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Research Report 14-B

Distribution of Load Stresses In Highway Bridges

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Research Report 14-B

Distribution of Load Stresses In Highway Bridges

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CONTENTS

Effect of Trucks upon a Few Bridge Floors in Iowa in 1922 and in 1948

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SYNOPSIS

INVESTIGATIONS for impact were made during the summers of 1922 to 1925 when the trucks were entirely different from those now in use. The present discussion is concerned not with definite values, but with various factors such as (1) the relation of the force of a wheel blow upon pavement to that of a similar blow upon a more flexible bridge floor and (2) the relation of the stress which is developed in the stringers and floor beams by a dynamic blow and by a static load of the same weight as the force of the blow.

Initial static readings for deformation were taken primarily as a basis for dynamic readings. These readings also showed the comparative deformations of the various longitudinal stringers, or in other words, the distribution of load among the stringers. They also pointed definitely to the action of the reinforced-concrete floor slab in relieving the steel stringers from much of the stress which would have been developed if they alone had been carrying all the load.

A brief exploratory research was undertaken in 1948 as a means of observing whether the composite action between the concrete floor slab and the steel stringers had remained after 28 years of service.

The results indicate that in a 34-ft. I-beam span the composite action was still effective; and that in a panel of a truss bridge, although the bond was apparently broken, the deformations and resulting stresses were less than if the steel alone was supporting the load.

The results also indicate that in the 20's and in 1948 the load transferred to the most loaded stringer was (for these closely spaced stringers) somewhat in line with the provisions of the AASHO specification.

● THE SIGNIFICANT results of the effect of trucks upon impact and stresses in the floors of 12 bridges in central Iowa, with which the writer has been connected, have been published (*1*, *2*, *3*, *4*, *5*). Four of the published reports are on the researches conducted from 1922 to 1925 and include the effects of both static and dynamic loads. Another of the publications deals only with a few tests from static loading which were made in 1948 as an exploratory research for the purpose of roughly approximating the effects of a quarter century of traffic upon the composite action between the reinforced-concrete floor slab and the steel stringers and floor beams.

The researches of the early 1920's were directed at the problem of impact in highway bridges. The available loads were Liberty trucks which weighed about 3 1/2 tons and, although rated for 5 tons of load, were loaded with gravel to a total of about 15 tons with 12 tons on the rear axle. The maximum speed attainable was 15 mph. Although 12 bridges were included in the study, the greater part of the data, especially for concrete floor on steel stringers, was secured from an 18-ft.-9-in. panel of a 150-ft. steel truss span, and a 32-ft.-8-in. beamspan, with 20-ft. roadway and a 24-ft. steel beam span with 24-ft. roadway. The dimensions of all of the spans and the loads are given in the report on the work (*3*).

The impact results of this and other available work were reviewed by the Committee on Impact in Highway Bridges of the American Society of Civil Engineers, of which the author was the chairman. A report was presented at the annual meeting in 1929 (*4*). All available data were based upon loads which seldom produced a static unit stress as great as 10,000 psi., and most stresses were far below that figure. Many impacts were reported of several hundred percent, but they were for light loads, largely unsprung, with the greatest dynamic unit stresses around 16,000 psi. and with the truck wheels going over artificial obstructions up to 2 by 4 in. The tires were well-worn solid rubber.

The recommendations of the committee in regard to impact in floors (*4*) are:

1. That stresses due to static loads and to impact are important, as regards the safety of the structure, only when they approach design values

2 That the percentage of impact increment decreases as the loads increase, and therefore as the unit stresses increase

3. That the larger impacts observed in the tests were produced by obstructions such as would be accidental and infrequent under actual traffic conditions.

4. That the actual occurrence, on a bridge, of loads having a magnitude corresponding to those used in the design of modern structures is infrequent.

5 That the simultaneous occurrence on a bridge floor of a maximum truck load and an accidental obstruction capable of producing high impact will be such a rare coincidence that presumbably the factor of safety usually will provide safety for this condition.

This committee recommends, therefore, that for the design of highway bridge floors and floor-beam suspenders, the impact increment of stress be assumed as 25% of the live load stress It should be used only when the floors are sufficiently smooth to conform to good modern practice, and unusual conditions should be provided for in accordance with the judgment of the individual designer The committee believes that this report contains information, with necessary precision, for guidance in unusual conditions.

The "information . . . for guidance in unusual conditions" is given in appendices. It may be separated for convenience into five steps: (1) the force of a blow of a truck wheel upon a concrete pavement; (2) the force of a blow of a truck wheel upon a bridge floor; (3) the relation between force of a blow upon pavement and the force of a similar blow upon the floor of a bridge; (4) the relation between the blow upon a bridge floor and the stress in a stringer or floor beam; and (5) the relation between the stress from a blow and the stress which would be developed by a static load of the weight of the force of a dynamic blow.

These five steps have been fairly well developed for the trucks and the bridges which were available in and around Ames, Iowa, from 1922 to 1925. The tires of the heavy trucks were solid rubber and well-worn (*3*). A limited amount of supplementary data is available from work done at Iowa State College, in 1927 (*4*) and from investigations of the effects of blows on pavement by the Bureau of Public Roads (*6, 7, 8, 9, 10*).

For additional studies of dynamic action upon bridge floors the force of a blow upon pavement for any given truck (step 1) could be obtained from any reliable observations such as those by the Bureau of Public Roads or from other sources.

TABLE 1

DYNAMIC FORCE OF WHEEL BLOWS IN KIPS ON PAVEMENT AND ON THREE BRIDGES AT 12 MPH

Structure*	Obs	Liberty Loaded		Liberty Empty		Light Hwy Truck	
		Force	%	Force	%	Force	%
Pavement	1"	48.0	100	37.6	100	9.8	100
S M	1"	—	—	28.5	76	8.3	85
S A	1"	36.0	75	34.2	91	8.8	90
C T	1"	43.2	90	—	—	—	—
Pavement	2"	64.6	100	56.5	100	18.3	100
S A	2"	51.0	79	48.1	85	16.2	89
C T	2"	60.9	94	—	—	—	—
Static Wheel - Load		8 800		3 350		1 350	
Unspring Wheel-Load		2 200		2 200		1 000	
Percent Unsprung		25.0		66		74	
Dynamic Unit Stresses Developed in one Stringer 2.5 ft South of Center Line							
S M	1"	11.2		5.0		—	
S A	1"	6.8		5.8		1.6	
S M	2"	15.6		—		—	
S A	2"	9.1		7.8		2.8	
C T	2"	4.9		—		—	

* S M Skunk River Main Span
S A Skunk River Approach
C T Campus Test

The force of a wheel blow upon a bridge floor (step 2) might be secured directly from available sources, by field observations, or it could be computed within a reasonable tolerance by the use of Equation 1, Appendix C of the reference (*4*).

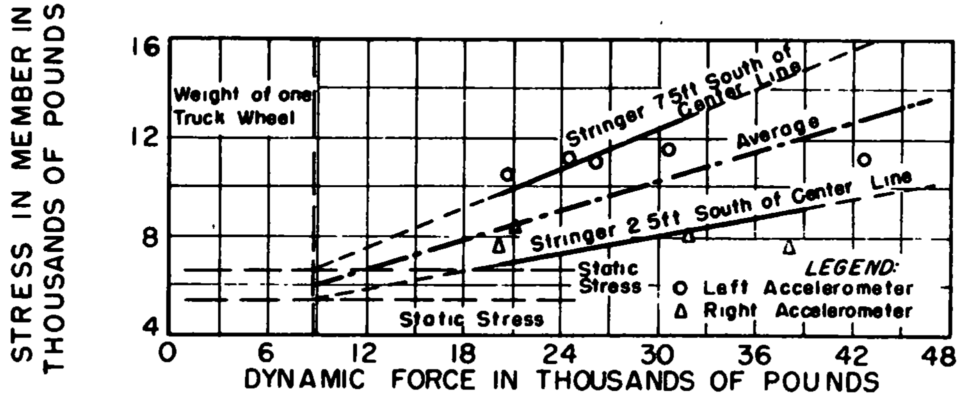

Figure 1. Relationship between dynamic force and simultaneous stresses in stringers 2 1/2 and 7 1/2 ft. south of centerline of Skunk River main span, Truck B; 1- by 2-in. obstruction.

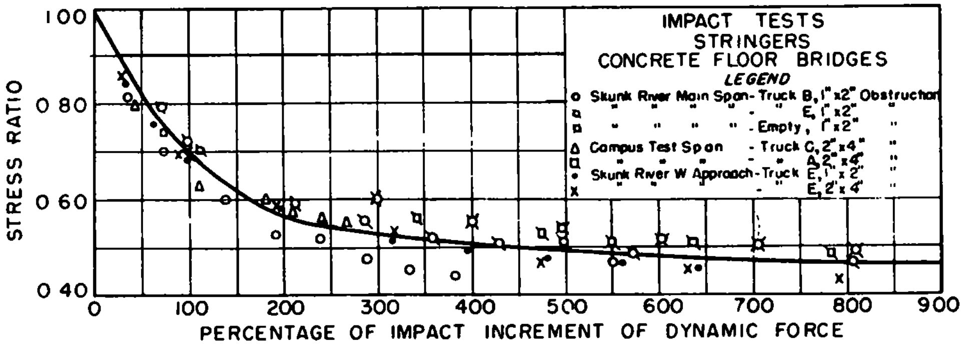

Figure 2. Relation of impact increment and stress ratio: Stringers of concrete-floor bridges.

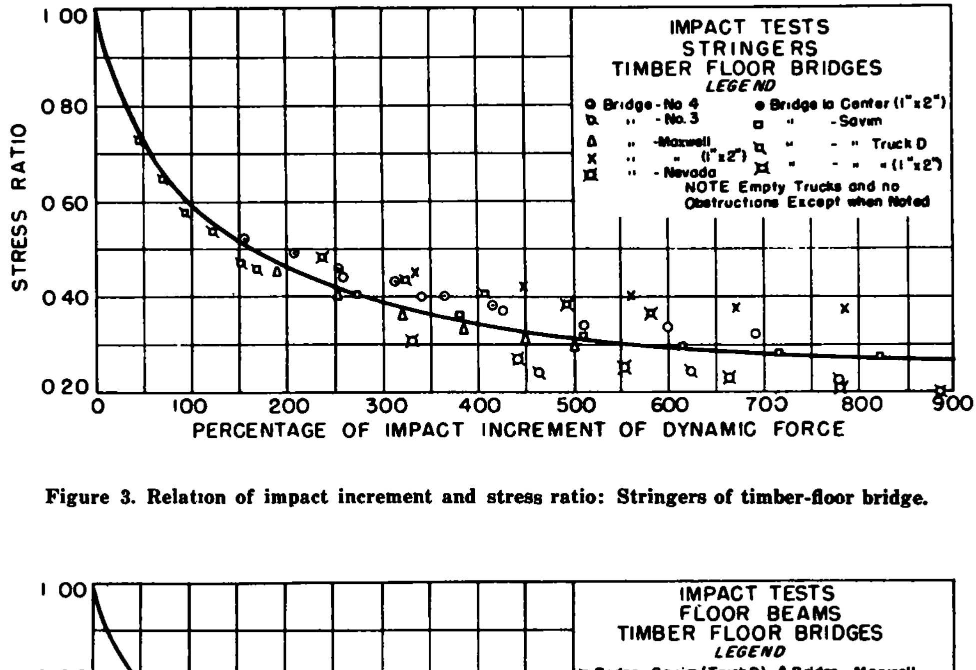

Figure 3. Relation of impact increment and stress ratio: Stringers of timber-floor bridge.

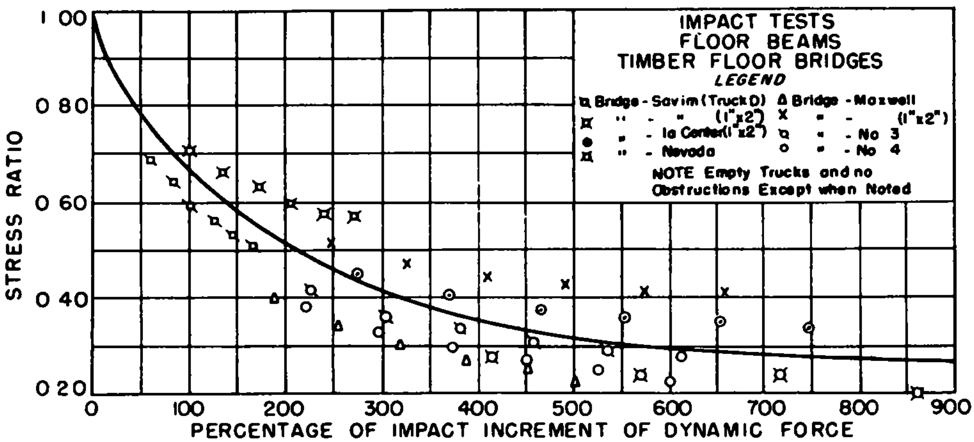

Figure 4. Relation of impact increment and stress ratio: Floor beams of timber-floor bridges.

The relation between a blow upon a pavement and upon a bridge floor (step 3) may be approximated by means described in Appendix D from which Table 1 has been prepared. The value of the data in the table for present trucks and tires is probably very small, but it is given to indicate a trend and to illustrate the possibilities of this type of analysis.

The relationship between dynamic force on bridge floors and simultaneous stresses (step 4) was given for the early trucks and structures by many diagrams (of which Fig. 1 is an example). From these diagrams a table was prepared to show the ratio between the *impact increment of dynamic force to the impact increment of simultaneous stresses* in the stringers and floor beams. These ratios were mostly above 2 1/2 in the stringers of three floors where concrete slabs were supported by steel stringers and more than four in the stringers and floor beams of six light truss bridges with timber floors on steel stringers.

Relationships between stress from a blow and the stress which would be developed by a static load which equals the force of a blow (step 5) were established as illustrated in Figures 2 to 4 by the use of the term *stress ratio,* which was defined as the ratio of the actual dynamic stress which was produced in a member to the stress that would have developed if a static load, equal in magnitude to the dynamic forces, had been applied to the place where the dynamic load was applied.

"The stress ratio diagram indicates a relation between the impact increment of dynamic force and the stress ratio for a variety of obstructions, loads and spans. The available information suggests that each relation is perfectly general in its field" (4). This statement was made in 1925 and was based upon researches conducted with vehicles with solid rubber tires. It needs checking for pneumatic-tired vehicles.

While the preceding five steps may give the basis for a rough indication of possible effect of present day equipment and structures, they are presented also as a basis for a question rather than as a definite guide. The question is: Would further researches under present or future conditions be justified, and would the results be useful in computing the impact upon and the stress in a bridge floor from any new trucks or special loads for which the force of a blow upon pavement would be established? Another question: Could useful information and statements be brought out such as that illustrated in the closing paragraph of Appendix B of Reference 4 ". . . impact is perceptibly greater when dual, rather than single tires are used and decidedly less when the rear load is carried on two axles (four wheels) rather than on the usual arrangement of one axle"?

Practically all of the available information on the behavior of bridge floors has been obtained in situations where the load was inadequate to develop stresses which even approached design values. All reported impacts, therefore, are too high to reflect the situation when overstressing is being approached, which, by the way, is the situation which reveals the true capacity of the floor to resist the unusual load without injury. Isn't it likely that the stresses, which might be developed by an occasional blow caused by an unusual obstruction, would be much less than usually suspected from observations upon the rigidity of an understressed floor?

No attempt has been made in this paper to bring together the effects of the force of a wheel blow and those of the speed of the truck. The variables appear to be too great to put into a mathematical statement; very few experimental results are available to show the relation between the force of a blow and speed of truck *on bridges.* The available reports on pavements, mostly by the U. S. Bureau of Public Roads, indicate the greatest force of the blow to be for a speed less than the maximum. A critical speed of around 15 mph. was indicated by much of the early equipment, while the latest information available suggests a heavier blow at speeds of 40 mph. than for greater speeds.

Static Effects

In the early studies the static stress in stringers and floor beams was determined as a reference point for impact. As the results were tabulated and plotted, they seemed to point to an interesting and perhaps valuable by-product in the form of the effect of the floor slab in distributing the load among the stringers. Figure 5 is one of seven similar plates which show the effect of one and of two trucks on each of four spans.

TABLE 2

TOTAL STRESS IN STRINGERS
ALL STRESSES ARE IN KIPS

Span	No of Trucks	Average of Observed Stresses	Computed stress: Stringers only	Computed stress: T-BM Action
West Panel	1	37.9	54.1	42.3
West Panel	2	75.2	104.0	82.7
West Approach	1	20.6	40.4	25.2
West Approach	2	42.9	78.2	48.8
Squaw Creek	2	44.2	113.5	44.5
Campus Test	1	8.5	20.5	13.0
Campus Test	2	17.5	41.0	26.0

The bottom figures at the left in Figure 5 give the sum of the observed stresses in the individual stringers in each of five positions of one truck. The bottom figures at the right indicate the sum of the stresses as computed on the assumption that the stringers

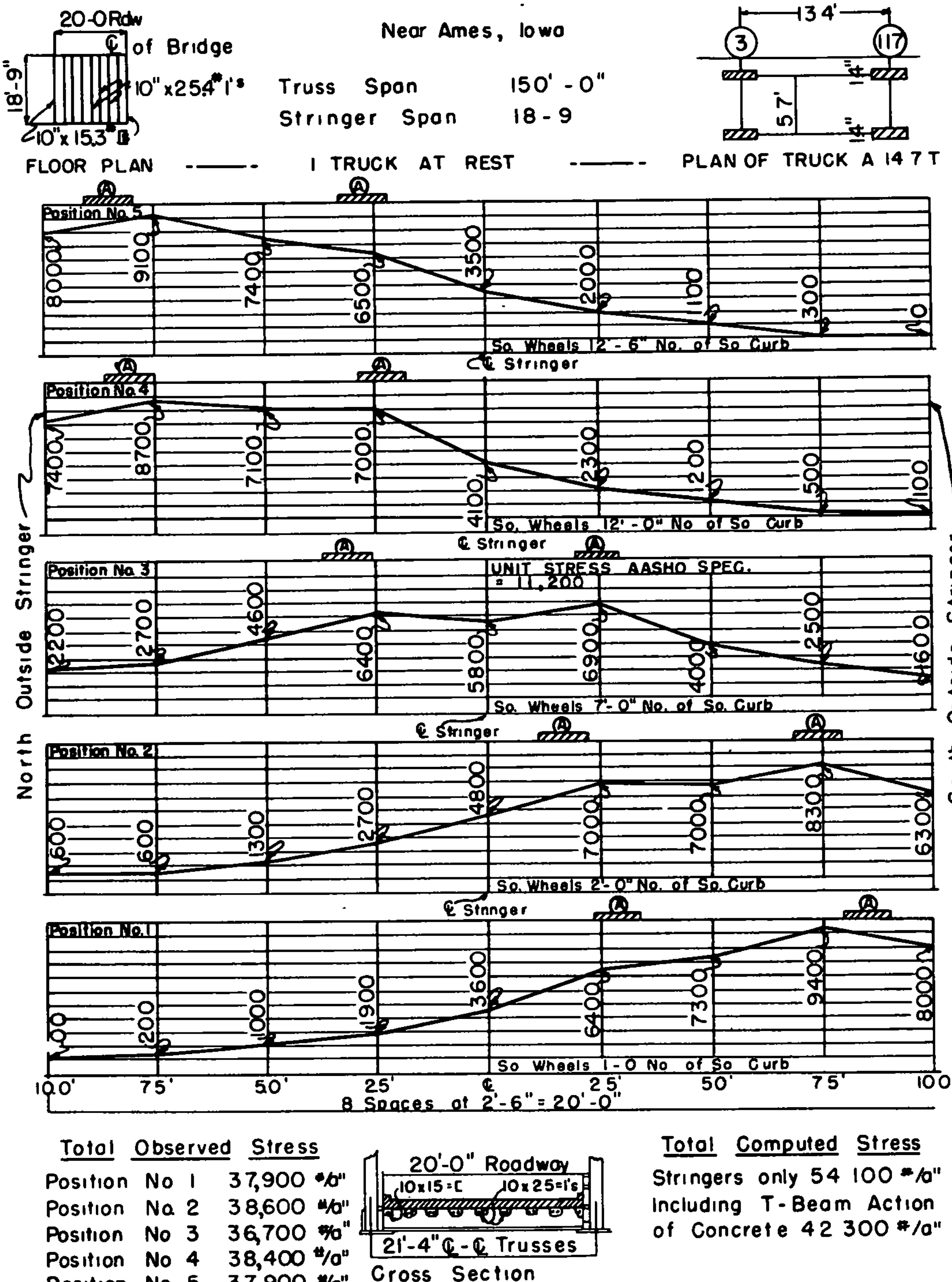

Figure 5. West panel of Skunk River Bridge: Distribution of static stress in stringers with one truck.

alone supported the entire load. They also show the sum of the stresses as computed on the basis of full composite action between the steel stringers and the reinforced floor slab.

Observed stresses from the seven plates under discussion were plotted by using the greatest stress in each stringer for any given span and load, regardless of the position of the load. A general tendency may be noticed for the greatest stressed stringer to be near the sides of the bridge for one truck and near the center when two trucks are on the span. This tendency is aggravated on the Skunk River Bridge where the outside stringers are lighter than the inside ones and cannot absorb as much load for a given deflection.

In each case the total observed stress is well below the stress which was computed under the assumption that the stringers alone supported the load. They are also somewhat below the computed stresses when full composite or T-beam action was assumed. This may indicate not only that fairly full composite action existed but also that additional restraint was supplied in some manner, possibly by an uncertain amount of fixity of the slab at the ends and from the stringer connections to the floor beams.

While the results were apparently accepted as representing the situation on each of the relatively new structures, much doubt has been expressed concerning the continuation of composite action after years of service, and therefore upon the advisability of recognizing such action in bridge specifications.

The 1948 Investigations

When one of the three structures which were the subjects of investigation in the 1920's, the Skunk River Bridge, was declared inadequate for modern traffic, largely because of clearances, and it was announced that the bridge was to be moved to another location on a secondary road, the possibility of securing limited information concerning the action after 25 years of further service was recognized.

The bridge consists of a 150-ft. through-riveted-steel span and a beam approach at each end with 20-ft. roadway There are nine lines of stringers. The stringers on the main span consist of seven 10-in. 25.4-lb. I-beams flanked on each side by one 10-in. 15.3-lb. channel. On the west approach span nine 15-in. I-beams at 42.9 lb. are used, for which the clear span is 32 ft. 8 in., and the distance center to center of end bearings is 34 ft.

The original reinforced concrete floor was 6 in. thick on steel stringers. In 1929 the floor thickness was increased by a 3-in. reinforced-concrete layer to 9-in. The bridge with original floor is considered typical of those of 30 years ago.

The Skunk River Bridge is on US 30, which was and still is an important transcontinental artery carrying heavy traffic, and the bridge has had an increasing volume of traffic over it in the 25 years between the tests. Therefore, it is of interest to see what effect age and traffic have had on the structure.

Although personnel, equipment, and budget for a comprehensive study were not available, a brief exploratory investigation was made in June 1948 by field measurements on three mornings between 4 and 8 o'clock. It was early recognized that high precision was impracticable, but sufficient checks were planned to catch mistakes.

The loads used in the 1948 study consisted of a heavy-duty truck trailer weighing 17,150 lb. carrying a 34,700 lb. track-laying tractor which could be moved on the truck to obtain various wheel concentrations. A skeleton outline of the truck is shown in Figure 6. The wheel concentrations are given in Table 3.

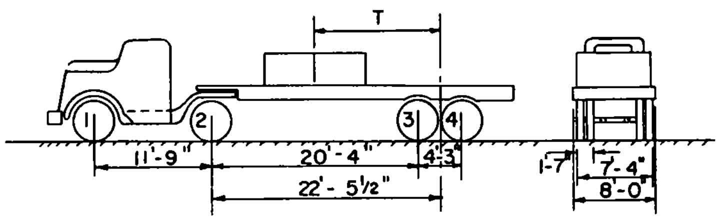

Figure 6. Skeleton outline of live load.

The length of the truck was such that only the wheels of the two rear axles were on a span. The position longitudinally was approximately that for maximum moment Transversely, the loads were in three different positions which were designated W, X, and Y. For Position W the north wheels were against the north curb For Position X the load was approximately on the center line, while for Position Y the south wheels were against the south curb. Each position thus had a definite notation (for example, III-X). Two or more separate placements were made for each load and position, for example, IV-W-1 and IV-W-2. Readings were taken both upon the west panel of the truss span and upon the west approach span as was done in 1922-25.

TABLE 3

AXLE LOADS FOR VARIOUS LIVE LOADS

Load	Load No	Distance T (Fig 6)	Axle-Weight in Pounds 1	2	3	4	Total
Truck Only	—	—	5,200	4,600	4,300	3,080	17,150
Truck	I	16 0	5,200	29,200	8,700	8,700	51,800
and	III	0	5,200	4,600	21,000	21,000	51,800
Tractor	IV	6 7	5,200	14,600	16,000	16,000	51,800
Tractor	—	—	—	—	—	—	34,700

Instruments. Strains were measured with two different types of electrical instruments attached to SR-4 strain gages Type A-1, and with direct reading extensometers. A Baldwin-Southwark portable strain indicator with switching unit for 22 connections and a Baldwin-Southwark-type of 6-channel oscillograph were connected with individual SR-4 gages. Five extensometers of 20-in. gage length with Last Word dials were connected to the beam flanges. Electric current was supplied for the electrical instruments by means of a motor-generator set on a truck.

Deflections were taken by means of eight Federal 0.001-in. dials working between the bottom flanges of the beams and a reference beam which was supported from the ground and independent of the staging for the operators.

Field Work. Individual readings for strain were taken at the centerline of each stringer on each side of the top of the bottom flange and, for a few stringers, on the bottom of the top flange.

The strain indicator, with attachments for 22 points at a time, provided the only means for heading all of the stringers. The six channels of the oscillograph and the five extensometers, distributed among the stringers and one floor beam served as checks upon the behavior of the strain indicator. The eight available deflection instruments were used in reading deflections for all loads

Computations The strains were translated into unit stresses by considering the modulus of elasticity as 29 million psi. Unit stresses were also computed from deflections under two extreme conditions, that the steel beam alone carried the entire load, and that the steel beam and concrete floor had full composite action. The correct stresses would naturally lie between these two extremes.

Results

Observed Stresses in Stringers. Individual stresses in the bottom flanges of the stringers were plotted on six plates A separate plate was used for each position for each load, but no attempt was made to distinguish each individual application of the load. All available results were plotted, but those from the strain indicator (the only instruments with connections to all stringers) were given the greatest weight in locating the points on the curves. The results from the other instruments are considered primarily as checks and as a means for appraising the general reliability of the work as a whole. Dotted lines connect simultaneous readings on opposite flanges of the same stringer. Two of these figures have been reproduced as Figures 7 and 8. The data on each curve are from a truck position near the curb on the north side of each span (Position W). The maximum ordinate and other data for each of the three positions on each of the two spans are given in Table 4. The stringer designations, A, B, etc., refer to the stringers, consecutively from the north side of the bridge.

The stresses which were computed from deflections under the assumption that the steel alone carries all the load are less than the ones which were computed from the strains, while the stresses which were based upon full composite action are greater than those from the strains. For the approach span the differences are not great, and the two sets of stresses from deflection may be considered as the limits of a band within which the actual stresses should lie (Fig 7). For the west panel the deflection stresses for steel alone are reasonably close to those from strains, while those computed under the assumption of full composite action (Fig. 8) are very much greater; in fact some of them fall beyond the limits of the sheet.

TABLE 4

RATIO OF LOAD CARRIED BY MAXIMUM STRESSED STRINGER TO THE TOTAL LOAD

Span	Load	Unit Stress in Bottom Flange of Stringers for 1-Truck Kips per Square Inch					
		1948			1925		
		Max	Total	Max /Total	Max	Total	Max /Total
West Panel	W	6 0	33 5	0 18	9 1	37 9	0 24
	X	6 2	36 4	0 17	6 9	36 7	0 19
	Y	6 5	34 7	0 19	9 4	37 9	0 25
West Appr	W	3 9	20 3	0 19	5 2	20 6	0 25
	X	3 2	21 0	0 15	3 4	20 5	0 17
	Y	3 5	18 8	0 19	5 0	21 4	0 23
	Spec n	—	—	0 21	—	—	0 21

Load to Maximum Stressed Stringer. In Table 4 is shown the ratio of the load which is transferred to the maximum stressed stringer to the total load. The ratio is based upon the unit stress in the one stringer as compared with the sum of the unit stresses in all the stringers. The stresses are taken from the curves (Figs. 7 and 8 and four similar ones). Similar results are also given in Table 4 for the 1925 studies on the same spans. For 1948, these ratios are 0.18 and 0.19 with the load on the side and 0 15 and 0.17 with the load on the centerline, while the value computed from the specifications is 0.21. In 1925 results (*3*, p. 51) show from 0 23 to 0.25 for the load on side and 0.17 and 0 19 for the load on the centerline. The ratio from the specification is the same as above or 0.21.

In 1925 the load to the maximum stressed stringer was greater than that allowed by the specification, while in 1948 it was less. The differences may be attributed to at least two causes, the use of a different

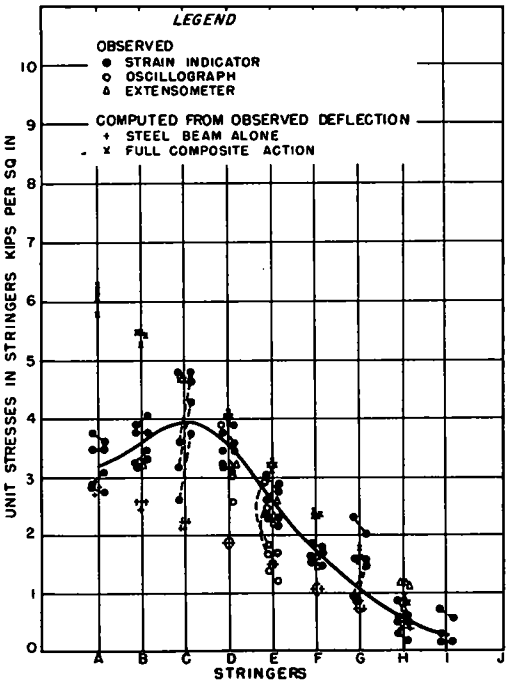

Figure 7. West-approach span, Load IV, Position W.

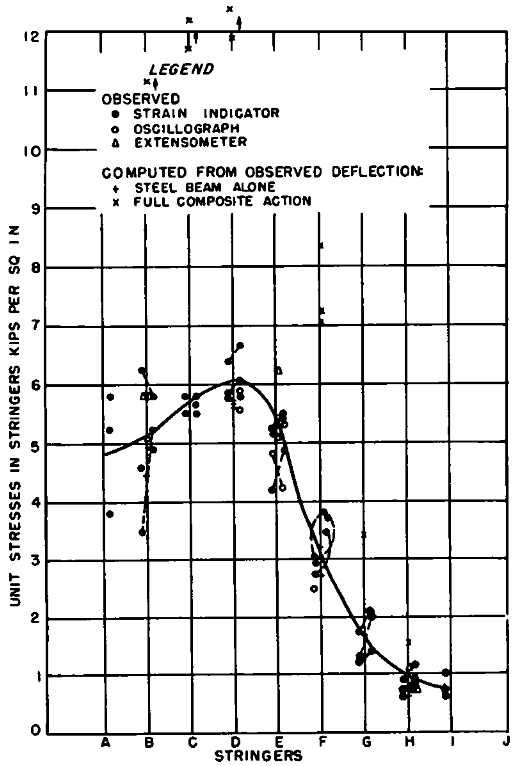

Figure 8. West panel, Load III, Position W.

type of truck, and the thickness of the concrete floor, which was 6 in. in 1925 and 9 in. in 1948.

The stringer spacing on the Skunk River Bridge, 2 ft. 6 in., is much less than for present practice. The present AASHO distribution of load is changed very slightly from the original which (as far as the author knows) first appeared in the 1923 specifications of the Iowa Highway Commission.

Computed Stress in Stringers. The computed stresses were based upon a distribution of load among the stringers according to the AASHO 1949 specifications. For the 1948 truck with two rear axles and four wheels on each axle, this distribution can be but a rough approximation at best.

TABLE 5

COMPARISON BETWEEN OBSERVED AND COMPUTED STRESSES

Type of Fiber Stress	Stringer in Approach Span	Stringer in West Panel, Main Span		
		Simple beam	Fixed ends	Probable end restraint
Computed				
Bottom flange,				
Steel only	9,800	15,600	6,100	11,500
Full composite action	4,900	5,400	2,100	4,000
Top fiber concrete,				
Full composite action	350	400	160	290
Horiz shear between steel and concrete, full composite action	90			300
Observed				
Bottom flange	3,500		6,200	
Top flange	±0		4,900	

A comparison between observed and computed unit stresses in stringers is made in Table 5. In the approach span the computed stress for steel alone is so much greater than the observed stress as to suggest definitely that the steel as a simple span did not carry the load. A very slight fixity exists, of course, from the fact that the supports are other than knife-edged. The greater part of the added resistance appears to come from composite action, which is the interaction between the steel beam and the concrete floor. (No mechanical bond was provided, but the concrete, 1 in. below the top of the steel, extended a little under the top flange.) The values for compression in concrete and horizontal shear between steel and concrete are within reasonable limits (even when dead load stresses are included) and do not exclude the possibility of full composite action. The fact that the computed stress in steel, even for full composite action, is so much greater than the observed stress (4,900 against 3,500) might be explained by the probability that the effective width of concrete, with the 9-in. thickness, was greater than the 30-in. (c-c stringers) which was used. The very small (0 ±) stress in the top flange suggests that the gravity axis was at about the top of the beam and that all the steel was in tension.

The computations for stresses in the stringers of the west panel were made under three assumptions: (1) simple beam, (2) beam with fixed ends, and (3) beam with end resistance which was computed from the reinforcement in the concrete floor and from the standard girder to floor beam connections. The first two represent extreme conditions which might be approached but could not be reached. For the third condition the steel was considered available up to the elastic limit (assumed conservatively as 30,000 psi.) with no help from the concrete. This might be a fairly reasonable value and will be used in the discussion.

In the west panel, although the stress in steel alone for simple span is higher than the observed stress, that for full composite action is less, the shear is too great, and the observed stress in the top flange approaches that in the bottom one. Therefore, any composite action must be small and may be mostly friction with the bond pretty well destroyed. It seems then that the composite action is small and that, therefore, the end restraint of the reinforced concrete floor contributes a partial continuity which is effective in reducing the stresses or, in other words, in increasing the capacity of the floor.

Floor Beam. The few observations which were made on one floor beam, when compared with computed values, suggest good composite action. The fact that the concrete floor is poured over the stringer flanges and the stringers are riveted to the floor beam apparently provides joint action between steel and concrete, which is independent of bond.

Deductions and Resulting Questions

This work has met with the same limitations which existed in many previous investigations—that of being unable to secure, or move over the highways, a load of sufficient weight to develop even full unit stresses in modern structures.

Within the limits of the available live load the present results point rather definitely to the fact that the stress in the steel stringers in each span was decidedly less than would have been developed in the steel action alone as simple beams. Although no mechanical bond was provided, the reinforced concrete floor evidently contributed in some manner to the value of the combined concrete floor and the steel stringers.

In the approach span the evidence points to definite (if not full) composite action, even though the span had been in use for twenty-eight years under heavy traffic. In the west panel the measured strains and deflections and the cracking of the concrete around the flanges of the stringers indicate but very little direct action between the concrete and the steel. Yet the low stresses in the stringers suggest that they are getting help from some source. The resisting moment from the steel reinforcement in the floor slab and from the stringer to floor beam connections could provide about the necessary help.

Although these deductions may reflect correctly the small unit stresses which were developed by the available live load, no information is at hand for extending the results to fully loaded structures.

Though no general conclusion should be drawn from an exploratory research as brief as the one under consideration, the present work might serve as a basis for possible further research.

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9. *Public Roads.* October 1927.
10. *Public Roads.* April 1931.

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Test on Rolled-Beam Bridge Using H20-S16 Loading

G. M. Foster, *Chief Deputy Commissioner, Michigan State Highway Department*

● IN ORDER to continue an investigation of the effectiveness of shear developers and to study certain lateral distribution features in bridge construction, the bridge engineer of the Michigan State Highway Department, in consultation with W. W. McLaughlin, testing and research engineer, proposed a testing program on a six-span bridge near Fennville, Michigan.

The general program was set up by E. A Finney, assistant testing and research engineer in charge of research Suggestions for the testing of certain features were made by G. S Vincent, Bureau of Public Roads, T. Y. Lin, Institute of Transportation and Traffic Engineering, University of California, E. C Hartman, Aluminum Research Laboratories, C. T. G. Looney, Yale University; G. B. Woodruff of Woodruff and Samson, Engineers, San Francisco; H. E. Hilts, Bureau of Public Roads, and others. Aids in testing methods were obtained from reports on the San Leandro Creek Bridge, Oakland, California, and the Paramata Bridge in New Zealand.

The field tests were supervised by L. D. Childs, physical research engineer M. Rothstein, bridge design engineer, analyzed the data. C. B Milroy, bridge project engineer, worked directly with the test crew in the field and expedited the work. V J. Spagnuolo, physical testing engineer, supervised the operation and maintenance of the recording equipment.

This report is a record of the progress to date. Testing of the structure will continue with a more detailed study of impact and vibration effects from rapidly moving vehicles.

Objectives of the Test Program

The general purpose of the investigation was to obtain stress and deflection data which could be correlated with theoretical values to accomplish efficiency and economy in the design of highway bridges. The information will also be used in a study of the live-load-carrying capacity of existing highway structures under loads imposed upon them by present-day, heavy, motor-transport units.

The specific objectives of the test program as proposed in the original outline were to (1) determine the stress distribution in the girder system under static, dynamic, and impact loading; (2) study the effect of diaphragm connection and method of spacing upon lateral distribution of loads; (3) measure the degree to which the concrete deck slab influences stress distribution to supporting members, (4) observe the differences in stress conditions in supporting steel members when deck slabs are anchored and unanchored to these members, (5) check design values with field data; (6) observe the effects of temperature upon stresses in the structure; (7) obtain vibration data on spans with different design features; (8) measure slippage between the deck slab and the supporting beams; (9) measure the midspan deflections of spans with different design features and under several load conditions, and (10) attempt to measure lateral stresses in the concrete deck both by surface gages and by gages attached to the reinforcing steel.

Although the specific objectives were not achieved in their entirety, due to limitations of equipment, some data was obtained for each phase of the study A continuation of the tests should supply sufficient additional information to fully accomplish all of the objectives.

The Structure

Fundamental dimensions of the structure are given on the plan in Figure 1. The bridge consists of six simple spans, each nominally 60 ft. in length with an overall deck width of 33 ft. 8 in. and a 90-deg angle of crossing. The deck is constructed of reinforced concrete with variable slab thickness to provide the required crown at the center and to allow for dead-load deflection of the beams. The deck is reinforced transversely with 5/8-in. deformed bars at 6-in. centers, top and bottom. It is supported by seven lines of 36-in. W. F. 182-lb. rolled beams spaced 5 ft 2 1/4 in on centers.

The six spans are alike except for the following features

Span 1. West end of beams embedded in concrete backwall, two rows of diphragms double-bolted to beams, actual span length from center to center of bearings is 58 ft. 5 in.

Span 2. Three rows of diaphragms double-bolted Span length 59 ft. 3 in.

Span 3. Composite construction using spiral shear developers. Two rows of diaphragms single-bolted. Span length 59 ft. 3 in.

Span 4. Three rows of diaphragms single-bolted Span length 59 ft. 3 in.

Span 5. Two rows of diaphragms. This span tested under three conditions:

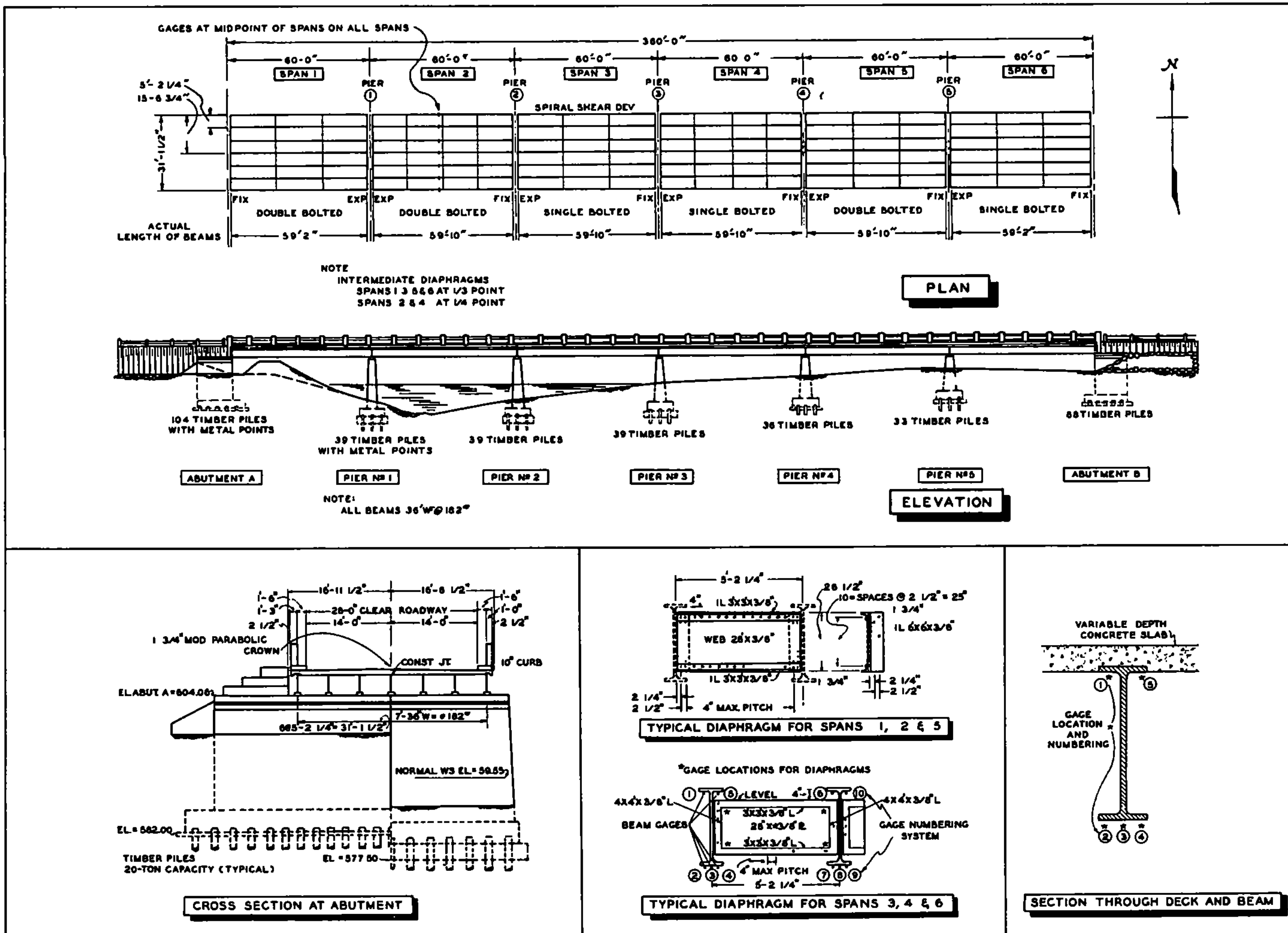


Figure 1. Fundamental details of structure.

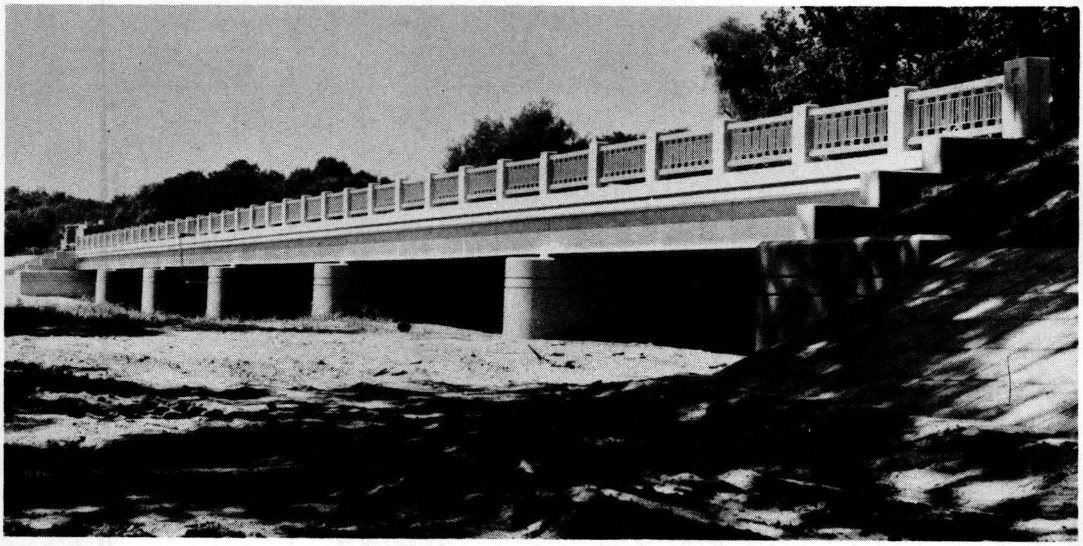


Figure 2. General view of bridge at time of test.

(a) with no diaphragm connections, (b) single-bolted, and (c) double-bolted. Span length 59 ft. 3 in.

Span 6. Two rows of diaphragms single-bolted. The east ends of the beams are embedded in the back-wall. Span length 58 ft. 5 in.

A general view of the bridge at the time of testing is shown in Figure 2. The field program was not begun until the water had subsided to its minimum level. At this stage, Spans 5 and 6 were dry, Spans 1 and 4 extended over water for about half their length, and Spans 2 and 3 were completely over water.

Several design features are illustrated in the accompanying photographs. A double-bolted diaphagm is pictured in Figure 3. Two rows of turned bolts fasten it rigidly to the beam web. In this illustration, the bolts on one side have been removed for the purpose of testing Span 5 under the "no-diaphragm" condition.

Figures 4 and 5 exhibit the placement of the reinforcing steel in the deck. Also, in Figure 4, the method of application of the strain gage to the reinforcing steel is shown. The spiral shear developer, which is welded to the tops of the beams of Span 3, may be seen in Figure 5.

Test Equipment

Loading Vehicles

A special test vehicle meeting the H20-S16 requirements was constructed by the Maintenance Division. A Walters truck was modified by extending the wheel base to 14 ft. and mounting a fifth wheel directly above the rear axle. A set of outside wheels was added to the rear axle to assure support for the 16-ton load without excessive overload on the tires. A semi-trailer was built with the distance between the truck and trailer axles also equal to 14 ft. The axle lengths were 6 ft. from center to center of wheel on the first and last axle, and 6 ft. 4 in. on the center one. These were sufficiently close to the measurements of the theoretical design vehicle to be used for direct comparison of design and field measured results.

Ballast blocks for loading the axles to the required 4, 16, and 16 tons respectively were made of plain concrete and were 1 by 2 by 4 ft. in size, with a weight of about 1,200 lb. each. They were cast in wood gang molds which were set up on the bridge deck. Before the concrete had set, a small amount of the mix was removed from the top of the block at the center and a U-shaped piece of reinforcing steel embedded at this point, with the bend flush with the surface. This provided a loop for the crane hook and facilitated handling without interfering with the stacking of the blocks.

Several photographs of the loading equipment are shown. Figure 6 is a view of the test vehicle loaded to meet H20-S16 requirements. Figure 7 exhibits the peculiar arrangement of the ballast necessary to produce proper load distribution. In Figure 8, several features may be seen. In the foreground are the gang molds in which the ballast blocks were cast. Behind these is the crane which loaded the blocks onto the test vehicle. To the right is the vehicle with the two heavy axles resting upon loadometers. Fortunately, the front axle 4-ton requirement was met without the use of ballast on the truck, so four loadometers were sufficient to check the load distribution.

After some testing with the single design vehicle, it was concluded that better results might be obtained with heavier loads. A second design vehicle was not available, but a standing load was readily constructed from beams and blocks. This was placed in the lane adjacent to the one used by the moving truck in such position as to produce maximum bending moment. Figure 9 shows this simulated vehicle and an actual test picture of both vehicles in use is shown in Figure 10.

Measuring Instruments

Strains and deflections were measured at midspan on all spans. The Baldwin SR-4 bonded strain gage was the heart of the instrumentation. These gages were cemented to the beams' flanges, to the diaphragms, to the bottom of the bridge deck, and on certain lateral reinforcing bars. They were also used on short thin cantilevers to make possible a permanent record of deflections.

The Type A-1 gages were used more than any other, although some AR-1 and A-8 gages were used in the diaphragm study, and A-9 gages were cemented to the bottom of the concrete deck in the study of lateral load distribution. Figure 11 is an installation of gages on a diaphragm, and the application of a gage to the reinforcing steel was shown in Figure 4.

Deflectometers were laboratory built. Figure 12 is an installation on a beam and an accompanying explanatory sketch. The device was constructed in such a way that depressing the beam actuated both a one-thousandth dial and the short cantilever to which the strain gage was attached. The dial permitted visual observation of the deflection and the cantilever transducer provided means of actuating an oscillograph galvanometer to provide a permanent record on sensitized paper. The combination of visual and electric indication made the calibration of the electrical record very simple.

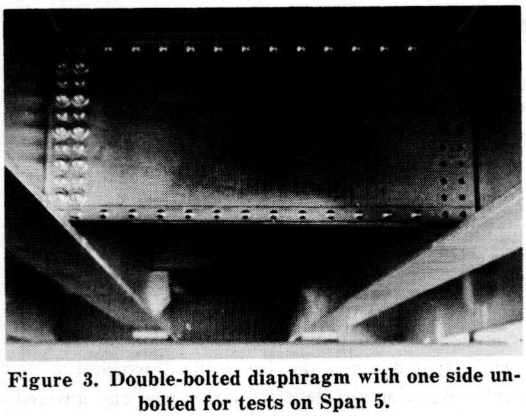


Figure 3. Double-bolted diaphragm with one side unbolted for tests on Span 5.

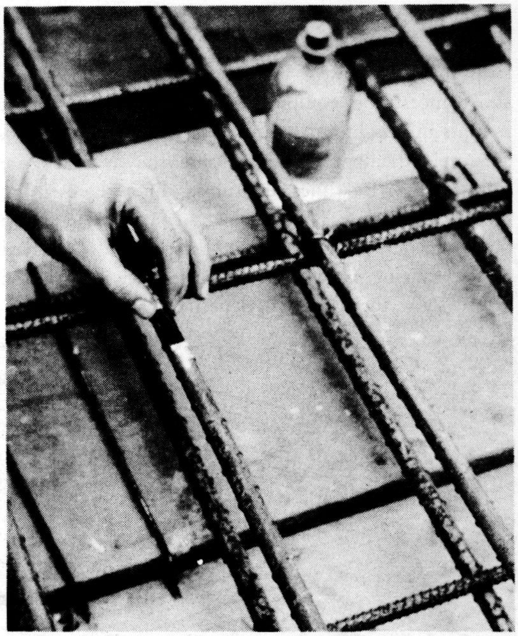


Figure 4. Reinforcement details and method of placing SR-4 gages.

The installation of gages and deflectometers under Span 3 is pictured in Figure 13. At the time this photograph was taken, the static tests had been completed and the wires to the middle gage at the bottom of each beam flange had been clipped. The gage heads were then attached for the dynamic tests. The operator was in the act of setting the deflectometer dials to the initial zero.

The position of the moving truck on the bridge deck was determined by the use of rubber tubes and pneumatic switches. The tubes were stretched across the lane at two locations. The first was at the point where the truck first entered the span and the second was at midspan. The switches actuated solenoid markers in the oscillograph and formed small pips on the record.

Slippage between the deck and supporting beams was read on dials sensitive to 0.0001 in. A dial mounted for this purpose is pictured in Figure 14.

Recording Devices

Two types of devices were used for recording the test data. For static tests, strains were measured by an SR-4 portable indicator and deflections were read directly from the dials. The indicator and Anderson switching units are seen in Figure 15. When moving load and impact tests were made, both strains and deflections were recorded upon a photosensitive paper strip in a Hathaway 12-channel oscillograph. This strain measuring equipment was mounted on shock mounts in a light truck, and is pictured in Figure 16.

Sample oscillograph records are shown in Figure 17. The vertical lines are timing lines representing 0.1-sec intervals. They enable a computer to figure the frequency of oscillation of the span and the speed of the moving vehicle. The pips at the top of the record show the truck wheel positions

strain gages were cemented to each beam at mid span in five locations. Two gages were placed on the under side of the upper flanges, and three were fastened to the lower face of the bottom flange. They were symmetrically placed so that the two upper gages were equidistant from the web, two of the lower gages were equidistant from the center, and the fifth gage was directly beneath the web This was illustrated in Figure 1.

When static tests were made, all of the gages were read. However, for dynamic testing it was possible

Figure 5. Spiral shear developers in reinforcement for Span 3.

to read only one gage per beam because of the limited number of channels on the oscillograph The static readings permitted the computation of the location of neutral axis of the beam whereas the dynamic record gave only maximum fibre stress on the lower surface of the beam.

Spans 1, 2, and 4 were tested with only two gages per beam. These gages were symmetrically located on the lower face of the bottom flange.

The deflectometers were clamped within a few inches of midspan and as close to the strain gages as possible. A fine steel cable was stretched tightly from the hinged plate on the deflectometer to a turnbuckle, and again from the turnbuckle to an anchor on the ground. Thus, the hook on the hinged plate which is at the upper end of the cable is always fixed with reference to the ground. The dial and cantilever were actuated when the beam upon which the assembly was clamped deflected under load and lowered

The strains and deflections were determined from the traces in the following manner the ratio of microinches per inch of strain to units of chart deflections was first computed from a calibration record. Then the maximum deviation of each trace from its zero line was multiplied by this factor to obtain maximum recorded strain. By this procedure, the strain magnitude at midspan on the lower surface of each beam was found from the upper seven traces on the record. Deflections were computed in a similar manner from the lower five traces. On Beams 6 and 7, the dial indicator readings were used directly because the recording equipment was limited to a total of 12 channels.

Outline of the Test Routine

Gage and Deflectometer Installation

After a period of preliminary tests and explorations on Span 6, the test settled down to a routine except for a few special features. On Spans 3, 5, and 6,

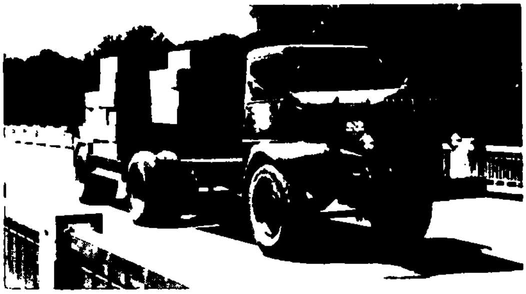


Figure 6. H20-S16 test vehicle.

the remainder of the deflectometer and forced the dial stem against the plate Reference again to Figure 12 clarifies this performance. On Span 2, due to the depth of the water and speed of the current, small wood piles were driven into the river bed to hold a beam under the line of gages. The deflectometer cables were fastened to this beam.

A pair of wires was soldered to each gage and a waterproofing material was applied over the gages and exposed soldered leads. The leads for the static tests ran directly to the static strain measuring equipment, which is pictured in Figure 15. For dynamic tests, the wires were soldered to gage heads which, in turn, were connected to the dynamic strain analyser by shielded cables.

Placement of the Load

In general, test results were obtained for the load in three or more positions on the bridge roadway. Reference is made to these locations with respect to the distance from the center line to the line of the left wheels of the vehicle Thus, Position 0 indicated that the left wheels were running on the center line. They were three feet from the center line in Position 3, and 4 ft. from the centerline in Position 4 A C L notation was used to indicate that the truck was straddling the centerline.

For the static studies, the truck was stopped upon the span when the lateral centerline of the span lay midway between the middle axle and the computed center of mass of the vehicle. Experimental placement to produce maximum strain proved that this position was not too critical. An error of 2 ft in either direction could not be detected on the recorder.

When the simulated truck was assembled upon the span, it was always placed in Position 4 in the left lane to represent a second vehicle overtaking and passing the first.

Moving load studies were made with the truck moving through Positions 0, 3, and 4 The speeds at which the vehicle was run are shown in the tabulated data.

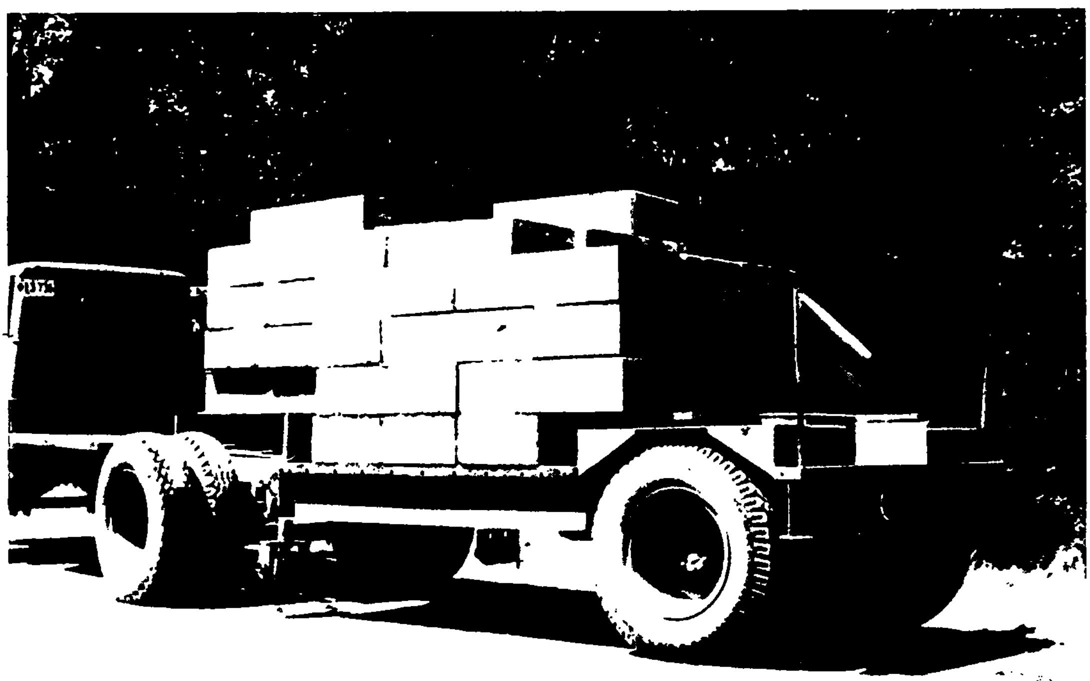


Figure 7. Details of load distribution to meet H20-S16 requirements.

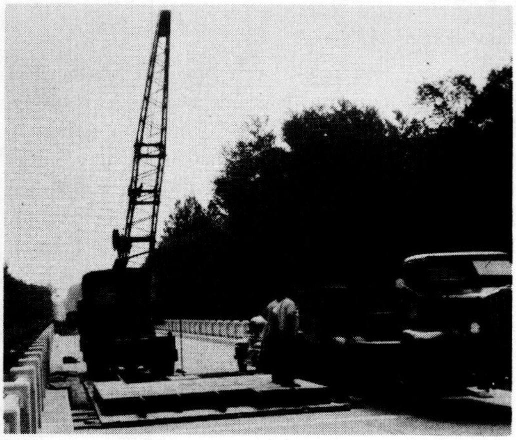


Figure 8. Molded ballast being placed on test vehicle.

Impact runs were all made through Position 4. Plates about 10 ft. long by 1 ft. wide were laid across the lane at midspan. These plates were of steel, and had thicknesses 1/4, 1/2, and 3/4 in. They were placed to cause maximum downward impact at the center of the span.

General Procedure

Before each test, the vehicle was moved back and forth across the span a number of times. The intent was to break in the structure and reduce the shear between the deck and the steel beams. However, test results indicated that a more severe break-in treatment should have been used.

Next, the gage circuits were balanced and deflectometers set to zero. For static tests, the bridge was loaded, the readings made, the truck removed, and final readings taken. This procedure was repeated to give three sets of readings for each position.

For dynamic tests, it was always necessary to run a calibration trace after the gage circuits were balanced in order to obtain the ratio of microinches per inch of strain or deflection to the chart deviation.

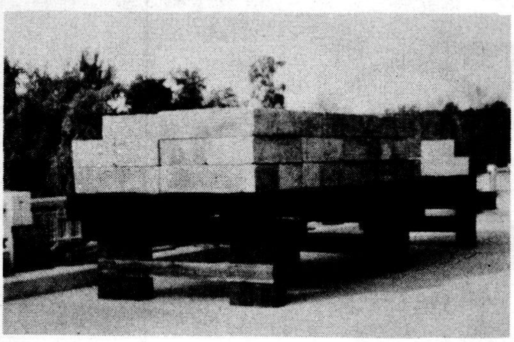


Figure 9. Simulated vehicle placed in south lane.

After this operation, the vehicle was driven across the span through the prescribed position. Again three records were made for each test.

Use of the Simulated Vehicle

After tests were run with a single vehicle, the standing load was placed on certain spans. Moving load and impact tests were then repeated with the design truck moving past the standing load.

Values representing deflections and strains caused by the combined loads of the simulated and mobile vehicles were obtained by an indirect method. The instruments were set at zero with the simulated vehicle on the span in position 4 in the south lane. The mobile vehicle was run past the simulated vehicle in the adjacent lane through Positions 0, 3, and 4. The recorded values were those in excess of the condition of deformation due to the standing load alone. The total deflections or strains for this two-vehicle state were the sums of these measured values and the values due to a single vehicle at Position 4.

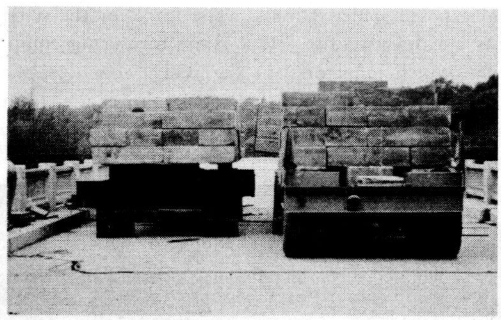


Figure 10. Method of obtaining two-truck-load conditions.

For impact tests, since the simulated load could not be moved to cause impact, a surcharge of 15 percent was added. This figure was derived from an inspection of an experimental impact record on Span 5. It was thought that the accumulated values of the strains due to the surcharged standing load plus the recorded values shown by the impact record of the design vehicle might more nearly approach the true impact effect which could be caused by two moving trucks. This method has evident shortcomings, since the increased load undoubtedly had some damping effect upon the slab vibrations.

Limitations

The scope of the investigation was limited by several factors, the first being the difficulty in obtaining heavy design vehicles. Although the H20-S16 vehicle satisfactorily fulfilled the requirements of a design vehicle for static and slow speed tests, its performance

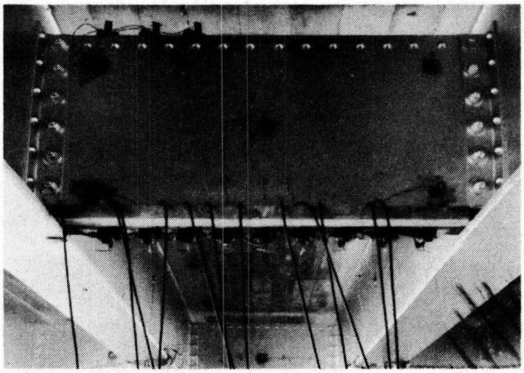


Figure 11. Rosettes on diaphragms wired to Hathaway gage heads

was somewhat limited with respect to speed and braking power. Also, a second vehicle would have been much preferred to the simulated truck used in the south lane. This would have made possible the dynamic measurement of total strains and deflections for various lane positions and truck arrangements, and actual impact results from two vehicles could have been obtained directly, obviating the necessity for the surcharge on the standing load.

A second limitation was the fact that it was almost impossible under the circumstances to drive the vehicle across the span at more than 12 mph. This was due to two facts: (1) the difficulty in attaining higher speeds without excessively long approach runs and (2) the room required to stop such a heavily loaded vehicle. There was no west approach to the bridge.

About 200 ft. of fill had been placed and gravel surfaced behind the west abutment, but this did not provide sufficient room in which to stop the truck at high speeds. It is probable that high-speed runs can be attempted after the road to the west has been completed.

Third is the fact that the recording equipment had 12-channel capacity, whereas there were 14 strains and deflections to be read. As a consequence, an attempt was made to watch the two deflection dials farthest from the load and note the sweep of the pointers.

Fourth, as in most tests, is the limitation of time. Some sort of a compromise must always be made between thoroughness of each test and the general scope of the project. Although three runs in rapid succession produced results with small variance, larger differences were noticed when similar groups of tests were performed later in the program. It would have been advantageous to have repeated all tests in both lanes and in both directions.

Listing of Tests and Presentation of Data

For an understanding of the scope of the investigation, a summary of all tests performed is given. These have been classified into four groups and are not listed in their chronological order:

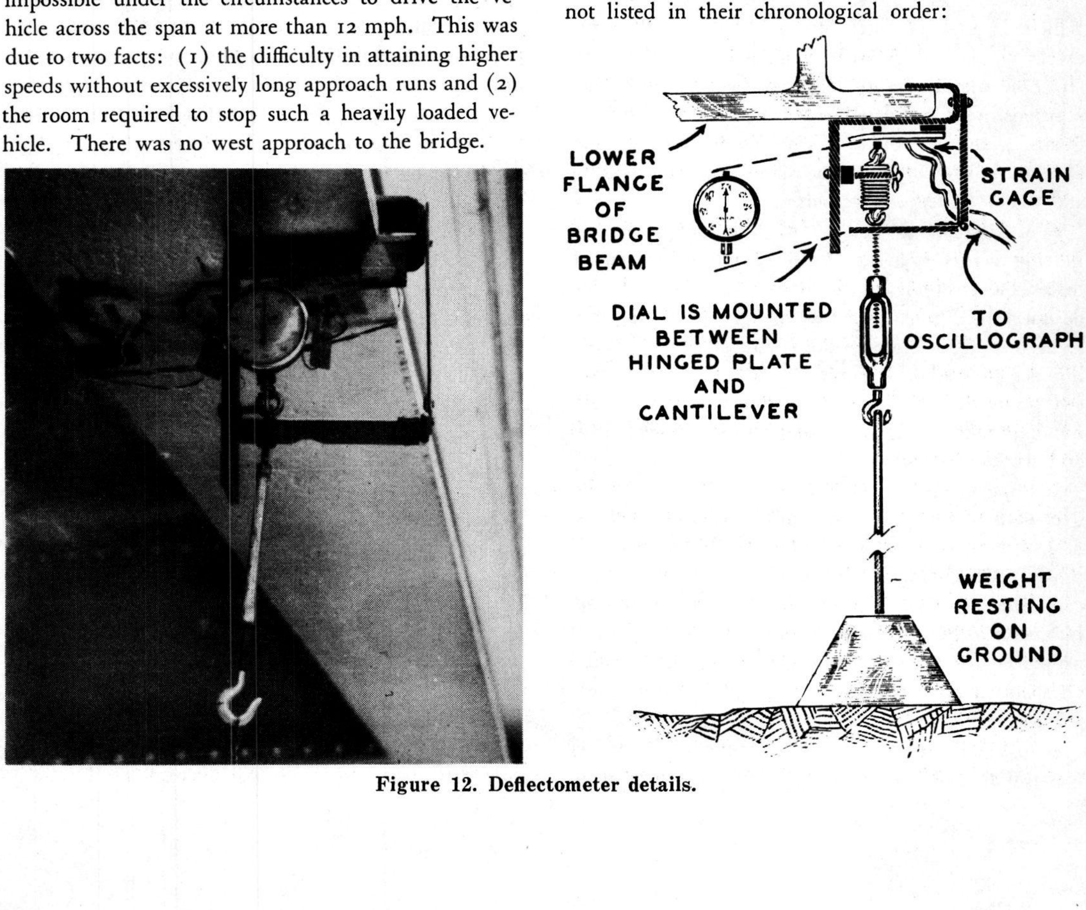


Figure 12. Deflectometer details.

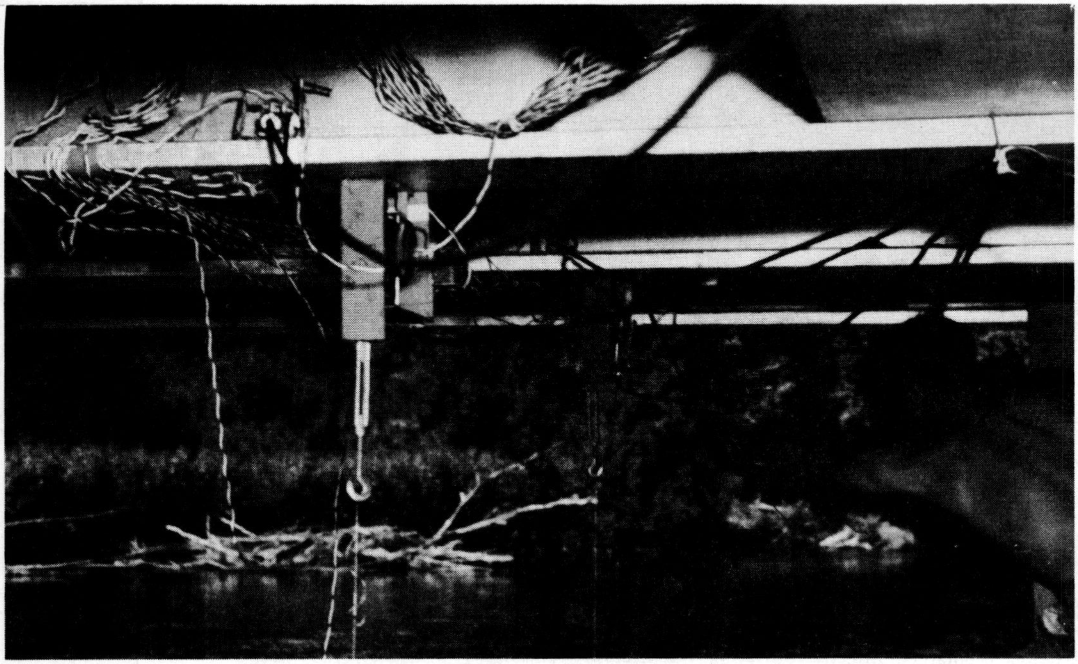


Figure 13. Installation of gages and deflectometers on span over water.

1. Static Load Tests. (a) One H20-S16 mobile vehicle in each of three lane positions on all spans except 6, 2, and 5 with single-bolted diaphragms. (b) One mobile design truck in each of three lane positions with simulated truck in adjacent lane on Spans 4 and 5. Span 5 was tested with no diaphragm bolts, single-bolted diaphragm connections, and double-bolted connections.

2. Moving Load Tests. (a) One design vehicle moving across span at 10 to 12 mph. in each of three lane positions on all spans except 6. (b) One design vehicle moving across span at 10 to 12 mph. in each of three lane positions with additional standing design load near center of adjacent lane. This test performed on Spans 3, 4, and 5. Span 5 with no diaphragm bolts, with diaphragms single-bolted, and also double-bolted.

3. Impact Tests. (a) One design vehicle moving over each of three sizes of impact plates on Spans 1, 2, 3, 4, and 5. (b) One design vehicle moving over impact plates with additional standing load in adjacent lane on Spans 3, 4, and 5. (c) One design vehicle over impact plates with standing load surcharged 15 percent in adjacent lane. This program executed on Spans 4 and 5, with Span 5 again in three diaphragm conditions.

4. Miscellaneous Tests. (a) A tandem-axle vehicle was run at speeds up to 30 mph. over an impact plate on Span 3 to note the effect of speed. (b) The mobile design vehicle was run at about 12 mph. over two impact plates at different locations and various spacings on Span 5 to explore for resonant frequency. (c) Several diaphragms were fitted with strain gages to find the lines of principal stresses. (d) Relative displacement of deck and beam was measured on Spans 3 and 5 to determine extent of slippage. (e) A record of temperatures was kept. (f) Physical data on the steel beams were obtained from the manufacturer, and flexure, compressive strength and static modulus tests were run on the bridge deck concrete.

Test Results

A complete tabulation of the data derived from the bridge loading studies is given in the table at the end of this report. Several apparent inconsistencies will be recognized in this tabulation. A possible explanation is the extent of reduction in shear between the deck and the beams. Graphs of the midspan deflections and stresses are included in Figures 18 through 22. The truck position is shown schematically for each graph, and the effect of this position upon the beam stresses is quite evident.

Comparison of Design Values and Field Data

Design stresses and deflections have been computed for each span, using the Michigan State Highway

Department's Standard Specifications for the Design of Highway Bridges. For live load and distribution of load, the Michigan specifications are the same as the AASHO. However, for impact, the Michigan specifications use the following formula.

$$I=\frac{L+20}{6L+20}$$

For the span length involved in this project, an impact factor of 21.1 percent is obtained, as compared with 27.1 percent using the current AASHO specifications. The results are compared directly with measured values in Table 1. In this summary, Spans 1 and 6 are grouped because they are end spans with a length slightly shorter than the others. Spans 2, 4, and 5 differ only in diaphragms. Span 3 has assured composite action by use of a shear developer. The shear developers consisted of the Porete Company Alpha-type spiral, which in this case was made of a 1/2 in. plain bar with a 4 1/2-in. mean diameter and a variable pitch, welded to the top of the beam flanges.

Maximum measured deflections and stresses under single vehicle loading usually occurred when the truck was moving with the inner wheels 4 ft from the bridge centerline (Position 4), and under two vehicle loading when the standing load was at Position 4 in one lane and the mobile vehicle passed along Position 0 in the adjacent lane. Impact stresses were maximum when the 3/4-in. plate was used. Under single truck loading, impact tests were made for the 4-ft position. This made possible the computation of impact effect on the basis of maximum measured deformation for a single truck. However, for two vehicles, impact was measured with both the mobile vehicle and the simulated truck at Position 4 Since maximum stresses and deflections were realized for two vehicles located at Positions 4 and 0 respectively, the effect of impact in this latter case was based upon deformations slightly less than maximum.

When the bridge was loaded with a single truck, the end spans were stressed to one third of the computed design stresses, but the measured deflections were only one sixth of the computed deflections. Spans 2, 4, and 5 developed slightly more than one third of the design stresses and about one fifth of the computed deflections. The trucks raised the measured stresses to almost one half of design, and gave deflections slightly more than one fourth of computed values.

Span 3 showed less than half the design stress under single truck loading, and about one fourth of the deflections Two vehicles produced slightly over half the design stress and between one fourth and one third of the computed deflections.

Lateral Distribution of Deflections and Stresses

The distribution of stresses and deflections laterally across each span is seen by the graphs of Figures 18 through 22. It is seen that the deflection or strain exhibited by each beam varies greatly across the span.

In order to readily compare the lateral distribution in the six spans an index was developed This index is the absolute sum of the deviations of the percent of total deflection or strain for each beam from 14 percent In other words, the strain index was formed by (1) summing the recorded strains for all seven beams under a certain load condition and designating this total as 100 percent; (2) denoting the strain on each beam as a percent of this total strain; (3) finding the numerical difference for each beam between the percent of total strain and 14 percent, since each beam would be strained slightly over 14 percent of the total strain if the distribution were perfect, and (4) summing these deviations without regard to sign to form the index. A similar index was formed from the deflection data. The average of the index for

TABLE 1

MEASURED LIVE LOAD DEFLECTIONS AND STRESSES COMPARED WITH DESIGN VALUES

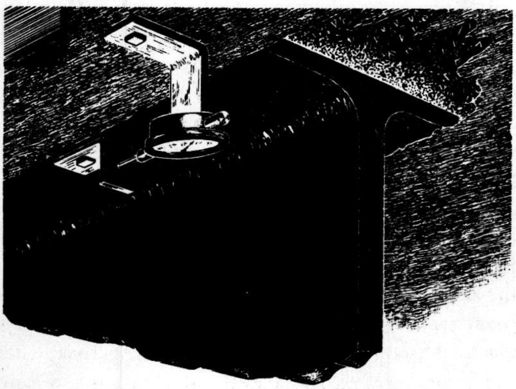
Load	Stress				Deflection			Dead Load	
	Spans	Design	Measured	% of Design	Design	Measured	% of Design	Stress Design	Deflection Design
		psi	psi	%	in	in	%	psi	in
One Vehicle No Impact	1 & 6	6,500	1,960	33	0 713	0 115	16	8,280	0 81
	2, 4 & 5	6,630	2,550	38	747	147	20	8,520	85
	3	4,690	2,030	43	314	087	28	8,520	85
One Vehicle 3/4 in Plate	1 & 6	7,880	2,320	29	864	116	13	8,280	81
	2, 4 & 5	8,030	2,670	33	904	145	16	8,520	85
	3	5,680	2,150	38	381	085	22	8,520	85
Two Vehicles No Impact	4 & 5	7,950	3,495	44	896	232	26	8,520	85
	3	5,630	3,190	57	377	116	31	8,520	85
Two Vehicles 3/4 in Plate	4 & 5	9,630	3,277	34	1 085	219	20	8,520	85
	4 & 5 W/S		3,683	38		229	21	8,520	85
	3	6,820	3,132	46	457	121	27	8,520	85

Note W/S indicates surcharge on standing load strain and the index for deflection was used as the lateral distribution index of the span. Table 2 presents these indices.

As an indication of the relative values involved, it may be pointed out that if perfect distribution were achieved, *i.e.*, all beams stressed or deflected the same amount, the index would be zero; and further, if no distribution were achieved, *i.e.*, only one beam taking all stress or deflection, the index would be 170. Further, using the AASHO design specification for distribution of the loading involved, the index would be 128. Thus it can be seen from Table 2 that for the six spans involved, the range in indices is very small, indicating little difference in lateral distribution. While in general the table shows that more distribution is obtained as the stiffness in a transverse direction is increased, even here there is some discrepancy as indicated by Span 5 with single-bolted diaphragms, which appears to have a lower index than with double-bolted diaphragms.

Assuming that the indices of Table 2, though small, are significant, the following is observed:

1. A comparison of the indices of Spans 1 with 6, and also Spans 2 with 4, shows that double bolting of the diaphragms offers slightly better lateral distribution than single bolting.



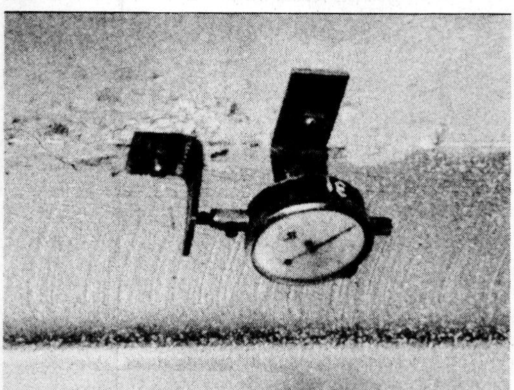


Figure 14. Dial indicator for measurement of slippage of deck on beams.

TABLE 2

INDICES FOR LATERAL DISTRIBUTION

Span	Diaphragms		Indices		Index of Lateral Distribution
	Rows	Bolting	Deflection	Strain	
1	2	double	48	46	47
2	3	double	48	42	45
3	2	single	48	52	50
4	3	single	52	48	50
5	0	none	50	48	49
5	2	single	40	46	43
5	2	double	50	44	47
6	2	single	55	45	50

2. The effect of the number of diaphragms is found by comparing indices for Spans 2 with 5 and Spans 4 with 3. Three rows double bolted offer a little better distribution than two rows double bolted, and three rows single bolted produce the same index as two rows single bolted.

3. Span 5, with no bolts, gave an index very slightly superior to that for Spans 3, 4, and 6. This might be interpreted to mean that the diaphragms do not aid materially in lateral distribution.

4. The index for Span 3 was one of the highest. This corroborates the fact that composite construction of deck and beams is not an aid in lateral distribution.

Factors in the Determination of Lateral Load Distribution

In an attempt to explain or predict the seemingly low values of stress and deflection obtained in the tests as compared to design values, it was deemed advisable to investigate and evaluate some of the basic factors influencing lateral load distribution. The two primary factors investigated were the load-distributing characteristics of the concrete slab and the composite or partial composite action found to exist between slabs and beams.

Although it is well known and adequately demonstrated in the testing that the actual distribution of load to the various stringers is quite complicated, it has been useful in analyzing test data and for design purposes to assign a definite proportion of each wheel load to each beam. The proportion assigned to each beam depends on the beam spacing and on the load distribution characteristics of the transverse members.

In previous analytical, experimental, and field testing work by others, it has been convenient to use a certain dimensionless ratio, usually denoted H, to represent the stiffness of the longitudinal beams relative to the stiffness of the slab in a transverse direction.

Extensive model testing and analytical work carried

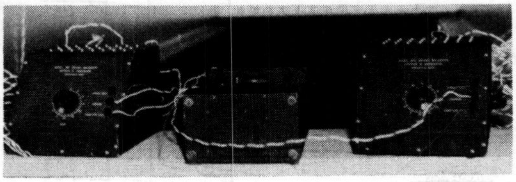


Figure 15. SR-4 indicator and Anderson switches for measurement of static load.

on at the Engineering Experiment Station of the University of Illinois by N. M. Newmark, S. P. Siess, and others is reported in the *Transactions* of the ASCE, Vol. 114, 1949. From analysis of data obtained from many model tests, it was found that the proportion of a wheel load carried by a beam, or in other words the width of lateral distribution of a wheel load, could be expressed as a function of the relative stiffness factor *H*.

It should be pointed out here that the concrete slab on the Fennville job is actually much thicker than the 7 in. considered in the design for the structure. The minimum slab thickness is increased by the incasement of the top flange, the transverse crown, and the amount added for dead load deflection. Thus, the slab thickness varies from about 9 in. at the fascia beam to more than 10 3/4 in. at the centerline beam.

It can be readily seen that because of the thicker slab involved on the test bridge, the relative stiffness of the beam's *H* will run comparatively low, and in fact varies from about 1.6 to 2.4 on the noncomposite spans and from 3.7 to 4.1 on the composite span. In the University of Illinois Experiment Station investigations, it was assumed that representative designs of a 60-ft. rolled beam span would have an *H* value of from 3 to 8 for noncomposite construction, and from 5 to 15 for composite construction. However, even though the *H* values for the Fennville structure are outside the range of values considered in the development of the formula for transverse distribution, the formula will be used later in making comparisons between predicted and field measurement values.

An additional complicating factor in these tests was the stiffening effect of the heavy safety curb. It is apparent, from a brief study of the tabulated test data, that the curb is acting with the slab in a transverse direction, resulting in a very stiff member. In many cases, the data shows the fascia beams are more

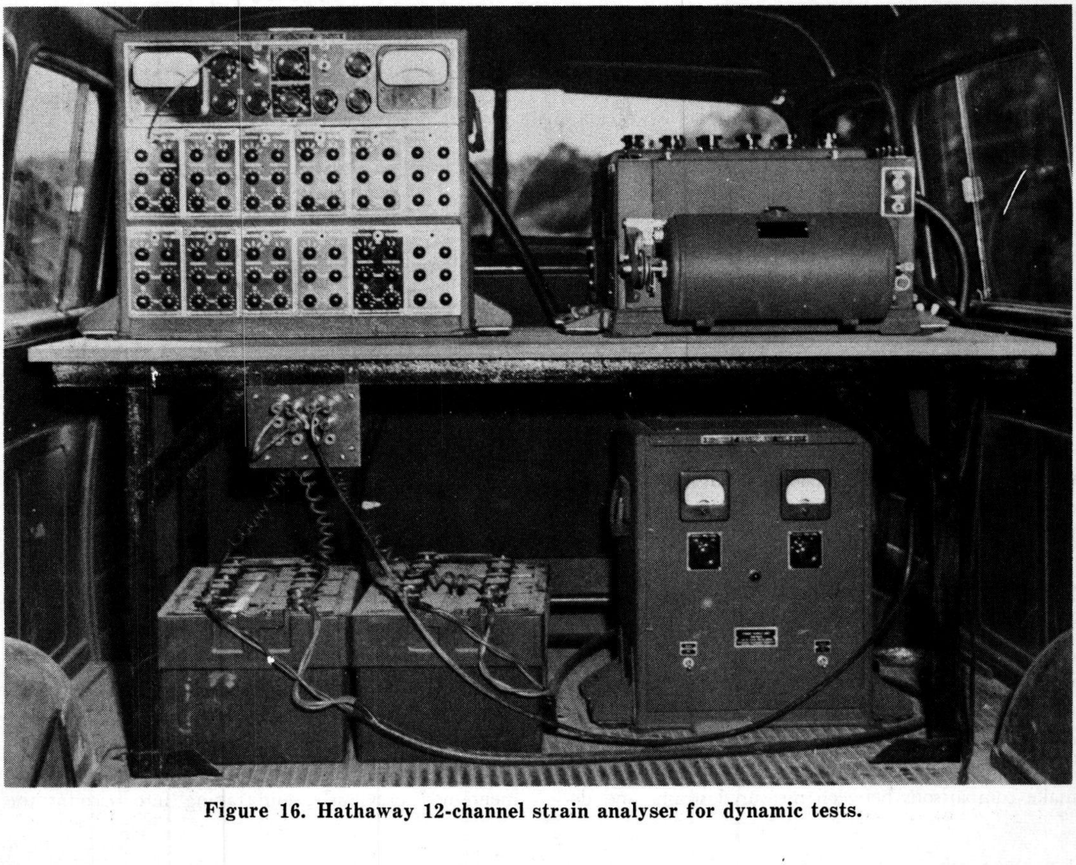


Figure 16. Hathaway 12-channel strain analyser for dynamic tests.

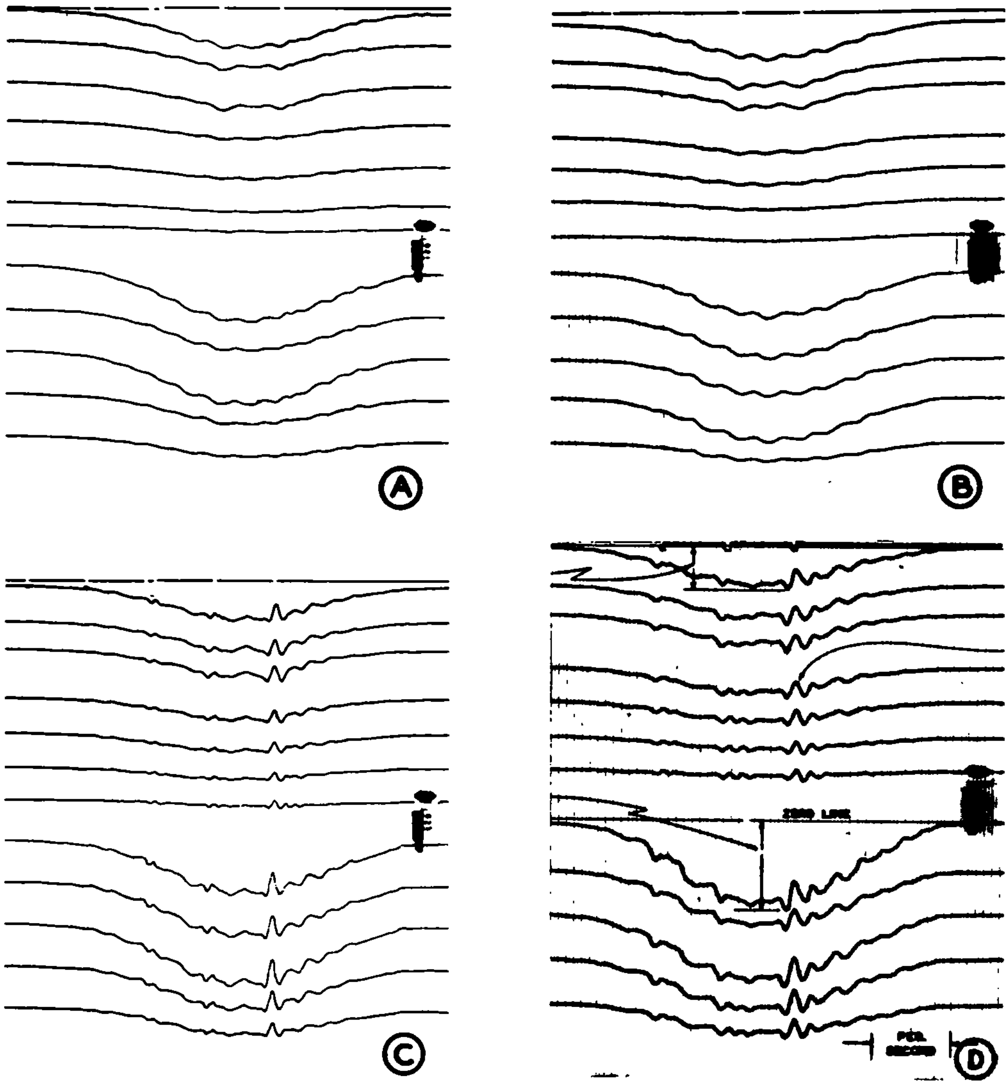


Figure 17. A: Span 1, single truck at 4-ft. position, no impact plate. B: Span 2, single truck at 3-ft. position, no impact plate, timing lines shown. C: Span 4, single truck over 3/4-in. impact plate. D: Span 5, truck moving over 3/4-in. impact plate, past standing load in adjacent lane.

highly stressed than the adjacent beams, even though the nearest line of wheels is over the first interior beam.

In the various series of static tests, where both bottom and top flange strains were recorded, it is, of course, possible to determine the location of the neutral axis of the beams The tests reveal that even in the five spans where no shear developers were used, a large amount of composite action exists as evidenced by the position of the neutral axis well above the middepth of the steel beam. In order to make comparisons between measured strains and deflections with design and predicted values, it was necessary to evaluate the effect of the partial composite action. Without attempting to fully analyze this action, it was believed that a fair basis of comparison of test data would be to use values for moment of inertia and section modulus determined by direct proportion between no composite action and full composite action as given by the location of the neutral axes.

Analyses were made, using a width of lateral distribution given by the formula of N. M Newmark, mentioned previously, and taking into account the

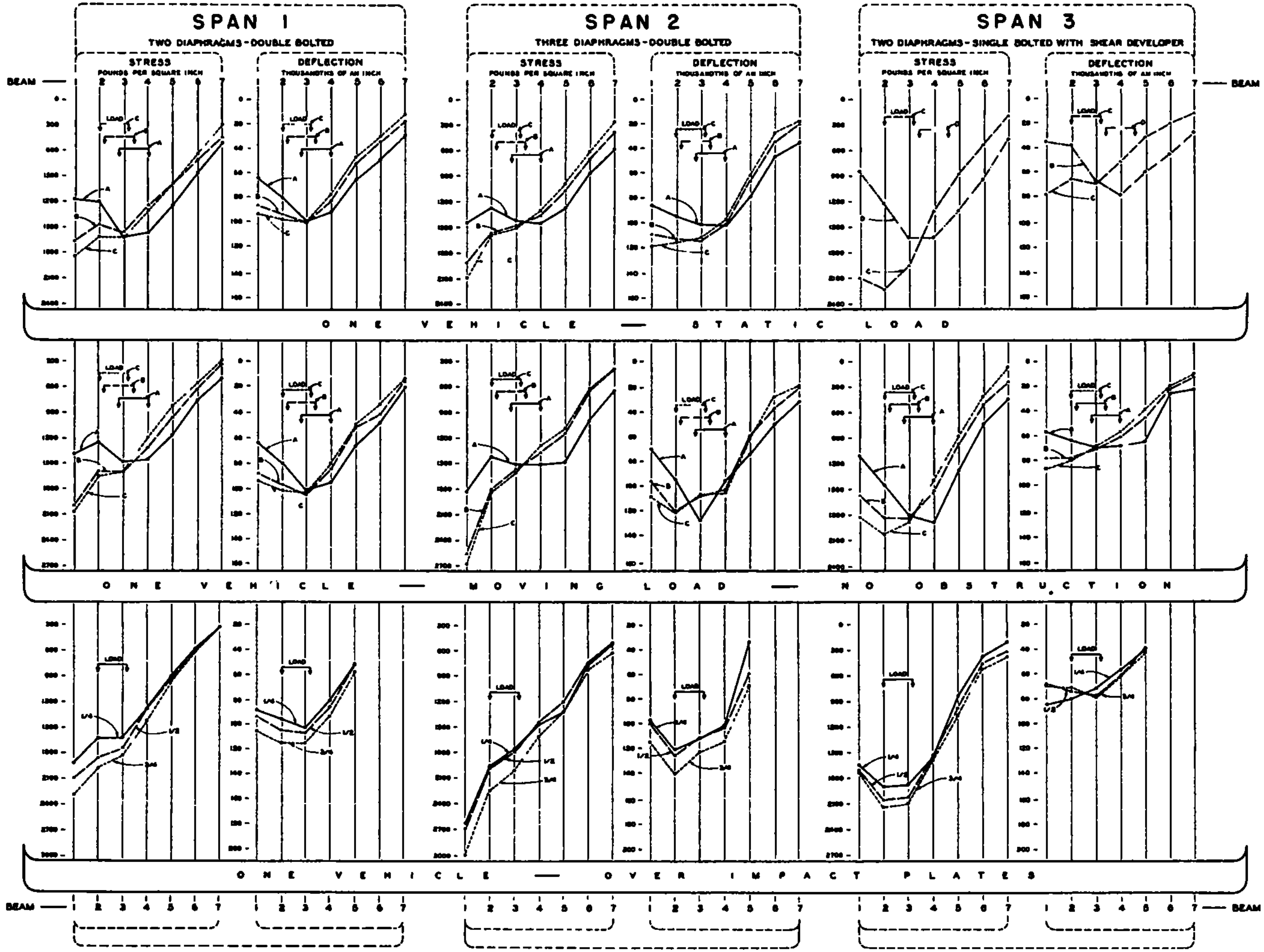


Figure 18. Distribution of stresses and deflections along lateral centerline of Spans 1, 2, and 3.

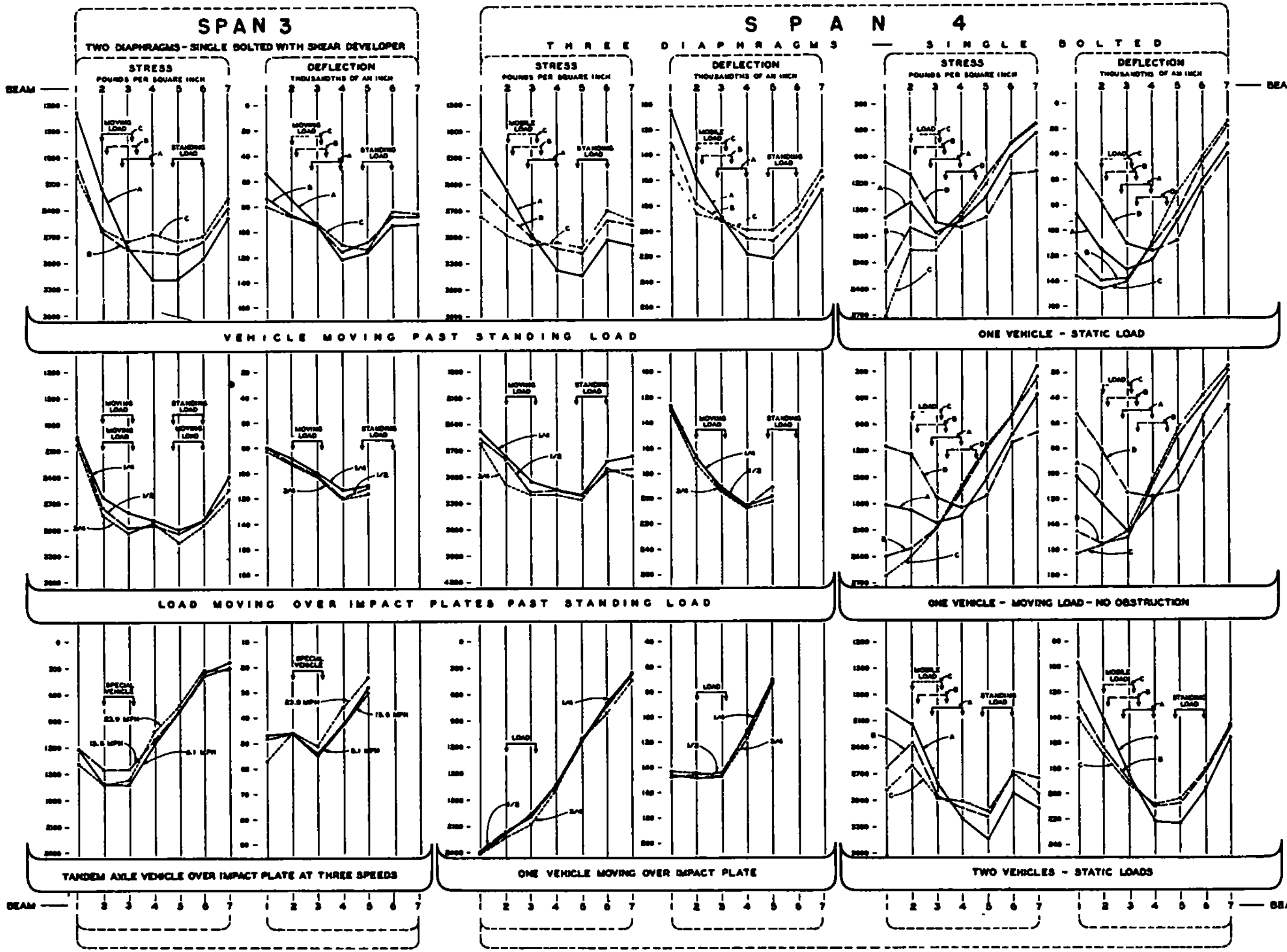


Figure 19. Distribution of stresses and deflections along lateral centerline of Spans 3 and 4.

partial composite action in the manner described above. To avoid complications from factors difficult to evaluate, only the results for the five center beams were considered. This eliminates the transverse stiffening effect of the curb and its further action as a composite section. Further, only the tests without impact were considered.

By formula, the width of lateral distribution for the noncomposite spans for a line of wheels is 6.5 ft. and 5.8 ft. for the full composite span. In seven series of tests on Span 5, the percent of composite action varied from 34 to 70, with an average of 46. The measured stresses varied from 60 to 72 percent of predicted, with an average of 66 percent, while the measured deflections ran from 48 to 57 percent, with an average of 53 percent.

Some justification for the method of considering partial composite action was given by a study of three series of tests on Span 3, the one with full composite section. Here, the measured stresses varied from 65 to 69 percent of predicted, with an average of 66 percent, while the deflections varied from 36 to 38 percent, with an average of 37 percent.
be predicted that in a wider bridge the effect of the curbs would be lessened on the beams near the center of the bridge.

Span Stiffness

Some consideration was given to the thought that the different diaphragm arrangements and fastening methods might affect the longitudinal stiffness of the spans. This stiffness was compared by noting the rank of numbers obtained by summing the deflections for all of the beams in each span, and also by comparing numbers representing the sum of the maximum strains for all of the beams in each span. These sums are tabulated in Table 3 for a single vehicle at Position 4.

TABLE 3

SUMS OF MAXIMUM STRAINS AND DEFLECTIONS OF BEAMS FOR ONE VEHICLE AT POSITION 4

Span	Diaphragms		Sum of Deflections	Rank	Sum of Strains	Rank
	Rows	Bolting	(10^{-2} in.)		(10^{-3} in./in.)	
1	2	double	47	2	28	2
2	3	double	55	4	32	5
3	2	single	36	1	30	3
4	3	single	68	7.5	37	8
5	0	none	68	7.5	35	6
5	2	single	56	5	31	4
5	2	double	66	6	36	7
6	2	single	53	3	27	1

Assuming the deflections and strains of equal importance, the values of total deflections must be weighed with those of total strain to arrive at a value for comparison. A simple average of ranks places the two end spans on the same level as Span 3 with the shear developer.

If the emphasis is placed upon deflections and the strain magnitudes are disregarded, we have the following pattern. (1) Span 3 with the shear developer is much stiffer than any other span. (2) Of the two end spans, 1 and 6, the span with double-bolted diaphragms is the stiffer. (3) Of the spans with three diaphragms, namely Spans 2 and 4, Span 2 with double-bolted connections is stiffer. (4) Span 2 with three diaphragms double bolted is stiffer than Span 5 with two diaphragms double bolted. (5) Span 5 with no diaphragms is of the same rank as Span 4 with three rows of single-bolted diaphragms, and the stiffness of Span 5 is only slightly improved by double bolting the diaphragm connections.

Effect of Impact upon Stresses and Deflections

In the impact study, the vehicle was run through Position 4, which was directly over Beams 2 and 3. For the single vehicle test, these two beams usually showed maximum values of deflections and strains under this load position, and for that reason the computation of impact factor was based upon these values.

The data for two vehicles usually showed highest values on Beams 4 and 5. It seemed logical to use these values for the computation of impact factor under the double load conditions.

Table 4 is a summary of the deflections and stresses resulting from tests made by running the design truck over the 3/4-in. impact plate at speeds from 10 to 12 mph. The average impact factor is the arithmetic average of the percent increase in deflection and the percent increase in stress. These increases are the differences between the values found when the truck was run over the plate, and the values recorded when no plate was used.

The impact factors are seen to vary from 0 to 23 percent. There seems to be no correlation between impact factor and span construction.

Reliability of data might be questioned because Span 4 showed no factor under single truck loading. This irregularity may be due to inaccuracies in load placement or drift in the electronic measuring equipment, or possibly the impact developed by the moving load without the plate was comparable to that when the plate was used. There certainly was some effect due to impact, because the record traces showed the usual pip just to the right of the center as illustrated in Figure 17. It is hoped that more successful tests may be performed at a later date, using heavier loads traveling at higher speeds.

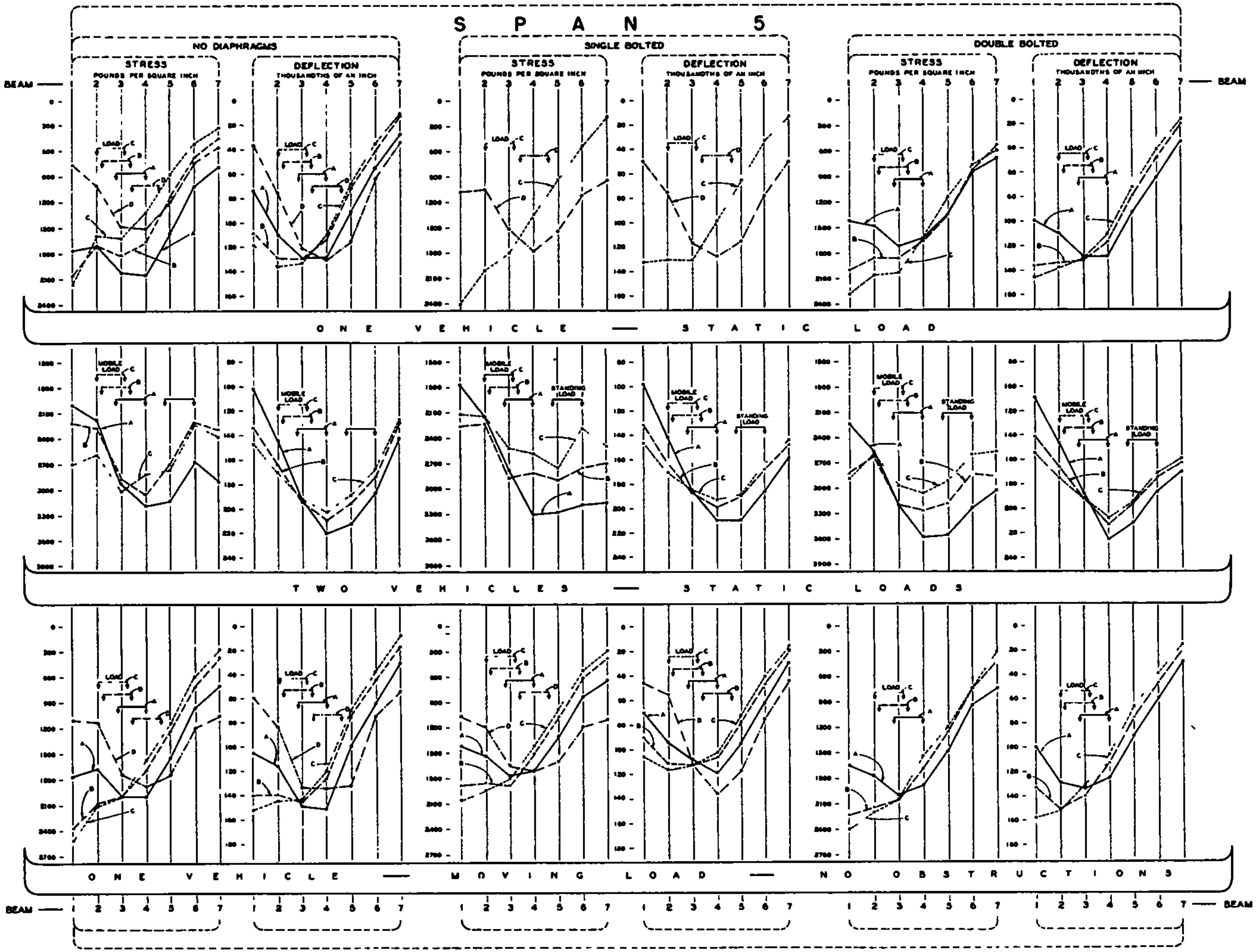


Figure 20. Distribution of stresses and deflections along lateral centerline of Span 5.

Vibration Characteristics

The undulations observed in Figure 17 are typical of all of the strain and deflection records. Although there is much variation in amplitude, there is regularity in frequency. The duration of vibration is limited to the interval that the span is loaded The rate of damping is so great that there is no evidence of vibration after the load has moved off the span

A tabulation of results is shown in Table 5. The data was taken from the deflection records for one vehicle at Position 4. The traces used were those for Beams 3 or 4, whichever exhibited the largest amplitude of vibration.

TABLE 4

EFFECT OF IMPACT UPON STRESSES AND DEFLECTIONS

(Single vehicle at position 4)

Span	Impact Plate	Defl	Stress	Defl	Stress	Defl	Stress	Av Impact Factor
		0.001 in	psi	0.001 in	psi	0.001 in	psi	%
1	none	102	1650	104	1590	103	1620	
	3/4 in	116	2000	116	1860	116	1930	16
2	none	121	1830	107	1600	114	1715	
	3/4 in	141	2230	123	2000	132	2115	20
3	none	80	2030	69	1890	75	1960	
	3/4 in	71	2150	79	2120	75	2135	5
4	none	157	2380	145	2060	151	2220	
	3/4 in	145	2260	147	2090	146	2175	—
5N*	none	145	2120	144	2000	144	2060	
	3/4 in	140	2290	146	2180	143	2235	4
5S	none	116	1940	112	1800	114	1870	
	3/4 in	145	2410	140	2180	142	2295	23
5D	none	152	2200	131	2060	141	2130	
	3/4 in	144	2380	143	2440	143	2410	7
(Two vehicles with surcharge on standing load)								
4	none	199	3130	199	3190	199	3160	
	3/4 in	222	3450	222	3570	222	3510	11
5N	none	210	2810	192	2780	201	2795	
	3/4 in	245	3330	228	3250	236	3290	17
5S	none	191	2870	182	2900	187	2885	
	3/4 in	222	3310	223	3390	222	3350	17
5D	none	227	2900	193	3160	210	3030	
	3/4 in	236	3800	234	3740	235	3770	18

* Diaphragm connections are designated as N = no connection, S = single bolted, and D = double bolted

TABLE 5

VIBRATION DATA

Span	1	2	3	4	5	6
Frequency (cps.)	2.25	2.25	2.85	2.12	2.12	2.50
Amplitude (0.00001 in.)	98	196	62	190	166	153

The record for Span 3 shows smaller amplitude and higher frequency then any other span. The end spans are next in order, with Span 1 showing lower amplitude and Span 6 giving higher frequency than Spans 2, 4, and 5.

Effect of Composite Deck Construction

The effects of the shear developer in Span 3 were noted in the previous discussions. A recapitulation of the relationship between Span 3 and the spans without shear developer is made, with reference to Tables 1, 2, 3, and 4

Design computations anticipated a relief of 29 percent in stress and 58 percent in deflections when the shear developer was incorporated in the span From Table 1, actual relief achieved under single truck loading was 20 percent in stress and 41 percent in deflections. Table 2 indicates no aid in lateral distribution from composite construction However, Span 3 ranks first in span stiffness with maximum deflections as listed in Table 3 being only 55 percent of those for the free spans. The vibration chart, Table 4, shows increased frequency and diminished amplitude for Span 3 from those of the comparative spans.

Supplementary Tests

As the opportunity presented itself, certain tests were made with the aim of supplementing the information gained in the regular testing program These studies included more impact runs, an attempt to find diaphragm stresses, measurements of strains in the deck steel and on the concrete, effects of temperature, and strain readings on deck beams subjected to the weight of the concrete deck.

Impact Effects Caused by Tandem Axles

The crane used by the Bridge Maintenance Section was capable of attaining higher speeds than the H20-S16 truck, and it was decided to attempt some tests with this vehicle running over the 3/4-in. impact plate. The vehicle was constructed with a single axle supporting 7,650 lb in front, a second axle 11.5 ft. from the front, and a third 4 ft from the second. The combined load on the second and third axles was 29,550 lb.

Runs were made at several speeds, and a final run without the plate was made for zero reference The strains registered maximum on Beam 2, with Beam 3 giving values very nearly as great. Deflections were largest on Beam 3. The deflection readings for Beam 2 were considerably smaller. A condensation of the data is given below in Table 6

TABLE 6

INFLUENCE OF VEHICLE SPEED UPON IMPACT EFFECTS

Run No	1*	2	3	4	5	6	7	8
Vehicle Speed, mph	8.1	12.8	13.4	14.5	15.6	17.7	23.9	8.7
Strain (10^{-6} in/in)	56	56	54	54	50	52	56	46
Deflection (.001 in.)	55	56	56	57	54	52	51	41

* NOTE On Run 1, the vehicle stopped with rear wheels on the span On Run 8 there was no impact plate

The results show a trend toward a minimum impact effect for this vehicle when it was driven at a

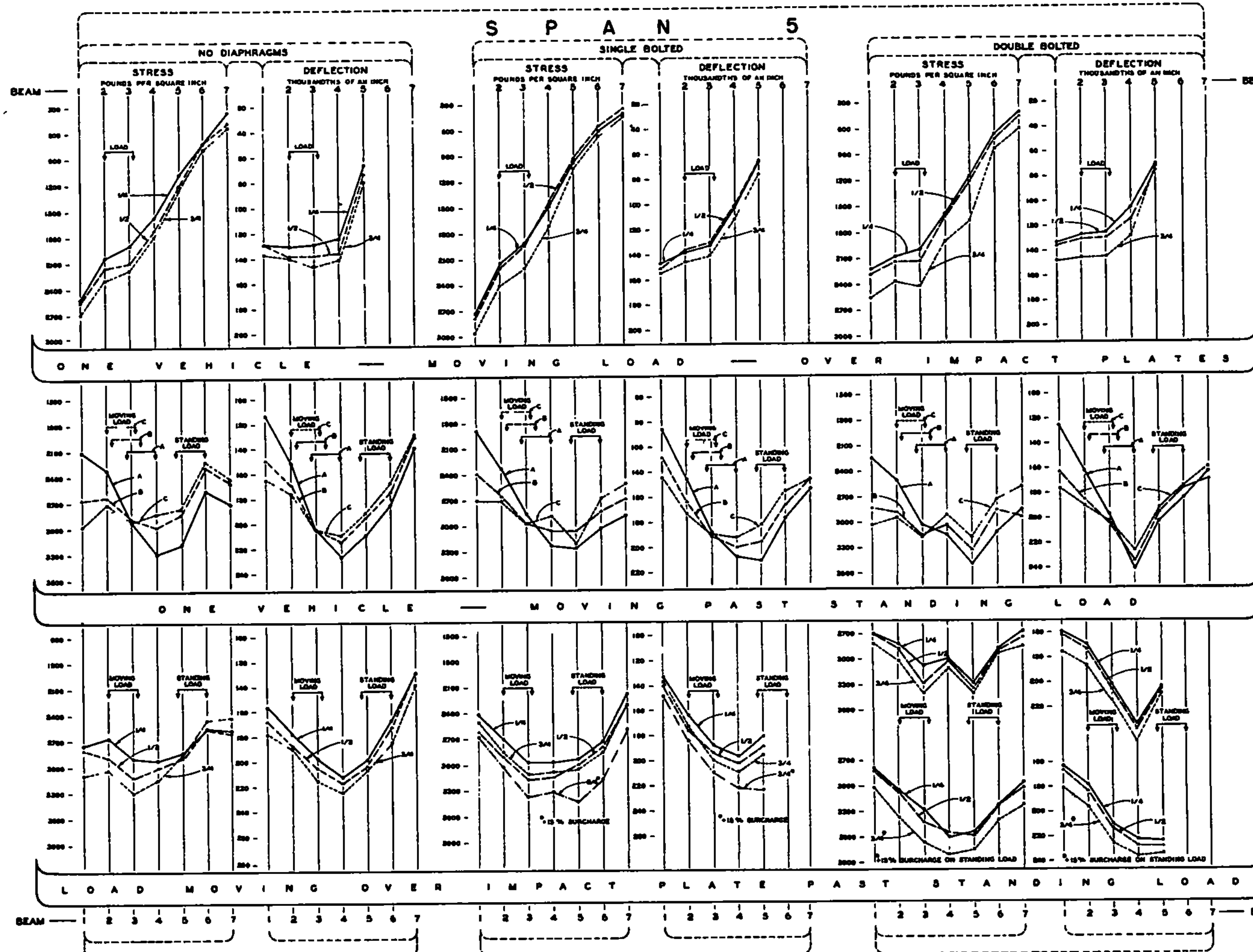


Figure 21. Distribution of stresses and deflections along lateral centerline of Span 5.

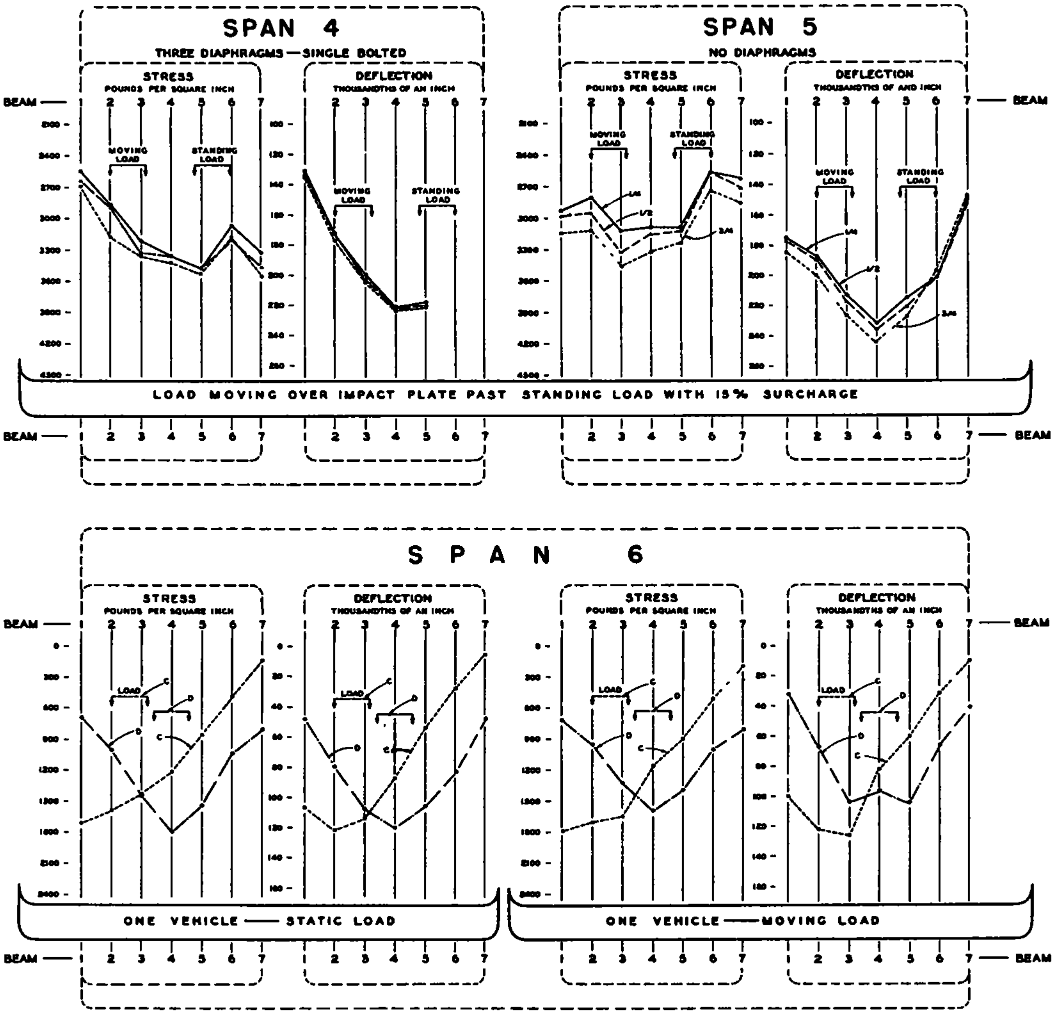


Figure 22. Distribution of stresses and deflections along lateral centerline of Spans 4, 5, and 6.

speed of 16 to 20 mph. The maximum impact factor was 39 percent, based upon deflections, and 22 percent, based upon strains.

Effect of Successive Impacts and Location of Impact Plates

Some exploratory testing for the effect of impact plate spacing was done on Span 5. The 3/4-in. plate and the 1/2-in plate were used. They were placed so that the H20-S16 truck first hit the 3/4-in. plate, and then the 1/2-in. plate, while the truck was traveling fully loaded at 11 mph. There were two series of tests made, first, with a 1-ft distance from the span center to the edge of the 1/2-in. plate, then distances of 1, 2, 3, 4, and 5 ft. between plates. The second series differed in that the distance from span center to the 1/2-in. plate was 3 1/2 ft. The same plate spacings were used.

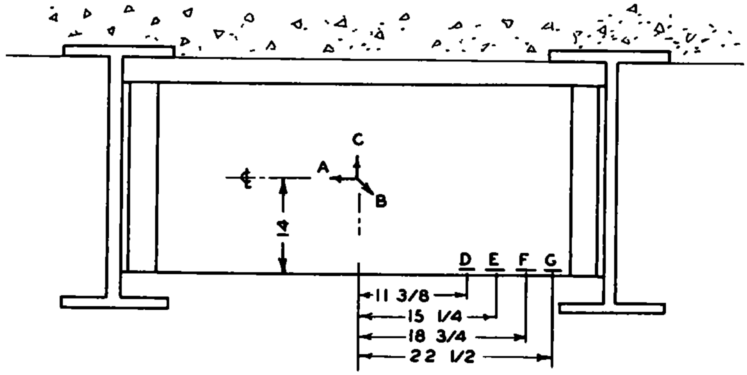
The record consistently showed maximum strain and deflection values at Beam 1. These maximums are given in Table 7.

TABLE 7

EFFECT OF SPACING OF IMPACT PLATES

	Strains					Deflections				
Spacing, ft	1	2	3	4	5	1	2	3	4	5
Series 1	97	99	97	94	94	178	179	173	167	174
Series 2	102	101	98	102	92	179	180	178	175	160
No plate	95					173				

It appears that highest values were obtained at 2-ft. spacing in Series 1, and at either 1- or 2-ft. spacing for Series 2. The effect seemed to fall off sharply at the 5-ft. spacing in Series 2. Since both the strain and deflection magnitudes for this distance were be-



GAGE LAYOUT FOR DIAPHRAGMS IN EAST LINE

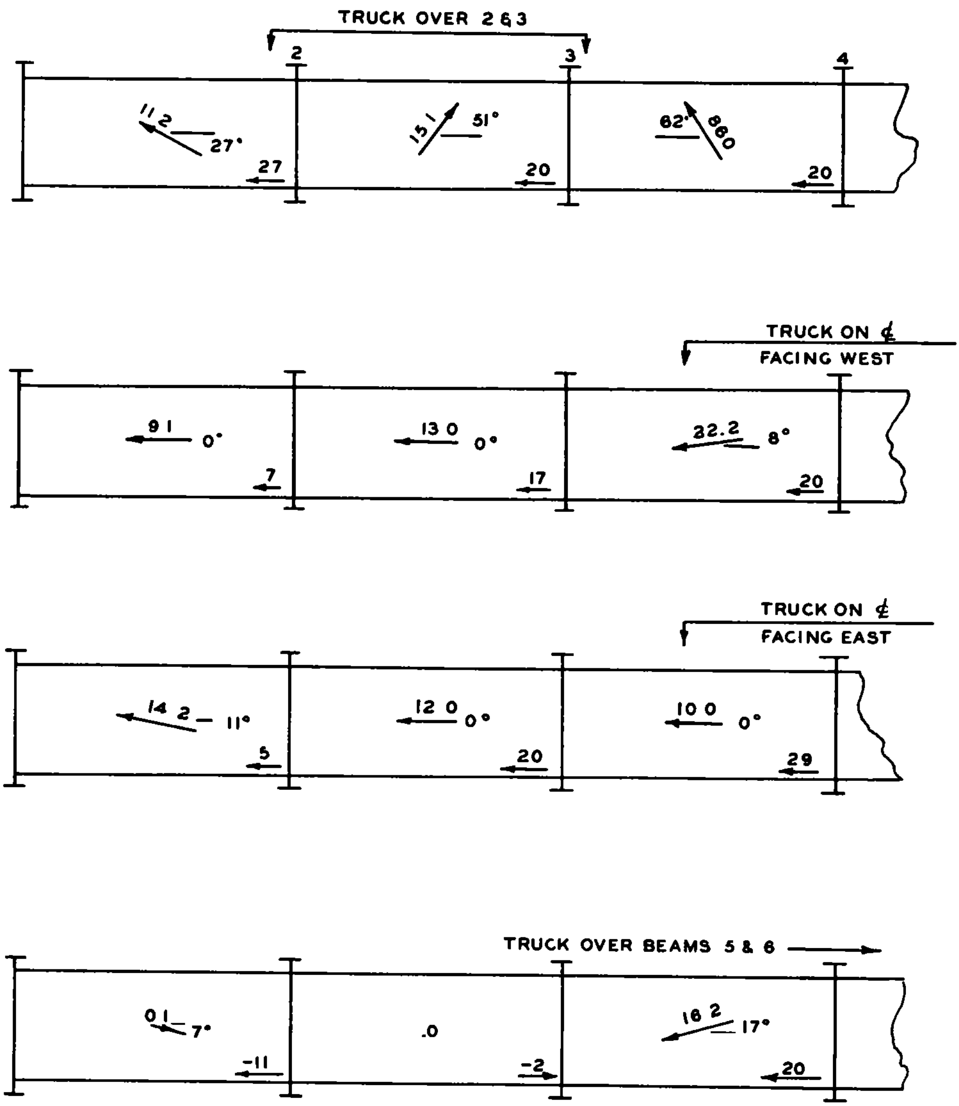


Figure 23. Strains in diaphragms, Span 6.

low those for the no-plate condition, it is possible that the vibrations were out of phase so that the downward impulse caused by the second plate occurred while the surge from the first impact was upward

Computing for critical plate spacing using vibration data for Span 5 from Table 5 and a truck speed of 11 mph (16.1 fps) we find that in the interval 1/2 times 12 sec, the truck traveled 7 6 ft Unfortunately, the maximum experimental spacing was 5 ft According to this method of computation, a spacing of 3 8 ft (1/2 times 7 6 ft.) should have caused a bucking action due to phase shift, and the recorded values for this plate spacing should be low Some reduction was evident in Series 1, but not in Series 2 at the 4-ft distance

Stresses in Diaphragms

Diaphragms on Span 6 were equipped with gages for the purpose of determining magnitude and direction of principal stresses while the span was subjected to load The gage layout is given in Figures 23 and 24, and the data is shown in Table 8. Three diaphragms were in the east row on Span 6, and were numbered from north to south The designations 1, 2, and 3 in Table 7 respectively indicate the diaphragms between Beams 1 and 2, 2 and 3, and 3 and 4 Diaphragm 4 is in the west row on Span 6 between Beams 3 and 4 The gage layout on this diaphragm is on Figure 24

Computations of principal strain magnitudes and directions from the readings of the rosette gages gave the results which are shown schematically in Figures 23 and 24 Most of the values on the diaphragm webs are small, although in the case of the diaphragm connecting Beams 3 and 4, a resulting strain of 86 microinches per in. was found In Figure 24, the largest value shown is 57 microinches per in In terms of steel with a modulus of elasticity of 30 million psi, these strains indicate stresses of 2,580 psi. and 1,710 psi respectively

The diaphragm directly beneath the load seems to be in the state of highest stress This is illustrated in the second drawing in Figure 24 Note also that one angle fillet stress is high The strain of 134 microinches per in is equivalent to 4,020 psi of stress

Measurement of Relative Movement Between Deck and Beam

Dial indicators were attached to the underside of the deck near the piers This detail was shown in Figure 14 Exploration on Span 6 proved that the greatest relative movement occurred at the ends, and movement at the center of the span was less than 0 001 in Readings at the ends of Spans 5 and 3, representing relative movements per half-span length, are tabulated in Table 9

TABLE 8

STRAINS IN DIAPHRAGMS

(Strains in 0 000001 in per in)

Gage Location (Fig 23)	Truck over 2 & 3 Diaphragm			Truck over C L (W) Diaphragm			Truck over 5 & 6 Diaphragm			Truck over C L (E) Diaphragm		
	1	2	3	1	2	3	1	2	3	1	2	3
A	5	12	30	9	13	22	0	—2	15	7	12	10
B	10	10	80	8	11	13	—5	0	5	5	10	0
C	—5	13	70	7	9	8	—8	2	3	0	8	—10
D	15	20	20	0	11	32	—5	10	13	—5	13	20
E	15	20	20	0	12	37	—8	0	13	3	17	20
F	22	18	20	5	15	38	—10	0	20	3	20	29

Fig 24	Truck over 2 & 3	Truck over 3 & 4	Truck over 4 & 5	Truck over C L
1	3	35	12	26
2	9	56	30	46
3	10	45	19	32
4	8	25	0	12
5	6	27	10	16
6	6	18	0	0
7	10	30	—7	12
8	18	45	5	28
9	18	43	8	26
10	4	28	11	22
11	0	24	16	—
12	—3	12	4	0
13	8	10	—17	—
14	6	6	—15	—
15	11	16	—5	—
16	120	134	39	—
17	—28	17	66	—
18	68	0	—66	—
19	—22	—11	0	—

It should be explained that the recorded movement for two vehicles is not a total movement, but is in reality an increment caused by a single truck. The readings were made from an assumed zero after the standing load had been placed. There is no method of accumulating these values, because the mobile truck was not run through the standing load positions, nor were dials attached to Beams 5 and 6.

The results indicate relative movement of 0 01 to 0 02 in. near the ends of the span for Span 5. No effort was made to determine where, along the span, slippage was sufficient to cause bond breakage.

The Span 3 data shows no movement as great as 0.001 in. This seems to be conclusive evidence of composite action.

Observations on Temperature Effects

The fact that the deflectometers used in this study behaved erratically when the reading interval was of a duration longer than half an hour led to a study of the effects of temperature upon these readings The sepcific objectives were to (1) observe the behavior of a free indicator under temperature fluctu-

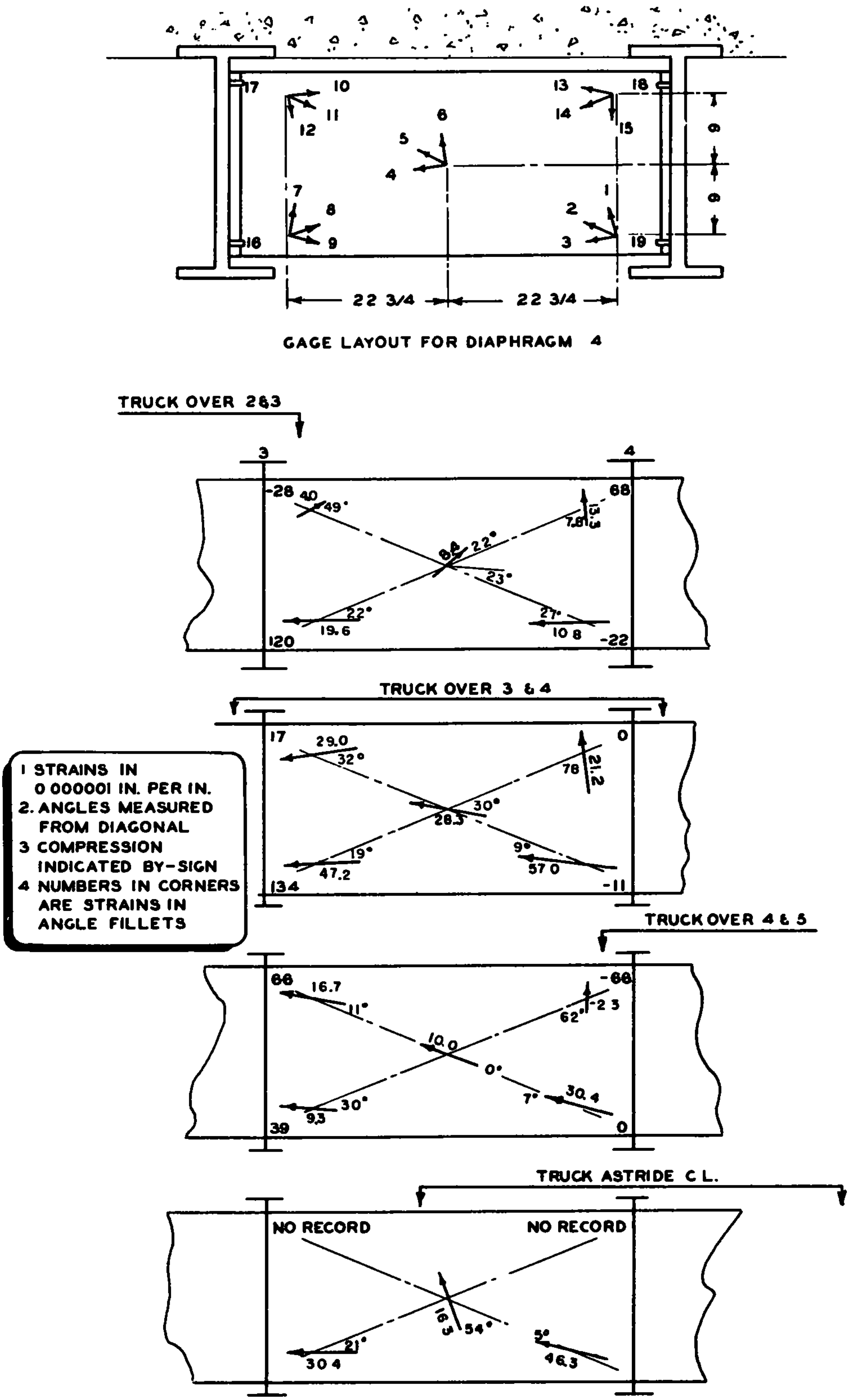


Figure 24. Strains in Diaphragm 4.

ations; (2) measure the vertical movement at the span center and try to correlate this movement with temperature; (3) observe the effects of temperature change upon relative movement between deck and beam; (4) measure variations in expansion joint width; and (5) check the reliability of the deflectometer reference system by comparing readings of the deflectometers using steel cables attached to anchors on the soil surface with the readings determined from dials supported by steel and wood columns

Indicator Reliability. The dial indicators were mounted in a position which would subject them to direct sunlight for a part of the day and to shadow for another part. They were allowed to remain here throughout a complete 24-hour cycle, with temperature fluctuations from 58F. to 95F The maximum variation in the reading was 0.001 in. This was sufficient proof of reliability, and it was concluded that the observed fluctuations on the bridge deflectometers were due to external causes.

Reference Check. Adjacent to deflectometer locations at Beam 4 and Beam 7 at the south fascia, columns were erected and dial indicators attached to the top with the stems resting against the bottoms of the respective beam flanges The center column was of wood, and the outside was a 1 1/2-in steel pipe. Although the dial readings varied throughout the test period, the fluctuations at the center beam were the same for both dials, and similarly for the dials at the outer beam. It was concluded that the steel cable method of maintaining a reference for the deflectometers was dependable.

Study of Vertical Movement of Unloaded Span. Indicator dials were installed atop steel columns to study the vertical movement of the beams of Span 5 at midspan. Three positions were selected, one at Beam 1 at the north face, a second at Beam 4, and a third at Beam 7 Readings were made on four consecutive days.

To supplement the dial readings, deck temperatures were read by means of surface thermocouples Table 10 includes these readings, together with those for the expansion joint width changes and relative movement between deck and beams.

The vertical movement of the span ranged from minus 0.055 in. on one side to plus 0.70 in on the other. The record does not seem to show any trend, but rather an unpredictable fluctuation. Daily temperatures seemed to have greater influence than the temperature differential in the deck However, the data makes evident the difficulties encountered in the measurement of deflections due to load when the time interval is large.

TABLE 9

MOVEMENT BETWEEN BRIDGE DECK AND STEEL BEAMS

(Relative movement in 0 0001 in)

Truck Position	SPAN 5												SPAN 3			
	ONE VEHICLE						TWO VEHICLES						Single Bolted Diaphragms			
	No Diaphragms		Diaphragms Single Bolted		Diaphragms Double Bolted		No Diaphragms		Diaphragms Single Bolted		Diaphragms Double Bolted		One Vehicle		Two Vehicles	
	Dial 2	Dial 3	Dial 2	Dial 3	Dial 2	Dial 3	Dial 2	Dial 3	Dial 2	Dial 3	Dial 2	Dial 3	Dial 2	Dial 3	Dial 2	Dial 3
0	99	138	95	135	111	139	112	171	148	218	99	115	5	8	4	7
3	108	139	110	141	132	138	109	203	107	216	96	121	4	8	6	8
4	106	138	112	132	136	128	108	202	178	182	115	122	5	7	6	9

NOTE: Dial 2—Read movement at Beam 2 Dial 3—Read movement at Beam 3.
Truck positions are distance in feet from C L to nearest wheel

Expansion Joint Width Changes. Two parallel lines were scribed upon each end of the metal plates of the expansion joint between Spans 5 and 6, for the purpose of measuring changes in joint width. Periodic readings of the distance between these lines gave the data shown in Table 10. The maximum width change was 0 06 in. for a temperature change of 22F. Since these joint width changes represent the expansion in a span length of approximately 60 ft., the measured value was only about two thirds of the predicted 0.10 in which should occur under free expansion.

Measurement of Strains in the Concrete Deck

Before the decks of Spans 3 and 4 were cast, gages were cemented to the lateral reinforcing steel as shown in Figure 13 There were two lines of gages on each span, one line being 5 ft. from the end and the other at the center. A plan of the installation on Span 4 is shown in Figure 25. Gages A, C, and E were on the bottom face of the lower reinforcing rod, and they were placed midway between the supporting beams The remaining gages were attached to the top of the upper rods, and were directly above the beams.

TABLE 10

EFFECTS OF TEMPERATURE CHANGES

Day of Month	Time	Deck Temp		Changes in Exp Jt Width		Vertical Movement of Span			Rel Movement of Slab & Beam	
						Beams *			Beams †	
		Top	Bottom	N	S	1	4	7	2	3
		F	F	in	in		0 001 in		0 0001 in	
18	4 00 p m	80	78	0	0	0	0	0	0	0
19	8 00 a m	66	66	— 01	— 01	62	43	—55	—5	—1
	11 00 a m	71	70	0	0	35	23	—8	—4	1
	2 00 p m	77	70	— 02	— 02	22	3	32	—2	1
	5 00 p m	80	75	— 04	— 04	18	8	14	—2	1
20	8 00 a m	58	58	02	02	46	58	—50	9	—11
	11 00 a m	67	67	01	02	62	43	—9	17	—10
	2 00 p m	76	70	0	0	68	38	62	17	6
	5 00 p m	80	75	— 01	0	70	48	51	17	0
21	8 00 a m	64	64	0	02	85	0	—20	17	—6

* A negative sign indicates an upward deflection
† Relative movement here is due to causes other than load

Span 3 was also equipped with gages, in a layout symmetrical to that of Span 4. The end gages were 5 ft from the east pier in this case

Readings were taken at the time of installation before the deck was placed, and at various times after pouring Final readings were made with the span loaded by the design vehicle The results are given in Table 11.

Analysis of the data on strains in reinforcing steel is complicated by the irregularity of the results. An inspection of the record prior to the loading tests suggests that some electrical disturbance other than change in gage resistance or creep in the bonding material affected the gages For example, the first line in Table 11 shows a strain of 1,500 microinches per in in the steel Since the steel is bonded to the concrete, a similar strain must be transferred to the surrounding concrete But concrete can resist only about 150 microinches per in of tensile strain without cracking, and no crack was seen at this point in the deck There are many entries over 150 microinches per in.

A second consideration is the divergence of the data for Span 4 at the center Instead of an increase

TABLE 11

STRAINS IN REINFORCING STEEL

(Strain indicator readings in 10^{-6} in per in)

Gage Location	After Set	Age 2 da	Age 2 wk	Age 1 mo	Age 2 mo	Load Stresses with Indicated Truck Positions				
						1	2	3	4	5
Span 3, E										
A	400	510	1585	1545	1500	—5	—5	10	—15	25
B	60	—10	150	35	—130	21	10	Gage Failed		
C	130	20	230	260	250	15	10	30	20	30
D	160	235	255	435	1385	27	0	Gage Failed		
E	93	—45	45	68	50	—10	0	25	—10	0
F	180	—150	360	463	1360	10	5	10	15	10
Span 3, ctr										
A	Gage Failed									
B	185	325	455	503	295	18	28	15	5	25
C	96	105	545	1065	685	25	5	10	25	40
D	—60	—140	—470	—943	60	12	15	Gage Failed		
E	70	230	290	620	375	0	5	8	10	10
F	150	60	160	525	335	5	5	20	10	10
Span 4, W										
A	—15	—32	—95	1535	4370	—8	—8	0	20	30
B	—15	—32	—305	—15	1095	5	15	15	—10	25
C	25	30	—120	—50	230	—2	—12	—3	—3	40
D	—125	—70	—285	25	—215	—55	—55	80	55	Gage Failed
E	150	185	15	445	1020	7	20	10	0	—5
F	—50	—57	—300	—295	—250	10	13	5	—10	—10
Span 4, ctr										
A	45	—15	115	—350	70	32	53	70	45	8
B	—88	—180	—1075	—1175	—805	—9	8	125	—25	—40
C	56	—30	—305	—465	0	27	18	60	—5	22
D	—12	—120	—1145	—1355	—1120	—10	15	75	10	—100
E	—74	60	—1530	—1400	—1725	—11	—43	75	5	80
F	18	—95	—960	—970	—660	19	17	65	0	—10

Position 1—Load over beams 2 & 3, middle axle over center line of gages
2— " " " " " end line of gages
3— " astride beam 4, " " " center line
4— " " " " " " end line
5— " " " 3, " " " center line

in tensile strain, almost all of the values here are compression

Under the loading study, no trend or pattern has been discovered Most of the values were very small, although one column of data on Span 4 contained larger strain values

It seems at present that the gage installation on reinforcing bars is of doubtful value.

Strains on the Deck Surface Due to Live Load

A brief investigation of strain magnitude on the lower surface of the concrete deck was made by cementing A-9 gages directly above the diaphragms The plan of Figure 25 shows the locations. Data from the study is given in Table 12

Most of the measured strains were very small The 70-microinch-per-in value on Gage 1 was the largest. This is equivalent to about 300 psi of stress, which is well below the modulus of rupture of the concrete.

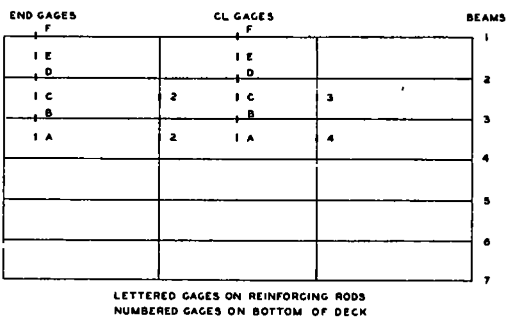


Figure 25. Gage layout for measurement of deck strains.

Tests on Materials

The bridge-deck materials were inspected and tested by the Pittsburgh Testing Laboratory and Michigan State Highway Department inspectors Table 13 is indicative of the quality of the materials used

Summary of Observations

From the foregoing discussion, certain facts are evident and others offer opportunity for discussion. Some of the evident facts are·

1. All spans were conservatively designed Except for Span 3 with composite action, the measured stresses were less than half the computed values, and measured deflections about one fourth those computed

2 Lateral distribution of load was not materially aided by diaphragms There seemed to be about the same degree of lateral distribution of load whether the diaphragms were single bolted, double bolted, or not bolted at all

TABLE 12

LATERAL STRAINS ON LOWER SURFACE OF CONCRETE DECK

Truck Position	Midaxle Location	Gage 1	Gage 2	Gage 3	Gage 4
		A SPAN 5—SINGLE BOLTED			
		0 000001 in per in			
Astride C L	F*	12	13	10	37
	W	21	37	15	28
Outer Wheels on C L	E	19	10	45	23
	W	70	20	27	24
Astride Beam 3	E	20	10	37	37
	W	48	30	30	20
		B SPAN 5—DOUBLE BOLTED			
Astride Beam 3	E	29	16	38	41
	W	57	34	31	23
Outer Wheels Over 3	E	19	11	22	8
	W	40	15	25	14

* E indicates east diaphragm line, W indicates west

3. The positive factors influencing relative span stiffness were limited to the composite action achieved by the shear developer and embedment of beams in abutments The apparent influence of diaphragms seemed to be nullified as the partial composite action was reduced

4. The effect of impact upon slab stresses and deflections was not studied sufficiently to provide a satisfactory value for impact factor Experimental values of this factor varied from 0 to 23 percent, and no cause for such variation was discovered.

5. The frequency of vibration of the spans was dependent upon the span stiffness. The stiffer spans vibrated at higher frequencies and lower amplitudes than the others.

6. The incorporation of shear developers in Span 3 produced a stiff span, but did not aid in lateral distribution of load Deflections of this span were only half of those found in the spans without composite action under the same loading conditions

7. Stresses in diaphragms were for the most part of small magnitude This fact is further corrob-

TABLE 13

TEST RESULTS ON MATERIALS

(a) Steel

Item	Yield psi	Ultimate psi	Elongation %	Chemical Analysis C	Mn	P	S
WF Beams	37 780	65,100	32 5	0 23	0 56	0 012	0 036
5/8 in def bar	48,029	81,152	18 6	39	42	010	035
1/2 in def bar	50,530	78 322	20 1	36	46	011	040

(b) Concrete

Aggregate		Postma 6A coarse, 2NS fine	
Cement	Span	6 Medusa A E	Percent Air 4 3
		5 Aetna A E	7 0
		3 Aetna Std + 3/8 oz Darex	6 4
		3 (corrected) + 1/4 oz Darex	4 4
		2 Aetna Std + 1/4 oz Darex	4 1

6 in x 6 in x 36 in Test Beam

Mod of Rupture 7 da	Mod of Rupture 28 da	Comp Strength 28 da	Mod of Elast 28 da
533 psi	650 psi	4,460 psi	4 83 x 10^6 psi

SUMMARIZED DATA FROM BRIDGE LOADING PROGRAM

Span	Date	Test Description	Truck Position	Strain 1	Strain 2	Strain 3	Strain 4	Strain 5	Strain 6	Strain 7	Defl. 1	Defl. 2	Defl. 3	Defl. 4	Defl. 5	Defl. 6	Defl. 7	Remarks
				Beams - Numbered North to South: Strains Micro In./In.							Deflections - .001							
SPAN 1	10-11-50	Static, 1 Vehicle	4	64	56	56	45	34	22	10	93	97	98	77	47	30	12	Av. of 2 gages on bottom of lower flange of each beam.
			3	57	51	54	43	34	24	10	86	93	100	82	52	34	17	
			0	41	42	56	54	43	30	18	64	80	99	91	65	49	29	
		Moving, 1 Vehicle	4	73	57	55	41	28	19	9	94	102	104	80	48	32	11	1 ga. bot. of each beam
			3	69	55	55	45	33	31	11	88	97	105	84	51	40	13	
			0	48	43	51	50	40	26	17	63	80	102	96	66	47	18	
		Impact, 1 Vehicle	4	80	69	64	50	34	22	12	106	116	116	94	58	—	—	3/4 in. plate at mid-span
			4	73	64	61	45	33	21	12	94	105	107	86	51	—	—	1/2 in plate at mid-span
			4	67	57	57	45	32	21	12	89	96	103	81	51	—	—	1/4 in. plate at mid-span
SPAN 2	10-10-50	Static, 1 Vehicle	4	72	55	52	45	34	21	9	119	116	112	98	62	28	18	Av. of 2 gages on bottom of lower flange of each beam.
			3	66	54	51	47	37	24	13	109	113	115	102	67	35	21	
			0	50	44	49	50	44	30	20	86	95	112	113	80	47	36	
		Moving, 1 Vehicle	4	91	63	55	44	37	21	13	109	121	107	106	62	28	19	1 gage on bottom of lower flange of each beam
			3	88	62	54	47	39	22	13	96	120	108	102	59	37	20	
			0	63	49	52	52	51	34	22	70	95	124	96	75	50	32	
		Impact, 1 Vehicle	4	103	77	69	55	45	28	21	115	141	123	115	70	—	—	3/4 in. plate at mid-span
			4	92	68	61	49	41	25	17	100	126	111	103	59	—	—	1/2 in. plate at mid-span
			4	90	67	60	50	45	26	17	97	121	112	101	57	—	—	1/4 in. plate at mid-span
SPAN 3	8-31-50	Static, 1 Vehicle	4.5	72	77	67	45	30	18	6	78	65	69	51	31	20	11	Av. of 2 ga. on bot. of bot. flange
				-1	-3	4	5	6	4	3								Av. of 2 ga. on bot. of top flange
			C.L.	29	42	56	56	45	32	16	35	38	67	79	60	45	27	Av. of 2 ga. on bot. of bot. flange
				6	3	0	-16	-6	-3	0								Av. of 2 ga. on bot. of top flange
	9-8-50	Moving, 1 Vehicle	4	63	70	65	48	30	14	7	87	80	69	57	39	20	10	1 ga. bot. of each beam
			3	54	63	63	52	33	17	8	78	78	71	61	46	23	13	
			0	38	50	62	65	44	25	15	57	64	70	69	55	27	23	
	9-6-50	Impact, 1 Vehicle	4	57	66	65	54	29	12	7	85	80	72	57	40	—	—	1/4 in. plate at mid-span
			4	59	71	70	53	34	16	11	68	74	77	62	39	—	—	1/2 in. plate at mid-span
			4	60	74	73	55	37	18	13	70	71	79	61	43	—	—	3/4 in. plate at mid-span
		Impact, Special	4	42	56	56	40	27	12	8	47	46	55	42	28	—	—	15 tons on tandems at 8.1 mph
				42	50	50	38	27	13	10	48	46	54	42	30	—	—	15 tons on tandems at 15.6 mph
				48	56	54	35	25	11	11	57	46	51	36	24	—	—	15 tons on tandems at 23.9 mph
	9-11-50	Moving, 2 Vehicles	4	69	90	95	92	95	93	78	80	87	93	110	114	84	86	Accumulated values of strains at bottom of each beam
			3	63	91	98	99	100	95	82	73	86	95	115	108	88	88	
			0	44	74	97	110	110	102	86	54	74	93	122	116	95	94	
		Impact, 2 Vehicles	4	67	90	96	99	103	99	82	80	89	100	114	110	—	—	1/4 in. plate - accumulated
			4	66	94	102	101	104	99	87	79	92	102	120	112	—	—	1/2 in. plate - accumulated
			4	69	97	104	100	108	101	91	83	93	103	121	117	—	—	3/4 in. plate - accumulated
SPAN 4	8 23 50	Static, 1 Vehicle	4.5	81	59	58	42	29	19	7	111	115	115	75	55	25	7	2 gages - bot. of each beam
			C.L.	33	38	57	59	55	38	37	48	77	111	116	108	67	40	(Av. of 2 directions)
	8 25 50	Moving, 1 Vehicle	4.5	87	66	65	48	36	19	9	126	132	123	99	63	34	12	1 ga. bot. of each beam
			C.L.	39	42	59	63	58	37	33	53	81	115	118	113	76	46	(Av. of 2 directions)
	10-9-50	Static, 1 Vehicle	4	94	68	68	55	42	26	18	136	146	140	110	70	42	17	1 ga. bot. of each beam
			3	76	59	63	53	39	26	18	118	139	138	112	75	46	19	
			0	55	49	61	56	47	31	22	87	115	131	123	91	61	32	
		Moving, 1 Vehicle	4	90	82	71	54	38	26	7	163	157	145	104	64	38	15	1 ga. bot. of each beam
			3	82	79	71	56	39	26	11	145	155	150	107	69	44	18	
			0	62	64	69	66	51	34	18	102	124	145	121	87	54	34	
		Impact, 1 Vehicle	4	84	76	68	57	39	24	12	146	148	147	111	71	—	—	1/4 in. plate at mid-span
			4	83	75	69	56	39	25	12	147	146	144	112	70	—	—	1/2 in. plate at mid-span
			4	84	78	72	58	38	28	15	143	145	147	117	74	—	—	3/4 in. plate at mid-span
	10-6-50	Static, 2 Vehicles	4	100	85	102	104	108	93	101	144	169	193	209	206	181	146	Accumulated values
			3	91	81	103	107	110	92	104	130	163	190	210	210	183	148	
			0	68	74	97	111	119	101	112	100	142	186	222	225	196	157	
		Moving, 2 Vehicles	4	98	105	109	108	110	95	99	146	186	191	199	199	182	152	Accumulated values
			3	87	97	106	110	112	99	102	129	178	190	205	207	189	157	
			0	71	87	105	119	121	107	109	102	158	189	218	221	198	168	
	10-6-50	Impact, 2 Vehicles	4	86	96	106	109	111	97	102	128	166	189	204	197	—	—	1/4 in. plate - accumulated
			4	89	97	110	109	111	102	107	130	164	191	205	199	—	—	1/2 in. plate - accumulated
			4	91	107	111	111	113	101	110	131	171	193	205	201	—	—	3/4 in. plate - accumulated
		Impact, 2 Vehicles with surcharge	4	89	100	112	117	121	107	116	131	172	200	221	218	—	—	1/4 in. plate - accumulated
			4	92	101	116	117	121	112	121	133	170	202	222	220	—	—	1/2 in. plate - accumulated
			4	94	111	117	119	123	111	124	134	177	205	222	222	—	—	3/4 in. plate - accumulated
SPAN 6	8-9-50	Static, 1 Vehicle	4.5	59	55	49	42	30	17	5	101	121	114	87	53	27	4	Av. of 3 ga. bot. of bot. flange
				-10	-36	-37	-28	-15	-5	-2								Av. of 2 ga. bot. of top flange
	8-10-50		C.L.	24	35	49	62	53	36	28	47	82	106	120	108	79	47	Av. of 3 ga. bot. of bot. flange
				-5	-21	-32	-31	-30	-16	-4								Av. of 2 ga. bot. of top flange
	8-17-50	Moving, 1 Vehicle	4.5	62	59	57	40	31	18	7	100	122	125	82	60	32	10	1 ga. bot. of bot. flange
			C.L.	25	33	46	55	48	35	28	32	67	105	96	104	66	41	

SUMMARIZED DATA (SHEET 2)

		TEST DESCRIPTION	TRUCK POSITION	BEAMS - NUMBERED NORTH TO SOUTH														REMARKS
				STRAINS MICRO IN./IN.							DEFLECTIONS - 001							
				1	2	3	4	5	6	7	1	2	3	4	5	6	7	
SPAN 5 NO DIAPHRAGM BOLTS	8-18-50	Static, 1 Vehicle	4.5	70	68	66	51	32	20	12	122	133	132	96	59	24	3	Av. of 3 bot. ga. - 2 directions
				-22	-40	-38	-20	-4	5	5								Av. of 2 ga. on bot. of top flange
			C.L.	27	35	52	53	42	26	20	37	76	121	131	116	64	34	Av. of 3 bot. ga.
				-5	-19	-32	-47	-39	-28	-10								Av. of 2 ga. on bot. of top flange
	9-27-50	Static, 1 Vehicle	4	76	56	57	46	31	18	12	118	136	133	110	68	35	11	Av. of 2 ga. bot. of bot. flange
				-19	-52	-52	-35	-18	- 9	-1								Av. of 2 ga. bot. of top flange
			3	72	60	64	58	40	24	16	108	128	129	113	71	41	16	Av. of 2 ga. bot. of bot. flange
				-18	-48	-48	-30	-14	-8	-1								Av. of 2 ga. bot. of top flange
			0	62	60	71	72	55	36	29	79	110	129	128	91	55	28	Av. of 2 ga. bot. of bot. flange
				-1	-30	-38	-38	-16	-7	-8								Av. of 2 ga. bot. of top flange
	9-21-50	Moving, 1 Vehicle	4.5	97	82	65	48	36	21	9	135	139	148	101	74	34	15	1 ga. bot. each beam
			C.L.	38	39	60	65	60	41	36	61	84	134	133	132	75	55	
	9-20-50	Moving, 1 Vehicle	4	87	73	69	54	39	20	9	152	145	144	120	71	39	15	1 ga. bot. each beam
			3	82	72	69	58	42	24	12	140	140	145	126	75	45	18	
			0	61	58	69	69	53	33	24	103	118	149	151	99	65	32	
		Impact, 1 Vehicle	4	88	70	65	54	37	24	12	129	131	129	124	65	—	—	1/4 in. plate at mid-span
			4	89	74	72	59	41	24	16	128	138	138	136	72	—	—	1/2 in. plate at mid-span
			4	93	79	75	62	43	27	18	137	140	146	141	78	—	—	3/4 in. plate at mid-span
	9-27-50	Static, 2 Vehicles	4	94	90	105	97	96	78	83	147	170	192	202	189	167	127	Accum. values, bot. of bot. flange
				-23	-49	-50	-54	-48	-49	-17								Accum. values, bot. of top flange
			3	77	79	100	105	94	77	80	134	164	194	209	195	173	129	Accum. values, bot. of bot. flange
				-32	-56	-56	-61	-56	-55	-23								Accum. values, bot. of top flange
			0	70	76	102	110	109	93	100	102	144	192	220	212	187	142	Accum. values, bot of bot. flange
				-7	-39	-56	-67	-55	-49	-10								Accum. values, bot. of top flange
		Moving 2 Vehicles	4	103	104	101	97	96	77	84	165	176	205	210	192	168	130	Accum. values, bot. of bot. flange
			3	92	91	100	102	98	80	86	150	169	205	215	196	175	131	
			0	73	80	101	113	110	89	95	114	152	206	228	210	186	140	
	9-28-50	Impact, 2 Vehicles	4	95	92	100	100	98	79	87	156	179	198	213	199	168	130	1/4 in. plate at mid-span
			4	97	100	108	104	100	81	90	167	188	206	219	204	173	130	1/2 in. plate at mid-span
			4	107	106	114	107	102	85	94	178	199	216	225	207	186	140	3/4 in. plate at mid-span
	9-29-50	Impact, 2 Vehicles with surcharge	4	109	108	120	115	112	95	99	185	200	227	245	228	197	147	3/4 in. plate at mid-span
				103	102	115	109	108	89	94	177	190	218	236	221	201	149	1/2 in. plate at mid-span
				102	97	108	107	107	89	91	176	188	214	233	215	201	151	1/4 in. plate at mid-span
SPAN 5 DIAPHRAGMS SINGLE BOLTED	9-23-50	Static, 1 Vehicle	4.5	83	69	62	46	32	18	6	131	128	129	97	65	32	12	Av. of 2 ga. on bot. of bot. flange
				-24	-29	-36	-17	-5	3	3								Av. of 2 ga. on bot. of top flange
			C.L.	36	36	52	61	53	37	32	48	74	116	126	114	77	49	Av. of 2 ga. on bot. of bot. flange
				0	-20	-36	-38	-32	-17	2								Av. of 2 ga. on bot. of top flange
		Moving, 1 Vehicle	4.5	84	69	57	48	37	20	9	130	135	127	114	70	35	12	1 ga. bot. of bot. flange
			C.L.	38	41	54	49	57	40	37	41	75	116	136	116	78	56	
		Moving, 1 Vehicle	4	71	67	62	49	34	18	10	106	116	112	102	72	40	15	1 ga. bot. of bot. flange
			3	65	64	65	52	37	21	13	63	111	112	106	77	46	19	
			0	49	53	61	59	45	29	22	28	94	108	119	95	62	29	
	9-23-50	Impact, 1 Vehicle	4	94	74	66	51	34	21	14	146	137	132	101	65	—	—	1/4 in. plate
			4	96	76	67	49	32	19	12	145	136	129	100	64	—	—	1/2 in. plate
			4	102	83	75	59	36	23	15	154	145	140	111	75	—	—	3/4 in. plate
			4	97	76	70	53	34	23	11	149	138	134	102	67	—	—	No plate
	9-22-50	Static, 2 Vehicles	4	73	74	87	89	95	79	86	147	171	186	193	189	164	144	Av. of 2 ga. bot of bot. flange
				-21	-43	-48	-49	-46	-34	-31								Av. of 2 ga. bot of top flange
			3	78	87	99	97	100	95	95	132	163	188	199	195	167	148	Av. of 2 ga. bot. of bot. flange
				-11	-32	-37	-42	-39	-37	-28								Av. of 2 ga bot. of top flange
			0	61	74	96	114	113	110	109	100	142	187	210	210	185	159	Av. of 2 ga. bot. of bot. flange
				-2	-19	-34	-46	-41	-34	-28								Av. of 2 ga. bot. of top flange
	9-21-50	Moving, 2 Vehicles	4	93	93	102	99	100	92	86	142	173	191	191	182	154	145	1 ga. bot of beam
			3	82	91	102	105	105	97	92	128	163	192	200	195	165	145	
			0	65	80	98	111	112	104	99	105	146	189	209	211	176	153	
	9-21-50	Impact, 2 Vehicles	4	83	93	102	102	101	94	94	133	164	189	197	181	—	—	1/4 in. plate at mid-span
			4	90	100	109	108	103	96	99	140	176	195	202	189	—	—	1/2 in. plate at mid-span
			4	86	98	107	106	104	98	98	138	173	201	209	196	—	—	3/4 in. plate at mid-span
		Impact, 2 Vehicles	4	92	104	116	114	117	109	112	148	184	211	222	223	—	—	3/4 in. plate at mid-span
SPAN 5 DIAPHRAGMS DOUBLE BOLTED	10-4-50	Static, 1 Vehicle	4	79	71	70	56	39	26	20	145	137	130	109	72	41	16	Av. 2 ga bot. of bot. flange
			4	-30	-49	-48	-27	-10	-2	3								Av. 2 ga bot. of top flange
			3	69	64	64	57	36	28	15	135	133	131	116	78	47	19	Av. 2 ga. bot. of bot. flange
				-31	-47	-48	-32	-14	-2	-1								Av. 2 ga. bot. of top flange
			0	49	51	59	56	46	29	23	99	109	128	128	92	64	34	Av. 2 ga. bot of bot. flange
				-21	-40	-49	-24	-12	-1	—								Av. 2 ga. bot. of top flange
		Moving, 1 Vehicle	4	83	76	71	53	41	25	10	158	152	131	106	67	40	16	1 ga. bot. of bot. flange
			3	77	74	71	58	43	25	14	133	150	140	113	76	47	20	
			0	57	61	69	65	51	32	25	100	129	134	125	91	60	30	
		Impact, 1 Vehicle	4	77	72	69	55	40	23	14	132	126	124	104	68	—	—	1/4 in. plate at mid-span
			4	79	65	74	56	42	25	16	134	129	128	113	74	—	—	1/2 in. plate at mid-span
			4	89	82	84	66	48	29	21	147	144	143	127	71	—	—	3/4 in. plate at mid-span (accum. value)
	10-3-50	Static, 2 Vehicles	4	99	89	102	105	100	89	88	155	175	192	208	194	171	159	Av. 2 ga. bot. of bot. flange
				-30	-52	-56	-54	-62	-59	-39								Av. 2 ga. bot. of top flange
			3	97	90	110	112	109	97	88	141	166	191	213	196	174	162	Av 2 ga. bot of bot. flange
				-18	-44	-49	-55	-58	-51	-32								Av. 2 ga. bot. of top flange
			0	77	88	110	123	122	111	104	109	147	188	225	212	186	170	Av. 2 ga. bot. of bot. flange
				-2	-27	-42	-58	-52	-54	-31								Av. 2 ga. bot. of top flange
		Moving, 2 Vehicles	4	104	101	108	100	109	94	89	177	191	203	227	193	175	160	1 ga. bot. of bot. flange (accum)
			3	97	99	108	104	113	98	91	163	188	205	235	196	176	160	
			0	77	85	104	108	120	106	97	126	163	198	243	204	185	164	
	10-6-50	Impact, 2 Vehicles	4	97	104	117	107	117	101	98	175	186	214	247	209	—	—	3/4 in. plate at mid-span
			4	93	99	113	104	115	100	94	161	173	205	237	204	—	—	1/2 in. plate at mid-span
			4	93	97	106	103	113	99	92	159	169	201	233	204	—	—	1/4 in. plate at mid-span
		Impact, 2 Vehicles with surcharge	4	96	105	113	124	122	111	102	163	178	210	222	223	—	—	1/4 in. plate at mid-span
			4	97	106	118	122	123	111	104	166	183	213	227	227	—	—	1/2 in. plate at mid-span
			4	104	116	126	131	129	117	111	181	196	225	236	234	—	—	3/4 in plate at mid-span

oration of the statement that diaphragms play a minor role in the lateral distribution of load.

8 Slippage measurements between deck and beam indicate bond breaks in spans without shear developer and composite action in the span with the shear developer. It is quite possible, however, that there could be considerable bond between deck and beam near the center of the spans. The limits of this area of effective bond were not measured.

Discussion of Results, Conclusions

Detailed study of the test results indicates that in general it is apparent that the type or number of diaphragms are not of great importance in lateral distribution of load. While it is true that in most test cases more lateral distribution was obtained with stiffer diaphragms, the amounts were small, and in some instances, as previously mentioned, the effect was just the opposite of that expected. The latter effect is undoubtedly explained by the fact that different amounts of partial composite action were obtained in the various tests, and in general, as expected, there was a gradual destruction of the partial composite action in the later tests

The change in the amount of composite action in the tests suggests that it would be wise in future tests to make an effort to reduce the composite action to a minimum, if possible, by means of heavy loadings and impacts. That some residual composite action, whether due to bond or friction, would remain can be predicted by results reported in the magazine *Civil Engineering*, Vol 21, No 7, of July 1951 of tests on the Skunk River Bridge in Iowa (see previous paper) These tests were made on a bridge that had been subjected to heavy traffic during its 28 years of service, and still showed partial composite action.

The failure of measured stresses to reach more than about two thirds of predicted values, even when thickened slab and partial composite action were taken into account, can be explained by the stiffening effect of the heavy safety curb and the fact that the 12-in -wide beam flanges, partially encased in the slab, introduce restraining moments at each beam It would be impossible from the test data available to evaluate each effect individually. Certainly, it can be predicted that in a wider bridge the effect of the curbs would be lessened on the beams near the center of the bridge. In the matter of the restraining effect of the wide beam flanges, it is possible that some reduction of this effect would be obtained by the heavy loading tests suggested above.

Of particular interest are the excellent results obtained on the span using the shear developers. The tests on slippage and stress and deflection indicate full composite action was obtained. From a general appraisal of the test results, it would appear that one possibility for future savings in bridge design would be to take advantage of the partial composite action known to exist and use less conservative methods in designing shear developers. Of course, further testing would be in order before taking such a step Certainly, the evidence from this test indicates that there is just cause for considering a revision of the AASHO specifications regarding distribution of loads to stringers.

In practically all cases where the specific objectives of the test program were not achieved, valuable information for future test projects was obtained in the matter of instrumentation and test procedure

Load Distribution between Girders on San Leandro Creek Bridge

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● IN THE spring of 1950, the Institute of Transportation and Traffic Engineering, University of California, in cooperation with the Bureau of Public Roads and the California Division of Highways, initiated an extensive program of strain and deflection measurements on a state freeway bridge crossing San Leandro Creek in Oakland. One of the main subjects under investigation was the distribution of the load between the girders. It is the purpose of this report to discuss briefly some of the results of the tests concerning load distribution as affected by (1) composite action of the concrete slab with the steel girders, (2) longitudinal and transverse position of the load, and (3) steel diaphragms.

These test results are compared with theoretical analysis and with AASHO specifications

Figure 1 indicates the framing of the tests spans and the locations of the principal gage stations. The bridge is composed of an 8-in. concrete deck with sidewalks, supported by three longitudinal steel girders on 11-ft. centers. There are two parallel structures of two lanes each; each structure having 23 spans. Every third span consists of a suspended span, hinge-supported on cantilever arms which are continuous over two spans on either side Diaphragms were placed at the quarter points and center of the continuous spans and near the hinges and center of the suspended spans. Two representative spans on one of the structures, Spans 19 and 20, were chosen for test; 19 being a typical suspended span, and 20 a typical continuous span.

The framing plan indicates the three supporting girders, designated as right, middle, and left The principal gage locations, designated as 19 5, 20.0, 20.5, and 21 0, are indicated by dotted lines.

Figure 2 shows the steel framing in the test spans and the installation of the numerous wires connecting the gages to the recording equipment About 350 SR-4 strain gages, 16 Carlson strain meters, and 8 induction-type deflectometers were mounted on the test spans. It will be noted that the exterior girders rest on the columns and the middle girder is supported by the cross-beams The hinge plates and diaphragms also appear in the photograph

Figure 3 shows the completed bridge with the Euclid test vehicle loaded to a gross weight of 67,000 lb. with sand and steel ingots The rear axle carried a load of 50,000 lb. and the front axle 17,000 lb The spacing between axles is 13 ft.

Figure 4 shows the five transverse positions of the test vehicle designated as left, half-left, center, half-right, and right. The locations of the SR-4 gages on the girders and the Carlson strain meters in the concrete are also shown.

In order to estimate the effect of composite action, the concrete deck was assumed to be divided into three sections; each section was considered as belonging to one girder. On the basis of composite action, assuming $n = 10$, it will be noted that the moments of inertia are three to four times larger than for the steel alone. The left girder has the highest composite moment of inertia because the slab was made thicker on that side to provide for transverse drainage.

In order to determine whether composite action existed, strain measurements for the three girders at Station 19.5 were plotted. These measurements were taken from oscillograph recordings of strain when the rear axle of the slowly moving vehicle was at midspan. Figure 5 shows the strains for each girder for two transverse positions of the load, the positions being those which produced the largest strains in the girder. It will be noticed that for each of the loading conditions, the four values of strain lie practically on a straight line

The theoretical neutral axes were computed on the basis of full composite action assuming the sections shown in previous Figure 4 It will be noted that the experimental neutral axes coincide closely with the theoretical axes for all three girders For the middle girder a strain diagram assuming no composite action has been added for comparative illustration. This shows a bottom flange tensile strain about 70 percent higher than the observed strain On the top flange, the assumption of no composite action resulted in high compression, whereas the observed strain was almost zero, as should be the case for full composite action. Since no shear connectors were

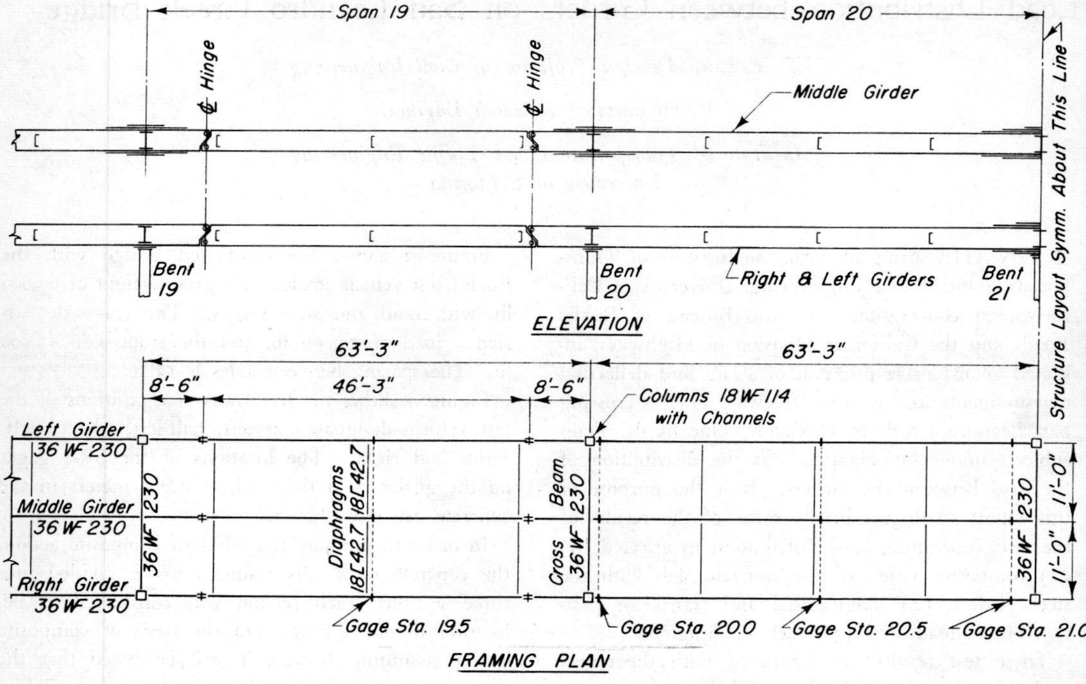


Figure 1. Steel layout of test spans.

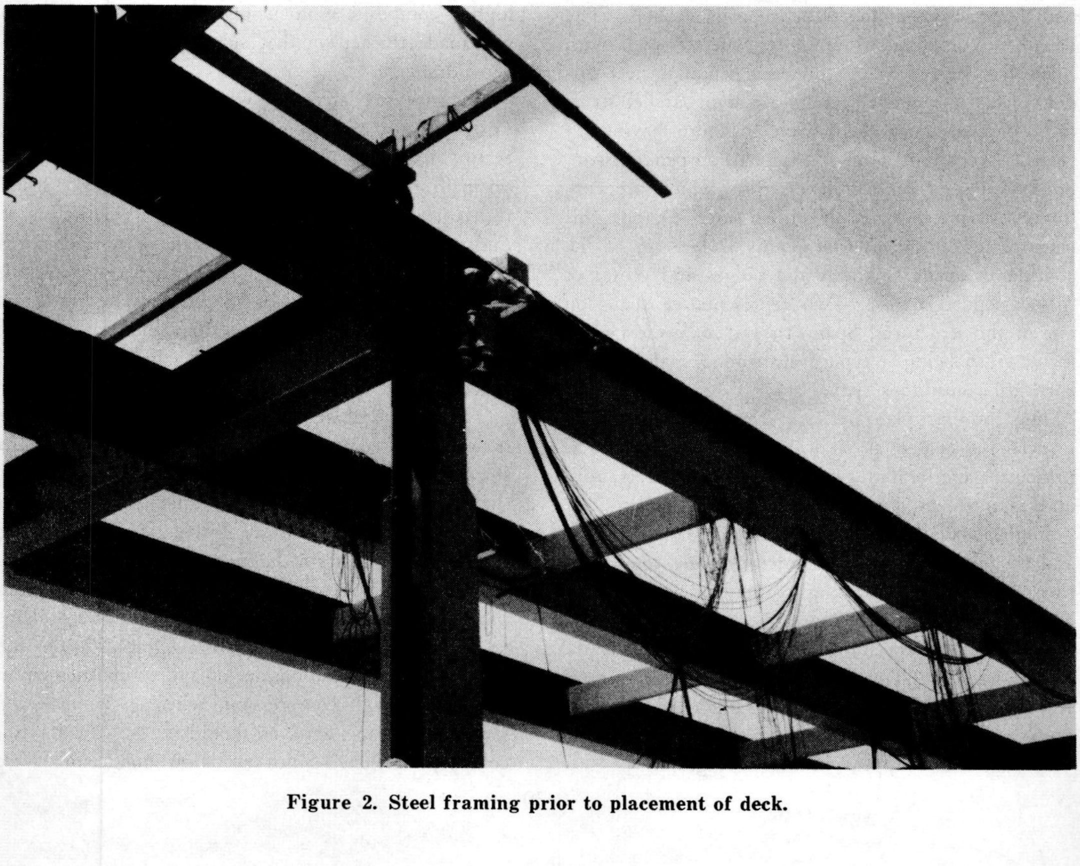


Figure 2. Steel framing prior to placement of deck.

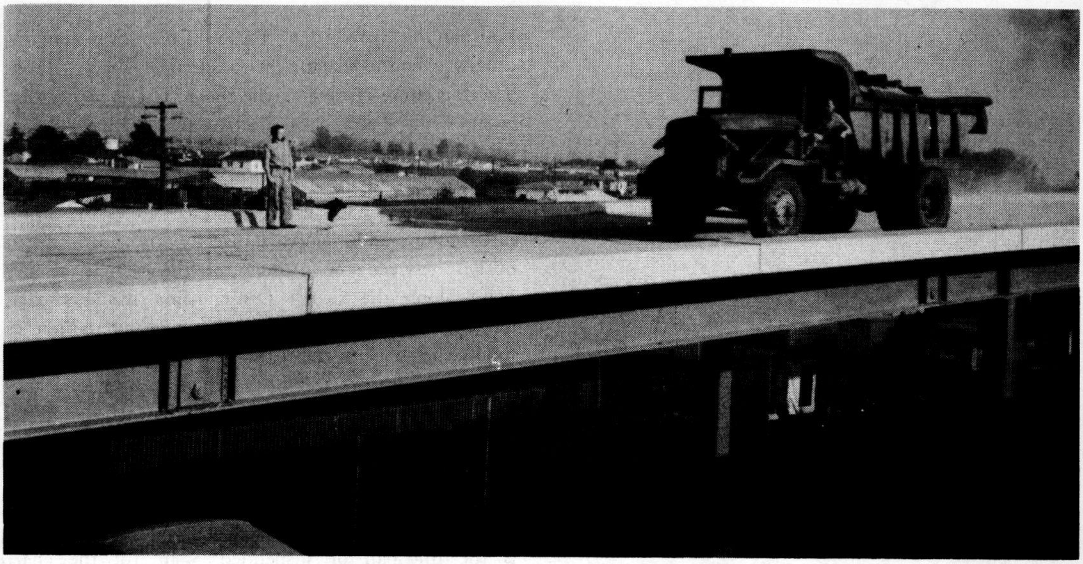


Figure 3. View of 67,000-lb. Euclid test vehicle on completed bridge.

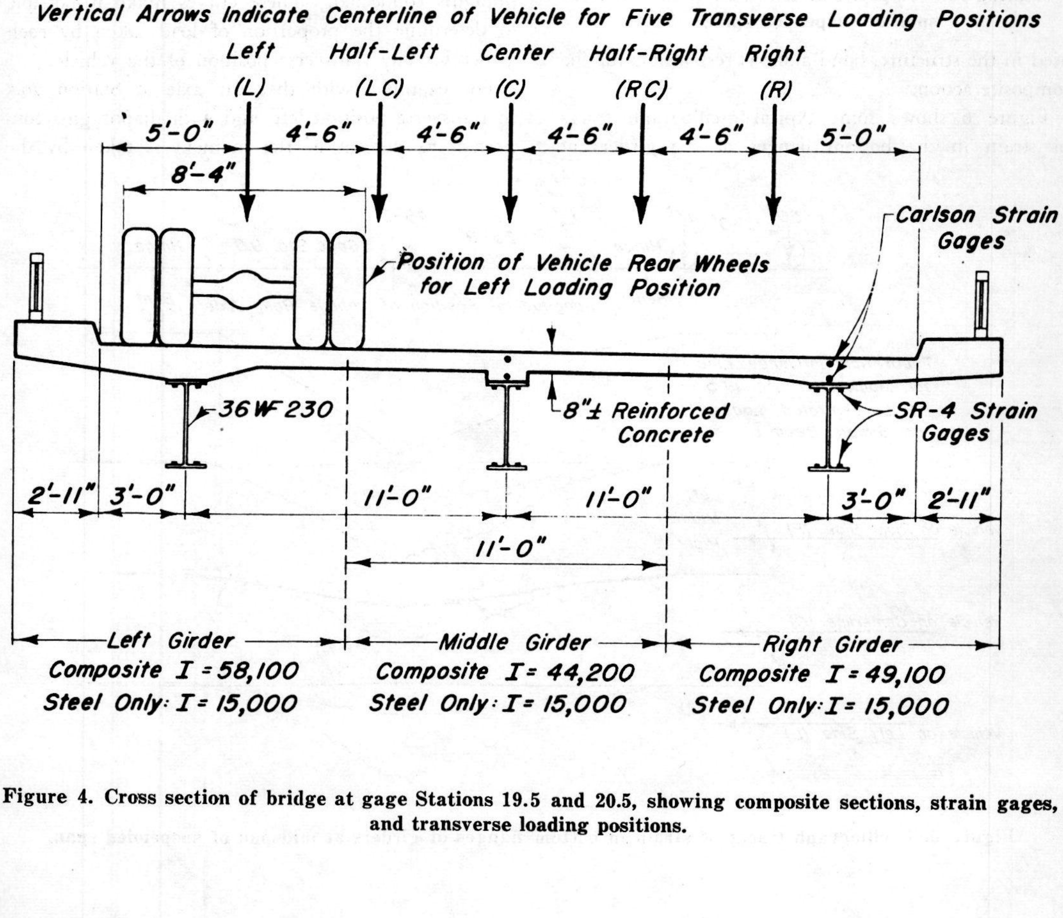


Figure 4. Cross section of bridge at gage Stations 19.5 and 20.5, showing composite sections, strain gages, and transverse loading positions.

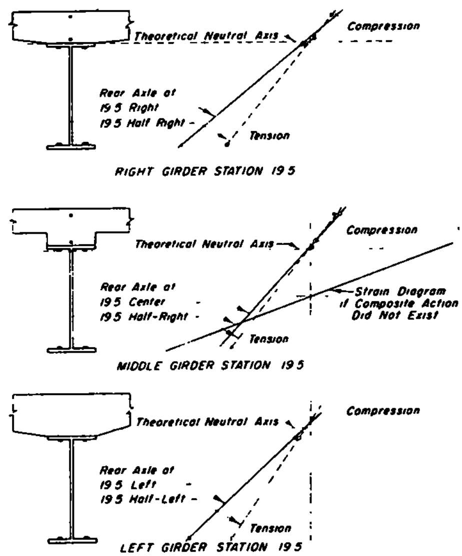


Figure 5. Representative cross sections of strain, indicating full composite action in all girders at midspan of suspended span.

used in the structure, bond alone is responsible for the composite action.

Figure 6 shows some typical oscillograph traces of strain in the bottom flanges of the girders at midspan of Span 19 as the vehicle moves longitudinally over the structure at a speed of about 3 mph The top curve represents the theoretical influence line of moment or strain for the two axle vehicle. Below this are the recorded traces of strain for each of the three girders in three transverse positions, right, center, and left Each group of traces gives the strain distribution and hence indirectly the load distribution between the girders for the vehicle at any point along the span. Disregarding minor oscillations, it will be noted that all the experimental curves follow the shape of the theoretical curve rather closely.

Figure 7 shows the distribution of the total moment among the three girders at two sections of the bridge, when the load is placed in various transverse positions The chart on the left hand side of the figure shows the influence lines for the girders when the rear axle of the vehicle is at Station 19 5, which is the midspan of suspended Span 19, the chart on the right indicates similar data when the rear axle is at Station 20 5, the midspan of continuous Span 20. The solid lines show the distribution with the diaphragms removed, and the dotted lines with diaphragms connected These curves make it possible to determine the proportion of load taken by each girder for any transverse position of the vehicle

For example, with the rear axle at Station 20.5 in transverse position left, and with diaphragms connected, 74 percent of the moment is taken by the

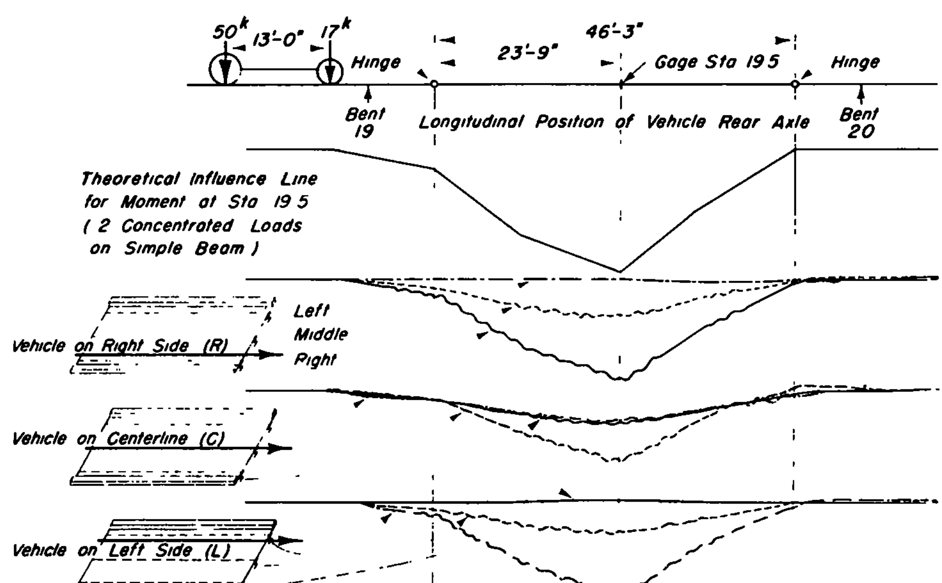


Figure 6. Oscillograph traces of strain in bottom flanges of girders at midspan of suspended span.

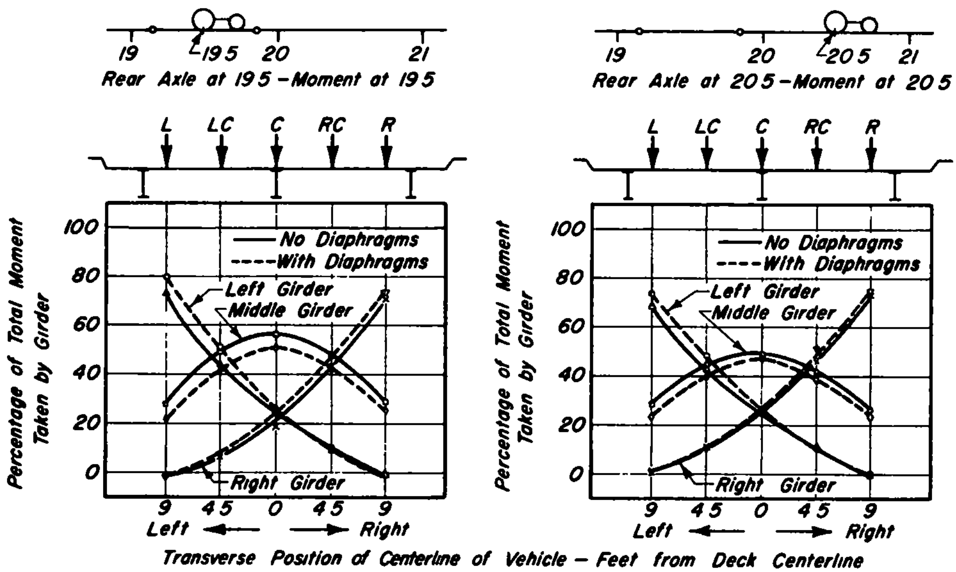


Figure 7. Experimental influence line showing percentage of total moment at section taken by each girder as transverse position varies.

left girder, 25 percent by the middle girder, and 1 percent by the right girder

It will be noted that the effect of the diaphragms on load distribution is rather small This is probably due to the fact that in this bridge the diaphragms are rather flexible compared to the transverse section of the concrete slab and the large composite section of the longitudinal girders.

In general the influence lines for the girders in the two spans are similar. However note that when the load is over the middle girder in the continuous span, more of the moment is distributed to exterior girders than is the case for the suspended span.

Figure 8 shows a comparison of theoretical and experimental distribution of load between girders when the vehicle is on suspended Span 19 In the chart on the left, experimental values of percentage of total moment taken by the middle girder for different transverse positions of the load are shown by the solid line. The dotted line represents the theoretical percentages computed by the use of Jensen's formulas (*Bulletin No 303*, University of Illinois Engineering Experiment Station). These formulas are not fully applicable to this bridge since the theory assumes a slab supported on three simple girders resting on unyielding end supports In our case, due to the deflections of the supporting cantilever girders and the crossbeams, the hinges settle differentially Thus there exist differential end sags among the girders. Jensen's formulas further assume that the slab is simply supported along the exterior girders. In the actual structure some torsional restraint is evidently exerted on the slab, producing partial fixity at the edges Computations by approximate methods have shown that allowance for both of these conditions will substantially increase the distribution of moment between girders. Points a, b, and c on the chart indicate the change in the peak of the middle girder influence line when (a), end sag, (b), half-fixity and sag, and (c), full fixity and sag, are taken into account It will be noted that, assuming half-fixity (Point b), the theoretical load distribution agrees closely with the experimental data This amount of torsional restraint is probably contributed by the expansion dams and diaphragms at the ends of the suspended span No confirmation of this idea has as yet been made.

Figure 9 shows experimental values of load distribution and stresses compared with values computed by the AASHO method using the Euclid vehicle in place of the standard AASHO truck. With the heavy axle at Station 19.5, transverse vehicle positions causing the largest moments in each of the three girders at this station are shown. For example, without the diaphragms, 0.73 of the total moment caused by the vehicle in the left lane, and 0.07 of that in the right lane are taken by the left girder, resulting in a total maximum moment of 0.80. With diaphragms the

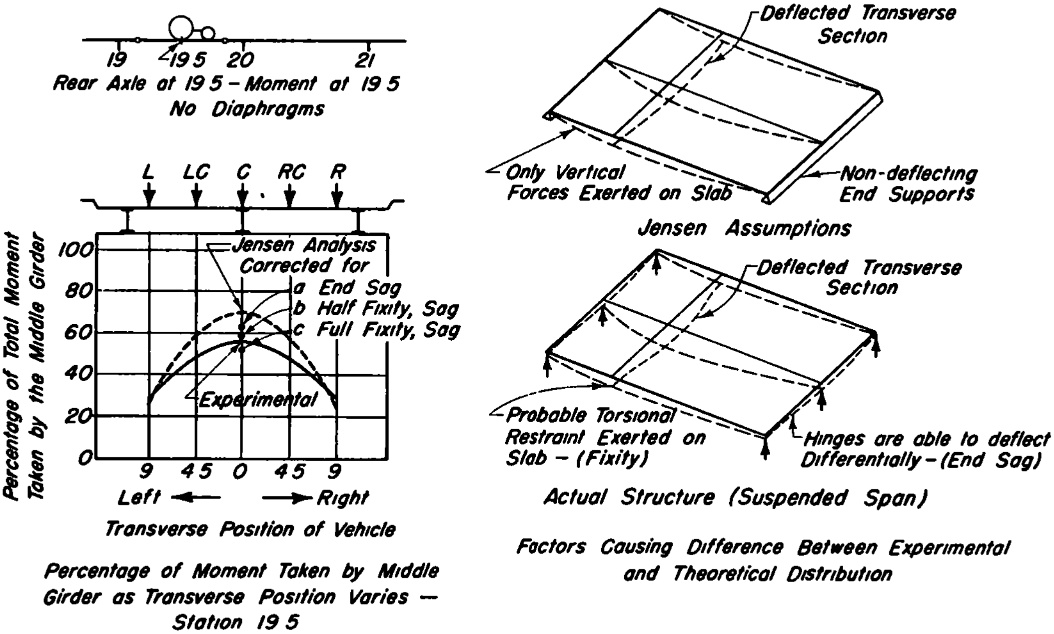
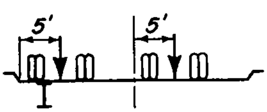
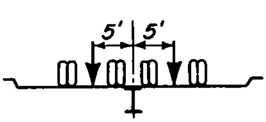
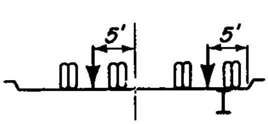


Figure 8. Distribution of moment between girders: Comparison of experimental results with theoretical analysis.

	Portion of Total Moment Taken by Girder for Indicated Vehicle Positions		Total No of Truck Loads Experimental	AASHO	Max. Steel Stress - p s i Experimental*	AASHO**
						
No Diaphragms	73	07	80	82	4,200	7,900
With Diaphragms	.80	.07	87		4,600	
			LEFT GIRDER — STATION 19 5			
						
No Diaphragms	47	47	94	1.00	5,800	9,600
With Diaphragms	40	41	.81		5,000	
			MIDDLE GIRDER — STATION 19 5			
						
No Diaphragms	05	.71	76	82	4,400	7,900
With Diaphragms	.06	74	80		4,700	
			RIGHT GIRDER — STATION 19 5			

* Computed by experimental distribution and composite section modulus
**Computed by AASHO distribution and steel section modulus.

Figure 9. Comparison of experimental maximum truck loading and stresses with AASHO specifications, mid-span of suspended span.

total is 0.87, whereas the AASHO method, assuming simple spans transversely and making no allowance for the diaphragms, yields 0.82. Likewise for the middle girder, the experimental values are 0.94 and 0.81 respectively, while the AASHO value is 1.00.* It will be noted that for this bridge the AASHO values appear to be conservative for the middle girder and agree fairly closely with the experimental values for the exterior girders.

On the right hand side of the table, the experimental values of maximum stress, taking into account both the effect of load distribution and of composite action, are compared with stresses computed by the AASHO method which does not consider composite action. The latter stresses range between 8,000 and 10,000 psi., while the experimental values are between 4,000 and 6,000 psi., or 40 to 50 percent lower.

* Factor of 1.00 is obtained as a result of AASHO Bridge Specification T 15(50), tentative revision adopted December 1950. 1949 Specification 3.3.1 resulted in a factor of 1.09 for the interior girder.

Field work on this project has been virtually completed. It is hoped that a complete report will be available for distribution early in 1953. The project was planned and carried out under the guidance of an advisory committee consisting of R. Archibald and H. R. Angwin of the U. S. Bureau of Public Roads, S. Mitchell, T. E. Stanton, and F. N. Hveem of the California Division of Highways, N. C. Raab of the Division of San Francisco Bay Toll Crossings, H. E. Davis, H. D. Eberhart, R. A. Moyer, T. Y. Lin and R. Horonjeff of the University of California, and G. B. Woodruff, consulting structural engineer, San Francisco. Collection of the basic data was made possible through the cooperation of the Bridge Department of the California Division of Highways, especially the resident engineers, W. C. Names and J. N. Perry, and their staffs. On the Institute staff, R. W. Clough, V. A. Plumb, and C. F. Scheffey contributed a great deal toward the success of the project.

Load Distribution on Highway Bridges Having Adequate Transverse Diaphragms

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● THE BRIDGE specifications of the American Association of State Highway Officials are now the design standard for highway bridges in the United States and are also the most widely used specifications in the other countries of the western hemisphere. These specifications have an empirical distribution of load to interior longitudinal girders, depending on the type of deck and the girder spacing For concrete decks and concrete girders the fractional wheel load applied to each girder is the girder spacing divided by 5 0. For exterior girders the live load is assumed to be the reaction from the panel of deck between the exterior and adjacent interior girders from the wheel load, regarding the deck panel as a simple beam. No consideration in the load distribution is given to the value of transverse-diaphragm beams connecting the longitudinal girders In the usual concrete girder-span diaphragm, beams are provided which have a stiffness comparable to the longitudinal girders These must have a very considerable effect on the transfer of load from one girder to another.

The AASHO specification results in a stronger interior girder than the exterior girders. In 1933 the Oregon State Highway Department made an investigation of a simple-span steel-girder bridge having a concrete deck The primary purpose was to check the composite action of the deck and girders, but it also allowed a comparison of the girder deflections under varying load positions. The investigation indicated that the exterior girders took as much, if not more, load than the interior girders. This led to the adoption by Oregon of a specification whereby the total assumed load on the span was divided equally between all girders when adequate diaphragm beams were provided.

In 1948 the state had occasion to build a simple-span concrete bridge over Oneonta Creek on the Columbia River Highway east of Portland. Figure 1 shows the structure loaded with two axles at midspan. This structure was selected for a full-size investigation to determine the load distribution to girders having an adequate diaphragm system. The investigational feature of the project was a cooperative undertaking by the U. S. Bureau of Public Roads and the Oregon State Highway Commission.

The structure has a span length of 48 ft. center to center of bearings. The east ends of the girders are supported on a bearing permitting angular rotation, but no horizontal movement. The west ends of the girders have 5 1/4-in. rockers permitting both rotation and longitudinal movement The alignment across the bridge is a tangent, the abutments are at right angles to the centerline, and the grade is level. The structure has a 26-ft -wide roadway with a 3-ft.-6-in. sidewalk on each side There are four 16 1/2- by 51-in. longitudinal girders at 7-ft -1/2-in centers with an 8-by-49-in diaphragm beam at midspan Beam and girder depths include the 6 1/2-in deck

Theoretical Distribution of Loads

The structure under discussion consists of four longitudinal girders connected at midspan by a diaphragm having a stiffness approximately equal to the girders The problem of distribution of load to the several girders is susceptible of analysis by a simple, although rather tedious, procedure provided certain assumptions are made These assumptions are (1) the slab acts as simple beams between girders in transferring wheel loads to girders and does not enter into the transfer of load from one girder to another and (2) the girders are not stiff enough in torsion to produce appreciable restraining moments at their connection to the diaphragms Both of these assumptions are open to question The slab is a continuous beam supported by all girders and plays some part in the transference of load In the usual concrete structure, however, the diaphragm depth is at least six times the slab depth and for equal widths is more than 200 times as stiff The most effective portion of the slab for load transference is in the area where the greatest deflection takes place The slab toward the girder support can contribute but little The contribution of the slab, while perhaps not a negligible factor, is probably minor. The torsional rigidity of the girder contributes in some measure to the stiffness of the diaphragm system For the very small angular change, this effect is probably a minor factor Both of these assump-

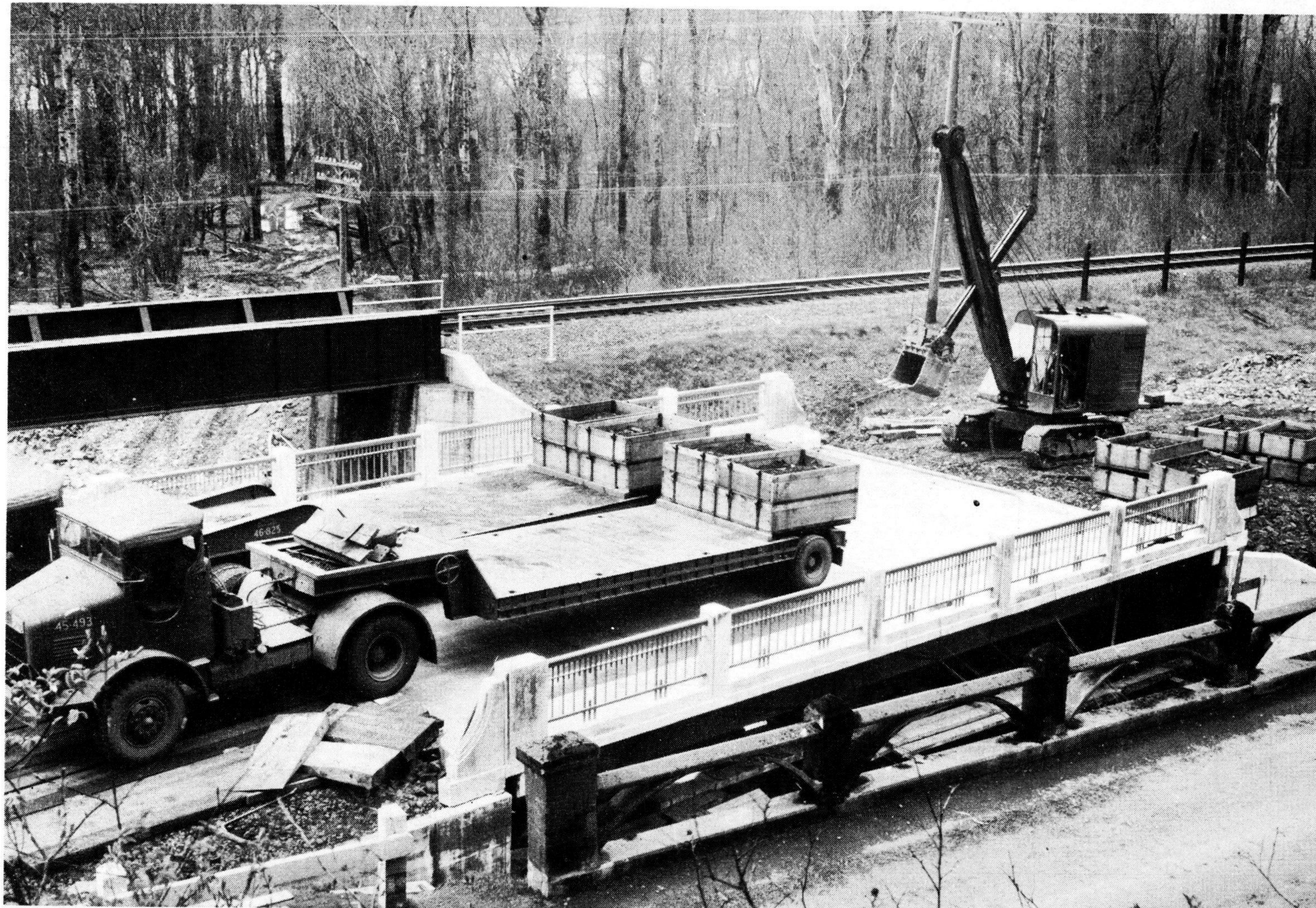

Figure 1.

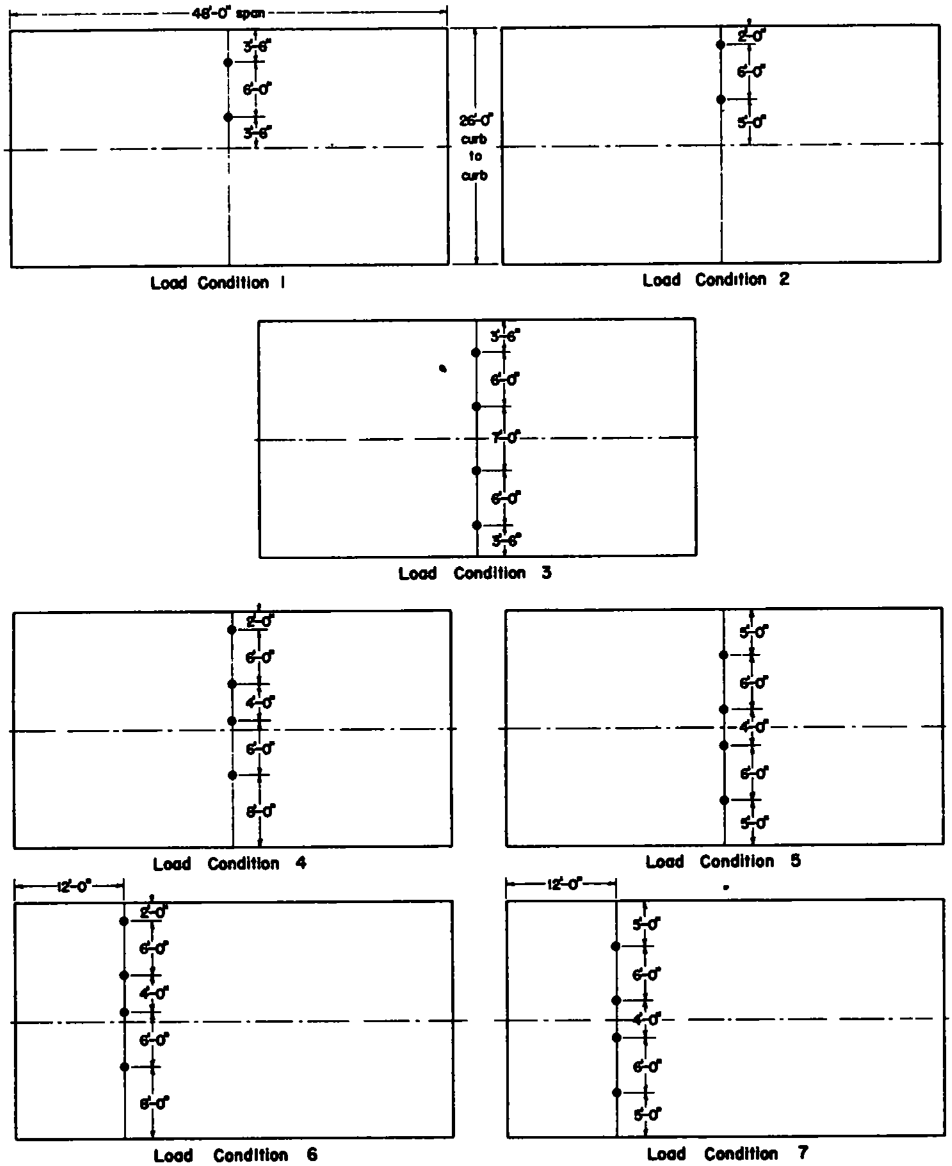

Figure 2. Locations of test loads.

tions are on the conservative side, and the actual distribution of load should be more than shown by the computations.

The method of computing the load transfer is the work of George S. Vincent, senior highway bridge engineer, Bureau of Public Roads. The wheel loads are distributed to the adjacent girders by the slab as though it were a simple beam. These loads deflect the girders and a part of the load is transferred to the diaphragms at their intersection with the girders. Since the diaphragm is in static equilibrium, the load transference at the outside girders may be regarded as reactions and at the interior girders as loads, and the deflection curve of the diaphragm set up in terms of the unknown load-transfer coefficient. The number of equations from this relationship is one less than the number of spans between girders, or two less than the number of intersections of girders and diaphragm Two additional equations are from the summation of vertical forces and the summation of moments These equations are sufficient for the determination of the unknown-load transfer coefficients

The load transfer depends on the relative stiffness of the members. Whether the concrete acts with the steel in resisting tension stresses (uncracked section) and whether the curbs and sidewalks act with the exterior girders have considerable effect. In the Oneonta Creek Bridge the testing was done before the bridge was opened to general traffic, and the test results indicated that the concrete was effective in tension and that the curbs and sidewalks acted with the exterior girders in resisting stress

For the Oneonta Creek Bridge with four equal beams at equal spacings and a single diaphragm at mid-span, the four simultaneous equations in the unknown load-transfer coefficients are

$$\frac{2D_1}{3}-(8R+1)D_2-7RD_3+\frac{D_4}{3}=\frac{2P_1}{3}-P_2+\frac{P_4}{3}$$

$$\frac{D_1}{3}-7RD_2-(8R+1)D_3+\frac{2D_4}{3}=\frac{P_1}{3}-P_3+\frac{2P_4}{3}$$

$$D_1+D_2+D_3+D_4=0$$

$$D_2+2D_3+3D_4=0$$

where D_1, D_2, D_3 and D_4 are the load transfer coefficients at the intersection of the diaphragm with each girder, P_1, P_2, P_3 and P_4 are the loads applied to each girder, and R is a ratio of the stiffness of the diaphragm to the stiffness of the girders. These four equations are sufficient for the determination of the load transfer coefficients. The derivation of the equa-

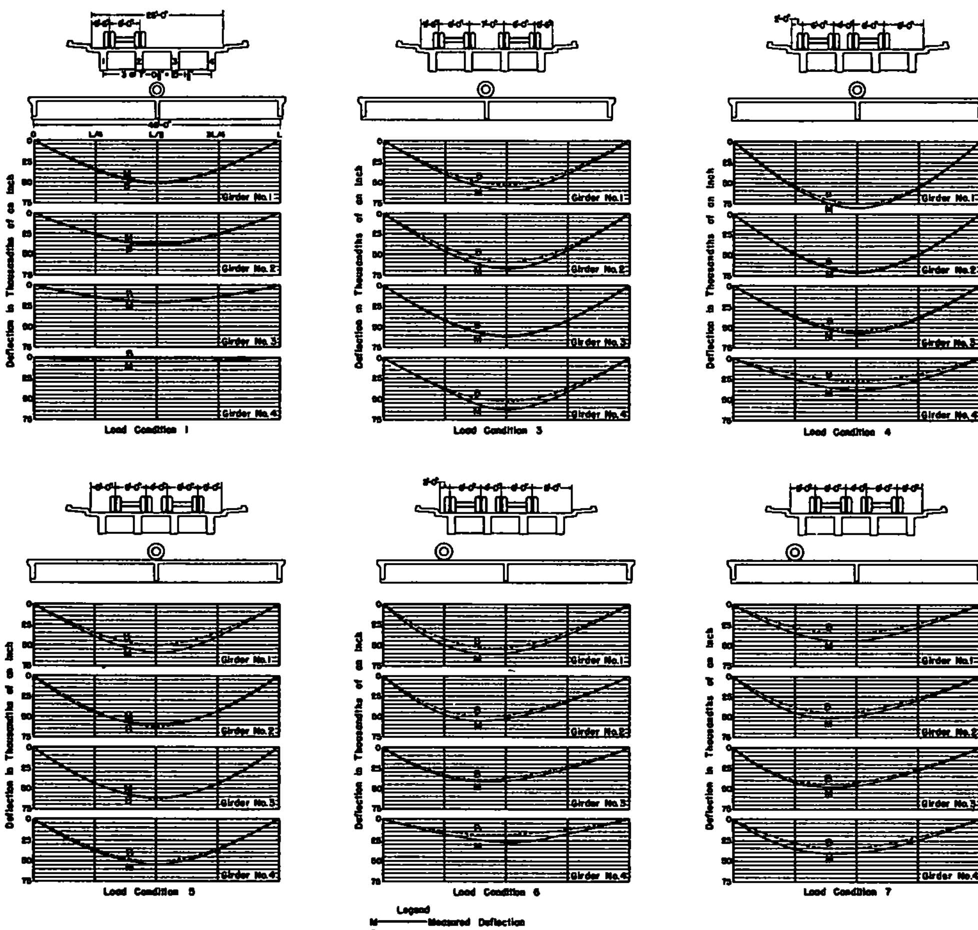

Figure 3. Deflection of girders.

tions is given in an appendix to this paper.

Instrumentation

The test installation was designed to furnish information on the problem from three approaches Gauge points were set in the bottoms of all girders at midspan and at quarter points The deflections under load were measured with inside micrometers from fixed points on the falsework below the girders

SR-4 strain gauges were installed on the metal reinforcing bars at points where knowledge of the stress might be informative. These points were as follows On the two exterior bars in the lower center of sidewalk at midspan On longitudinal bars in the face of the roadway curbs at midspan The SR-4 gauges were placed in pairs on opposite sides of the bars and connected in series to correct for any eccentricity of loading The gauges were waterproofed with adhesive tape and petrosene wax. The gauges were placed on the bars and enclosed in a sheet metal housing so that no concrete came in direct contact with the gauge. Lead wires were brought from the gauges to a central station where all readings were made

The reactions under each end of each girder were measured by individual weighing devices. These

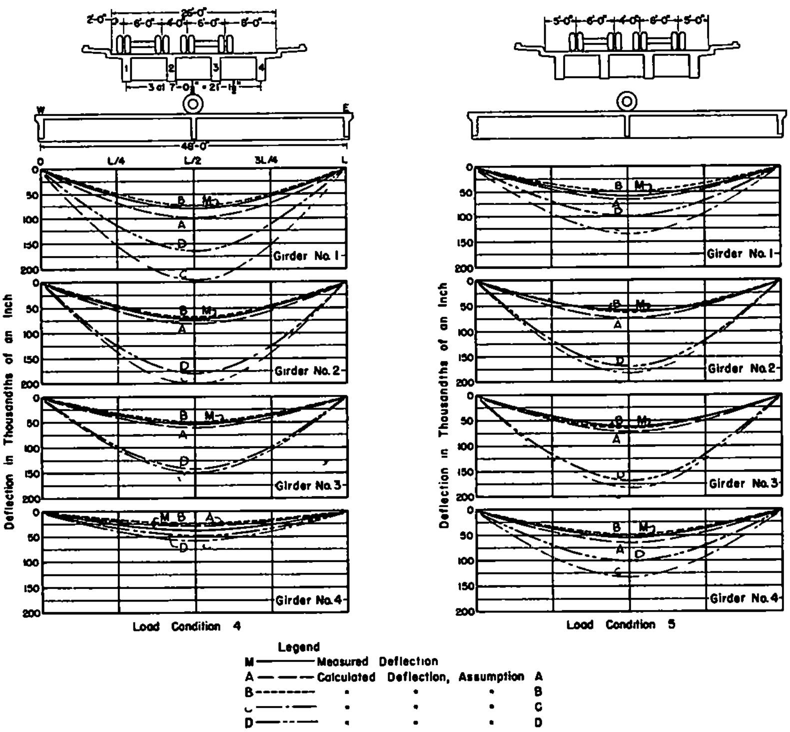

Figure 4. Deflection of girders, load Conditions 4 and 5.

layer of the main tension steel in the bottom of each girder at midspan and at the quarter points On the tension steel in the bottom of the diaphragm beam at the point of intersection with each main girder. On longitudinal bars in the slab above each girder at midspan and at quarter points On longitudinal bars in the top of the deck slab midway between girders at midspan and at quarter points. On five transverse bottom deck bars symmetrically placed about one quarter point. On longitudinal bars in consisted of a short section of an aluminum alloy cylinder with SR-4 gauges at each quadrant. The opposite gauges were connected in series to correct for eccentricity The aluminum cylinders were calibrated on a testing machine and stress-strain curves plotted for each cylinder The girder loads were applied to the cylinders through a ball joint to decrease eccentricity to the minimum The cylinders were supported on the abutments by parallel plates and leveling screws to level the support and to equalize the dead load on the girders prior to loading for deflection and stress measurements. At the conclusion of the test program the cylinders were replaced with bearing plates and rockers.

Loading

The loads were single-axle, flat-bed trailers towed by tractors with a spacing of 25 ft between the rear tractor axle and the trailer axle. This arrangement allowed the trailer axle loads to be placed at any point on the span with the tractor loads off the span. The trailers had been especially built for heavy hauling, and the wheel spacing on the axles did not match the spacing usually assumed for bridge design. The axle load was therefore applied by a beam supported on the deck by blocks the size of a loaded tire imprint and at the conventional spacing Two loaded trailers were used to produce the desired loading arrangements

Seven load arrangements were used These arrangements are shown in Figure 2. In the first two a single trailer was used, in one instance with the trailer in the normal position in one traffic lane, and then with the trailer placed as close as practical to one curb Three arrangements were used with the two trailer axles at midspan, with each axle in the normal position in its traffic lane, with the two axles placed as nearly as practical to one curb, and with the two axles symmetrically placed about the center line and as near together as was practical. Two arrangements with both trailer axles at a quarter point were used. In one arrangement the axles were placed as near to one curb as practical and in the second the two axles were symmetrically placed about the bridge

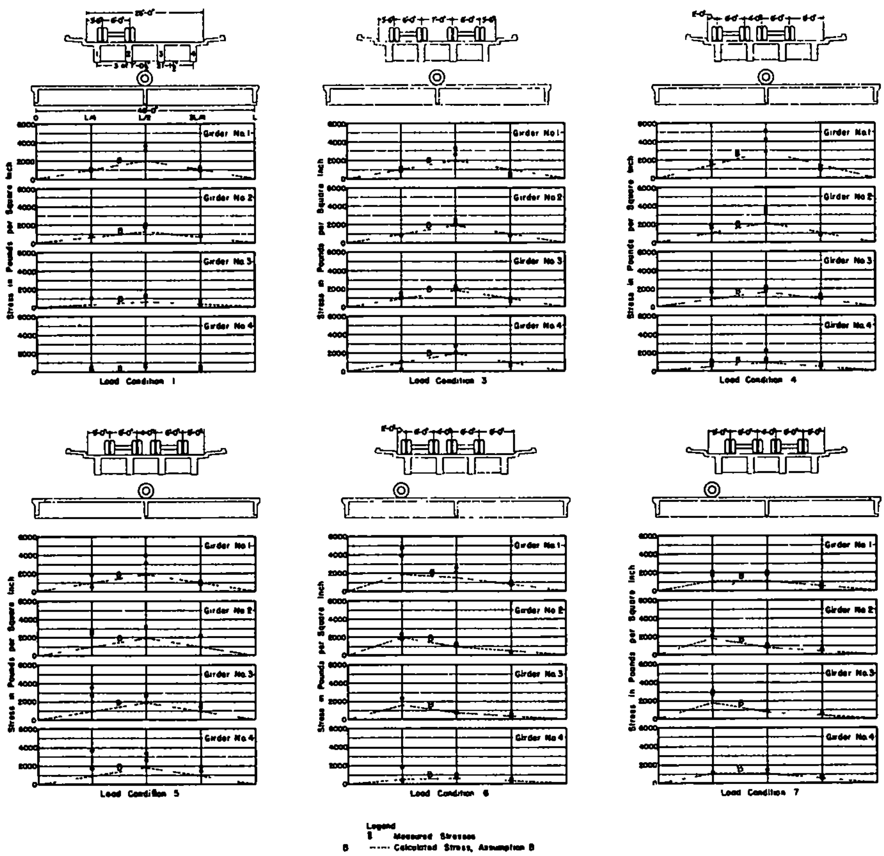

Figure 5. Stress in girder tension steel.

center line and as close together as practical operation would permit. All loadings were made with 48,000 lb on each axle which applied loads through the blocks corresponding to a 24,000-lb. wheel load. These loads are, of course, more than the structure was designed for, but were chosen to give deflections and stresses that could be easily measured.

Test Data and Analysis

As mentioned before, the assumptions as to whether the concrete acts as a cracked or an uncracked section and as to the effectiveness of the sidewalks and curbs in acting with the outside girders play a large part in the calculated values for both deflection and stress. The testing at the Oneonta Creek bridge was done immediately after the completion of the structure and before it was opened to traffic. As would be expected, the structure acted as though the concrete were acting with the steel in resisting tension stresses The test results also showed that the sidewalks and curbs acted with the exterior girders.

Calculations for deflection and stress were made for all load positions under each of the following assumptions· (A) uncracked concrete section without considering the sidewalks or curbs as effective, (B) uncracked concrete section with sidewalks and curb, (C) cracked concrete section without considering the sidewalks or curbs as effective, and (D) cracked concrete section with sidewalks and curbs.

Since the condition of the structure at the time of test and the test results themselves indicate that the structure was acting as uncracked concrete with the sidewalks and curbs effective, the comparison between calculated deflection and stress and field measurements is made under Assumption B except for load Positions 4 and 5 where all four assumptions are shown. Eventually the concrete on the tension side of the girders will crack and no longer act in tension, and the deflection and stress will approach those of Assumption D.

Deflections

The calculated deflections for Assumption B and the measured deflections for all load conditions except load Condition 2 are shown in Figure 3. The instrumentation failed on load Condition 2, which is for a single-axle load near one curb. Since this is not a critical load condition, this test was not repeated. It will be noted that there is a remarkable correspondence between the measured deflections and those calculated under Assumption B, the uncracked concrete section. Attention is particularly called to the graph showing load Condition 4. With two axles placed as near to one curb as is practical, this loading produces the greatest deflection and stress. The measured deflections and the calculated deflections for the uncracked section are in good agreement. In general, the measured deflections are slightly more than should occur if the concrete were entirely effective. A very small amount of initial cracking could easily account for the differences.

Figure 4 shows the deflections of the four girders under load Conditions 4 and 5 and under all four assumptions. The measured and calculated deflections are given in Table 1.

TABLE 1

DEFLECTIONS—LOAD CONDITION 4

Position on Bridge	Girder Number	Measured	DEFLECTIONS Calculated A	B	C	D
		in	in	in	in	in
L/4	1	0 054	0 069	0 053	0 153	0 114
L/4	2	049	057	048	139	125
L/4	3	037	041	037	102	099
L/4	4	022	021	018	041	034
L/2	1	081	101	076	222	166
L/2	2	072	083	070	202	182
L/2	3	057	060	053	150	143
L/2	4	037	030	027	059	049
3L/4	1	055	069	053	153	114
3L/4	2	049	057	048	139	125
3L/4	3	039	041	037	102	099
3L/4	4	025	021	018	041	034
DEFLECTIONS—LOAD CONDITION 5						
L/4	1	0 039	0 044	0 035	0 092	0 070
L/4	2	046	050	043	126	117
L/4	3	018*	050	043	126	117
L/4	4	037	044	035	092	070
L/2	1	059	064	050	134	102
L/2	2	059	073	063	183	170
L/2	3	062	073	063	183	170
L/2	4	055	064	050	134	102
3L/4	1	039	044	035	092	070
3L/4	2	043	050	043	126	117
3L/4	3	044	050	043	126	117
3L/4	4	035	044	035	092	070

* Erroneous gauge reading

The measured deflections match the calculated deflections under Assumption B surprisingly well The measured deflection of the exterior girder was 0.081 in, while the adjacent interior girder deflected 0.072. This occurred even though the exterior girder with the sidewalk and curb has a moment of inertia of 578 $in.^4$ and the interior girder has a moment of inertia of 402 $in.^4$ The deflection of the exterior girder of 0.081 in. was the greatest deflection under any girder for any load condition. Load Condition 5, with the two axles symmetrically placed about the longitudinal centerline and as near together as practical, gives the greatest load on the interior girder. The measured deflection of the interior girders averaged 0.061 under this loading. A comparison of Curves A or C for the two loadings, where the girders have equal moments of inertia, shows the relative deflections under loadings which give the maximum deflections of the exterior and interior girders. Under load Condition 4, with the two axles crowded toward the curb, the maximum deflection is in the exterior girder and was 0.081 in Under load Condition 5, with the two axles as near the center line as practical, the maximum deflection is in the two interior girders and averaged 0 061 in This indicates that the exterior girders should be at least as strong as the interior girders

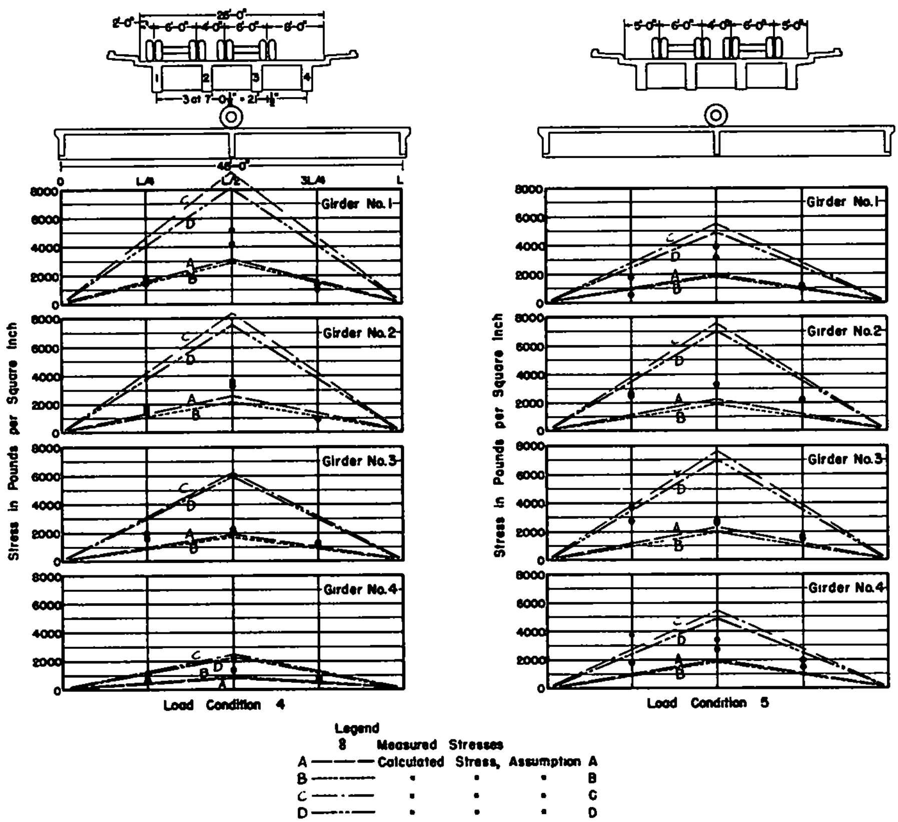

Figure 6. Stress in girder tension steel, load Conditions 4 and 5.

Stress

The stress measurements with the SR-4 gauges attached to the tension steel of the girders are not as consistent as the deflection measurements Even though every practical precaution in the installation and protection of the gauges was taken, the results were rather erratic.

The measured stresses and the calculated stresses under Assumption B, for all load conditions except Condition 2, are shown in Figure 5. The measured stresses in general are higher than the calculated stresses, probably due to the effect of initial cracking of the concrete

The measured and calculated stresses for load Conditions 4 and 5 are given in Table 2 and the plotted data in Figure 6

The highest stress was found in the exterior girder under load Condition 4 when the measured stress was 4,650 psi. in the reinforcing steel Under load Condition 5, which should produce maximum stress in the interior girders, the steel stresses were 3,525 psi. and 2,775 psi., an average of 3,150 psi. These measurements, while subject to considerable question quantitatively, support the deflection measurements in indicating that the exterior girders can be subjected to heavier loads than the interior girders.

An examination of Figure 6 shows that in general the measured stresses are between the values which the Vincent analysis gives for the cracked and the uncracked sections It is probable that the concrete immediately adjacent to the gauges was only partially effective in resisting tension

Reactions

The weighing of the reactions at the ends of the girders was not entirely satisfactory. In moving the loaded trailer axle on and off the span it was impossible to prevent slight movements of the span which affected the loading on the alloy cylinders. There was also some friction between the span and the backwalls of the abutments that affected the results In every case the total load shown by the weighing devices was less than the applied load In a few cases one weighing device would show an unreasonably large proportion of the total load In general, however, the reactions were fairly well in line with the predictions of the Vincent analysis. Table 3 gives the measured and computed reactions for load Condition 4 in which the two axles were crowded to one side of the structure In this table a column headed "Adjusted Value" has been added in which the actual measurements have been proportionately increased so that the total equals the applied load.

Conclusions

Because of the questions as to the action of the concrete as a cracked or an uncracked section and as to the amount the sidewalks and curbs contribute to the moment of inertia of the exterior girders, the test results should not be used quantitatively. The comparisons between the several load conditions and between the exterior and interior girders do give a true picture of the effect of diaphragm beams in distributing the loads.

TABLE 2

STRESS—LOAD CONDITION 4

Position on Bridge	Girder Number	Live Load Stress: Measured	Calculated A	Calculated B	Calculated C	Calculated D
		lb /in 2	lb /in 2	lb /in 2	lb /in 2	lb /in 2
L/4	1	1,575	1,559	1,428	4,601	4,023
L/4	2	1,725	1,300	1,083	4,209	3,787
L/4	3	1,800	936	827	3,115	2,984
L/4	4	975	466	497	1,228	1,189
L/2	1	4,650	3,117	2,856	9,202	8,046
L/2	2	3,450	2,601	2,166	8 418	7,575
L/2	3	2,100	1,872	1,654	6,229	5,969
L/2	4	1,800	932	994	2,456	2,378
3L/4	1	1,125	1,559	1,428	4,601	4,023
3L/4	2	900	1,300	1,083	4,209	3,787
3L/4	3	1,050	936	827	3,115	2,984
3L/4	4	675	466	497	1,228	1,189
STRESS—LOAD CONDITION 5						
L/4	1	1,200	993	944	2,765	2,472
L/4	2	2,625	1,138	977	3,811	3,550
L/4	3	3,150	1,138	977	3,811	3,550
L/4	4	2,775	993	944	2,765	2,472
L/2	1	3,525	1,985	1,887	5,530	4 944
L/2	2	3,300	2,275	1,955	7,622	7,099
L/2	3	2,775	2,275	1,955	7,622	7,099
L/2	4	3,075	1,985	1,887	5,530	4,944
3L/4	1	975	993	944	2,765	2,472
3L/4	2	2,250	1,138	977	3,811	3,550
3L/4	3	1,575	1,138	977	3,811	3,550
3L/4	4	1,725	993	944	2,765	2,472

TABLE 3

REACTIONS—LOAD CONDITION 4

Girder Reaction	Weight as Measured	Adjusted Value	Calculated A	Calculated B	Calculated C	Calculated D
No 1 W	17,043	18 322	17,490	19,412	16,780	17,964
No 2 W	13,426	14,433	14 700	12 381	15,369	13,830
No 3 W	9,043	9,721	10,579	9,453	11,373	10,898
No 4 W	5,591	6,011	5,231	6,754	4,478	5,308
No 1 E	17,749	19,081	17,490	19,412	16,780	17 964
No 2 E	11,055	11,885	14,700	12,381	15,369	13,830
No 3 E	10,584	11,378	10,579	9,453	11,373	10,898
No 4 E	4,808	5,169	5,231	6,754	4,478	5,308
TOTAL	89,299	96,000	96,000	96,000	96,000	96,000

The results from the deflection and stress measurements correspond with the calculated values by the Vincent method so closely that this method can be used with confidence when a close approximation of the actual load distribution is of enough importance to justify the labor involved

The present AASHO specification for load distribution to concrete girders in spans having adequate diaphragm beams is faulty in that it results in assigning more load to the interior girders than to the exterior girders In the usual structure the exterior girders carry as much load as the interior girders and, under some girder arrangements and load positions, may carry even more

For structures having adequate transverse diaphragms, a loading assumption is suggested in which the entire deck width is loaded with axle loads and fractions of axle loads and the total load divided equally to all the girders. This is a simple specification, easily and quickly applied, and, in view of the many uncertainties inherent in design, is accurate enough. Certainly it is more accurate than the present procedure

The Oneonta Creek Bridge was built under contract with Marshall Dresser as resident engineer The planning of the investigation was done by Richard Rosecrans, structural research engineer. The installation of gauges and making of tests was under the supervision of Oscar White, assistant engineer of materials and tests. The analysis of test data was by Roy Edgerton, structural research engineer.

APPENDIX

Vincent Method of Computing Load Distributions

This analysis sets up equations for the deflections of the girders and the diaphragm with respect to their dead load positions and for the force distribution necessary to produce these deflections. The individual girder is deflected by the applied wheel loads and the forces transmitted to it by the diaphragm, whether upward or downward at the particular girder. The diaphragm acts as a continuous beam over yielding supports or, more accurately stated, as an elastic member in space in equilibrium under the action of forces applied at its intersections with the various girders Its deflection under the action of these forces can be readily expressed; for convenience in this analysis its deflection is expressed with respect to the chord connecting its intersections with the two outside girders

In this analysis the torsional rigidity of the girders is neglected, *i e*, it is assumed that the girders are not stiff enough in torsion to produce appreciable restraining moments at the ends of the diaphragm or at its connections to the intermediate girders. This assumption is important in its effects. For example if it were assumed that the girders were so stiff in torsion as to fully fix the diaphragm at the ends and at the various interior girders then no diaphragm moment would be carried past any girder and each segment of diaphragm between adjacent girders would be subjected to reversed moments of equal magnitude at its two ends, these moments and the resulting shear transferred from one girder to the other being determined by the relative deflections of the adjacent girders and the stiffness of the diaphragm segment between them. Under this assumption of relatively great torsional rigidity the individual girder stems would remain vertical even under extreme eccentric loading and the diaphragm would deflect in a series of reverse curves. There can be little doubt that the torsional rigidity of the individual girder stem is nearly negligible in so far as its capacity to develop fixed end moments in the diaphragm is concerned and it is much nearer the truth to neglect this torsional resistance than to assume fixed end conditions. Furthermore, the neglect of any factor such as torsional rigidity which tends to stiffen the diaphragm is on the conservative side, indicating somewhat less distribution of load than occurs.

This analysis neglects also the effect of the slab in distributing loading between girders This effect is far from negligible in the case of girder spans without diaphragms as shown by theoretical analysis and model tests at the University of Illinois. However, when diaphragms as deep as the girders are used, their stiffness is great in comparison with that of the slab and they therefore assume the major portion of the task of distributing the load This is especially true if several diaphragms are used or if a single diaphragm is used at the center of a span of such length that the moment is due almost entirely to the rear truck wheels placed at or near the center of the span

Though the method is of general application, the equations are developed for the case of a four-girder bridge with a diaphragm at midspan and with the live loads applied at midspan.

Figure A shows the span layout and the forces

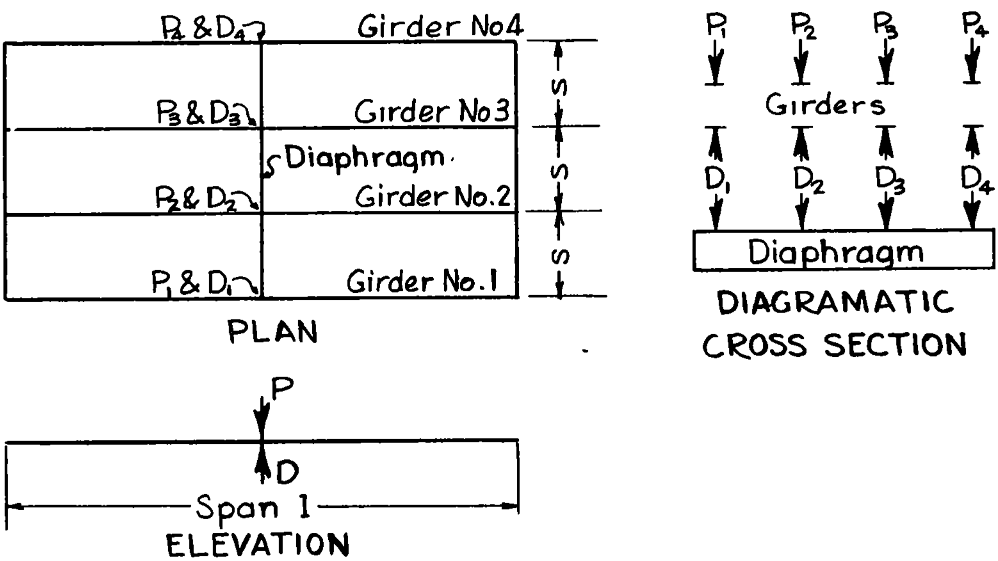

Figure A.

acting on its various elements

P_1, P_2, etc., are the wheel loads distributed to each girder, assuming simple beam action between girders. The final equations are developed in terms of these general loads, thus the effects of various transverse positions of the wheel loads can be determined by substituting the proper values for P_1, P_2, etc., computed for the desired wheel load positions. D_1, D_2, etc., are forces transferred from the girders to the diaphragm. The convention is used that a positive D acts upward on the girder and downward on the diaphragm. Since the diaphragm is supported only by the girders, the laws of equilibrium require that the summation of all forces, D, be zero and some will be negative in sign and thus reversed in direction from that shown in the sketches.

The case of equal moments of inertia for all girders ($I_g=I_1=I_2=I_3=I_4$) will first be developed.

The net load of a typical girder is $P - D$ and the deflection at the center is

$$\Delta=\frac{(P-D)l^3}{48\,E_gI_g} \qquad (1)$$

wherein E_g is the modulus of elasticity and I_g is the moment of inertia of a girder.

The movement of the diaphragm in space under some combination of loads P_1, P_2, etc., on the bridge is illustrated by Figure B, which shows also the de-

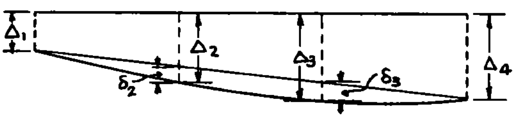

Figure B.

flections of points on the diaphragm with respect to the chord joining its ends. It should be noted that

$$\Delta_2=\frac{2}{3}\Delta_1+\frac{1}{3}\Delta_4+\delta_2 \text{ and} \qquad (2)$$

$$\Delta_3=\frac{1}{3}\Delta_1+\frac{2}{3}\Delta_4+\delta_3. \qquad (3)$$

Since the diaphragm is a beam in equilibrium under the action of forces D, we may choose to consider any of these forces as reactions and the others as loads. We must recognize that the actual signs of some of these forces will be negative and be prepared, therefore, to find in the final solution that some of our assumed reactions act downward and some of our assumed loads act upward. The diaphragm can be represented as a conventional simple beam by showing $-D_1$ and $-D_4$ as upward acting forces as shown in Figure C.

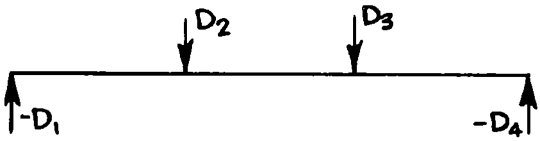

Figure C.

The deflection of the diaphragm at each girder intersection under these loads can be computed by various methods. It is perhaps easiest to use the formula

$$\Delta_x=\frac{Pbx}{6EI}(l^2-b^2-x^2) \qquad (4)$$

applying to Figure D. In applying this formula, the

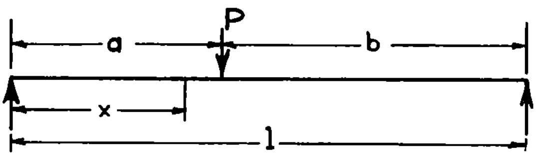

Figure D.

diaphragm deflection at Girder 2, first due to D_2, then due to D_3, are determined and added. By this method

$$\delta_2=\frac{8D_2s^3}{18E_dI_d}+\frac{7D_3s^3}{18E_dI_d}=\frac{s^3}{18E_dI_d}(8D_2+7D_3) \qquad (5)$$

$$\delta_3=\frac{s^3}{18E_dI_d}(7D_2+8D_3) \qquad (6)$$

wherein E_d is the modulus of elasticity and I_d is the moment of inertia of diaphragm.

We now introduce $K=\frac{l^3}{48E_gI_g}$ and $N=\frac{s^3}{18E_dI_d}$

Substituting these values in Equation 2

$$K(P_2-D_2)=\frac{2}{3}K(P_1-D_1)+\frac{1}{3}K(P_4-D_4) +N(8D_2+7D_3) \qquad (7)$$

Introducing $R=\frac{N}{K}$

$$(P_2-D_2)=\frac{2}{3}(P_1-D_1)+\frac{1}{3}(P_4-D_4)+R(8D_2+7D_3) \quad (8)$$

$$\frac{2}{3}D_1-(8R+1)D_2-7RD_3+\frac{1}{3}D_4=\frac{2}{3}P_1-P_2+\frac{1}{3}P_4 \quad (9)$$

$$\frac{1}{3}D_1-7RD_2-(8R+1)D_3+\frac{2}{3}D_4=\frac{1}{3}P_1-P_3+\frac{2}{3}P_4 \quad (10)$$

From the conditions of static equilibrium of the diaphragm under forces D_1, D_2, D_3 and D_4, two additional equations can be written.

$$\Sigma F_v=D_1+D_2+D_3+D_4=0 \text{ and} \quad (11)$$

$$\Sigma M_1=D_2+2D_3+3D_4=0 \quad (12)$$

In the simultaneous solution of Equations 9, 10, 11, and 12 for any particular bridge, it is best to introduce the computed value of R, but the values of P_1, P_2, P_3 and P_4 should be left in general terms so that effect of any transverse position of wheel load can be determined without solving additional sets of equations.

If the moments of inertia of the girders of a structure differ enough to warrant consideration in the computation, separate values of K_1, K_2, etc, are introduced and Equations 9 and 10 become:

$$\frac{2}{3}K_1D_1-(8N+K_2)D_2-7ND_3+\frac{1}{3}K_4D_4=\frac{2}{3}K_1P_1-K_2P_2+\frac{1}{3}K_4P_4 \quad (13)$$

$$\frac{1}{3}K_1D_1-7ND_2-(8N+K_3)D_3+\frac{2}{3}K_4D_4=\frac{1}{3}K_1P_1-K_3P_3+\frac{2}{3}K_4P_4 \quad (14)$$

This same general method can be applied to spans with greater numbers of girders and diaphragms. It will be noted that the number of simultaneous equations will equal the number of D forces which, in turn, will equal the number of girder-diaphragm intersection. To develop equations for conditions involving loadings other than midspan, numerical coefficients must be determined for P_1, P_2, etc., in Equation 1.

Distribution of Loads to Girders in Slab-and-Girder Bridges: Theoretical Analyses and Their Relation to Field Tests

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SYNOPSIS

THE object of this paper is to present a picture, based on theoretical analyses, of the manner in which loads on slab-and-girder highway bridges are distributed to the supporting girders. The discussion is restricted to simple-span, right bridges consisting of a slab of constant thickness supported on five girders, spaced equidistantly, and having equal flexural stiffnesses but no torsional stiffness

The numerous variables influencing the behavior of this type of structure are listed, and the effects of the following are considered in detail· (1) the relative stiffness of girders and slab, H, (2) the ratio of girder spacing to span of bridge, b/a; (3) the number and arrangement of the loads on the bridge; and (4) the effect of diaphragms, their stiffness, number, and location on the structure Particular emphasis is placed on the relative magnitudes of the maximum moments in interior and exterior girders.

It is shown that when the slab is fairly flexible in comparison to the girders, the maximum moment in an interior girder will usually be larger than the corresponding maximum moment in an exterior girder, if the loads in each case are arranged so as to produce maximum effects in the girder considered. This condition of maximum moment in an interior girder is found to be typical for reinforced-concrete T-beam brides having no diaphragms. However, if the transverse stiffness of the structure is fairly large in comparison with the stiffness of the girders, then the maximum moment in the exterior girder will generally be the greatest. Such conditions will usually be encountered for typical I-beam bridges and for concrete-girder bridges having adequate transverse diaphragms.

For those arrangements of loads which are critical in design, an increase in relative stiffness of the slab and the girders (decrease in H) will generally reduce the maximum moment in the interior girders. For exterior girders, a corresponding decrease in H may either increase or decrease the maximum moment.

A change in the ratio b/a affects the distribution of loads to the girders in much the same way as a change in H, since both of these quantities are measures of the relative stiffness of the slab and girders. Thus, a decrease in b/a improves the load distribution in about the same manner as a decrease in H

The behavior of a slab-and-girder bridge under a single wheel load is found to be different from the behavior of the same structure under multiple wheel loads Unless the performance of the structure and the effects of the numerous variables affecting its behavior are investigated for all possible conditions of loading to which the bridge may be subjected, certain aspects of the action of the structure may be overlooked.

The addition of diaphragms in slab-and-girder bridges supplements the capacity of the roadway slab to distribute loads to the supporting girders The manner and extent to which diaphragms modify the distribution of load depends on such factors as the stiffness of the diaphragm, the number employed, their longitudinal location, and also on all those parameters influencing the behavior of slab-and-girder bridges without diaphragms. Diaphragms will almost always reduce the maximum moment in an interior girder but they will usually increase the maximum moment in an exterior girder. These effects, which are a function of the many variables referred to above, may be beneficial or harmful depending on whether the moment controlling design occurs in an interior or exterior girder. The conditions under which diaphragms will increase or decrease the controlling design moments are described in the body of the report.

The simplifying assumptions involved in the analyses and the limitations imposed by these assumptions are discussed in detail, and consideration is given to the probable effects of the neglected variables.

The relationship between thoretical analyses and the behavior of actual structures is also considered, and the paper concludes with a discussion of the manner in which theoretical analyses can best be used in planning field tests on slab-and-girder bridges, and in interpreting the results obtained.

The slab-and-girder highway bridge is a structure for which neither theoretical analyses nor laboratory or field tests alone can be expected to yield a complete and trustworthy description of its action. Only by considering together the results of both analyses and tests can we hope to understand a type of structure whose behavior depends on so many variables.

● THE slab-and-girder highway bridge as considered in this paper consists essentially of a reinforced-concrete slab supported by a number of parallel steel or concrete girders extending in the direction of traffic. The wide use of such bridges, together with an increasing awareness of their inherent complexity, has emphasized the need for a better understanding of the way in which they function. Of particular interest has been the manner in which wheel loads from vehicles are distributed to the supporting beams.

Studies of slab-and-girder bridges were begun in 1936 at the University of Illinois in cooperation with the Illinois Division of Highways and the U S. Bureau of Public Roads. The results of these studies have been presented in several publications (*1, 2, 3, 4, 5, 6*). Included in this program were extensive theoretical analyses in which the effects of several important variables were studied, and a rather complete picture of the behavior of such structures was obtained. In addition, numerous laboratory tests on scale-model I-beam bridges were made to determine the accuracy of certain assumptions in the analyses and to study the behavior of the bridges at ultimate loads.

The object of this paper is to present a picture, based on theoretical analyses, of the manner in which loads are distributed to the girders in slab-and-girder bridges. The scope of these analyses, and thus also the scope of this paper, has been limited to the behavior of the bridge under working loads. This is an important limitation, since both the ultimate strength of the structure and its behavior at loads producing yielding are factors which should be given great weight in the selection of design methods.

A second purpose of this paper is to consider the relationship between the results obtained from theoretical analyses and those obtained from tests of actual structures. This is a two-way relationship; neither approach to the problem can be considered alone and each can benefit from a study of the other. The theoretical approach cannot be accepted with entire confidence until its predictions have been verified by comparison with the behavior of real bridges. On the other hand, no field test can give the full picture, since the number of variables that can be considered is necessarily quite limited. Only by considering the two together can we obtain a complete and generally applicable solution to the problem.

Analyses of Slab-and-Girder Bridges

Variables

The slab-and-girder bridge is a complex structure, and an exact analysis can be made only by relatively complex means. In essence, this structure consists of a slab continuous in one direction over a series of flexible girders. The presence of the slab as a major element of the structure is, of course, one complicating factor. However, the complexity of the structure is further increased by the continuity of the slab and by the deflections of the supporting girders.

The problem of studing analytically the slab-and-girder bridge is further complicated by the larger number of variables that may conceivably affect its behavior. The more significant variables may be listed as follows:

Variables relating to the geometry of the structure: (1) Whether girders are simply supported, continuous, or cantilevered; (2) whether the bridge is right or skewed; (3) the number of girders; (4) the span length of the girders; (5) the spacing of the girders, and whether or not it is uniform; and (6) the number and locations of diaphragms.

Variables relating to the stiffness of the bridge elements: (7) The flexural stiffness of the girders (this may or may not be the same for all girders and may vary along the span); (8) the torsional stiffness of the girders (this enters only when the girders are attached rigidly to the slab or diaphragms); (9) the stiffness of the slab (this depends primarily on the slab thickness and may or may not be uniform); and (10) the stiffness of the diaphragms, if present, and the efficiency of their connections to the girders.

Variables relating to the loading. (11) Number of wheel loads or truck loads considered; (12) transverse location of the load or loads on the bridge; and (13) longitudinal location of the load or loads on the bridge, especially with reference to the location of diaphragms.

The method of analysis used herein is that developed by N. M. Newmark (*1*) and is capable of taking into account all of the variables listed above except the effects of skew. However, since the amount of work involved in considering all of these variables over an appropriate range would be prohibitive, it was necessary to limit either the number of variables considered or the range over which they were assumed to vary. The first alternative was chosen and the analyses were made for a simplified structure obtained by restricting several of the variables to a single value This permitted the remaining variables to be considered for a relatively large range of values.

Scope of Analyses

The structures analyzed were all simple-span bridges consisting of a slab having constant thickness supported on five girders, spaced equidistantly, and having equal flexural stiffnesses and zero torsional stiffness Loadings considered included single concentrated loads as well as combinations of trucks placed so as to produce maximum moments in the various beams. The results of these analyses have been reported (*2*, *3*, *6*).

Additional analyses for bridges with one, two, or three diaphragms (*7*) and for bridges with only three girders (*8*, *9*) have also been made. The results of these studies have been considered in the discussions which follow, but for the most part this paper is based on the results of analyses reported in Reference *2*

For the simplified structures analyzed, the remaining variables are the loading conditions and the following properties of the bridge

Span of girders, a

Spacing of girders, b.

Flexural stiffness of each girder, E_gI_g, where

E_g=modulus of elasticity of material of girder.

I_g=moment of inertia of girder.

Flexural stiffness of slab, $N=\frac{E_sI_s}{1-\mu^2}$, where

E_s=modulus of elasticity of material of slab

I_s=moment of inertia of slab per unit of width

μ=Poisson's ratio for material of slab.

(For reinforced concrete slabs it is convenient and sufficiently accurate to assume μ=0 and to compute I_s on the basis of the gross concrete section; thus $N=\frac{E_st^3}{12}$ where t is the thickness of the slab.)

The conditions of the analysis are such that the variables listed above do not enter separately but can be combined into dimensionless ratios as follows:

b/a=ratio of girder spacing to span,

$H=\frac{E_gI_g}{aN}$ = ratio of girder stiffness to the stiffness of a width of slab equal to the span of the bridge.

The quantity H relates the longitudinal stiffness of a girder to the transverse stiffness of the slab. Since the quantity N is the slab stiffness per unit of width, it is necessary to multiply N by some width in order to make H a dimensionless ratio The term a introduced in the denominator serves this purpose, but it should not be inferred that the analysis involves the assumption that the slab has an "effective width" equal to a. The analysis requires no such assumption, since it treats the slab as a slab without recourse to equivalent beams; the quantity H is simply a convenient dimensionless parameter.

The scope of the analyses made at the University of Illinois included five-girder bridges having values of b/a=0 1, 0.2, and 0.3 and values of H ranging from 0.5 to 20, with H=infinity considered also as a limiting case. For each of these structures, moments and deflections were computed for a single concentrated load placed at various positions, both transversely and longitudinally on the bridge. These calculations yielded influence lines or influence surfaces for moments and deflections and thus permitted the determination of maximum effects for various combinations of loads representing, usually, two trucks on the bridge. When truck loads are considered it is necessary to assign numerical values to the beam spacing b, and in these studies the range in b was 5 to 8 ft. Corresponding span lengths, a, ranged from 17 to 80 ft., depending on the value of b/a.

It should be mentioned that the program of research described above involved extensive studies of slab moments as well as girder moments and deflections. However, the scope of this paper is limited to those portions of the analyses concerned with moments or deflections of the girders.

When the slab acts as a transverse distributing member as described in (2) above, it performs essentially the same function as a diaphragm, except that the nature of the loading transferred to the girders is quite different. For a diaphragm, the loads carried to the girders are concentrated loads applied at the points where the diaphragm is attached to the girders. The loads transmitted by the slab are not concentrated but are distributed along the girders in a manner illustrated in Figure 1. The moment diagram for each beam is shown for a concentrated load

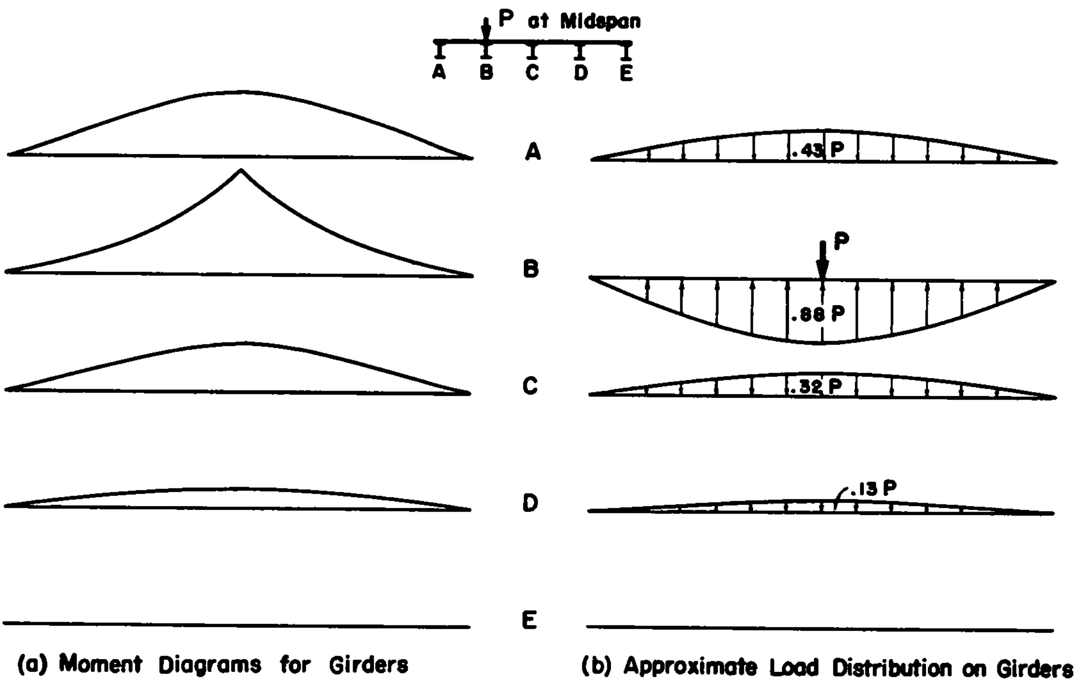

Figure 1. Nature of distribution of load along girders (H=5 and b/a=0.1).

P on Beam B. The loading curves corresponding to these moment diagrams are also shown. The concentrated load applied to Beam B is distributed to the other beams as shown, leaving a load on Beam B made up of two parts a downward concentration equal to P and an upward load distributed along the beam.

It is evident from the curves in Figure 1 that the distribution of load along the beams may be quite different for the various beams. Consequently, the relation between total load and moment or deflection will not be the same for all beams.

The amount and character of the transverse load distribution provided by the slab depends on the values of b/a, H, and the character of the loading.

Action of Slab in Distributing Loads

General

As might be expected, the action of the slab in a slab-and-girder bridge is rather complex. However, as an aid to visualizing the behavior of the structure, the slab may be considered to have two major functions: (1) The slab acts as a roadway and provides a deck spanning between girders and supporting the wheel loads from vehicles. In this function, the slab serves to transfer wheel loads to the adjacent girders, when such loads are applied at positions between the girders; (2) Because of its transverse stiffness and continuity, the slab acts to equalize deflections of the girders and thus to distribute load among them.

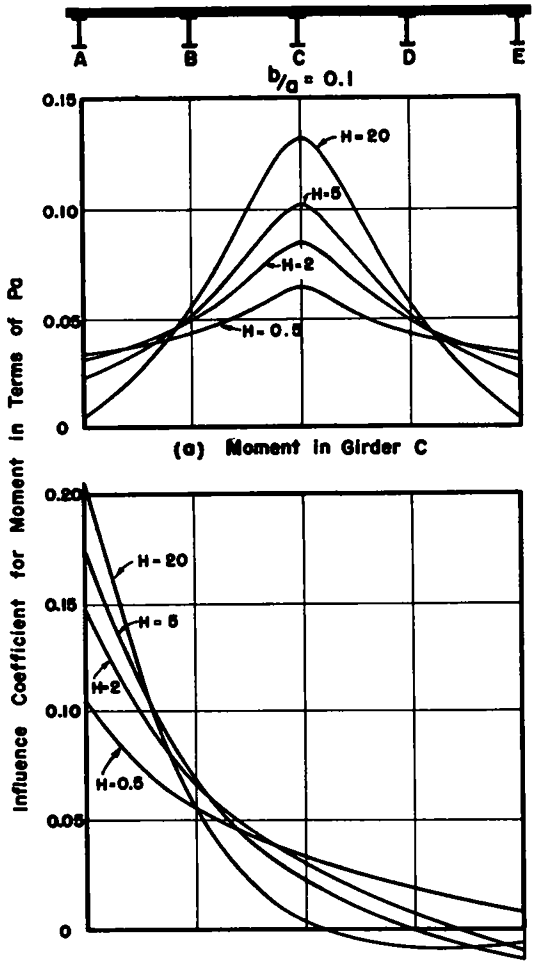

Figure 2. Influence lines for moment in girders at midspan for load moving transversely across bridge at midspan.

The effects of these variables are discussed in the following sections of this paper

Effect of Relative Stiffness H

The relative stiffness of the girders and the slab, as expressed by the ratio H, is one of the most important variables affecting the load distribution to the girders. The effectiveness of the slab in distributing loads will increase as its stiffness increases. Moreover, a slab of a given stiffness will be more effective when the potential relative deflections of the girders are large, that is, when the girder stiffness is small. Thus the distribution of load will generally become greater as the value of H decreases, whether the change is due to a decrease in girder stiffness or to an increase in slab stiffness.

The effects of variations in H can best be illustrated by means of examples taken from the analyses of five-girder bridges. Typical influence lines for moment at midspan of the girders are shown in Figure 2 for a structure with $b/a=0.1$ and for various values of H

Figure 2(a) shows the influence lines for the center girder. For small values of H, corresponding to a relatively stiff slab, the curves are rather flat, indicating that the slab is quite effective in distributing the moment among the girders. As the value of H increases, the moment becomes more and more concentrated in the loaded girder, and for $H=$infinity, would theoretically be carried entirely by that girder.

Figure 2(b) shows influence lines for an edge girder. Although the shape of these curves is quite different, owing to the location of the girder, the trends with changes in H are similar to those for Figure 2(a).

It may also be seen from the influence lines in Figure 2 that the effects of a concentrated load on the more distant girders is relatively small. Thus, the addition of more girders on either side in Figure 2(a), or on the side opposite the load in Figure 2(b), would obviously have little effect on the character or magnitudes of the influence lines. Although this

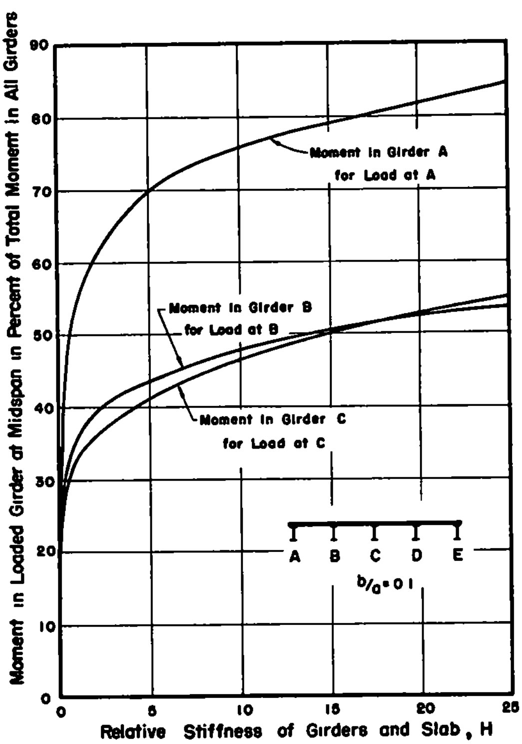

Figure 3. Variation of moment in loaded girder as a function of H for concentrated load at midspan.

conclusion does not apply without reservation for all possible values of H and b/a, it is reasonably valid for practically all structures having the proportions considered in the analyses This observation then provides justification for extending the results of the analyses to bridges having more than five girders, and possibly also in some cases to bridges having only four girders.

The effects of changes in the relative stiffness H may be shown more directly by the curves of Figure 3 for a bridge having $b/a=0.1$. Relative moments at midspan of girders A, B, and C for a single, concentrated load directly over the girder at midspan are shown as a function of H The moments are given in percent of the total moment in all the girders, that is, neglecting the portion of the static moment carried directly by the slab.[1]

The close agreement between the curves for Girders B and C suggests that the behavior of all interior girders is much the same regardless of their location. It also provides further justification for extending the results of these analyses to bridges having more than five girders or to bridges having only four girders.

It can also be seen from Figure 3 that relatively much less distribution of moment occurs for a concentrated load over an edge beam than for a load over an interior beam. When a load is applied over Beam A, the slab, no matter how stiff, cannot transfer the load effectively to the more distant girders, which are relatively farther away for this loading than for a load over Beam C. Such a reduction in the degree of distribution is evident also from Figure 2(b).

A further illustration of the way in which the moments resulting from a single, concentrated load are distributed among the beams is provided by Figure 4 for a bridge having five girders and $b/a=0.1$. Relative moments in all girders for a load over Girder B are plotted as a function of H in this figure. The curve for moment in Girder B is the same as that on Figure 3. For this girder the moment increases continuously as the value of H increases For an infinitely stiff slab, corresponding to $H=0$, all girders participate equally in carrying the load, while for $H=$infinity all of the moment is carried by the loaded girder. A study of the variation of moment in the remaining girders as H decreases from near infinity to zero in Figure 4 gives further insight into the behavior of this type of structure Consider first the moments in Girder A. At H equals infinity this

[1] The portion of the longitudinal moment carried by the slab is usually quite small An approximate expression for determining this moment is given on pp 24 25 of Reference 2

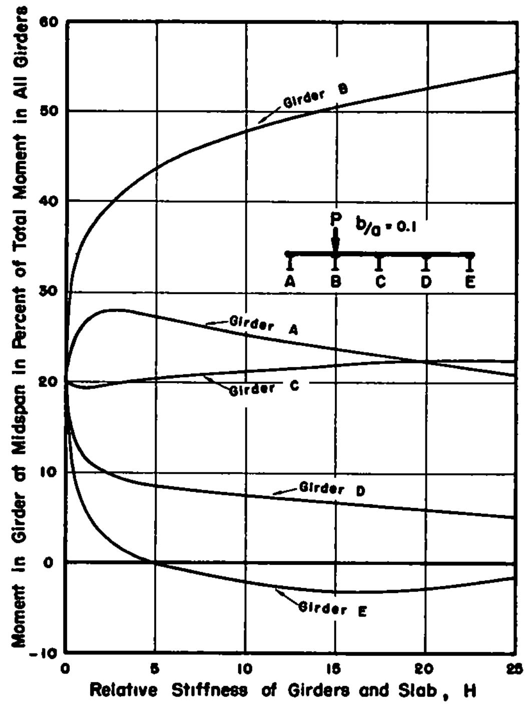

Figure 4. Variation of moment in girders as a function of H for a concentrated load over Girder B at midspan.

moment is zero As the slab becomes stiffer and H decreases, this moment gradually increases until a value of $H=2$ or 3 is reached. At this point, the moment in Girder A begins to decrease with further decrease in H and finally reaches a value of 20 percent at $H=0$. This rather interesting behavior can be explained in terms of the increasing ability of the slab to distribute moment to the more distant girders as its stiffness increases. Note first that the moment in Girder C changes very little for the range of H on the figure. For values of H greater than about 5, the moments in Girders D and E are relatively small and do not change rapidly with H, indicating that in this range the stiffness of the slab is not sufficient to transfer an appreciable portion of the load to these more distant girders. Consequently, most of the decrease in moment in Girder B as H decreases is accomplished by transfer of moment to Girder A. However, for values of H less than 5 in Figure 4 the stiffness of the slab becomes great enough to increase appreciably the participation of girders D and E, and the moment in these girders begin to increase more rapidly as H decreases In this stage the load applied over Girder B is more widely distributed and the adjacent Girder A is no

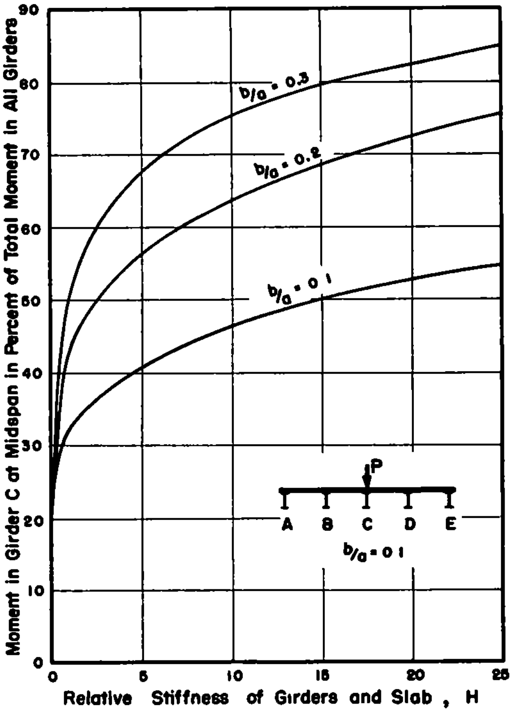

Figure 5. Effect of b/a on midspan moment in loaded Girder C for concentrated load at midspan.

longer required to resist as much moment as before. Thus the moment in Girder A ceases to increase and actually decreases to its final value of 20 percent at $H=0$. The nature of the curve for Girder A in this figure is generally typical of those for this loading condition and for other values of b/a. However, as b/a increases, the maximum moment in girder A occurs for smaller values of H than that shown in Figure 4 for $b/a=0.1$.

Effect of Ratio b/a

The second major variable included in the analyses is the ratio of girder spacing to span, b/a. A change in the relative span lengths of the slab and the girders, as represented by a change in b/a, causes a corresponding change in the relative stiffnesses of these two elements; that is, an increase in b/a corresponds to a decrease in the transverse stiffness of the bridge. Thus, in general, the effect of increasing b/a is similar to that of increasing H. This is illustrated in Figure 5 which contains curves of relative moments at midspan of Girder C for a concentrated load over Girder C. The variation of moment with H is shown for structures having $b/a=0.1$, 0.2, and 0.3. The relative effects of changing b/a and H are easily seen from this figure For example, an increase of b/a from 0.1 to 0.2 produces an increase in moment in Girder C approximately equal to that resulting from about a sixfold increase in H. That is, a change from $b/a=0.1$, $H=4$ to $b/a=0.2$, $H=4$ is equivalent to a change from $b/a=0.1$, $H=4$ to $b/a=0.1$, $H=25$ Similar relations hold for an increase in b/a from 0.2 to 0.3 but the equivalent change in H in this case is less than threefold.

Although an increase in b/a will always result in less distribution of load, the effect for an actual slab-and-girder bridge will usually be less than indicated in Figure 5 because of changes in H that occur as a result of changes in b/a For example, if b/a is increased by shortening the span a, the change in span results in smaller and less stiff girders and thus causes a decrease in H which partially offsets the effects of increasing b/a. Similarly, if b/a is increased by making the girder spacing b larger, changes in H are again produced, chiefly because of increase in slab thickness which usually results from the changed span of the slab Although the girder stiffness may also be increased as a result of the wider spacing, the net result is usually a decrease in H, since the slab stiffness varies as the cube of the thickness and may be increased a fairly large amount.

Effect of Loading

The preceding discussions of the manner in which load distribution depends on H and b/a have been confined to the case of a single, concentrated load on the structure This loading condition was chosen partly for its simplicity but also because all of the effects discussed are greater for a single, concentrated load than for multiple loads. For this reason it is necessary to discuss also the behavior of the structure for the case of more than one load applied at a given section, since highway bridges are always subjected to multiple loads. In some cases, two loads corresponding to a single truck may be considered, but more commonly the loading will consist of four loads representative of two trucks.

The curves in Figure 6 show the variation with H of the maximum moments in Girders A and C of a five-girder bridge having $b/a=0.1$. In each case the loads are placed transversely in the position to produce maximum moment in the girder considered The spacing of the loads corresponds to the spacing of truck wheels on a bridge having a girder spacing of 6 ft.

Consider first the curve for Girder C in Figure 6. This curve is very similar to that for the same girder in Figure 3, except that the decrease in moment with a decrease in H is much less. For a concentrated load (Fig. 3), the moment decreases from 54 percent of the total moment at $H=25$ to only 20 percent at $H=0$. However, for four loads (Fig 6), the moment in Girder C for $H=25$ is only about 30.3 percent of the total, since the application of four loads provides in itself a better distribution of total moment among the girders. Since this girder must resist 20 percent of the moment at $H=0$, it is evident that a decrease in H can produce much less reduction in moment for multiple loads than for a single load.

The curve for Girder A in Figure 6 is quite different from that for Girder C, in that there is a range of H in which the moment increases as H decreases This phenomenon was observed also in the curve for moment in Girder A for a single load over Girder B (Fig. 4). The similarity between these two curves is to be expected since the center of gravity of the four loads in Figure 6 is very close to Girder B. Thus, the explanation for the peculiarities of this curve are the same as those given in the discussion of Figure 4.

It can be seen from Figure 6 that for H less than about 10 the moment in the edge girder is the greater while for H greater than 10 the opposite is true. This condition is fairly typical for other structures with a load over the edge girder as shown in Figure 6, but the value of H at which the two curves cross will depend on the values of other variables, such as b/a and the spacing of the wheel loads relative to the spacing of the girders. Obviously, the magnitude of the moment in an edge girder will be decreased if the loads are shifted away from it. If conditions are such that the outer wheel load cannot be placed directly over the edge girder or sufficiently close to it, the moment in the edge girder may be less than that in an interior girder for all values of H

Another difference in the behavior of edge and interior girders is the way in which the moments vary with H. For an interior girder, the maximum moment always decreases as H becomes smaller and this trend is independent of the type or number of loads. However, the moment in an edge girder first increases and then decreases as H is made smaller. The value of H at which this change takes place depends somewhat on the other variables not shown in Figure 6.

Another characteristic of the structure loaded with several loads is worthy of mention although it is not illustrated in Figure 6. As the number of loads increases, the distribution of load *along* the girders becomes more nearly alike for the several girders. Consequently, the differences between relative loads, moments, and deflections become less. For example, consider a structure having $b/a=0.1$ and $H=5$. For a concentrated load over Girder C the moment in that girder is 2.05 times the average moment for all the girders, while the deflection of Girder C is only 1.55 times the average. However, for four loads placed as in Figure 6, the corresponding ratios of maximum to average are 1.28 for moment and 1.23 for deflection. This relatively close agreement between the distribution of moment and deflection for a practical case of loading is quite convenient in that it makes it possible to use the same assumptions for the computation of moments and deflections in the design of slab-and-girder bridges.

Action of Diaphragms in Distributing Loads

Diaphragms or other kinds of transverse bracing between the girders are often used in slab-and-girder bridges, in an attempt to improve the distribution of loads among the girders. The results of analyses show, however, that the addition of diaphragms does not always accomplish this aim since in certain cases it may actually increase the maximum moment in a girder. The conditions which determine whether diaphragms will decrease or increase the moment in a particuler girder can best be described by considering two typical examples.

First, consider a five-girder bridge with four loads

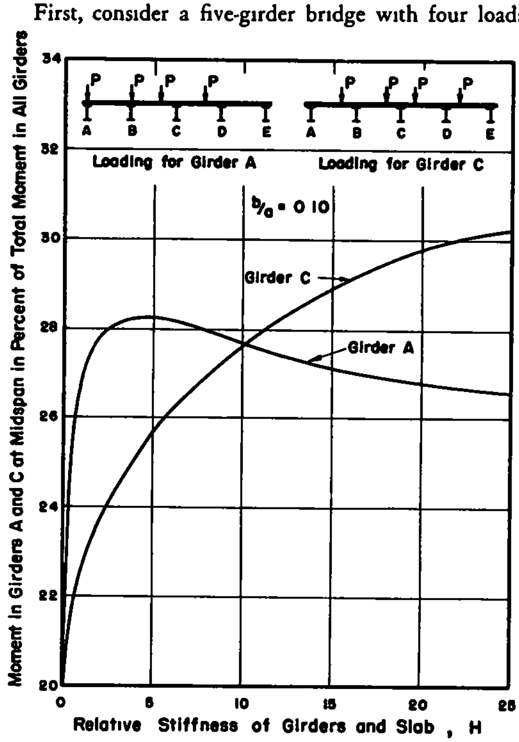

Figure 6. Variation with H of maximum moment in exterior and interior girders for four wheel loads at midspan.

placed to produce maximum moment in the center girder. The moments in this girder as a function of H are shown in Figure 6. Note that the loads are located symmetrically about the longitudinal centerline of the structure, and that it is the moment in Girder C that is being considered. If no diaphragms are present, the effect of increasing the transverse stiffness by increasing the stiffness of the slab causes a continuous decrease in moment as illustrated by the curve in Figure 6 for decreasing values of H. When the slab becomes infinitely stiff ($H=0$), the load and moment is distributed equally to all of the girders, and the maximum distribution is thus obtained. Now consider the same structure, having a slab with a stiffness corresponding to say $H=20$, but having a diaphragm added at midspan. If the diaphragm is assumed to be infinitely stiff, the load and moment will be distributed uniformly among the girders, since the applied loads are placed symmetrically about the longitudinal centerline of the bridge. The effect of providing infinite transverse stiffness is therefore the same whether the added stiffness is provided in the slab or by means of a diaphragm. It is reasonable to assume, therefore, that this equivalence in effect of slab and diaphragm will hold also for intermediate diaphragm stiffnesses, and analysis has shown this to be true. Thus, for a symmetrically loaded bridge, the addition of transverse stiffness by means of diaphragms produces a reduction in the maximum girder moments in much the same manner as would an increase in slab stiffness (decrease in H).

Consider next the other loading condition illustrated in Figure 6 with loads placed eccentrically in the transverse direction so as to produce maximum moments in an exterior girder. In the structure without diaphragms, the effect of increasing the slab stiffness is shown by the curve in Figure 6 as H decreases. At first, the moment in the edge girder increases. Then, as the stiffness becomes very great (H small), the moment begins to decrease. And finally, for infinite slab stiffness ($H=0$), the load and moment is again distributed uniformly to all of the girders just as it was for symmetrically placed loads. This ability of an infinitely stiff slab to provide uniform distribution of load for any arrangement of the loads results from the torsional stiffness of the slab which, in theory, becomes infinite when the transverse stiffness does. This property of the slab is not possessed by a diaphragm. Thus, if the transverse stiffness is increased by the addition of a diaphragm at midspan the behavior of the bridge is quite different from that produced by an increase in slab stiffness. Consider the limiting case of an infinitely stiff diaphragm. For this condition, the deflection of the girders, and thus the distribution of load to equally stiff girders, becomes linear, but not uniform. In other words, the structure tilts because of the eccentricity of the loading, and the moment in Girder A becomes something greater than 20 percent. Actually, for the loading arrangement shown in Figure 6, the moment in Girder A for an infinitely stiff diaphragm is theoretically equal to 33.3 percent. Thus, if the load is eccentrically located on the bridge, the addition of diaphragms may result in an appreciable increase in the edge-girder moment.

Magnitude of Effects

The foregoing discussion has shown clearly that beneficial effects are not always produced by the addition of diaphragms. It is important, therefore, to know under which conditions a diaphragm is able to exert its greatest effects and to have some idea of how great these effects might be. Since a diaphragm, like the slab, derives its effectiveness in transferring load from its ability to resist relative deflections of the girders, any condition leading to large relative deflections, or to more nonuniform distribution of load or moment, will provide the diaphragm with a better opportunity to transfer loads. Thus, the following conditions should lead to the greatest effects of diaphragms: large values of H; large values of b/a, or a decrease in the number of loads. The effects of these variables, as well as others, are discussed in the sections following.

Effect of H *and Diaphragm Stiffness*

The relative stiffnesses of the slab, the diaphragms, and the girders are all related in their effect on the load distribution. It is convenient to combine these three stiffnesses in two dimensionless ratios. One of these is, of course, H, which relates the stiffness of the girders to the stiffness of the slab. The other is defined as

$$k = \frac{E_d I_d}{E_g I_g}$$

where $E_d I_d$ and $E_g I_g$ are the moduli of elasticity and moments of inertia of a diaphragm and a girder, respectively.

It is obvious that the effectiveness of the diaphragm is a function of its stiffness, and that it increases with an increase in k. However, the change in moment produced by the addition of a diaphragm of given stiffness depends on the stiffness of the slab already present. This can best be illustrated by reference to the moment curve for Girder C in Figure 6. The structure considered in this figure is representative

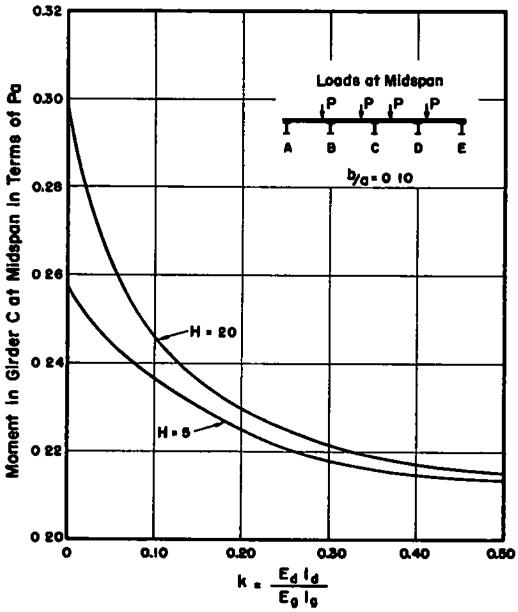

Figure 7. Effect of adding diaphragm at midspan of bridge on moments at midspan.

of a bridge having a girder spacing of 6 ft. and a span of 60 ft. A concrete-girder bridge of these dimensions would have a value of H in the neighborhood of 20 to 50, while a noncomposite I-beam bridge would have an H of about 5 Since results of analyses are available for values of $H=5$ and 20, these will be used for comparisons; they can be considered roughly typical of the two types of bridges mentioned First consider the larger value of H. The moment in Girder C for no diaphragm is found to be 0.298 *Pa* If a diaphragm is now added at midspan with a stiffness corresponding to $k=0.40$, a fairly large value, the moment in Girder C at midspan is reduced to 0.217. The reduction in this case is 27 percent Now consider a bridge having $H=5$, and add the same diaphragm. For no diaphragm the moment in C is 0.256 *Pa,* and with a diaphragm having $k=0.40$ it becomes 0.215. The reduction in this case is only 16 percent, or a little more than half as much as for the other bridge. The reason for this becomes evident if it is noted that the moment after the diaphragm was added was approximately the same in both structures, 0.217 and 0.215. This means that the action of a diaphragm of this stiffness dominates the action of the slab and leads to about the same result in the two cases However, since the bridge with $H=5$ initially has a somewhat smaller moment than the bridge with $H=20$, the change produced by the diaphragm is correspondingly less. The relations just discussed are illustrated better in Figure 7 which gives moments for the same structure and loading as in Figure 6. The moment in Girder C for symmetrical loading is shown as a function of k for the two values of H. It is easily seen from this figure that a given diaphragm stiffness provides a much greater reduction of moment if $H=20$ than if $H=5$

Figure 8 is similar to Figure 7, except that the moment given is that in Girder A for the eccentric load arrangement shown Again, the bridge and loading are the same as in Figure 6. In Figure 8, the maximum moment in an edge girder increases as the diaphragm stiffness increases, for the reasons given previously. Comparisons can be made as before for structures having values of $H=5$ and 20. For $H=20$, the addition of a diaphragm with $k=0.4$ increases the moment from 0.268 *Pa* to 0.319 *Pa,* an increase of 19 percent. For $H=5$, the corresponding increase is from 0.283 to 0.302, or only 7 percent. Thus in this case also, the effect of adding a diaphragm is greater for the larger value of H.

Figures 7 and 8 show also that the diaphragm has a diminishing effect as its stiffness increases; that is the moment curves tend to flatten out as k increases. For example, for Girder C and $H=20$ in Figure 7, an increase in k from 0 to 0.40 reduces the moment 27 percent, while a further increase in k from 0.40 to infinity would produce an additional decrease of only about 6 percent in terms of the moment for $k=0$.

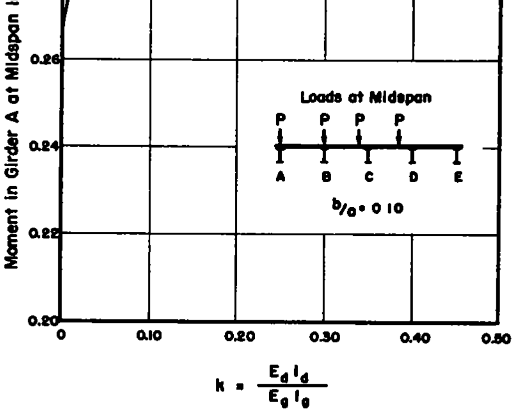

Figure 8. Effect of adding diaphragm at midspan of bridge on moments at midspan.

The comparisons in the preceding paragraphs have been presented only to give a picture of the relative effects of adding diaphragms to structures having different values of *H*. The numerical values are applicable only to the particular structures considered and no general conclusions regarding the absolute effects of diaphragms can be drawn from them, since there are several other variables whose effects have not yet been considered.

It is also important to note that the theoretical analyses on which the foregoing discussions are based involve the assumption that the longitudinal girders have no torsional stiffness. If such stiffness is present, the action of a diaphragm for eccentric loading approaches more nearly that of the slab. However, a relatively high degree of torsional stiffness and a fairly stiff connection between diaphragms and girders is required before this effect becomes appreciable. These conditions are more likely to be present in bridges with concrete girders and diaphragms than in the I-beam-type of bridge.

Effect of b/a

The relative deflections of the girders in a bridge without diaphragms become greater as the value of *b/a* increases. Therefore, the effects of the diaphragms, which are dependent on the relative deflections, will tend to be greater for larger values of *b/a*. The actual effects will be similar to those discussed in the preceding sections; that is, the moment in an interior girder for symmetrical loading will be decreased, while the moment in an exterior girder will be increased if the loads are placed eccentrically with respect to the longitudinal centerline of the bridge. In either case, the changes in moment will be greater for larger values of *b/a*.

Effect of Number of Loads

The effects produced by adding diaphragms will depend on the number of loads considered to act on the structure at a given transverse section. The choices in either analyses or test programs are normally three: (1) a single concentrated load; (2) two loads, representing a single truck; or (3) four loads, representing two trucks. Data have been presented previously to show that the distribution of load and the deflections of the girders tend to become more uniform as the number of loads is increased. Obviously then, added diaphragms will be more effective for a single load than for two or four loads.

Effect of Transverse Location of Loads

If the loads are placed symmetrically with respect to the longitudinal centerline of the bridge, the addition of diaphragms will always produce a more uniform distribution of load, and the largest girder moment, occurring for this case in an interior girder, will be decreased. However, if the loads are shifted transversely toward one side of the bridge, the largest moment may occur in the edge girder, and will be increased by the addition of diaphragms.

The practical significance of an increase in edge-girder moment depends on the relative magnitudes of the moments in edge and interior girders, the loads being placed in each case to produce maximum moments in the girder being considered. If truck loads can be placed on the bridge with one wheel load directly over or very close to an edge girder and if the value of *H* is relatively small, the moment in an edge girder will usually be greater than that in an interior girder when each is loaded for maximum effect (see Fig. 6). In this case, the addition of diaphragms will increase the moment in the edge girder, while decreasing the moment in the interior girder. The governing moment is thus increased and the effect of adding diaphragms may be considered to be harmful for these conditions On the other hand, if the layout of the bridge and the locations of the curbs are such that a large transverse eccentricity of load is not possible, or if *H* is large, the governing moment will usually be that in an interior girder. The addition of diaphragms will again cause a decrease in moment in the interior girder and an increase in moment in the exterior girder. If the final result is equal moments in the two girders, each for its own loading condition, the effect of diaphragms is beneficial, since the governing moment has been reduced. However, the diaphragms may change the moments so much that the edge-girder moment is the greater, and may even produce the condition in which the edge-girder moment with diaphragms is greater than the interior-girder moment without them. In this case, the effect of the diaphragms is again harmful.

It is evident from the foregoing discussion that the transverse location of the loads has an important bearing on whether the effect of adding diaphragms is to increase or decrease the governing moment in the girders. However, the effects of the other variables affecting the behavior of the structure should not be ignored. Whether the governing moments in a given bridge will be increased or decreased, and to what degree, will depend also on the values of *H*, *b/a*, *k*, and on the longitudinal location of the diaphragms as discussed in the following sections. This phase of the action of bridges with diaphragms is quite complex and the theoretical studies are still too limited in scope to state, in terms of all the variables, the conditions under which added diaphragms will be beneficial or harmful.

Effect of Longitudinal Location of Diaphragms Relative to Load

It is almost obvious that a diaphragm will be most effective when it is located in the structure at the same longitudinal location as the loads being considered. However, in a highway bridge the loads may be applied at any point along the girders, while diaphragms can be placed at only a few locations Since maximum moments in a bridge will usually be produced by loads applied in the neighborhood of midspan, a diaphragm or diaphragms located at or near midspan should be most effective. Consider the examples given previously for the structures and loadings shown in Figures 6, 7, and 8. In this case, the loads and moments are at midspan, and the effects of adding a single diaphragm at midspan have been discussed. If, instead, two diaphragms had been added at the third points, each having a stiffness corresponding to $k=0.40$, the results would have been somewhat different. For example, for the interior girder, the addition of *two* diaphragms at the third points would decrease the moment by 9 and 23 percent, respectively, for $H=5$ and 20, as compared to reductions of 16 and 27 percent for a single diaphragm at midspan. Similarly, the moment in Girder A would be increased 3 and 13 percent, respectively, for $H=5$ and 20, by the addition of diaphragms at the third points, as compared to increases of 7 and 19 percent for a diaphragm at midspan It should be noted that although the total diaphragm stiffness is twice as great in one case as in the other, the effect is still reduced significantly because of the less advantageous location with respect to the load. Of course, if loads were applied at a third point of the span the diaphragm at this location would be quite effective, but the girder moments produced for this location of the load would not be significant in design.

Analyses have shown also that if a diaphragm has been added at midspan, the addition of other diaphragms, say at the quarter points, will have little effect for loads at or near midspan. This can be explained by the fact that the relative deflections of the girders at the quarter points have been decreased by the addition of a diaphragm at midspan

It has been shown that if the loads are applied at midspan, the effectiveness of diaphragms will decrease the more distant they are from the loads Conversely, if a diaphragm is located at midspan, its effectiveness will decrease as the loads move away from midspan. Analyses have shown that the maximum girder moments in a bridge with a diaphragm at midspan will be obtained for loads placed a short distance from midspan. The exact location of the loads for maximum moment will depend on the values of H, k, b/a, and the number of loads on the structure For the bridges and loading of Figures 6, 7 and 8, and for a single diaphragm at midspan having $k=0.40$, the maximum moments in Girder C for loads off midspan are 2 and 6 percent greater, respectively for $H=5$ and 20, than the moments for loads at midspan. The magnitude of this increase depends on a number of factors and the above values should be considered only illustrative. Since the moment in Girder A is increased by the addition of a diaphragm, it will be a maximum for loads applied at the location of the diaphragm.

The foregoing remarks may be summarized as follows: Diaphragms, unlike the slab (which acts at all points along the girders), can be added only at discrete points; their effectiveness is therefore not equal at all locations but extends only for some distance either side of the diaphragm. Consequently, for greatest effectiveness, diaphragms should be placed near the locations at which loads will be placed for maximum moments, usually near midspan. Furthermore, since maximum moments do not decrease greatly as the loads are moved away from midspan, analyses have shown that in many cases the optimum arrangement will consist of two diaphragms placed a short distance either side of midspan.

Flexibility of Diaphragm Connections

All of the analyses used as a basis for the foregoing discussions of the effects of diaphragms involve the assumption that the diaphragms are continuous members extending across the full width of the bridge. However diaphragms in I-beam bridges commonly consist of short sections of rolled beams or of transverse frames spanning between adjacent girders. In such cases, the continuity of the diaphragm is derived solely from the rigidity of its connections to the girders. If these connections are not sufficiently rigid to provide flexural stiffness equal to that of the diaphragms proper, the effective stiffness of the diaphragm, and thus its ability to distribute load, will be decreased.

It seems reasonable to assume that the condition of a fully continuous diaphragm is approached more closely where reinforced-concrete beams are used for diaphragms, as is the case in concrete-girder bridges and in some I-beam bridges.

The problem of determining the effective rigidity of a diaphragm, taking into account the flexibility of the connections, and the problem of evaluating the stiffness of framed bracing are outside the scope of this paper. Nevertheless, it is one of the most important problems confronting the designer who wishes to use diaphragms as an aid to load distribution.

Another problem of similar nature is represented by the skew bridge in which the diaphragms are frequently staggered longitudinally and thus depend on the torsional rigidity of the girders as well as on the rigidity of the connection to provide continuity across the bridge. This problem is also outside the scope of this paper.

Limitations of Analyses

The applicability of the analyses described in this paper is necessarily limited by the simplifying assumptions that have been made and by the fact that not all of the variables affecting the behavior of slab-and-girder bridges have been considered. Consequently, close agreement between the predictions of the analyses and the real behavior of actual bridges should not be expected unless the properties and characteristics of the structure are reasonably similar to those assumed in the analyses. It becomes desirable, therefore, to consider the assumptions of the analyses and the limitations imposed by those assumptions, and to consider so far as possible the effects of the neglected variables.

Properties of Materials

A basic assumption in the analyses is that the slab is homogeneous, elastic, and isotropic. Although a reinforced-concrete slab satisfies none of these conditions, especially after cracking has occurred, the results of tests on scale-model I-beam bridges have shown that the distribution of load to the girders is predicted very closely by an elastic analysis. This conclusion, of course, does not apply after extensive yielding of the slab reinforcement has occurred.

Ultimate Strength

Another basic assumption is that the entire structure—slab, girders, and diaphragms—behaves elastically; that is, deflections, moments, and shears are linear functions of load, and thus, superposition of effects is possible. Obviously, this condition is not satisfied after significant yielding has taken place in any element of the bridge, and these analyses are therefore not suitable for predicting ultimate capacities which are attained usually only after considerable inelastic acion.

Values of b/a

Of the several variables relating to the geometry of the structure, only the ratio of girder spacing to span, b/a, has been considered in the analysis, and this only for values of 0.1, 0.2, and 0.3. This range of values includes a majority of actual structures, and some extrapolation is possible, especially to lower values of b/a since the load distribution for $b/a=0$ is theoretically uniform.

Number of Girders

Although only bridges having five girders have been considered, it has been pointed out in a previous section that the influence lines for moments in the girders (Fig. 2) may be used for bridges with more than five girders and even, in some cases, for bridges with only four girders. Analyses have also been made for a three-girder structure; some of these have been published (*8*), while the others have not (*9*).

Continuous Bridges

A further limitation of the analyses is that only simple-span bridges have been considered. However, some analyses, and fairly extensive tests on scale models (not yet published), have shown that the distribution of moment to the girders in a continuous bridge is approximately the same as that in a simple-span structure having values of H and b/a corresponding to those for the continuous bridge using for a the span between points of contraflexure. This similarity extends also to the distribution of girder moments over an interior support.

Skew Bridges

Only right bridges have been considered, and no analyses for skew bridges are available. However, tests on scale models (*5*) have indicated that for angles of skew up to about 30 deg. the distribution of load is very similar to that for a right bridge. For larger angles of skew, the distribution of load is affected adversely, however, at the same time, the total moment in the girder is decreased in such a manner that the maximum girder moment is also decreased in spite of the changed distribution (*5*, *6*). The effects of diaphragms in skew bridges have not been studied.

Nonuniform Girder Spacing

It has been assumed in all of the analyses that the girder spacing b is uniform. If this spacing varies slightly it is probable that the use of an average value when computing b/a will be satisfactory. However, this approximation may not be valid if the variation in b is great; fortunately this condition is not common in slab-and-girder bridges.

Stiffness of Slab

Some uncertainty always exists regarding the absolute stiffness of a reinforced-concrete slab, since it is affected by the degree and extent of cracking. However, the tests of scale-model bridges (4) showed an excellent correlation between the results of analyses and tests when H was based on a slab stiffness computed for the gross concrete section, neglecting the reinforcement, and taking Poisson's ratio equal to zero. Whether a similar approximation will also be satisfactory when applied to actual structures can be determined only by studying the results of field tests.

Stiffness of Girders

The other quantity entering into the expression for H is the stiffness of the girders, and this too is subject to some uncertainty. For I-beam bridges the major problem is estimating the degree of composite action which exists between the slab and the girders of the bridge in question. If no composite action exists, the girder stiffness is easily determined. If composite action is provided by means of positive anchorage between the slab and girder, the stiffness of the composite T-beam may be computed easily by including a width of slab extending half the distance to the adjacent girder on each side. Tests in the laboratory as well as in the field have shown that some degree of interaction probably exists in most actual bridges, even if positive shear connection is not provided. The source of shear transfer in these structures is either bond or friction between the slab and I-beam, or perhaps both Since the stiffness of an I-beam is increased markedly by the existence of even a small amount of interaction, the value of girder stiffness, and thus of H, may be quite indeterminate in a real bridge. For this reason, it is desirable that tests on such structures include strain measurements on both top and bottom flanges of the I-beams, so that the position of the neutral axis can be determined and the degree of interaction estimated.

The absolute stiffness of reinforced-concrete girders is also uncertain because of the indeterminate effects of cracking. It is customary in reinforced-concrete frames to compute relative stiffnesses on the basis of the gross concrete sections of the various members. This procedure may be used also for computing H when both the girder and the slab are reinforced concrete However, the possibility should not be overlooked that the absolute stiffnesses of these two members may be affected differently by cracking and that their relative stiffnesses may be changed Thus, again there may be some uncertainty regarding the real value of H for a particular bridge However, the value of H will usually be fairly large for concrete-girder bridges and the moments in the girders are not especially sensitive to variations in H when H is large (Figs 3 to 6)

Unequal Girder Stiffnesses

Only bridges in which all girders have the same stiffness have been considered in this paper. This condition, however, is frequently not satisfied in actual structures In concrete-girder or composite I-beam bridges, the edge girders may have an increased stiffness because of the greater cross section of the curbs or sidewalks as compared to the slab proper. Also, some I-beam bridges have been designed with the edge beams smaller than the interior beams.

The effects of unequal girder stiffnesses have been studied analytically for one bridge having edge girders 20 percent stiffer than the interior girders (2, 9) These effects have also been observed in tests of scale-model I-beam bridges in which the edge beams were less stiff than the interior beams. In both cases the bridges had five girders Although these data are not sufficient to permit precise statements regarding the behavior of bridges with girders of unequal stiffness, some idea can be given of how such a bridge will behave. Consider a structure in which the edge girders are stiffer than the interior girder, since this is a fairly common condition in actual highway bridges. In this case, the stiffer girders attract additional load, the amount of which depends on how much stiffer these girders are in comparison to the others, as well as on the transverse stiffness of the slab or diaphragms, through which loads reach the girders

The limited data available indicate that the increase in load is not as great as the increase in stiffness Thus, the deflections of the stiffer girder will not be increased An increase in load produces also an increase in moment in about the same proportion, however, this does not necessarily lead to an increase in stress, since the section modulus is usually increased by the same factors which cause the increase in stiffness. Whether or not the stresses will be increased in any given case will depend on the relative magnitudes of the increases in moment and section modulus.

Torsional Stiffness of Girders

The torsional stiffness of the girders has been neglected in all of the analyses described herein. This is on the side of safety, since such stiffness always contributes to a more-uniform distribution of load. The torsional stiffness of noncomposite I-beams is negligible compared to the flexural stiffness of the slab, and even for composite I-beams the effect may still be small However, the torsional stiffness of concrete girders may be appreciable and may produce noticeable improvements in the load distribution, especially as it reduces the harmful effects of stiff diaphragms. If H is large and the diaphragm is relatively stiff, the contribution of the slab will be relatively small and the structure may be analyzed relatively easily, but with fairly good accuracy, by means of a crossing-beam or grid analysis, including the effects of torsion but neglecting the presence of the slab.

Stiffness of Diaphragms

A major uncertainty will always exist regarding the stiffness of the diaphragms If rolled sections or framed bracing are used, the rigidity of the connections at the girders is the major problem. If reinforced-concrete diaphragms are used, the effect of cracking must be evaluated. This latter is particularly important where concrete diaphragms are used in a bridge with steel stringers, since the relative stiffness of diaphragms and girders, k, becomes quite uncertain, because of the two different materials involved. However, for these conditions the value of k is likely to be relatively large, and variations in k will consequently be less important (see Figs 7 and 8).

Use of Analyses in Planning and Interpreting Field Tests

An important use of the results of analyses is in the planning of field tests to yield significant results, and in the interpretation of field tests to provide the greatest amount of useful information.

Load, Moment, and Deflection

Frequent reference has been made in this paper to the distribution of load. However, since the girders are designed for moment and shear, not load itself, a knowledge of the distribution of total load to the girders is of little value to the designer unless he knows also how the load is distributed along the length of each girder. For this reason, the measurement of load itself, for example, by measuring reactions, may provide little useful information except as a check on other measured quantities.

Since moments are of primary interest to the designer, it is certainly desirable that they be determined in field tests, if at all possible. Although moment cannot be measured directly, it can usually be computed from measured strains. In reinforced-concrete girders, the determination of moments from measured strains is usually a difficult problem because of the effects of cracking on the moment-strain relation. The calculation of moments from measured strains may be somewhat easier in the case of steel stringers, but even here the effective section modulus may not be known exactly, because of the existence of a partial interaction between the slab and girders in bridges without mechanical shear connectors. However, if strains are measured on both the top and bottom flange of the beam so as to locate the position of the neutral axis, the degree of interaction can be determined approximately and the effective section modulus and moment of inertia for the composite beam can be estimated from the theory of partial interaction presented in Reference 10

Measurements of deflection in tests of slab-and-girder bridges are always of value since the deflections are of interest in themselves. However, the assumption should not be made that the distribution of load or moment among the girders is the same as the distribution of deflection Although these distributions may be nearly the same under certain conditions, they may be greatly different under others Obviously, if the girders are of different stiffnesses, the distribution of deflection will depend on the relative stiffnesses of the girders as well as on the loads that they carry. Moreover, even if the girders are of equal stiffnesses, the distribution of deflection may not be the same as the distribution of moment, or even of total load, since the longitudinal distribution of load along the various girders may be quite different (Fig. 1). This difference will be especially pronounced if only a single concentrated load is used in the test, and comparisons of moments and deflections for this case have been given elsewhere in this paper. If several loads are applied to the bridge, the distribution of deflection and moment will become more nearly alike, and in many tests advantage may be taken of this relation if it is not possible or convenient to determine moments from measurements of strain.

Loading

The analyses have shown that the effects of variations in H, b/a, diaphragm stiffness, or diaphragm location will depend to a considerable extent on both the number and locations of the loads used in a test.

The loading considered in the design of a bridge usually consists of not less than two trucks for a two-lane bridge, the most common type, and it is the behavior of the bridge under this loading that is of greatest interest. Frequently, however, it is not possible to make field tests with two trucks, and only a single-truck loading is used. For this case, the maximum moments, the distribution of moment or deflection, and the effect of adding diaphragms will be different than for a two-truck loading. Moreover, the distribution of moment will be different from the distribution of deflection. These differences present certain difficulties in interpreting the results but they can be overcome partially by obtaining data for various transverse positions of the single truck and combining the results to simulate the effects of two trucks on the bridge. Such superposition of effects is valid only if all of the observed phenomena are linear functions of load, this condition will usually be satisfied, however, except possibly for concrete-girder bridges in which the degree and extent of cracking may increase as successive tests are made. In such bridges, it is usually desirable to load the structure at all of the test locations at least once before any measurements are made. A similar problem may be encountered in I-beam bridges in which the degree of composite action may change during the tests

In some cases it may be more convenient to test the bridge under a single, concentrated load The various phenomena observed for this loading will be greatly different from those corresponding to a load consisting of two trucks, and the results can be interpreted correctly only by obtaining influence lines, or an influence surface, for the desired quantity by placing the single load at several different transverse and longitudinal locations on the bridge. The problem of superposition is even more acute in this case than for single-truck loading, and special care should be taken to determine if the relation between load and moment or deflection is truly linear over the range necessary to permit addition of effects.

The transverse location of the loads at any section has been shown to have an appreciable effect on the maximum moments in the girder, especially if diaphragms are present. Consequently, an effort should be made in any field test to place the loads as eccentrically as permitted by the spacing and clearance requirements of the specifications. If this is not done, an erroneous concept of the action of diaphragms may be obtained.

The longitudinal location of the test loads will usually be that producing maximum moments in the bridge If the bridge does not have diaphragms, the maximum moment in a simple span will occur under the rear axle of the truck or trucks when that axle is located a short distance from midspan However, since the moment at midspan for the rear axle at midspan is only slightly less than the maximum, it is frequently more convenient to measure strain or deflection at midspan with the rear-axle loads at midspan. This procedure should prove entirely satisfactory if no diaphragms are present. However, if a diaphragm is present at midspan, the moments and deflections at midspan for load at midspan may be significantly less than those which may be found under a load placed a short distance away from the diaphragm. Obviously, such shifting of the locations at which the load is placed and measurements are made adds much to the complexity of the test. However, it is important to recognize that the effect of diaphragms depends on the longitudinal location of the load, and this variable should either be included in the test program or its effect should be evaluated theoretically.

Other factors influencing the results of tests are H and b/a. Although these quantities are not likely to vary in a single test structure, it is necessary to recognize that a concrete-girder bridge having a large value of H will not behave the same as an I-beam bridge having a small value of H The same is true of bridges having different values of b/a Obviously, then, tests made on a single bridge cannot be generalized to apply to all slab-and-girder bridges. Even tests on a number of bridges are not capable of giving a complete or general picture of the behavior of such bridges, since such a complex structure does not lend itself readily to a purely empirical study. The importance and usefulness of theory becomes evident at this point. If field tests can be planned and carried out so as to yield significant comparisons with the predictions of the analyses, and if these comparisons show reasonable agreement, the theory then becomes a tool which can be used with confidence to understand and predict the behavior of slab-and-girder bridges. Without verification from field tests, the theory is of limited value; and without the aid of the theory, field tests, unless very great in number, cannot give a general picture applicable to the full range of the variables

Conclusion

The numerous variables affecting the distribution of load to girders in slab-and-girder bridges have been discussed solely on the basis of the results of theoretical analyses. The following major variables have been considered: (1) Relative stiffness of girders and slab, H, (2) ratio of girder spacing to span, b/a; (3) number and arrangement of loads; and (4) diaphragms, including effect of diaphragm stiffness and longitudinal location The discussion has been limited throughout to simple-span, right bridges having five girders spaced equidistantly and all having the same stiffness Torsional stiffness of the girders has been neglected.

The slab-and-girder bridge is a complex structure. Nevertheless, its behavior can be predicted and understood with the aid of theoretical analyses involving a number of the more important variables. The addition of diaphragms still further complicates the action of this type of bridge, but even here some insight into the effect of diaphragms can be obtained from analyses. This phase of the problem, however, has not yet been studied as fully as the action of the slab and girders alone.

Of course, an understanding of the theoretical behavior of this type of bridge is not enough. What we really desire is the ability to understand and predict the behavior of actual slab-and-girder bridges. To this end, the predictions of the analysis must be compared with the results of field tests; only in this way can we hope to understand a type of structure whose behavior depends on so many variables.

Acknowledgment

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All of the analyses were made under the direction of N. M. Newmark, research professor of structural engineering, who planned and guided the work at all stages.

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Reactions of a Two-Span, Skewed, Rigid-Frame Bridge

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● THE MODERN highway designer, faced with an unprecedented high-speed traffic load, finds solutions to many congested-intersection problems with grade-separation structures. Traffic-count limits have been firmly established which indicate the advisability of such a solution. However, since highways frequently intersect obliquely, the use of skewed structures becomes necessary in order to avoid serious traffic hazards.

In recent years, the rigid-frame bridge has had application to the grade-separation problem, because of its marked advantages in meeting limited headroom conditions, its adaptability to architectural treatment, and its economy typical of continuous structures. With the longer spans required by improved highway design, the two-span rigid frame finds potential application to grade separations, the center leg being accommodated by the medial dividing strip. However, it is imperative that one consider the deterring factors to such a design. The demands made upon the designer by increase in design time, together with the decreased fee inherent in a more economical design, presents a primary obstacle to such a solution. This is the paradox that confronts any conscientious designer. This inconsistency can be partially alleviated by an enlightened approach to the design problem, but perhaps more forcefully by a reappraisal of the system of determination of fees on a construction cost basis—a system which actually places a penalty on a more intricate and lengthy design procedure—even though it produces a more economical design. The study herein reported was made in an endeavor to contribute to a simplified approach to the skewed, rigid-frame problem and to indicate areas where continued study is advisable.

Where highway intersections are nonrectangular, structures must be skewed in order to maintain road alignment essential to safety. Skewed rigid frames, however, present some difficulties with which early designers could not cope. Some failures have occurred which have been attributed to fallacious design procedure based on lack of knowledge of the forces acting on and within a skewed frame.

In 1924, J. Charles Rathbun presented the first logical mathematical analysis (*1*) of skewed frames which correctly included the effects of torsion on the reactions of such structures. In an attempt to test the validity of Rathbun's solution, the late George E. Beggs conducted a series of model tests at the specific request of the American Society of Civil Engineers. Beggs' conclusions were in substantial agreement with the theoretical solution; consequently the sponsoring committee reported (*2*) that the results of the limited number of tests seemed to indicate that the theory might be safely applied to skewed structures.

The Rathbun analysis has been used successfully, and important simplifications of his work have been made by Richard M. Hodges in 1944 (*3*) and Maurice Barron in 1950 (*4*).

Description of the Investigation

The investigation herein presented is a continuation of a model analysis program originally conceived and recently reported by Walter C Boyer (*5*) which was intended to further dispel the doubts surrounding the Rathbun theory The results of Boyer's work on single-span skewed frames were in good agreement with the Rathbun solution.

The theoretical analysis is based on the same assumptions that were used by Rathbun in his original solution (*6*) The general form of the solution was suggested by the work of Barron (*7*), involving unit deflections, but required the solution of two sets of simultaneous elastic equations

The basic structure studied was a hinged, two-span rigid frame of two equal square spans 100 ft long with a leg height of 22 ft, which was satisfactory for model analysis when scaled down. Effect of skew on reactions was investigated for skew angle variation from 0 to 50 deg, in increments of 10 deg The deck was flat, constant in thickness, and 40 ft. wide.

Reactions inherent in the skewed frame and the corresponding nomenclature are shown in Figure 1 It should be carefully noted that the primary effect of skew angle is to create a couple with the eccentric horizontal reactions, R_x, and that this couple must, for equilibrium, be resisted by an equal and opposite couple involving the cross shears, R_z It is this reaction component, R_z, which is peculiar to the skew frame and which contributes largely to the torsional stresses in the deck slab.

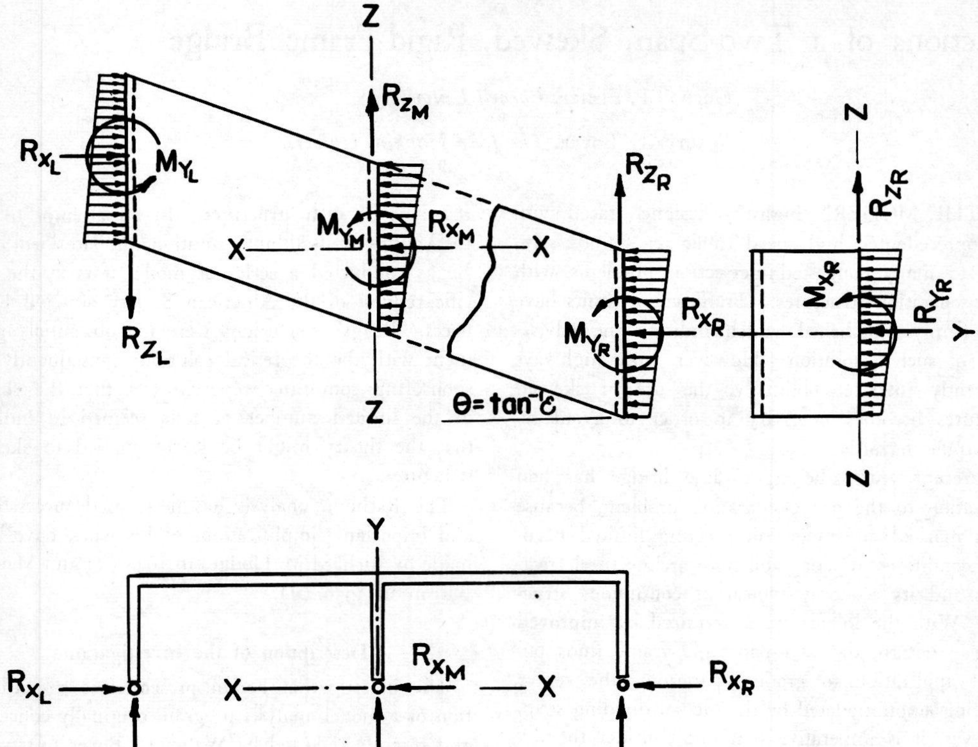

Figure 1. Reactions for a hinged, two-span, skewed frame.

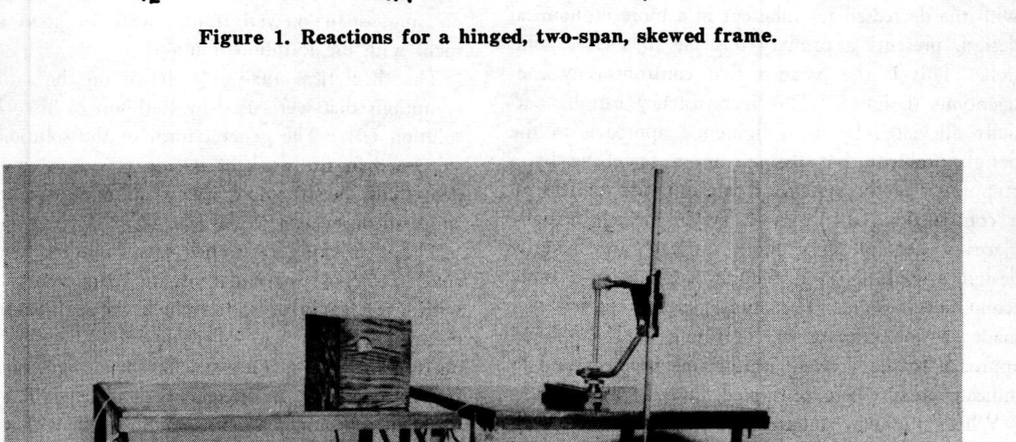

Figure 2. Test arrangement.

Procedure and Equipment

The experimental analysis used in this investigation is based on the deformeter method developed by Beggs (*8*), by means of which influence lines are obtained through the use of controlled deflections rather than applied loads. A modification of the Beggs method as proposed by William J. Eney (9), employing reasonably large deflections of an order that can be read with the unaided eye, has been used for this study.

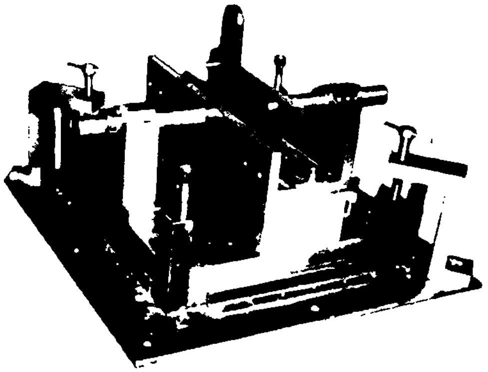

Figure 3. Movable abutment.

The test arrangement shown in Figure 2 consisted of two movable abutments, one at the right end and the other at the center of the bridge; a stationary abutment at the left end; deflection gage and circuit detector system; and two independent datum plates mounted on leveling stands.

The movable abutment used to induce the controlled deflections is shown in Figures 3 and 4. Vertical uplift for the function, R_y, is achieved by raising the whole abutment off the lower base plate by means of the uplift screws, and placing under the abutments shim blocks of thickness corresponding to the desired deflection. Horizontal displacement along the *x* axis for function R_x is obtained by sliding the abutment between guide angles (B) and placing shim blocks between the abutment and guide angle (A). Likewise, movement along the *z* axis for the cross-shear function, R_z, is applied by sliding the abutment between guide angles (A) and shimming against guide angle (B). Torsional moment, M_x, requires a rotation of the channel section about the shaft forming the *x-x* axis, controlled in magnitude by pinning into calibrated radial holes in the butt stop or by other suitable means to obtain a required angle of twist. Rotation for horizontal moment, M_y, is applied in the horizontal plane about the center pin and is controlled by pinning into base-plate holes arranged on a previously calibrated, fixed radial pitch.

To measure the vertical slab deflections induced by such controlled displacements, a converted hydraulic point gage, accurate to 0.003 in., was used and arranged to move freely on the fixed and independent datum. Deflection readings for each function were taken at each of the grid points shown in Figure 5.

An electric circuit making use of an electronic circuit detector, and a coating of conducting silver paste on the slab surface completed the test setup.

Models were constructed to a scale of 1 in. equals 5 ft. of grade XXX paper-base phenolic-resin sheet. The legs were twice as thick as the slab, and were joined to the slab by steel clamps to form a rigid knee. Hinged supports were made of ordinary cabinet hinges, carefully selected, reamed, and repinned to provide frictionless rotation without excessive play.

Test Results

Model results, in terms of influence ordinates, are compared with corresponding theoretical values in Tables 1 and 2.

In Tables 1 and 2, inflence ordinates for centerline loading are given for representative reactions, in this instance R_{yr} and R_{xr} at the abutment and R_{ym} and R_{xm} at the center pier.

Values for off-center loading for a 30-deg. skew bridge are given in Tables 3 and 4 for the rectangular functions R_{yr} and R_{xr} at the abutment. Off-

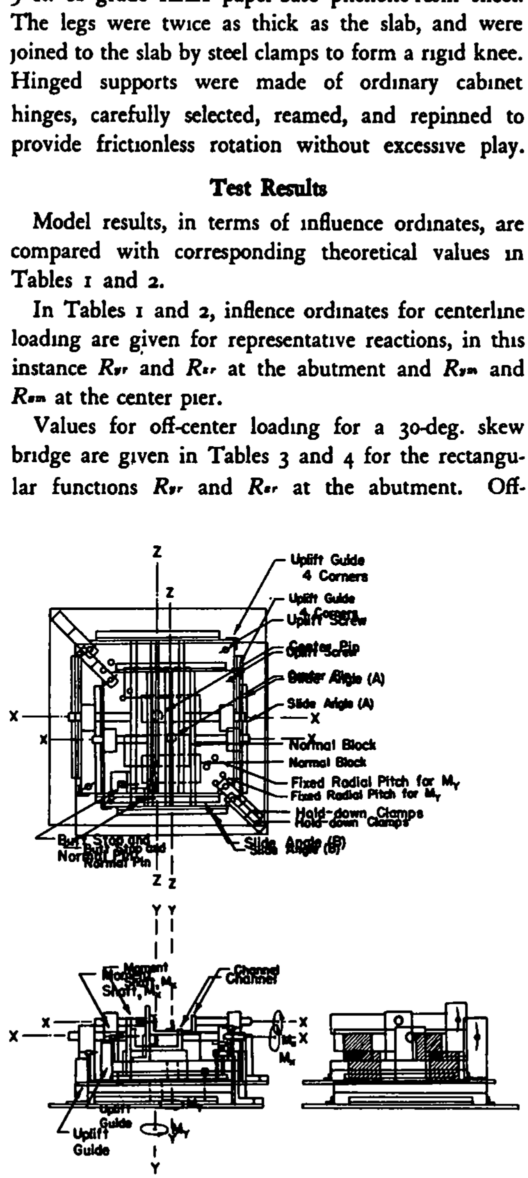

Figure 4. Schematic diagram of movable abutment.

center loading ordinates for the functions R_{vm} and R_{hm} at the center pier are not given, but they showed even closer agreement between experimental and theoretical values than the abutment reactions.

Influence contours for visual comparison of typical functions are shown in Figures 6 to 9 for a 30-deg. skew.

Model limitations precluded the reporting of experimental results for R_s, the cross shear. Theoretical analysis, however, shows R_s to be almost precisely equal to the product of the tangent of the skew angle and the corresponding horizontal thrust, *i.e.*, R_s equals R_h tan θ. This coincides with both theoretical and experimental observations for the single-span

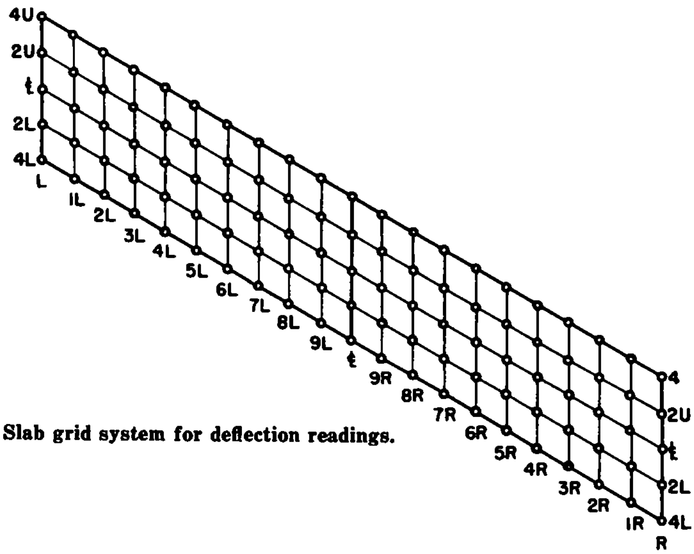

Figure 5. Slab grid system for deflection readings.

bridge and the work of other investigators mentioned heretofore. The double-span bridge gave no evidence to the contrary, and there seems to be little reason to doubt the relationship given.

Experimental results for cross-shear functions, R_{sr} and R_{sm}, are not reported. The inherent stiffness of the bridge in resisting the action of the cross-shear against the full width of the slab, and the consequent limitations of the model equipment, obviated the possibility of measuring reliable values for these functions. Redesign of the equipment was not warranted for the purpose of this program.

Conclusions

Study of the results leads to the immediate conclusion that the vertical reactions, R_v, and the horizontal thrusts, R_h, are essentially independent of the skew for centerline loading. This means, in effect, that R_v and R_h for any angle of skew are the same as for a similar right frame of the same square span. The skew has some effect on these reactions for off-center loading, but is not considered practically important in view of the fact that centerline loading gives the greater stresses in the bridge and is ordinarily used in design. The independence of these reactions follows the similar conclusion for the single span bridge reported by Boyer (*10*), and is given further support by the simplified theories of R. M. Hodges (*11*), and M. Barron (*12*).

The horizontal-plane moments, M_v, proved to be negligible by both model and theoretical analysis, and can be neglected safely in design without serious error. Slab deflection of the model bridges for this function in nearly all cases was too small to be measured with the device used.

A serious discrepancy between experiment and theory has been found for the torsional moments, M_t. For centerline loading, the theory gives negligible values for both M_{tr} and M_{tm}, whereas the model study shows centerline ordinates of considerable magnitude, with the difference increasing with skew angle. Since the usual method of design is based on centerline loading, design moments as given by the theory are apparently much smaller than those which actually exist, and therefore on the unsafe side. Off-center loading for M_{tr} again shows model results to be generally higher than the theoretical, but the discrepancy is neither so obvious nor serious as for the centerline.

Experimental off-center values for the torsional moment, M_{vm}, are at variance completely with the theoretical values and indicate a basically different type of slab action. Equilibrium in both analyses checks reasonably well, and each appears to be a rational action of the bridge. The authors consider the experimental result to be closer to the true action of the bridge on the basis that it consistently occurred for all angles of skew.

The question naturally arises as to why such discrepancy exists. It has been shown that the major differences occur for the torsional moment functions, M_v, whereas the results for the rectangular functions are in substantial agreement with theory. It has been further shown by Barron (*13*) that, for all practical purposes, the rectangular and torsional systems are independent of each other. This gives support to the possibility of the situation existing in this investigation, namely, that there could be a major discrepancy in the torsional moments, and simultaneously agreement in the rectangular functions.

In view of the fact that differences occur only in this isolated instance, suspicion is cast accordingly upon the use of the torsional factor, F. For the purposes of this investigation, it was assumed that

$$F=\frac{bt^3}{35{\cdot}78},$$

an empirically obtained factor (*14*) for torsion of concrete beams with 1-to-4 depth-to-width ratio. This factor agrees reasonably well with the Saint Venant value for such sections. The usual ratio for bridge slabs, however, is 1-to-15 or greater. The Saint Venant factor,

$$F=\frac{bt^3}{3},$$

for unskewed plates with large width-thickness ratios appears to be valid from the work of many investigators (Foppl, Stussi, Bach, etc.) and would have been a more satisfactory value to use here; however, the large differences in the torsional moments cannot be accounted for by this fact alone. The skewed plate presents a problem in combined torsion and bending and there is considerable doubt that the Saint Venant factor can be used without modification accounting for possible interaction of the plate's flexural rigidity

The Saint Venant factor is defined for the square

TABLE 1

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR REACTION R_{Y_R}

Part A Experimental Values

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	034	043	035	013	− 013	− 039	− 054	− 056	− 039	0
10°	0	028	039	028	012	− 016	− 043	− 059	− 067	− 047	0
20°	0	028	039	028	016	− 016	− 035	− 055	− 059	− 043	0
30°	0	035	047	039	020	− 008	− 039	− 055	− 059	− 043	0
40°	0	032	043	035	012	− 020	− 047	− 067	− 063	− 039	0
50°	0	039	055	055	028	− 008	− 035	− 063	− 067	− 039	0

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
0°	058	141	239	352	473	601	720	828	928	1 000
10°	055	137	239	353	474	603	717	835	928	1 000
20°	051	134	236	354	480	606	732	838	932	1 000
30°	059	133	239	349	478	600	725	838	928	1 000
40°	055	130	232	347	472	508	720	830	925	1 000
50°	055	118	217	339	477	618	752	858	941	1 000

Part B Theoretical Values

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	0 034	0 045	0 038	0 020	−0 003	−0 026	−0 043	−0 049	−0 036	0
10°	0	0 034	0 045	0 038	0 021	−0 003	−0 026	−0 043	−0 049	−0 036	0
20°	0	0 034	0 045	0 039	0 021	−0 003	−0 026	−0 043	−0 049	−0 036	0
30°	0	0 035	0 046	0 039	0 021	−0 002	−0 026	−0 043	−0 048	−0 036	0
40°	0	0 035	0 046	0 040	0 022	−0 002	−0 025	−0 043	−0 048	−0 036	0
50°	0	0 035	0 047	0 040	0 022	−0 001	−0 025	−0 043	−0 048	−0 036	0

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
0°	0 064	0 151	0 257	0 374	0 497	0 620	0 738	0 845	0 934	1 000
10°	0 064	0 151	0 257	0 374	0 497	0 621	0 738	0 845	0 934	1 000
20°	0 064	0 151	0 257	0 374	0 497	0 621	0 739	0 845	0 934	1 000
30°	0 064	0 152	0 257	0 374	0 498	0 621	0 739	0 846	0 935	1 000
40°	0 064	0 152	0 257	0 375	0 498	0 622	0 740	0 846	0 935	1 000
50°	0 064	0 152	0 257	0 375	0 499	0 622	0 740	0 847	0 935	1 000

TABLE 2

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR REACTION R_{X_R}

Part A Experimental Values

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	082	102	085	039	— 019	— 076	— 112	— 121	— 085	0
10°	0	074	092	074	029	— 024	— 068	— 110	— 118	— 092	0
20°	0	076	095	076	039	— 018	— 068	— 108	— 121	— 087	0
30°	0	087	120	104	059	— 006	— 069	— 112	— 122	— 087	0
40°	0	095	128	112	059	— 006	— 073	— 120	— 130	— 093	0
50°	0	098	149	136	079	0	— 079	— 135	— 147	— 100	0

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
0°	121	245	369	466	548	581	560	467	281	0
10°	100	220	336	438	509	538	514	422	241	0
20°	102	215	331	428	504	536	515	420	249	0
30°	116	228	337	433	508	539	518	425	250	0
40°	100	201	309	414	500	543	533	449	331	0
50°	102	193	299	400	492	543	537	452	270	0

Part B Theoretical Values

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
0°	0	0 077	0 100	0 085	0 045	—0 009	—0 059	—0 098	—0 110	—0 081	0
10°	0	0 077	0 100	0 085	0 046	—0 007	—0 059	—0 098	—0 110	—0 081	0
20°	0	0 077	0 101	0 086	0 046	—0 007	—0 059	—0 098	—0 111	—0 081	0
30°	0	0 078	0 103	0 087	0 047	—0 006	—0 058	—0 098	—0 109	—0 081	0
40°	0	0 078	0 104	0 088	0 049	—0 005	—0 057	—0 097	—0 109	—0 081	0
50°	0	0 079	0 105	0 090	0 050	—0 004	—0 056	—0 097	—0 109	—0 081	0

GRID POINTS ALONG CENTERLINE OF SLAB

Skew Angle	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
0°	0 121	0 250	0 376	0 481	0 555	0 585	0 559	0 460	0 279	0
10°	0 121	0 250	0 376	0 481	0 557	0 586	0 559	0 460	0 279	0
20°	0 121	0 251	0 376	0 481	0 557	0 586	0 560	0 461	0 279	0
30°	0 121	0 251	0 376	0 482	0 558	0 587	0 561	0 463	0 280	0
40°	0 121	0 251	0 377	0 483	0 559	0 589	0 562	0 464	0 280	0
50°	0 121	0 251	0 377	0 484	0 560	0 590	0 564	0 465	0 281	0

TABLE 3

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR OFF-CENTER LOADING FOR REACTION R_{Y_R}, 30-DEG SKEW

Model Results

TRANSVERSE SECTION

Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	039	071	078	067	047	012	— 016	— 035	— 035	0
c	0	035	047	039	020	— 008	— 039	— 055	— 059	— 043	0
4L	0	024	020	0	— 031	— 067	— 082	— 094	— 074	— 039	0

Long Section	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
4U	051	118	200	298	410	545	678	800	917	1 000
c	059	133	239	349	478	600	725	838	928	1 000
4L	070	165	275	403	533	655	764	858	944	1 000

Theoretical Results

TRANSVERSE SECTION

Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	0 057	0 085	0 089	0 080	0 059	0 033	0 008	—0 009	—0 014	0
c	0	0 035	0 046	0 039	0 021	—0 002	—0 026	—0 043	—0 048	—0 036	0
4L	0	0 012	0 006	—0 013	—0 038	—0 064	—0 085	—0 095	—0 088	—0 059	0

Long Section	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
4U	0 041	0 112	0 205	0 315	0 436	0 562	0 687	0 806	0 912	1 000
c	0 064	0 152	0 257	0 374	0 498	0 621	0 739	0 846	0 935	1 000
4L	0 086	0 191	0 308	4 433	0 559	0 680	0 791	0 885	0 957	1 000

section only, that is, for a section perpendicular to the bridge centerline, whereas the theoretical analysis assumes that it applies equally well to the skewed section. Such assumption is highly questionable, and the matter cannot be resolved simply by splitting M_s into components along the centerline and at right angles thereto because of the difficulties arising from the triangular portions at each end of the slab. Partial edge clamping at the bridge knee, perhaps nonuniform in character, adds to the complexity.

It is conceivable, therefore, that the action of a skewed plate in torsion should be considerably different from that for a beam or unskewed plate, and that the apparent "torsional factor" for such plate should be likewise different from the corresponding Saint Venant value. Certainly there seems to be little basis for confidence in the torsional factor as currently used in skew bridge analysis. Fundamental theoretical and experimental study of the torsional action of skewed plates of large width-to-thickness ratio is felt to be highly desirable.

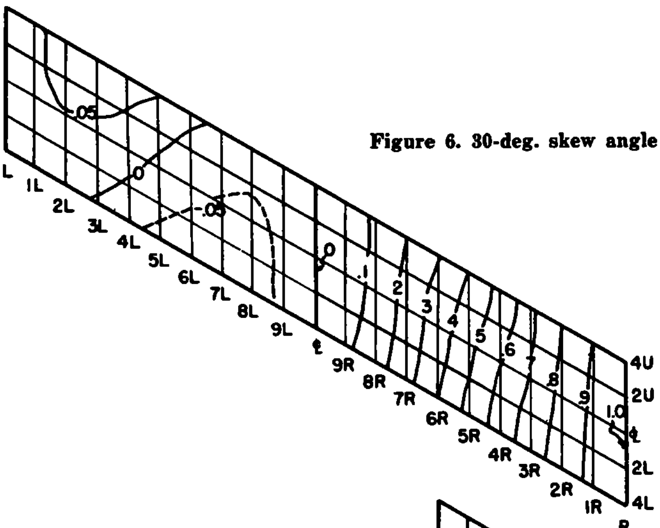

Figure 6. 30-deg. skew angle: Influence contours for B_{y_r}

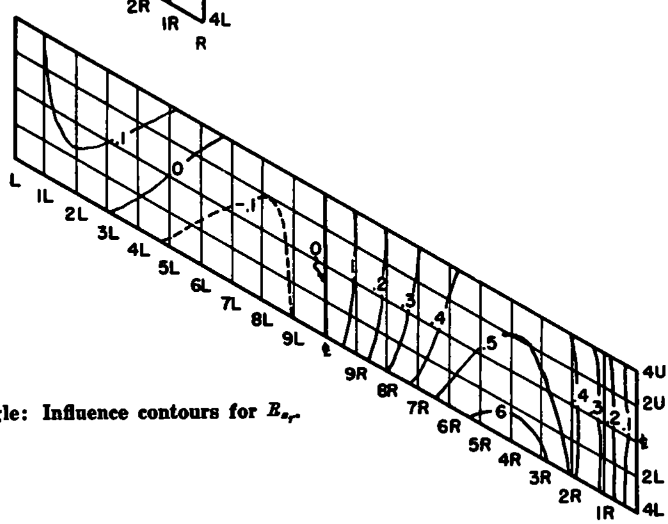

Figure 7. 30-deg. skew angle: Influence contours for B_{x_r}

In conclusion, it is felt that the experimental results reported essentially prove the validity of the theory, and that the theory should provide safe design values once the torsional factor has been redefined for skew plates.

Summary

The increasing use of high speed, divided highways has provided an excellent application for the double-span rigid-frame bridge as a grade-separation structure. However, highways frequently do not intersect at right angles and the use of a skewed structure becomes necessary.

This investigation seeks to add credence to the theory in current use, and to point out such limitations as may exist, by means of experimental correlation of reactive forces.

The following facts have been brought out:

(1) The vertical reactions R_v and the horizontal thrusts R_h are essentially independent of skew angle for centerline loading and vary with skew only slightly for off-center loading.

(2) For practical design purposes, R_h and R_v are the same as for a similar right frame of the same square span. The model study completely substantiates the theory for these reactions.

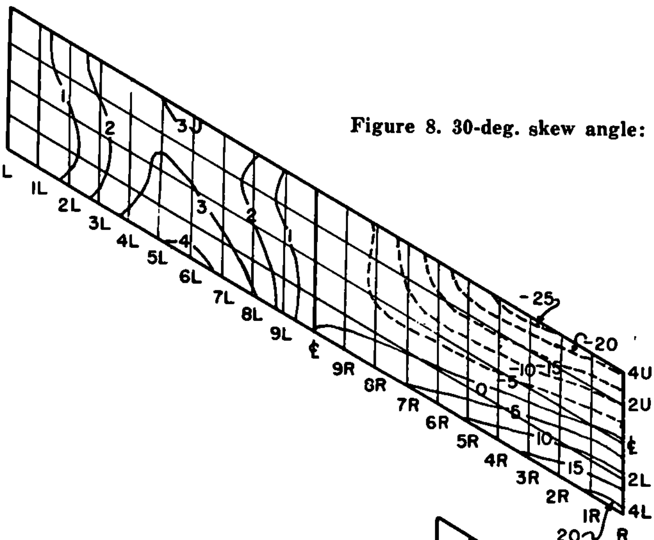

Figure 8. 30-deg. skew angle: Influence contours for M_{x_r}.

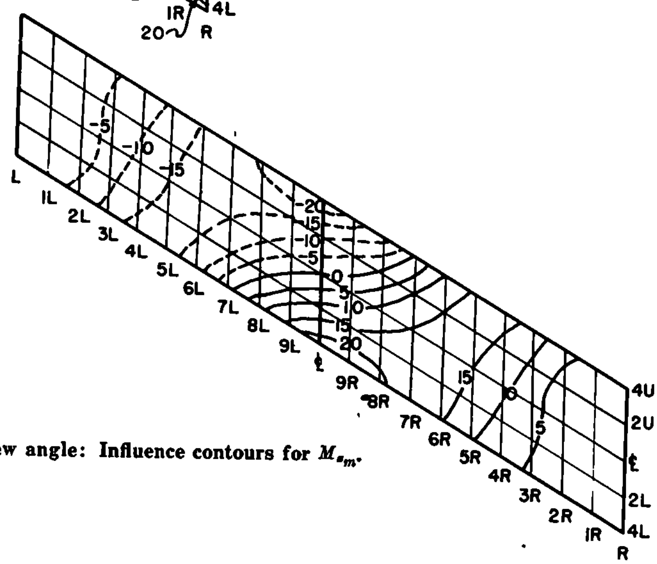

Figure 9. 30-deg. skew angle: Influence contours for M_{x_m}.

Following many early uncertainities, J. Charles Rathbun presented in 1924 a logical three-dimensional analysis of the skewed rigid-frame bridge, but his procedure was received by the profession with hesitation and suspicion. Important theoretical simplifications have been made by Hodges and Barron.

(3) Cross-shear, R_s, is equal to the tangent of the skew angle multiplied by the corresponding thrust R_x; model results are not reported for this reaction because of model limitations, but ample evidence exists to substantiate this fact.

TABLE 4

COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR OFF CENTER LOADING FOR REACTION R_{x_R} 30-DEG SKEW

Model Results

TRANSVERSE SECTION

Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	104	171	191	167	114	043	— 026	— 067	— 069	0
c	0	087	120	104	059	— 006	— 069	— 112	— 122	— 087	0
4L	0	065	057	014	— 051	— 118	— 165	— 187	— 163	— 095	0

	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
4U	110	210	301	384	445	478	468	394	246	0
c	116	228	337	433	508	539	518	425	250	0
4L	140	289	431	545	622	687	604	484	272	0

Theoretical Results

TRANSVERSE SECTION

Long Section	Left	1L	2L	3L	4L	5L	6L	7L	8L	9L	c
4U	0	0 128	0 191	0 203	0 180	0 132	0 075	0 018	—0 021	—0 031	0
c	0	0 078	0 103	0 087	0 047	—0 006	—0 058	—0 098	—0 109	—0 081	0
4L	0	0 027	0 014	—0 030	—0 085	—0 145	—0 191	—0 214	—0 198	—0 131	0

	9R	8R	7R	6R	5R	4R	3R	2R	1R	Right
4U	0 071	0 162	0 260	0 349	0 419	0 455	0 444	0 374	0 229	0
c	0 121	0 251	0 376	0 482	0 558	0 587	0 561	0 463	0 280	0
4L	0 171	0 339	0 492	0 615	0 696	0 720	0 677	0 551	0 330	0

(4) Horizontal moment M_y is negligible and may be neglected in design.

(5) Torsional moments, M_x, show serious discrepancy with theory, with theoretical values apparently on the unsafe side. It is believed that the Saint Venant torsional factor is not applicable to skewed plates having the proportions of bridge slabs, and fundamental investigation of this factor is considered to be desirable.

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Discussion

JASON PLOWER and HERBERT GEE, *California State Division of Highways*—We feel that the information is well presented and the report is a noteworthy contribution. It is particularly satisfying to note that our general practice in design of this kind of structure agrees closely with most of their findings The only uncertainty is in the torsional moments, mainly because actual comparative data are lacking

The authors state in their first paragraph that the design of skewed structures often becomes necessary because highways cross each other obliquely. How true this is. Not only do designers of highway-separation structures find themselves confronted with skewed layouts, they often find that their problems are even more complex. For instance, at many complicated intersections a layout may require structures that are skewed by a varying amount at each bend. In addition, the superstructure may be on a curve of fairly sharp radius Therefore bridge designers welcome any experimental studies which will give them a better understanding of the many complex design problems confronting them.

Possibly a greater value would have been derived from the results had a model of more usual proportions been used for the test. From its description, the basic structure studied, apparently, was a two-span, rigid-frame, flat-slab bridge of 100-ft. spans. Generally, for spans of this size, it would have been uneconomical to build as a true flat-slab type. We have found the economical span limit to be about 55 ft.

It is further noted that the model used has legs which are twice the thickness of the slab and of constant thickness. Our experience has shown that in average slab type designs, the abutment thickness at the top is usually about 0.8 or 0 9 of the depth of the slab and tapers towards the footing. The depth of slab to span ratio is approximately 0.060. The center pier, being symmetrically located, is of lesser importance, and its size, shape, and other features depend upon its aesthetic and economical requirements.

The authors have not reported on the earth pressure at the back of the abutments which is invariably present. This pressure, though relatively small when used in conjunction with forces created by a 100-ft span with skews under 25 deg, should nevertheless be taken into account. On bridges with larger skews, the earth pressure may have an important influence upon the structure due to its eccentric application

In actual practice, 100-ft. spans would call for a T-beam or box-girder superstructure construction and numerous structures of this span and type have been built by the California Division of Highways. Factors such as width of structure and lengths of wingwalls may become paramount in the economical determination of the type to be chosen. For two-span rigid frames, the thickness of solid, slab-type abutments at the top varies between 0.6 and 0.8 of the depth of the girders and tapers towards the footing. The depth of girder to span ratio is usually from 0.065 to 0.080 for T-beams and from 0.055 to 0.070 for box girders. Skews up to 60 deg. have been used in some instances. Generally, any two-span, rigid-frame bridge with skew of 45 deg or over should have strong arguments in its favor if it is to be selected, otherwise, a free ended span on self supporting abutments or open end type of spans should be used.

With regards to the torsional action in bridges with large skews, the slab-and-girder construction has the ability to deform and adjust itself. The acute corners between girders and abutments are heavily reinforced with additional reinforcements and diaphragms to distribute the corner loads so as to make them act more like rectangular structures. Experience has indicated that our treatment of the acute corners of skewed bridges is on the safe side as attested by the many structures of this type in use.

We believe that the experiment is an advancement in the direction of proving the validity of the accepted theory and further investigations should be encouraged However, any future research would be more beneficial if models used are more within the proportions of usual designs. Comparative data between flat-slab bridges and slab-and-girder structures may reveal results that are exceedingly valuable and certainly any information on torsional moment in slab-and-girder construction is most welcome.

E. L. Erickson, *Bureau of Public Roads*—This paper describes tests on phenolic models of a 2100-ft.-span rigid frame with 22-ft. legs. The span was measured at right angles to the abutments. The thickness was constant and the width 40 ft. The Maxwell theory of reciprocal deflections was used instead of direct loading. The method is similar to that of the Beggs deformeter gage except that much larger deflections were used, thus obviating the need for microscopes. Skews of 0 to 50 deg. were studied but the authors do not state how they vary the skew. Probably they used a number of models. The abutments and piers were hinged at the footings One abutment was subjected to various deflections and rotations and then deflections measured at various points on the body of the frame.

The authors give tables showing the comparison of tests and computations using (presumably) the Rathbun analysis. On the whole the tables show that the tests agree reasonably well with theory. No comparison is shown, however, between test and theory for torsional moments but the authors state that the agreement here was very poor and that theory erred on the unsafe side, assuming the tests to be correct.

Not only do the tests corroborate the Rathbun theory, with the exceptions noted, but they also indicate the feasibility of using certain short cuts previously suggested by Maurice Barron.

If it were not for two loose ends, engineers could design skew arches and frames with a good deal of confidence. We still lack sufficient knowledge to properly evaluate the effect of a combination of transverse shear and torsion and we do not know how the width of the structure effects the stresses. As regards the first point, Hayden and Barron in their book, *Rigid Frame Bridge,* suggest a rule of thumb for determining the transverse steel, which would indicate that they consider the matter of minor importance. It would seem, however, that the second point, *viz.*, the effect of width of structure, can be of vital importance. Common sense would indicate that a skew bridge wider than its square span would act more like a square bridge than a skew bridge.

Fisher and Boyer also studied the effect of eccentric loads. The data obtained is valuable and is related to width of structure, but unfortunately the width of the models was quite small in relation to the span and so did not show up the effect of width sufficiently.

It is hoped, therefore, that further tests will be made to throw more light on these loose ends, *viz.*, the rational design of the transverse steel and the effect of width of structure on the stresses due both to concentric and eccentric loads.

G. P. Fisher and W. C Boyer, *Closure*—The authors wish to thank the discussers for the interest and effort put forth in reviewing this paper, and are pleased to find such favorable acceptance.

Most of the questions raised by the discussers deal with the selected proportions and material of the model bridges tested. It was necessary that the material used for the models be elastic and reasonably isotropic within the range of deflection desired, have close tolerance on thickness, and be applicable to high humidity conditions. Phenolic laminate (Formica) was finally selected as most nearly fulfilling these requirements. As the "deformeter" method of model analysis makes use of *ratios* of deflections, the kind of material used is not of prime importance and may well be different from that of the prototype. As a matter of fact, it is desirable that the model material have as low an elastic modulus as possible so as to allow measurable deflections with rather light loading.

The information provided by Plower and Gee relating to economical proportions of rigid-frame bridges is highly useful, especially for the ribbed-slab types. The unusually long span of the model bridges was chosen purposely in order to exaggerate the effect of the cross-shears, R_v, and the leg height of 22 ft. was selected as typical of grade separation structures. Roadway width, questioned by Erickson, was not varied in this series of tests, as it appears to have minor influence on the basic action of the structure, as indicated by a previous investigation of the single-span bridge by Boyer (see Ref. 5). Variation of skew was accomplished by use of a number of models. Constant thickness of slab and legs was necessary for ease of manufacture of the models. It is thought that the selected proportions do not invalidate in any way the results obtained or their significance for bridges of more usual proportions.

Erickson raises a valid question with regard to evaluation of stresses resulting from combined torsion and transverse shear, and this is a phase of design which requires thorough investigation. While this investigation was concerned only with the evaluation of total forces, it cannot be ignored that the distribution of these forces constitutes a major problem. The authors believe, at the moment, that the torsional shear stresses as computed by St. Venant theory may be simply superimposed on the flexural shears (the latter assuming the slab as a beam of depth equal to the roadway width) provided the stresses are computed for the square width of roadway and not the skew width. As pointed out in the paper, the use of the St. Venant torsional analysis (and indeed to the one used herein for comparison) is a highly questionable practice and possibly nonsensical, not only for the determination of the total reactions but also for the distribution over a given design section.

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