Soil Investigation Employing A New Method of Layer-Value Determination for Earth Resistivity Interpretation

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● IN an effort to improve methods of making soil investigations of proposed borrow sites and highway construction the Michigan State Highway Department is now employing the "earth resistivity" method as a means of obtaining information. The objective in adopting this method is to eliminate, or at least reduce, the chances of costly errors in estimates of earth quantities and quality of earth borrow due to the lack of adequate information. Until this resistivity instrument was acquired nearly all investigations were made by hand augering with the occasional assistance of jet borings when the importance of the information warranted its cost of operation. These methods are laborious and in most cases, give inadequate data. It is impossible to auger into a granular material which lies below water table without the use of power drilling and some form of casing. Although a soils engineer can determine the source of good granular borrow, for example, from a few hand borings and trained observations, it is very difficult to estimate the size and location of the deposit or to detect a hidden clay stratum even if its presence is suspected. With the purchase of the resistivity instrument it was the intent of the Department to develop a procedure that would give more detailed and accurate information of soil conditions.

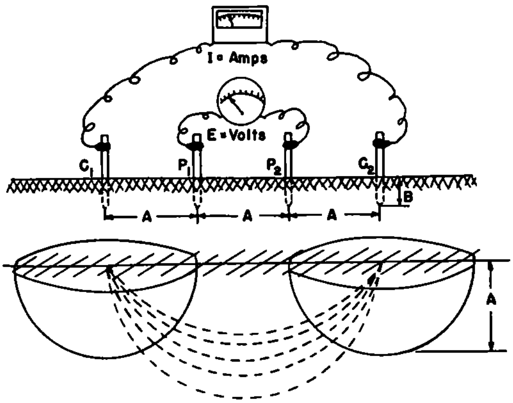
It has now been about two years since the instrument was purchased during which time considerable experimentation has been carried on with the result that detailed information on types, quantities, and locations of certain soil materials can now be determined with an accuracy which is considered to be within practical limits.

BACKGROUND AND METHODS OF USE

Instruments for measuring earth resistivity have been used for many years by geologists and geophysicists in their attempts to prospect and explore the earth's crust in search of oil, minerals, etc. In the course of years much research has been done to improve the techniques, instruments, and interpretation of results to obtain better detail and accuracy. It is not the writer's intention to go into an explanation of the numerous methods used by various groups of geophysicists and engineers other than to give a partial list of the more common ones as follows: Porous Pot, direct method; Gish-Rooney[1] method; "Megger" method; Single Probe method.

After considerable study and experimentation to determine the advantages and disadvantages of various methods with respect to the type of information desired from soil investigations, the Gish-Rooney method was selected. One of the main advantages of this method is the elimination of the effects of ground and stray currents by the use of an alternating, or more correctly, commutated circuit. Voltages and currents are read separately from which the apparent average resistivity of the soil is computed. The arrangement of four electrodes in a straight line spaced an equal distance from each other is used almost exclusively. This arrangement

[1]Gish, O H , "Improved Equipment for Measuring Earth-Current Potentials and Earth Resistivity". National Research Council, Bulletin, Nov 1926, Vol II, Pt 2, No 56.



* Wenner's equation for the average resistivity of soil

$$\rho = \frac{4\pi AR}{1 + \frac{2A}{(A^2 + 4B^2)^{\frac{1}{2}}} - \frac{A}{(A^2 + B^2)^{\frac{1}{2}}}}$$

When B is small compared to A, the equation simplifies to:

$$\rho = 2\pi A \frac{E}{I}$$

*Wenner, U.S. Bureau of Standards Scientific Paper No. 258

Figure 1. Wenner's configuration in the spacing of electrodes used in the Gish-Rooney method for measuring earth resistivity, illustrating the equipotential-bowl theory.

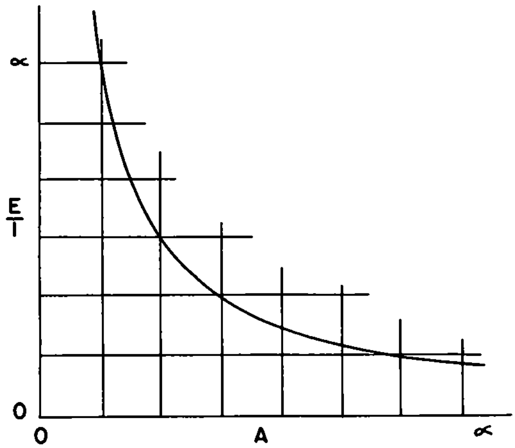


Figure 2.

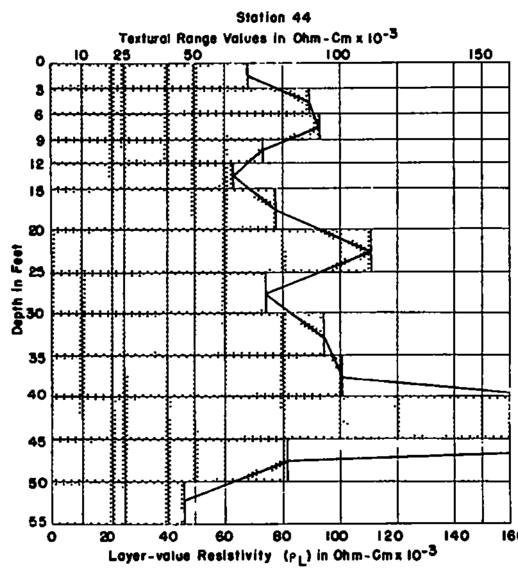


Figure 3.

is generally known as Wenner's[2] configuration. By using this arrangement the spacing between electrodes is equal to the depth of soil investigated as shown in Figure 1. As with any tool being applied to a new field, there is a stage of development during which different approaches and practices are studied, tried, revised, discarded or improved, and finally a definite procedure embracing the limitations of the tool is adopted as standard practice. The procedure adopted by the Department as standard practice, at least for the present time, consists of making depth-profile measurements at selected stations along one or more lines of traverse. The distance between stations and the number of traverse lines selected depend upon the size and depth of the soil body for which information is desired and the time allowed to make the investigation. Naturally there are exceptions made to the standard practice for those cases requiring specific and particular information. In general, traverse lines are made not more than 100 feet apart and the distance between stations is held to not more than 100 feet. In measuring depth profiles, it is considered good practice to use 3-foot intervals of layer thickness for depths up to 15 or 21 feet and 5-foot intervals for depths of investigation greater than this 15 or 21 feet. The advantages obtained by measuring several shallow layers in preference to fewer layers of greater thickness will be appreciated when the interpretation of field results as developed and used by the Department is understood.

[2]Wenner, Frank, "Method of Measuring Earth Resistivity". U S Bureau of Standards, Scientific Paper No. 258, Bulletin, Vol 12-No 3, 1915-16

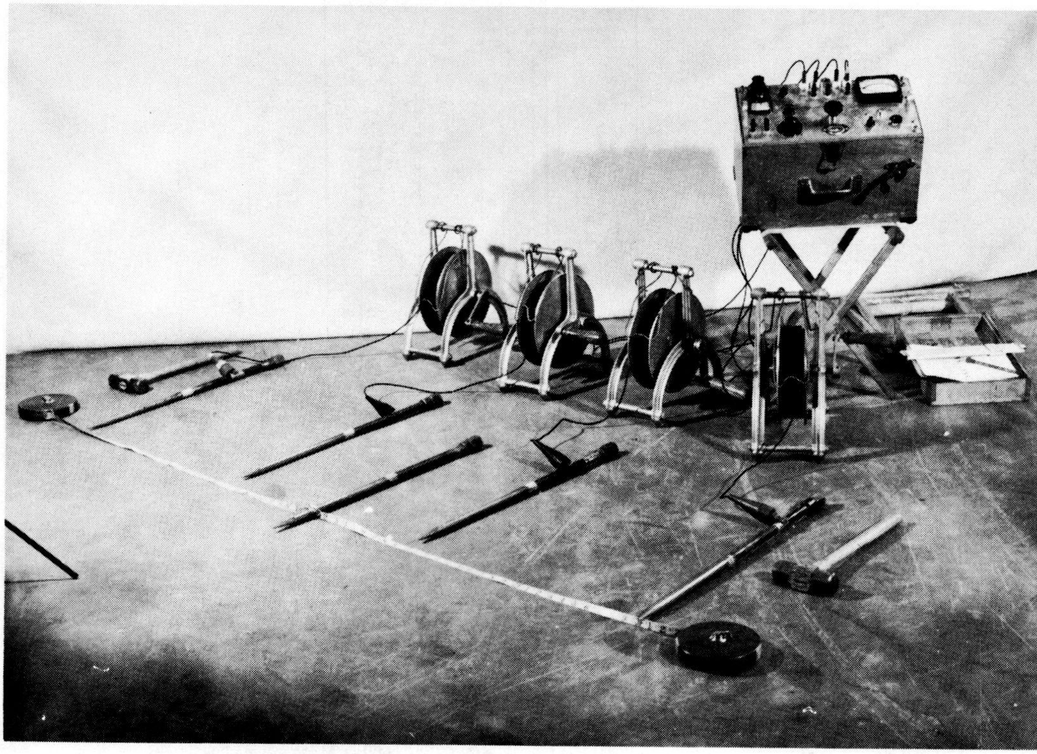


Figure 4. Assembly of equipment for earth-resistance survey.

INTERPRETATIONS OF FIELD MEASUREMENTS

The interpretation of field measurements from which reliable deductions can be made presented a most difficult problem. A study was made of the several different methods of interpretations as presented in various published bulletins and papers, some of which are based on theoretical and mathematical considerations and at least one of which is based upon purely empirical considerations.

In general, theoretical and mathematical methods require such a great volume of computations that the amount of time required to obtain the desired information would defeat the purpose of using the resistivity instrument inasmuch as time and costs of obtaining accurate information are prime considerations. On the other hand, after many attempts to apply empirical methods, it was found that even the more recent methods of empirical interpretation were somewhat inadequate and not sufficiently reliable.

Therefore, it was felt that a method of interpretation might be developed which would give the particular type of detailed and reliable information such as required by the Department if only on a comparative basis. As a result of much field work and calculation of electrical measurements a method of interpreting field data has been developed on the premise that Wenner's formula is a truly fundamental expression for determining the average apparent resistivity of any thickness of an earth mass.

EQUATION FOR DETERMINING LAYER VALUE

Wenner's formula[3] for the 4-electrode, equal spacing configuration is given as:

$$\rho = 2\pi A\frac{E}{I} \qquad (1)$$

where ρ = average specific resistivity of depth A in ohm-cms

A = spacing of electrodes and depth investigated in cms

[3]op cit.

E = potential differential across the inner two electrodes through "A" depth of earth in volts

I = current carried through the mass as introduced through the outer electrodes in amperes

See Figure 1 for Wenner's formula and a sketch illustrating the equi-potential bowl theory.

Inasmuch as A is a variable, then in order that ρ remain constant for different thicknesses of a homogeneous soil, the ratio of E/I must vary inversely with A. The curve in Figure 2 shows the relationship of E/I to A.

The equation for determining layer values which is being presented at this time is based on the hypothesis that layers of earth are analogous in behavior to parallel electrical resistances.

On the basis of this hypothesis, each layer of a two or more layer system will have its particular value of resistance as illustrated in the following sketch for a three-layer system:

A'	R_1	Layer 1	Three layers of non-homogeneous soil.
A'	R_2	Layer 2	
A'	R_3	Layer 3	

A' = thickness of layer interval

R = average resistance of layer

For the above condition the average

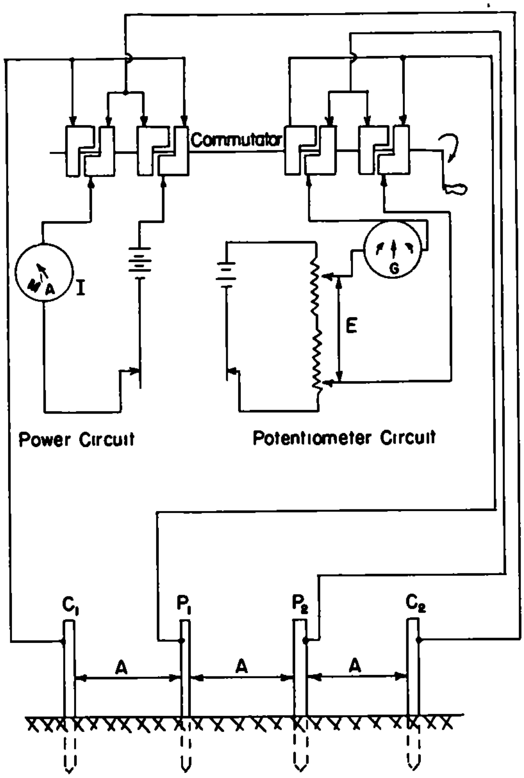


Figure 5. Schematic circuit diagram of earth-resistivity equipment.

resistivity values obtained by the earth resistivity equipment would be ρ_1 for depth A', ρ_2 for depth 2A', and ρ_3 for depth 3A', etc. It is recognized that the

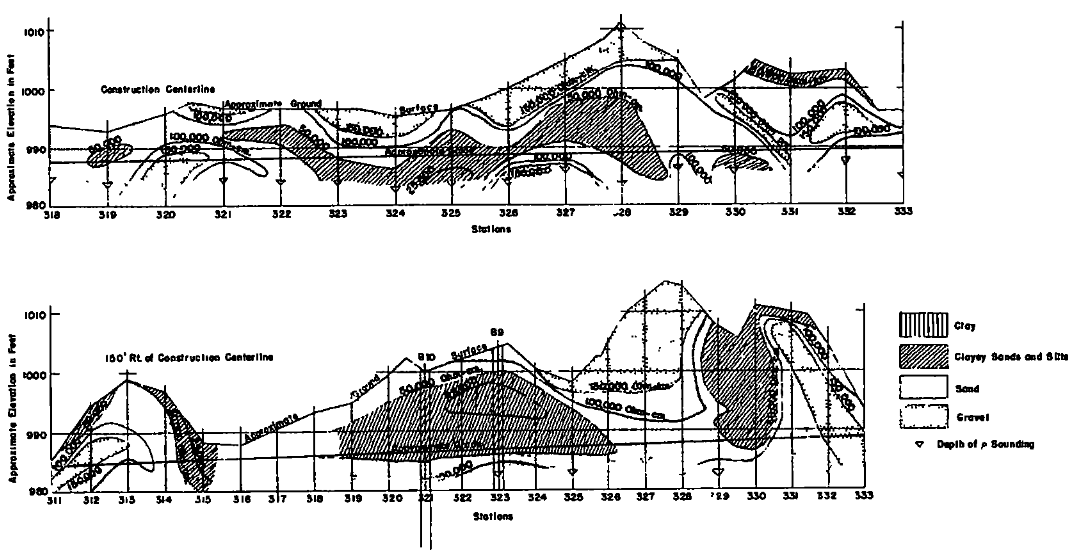


Figure 6. Profile contours, Stations 311 to 333.

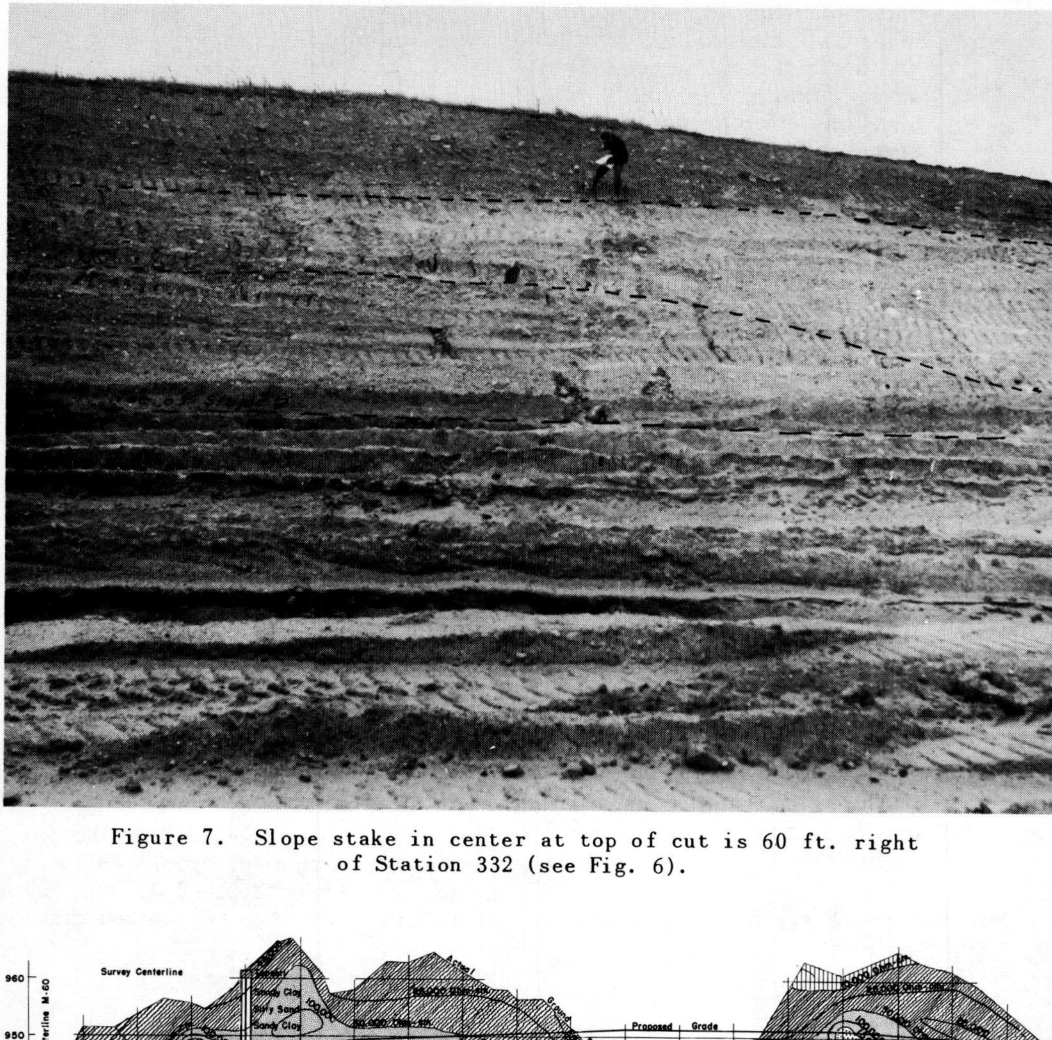


Figure 7. Slope stake in center at top of cut is 60 ft. right of Station 332 (see Fig. 6).

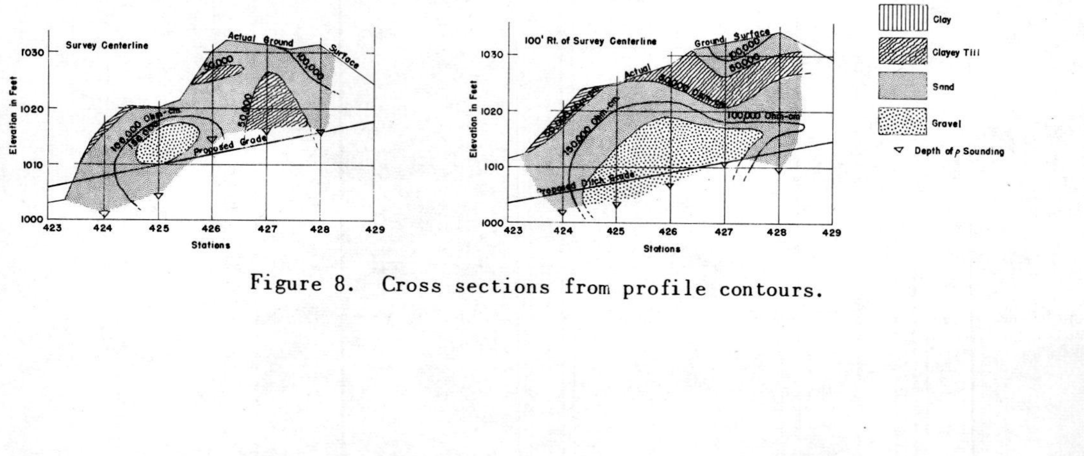


Figure 8. Cross sections from profile contours.

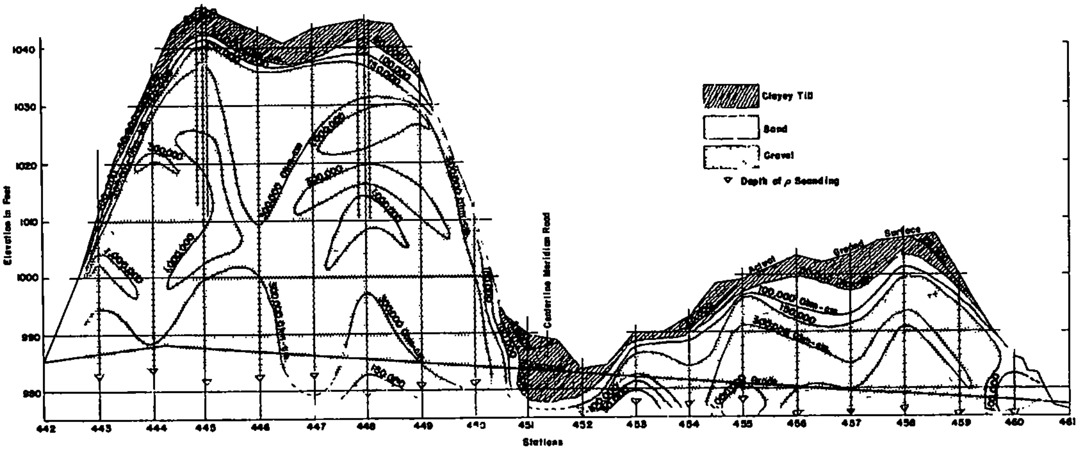
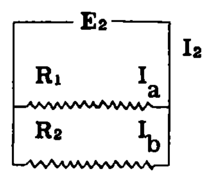


Figure 9. Cross section from profile contour.

value of $\frac{E}{I}$ in Wenner's formula (Eq. 1) may give only an approximate value of resistance for the soil because the equipotential bowl theory does not take into consideration the warping effect caused by the varied paths taken by the current through heterogeneous materials. Nevertheless, it serves as a comparative value with which different types of soil may be differentiated from each other. Considering now the value of resistance for the first layer, in the sketch above, it may be assumed that A' represents a layer of homogeneous soil and, therefore, the value of resistance is equal to the quotient obtained by dividing the potential differential by the current carried as read from the resistivity instrument.

Thus: $R_1 = \frac{E_1}{I_1}$, or the average specific resistance for Layer 1. If E_2 and I_2 are the values read when investigating the depth 2A' and the assumption is made that Layers 1 and 2 act as parallel resistance of different values through which the current is pushed, then this condition may be illustrated by the following analogy:



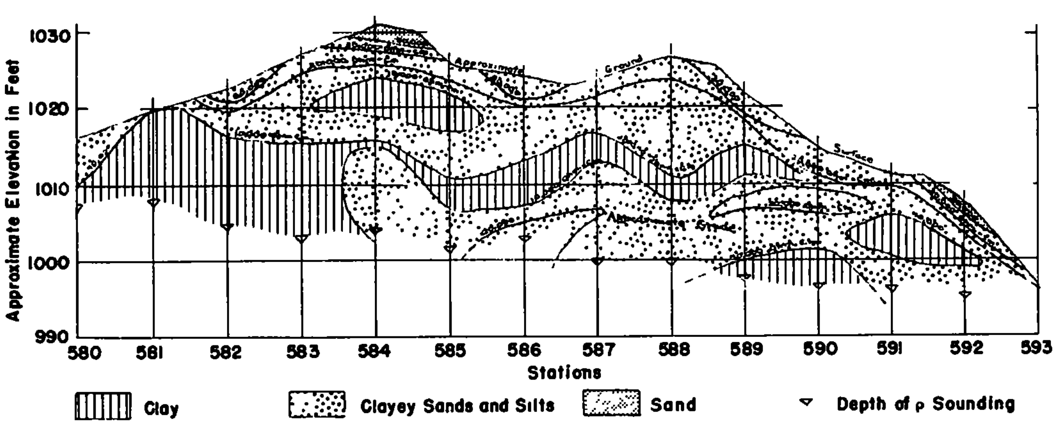


Figure 10. Profile contours taken on construction centerline.

Figure 11. Slope stake at top of cut is 50 ft. left of Station 586+50 (see Fig. 10).

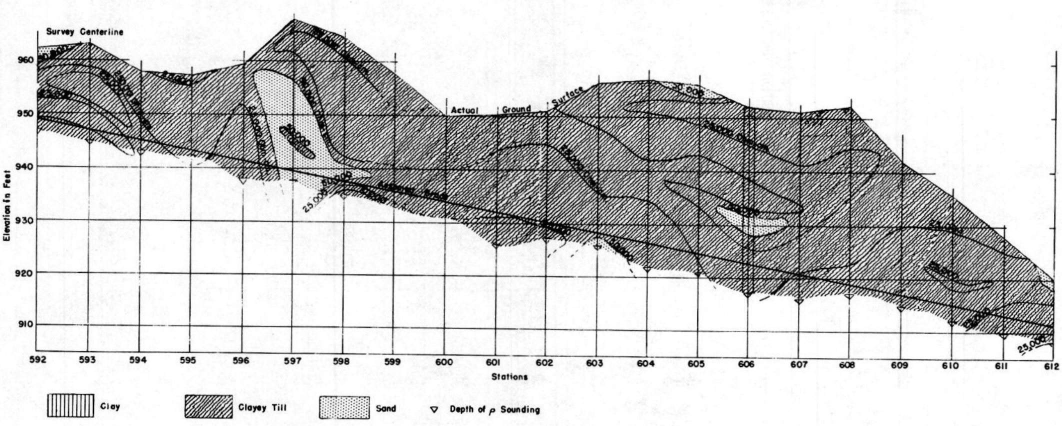


Figure 12. Cross sections from profile contours.

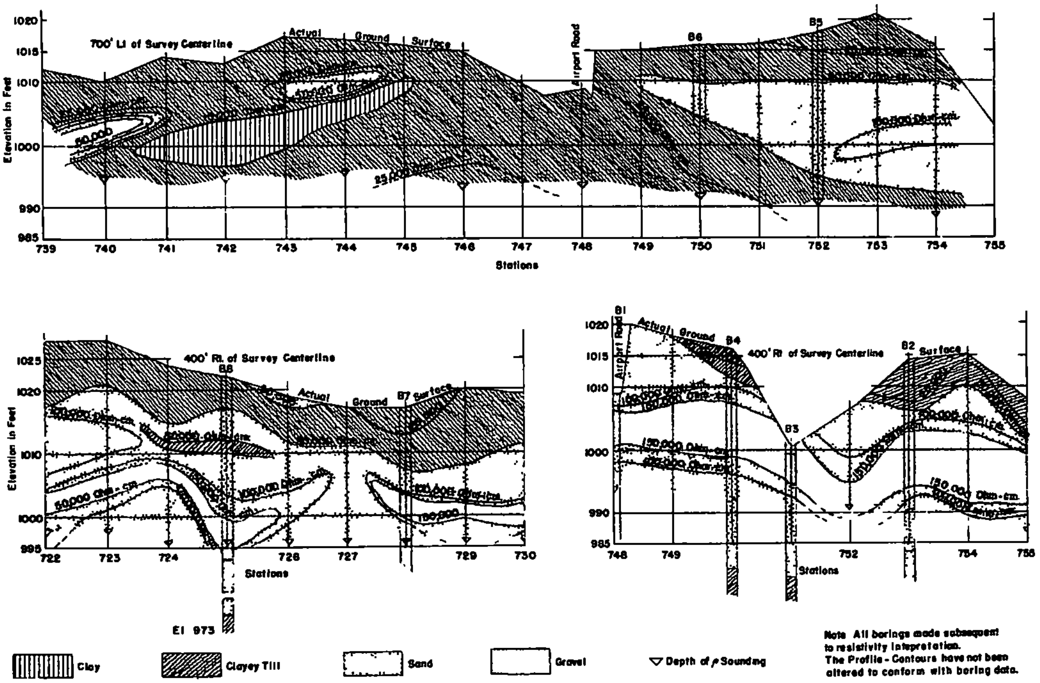


Figure 13. Cross sections from profile contours.

The unknown value of R_2 in the above analogy is determined as follows:

Step 1) $R_1 = \frac{E_1}{I_1}$ (known)

2) $I_a = \frac{E_2}{R_1}$ (known)

3) $I_b = \frac{E_2}{R_2}$

4) $I_2 = I_a + I_b$

5) $I_2 = \frac{E_2}{R_1} + \frac{E_2}{R_2}$

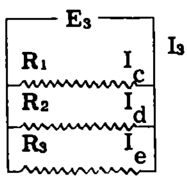
6) $\frac{E_2}{R_2} = I_2 - \frac{E_2}{R_1}$

7) $R_2 = \frac{E_2}{I_2 - \frac{E_2}{R_1}}$

Substituting R_2 for $\frac{E}{I}$ in Wenner's equation, the value of resistivity, ρ_{L2}, for Layer No. 2 will be

$$\rho_{L2} = 2\pi A R_2 \qquad (2)$$

Using the same analogy and principles as used above for R_2 the value of R_3 for the third layer may be found as follows where E_3 and I_3 are the respective potential differential and current values given by the resistivity instrument for the 3A' depth.



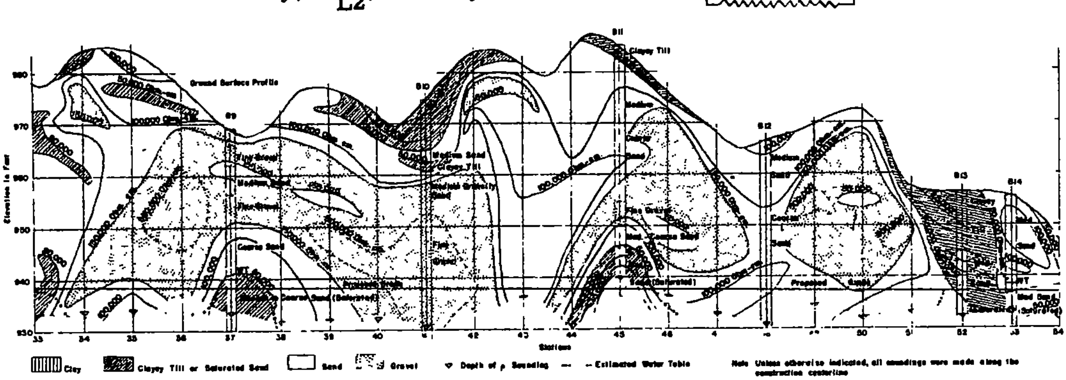


Figure 14. Cross sections from profile contours.

Figure 15. Cut partially excavated, 60 ft. left of Station 41+50.

8) $I_c = \frac{E_3}{R_1}$ (known)

9) $I_d = \frac{E_3}{R_2}$ (known)

10) $I_e = \frac{E_3}{R_3}$

11) $I_3 = I_c + I_d + I_e$

12) $I_3 = \frac{E_3}{R_1} + \frac{E_3}{R_2} + \frac{E_3}{R_3}$

13) $\frac{E_3}{R_3} = I_3 - \left(\frac{E_3}{R_1} + \frac{E_3}{R_2}\right)$

14) $R_3 = \frac{E_3}{I_3 - \left(\frac{E_3}{R_1} + \frac{E_3}{R_2}\right)}$

All of the values in Step 14) are known except R_3 which, therefore, can be determined. This equation may, of course, be used for any number of layers and will take the general form for any number of layers n as:

$$R_n = \frac{E_n}{I_n - \left(\frac{E_n}{R_1} + \frac{E_n}{R_2} + \ldots\ldots \frac{E_n}{R_{n-1}}\right)} \quad (3)$$

The use of Equation 3 becomes rather laborious when it is desired to determine the value of resistivity for a layer located several depth-intervals below the surface. However, it can be proven that the term $\left(\frac{E_n}{R_1} + \frac{E_n}{R_2} + \ldots\ldots \frac{E_n}{R_{n-1}}\right)$ equals the term $\frac{E_n}{\bar{R}_{n-1}}$. The substitution of the latter term in Equation 3 then renders ths solution of the layer values of resistivity much more expedient.

Proof of the identity of the above terms is given as follows with reference being made to the three-layer case: Let R designate the average value of resistance for an individual layer of material, and let $\bar{R}$ designate the average value of resistance for any depth of soil measured from the surface as given by the ratio of $\frac{E}{I}$. It is evident that for the first layer $R_1 = \bar{R}_1 = \frac{E_1}{I_1}$, but for subsequent layers the equality does not hold. Therefore, $\bar{R}_{(n-1)}$ will represent the average resistance value for the depth of n number of layers minus one, or

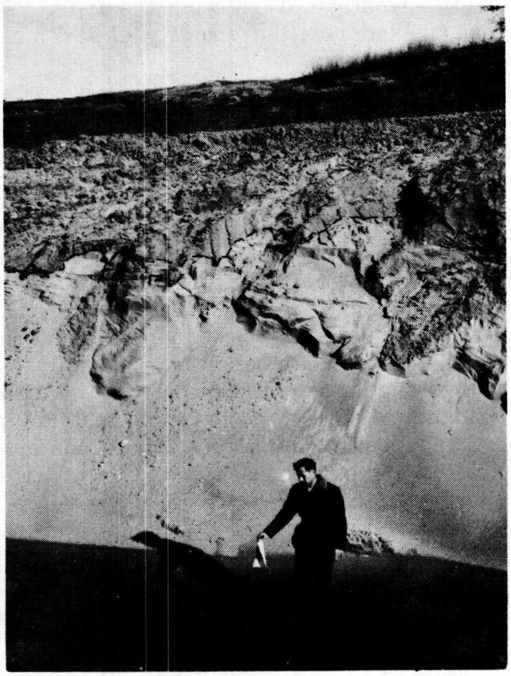


Figure 16. Station 45 ℄.

$$\bar{R}_{n-1} = \frac{E_{n-1}}{I_{n-1}}$$

$$R_3 = \frac{E_3}{I_3 - \frac{E_3}{\bar{R}_2}} = \frac{E_3}{I_3 - \left(\frac{E_3}{R_1} + \frac{E_3}{R_2}\right)} \text{ (from Step 14)}$$

$$\text{where } \bar{R}_2 = \bar{R}_{n-1} = \frac{E_2}{I_2}$$

$$R_2 = \frac{E_2}{I_2 - \frac{E_2}{R_1}} = \frac{E_2}{I_2 - \frac{E_2 I_1}{E_1}} = \frac{E_2 E_1}{E_1 I_2 - E_2 I_1} \text{ (from Step 7)}$$

If,

$$15)\ \frac{E_3}{\bar{R}_2} = \frac{E_3}{R_1} + \frac{E_3}{R_2}$$

Then substituting $\frac{E}{I}$ for respective $\bar{R}$s and Rs,

$$16)\ \frac{E_3 I_2}{E_2} = \frac{E_3 I_1}{E_1} + \frac{E_3 E_1 I_2 - E_3 E_2 I_1}{E_2 E_1}$$

$$17)\ \frac{E_3 I_2}{E_2} = \frac{E_3 I_1}{E_I} + \frac{E_3 E_1 I_2}{E_2 E_1} - \frac{E_3 E_2 I_1}{E_2 E_1}$$

$$18)\ \frac{E_3 I_2}{E_2} = \frac{E_3 I_1}{E_1} + \frac{E_3 I_2}{E_2} - \frac{E_3 I_1}{E_1}$$

$$19)\ \frac{E_3 I_2}{E_2} = \frac{E_3 I_2}{E_2}$$

Equation 3 can now be expressed as,

$$R_n = \frac{E_n}{I_n - \frac{E_n}{\bar{R}_{n-1}}} \quad (4)$$

If in the three layer case all of the soil is considered to be homogeneous, then $R_1 = R_2 = R_3$. Now, referring to Figure 2, the question arises as to whether the layer Equations 3 and 4 take into consideration the fact that for a homogeneous material the ratio of $\frac{E}{I}$ or $\bar{R}$, varies inversely with the depth.

If the layer equations do take into consideration this variation, then it can be proved, when $R_1 = R_2 = R_3$, that $\bar{R}_3 = \frac{\bar{R}_1}{3}$, or that $\bar{R}_n = \frac{\bar{R}_1}{n}$

$$14)\ R_3 = \frac{E_3}{I_3 - \frac{E_3}{R_1} + \frac{E_3}{R_2}} \text{ or } \frac{E_3}{I_3 - \frac{E_3}{\bar{R}_2}}$$

Since $R_2 = R_1$

$$20)\ \frac{E_3}{R_3} = I_3 - \frac{2E_3}{R_1}$$

Also $R_3 = R_1$

$$21)\ I_3 = \frac{3E_3}{R_1}$$

$$22)\ R_1 = \frac{3E_3}{I_3} = 3\bar{R}_3$$

$$23)\ \bar{R}_3 = \frac{R_1}{3}, \text{ or}$$

$$\bar{R}_n = \frac{R_1}{n} \quad (5)$$

THE USE OF THE LAYER EQUATION PRACTICE

In order to classify the types of soils encountered, a system of recognition is provided based upon ranges of layer-value resistivities determined from experience.

For the types of soils existing in the lower Peninsular of Michigan the following table has been developed:

ρL	Soil Types
0 - 10,000	Clay and Saturated Silt
10,000 - 25,000	Sandy Clay and Wet Silty Sand
25,000 - 50,000	Clayey Sand and Saturated Sand
50,000 - 150,000	Sand
150,000 - 500,000	Gravel

When the value of the layer resistivity is greater than 500,000 ohm-cm the interpretation of soil must be augmented with boring information. The reason for this is that a number of conditions can exist which will show high resistivity values, and these conditions range from dry loose sand and gravel to weathered rock and bedrock.

Inasmuch as the thickness of the layer is an arbitrary selection, the layer-value of resistivity must represent the average resistivity of all the soil types lying within the boundaries of any particular layer.

After all of the layer-values have been calculated they are plotted in bar-graph fashion against their respective intervals of depth as shown on Figure 3. The values for the layers are then connected to each other by lines drawn from the middle of each layer. The intersection of the various range values with the resistivity connecting lines will determine the elevation limits for the soil types. These intersection points can then be connected from station to station to from contour boundaries which, in effect, gives a cross-sectional view of the soil profile to any depth investigated showing the type, location, and relative quantity of soil materials.

CONCLUSION

It is the writer's opinion that investigations of borrow and proposed cut-sections of considerable size can be made faster and provide greater accuracy and detail by the resistivity method than by such methods as hand augering and soil borings. For example, there have been a number of occasions when the analysis of soil deposits by the resistivity method has indicated the presence of materials not apparent from surface conditions and shallow borings usually employed. Although this method is still in the development stage, subsequent borings and pit excavations proved the analyses to be correct. Thus the method of interpreting the field data by the layer-value determination equation has been successful to date.

It is felt that the layer-value determination as outlined here is not seriously affected, if at all, by the warping of the equipotential bowl which necessarily must take place to conform to the various resistances of the heterogeneous layers of material. Therefore, it is the writer's opinion that as more experience is obtained and with further laboratory study, the method will prove to be sufficiently accurate and reliable to satisfactorily predict the soil characteristics and conditions as required by the Department.