

Reactions of a Two-Span, Skewed, Rigid-Frame Bridge

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● THE MODERN highway designer, faced with an unprecedented high-speed traffic load, finds solutions to many congested-intersection problems with grade-separation structures. Traffic-count limits have been firmly established which indicate the advisability of such a solution. However, since highways frequently intersect obliquely, the use of skewed structures becomes necessary in order to avoid serious traffic hazards.

In recent years, the rigid-frame bridge has had application to the grade-separation problem, because of its marked advantages in meeting limited headroom conditions, its adaptability to architectural treatment, and its economy typical of continuous structures. With the longer spans required by improved highway design, the two-span rigid frame finds potential application to grade separations, the center leg being accommodated by the medial dividing strip. However, it is imperative that one consider the deterring factors to such a design. The demands made upon the designer by increase in design time, together with the decreased fee inherent in a more economical design, presents a primary obstacle to such a solution. This is the paradox that confronts any conscientious designer. This inconsistency can be partially alleviated by an enlightened approach to the design problem, but perhaps more forcefully by a reappraisal of the system of determination of fees on a construction cost basis—a system which actually places a penalty on a more intricate and lengthy design procedure—even though it produces a more economical design. The study herein reported was made in an endeavor to contribute to a simplified approach to the skewed, rigid-frame problem and to indicate areas where continued study is advisable.

Where highway intersections are nonrectangular, structures must be skewed in order to maintain road alignment essential to safety. Skewed rigid frames, however, present some difficulties with which early designers could not cope. Some failures have occurred which have been attributed to fallacious design procedure based on lack of knowledge of the forces acting on and within a skewed frame.

In 1924, J. Charles Rathbun presented the first logical mathematical analysis (1) of skewed frames which correctly included the effects of torsion on the

reactions of such structures. In an attempt to test the validity of Rathbun's solution, the late George E. Beggs conducted a series of model tests at the specific request of the American Society of Civil Engineers. Beggs' conclusions were in substantial agreement with the theoretical solution; consequently the sponsoring committee reported (2) that the results of the limited number of tests seemed to indicate that the theory might be safely applied to skewed structures.

The Rathbun analysis has been used successfully, and important simplifications of his work have been made by Richard M. Hodges in 1944 (3) and Maurice Barron in 1950 (4).

Description of the Investigation

The investigation herein presented is a continuation of a model analysis program originally conceived and recently reported by Walter C. Boyer (5) which was intended to further dispel the doubts surrounding the Rathbun theory. The results of Boyer's work on single-span skewed frames were in good agreement with the Rathbun solution.

The theoretical analysis is based on the same assumptions that were used by Rathbun in his original solution (6). The general form of the solution was suggested by the work of Barron (7), involving unit deflections, but required the solution of two sets of simultaneous elastic equations.

The basic structure studied was a hinged, two-span rigid frame of two equal square spans 100 ft long with a leg height of 22 ft, which was satisfactory for model analysis when scaled down. Effect of skew on reactions was investigated for skew angle variation from 0 to 50 deg, in increments of 10 deg. The deck was flat, constant in thickness, and 40 ft. wide.

Reactions inherent in the skewed frame and the corresponding nomenclature are shown in Figure 1. It should be carefully noted that the primary effect of skew angle is to create a couple with the eccentric horizontal reactions, R_x , and that this couple must, for equilibrium, be resisted by an equal and opposite couple involving the cross shears, R_z . It is this reaction component, R_z , which is peculiar to the skew frame and which contributes largely to the torsional stresses in the deck slab.

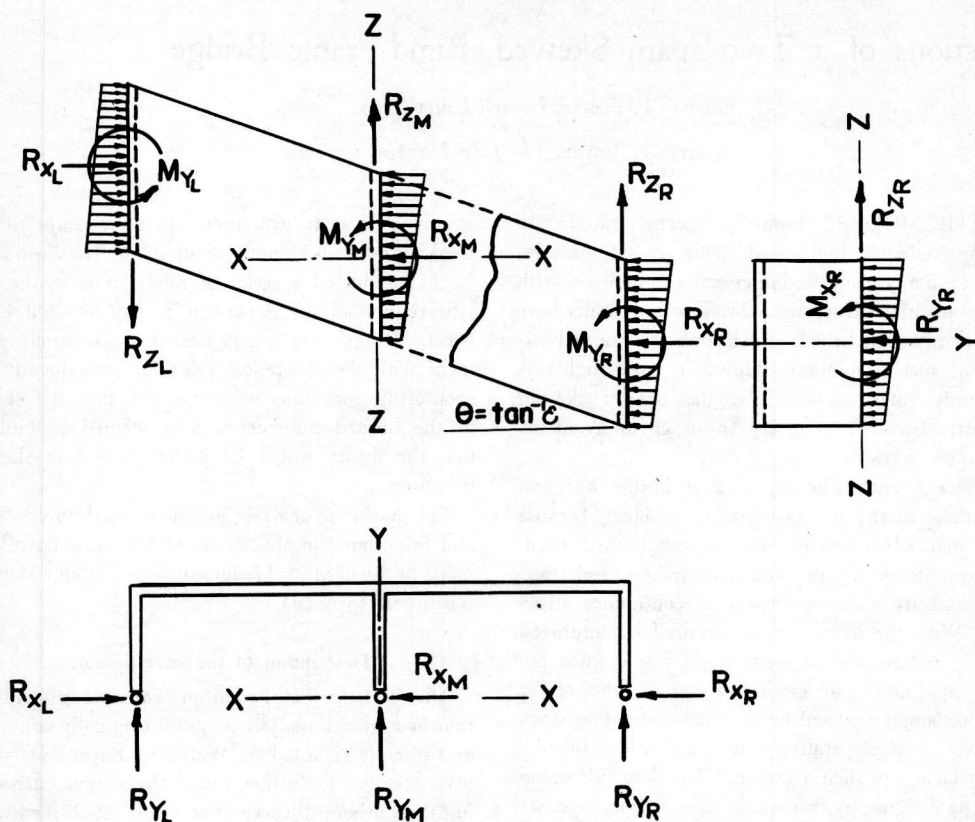


Figure 1. Reactions for a hinged, two-span, skewed frame.

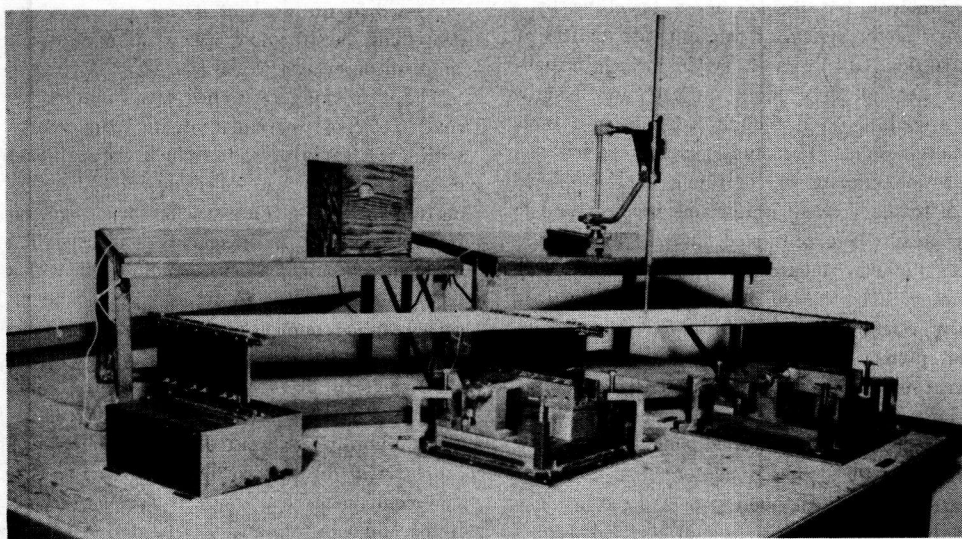


Figure 2. Test arrangement.

Procedure and Equipment

The experimental analysis used in this investigation is based on the deformeter method developed by Beggs (8), by means of which influence lines are

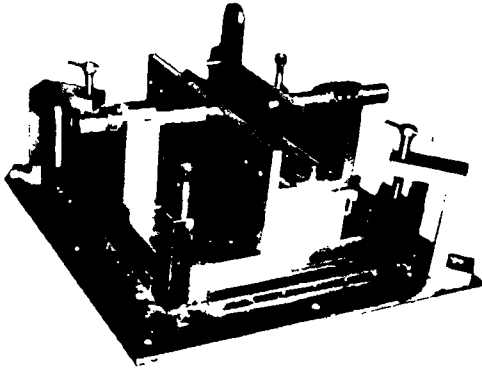


Figure 3. Movable abutment.

obtained through the use of controlled deflections rather than applied loads. A modification of the Beggs method as proposed by William J. Eney (9), employing reasonably large deflections of an order that can be read with the unaided eye, has been used for this study.

The test arrangement shown in Figure 2 consisted of two movable abutments, one at the right end and the other at the center of the bridge; a stationary abutment at the left end; deflection gage and circuit detector system; and two independent datum plates mounted on leveling stands.

The movable abutment used to induce the controlled deflections is shown in Figures 3 and 4. Vertical uplift for the function, R_v , is achieved by raising the whole abutment off the lower base plate by means of the uplift screws, and placing under the abutments shim blocks of thickness corresponding to the desired deflection. Horizontal displacement along the x axis for function R_x is obtained by sliding the abutment between guide angles (B) and placing shim blocks between the abutment and guide angle (A). Likewise, movement along the z axis for the cross-shear function, R_z , is applied by sliding the abutment between guide angles (A) and shimming against guide angle (B). Torsional moment, M_x , requires a rotation of the channel section about the shaft forming the x - x axis, controlled in magnitude by pinning into calibrated radial holes in the butt stop or by other suitable means to obtain a required angle of twist. Rotation for horizontal moment, M_z , is applied in the horizontal plane about the center pin and is controlled by pinning into base-plate holes arranged on a previously calibrated, fixed radial pitch.

To measure the vertical slab deflections induced by such controlled displacements, a converted hydraulic point gage, accurate to 0.003 in., was used and arranged to move freely on the fixed and independent datum. Deflection readings for each function were taken at each of the grid points shown in Figure 5.

An electric circuit making use of an electronic circuit detector, and a coating of conducting silver paste on the slab surface completed the test setup.

Models were constructed to a scale of 1 in. equals 5 ft. of grade XXX paper-base phenolic-resin sheet. The legs were twice as thick as the slab, and were joined to the slab by steel clamps to form a rigid knee. Hinged supports were made of ordinary cabinet hinges, carefully selected, reamed, and repinned to provide frictionless rotation without excessive play.

Test Results

Model results, in terms of influence ordinates, are compared with corresponding theoretical values in Tables 1 and 2.

In Tables 1 and 2, influence ordinates for centerline loading are given for representative reactions, in this instance R_v and R_z at the abutment and R_v and R_z at the center pier.

Values for off-center loading for a 30-deg. skew bridge are given in Tables 3 and 4 for the rectangular functions R_v and R_z at the abutment. Off-

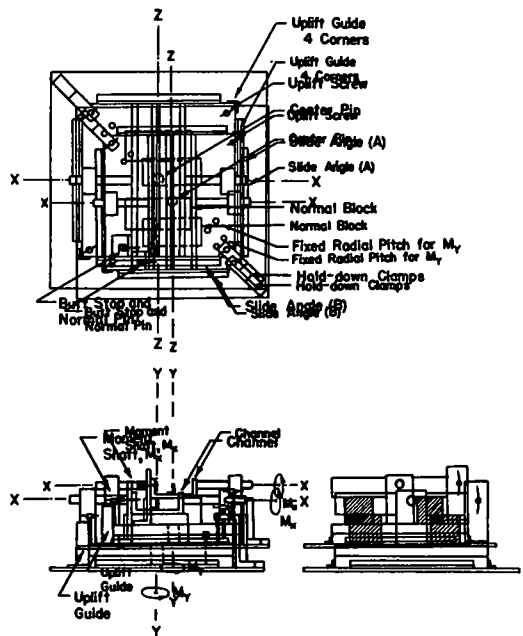


Figure 4. Schematic diagram of movable abutment.

center loading ordinates for the functions R_{vm} and R_{vm} at the center pier are not given, but they showed even closer agreement between experimental and theoretical values than the abutment reactions.

Influence contours for visual comparison of typical functions are shown in Figures 6 to 9 for a 30-deg. skew.

Model limitations precluded the reporting of experimental results for R_v , the cross shear. Theoretical analysis, however, shows R_v to be almost precisely equal to the product of the tangent of the skew angle and the corresponding horizontal thrust, i.e., R_v equals $R_h \tan \theta$. This coincides with both theoretical and experimental observations for the single-span

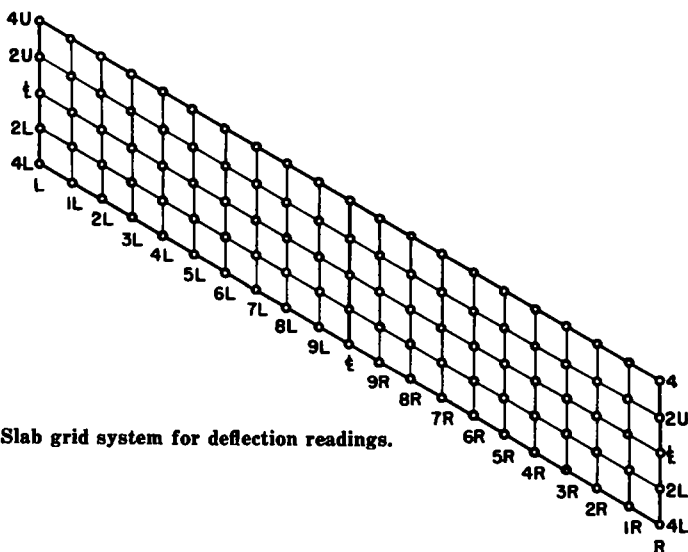


Figure 5. Slab grid system for deflection readings.

Experimental results for cross-shear functions, R_v and R_{vm} , are not reported. The inherent stiffness of the bridge in resisting the action of the cross-shear against the full width of the slab, and the consequent limitations of the model equipment, obviated the possibility of measuring reliable values for these functions. Redesign of the equipment was not warranted for the purpose of this program.

Conclusions

Study of the results leads to the immediate conclusion that the vertical reactions, R_v , and the horizontal thrusts, R_h , are essentially independent of the skew for centerline loading. This means, in effect, that R_v and R_h for any angle of skew are the same as for a similar right frame of the same square span. The skew has some effect on these reactions for off-center loading, but is not considered practically important in view of the fact that centerline loading gives the greater stresses in the bridge and is ordinarily used in design. The independence of these reactions follows the similar conclusion for the single span bridge reported by Boyer (10), and is given further support by the simplified theories of R. M. Hodges (11), and M. Barron (12).

bridge and the work of other investigators mentioned heretofore. The double-span bridge gave no evidence to the contrary, and there seems to be little reason to doubt the relationship given.

The horizontal-plane moments, M_h , proved to be negligible by both model and theoretical analysis, and can be neglected safely in design without serious error. Slab deflection of the model bridges for this function in nearly all cases was too small to be measured with the device used.

A serious discrepancy between experiment and theory has been found for the torsional moments, M_t . For centerline loading, the theory gives negligible values for both M_{tm} and M_{tm} , whereas the model study shows centerline ordinates of considerable magnitude, with the difference increasing with skew angle. Since the usual method of design is based on centerline loading, design moments as given by the theory are apparently much smaller than those which actually exist, and therefore on the unsafe side. Off-center loading for M_{tm} again shows model results to be generally higher than the theoretical, but the discrepancy is neither so obvious nor serious as for the centerline.

Experimental off-center values for the torsional moment, M_{τ} , are at variance completely with the theoretical values and indicate a basically different type of slab action. Equilibrium in both analyses checks reasonably well, and each appears to be a rational action of the bridge. The authors consider the experimental result to be closer to the true action of the bridge on the basis that it consistently occurred for all angles of skew.

The question naturally arises as to why such discrepancy exists. It has been shown that the major differences occur for the torsional moment functions, M_{τ} , whereas the results for the rectangular functions

poses of this investigation, it was assumed that

$$F = \frac{bt^3}{35.78},$$

an empirically obtained factor (14) for torsion of concrete beams with 1-to-4 depth-to-width ratio. This factor agrees reasonably well with the Saint Venant value for such sections. The usual ratio for bridge slabs, however, is 1-to-15 or greater. The Saint Venant factor,

$$F = \frac{bt^3}{3},$$

TABLE I
COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR REACTION R_{YR}

Part A Experimental Values											
GRID POINTS ALONG CENTERLINE OF SLAB											
Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	0.34	0.43	0.35	0.13	— 0.13	— 0.39	— 0.54	— 0.56	— 0.39	0
10°	0	0.28	0.39	0.28	0.12	— 0.16	— 0.43	— 0.59	— 0.67	— 0.47	0
20°	0	0.28	0.39	0.28	0.16	— 0.16	— 0.35	— 0.55	— 0.59	— 0.43	0
30°	0	0.35	0.47	0.39	0.20	— 0.08	— 0.39	— 0.55	— 0.59	— 0.43	0
40°	0	0.32	0.43	0.35	0.12	— 0.20	— 0.47	— 0.67	— 0.63	— 0.39	0
50°	0	0.39	0.55	0.55	0.28	— 0.08	— 0.35	— 0.63	— 0.67	— 0.39	0
GRID POINTS ALONG CENTERLINE OF SLAB											
Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right	
0°	0.58	1.41	2.39	3.52	4.73	6.01	7.20	8.28	9.28	1.000	
10°	0.55	1.37	2.39	3.53	4.74	6.03	7.17	8.35	9.28	1.000	
20°	0.51	1.34	2.36	3.54	4.80	6.06	7.24	8.38	9.32	1.000	
30°	0.59	1.33	2.39	3.49	4.78	6.00	7.25	8.38	9.28	1.000	
40°	0.55	1.30	2.32	3.47	4.71	5.98	7.20	8.30	9.25	1.000	
50°	0.55	1.18	2.17	3.39	4.77	6.18	7.52	8.58	9.41	1.000	
Part B Theoretical Values											
GRID POINTS ALONG CENTERLINE OF SLAB											
Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	0.034	0.045	0.038	0.020	— 0.003	— 0.026	— 0.043	— 0.049	— 0.036	0
10°	0	0.034	0.045	0.038	0.021	— 0.003	— 0.026	— 0.043	— 0.049	— 0.036	0
20°	0	0.034	0.045	0.039	0.021	— 0.003	— 0.026	— 0.043	— 0.049	— 0.036	0
30°	0	0.035	0.046	0.039	0.021	— 0.002	— 0.026	— 0.043	— 0.048	— 0.036	0
40°	0	0.035	0.046	0.040	0.022	— 0.002	— 0.025	— 0.043	— 0.048	— 0.036	0
50°	0	0.035	0.047	0.040	0.022	— 0.001	— 0.025	— 0.043	— 0.048	— 0.036	0
GRID POINTS ALONG CENTERLINE OF SLAB											
Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right	
0°	0.064	0.151	0.257	0.374	0.497	0.620	0.738	0.845	0.934	1.000	
10°	0.064	0.151	0.257	0.374	0.497	0.621	0.738	0.845	0.934	1.000	
20°	0.064	0.151	0.257	0.374	0.497	0.621	0.739	0.845	0.934	1.000	
30°	0.064	0.152	0.257	0.374	0.498	0.621	0.739	0.846	0.935	1.000	
40°	0.064	0.152	0.257	0.375	0.498	0.622	0.740	0.846	0.935	1.000	
50°	0.064	0.152	0.257	0.375	0.499	0.622	0.740	0.847	0.935	1.000	

are in substantial agreement with theory. It has been further shown by Barron (13) that, for all practical purposes, the rectangular and torsional systems are independent of each other. This gives support to the possibility of the situation existing in this investigation, namely, that there could be a major discrepancy in the torsional moments, and simultaneously agreement in the rectangular functions.

In view of the fact that differences occur only in this isolated instance, suspicion is cast accordingly upon the use of the torsional factor, F . For the pur-

for unskewed plates with large width-thickness ratios appears to be valid from the work of many investigators (Foppl, Stussi, Bach, etc.) and would have been a more satisfactory value to use here; however, the large differences in the torsional moments cannot be accounted for by this fact alone. The skewed plate presents a problem in combined torsion and bending and there is considerable doubt that the Saint Venant factor can be used without modification accounting for possible interaction of the plate's flexural rigidity

The Saint Venant factor is defined for the square

LOAD STRESS IN BRIDGES

TABLE 2
COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR REACTION R_{XB}

Part A Experimental Values

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	082	102	085	039	— 019	— 076	— 112	— 121	— 085	0
10°	0	074	092	074	029	— 024	— 068	— 110	— 118	— 092	0
20°	0	076	095	076	039	— 018	— 068	— 108	— 121	— 087	0
30°	0	087	120	104	059	— 006	— 069	— 112	— 122	— 087	0
40°	0	095	128	112	059	— 006	— 073	— 120	— 130	— 093	0
50°	0	098	149	136	079	0	— 079	— 135	— 147	— 100	0

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
0°	121	245	369	466	548	581	560	467	281	0
10°	100	220	336	438	509	538	514	422	241	0
20°	102	215	331	428	504	536	515	420	249	0
30°	116	228	337	433	508	539	518	425	250	0
40°	100	201	309	414	500	543	533	449	331	0
50°	102	193	299	400	492	543	537	453	270	0

Part B Theoretical Values

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	0 077	0 100	0 085	0 045	— 0 009	— 0 059	— 0 098	— 0 110	— 0 081	0
10°	0	0 077	0 100	0 085	0 046	— 0 007	— 0 059	— 0 098	— 0 110	— 0 081	0
20°	0	0 077	0 101	0 086	0 046	— 0 007	— 0 059	— 0 098	— 0 111	— 0 081	0
30°	0	0 078	0 103	0 087	0 047	— 0 006	— 0 058	— 0 098	— 0 109	— 0 081	0
40°	0	0 078	0 104	0 088	0 049	— 0 005	— 0 057	— 0 097	— 0 109	— 0 081	0
50°	0	0 079	0 105	0 090	0 050	— 0 004	— 0 056	— 0 097	— 0 109	— 0 081	0

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
0°	0 121	0 250	0 376	0 481	0 555	0 585	0 559	0 460	0 279	0
10°	0 121	0 250	0 376	0 481	0 557	0 586	0 559	0 460	0 279	0
20°	0 121	0 251	0 376	0 481	0 557	0 586	0 560	0 461	0 279	0
30°	0 121	0 251	0 376	0 482	0 558	0 587	0 561	0 461	0 280	0
40°	0 121	0 251	0 377	0 483	0 559	0 589	0 562	0 461	0 280	0
50°	0 121	0 251	0 377	0 484	0 560	0 590	0 564	0 465	0 281	0

TABLE 3

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR OFF-CENTER LOADING FOR REACTION R_{YH} , 30-DEG SKEW

Model Results

TRANSVERSE SECTION

Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	039	071	078	067	047	012	— 016	— 035	— 035	0
c	0	035	047	039	020	— 008	— 039	— 055	— 059	— 043	0
4L	0	024	020	0	— 031	— 067	— 082	— 094	— 074	— 039	0
	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right	
4U	051	118	200	298	410	545	678	800	917	1 000	
c	059	133	239	349	478	600	725	838	928	1 000	
4L	070	165	275	403	533	655	764	858	944	1 000	

Theoretical Results

TRANSVERSE SECTION

Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	0 057	0 085	0 089	0 080	0 059	0 033	0 008	— 0 009	— 0 014	0
c	0	0 035	0 046	0 039	0 021	— 0 002	— 0 026	— 0 043	— 0 048	— 0 036	0
4L	0	0 012	0 006	— 0 013	— 0 038	— 0 064	— 0 085	— 0 095	— 0 088	— 0 059	0
	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right	
4U	0 041	0 112	0 205	0 315	0 436	0 562	0 687	0 806	0 912	1 000	
c	0 064	0 152	0 257	0 374	0 498	0 621	0 739	0 846	0 935	1 000	
4L	0 086	0 191	0 308	0 433	0 559	0 680	0 791	0 885	0 957	1 000	

section only, that is, for a section perpendicular to the bridge centerline, whereas the theoretical analysis assumes that it applies equally well to the skewed section. Such assumption is highly questionable, and the matter cannot be resolved simply by splitting M into components along the centerline and at right angles thereto because of the difficulties arising from

that the apparent "torsional factor" for such plate should be likewise different from the corresponding Saint Venant value. Certainly there seems to be little basis for confidence in the torsional factor as currently used in skew bridge analysis. Fundamental theoretical and experimental study of the torsional action of skewed plates of large width-to-thickness

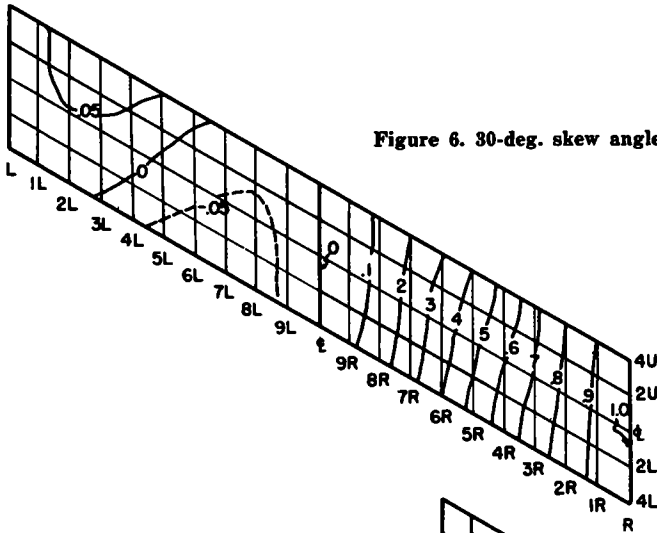


Figure 6. 30-deg. skew angle: Influence contours for $E_{\eta\eta}$.

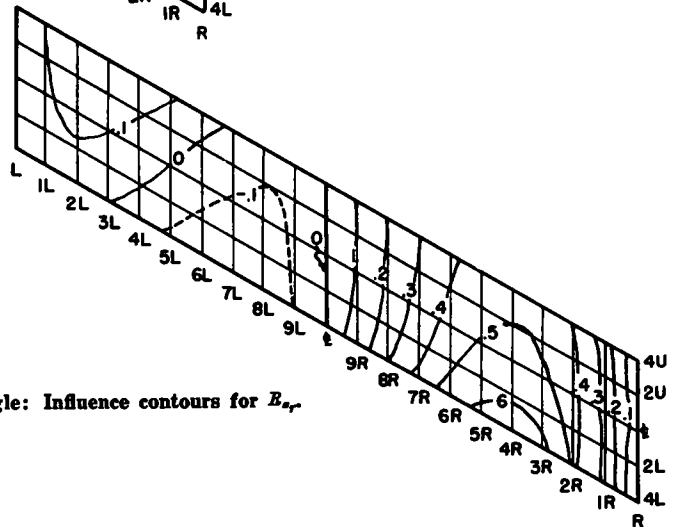


Figure 7. 30-deg. skew angle: Influence contours for $E_{\xi\xi}$.

the triangular portions at each end of the slab. Partial edge clamping at the bridge knee, perhaps nonuniform in character, adds to the complexity.

It is conceivable, therefore, that the action of a skewed plate in torsion should be considerably different from that for a beam or unskewed plate, and

ratio is felt to be highly desirable.

In conclusion, it is felt that the experimental results reported essentially prove the validity of the theory, and that the theory should provide safe design values once the torsional factor has been redefined for skew plates.

(3) Cross-shear, R_s , is equal to the tangent of the skew angle multiplied by the corresponding thrust R ; model results are not reported for this reaction because of model limitations, but ample evidence exists to substantiate this fact.

6. *Loc. cit.*, RATHBUN, "Stresses in Ring of Concrete Skew Arch," p. 611.
7. *Loc. cit.*, BARRON, "Reinforced Concrete Skewed Bridges," p. 2.
8. GEORGE E. BEGGS, "The Use of Models in the Solution of Indeterminate Structures," *Jour-*

TABLE 4
COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR OFF CENTER LOADING FOR
REACTION R_{XR} 30-DEG SKEW

Model Results											
TRANSVERSE SECTION											
Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	104	171	191	167	114	043	— 026	— 067	— 069	0
c	0	087	120	104	059	— 006	— 069	— 112	— 122	— 087	0
4L	0	065	057	014	— 051	— 118	— 165	— 187	— 163	— 095	0
Theoretical Results											
TRANSVERSE SECTION											
Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	0 128	0 191	0 203	0 180	0 132	0 075	0 018	— 0 021	— 0 031	0
c	0	0 078	0 103	0 087	0 047	— 0 006	— 0 058	— 0 098	— 0 109	— 0 081	0
4L	0	0 027	0 014	— 0 030	— 0 085	— 0 145	— 0 191	— 0 214	— 0 198	— 0 131	0
Theoretical Results											
TRANSVERSE SECTION											
Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0 071	0 162	0 260	0 349	0 419	0 455	0 444	0 374	0 229	0	
c	0 121	0 251	0 376	0 482	0 558	0 587	0 561	0 463	0 280	0	
4L	0 171	0 339	0 492	0 615	0 696	0 720	0 677	0 551	0 330	0	

(4) Horizontal moment M_v is negligible and may be neglected in design.

(5) Torsional moments, M_s , show serious discrepancy with theory, with theoretical values apparently on the unsafe side. It is believed that the Saint Venant torsional factor is not applicable to skewed plates having the proportions of bridge slabs, and fundamental investigation of this factor is considered to be desirable.

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Discussion

JASON PLOWER and HERBERT GEE, *California State Division of Highways*—We feel that the information is well presented and the report is a noteworthy contribution. It is particularly satisfying to note that our general practice in design of this kind of structure agrees closely with most of their findings. The only uncertainty is in the torsional moments, mainly because actual comparative data are lacking.

The authors state in their first paragraph that the design of skewed structures often becomes necessary

because highways cross each other obliquely. How true this is. Not only do designers of highway-separation structures find themselves confronted with skewed layouts, they often find that their problems are even more complex. For instance, at many complicated intersections a layout may require structures that are skewed by a varying amount at each bend. In addition, the superstructure may be on a curve of fairly sharp radius. Therefore bridge designers welcome any experimental studies which will give them a better understanding of the many complex design problems confronting them.

Possibly a greater value would have been derived from the results had a model of more usual proportions been used for the test. From its description, the basic structure studied, apparently, was a two-span, rigid-frame, flat-slab bridge of 100-ft. spans. Generally, for spans of this size, it would have been uneconomical to build as a true flat-slab type. We have found the economical span limit to be about 55 ft.

It is further noted that the model used has legs which are twice the thickness of the slab and of constant thickness. Our experience has shown that in average slab type designs, the abutment thickness at the top is usually about 0.8 or 0.9 of the depth of the slab and tapers towards the footing. The depth of slab to span ratio is approximately 0.060. The center pier, being symmetrically located, is of lesser importance, and its size, shape, and other features depend upon its aesthetic and economical requirements.

The authors have not reported on the earth pressure at the back of the abutments which is invariably present. This pressure, though relatively small when used in conjunction with forces created by a 100-ft span with skewness under 25 deg., should nevertheless be taken into account. On bridges with larger skewness, the earth pressure may have an important influence upon the structure due to its eccentric application.

In actual practice, 100-ft. spans would call for a T-beam or box-girder superstructure construction and numerous structures of this span and type have been built by the California Division of Highways. Factors such as width of structure and lengths of wingwalls may become paramount in the economical determination of the type to be chosen. For two-span rigid frames, the thickness of solid, slab-type abutments at the top varies between 0.6 and 0.8 of the depth of the girders and tapers towards the footing. The depth of girder to span ratio is usually from 0.065 to 0.080 for T-beams and from 0.055 to 0.070 for box girders. Skews up to 60 deg. have been used in some instances.

Generally, any two-span, rigid-frame bridge with skew of 45 deg or over should have strong arguments in its favor if it is to be selected, otherwise, a free ended span on self supporting abutments or open end type of spans should be used.

With regards to the torsional action in bridges with large skewness, the slab-and-girder construction has the ability to deform and adjust itself. The acute corners between girders and abutments are heavily reinforced with additional reinforcements and diaphragms to distribute the corner loads so as to make them act more like rectangular structures. Experience has indicated that our treatment of the acute corners of skewed bridges is on the safe side as attested by the many structures of this type in use.

We believe that the experiment is an advancement in the direction of proving the validity of the accepted theory and further investigations should be encouraged. However, any future research would be more beneficial if models used are more within the proportions of usual designs. Comparative data between flat-slab bridges and slab-and-girder structures may reveal results that are exceedingly valuable and certainly any information on torsional moment in slab-and-girder construction is most welcome.

E. L. ERICKSON, *Bureau of Public Roads*—This paper describes tests on phenolic models of a 2100-ft.-span rigid frame with 22-ft. legs. The span was measured at right angles to the abutments. The thickness was constant and the width 40 ft. The Maxwell theory of reciprocal deflections was used instead of direct loading. The method is similar to that of the Beggs deformeter gage except that much larger deflections were used, thus obviating the need for microscopes. Skews of 0 to 50 deg. were studied but the authors do not state how they vary the skew. Probably they used a number of models. The abutments and piers were hinged at the footings. One abutment was subjected to various deflections and rotations and then deflections measured at various points on the body of the frame.

The authors give tables showing the comparison of tests and computations using (presumably) the Rathbun analysis. On the whole the tables show that the tests agree reasonably well with theory. No comparison is shown, however, between test and theory for torsional moments but the authors state that the agreement here was very poor and that theory erred on the unsafe side, assuming the tests to be correct.

Not only do the tests corroborate the Rathbun

theory, with the exceptions noted, but they also indicate the feasibility of using certain short cuts previously suggested by Maurice Barron.

If it were not for two loose ends, engineers could design skew arches and frames with a good deal of confidence. We still lack sufficient knowledge to properly evaluate the effect of a combination of transverse shear and torsion and we do not know how the width of the structure effects the stresses. As regards the first point, Hayden and Barron in their book, *Rigid Frame Bridge*, suggest a rule of thumb for determining the transverse steel, which would indicate that they consider the matter of minor importance. It would seem, however, that the second point, *viz.*, the effect of width of structure, can be of vital importance. Common sense would indicate that a skew bridge wider than its square span would act more like a square bridge than a skew bridge.

Fisher and Boyer also studied the effect of eccentric loads. The data obtained is valuable and is related to width of structure, but unfortunately the width of the models was quite small in relation to the span and so did not show up the effect of width sufficiently.

It is hoped, therefore, that further tests will be made to throw more light on these loose ends, *viz.*, the rational design of the transverse steel and the effect of width of structure on the stresses due both to concentric and eccentric loads.

G. P. FISHER and W. C. BOYER, *Closure*—The authors wish to thank the discussers for the interest and effort put forth in reviewing this paper, and are pleased to find such favorable acceptance.

Most of the questions raised by the discussers deal with the selected proportions and material of the model bridges tested. It was necessary that the material used for the models be elastic and reasonably isotropic within the range of deflection desired, have close tolerance on thickness, and be applicable to high humidity conditions. Phenolic laminate (Formica) was finally selected as most nearly fulfilling these requirements. As the "deformeter" method of model

analysis makes use of *ratios* of deflections, the kind of material used is not of prime importance and may well be different from that of the prototype. As a matter of fact, it is desirable that the model material have as low an elastic modulus as possible so as to allow measurable deflections with rather light loading.

The information provided by Plower and Gee relating to economical proportions of rigid-frame bridges is highly useful, especially for the ribbed-slab types. The unusually long span of the model bridges was chosen purposely in order to exaggerate the effect of the cross-shears, R_x , and the leg height of 22 ft. was selected as typical of grade separation structures. Roadway width, questioned by Erickson, was not varied in this series of tests, as it appears to have minor influence on the basic action of the structure, as indicated by a previous investigation of the single-span bridge by Boyer (see Ref. 5). Variation of skew was accomplished by use of a number of models. Constant thickness of slab and legs was necessary for ease of manufacture of the models. It is thought that the selected proportions do not invalidate in any way the results obtained or their significance for bridges of more usual proportions.

Erickson raises a valid question with regard to evaluation of stresses resulting from combined torsion and transverse shear, and this is a phase of design which requires thorough investigation. While this investigation was concerned only with the evaluation of total forces, it cannot be ignored that the distribution of these forces constitutes a major problem. The authors believe, at the moment, that the torsional shear stresses as computed by St. Venant theory may be simply superimposed on the flexural shears (the latter assuming the slab as a beam of depth equal to the roadway width) provided the stresses are computed for the square width of roadway and not the skew width. As pointed out in the paper, the use of the St. Venant torsional analysis (and indeed to the one used herein for comparison) is a highly questionable practice and possibly nonsensical, not only for the determination of the total reactions but also for the distribution over a given design section.