Roadside Design for Safety

K. A. STONEX
Assistant Director, General Motors Proving Ground, Milford, Mich.

A safety engineering study at the General Motors Proving Ground revealed that off-the-road accidents were the most prevalent type and concluded that this was the greatest potential hazard in the operation. Comparison with accident statistics and the physical characteristics of roadsides on public highways leads to the same conclusions.

Between 30 and 35 percent of highway fatalities occur in off-the-road accidents, year after year. An objective look at many public highway roadsides shows that they offer few safeguards in the event of vehicle malfunction or human malfunction; in the eyes of the industrial safety engineer, much of the roadside is deficient in this respect. The objective of this study is to develop criteria for roadside design which would remove these deficiencies.

The severity of operation through roadside and median ditches as a function of speed and cross-section detail is measured in terms of accelerations along the principal axes as a test car is driven through the ditch. Measured values are correlated with severity as gauged by the driver. Cross-section design criteria are developed such that the severity of accidents at legal road speeds can be kept within the tolerable range.

Guardrails are recognized as a feature which must be resorted to on occasions, and thus must be considered as a part of roadside design. Full-scale test results of guardrail installations, emphasizing modified end treatments, are given. The hazards of striking standard traffic signs and light poles are indicated and practical solutions are suggested.

Analysis and measurement of ground surface reaction "coefficient" relates passenger car stability factors to roadside slope values such that criteria can be developed for significant design factors. The comparative costs of roadsides designed for safety are developed for specific examples of roads passing through both level and wooded hilly terrain.

• THE SIGNIFICANCE of the roadside in the highway safety problem is apparent from the National Safety Council's statistics. Year after year, between 30 and 35 percent (12,000 to 13,000) of the highway fatalities in the United States occur in noncollision accidents, most of which involve the vehicles leaving the roadway.

This factor is second only to the two-vehicle collision as the most important and deadly. A great deal of engineering design effort is being devoted constructively to reduction and elimination of the two-car collision. The divided highways of the Interstate System, the turnpikes and the expressways, grade separations, one-way streets, and signal installations and stop signs on intersections at grade are all attempts to eliminate this type of accident. Intensive driver training, public educational and enforcement programs are corresponding efforts in other fields.

In the policies on geometric design of the Interstate System, recognition is given to the importance of roadside hazards by the adoption of enlightened standards of roadside design with respect to slope, ditch cross-sections, and elimination of obstacles. In reconstruction and modernization of existing roads, some attention is given to this problem, but one cannot help but feel that the minimum standards are applied all too frequently in construction of portions

of the Interstate System; one cannot help but feel that too little emphasis is given to the elimination of roadside hazards on primary and secondary and rural roads, even when they are being reconstructed.

It is the intention of this paper to discuss the problem in general and to give a number of specific examples of roadside hazards drawn in large part from experience with the Proving Ground road system. It is the further intention to discuss some research results from which specific design criteria for roadside slopes and ditches are derived. The stability factors of passenger cars are related to observed roadside characteristics, especially with respect to the slope of the roadsides and the coefficient of friction of the roadside surfaces. Some discussion is given of the hazards involved in conventional light poles and sign posts, with suggestions as to how this type of hazard may be reduced or eliminated. In addition, further observations on full-scale guardrail impact tests are given to supplement earlier publications on this subject (*1*). Particular emphasis is given to guardrail end installations.

Some of the examples of roadside hazards have been discussed previously; they are repeated here as background and to give appropriate emphasis to the problem (*2*, *3*).

The problems of safe roadside design have become of great importance to the management of General Motors Proving Ground, with the increasing emphasis placed on the safety of the employee.

The Proving Ground was established in 1925 as an outdoor road test laboratory when a private road system was built for development work on the company's products. From a small operation it was expanded rapidly as required, and facilities have kept pace with the increasing complexity of automotive design.

Today the road system includes about 62 miles of all types of surfaces common to those of the public highway; portions of this system with the highest traffic volume were completed by the mid-1930's. During the past 35 years, more than 200 million test miles have been operated. The current rate is about 40,000 mi per day, or 10 million miles per year.

At the Proving Ground the normal standards of shop safety have always been employed. Originally the attitude about industrial accidents, quite generally, was that they were bound to happen, and one had to learn to live with them. The industrial safety engineer has shown that that is not so. He has shown that accidents are preventable, and that accidents in the plant usually come about because of some human malfunction. Recognizing that, while accidents are preventable, some will not be prevented, he provides all safeguards he can imagine for all types of carelessness and inattentiveness which people may show so that the effect of an accident may be minimized. When, a few short years ago, the concepts of the industrial safety engineer were applied in consideration of the Proving Ground safety program, it became evident that the most serious potential hazards lie in the operation of the vehicles on the road system, because both the masses and the speeds of cars exceed those of any of the other machines being used.

Generally speaking, the Proving Ground road system was quite well developed by 1940. In construction, the design standards which prevailed generally at the time of construction were used, and the practices used were comparable with those of the state highway departments. The basic elements of highway safety were considered with admirable foresight in the design of the road system and one-way traffic, limited access, and avoidance of intersections of main test roads at grade has always been the policy. Thus, the basic design of the system avoided many of the types of accident which are now causing such great concern in the public transportation system. At the time, this approach was unique in its enlightenment and progressiveness.

As the accident statistics were reviewed it was found that during the calendar years 1953–1958, inclusive, covering approximately 65 million test

miles, there was a total of 236 accidents, of which 72 percent were off-the-road.

The first and most obvious concern was to determine the reasons why drivers left the road. In many cases the driver went to sleep. In other cases he was obviously inattentive. Educational programs were undertaken, reprimands were given, and in the more flagrant cases drivers were discharged.

In spite of all of this, it became evident that drivers do leave the road—infrequently it is true, but all too often simply because they are people and suffer normal human fallibility.

A further consideration led to the conclusion that it was not possible to keep all drivers on the paved surface all the time. One of the fundamental principles of safety engineering is to anticipate every possible type of accident which may occur because of machine failure or human failure and then to establish safeguards to minimize the hazards or injury which may result when such a failure occurs. When the road system was analyzed from the same point of view, it was disturbing to find that the design standards provided little or no safeguard in the event of a failure of some type. The originators had pioneered in safety engineering by taking fundamental steps to avoid accidents, but they did not apply the second concept of the industrial safety engineer—to provide all safeguards in the event that an accident occurs because of human fallibility.

When it was realized that the roadside design did not incorporate the safety features common to the machine shop, garage and maintenance tools, an attempt was made to make amends at the earliest possible time by using the experience of others. Immediate comparisons were made with public highways in the adjacent area: it was found that the public highway system repeated most of the shortcomings of Proving Ground road system. Highway construction was observed in other states, at the turnpikes, even the newest components of the Interstate System, and even there it was found that there is a lack of safeguards which would not be tolerated in any modern industrial operation. Almost every mile on any of these roads has one or more places where the occupants would suffer serious or fatal injury if the vehicle left the road at normal highway speeds.

An approach to the problem of safe roadside design from the attitude of the industrial safety engineer became the primary interest in providing the proper safeguards to employees operating on the Proving Ground road system. Every roadway accident includes as factors the driver, the vehicle, and the highway. These factors were considered properly in the approach to the problem.

The Proving Ground drivers are adult males in good health, selected on the basis of average or above competence, and on the characteristics of desirability in an employee. They are all qualified drivers with several years of experience in normal highway driving and it is fair to assume that they compare with the upper strata of drivers in the traffic stream on the basis of these characteristics. They drive on a closed road system with favorable geometry, relatively low traffic volumes, controlled access, one-way operation, and under relatively close supervision. Extensive training and educational programs have been conducted. The possibility of more effective ways of improving driver performance has not been overlooked—the obvious solutions have been exhausted; some which are not obvious have been tried. In spite of this, during 1953–8, inclusive, there were 170 off-the-road accidents.

Scrupulous attention is given to safety in design and manufacture, and current automobiles well maintained are practically free from failure in service.

Confidence in the quality of the vehicles and experience with the fallibility of the drivers resulted in the firm conviction that the major deficiency in the effort to eliminate accidents was failure to devote sufficient attention to the road itself. In this case, the concept of the original traffic pattern practically eliminated two-vehicle collisions and confined the problem almost entirely to the roadside.

Figure 1. Proving Ground car-tree accident.

Figure 2. 35-Mph car-tree impact under remote control.

Figure 3. Trees adjacent to modern road.

COMMON ROADSIDE HAZARDS

When one adopts the safety engineer's attitude and concepts, the most dangerous situation perceived immediately is that of obstacles adjacent to the road; that is, close enough to the traveled surface that the driver who has lost control of his vehicle temporarily will strike the obstacle before he has an opportunity to regain control. On the Proving Ground road system, and on nearby public highways, the most obvious obstacles are trees. Trees are desirable and beautiful; in the early days of

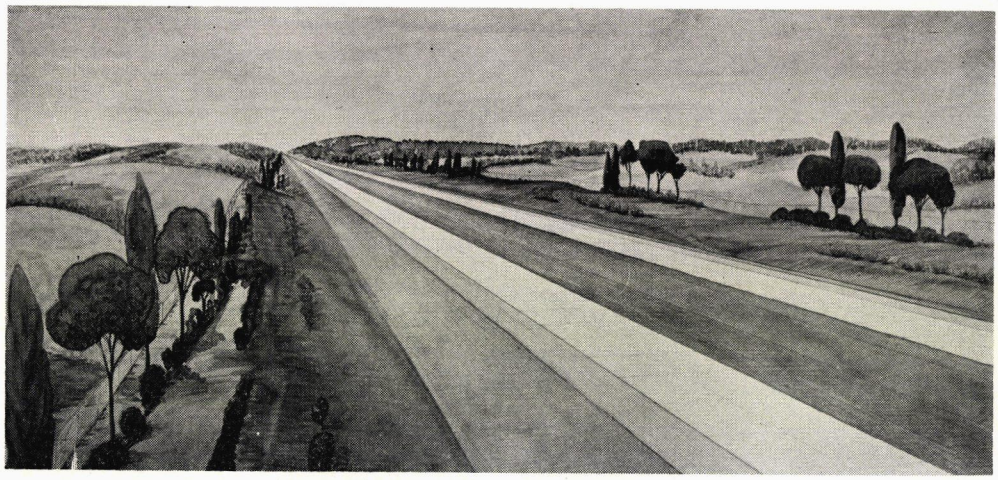

Figure 4. Artist's concept of safe roadside.

Figure 5. Man-made roadside obstacle.

the Proving Ground the alignment frequently was modified so that a large beautiful oak might be saved. However, a review of any newspaper shows that trees contribute almost every day to the statistics of injury and death.

Trees have played a part in the more serious Proving Ground accidents (Fig. 1). The severity of an impact at normal speeds is indicated in Figure 2. This car was driven into the tree at 35 mph by remote control; it was damaged seriously and it was evident to anyone who witnessed the accident that occupants of the car would have been seriously injured and probably killed. To the safety

Figure 6. Roadside sign.

engineer, the immediate conclusion is that trees close to the roadside must be eliminated systematically. Fortunately, with modern equipment this is practical and relatively inexpensive. After the trees are removed and the roadside is smoothed with a grader, no possibility of serious accident in this area remains.

This situation has its direct counterpart in the public highway system; on some major highways of relatively modern design there are trees so close to the road as to be lethal obstacles (Fig. 3) and, in fact, on some of the newest highways, small trees are being planted as part of the landscaping program. As a result, in all too few years, the small trees will grow into big ones—significant hazards being cultivated immediately adjacent to the edge of the road.

Beauty is possible without large trees, or with trees well in the background; in exceptional cases, well designed guardrail installations may be used if speed is well regulated. Figure 4 shows how shrubbery can be used.

On many city streets, and even some rural highways, there are man-made obstacles in the form of utility poles and light poles immediately adjacent to the paved surface (Fig. 5) and signs to guide the traveler (Fig. 6).

Figure 7. Comparison of low-impact and conventional light poles.

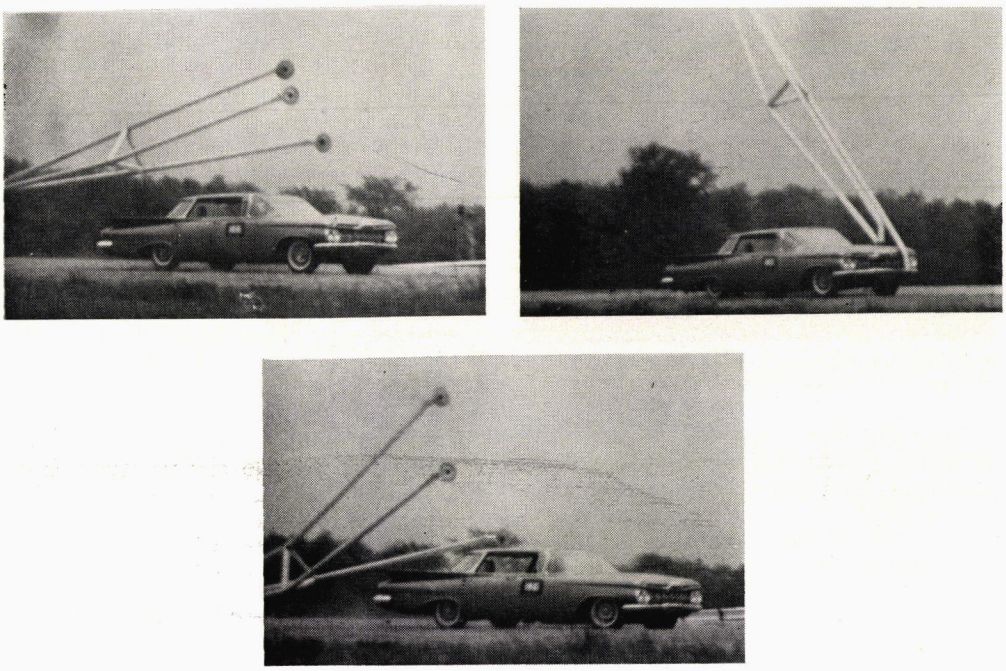

Figure 8. Full-scale test of low-impact pole.

Figure 9. Damage from collision at 40 mph with traffic sign mounted at 42 in.; car runs into sign detached from post by impact.

Figure 10. Full-scale test at 40 mph; traffic sign mounted 60 in. above pavement allows car to run beneath harmlessly.

Conventional light poles are self-supporting structures normally erected on a concrete base and designed to withstand winds of hurricane velocity. Unfortunately, they become roadside obstacles of important dimensions. As a compromise, a tripod structure constructed of light tubular material with shear mounts flush with the base has been proposed. Figure 7 shows such a pole in contrast with a conventional pole in a parking lot.

This was evaluated in a full-scale test. Figure 8 shows three frames from the motion picture record of this test. The car passed through beneath the pole with negligible impact and only superficial damage; one leg, whipping during the collision, would have injured occupants of a convertible, but the design can be modified to control this.

Standard roadside signs, mounted at 42 in. above the pavement, are also a hazard. Figure 9 shows that in a collision at 40 mph a sign of this type pierces the windshield partially and showers the front passenger compartment with glass. At higher speeds, the sign would not drop so far and occupants would find themselves running into a 25-lb sign at whatever speed the car was traveling.

When the sign is mounted at 60 in., the car passes beneath harmlessly (Fig. 10). Road signs at the Proving Ground are being relocated at 66 in.

DITCHES

In all parts of the United States it is necessary to provide some type of drainage system along the road to carry

Figure 11. Typical ditch on secondary road.

Figure 12. Sharp V-ditch on modern superhighway.

off surface water. These ditches are effective for carrying off the water, but they may present a serious hazard.

Figure 11 shows a typical situation on many miles of rural road; it does not take a safety engineer to recognize the seriousness of the inevitable accident when some driver becomes inattentive or falls asleep and leaves the road at this point.

Unfortunately, these practices are carried over to new roads (Fig. 12); sharp V-ditches are still being graded almost immediately adjacent to the traveled path of roads being built according to the Interstate System standards.

In some cases, careless inspection procedures or lack of detail in construction leaves a mound unnecessarily (Fig.

Figure 13. Abrupt unnecessary roadside mound.

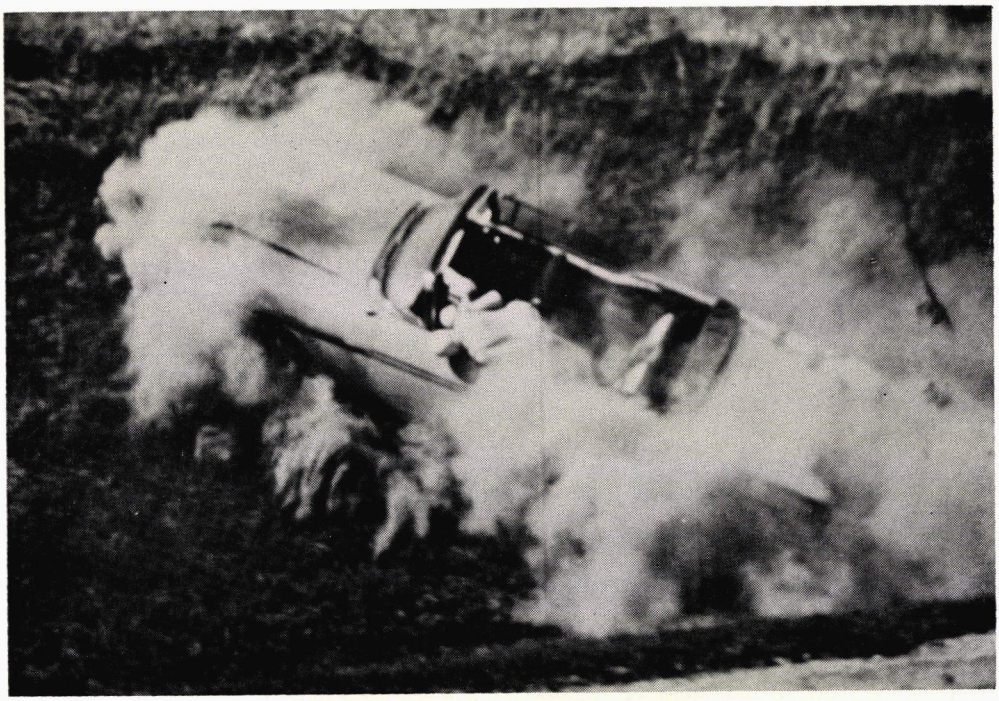

Figure 14. Impact test, ditch with 2:1 backslope.

13). On rural roads, additional right-of-way must be procured or agreements with the abutting landowners made to abolish such ditches and banks. However, on portions of a modern road system where adequate right-of-way is available, such construction is intolerable to the safety engineer.

The severity of ditch impacts is indicated by Figure 14, which shows a remotely controlled car driven off a road through a ditch with a 2:1 backslope. The car was severely damaged and it is evident that the occupants would have suffered serious injury, at least.

Figure 15 shows a much milder degree

Figure 15. Impact test, ditch with 4:1 backslope.

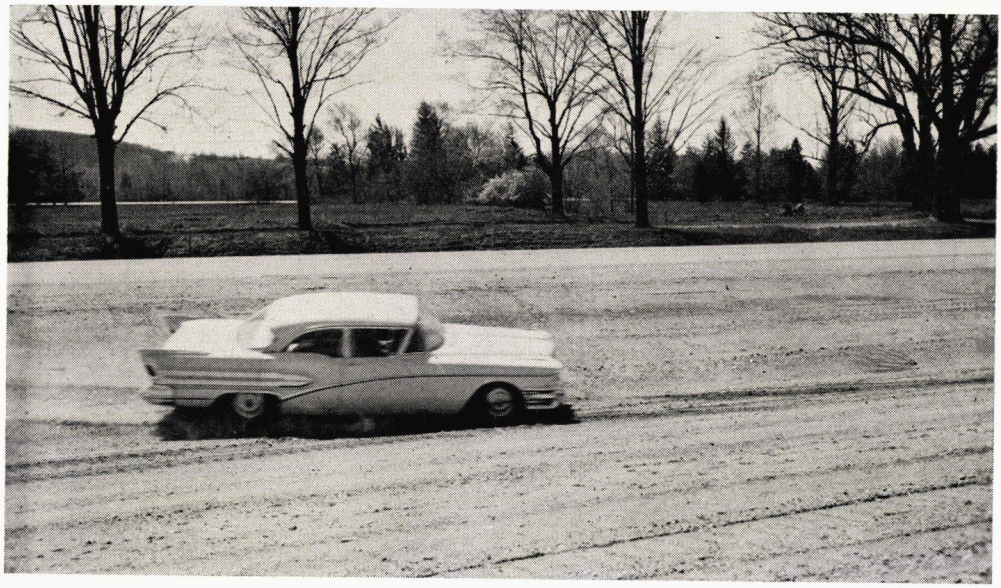

Figure 16. 60-Mph test, driving through flat-bottom ditch.

of severity when the backslope is 4:1. In this case the car climbed the bank with a rather minor impact and the injury and damage would have been negligible.

The desirability of ditches with flat bottoms and smooth contours and flat slopes has been discussed before, but no design criteria have been given. Some preliminary experimentation showed that a car could be driven through a flat ditch with a wide rounded bottom at 60 mph with ease and comfort (Fig. 16).

DITCH TESTS

In the foregoing test it was not clear whether the value of the slope and backslope, the width of the bottom ditch, or the depth of the ditch contributed most significantly to the severity of operation.

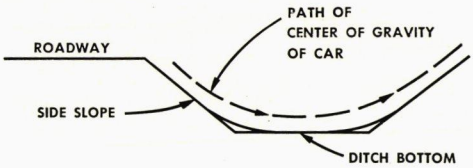

Figure 17. Schematic path of car center of gravity through ditch.

A series of tests was run to evaluate the severity of crossing a ditch as a function of speed and ditch cross-section elements. In the initial series of tests, a ditch was dug in conformity with the Michigan State Highway Department standards for a median ditch on a divided road. Cars were driven through the ditch at moderate speeds; the driver noted the subjective severity as speed was increased and measurements of "vertical" accelerations were made so that the numerical values could be correlated with the driver's sensations up to the point where the operation became so severe it was unsafe.

Intuition suggests that a ditch cross-section should be of some curved form to minimize impact; as the suspension system deflects under impact, the unsprung mass of the car will follow a curved path. If the transition from the side slope to the bottom of the ditch is gentle enough that the bumper does not dig in, the unsprung weight and the sprung mass of the car should have a continuous curvilinear motion. The simplest to consider would be a circular motion as indicated schematically in Figure 17.

If the ditch cross-section is circular

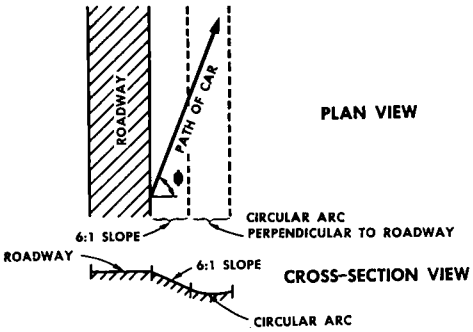

Figure 18. Plan and section, ditch test.

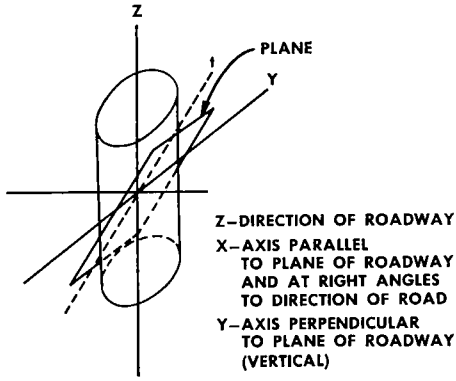

Figure 19. Projection of circular cross-section on car path through ditch.

with radius r, the projection on the path at which the car may run through the ditch becomes elliptical in form; the path will make some angle with the axis of the road, possibly up to 20° or more. The projection of a circular cross-section on a path at angle of $90^\circ - \phi$ from the axis of the road has the form:

$$\frac{t^2}{r^2/\cos^2\phi} + \frac{y^2}{r^2} = 1 \qquad (1)$$

which is an ellipse in the y-t plane, with major axis of $r/\cos\phi$ and minor axis of r, where $90^\circ - \phi$ = angle between path of car and axis of ditch. The derivation is given in the Appendix and Figures 18 and 19 indicate how this projection is made.

With a given ditch cross-section, the radius of curvature may be estimated graphically, and with the speed arbitrary, the value of the radial acceleration can be computed.

To verify the analysis and to develop

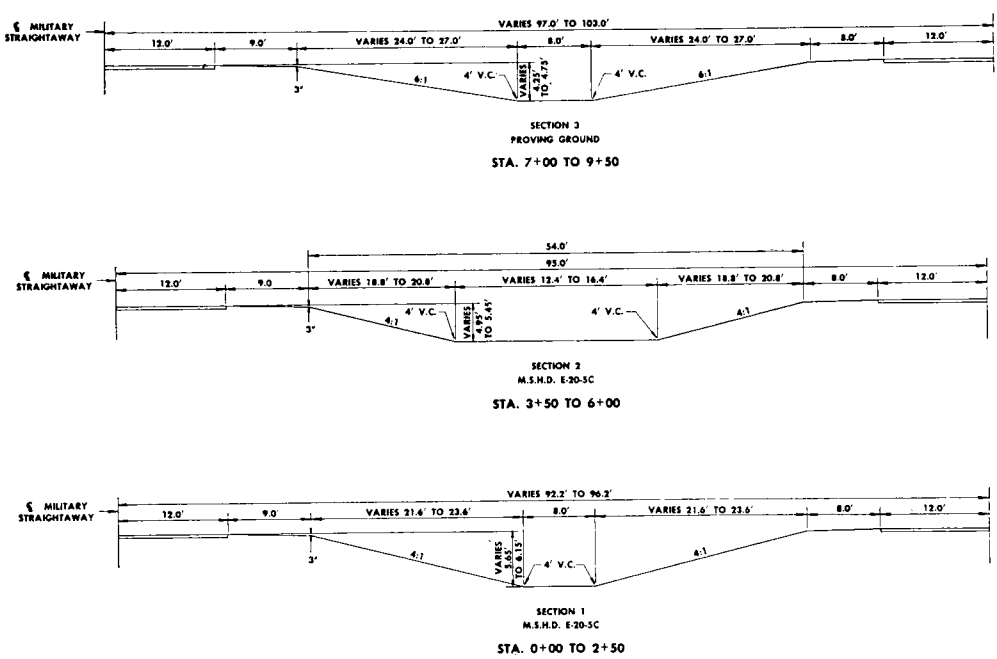

Figure 20. Test ditch cross-sections.

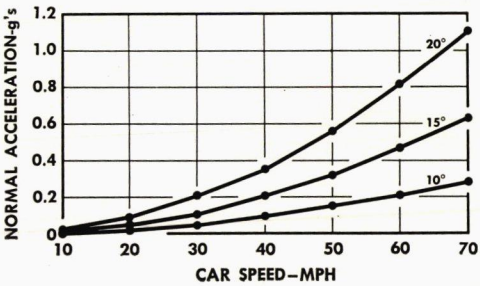

Figure 21. Normal ("vertical") acceleration computed from test section design.

values of radial acceleration or severity which could be tolerated, three test ditch sections were constructed as shown in Figure 20. Two sections with a 4:1 slope are taken from the Michigan State Highway Department standards for a median ditch; these have ditch slopes of 4:1 with varying width of the bottom and varying depth to provide longitudinal drainage. Section 3 has the slope and backslope of 6:1 and the depth varying from 4¼ to 4¾ ft, with an 8-ft wide bottom.

Typical values of the normal or vertical accelerations were computed from the test sections of 6:1 slope (Fig. 21).

The tests were conducted by laying out angles between the car path and axis of the ditch of 10°, 15°, and 20° and by driving the car through the ditch at increasing increments of speed. During each test, recordings were made of the normal acceleration (that is, the radial or vertical acceleration) and the driver's opinion of the severity was noted. Tests were conducted up to the point of extreme discomfort, and an estimate was made of the tolerable value of normal acceleration. It should be noted here that considerable training was involved and the test engineer developed a considerable resistance to this type of operation. This is an experience of considerable severity at the higher speeds, and it may be anticipated that the unwary driver will suffer severe psychological damage before he suffers physical injury. Because of his natural alarm, he is apt to lose control of the car and precipitate an even more serious accident. Figures 22 and 23 show typical test scenes.

After the practical limit of driver tolerances had been reached, test cars equipped with remote control devices were operated in a limited series of tests to determine, if possible, the severity at which structural damage began to ap-

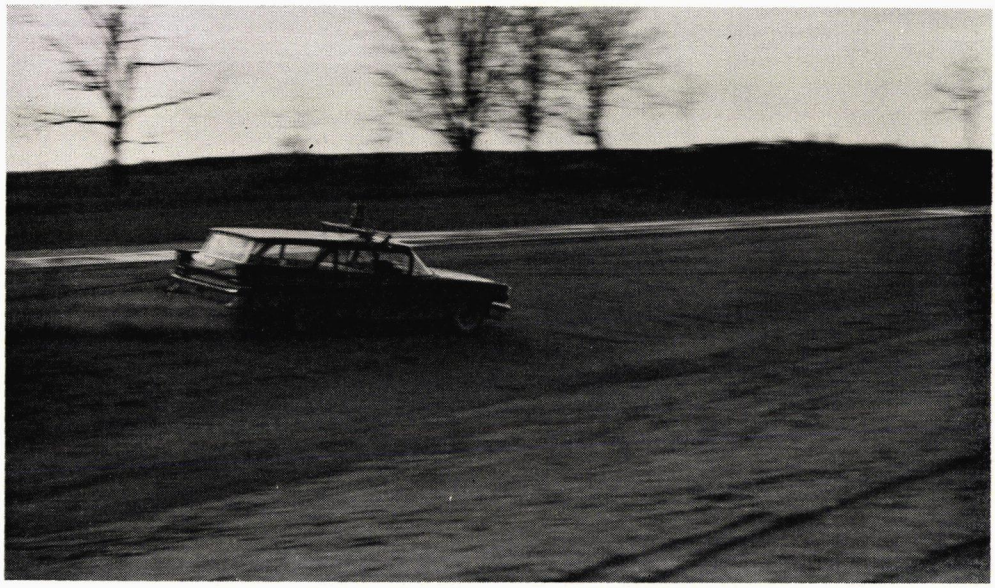

Figure 22. Car entering ditch on test.

Figure 23. Car passing through ditch; bumper strikes ground here.

pear. It was intended to continue the tests up to the point where it was assumed that serious injury would result to the passengers. A limited number of tests was made; these are not considered significant, because there should be no serious interest in a ditch section where the severity is beyond the driver's tolerance.

The test data consisted of the values of acceleration measured by a transducer carried on the car such that it measured the accelerations approximately normal to the longitudinal axis of the car and recorded them on an oscillograph; car speed was recorded simultaneously.

Figure 24 is a typical oscillogram from these tests. The upper trace indicates car speed; in this test it was 51 mph. The second trace from the top shows an indication of the time during which the car was passing through the ditch as noted by the driver; this is indicated by the slightly elevated portion of the second line from the top extending across the middle two-thirds of the chart.

The bottom trace shows the acceleration recorded; as the car enters the ditch, the acceleration is slightly above the zero line, or negative, probably from the effect of going over the vertical curve. The acceleration increases fairly rapidly and during the most severe portion of the passage through the ditch it has a fairly high level, which persists for approximately 0.2 sec. The mean value indicated during this portion of the passage through the ditch was approximately 0.75 *g*. The value calculated by assuming the reasonable path of curvature from the projected cross-section of the ditch was 0.32 *g*.

This relationship between the calculated value and the typical value measured is representative of the conditions found in most of these tests. This chart indicates that, during such tests, observed values were in the range of those calculated, but values also were observed persisting for an appreciable length of time which are considerably higher than those calculated; in the severe cases, of which this is an example, the observed values are nearly twice the calculated values.

This is easy to understand; the suspension system of an automobile will "bottom" under values of vertical acceleration which are relatively mild in the framework of reference of these tests. When the suspension system bottoms, vertical forces are transmitted through the rubber bumpers, which have a much

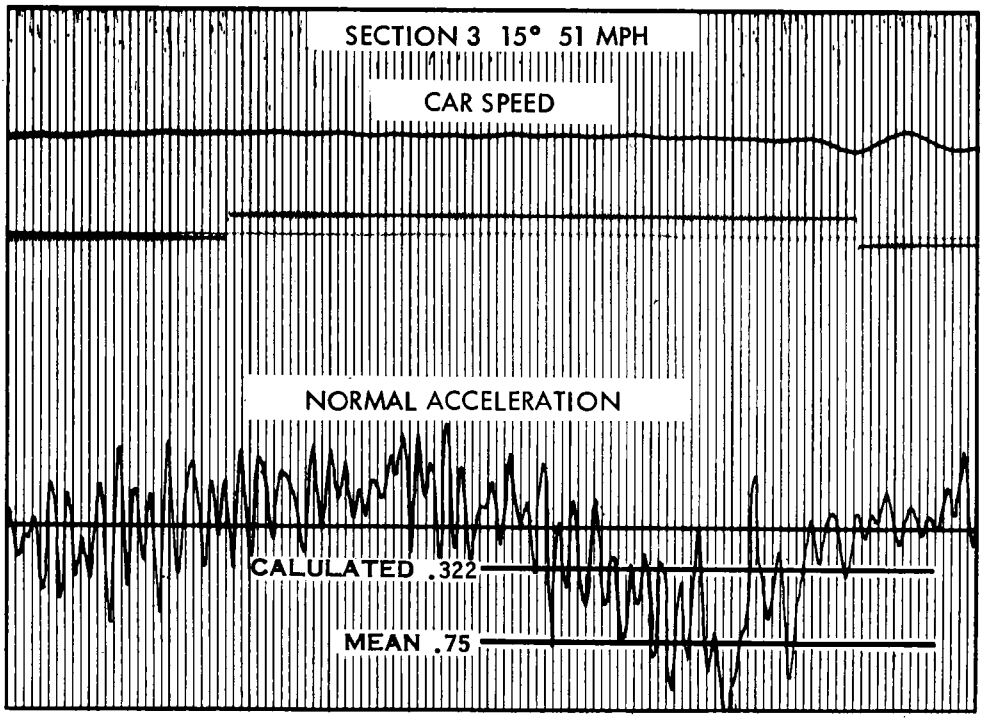

Figure 24. Oscillogram showing ditch test results.

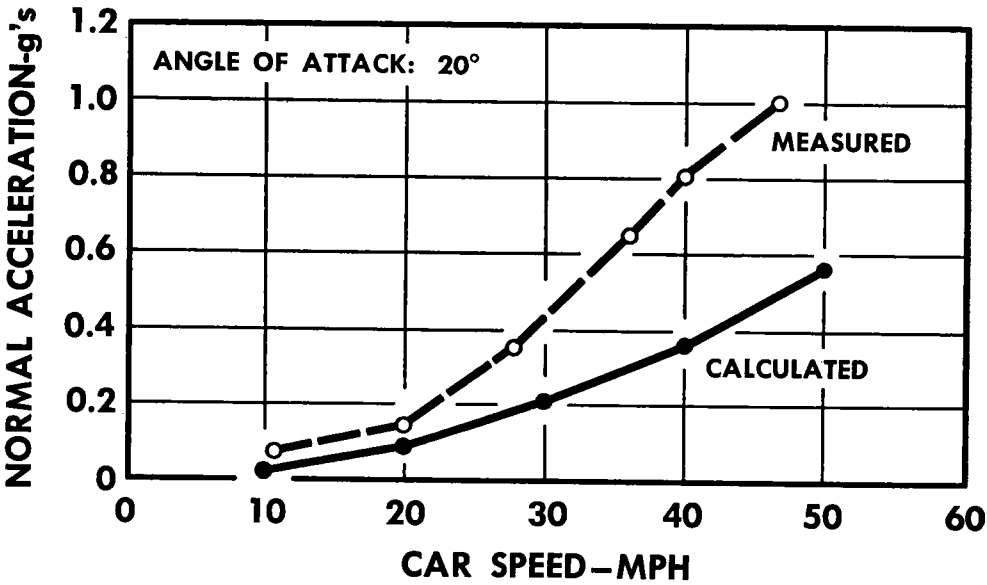

Figure 25. Comparison of calculated and measured accelerations; ditch section 3, slope 6:1, 8-ft bottom.

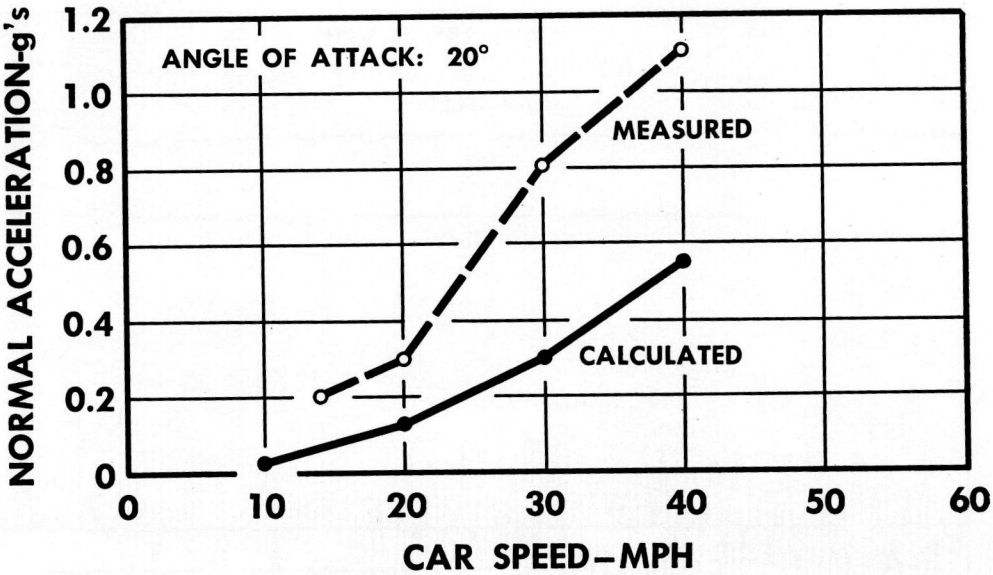

Figure 26. Comparison of calculated and measured accelerations; ditch section 1, slope 4:1, 8-ft bottom.

higher rate than the car springs and a rate which may increase considerably as the deflection is increased; consequently, an impact severe enough to bottom the suspension system will introduce non-linearities for which no provision is made in the calculations and, indeed, which would be difficult or impossible to compute directly by simple means.

Figures 25 and 26 show the comparison of calculated and measured values on ditch sections 3 and 1, respectively, as a function of speed for a 20° angle of attack.

It will be noted that at the relatively mild conditions at 10 mph there is reasonably close agreement between the observed and calculated values but that the difference between them increases rapidly as the speed and, consequently, severity of the test increases. It may be noted also that the values of both the computed and measured decelerations increase as the angle of attack increases; thus, for a 20° angle of attack at 50 mph where the computed value is approximately 0.56 *g*, the observed value is approximately 1.1 *g* on the tests on ditch section 3 (Fig. 25).

Somewhat comparable results are shown in Figure 26 in the tests on ditch section 1; the differences between the observed and computed values are even greater. The observed severity of tests was somewhat greater on ditch section 1 and the limiting speed was approximately 40 mph, compared with approximately 50 mph on ditch section 3.

The values (Table 1) indicate that the observed severity is approximately twice that of the calculated value under the more severe conditions.

Thus it is apparent that none of the three ditch sections tested would be acceptable for a primary road or a road where speeds above 50 mph might be anticipated.

The ditch section shown under test in Figure 16 was re-evaluated more carefully to provide a better estimate of its capacity by using a remotely controlled car at elevated speeds. The cross-section is shown in Figure 27.

Driving experience at moderate speeds indicates that the severity of impact begins to become uncomfortable at approximately the point where the suspension bottoms; it approaches the intolerable level when the bumper strikes the ground. At higher speeds, it would be expected that even minor contact

TABLE 1

CALCULATED AND MEASURED VALUES OF g FOR VARIOUS CROSS-SECTIONS, ANGLES OF ATTACK, AND SPEEDS

Angle of Attack, deg	Value of g														
	10 MPH			20 MPH			30 MPH			40 MPH			50 MPH		
	Meas.	Calc.	%[a]	Meas.	Calc.	%[a]	Meas.	Calc.	%[a]	Meas.	Calc.	%[a]	Meas.	Calc.	%[a]
							(*a*) SECTION 1								
10	0.09	0.009	10	0.14	0.036	26	0.25	0.081	32	0.39	0.143	37	—	—	—
15	0.07	0.020	29	0.15	0.079	53	0.36	0.178	50	0.54	0.316	59	—	—	—
20	0.14	0.034	24	0.30	0.137	46	0.80	0.309	39	1.10	0.548	50	—	—	—
							(*b*) SECTION 3								
10	0.05	0.006	12	0.10	0.023	23	0.16	0.052	32	0.19	0.096	50	0.25	0.145	58
15	0.05	0.013	26	0.14	0.052	37	0.30	0.116	39	0.39	0.205	53	0.71	0.322	45
20	0.07	0.025	28	0.15	0.100	67	0.42	0.224	53	0.80	0.398	50	1.10	0.645	59

[a] $\frac{\text{Calculated}}{\text{Measured}} \times 100.$

with the ground would be injury-producing; therefore, this is a condition which the ditch design should avoid. There is some evidence that suspension systems will bottom heavily under normal vertical accelerations in the order of 0.5 g, which appears to be in the range in which the calculated severity of operation is a first approximation to the average values observed.

Design criteria might therefore be based on the development of ditch cross-sections which, when projected at reasonable angles of attack, would yield path profiles such that the curvature of the path of the center of gravity could

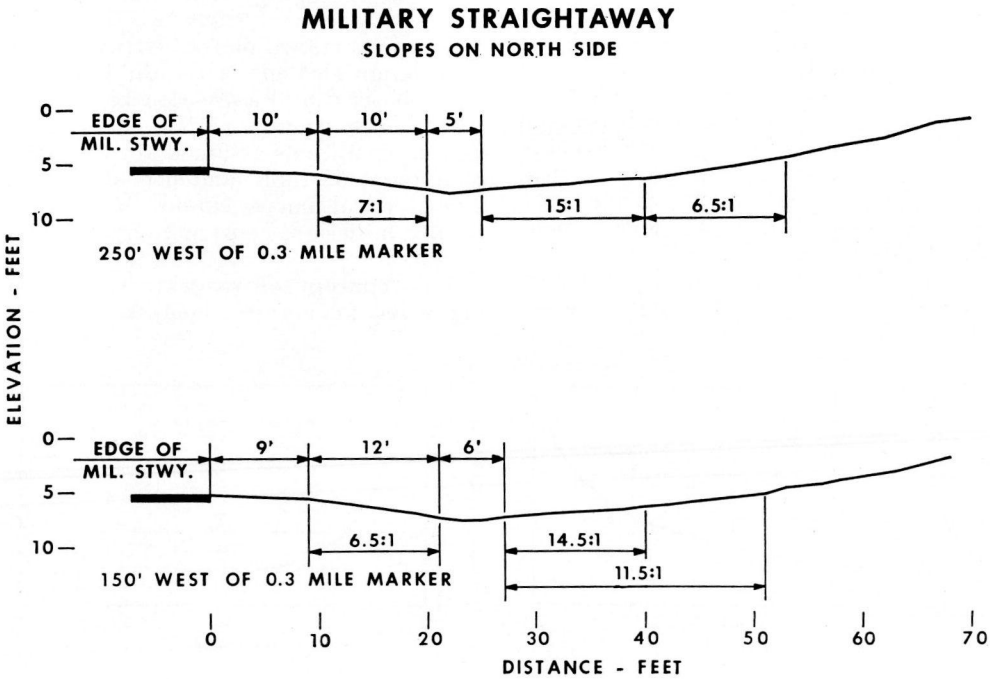

Figure 27. Test at 75 mph in first experimental ditch.

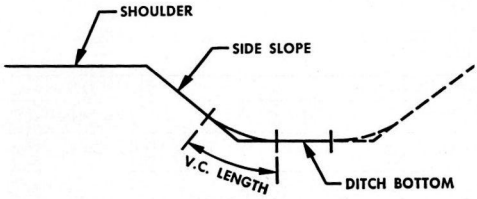

Figure 28. Elements considered in the development of ditch design criteria.

be estimated reasonably accurately and first-order approximation to vertical acceleration computed. Conservative design criteria should provide that calculated values of vertical accelerations should not exceed 0.5*g* for a car passing through a ditch at an angle of 15° under the anticipated speeds of operation. This would assure reasonably comfortable operation at the design speed and provide a slight margin of safety for the driver who may have been unwise enough to exceed the design speed and unfortunate enough to leave the paved surface at 15° or even some greater angle.

The most important element of the ditch section design in controlling the severity is the length of the vertical curve between the side slopes and the ditch bottom. Obviously the radius of curvature is the controlling element. For design purposes, however, it is much simpler to use a circular vertical curve and employ criteria based on vertical curve length. Figure 28 defines the elements considered in the development of ditch design criteria.

Figure 29 shows the relation between vertical curve length and the ditch slope

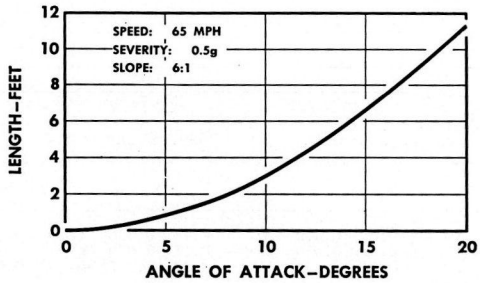

Figure 30. Vertical curve length as a function of angle of attack for severity of 0.5g on a slope of 6:1.

for the arbitrary conditions of the speed of 65 mph and angle of attack of 15°, which will provide a severity or normal acceleration of 0.5*g*. To be noted is the rapid increase in required length of vertical curve as the slope increases.

Figure 30 shows the influence of the angle of attack on vertical curve length for the arbitrary conditions of speed of 65 mph and a ditch with a slope of 6:1 which will produce a severity or minimum acceleration of 0.5*g*. From this it will be noted that the severity increases much more rapidly than the angle of attack.

The importance of the vertical curve as a design element of the ditch section is emphasized by Figure 31, which shows variation in severity with vertical curve length; conditions assumed are 15° angle of impact, 65 mph, and 6:1 side slope. The vertical curve cannot be left to chance in design, construction, or maintenance.

This concept of roadside safety for highways at current operating speeds

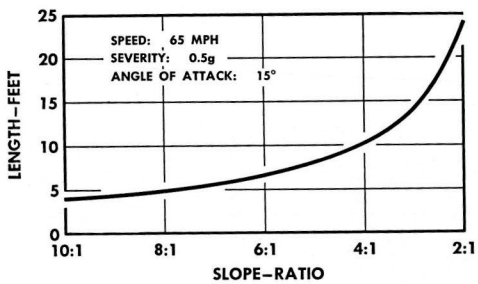

Figure 29. Vertical curve length as a function of ditch slope for severity of 0.5g at a 15° angle of attack.

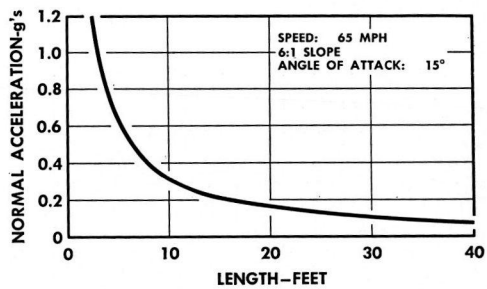

Figure 31. Variation in severity with ditch vertical curve length.

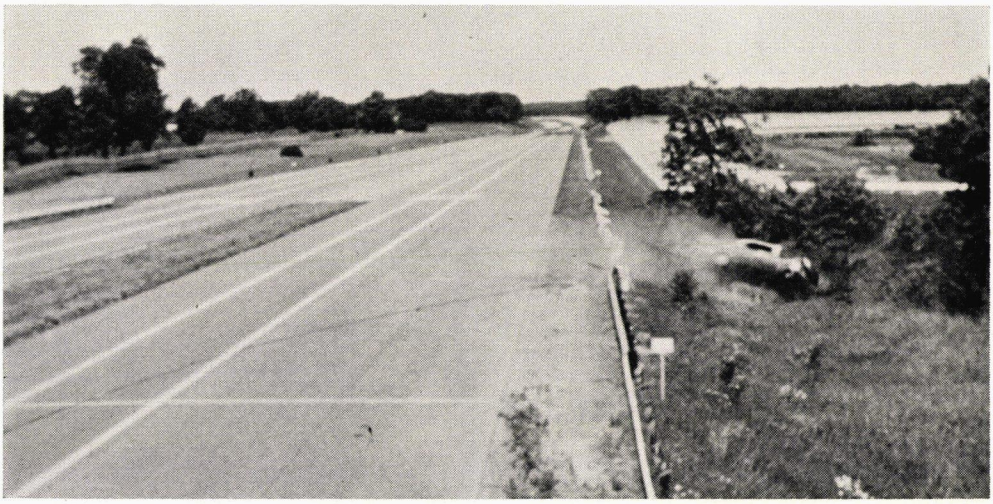

Figure 32. Car on test at 65 mph rolls over at toe of 4:1 slope.

provides that obstacles are cleared for a reasonable distance from the edge of the pavement to provide maneuver room for the driver who leaves the pavement, and that the roadside is traversable so he can maneuver satisfactorily. Obviously the driver will be unable to control his car if the roadside is so severe that the car is damaged, or if he is injured or even severely shaken up or alarmed.

ROADSIDE SLOPES

There are few data of the value of the side slopes on fill sections upon which to base design criteria. In some cases it is obvious (Fig. 32) that the slope is too steep, or that the transition from the side slope to natural grade is too abrupt. In many cases, on relatively flat gentle slopes the car slides rather than rolls

Figure 33. Car slides on 6:1 slope.

Figure 34. Car rolls over on side slope.

over (Fig. 33); but in certain cases the car may roll over (Fig. 34).

The force and moment systems on a car sliding down a slide slope are indicated in Figure 35, in which W is the weight of the car, T is the tread, H is the center of gravity height, θ is the angle of inclination of the slope, and F is the sum of the gravitational component and ground reaction or impact reaction against an obstacle at the point where the weight on the upper wheel, W_u, approaches zero. The equilibrium of force components parallel to the slope of the plane and normal to the plane are given by Eqs. 2 and 3, respectively; the equilibrium of moments by Eq. 4.

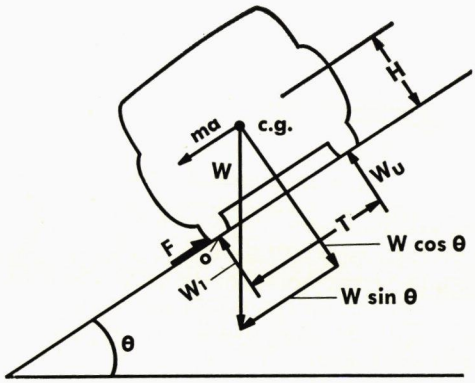

Figure 35. Force and moment relations when car sliding down side slope is decelerated.

$$\Sigma F_x = F - (m\,a + W \sin\theta) = 0 \qquad (2)$$

$$\Sigma F_y = W_1 - W \cos\theta \qquad (3)$$

$$\Sigma M_0 = \frac{T}{2} W \cos\theta - H(m\,a + W \sin\theta) = 0. \qquad (4)$$

From Eq. 2,

$$F = m\,a + W \sin\theta. \qquad (5)$$

From Eqs. 5 and 4,

$$\frac{T}{2} W \cos\theta - HF = 0. \qquad (6)$$

$$F = \frac{T}{2H} W \cos\theta \qquad (7)$$

and the coefficient of friction (or coefficient of ground reaction) is

$$f = \frac{F}{W} = \frac{T}{2H}, \quad \text{where } \theta = 0. \qquad (8)$$

Thus, on a level road the coefficient of ground reaction which will balance the

car about the reacting wheel is equal to the ratio $T/2H$, where T is the tread width and H is the center of gravity height.

Eq. 7 shows that on a car sliding down a slope the ground reaction force necessary to tip it over is proportional to the cosine of the angle of inclination of the slope, consequently the reaction force against the wheel required to tip the car over decreases as the angle of inclination of the slope increases. It should be noted, further, that the ratio of the horizontal and normal forces, F/W, may be a coefficient of friction or a coefficient of ground reaction required to give this equilibrium of overturning moments and that the value of this reaction is dependent on the tread width and the center of gravity height as expressed in Eq. 8. On a level road, for example, the car will overturn when the coefficient of friction exceeds the ratio of the tread and twice the center of gravity height.

The value of the deceleration which must be provided by the friction or ground reaction on the side slope to overturn the vehicle may be determined from the foregoing relations.

Solution for the deceleration, a, gives:

$$m\,a=\frac{T}{2H}W\cos\theta-W\sin\theta \qquad (9)$$

$$\frac{Wa}{g}=\frac{T}{2H}W\cos\theta-W\sin\theta \qquad (10)$$

$$\frac{a}{g}=\frac{T}{2H}\cos\theta-\sin\theta \qquad (11)$$

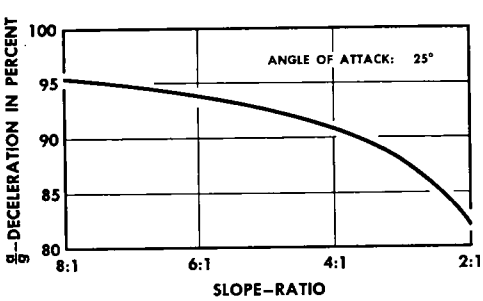

Figure 36. Effect of roadside slope on "tripping" deceleration.

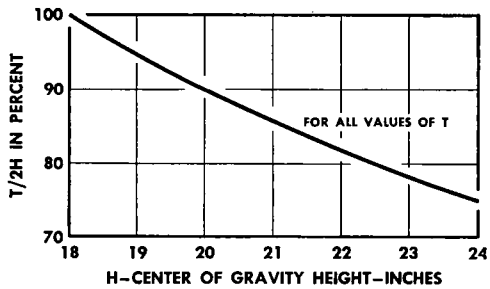

Figure 37. T/2H as function of H and T.

a/g is the deceleration in gravity units; it is in the same units and magnitude as f in Eq. 8.

The effect of roadside slope on the deceleration provided by the friction or ground reaction on the deceleration required to trip the car is shown in Figure 36 for an arbitrary angle of attack of 25°, where a value of $T/2H$ representative of current automobiles is considered. It shows, for example, that deceleration provided by the ground reaction and the coefficient of friction of the slope required to tip the car over is reduced by about 6 percent below that required on a level road on a slope of 6:1; on a roadside slope of 4:1, the tripping deceleration required to overturn the car is reduced by 9 percent; on a 3:1 slope by 13 percent; on a 2:1 slope by 20 percent. Figure 36 means, then, that it is easier to tip a car over on a steep slope than on a relatively flat slope.

As computed approximately for static conditions, current automobiles have an average stability factor of about 1.4 with some small variations related to different design approaches.

The effect on the stability factor, $T/2H$, of variation in T and H, the tread width and center of gravity height respectively, is shown in Figure 37 for all treads in the practical range and a range in center of gravity height from 18 in. to 24 in. The effect of lowering the height of the center of gravity is of more importance on this variation than is the effect of changing the tread. Figures 38 and 39 show the rates of change of f with T and H, respectively. The derivative of this function with respect to T,

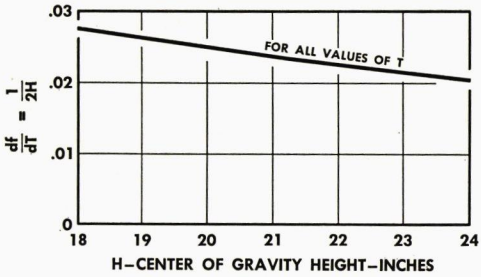

Figure 38. Rate of change of f with respect to T.

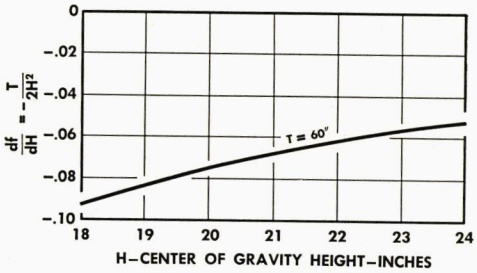

Figure 39. Rate of change of f with respect to H.

$(df)/(dT) = (1)/(2H)$, is independent of T, indicating that f decreases as the reciprocal of H of all values of T. On the other hand, the derivative of f with respect to H, $(df)/(dH) = -(T)/(2H^2)$, is inversely proportional to the square of H, the center of gravity height, so that the contribution of H to the stability index varied as the negative reciprocal of H^2.

The relative stability of the current cars has been achieved largely by virtue of low center of gravity height. In passing, it should be noted that reduction in the center of gravity height means inevitably that the driver's position in the car will be lowered and consequently that the driver's eye height is also lowered.

The trend in driver's eye height and its relation to crest vertical curve passing distance is discussed elsewhere (*4*, *5*, *6*). It should be clear, however, that the reduction in eye height occurs somewhat in proportion to the decrease in center of gravity height and that the stability of the car is increased approximately as the inverse square of the reduction in driver's eye height; that is, as the square of the decrease in height of the center of gravity.

The significance of the stability factor, $T/2H$, is that this factor is equal to the coefficient of friction of the surface on which the car will overturn when it is sliding sideways or "coefficient" of ground reaction; the units of stability factor are the same as those of coefficient of friction.

To determine the significance of the values of the stability factor, or the tripping deceleration rate, which must be provided by ground reaction or coefficient of friction of the ground surface, a car was dragged sideways on several types of surface. The method is shown in Figure 40 and oscillograms of the results of tests on representative surfaces are shown in Figure 41. The recorded test results were measured

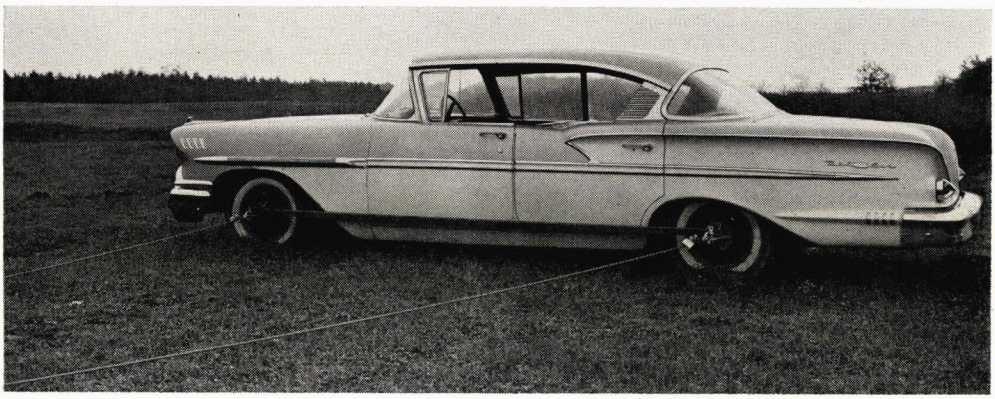

Figure 40. Method of measuring lateral ground reaction.

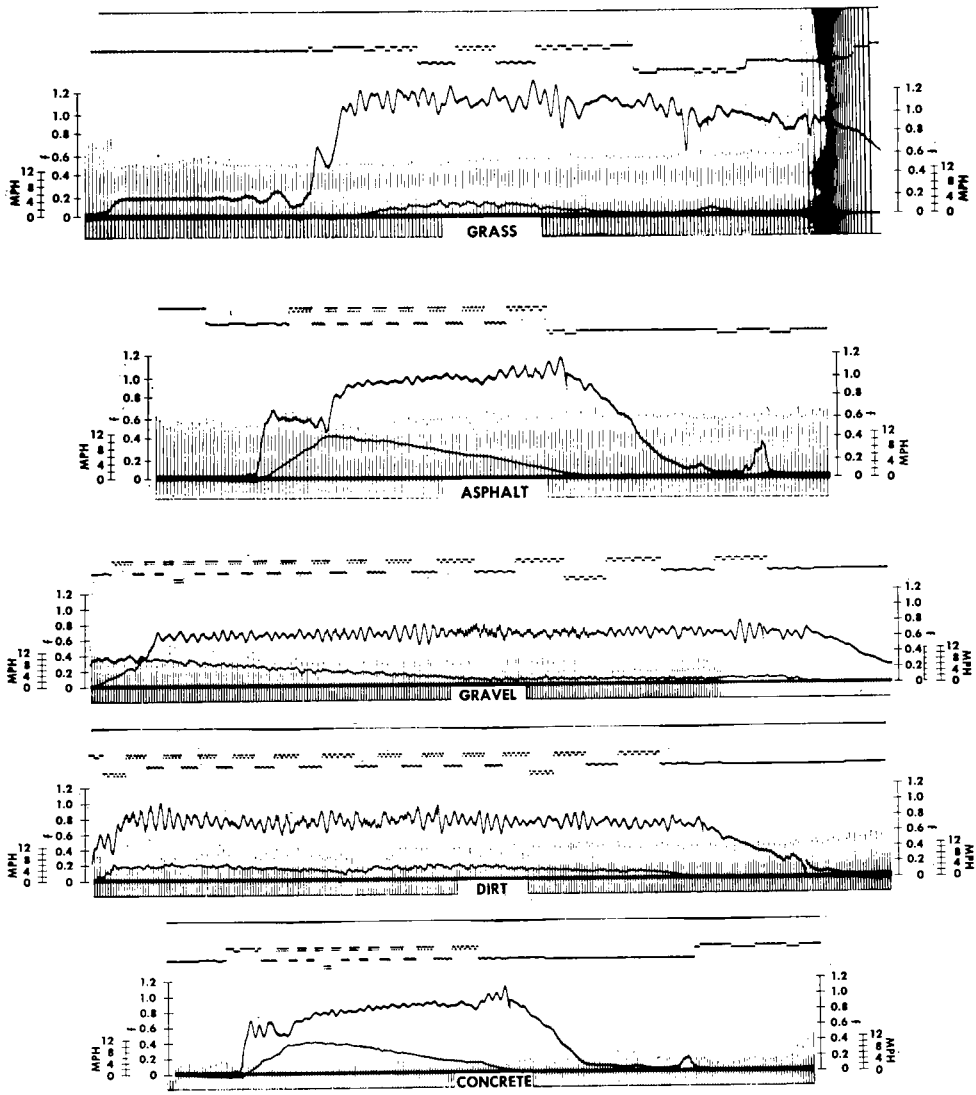

Figure 41. Coefficients of lateral ground reaction.

over a range of low speeds. The grass surface, was dry, firm, and typical of the mowed grass in a field or representative of sodded roadside surfaces in Michigan. The ground was firm and dry, representative of typical summer conditions. Tests were repeated in late October when the fall rains had moistened the ground thoroughly (Fig. 42). The differences between dry and firm and wet sod are not large and are generally less than the effects of local variations or protuberances during each of the tests.

The side force ground reaction on dry sod (Fig. 41) ranges primarily between 1.0 and 1.2 as expressed in units of coefficient of friction, with local variations which may be taken as indicative of the effect of small local protuberances. Although the range of speed was low, there is little variation in the value of "coefficient" with speed. Although this factor is expressed as if it were a coeffi-

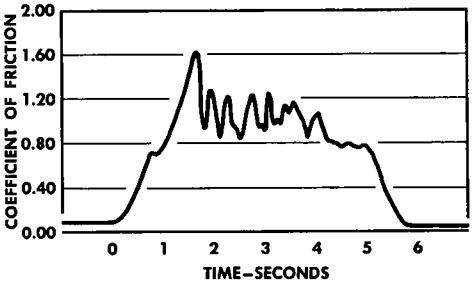

Figure 42. Coefficient of lateral ground reaction, wet sod.

cient of friction, it is probably not a true coefficient and values are probably dependent on surface irregularities.

The values developed on a bituminous concrete surface (Fig. 41) show that at lower speeds the coefficient of friction also falls in the range between 1.0 and 1.2, and at speeds of approximately 10 mph the coefficient falls below 1.0; that is, in this case, a decrease of coefficient was observed with an increase of speed.

The results of tests made on a gravel road surface (Fig. 41) show that the coefficient of friction varies between 0.6 and 0.8, with a typical value of possibly 0.7. Little effect is shown over a speed range of approximately 11 mph to nearly 0 mph.

The results of tests made on a dirt surface (Fig. 41) show that the typical values of speeds of the order of 4 mph or less are generally speaking below 0.8, although the coefficient reaches 1.0 locally. The effect of speed over a range up to approximately 5 mph appears to be negligible.

The coefficient of friction measured by dragging a car sideways on a portland cement concrete road surface (Fig. 41) shows that for speeds of approximately 10 mph the coefficient is somewhat below 0.8. At lower speeds (8 mph and lower) the coefficient rises above 0.8 and exceeds 1.0 at creeping speeds.

On paved surfaces the coefficient develops higher values at lower speeds, but this condition is apparently not found on sod, gravel or dirt surfaces.

Figure 43 is a summary of average values of lateral coefficient of friction or ground reaction shown on the oscillograms in Figures 41 and 42.

Although it was not possible in this series of tests to observe values of ground reaction at practical road speeds, the oscillograms made in the range from 0 to 12 mph do not suggest that there is an important variation in speed on either wet or dry sod; the values on wet and dry sod are essentially the same.

It seems obvious that the maximum practical values of coefficient of friction or ground reaction on a side slope with firm, dry sod will occur when there are irregularities in the surface, protuberances or ruts which the car wheel may strike, so that relatively high values of impact resistance occur which may trip the car, or when the ground is soft enough that the lower wheel can dig in and develop a relatively large shear force against the edge of the groove in the ground.

In view of the relatively small reserve of stability provided by current automobiles with low center of gravity height, careful design and construction of the roadside is a matter of great significance in the design for roadside safety. This leads to the suggestion that more sophisticated design and construction practices for roadside surfaces should provide for compact smooth surfaces and that maintenance practices should give much more emphasis to preserving this smoothness. The importance of smooth, firm, low coefficient roadside surfaces can hardly be overemphasized in the consideration of roadside safety.

The effect of roadside slope in reducing the tripping deceleration level is of first order of significance; the 6 to 20 percent

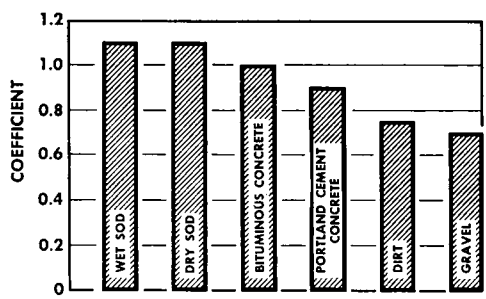

Figure 43. Summary of average values of lateral coefficient of friction.

reductions noted in Figure 36 when the car is sliding down the slopes of 6:1 and 2:1 at a 25° angle may indeed be of great significance.

The value of the slope also has secondary effects, because the steeper the slope, the longer the velocity of the car will be maintained and thus the greater will be the possibility of striking some protuberance which will trip it. Furthermore, the steeper the slope, the greater is the weight transfer from the upper to the lower wheel and the greater the indentation into the ground will be and the larger the shear forces may be.

It must be concluded that for safe roadside design the slopes must be as flat as possible, not steeper than 6:1 and preferably flatter. They must be as smooth and firm as possible and provide the lowest possible reaction against a car sliding sideways down them.

Unfortunately, there is no manner of specifying roadside smoothness adequately. Tentatively it may be said that slopes should be free from stumps, firmly embedded stones, and erosion channels, and smooth enough to be mowed comfortably. The apparent margin of stability factor of even the current automobiles with low center of gravity height is such that relatively small improvements in the flatness and the smoothness of the roadside slopes would make significant reduction in roadside hazards.

GUARDRAILS

Under some circumstances, it will be impossible to eliminate the obstacles from beside the road; bridge piers must be relatively close, and in mountainous terrain it will be impossible to have side slopes constructed according to the ideal previously discussed. In other cases, on high fill, the slope of the natural ground will be such that it will be impossible to build a flat, gentle side slope, and a steep fill will be required. In these cases, some use of guardrail must be made to protect against the more serious obstacles.

Lundstrom and Skeels have reported on a series of full-scale guardrail tests conducted at the Proving Ground (*1*). A major conclusion of their paper was that there was no such thing as a perfect guardrail, that a guardrail was a last resort, and that it should be used only when no other solution was possible. Beaton (*7*) reported on a series of tests of median barrier installations comparable with guardrails. However, because the Lundstrom and Skeels report was incomplete, some additional tests were conducted with particular reference to the design of the end installation. Figure 44 shows results of a full-scale impact of a car against the end of a standard guardrail. This produces a shocking direct collision with an obstacle; it is a completely undesirable installation. An

Figure 44. Collision with standard guardrail end installation.

Figure 45. Test of 25-ft ramp end installation at 50 mph.

improvement on this was sought by ramping the end sections down to the ground to allow the car to slide upward. Figure 45 shows a car striking the end of such a ramped section at 50 mph. The impacts were rather moderate, and this approach appeared to be rather promising.

A second test was made at 60 mph on an installation having a somewhat longer ramp (Fig. 46). In this case, it is obvious that the ramp was too steep and the car was pitched violently up in the air. A third installation was made with a still longer ramp and with closely spaced posts extending 6 in. above the rail (Fig. 47). The results were somewhat more favorable, but the impact was severe.

There may be other and better solutions to this problem: Figure 48 shows probably a nearly ideal condition where the back slope of the ditch was approximately 30 in. above the pavement surface and the guardrail was taken across the ditch and started at approximately the top of the bank so that the end is protected completely. This solution can be applied to equal advantage where the back slope extends well above the level of the pavement, provided there is a shallow ditch of good design.

In locations where there is no convenient ditch and back slope, a long low

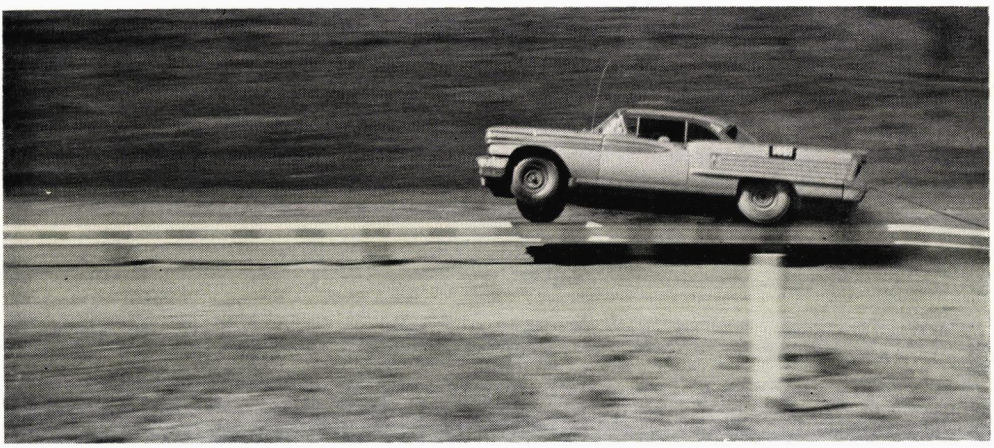

Figure 46. 60-Mph test of 37½-ft ramp end installation.

Figure 47. Test of guardrail end ramp with 6-in. post exposed.

earth mound ahead of the end of the guardrail (Fig. 49) would appear to have great advantage. As shown, provision is made that one or the other of the wheels might run up on the bank, and when the car reaches the guardrail it will simply slide along the top. If the car strikes the approach ramp dead center, it will simply slide up the long gentle ramp with very low impact values.

A variation of this design might be to build a somewhat wider ramp, falling away more slowly as the end of the guardrail is reached, so that the whole car would drive up the ramp and the flat departure slope of the ramp would allow the car to settle down on top of the guardrail gently. There has been no opportunity to evaluate the design suggested in Figure 49. One is left with the impression that it should be a satisfactory solution to the problem for most installations. It seems clear that almost anything is better than no end treatment at all.

As noted also by Lundstrom and Skeels (7), there is still uncertainty as to the type of installation which will produce the minimum hazard to the occupants of the car. There is some question about the compromise between minimum hazard to the occupants and to the other travelers on the highway. There is some question remaining about the optimum

Figure 48. Guardrail end installation in ditch back slope.

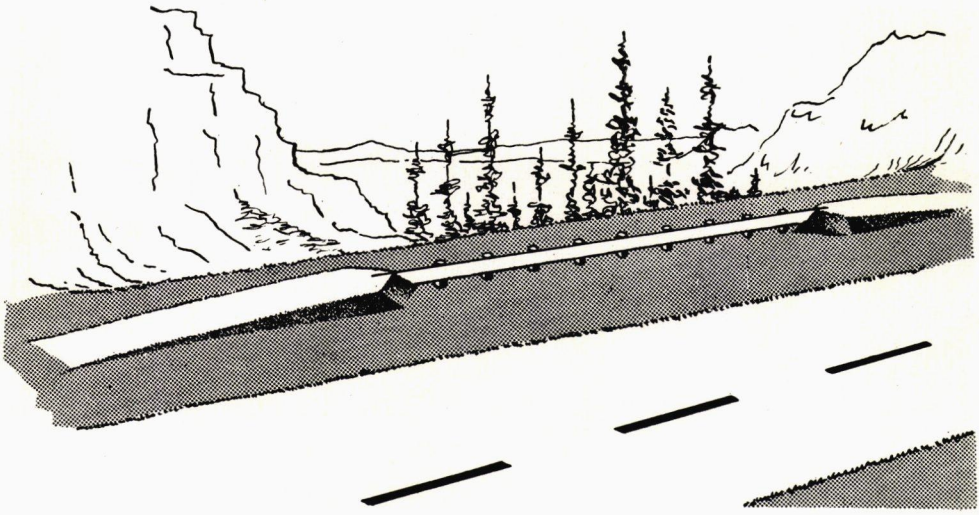

Figure 49. Artist's concept of buried guardrail end installation.

type of guardrail, whether it be a beam-type, cable-type, net-type, or some fixed impassable barrier. More information is needed on the best type of material, which may be steel, aluminum, fiberglass or wood. There is some question still about the optimum size of posts, their spacing, and the material of which they are made. There are still questions about the best way of mounting the guardrail, whether it should be mounted directly to the post, mounted with a spring connection, or with a solid block.

Although it appears that there is a great deal not known about guardrail design, it is clear that hitting a guardrail is an accident, and that installation of guardrails should be avoided wherever possible.

IDEAL EXAMPLE

Figure 50 shows how these concepts of safe roadside design have been applied to the latest Proving Ground test road, built in 1958. There are no obstacles within 100 ft of the pavement, all slopes are gradual, and all ditch bottoms are wide and gently rounded. The cost of construction was only about $9,500 per mile above what the cost of standard highway design would have been. The terrain was favorable, but part of the area was heavily wooded and drainage requirements were unusually expensive.

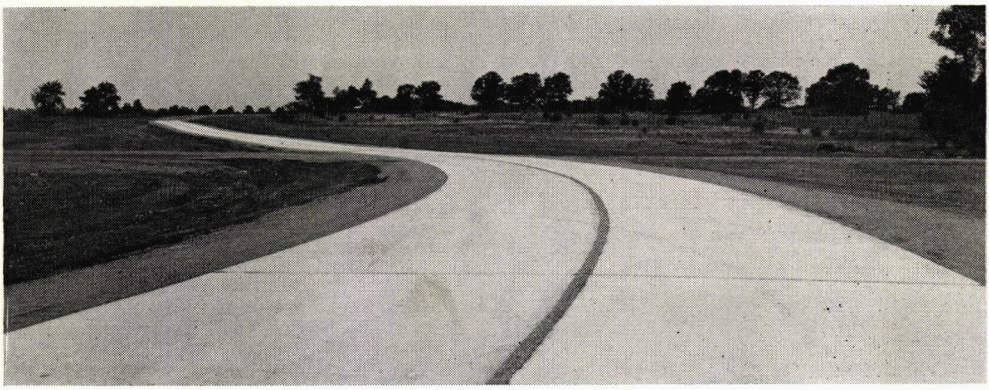

Figure 50. "Ideal" roadside design, Proving Ground R and H Loop.

Figure 51. Typical scene on Proving Ground Hill Road. 1926 standard.

Figure 51 shows a typical scene on the sharply curved alignment (Fig. 52) of Hill Road, part of which was completed in 1927. It covers a total distance of 2.37 mi over fairly rough country, and has numerous short steep grades and a total rise and fall of 6.10 ft per 100 ft. It was constructed originally according to 1926 standards; the typical cross-section is shown in Figure 53. For comparison, the somewhat improved cross-section designated as the 1940 standard (Fig. 54) is possibly typical of many of the rural roads, particularly secondary roads, now being constructed. The primary differences are that the shoulder width has been increased from 3 ft to 6 ft and the ditch slope has been flattened from 1.5:1 to 3:1.

The typical section required by the 1960 standard is shown in Figure 55. The differences here are that the lane width has been increased to 12 ft, the shoulder

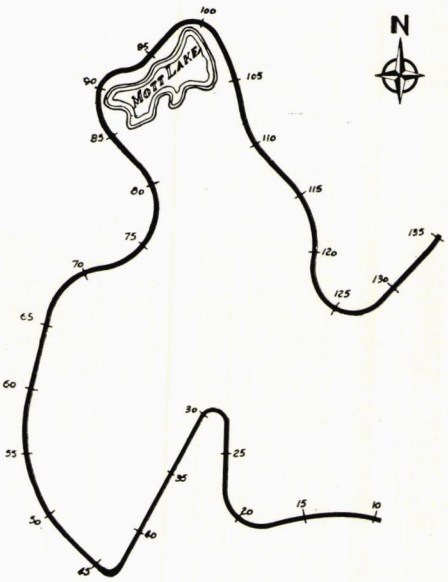

Figure 52. Hill Road layout.

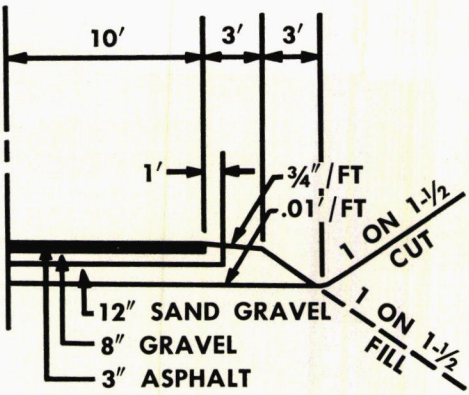

Figure 53. 1926 standard cross-section.

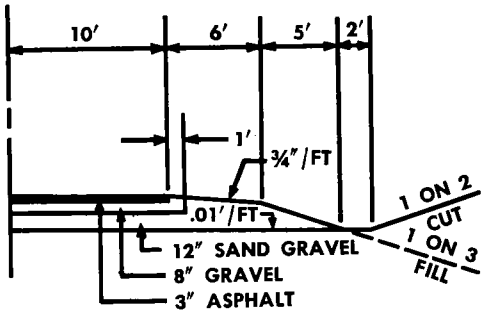

Figure 54. 1940 standard cross-section.

width has been increased to 10 ft, the maximum slopes have been decreased to 6:1, 6.5-ft vertical curves are incorporated in the ditch bottoms, and obstacles are cleared from each side of the center line to a minimum distance of 100 ft.

The cost details according to the three standards are compared in Table 2 and summarized in Table 3.

The relative costs of construction estimated at current unit prices are: for the 1926 standard, $48,800 per mile; for the 1940 standard, $54,000 per mile; and

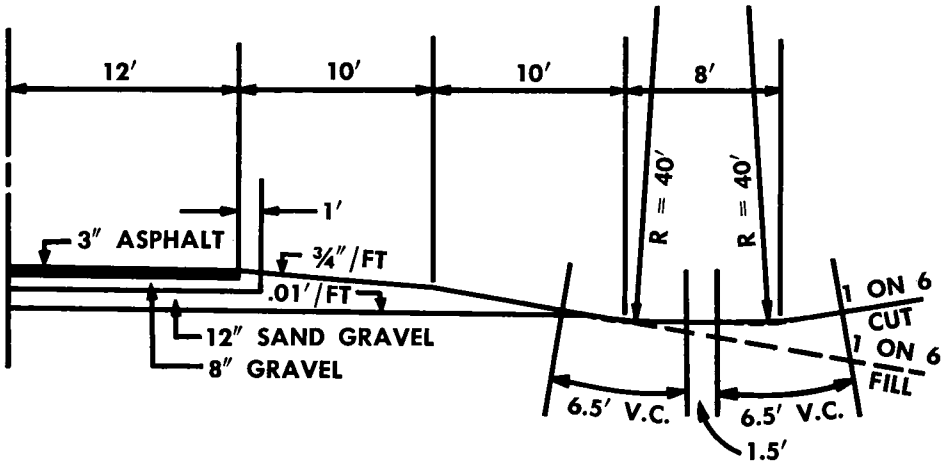

Figure 55. 1960 standard cross-section.

TABLE 2

COST COMPARISON, 2.37 MILES OF HILL ROAD

Items of Work	Unit	Unit Price, $	Quantities 1926 Std.	Quantities 1940 Std.	Quantities 1960 Std.	Cost, $ 1926 Std.	Cost, $ 1940 Std.	Cost, $ 1960 Std.	Cost Diff., $ 1940 Minus 1926	Cost Diff., $ 1960 Minus 1926	% Increase Above 1926 1940	% Increase Above 1926 1960
1 Clearing	Acre	500.00	2.5	3.5	10.5	1,250	1,750	5,250	500	4,000	+0.43	+3.46
2 Tree Removal	Tree	50.00	20	45	120	1,000	2,250	6,000	+1,250	+5,000	+1.08	+4.33
3 Excavation	Cu yd	0.35	45,000	62,500	123,200	15,750	21,875	43,200	+6,125	+27,450	+5.30	+23.75
3a Overhaul	C c y m[a]	0.15	1,600	3,500	13,000	240	525	1,950	+285	+1,710	+0.25	+1.48
4 Sand Subbase	Cu yd	0.45	16,250	23,200	30,000	7,320	10,440	13,500	+3,120	+6,180	+2.70	+5.35
5 Gravel	Cu yd	0.75	6,800	6,800	8,100	5,100	5,100	6,075	0	+975	0.00	+0.84
6 Purchase Gravel	Cu yd	1.50	8,500	8,500	10,000	12,750	12,750	15,000	0	+2,250	0.00	+1.95
7 Drainage		Varies				5,066	5,500	7,630	434	+2,564	0.38	+2.21
8 Asphalt	Ton	12.50	4,375	4,375	5,250	54,700	54,700	65,600	0	+10,900	0.00	+9.43
9 Topsoil	Cu yd	0.50	1,600	2,600	5,900	800	1,300	2,950	500	+2,150	0.43	+1.86
10 Seeding	Acre	100.00	4	6.5	14.5	400	650	1,450	250	+1,050	0.22	+0.91
11 Guardrail	Lin ft	3.50	3,200	3,200	2,500	11,200	11,200	8,750	0	−2,450	0.00	−2.12
Total						115,576	128,040	177,355	12,464	61,779	+10.78	+53.45

[a] 100 cu yd-mi.

TABLE 3

COST COMPARISON, PER MILE

Standard	Cost per Mile, $	Cost Diff., $
1926	48,800	—
1940	54,000	—
1960	75,000	—
1940–26	—	5,200
1960–40	—	21,000
1960–26	—	26,200

for the 1960 standard $75,000 per mile. Thus, the difference in cost per mile between the 1960 and the 1926 standards is only approximately $26,000 per mile. This means that for $26,000 per mile this primitive one-way rural road can be transformed into a highway with roadside design standards surpassing those of the New York Thruway.

SUMMARY

Roadside hazards are a significant part of the highway accident problem; these hazards can and should be reduced significantly by eliminating obstacles adjacent to the roadside including trees, light poles, and sign posts.

Ditch sections should be shallow and wide. A ditch with a 4:1 slope and back slope and an 8-ft bottom will produce verticle accelerations exceeding 1.0 *g* at 40 mph. The maximum intensity of operation short of driver injury is approximately 2.0 *g*, which occurs in a ditch of this type at an angle of attack of 20° at a speed of approximately 50 mph. The vertical or normal accelerations produced can be estimated by the calculation under rather mild conditions up to the point where the suspension system bottoms at values of normal accelerations of possibly 0.5 *g*. Beyond these values the non-linearities of the suspension system make it impossible to calculate normal accelerations by simple, direct methods. For high-type roads where practical speeds above 65 mph may be anticipated, a ditch section having slopes of 6:1, bottom width of at least 6.5 ft, with vertical curves 6.5 ft long on each side, is the most severe that should be used for the desirable standards of roadside safety. This section will give computed values of normal acceleration of 0.5*g* at 15° angle of attack.

Side slopes have a first-order influence on the probability of the car rolling over. The stability factors of even the current automobiles with low center of gravity height have a margin of reserve beyond the frictional reaction of smooth, firm, sodded slopes such that even small losses in effective stability may be significant. These losses may be produced by steep roadside slopes or roughness of the surface. On the other hand, even the rather small numerical improvement achieved by reducing the roadside slope and smoothing the irregularities of the roadside surface would make significant gains in roadside safety. The slope should be no steeper—and preferably flatter—than 6:1. The roadside should be smooth and clear of small obstacles and protuberances, and should be constructed of firm material.

Guardrails should be eliminated wherever possible. Conventional end installations are serious man-made obstacles and improvements in design of these installations have been suggested. A straight ramp reduces the hazards materially; where possible the end may be buried in the ditch back slope or a long low artificial mound may be built to cover the end.

The cost of construction according to the 1960 roadside standard over the primitive 1926 standard is approximately $9,000 per mile on level terrain and $26,000 per mile in hilly wooded country on specific portions of the Proving Ground road system.

ACKNOWLEDGMENT

The contributions of the many participants in this project are acknowledged gratefully. Of particular value have been those of Mrs. Arline Marshall, Prof. A. H. Easton during 1959 summer employment, and A. H. Kelly, Jr., for their analytical work. Also to P. C. Skeels and his staff, who conducted the tests.

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APPENDIX

If a car leaves the road at some angle ϕ (Fig. 18) and travels along the side slope and curvilinear ditch at this angle, the slope and vertical curve of the path of travel depend on this angle ϕ and the cross-section of the side slope and ditch.

Uniform Side Slope

Along the uniform side slope section, let the side slope equal $\tan\alpha$ where α is the angle of the slope. Then the car travels on a grade equal to $\tan\alpha \cos\phi$ as shown below.

Let α = angle of side slope;
ϕ = angle at which car leaves the roadway; and
θ = angle of grade of path of travel.

The side slope is represented in Fig. 56 At a distance of x feet from the roadway, the ground has dropped y feet and

$$\tan\alpha = y/x \qquad (12)$$

The grade of the path of travel is represented in Fig. 57, when d and y are the horizontal and vertical projections of the car's path and

$$\tan\theta = y/d \qquad (13)$$

The y in Eqs. 12 and 13 is the same. Solving simultaneously,

$$\tan\theta = \frac{x}{d}\tan\alpha \qquad (14)$$

Viewing x and d in the horizontal plane (Fig. 58),

$$\cos\phi = \frac{x}{d} \qquad (15)$$

Substituting Eq. 15 in Eq. 14,

$$\tan\theta = \tan\alpha \cos\phi \qquad (16)$$

Circular Ditch Bottoms

Figure 18 represents a road with a 6:1 side slope and a ditch bottom formed like the arc of a circle. If a car leaves the

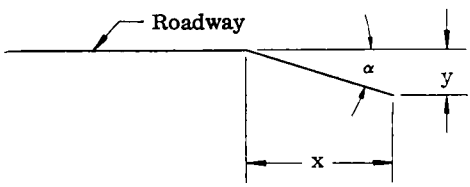

Figure 56.

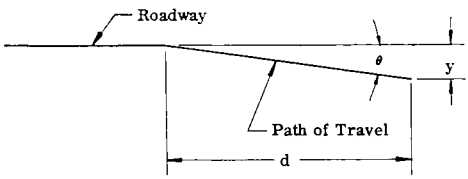

Figure 57.

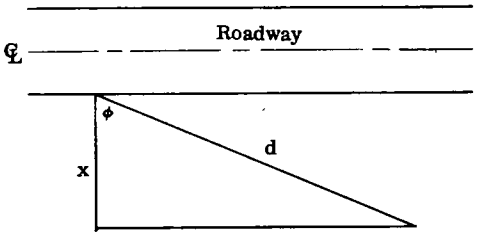

Figure 58.

road at some angle, ϕ, the slope it travels is less than 6:1 based on $\cos\phi$ and the curve it goes through is something other than a circular arc. The path corresponds to the curve formed by the intersection of a plane with a cylinder.

In Figure 19, the plane has one line in common with the y-axis and rotates about the y-axis making an angle ϕ with the x-axis. This angle corresponds to ϕ in Figure 18. Let t represent the second axis of the plane. The equation for the curve where the plane and cylinder intersect will be in the $y-t$ plane and is the result of treating the equations for the cylinder and the plane as simultaneous equations.

The equation for the cylinder (Fig. 59) is

$$x^2+y^2=r^2 \tag{17}$$

where r is the radius. The equation for

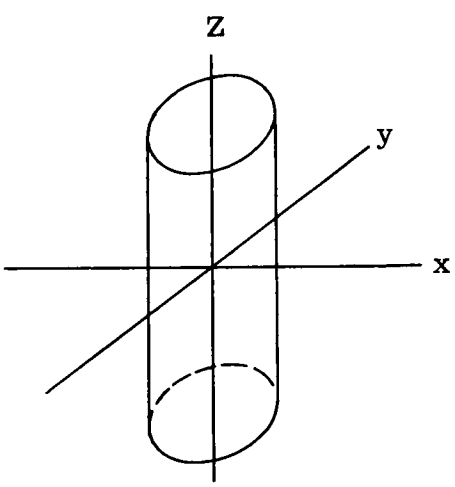

Figure 59.

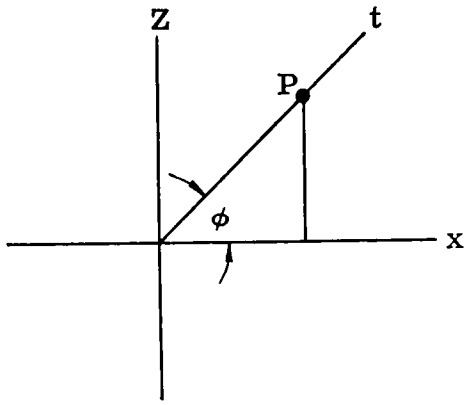

Figure 60.

the plane (Fig. 60) is

$$Z=x\tan\phi \tag{18}$$

A point P on the plane has a distance from the y-axis of

$$t=\sqrt{Z^2+x^2} \tag{19a}$$

$$t^2=Z^2+x^2 \tag{19b}$$

$$t^2=x^2\tan^2\phi+x^2=x^2(\tan^2\phi+1)=x^2/\cos^2\phi \tag{19c}$$

Transposing Eq. 17 and substituting in Eq. 19c gives

$$t^2=(r^2-y^2)/\cos^2\phi \tag{20a}$$

$$\frac{t^2}{1/\cos^2\phi}=r^2-y^2 \tag{20b}$$

$$\frac{t^2}{1/\cos^2\phi}+y^2=r^2 \tag{20c}$$

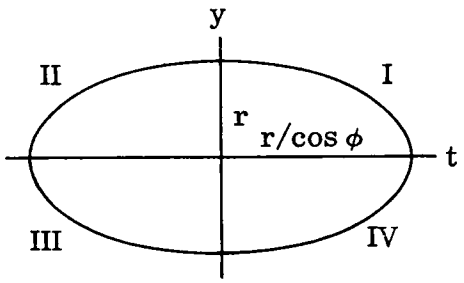

Figure 61.

$$\frac{t^2}{r^2/\cos^2\phi}+\frac{y^2}{r^2}=1 \qquad (20d)$$

Eq. 20*d* is an ellipse in the plane $y-t$ (Fig. 61) with major axis $=\pm r/\cos\phi$ and minor axis $=\pm r$.

The portion of the ellipse usable for this problem lies in quadrants III and IV, symmetrical about the y-axis and limited by slope of $\cos\phi$ times the side slope of the road from Figure 56.

For the ellipse in Eq. 20*d* solve for y, y', y'', and the radius of curvature, R. This is the vertical radius that the car would travel along its path.

$$t^2\cos^2\phi+y^2=r^2 \qquad (21a)$$

$$y^2=r^2-t^2\cos^2\phi \qquad (21b)$$

$$y=\sqrt{r^2-t^2\cos^2\phi} \qquad (21c)$$

Differentiating y with respect to t,

$$y=\frac{dy}{dt}=1/2(r^2-t^2\cos^2\phi)^{-\frac{1}{2}}(-2t\cos^2\phi) \qquad (22a)$$

$$y'=\frac{dy}{dt}=(r^2-t^2\cos^2\phi)^{-\frac{1}{2}}(-t\cos^2\phi) \qquad (22b)$$

$$y''=\frac{d^2y}{dt^2}=[r^2-t^2\cos^2\phi]^{-\frac{1}{2}}[-\cos^2\phi]+[-t\cos^2\phi]\times[-1/2(r^2-t^2\cos^2\phi)^{-\frac{3}{2}}(-2t\cos^2\phi)] \qquad (23)$$

$$y=(r^2-t^2\cos^2\phi)^{\frac{1}{2}} \qquad (24)$$

$$y'=\frac{-t\cos^2\phi}{(r^2-t^2\cos^2\phi)^{\frac{1}{2}}} \qquad (25)$$

$$y''=\frac{-\cos^2\phi}{(r^2-t^2\cos^2\phi)^{\frac{1}{2}}}-\frac{(t\cos^2\phi)(t\cos^2\phi)}{(r^2-t^2\cos^2\phi)^{\frac{3}{2}}} \qquad (26)$$

The radius of curvature is

$$R=\frac{[1+(y')^2]^{\frac{3}{2}}}{y''} \qquad (27a)$$

$$=\left[1+\frac{t^2\cos^4\phi}{r^2-t^2\cos^2\phi}\right]^{\frac{3}{2}}\Bigg/\frac{(-\cos^2\phi)(r^2-t^2\cos^2\phi)-t^2\cos^4\phi}{(r^2-t^2\cos^2\phi)^{\frac{3}{2}}} \qquad (27b)$$

$$=\frac{[(r^2-t^2\cos^2\phi)+t^2\cos^4\phi]^{\frac{3}{2}}}{(-\cos^2\phi)(r^2-t^2\cos^2\phi)-t^2\cos^4\phi} \qquad (27c)$$

Maximum R is at $t=0$:

$$R=\pm\left(\frac{r^3}{-r^2\cos^2\phi}\right)=\frac{\pm r}{\cos^2\phi} \qquad (28)$$

Minimum R is at $t=r/\cos\phi$:

$$R=\frac{(r^2\cos^2\phi)^{\frac{3}{2}}}{-r^2\cos^2\phi}=\frac{\pm(r^3\cos^3\phi)}{-r^2\cos^2\phi}=\pm r\cos\phi \qquad (29)$$

For the general case, where $\tan\alpha=$ slope, the limiting slope of the ellipse is $\tan\alpha\cos\phi$.

$$y'=\tan\alpha\cos\phi=\frac{-t\cos^2\phi}{(r^2-t^2\cos^2\phi)^{\frac{1}{2}}} \qquad (30)$$

Find t and solve Eq. 27*c* for R:

$$\frac{-t\cos\phi}{(r^2-t^2\cos^2\phi)^{\frac{1}{2}}}=\tan\alpha \qquad (31)$$

$$t^2\cos^2\phi=\tan^2\alpha(r^2-t^2\cos^2\phi) \qquad (32)$$

$$=r^2\tan^2\alpha-\tan^2\alpha(t^2\cos^2\phi) \qquad (33)$$

$$t^2\cos^2\phi(1+\tan^2\alpha)=r^2\tan^2\alpha \qquad (34)$$

$$t^2\cos^2\phi=\frac{r^2\tan^2\alpha}{1+\tan^2\alpha}=r^2\tan^2\alpha\cos^2\alpha \qquad (35)$$

$$t=\frac{r\tan\alpha\cos\alpha}{\cos\phi} \qquad (36)$$

Substituting Eq. 36 in Eq. 27*c*,

$$R=\frac{[(r^2-r^2\tan^2\alpha\cos^2\alpha)+r^2\tan^2\alpha\cos^2\alpha\cos^2\phi]^{\frac{3}{2}}}{(-\cos^2\phi)(r^2-r^2\tan^2\alpha\cos^2\alpha)-r^2\tan^2\alpha\cos^2\alpha\cos^2\phi} \tag{37a}$$

$$=\frac{[r^2-r^2\tan^2\alpha\cos^2\alpha(1-\cos^2\phi)]^{\frac{3}{2}}}{-r^2\cos^2\phi} \tag{37b}$$

$$=\frac{[r^2(1-\tan^2\alpha\cos^2\alpha\sin^2\phi]^{\frac{3}{2}}}{-r^2\cos^2\phi} \tag{37c}$$

$$=\frac{\pm r[1-\tan^2\alpha\cos^2\alpha\sin^2\phi]^{\frac{3}{2}}}{-\cos^2\phi} \tag{37d}$$

Sample Calculation

Calculate accelerations through the ditch bottom along path of travel for

$\phi=80°$; $90-\phi=10°$
$\phi=75°$; $90-\phi=15°$
$\phi=70°$; $90-\phi=20°$

and $V=30$ mph and 40 mph on the section shown in Figure 62, where $\tan\alpha=0.25$, $\sin\alpha=0.24254$, $\cos\alpha=0.97014$, $2r\sin\alpha=12$, $r\sin\alpha=6$, and $r=24.74$ ft. Then the radius of curvature of path of travel is (Eq. 37*d*)

$$R=\frac{24.74[1-(0.25)^2(0.97014)^2\sin^2\phi]^{\frac{3}{2}}}{\cos^2\phi} \tag{38a}$$

$$=\frac{24.74[1-0.05882\sin^2\phi]^{\frac{3}{2}}}{\cos^2\phi} \tag{38b}$$

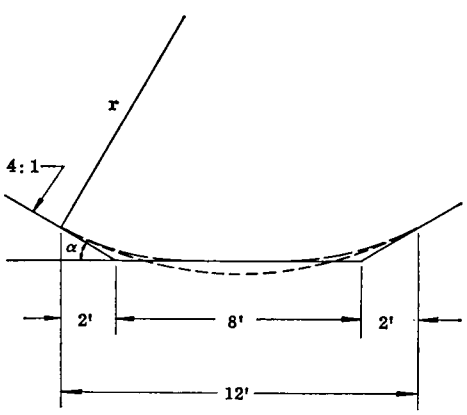

Figure 62.

Calculate radius of curvature for paths of 10°, 15°, and 20°, giving $R=751.34$, 339.33, and 195.23, respectively.

$\tan\alpha\cos\phi=$

$\tan\theta$	.04341	.06470	.08550
$\cos\theta$	.99906	.99792	.99637
$\sin\theta$	.04337	.06456	.08519

Calculate acceleration, a/g, for these paths at 30 and 40 mph, giving at 30 mph 0.08, 0.18, and 0.30, respectively, and at 40 mph 0.14, 0.32, and 0.54, respectively.

Practical Design Approach

Assume $V=65$ mph; $A=0.5g=16.08$ ft/sec^2; $\phi=75°=90°-15°$; $\sin\phi=0.96593$; and $\cos\phi=0.25882$.

Then

$$\frac{V^2}{R}=16.08$$

and

$$R=\frac{(65)^2(2.1511)}{16.08}=565.2 \text{ ft.}$$

Using $R=565.2$ and Eq. 37*d*,

$$565.2=\frac{r[1-\tan^2\alpha\cos^2\alpha(0.93302)]^{\frac{3}{2}}}{0.06699} \tag{39}$$

By assuming a value of $\tan\alpha$ which is the side slope, the radius of curvature, r, of the ditch bottom and the length of the vertical curve needed can be calculated. Two examples are shown:

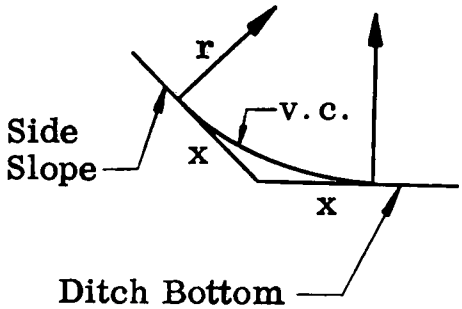

Figure 63.

6:1 Slope

Let $\tan\alpha = 0.16667$, $\sin\alpha = 0.16440$, and $\cos\alpha = 0.98639$.

Then (Eq. 39)

$$565 = \frac{r[1-(0.16667)^2(0.98639)^2(0.93302)]^{\frac{3}{2}}}{0.06699}$$

and

$$r = \frac{565(0.06699)}{0.96241} = 39.33 \text{ ft},$$

from which

$$\tan\tfrac{1}{2}\alpha = 0.08247 = x/r;$$

and

$$x = 0.08247(39.33) = 3.24 \text{ ft}$$

(Fig. 63).

4:1 Slope

Let $\tan\alpha = 0.25$, $\sin\alpha = 0.24254$, and $\cos\alpha = 0.97014$.

Then (Eq. 39)

$$565 = \frac{r[1-(0.25)^2(0.97014)^2(0.93302)]^{\frac{3}{2}}}{0.06699}$$

and

$$r = \frac{565(0.06699)}{0.91882} = 41.19 \text{ ft},$$

from which

$$\tan\tfrac{1}{2}\alpha = 0.12310 = \frac{x}{r};$$

and

$$x = 0.12310(41.19) = 5.07 \text{ ft}$$

(Fig. 63).