Use of Field, Laboratory and Theoretical Procedures for Analyzing Landslides

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SYNOPSIS

Mathematical methods for analyzing the stability of many slopes have been available for a number of years. These methods stand ready to be proved, modified, or refuted. There is, therefore, a need for field and laboratory test data taken from actual landslides. Unfortunately, insufficient information of this nature can be found in the literature. There is an abundance of written material concerning landslides available but the bulk of this information is descriptive and is of limited value to the engineer faced with landslide problems.

This paper presents field and laboratory data obtained from three actual landslides. The study was confined to a two-dimensional analysis of a shear-type failure in shallow deposits of unconsolidated materials. The data were used to check the validity of the circular-arc method of slope analysis. The soil strength required for stability, as determined from this method of analysis, was compared with the strength of the soil as measured by laboratory tests. The data are insufficient to indicate definitely the range of applicability of the circular-arc method. However, when combined with similar data from previous studies the results indicate the limited applicability of this approach and point to the area where further study is needed before it can be used to obtain quantitative answers to the problem of prevention and correction of highway landslides.

SINCE THE BEGINNING of time the shape and form of land masses have been undergoing changes. Large portions of the earth have been lifted above their respective surrounding areas while other portions have been depressed. Where these differences in relief occur, even though they may be small, one finds the forces of nature busy at work hewing down or building up these areas to a common level. In parts of the world where conditions are favorable, landslides have been one of the most active forces for changes in appearances of the earth's surface.

Landslides change the surface features of the earth and, in so doing, cause great damage in many areas of the world. It is quite difficult, perhaps impossible, to accurately determine the cost to the public of the slides that occur in a given area in any one year. Even though this problem is not easily resolved, estimates have been made from time to time. About 15 years ago Ladd (1) estimated that the area embraced by western Pennsylvania, southern and eastern Ohio, northern and eastern Kentucky, and western West Virginia suffered an annual damage of about $10,000,000. This estimate is probably based upon the cost of extra construction and maintenance work required by the highways, railroads, and public utilities in this area. This same estimate may, but probably does not, include loss of life, loss of property by individuals, and the wasting of agricultural lands.

Man has always been affected by landslides, but only during the last 100 years has the attention of engineers and geologists been focused on this phenomenon to any great extent. The last 40 years of this period appear to have been the most fruitful from the standpoint of the engineer and geologist. The building of our railroads, the Panama Canal, larger and larger earth dams and our modern highway systems have done much to focus attention upon slides.

Early studies of landslides were concerned primarily with descriptions and methods of classifications. Evidence in the engineering and geological literature indicates that there has been a recent trend away from this early approach to the landslide problem. The emphasis seems to be shifting away from descriptions and classifications. A greater effort is being made to obtain a better fundamental understanding of the causes and methods of prevention and control of landslides. A better understanding of soil action is being sought. Several mathematical solutions have been proposed and in a few instances attempts have been made to justify one or more of these solutions with field and laboratory test data.

Although this trend appears to be in the proper direction and some progress has been made, much remains to be learned about landslides. Much remains to be learned concerning the shear-strength characteristics of the soils commonly involved in slides. In fact, this phase of the work seems to lag behind the existing mathematical methods of analysis.

PREVIOUS THEORETICAL INVESTIGATIONS

In connection with existing mathematical studies, Carrillo (2) points out that every theoretical investigation in this field assumes that the shearing strength of the soil is governed by Coulomb's empirical equation $s = c + n \tan \phi$ wherein s is the unit shearing strength, c the unit cohesion or unit shearing strength when no confining stress is applied to the soil, n the applied normal stress acting on the surface of failure, and ϕ the effective angle of internal friction. The values of c and ϕ are not constants for any given soil. In the case of clay-like soils they should be regarded as variable coefficients. A detailed discussion of the Coulomb equation and the shearing strength of clay-like soils is beyond the scope of this paper. Nevertheless, the importance of exercising the utmost care in the application of Coulomb's equation cannot be overstressed.

It is also assumed that the soil is homogeneous. Since soils are not perfectly homogeneous no solution can be accepted as entirely reliable.

Carrillo indicates that practically all attempts to apply mathematics to the problem of slope stability can be placed into one of two categories: (1) studies based on the state and distribution of stress in the soil mass at the instant of failure and (2) studies based on the assumption of a potential surface of plastic failure wherein the nature of the stresses in the sliding mass other than along this surface are usually disregarded.

Studies based upon the stress distribution in a slope of perfectly elastic, homogeneous, and isotropic material have been made primarily by French scientists. Resal (3), Frontard (4) and Caquot (5) have made noteworthy attempts to obtain a solution by this method. An exact solution has never been obtained.

Resal based his solution on a generalization of Rankine's state of stress. He worked with a semi-infinite mass of homogeneous material bounded by an inclined plane top surface, subject to its own weight and in equilibrium. The stresses that act on any plane parallel to the top surface are assumed to be vertical and directly proportional to the depth below the top surface. Terzaghi (6) very ably describes this method and offers valuable criticism. Taylor (7) points out that on one important matter the Rankine-Resal method agrees with conditions that are met in the field by indicating that the upper part of the mass is in tension.

Frontard assumed that a Rankine-Resal state is applicable to a slope of finite extent. He conceived a surface of failure for a slope of critical height to be composed of a vertical tension crack, an arc of active failure, and an arc of passive failure. He solved the differential equations for the curved portion of this sliding surface and found them to be deformed hypocycloids. Terzaghi discusses the Frontard solution and points to a number of inconsistencies in this method.

Carillo states that "Caquot abandoned the conception of the infinite slope and assumed, instead, that the stresses at any plane parallel to the slope are uniform and make a constant angle $j < i$ with the normal. This assumption introduces no improvement (over the Resal-Frontard method), for while the case $j = i$ may be sustained on the assumption of an infinite slope, the case of $j < i$ is an arbitrary conception." The angle i that Carillo refers to here is the angle between the inclined slope and the horizontal.

Others besides Resal, Frontard, and Caquot have attempted to solve this problem from the standpoint of stress distribution in the mass but a method free of contradictions is still unknown.

The second avenue of approach to this problem assumes a potential surface of plastic failure wherein the nature of the stresses in the sliding mass other than along this surface are usually disregarded. This method appears to be more promising at the present time.

Noteworthy contributors to the surface of sliding concept have been Culmann (8), K. E. Peterson of Sweden, Fellenius (9), Krey (10), Glennon Gilboy, Rendulic (11), Taylor (7), Terzaghi (6), and Jåky (12).

Culmann was the first of this group to attempt the analysis of a slope. He assumed that failure would occur along a plane surface through the toe of the slope. However, it is now known that many slope failures occur along a curved surface that sometimes passes below and beyond the toe. The inconsistencies of this method usually lead to results that appear to be unsafe. Today it is of historical interest only.

Sharpe (13) stated that Molitor noted in 1894 that rupture takes place not on a plane but on a curved surface which approaches the form of a hyperbola. Peterson was probably the first to suggest that a circular arc should be used to approximate this curved surface of failure. His assumptions resulted from a study of a quay wall failure in Goeteborg, Sweden in 1916. His observations were supported by the Swedish Geotechnical Commission (14) which subsequently studied a large number of landslides in that country. The circular-arc method of analysis was an outgrowth of these studies. This method has been accepted as satisfactory by many engineers interested in the problem. It requires the use of a number of trial circles to determine the least stable condition. The practical limitations and range of applicability of this method have never been too well defined by experimental data, however.

As a result of the commission's findings, several procedures for analyzing stability based on the circular arc have been advanced. One of these is the method of slices, which was developed by the Geotechnical Commission. A circular arc of failure is assumed and to make the problem statically determinate it is assumed that the forces acting on opposite sides of each vertical slice through the mass are equal, opposite, and colinear.

Another procedure is the ϕ-circle or friction-circle method. This is based upon a method devised by Krey for the analysis of the bearing-capacity problem. It was later applied to the problem of slope stability by Gilboy. It assumes that the failure surface can be represented by a circular arc and that the line of action of the resultant of the friction forces acting on this arc is tangent to a small circle with the same center as the failure circle..

Still another procedure has been proposed by Jåky. He assumed that all points in the sliding mass are on the verge of failure along circular arcs. Carrillo states that "as a consequence of these assumptions, a system of external normal stresses is required at the supposedly free surface."

Rendulic proposed a method in which he assumed that failure occurs along the arc of a logarithmic spiral rather than a circular arc. He assumed that the resultant force acting along this sliding surface is inclined at an angle equal to ϕ from the normal and that it will pass through the origin of the spiral. No additional assumptions were required.

Fellenius seems to have been the first to assume that the angle of internal friction, ϕ, for a sliding mass of saturated clay is zero. This method is becoming widely used, yet the number of cases in which it has been checked against actual failures is still rather small. Skempton (15) discussed this method and cited a number of problems for which it is applicable. He indicated that in cases where the rate of loading of the soil mass is slow enough, ϕ is not equal to zero and the assumption is no longer valid. He pointed out that clays which are not fully saturated do not have an angle of internal friction equal to zero when tested under conditions of no water-content change. Thus in the case of many man-made embankments, the $\phi = 0$ analysis does not apply. Finally, he indicated that even in the case of fully saturated clays that are tested under conditions of no water-content change, the true angle of internal friction, ϕ, of the clay is not equal to zero. Evidently, there is a difference between the angle of internal resistance, ϕ, and the true angle of internal resistance, ϕ_f, but this difference is not clearly defined by Skempton. In any case he believes that the ϕ equal zero analysis will not, in general, lead to a correct location of the actual failure surface nor will it give a theoretically correct factor or safety if the test values of cohesion are applied to an actual failure surface.

All of these methods are described and discussed in greater detail in present-day textbooks and literature treating the subject of soil mechanics. In addition to these solutions, others have been advanced but in general they are thought to be either too complicated or too specialized to describe herein.

By confining their studies to embankments of homogeneous materials with a constant angle of slope, a level top-surface, neglecting the effects of seepage water, and assuming that the shearing resistance is constant along the failure surface, Taylor and Fellenius have obtained solutions for a large number of cross-sections in terms of the height of slope and physical properties of the soil. Charts are now available in many soil-mechanics textbooks that enable one to analyze quickly slopes of this type. Unfortunately, many of our slides are not of the type assumed and the charts are, therefore, of little value in many instances. Nevertheless, much interesting and valuable information concerning slides was uncovered as a result of the investigation of these two men. By comparing the factors of safety that are obtained when applying the solutions mentioned above (log spiral, ϕ-circle, Culmann plane, etc.) to a particular slope

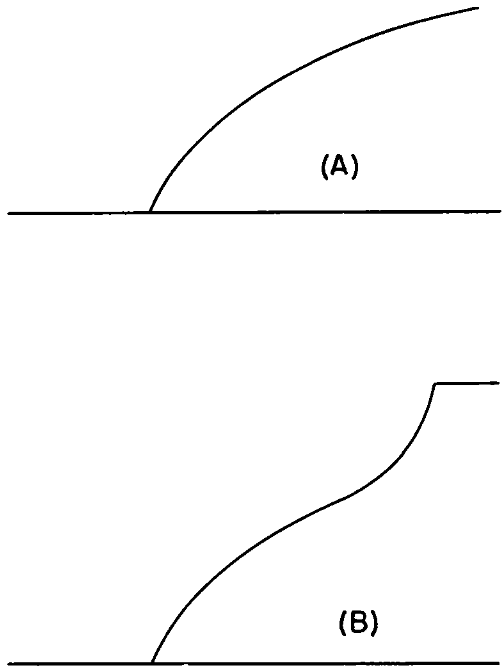

Figure 1. Two profiles of common slope formation of cohesive soils as found in nature.

it became apparent that several of the methods gave approximately the same results. The Resal-Frontard method seemed to produce conservative results while the Culmann plane method gave values that appeared to be unsafe. Even so, the disagreement between the Culmann and the other methods was reduced considerably in the case of steeper slopes. These comparative studies indicated that the ϕ-circle and log spiral methods gave almost identical results. There was even a close agreement between the positions of the two critical failure surfaces as obtained by these two methods.

Taylor also showed that for one slope, at least, the location of the center for the critical circle could be moved about considerably without changing the factor of safety more than 3 or 4 percent. Movement of the center in a direction roughly parallel to the slope was more critical than movement in a direction approximately perpendicular to the slope. These findings are true only for the type of slope studied. However, it appears reasonable to believe that they may also be true for more complex-shaped sections.

The effects of seeping water in slopes has also been studied by a number of investigators. Several have employed the flow net for this purpose. However, it has been pointed out on a number of occasions that the stress carried by the water at the time of failure is unknown, in spite of what the flow net may indicate.

In conclusion, it might be stated that of the two methods of solution (stress distribution in the soil mass and an assumed surface of sliding) the assumed-surface-of-sliding method currently appears to offer the more promising avenue of approach. Several different solutions employing various sliding surfaces have been developed. For certain types of slides there is evidence that indicates that it matters little whether one chooses the log spiral, φ circle, or slices method to solve a slope stability problem. For practical purposes the final results will all be approximately the same.

Frontard (16), in a slightly different approach to the problem, has suggested that our present method of profiling embankments could be improved. He notes that in every country embankments are constructed by using plane surfaces. At times terraces with plane surfaces are used. He believes that this is proper in case of cohesionless soils. In the case of cohesive soils he believes that it is in direct opposition to everything that can be observed in nature. He thinks that engineers should use more astuteness in determining the shapes of earth slopes. He notes that he has never seen slopes with plane surfaces in hilly areas of clay-like soils. Shown in Figure 1 are two curved profiles that he has encountered in nature. By using embankments with surfaces that have curved profiles rather than plane profiles, he claims that considerably greater heights can be attained without the risk of sliding. He cited examples where profiling methods have been used successfully. In another paper (17) he summarized his mathematical solutions of curvilinear profiles for several slopes. This approach seems to offer possibilities, but further studies are called for.

PREVIOUS FIELD AND LABORATORY STUDIES

Berger (18) recently made a thorough study of this phase of the work. In reporting the results of his work, he noted: "Hundreds of well documented slides are described in the literature. The writer originally expected to find a sufficient number having reliable test data to permit some statistical correlation of the results. However, only 15 slides were actually found having strength data of sufficient reliability to be included in this study. The data for six of these slides were unpublished . . . the slides were all influenced by the presence of a lower critical stratum of soft plastic clay and were generally overlain by stronger clays, or cohesionless material. All slopes were made of non-homogeneous soils which prevented use of the stability number or the location of the critical center by Taylor's or Fellenius's charts."

As Berger pointed out, nine of these slides had been analyzed and reported in the literature. Nevertheless, he made an independent analysis of each of these slides, and attempted to determine and compare the rélationship between the true factor of safety, which he said should be one or less at the time of failure, and the factors of safety that were predicted from the various laboratory test data.

Since there is no standardized laboratory-test procedure available for determining the shearing strength of clay-like soils, Berger encountered many difficulties in his attempts to compare results. In spite of these difficulties Berger did make some comparisons. He found that the factors of safety computed for eight slides for which average unconfined strength data were available ranged from 0.96 to 3.42. Six of these slides showed a very good agreement having factors of safety ranging from 0.96 to 1.25 with an average of 1.14. The other two slides gave values of 1.75 and 3.42. Numerous slickensided surfaces were noted in the soils for the first of these two slides, but no reason could be advanced for the safety factor of 3.42 that was computed for the second slide.

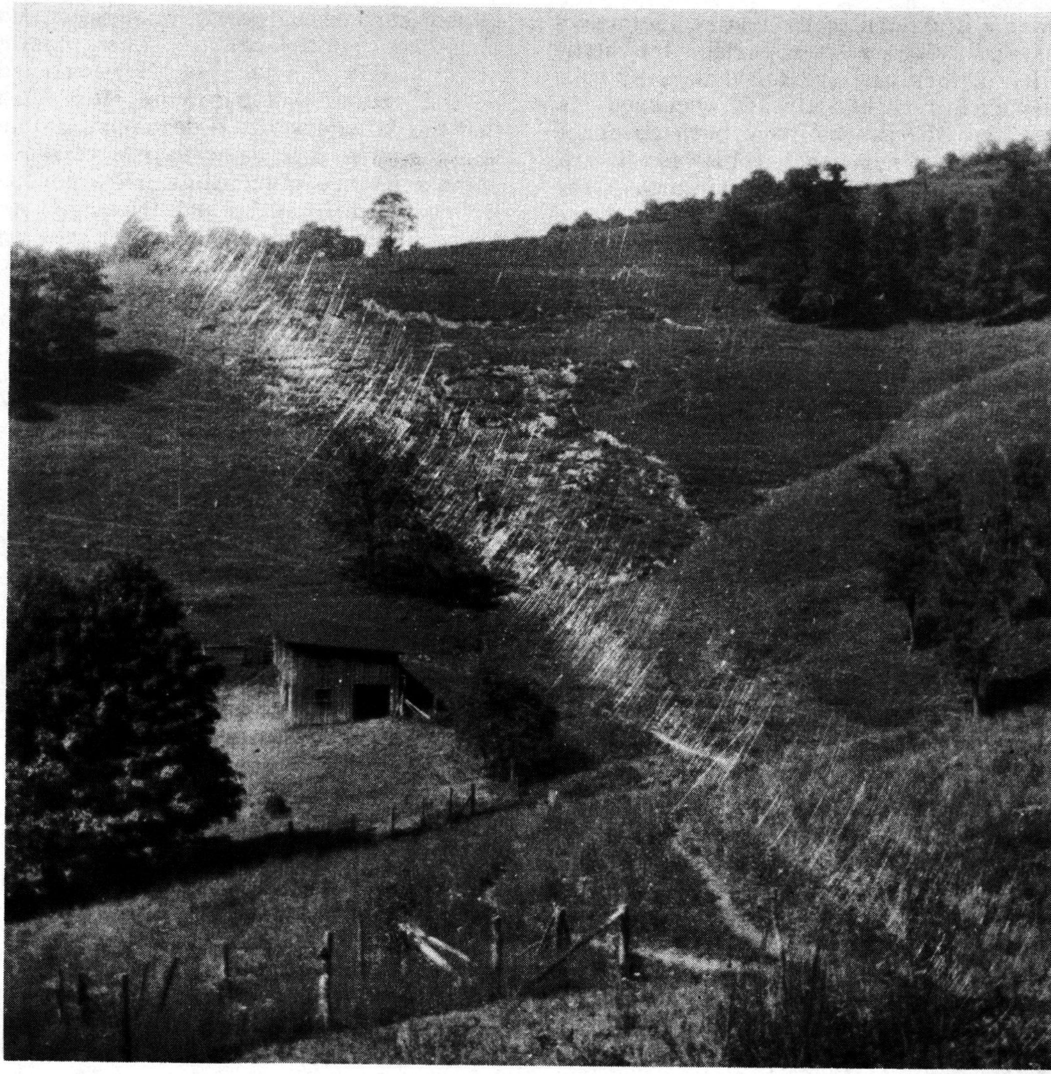

Figure 2. Landslide near Salem, West Virginia.

By using the minimum values of strength as determined by the unconfined compression test, he found that the computed safety factors showed a variation from 0.53 to 1.48 with an average of 0.97.

Factors of safety were computed for four slopes using strength data obtained by the undrained direct-shear test. Three of these resulted in values ranging between 0.96 and 1.11 while the fourth gave a value of 3.81. He said that "no reason can be presented for this extreme value." He indicated that the strength tests were carefully performed in one of the better laboratories in this country.

Triaxial-strength test-data were available for several of these same slides and similar comparisons of stability were made using the data. The results of these comparisons gave safety factors that were, in general, greater than those obtained for the unconfined test.

This one report embodies much of the currently available information concerning this phase of the work. In the opinion of the writer it clearly shows the meagerness of reliable, factual information concerning slope analysis. It indicates that there is considerable scattering of values of calculated factors of safety. Generally speaking, the stability factors are greater than unity even though failure has already occurred. A number of reasons have been advanced for these discrepancies between actual and calculated stability factors. The writer is not prepared to explain these differences. Only after a considerably greater quantity of reliable test data are reported will these questions be resolved.

The type of slide studied by Berger probably differed somewhat from the type analyzed by the writer. The slides that are discussed by the writer occurred in shallow deposits of unconsolidated residual soils. Failures in these soils usually occur along a rather well-defined surface and are often accompanied by a flow. These shallow deposits of soil usually rest above beds of shale, sandstone, and limestone from which they were formed. Quite often these parent beds have been weathered in such a manner that the surface upon which the soils rest is inclined considerably to the horizontal. Water that is carried by the porous beds of sandstone makes its way along this inclined surface of partly weathered and more-impervious shale. The plastic soils in this zone are usually weakened by this water, and it is quite often here that the failure surface develops. In the early spring the upper layers of these same soils are often made more porous by repeated freezing and thawing. An abundant supply of surface water is often available during this same period and there is a tendency for this water to saturate the upper layers of this porous soil. The writer believes that a majority of the slides studied by Berger occurred in rather deep deposits of unconsolidated nonresidual material. Slides in this type of material usually develop along well—defined surfaces and the amount of flow is usually negligible. Spreading sometimes accompanies this type of failure however.

PRESENT STUDIES

Three landslides were studied and analyzed by the writer. One of these slides occurred in a hillside field near Salem, West Virginia. The other two

Figure 3. Landslide on Indiana Route 62. Movement of the embankment undercuts pavement in the right lane.

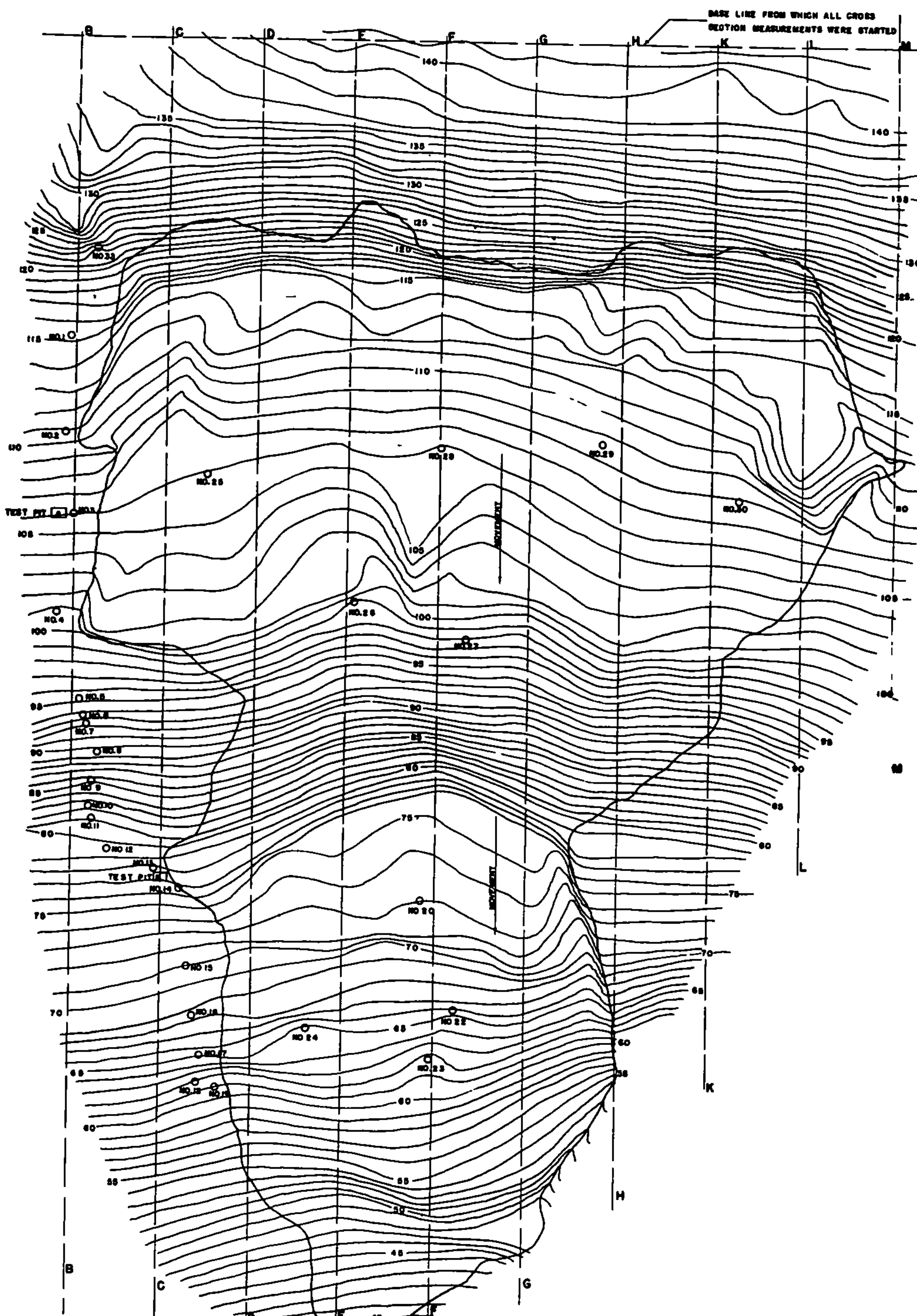

Figure 4. Topography for the Salem slide.

TABLE 1

Slide Location	Liquid Limit %	Plastic Limit %	P I %	Specific Gravity of Solids	Mass Unit Weight lb per cu ft
Salem	57 2	26 9	30 3	2 80	130
English	50 4	20 3	30 1	2 67	126
S H 62	91.0	24 9	66 1	2 60	113

occurred along highways located in Indiana. Photographs of two of these slides and a brief description of the methods that were employed to obtain pertinent data are shown and briefly described below.

Data for the slide near Salem, West Virginia, were obtained by the writer. All subsurface borings were made with a hand auger. Borings into the underlying rock formation were not possible with this equipment. The soil samples obtained from borings were used for preliminary soil identification purposes and for the location of test pits.

Soil samples for testing purposes were taken from test pits along the east edge of the slide. These pits were placed outside of the sliding area. The soil samples from these pits were used for specific gravity, unit weight, Atterberg Limits, and unconfined compressive-strength determinations.

Profiles were plotted and assumed failure surfaces were drawn for several longitudinal sections through the slide. The shearing resistance of the soil was assumed to be a constant and equal to one half of the unconfined compressive strength. The shearing resistance was assumed to be uniformly distributed

TABLE 2

Slide	Section	Factor of Safety with Respect to Sliding
Salem	B-B	4 22
	D-D	3 22
	F-F	2 33
	G-G	2 85
	K-K	3 34
English	163+00	4 88
	163+25	5 08
	163+50	5 45
	163+75	7 29
S H 62	1275+00	1 88

along the sliding surface. A stability analysis was made for each sliding surface. The strength requirements of the soil for stability were then compared to the available strengths as indicated by the results of the unconfined compression tests.

Profiles, cross-sections, and boring records for the slide on Indiana State Highway 37 near English, Indiana, and the slide along Indiana State Highway 62 one mile west of Indiana State Highway 145 were furnished by the State Highway Commission of Indiana. Soil samples for testing purposes were taken from test pits adjacent to each of these slides by the writer. The testing procedures and methods of analysis for these two slides were similar to those employed for the Salem slide.

TABLE 3

Slide Location	Factor of Safety with respect to sliding for critical section and critical circle based upon the soil strength as sampled		Factor of Safety with respect to sliding for critical section and critical circle based upon the soil strength of the soaked sample
	Avg Str	Min Str	Avg Strength
Salem	2 33	1 62	1 25
English	4 96	3.85	2 39
S H 62	1 88	1 67	1 08

RESULTS

Shown in Table 1 are certain index properties of the soils for the three slides studied. These are the properties of the soils in the immediate vicinity of the sliding surface. These average index properties show that the soil near the sliding surface for each slide was a highly plastic clay. These clays were only moderately sensitive to remolding.

Shown in Table 2 are the calculated safety factors with respect to sliding required for stability for each of the longitudinal sections. These factors of safety are based upon average values of shearing resistance as determined by laboratory tests. They are minimum factors of safety for that section.

It is interesting to note that for the Salem slide, where data for numerous

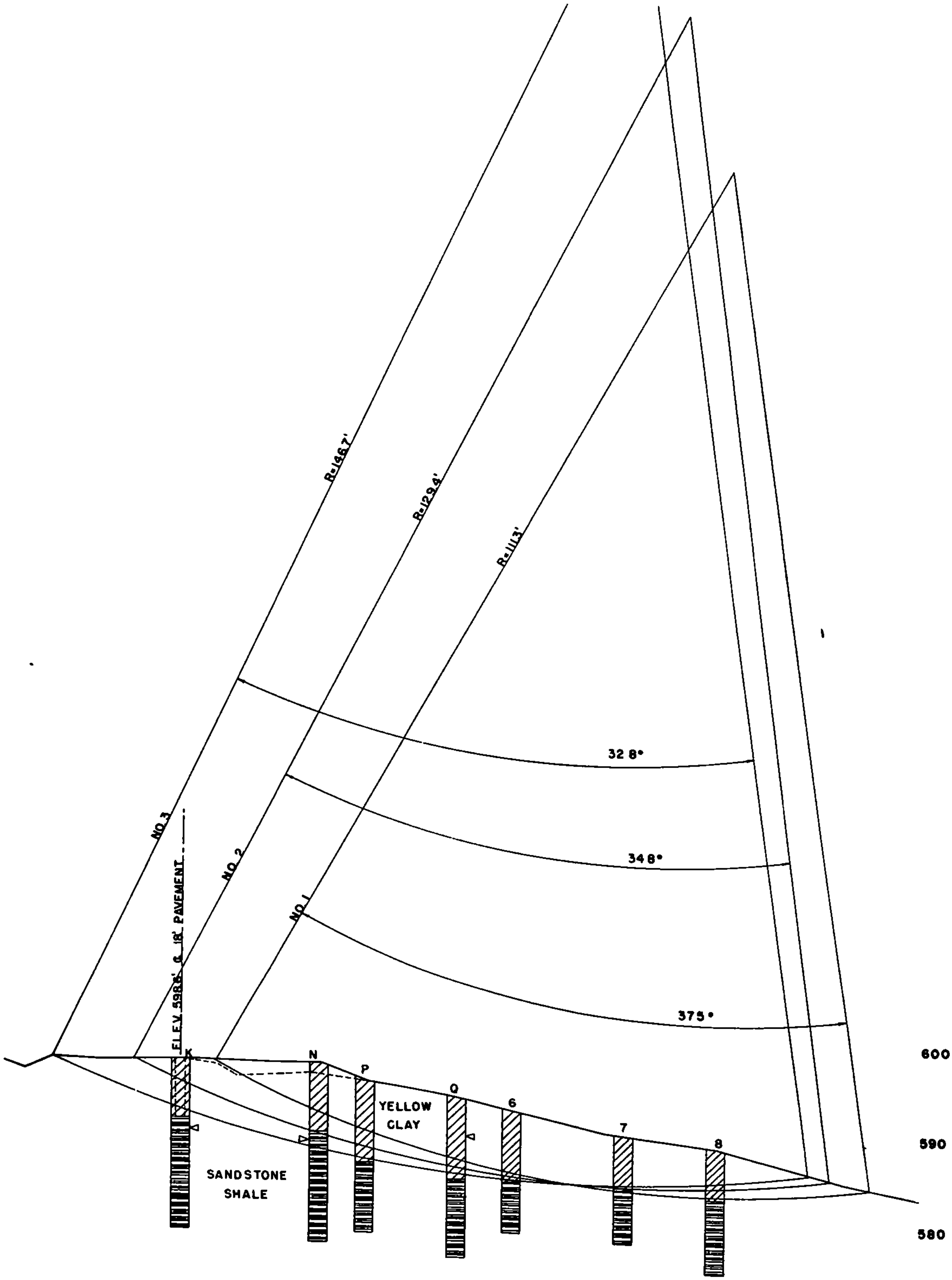

Figure 5. Typical section showing assumed failure surfaces.

sections were available, the factors of safety decrease as we move from the edges of the slide toward the center. These data indicate that the factor of safety varies from section to section. Therefore, it is probable that for this type of slide there is a critical area in which failure commences and from which it then spreads to the adjacent soil mass. Local geological features probably play a large part in the location of these critical areas.

Several circles were analyzed at each station or section. The centers for these circles were often displaced from one another considerably. Nevertheless, in many instances the factor of safety with respect to sliding did not change appreciably for any one set of circles. This is in agreement with Taylor's belief that the exact location of the critical circle is often not exceedingly important.

Shown in Table 3 are factors of safety that were critical for each slide. One column of factors is based upon the average strength of the samples tested. Another column gives factors of safety based upon the minimum shearing resistance of the tested specimens. The last column shows safety factors based upon the shearing strength of a number of samples; these samples were soaked in water since it was apparent that the consistency of the soils as sampled was not the same as one usually encounters for slides of this type at the time of failure.

DISCUSSION OF RESULTS AND CONCLUSIONS

The factors of safety for these three slides based upon the average available shearing strengths are too high. Minimum available strength values give factors of safety that are somewhat lower, but they are still high. This agrees in many respects with the results of work done previously by Berger (18). Soaked samples give strengths that result in factors of safety that are more nearly equal to unity for the Salem and State Highway 62 slides, but are still high in the case of the slide at English. The writer does not advocate the use of soaked samples for strength determinations at this time. Nevertheless, soaking may be justified in cases where the soils involved in a slide change consistency rather quickly and it is evident that the consistency of the sampled soil is not similar to the consistency of the soil in the embankment at the time of failure. The number of slides for which these conditions exist is, of course, limited.

It appears that many slides may always defy mathematical analysis. Nevertheless, the inconsistencies that are present in our current methods of analysis, as evidenced by this work and that of Berger's, may not become apparent until we learn more concerning the shearing strength of clay-like soils, progressive failures, and the effects of seepage forces, tension cracks and impact loads. A better understanding of these factors rather than a new method of analysis holds the key to the embankment problem at the present time.

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