throst Diagrams



Figure 19. Moment and thrust diagrams, flexible pipe conduits.


Figure 20. Calculated flexural and compressive stresses, experiment 1.

During the course of approximately 25 years of study of the flexible pipe problem, the writer has observed a number of situations that provide a background for design concepts in this field. One of these situations involves a $60-\mathrm{in} .10 \mathrm{ga}$. pipe in eastern Iowa, which was installed under about 9 ft of cover in the Fall of 1939. The pipe replaced a 16 -ft span wood stringer bridge with pile and timber plank bulkheads as shown in Figure 21. When the pipe was installed, the timber deck was removed, but the bulk-
heads were left in place. The soil between the pipe and the bulkheads was pneumatically tamped up to the top of the pipe, but it consisted of top soil containing considerable orgenic matter and was not high-quality material.


Figure 21. Culvert 1497, Linn County, Iowa.

The pipe was "shop strutted"; that is, horizontal tie rods and turnbuckles were installed on the horizontal diameter at 2 - ft intervals throughout the length of the pipe. The turnbuckles were drawn up so that the initial vertical diameter before placement of sidefills was approximately 63.4 in . and the initial horizontal diameter was 58.2 in. Diameters were measured between points marked at the top, bottom and two sides at stations 10 ft apart throughout the length of the culvert. Repeat measurements of diameters were made at frequent intervals during the first 14 months after completion of the fill and at infrequent intervals since that time. The most recent measurements were made on September 26, 1961. Graphs of diameter changes at Sta. 0+30, which is under the center of the roadway, are shown in Figure 22. A photograph of the interior of the pipe at this same station is shown in Figure 23. One can practically "see" the bending moment in the pipe wall in this picture.

This case history is presented, not because it is a typical or usual situation. In fact, it is very unusual. It is presented to demonstrate and emphasize the fact that the pipe undergoes bending and deflection and in so doing must be able to develop sufficient passive resistance pressure in the sidefill soil to hold it in equilibrium, or deflection will be excessive.

One can only hypothesize as to the details of the cause of the unusually large deflection of this pipe. An hypothesis is that the timber plank bulkheads have probably rotted away through the years and permitted the sides of the pipe to move outward, even though columns of soil $5 \frac{1}{2} \mathrm{ft}$ wide intervened between the pipe and the bulkheads. How-


Figure 22. Culvert 1497, Linn County, Iowa.
ever, the sides of the pipe have moved outward more than the thickness of the timber planking, which was probably 3 in . and certainly not more than 4 in ., whereas the horizontal diameter increased by a total of 13.4 in ., or an average of 6.7 in . on each side. It appears that there was considerable compression strain in the sidefill soil columns in addition to the decay of the timber planking.

A $96-\mathrm{in}$. standard corrugated metal pipe which completely collapsed is shown in Figure 24. This pipe was also shop strutted. Unfortunately the tie rods were left in place during construction of the fill. The pipe was not permitted to deflect in a normal manner and it is believed this caused abnormally high stresses in the pipe ring and led to the collapse. Many of the tie rods were actually pulled in two in the region of the failure. An additional fact was that tests of the sidefill soil indicated that it was soft, yielding material, even when compacted. However, because the sides of the pipe were not permitted to move outward an appreciable amount against the soil, the poor quality of the material is not thought to have been a major factor in the failure.

Figure 25 is a view of a $144-\mathrm{in}$. structural plate pipe taken from inside looking toward one end. The pipe is on


Figure 23. Excessive deflection of a $60-\mathrm{in}$. 10 ga . corrugated metal pipe culvert under 9 ft of fill.


Figure 24. Failure of 96 -in. 8 ga. corrugated metal pipe.


Figure 25. End view of $144-\mathrm{in}$. structural plate pipe on shrap skew.
a rather sharp skew, and the ends were beveled to conform with the side slopes of the embankment. As a result earth pressure acted against one side of the pipe while there was no compensating or balancing pressure on the opposite side and the inward bulging action resulted. This is not unusual when sharply skewed pipes are beveled at the ends in this manner. A more satisfactory treatment would be to carry the pipe beyond the side slope to a square end and then build a berm out from the embankment to envelop the pipe.

A partial seam failure in this same pipe is shown in Figure 26, and, again, one can "see" the bending moment in the pipe wall. This condition is thought to have been agravated by the fact that the plates in the bottom of the pipe were thicker than those in the sides and the crown. The seam shown is at the junction of the thick and thinner plates, and the sudden change in plate stiffness appears to have caused a concentration of bending moment stress with the result shown.

Another situation that came to light in connection with this structure is the fact that apparently the pipe was bedded on a flat surface without being shaped to fit the curvature of the bottom plates. The soil at the sides was tamped under the haunches of the pipe but did not fill the triangular wedge-shaped space between the pipe and bedding, as shown in Figure 27. It was possible to identify this unfilled area by tapping on the bottom of the pipe with a ball peen hammer. The open space appeared to be from 12 to 18 in . wide on each side at the bottom.

The failure to fill the space undoubtedly caused the bottom reaction to be concentrated on a relatively narrow longitudinal element of the pipe, resulting in greater deflection and bending moments than there would have been if the bedding had been shaped to fit the pipe. Also, it is possible that this open space may provide a passageway for water outside the pipe in time of heavy flow and could lead to erosion and failure of the embankment.

Figure 28 shows line diagrams of two corrugated metal pipes in the loess region of western Iowa which have settled an excessive amount due to compression strain in the underlying soil under the weight of the overlying embankment. These pipes were constructed on a flat uniform grade from inlet to outlet. The maximum sag


Figure 26. Partial seam failure in 144-in. structural plate pipe.
in the flow line of these culverts amounts to 33 to 40 percent of the pipe diameter. This has resulted in extensive accumulation of water and silt inside the pipes and is generally unsatisfactory.

The analytical and experimental studies referred to plus observation of the performance of some actual structures in the field, lead the writer to offer the following series of "Do's and Dont's" in the design and construction of flexible pipe culverts:

1. Investigate the probable deflection of the pipe and hold it within safe limits. Do not rely solely on the tangential thrust in the pipe wall as a basis for design.
2. Remember that a flexible pipe fails primarily by excessive deflection rather


Figure 27.


## 96 in. Corrugated Metal Pipe Culvert



90 in. Corrugated Metal Pipe Culvert

Figure 28. Settlement of two flexible pipe culverts in loess region of western Iowa.
than by rupture of the pipe wall. Do not design the pipe on the basis of maximum bending stress.
3. Watch a pipe which has been pre-deformed by timber struts or tie-rods during construction of the overlying embankment. Be prepared to remove them at once if the pipe fails to deform in a normal manner or shows signs of distress and local bending. Do not leave pre-deforming devices in place very long after pipe has been backfilled and fill has been well started.
4. Select good compactible soil to be placed at the sides of the pipe. Do not use just any old "dirt" which is most readily avallable for the sidefill soil.
5. Compact the sidefill soll by acceptable controlled methods for a distance on each side of at least two pıpe diameters; more if feasible. Do not merely run a tractor back and forth parallel to the pipe a few times and consider that the soil has been well compacted.
6. Place and compact the sidefill soll evenly on both sides of the pipe throughout its whole length. Do not let the fill on one side of the pipe get to be more than 6 in . higher than the fill on the other side at any time.
7. When a flexible pipe drainage structure consists of multiple parallel lines, space the pipes far enough apart to permit hauling and compacting equipment to operate between them. Do not space the pipes so close together that it is difficult to place and compact the sidefill soil properly.
8. Shape the soil bedding to fit the bottom of the pipe for approximately $60^{\circ}$ to $90^{\circ}$. Do not lay the pipe on a flat bed of soll.
9. Change plate thickness gradually, if plates of various gages are used. Do not bolt thin plates directly to thick plates, especially in regions of high bending moment.
10. Carry skewed pipes out a sufficient distance beyond the embankment slope to per-
mit the end bevel to be at right angles to the pupe barrel. Do not bevel a skewed pipe parallel to the embankment slope.
11. Estimate the amount of settlement of the natural ground surface under an embankment and camber the invert grade of the pipe culvert a sufficient amount to insure against a sag or backslope in the line. Do not construct the pipe on a uniform grade from inlet to outlet in regions where substantial subsidence of the supporting soil may be expected.
12. Remember that flexible ripes usually contmued to deflect long after the embankment has been completed. Do not assume that the pipe deflection at the time the embankment is completed is the maximum deflection that will develop.

## REFERENCE

9. Watkins, R.K., and Spangler, M. G., "Some Characteristics of the Modulus of Passıve Resistance of Soll: A Study of Similitude." HRB Proc., 37:576 (1958).
H. L. WHITE, Chief Sales Engineer, Metal Products Division, Armco Steel Corporation, Middletown, Ohio-Much research has gone into both laboratory and field experimentation in respect to flexible metal culverts in the past five years by Metal Products Division of Armco Steel Corporation. (This organization untıl the first of the year was known as Armco Drainage \& Metal Products, Inc., a subsidiary of Armco Steel Corporation.)

One fact has been well proven by these most recent experiments and actual jobsflexible culverts properly backfilled do not deflect much more than 1 percent and usually less than 2 percent when the fill load is placed on them. By proper regulation of the compaction procedure durıng backfilling, the structure can be elongated vertically from its assembled size approximately 2 percent before the fill load is placed on it. On the placement of the fill load above it, the structure will then return to approximately its fabricated size.

The recent installation of a 10 percent vertically-ellipsed 24 -ft diameter structural plate pipe in the DeCoursey Yards of the L\&N Rallroad south of Cincinnati, demonstrated this most dramatically. This structure and some 25 others ranging in size from 15 to 21 ft have been installed in the past few years by proper backfilling and adequate compaction methods, using material of good quality, and each of them has proven this point.

The installation at Cullman, Ala., of three 7 - ft diameter tubes, each 512 ft long, under 134 ft of cover, was one of the first installations to show that properly installed flexible culverts act prımarily as compression rings. The greatest deflection observed in these structures under the 134 ft of fill was 2 percent from their 3 percent vertical elongation. A technical report on these structures was presented to the Highway Research Board at the 1952 annual meeting.

This then is the hypothesis on which Armco Drannage \& Metal Products issued the method of calculating these structures now known as the "ring compression method" (9). It is well recognized that this is a method that predicates that the pipe be subjected to little or no bending stress. It is also well recognized that to take full advantage of this particular method of calculating the necessary strength of the structure, quality and compaction of the backfill must be such as to maintain the structure very close to its fabricated shape.

The work done by Mr. Koepf in investigating the action of culverts of aluminum material is very excellent and thoroughly instrumentated and well analyzed. However, it should be recognized that the trend is away from obtaining compaction at only 85 Proctor on the backfill material. Though it is true that this compaction can be obtained under careless circumstances, or what many people have been led to believe is average installation practice, it has definitely been proven that higher compaction is usually obtained.

In one installation of twin 14-ft diameter sectional plate pipe (not Armco pipe) on Ohio 674, two miles south of Canal Winchester, field compaction tests were taken on
each lift of the backfill. Compaction was attained by the use of both pneumatic and vibratory compactors. The average compaction was 97 percent AASHO. The structure was installed in March and April 1955, and no progressive deflections have occurred.

More and more engineers and contractors are becoming aware of and are using the method of well-compacted backfill to maintain flexible structures with very minor vertical deflections under full loads. For this reason, the author believes that the work done by Mr. Koepf should come under the heading of "Investigation of the Action of Flexible Culverts Having a Compacted Backfill of 85 Percent Proctor" and should not be considered as covering the entire range of the possibilities of installation of such culverts.

Professor Spangler's remarks in respect to the hypothesis made 25 years ago for the magnitude and distribution of loads and pressures on flexible pipes under earth cover are as true now as they were then-they are still approximate assumptions made for the convenience of mathematically analyzing a pipe based on these assumptions. They may or may not pertain. It is rather doubtful if a pressure diagram such as has been approximated by this hypothesis could have been obtained on the Lafleche, Saskatchewan, culvert. This was a 72 -in. diameter culvert in a $1 / 2$ - by $2 / 3$-in. corrugation ellipsed vertically during installation some 3 percent and failed in a compression type of buckle by 34 ft of cover in a dam installation. The gage of this particular structure was 20 -approximately as thick as a well-worn dime-and it is doubtful if the moment strength of this culvert could alter a pressure diagram of the surrounding earth which had to be approximately uniform radially to maintain this culvert after it had taken the 2 percent deflection under cover.


Figure 29. Twin 17 1/2-ft diameter, 7 gage MULTI-PLATE Pipes carrying highway under industrial railroad tracks at Parrish, Ala. Fill is 25 ft high.


Figure 30. A 20-ft diameter, 1 gage MULTI-PLATE culvert under mainline of L\&N RR near Independence, Ky.

To cite another example, an $18-\mathrm{ft}$ dia-


Figure 31. A 24-ft diameter, 10 percent vertically elongated structure to serve as locomotive underpass under hump at new classification yard, DeCoursey, Ky. Structure, after dead load had been applied, came to $1 / 2 \mathrm{in}$. of designed diamter. meter 5 percent vertically elongated structural plate conduit of 10 gage steel was placed in Greene County, Ohio, and so compacted that the $9-\mathrm{ft}$ height of cover deadload still allowed the culvert to remain approximately 1 in . above the vertical formed dimension. Again, the moment strength of this structure is very slight as compared to the compression strength, and because the culvert remained in practically the shape in which it was fabricated that none of this small amount of moment strength was used. It is the writer's opinion that there is much more to be gained economically by operating on the backfill to obtain proper passive resistance and proper shape of culvert than there is to operate on the culvert itself in terms of moment strength.

Sufficient moment strength is required in such culverts to enable them to be handled, be backfilled, and contain adequate provisions against buckling of the wall under compressive stress. The best-known data in existence for how much this needs to be is contained in the presently published tables for gages of flexible steel culverts. Utilizing these gage tables as the accumulated data from many thousands of installations,


Figure 32. Upper structure is $18-\mathrm{ft}$ diameter, 8 gage MULTI-PLATE under 17 ft of cover; lower one is $10-\mathrm{ft}$ diameter, 1 gage under 70 ft of cover; located at Ice Harbor Dam, Wash.


Figure 33. Triple 20 - by $10-\mathrm{ft}$ MULTI-PLATE Arch located under Interstate 90 in Cleveland, Ohio. Maximum fill on this l, $000-\mathrm{ft}$ long culvert is 20 ft , yet deflection is practically negligible.
criteria can be and have been established for the flexibility factor of such structures. These criteria are available from Metal Products Division of Armco Steel Corporation in Middletown, Ohio. They have been published in papers presented to engineers throughout the country.

Adequate safety factors are employed in the ring compression design of flexible structures to provide safety against column buckling until this particular point can be brought to a finer mathematical conclusion than now exists. Work is being carried on in university laboratories to determine the relationship between the quality of the backfill and the required column properties of the culvert wall.

In conclusion, the writer would like to submit that there are many more culverts in existence performing with deflections of less than 2 percent vertically than there are of those that were so unfortunate as to have inadequate backfill applied to them. The ratio is probably many thousand to one. Figures 29 through 33 are examples of a few of such culverts in larger sizes and various shapes. The design and installation of flexible pipe culverts in these figures has progressed to a very conclusive point in just the past few years although no material available in today's text books adequately describes the science.
R. L. BROCKENBROUGH and J. ALAN MYERS, United States Steel Corporation, Pittsburgh, Pa. - Mr Koepf's paper is of interest to those associated with the design of flexible underground conduits because it presents a summary of the results of a test made on corrugated aluminum alloy pipe under a $30-\mathrm{ft}$ embankment. The test has contributed to expanding the limited knowledge and experience associated with the use of corrugated aluminum pipe for the drainage application. On the basis of an evaluation of the data collected, the author presents a table of recommended maximum fill heights up to 50 ft for certain combinations of sheet thickness and pipe diameter. The writers
believe several of the assumptions the author has made in evaluating the collected test data require critical examination.

Figure 7 shows the distribution of earth pressures across the top of a test culvert under 30 ft of earth fill. The vertical earth pressure directly over the center of the pipe is only about one-fourth the vertical earth pressure measured in the regions adjacent to the sides of the prpe. (This loading is similar to a "ditch condition" loading.) Therefore, as mentioned in the paper, the actual pressure on the culvert is considerably below the pressure that would be predicted by assuming a settlement ratio of zero. (The settlement ratio is an abstract quantity which, when combined with other terms, may be used to calculate the load on a culvert. When the settlement ratio is zero, the load on the culvert is equal to the total weight of the column of earth directly over 1t.) The loading condition attained in the test leads the author to state:

> Normal design tables for aluminum culvert herein are then based on the settlement ratio of zero in conformance with standard practice on other materials. Actually, such a decision builds into the aluminum culvert a safety margin somewhat higher than for other materials due to the strong tendency toward ditch condition loading. This margin has been considered in later selection and evaluation of safety factors on pipe stress.

The writers cannot agree with this statement because it implies ditch condition loading is a particular characteristıc of corrugated aluminum alloy culverts. The strong tendency toward the ditch condition loading experienced in the test probably resulted from the type of soil that was used and the construction procedure followed in making the embankment. The pressure distribution noted probably would have occurred for a corrugated steel culvert or any other flexible underground conduit under the same installation conditions. The embankment, if made from another type of soil, might not have undergone ditching action to the same extent. Consequently, it is erroneous to assume there is a higher safety margin inherent in the use of aluminum alloy culverts.

The pressure distribution noted is important, however, because it once again confirms that the tendency toward ditch condition loading is a characteristic of all conduits flexible in nature, regardless of the kind of material from which the structure walls are made.

The author used a safety factor of 2.50 against yielding in determining the allowable design stress for combined bending and axial compression of the aluminum culvert material. The resulting maxımum allowable stress was used in developing the recommended fill heights from the strain data collected. The use of the 2.50 safety factor was based on the "ASCE Specification for Structures of Aluminum Alloy 6061-T6." Although this factor may be adequate for ordinary aluminum engineering structures, experience gained from the use of other materials indicated a higher factor of safety should be used for a flexible metal culvert. This is true because exact loadings on underground conduits are difficult to determine. Soft spots in an embankment or undermining of a culvert can cause unpredictably high bending moments with resulting stresses considerably above the stresses that might be ordinarily encountered. The increased tendency toward the ditching condition claimed for corrugated aluminum alloy culverts should not give confidence to the choice of the safety factor, because the writers believe the claim is not necessarily valid for reasons already mentioned.

Actually, an over-all factor of safety of 2.50 was not adequately demonstrated. In the tests performed on the full circle aluminum alloy culverts, the $30-\mathrm{ft}$ embankment loading produced average measured vertical pressures from 1.0 to 1.4 times the pressures that might be encountered under the recommended fill helghts for other soll and field installation conditions. For example, in Table 1, the author recommends a maximum fill height of 14 ft for a $60-\mathrm{in}$. full circle aluminum culvert. A design earth pressure of about 10 psi (assuming a settlement ratio of 0 - and 100 -pcf material) would ordinarily be used for this height. The mean earth pressure above the $60-\mathrm{in}$. test culvert was 14.4 psi at maximum test load. To demonstrate experimentally that this cul-
vert actually had a safety factor of 2.50 against failure, it would have been necessary to load it to a fill height that would develop a measured pressure of 25 psi ( 2.5 times the assumed design pressure of 10 psi ). The author implies that the safety factor exists because the proposed table has been designed so that the maximum stress developed by a pressure 2.5 times the usual design pressure (presumably 25 psi for the $60-\mathrm{in}$. diameter culvert) is approximately equal to the material yield stress. This would only be true, of course, if it could be safely assumed that ring buckling does not occur before yielding of the wall material. The validity of this assumption was not established theoretically in the paper nor was it demonstrated experimentally by the test.

The paper makes only slight mention of ring buckling as a structural design consideration for flexıble metal pıpe. It appears no account has been taken of this phenomenon in establishing the recommended fill heights shown in the proposed gage vs fill herght table. The writers believe buckling cannot be overlooked in designing an aluminum alloy culvert. Reynold K. Watkins (10) describes a study he conducted in an attempt to account for the possible influence of ring stiffness and the effect of elastic deformation on the load that will cause ring buckling. The results of the study led Watkins to conclude: 'Buckling of the ring is a critical consideration in the design of flexible conduits." In other words, for certain combinations of soll type, loading, diameter, and wall stıffness, a flexible conduit may fall due to elastic instability of the wall, rather than crushing of the joint, excessive deformation, or yielding of the wall material. Buckling wall become increasingly significant as one tries to refine existing flexıble metal culvert designs to make more efficient use of materials. Because of aluminum's relatively low modulus of elasticity, ring buckling (which is dependent on wall stiffness, EI) possibly will be, in many cases, the critical consideration in the structural design of corrugated aluminum alloy culvert.

## REFERENCE

10. Watkins, R.K., "Failure Conditions of Flexible Culverts Embedded in Soil. " HRB Proc., 39:361-371 (1960).
A. H. KOEPF, Closure-Discussions by M. G. Spangler, H. L. White, R. L. Brockenbrough, and J. Alan Myers, have done an excellent job of further expanding the concepts by which flexible metal culverts might be evaluated. Emphasis is placed on the ability of flexible culvert to reduce its loading by deflection, thus extending the limits of design in the higher fills. Mr. White's indication that higher fills can be attained when the soll is well compacted under controlled conditions is well supported. In this instance, bending stress stays relatively low compared to joint strength. It was the intent of this study to establish a conservative floor under design limits and consider the effect of many potential methods of fallure, selecting for a design limit that method of most importance for each size of pipe and gage. Certainly where conditions prevail which are better than the base and can be constantly substantiated in the field, the design values can be reviewed and design limits revised.

In the instance of the discussion by Mr. Brockenbrough and Mr. Myers, it should be pointed out that the selection of 30 ft as the maximum fill height was considered sufficient to demonstrate the pattern of strain, pressure, and deflection; and was not selected to demonstrate a condition of imminent failure. It is felt that if the former follows a rational behavior, the latter can be demonstrated by extrapolation of data.
M. G. SPANGLER, Closure-Mr. White and the writer are in complete agreement on the proposition that high-quality backfilling material, properly placed and compacted around a flexıble pipe culvert, will produce an installation that is highly satisfactory from a structural standpoint. We disagree concerning design methods for taking
into account the role that the backfill material plays in establishing the structural equilibrium of the flexible pipe.

Mr . White approaches the design of a flexible pipe culvert by assuming that the soil around the structure is of such high quality that deflection of the pipe can be ignored in the design procedure. He has demonstrated this idea by means of a lightweight tin cylinder confinded between two wooden blocks. These blocks inhibit and minimize the deflection of the thin ring so that it will carry a relatively tremendous load. He has described this wooden block support as a "perfect soil" and this description is very appropriate.

Unfortunately only a small percentage of actual soil backfills, placed and compacted by human beings, can be said to approach this state of perfection, although some do. For example, the Cullman County, Ala., installation referred to, has a near-perfect backfill environment and the deflection of the $84-\mathrm{in}$. pipes has been negligible. On the other hand, there are numerous installations wherein the soil backfill has not been of such high quality, and pipe deflection has been a factor.

In contrast to this philosophy, the writer believes that the quality of a soil backfill can be expressed quantitatively and that this quantitatively expressed quality has a direct bearing on the required stiffness of the pipe ring and on the pipe deflection that will develop during its functional life.

Mr. White's concluding paragraph in which he states that the ratio of culverts in service with deflections less than 2 percent to those with greater deflections is probably "many thousand to one, " is far two optimistic. In 1943 a leading flexible pipe manufacturer conducted a survey of culvert pipe deflections which embraced 239 run-of-themine structures in widely dispersed geographical locations. These pipes were all of the "structural plate" type and ranged in size from 60 to 180 in . The heights of fill cover ranged from 0 to 90 ft .

The deflection of one of these culverts was described as "excessive-not used." The deflections of the remaining 238 culverts ranged from -4.97 to +12.10 percent of the nominal diameter and the average deflection was +2.32 percent. It is believed that those plpes with negative and very small positive deflections were probably strutted or fabricated to an initial out-of-roundness, but specific information on this matter is not available.

Using the 2 percent break-off point it is noted that 131 pipes deflected less than 2 percent of the nominal diameter, whereas 108 deflected 2 percent or more. The ratio is 1.21 to 1 . Granted that installation know-how and procedures have improved during the past 20 years, it is doubtful if the ratio of "many thousand to one" is valid.


[^0]:    ${ }^{a}$ Based on measured $P_{B}, P_{A}^{\prime}$, stress.

[^1]:    ${ }^{a}$ Rivet sizes are 5/16-1n. diameter for 0.060 - and 0.075-1n. sheet and 3/8-1n. diameter for 0. 105- and 0. 135-1n. sheet.
    ${ }^{\mathrm{b}}$ See (1) for derivation.
    ${ }^{\mathrm{c}}$ See (I) for derivation. Safety factor of 3.3 or published value used.

