Spherical Lens Optics Applied to Retrodirective Reflection

JAMES H. HAVENS, Research Chemist, and
ALLIE C. PEED, JR., Research Engineer,
Kentucky Department of Highways

SYNOPSIS

THIS paper describes some expedient applications of elementary optical principles to the evaluation of glass-bead reflectorizing systems for highway signs and markers. The optical function of spherical lenses in achieving reflex reflection is illustrated both photographically and diagrammatically. Various optical designs are discussed and analyzed. By simple geometric optics, the efficiency of these systems is correlated with the practical performance criteria for retrodirective reflectorization.

During the past several years various commercial interests have developed or proposed numerous devices as systems for reflectorizing highway signs and markers. Some of them have gained widespread acceptance while others have been rejected or replaced by more recent innovations. The commensurate efforts on the part of highway agencies have been directed toward evaluations, specifications, and performance criteria. As a partial consequence of this, most of the literature on the subject deals more with those particular phases rather than with the theoretical aspects of reflectorization (1, 2, 3).

Perhaps this omission has been due to the fact that most of the basic optical principles of reflectorization can be worked out from information found in any good college physics book. It is, however, the contention here that this knowledge and understanding is essential to the judicious selection of reflective systems. This paper is, then, dedicated to the documentation of some of the theoretical aspects relating spherical lens optics to retrodirective reflectorization.

Performance Criteria -- The performance criteria for reflective highway signs are dictated largely by practical trigonometry. Very simply, the tangent of the angle between the beam of light from an automobile's headlamp and the line of sight of the driver to a distance ahead is equal to the vertical distance between the driver's eyes and the headlamps divided by the distance ahead. If the driver's eyes are 2 ft. above the headlamps and a sign is 400 ft. ahead, the angle is approximately $\frac{1}{4}$ deg., whereas to a distance of 25 ft. ahead, the angle increases to approximately $4\frac{1}{2}$ deg. Only the light reflected back through this conical divergence angle is of any use to the driver.

Due to the fact that signs are set off from the path of the vehicle, the angle between the incident beam and the normal to the surface of the sign may be as little as 1 deg. at 400 ft. and increase to 30 deg. at 25 ft. Thus, the angularity requirements for a retrodirective sign surface are two-fold: it must return a substantial portion of the light backward along the incident beam, and it must preserve that property even through large angles of orientation. The redundant but necessarily descriptive terms "reflex" or "retrodirective" reflection have been ascribed to reflective systems capable of exhibiting those characteristics.

Reflector Categories -- Basically, reflectors are a subclassification of secondary luminous sources which may be defined as "any source that is luminous by virtue of reflection or transmission of luminous flux from or through the surface" (4). Reflection, excluding the reflex type, has been further divided into two general types: diffuse and specular. Accordingly:

> "An ideal diffuse reflecting surface reflects all luminous flux in such a geometric manner that the brightness of the object is constant (with respect to the viewing angle)."

> "An ideal specular surface reflects all luminous flux received by it at an angle of reflection equal to the angle of incidence" (4).

Figure 1 gives a somewhat-idealized generalization of the difference in reflection patterns for the three fundamental categories. In both the diffuse and specular types the pattern of reflection is symmetrical about the normal to the surface. In the reflex type, however, the pattern is symmetrical about the line of incidence from the source.

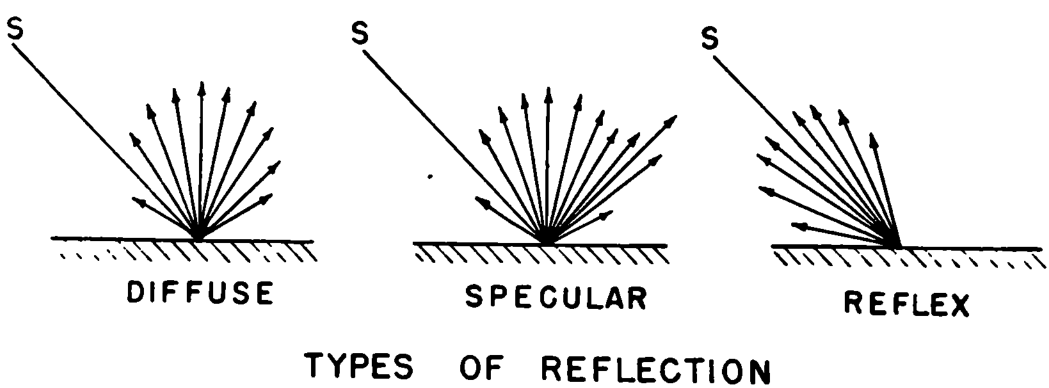

Figure 1. Vectorial illustration of three basic categories of Reflection Characteristics.

One example of a diffusing-type sign is a weathered or chalking painted surface. Since it exhibits a hemispherical pattern of reflection, the intensity of reflection at any distance in front of the sign varies inversely as the square of that distance. Thus the reflection intensity would be about 250 times greater at 25 ft. than it would be at 400 ft.

In the second case, or the specular type, the specular vectors are shown as simply added to, or superimposed upon, the pattern of diffuse reflection. This is a condition approached by a fresh, unweathered enamel surface. By assuming unit illumination at the surface, a fractional relationship must exist between the degree of diffusivity and the degree of specularity in the reflected light. Greater reflection in a particular direction can be achieved only by sacrificing reflection in other directions. The rougher the texture of the surface, the more it scatters the light. The smoother the surface, the more it tends to reflect an image of the source.

Glass-Bead Reflectorization -- Figure 2 shows a photomicrograph of a typical reflecting surface in which minute spherical lenses or glass beads are used to achieve reflex characteristics. These glass beads are in the order of 0.15 mm. to 0.05 mm. in diameter and are imbedded to their equatorial plane in a pigmented resinous or plastic matrix adhering to the sign stock. Each of these minute glass beads acts as a lens which gathers in the incident light, focuses it upon an underlying surface, and returns the reflection back toward the source. Since the sizes of the beads are below

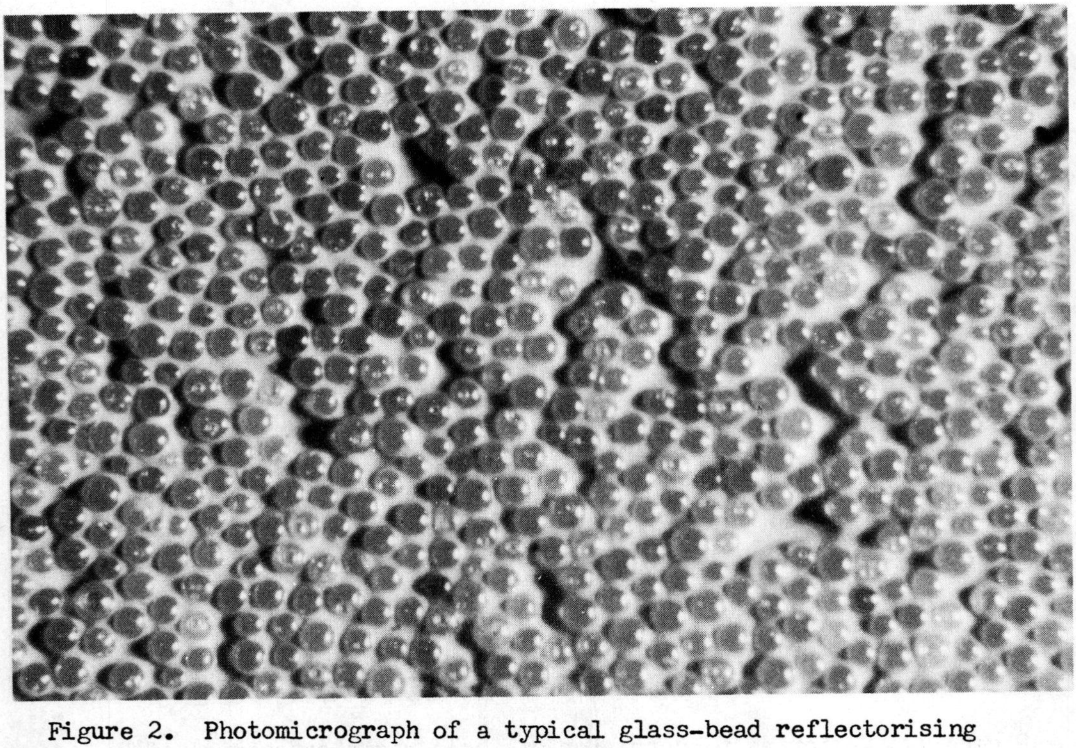

Figure 2. Photomicrograph of a typical glass-bead reflectorising System.

the normal resolution of the eye, viewed even at arms length, the surface seems to luminesce when viewed within the cone of reflex reflection. This is particularly advantageous from the standpoint of legibility. The legend is simply painted over the beads giving a large luminous area as the background for dark, nonreflecting letters and design. Other lens systems, such as reflector buttons, are large enough to be resolved individually as points of high glare, even under distant viewing conditions.

Elementary Optics -- As a practical approach to the theoretical analysis of this optical system, Figure 3 shows an actual photograph of a single glass sphere in the path of a restricted but collimated beam of light incident from the left. It is clearly shown that the light is converged to a focal point behind the sphere. The position of the focal point for a spherical lens is governed by the refractive index of the particular glass. The convention is to express focal lengths in terms of the radius and with respect to the optical center of the lens. Fortunately, for a complete sphere, its optical center is coincident with the center of the sphere. Focal lengths are related to the radius of curvature and refractive index by the simplified equation:

$$f = \frac{u'r}{2(u'-1)}$$

in which f = focal length measured from center of the sphere
u' = refractive index of the glass with respect to air
r = radius of the sphere

Assuming a refractive index of 1.5, which is an approximate value for ordinary glasses, f is calculated to be 3/2 times r or a distance equal to $\frac{1}{2}$ r behind the trailing surface. This is approximately the condition shown photographically in Figure 3.

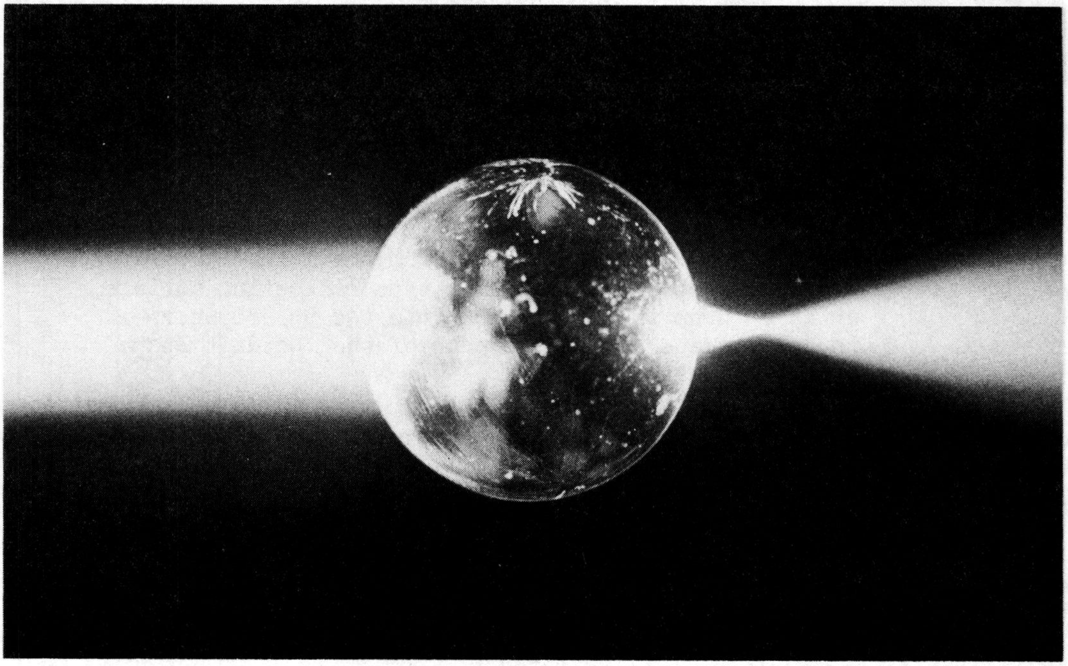

Figure 3. Photograph showing convergence of light by a single Glass Sphere. A bem of parallel light rays is incident from the left.

If a reflecting surface is positioned at the focal point and normal to the incident beam, then the light is reflected back into the sphere and is re-collimated back through the incident beam. This condition is also illustrated photographically in Figure 4. In the photograph, however, the light going into the latter lies inside the boundaries of the incident beam. The reflecting surface used here was a piece of aluminum foil having high specular characteristics. Some scattering or diffusion, however, may be noted at the surface in the photograph.

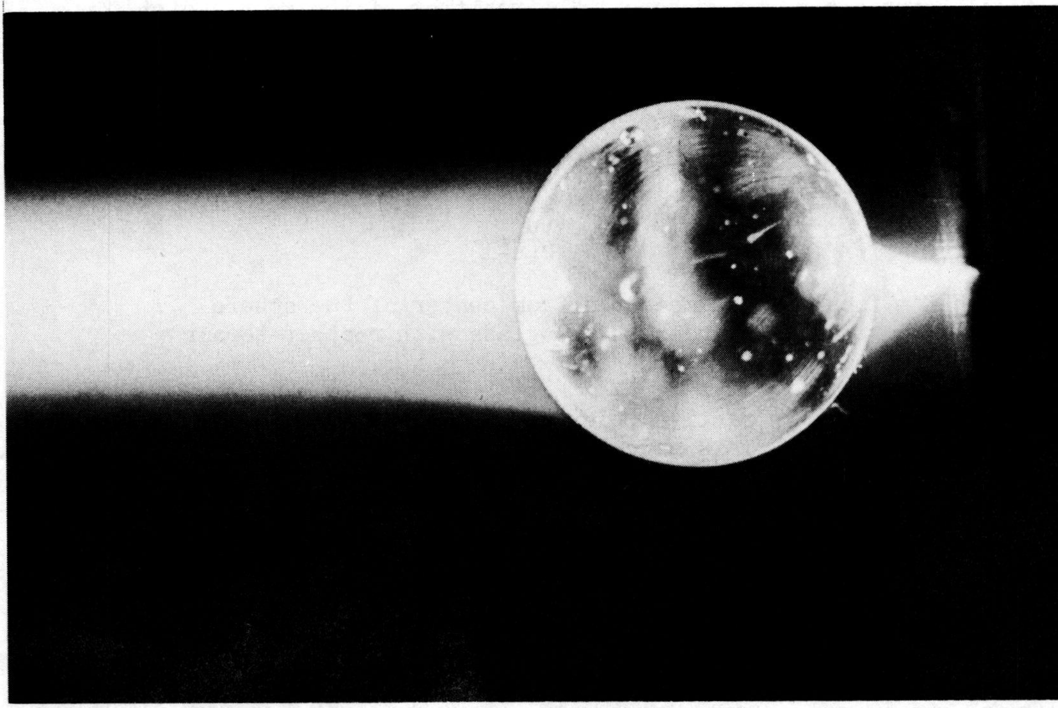

Figure 4. Photograph illustrating the function of a spherical lens in reflex-reflection. Here again, a beam of parallel light rays is incident from the left. The rays are converged onto a specular-type reflecting surface which returns the light back into the sphere where it is recollimated along the path of the incident beam.

If a high degree of retrodirection is to be achieved, the position of the reflector with respect to the focal point is very critical. If positioned behind the focal point, then it receives diverging light from the lens and, due to the greater angle of incidence upon the surface, a large portion of the light reflected may miss the sphere entirely. The system therefore loses part of its former efficiency. Considering the opposite extreme, with the reflecting surface positioned in front of the focal point, converging light strikes the surface and is returned through the sphere in a widely diverging cone, as shown in Figure 5. In this case, a large circular area of the surface is illuminated by converging light which is reflected into the lens rather than away from it as before. Substituting a diffusing-type reflecting surface does not alter the operation of the system appreciably. A greater portion of the light may be lost through scattering, but the mechanics of operation are essentially the same. If the lens is improperly positioned with respect to the reflecting surface, diffusion may compensate to some degree for that imperfection.

With further reference to the relationship between focal lengths and radius of the sphere, in the equation $f = u'r/2(u'-1)$, f increases as r increases. Obviously larger lenses must be spaced at a greater distance from the reflecting surface. Further, as the refractive index of the glass approaches 2.0, f approaches r. If f is equal to r, then the focal point is coincident with the trailing surface of the sphere. As the refractive index increases above 2.0, the focal point moves inside the sphere; which means, of course, that it is impossible to position a reflecting surface there. It also means that all the refraction is produced by the front surface only, and the above equation becomes invalid. A similar equation may be resolved for lenses having front surfaces only; i.e., $f = r/u-1$. Here, also, when $u' = 2.0$, $f = r$.

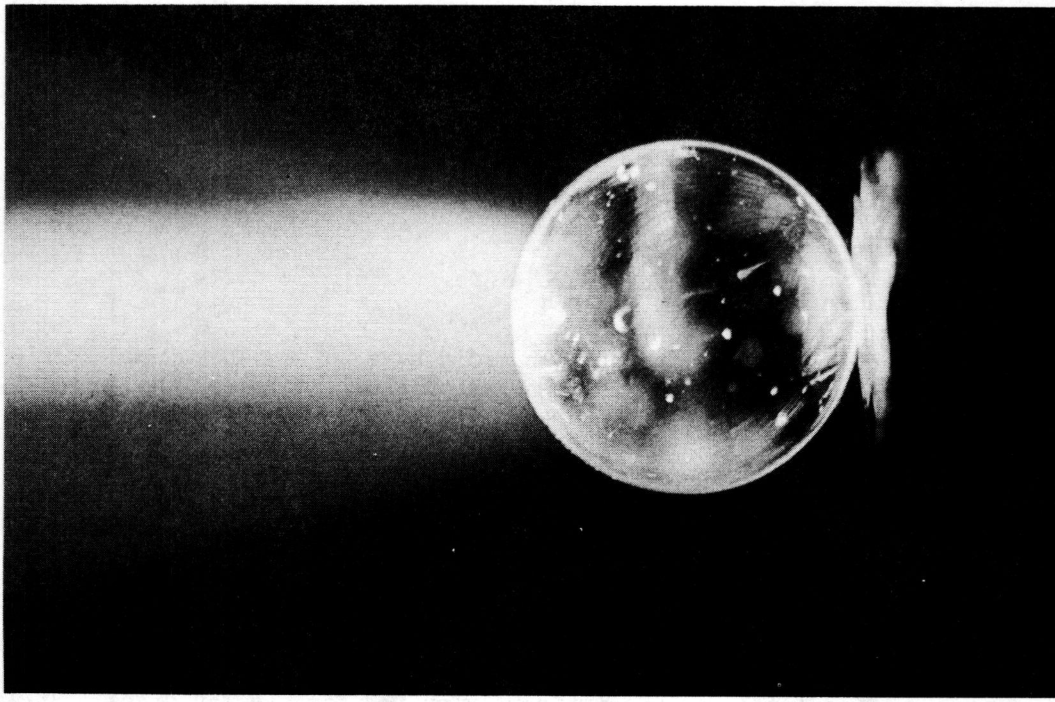

Figure 5. This photograph illustrates the divergent character of light returned through the sphere when the reflecting surface is positioned in front of the focal point.

Geometric Optics -- In the preceeding discussion and in the previous photographs, considerable importance has been attributed to focal-point calculations and the spacing of the reflecting surface. It is important to call attention here to the fact that these focal-length formulas have been resolved for paraxial rays (those which pass undeviated through the center of curvature). They are, therefore, based upon a series of assumptions which introduce appreciable error with the greater obliquity of the rays. In other words, there is no discrete point of convergence for all the rays entering the sphere. This optical imperfection in lenses is called spherical aberration. Consequently the only straight forward method of analyzing the optical systems for reflectorization, to be considered subsequently, is to trace the path of the light through the systems, applying Snell's law of refraction to each surface.

In Figures 3, 4, and 5, both the front and rear surfaces are refracting with respect to air. Actually, they are suspended in air, and only by assigning a refractive index of unity to the air is it possible to resolve the simplified equations above. By using Snell's law any ray of light may be traced through any series of reflecting media by geometric construction, or more specifically, by geometric optics. Snell's law may be written as:

$$\frac{\sin I}{\sin R} = \frac{u'}{u}$$

where I = angle of incidence, with respect to the normal to the boundary-surface

R = angle of refraction, or the angle made with respect to the normal after crossing the boundary

u′ = refractive index of the medium the ray is entering

u = refractive index of the medium the ray is leaving.

Of course, when u is equal to unity, as for air, the equation reduces to its more simple form:

$$\frac{\text{Sin } I}{\text{Sin } R} = u'$$

Refractive indices for unknown media have to be determined experimentally by standard methods. In these theoretical analyses, the appropriate values for refractive indices are assumed and do not represent any specific material.

Optical Design of Reflex Reflectors -- The design of a reflex reflecting system is limited in a practical way by the mechanics involved in actually fabricating the system for use on a sign. Obviously the spheres can not be suspended in front of the sign as they are shown in Figures 3, 4, and 5. It is practical, both optically and mechanically, to imbed them to at least their equatorial plane in a suitable binder, incorporating them as an integral part of the surface. Figure 6 illustrates one of the simplest and most practical designs imminently suited for highway signs. Structurally, at least, this cross-sectional view is comparable to the surface shown by the photomicrograph in Figure 2. Otherwise, it describes a general category of optical designs. The most significant feature in the geometric construction of this optical system is the angle d, which represents the deviation of the returning ray from true parallel reflection. It will be recalled from earlier treatment, based on performance criteria, that only the light returned within a divergence angle of 4½ deg. can be of any use to the driver. In this system that angle is d. Fortunately, due to the simplicity of the system, angle d can be equated in terms of I and u′, and it is otherwise independent of the radius and focal length of the sphere. Accordingly, the efficiency of the system may be tested with respect to the specific property of the glass, u′. From the development shown in Figure 6, where

$$d = 2I - 4\sin^{-1}\left(\frac{\sin I}{u'}\right)$$

d may be calculated for any value of I and u′.

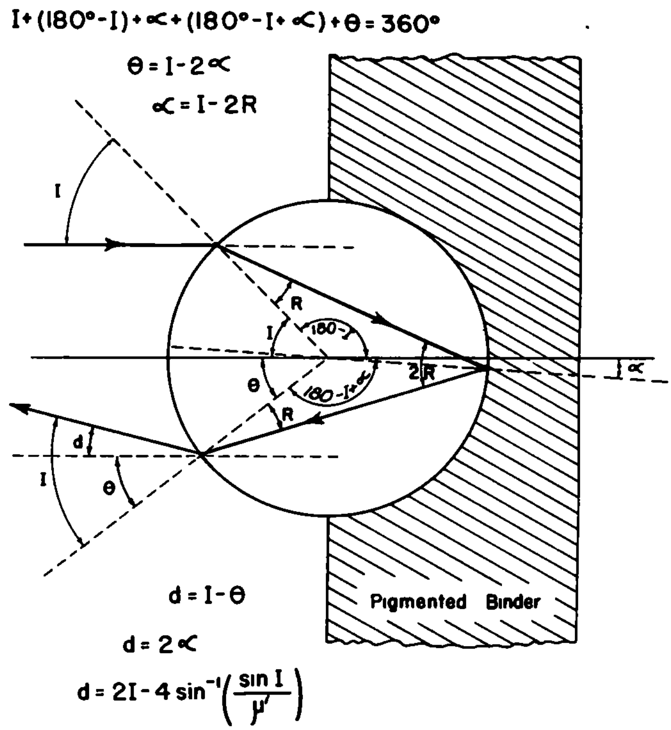

Figure 6. Schematic diagram showing cross-sectional view of a glass sphere imbedded in a pigmented binder and the geometric construction of the path of a single ray of light through the system.

Figure 7 shows a group of theoretically calculated curves relating d to I for selected values of u′ ranging from 1.50 to 2.00. It is significant to note that the angles of incidence contributing useful reflection are those angles corresponding to that portion of the respective curves lying within the bracketed region of $4\frac{1}{2}$ deg. positive or negative divergence. Of course, the sign of the divergence is extraneous to the utility of the light. From further examination of the curves, it seems that minimum divergence is achieved when u′ = 2.00 which is favorable to extremely long viewing conditions where the useful divergent angle is $\frac{1}{2}$ deg. or less. However, by sacrificing some of the efficiency for those extreme conditions, a somewhat greater portion of the lens surface becomes useful and the curves cross the zero-divergence line at two angles of incidence.

In this interpretation of the curves, two other features of spherical lenses must be considered. First: 75 percent of the equivalent normal surface of the sphere lies within the 60-deg. angle of incidence. Second: the fraction of incident light reflected without entering the surface of the lens remains fairly constant to 60 deg. incidence but increases sharply at greater angles. Within these boundary angles, even the influence of u, within the range of 1.50 to 2.00, on the fraction of light lost by surficial reflection is less than one tenth of all the light received. These two features establish the boundaries of useful lens surface at approximately 60-deg. incidence. Therefore, those portions of the curves for incidence angles greater than 60 deg. should be disregarded. The angle I in all of the foregoing discussion refers to the angle a ray of light makes with respect to the normal to the surface of the sphere and should not be confused with the angle which the driver's eyes and the headlamps of his car make with the plane of the sign on the highway. That angle is to be discussed in the following paragraph.

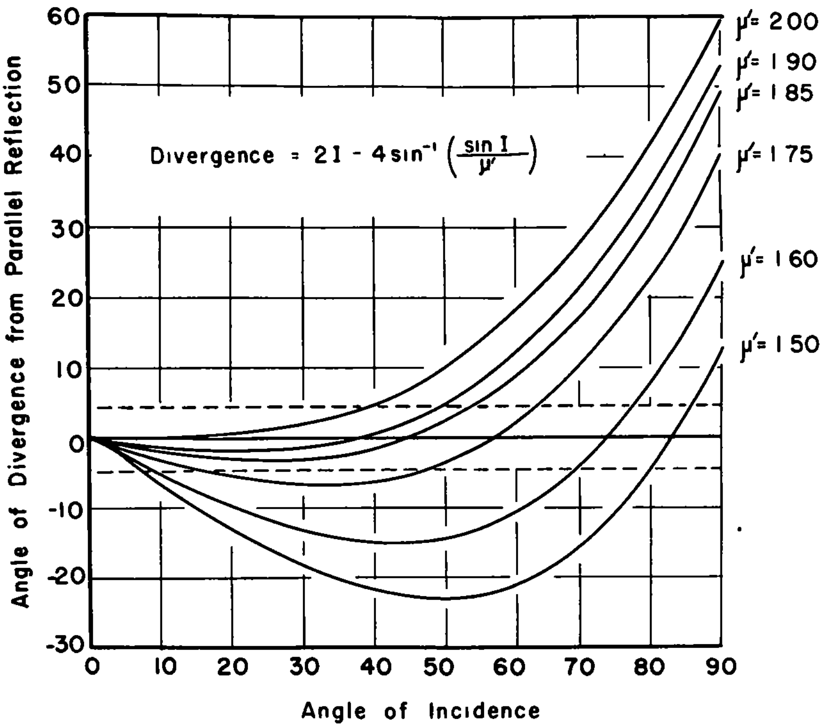

Figure 7. Typical curves showing the relationship between d, I, and u′ for the system illustrated by Figure 6.

From further inspection of the diagram in Figure 6, it may be noted that the central axis, there shown as normal to the plane of the sign, may be rotated about the center of curvature of the sphere until the 60-deg. maximum incidence angle just grazes the binder without impairing the efficiency of the system at all. This means, of course, that the plane of the sign may be rotated through an angle of 30 deg. to the driver and headlamps without sacrificing any of the reflex efficiency of the sign. At angles greater than 30 deg., the binder obscures more and more of the useful aperture of the lens.

In contrast to the system already described, Figure 8 illustrates another system which utilizes the longer focal length lenses and which is functionally comparable to the optical system illustrated by Figure 4. In this diagram, the sphere is envisaged as being imbedded in a transparent medium and properly spaced in front of the reflecting surface shown by the shaded area in the drawing. Here the incident ray is first refracted at the air-glass interface, then at the glass-spacer interface, is reflected and returns in a nonsymmetrical manner back across the two refracting interfaces. This system is complicated by a multiplicity of dependent variables which defy resolution and simplification. When u′ for the glass and u" for the spacing medium are known, any incident ray may be traced through the system by geometric construction as shown in the figure, regardless of the inclination of the central axis through the sphere to the reflecting surface which is in the plane of the sign. That is true only for fixed values of r and s. Again from earlier consideration, it will be recalled that, for zero inclination of the central axis, r + s should be approximately equal to the focal length of the sphere. However, due to spherical aberration, greater efficiency is realized for zero inclination when r + s is slightly less than the focal length. Each inclination then introduces an entirely different set of circumstances.

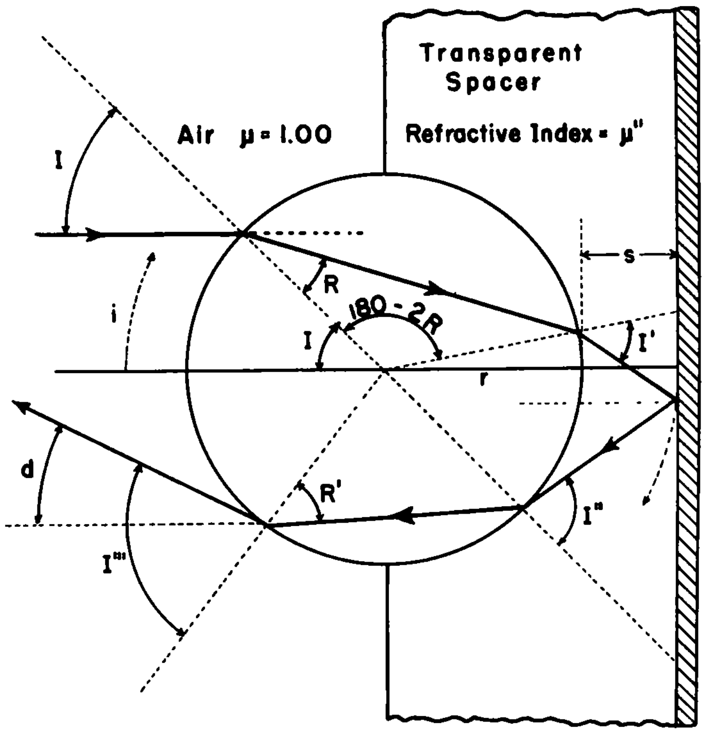

Figure 8. Cross-sectional diagram showing a single glass sphere imbedded in a transparent spacing medium overlying a plane reflecting surface. The geometric construction of the path taken by a single ray of light through the system is also shown.

In general, the design illustrated in Figure 8 seems to be more efficient for near normal orientation of the driver and headlamps with respect to the plane of the sign. Also, it is only by the use of a diffusing-type reflecting surface behind the spheres that it is possible to preserve reflex characteristics through greater angles of inclination. These fundamental imperfections arise from the association of plane and spherical surfaces. In this particular design, the use of the plane reflecting surface is recognized as a construction expediency. It may be regarded, then, as a practical modification of a more fundamental system in which the plane reflecting surface replaces a spherical surface having a radius approximately equal to r + s. Accordingly, a similar system having a spherical reflecting surface would be capable of accommodating almost any angle of inclination, i, without disrupting the symmetry of the system. This corresponding fundamental design is illustrated by Figure 9 where it is shown that the former complexity of variables has been eliminated.

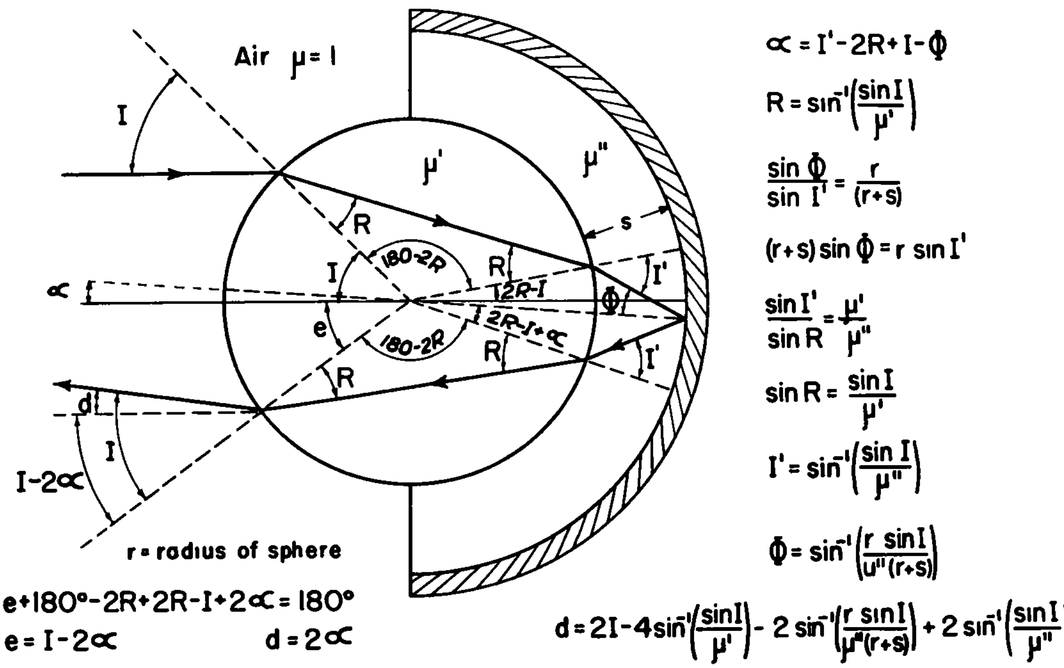

Figure 9. Diagram showing the basic optical design corresponding to the system illustrated in Figure 8.

Now, the divergence angle, d, may be equated for this system too. Accordingly:

$$d = 2I - 4\sin^{-1}\left(\frac{\sin I}{u'}\right) - 2\sin^{-1}\left(\frac{r\sin I}{u''(r+s)}\right) + 2\sin^{-1}\left(\frac{\sin I}{u''}\right)$$

When $u' = u''$

$$d = 2I - 2\sin^{-1}\left(\frac{\sin I}{u'}\right) - 2\sin^{-1}\left(\frac{r\sin I}{u'(r+s)}\right)$$

Also, when $s = 0$

$$d = 2I - 4\sin^{-1}\left(\frac{\sin I}{u'}\right)$$

which is the basic equation for the first design illustrated by Figure 6.

Horizontal Surfaces -- Reflectorization is an effort to compensate for some of the inadequacies connected with night-driving visibility. It is borne out by experience that reflectorized horizontal surfaces such as centerline stripes suffer considerable loss in efficiency during moderate to heavy rains. This loss is unfortunate because it occurs under critical conditions of visibility when drivers need to be compensated the most. The possibility of incorporating additional compensation into the optical design of the reflectorizing systems offers an interesting and practical application of the previously discussed theories.

In Figure 6, the glass sphere is considered to be refracting with respect to air. By imagining such a surface oriented horizontally and the sphere completely inundated by a film of water, the sphere would no longer be refracting with respect to a medium where $u = 1$ but where $u = 1.33$. In order to preserve the same refractive efficiency of the glass sphere, the ratio the sines of I and R would have to remain constant.

Suppose, for instance, that the optimum ratio of sin I to sin R is taken as 1.90; then u'/u must equal 1.90. If u' is the refractive index of the glass and u the refractive index of the water, 1.33; then u' would have to be equal to 2.50.

Contrasting these theoretically ideal conditions with a reflectorizing system using ordinary glass have a refractive index of approximately 1.50 with respect to air; when inundated by water, the ratio of sin I to sin R is no longer equal to 1.50 but is 1.13 which is little, if any, better than no refraction at all. So, even to preserve a ratio of 1.50 to compensate the system for water inundation, the refractive index of the glass would have to be increased to 2.00.

Theoretically, at least, horizontal surfaces could be compensated for inundation by incorporating some of those more highly refractive glasses with those considered optimum for normal weather conditions; provided, of course, the more highly refractive glasses were available. Such theoretical conjectures as this exemplify the possibilities for further modification and refinement of reflex optical systems.

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