

## Rapid Methods for Determining Liquid Limits of Soils

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Engineers responsible for the routine testing of large numbers of soil samples for highway construction and maintenance need rapid methods for testing. In this study, two simplified methods of determining the liquid limit of soil are compared with the standard method to indicate their reliability and accuracy. Both are one-run methods, i. e., require a single determination of moisture and number of blows for groove closure to calculate the liquid limit. The methods discussed—(1) a so-called chart method developed by the Washington State Highway Department and (2) a modification of the chart method by use of a slide rule—have been compared for a wide range of soils found in many of the states. Both of these methods are rapid and reduce the time normally required for testing as much as 30 to 70 percent. They were found to have good reproducibility and to have sufficient accuracy, within limits, to be acceptable as alternate test procedures for the standard method used for the determination of the liquid-limit constant.

● IN 1926 the lower liquid limit, developed by Albert Atterberg,<sup>1</sup> and described in the International Reports on Pedology, 1911, was suggested as one of the soil constants that could be used to evaluate a soil as a road-building material.

This soil constant is now widely used by highway and airport engineers to evaluate soils for use as subgrade, base course, and embankment material. High liquid limits usually indicate heavy clays and low values indicate friable soils, silts, and clayey sands.

The lower liquid limit, now known as the liquid limit, is defined as the percent of moisture, based upon the dry weight of soil, at which a soil will just begin to flow when jarred slightly, and according to this definition, soils at their liquid limit have a small but definite shear resistance which can be overcome by the application of very little force. The cohesion of the soil-water mixture at the liquid limit is practically zero.

The determination of this soil constant as originally proposed was made by a hand method. In this method, the operator adjusted the moisture content by trial and error until exactly 10 blows closed a standard width groove made in the soil sample. The moisture content for this groove closure

was taken as the liquid limit of the soil. This method required considerable skill and judgment by the operator to obtain duplicate test results.

As soil testing became more widely used, the need for a more practical test procedure became apparent and a mechanical device<sup>2</sup> was developed which eliminated personal judgment in estimating the intensity of the blows which cause closure of the groove. Although the liquid limit machine was found to give good reproducibility of results and has been adopted as a standard test procedure<sup>3</sup>, it requires more time than the hand method.

For example, the standard method required that three random trials be made, each at a different moisture content and number of blows; that the percent moisture and number of blows be plotted on semilog paper and a flow curve drawn; and that the liquid limit be taken as the moisture content corresponding to the point where the flow curve intersects the 25-blow line. This procedure has less probable error than a single determination by the hand method and the machine can be used by operators of less skill, but the additional trials, weigh-

<sup>2</sup> Research on the Atterberg Limits of Soils, by Arthur Casagrande, PUBLIC ROADS, Vol. 13, No. 8, Oct. 1932.

<sup>3</sup> Standard Method of Determining the Liquid Limit of Soils, AASHTO Designation T89-49, Part II, Standard Specifications for Highway Materials and Methods of Sampling and Testing, published by The American Association of State Highway Officials, 1950.

<sup>1</sup> Adaptation of Atterberg Plasticity Tests for Subgrade Soils, by A. M. Wintermyer, PUBLIC ROADS, Vol. 7, No. 6, Aug. 1926.

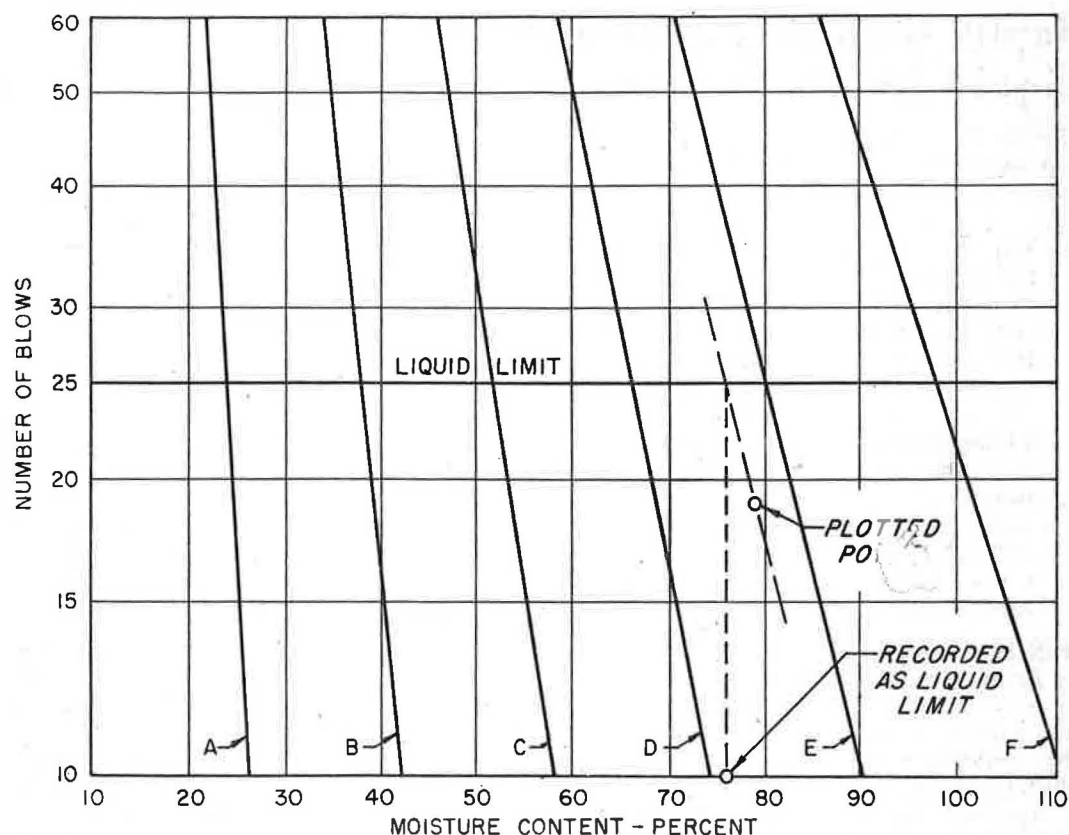


Figure 1. Relation between slope of typical flow curves and liquid limit.

ings, and computations require considerably more time than that required by skilled operators familiar with the use of the one-point hand method.

It is reasonable to expect that engineers responsible for routine testing of soils would be reluctant to accept a longer test procedure without carefully examining the method to determine if modifications could be made which would permit more rapid testing of soils.

One of the first modifications to be tried was the use of the mechanical liquid-limit device to furnish the standard force used in the hand method. This was accomplished by adjusting the soil-water consistency by a trial-and-error method until exactly 25 blows by the liquid-limit machine closed the standard-width groove.

This modification of the machine method, usually referred to as the "moisture-adjustment method," can be developed into an accurate rapid method by skilled operators if calibrated with the standard meth-

od, and it can be used effectively to reduce the time required for the determination of the liquid limit of a soil. However, it is difficult for inexperienced or less-skilled

TABLE 1

COMPARISON OF DEVIATIONS IN VALUES OF LIQUID LIMIT OBTAINED BY THE COOPER AND JOHNSON CHART METHODS FROM THOSE OBTAINED BY STANDARD TESTS<sup>a</sup>

Range in deviations <sup>b</sup>	Original Chart Method <sup>c</sup>			Revised Chart Method <sup>d</sup>		
	# tests	% total	Cum. %	# tests	% total	Cum. %
0	4	6	6	8	9	9
0.1 - 0.5	28	38	44	41	48	57
0.6 - 1.0	19	26	70	26	31	88
1.1 - 1.5	6	8	78	7	8	96
1.6 - 2.0	6	8	86	3	4	100
2.1 - 2.5	6	8	94	-	-	-
2.6 - 3.0	1	1	95	-	-	-
3.1 - 3.5	2	3	98	-	-	-
3.6 - 4.0	0	0	98	-	-	-
over 4.0	2	2	100	-	-	-
Total	74	100		85	100	

<sup>a</sup> Duplicate samples tested, one at random blows and the other by the standard test procedure used in the laboratory.

<sup>b</sup> Deviations  $\pm$  from the value of liquid limit determined by test.

<sup>c</sup> Chart (Fig. 1) based on groove closure within range of 15 to 40 blows.

<sup>d</sup> Chart (Fig. 2) based on groove closure within range of 17 to 36 blows.

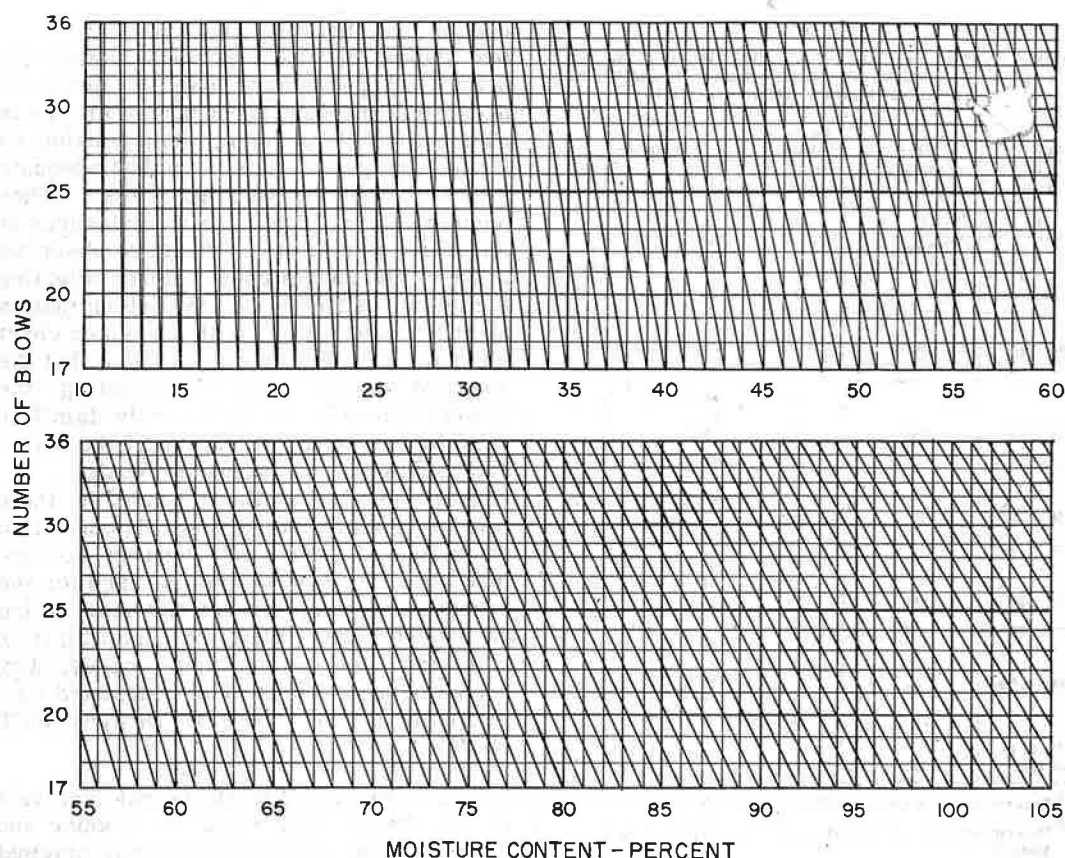


Figure 2. Chart developed by Washington State Highway Department for the calculation of the liquid limit.

operators to adjust the consistency of the soil-water mixture to obtain groove closure at exactly 25 blows.

#### CHART METHOD FOR DETERMINING LIQUID LIMIT

A rapid method for determining the liquid limit has been devised by Cooper and Johnson, of the Washington State Highway Department,<sup>4</sup> for the routine testing of soils in their laboratory. It is called the "chart method" and is based upon the relation between the slopes of flow curves obtained for soils with different liquid-limit test values.

Cooper and Johnson observed, in their initial study of soils taken from various

parts of Washington, that the flow index<sup>5</sup> computed from the flow curves obtained by the Standard AASHTO Method of Test increased uniformly with the liquid limit of the soils. From these data, six typical flow curves, A through F, shown in Figure 1, were developed.

Cooper and Johnson established the validity of the six typical curves by plotting the number of blows and moisture contents for single points representing random trial runs on each of 27 different soils. In this series of tests, the number of blows for groove closure was varied within a range of 15 to 40. The liquid limit was determined for each soil by drawing a line through the plotted point, which is parallel to the nearest typical flow curve (A through F) and recording as the liquid limit, the moisture content at the intersection of this line with the 25-blow line.

<sup>4</sup>A Rapid Method of Determining the Liquid Limit of Soils, by John H. Cooper and Kenneth A. Johnson, Materials Laboratory Report No. 83 March 1950, Washington State Highway Department.

<sup>5</sup>Flow index is the range in moisture content represented by the number of blows or shocks in one cycle of the logarithmic scale of a flow curve.

TABLE 2  
FREQUENCIES OF DIFFERENT SIZE DEVIATIONS OF  
CHART AND SLIDE RULE LIQUID LIMITS FROM THOSE  
BY THE STANDARD AASHO METHOD

Range of liquid limit	Number of one-point tests	Deviation from Standard LLa	Frequency of deviations	
			Chart <sup>b</sup> Percent (4)	Slide rule <sup>c</sup> Percent (5)
(1)	(2)	(3) <sup>d</sup>		
15 - 25	171	0	76.6	84.8
		1	23.4	15.2
		2	0	0
26 - 40	209	0	76.1	77.0
		1	23.4	23.0
		2	0.5	0.0
		3	0	0
41 - 55	129	0	69.0	70.6
		1	27.1	27.9
		2	3.9	1.6
		3	0	0
56 - 70	82	0	58.6	58.6
		1	40.2	40.2
		2	1.2	1.2
		3	0	0
71 - 85	94	0	44.7	45.7
		1	47.9	48.9
		2	7.4	5.3
		3	0	0
86 - 100+	74	0	39.2	35.2
		1	37.8	43.2
		2	20.2	18.9
		3	2.7	2.7
		4	0	0
Total No. of tests	759			

<sup>a</sup> Determined by AASHO Standard Method T 89-49

<sup>b</sup> Determined by the Washington State Highway Dept. Chart Method

<sup>c</sup> Determined by the Bureau of Public Roads Slide Rule Method

<sup>d</sup> Liquid limit values are reported to the nearest whole number, therefore, deviations from the standard values within the ranges of  $\pm(0$  to  $0.49)$ ,  $\pm(0.50$  to  $1.49)$ ,  $\pm(1.50$  to  $2.49)$ ,  $\pm(2.50$  to  $3.49)$ , and  $\pm(3.50$  to  $4.49)$  are grouped under the nearest values of 0, 1, 2, 3, and 4, respectively.

These values were compared with the liquid limit determined by the standard laboratory method and good agreement was found between the computed values obtained by this chart method and those determined by the standard test procedure.

Cooper and Johnson made a further simplification of the chart method by subdividing the original chart to show typical flow curves for each percentage of moisture so that any point selected from a single moisture content and corresponding number of blows for groove closure could be projected visually to the 25-blow line to obtain the liquid limit.

This improved chart, shown in Figure 2, was checked by Cooper and Johnson by using 84 duplicate tests of 73 soil samples. In each case, the liquid limit was determined by the "moisture-adjustment method" and by the chart method. This

comparison between the two methods of test showed that discrepancies ranged from 0.0 to 1.8 percentage points. However, because of the low frequency of errors in excess of 1.0 percent, they considered the accuracy of the chart method adequate for soil-classification purposes. They pointed out that two separate closures of the soil-water mixture should be observed to check its consistency before selecting a moisture sample for the determination of the liquid limit by this revised chart method. It was their opinion that the operator should, after completing the second closure, be sufficiently familiar with the flow characteristics of the material to detect any erroneous results.

Although this method is used in their routine identification and classification of soils, Cooper and Johnson recommend that it should not be used for the acceptance or rejection of materials which have borderline values of liquid limit or plasticity index. In these cases, they recommend the use of the standard mechanical method (AASHO Designation T 89-49).

The results of a statistical analysis of the basic data made by Cooper and Johnson in the preparation of their original and revised chart methods are shown in Table 1. These data show that the reduction from the 15-to-40 to the 17-to-36 range in the number of blows for groove closure increased the accuracy of the chart method. The maximum deviation in the revised chart method for the 17-to-36 blow range was two points. For 88 percent of the tests, the deviation was less than one as compared to 70 percent for the 15-to-40 range.

TABLE 3

VALUES OF THE DENOMINATOR OF EQUATION 1 CORRESPONDING TO THE NUMBER OF BLOWS REQUIRED FOR GROOVE CLOSURE IN THE LIQUID LIMIT TEST

S Blows	$1.419 - 0.3 \log S^a$	S Blows	$1.419 - 0.3 \log S^a$
15	1.066	28	0.985
16	1.059	29	0.980
17	1.050	30	0.976
18	1.043	31	0.972
19	1.036	32	0.968
20	1.029	33	0.964
21	1.023	34	0.960
22	1.017	35	0.956
23	1.011	36	0.952
24	1.005	37	0.948
25	1.000	38	0.945
26	0.995	39	0.942
27	0.990	40	0.939

<sup>a</sup> Value of the denominator of Equation 1.

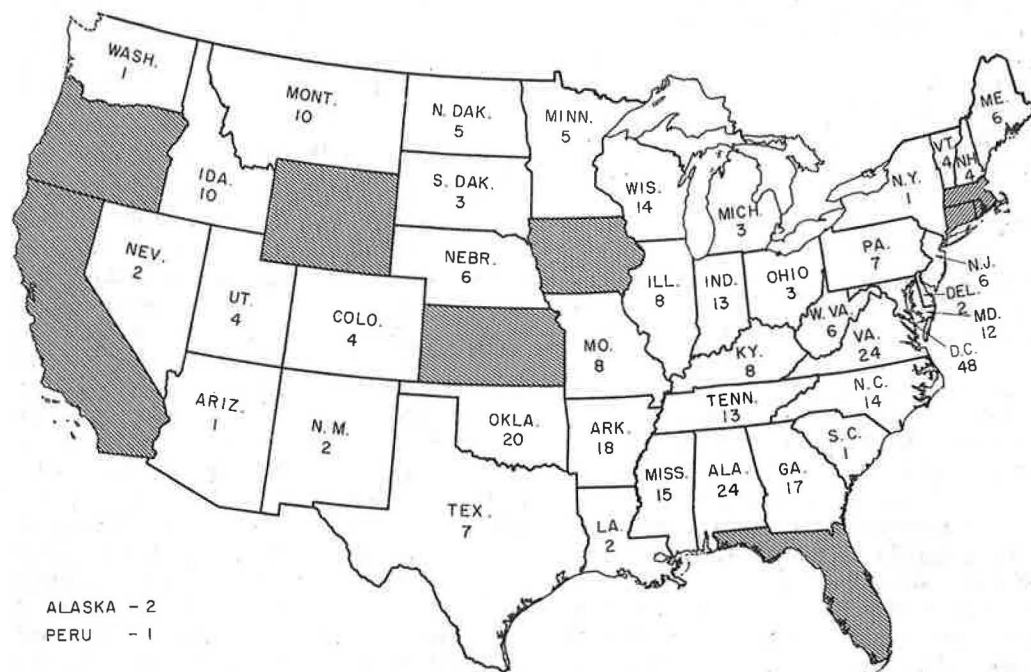
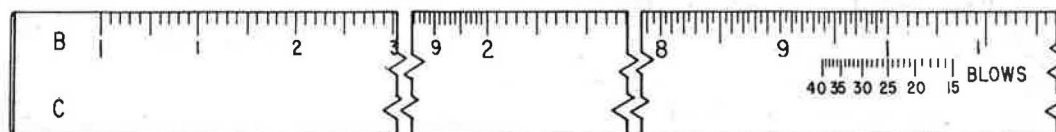
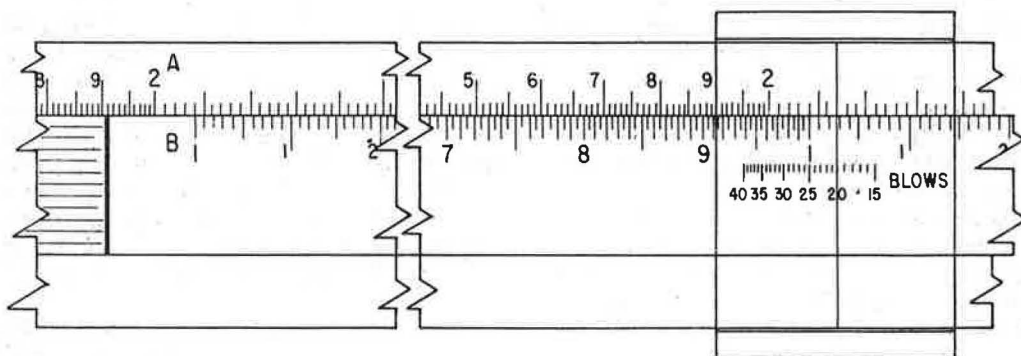


Figure 3. Map showing the number of soil samples from various areas used by the Bureau of Public Roads in testing the relation between the one-point liquid limit tests and the standard AASHTO method (Designation T 89-49).



A - LOCATION OF SPECIAL SCALE (BLOWS) WITH RESPECT TO B SCALE OF SLIDE RULE



B - SLIDE RULE SET FOR 21.4 PERCENT MOISTURE AT 20 BLOWS, INDICATING CALCULATED LIQUID LIMIT OF 20.8

Figure 4. Slide-rule with special scale for the calculation of liquid limit.



### CHART METHOD INVESTIGATED AND COMPARED WITH STANDARD METHOD

Since only soils from Washington had been studied by Cooper and Johnson, and the moisture-adjustment method was the yardstick of comparison, the Bureau of Public Roads widened the scope of their investigation by checking the new method against the standard method of liquid limit (AASHTO Designation T 89-49) for 364 soil samples selected to represent soils from all sections of the United States. Figure 3 shows the areas from which these samples were obtained. The soils selected have liquid limits ranging from 15 to 104.

Data for checking the accuracy of the chart method were taken from laboratory records and two points with the highest and lowest number of blows were selected from each standard flow curve for each of the 364 soil samples. The moisture contents corresponding to these two numbers of blows for these two points were used for calculating the liquid limit from the revised chart.

To compare the accuracy of these results with those obtained by the standard test method, the data were subdivided according to their liquid limit values into six groups. The first group was terminated at 25 since it is the maximum liquid limit allowed for base course materials by the standard specifications of AASHTO, and it is also the upper limit for A-1 soils in the AASHTO system of soil classification.<sup>6</sup>

The second group was terminated at 40, the maximum liquid limit for A-4 and A-6 soil groups in the AASHTO system of soil classification. The liquid limit values above 40 were arbitrarily subdivided into four 15-point ranges. A tabulation of the deviations of 759 liquid-limit values calculated by the chart method from those determined by the Standard AASHTO Machine Method is shown in Table 2. Since the groups do not contain the same number of tests, the frequencies of the deviations, Column 4, are expressed in the form of percentages to facilitate comparisons.

These data obtained for 364 soil samples tested by the Bureau of Public Roads show a range in deviations from the standard values of 0.0 to about 2.0. This range

<sup>6</sup>The Classification of Soils and Soil-Aggregate Mixtures for Highway Construction Purposes, AASHTO Designation M 145-49, Part I, Standard Specifications for Highway Materials and Methods of Sampling and Testing, 6th Edition 1950, published by the American Association of State Highway Officials.

of deviations compares quite favorably with the range 0 to 1.8 reported by Cooper and Johnson for test data obtained for 73 soil samples taken from Washington.

### SLIDE-RULE METHOD DEVELOPED

The preceding investigation of the accuracy of the chart method for calculation of the liquid limit suggested the possibility of eliminating the use of the chart by the development of a special scale which could be inscribed on a slide rule.

This was accomplished by deriving an equation for the family of typical curves used by Cooper and Johnson in the preparation of their chart. A study of the typical curves in Figure 1, indicated a point of convergence near the zero-moisture-content axis, and while it would be possible to derive an equation for such a family of curves, the problem was simplified by arbitrarily moving the point of convergence to the zero axis. This point is so far from the 25-blow line that the consequent changes in the slopes of the flow curves are negligible within the 17-to-36 blow range used. The formula derived for this new family of curves is:

$$LL = \frac{W}{1.419 - 0.3 \log S} \quad (1)$$

where

LL = liquid limit

S = number of blows (or shocks) required to close groove

W = percent moisture contained by the soil when S-blows close the groove

The use of this formula is facilitated by computing a table of the denominator ( $1.419 - 0.3 \log S$ ) for different values of S. The data in Table 3 show values of this denominator for values of S and the liquid limit for any moisture content at groove closure between 15 and 40 blows is obtained by dividing the percent moisture by the value of the denominator in Table 3 corresponding to the number of blows required to close the groove.

A more-convenient method for using the data in Table 3 is to inscribe a special scale in the blank space below the B scale of a slide rule to indicate the number of blows required for groove closure corresponding to the values of the denominator of Equation 1.

Figure 4a shows a section of a 20-inch

polyphase slide rule with this special scale inscribed for the number of blows between 15 and 40. A study of this special scale with respect to the normal B scale of the slide rule shows that it is constructed by making a mark in the blank section located below the B scale of the slide rule for each value of the denominator of

### COMPARISON OF THE SLIDE RULE AND CHART METHOD

The reproducibility of the slide-rule method was checked against the standard method for the same data that were used to check the revised chart method developed by Cooper and Johnson (see Table

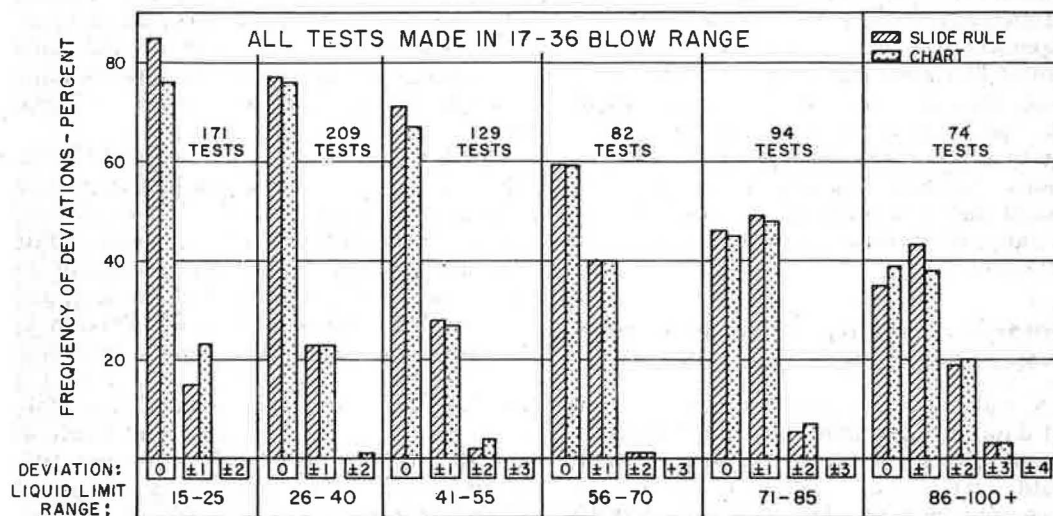


Figure 5. Percentage of chart and slide-rule tests deviating from the AASHTO standard liquid limit by 0,  $\pm 1$ ,  $\pm 2$  or more, in six ranges of liquid limit. (All tests made between 17 and 36 blows and liquid limit values reported in whole numbers).

Equation 1. For example, the 20-blow mark on the special scale corresponds to a B-scale reading of 1.029, the 25-blow mark to a value of 1.000 (the midpoint of the B scale) the 30-blow mark reading of 0.976, etc.

The calculation of the liquid limit by the slide-rule method is easily made by setting the indicator line over the percent of moisture found in the soilpat on the A scale of the slide rule and adjusting the special scale until the corresponding number of blows coincides with the indicator line. With this setting of the slide rule, the liquid limit is read on the A scale above the end index of the B scale, or on the A scale directly above the 25-blow mark on the special scale.

In the example, shown in Figure 4b, 20 blows were required to close the groove and the corresponding moisture content was 21.4 percent. The calculated liquid limit shown above the end index of the B scale, on the A scale, is 20.8.

2, Column 4). The percentage frequency for the same deviations, Column 3, is shown in Column 5, Table 2, so that comparisons can be made with the chart method. Figure 5 shows the comparison of the accuracy of the slide rule and the revised chart methods in the form of a bar graph for the ranges of liquid limits selected for this study. The number of tests deviating by  $\pm 0$ ,  $\pm 1$ , and  $\pm 2$  from the liquid limits determined by the standard method is expressed on a percentage basis to facilitate the comparison of the accuracy of the two methods.

It is shown by these comparisons that within the range of liquid limits most generally encountered, the chart and slide-rule methods may deviate from correct values by as much as 2 percentage points. Therefore it is concluded that dependence should not be placed on results obtained by the chart or slide-rule methods for the acceptance or rejection of materials when the liquid limit as determined by either of

these methods is within 2 percentage points of the specification limits.

The data in Figure 5 show that for the liquid limit range of 15 to 40, over 75 percent of the calculated values for both methods are within  $\pm 1$  point and practically all are within  $\pm 2$  points of the values determined by the standard test procedure.

However, for the acceptance or rejection of soil-aggregate materials based on a specification having a maximum liquid limit of 25, a deviation of  $\pm 0.5$  from that value, or 2 percent of 25, is the maximum that can be permitted. In order to provide this accuracy, the one-point method needed further improvement. It was thought that this could be accomplished if the range of blows for groove closure was decreased.

#### ACCURACY OF METHODS IMPROVED BY NARROWING THE RANGE IN BLOWS

A statistical analysis was made of the test data to determine the range in number of blows for groove closure which would give results within 2 percent of those obtained by the standard AASHTO method. The results of this analysis are shown in Table 4 and a study of these data indicates that the desired level of accuracy can be accomplished by narrowing the range of the acceptable number of blows for groove closure from a range from 17 to 36 to a range from 22 to 28.

Three ranges in number of blows for

groove closure 29 to 35, 22 to 28, and 17 to 21 are indicated in Column 1 of Table 4 to show the reproducibility obtained by chart and slide rule methods for six ranges in liquid limit, Column 2. The number of tests studied in each of these ranges is shown in Column 3.

To facilitate comparison of the accuracy of the methods for each of the three ranges in blows for groove closure, the number of cases having less than the indicated deviations is expressed on a percentage basis for each of the six ranges in liquid limit.

Within each of the ranges of liquid limit, the number of cases having less than each consecutive larger deviation is expressed on a cumulative percentage basis. For example, under the chart method for 29 to 35-blow range, Column 1 of Table 4, and a liquid limit range of 15 to 25, Column 2, the percentages of test results deviating less than 0.3, 0.5, 0.8, 1.1, 1.4, 1.7 and 2.0+ from the true values, (see Columns 4, 5, 6, 7, 8, 9 and 10 of Table 4) are 42, 74, 90, 100, 100, 100 and 100, respectively. The underlined values of 42 and 74 are the percentages of test results deviating less than 0.3 and 0.5, and are within 2 percent of the maximum and minimum values of liquid-limit range of 15 to 25. Similarly, other comparative values for any designated range in number of blows or liquid limits can be determined for either the chart or slide-rule methods.

A study of these comparative data in

TABLE 4  
COMPARISON OF ACCURACY OF CHART AND SLIDE RULE METHODS

Range of blows for LL tests (1)	Liquid limit ranges (2)	No. of tests (3)	Frequency deviations from std. test values in cum. percentages													
			Chart Method							Slide Rule Method						
			Deviations $\pm$							Deviations $\pm$						
			.3-	.5-	.8-	1.1-	1.4-	1.7-	2.0+	.3-	.5-	.8-	1.1-	1.4-	1.7-	2.0+
			(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
29-35	15-25	31	42	74	90	100	100	100	100	81	97	100	100	100	100	100
	26-40	37	51	84	95	100	100	100	100	55	87	100	100	100	100	100
	41-55	37	38	54	92	97	100	100	100	43	73	95	100	100	100	100
	56-70	22	32	41	86	96	96	100	100	23	50	85	100	100	100	100
	71-85	31	23	29	61	81	90	97	100	23	36	74	87	97	97	100
	86-100+	20	15	25	30	40	85	95	100	20	25	35	50	70	85	100
22-28	15-25	34	88	100	100	100	100	100	100	91	100	100	100	100	100	100
	26-40	27	85	100	100	100	100	100	100	93	100	100	100	100	100	100
	41-55	23	65	91	100	100	100	100	100	65	93	100	100	100	100	100
	56-70	13	62	92	100	100	100	100	100	69	92	100	100	100	100	100
	71-85	28	68	82	96	96	100	100	100	82	96	100	100	100	100	100
	86-100+	20	30	45	85	85	95	100	100	35	45	70	85	95	100	100
17-21	15-25	37	51	81	97	97	100	100	100	73	95	97	100	100	100	100
	26-40	43	56	77	100	100	100	100	100	58	84	98	100	100	100	100
	41-55	25	52	80	88	96	100	100	100	40	68	84	88	96	100	100
	56-70	22	36	50	73	91	100	100	100	36	50	68	82	91	100	100
	71-85	29	38	41	66	86	93	93	100	38	59	69	90	93	93	100
	86-100+	22	9	23	41	50	84	73	100	9	18	36	46	55	59	100

Note: Underlined values are the percentage of tests with deviations less than 2 percent of the minimum and maximum liquid limit range shown in Column 2.



Table 4 clearly shows that the greatest degree of accuracy for both methods is obtained in the 22-to-28-blow range and the least accuracy occurs in the 17-to-21 blow range. Within the 22-to-28 blow range there are no test values (see underlined results in Table 4) calculated by either the chart or the slide-rule methods which exceed the maximum deviations based upon 2 percent limit of error in the liquid-limit values determined by the standard AASHTO method. The only exception was the deviation of less than 0.3, which is 2 percent of the minimum value (15) in the 15-to-25 liquid-limit range, which shows 88 percent for the chart and 91 percent for the slide-rule methods. However, this is insignificant, since it is the usual practice to report the liquid-limit value to the nearest whole number. For this reason, the 22-to-28-blow range was selected as the limits to be used to obtain test results meeting the tolerance specified.

#### SUMMARY

Thus, on the basis of the data obtained in this investigation, both the chart and slide-rule methods should be acceptable

as alternates for the standard methods, provided: (1) procedure for the preparation of the soil pat for test be that prescribed for the standard method, (2) the acceptable number of blows for groove closure be within the 22-to-28 blow range, and (3) at least two consecutive closures be observed before selecting the moisture sample for calculation of the liquid limit.

Under these conditions, the results of the slide-rule method have been consistently checked with those obtained by the AASHTO Standard Method in the Bureau of Public Roads Soils laboratory for more than 2 years. There appears to be no difficulty for technicians in the adjustment of the consistency within the 22-to-28 blow range.

It is suggested that other laboratories make similar comparative check studies, using local soils, to determine whether the accuracy of this improved one-point method of calculation can be established for a range of conditions wider than those included in this study. If comparable accuracy is obtained by these proposed check tests, it would warrant the use of this method as a standard AASHTO and ASTM method.

#### Discussion

ROGER V. LeCLERC, Senior Materials Engineer, Washington Department of Highways — The one-shot method of liquid-limit determination, as described in the Washington Highway Department Laboratory Report 83, "A Rapid Method of Determining the Liquid Limit of Soils," by Cooper and Johnson, has been used in routine testing in our laboratory for approximately 4 years. It is not used as a basis of acceptance or rejection of materials, the mechanical method of ASTM Designation D423 being used in these cases. We find that the rapid method is quite acceptable for classification purposes and that it promotes an appreciable saving in time.

For quite some time it had been our contention that it would be possible to convert the chart developed by Cooper and Johnson into an equation which could be solved by means of a slide rule. The paper by Olmstead and Johnston has ably demonstrated that this is so and, also, that the one-shot method of liquid-limit determination is applicable to soils else-

where throughout the United States. The equation that they have developed to represent the relationship between flow curves and liquid limits and their suggestion for converting an ordinary slide rule into a liquid-limit slide rule are most commendable.

Some investigation along similar lines was begun in our laboratory in the spring of 1953, but the investigation was short-lived, due to the press of an extra-heavy work load and did not, therefore, proceed to any conclusion. The results of our minor investigation and comments are offered for whatever worth they may be in supplementing the work so ably presented by the authors.

The work done in our laboratory was primarily a re-analysis of data presented in our Laboratory Report 83, the report by Cooper and Johnson, in an attempt to find an equation for the chart. No new data were involved. Our first efforts indicated two possible approaches. The following equation was developed from

that approach which appeared more accurately to fit the data:

$$(LL - 10) = \frac{(W - 10)}{1.48 - .343 \log S} \quad (A)$$

The terminology is the same as given in the paper by Olmstead and Johnston. This equation would argue that the curves as presented in the chart tend to converge at the 10-percent-moisture-content line. It was at this point that our work had to be abandoned with only a cursory attempt to check the accuracy of the equation.

After receipt of the authors' paper, curiosity led us to follow through on the second possibility for an equation. We found that the equation which developed from this latter approach was identical with that developed by the authors. It is quite apparent that we were somewhat amiss in our evaluation of which approach would give the better results.

It might be pointed out that work of a similar nature on the liquid limit determination has been reported by the Corps of Engineers, U.S. Army, Waterways Experiment Station at Vicksburg, Mississippi, in their Technical Memorandum 3-286, entitled "Simplification of the Liquid Limit Test Procedure." They, too, developed an equation which was based on the fact that a logarithmic plot of moisture content versus number of blows in the liquid-limit test produced flow curves with approximately the same slope, at least within a limited range of blows. They stated that their equation, which is shown below, was considered strictly applicable only to those soils which they tested (inorganic clays from the Alluvial Valley of the Mississippi River and the East and West Gulf Coastal Plains).

$$LL = W (S/25)^{0.121} \quad (B)$$

The terminology, once again, is that used by the authors in their paper.

Analysis of the liquid limit data contained in the report by Cooper and Johnson in a manner similar to that used by the Army Engineers showed that their liquid limit chart may also be represented quite closely by the equation:

$$LL = W (S/25)^{0.130} \quad (C)$$

Within a 17-to-36 range in the number of blows, the above equation and that of the authors give close results. Both of these equations may easily be converted to slide-

rule form in the manner demonstrated by Olmstead and Johnston. We found that a 10-inch polyphase duplex slide rule with the special scale inscribed below the folded C scale (CF) works well. The C, D, CF, and DF scales are used and the accuracy should be comparable to that of the A and B scales on the 20-inch slide rule used by the authors.

A comparison of deviations in values of the liquid limit as determined by the chart method, by the authors' equation, and by Equations A and C given previously shows no significant difference in accuracy for a range of blows between 17 and 36 when applied to data on Washington soils. We would be curious to know if this same comparison of accuracy would prevail on a wider range of soils.

In conclusion, we feel that Olmstead and Johnston have contributed greatly to simplifying the routine task of liquid limit determination in soils testing. The data they have presented should provide the impetus for further substantiation of their conclusions and the eventual use of a rapid method as a standard for the determination of the liquid limit of soils.

W.J. EDEN, Division of Building Research, National Research Council, Ottawa, Canada — The authors are to be commended in their effort to summarize the results of liquid-limit determination of soils in the United States. Perhaps it would be of interest to add to the data presented, the results of 150 tests on two Canadian soils. The two soils in question are the "Leda" clay, a marine clay, which occurs in the vicinity of Ottawa, and varved clay occurring at Steep Rock Lake in northwestern Ontario.

The data were originally treated in the manner suggested by the U. S. Army Corps of Engineers, Waterways Experiment Station. Briefly, this method assumes that (1) flow lines plotted on logarithmic paper will be straight lines and (2) soils of the same geological origin will have a constant flow lines slope. With these assumptions, the flow line may be expressed by the equation:

$$L. L. = W_n \frac{N^{\tan B}}{25}$$

where:

N = no. of blows

W<sub>n</sub> = water content at N blows

TABLE A  
VALUES OF TAN B FOR VARIOUS SOILS

Soil Type and Location	No. of Tests	Tan B
Leda Clay — Ottawa, Canada	100	0.1003
Dark Laminae, varved clay — Steep Rock Lake, Ontario, Canada	31	0.1400
Light Laminae, varved clay — Steep Rock Lake, Ontario, Canada	19	0.0982
Average for Three Canadian Soils	150	0.1082
Alluvial and Coastal Soils — Southern USA (Reported by Vicksburg Waterways Exp. Station)	767	0.121
Soils from Various Locations in USA (Reported by Olmstead)	759	0.1351

TABLE B  
DATA WITHIN RANGE OF TAN B  $\pm 6$   
(STANDARD DEVIATION)

Soil Type	Tan B	Standard Deviation $\pm 6$	% of Total Observations Within Tan B $\pm 6$
Leda Clay	0.1003	.0516	82%
Dark Laminae — Varved Clay	0.1400	.0404	74%
Light Laminae — Varved Clay	0.0982	.0265	84%
Average	0.1082	.0497	78%

tan B = slope of the flow line when plotted on logarithmic paper

At any particular number of blows N, the equation may be expressed as  $L. L. = W_n \times K$ . This is a reciprocal relation to the equation expressed by the authors.

$$L. L. = \frac{W_n}{1.419 - 0.3 \log N} = \frac{W_n}{K}$$

Data for 100 tests on Leda clay, 31 tests on the dark laminae of varved clay, and 19 tests on the light laminae of varved clay were collected, and values of tan B shown in Table A were computed.

As can be seen from the values of tan B, the values obtained are close. In the writer's opinion, the geological origin of the soil need not be considered, because it was found in the analysis, that the variation in values of tan B for soils of any particular geological formation was greater than the variations shown in Table A.

To show the errors inherent in this method, take for example a soil with a water content of 50 percent at 20 blows. If the liquid limit is calculated using the various values of tan B shown in Table A, the results are as follows:

Tan B	L. L.
0.1003	48.9
0.1400	48.4
0.0982	48.9
0.1082	48.8
0.1210	48.7
0.1351	48.7

In conclusion, the writer suggests that the rapid methods for determining liquid limit are sufficiently accurate for all but special correlative purposes, certainly for classification purposes. Before a one-point method is adopted, it would be well as suggested by the authors, to make an effort to collect existing data in order to arrive at a truly representative value of tan B or other constant. In this regard, since the authors have already done so much toward this to date, the effort towards standardization of the method could well be sponsored by the Bureau of Public Roads.

F. R. OLMSTEAD and C. M. JOHNSTON, Closure — It is gratifying to note (from information on the three Canadian soils) that all six of the liquid-limit values Eden used to illustrate the inherent error of the method differ from one another by less than the tolerance we specified as the maximum limit consistent with the reproducibility of the test.

We have conducted many cooperative check tests with a large number of state highway laboratories using different soils at various times. The results of these tests have shown that experienced operators working in the same or different laboratories cannot be expected to check closer than  $\pm 2$  percent of the liquid limit of the soil involved, even when using the standard method. Of course, some operators check perfectly, but not every time; therefore, the 2-percent rule was selected to embrace as large a number of operators as possible while still maintaining safe classification of soils for highway use.

Based on this evidence, we feel that a search for a more-perfect series of slopes for the family of typical flow curves would serve no practical purpose, unless we find some soils that invariably fall outside the  $\pm 2$ -percent limit.

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