Load Distribution on Highway Bridges Having Adequate Transverse Diaphragms

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● THE BRIDGE specifications of the American Association of State Highway Officials are now the design standard for highway bridges in the United States and are also the most widely used specifications in the other countries of the western hemisphere. These specifications have an empirical distribution of load to interior longitudinal girders, depending on the type of deck and the girder spacing For concrete decks and concrete girders the fractional wheel load applied to each girder is the girder spacing divided by 5 0. For exterior girders the live load is assumed to be the reaction from the panel of deck between the exterior and adjacent interior girders from the wheel load, regarding the deck panel as a simple beam. No consideration in the load distribution is given to the value of transverse-diaphragm beams connecting the longitudinal girders In the usual concrete girder-span diaphragm, beams are provided which have a stiffness comparable to the longitudinal girders These must have a very considerable effect on the transfer of load from one girder to another.

The AASHO specification results in a stronger interior girder than the exterior girders. In 1933 the Oregon State Highway Department made an investigation of a simple-span steel-girder bridge having a concrete deck The primary purpose was to check the composite action of the deck and girders, but it also allowed a comparison of the girder deflections under varying load positions. The investigation indicated that the exterior girders took as much, if not more, load than the interior girders. This led to the adoption by Oregon of a specification whereby the total assumed load on the span was divided equally between all girders when adequate diaphragm beams were provided.

In 1948 the state had occasion to build a simple-span concrete bridge over Oneonta Creek on the Columbia River Highway east of Portland. Figure 1 shows the structure loaded with two axles at midspan. This structure was selected for a full-size investigation to determine the load distribution to girders having an adequate diaphragm system. The investigational feature of the project was a cooperative undertaking by the U. S. Bureau of Public Roads and the Oregon State Highway Commission.

The structure has a span length of 48 ft. center to center of bearings. The east ends of the girders are supported on a bearing permitting angular rotation, but no horizontal movement. The west ends of the girders have 5 1/4-in. rockers permitting both rotation and longitudinal movement The alignment across the bridge is a tangent, the abutments are at right angles to the centerline, and the grade is level. The structure has a 26-ft-wide roadway with a 3-ft.-6-in. sidewalk on each side There are four 16 1/2- by 51-in. longitudinal girders at 7-ft-1/2-in centers with an 8-by-49-in diaphragm beam at midspan Beam and girder depths include the 6 1/2-in deck

Theoretical Distribution of Loads

The structure under discussion consists of four longitudinal girders connected at midspan by a diaphragm having a stiffness approximately equal to the girders The problem of distribution of load to the several girders is susceptible of analysis by a simple, although rather tedious, procedure provided certain assumptions are made These assumptions are (1) the slab acts as simple beams between girders in transferring wheel loads to girders and does not enter into the transfer of load from one girder to another and (2) the girders are not stiff enough in torsion to produce appreciable restraining moments at their connection to the diaphragms Both of these assumptions are open to question The slab is a continuous beam supported by all girders and plays some part in the transference of load In the usual concrete structure, however, the diaphragm depth is at least six times the slab depth and for equal widths is more than 200 times as stiff The most effective portion of the slab for load transference is in the area where the greatest deflection takes place The slab toward the girder support can contribute but little The contribution of the slab, while perhaps not a negligible factor, is probably minor. The torsional rigidity of the girder contributes in some measure to the stiffness of the diaphragm system For the very small angular change, this effect is probably a minor factor Both of these assump-

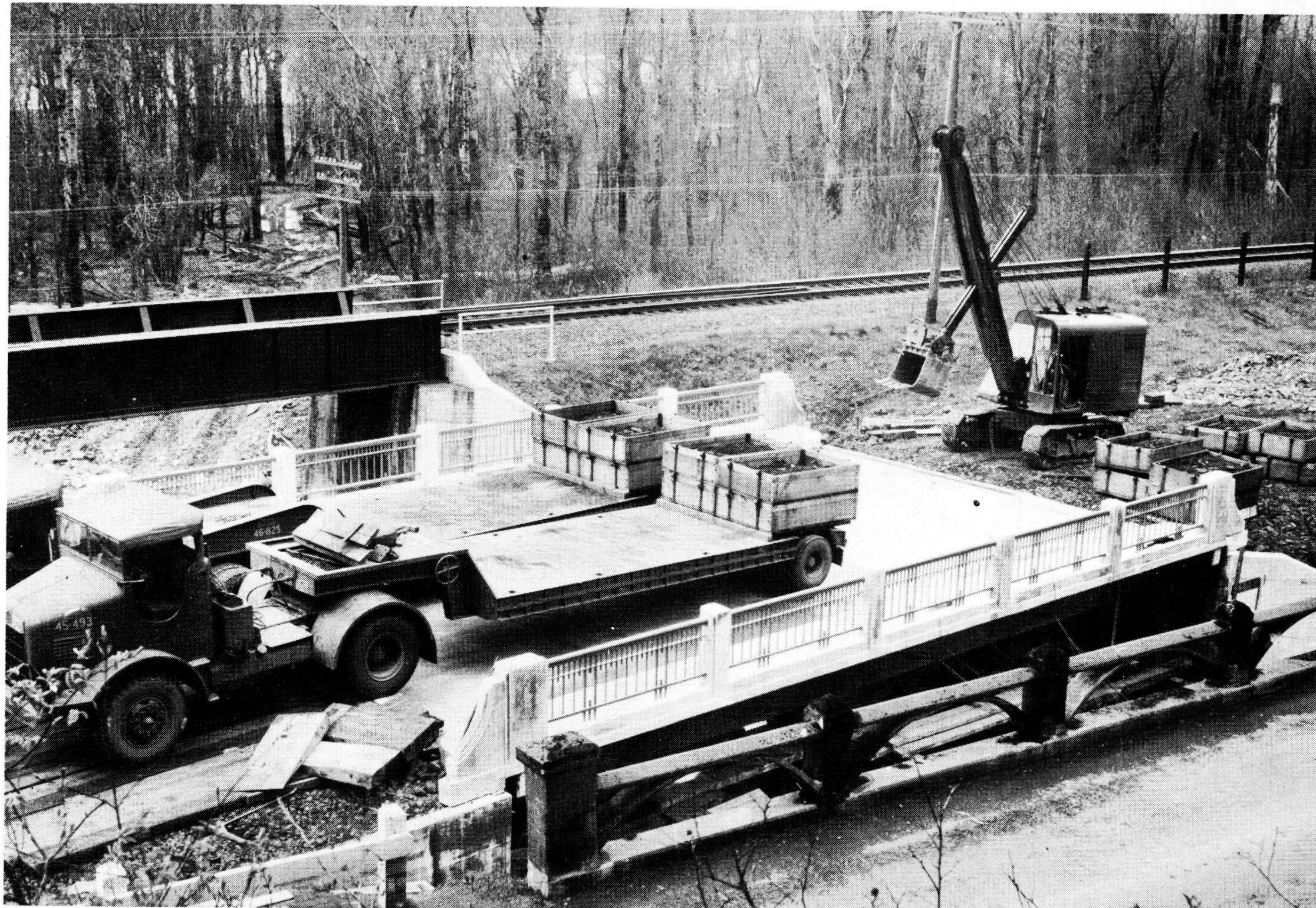

Figure 1.

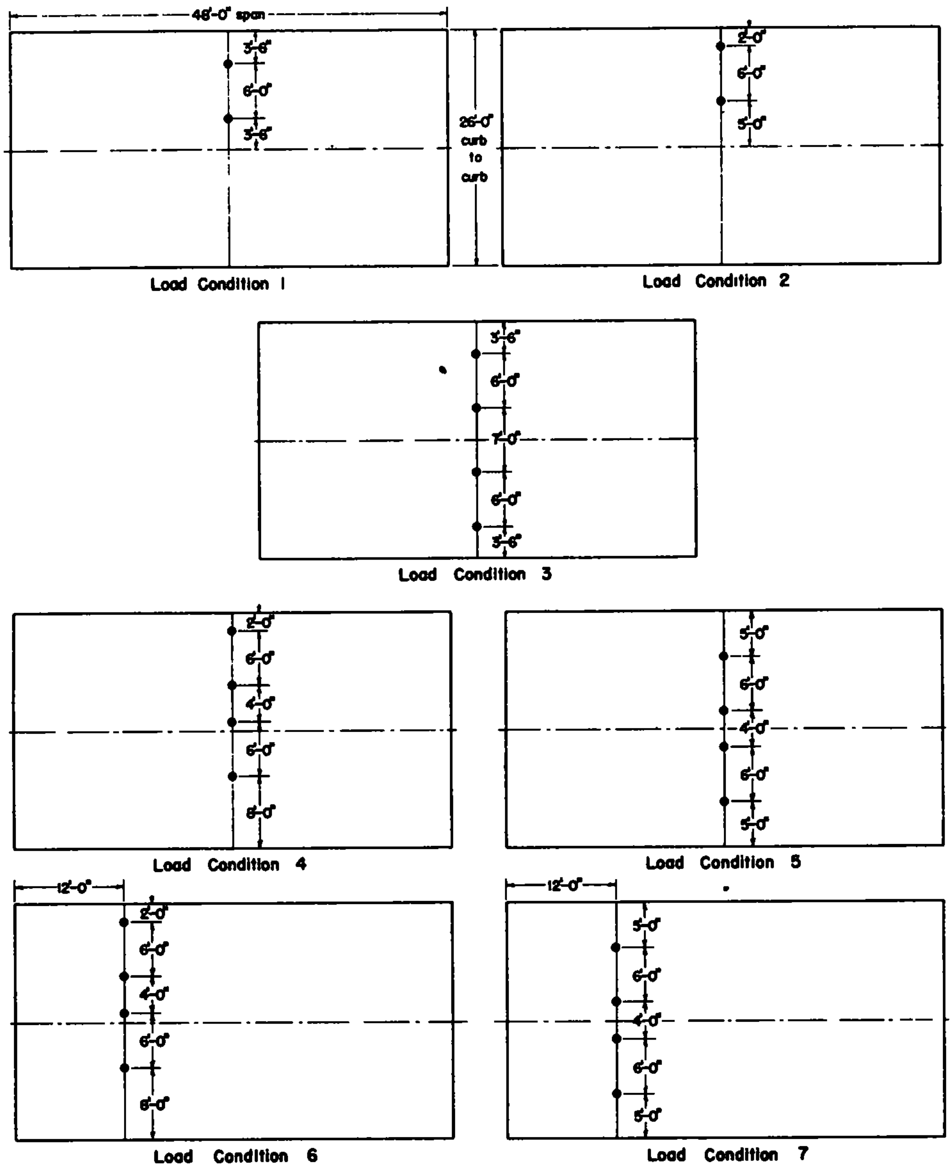

Figure 2. Locations of test loads.

tions are on the conservative side, and the actual distribution of load should be more than shown by the computations.

The method of computing the load transfer is the work of George S. Vincent, senior highway bridge engineer, Bureau of Public Roads. The wheel loads are distributed to the adjacent girders by the slab as though it were a simple beam. These loads deflect the girders and a part of the load is transferred to the diaphragms at their intersection with the girders. Since the diaphragm is in static equilibrium, the load transference at the outside girders may be regarded as reactions and at the interior girders as loads, and the deflection curve of the diaphragm set up in terms of the unknown load-transfer coefficient. The number of equations from this relationship is one less than the number of spans between girders, or two less than the number of intersections of girders and diaphragm Two additional equations are from the summation of vertical forces and the summation of moments These equations are sufficient for the determination of the unknown-load transfer coefficients

The load transfer depends on the relative stiffness of the members. Whether the concrete acts with the steel in resisting tension stresses (uncracked section) and whether the curbs and sidewalks act with the exterior girders have considerable effect. In the Oneonta Creek Bridge the testing was done before the bridge was opened to general traffic, and the test results indicated that the concrete was effective in tension and that the curbs and sidewalks acted with the exterior girders in resisting stress

For the Oneonta Creek Bridge with four equal beams at equal spacings and a single diaphragm at mid-span, the four simultaneous equations in the unknown load-transfer coefficients are

$$\frac{2D_1}{3}-(8R+1)D_2-7RD_3+\frac{D_4}{3}=\frac{2P_1}{3}-P_2+\frac{P_4}{3}$$

$$\frac{D_1}{3}-7RD_2-(8R+1)D_3+\frac{2D_4}{3}=\frac{P_1}{3}-P_3+\frac{2P_4}{3}$$

$$D_1+D_2+D_3+D_4=0$$

$$D_2+2D_3+3D_4=0$$

where D_1, D_2, D_3 and D_4 are the load transfer coefficients at the intersection of the diaphragm with each girder, P_1, P_2, P_3 and P_4 are the loads applied to each girder, and R is a ratio of the stiffness of the diaphragm to the stiffness of the girders. These four equations are sufficient for the determination of the load transfer coefficients. The derivation of the equa-

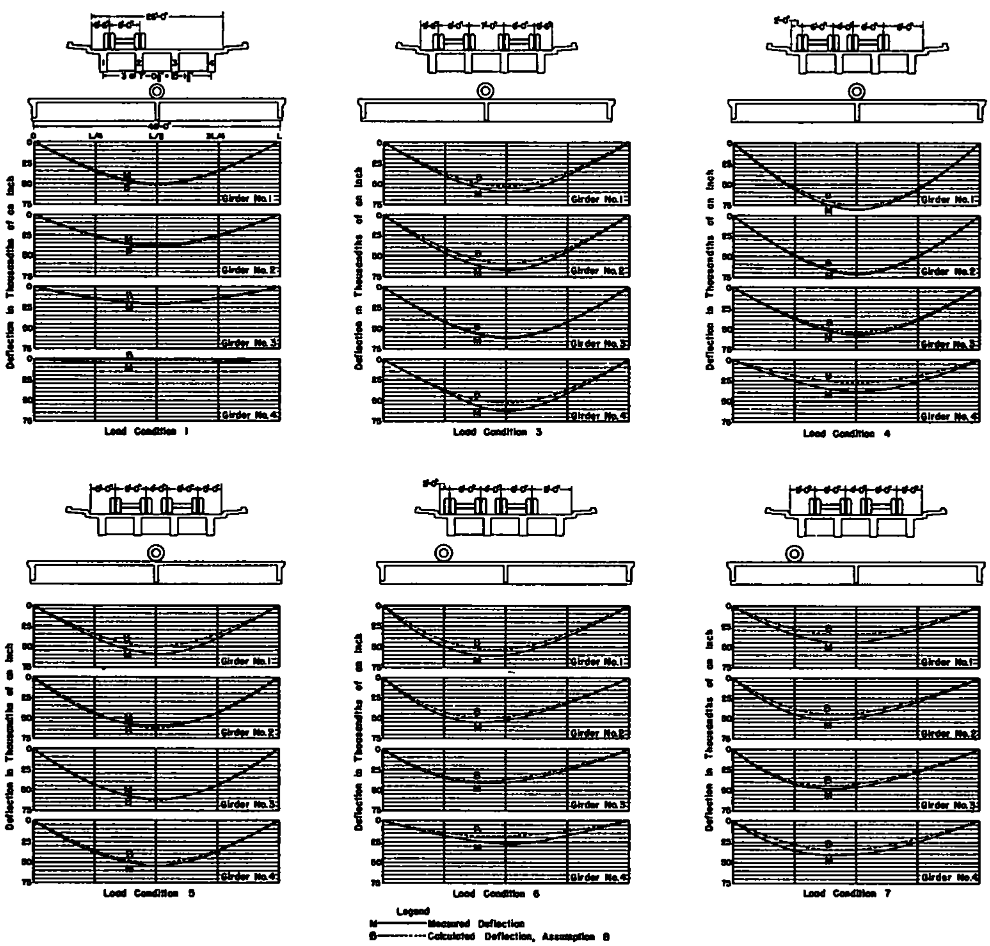

Figure 3. Deflection of girders.

tions is given in an appendix to this paper.

Instrumentation

The test installation was designed to furnish information on the problem from three approaches Gauge points were set in the bottoms of all girders at midspan and at quarter points The deflections under load were measured with inside micrometers from fixed points on the falsework below the girders

SR-4 strain gauges were installed on the metal reinforcing bars at points where knowledge of the stress might be informative. These points were as follows On the two exterior bars in the lower layer of the main tension steel in the bottom of each girder at midspan and at the quarter points On the tension steel in the bottom of the diaphragm beam at the point of intersection with each main girder. On longitudinal bars in the slab above each girder at midspan and at quarter points On longitudinal bars in the top of the deck slab midway between girders at midspan and at quarter points. On five transverse bottom deck bars symmetrically placed about one quarter point. On longitudinal bars in center of sidewalk at midspan On longitudinal bars in the face of the roadway curbs at midspan The SR-4 gauges were placed in pairs on opposite sides of the bars and connected in series to correct for any eccentricity of loading The gauges were waterproofed with adhesive tape and petrosene wax. The gauges were placed on the bars and enclosed in a sheet metal housing so that no concrete came in direct contact with the gauge. Lead wires were brought from the gauges to a central station where all readings were made

The reactions under each end of each girder were measured by individual weighing devices. These consisted of a short section of an aluminum alloy cylinder with SR-4 gauges at each quadrant. The opposite gauges were connected in series to correct for eccentricity The aluminum cylinders were calibrated on a testing machine and stress-strain curves plotted for each cylinder The girder loads were applied to the cylinders through a ball joint to decrease eccentricity to the minimum The cylinders were supported on the abutments by parallel plates and leveling screws to level the support and to equal-

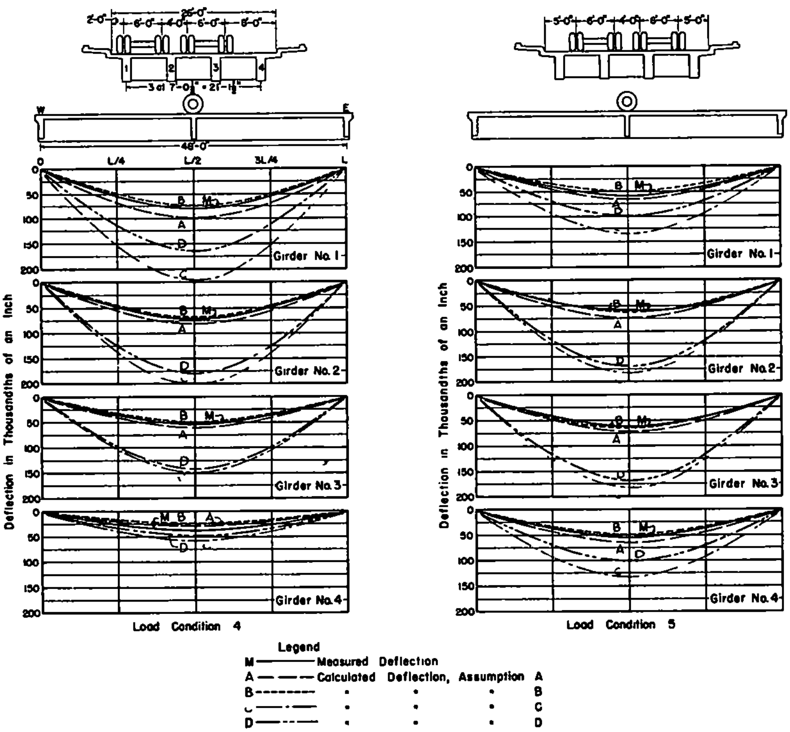

Figure 4. Deflection of girders, load Conditions 4 and 5.

ize the dead load on the girders prior to loading for deflection and stress measurements. At the conclusion of the test program the cylinders were replaced with bearing plates and rockers.

Loading

The loads were single-axle, flat-bed trailers towed by tractors with a spacing of 25 ft between the rear tractor axle and the trailer axle. This arrangement allowed the trailer axle loads to be placed at any point on the span with the tractor loads off the span. The trailers had been especially built for heavy hauling, and the wheel spacing on the axles did not match the spacing usually assumed for bridge design. The axle load was therefore applied by a beam supported on the deck by blocks the size of a loaded tire imprint and at the conventional spacing. Two loaded trailers were used to produce the desired loading arrangements.

Seven load arrangements were used. These arrangements are shown in Figure 2. In the first two a single trailer was used, in one instance with the trailer in the normal position in one traffic lane, and then with the trailer placed as close as practical to one curb. Three arrangements were used with the two trailer axles at midspan, with each axle in the normal position in its traffic lane, with the two axles placed as nearly as practical to one curb, and with the two axles symmetrically placed about the center line and as near together as was practical. Two arrangements with both trailer axles at a quarter point were used. In one arrangement the axles were placed as near to one curb as practical and in the second the two axles were symmetrically placed about the bridge

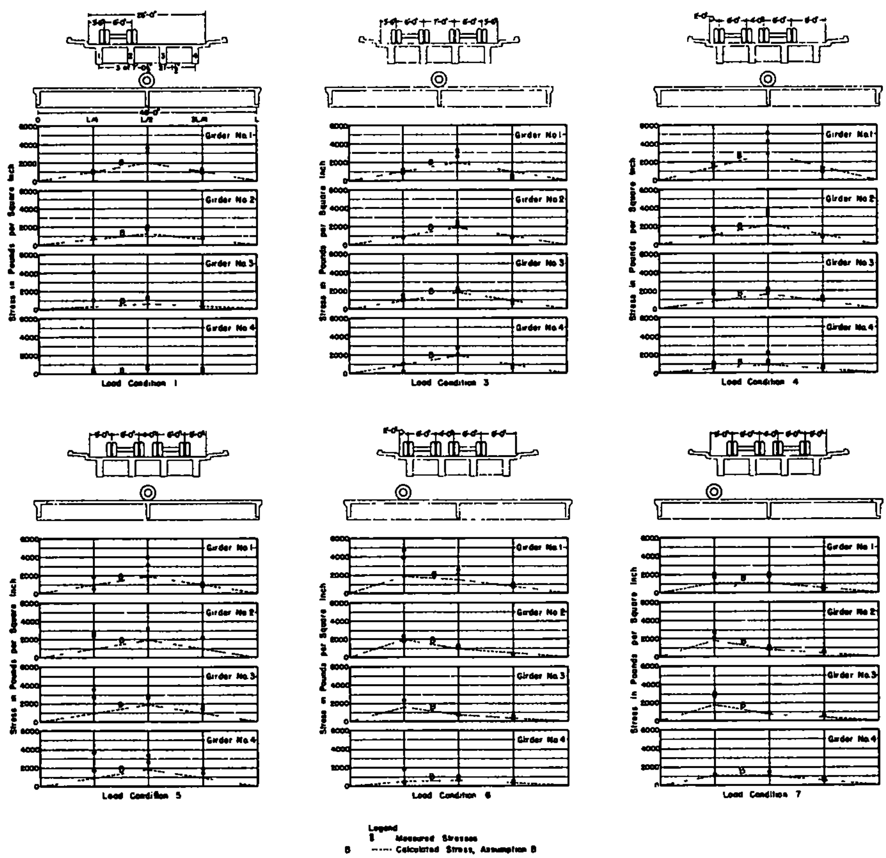

Figure 5. Stress in girder tension steel.

center line and as close together as practical operation would permit. All loadings were made with 48,000 lb on each axle which applied loads through the blocks corresponding to a 24,000-lb. wheel load. These loads are, of course, more than the structure was designed for, but were chosen to give deflections and stresses that could be easily measured.

Test Data and Analysis

As mentioned before, the assumptions as to whether the concrete acts as a cracked or an uncracked section and as to the effectiveness of the sidewalks and curbs in acting with the outside girders play a large part in the calculated values for both deflection and stress. The testing at the Oneonta Creek bridge was done immediately after the completion of the structure and before it was opened to traffic. As would be expected, the structure acted as though the concrete were acting with the steel in resisting tension stresses The test results also showed that the sidewalks and curbs acted with the exterior girders.

Calculations for deflection and stress were made for all load positions under each of the following assumptions: (A) uncracked concrete section without considering the sidewalks or curbs as effective, (B) uncracked concrete section with sidewalks and curb, (C) cracked concrete section without considering the sidewalks or curbs as effective, and (D) cracked concrete section with sidewalks and curbs.

Since the condition of the structure at the time of test and the test results themselves indicate that the structure was acting as uncracked concrete with the sidewalks and curbs effective, the comparison between calculated deflection and stress and field measurements is made under Assumption B except for load Positions 4 and 5 where all four assumptions are shown. Eventually the concrete on the tension side of the girders will crack and no longer act in tension, and the deflection and stress will approach those of Assumption D.

Deflections

The calculated deflections for Assumption B and the measured deflections for all load conditions except load Condition 2 are shown in Figure 3. The instrumentation failed on load Condition 2, which is for a single-axle load near one curb. Since this is not a critical load condition, this test was not repeated. It will be noted that there is a remarkable correspondence between the measured deflections and those calculated under Assumption B, the uncracked concrete section. Attention is particularly called to the graph showing load Condition 4. With two axles placed as near to one curb as is practical, this loading produces the greatest deflection and stress. The measured deflections and the calculated deflections for the uncracked section are in good agreement. In general, the measured deflections are slightly more than should occur if the concrete were entirely effective. A very small amount of initial cracking could easily account for the differences.

Figure 4 shows the deflections of the four girders under load Conditions 4 and 5 and under all four assumptions. The measured and calculated deflections are given in Table 1.

TABLE 1

DEFLECTIONS—LOAD CONDITION 4

Position on Bridge	Girder Number	DEFLECTIONS: Measured	Calculated: A	B	C	D
		in	in	in	in	in
L/4	1	0 054	0 069	0 053	0 153	0 114
L/4	2	049	057	048	139	125
L/4	3	037	041	037	102	099
L/4	4	022	021	018	041	034
L/2	1	081	101	076	222	166
L/2	2	072	083	070	202	182
L/2	3	057	060	053	150	143
L/2	4	037	030	027	059	049
3L/4	1	055	069	053	153	114
3L/4	2	049	057	048	139	125
3L/4	3	039	041	037	102	099
3L/4	4	025	021	018	041	034
DEFLECTIONS—LOAD CONDITION 5						
L/4	1	0 039	0 044	0 035	0 092	0 070
L/4	2	046	050	043	126	117
L/4	3	018*	050	043	126	117
L/4	4	037	044	035	092	070
L/2	1	059	064	050	134	102
L/2	2	059	073	063	183	170
L/2	3	062	073	063	183	170
L/2	4	055	064	050	134	102
3L/4	1	039	044	035	092	070
3L/4	2	043	050	043	126	117
3L/4	3	044	050	043	126	117
3L/4	4	035	044	035	092	070

* Erroneous gauge reading

The measured deflections match the calculated deflections under Assumption B surprisingly well The measured deflection of the exterior girder was 0.081 in, while the adjacent interior girder deflected 0.072. This occurred even though the exterior girder with the sidewalk and curb has a moment of inertia of 578 in.^{4} and the interior girder has a moment of inertia of 402 in.^{4} The deflection of the exterior girder of 0.081 in. was the greatest deflection under any girder for any load condition. Load Condition 5, with the two axles symmetrically placed about the longitudinal centerline and as near together as practical, gives the greatest load on the interior girder. The measured deflection of the interior girders averaged 0.061 under this loading. A comparison of Curves A or C for the two loadings, where the girders have equal moments of inertia, shows the relative deflections under loadings which give the maximum deflections of the exterior and interior girders. Under load Condition 4, with the two axles crowded toward the curb, the maximum deflection is in the exterior girder and was 0.081 in Under load Condition 5, with the two axles as near the center line as practical, the maximum deflection is in the two interior girders and averaged 0 061 in This indicates that the exterior girders should be at least as strong as the interior girders

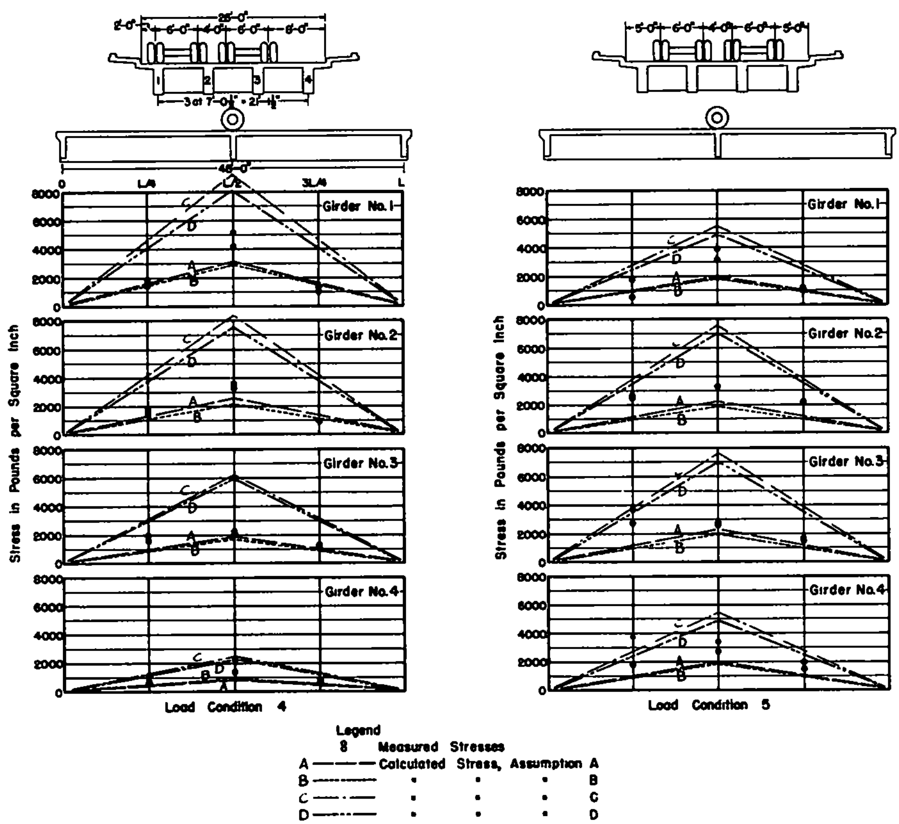

Figure 6. Stress in girder tension steel, load Conditions 4 and 5.

Stress

The stress measurements with the SR-4 gauges attached to the tension steel of the girders are not as consistent as the deflection measurements Even though every practical precaution in the installation and protection of the gauges was taken, the results were rather erratic.

The measured stresses and the calculated stresses under Assumption B, for all load conditions except Condition 2, are shown in Figure 5. The measured stresses in general are higher than the calculated stresses, probably due to the effect of initial cracking of the concrete

The measured and calculated stresses for load Conditions 4 and 5 are given in Table 2 and the plotted data in Figure 6

The highest stress was found in the exterior girder under load Condition 4 when the measured stress was 4,650 psi. in the reinforcing steel Under load Condition 5, which should produce maximum stress in the interior girders, the steel stresses were 3,525 psi. and 2,775 psi., an average of 3,150 psi. These measurements, while subject to considerable question quantitatively, support the deflection measurements in indicating that the exterior girders can be subjected to heavier loads than the interior girders.

An examination of Figure 6 shows that in general the measured stresses are between the values which the Vincent analysis gives for the cracked and the uncracked sections It is probable that the concrete immediately adjacent to the gauges was only partially effective in resisting tension

Reactions

The weighing of the reactions at the ends of the girders was not entirely satisfactory. In moving the loaded trailer axle on and off the span it was impossible to prevent slight movements of the span which affected the loading on the alloy cylinders. There was also some friction between the span and the backwalls of the abutments that affected the results In every case the total load shown by the weighing devices was less than the applied load In a few cases one weighing device would show an unreasonably large proportion of the total load In general, however, the reactions were fairly well in line with the predictions of the Vincent analysis. Table 3 gives the measured and computed reactions for load Condition 4 in which the two axles were crowded to one side of the structure In this table a column headed "Adjusted Value" has been added in which the actual measurements have been proportionately increased so that the total equals the applied load.

Conclusions

Because of the questions as to the action of the concrete as a cracked or an uncracked section and as to the amount the sidewalks and curbs contribute to the moment of inertia of the exterior girders, the test results should not be used quantitatively. The comparisons between the several load conditions and between the exterior and interior girders do give a true picture of the effect of diaphragm beams in distributing the loads.

TABLE 2

STRESS—LOAD CONDITION 4

Position on Bridge	Girder Number	Live Load Stress: Measured	Calculated A	Calculated B	Calculated C	Calculated D
		lb /in²	lb /in²	lb /in²	lb /in²	lb /in²
L/4	1	1,575	1,559	1,428	4,601	4,023
L/4	2	1,725	1,300	1,083	4,209	3,787
L/4	3	1,800	936	827	3,115	2,984
L/4	4	975	466	497	1,228	1,189
L/2	1	4,650	3,117	2,856	9,202	8,046
L/2	2	3,450	2,601	2,166	8 418	7,575
L/2	3	2,100	1,872	1,654	6,229	5,969
L/2	4	1,800	932	994	2,456	2,378
3L/4	1	1,125	1,559	1,428	4,601	4,023
3L/4	2	900	1,300	1,083	4,209	3,787
3L/4	3	1,050	936	827	3,115	2,984
3L/4	4	675	466	497	1,228	1,189
STRESS—LOAD CONDITION 5						
L/4	1	1,200	993	944	2,765	2,472
L/4	2	2,625	1,138	977	3,811	3,550
L/4	3	3,150	1,138	977	3,811	3,550
L/4	4	2,775	993	944	2,765	2,472
L/2	1	3,525	1,985	1,887	5,530	4 944
L/2	2	3,300	2,275	1,955	7,622	7,099
L/2	3	2,775	2,275	1,955	7,622	7,099
L/2	4	3,075	1,985	1,887	5,530	4,944
3L/4	1	975	993	944	2,765	2,472
3L/4	2	2,250	1,138	977	3,811	3,550
3L/4	3	1,575	1,138	977	3,811	3,550
3L/4	4	1,725	993	944	2,765	2,472

TABLE 3

REACTIONS—LOAD CONDITION 4

Girder Reaction	Weight as Measured	Adjusted Value	Calculated A	Calculated B	Calculated C	Calculated D
No 1 W	17,043	18 322	17,490	19,412	16,780	17,964
No 2 W	13,426	14,433	14 700	12 381	15,369	13,830
No 3 W	9,043	9,721	10,579	9,453	11,373	10,898
No 4 W	5,591	6,011	5,231	6,754	4,478	5,308
No 1 E	17,749	19,081	17,490	19,412	16,780	17 964
No 2 E	11,055	11,885	14,700	12,381	15,369	13,830
No 3 E	10,584	11,378	10,579	9,453	11,373	10,898
No 4 E	4,808	5,169	5,231	6,754	4,478	5,308
TOTAL	89,299	96,000	96,000	96,000	96,000	96,000

The results from the deflection and stress measurements correspond with the calculated values by the Vincent method so closely that this method can be used with confidence when a close approximation of the actual load distribution is of enough importance to justify the labor involved

The present AASHO specification for load distribution to concrete girders in spans having adequate diaphragm beams is faulty in that it results in assigning more load to the interior girders than to the exterior girders In the usual structure the exterior girders carry as much load as the interior girders and, under some girder arrangements and load positions, may carry even more

For structures having adequate transverse diaphragms, a loading assumption is suggested in which the entire deck width is loaded with axle loads and fractions of axle loads and the total load divided equally to all the girders. This is a simple specification, easily and quickly applied, and, in view of the many uncertainties inherent in design, is accurate enough. Certainly it is more accurate than the present procedure

The Oneonta Creek Bridge was built under contract with Marshall Dresser as resident engineer The planning of the investigation was done by Richard Rosecrans, structural research engineer. The installation of gauges and making of tests was under the supervision of Oscar White, assistant engineer of materials and tests. The analysis of test data was by Roy Edgerton, structural research engineer.

APPENDIX

Vincent Method of Computing Load Distributions

This analysis sets up equations for the deflections of the girders and the diaphragm with respect to their dead load positions and for the force distribution necessary to produce these deflections. The individual girder is deflected by the applied wheel loads and the forces transmitted to it by the diaphragm, whether upward or downward at the particular girder. The diaphragm acts as a continuous beam over yielding supports or, more accurately stated, as an elastic member in space in equilibrium under the action of forces applied at its intersections with the various girders Its deflection under the action of these forces can be readily expressed; for convenience in this analysis its deflection is expressed with respect to the chord connecting its intersections with the two outside girders

In this analysis the torsional rigidity of the girders is neglected, *i e*, it is assumed that the girders are not stiff enough in torsion to produce appreciable restraining moments at the ends of the diaphragm or at its connections to the intermediate girders. This assumption is important in its effects. For example if it were assumed that the girders were so stiff in torsion as to fully fix the diaphragm at the ends and at the various interior girders then no diaphragm moment would be carried past any girder and each segment of diaphragm between adjacent girders would be subjected to reversed moments of equal magnitude at its two ends, these moments and the resulting shear transferred from one girder to the other being determined by the relative deflections of the adjacent girders and the stiffness of the diaphragm segment between them. Under this assumption of relatively great torsional rigidity the individual girder stems would remain vertical even under extreme eccentric loading and the diaphragm would deflect in a series of reverse curves. There can be little doubt that the torsional rigidity of the individual girder stem is nearly negligible in so far as its capacity to develop fixed end moments in the diaphragm is concerned and it is much nearer the truth to neglect this torsional resistance than to assume fixed end conditions. Furthermore, the neglect of any factor such as torsional rigidity which tends to stiffen the diaphragm is on the conservative side, indicating somewhat less distribution of load than occurs.

This analysis neglects also the effect of the slab in distributing loading between girders This effect is far from negligible in the case of girder spans without diaphragms as shown by theoretical analysis and model tests at the University of Illinois. However, when diaphragms as deep as the girders are used, their stiffness is great in comparison with that of the slab and they therefore assume the major portion of the task of distributing the load This is especially true if several diaphragms are used or if a single diaphragm is used at the center of a span of such length that the moment is due almost entirely to the rear truck wheels placed at or near the center of the span

Though the method is of general application, the equations are developed for the case of a four-girder bridge with a diaphragm at midspan and with the live loads applied at midspan.

Figure A shows the span layout and the forces

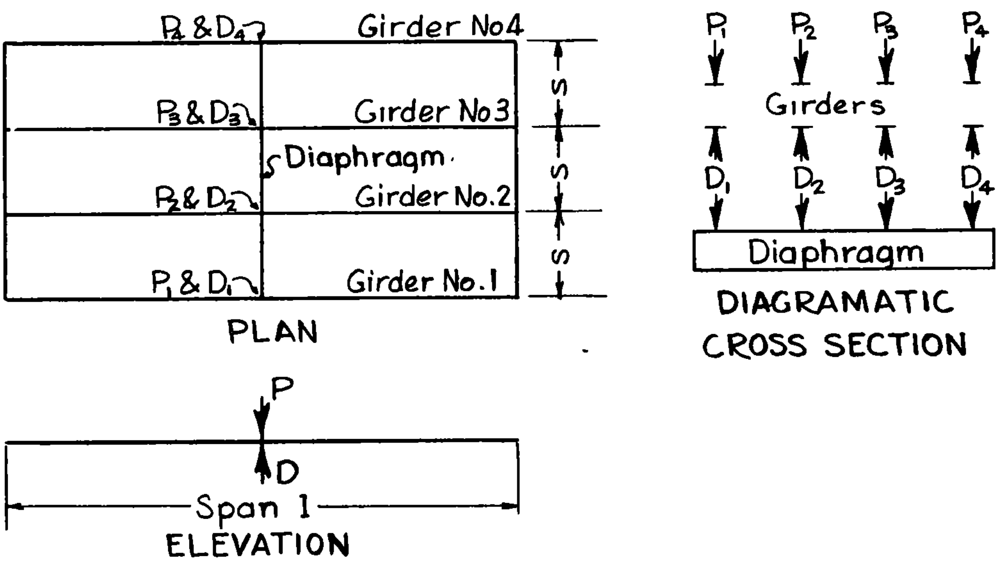

Figure A.

acting on its various elements

P_1, P_2, etc., are the wheel loads distributed to each girder, assuming simple beam action between girders The final equations are developed in terms of these general loads, thus the effects of various transverse positions of the wheel loads can be determined by substituting the proper values for P_1, P_2, etc., computed for the desired wheel load positions D_1, D_2, etc., are forces transferred from the girders to the diaphragm The convention is used that a positive D acts upward on the girder and downward on the diaphragm. Since the diaphragm is supported only by the girders, the laws of equilibrium require that the summation of all forces, D, be zero and some will be negative in sign and thus reversed in direction from that shown in the sketches.

The case of equal moments of inertia for all girders ($I_g = I_1 = I_2 = I_3 = I_4$) will first be developed.

The net load of a typical girder is $P - D$ and the deflection at the center is

$$\Delta = \frac{(P - D)l^3}{48\, E_g I_g} \qquad (1)$$

wherein E_g is the modulus of elasticity and I_g is the moment of inertia of a girder.

The movement of the diaphragm in space under some combination of loads P_1, P_2, etc, on the bridge is illustrated by Figure B, which shows also the de-

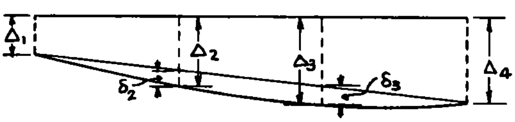

Figure B.

flections of points on the diaphragm with respect to the chord joining its ends It should be noted that

$$\Delta_2 = \frac{2}{3}\Delta_1 + \frac{1}{3}\Delta_4 + \delta_2 \text{ and} \qquad (2)$$

$$\Delta_3 = \frac{1}{3}\Delta_1 + \frac{2}{3}\Delta_4 + \delta_3. \qquad (3)$$

Since the diaphragm is a beam in equilibrium under the action of forces D, we may choose to consider any of these forces as reactions and the others as loads. We must recognize that the actual signs of some of these forces will be negative and be prepared, therefore, to find in the final solution that some of our assumed reactions act downward and some of our assumed loads act upward. The diaphragm can be represented as a conventional simple beam by showing $-D_1$ and $-D_4$ as upward acting forces as shown in Figure C.

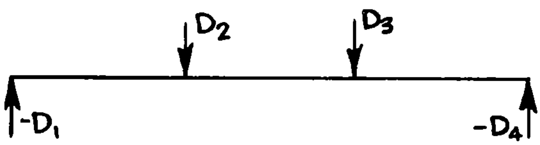

Figure C.

The deflection of the diaphragm at each girder intersection under these loads can be computed by various methods. It is perhaps easiest to use the formula

$$\Delta_x = \frac{Pbx}{6EI}(l^2 - b^3 - x^3) \qquad (4)$$

applying to Figure D. In applying this formula, the

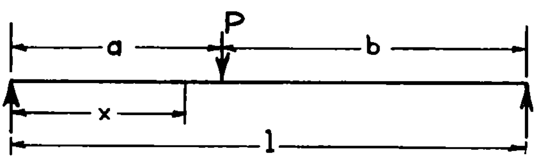

Figure D.

diaphragm deflection at Girder 2, first due to D_2, then due to D_3, are determined and added By this method

$$\delta_2 = \frac{8D_2 s^3}{18E_d I_d} + \frac{7D_3 s^3}{18E_d I_d} = \frac{s^3}{18E_d I_d}(8D_2 + 7D_3) \qquad (5)$$

$$\delta_3 = \frac{s^3}{18E_d I_d}(7D_2 + 8D_3) \qquad (6)$$

wherein E_d is the modulus of elasticity and I_d is the moment of inertia of diaphragm.

We now introduce $K = \frac{l^3}{48E_g I_g}$ and $N = \frac{s^3}{18E_d I_d}$

Substituting these values in Equation 2

$$K(P_2 - D_2) = \frac{2}{3}K(P_1 - D_1) + \frac{1}{3}K(P_4 - D_4) + N(8D_2 + 7D_3) \qquad (7)$$

Introducing $R = \frac{N}{K}$

$$(P_2-D_2)=\frac{2}{3}(P_1-D_1)+\frac{1}{3}(P_4-D_4)+R(8D_2+7D_3) \tag{8}$$

$$\frac{2}{3}D_1-(8R+1)D_2-7RD_3+\frac{1}{3}D_4=\frac{2}{3}P_1-P_2+\frac{1}{3}P_4 \tag{9}$$

$$\frac{1}{3}D_1-7RD_2-(8R+1)D_3+\frac{2}{3}D_4=\frac{1}{3}P_1-P_3+\frac{2}{3}P_4 \tag{10}$$

From the conditions of static equilibrium of the diaphragm under forces D_1, D_2, D_3 and D_4, two additional equations can be written.

$$\Sigma F_v=D_1+D_2+D_3+D_4=0 \text{ and} \tag{11}$$

$$\Sigma M_1=D_2+2D_3+3D_4=0 \tag{12}$$

In the simultaneous solution of Equations 9, 10, 11, and 12 for any particular bridge, it is best to introduce the computed value of R, but the values of P_1, P_2, P_3 and P_4 should be left in general terms so that effect of any transverse position of wheel load can be determined without solving additional sets of equations.

If the moments of inertia of the girders of a structure differ enough to warrant consideration in the computation, separate values of K_1, K_2, etc, are introduced and Equations 9 and 10 become:

$$\frac{2}{3}K_1D_1-(8N+K_2)D_2-7ND_3+\frac{1}{3}K_4D_4=\frac{2}{3}K_1P_1-K_2P_2+\frac{1}{3}K_4P_4 \tag{13}$$

$$\frac{1}{3}K_1D_1-7ND_2-(8N+K_3)D_3+\frac{2}{3}K_4D_4=\frac{1}{3}K_1P_1-K_3P_3+\frac{2}{3}K_4P_4 \tag{14}$$

This same general method can be applied to spans with greater numbers of girders and diaphragms. It will be noted that the number of simultaneous equations will equal the number of D forces which, in turn, will equal the number of girder-diaphragm intersection. To develop equations for conditions involving loadings other than midspan, numerical coefficients must be determined for P_1, P_2, etc., in Equation 1.