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Research Report No. 12-F

STRESS DISTRIBUTION IN A HOMOGENEOUS SOIL

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STRESS DISTRIBUTION IN A HOMOGENEOUS SOIL

PRESENTED AT THE TWENTY-NINTH ANNUAL MEETING
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HIGHWAY RESEARCH BOARD
DIVISION OF ENGINEERING AND INDUSTRIAL RESEARCH
NATIONAL RESEARCH COUNCIL

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STRESS DISTRIBUTION IN A HOMOGENEOUS SOIL

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SYNOPSIS

The Corps of Engineers, Department of the Army, is conducting a long-range study of the distribution of stresses and strains in flexible pavements under airplanes to obtain data which it is hoped will be useful in developing a theoretical method of flexible pavement design. As a part of this study, a homogeneous clayey-silt test section containing 37 earth pressure cells and 5 deflection gages was built and tested by the Waterways Experiment Station at Vicksburg, Mississippi. The program of tests included measurements of stresses in the vertical, two horizontal, and two diagonal directions and measurements of deflections in the vertical direction. This paper describes the physical features of the test section and the testing equipment and also presents some of the results that were obtained. The test results are discussed and comparisons are made with values computed from the theory of elasticity. Data are presented showing comparisons between field stress-strain relations obtained in the test section with those obtained in laboratory "quick" triaxial tests. The subject matter of this paper is abstracted from a report which will be published in the early part of 1950.

In the design of flexible pavements, one requirement of primary importance is the total thickness of base and pavement. The Corps of Engineers uses the California Bearing Ratio (CBR) method (1)[1] to determine the total thickness of flexible pavement at military airfields. The CBR method is an empirical method developed by the California Highway Department. This method uses an index (CBR) of strength obtained in a penetration-type shear test and a family of curves derived from service behavior observations to determine the thickness for a given CBR and load condition. The adoption of this method and its adaptation to airfield pavements are described in a symposium (2) recently published in the *Proceedings* of the American Society of Civil Engineers.

The CBR method of design is subject to the limitations of any empirical test. Extensions to conditions outside the range for which the test has been correlated are difficult and can be validated only by "hindsight" observations of pavements already constructed or by relatively expensive full-scale tests. It is also difficult to evaluate to what extent the index obtained in the penetration-type shear test follows the true strength in the prototype. These criticisms of an empirical test are not intended to belittle the value of existing empirical procedures. It is well known that methods based on empirically derived values have proved quite satisfactory not only in the field of soil mechanics but in other fields as well. It is felt however that a design method which has a theoretical basis can be applied to new conditions with a much greater degree of confidence than one which is based entirely on experimental values.

Studies conducted at the time the Corps of Engineers adopted the CBR method of design (1941) indicated that no satisfactory method using a theoretical formula was then available and that an empirical method would have to be used at least for the immediate future. It was recognized, however, that information on the behavior of flexible pavements under loads was sorely needed, and provisions were made for obtaining such data. Pressure cells and deflection gages were included in accelerated

[1]Figures in parentheses refer to references listed at the end of this paper.

traffic tests conducted at Stockton, California, in 1942 (3), at Barksdale Field, Louisiana, in 1942-1943 (4), and in the test section at Marietta, Georgia, in 1943-1944 (5), and readings of the induced pressures and deflections were made under varying conditions of load. When the Flexible Pavement Laboratory was established at the Waterways Experiment Station in 1943, it was assigned a continuing project of studying the distribution of stresses and deflections under airplane wheel loads. The first task was the theoretical computation of stresses and deflections and the comparison of these computed values with observed values from the test sections referred to above. In some instances the computed and observed values were in reasonable agreement, and in others there was wide deviation. It was soon realized that the behavior of flexible pavements under load was extremely complex and that a basic understanding of this behavior was not available.

In 1946 a comprehensive study of stress distribution under airplane wheel loads was started. It was concluded from the previous experience that it would be unwise to set up a program with the sole purpose of developing a truly theoretical method of design. Instead, the studies were directed more toward increasing the basic understanding of the behavior of pavements, bases, and subgrades when subjected to wheel loads. It was expected that the results of these studies would be useful in solving certain problems in the application of the empirical method. This expectation has materialized as evidenced by the theoretical resolution of the single-wheel design curves into curves for multiple-wheel assemblies and curves for high-pressure tires (2). These studies, if continued, will provide the Corps of Engineers and other interested agencies with much useful information for the application of empirical procedures and it is entirely possible that as a result, a truly theoretical design method for flexible pavements will be developed.

One of the first phases of the comprehensive program on stress distribution was the study of the simplest possible case, a homogeneous material. Such a test section was constructed in 1947 and tested in 1947 and 1948. This paper presents some of the more interesting results of this study. Plans are currently under way to study the next step, a two-layered system consisting of a base and a subgrade. The long range program also includes three-layered systems. It is contemplated that more than one type of subgrade and base will be studied.

TEST PROGRAM

The study of the homogeneous case was accomplished by applying static loads to the surface of a test section composed entirely of one type of soil and measuring the resulting pressures and deflections. The CBR of the soil was 11 percent. Figure 1 shows a schematic layout of the testing setup. A unique feature of the test section was that the gages and cells (except for a few supplemental cells) were installed at one elevation and readings for different depths of soil above the cells were obtained by cutting off successive 1-ft. lifts and repeating the program of loading. This feature eliminated some of the variations that occur when different cell and gage installations are used to obtain a variation with depth.

The majority of the stresses measured in this investigation were those produced by the applied loads, termed induced stresses. An attempt was made to measure residual stresses, those present after the load is removed, but the pressure cells were not well designed for this type of measurement. The data that were obtained on residual stresses together with the data obtained from Stockton Test Section No. 2 (6) indicate the possibility that flexible pavements subjected to repeated loads may have residual stresses well in excess of those caused by the weight of the soil and pavement. Future installations will be made with cells that are adapted to the measurement of residual stresses. It is emphasized that all stresses presented in this paper are induced stresses, those caused by the applied loads.

Soil - The soil used in constructing the test section is a weathered loess native to Vicksburg area. The liquid limit averages 36 percent, and the plasticity index averages 12 percent. The material shows a mild reaction to the "shaking" test and therefore is designated as a clayey silt, although its behavior characteristics are more nearly those of a lean clay. The material is classified as ML to CL by the Corps of Engineers' classification and as A-4 by the classification of the Bureau of Public Roads. From the results of laboratory tests on compacted samples and from previous experience it was believed that the design CBR of 10 percent could be obtained at a moisture content of about 18 percent and a density of about 105 lb. per cu. ft. In constructing the test section the soil was processed to a uniform moisture content, placed in 4-in. lifts, and rolled with a sheepsfoot roller. The average value of 25 CBR tests made in the area of the cell installation was 10.8 percent; all but three of the tests fell between 8 and 13 percent. Moisture, density, plate bearing, and triaxial tests were also made, but the results of such tests are not presented in this paper except for a portion of the triaxial results presented with certain analyses in subsequent paragraphs. Results of all tests will be presented in detail in a report now being prepared by the Waterways Experiment Station. After construction was completed in May 1947, the test section was covered with asphalt-treated burlap. No significant changes occurred in the moisture and density throughout the period of testing (May 1947 through March 1948), and no significant changes in CBR occurred through December 1947. However, tests made in March 1948, after testing had been completed, showed that the CBR had increased to a range of 13 to 19 percent with an average of about 16 percent.

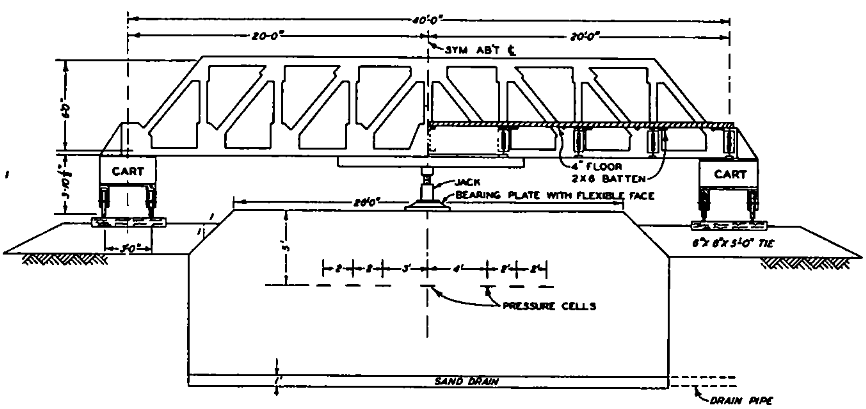


Figure 1. Loading Truss and Carts - Cross Section of Test Section

Pressure Cell and Deflection Gage Installation - WES 12-in. diameter pressure cells were used in the primary installation. Readings of all the cells were made practically simultaneously with a Baldwin Southwark automatic recorder. The pressure cells and appurtenant apparatus are described elsewhere (3, 6). The gages used for the major program of deflection measurements utilized a unique application of a pair of selsyn motors. Figure 2 is a schematic diagram of the deflection gage installation. A reference rod was seated in the soil about 20 ft. below the gage installation so that it would not move during

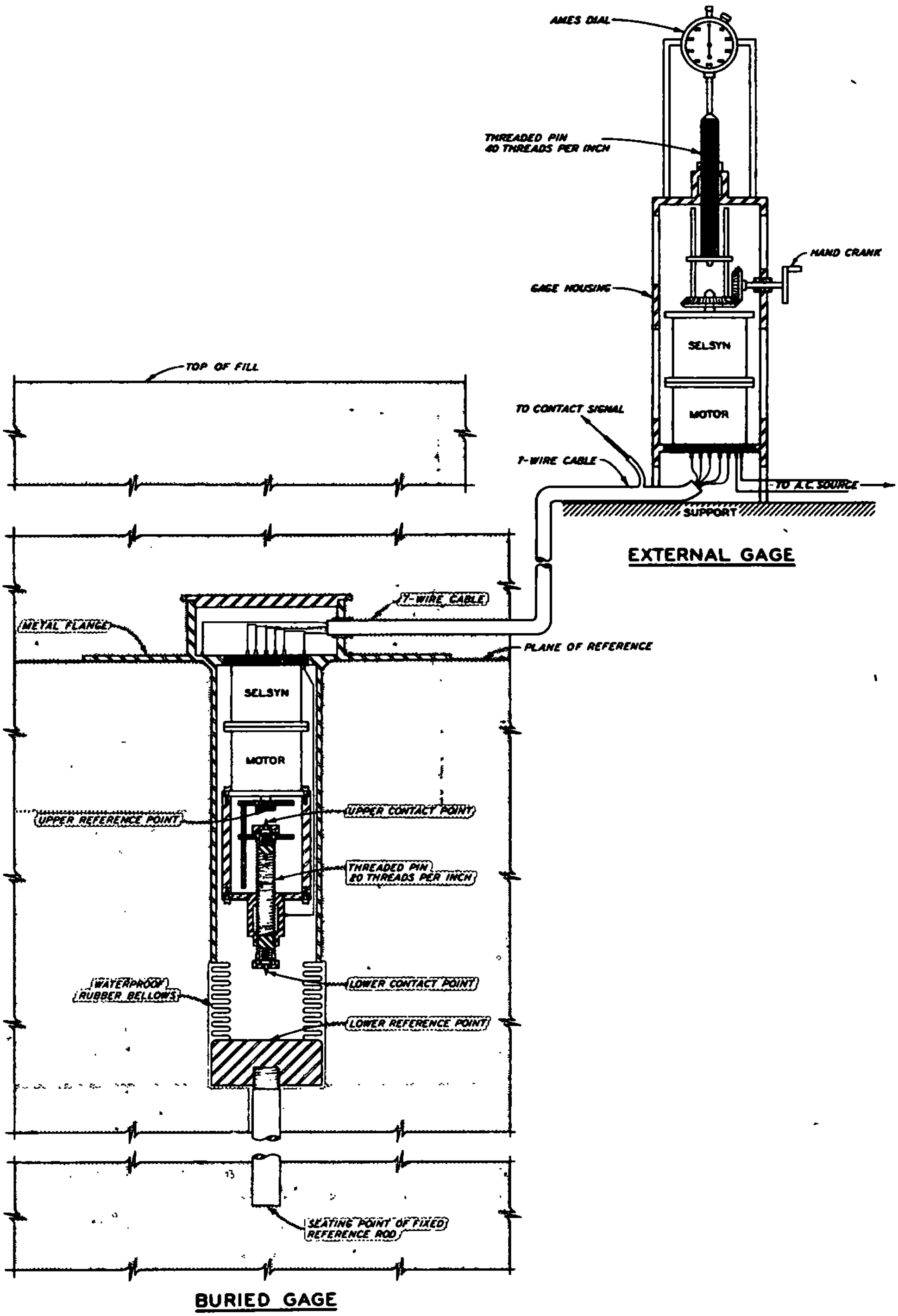


Figure 2. External and Buried Selsyn Deflection Gages

load applications. A metal flange was seated on the plane of reference. Differential movement between the reference rod and the plane of seating was measured with a simple micrometer. The micrometer screw in the buried gage was rotated by a selsyn motor connected to a second motor at an external location. Since the selsyn motors are "geared together electrically," any rotation at the external gage was duplicated on the buried gage. A secondary electrical system indicated when the micrometer in the buried gage contacted the reference rod. Other sizes and types of deflection gages and pressure cells were installed for special purposes, but the results are not treated in this paper.

The pressure cell layout shown in Figure 3 was designed to give readings that would permit the resolution of the major and minor principal stresses on planes of symmetry under single and dual loads. Cells were installed in the vertical, two horizontal, and two diagonal directions. The cells were spaced along lines U, V, X, Y, and Z, at intervals so that when loads were applied at the intersection of these lines and lines A and B, readings would be obtained directly under the center of the load and at offset distances up to 9 ft. The deflection gages were spaced in a similar manner along line W. Deflections were measured in the vertical direction only.

The primary installation was made at a depth of 5 ft. below the elevation of the finished grade of the test section. A few cells were installed at the 7-ft. depth. In each case, after construction had proceeded to an elevation about 1 ft. above the elevation at which the cells were to be installed, a small, shallow pit was excavated and the cell embedded carefully. Backfill was placed around the cells and tamped with air tamps. Care was taken not to damage the cells and continual check readings were made during installation to determine if damage occurred. During tamping numerous density samples were taken to insure that the tamped material was compacted to the same density as the surrounding soil. Figure 4 is a view of operations during placement of the cells.

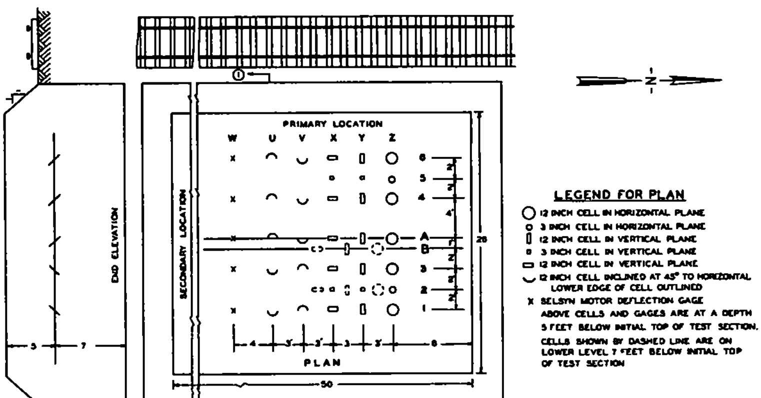


Figure 3

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS HOMOGENEOUS TEST SECTION

PLAN, ELEVATION AND CROSS SECTION

Loading Equipment - The loads were applied by jacking against a steel truss which spanned the test section. The truss was mounted on wheels so that it could be moved the entire length of the test section. The jacking equipment consisted of an electrically driven pumping mechanism which applied pressure in controlled amounts to one or more of four hydraulic jacks. Loads were applied through circular bearing

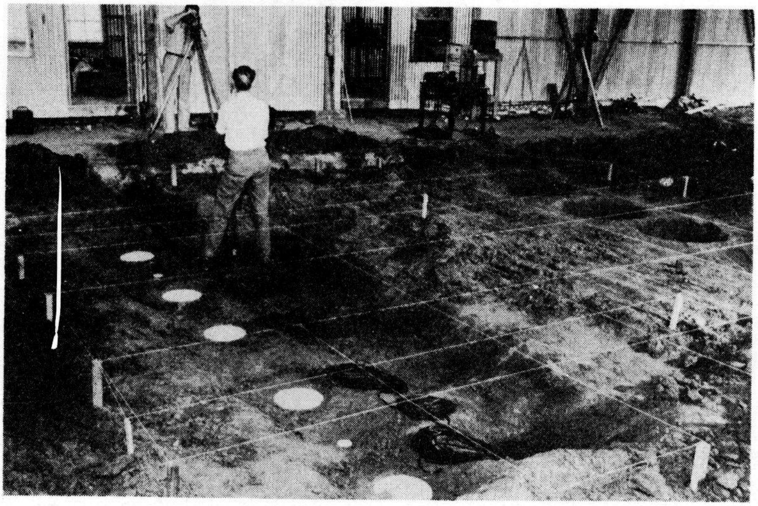


Figure 4. Installation of Horizontal Cells

plates developed for this project from basic ideas supplied by the Bureau of Public Roads. The ground-contacting faces of these plates are water-inflated rubber diaphragms. Figure 5 shows

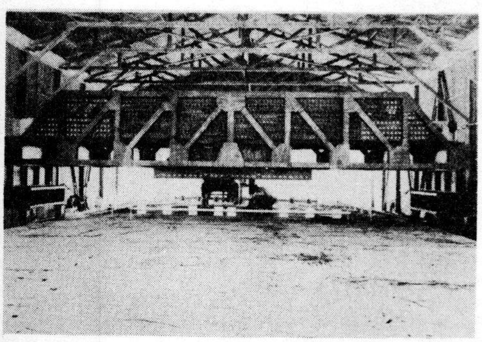


Figure 5. Steel Truss for Load Reaction

the loading truss, the jacks, and the bearing plates.

Tests Performed - Single and dual loads of 15,000, 30,000, 45,000 and 60,000 lb. were applied to the surface of the test section in the order named. The single loads were applied with a 1,000-sq. in. plate and the dual loads were applied with two 500-sq. in. plates. Since the total contact area for both single and dual loads was 1,000 sq. in., the contact pressures were 15, 30, 45, and 60 psi. respectively. Loads were applied with the dual loads spaced 3.0, 4.5, 6.0, and 7.5 ft. center to center. Pressures and deflections were measured at points in vertical planes which were symmetrical with respect to the loaded areas. After the program was completed on the surface of the test section, the top 1 ft. of the test section was cut off and the program was repeated. This procedure was continued so that measurements were made with heights of fill of 5, 4, 3, 2, and 1 ft. In most cases each load was applied and pressures and deflections were measured three times in order to obtain good average values. Approximately 30,000 pressure readings and 3,000 deflection readings were obtained. Only typical cases and summarizations are presented in this paper. Results will be available in detail in the report previously mentioned.

Stress Notation - In any presentation of pressure cell readings it is practically necessary to adopt symbols. In accordance with the usually accepted standards, the Greek letter sigma (σ) has been used for normal stresses and tau (τ) for shearing stresses. Subscripts are used to indicate the particular stress. Figures 6 and 7 include the terminology used for pressure cell readings and the terminology applied to both normal and shearing stresses on an elemental cube. It can be seen on

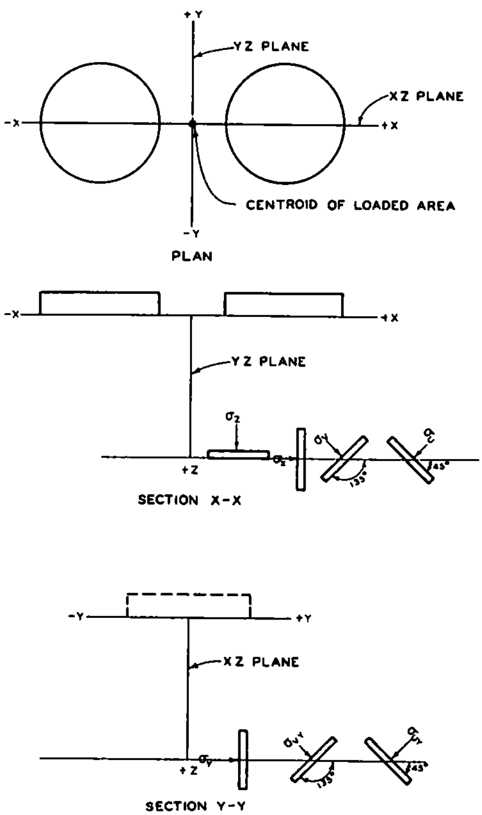
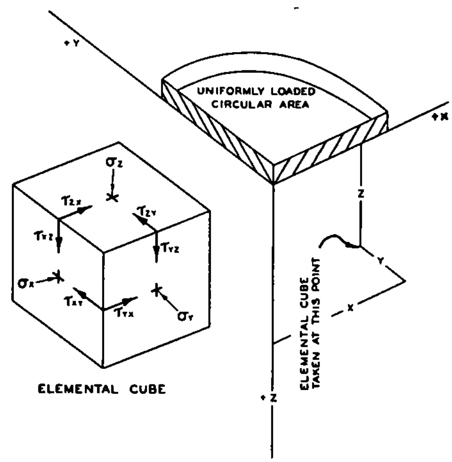


Figure 6. Orientation of Pressure Cells

Figure 7 that the stress on a horizontal plane has been designated sigma z (σ_z); correspondingly, on Figure 6 the stress measured by the pressure cells placed in a horizontal plane is also termed σ_z. Stresses on the two vertical planes are designated σ_x and σ_y. There is no accepted terminology for stresses on diagonal planes, and these have been arbitrarily designated as σ_u and σ_v. In the designation of stresses, especially shear stresses, it is necessary to assume values in one direction as positive and in the other as negative. Figure 7 shows the direction assumed as positive for the various stresses.

Presentation of Results - The reduction of the 30,000 stress measurements and 3,000 deflection measurements followed a routine procedure which is described very briefly. The pressure cell readings were obtained from the recorder charts in terms of resistance change in the electrical circuit and were converted to pounds per square inch by means of calibration charts prepared



NOTE ALL STRESSES INDICATED ARE POSITIVE

Figure 7. Schematic Drawing of Stresses on an Elemental Cube

in advance for each pressure cell. Multiple readings (usually triplicate) for the same test setup were averaged and plotted on diagrams of stress versus horizontal distance from the centroid of the loaded area. Figures 8 through 12 are typical plots for vertical, horizontal, and diagonal stresses at a depth of 1 ft. Pressure readings under the single bearing plate are shown in the upper left plot; pressures under dual plates spaced 3.0, 4.5, 6.0, and 7.5 ft. center to center are shown in the other plots. Similar plots were made for depths of 2, 3, 4, and 5 ft. but are

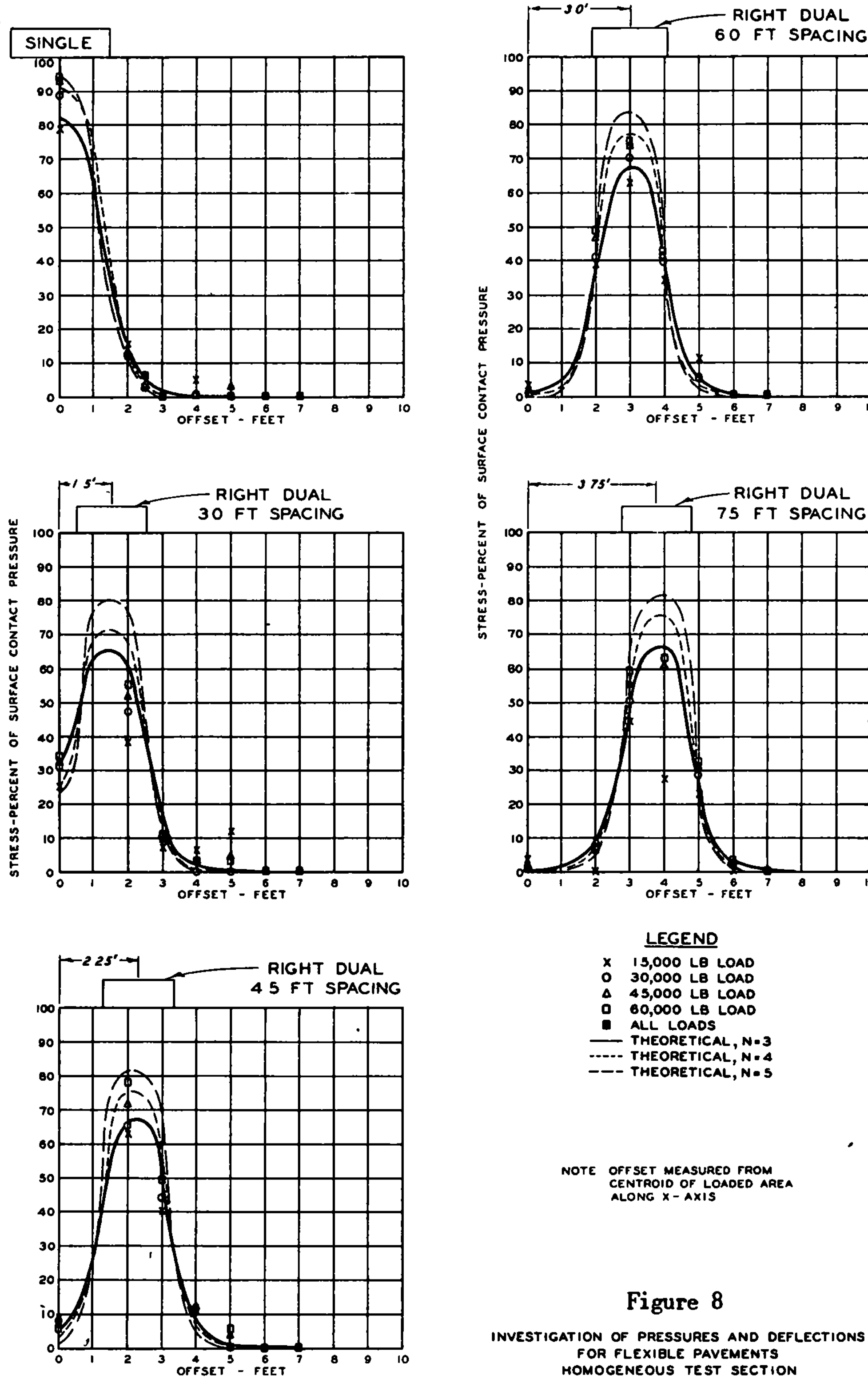


Figure 8

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_z AT 1 FT DEPTH

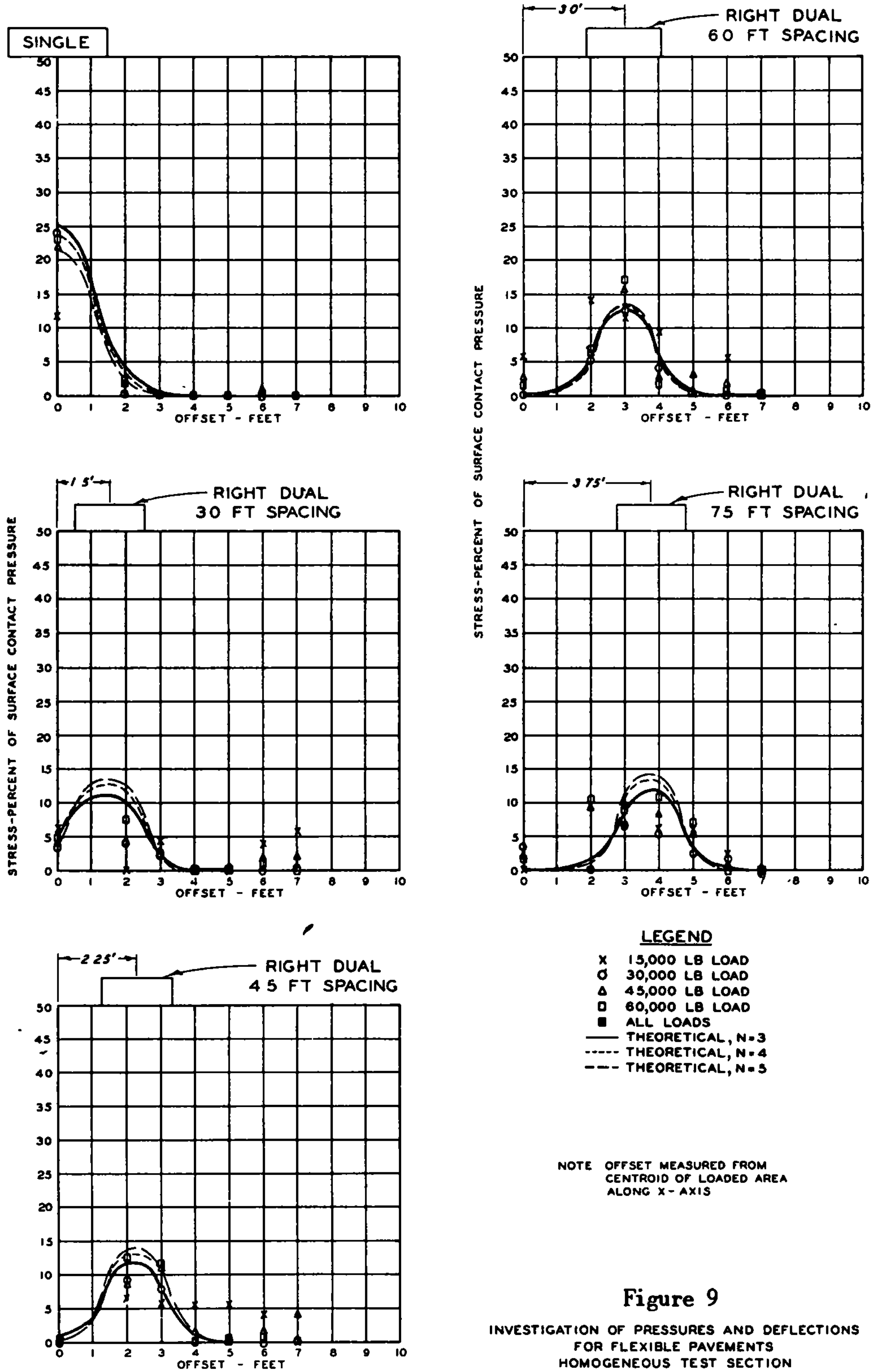


Figure 9

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_Y AT 1 FT DEPTH

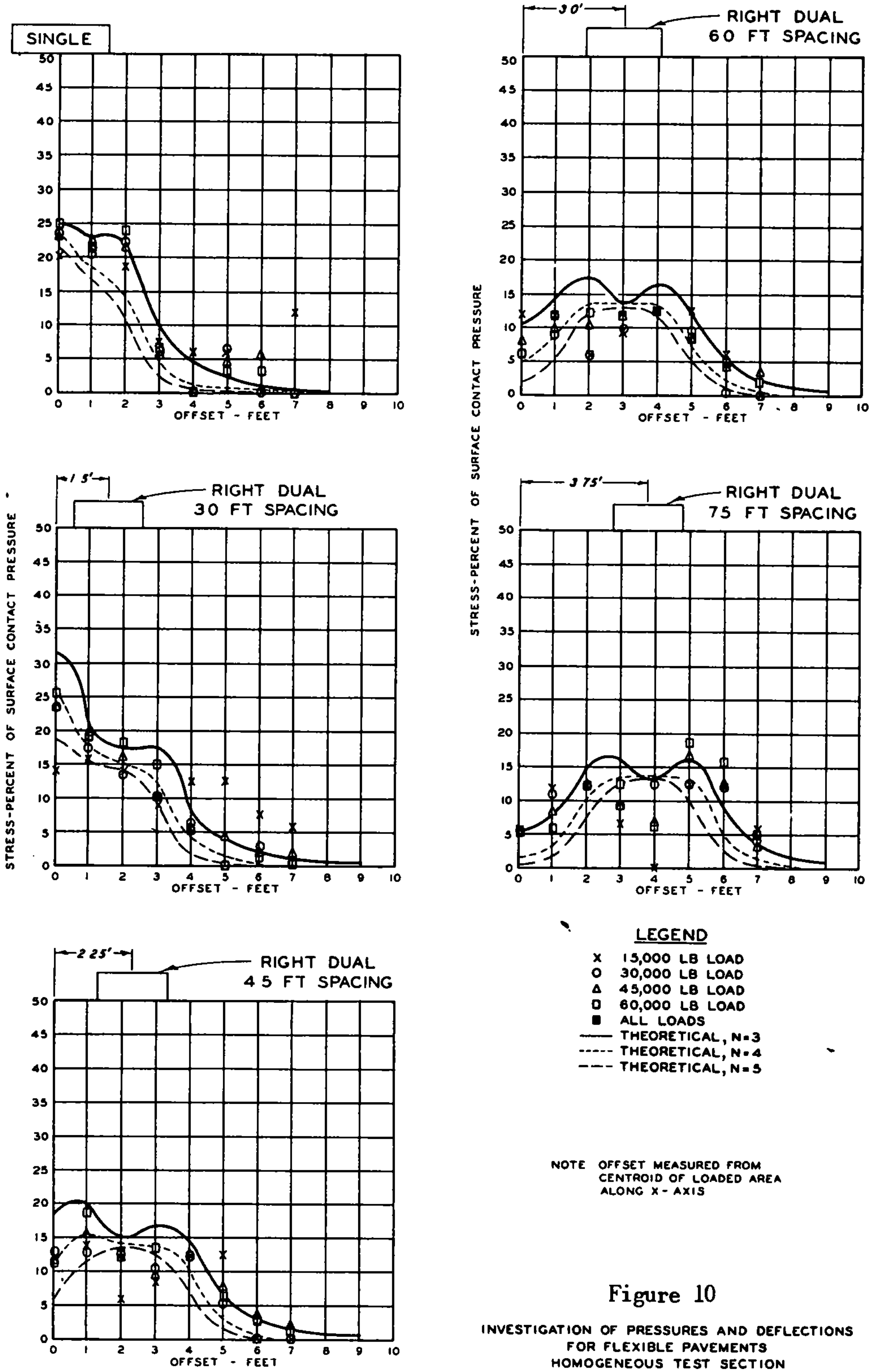


Figure 10

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_x AT 1 FT DEPTH

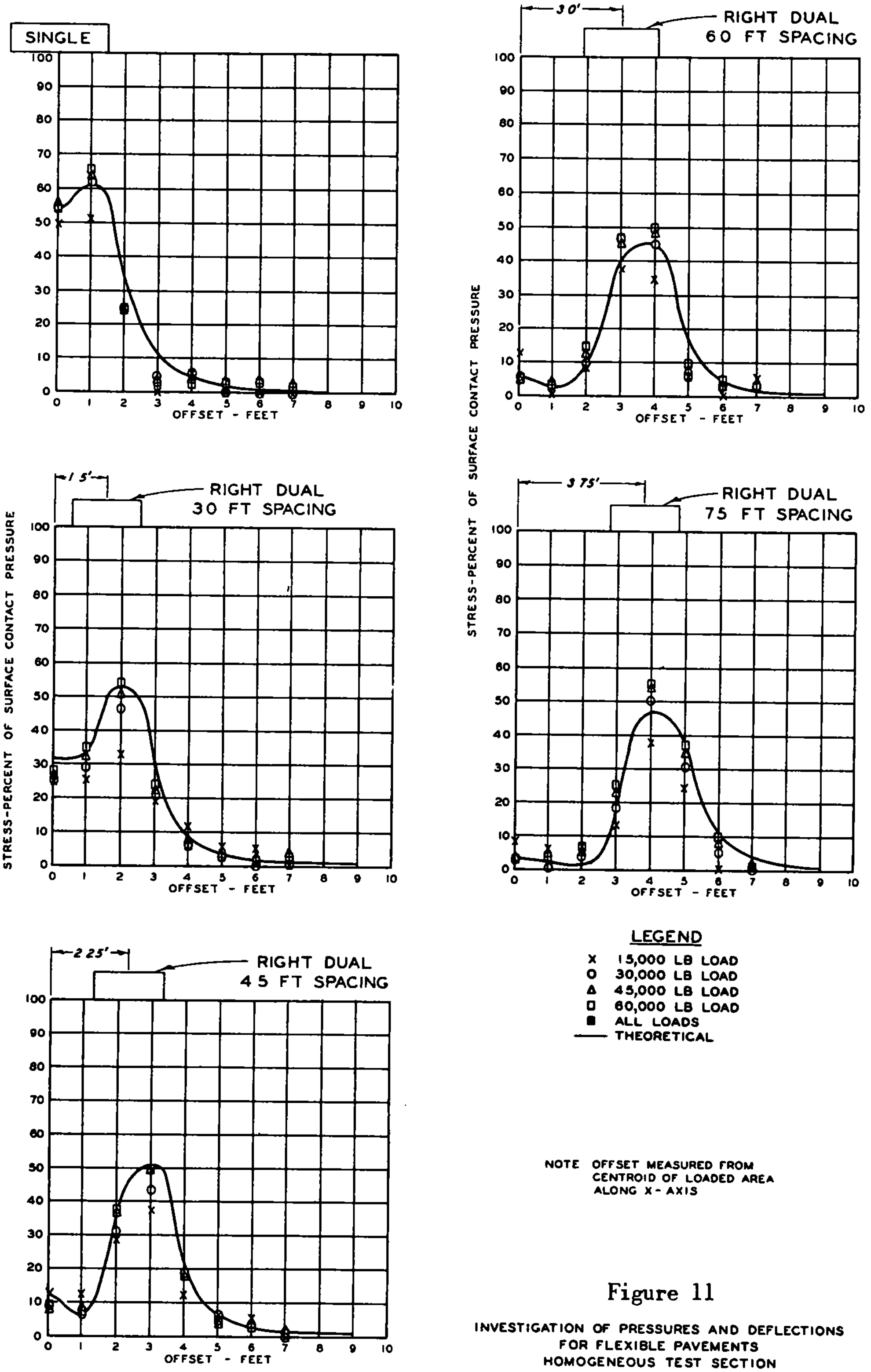


Figure 11

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_v AT 1 FT DEPTH

not shown. In all cases the measured data are shown by plotted points with a separate symbol for each load. All stresses are plotted as a percentage of the contact area. This method of presentation has the distinct advantage of permitting a rapid comparison of the values obtained under the different total loads. According to elastic theory, the stress at a point is a direct multiple of the contact pressure, therefore, where the stress is expressed as a percentage of the contact pressure the values for the four loads should plot at the same point. The method has a minor disadvantage in that it tends to exaggerate deviations where the readings are small. Curves are shown on the plots which represent the stresses computed from elastic theory. Curves for a concentration factor (7) of n = 3 are straight elastic theory; curves for concentration factors of n = 4 and n = 5 are shown except on Figure 12.

Figure 13 is a typical plot of shearing stresses. This example shows the values for the shearing stress τ_{xz} at a depth of 1.0 ft. Shearing stresses actually were not measured but were computed from measured normal stresses. The plotted points on Figure 13 were computed from the diagonal stresses

$$\left(\tau_{xz} = \frac{\sigma_v - \sigma_u}{2} \right).$$

Curves of the stresses as computed from elastic theory also are shown on Figure 13. The distinction between stresses computed from measured values and those computed from elastic theory should be noted since stresses computed from measured values are considered in the same light as basic data.

The primary reason for installing pressure cells at four different angles as in this test section was that it would permit the determination of the major (σ_1) and minor (σ_3) principal stresses on planes of symmetry. The method for computing the major and minor principal stress from pressures on horizontal, vertical, and diagonal planes has been described elsewhere (8) and is not presented here. Figures 14 through 18 present the values of the major principal stress as computed from the measured pressures for the various conditions of loading and for depths of 1 through 5 ft. Figures 19 through 21 present values of the minor principal stress at depths of 1 through 3 ft. Values at 4-and 5-ft. depths were very small and are not presented. Isobars of the major and minor principal stresses are shown on Figures 22 and 23 for the 45,000- and 60,000-lb. -wheel load. Isobars of the maximum shearing stress

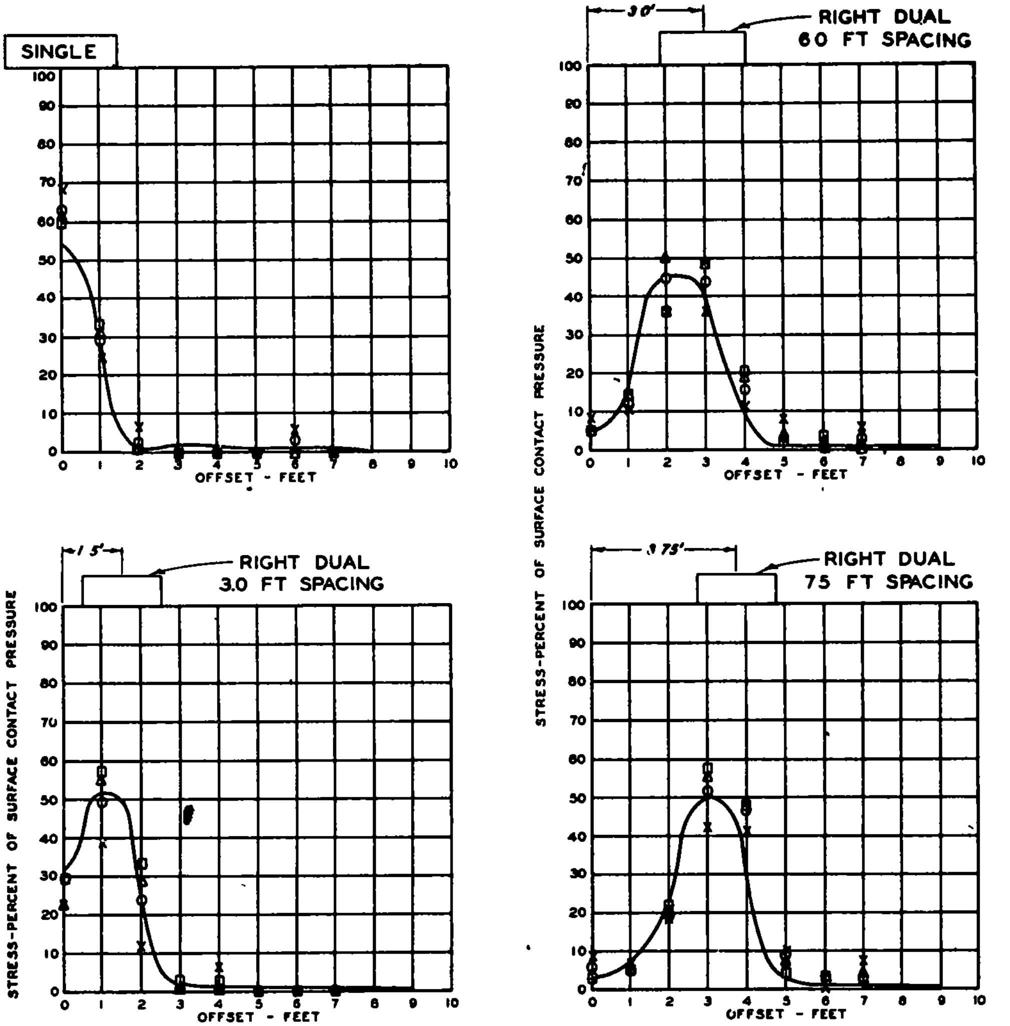
$$\left(\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \right)$$

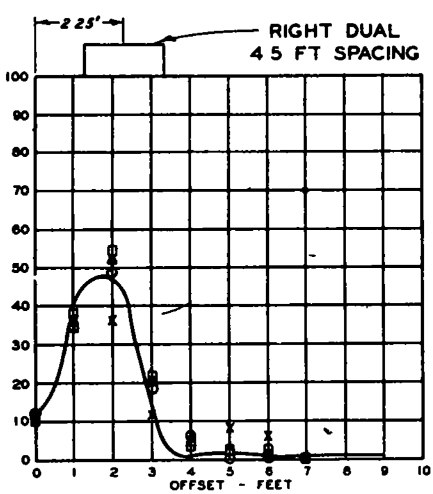
are shown on Figures 22 and 23 also. Stresses computed from measured values are shown as solid lines; stresses computed from elastic theory are shown by dashed lines.

The reduction of deflection measurements followed the same general procedure as the reduction of pressure cell readings. Deflections were measured directly in inches. Multiple readings (usually duplicate or triplicate) for the same setup were averaged and plotted against offset distance. As in the case of stresses, the deflections were plotted in ratio form so that the theoretical deflections for all four loads would plot as one point. Figures 24 through 28 show the measured deflections for all conditions of loading and depth. As an example of the actual deflection in inches, the values at a depth of 1 ft. under the center of the single plate (Fig. 24 upper left) were 0.036 in., 0.082 in., 0.136 in., and 0.220 in. for the 15,000-, 30,000-, 45,000-, and 60,000-lb. loads, respectively. The curves shown on Figures 24 through 28 were plotted from values computed from elastic theory. In order to develop curves that would cover the range of the measured deflections, values of 5,000, 10,000, and 25,000 psi. were assumed for the elastic modulus.

ANALYSIS

Consistency and Accuracy of Pressure Cell Readings - One of the most intriguing aspects of any study of stresses is whether or not the pressure cell readings actually represent the stresses





LEGEND

- X 15,000 LB LOAD
- O 30,000 LB LOAD
- Δ 45,000 LB LOAD
- □ 60,000 LB LOAD
- ■ ALL LOADS
- —— THEORETICAL

NOTE OFFSET MEASURED FROM CENTROID OF LOADED AREA ALONG X-AXIS

Figure 12

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE

σ_v AT 1 FT DEPTH

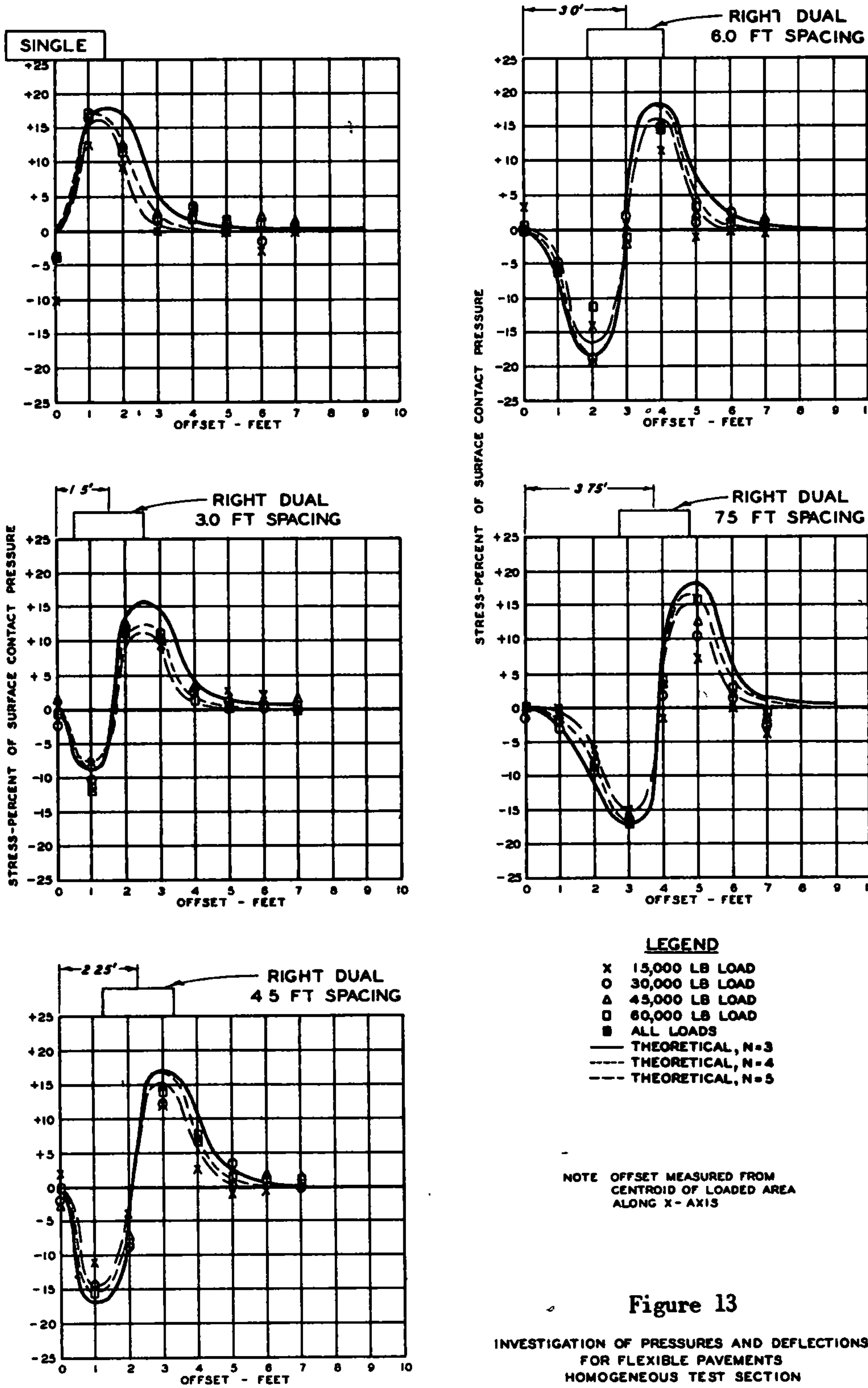


Figure 13

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
τ_{xz} AT 1 FT DEPTH

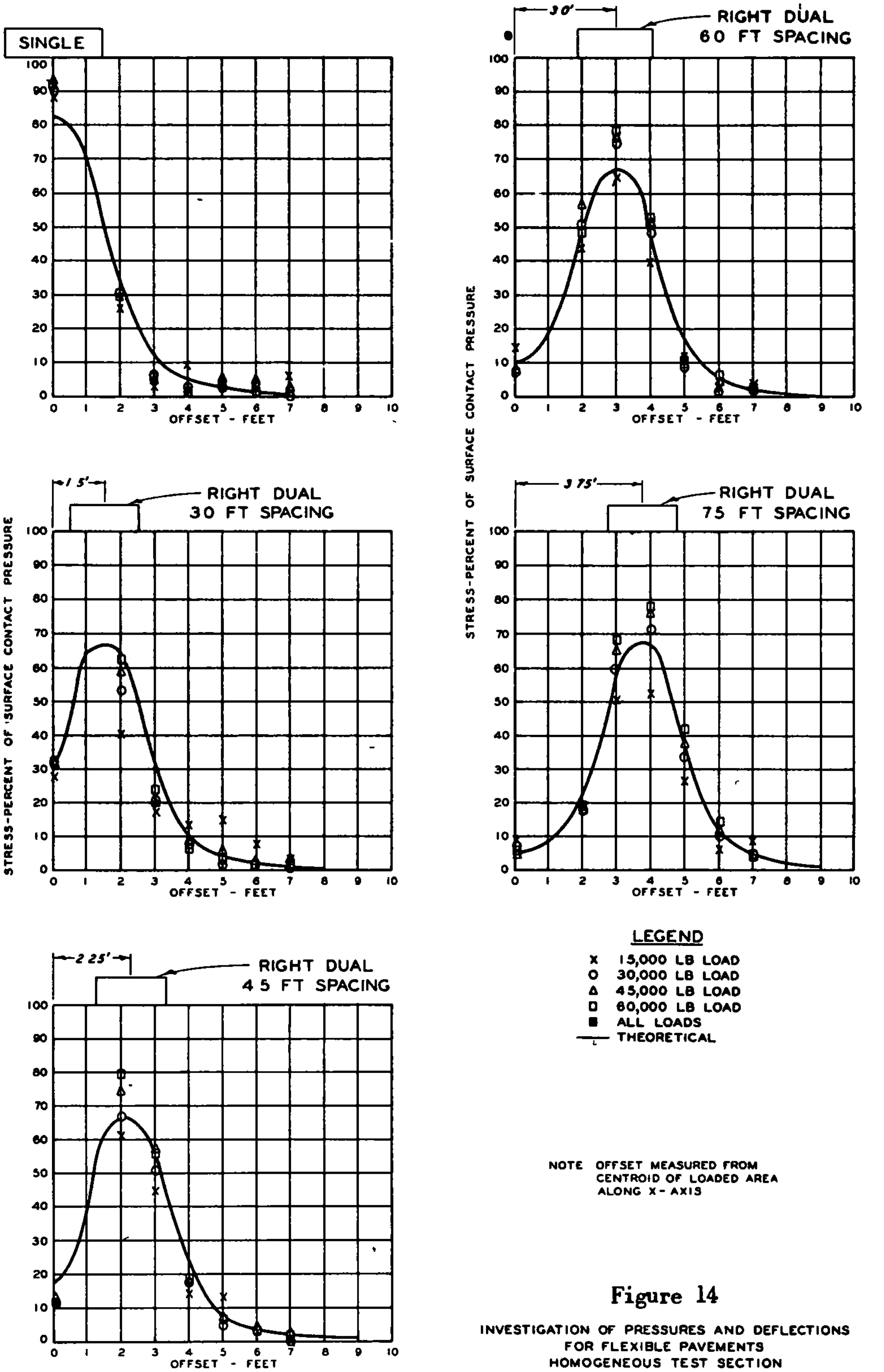


Figure 14

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_1 AT 1 FT DEPTH

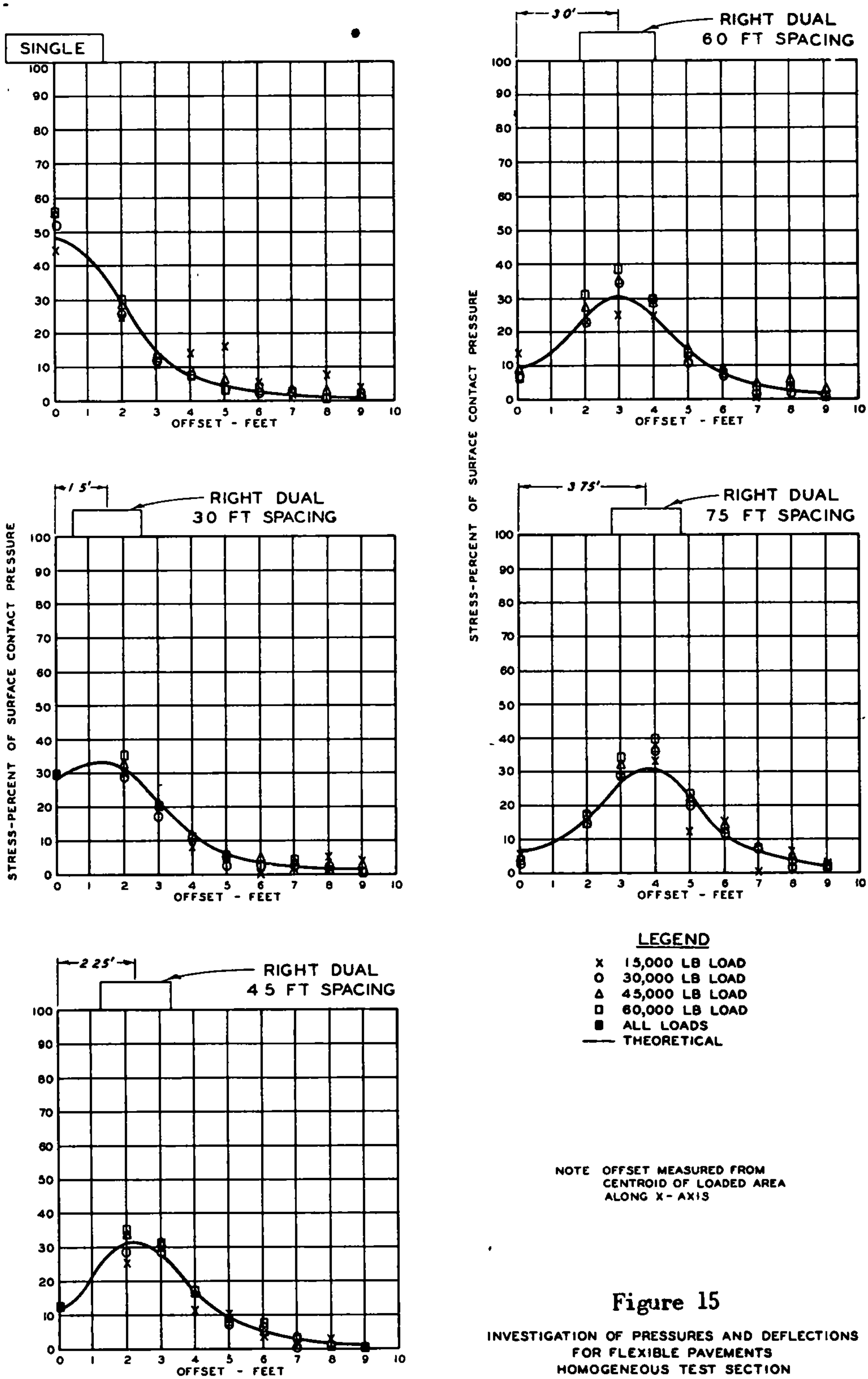


Figure 15

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_1 AT 2 FT DEPTH

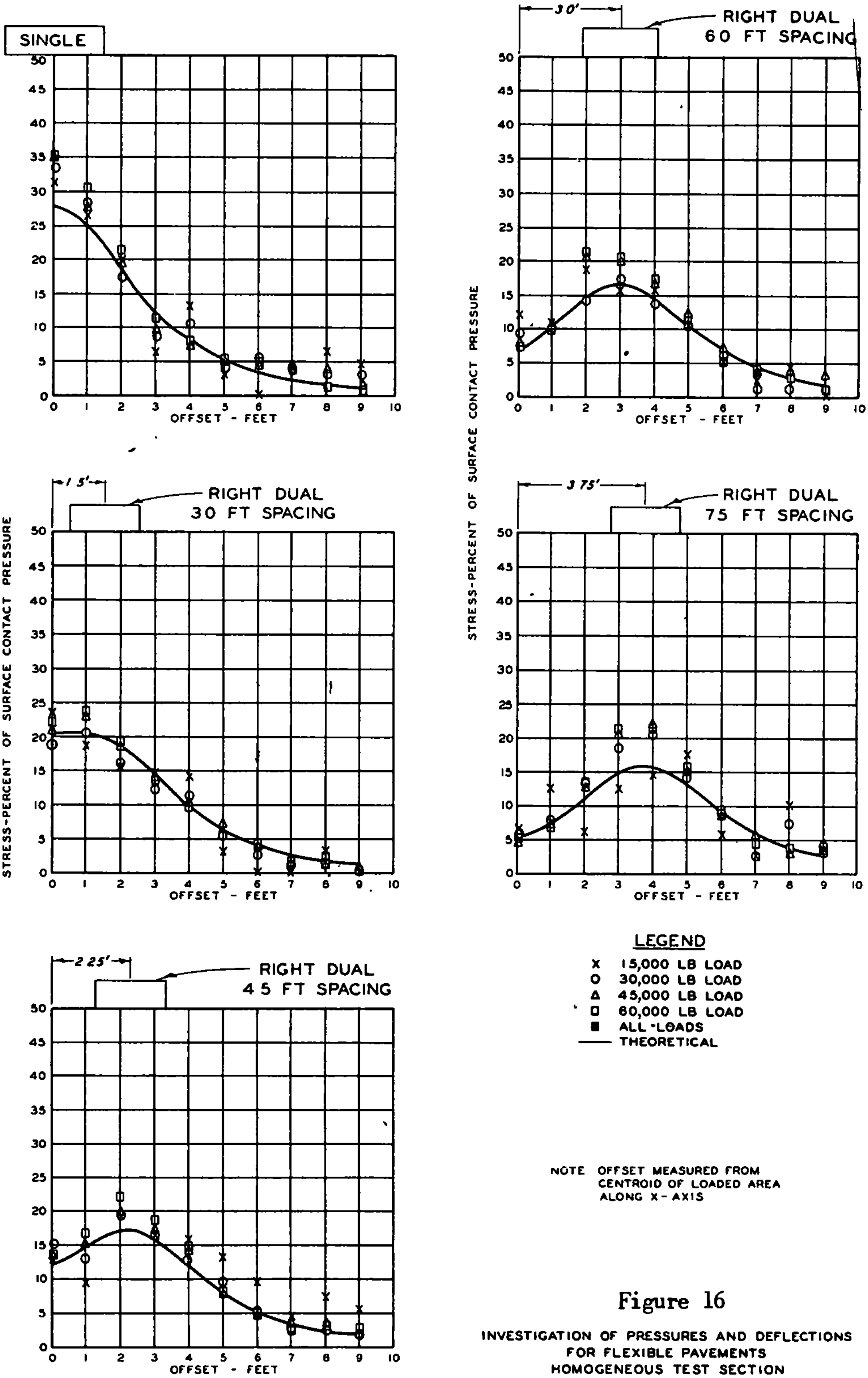


Figure 16

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_1 AT 3 FT DEPTH

present in the soil mass. An effort was made to answer this question by applying certain checks to the pressure cell readings. The majority of the checks were made as a part of a separate study on the development of earth pressure cells and they are not presented in detail. They included a study of the precision of the instrumentation, comparison of stresses measured by pressure cells at homologous points, and summations based on the requirements of static equilibrium. The results are summarized as follows:

1. The precision of the instrumentation including the pressure cells was such that a maximum error of 1.0 psi. plus or minus could occur from this source but the average error from this source probably was of the order of 0.5 psi.

2. Stresses measured by pressure cells at homologous points showed a deviation from the average of less than 1 psi. in 95 percent of the cases.

3. Summations based on the laws of statics indicated that the total loads applied at the surface were being registered on the pressure cells definitely within 25 percent and probably within 10 percent with no consistent over- or under-registration for the system as a whole.

Comparison of Measured and Computed Stresses - According to elastic theory when stresses are expressed as a percentage of the contact pressure, the individual points for the 15,000-, 30,000-, 45,000-, and 60,000-lb. loads, which have contact pressures of 15, 30, 45, and 60 psi., respectively, should plot as one point at any one condition of loading, depth, and offset. A study of the vertical stress measurements at 1-ft. depth (Fig. 8) shows that where the recorded stress was less than about 10 percent of the contact pressure there is no systematic order, and the variations represent a normal scattering of data. However, where the recorded stresses were greater than about 10 percent of the contact pressure, there is a consistent tendency for the smallest load to give the smallest stress and the largest load to give the largest stress. Where the measured stress was in the order of 25 percent of the contact pressure, this deviation was in the order of five percentage points; where the measured stress was above 25 percent of the contact pressure, the deviation ranged up to about 20 percentage points. It is difficult to visualize the magnitude of this deviation and a numerical example is given. On Figure 8 under dual loads spaced 3.0 ft. (center plot on left) at 2.0 ft. offset, the vertical stresses were 38, 48, 52, and 56 percent of the contact pressure for the four loads, respectively, which represents an 18-point deviation. The actual pressures were 5.7, 14.3, 23.5, and 33.3 psi. for the 15,000-, 30,000-, 45,000-, and 60,000-lb. loads, respectively. If it is assumed that the midpoint of the range was correct (47 percent), then pressures of 7.1, 14.1, 21.2, and 28.2 psi. would be required to plot as a single point. The lowest value would be increased 25 percent, and the highest value decreased 15 percent. The percentage change for the five-point deviation for values between 10 and 25 percent of the contact pressure would be about the same since the actual pressures in this range were lower. This same trend was repeated in all the measurements of σ_z at other depths where values above 10 percent of the contact pressure were recorded. As noted on Figures 9 and 10, the recorded stresses of σ_x and σ_y were generally small; even where they exceeded 10 percent of the contact pressure, there is no precise trend for the points to plot in order of load. Recorded values of σ_u, σ_v, and τ_{xy} show a definite trend to plot in accordance with load. Values for the major principal stress also follow this trend, but the values for the minor principal stress are small and the trend is not exhibited.

In Figures 8, 9, 10, and 13, theoretical curves are shown for concentration factors of n = 3, 4, and 5. Where the stresses are below about 10 percent of the contact pressure, the theoretical curves show very little difference for the three concentration factors, and any of the curves represent a good

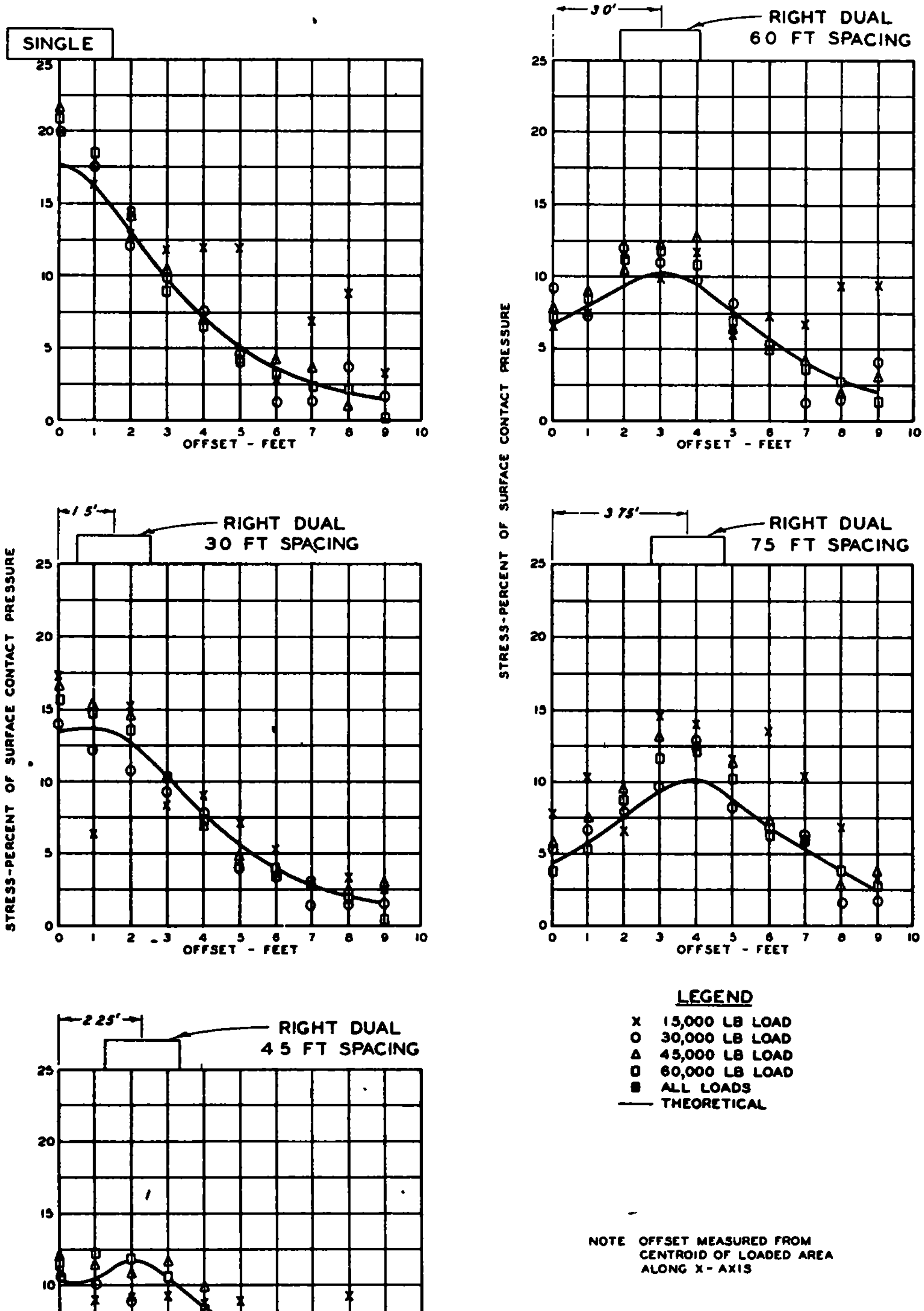


Figure 17

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_1 AT 4 FT DEPTH

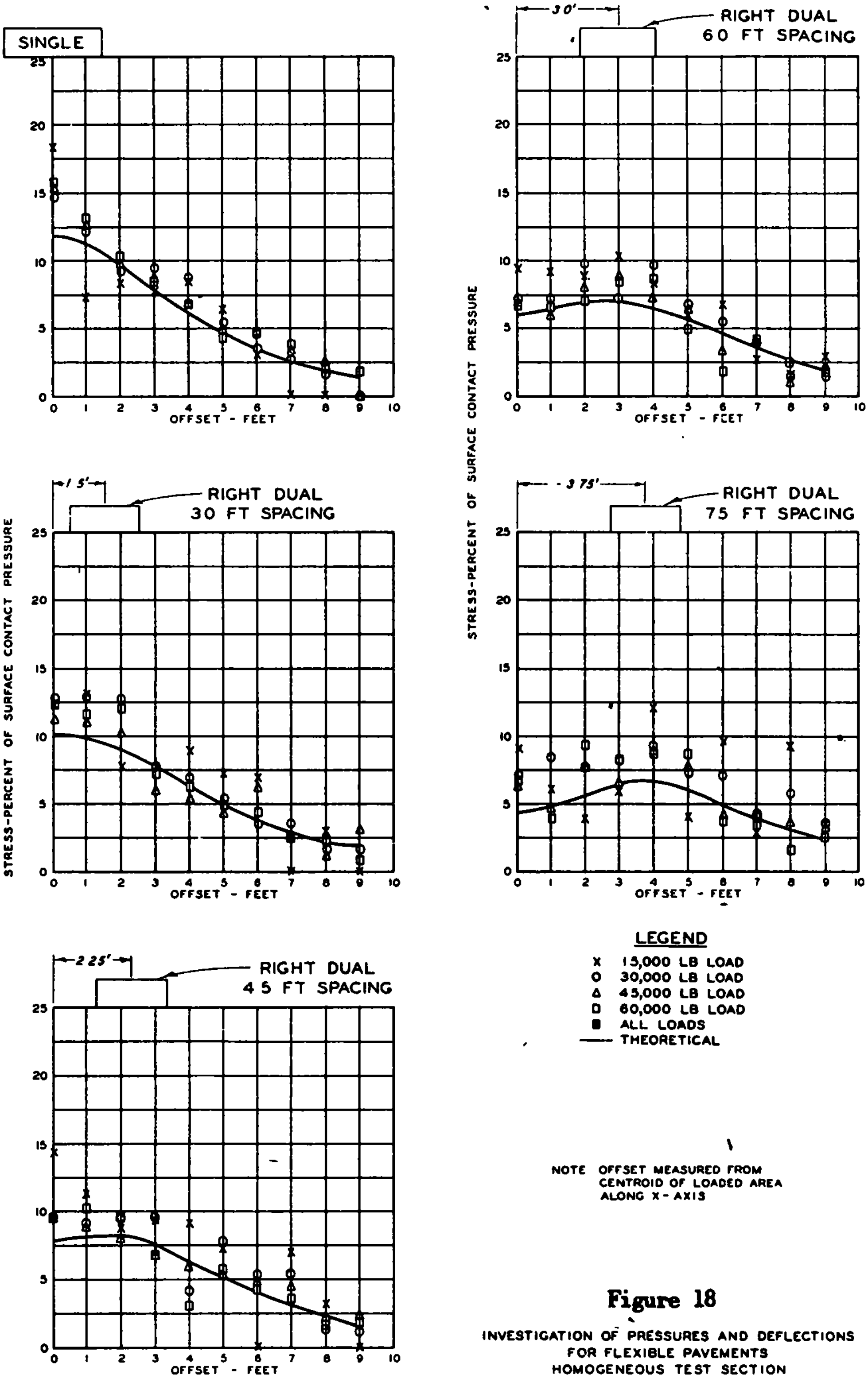


Figure 18

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_1 AT 5 FT DEPTH

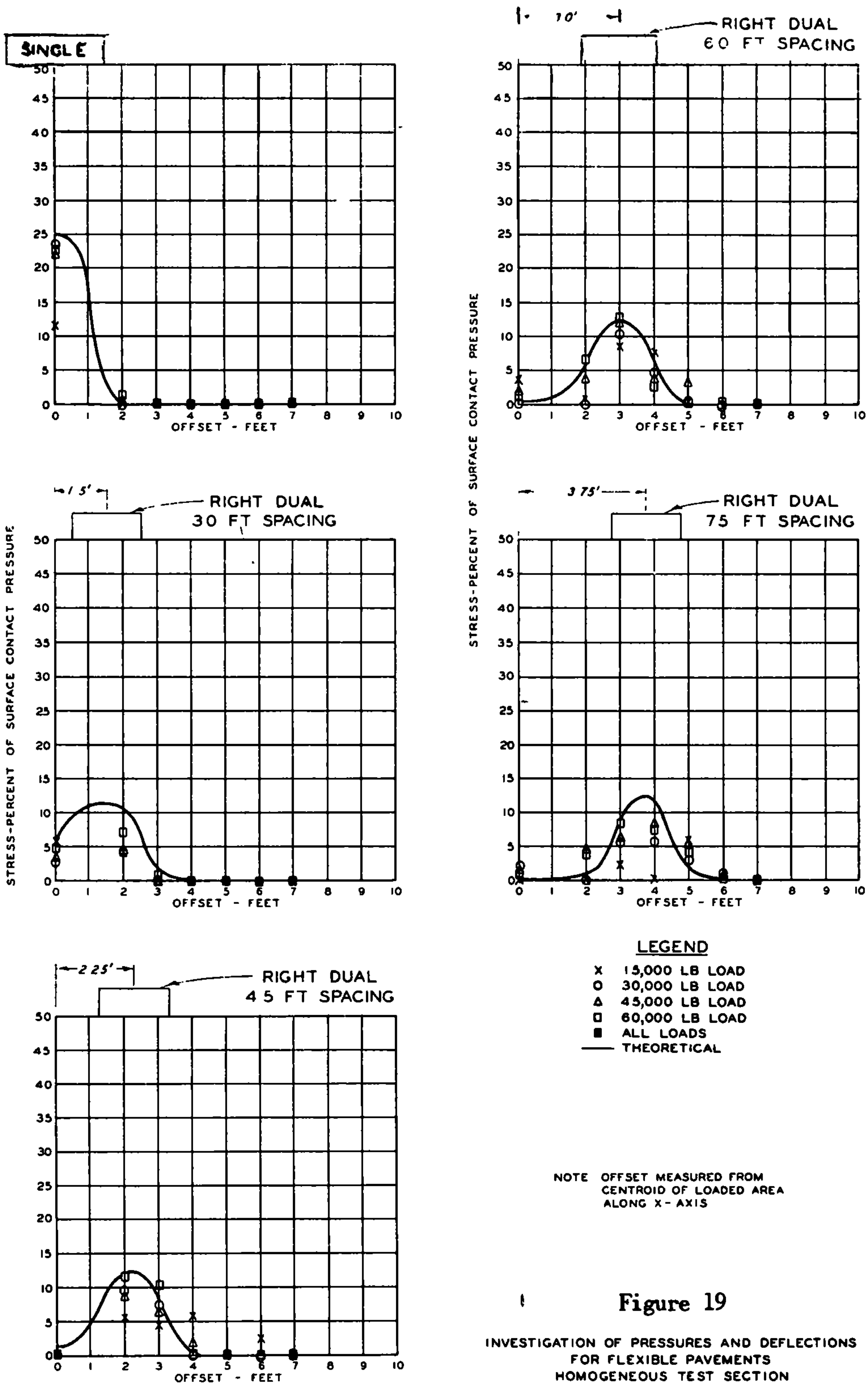


Figure 19

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_3 AT 1 FT DEPTH

curve through the recorded data in this zone. For the higher stresses, the curves for the different concentration factors show sufficient difference to be considered. In a few instances (top plots on Fig. 8) the curve for n = 3 agrees with the measured stresses for the 15,000-lb. load, and the curves for the higher concentration factors agree best with the higher loads. In the majority of the cases for vertical stress (including data from other depths) the best over-all agreement in the zones above 10 percent of the contact pressure is obtained with the theoretical curve for n = 3; curves for n = 4 and n = 5 usually show values higher than the recorded stresses. Analysis of measurements of the other stresses shows that they follow this same trend except for minor deviations. In general, if an average of the four points at each location is considered, the theoretical curve for n = 3 represents the best systematic curve that could be drawn through the data if all conditions of load and depth and if all the stresses are considered.

A study of the isobars of stress (Figs. 22 and 23) shows a marked similitude between the distribution patterns developed from actual test data and those computed from theory for a concentration factor of 3. There is a consistent trend for the measured stresses to be greater than the theoretical stress at points beneath the load and to be approximately equal to or less than the theoretical stress at other points. The trend is more marked in the plots for the 45,000- and 60,000-lb. loads than in the plots for the 15,000- and 30,000-lb. loads which are not shown.

Comparison of Measured and Computed Deflections - Examination of Figures 24 through 28 reveals that values for the ratio of measured deflection to contact pressure tend to plot in order of contact pressure with noticeable consistency even where the deflections were small. This trend indicates that the stress-strain relationship for this soil is not constant as in an elastic body but varies with stress. A comparison of the plotted points with the theoretical curves also shows that the stress-strain relationship is not a constant. In general the points tend to fall on the theoretical curve computed with an elastic modulus of 25,000 psi. where the deflections were small and on the curve for an elastic modulus of 10,000 psi. where the deflections were large. Since the elastic modulus is the slope of the stress-strain curve, it is seen that the stress-strain relationship is not a constant. It also should be noted that under the dual loads the maximum deflection occurs directly beneath one of the loaded areas at shallow depths, but at greater depths the location of the maximum deflection tends to shift toward the centroid of the loaded area. This behavior has been noted in previous studies (2) and has been used to determine the effect of dual- as compared to single-wheel loads.

Stress-Strain Relationship - Probably the least known quantity in soil mechanics is the true, or in situ, stress-strain relationship. Throughout the history of soil mechanics, investigators have devised shear tests to duplicate prototype conditions culminating in the triaxial compression test, but none of the investigators has had a true, or in situ, stress-strain curve to compare with the test curve. In the usual triaxial test, the stress-strain curve is a plot of the deviator stress ($\sigma_1 - \sigma_3$, in other words the difference between the vertical and horizontal pressure) versus vertical strain. The test data may be expressed in similar form by assuming a value of 0.5 for μ (Poisson's ratio) in the following equation from the theory of elasticity

$$E_m = \frac{\sigma_z - \mu\,(\sigma_y + \sigma_x)}{\epsilon_z}.$$

This equation is an expression of the stress-strain relationship in an elastic body and is independent of the loading conditions or location. To obtain the values for the vertical strain (ϵ_z) the measured vertical deflections at a given offset were plotted versus the depth, and a curve drawn through the points. The slope of this curve at any

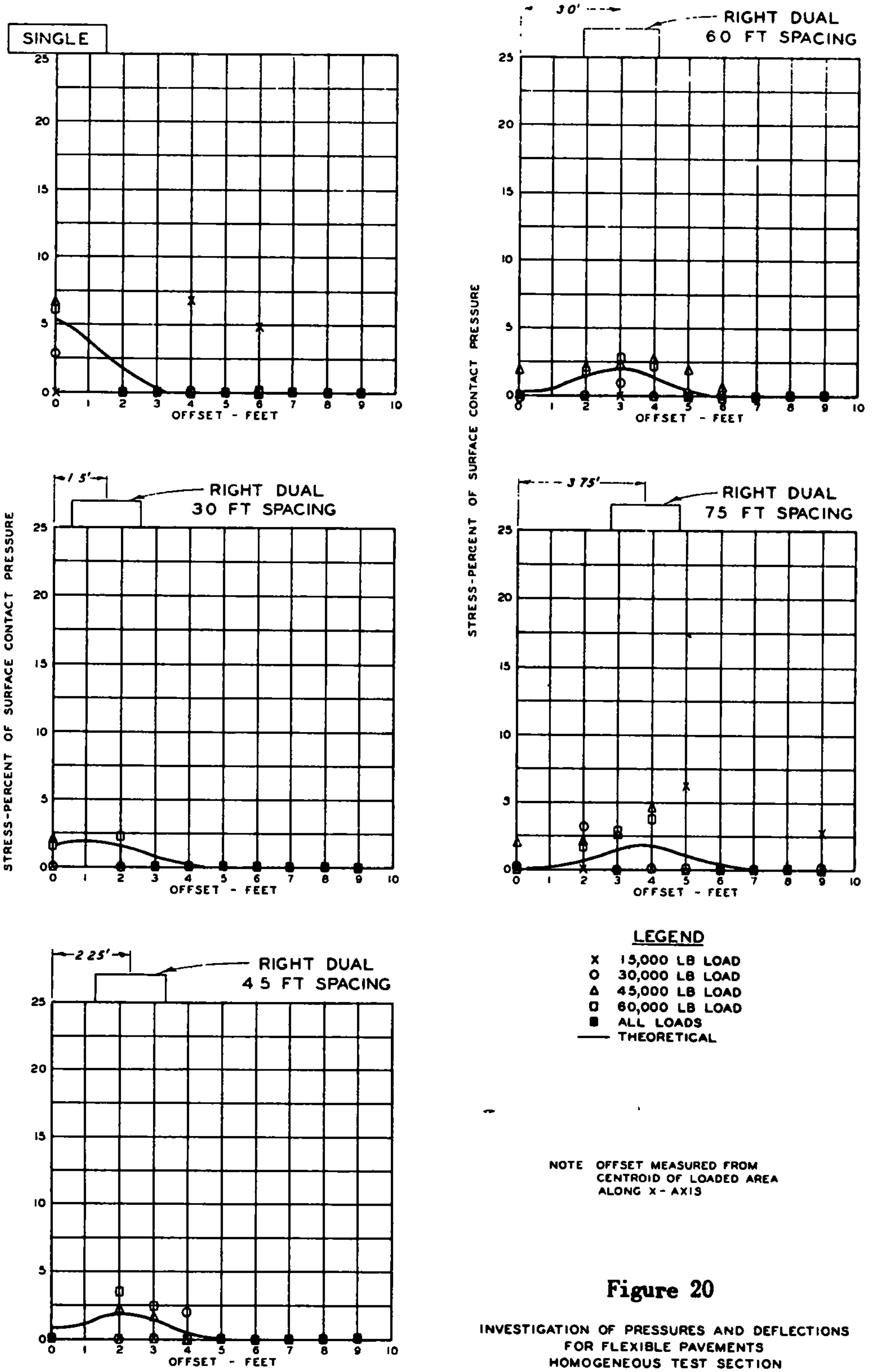


Figure 20

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

STRESS VS OFFSET DISTANCE
σ_3 AT 2 FT DEPTH

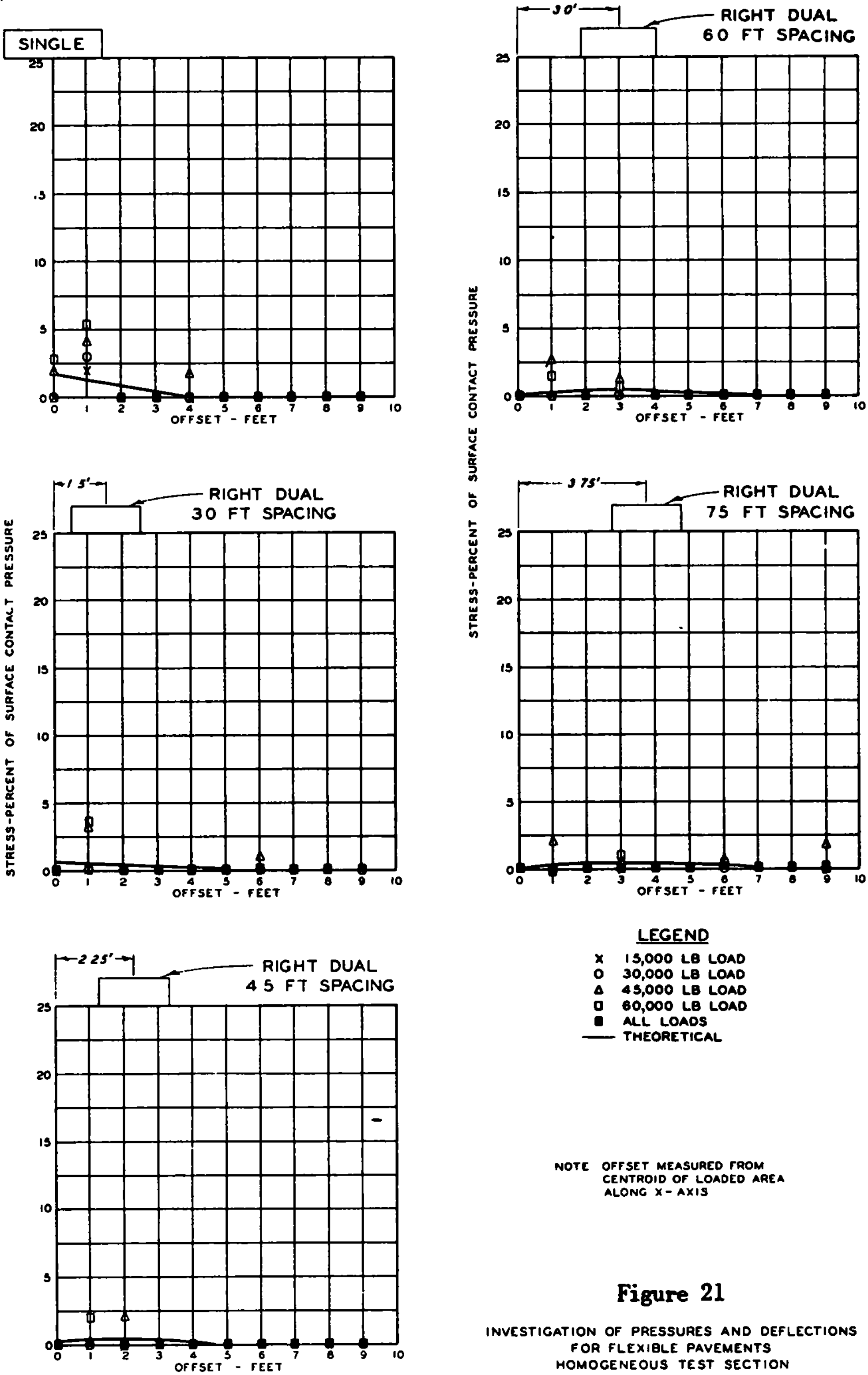
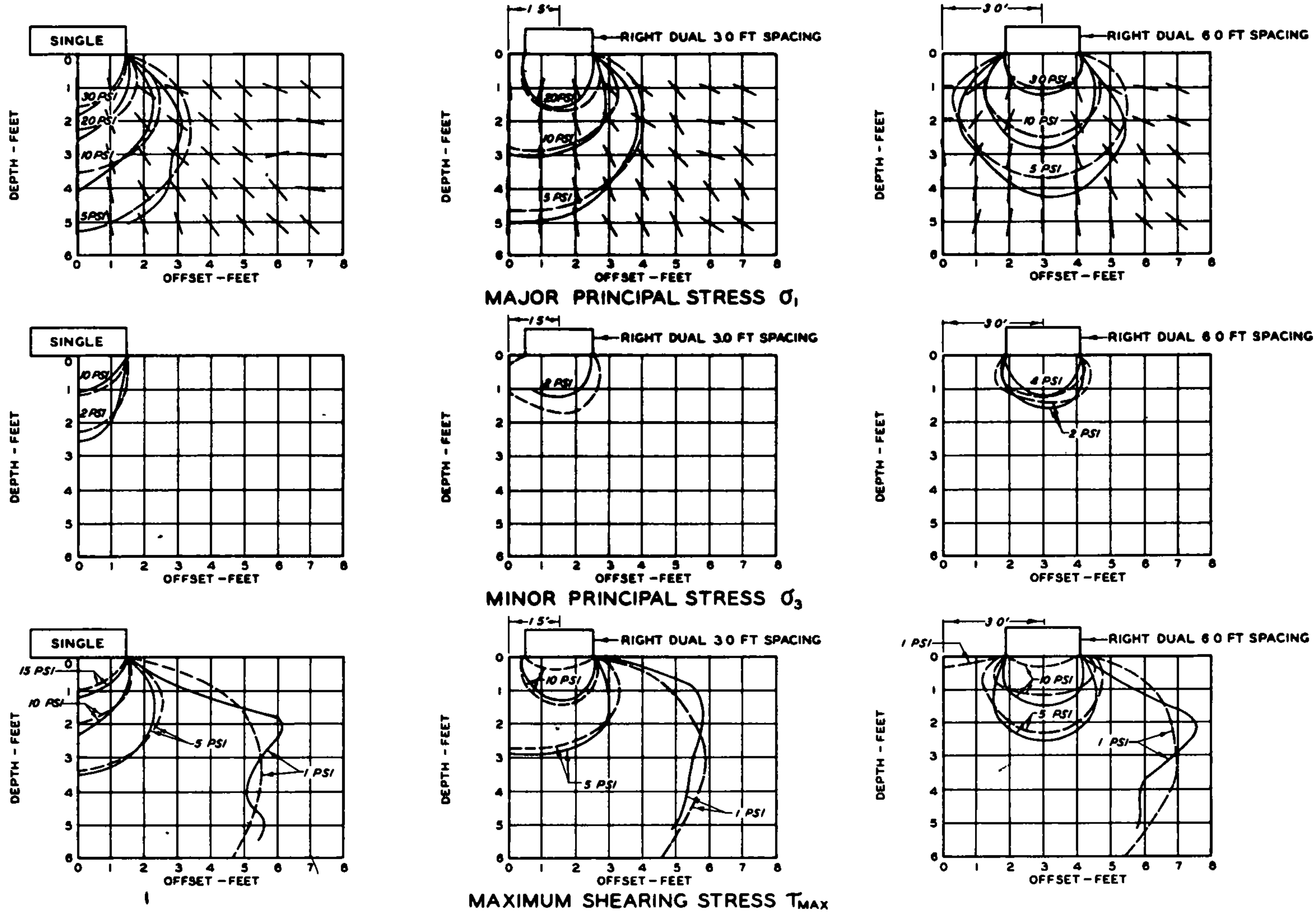


Figure 21

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

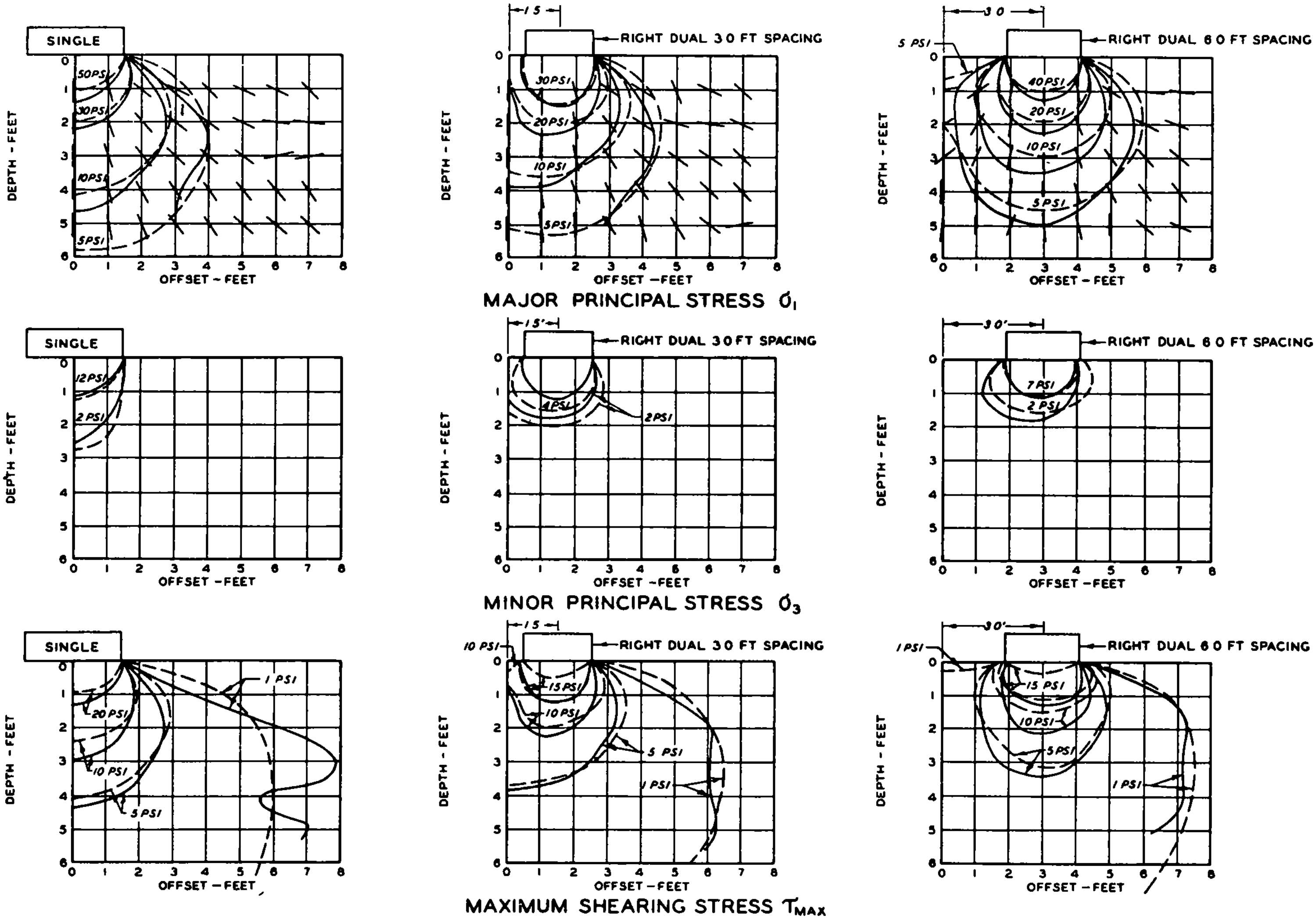
STRESS VS OFFSET DISTANCE
σ_3 AT 3 FT DEPTH



NOTE SOLID LINES ARE TEST DATA, DASHED LINES ARE THEORY, ALL DIRECTION LINES ARE FROM TEST DATA
OFFSET MEASURED FROM CENTROID OF LOADED AREA ALONG X-AXIS

ISOBARS OF STRESS
45,000 LB LOAD

Figure 22



NOTE SOLID LINES ARE TEST DATA, DASHED LINES ARE THEORY, ALL DIRECTION LINES ARE FROM TEST DATA
OFFSET MEASURED FROM CENTROID OF LOADED AREA ALONG X-AXIS

ISOBARS OF STRESS
60,000 LB LOAD

Figure 23

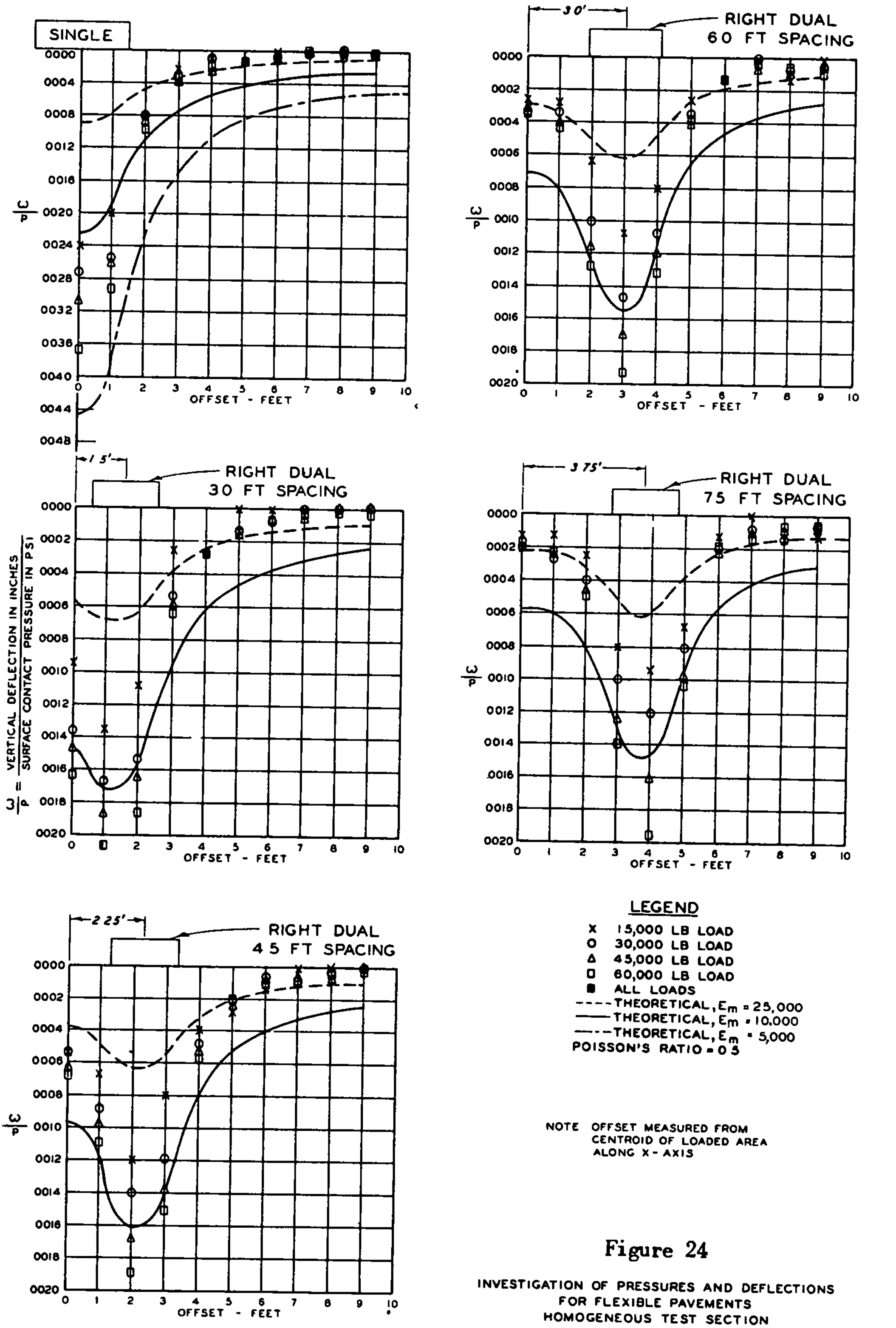


Figure 24

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS HOMOGENEOUS TEST SECTION

DEFLECTION VS OFFSET DISTANCE ω AT 1 FT DEPTH

specific point represents the vertical strain at that point. Values for the vertical strain and for the stress difference were obtained from all single and dual loads at all depths and at offsets of 0, 1, 2, and 3 ft. Figure 29 shows a plot of stress versus strain developed from the test data in this manner. Although there is quite a wide scattering of values, it is evident that a reasonable curve as shown by the dashed line can be drawn through the plotted points. Also shown on Figure 29 are the initial parts of stress-strain curves from triaxial tests made on undisturbed samples taken from the test section. The triaxial tests were intended to duplicate the loading that occurred in the test section. Tests were conducted at the natural moisture content. They were of the "quick" type in that lateral loads were applied and then the sample was sheared in a relatively short time. Curves are shown for lateral pressures of 6.9, 13.9, 27.8, and 55.6 psi.

At strains below 1.0 percent (.010 in. per in.) all of the curves from the laboratory test data plot below the test section curve. At a strain of approximately 1.0 percent, the curves for the two higher lateral pressures are in fair agreement with the test section data. The value of this agreement, however, is subject to question, since the highest lateral pressure experienced in the test section was approximately 15 psi. Other tests have been conducted in an effort to reach better agreement between the laboratory and field data. These included a confined compression test in a consolidation device and closed system triaxial tests with preconsolidation cyclic loading. So far the agreement has been no better than for the usual quick triaxial test. The problem is still being studied.

It should be noted that the stress-strain curve (dashed line) shown on Figure 29 has been determined by points plotted from the test data for the single and all dual loads, for five different depths, and for offset distances of 0, 1, 2, and 3 ft. Other plots (not shown) also were made in which the values were segregated by loads, by depths, by offset distances, and by lateral pressure groups. These plots indicate that the vertical strain produced by a given stress-difference is not affected by the total load, the contact pressure, or the offset distance but is very slightly affected by the lateral pressure and is somewhat more affected by the depth. These deviations are all very small and are not regarded as definite trends until more exact information is obtained concerning such other factors as Poisson's ratio or residual stresses. It appears, therefore, that in this test section the strain varied primarily with the stress-difference.

CONCLUSIONS

The following conclusions appear justified from this study:

1. The distribution pattern of the measured stresses follows the same general shape as that computed from the theory of elasticity for a homogeneous, isotropic material.
2. Where the stress values were small, the ratio of the measured stresses to the contact pressure was approximately a constant.
3. Where the stress values were greater than about 10 percent of the contact pressure, the ratio of the measured stress to the contact pressure varied with the total load, the contact pressure, and the area of contact.
4. The use of the Frohlich-Griffith concentration factor did not improve the agreement between the measured and the theoretical stresses.
5. In all cases the ratio of the measured vertical deflection to the contact pressure varied directly with the contact pressure.
6. In general the stress-strain curve developed from the test data has a shape similar to that obtained from "quick" triaxial tests performed on undisturbed samples.

ACKNOWLEDGMENTS

The studies reported in this paper, except the checks of the pressure cells, were made by the Flexible Pavement

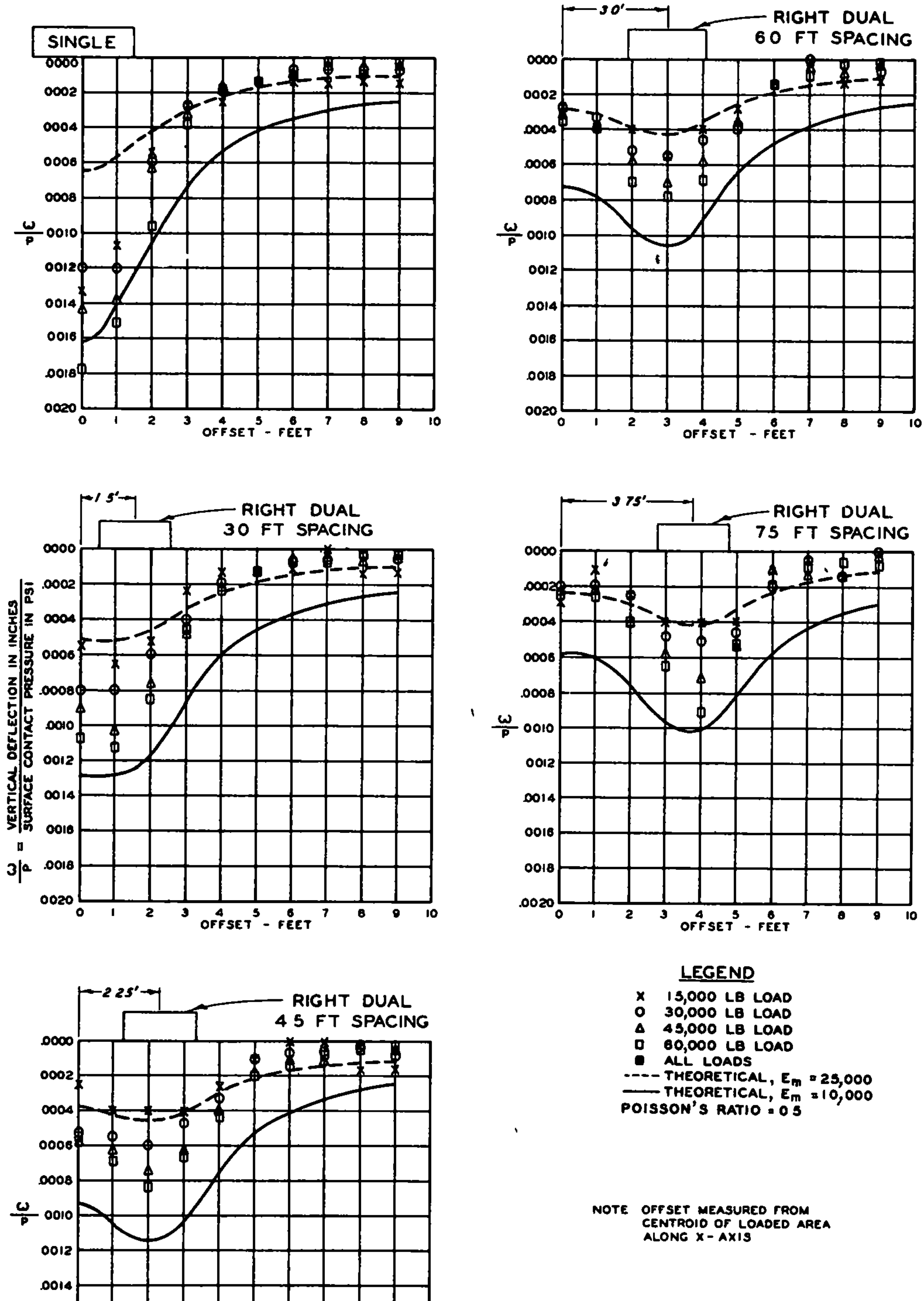


Figure 25

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS HOMOGENEOUS TEST SECTION

DEFLECTION VS OFFSET DISTANCE ω AT 2 FT DEPTH

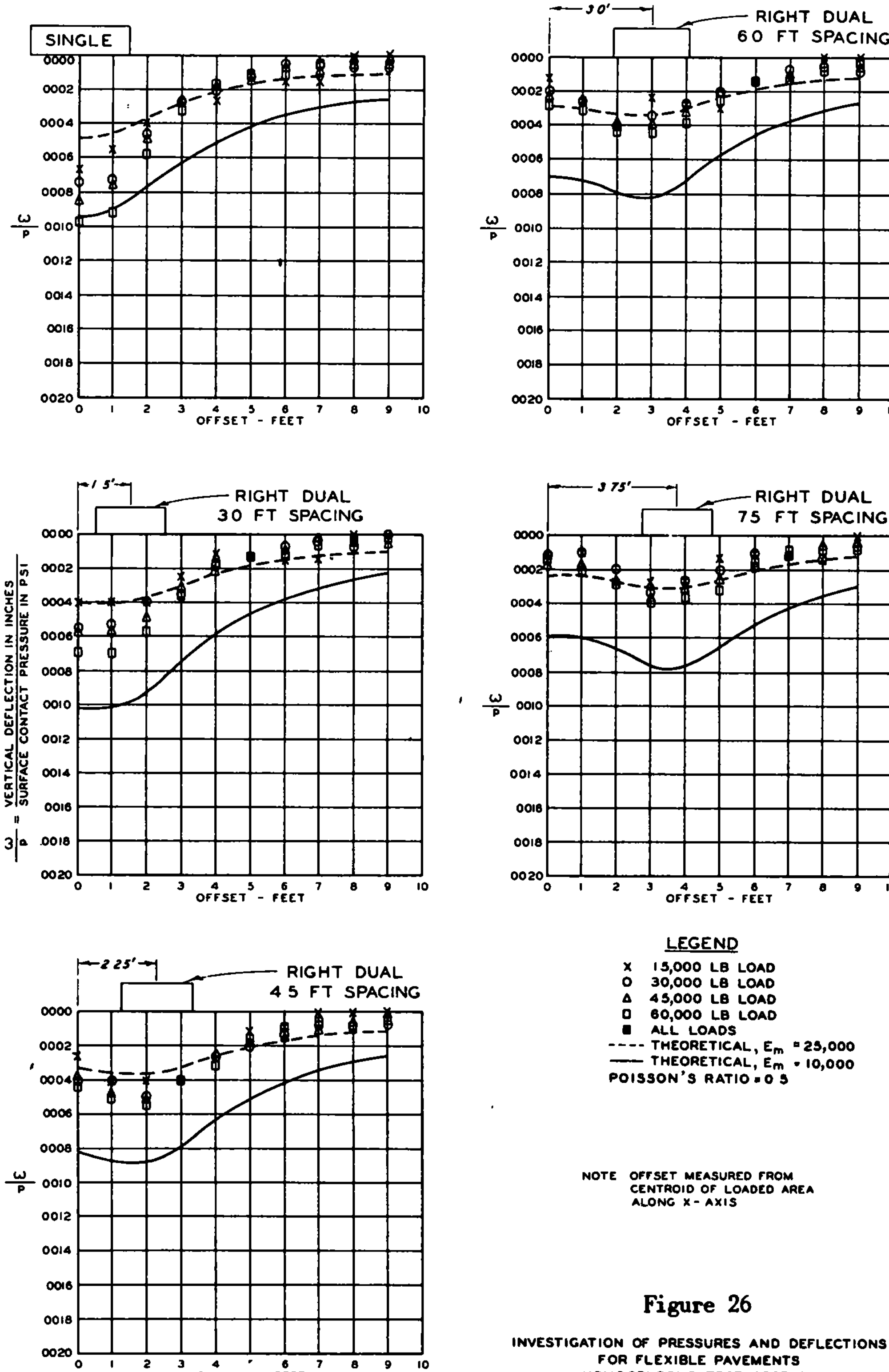


Figure 26

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS HOMOGENEOUS TEST SECTION

DEFLECTION VS OFFSET DISTANCE
ω AT 3 FT DEPTH

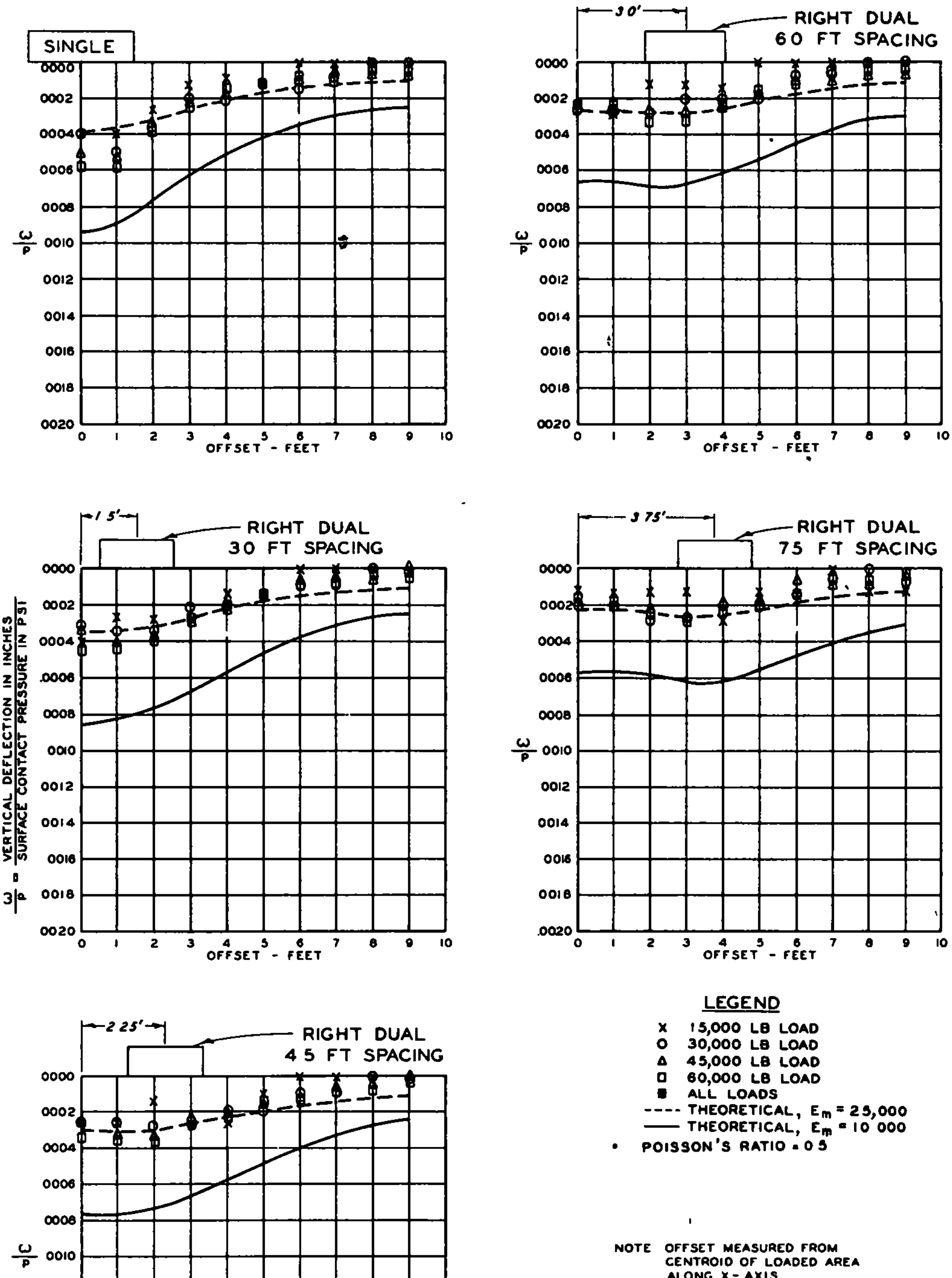


Figure 27

INVESTIGATION OF PRESSURES AND DEFLECTIONS
FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

DEFLECTION VS OFFSET DISTANCE
ω AT 4 FT DEPTH

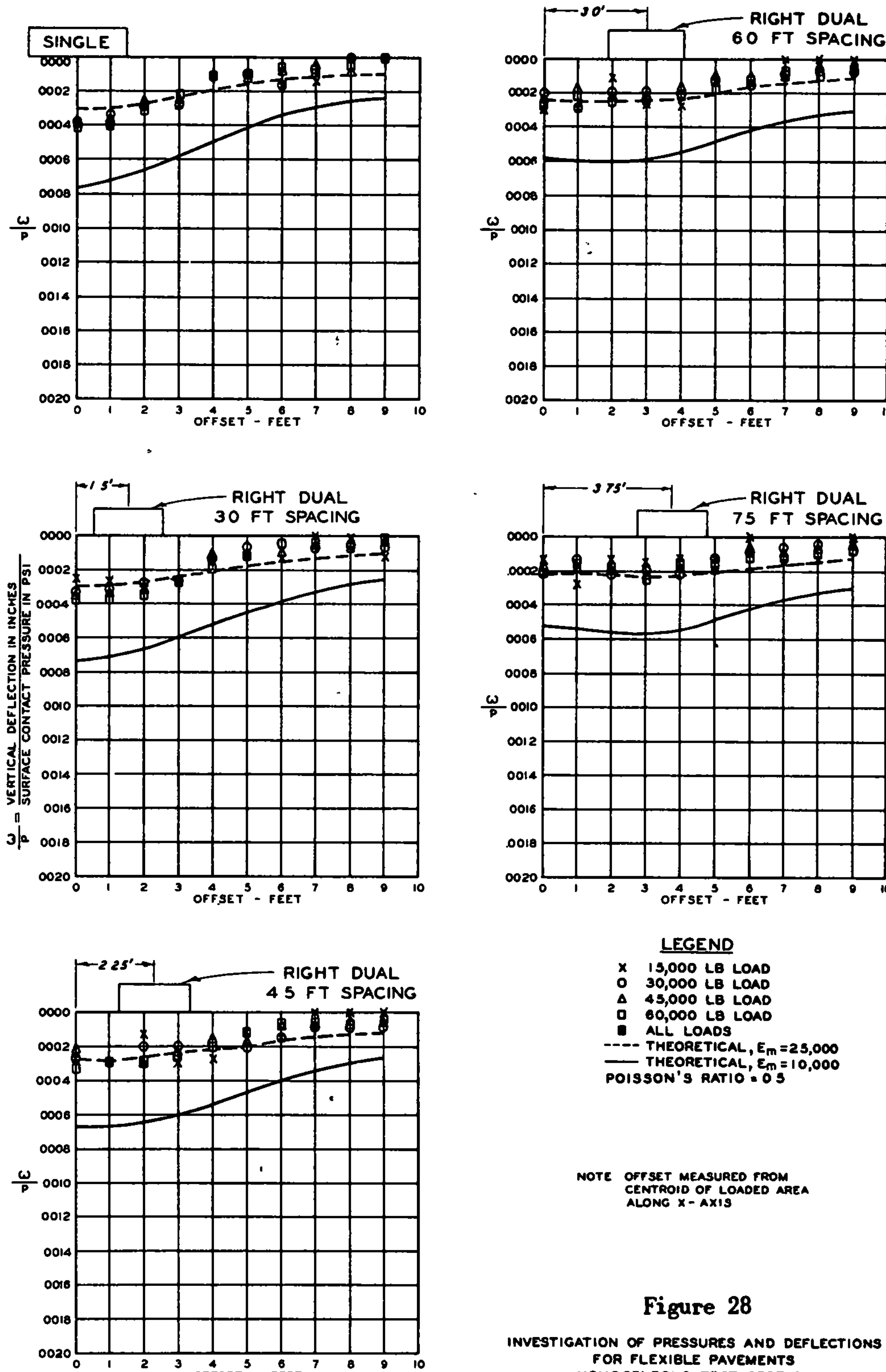
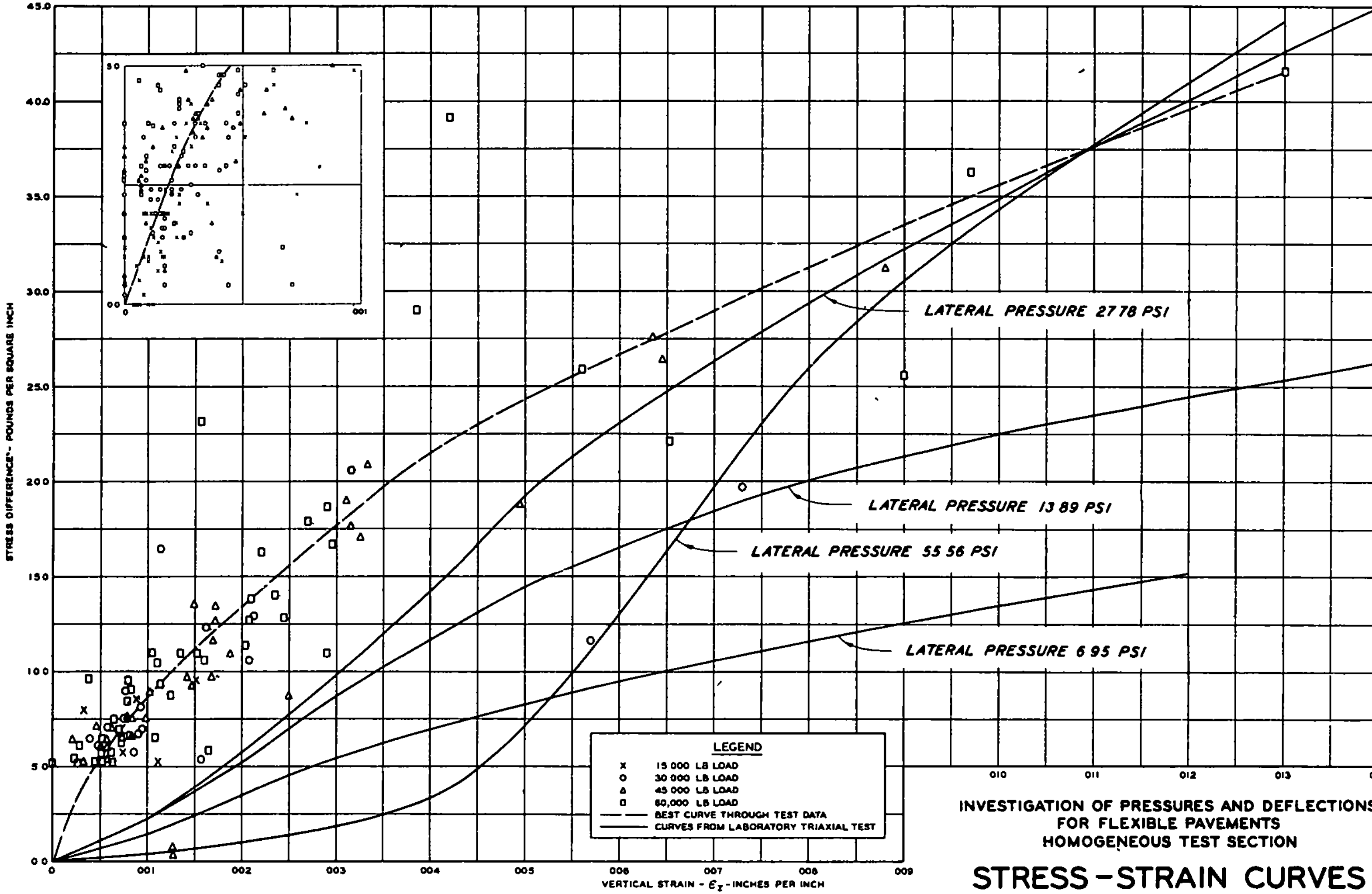


Figure 28

INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS
HOMOGENEOUS TEST SECTION

DEFLECTION VS OFFSET DISTANCE
ω AT 5 FT DEPTH



INVESTIGATION OF PRESSURES AND DEFLECTIONS FOR FLEXIBLE PAVEMENTS

HOMOGENEOUS TEST SECTION

STRESS-STRAIN CURVES

TEST SECTION DATA COMPARED WITH LABORATORY TRIAXIAL DATA

Figure 29

Branch, Soils Division, Waterways Experiment Station, CE, Vicksburg, Mississippi, as a part of the investigational program on airfield pavement design being conducted by the Airfields Branch, Military Construction, Office, Chief of Engineers. The checks of the pressure cells were made by the Flexible Pavement and Embankment and Foundation Branches of the Soils Division as a part of the pressure cell development program being conducted by the Soils Branch, Civil Works, Office, Chief of Engineers.

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DISCUSSION

D. P. KRYNINE, *University of California* - As all reports of the Army Engineers, this paper contains detailed descriptions of tests and results and carefully plotted data. Apparently, the authors performed many tedious computations and they are to be commended for their good work. The paper thus merits a detailed study, and I am going to comment on it in a general way only.

The measuring devices were installed at a depth of five feet from the upper surface, and readings for different depths were obtained by cutting off successive one foot lifts and repeating the program of loading. In all probability, the experimental mass was gradually densified by the tests themselves. An indirect proof of the veracity of this suggestion is the increase of the CBR from about 10 percent at the start of the tests to an average of 16 percent at the end. Moreover, due to the order of load applications at each cycle starting by the light loads, the characteristics of the mass probably were not the same under different loads. The examination of the pressure under the center line of a single load furnishes readings larger than those of the elastic theory (with one exception of the light load in Figure 15), and this is the sign that the given earth mass did not follow Hooke's Law. In this case, the principal of superposition or simple addition of stresses does not hold. Generally speaking, if the concentration factor is used, the corresponding stress

NOTE:

Closing remarks by Messrs. Foster and Fergus concerning the discussions of their paper by Krynine and Scrivner may be found, along with their second report on this project, in Volume 30, PROCEEDINGS of the Highway Research Board.

computations may be valid for a concentrated load, but not for distributed loads.

The writer was very much interested in the fact of the maximum deflection under one of the dual loads and not at the centroid as occurred in some instances. The writer is wondering whether the fact may be ascribed to accidental eccentricity of loading.

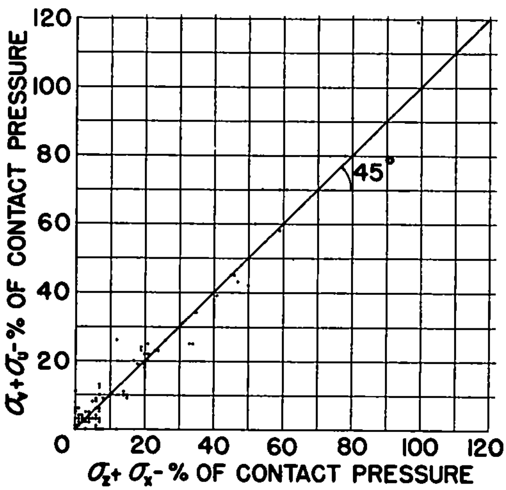

Figure A. Comparison of $\sigma_z + \sigma_x$ with $\sigma_v + \sigma_u$ at Depth of One Foot (Data from Figures 8, 10, 11 and 12)

F. H. SCRIVNER, *Texas Highway Department* - To all those who have been working toward the development of reliable methods for the design of flexible bases, this report should be highly significant. Apparently stresses in soils can be measured with reasonable accuracy, and apparently the theory of elasticity is usually sufficiently accurate for computing stresses in homogeneous soils. We await with interest the results of the measurements to be made in layered systems.

The authors mention that summations based on the laws of statics showed that the pressure cells were registering the applied loads within a probable error of ten percent. Another rather obvious check, based on statics and on the assumption that the soil is homogenous, is illustrated in Figure A, the data for which was taken from Figures 8, 10, 11, and 12. If the soil were perfectly homogeneous and isotropic, and if the pressure cells functioned perfectly, all the points in this figure would fall on the 45 degree straight line through the origin. Considering the practical difficulties involved in compacting soil to a homogeneous, isotropic condition, and in measuring stresses correctly, it would appear that the correlation shown in this figure is remarkably good.

E. S. BARBER, *University of Maryland* - For the soil tested, this report shows a remarkable agreement between stresses calculated for a homogeneous isotropic elastic support and measured stresses even close to the loaded area.

It is noted that smaller stress-strain moduli are indicated below the center of the loaded area where the intermediate and minor principal stresses are equal than at either side where these stresses are generally unequal. Since the modulus depends on the intermediate as well as the minor principal stress, it appears that higher moduli on either side of the load may be due to the effect of the intermediate principal stress. It is desirable to make some tests with three independent principal stresses, especially when frictional materials are involved.

S. M. FERGUS, *Closure* - Mr. E. S. Barber points out that the values of the stress-strain modulus found at points under the center of the load are smaller than the corresponding values at points away from the center and suggests that this difference may be due to the equality or inequality of the intermediate and minor principal stresses. The laboratory tests with three independent principal stresses suggested by Mr. Barber may well be included in future work; however, the analysis of the test data made by the authors (Fig. 29) indicates that the value of the stress-strain modulus varies with the total value of the stress difference

$$[\sigma_z - \mu(\sigma_y + \sigma_x)]$$

rather than with the relative values of the principal stresses. The authors desire to point out that the strains in this test section were obtained from plots of deflection versus depth rather than with strain gages, and the conclusions drawn therefrom should be regarded as preliminary and subject to such modifications as later studies may indicate.

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